Three Essays on Optimal Tax and Monetary Policy Rules

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Submitted to the Department of Economics and the
Graduate Faculty of the University of Kansas
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

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Date defended: May 05, 2015

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Date approved: May 05, 2015

Abstract

This dissertation consists of three essays which are organized as chapters. On the first chapter, I compute welfare-maximizing monetary and fiscal policy feedback rules using a calibrated DSGE model with sticky prices, monopolistic competition in the intermediate goods market, and tax distortions. The exogenous government spending is considered to be productive. Therefore, the preferences of the representative household and the technology of the production function include government spending which is decomposed into public consumption and investment. The optimal policy rules under which the nominal interest rate reacts to inflation, and the tax rates responds to public debt are associated with the highest level of welfare which is surprisingly equal to richer and baseline rules. The inflation volatility is closed to zero.

On the second chapter, I evaluate the impact of tax and monetary policy rules with disaggregated government purchases on welfare, real exchange rate and business cycle in a small open economy using a new-Keynesian dynamic stochastic general equilibrium frame work. The model predicts that the government consumption has more impact than government investment on both private consumption and investment, but less impact on the real GDP. Besides, the government purchases-real exchange rate puzzle is generated by the model. In this sense, the government consumption contributes more on generating the puzzle than the government investment. Moreover, both government consumption and investment have positive impact on welfare for any policy rules. The optimized policy rules have a pronounced anti-inflation stance and entail significant nominal and real exchange rate volatility for monetary policy. For tax policy rules, the

public debt stance is the optimized rules.

The last chapter tries to answer the following question: what are the effects of liquidity facilities (unconventional credit policy) on macroeconomic and financial variables when a small open economy faces a liquidity shock? To answer this question, we introduction the external sector with foreign private paper and bonds in a DSGE model with both nominal and real rigidities. The main result of this paper is that both unconventional credit policy has large quantitative effects on macroeconomic and financial variables. In fact, with the quantitative easing, output, consumption and investment stops to decrease after two quarters and then become positive. However, without the liquidity facilities, output, consumption and investment would have dropped continuously up to 10%, 15% 10%, respectively. This result is closely related to DEFK 's finding in terms of sign. Besides, the domestic inflation, the objective of the conventional monetary policy, falls and becomes negative after four quarters. Then, it raises and becomes positive up to two percent (2%) after eight quarters. Furthermore, a negative liquidity shock under the quantitative easing has positive impact on employment. Nominal and real exchange rates depreciate due to a negative impact of liquidity shock. Finally, the liquidity shock has positive impact on the financial variable (domestic and foreign spreads). The domestic and foreign spreads increase up to 100 and 120 basis points, respectively.

Acknowledgements

I would never have been able to finish my dissertation without the guidance of my committee members, advise from my former professor, help from friends, and support from my family and wife.

I would like to express my deepest gratitude to my advisor, Professor William Barnett, for his excellent guidance, care, patience, and for providing me with an excellent atmosphere for doing research.

I would like to thank Professor Shu Wu for guiding my research for the past several months and helping me to develop my background in macroeconomic modelling. I would also like to thank Professor John Keating who was always willing to help and give his best suggestions. Special thanks go to Professors Jian Zhang and Bozenna Pasik-Ducan for being in my dissertation committee.

Besides, I thank Professors Max Gillman and Robert Kollmann, Dr. Andrew Lee Smith, Dr. Febrio Kacaribu, Andrew Keinsley, and all other participants of Macroeconomics Research Seminar at the University of Kansas for their comments and suggestions. I express my thank to Dr Benoit Lukongo, Assani Amuzati Jr, André Lubaga, Fiston Vuvu, and Rachal Etshim. They patiently corrected my writing and provide useful comments.

Furthermore, I would like also to express my gratitude to mentor at the University of Kinshasa, Professor Kalonji Ntalaja, for his advises which helped me to undertake graduate studies in USA. Without his supports, guidance, and encouragements, my research would have not been conducted.

Moreover, I would also like to thank my parents Boniface Yemba and Micheline Omoy, my sisters and brothers, my uncle Christophe Alimasi and his wife Sophie Siniki. They were always supporting me and encouraging me with their best wishes.

Finally, I express my deepest gratitude to my lovely wife, Sylvie Paluku, and my children (Alvaro, Carmen, Michelle, and Gabrielle) without their unmeasured sacrifices and supports, I will not be able to complete my research.

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Maximizing Monetary and Tax Policy Rules in a Closed Economy with Dissaggregate Government Purchases

Abstract

This paper computes welfare-maximizing monetary and fiscal policy feedback rules using a calibrated DSGE model with sticky prices, monopolistic competition in the intermediate goods market, and tax distortions. The exogenous government spending is considered to be productive. Therefore, the preferences of the representative household and the technology of the production function include government spending which is decomposed into public consumption and investment. The optimal policy rules under which the nominal interest rate reacts to inflation, and the tax rates responds to public debt are associated with the highest level of welfare which is surprisingly equal to richer and baseline rules. The inflation volatility is closed to zero.

JEL Code : E52; E61; E63

Keywords: Monetary Policy, Fiscal Policy, Welfare, Productive Government Spending.

1.1 Introduction

The aftermath of the international financial crises occurred in 2007 and 2008 have put in light the role of fiscal policy to stabilize the output and boost the economy through the fiscal stimulus for many developed and emerging economies, whereas, the monetary policy operated with zero lower bound on nominal interest rate. The success of the fiscal stimulus is more questionable when we consider the mitigate impact of the government spending on the major macroeconomic variables such as output, private consumption and investment that predict theoretical and empirical standard macroeconomic models. Most of time, the results depend either on financing decisions of government spending or how the government spending is used in the model.

In one hand, the standard neoclassical models predict that the government spending shock has a positive impact on output and hours of work, but has negative impact on private consumption and investment. Baxter and King (1993), using a general equilibrium model, which allowed the preferences and technology to depend both on private and government variables, found that the government spending has positive impact on output and negative impact on private consumption and capital stock because of the wealth effect that lower the disposable income. Besides, they found that the financing decision is quantitatively much more important than the direct resource cost of government spending. In fact, the impact is negative on output if higher government spending is financed by the general income tax. However, the lumps-sum tax is always associated with a positive impact on output because of the wealth effect.

In other hand, the modification of neoclassical models through the New Keynesian Dynamic stochastic general equilibrium (NK-DSGE) model generates positive impact of government spending on private consumption and capital stock. Linnemann and Schabert (2003) build a model with nominal rigidities (sticky prices) and found that the government spending shock has a positive impact on private consumption due to the rise of real wages. Also, Linnemann (2006) modified the preferences of the representative household by not allowing consumption and leisure to be separated, found a positive response of consumption consecutive to a government spending shock because of substitute effect between consumption and leisure. This explanation was ruled out by Bilbiie (2006) who found that one of the goods (consumption or leisure) must be inferior to predict that. Gali, Lopez-Salido and Valles (2007) build a NK-DSGE model with nominal rigidities (prices) and real rigidities (rule-of-thumb behavior by some households) which generate an increase in consumption in response to a rise in government spending. In fact, the rule-of-thumb consumers partly insulate aggregate demand from the negative wealth effects generated by the higher levels of taxes needed to finance the fiscal expansion, while making it more sensitive to current disposable income. However, the sticky prices make it possible for real wages to increase even in the face of a drop in the marginal product of labor, as the price markup may adjust sufficiently downward to absorb the resulting gap. Furthermore, Escolani (2007) developed a NK-DSGE model which allows the preferences to depend both on private and government consumptions and also considered two types of households (optimizing and non-optimizing), and nominal rigidities (prices and wages) to analyze the relationship between private and government consumptions. He found a complementarity relationship between them through the channel of the marginal utility of consumption. Hence, the private consumption will rise after a positive shock on government spending.

Moreover, Woodford (2010) summarized the mechanisms behind the above models associated with the zero lower bound of nominal interest rate is used by the Central Bank. During the recession, the expansionary fiscal policy is very relevant and effective. When the Central Bank does not act against the government, the expected inflation is higher than the current one which causes the real interest rate to fall and multiplies a positive impact of government spending on output.

This paper evaluates the effects of monetary and fiscal policy rules on Welfare and Business cycles for a closed economy using a dynamic stochastic general equilibrium (DSGE) model with nominal rigidities (sticky prices), Tax distortion (consumption tax, labor income tax, and capital income tax), and monopoly competition in intermediate goods markets in the New Keynesian framework. I expend the model developed by Kollmann (2010 and 2008) and Baxter and King (1993) with productive government spending. As Baxter and King (1993), I allow the preferences of the representative household to include not only the private consumption, but also the consumption part of the government spending. Besides, the Cobb-Douglas production function of the intermediate firms combine both private and public capital accumulation. The later production function is the same as Baxter and King (1993) and Kollamann (2010). Also, the paper completes the work of Schitt-Grohe and Uribe (2007, 2004).

This paper differs to Kollmann's paper(2008) in two ways. The first difference is the decomposition of the government spending into consumption and investment based on the roles they played in economic activities, whereas Kollmann's paper consider the government spending as 'useless' in the sense that it doesn't not affect directly the preference of the representative household and the technology of the intermediate goods' producers. The second difference is related to the government funding. In this paper, I consider the variety of income tax and consumption tax while Kollmann considered the income tax only. This paper will also evaluate the dynamic of the government purchases on key macroeconomic variables and household welfare. Besides, the effect of government spending on macro variables is evaluated here using welfare-maximization operational feedback rules in addition to others macroeconomic variables such as the private capital accumulation and output that others papers have not taken into account so far. The welfare evaluation is based on the works of Kollmann (2002 and 2008) that is more realistic than the Ramsey approach. In fact, the simple feedback rules link monetary and fiscal policy to small sets of easily observable macro variables (Kollmann 2002). The model features the exogenous shocks to productivity, public consumption and investment. The tax rates which describe the fiscal policy are set as a function of real public debt, productivity, public consumption , public investment, and inflation. However, the monetary policy follows a Taylor-style nominal interest rate rule which is set as a function of inflation, output, and government deficit. In order to compare the results with Kollmann (2008), I calibrate my model using the quarterly data (1990:1 to 2012:4) of OECD countries .

The results show that the introduction of the government spending in the both preferences and production function changes the channel by which the government spending shocks affect the main macroeconomic variables such as the output, the private consumption and the capital accumulation. In fact, the government spending shock (globally) has positive effect on output, private consumption, and private capital accumulation. Both richer and simpler rules show that the government consumption is complement to the private consumption. Furthermore, the public capital accumulation is complement to the private capital accumulation which contradicts Baxter and King (1993) work. Hence, those results point out the role of each component of the government spending on economic activities.

Under the price stickiness, monopolistic competition, income tax, and the government spending in both preferences and production function, the optimized monetary policy is full inflation stabilization (Kollamann 2002, 2008; Erceb et al. 2000; Rotemberg and Woodford). This paper finds the same results. Besides, the optimized fiscal policy implies that an increase in the stock of the debt by an amount equal to (quartly) steady state GDP raises the capital, labor income, and consumption tax rates by 2.2257, 1.0003, and 0.0314 percentage points which is relative lower than what Kollmann (2008) found in his paper without government spending in the preferences and production function (9.09) for the proportional tax rate.

The rest paper will be organized as follow. I will present the model in details in section 2 including the welfare evaluation and the solution method. The results are presented in section 3 and will conclude the paper in section 4.

1.2 The Model

I consider a model with the representative household, the firms, government and the central bank. I assume that the representative household owns all the means of productions. Therefore, the household rents the capital stock to firms and supplies the labor force in competitive markets for labor and capital stock. The economy produces two types of goods. The single final good is produced by combining the continuum of intermediate goods indexed by $s \in [0,1]$. The assumption of the perfect competitive market holds only for the final good. The intermediate goods producers operate in a monopolistic competitive market. Finally, the government finances its spending by levitating income tax (on labor, rental capital) and consumption tax. Also, the government issues nominal bonds with maturity in period $t+1$.

1.2.1 The Representative Household

The representative household maximizes his preferences through

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, GC_t, L_t), \tag{1.1}$$

where E_t represents the mathematical expectation condition upon the complete information pertaining to period t and earlier. Also, $0 < \beta < 1$ is the subjective discount factor. Finally, C_t, L_t , and GC_t represent respectively the consumption, labor effort, and government consumption at time period t. The utility function is given by

$$U(C_t, GC_t, L_t) = ln(C_t + \gamma GC_t) - \psi L_t \tag{1.2}$$

Since the household holds all factors of production, the law of motion of capital stock is given by

$$PK_{t+1} = PK_t + (1-\delta) + IP_t \tag{1.3}$$

where PK_{t+1}, PK_t, IP_t, and δ represent respectively the private capital stock at period $t+1$, the private capital stock at time t, the gross private investment at time t, and the depreciation rate of the capital $(0 < \delta < 1)$.

Besides, A_{t+1} represents the riskless one-period bonds paying one unit of the numéraire in the following period $t+1$ while A_t is the quantity of bonds carried over from period $t-1$. Therefore, the household budget constraint at period t is

$$A_{t+1} + P_t(C_t + I_t) = R_{t-1}A_t + R^k_{t-1}PK_{t-1} + \int_0^1 \pi(s)d_s + W_tL_t - T_t \tag{1.4}$$

where R_{t-1} is the gross interest rate on bonds. Also, R^k_t, W_t, T_t, and G_t represent, respectively, the rental rate of capital, the rental rate of labor, the total taxes received by the government from the household, and the government spending. Finally, τ^C_t, τ^K_t, and τ^L_t are the consumption tax rate, the capital tax rate, and the labor income tax rate.

$$T_t = \tau^C_t C_t + \tau^K_t R^k_t K_{t-1} + \tau^L_t W_t L_t - \delta\tau^k_t P_t K_{t-1} \tag{1.5}$$

Therefore, the household maximizes (2) subject to (3), (4) and (5) from which the first order conditions are derived:

$$E_t\rho_{t,t+1}\left(\frac{(1+\tau^C_t)}{(1+\tau^C_{t+1})}\right)R_t\left(\frac{P_t}{P_{t+1}}\right) = 1 \tag{1.6}$$

$$E_t \rho_{t,t+1} \left(\frac{(1+\tau_t^C)}{(1+\tau_{t+1}^C)} \right) \left[(1+\tau_{t+1}^K) R_t^K + \delta \tau_{t+1}^K + (1-\delta) \right] = 1 \tag{1.7}$$

$$\left(\frac{1}{(C_t + \gamma G C_t} \right) \left(\frac{(1-\tau_t^L)}{(1+\tau_t^C)} \right) W_t = \psi \tag{1.8}$$

where $\rho_{t,t+1} = \beta \left(\frac{(C_t + \gamma G C_t)}{(C_{t+1} + \gamma G C_{t+1})} \right)$ is the stochastic discount factor.

1.2.2 The Final Good Firms

The economy produce the final good according the aggregate technology

$$Z_t = \left[\int_0^1 y_t^{\frac{\vartheta}{(\vartheta-1)}}(s) ds \right]^{\frac{(\vartheta-1)}{\vartheta}} \tag{1.9}$$

with $\vartheta > 1$.

Where Z_t is the final good output at period t. Besides, $y_t(s)$ is the quantity of type s intermediate goods used to produce the final good. Assume that$p_t(s)$ is the prices of intermediate goods. Then, the cost minimization in final good production implies

$$y_t(s) = (p_t(s)/P_t)^{-\nu} Z_t \tag{1.10}$$

with $P_t = \left(\int_0^1 p_t^{1-\nu}(s) ds \right)^{\frac{1}{1-\nu}}$ and where P_t is the price of final good. Also, P_t represents the marginal cost since the final good is used by perfectly competitive firm.

1.2.3 The Intermediate Goods Firms

The domestic intermediate good s is produced using the technology

$$y_t(s) = \theta_t K_t^{\alpha}(s) L_t^{1-\alpha}(s), \tag{1.11}$$

where θ_t, and K_t are respectively exogenous productivity and aggregate capital stock that are identical for all domestic producers; y_t is the output at period t.

$$K_t = GK_t + PK_t, \tag{1.12}$$

where GK_t represents the public capital stock that is exogenous and identical for all domestic producers.

As for the private capital stock, the law of motion of the public capital stock is given as

$$GK_t = (1-\delta)GK_{t-1} + Ig_t, \tag{1.13}$$

where Ig_t is the public investment.

The cost minimization of (11) subject to the total cost $\left(W_t L_t(s) + R_t^k K_t(s)\right)$ from which the first order conditions are derived and implied

$$\frac{L_t(s)}{K_t(s)} = \left(\frac{(1-\alpha}{\alpha}\right)\left(\frac{R_t^k}{W_t}\right) \tag{1.14}$$

,and the marginal cost is given by

$$MC_t = \left(\frac{1}{\theta_t GK_t^{\omega}}\right)\left[(R_t^k)^{\alpha} W_t^{1-\alpha} \alpha^{-\alpha} (1-\alpha)^{\alpha-1}\right] \tag{1.15}$$

The profits of domestic intermediate good producer, π_t is

$$\pi_t\left(p_t(s)\right) = \left(p_t(s) - MC_t\right)\left(\frac{p_t(s)}{P_t}\right)^{-\nu} Z_t \tag{1.16}$$

The price is set according to Calvo (1983). The intermediate firms are not allowed to adjust their prices optimally in each period. There is a constant probability $1-d$ where $d \in (0,1)$ that the firm can adjust its price randomly. Otherwise the price remained unchanged. The expected time between price adjustments is $\frac{1}{1-d}$. The firms are assumed to meet all demand at posted prices. They maximize the value of their profit stream, net of the income tax paid by the household on profits. Therefore, at date t, the firm sets a new price, $p_{t,t}$. The firm sets

$$p_{t,t} = ArgMax_p \sum_{j=0}^{\infty} d^j E_t[\rho_{t,t+j}(1-\tau^{\pi}_{t+j})\pi_{t+j}(p)|P_{t+j}] \tag{1.17}$$

$\rho_{t,t+j}$ is the stochastic discount factor price that equals the household's marginal rate of substitution between consumption at t and $t+j$. The solution of the maximization problem regarding $p_{t,t}$ is

$$p_{t,t} = \frac{\nu}{\nu-1}\left[\frac{\sum_{j=0}^{\infty} d^j E_t \Xi_{t,t+j}(1-\tau^{\pi}_{t+j})MC_{t+j}}{\sum_{j=0}^{\infty} d^j E_t \Xi_{t,t+j}(1-\tau^{\pi}_{t+j})}\right] \tag{1.18}$$

where $\Xi_{t,t+j} = \rho_{t,t+j}(P_{t+j})^{\nu-1}Z_{t+j}$

Then, the price level of the final good in period t is

$$P_t^{1-\nu} = dP_t^{1-\nu} + (1-d)p_{t,t}^{1-\nu} \tag{1.19}$$

1.2.4 The Government

The government alleviates tax on all sources of income for households. Also, the government issues one period debt, D_t, that matures at t, and balances its budget at each period. I consider the government final good purchases, G_t, as exogenous.

$$P_t G_t + D_{t-1} R_{t-1} = D_t + T_t \tag{1.20}$$

I define the real government debt normalized by steady state real GDP as $B_t \equiv \left(\frac{D_t}{P_t Y_{ss}}\right)$.

The Government spending is decomposed into

$$G_t = GC_t + Ig_t \tag{1.21}$$

where GC_t and Ig_t are the government consumption and investment in period t.

Therefore, the tax policy rules are set as

$$\tau_t^C = \tau_{ss}^C + \phi_C^B \hat{B}_t + \phi_C^{GC} \hat{GC}_t + \phi_C^{Ig} \hat{Ig}_t + \phi_C^{\theta} \hat{\theta}_t + \phi_C^{\Pi} \hat{\Pi}_t \tag{1.22}$$

$$\tau_t^K = \tau_{ss}^K + \phi_K^B \hat{B}_t + \phi_K^{GC} \hat{GC}_t + \phi_K^{Ig} \hat{Ig}_t + \phi_K^{\theta} \hat{\theta}_t + \phi_K^{\Pi} \hat{\Pi}_t \tag{1.23}$$

$$\tau_t^L = \tau_{ss}^L + \phi_L^B \hat{B}_t + \phi_L^{GC} \hat{GC}_t + \phi_L^{Ig} \hat{Ig}_t + \phi_L^{\theta} \hat{\theta}_t + \phi_L^{\Pi} \hat{\Pi}_t \tag{1.24}$$

where $\hat{B} = \left(\frac{B_t - B_{ss}}{B_{ss}}\right)$, $\hat{GC} = \left(\frac{GC_t - GC_{ss}}{GC_{ss}}\right)$, $\hat{Ig} = \left(\frac{Ig_t - Ig_{ss}}{Ig_{ss}}\right)$, $\hat{\theta} = \left(\frac{\theta_t - \theta_{ss}}{\theta_{ss}}\right)$, $\hat{\Pi} = \left(\frac{\Pi_t - \Pi_{ss}}{\Pi_{ss}}\right)$, and $\Pi_t = \left(\frac{P_t}{P_{t+1}}\right)$.

It is important to indicate that the variables with subscripts ”$_{ss}$” denote the steady state values. Besides, I distinguish, as Kollmann(2008) three types of feedback policy rules :

- the "simpler rules" which stipulate that the any tax rate above reacts only to the real government debt;
- the "reacher rules" which link any tax rate to real government debt, government consumption, government investment, productivity shock, and inflation; and
- the "baseline rules" which link any tax rate to real government debt, government consumption, government investment, and productivity shock.

I assume that the government commits to setting the policy parameters ϕ_C^B, ϕ_C^{GC}, ϕ_C^{Ig}, ϕ_C^{θ}, and ϕ_C^{Π} at values that maximize the unconditional expected value of household utility subject to the restriction that the unconditional mean of real debt has to close to its steady state value (Kollmann 2008) as

$$|EB_t - B_{ss}| < 0.01. \tag{1.25}$$

The equation (24) is set to rule out the long-run values of debt and taxes that differ greatly from the values observed in reality which has been showed by Aiyagari et al.(2002) from the optimal Ramsey fiscal policy.

1.2.5 The Central Bank

Following Taylor (1993, 1999), the central bank set the short-run nominal interest rate, R_t, in response to inflation and output gaps.

$$R_t = R_{ss} + \phi_R^{\Pi}\hat{\Pi}_t + \phi_R^{y}\hat{Y}_t + \phi_R^{Def}\hat{Def}_t \tag{1.26}$$

where $\hat{Y}_t = \left(\frac{Y_t - Y_{ss}}{Y_{ss}}\right)$, and $\hat{Def}_t = \left(\frac{Def_t - Def_{ss}}{Def_{ss}}\right)$.

Def_t represents the government primary deficit which is defined as

$$Def_t = B_t - B_{t-1}$$

Therefore, two policy rules will be discussed in this paper as in Kollmann (2008):

- the "simpler rules" is defined as the nominal short-run interest rate reacts only to inflation gap and
- the "reacher rules" link the nominal short-run to inflation gap, output gap and government deficit.

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As the government, the central bank also commits to setting the policy parameters ϕ_R^{Π},ϕ_R^{Y}, and ϕ_R^{Def} at the values that maximize the unconditional expected value of household utility.

1.2.6 Market Clearing Conditions

The intermediate goods markets clear as firms meet all demand at posted prices. The markets for final good, labor, and capital rental clear when:

$$Z_t = C_t + I_t + G_t, L_t = \int_0^1 L_t(s) d_s, K_t = \int_0^1 K_t(s) d_s, and \pi_t = \int_0^1 \pi_t(s) d_s \tag{1.27}$$

The bond market is cleared when

$$A_t - B_t = 0. \tag{1.28}$$

1.2.7 Exogenous Variables

I consider three shocks in this model: the productivity and the different types of government spending.

$$\hat{\theta}_t = \rho^{\theta}\hat{\theta}_{t-1} + e_t^{\theta}, \tag{1.29}$$

where $0 \leq \rho^{\theta} < 1$, and $e_t^{\theta} N(0, \sigma_{e\theta}^2)$.

$$\hat{GC}_t = \rho^{GC}\hat{GC}_{t-1} + e_t^{GC} \tag{1.30}$$

where $0 \leq \rho^{GC} < 1$ and $e^{GC} N(0, \sigma_{e\theta}^2)$

$$\hat{Ig}_t = \rho^{Ig}\hat{Ig}_{t-1} + e_t^{Ig} \tag{1.31}$$

where $0 \leq \rho^{Ig} < 1$ and $e_t^{Ig} N(0, \sigma_{e\theta}^2)$

1.3 Calibration and Solution Method

In order to compare the results with Kollamann (2008), I solve my model using Sims' (2000) second-order accurate method. The welfare is evaluated through a second-order Taylor expansion of the utility function around the steady state which gives $E(U(C_t, GC_t, L_t)) \cong U(C_t, GC_t, L_t) + E(\hat{C}_t + \gamma\hat{GC}_t)) - LE(\hat{L}_t) - Var(\hat{C}_t + \gamma\hat{GC}_t))$, where $Var(\hat{C}_t + \gamma\hat{GC}_t)$ is the variance of $\hat{C}_t + \hat{GC}_t)$. By expressing the welfare as the permanent relative change in private consumption and government consumption (compared to the stady state), ξ, which gives

$$E(U(C_t, GC_t, L_t)) : U((1+\xi)(C_t, GC_t), L_t) \cong U(C_t, GC_t, L_t) + E(\hat{C}_t + \gamma\hat{GC}_t) - LE(\hat{L}_t) - Var(\hat{C}_t + \gamma\hat{GC}_t)$$

. Hence, the welfare ξ can be decomposed into two components ξ^m and ξ^v as following:

$$U((1+\xi^m)(C_t, GC_t), L_t) = U(C_t, GC_t, L_t) + E(\hat{C}_t + \gamma \hat{GD}_t) - LE(\hat{L}_t)$$

,

$$U((1+\xi^v)(C_t, GC_t), L_t) = U(C_t, GC_t, L_t) - Var(\hat{C}_t + \gamma \hat{GC}_t)$$

. ξ^m and ξ^v represent the mean of private consumption plus non-investment government spending, and hours worked, and variance of private consumption plus non-investment government spending. By applying the equation (2) into the previous equations yields

$$\ln(1+\xi) = E(\hat{C}_t + \gamma \hat{GC}_t) - LE(\hat{L}_t) - Var(\hat{C}_t + \gamma \hat{GC}_t), \tag{1.32}$$

$$\ln(1+\xi^m) = E(\hat{C}_t + \gamma \hat{GC}_t) - LE(\hat{L}_t), \tag{1.33}$$

$$\ln(1+\xi^v) = Var(\hat{C}_t + \gamma \hat{GC}_t). \tag{1.34}$$

Therefore,

$$(1+\xi) = (1+\xi^m)(1+\xi^v). \tag{1.35}$$

The model is calibrated to quarterly data (1990:1 to 2012:4) of OECD countries. The parameters are set as shown in the table 1. The parameters are consistent with previous studies on OECD countries (Kollmann (2008), Baxter and King (1993), Schitt-Grohe(2004, 2007), Gali et al. (2007)).

Table 1.1: **Parameters' values**

Parameter	Value
β	.99
α	.24
γ	.38
δ	.025
ν	6
d	.75
r	1.01
ρ_θ	.95
ρ_{GC}	.95
ρ_{Ig}	.95
ω	.39
σ^θ	0.01
σ^{GC}	0.01
σ^{Ig}	0.01

1.4 Results

The simulations results are reported in table 2, table 3 and figures 1 to 3. The table 1 reports the results for the optimized policy rules. However, the table 3 reports the optimized parameters rules. The figures 1 to 3 present the dynamic response of macroeconomic variables to the shocks. The columns 1 to 4 in table 1 show the results under sticky prices. Besides, all variables are expressed in terms of deviation from the steady state.

The results show that the introduction of the government spending in the both preferences and production function changes the channel by which the government spending shocks affect the main macroeconomic variables such as the output, the private consumption and the capital accumulation. In fact, the government spending shock (globally) has positive effect on output, private consumption, and private capital accumulation. Both richer and simpler rules show that the government consumption is complement to the private consumption and positive correlated with private capital accumulation, hours worked and output. However, the public capital accumulation is also complement to the private capital accumulation which contradicts Baxter and King (1993) work using the distortion tax. Hence, those results point out the role of each component of the government spending on economic activities.

Under the price stickiness, monopolistic competition, income tax, consumption tax and the government spending in both preferences and production function, the optimized monetary policy is full inflation stabilization (Kollamann 2002, 2008; Erceb et al. 2000; Rotemberg and Woodford). In fact, the optimized policy rule implies that an increase in the inflation gap by one percent increases the nominal interest rate by 1.04 percent. Besides, the optimized fiscal policy implies that an increase in the stock of the debt by an amount equal to (quarterly) steady state GDP raises the consumption, capital income , and labor income tax rates by .03, 2.23, and 1.003 percentage points which is relative lower than what Kollmann (2008) found in his paper without government spending in the preferences and production function (9.09).

The real debt fluctuations are highly persistent (autocorrelation 0.99) and volatile (standard deviation of real debt 1.37%). The inflation and the gross return on capital fluctuations are also highly persistent (autocorrelation .97 and .97 respectively) and less volatile (standard deviation 0.08% and 0.1%). Other variables' fluctuations are highly persistent (autocorrelation .95 in average) and highly volatile (standard deviation 5.83% in average). In fact, the most volatile variable is private investment (standard deviation 35.49%). Besides, the consumption tax rate which undergoes anticyclical fluctuations (correlation with the GDP is -.39) is the second highest volatile variable (standard deviation 18.58%.

The unconditional mean value of the output exceeds its steady state value by 0.31%. The mean of the consumption also exceeds its steady value by .32%. Besides, the conditional mean of the inflation and real debt are slightly below their steady state by 0.02%. This confirms that the optimized monetary policy entails full inflation stabilization. Moreover, the mean value of the capital stock exceeds slightly its steady state by 0.07% which may be viewed as reflecting precautionary saving (also confirmed by Kollamann 2008). Furthermore, the conditional welfare is higher than steady state welfare measure, $\xi = 1.308076\%$, based on all the shocks. The mean of the welfare contributes positively, $\xi^m = 0.39.976\%$. Its variance contributes mostly with $\xi = 0.917100\%$ (This level of welfare is higher than Kollmann 2008). When, we consider the shock individually, government consumption and investment have bigger effect on welfare than the productivity. This can be explainable by the fact that the government spending is included in the preferences and production function.

Under the flexible prices, the welfare ($\xi = 1.706702\%$) is greater than under sticky price ($\xi = 1.308076\%$) which is consistent with current literature (Obdtfeld and Rogoff (1995, 1998, 2000, and 2001), Betts and Devereux 1996, Devereux and Engel 2003 and 2006, Kollmann 2002, and Engel 2013). In fact, the literature predicts high welfare under flexible prices than under sticky prices.

The productivity shock is the major source of volatility of macroeconomic variables.In fact, the productivity shock explains the variances of the output, consumption, capital stock, hours worked, inflation and nominal interest rate by 55.58%, 75.91%, 81.14%, 61.27%, 85.96% and 85.54% respectively when I take into account all shocks. This finding is consistent with Kollmann (2008).

Under the optimized policy, the paper finds that a positive productivity shock raises the output, the consumption, the capital stock, the investment, and the hours worked. However, the inflation and the income tax rate are negative affected by the productivity shock. Besides, a positive government consumption shock increases the output, the consumption, the capital accumulation and the tax rates. Moreover, a positive public capital stock shock increases all key macro variables (consumption, output, capital stock, investment), but lowers the gross return on capital. Finally, the richer policy rules are slightly associated with the same welfare gain as the simpler rules (positive gain) which is the same as the work of Kollmann (2008). Therefore, the single-minded pursuit of price stability by many central banks in OCDE countries seems effective if we consider the welfare policy evaluation criterion. This is also confirmed by the finding of Kollmann (2008).

1.5 Conclusion

This paper computers welfare-maximizing monetary and fiscal policy feedback rules in calibrated DSGE model with sticky prices, monopolistic competition in the intermediate goods markets, and tax distortions. The government spending shock is decomposed into public investment and consumption. Also, the model allows the government consumption in the preferences of the representative household and the public capital stock in the production function of the intermediate goods' producer. The simpler optimized policy rules are not only full inflation stabilization for monetary policy, but also public debt stance for the fiscal policy.

The future works in this paper will expand the model to an open economy and will compare the policy rules under sticky prices with flexible prices, tax distortion and lap sum tax. Also, the model will include more frictions such as the adjustment costs, the consumption habit, and the non-optimized consumers.

Table 1.2: **Optimized policy rule**

variable	Sticky Prices				Flexible Prices
	Baseline rules	Baseline rules	Richer Rules	Simpler Rules	Baseline Rules
	θ, GC, Ig	GC, Ig	θ, GC, Ig	θ, GC, Ig	θ, GC, Ig
		Standard deviations(in %)			
K	5.42	5.9	5.99	5.35	0.33
IP	34.43	0.69	15.03	35.49	2.67
GK	0.18	0.18	0.18	0.18	0.10
C	3.86	4.69	4.69	3.8	0.23
Π	0.29	0.02	0.1	0.08	0.50
G	0.23	0.23	0.23	0.23	0.39
GC	0.32	0.32	0.32	0.32	0.32
Ig	0.32	0.32	0.09	0.32	0.22
B	8.1	6.28	2.28	1.37	0.77
Def	1.72	0.97	.97	1.8	0.62
R	5.42	0.9	3.97	5.41	0.34
		Correlations with GDP			
R	-0.01	-0.01	-0.01	-0.04	-0.91
G	0.03	0.1	0.1	0.10	0.28
GC	0.002	0.15	0.0012	0.11	0.25
Ig	0.04	0.016	0.02	0.0156	0.13
τ^k	-0.32	-0.38	-0.36	-0.39	-0.73
τ^c	-0.71	-0.72	-0.95	-0.39	-0.68
τ^l	-0.12	-0.12	-0.12	-0.39	-0.77
		Autocorrelations			
Y	0.76	0.80	0.93	0.92	0.82
R	0.97	0.97	0.98	0.97	0.82
τ^k	0.97	0.97	0.98	0.97	0.85
τ^c	0.93	0.94	0.95	0.93	0.81
τ^l	0.98	0.99	0.98	0.97	0.90
B	0.99	0.98	0.98	0.99	0.83
		Means (in %)			
Y	-0.02	-0.02	-1.28	-0.02	1.13
C	-0.23	-0.23	-1.81	-1.23	0.26
L	-0.19	-0.19	-1.55	-1.19	0.71
K	-0.31	-0.31	-0.42	-0.31	2.43
τ^k	-1.31	-1.31	-1.19	-1.31	-8.30
τ^c	-0.59	-0.59	-18.89	-0.59	-1.04
τ^l	-0.60	-0.60	-6.81	-0.60	-2.99
B	0.56	0.56	-9.4	0.56	-4.15
		Welfare (in %)			
ξ	0.000172	0.000172	1.099802	1.308076	1.706702
ξ^m	0.000002	-0.000004	0.182702	0.390976	0.789603
ξ^v	0.000170	0.000176	0.917100	0.917100	0.917099

Table 1.3: **Policy Parameters**

variable	Sticky Prices			
	Baseline rules	Baselinerules	Richer Rules	Simpler Rules
	θ, GC, Ig	GC, Ig	θ, GC, Ig	θ, GC, Ig
ϕ_R^{Π}	1.025	1.025	1.025	1.03
ϕ_R^{Y}	0.7215	0.7215	0.7215	—
ϕ_R^{Def}	—	—	-0.6745	—
ϕ_K^{B}	2.2189	2.2189	2.2189	2.2257
ϕ_K^{GC}	-0.6367	-0.6367	-0.6367	—
ϕ_K^{Ig}	-0.6023	-0.6023	-0.6023	—
ϕ_K^{θ}	-0.6396	-0.6396	-0.6396	—
ϕ_K^{Π}	-0.4859	-0.4859	-0.4859	—
ϕ_C^{B}	0.0669	0.0669	0.0669	0.0314
ϕ_C^{GC}	-0.6432	-0.6432	-0.6432	—
ϕ_C^{Ig}	-0.5225	-0.5225	-0.5225	—
ϕ_C^{θ}	-0.6786	-0.6786	-0.6786	—
ϕ_C^{Π}	-0.4116	-0.4116	-0.4116	—
ϕ_L^{B}	1.0113	1.0113	1.0113	1.003
ϕ_L^{GC}	-0.3524	-0.3524	-0.3524	—
ϕ_L^{Ig}	-0.5799	-0.5799	-0.5799	—
ϕ_L^{θ}	-0.6123	-0.6123	-0.6123	—
ϕ_L^{Π}	-0.6072	-0.6072	-0.6072	—

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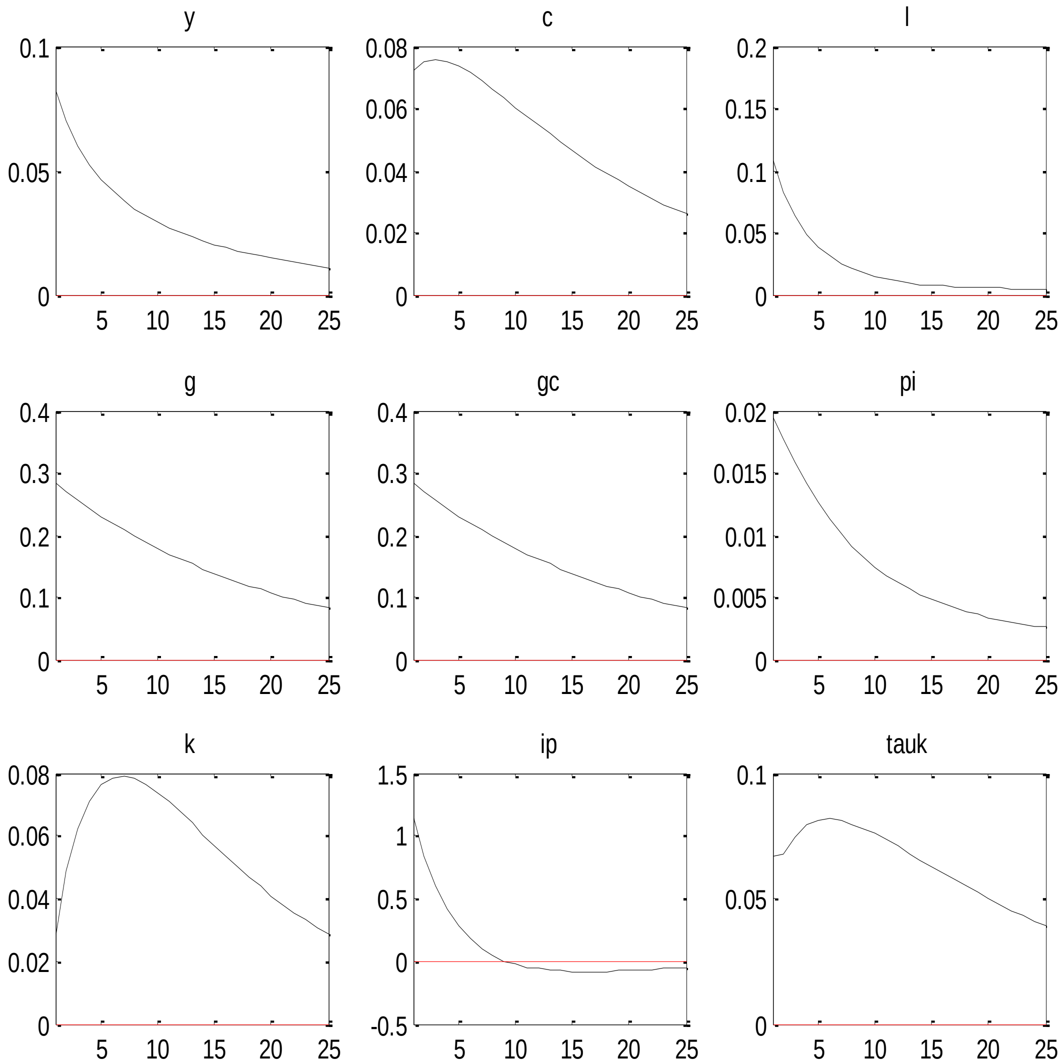
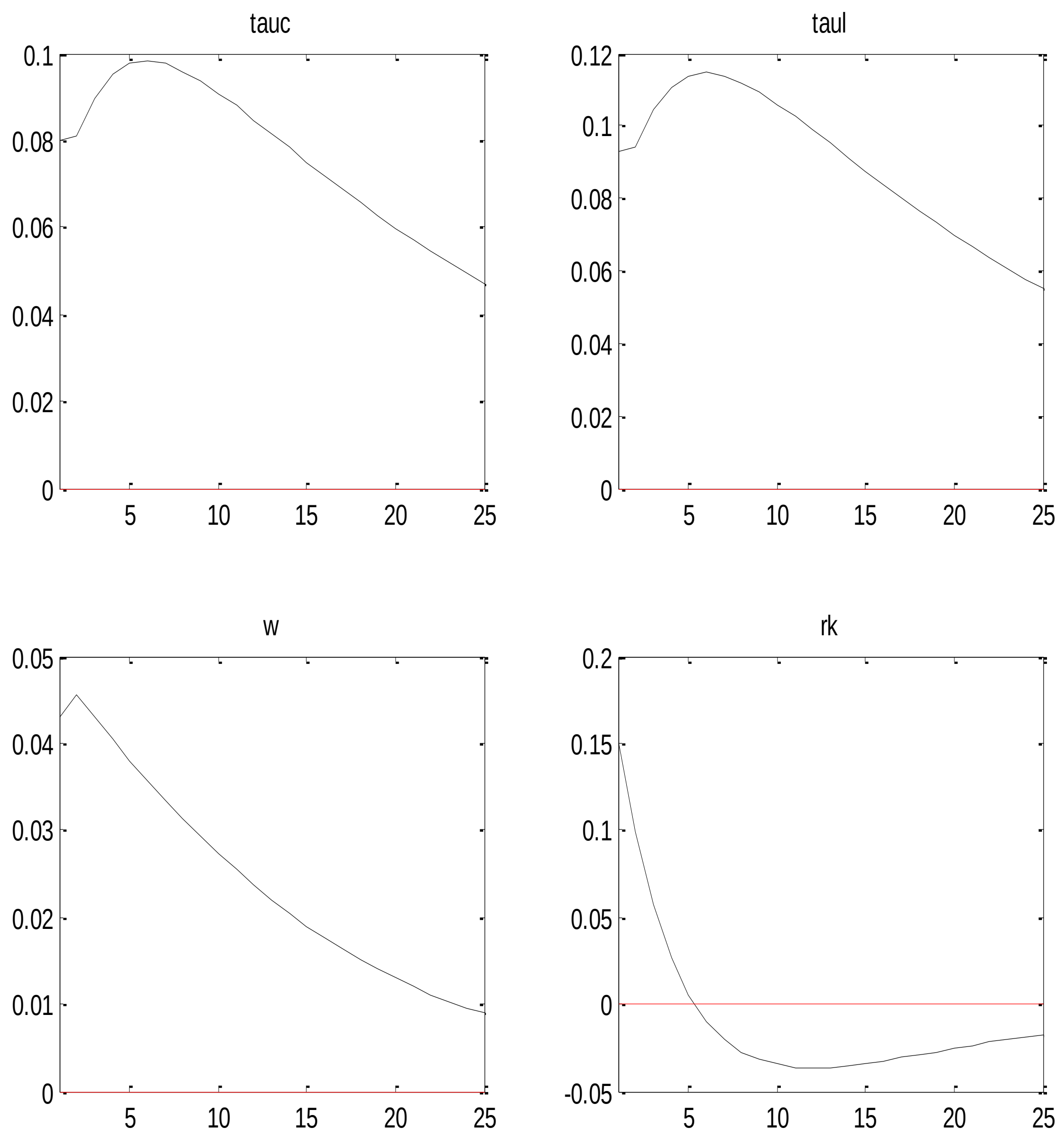
Appendices

A. Impulse Response Functions

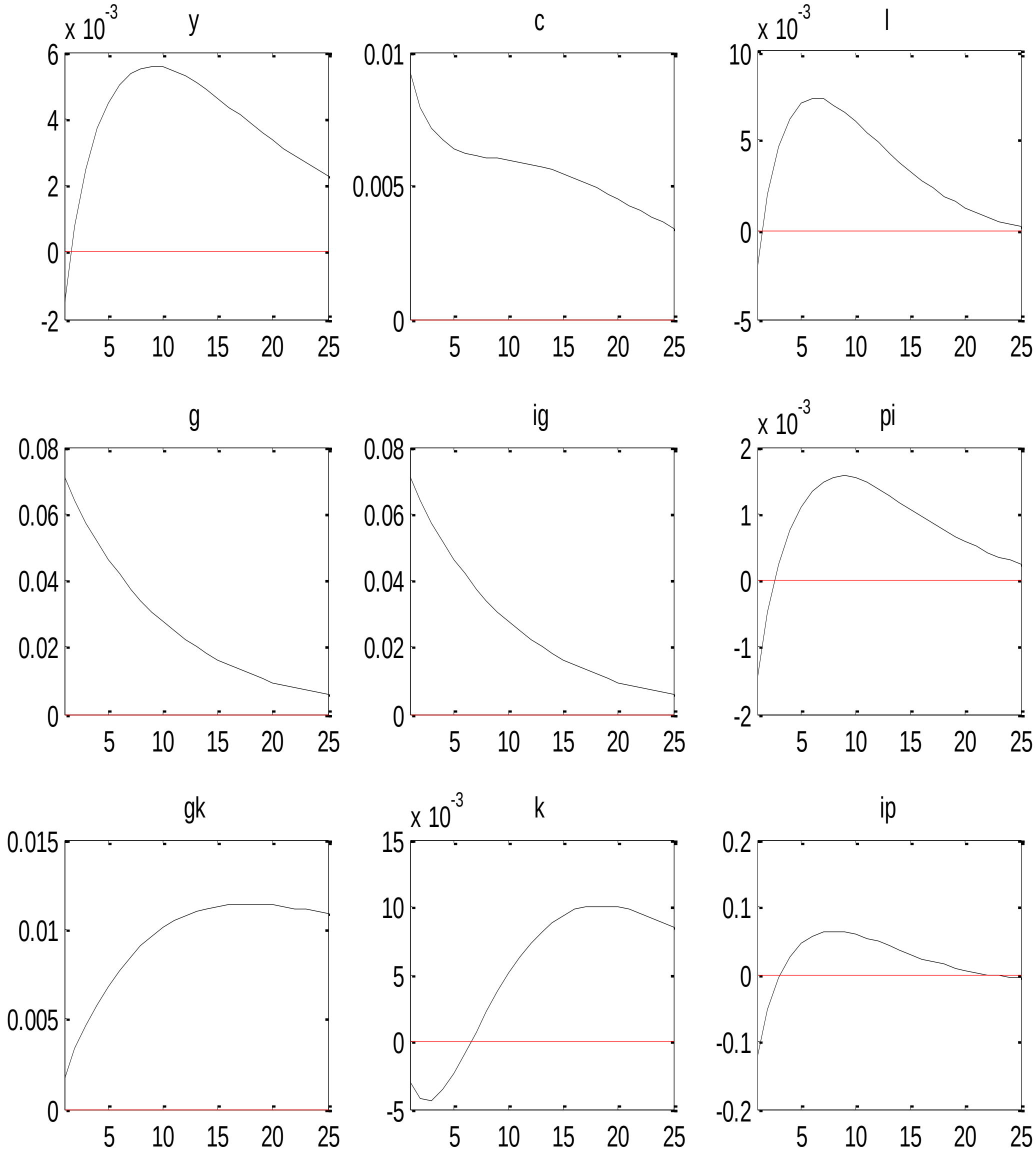
A.1. Impact of 1% increase on Government Consumption



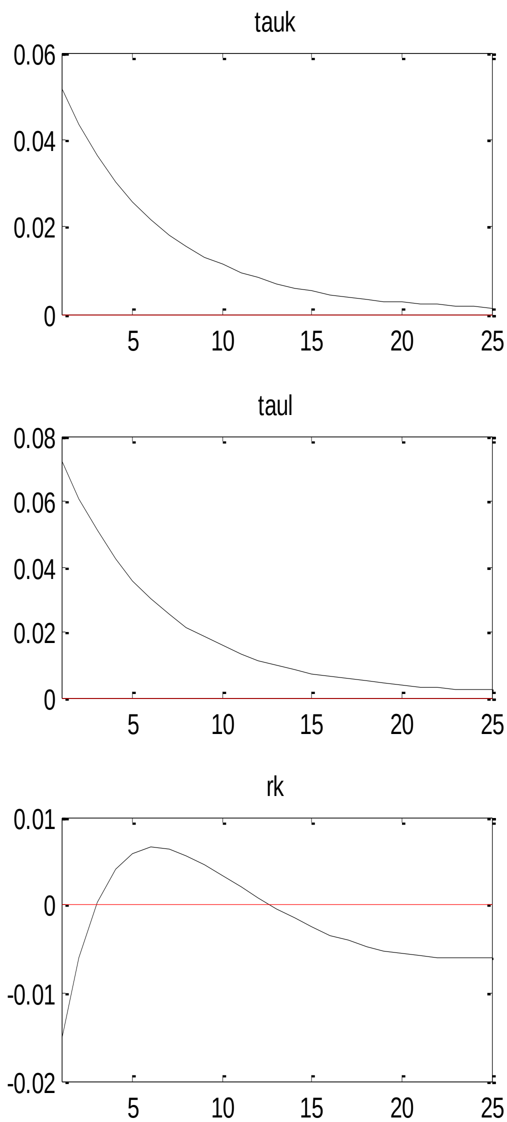
A.2. Impact of 1% increase on Government Consumption



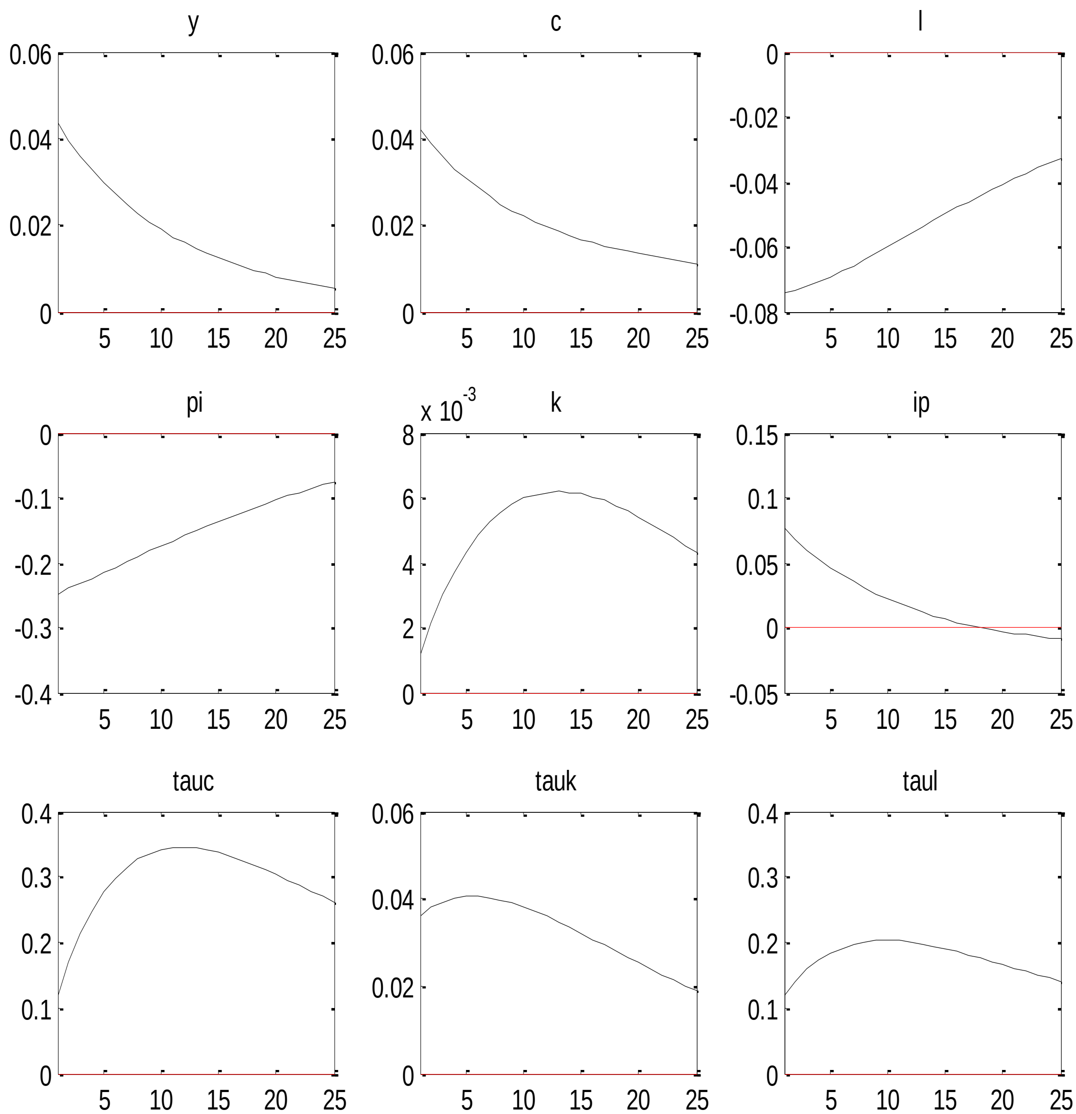
A.3. Impact of 1% increase on Government Investment



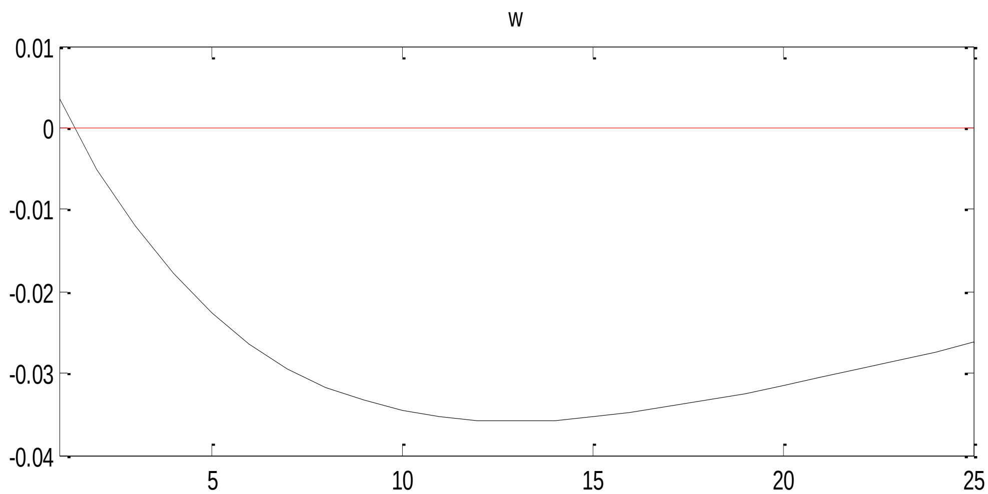
A.4. Impact of 1% increase on Government Investment

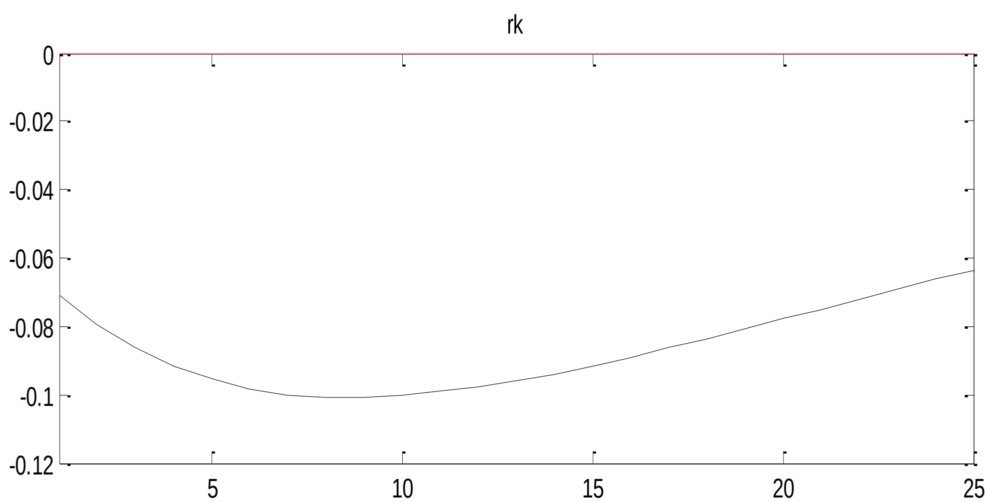


A.5. Impact of 1% increase on Productivity



A.6. Impact of 1% increase on Productivity





B. **The Model's Non-Linear Equilibrium Conditions**

$$Y_t = C_t + I_t + G_t \tag{36}$$

$$G_t = GC_t + Ig_t \tag{37}$$

$$GK_t = Ig_t + (1-\delta)GK_{t-1} \tag{38}$$

$$PK_t = I_t + (1-\delta)PK_{t-1} \tag{39}$$

$$K_t = PK_t + GK_t \tag{40}$$

$$\beta E_t \left[\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \frac{\left(1+\tau_t^C\right)}{\left(1+\tau_{t+1}^C\right)} \left(\frac{R_t}{\Pi_{t+1}} \right) \right] = 1 \tag{41}$$

$$\beta E_t \left[\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \frac{\left(1+\tau_t^C\right)}{\left(1+\tau_{t+1}^C\right)} \left((1-\tau_{t+1}^K)R_t^K + \delta \tau_{t+1}^K + (1-\delta) \right) \right] = 1 \tag{42}$$

$$\frac{1}{C_t + \gamma GC_t} \frac{\left(1-\tau_t^L\right)}{\left(1+\tau_t^C\right)} W_t = \psi \tag{43}$$

$$Y_t = \theta^t GK_t^{\omega} K_t^{\alpha} L_t^{1-\alpha} \tag{44}$$

$$W_t = (1-\alpha)MC_t \frac{Y_t}{L_t} \tag{45}$$

$$R_t^K = \alpha MC_t \frac{Y_t}{K_t} \tag{46}$$

$$N_t = Y_t MC_t + \eta \beta E_t \left[\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \Pi_{t+1}^{\nu} N_{t+1} \right] \tag{47}$$

$$J_t = \frac{\nu}{\nu - 1} N_t \tag{48}$$

$$P_t^* = \left(\frac{1 - \eta \Pi_{t+1}^{\nu-1}}{1-\eta} \right)^{\frac{1}{1-\nu}} \tag{49}$$

$$J_t = Y_t P_t^* + \eta \beta E_t \left[\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \left(\frac{P_t^*}{P_{t+1}^*} \right) \Pi_{t+1}^{\nu-1} J_{t+1} \right] \tag{50}$$

$$G_t + R_{t-1}B_{t-1} = B_t + T_t \tag{51}$$

$$T_t = \tau_t^C C_t + \tau_t^K (R_t^K - \delta) K_t + \tau_t^L W_t L_t \tag{52}$$

$$log(GC_t) = \rho_{gc} log(GC_{t-1}) + e_{gct} \tag{53}$$

$$log(Ig_t) = \rho_{Ig} log(Ig_{t-1}) + e_{Igt} \tag{54}$$

$$log(\theta_t) = \rho_{\theta} log(\theta_{t-1}) + e_{\theta t} \tag{55}$$

$$\left(\frac{R_t}{R_{ss}}\right) = \left(\frac{\Pi_t}{\Pi_{ss}}\right)^{\phi_R^{\Pi}} \left(\frac{Y_t}{Y_{ss}}\right)^{\phi_R^{Y}} \left(\frac{Def_t}{Def_{ss}}\right)^{\phi_R^{Def}} \tag{56}$$

$$\left(\frac{\tau_t^C}{\tau_{ss}^C}\right) = \left(\frac{\Pi_t}{\Pi_{ss}}\right)^{\phi_C^{\Pi}} \left(\frac{B_t}{B_{ss}}\right)^{\phi_C^{B}} \left(\frac{GC_t}{GC_{ss}}\right)^{\phi_C^{GC}} \left(\frac{Ig_t}{Ig_{ss}}\right)^{\phi_C^{Ig}} \left(\frac{\theta_t}{\theta_{ss}}\right)^{\phi_C^{\theta}} \tag{57}$$

$$\left(\frac{\tau_t^L}{\tau_{ss}^L}\right) = \left(\frac{\Pi_t}{\Pi_{ss}}\right)^{\phi_L^{\Pi}} \left(\frac{B_t}{B_{ss}}\right)^{\phi_L^{B}} \left(\frac{GC_t}{GC_{ss}}\right)^{\phi_L^{GC}} \left(\frac{Ig_t}{Ig_{ss}}\right)^{\phi_L^{Ig}} \left(\frac{\theta_t}{\theta_{ss}}\right)^{\phi_L^{\theta}} \tag{58}$$

$$\left(\frac{\tau_t^K}{\tau_{ss}^K}\right) = \left(\frac{\Pi_t}{\Pi_{ss}}\right)^{\phi_L^{\Pi}} \left(\frac{B_t}{B_{ss}}\right)^{\phi_K^{B}} \left(\frac{GC_t}{GC_{ss}}\right)^{\phi_K^{GC}} \left(\frac{Ig_t}{Ig_{ss}}\right)^{\phi_K^{Ig}} \left(\frac{\theta_t}{\theta_{ss}}\right)^{\phi_K^{\theta}} \tag{59}$$

C. **Steady State**

I assume that the steady state values of inflation are $\Pi_{ss} = 1$. Besides, I fix the steady state values of the exogeneous variables are set as $\theta_{ss} = 1$, $GC_{ss} = 0.1$, and $Ig_{ss} = 0.1$. It is very easy to fix the worked hours as $L_{ss} = \frac{1}{3}$ and $B_{ss} = 0$ in order to solve numerecally with dynare for the steady state values of remained variables.

$$MC_{ss} = \frac{\nu - 1}{\nu} \tag{60}$$

$$\Pi_{ss} = 1 \tag{61}$$

$$\theta_{ss} = 1 \tag{62}$$

$$Y_{ss} = C_{ss} + I_{ss} + G_{ss} \tag{63}$$

$$G_{ss} = GC_{ss} + Ig_{ss} \tag{64}$$

$$Ip_{ss} = \delta PK_{ss} \tag{65}$$

$$GK_{ss} = \frac{Ig_{ss}}{\delta} \tag{66}$$

$$R_{ss} = \frac{\Pi_{ss}}{\beta} \tag{67}$$

$$\beta\left[(1 - \tau_{ss}^K) + \alpha\tau_{ss}^K + (1 - \delta)\right] = 1 \tag{68}$$

$$\left[\frac{1}{C_{ss} + \gamma GC_{ss}} \frac{(1 - \tau_{ss}^L)}{(1 + \tau_{ss}^C)} W_{ss}\right] = \psi \tag{69}$$

$$N_{ss} = \frac{Y_{ss} MC_{ss}}{1 - \eta\beta} \tag{70}$$

$$J_{ss} = \frac{\nu}{\nu - 1} N_{ss} \tag{71}$$

$$P^*_{ss} = 1 \tag{72}$$

$$J_{ss} = \frac{Y_{ss}}{1 - \eta\beta} \tag{73}$$

$$T_{ss} = \tau^C_{ss} C_{ss} + \tau^K_{ss} (R^K_{ss} - \delta) K_{ss} + \tau^L_{ss} W_{ss} L_{ss} \tag{74}$$

$$G_{ss} = T_{ss} \tag{75}$$

$$Y_{ss} = \theta_{ss} K^{\alpha}_{ss} L^{1-\alpha}_{ss} \tag{76}$$

Chapter 2

Tax and Monetary Policy Rules in a Small Open Economy with Dissaggregate Government Purchases

Abstract

This paper aims to evaluate the impact of tax and monetary policy rules with dissaggregate government purchases on welfare, real exchange rate and business cycle in a small open economy using a new-Keynesian dynamic stochastic general equilibrium frame work. The model predicts that the government consumption has more impact than investment on both private consumption and investment, but less impact on the real GDP. Moreover, the government purchases-real exchange rate puzzle is generated by the model. In this sense, the government consumption contributes more on generating the puzzle than the investment. Moreover, the productive and complement government purchases have positive impact on welfare for any policy rules. The optimized policy rules have a pronounced anti-inflation stance and entail significant nominal and real exchange rate volatility for monetary policy. For tax policy rules, the public debt stance is the optimized rules.

JEL classification : E32; E43; E52; E62; F31; F41

Keywords : Open Economy; Fiscal Policy; Monetary Policy; Determination of Interest Rate Rules; Business Cycles

2.1 Introduction

This paper aims to evaluate the impact of tax and monetary policy rules with decomposed government purchases (consumption and investment) on welfare, real exchange rate and business cycle for an small open economy using a dynamic stochastic general equilibrium (DSGE) model with nominal rigidities: monopoly competition and sticky prices à la Calvo for intermediate producers. The model also takes into account seven chocks due to : (i) the domestic productivity, (ii) the world interest rate, (iii) the uncovered interest parity condition (UIP) (Fama 1984), (iv) the world output, (v) the government consumption, (vi) government investment, and (vii) the world inflation. In fact, this paper adds the government sector to the model developed by Kollmann 2002. Many papers have analyzed the impact of government purchases on private consumption, output, and real exchange rate (Ravn, Schmitt-Grohé, and Uribe, 2007, Monacelli and Perotti, 2010; Kollmann, 2010; Basu and Kollman, 2013). The latter paper finds that a rise in government purchases not only increases the private consumption, but also depreciates the real exchange rate due to the introduction of productive government investment if these purchases increase domestic private sector, and labor supply is highly elastic. These main results raise two important puzzles: (i) the government purchases-consumption puzzle (Sanchez, 2001; Linnemann and Schabert, 2003; Blanchard and Perotti 2002; Linnemann, 2008; Gali et Al. 2007; Escolani, 2007; Mountford and Uhlig, 2009; Woodford, 2010) and (ii) government purchases-real exchange rate puzzles (Sanchez, 2001; Forni and Pisani, 2010; Monacelli and Perotti, 2010; Kollmann, 2010; Basu and Kollmann, 2013). This paper aims to analyze the effect of decomposed the government purchases (consumption and investment)on a small open economy. Which government purchases contribute more to the depreciation of the real exchange rate, the household's welfare and the business cycle? This question will help to understand the mechanism by which the decomposed government purchases affect an small open economy with respect to optimized tax and monetary policy rules.

The puzzles are due to the fact that the main results mentioned above contradict the predictions of the standard neoclassic models which find that the rise in government purchases (i) decreases private consumption (Baxter and King, 1993; Backus et Al. 1994; Ramey and Shapiro 1998; Ramey, 2008), and (ii) appreciates the real exchange due to the wealth effect (Kollmann, 1991, 1995; Backus et al. 1993, 1994) which lowers private consumption and increases hours of work and output. Therefore, if the consumption risk is internationally shared through complete financial markets, the rise in the home marginal utility of consumption is accompanied by an appreciation of the home real exchange rate. However, the model developed in this paper generates the government purchases-real exchange rate puzzle through marginal product of both labor and capital.

The introduction of the government purchases in both utility and production functions follows the seminal work of Baxter and King (1993) in which the private and government consumption in the utility function of the representative agent is separated. However, this paper will consider a non-separate formulation between the private and government consumption which should sustain their complementary. Besides, most of macroeconomic models have neglected to analyze the effect of the decomposed government purchases in the entire economy and the literature related to the question is not abundant. This paper aims to enrich the literature by analyzing the effect of the decomposed government purchases in a small opened economy through the optimized tax and monetary policy rules.

As the model deals with many relative prices, the paper takes into account the departure from the law of one price (LOP) which implies limited exchange rate pass-through and price to market (PTM) behavior of producers (Knetter, 1993; Betts and Devereux, 2000;Kollmann, 2002; Devereux and Engel,1998, 2002; Devereux and Yetman, 2014). In fact, the price to market assumption means that the producers will charge price in the currencies of their costumers. Therefore, the monetary authority set a rule which links the nominal interest rate to gross domestic PPI inflation. This rule will be compared to alternative measures of inflation such as the producer currency pricing, gross consumer price index (CPI) inflation,and exchange rate peg.

Furthermore, the financial market for bonds is assumed to be incomplete because the world households do not buy bonds issued in small open economy currency. Only the households in small open economy who hold both bonds in domestic and foreign currencies. Therefore, the consumption risk is not efficiently shared internationally (e.g., Kollmann 2010). Any rise in the government purchases will depreciate the real exchange rate.

I assume here that the government consumption is complement to the private consumption (e.g., Bouakez and Rebei, 2007; Gali et al., 2007; Monacelli and Perotti, 2010). This channel works perfectly through the international risk sharing of the consumption growth between the domestic and the foreign economies to explain the depreciation of real exchange. Besides, government consumption contributes to increase the aggregate demand which has positive impact on final goods. However, the government investment is productive in the sense that it increases the productivity of both labor and the effective capital stock through public capital stock. Therefore, the government investment increases output and depreciates the real exchange rate (the second channel). So far in the literature, the role of the government consumption to generate the depreciation of real exchange rate has never been studied. This paper investigates the impact of decomposed government purchases not only on real exchange rate, but also on welfare-optimized tax and monetary policy rules and business cycle.

The paper finds that the government consumption has more impact than investment on both private consumption and investment, but less impact on the real GDP. Moreover, the government purchases-real exchange rate puzzle is generated by the model. In this sense, the government consumption contributes more on the generate the puzzle than the investment. Besides, both government consumption and investment have positive impact on welfare for any policy rules. The optimized policy rules have not only a pronounced anti-inflation stance and entail significant nominal and real exchange rate volatility for monetary policy, but also a public debt stance for tax policy rules.

The mechanism by which the decomposed government purchases-exchange rate puzzle is generated by the model is different for each component of the government purchases. An increase in the government investment shock improves both marginal productivity of labor and capital stock which have a positive impact on labor supply, private capital stock, and output. This supply effect boosts the private consumption and wealth which depreciate the real exchange rate (first channel). The second and the third channels through which the government consumption deteriorates the real exchange rate is the international risk sharing. If the real exchange rate is expressed in terms relative marginal utility of domestic and foreign consumption, an increase in government consumption will depreciate the real exchange rate through two channels. The first channel is a direct effect of the government consumption on real exchange rate. The second channel is the complement relationship between private consumption and government consumption.

The rest paper will be organized as follow. I will present the model in details in section 2. The results are presented in section 3 and will conclude the paper in section 4.

2.2 The Model

The model describes here is a New-Keynesian DSGE model for a small open economy with a representative household, firms, a government, a central bank, and rest of the world. The model adds the government agent to the model developed by Kollmann (2002).

The representative household maximizes his utility under the budget constraint and the law of motion of the private capital. Also, he holds all means of production (labor and private capital stock) that are rent/supply to firms in a perfect competitive market. From the revenues that he receives, he consumes, invests in capital stock, and saves in terms of bonds (domestic and foreign).

There are two types of firms : the firms which produce the final goods and the ones which produce the intermediate goods. I assume that the final composite good is not tradable in the world market. Only the intermediate goods are tradable between the small open economy and the rest of the world. The composite final good is produced by combining the intermediate goods from the domestic and import foreign firms. There is a continuum of intermediate goods indexed by $s\varepsilon[0,1]$.

The government sector collects taxes, issues the debt through the central bank and finances its consumption and investment purchases. The latter is used to produce the domestic intermediate goods and the former to be consumed by the representative household. This is one the contribution of this paper.

The central bank issues bonds for domestic government and set the nominal interest according to the Taylor rules.

Finally, the rest of the world imports the intermediate goods which are used to produce the composite final domestic goods. Also, it sells the foreign bonds to the representative household.

2.2.1 The representative household

The representative household maximizes his preferences through

$$E_0 \sum_{t=0}^{t=\infty} \beta^t U(C_t, GC_t, L_t). \tag{2.1}$$

where E_0 represents the mathematical expectation conditional upon the complete information pertaining to period t and earlier. Also, $0 < \beta < 1$ is the subjective discount factor. $U(.)$ is the utility function which is monotone and continuous. Finally, C_t, L_t, and GC_t represent respectively the private consumption, labor effort, and government consumption at time period t. Let the utility function take the following form:

$$U(C_t, GC_t, L_t) = ln(C_t + \gamma GC_t) - \psi L_t. \quad (2.2)$$

I assume here that the government consumption is complement to the private one. This assumption is important to generate the government purchases-real exchange rate puzzle.

Since the household holds all domestic producers and accumulates physical capital. The law of motion of the capital stock is given by

$$PK_t = (1-\delta)PK_{t-1} + Ip_t \quad (2.3)$$

where PK_t, PK_{t-1}, Ip_t, and $0 < \delta < 1$ represent respectively the private capital stock at period t , the private capital stock at time $t-1$, the gross investment at time t, and the depreciation rate of the capital stock.

Besides, the household's budget constraint at period t is defined as

$$A^d_{t+1} + S_t A_{t+1} + P_t(C_t + Ip_t) = R_{t-1}A^d_t + S_t \varphi_{t-1} Rf_{t-1} A_t + R^K_{t-1} PK_{t-1} + DIV_t + W_t L_t - T_t \quad (2.4)$$

where

$$T_t = \tau^C_t P_t C_t + \tau^K_t R^K_t PK_{t-1} + \tau^L_t W_t L_t - \tau^K_t P_t PK_{t-1} \quad (2.5)$$

where A^d_t and A_t are the net stocks of risk-free domestic and foreign currency bonds that mature in period t. R_{t-1} is the interest rate on domestic currency bonds and Rf_{t-1} is the interest rates on foreign currency bonds adjusted by a risk premium, φ_{t-1}. This risk premium is a endogenous function which is a decreasing function of stationary holding of foreign assets of the entire domestic economy, and a increasing function of risk premium shock, χ_{t-1}. The endogenous risk premium function has the following form

$$\varphi_{t-1} = exp(-\left(\frac{A_{t-1}}{A_{ss}}\right) + \chi_{t-1}) \tag{2.6}$$

where

$$ln\chi_{t-1} = \rho_{\chi} ln\chi_{t-2} + \varepsilon^{\chi}_{t-1}. \tag{2.7}$$

χ_{t-1} can be interpreted as bias in the household's date $t-1$ forecast of the date t exchange rate, S_t, based on yesterday information about interest rate (Kollmann, 2002). Besides, the stock of bonds from domestic government is assumed to be non-negative. In fact, the household is not allowed to borrow from the government. The foreign bonds are bought from the world market. Here, the model allows this stock of bonds to be negative or positive. If the stock is negative, it means that the household is a net borrower and a net lender, otherwise.

Moreover, R^K_t, W_t, T_t, DIV_t, and S_t represent, respectively, the rental rate of capital, the rental rate of labor, the total taxes received by the government from the household, the dividend received by household and the nominal exchange rate. Finally, τ^C_t, τ^K_t, and τ^L_t are the tax rates on consumption (generally known as the value added tax in most of European countries), capital, the and labor income respectively.

Therefore, the household maximizes (1) subject to (4) and (5) from which the first order conditions are derived which respect to domestic currency bonds, international currency bonds, private capital stock at time t, and labor hours:

$$1 = \beta E_t \left[\left(\frac{C_t + \gamma G C_t}{C_{t+1} + \gamma G C_{t+1}} \right) \left(\frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} \right) \frac{1}{\pi_{t+1}} R_t \right], \tag{2.8}$$

$$1 = \beta E_t \left[\left(\frac{C_t + \gamma G C_t}{C_{t+1} + \gamma G C_{t+1}} \right) \left(\frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} \right) \frac{1}{\pi_{t+1}} e_{t+1} \varphi_t R f_t \right], \tag{2.9}$$

$$1 = \beta E_t \left[\left(\frac{C_t + \gamma G C_t}{C_{t+1} + \gamma G C_{t+1}} \right) \left(\frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} \right) \left(R_t^K (1 - \tau_{t+1}^K) + \delta \tau_{t+1}^K + (1 - \delta) \right) \right], \tag{2.10}$$

$$\psi = \left[\left(\frac{1}{C_t + \gamma G C_t} \right) \left(\frac{1 - \tau_t^L}{1 + \tau_t^C} \right) W_t \frac{1}{\phi_t} \right], \tag{2.11}$$

e_{t+1} is the depreciation factor of the nominal exchange rate between period $t+1$ and t which is defined as:

$$e_{t+1} = \frac{S_{t+1}}{S_t}. \tag{2.12}$$

2.2.2 Uncovered Interest Parity

Combining equations (6) and (7) yields

$$R_t = E_t e_{t+1} \varphi_t R f_t. \tag{2.13}$$

Then, the log-linearization around the steady state of (13) gives the uncovered interest parity (UIP) condition with an endogenous risk premium, φ_t :

$$E_t \hat{S}_{t+1} - \hat{S}_t = \hat{R}_t - \hat{R f}_t - \hat{\varphi}_t. \tag{2.14}$$

I define $\hat{S}_t = \frac{S_t - S_{ss}}{S_{ss}}$, $\hat{R}_t = \frac{R_t - R_{ss}}{R_{ss}}$, $\hat{Rf}_t = \frac{Rf_t - Rf_{ss}}{Rf_{ss}}$, and $\hat{\varphi}_t = \frac{\varphi_t - \varphi_{ss}}{\varphi_{ss}}$ where the subscript ss means steady state value. I assume here that φ_t is one at steady state in order to solve the model. the the The departure of the UIP condition was first empirically observed by Fama (1984) is measured here by φ_t. This endogeneous risk premium function is introduced following Schmitt-Grohé and Uribe (2003) in order to make the small open economy model stationary.

2.2.3 The firms

- **Final good**

The economy produce the final good according the aggregate technology

$$Z_t = \left[(1-\alpha^m)^{\frac{1}{\vartheta}} (Y_t^d)^{\frac{(\vartheta-1)}{\vartheta}} + (\alpha^m)^{\frac{1}{\vartheta}} (Y_t^m)^{\frac{(\vartheta-1)}{\vartheta}} \right]^{\frac{\vartheta}{(\vartheta-1}} \tag{2.15}$$

where Z_t is the final good output at time t; Y_t^d is the quantity of domestic intermediate goods used to produced the final good output; Y_t^m is the quantity of imported intermediate goods used to produce the final good output;$\vartheta > 0$ is the elasticity of substitution between the imported and domestic intermediate goods in the production of the final good output; and α^m is the share of imported intermediate good in the production of the composite final good. Also, it determines the steady state degree of openness (here is expressed in terms of imports to GDP ratio) of the country to the rest of the world.

Besides, I assume the firm operates in a perfectly competitive market. It takes the price of output, P_t, as given. Therefore, if P_t^d and P_t^m are prices of domestic and imported intermediate goods sold in the domestic market, the problem of the firm consists of cost minimizing of the production of the final composite output by combining the inputs (intermediate goods) subject to (15) above which yields the following demands for intermediate goods required to produce the final composite good:

$$Y_t^m = \alpha^m \left(\frac{P_t^m}{P_t} \right)^{-\vartheta} Z_t, \tag{2.16}$$

and

$$Y_t^d = (1 - \alpha^m) \left(\frac{P_t^d}{P_t} \right)^{-\vartheta} Z_t, \tag{2.17}$$

where

$$P_t = \left[(1 - \alpha^m)(P_t^d)^{1-\vartheta} + \alpha^m (P_t^m)^{1-\vartheta} \right]^{\frac{1}{(1-\vartheta)}}. \tag{2.18}$$

Since the firm operates in a perfectly competitive market, the price, P_t, is also its marginal cost. Finally, P_t is the consumer price index (CPI).

- **The Domestic Intermediate goods**

The domestic intermediate good s is produced using the technology

$$Y_t(s) = \theta_t K_{t-1}(s)^{\alpha} L_t(s)^{1-\alpha} \tag{2.19}$$

where Y_t is the final good output at period t, and θ_t is the exogenous domestic productivity shock which is identical to all s firms.

$$K_t(s) = GK_t + PK_t(s), \tag{2.20}$$

K_t is the effective (total) capital stock and GK_t is the exogenous government capital stock available at time t and is identical to all firms.

Assume in the first stage that the wage rate is W_t and the rental capital rate is R_t^K. Then, the problem for the firms consists of choosing L_t and K_t, taking W_t and R_t^K as given, which maximize their profit function:

$$\Pi^d_{t+j}(s) = P_t^{\star d}(s)Y_{t+j}(s) - R^K_{t+j}K_{t+j}(s) - W_t L_{t+j} \tag{2.21}$$

subject to (19) whose first order conditions are:

$$W_t = (1-\alpha)\frac{MC_t Y_t(s)}{L_t(s)} \tag{2.22}$$

where MC_t is the marginal cost and $P_t^{\star d}$ is the price of the intermediate goods in domestic currency,

$$R_t^K = \alpha \frac{MC_t Y_t(s)}{K_t(s)}. \tag{2.23}$$

The total domestic production is used domestically, Y_t^d or exported to the world market, Y_t^x, so that

$$Y_t = Y_t^d + Y_t^x, \tag{2.24}$$

where Y_t^x is the total export for the small open country whose demand function in the world market is

$$Y_t^x = \alpha^x \left(\frac{P_t^x}{P_t^\star}\right)^{-\eta} Z_t^\star. \tag{2.25}$$

P_t^x represents the price of the export intermediate good in the world market, $P_t\star$ the exogenous world price level, α^x the share of the small open economy's export in the world market, and $Z_t^\star$ the world output level. $Z_t^\star$ is exogenous in the small open economy.

Since the intermediate goods' firms operate in a monopolistic competition market, they gain profits. Then, in the second stage the firms set the price à la Calvo (1983) that maximizes the expected discounted real profits. Therefore, in each period, a fraction of $(1-\lambda)$ firms are able to reset their prices, while others keep unchanged their prices. Usually, the period from which the price is unchanged is $\frac{1}{(1-\lambda)}$.

Thus, the firms solve the following problem

$$\max_{P_t^{\star d}(s)} E_t \frac{\sum_{j=0}^{\infty}(\beta\lambda)^j \left(\frac{C_t+\gamma GC_t}{C_{t+j}+\gamma GC_{t+j}}\right)\Pi_{t+j}^d(s)}{P_{t+j}^d}$$

subject to (19).

The following demand function is derived as the solution of the above problem:

$$Y_{t+j}^d(s) = \left(\frac{P_t^{\star d}(s)}{P_{t+j}^d}\right)^{-\nu} Y_{t+j}^d \tag{2.26}$$

whose first order condition with respect to $P_t^{\star d}$ is

$$P_t^{\star d} = \frac{\nu}{\nu-1}\frac{E_t\sum_{j=0}^{\infty}(\beta\lambda)^j\left(\frac{C_t+\gamma GC_t}{C_{t+j}+\gamma GC_{t+j}}\right)Y_{t+j}(s)MC_{t+j}}{E_t\sum_{j=0}^{\infty}(\beta\lambda)^j\left(\frac{C_t+\gamma GC_t}{C_{t+j}+\gamma GC_{t+j}}\right)Y_{t+j}(s)/P_{t+j}^d}. \tag{2.27}$$

The import price index is

$$(P_t^d)^{1-\nu} = \lambda(P_{t-1}^d)^{1-\nu} + (1-\lambda)(P_t^{\star d})^{1-\nu}. \tag{2.28}$$

Analogously, the firms can choose a new export price at time t by solving the following problem

$$\max_{P_t^{\star x}(s)} E_t \frac{\sum_{j=0}^{\infty}(\beta\lambda)^j \left(\frac{C_t+\gamma GC_t}{C_{t+j}+\gamma GC_{t+j}}\right) \Pi_{t+j}^x(s)}{P_{t+j}^x}$$

above subject to (19) and the demand function

$$Y_{t+j}^x(s) = \left(\frac{P_t^{\star x}(s)}{P_{t+j}^x}\right)^{-\nu} Y_{t+j}^x. \tag{2.29}$$

The solution gives $P_t^{\star x}$ as

$$P_t^{\star x} = \frac{\nu}{\nu-1} \frac{E_t \sum_{j=0}^{\infty}(\beta\lambda)^j \left(\frac{C_t+\gamma GC_t}{C_{t+j}+\gamma GC_{t+j}}\right) Y_{t+j}(s) MC_{t+j}^x}{E_t \sum_{j=0}^{\infty}(\beta\lambda)^j \left(\frac{C_t+\gamma GC_t}{C_{t+j}+\gamma GC_{t+j}}\right) Y_{t+j}(s)/P_{t+j}^x}. \tag{2.30}$$

where

$$MC_t^x = \frac{P_t^d}{P_t^x S_t}, \tag{2.31}$$

and S_t is the nominal exchange rate expressed here as the domestic currency price of foreign currency.

The import price index is

$$(P_t^x)^{1-\nu} = \lambda (P_{t-1}^x)^{1-\nu} + (1-\lambda)(P_t^{\star x})^{1-\nu}. \tag{2.32}$$

- **The Imported Intermediate goods**

As for the domestic intermediate goods, the imported intermediate producers set their price à la Calvo (1983).

The importer s sets its price, $P_t^{\star m}(s)$, that maximizes the expected discounted profits below, taking as given the nominal exchange rate, S_t, and the world price level, $P_t^\star$,

$$\max_{P_t^{\star m}(s)} E_t \frac{\sum_{j=0}^{\infty}(\beta\lambda)^j \left(\frac{C_t+\gamma GC_t}{C_{t+j}+\gamma GC_{t+j}}\right)\Pi_{t+j}^m(s)}{P_{t+j}^m}$$

subject to

$$Y_{t+j}^m(s) = \left(\frac{P_t^{\star m}(s)}{P_{t+j}^m}\right)^{-\nu} Y_{t+j}^m. \tag{2.33}$$

where

$$\Pi_t^m = (P_t^{\star m}(s) - MC_t^m)\left(\frac{P_t^{\star m}(s)}{P_t^m}\right)^{-\nu} Y_t^m, \tag{2.34}$$

where the marginal cost for importer is

$$MC_t^m = \frac{S_t P_t^\star}{P_t^m}. \tag{2.35}$$

The solution for $P_t^{\star m}$ implies the following first order condition

$$P_t^{\star m} = \frac{\nu}{\nu-1}\frac{E_t \sum_{j=0}^{\infty}(\beta\lambda)^j \left(\frac{C_t+\gamma GC_t}{C_{t+j}+\gamma GC_{t+j}}\right) Y_{t+j}^m(s) MC_{t+j}^m}{E_t \sum_{j=0}^{\infty}(\beta\lambda)^j \left(\frac{C_t+\gamma GC_t}{C_{t+j}+\gamma GC_{t+j}}\right) Y_{t+j}^m(s)/P_{t+j}^m}. \tag{2.36}$$

The import price index is

$$(P_t^m)^{1-\nu} = \lambda(P_{t-1}^m)^{1-\nu} + (1-\lambda)(P_t^{\star m})^{1-\nu}. \tag{2.37}$$

2.2.4 The Government

The government alleviates tax on all sources of income for households. Also, the government issues one period debt, D_t, that matures at t, and balances its budget at each period. I consider the government final good purchases, G_t, as exogenous.

$$P_t G_t + D_{t-1} R_{t-1} = D_t + T_t \tag{2.38}$$

where $T_t = \tau_t^C P_t C_t + \tau_t^K R_t^K PK_{t-1} + \tau_t^L W_t L_t - \tau_t^K P_t PK_{t-1}$.

I define the real government debt normalized by steady state real GDP as $B_t \equiv \left(\frac{D_t}{P_t Z_{ss}}\right)$.

The Government spending is decomposed into

$$G_t = GC_t + Ig_t \tag{2.39}$$

where GC_t and Ig_t are the government consumption and investment at period t. I assume here that, the public capital evolves according to

$$GK_{t+1} = Ig_t + (1-\delta) GK_t. \tag{2.40}$$

Therefore, the tax policy rules are set as

$$\frac{\tau_t^C}{\tau_{ss}} = \left(\frac{B_t}{B_{ss}}\right)^{\gamma_C^B} \left(\frac{GC_t}{GC_{ss}}\right)^{\gamma_C^{GC}} \left(\frac{IG_t}{IG_{ss}}\right)^{\gamma_C^{Ig}} \left(\frac{\theta_t}{\theta_{ss}}\right)^{\gamma_C^{\theta}} \left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_C^{\theta}}, \tag{2.41}$$

$$\frac{\tau_t^K}{\tau_{ss}} = \left(\frac{B_t}{B_{ss}}\right)^{\gamma_K^B} \left(\frac{GC_t}{GC_{ss}}\right)^{\gamma_K^{GC}} \left(\frac{IG_t}{IG_{ss}}\right)^{\gamma_K^{Ig}} \left(\frac{\theta_t}{\theta_{ss}}\right)^{\gamma_K^{\theta}} \left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_K^{\theta}}, \tag{2.42}$$

$$\frac{\tau_t^L}{\tau_{ss}} = \left(\frac{B_t}{B_{ss}}\right)^{\gamma_L^B} \left(\frac{GC_t}{GC_{ss}}\right)^{\gamma_L^{GC}} \left(\frac{IG_t}{IG_{ss}}\right)^{\gamma_L^{Ig}} \left(\frac{\theta_t}{\theta_{ss}}\right)^{\gamma_L^{\theta}} \left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_L^{\theta}}. \tag{2.43}$$

It is important to indicate that the variables with subscripts "$_{ss}$" denote the steady state values. Besides, I distinguish, as Kollmann(2008) three types of feedback policy rules :

- the "simpler rules" which stipulate that the any tax rate above reacts only to the real government debt;
- the "reacher rules" which link any tax rate to real government debt, government consumption, government investment, productivity shock, and inflation; and
- the "baseline rules" which link any tax rate to real government debt, government consumption, government investment, and productivity shock.

I assume that the government commits to setting the policy parameters ϕ_C^B, ϕ_C^{GC}, ϕ_C^{Ig}, ϕ_C^{θ}, and ϕ_C^{Π} at values that maximize the unconditional expected value of household utility subject to the restriction that the unconditional mean of real debt has to close to its steady state value (Kollmann 2008) as

$$|EB_t - B_{ss}| < 0.01. \tag{2.44}$$

The equation (44) is set to rule out the long-run values of debt and taxes that differ greatly from the values observed in reality which has been showed by Aiyagari et al.(2002) from the optimal Ramsey fiscal policy.

2.2.5 The Central Bank

Following Taylor (1993, 1999), the central bank set the short-run nominal interest rate, R_t, in response to inflation and output gaps.

$$\frac{R_t}{R_{ss}} = \left(\frac{\pi_t^d}{\pi_{ss}^d}\right)^{\phi_\pi} \left(\frac{Z_t}{Z_{ss}}\right)^{\phi_Z} \left(\frac{e_t}{e_{ss}}\right)^{\phi_e} \tag{2.45}$$

Therefore, two policy rules will be discussed in this paper:

- the "simpler rules" is defined as the nominal short-run interest rate reacts to inflation and output gap (e.g., Taylor, 1993, 1999; Kollmann, 2002, 2008)
  and
- the "reacher rules" link the nominal short-run to inflation, output and deprecation factor of the nominal exchange rate gap.

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As the government, the central bank also commits to setting the policy parameters ϕ_π,ϕ_Z, and ϕ_e at the values that maximize the unconditional expected value of household utility.

2.2.6 Relative Prices and International Risk Sharing

Since the model is solved in real terms, I have defined some relative prices in order to simplify many things.

$$\psi_t^{lop} = MC_t^m = \frac{S_t P_t^\star}{P_t^m} = e_t \frac{\pi_t^\star}{\pi_t^m} \psi_{t-1}^{lop} \tag{2.46}$$

describes the deviation from the law of one price (LOP) because I impose that ψ_t^{lop} is different from one. However, in steady state psi_{ss}^{lop} must be equal to one.

$$\psi_t^\star = \frac{P_t^\star}{P_t} = \frac{\pi_t^\star}{\pi_t} \psi_{t-1}^\star \tag{2.47}$$

$$\psi_t^x = \frac{P_t^x}{P_t^\star} = \frac{\pi_t^x}{\pi_t^\star} \psi_{t-1}^x \tag{2.48}$$

$$\psi_t^d = \frac{P_t^d}{P_t} = \frac{\pi_t^d}{\pi_t} \psi_{t-1}^d \tag{2.49}$$

$$\psi_t^m = \frac{P_t^m}{P_t} = \frac{\pi_t^m}{\pi_t}\psi_{t-1}^m. \quad (2.50)$$

Let now define the real exchange rate and the international risk sharing respectively as

$$RER_t = \frac{S_t P_t^\star}{P_t} = S_t \psi_t^\star. \quad (2.51)$$

$$RER_t = \frac{\left(\frac{C_t^\star + \gamma GC_t^\star}{C_{t+1}^\star + \gamma GC_{t+1}^\star}\right)}{\left(\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}}\right)} \quad (2.52)$$

2.2.7 Market clearing conditions

The intermediate goods markets clear as firms meet all demand at posted prices. The market for final good, labor, and capital rental clear when:

$$Z_t = C_t + Ip_t + G_t, \quad (2.53)$$

$$L_t = \int_0^1 L_t(s)ds, \quad (2.54)$$

and

$$K_t = \int_0^1 K_t(s)ds. \quad (2.55)$$

The bond market clearing requires as Kollmann (2002), I assume that foreigners do not hold bonds denominated in the currency of small open economy. Therefore, the bonds markets clear:

$$A_t^d = 0, \quad (2.56)$$

and

$$\psi_t^{\star}\psi_t^{x}Y_t^{x} - \psi_t^{\star}Y_t^{m} = A_t - \varphi_t Rf_{t-1}A_{t-1} \tag{2.57}$$

The equation (56) determines the evolution of the net foreign assets following Schmitt-Grohé and Uribe (2003). It represents the balance of payments that includes the risk premium on foreign assets holding by households. The left hand side of equation (56) represents the balance of trade, while the right hand side represents the balance of capital. Finally, I define the net asset position as $NFA_t = \frac{A_t}{P_t^{\star}Z_{ss}}$

2.2.8 Exogenous variables

I consider nine shocks in this model: the domestic productivity, the world inflation, the world interest rate, the world output, the UIP, the government consumption, the government investment, the preference, and the labor supply which are expressed as following :

$$logZ_t^{\star} = \rho_{Z^{\star}} logZ_{t-1}^{\star} + \varepsilon_{Z^{\star}} \tag{2.58}$$

$$log\pi_t^{\star} = \rho_{\pi^{\star}} log\pi_{t-1}^{\star} + \varepsilon_{\pi^{\star}} \tag{2.59}$$

$$logIg_t = \rho_{Ig} logIg_{t-1} + \varepsilon_{Ig} \tag{2.60}$$

$$logGC_t = \rho_{GC} logGC_{t-1} + \varepsilon_{GC} \tag{2.61}$$

$$log\chi_t = \rho_{\chi} log\chi_{t-1} + \varepsilon_{\chi} \tag{2.62}$$

$$logRf_t = \rho_{Rf} logRf_{t-1} + \varepsilon_{Rf} \tag{2.63}$$

$$log\theta_t = \rho_\theta log\theta_{t-1} + \varepsilon_\theta \tag{2.64}$$

2.2.9 Solution method and Welfare measures

I solve my model using Sims' (2000) second-order accurate method. The welfare is evaluated through a second-order Taylor expansion of the utility function around the steady state which gives

$$E(U(C_t, GC_t, L_t)) \cong U(C_t, GC_t, L_t) + E(\hat{C}_t + \gamma \hat{GC}_t)) - L_{ss}E(\hat{L}_t) - Var(\hat{C}_t + \gamma \hat{GC}_t)) \tag{2.65}$$

where $Var(\hat{C}_t + \gamma \hat{GC}_t)$ is the variance of $(\hat{C}_t + \hat{GC}_t)$.

By expressing the welfare as the permanent relative change in private consumption and government consumption (compared to the stady state), ξ, which gives

$$E(U(C_t, GC_t, L_t)) : U((1+\xi)(C_t, GC_t), L_t) \cong U(C_t, GC_t, L_t) + E(\hat{C}_t + \gamma \hat{GC}_t) - L_{ss}E(\hat{L}_t) - Var(\hat{C}_t + \gamma \hat{GC}_t) \tag{2.66}$$

Hence, the welfare ξ can be decomposed into two components ξ^m and ξ^v as following:

$$U((1+\xi^m)(C_t, GC_t), L_t) = U(C_t, GC_t, L_t) + E(\hat{C}_t + \gamma \hat{GD}_t) - L_{ss}E(\hat{L}_t) \tag{2.67}$$

$$U((1+\xi^v)(C_t, GC_t), L_t) = U(C_t, GC_t, L_t) - Var(\hat{C}_t + \gamma \hat{GC}_t). \tag{2.68}$$

ξ^m and ξ^v represent the mean of private consumption plus non-investment government spending, and hours worked, and variance of private consumption plus non-investment government spending.

By applying the equation (2) into the previous equations yields

$$\ln(1+\xi) = E(\hat{C}_t + \gamma\hat{G}C_t) - L_{ss}E(\hat{L}_t) - Var(\hat{C}_t + \gamma\hat{G}C_t) \tag{2.69}$$

$$\ln(1+\xi^m) = E(\hat{C}_t + \gamma\hat{G}C_t) - L_{ss}E(\hat{L}_t) \tag{2.70}$$

$$\ln(1+\xi^v) = -Var(\hat{C}_t + \gamma\hat{G}C_t) = Var(\hat{C}_t) + \gamma^2 var(\hat{G}C_t) + 2cov(\hat{C}_t, \hat{G}C_t) \tag{2.71}$$

Therefore,

$$(1+\xi) = (1+\xi^m)(1+\xi^v). \tag{2.72}$$

2.2.10 Parameters (non-policy)

Most of non-policy parameters is calibrated to quarterly data for France, Germany, the U.K and Netherlands from 1977 to 2007. The period is chosen because of the availability of quarterly data. Other parameters follow the works of Bouakez and Rebei (2007, Kollmann (2001),and Baxter and King (1993), and . The real domestic and foreign interest rate are set to be equal at steady state which are derived from the first order conditions with respect to domestic and foreign bonds, $R_{ss} = Rf_{ss} = \frac{\pi_{ss}}{\beta}$. The inflation (CPI) is set to 1.005 which means that is 2% annually. Therefore, $\beta = \frac{\pi_{ss}}{R_{ss}}$.

The indexation of prices, ν is set to 6 which corresponds to Kollmann (2001)'s the price-marginal cost steady state markup factor for intermediate goods $\frac{\nu}{(\nu-1)} = 1.2$. The price elasticities of substitution between import and domestic intermediate goods, in one hand, and between the export and foreign intermediate goods are set $\vartheta = \eta = 0.6$, in other hands. Moreover, the elasticity of output with respect to capital, α, is set to 0.24 which is consistent also with Kollmann (2002) and Bouakez and Rebei (2007). The depreciation rate of the capital stock, δ, is set to 0.025 which corresponds to many previous studies (e.g., Bouakez and Rebei, 2007; Kollmann, 2002, 2008, and 2010.....). Following Kollmann (2002), $\lambda = 0.75$, the average interval that producers of intermediate goods change their prices à la Calvo.

Furthermore, the risk premium parameter, α^A, the steady state import/Z_{ss} ratio, α^m, the steady state export/$Z^{\star}_{ss}$ ratio, and the degree of complementarity (substitution) between private and government consumptions, γ, are set to 0.8, 0.3, 0.0075, and 0.39 respectively. The last parameter is consistent with Baxter and King (1993).

Finally, the parameters' values related to all shocks ($rho_{..}$ and variances $\varepsilon^{...}$) are included in the appendix C.

2.3 Results

The main results of the simulation are reported in table 1 according to the baseline model (sticky prices with optimized policy) and all other alternative models (flexible prices, producer currency pricing, and fixed exchange rate regime). The tables 2 and 3 report the variation decomposition of main variables and the predicted standard deviations and mean values of important variables. The impulse response functions are shown in the appendix A.

As for Kollmann (2002), the statistics/responses for the domestic interest rate, NFA_t, and different tax rates refer to differences of these variable from steady state values. However, the statistics of the remaining variables are referred to relative deviations from steady state values.

2.3.1 Results for the baseline model (sticky prices)

The results are reported on the table 1 column (1). The optimized policy rules' coefficients are the following:

- Monetary Policy rule: $R_t = R_{ss} + 3.857\hat{\pi}_t^d - 0.024\hat{Z}_t - 0.001531\hat{e}_t$
- Income tax rate rule: $\tau_t^C = \tau_{ss}^C + 1.5\hat{B}_t - 0.25\hat{GC}_t - 0.02\hat{IG}_t - 0.1\hat{\theta}_t + 0.1\hat{\pi}_t - 0.2\hat{e}_t$
- Capital tax rate rule: $\tau_t^K = \tau_{ss}^K + 1.25\hat{B}_t - 0.64\hat{GC}_t - 0.5\hat{IG}_t - 0.1\hat{\theta}_t + 0.1\hat{\pi}_t - 0.2\hat{e}_t$
- Income tax rate rule: $\tau_t^L = \tau_{ss}^L + 1.75\hat{B}_t - 0.5\hat{GC}_t - 0.4\hat{IG}_t - 0.45\hat{\theta}_t + 0.2\hat{\pi}_t - 0.1\hat{e}_t$.

The simple rules which link the nominal gross interest rate on bonds to both output and inflation gap for monetary policy and each tax rate to public debt give the welfare level closed to above rules. This optimized policy rule has a strong stance on inflation. The central bank rises the nominal interest rate in response to an increase in domestic PPI inflation. Following Kollmann 2002, there is a contrast result for the coefficient of output gap which is -0.024 (Kollmann calibrated -0.01). However, this coefficient is closed to zero.

The negative sign of output gap is due to UIP shock which increases the mean of foreign asset holdings, imports of goods and services, private consumption, and welfare. At the same time, it decreases the domestic output for the first 20 quarters, exports for the first 15 quarters, and the domestic nominal interest rate for the first 5 quarters. Besides, the UIP shock appreciates both nominal and real exchange rate. As the optimal policy rule is based on the household's welfare criterion which is dominated by the mean of private consumption, the UIP condition (equation 2.16) forces the central bank to decrease the nominal interest after any increase in output gap.

The welfare level for the richer rule (as given on the table 2.1) is $\zeta = 0.760664\%$ against for the simple rule of $\zeta = 0.734483\%$. Besides, when I consider the monetary policy rule without the factor of depreciation rate of exchange rate ($\hat{e}_t$), the welfare drop from $\zeta = 0.760664\%$ to

$\zeta = 0.757956\%$ which corresponds to a gap of 0.002708%. Therefore, when the central bank adjust the nominal rate to the factor of depreciation rate of exchange rate, the welfare gain is 0.002708%. This result is consistent with Kollmann (2002) who finds a gain of 0.002%. The level of the welfare is decomposed into mean ($\zeta^m = 0.084656\%$) and variance ($\zeta^v = 0.760664\%$).

The standard deviation for the domestic PPI inflation is 0.7818% for the richer rule with $\hat{e}$ against 0.6375% for the simple rule. However, the standard deviation for the CPI is lower than the domestic PPI 0.6345%. Furthermore, the standard deviation for output, consumption, and private investment are 1.7317%, 1.5245%, and 11.8239% respectively. The predicted standard deviation for the private investment is very higher than other variables. Standard deviation for private consumption and output is relatively closed to Kollmann (2002) which were about 2%. The standard deviation of net foreign asset and the real exchange rate are 3.35504% and 2.0903% respectively.

The mean of the hours worked, output, inflation (CPI), inflation (PPI), private capital stock and total capital stock are respectively 0.738%, 0.9124%, 0.0102%, 0.0123%, 2.0722% and 1.4645% below their steady state. However, the mean of the private consumption is about 0.1531% above its steady state. Furthermore, the mean of the stock of foreign asset is above its steady state by an amount which is about 0.2957% of the steady state of the real GDP. Moreover, the mean of the real exchange rate shows an appreciation of 0.0966% with respect to its steady state and the mean of nominal exchange exhibits a depreciation of 0.1073% with respect to its steady state. Finally, the mean of import of intermediate goods is about 8.7088% below its steady state.

The government purchases are positive correlated with private consumption, real output, private capital stock, and hours worked. However, there negatively correlated with both real and nominal exchange rate. More specifically, the coefficient of correlation between the government purchases and the macroeconomic variable listed above are respectively 0.0078, 0.046, 0.0121, 0.0477, -0.1404 and -0.2038 for private consumption, real output, private capital stock, hours worked, nominal and real exchange rate.

When I consider the decomposed government purchases, the government consumption is highly correlated to major macroeconomic variables relative to government investment. In fact, the coefficient of correlation between government consumption and other macroeconomic variables are respectively 0.0073, 0.0466, 0.0112, 0.0479, -0.1406 and -0.2056. However, the coefficient of correlation between government investment and other macroeconomic variables are respectively 0.0035, 0.0010, 0.0063, 0.0029, -0.0108 and -0.0079.

The variance decomposition of the major macroeconomic variables indicates that the productivity shock is the main source of volatility of these variables. In fact, the productivity shock explains 78.78% of the variance of real output, 77.69% of private consumption, 85.23% of hours worked, 80.23% of private investment, 83.30% of the CPI inflation, 86.96% of producer domestic inflation, 50.31% of real exchange rate, and 65.10% of nominal exchange rate. Besides, the world inflation shock constitutes the second main source of volatility after the productivity shock.

The dynamic responses to the different shocks show that the real GDP and private consumption raises in response to one percent increase in world output shock, world inflation shock, and domestic productivity shock. However, the world interest rate, and the risk premium lower the real GDP after two first quarters. The private consumption lowers in risk premium shock and world interest rate. The real exchange rate depreciates in response to an increase in world output, risk premium shock, and world interest rate. However, the real exchange rate appreciates after an increase in world inflation shock(after the first two quarters) and productivity shock. Furthermore, the nominal exchange rate appreciates due to an increase in world output shock(after the first two quarters), and world inflation (after 7 quarters). Nevertheless, the nominal exchange rate depreciates after a positive shock to the risk premium shock, world interest rate, and domestic productivity. Finally, a positive world interest rate shock (for the first 10 quarters), domestic productivity shock (after the first 7 quarters), risk premium shock (for the last thirteen quarters), and world inflation increase the nominal gross domestic interest rate. However, the nominal gross domestic interest rate falls after a positive world output (after the first ten quarters), world interest rate for the first fifteen quarters, and the domestic productivity shock (the first eight quarters). These results are confirmed by Kollmann, 2002.

2.3.2 Results for Flexible price

Under the richer rule, the welfare level is $\zeta = 2.404015\%$ whose gain is driven mostly by its mean component, $\zeta^m = 2.182918\%$. Besides, the contribution of the variance component is not negligible , $\zeta^v = 0.221098\%$. This result seems confirm the literature (Obdtfeld and Rogoff (1995, 1998, 2000, and 2001), Betts and Devereux 1996, Devereux and Engel 2003 and 2006, Kollmann 2002, and Engel 2013). In fact, the literature predicts high welfare under flexible price than under sticky price.

The standard deviation for the domestic PPI inflation is 0.2196% which is slightly lower than the CPI inflation which is 0.3850%. Moreover, the standard deviation for output, consumption, and private investment are 0.2247%, 0.3790%, and 7.9329% respectively. The predicted standard deviation for the private investment is very higher than other variables which means that the private investment is more volatile than other variables. Finally, the standard deviation of net foreign asset and the real exchange rate are 0.7389% and 0.7161% respectively. In comparison to the sticky price model, this model is less volatile which confirm the literature(Chari, Kehoe, and McGrattan, 2000).

The mean of hours worked and output are respectively 0.5494% and 0.1334% below their steady state. However, the mean of the private consumption, private investment, inflation (CPI), inflation (PPI), private capital stock and total capital stock are respectively 1.0825%, 1.6750%, 0.0494%, 0.1502%, 1.6750%, and 1.1838% above their steady states. Moreover, the mean of the stock of foreign asset is above its steady state by an amount which is about 0.5523% of the steady state of the real GDP. Furthermore, the mean of the real exchange rate shows a depreciation of 3.8717% with respect to its steady state and the mean of nominal exchange exhibits a depreciation of 2.8836% with respect to its steady state. Finally, the mean of import of intermediate goods is about 3.7675% above its steady state.

2.3.3 Results for Producer Currency Pricing

The previous models assume that the intermediate goods are sold in consumers' currencies which implies the deviation from the law of one price and full exchange rate pas-through. In the subsection, I assume that the LOP holds. Therefore, under the optimized rule, the welfare is $\zeta = 2.458651\%$ whose gain is driven mostly by its mean component, $\zeta^m = 2.232015629\%$. Besides, the contribution of the variance component is not negligible , $\zeta^v = 0.226635783\%$. As I mentioned in the previous subsection, the literature has predicted that the producer currency pricing generates the welfare level higher than the consumer currency pricing, especially under the flexible exchange rate regime which is under analysis (e.g, Devereux and Engel 1998, Engel 2013).

The standard deviation for the domestic PPI inflation,0.1615%, is slightly lower than the CPI inflation which is 0.2039%. Moreover, the standard deviation for output, consumption, and private investment are 0.2229%, 0.3765%, and 7.6400% respectively. The predicted standard deviation for the private investment is very higher than other variables which means that the private investment is more volatile than other variables. Finally, the standard deviation of net foreign asset and the real exchange rate are 0.5436% and 0.7261% respectively.

In comparison to the price to market pricing, the standard deviation of the domestic PPI inflation is lower than the domestic CPI inflation with lowest for PCP (.1615%) against 0.2196% for flexible price. Therefore, the welfare maximizing monetary and fiscal policy rules entails the stabilization of the domestic PPI inflation which also confirms by previous works (e.g., Devereux and

Engels, 1998; Betts and Devereux, 2000; Kollmann, 2002; Senay and Sutherland, 2010).

2.3.4 Results for fixed Exchange Rate

Under the richer rules, the welfare level is $\zeta = 0.034306\%$ which is mostly dominated by its variance component, $\zeta^v = 0.033085\%$. The contribution of the mean component is negligible, $\zeta^m = 0.001221\%$. Therefore, by pegging the exchange rate, the welfare is reduced significantly in comparison to the flexible exchange rate regime ($\zeta = 0.760664\%$). These results are confirmed by the previous studies (Engel 2013; Devereux and Yetman 2012; Kollamnn 2002).

Besides, the volatility of the main macroeconomic variable such as nominal and real exchange rate, private consumption, output, net foreign assets, and private investment is high under this regime than the flexible one. In fact, the standard deviation of nominal and real exchange rate, private consumption, output, net foreign assets, and private investment are respectively 6.52%, 4.26%, 1.81%, 3.03%, 8.23%, and 25.02%.

Moreover, the means of private consumption, hours worked, private capital stock, import of intermediate goods, and export of intermediate output are below their steady state, respectively, by 2.76%, 2.64%, 1.54%, 0.11%, and 12.48%. However, the means of output and domestic intermediate goods are above their steady state values by 1.57% and 0.63% respectively. Furthermore, the mean of the real exchange rate shows a depreciation of 1.65% with respect to its steady state value. The net foreign assets is above its steady state by an amount which is about 0.46% of the steady state of the real GDP.

2.3.5 Impact of Decomposed Government purchases on Real Exchange Rate and other macroeconomic variables

Following Kollmann 2002 and 2010, and Basu and Kollmann 2013, the model generates the government purchases-exchange rate puzzles for all the models. Specifically, the impact of government purchases on both the real exchange rate is reported in the different impulse response functions. Based on the sticky price model (baseline model), a positive government consumption contributes more to the depreciation of both nominal and real exchange rates than a positive government investment. In fact, the nominal exchange rate depreciates by 5% immediately, then attains a peak of 13% after ten quarters, and falls at 4% by the end of the twenty fifth quarter due to a positive government consumption shock. Nevertheless, the depreciation of nominal exchange rate after a positive government investment is not only about 0.4% immediately, but also does not exceed 1.5% for the entire period of simulation. Moreover, the depreciation of real exchange rate attains 9% due to a positive government consumption, while a positive government investment depreciate the real exchange rate by no more than 1% for the all quarters.

For the flexible prices' model, the results seems little different from the sticky price one. A positive government consumption shock induces a depreciation of both real and nominal exchange rates by 11% in high. In contrary, the results are ambiguous for a positive government investment shock. The depreciation is confirmed fully for the real exchange rate by 0.2% in high. But it does not generated the depreciation of nominal exchange after seven quarters. In fact, any positive government investment will appreciate the latter in long run (after the first seven quarters).

The transmission mechanism through which the government purchases generate the depreciation of both nominal and real exchange rates does not differ much of the previous works by Kollmann (1995, 2010) and Basu and Kollmann (2013). In fact, in the literature, so far, there are two main links : the international risk sharing and marginal productivity of public capital stock. This paper adds two additional links through which the decomposed government purchases depreciate the real exchange rate: marginal utility of private consumption and marginal productivity of private capital stock.

An increase in the exogenous government consumption has positive impact on private consumption. This result is a puzzle because it contradicts the prediction of both classical and Keynesian. It well known as the crowdind-out of private consumption assumption. By this mechanism, the decrease in private consumption increase the relative consumption on the definition of the international risk sharing which appreciate the real exchange rate. However, when there is crowdind-in of the private consumption, the relative consumption will decrease, then the real exchange rate will depreciate. Another way to see it is through the marginal utility of consumption.

Moreover, the previous work on the topic by Kollmann (1995, 2010) and Kollmann and Basu (2013) predict the link between the real exchange rate and government purchases through the marginal productivity of labor which increases the output. This paper finds another channel, the marginal productivity of the private capital stock since the latter works did not include the private capital. By this channel, the government investment has a double impact on output: it boosts not only the marginal productivity of total capital through the productivity public capital, but also the marginal productivity of labor which both have a strong impact on output.

The last prediction of the previous works is the completeness of the market of asset market. Since in the paper I consider the case of incomplete asset market in the sense that the domestic bonds are not allowed to be purchased by the foreign households. Therefore, the latter limits the international risk sharing which plays an important role on generating the depreciation of the real exchange rate after a raise on government consumption through the crowding-in of private consumption (see Kollmann, 2010).

Finally, a raise on both government consumption and invest has a positive impact of output with a delay of two quarters, private consumption and investment in long run, hours worked, gross return on bonds, the next foreign asset position, the export of domestic intermediate goods, domestic intermediate goods, and imports. However, it creates inflation (both CPI and PPI). This is the main channel through which the government spending-real exchange rate puzzle is generated. In fact, the crowd-in on both private consumption and investment implies high output and price level due to a rightly shift of the aggregate demand on the market of goods and services. Thus, on the money market, the increase in both real output and price level will imply high interest rate and quantity of money. From UIP condition (equation 2.14), an increase in domestic nominal interest will result to the depreciation of nominal and real exchange rate.

2.4 Conclusion

This paper aims to evaluation the impact tax and monetary policy rules with decomposed government purchases on welfare, real exchange rate and business cycle in a small open economy using a new-Keynesian dynamic stochastic general equilibrium frame work. The model predicts that the government consumption has more impact than investment on both private consumption and investment, but less impact on the real GDP. Moreover, the government purchases-real exchange rate puzzle is generated by the model. In this sense, the government consumption contributes more on generating the puzzle than the investment. Moreover, the productive and complement government purchases have positive impact on welfare for any policy rules. The optimized policy rules have a pronounced anti-inflation stance and entail significant nominal and real exchange rate volatility for monetary policy. For tax policy rules, the public debt stance is the optimized rules.

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Appendices

A. The complete Non-Linear Model

$$1 = \beta E_t \left[\left(\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left(\frac{1+\tau_t^C}{1+\tau_{t+1}^C} \right) \frac{1}{\pi_{t+1}} R_t \right] \tag{73}$$

$$1 = \beta E_t \left[\left(\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left(\frac{1+\tau_t^C}{1+\tau_{t+1}^C} \right) \frac{1}{\pi_{t+1}} e_{t+1} \varphi_t R f_t \right] \tag{74}$$

$$1 = \beta E_t \left[\left(\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left(\frac{1+\tau_t^C}{1+\tau_{t+1}^C} \right) \left(R_t^K (1 - \tau_{t+1}^K) + \delta \tau_{t+1}^K + (1-\delta) \right) \right] \tag{75}$$

$$\psi = \left[\left(\frac{1}{C_t + \gamma GC_t} \right) \left(\frac{1-\tau_t^L}{1+\tau_t^C} \right) W_t \frac{1}{\phi_t} \right] \tag{76}$$

$$1 = (1-\alpha^m)(\psi_t^d)^{1-\vartheta} + \alpha^m (\psi_t^m)^{1-\vartheta} \tag{77}$$

$$J_t^d = \frac{\nu}{\nu - 1} N_t^d \tag{78}$$

$$P_t^{\star d} = \left[\frac{1 - \lambda \pi_{t+1}^{d\nu - 1}}{1-\lambda} \right]^{\frac{1}{1-\nu}} \tag{79}$$

$$J_t^d = y_t^d P_t^{\star d} + \lambda \beta E_t \left[\left(\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left(\frac{P_t^{\star d}}{P_{t+1}^{\star d}} \right) \pi_{t+1}^{d\nu - 1} J_t^d \right] \tag{80}$$

$$N_t^d = y_t^d MC_t + \lambda \beta E_t \left[\left(\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \pi_{t+1}^{d\nu - 1} N_t^d \right] \tag{81}$$

$$J_t^x = \frac{\nu}{\nu - 1} N_t^x \tag{82}$$

$$P_t^{\star x} = \left[\frac{1 - \lambda \pi_{t+1}^{x\nu - 1}}{1 - \lambda} \right]^{\frac{1}{1-\nu}} \tag{83}$$

$$J_t^x = y_t^x P_t^{\star x} + \lambda \beta E_t \left[\left(\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left(\frac{P_t^{\star x}}{P_{t+1}^{\star x}} \right) \pi_{t+1}^{x\nu - 1} J_t^x \right] \tag{84}$$

$$N_t^x = y_t^x MC_t^x + \lambda \beta E_t \left[\left(\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \pi_{t+1}^{x\nu - 1} N_t^x \right] \tag{85}$$

$$J_t^m = \frac{\nu}{\nu - 1} N_t^m \tag{86}$$

$$P_t^{\star m} = \left[\frac{1 - \lambda \pi_{t+1}^{m\nu - 1}}{1 - \lambda} \right]^{\frac{1}{1-\nu}} \tag{87}$$

$$J_t^m = y_t^m P_t^{\star m} + \lambda \beta E_t \left[\left(\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \left(\frac{P_t^{\star m}}{P_{t+1}^{\star m}} \right) \pi_{t+1}^{m\nu - 1} J_t^m \right] \tag{88}$$

$$N_t^m = y_t^m \left(\frac{RER_t}{\psi_t^m} \right) + \lambda \beta E_t \left[\left(\frac{C_t + \gamma GC_t}{C_{t+1} + \gamma GC_{t+1}} \right) \pi_{t+1}^{m\nu - 1} N_t^m \right] \tag{89}$$

$$PK_t = (1 - \delta) PK_{t-1} + Ip_t \tag{90}$$

$$GK_t = (1 - \delta) GK_{t-1} + Ig_t \tag{91}$$

$$K_t = PK_t + GK_t \tag{92}$$

$$Y_t = \theta_t K_t^{\alpha} L_t^{1-\alpha} \tag{93}$$

$$Y_t = Y_t^d + Y_t^x \tag{94}$$

$$Y_t^d = (1-\alpha^m)(\psi_t^d)^{-\vartheta} Z_t \tag{95}$$

$$Y_t^m = \alpha^m (\psi_t^m)^{-\vartheta} Z_t \tag{96}$$

$$Y_t^x = \alpha_x (\psi_t^x)^{-\eta} Z_t^{\star} \tag{97}$$

$$\psi_t^{lop} = MC_t^m = \frac{RER_t}{\psi_t^m} \tag{98}$$

$$\psi_t^{\star} = \frac{\pi_t^{\star}}{\pi_t} \psi_{t-1}^{\star} \tag{99}$$

$$\psi_t^x = \frac{\pi_t^x}{\pi_t^{\star}} \psi_{t-1}^x \tag{100}$$

$$\psi_t^d = \frac{\pi_t^d}{\pi_t} \psi_{t-1}^d \tag{101}$$

$$\psi_t^m = \frac{\pi_t^m}{\pi_t} \psi_{t-1}^m \tag{102}$$

$$MC_t^x = \frac{\pi_t^d}{\pi_t^x} \frac{1}{e_t} MC_{t-1}^x \tag{103}$$

$$Z_t = C_t + Ip_t + G_t \tag{104}$$

$$G_t = GC_t + Ig_t \tag{105}$$

$$W_t = (1-\alpha) MC_t Y_t \left(\frac{1}{L_t} \right) \tag{106}$$

$$R_t^K = \alpha MC_t Y_t \left(\frac{1}{K_t} \right) \tag{107}$$

$$\psi_t^\star \psi_t^x Y_t^x - \psi_t^\star Y_t^m = A_t - \varphi_t Rf_{t-1} A_{t-1} \tag{108}$$

$$G_t + D_{t-1} R_{t-1} = D_t + \tau_t^C C_t + \tau_t^K R_{t-1}^K K_{t-1} + \tau_t^L W_t L_t + \delta \tau_t^K K_{t-1} \tag{109}$$

$$\varphi_t = exp(-\left(\frac{A_t}{A_{ss}} \right) + \chi_t) \tag{110}$$

$$e_t = \frac{S_t}{S_{t-1}} \tag{111}$$

$$B_t = \frac{D_t}{Z_{ss}} \tag{112}$$

$$RER_t = S_t \frac{P_t^{\star}}{P_t} = S_t \psi_t^{\star} \tag{113}$$

$$\frac{R_t}{R_{ss}} = \left(\frac{\pi_t^d}{\pi_{ss}^d}\right)^{\phi_\pi} \left(\frac{Z_t}{Z_{ss}}\right)^{\phi_Z} \left(\frac{e_t}{e_{ss}}\right)^{\phi_e} \tag{114}$$

$$\frac{\tau_t^C}{\tau_{ss}} = \left(\frac{B_t}{B_{ss}}\right)^{\gamma_C^B} \left(\frac{GC_t}{GC_{ss}}\right)^{\gamma_C^{GC}} \left(\frac{IG_t}{IG_{ss}}\right)^{\gamma_C^{Ig}} \left(\frac{\theta_t}{\theta_{ss}}\right)^{\gamma_C^{\theta}} \left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_C^{\theta}} \tag{115}$$

$$\frac{\tau_t^K}{\tau_{ss}} = \left(\frac{B_t}{B_{ss}}\right)^{\gamma_K^B} \left(\frac{GC_t}{GC_{ss}}\right)^{\gamma_K^{GC}} \left(\frac{IG_t}{IG_{ss}}\right)^{\gamma_K^{Ig}} \left(\frac{\theta_t}{\theta_{ss}}\right)^{\gamma_K^{\theta}} \left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_K^{\theta}} \tag{116}$$

$$\frac{\tau_t^L}{\tau_{ss}} = \left(\frac{B_t}{B_{ss}}\right)^{\gamma_L^B} \left(\frac{GC_t}{GC_{ss}}\right)^{\gamma_L^{GC}} \left(\frac{IG_t}{IG_{ss}}\right)^{\gamma_L^{Ig}} \left(\frac{\theta_t}{\theta_{ss}}\right)^{\gamma_L^{\theta}} \left(\frac{\pi_t}{\pi_{ss}}\right)^{\gamma_L^{\theta}} \tag{117}$$

$$logZ_t^{\star} = \rho_{Z^{\star}} logZ_{t-1}^{\star} + \varepsilon_{Z^{\star}} \tag{118}$$

$$log\pi_t^{\star} = \rho_{\pi^{\star}} log\pi_{t-1}^{\star} + \varepsilon_{\pi^{\star}} \tag{119}$$

$$logIg_t = \rho_{Ig} logIg_{t-1} + \varepsilon_{Ig} \tag{120}$$

$$logGC_t = \rho_{GC} logGC_{t-1} + \varepsilon_{GC} \tag{121}$$

$$log\chi_t = \rho_\chi log\chi_{t-1} + \varepsilon_\chi \tag{122}$$

$$logRf_t = \rho_{Rf} logRf_{t-1} + \varepsilon_{Rf} \tag{123}$$

$$log\theta_t = \rho_\theta log\theta_{t-1} + \varepsilon_\theta \tag{124}$$

$$log\zeta_t = \rho_\zeta log\zeta_{t-1} + \varepsilon_\zeta \tag{125}$$

$$log\phi_t = \rho_\phi log\phi_{t-1} + \varepsilon_\phi \tag{126}$$

B. Log Linear Model

$$\left(\frac{C_{ss}}{C_{ss}+\gamma GC_{ss}}\right)(\hat{C}_{t+1}-\hat{C}_t)+\gamma\left(\frac{C_{ss}}{C_{ss}+\gamma GC_{ss}}\right)(\hat{GC}_{t+1}-\hat{GC}_t)+\left(\frac{\tau^C_{ss}}{(1+\tau^C_{ss})}\right)(\hat{\tau}^C_{t+1}-\hat{\tau}^C_t)+\hat{\pi}_{t+1}-\hat{R}_t=0 \quad (127)$$

$$\left(\frac{C_{ss}}{C_{ss}+\gamma GC_{ss}}\right)(\hat{C}_{t+1}-\hat{C}_t)+\gamma\left(\frac{C_{ss}}{C_{ss}+\gamma GC_{ss}}\right)(\hat{GC}_{t+1}-\hat{GC}_t)+\left(\frac{\tau^C_{ss}}{(1+\tau^C_{ss})}\right)(\hat{\tau}^C_{t+1}-\hat{\tau}^C_t)+\hat{\pi}_{t+1}-\hat{Rf}_t-\hat{e}_{t+1}-$$

$$\left(\frac{C_{ss}}{C_{ss}+\gamma GC_{ss}}\right)(\hat{C}_{t+1}-\hat{C}_t)+\gamma\left(\frac{C_{ss}}{C_{ss}+\gamma GC_{ss}}\right)(\hat{GC}_{t+1}-\hat{GC}_t)+\left(\frac{\tau^C_{ss}}{(1+\tau^C_{ss})}\right)(\hat{\tau}^C_{t+1}-\hat{\tau}^C_t)=\left(\frac{1}{(R^K_{ss}(1-\tau^K_{ss})+\delta}\right.$$

$$\left(\frac{C_{ss}}{C_{ss}+\gamma GC_{ss}}\right)\hat{C}_t+\gamma\left(\frac{C_{ss}}{C_{ss}+\gamma GC_{ss}}\right)\hat{GC}_t+\left(\frac{\tau^C_{ss}}{(1+\tau^L_{ss})}\right)(\hat{\tau}^C_{t+1}-\hat{\tau}^C_t)-\left(\frac{\tau^L_{ss}}{(1-\tau^L_{ss})}\right)(\hat{\tau}^L_{t+1}-\hat{\tau}^L_t)-\hat{W}_t)=0 \quad (130)$$

$$\hat{PK}_t=(1-\delta)\hat{PK}_{t-1}+\delta\hat{Ip}_t \quad (131)$$

$$\hat{GK}_t=(1-\delta)\hat{GK}_{t-1}+\delta\hat{Ig}_t \quad (132)$$

$$\hat{K}_t=\frac{PK_{ss}}{K_{ss}}\hat{PK}_t+\frac{GK_{ss}}{K_{ss}}\hat{GK}_t \quad (133)$$

$$\hat{Y}_t=\hat{\theta}_t+\alpha\hat{K}_{t-1}+(1-\alpha)\hat{L}_t \quad (134)$$

$$\hat{Y}_t=\frac{Y^d_{ss}}{Y_{ss}}\hat{Y}^d_t+\frac{Y^x_{ss}}{Y_{ss}}\hat{Y}^x_t \quad (135)$$

$$\hat{Y}^d_t=-\vartheta\hat{\psi}^d_t+\hat{Z}_t \quad (136)$$

$$\hat{Y}_t^m = -\vartheta\,\hat{\psi}_t^m + \hat{Z}_t \tag{137}$$

$$\hat{Y}_t^x = -\eta\,\hat{\psi}_t^x + \hat{Z}_t^\star \tag{138}$$

$$\hat{\psi}_t^{LOP} = \hat{MC}_t^m = R\hat{E}R_t - \hat{\psi}_t^m \tag{139}$$

$$\hat{\psi}_t^\star = \hat{\pi}_t^\star - \hat{\pi}_t + \hat{\psi}_{t-1}^\star \tag{140}$$

$$\hat{\psi}_t^x = \hat{\pi}_t^x - \hat{\pi}_t + \hat{\psi}_{t-1}^x \tag{141}$$

$$\hat{\psi}_t^d = \hat{\pi}_t^d - \hat{\pi}_t + \hat{\psi}_{t-1}^d \tag{142}$$

$$\hat{\psi}_t^m = \hat{\pi}_t^m - \hat{\pi}_t + \hat{\psi}_{t-1}^m \tag{143}$$

$$\hat{MC}_t^x = \hat{\pi}_t^d - \hat{\pi}_t^x - \hat{e}_t + \hat{MC}_{t-1}^x \tag{144}$$

$$\hat{Z}_t = \frac{C_{ss}}{Z_{ss}}\hat{C}_t + \frac{Ip_{ss}}{Z_{ss}}\hat{Ip}_t + \frac{G_{ss}}{Z_{ss}}\hat{G}_t \tag{145}$$

$$\hat{G}_t = \frac{GC_{ss}}{G_{ss}}\hat{GC}_t + \frac{Ig_{ss}}{G_{ss}}\hat{Ig}_t \tag{146}$$

$$\hat{W}_t = \hat{MC}_t + \hat{Y}_t - \hat{L}_t \tag{147}$$

$$\hat{R}_t^K = \hat{MC}_t + \hat{Y}_t - \hat{K}_t \tag{148}$$

$$\frac{\hat{A}_t}{(1 - Rf_{ss}\varphi_{ss})} \tag{149}$$

$$-\frac{Rf_{ss}\varphi_{ss}}{(1 - Rf_{ss}\varphi_{ss})}\hat{A}_{t-1} \tag{150}$$

$$-\left(\frac{\psi_{ss}^x Y_{ss}^x}{(\psi_{ss}^x Y_{ss}^x - \psi_{ss}^\star Y_{ss}^m)}\right)(\hat{Y}_t^x + \hat{\psi}_t^x) \tag{151}$$

$$+\left(\frac{\psi_{ss}^\star Y_{ss}^m}{(\psi_{ss}^x Y_{ss}^x \psi_{ss}^\star Y_{ss}^m)}\right)(\hat{Y}_t^m + \hat{\psi}_t^\star) = 0 \tag{152}$$

$$\hat{GC}_t + \hat{I}g_t + \frac{1}{\beta}(\hat{D}_t + \hat{R}_{t-1} - \hat{\pi}_t) \tag{153}$$

$$= \hat{D}_t + \frac{\tau_{ss}^C C_{ss}}{G_{ss}}(\hat{\tau}_t^C + \hat{C}_t) \tag{154}$$

$$+\frac{\tau_{ss}^C R_{ss}^K K_{ss}}{G_{ss}}(\hat{\tau}_t^K + \hat{R}_{t-1}^K + \hat{K}_{t-1}) \tag{155}$$

$$+\frac{\tau_{ss} W_{ss} L_{ss}}{G_{ss}}(\hat{\tau}_t^L + \hat{W}_t + \hat{L}_t \tag{156}$$

$$-\frac{\delta \tau_{ss}^K}{G_{ss}}(\hat{\tau}_t^K + \hat{K}_{t-1}) \tag{157}$$

$$R\hat{E}R_t = \hat{S}_t + \hat{\psi}_t^\star \tag{158}$$

$$\hat{e}_t = \hat{S}_t - \hat{S}_{t-1} \tag{159}$$

$$\hat{B}_t = \hat{D}_t - \hat{\pi}_t \tag{160}$$

$$\hat{\pi}_t^d = \beta E_t \hat{\pi}_t^d + \frac{(1-\beta\lambda)(1-\lambda)}{\lambda} \hat{MC}_t \tag{161}$$

$$\hat{\pi}_t^m = \beta E_t \hat{\pi}_t^m + \frac{(1-\beta\lambda)(1-\lambda)}{\lambda} (R\hat{E}R_t - \hat{psi}_t^m) \tag{162}$$

$$\hat{\pi}_t^x = \beta E_t \hat{\pi}_t^x + \frac{(1-\beta\lambda)(1-\lambda)}{\lambda} \hat{MC}_t^x \tag{163}$$

$$\hat{\varphi}_t = (-\alpha^A \hat{A}_t + c\hat{h}i_t) \tag{164}$$

$$\hat{\pi}_t = (1-\alpha^m)(\psi_{ss}^d)^{(\vartheta-1)} \hat{\pi}_t^d + \alpha^m (\psi^m)^{(\varphi-1)} \hat{\pi}_t^m \tag{165}$$

$$\hat{R}_t = \phi_\pi \hat{\pi_t^d} + \phi_Z \hat{Z}t + \phi_e \hat{e}t \tag{166}$$

$$\hat{\tau}_t^C = \gamma_C^B \hat{B}_t + \gamma_C^{GC} \hat{GC}_t + \gamma_C^{Ig} \hat{Ig}_t + \gamma_C^\theta \hat{\theta}_t + \gamma_C^\pi \hat{\pi}_t + \gamma_C^e \hat{e}_t \tag{167}$$

$$\hat{\tau}_t^K = \gamma_K^B \hat{B}_t + \gamma_K^{GC} \hat{GC}_t + \gamma_K^{Ig} \hat{Ig}_t + \gamma_K^\theta \hat{\theta}_t + \gamma_K^\pi \hat{\pi}_t + \gamma_K^e \hat{e}_t \tag{168}$$

$$\hat{\tau}_t^L = \gamma_L^B \hat{B}_t + \gamma_L^{GC} \hat{GC}_t + \gamma_L^{Ig} \hat{Ig}_t + \gamma_L^\theta \hat{\theta}_t + \gamma_L^\pi \hat{\pi}_t + \gamma_L^e \hat{e}_t \tag{169}$$

$$\hat{Z}_t^\star = \rho_{Z^\star} \hat{Z}_{t-1}^\star + \varepsilon_t^{Z^\star} \tag{170}$$

$$\hat{\pi}_t^{\star} = \rho_{\pi^{\star}} \hat{\pi}_{t-1}^{\star} + \varepsilon_t^{\pi^{\star}} \tag{171}$$

$$\hat{GC}_t = \rho_{GC} \hat{GC}_{t-1} + \varepsilon_t^{GC} \tag{172}$$

$$\hat{Ig}_t = \rho_{Ig} \hat{Ig}_{t-1} + \varepsilon_t^{Ig} \tag{173}$$

$$\hat{Rf}_t = \rho_{Rf} \hat{Rf}_{t-1} + \varepsilon_t^{Rf} \tag{174}$$

$$\hat{\theta}_t = \rho_{\theta} \hat{\theta}_{t-1} + \varepsilon_t^{\theta} \tag{175}$$

$$\hat{\chi}_t = \rho_{\chi} \hat{\chi}_{t-1} + \varepsilon_t^{\chi} \tag{176}$$

C. Sticky Price Model’s Impulse Responses

Figure 1: Impulse Response for 1% increase in World Output

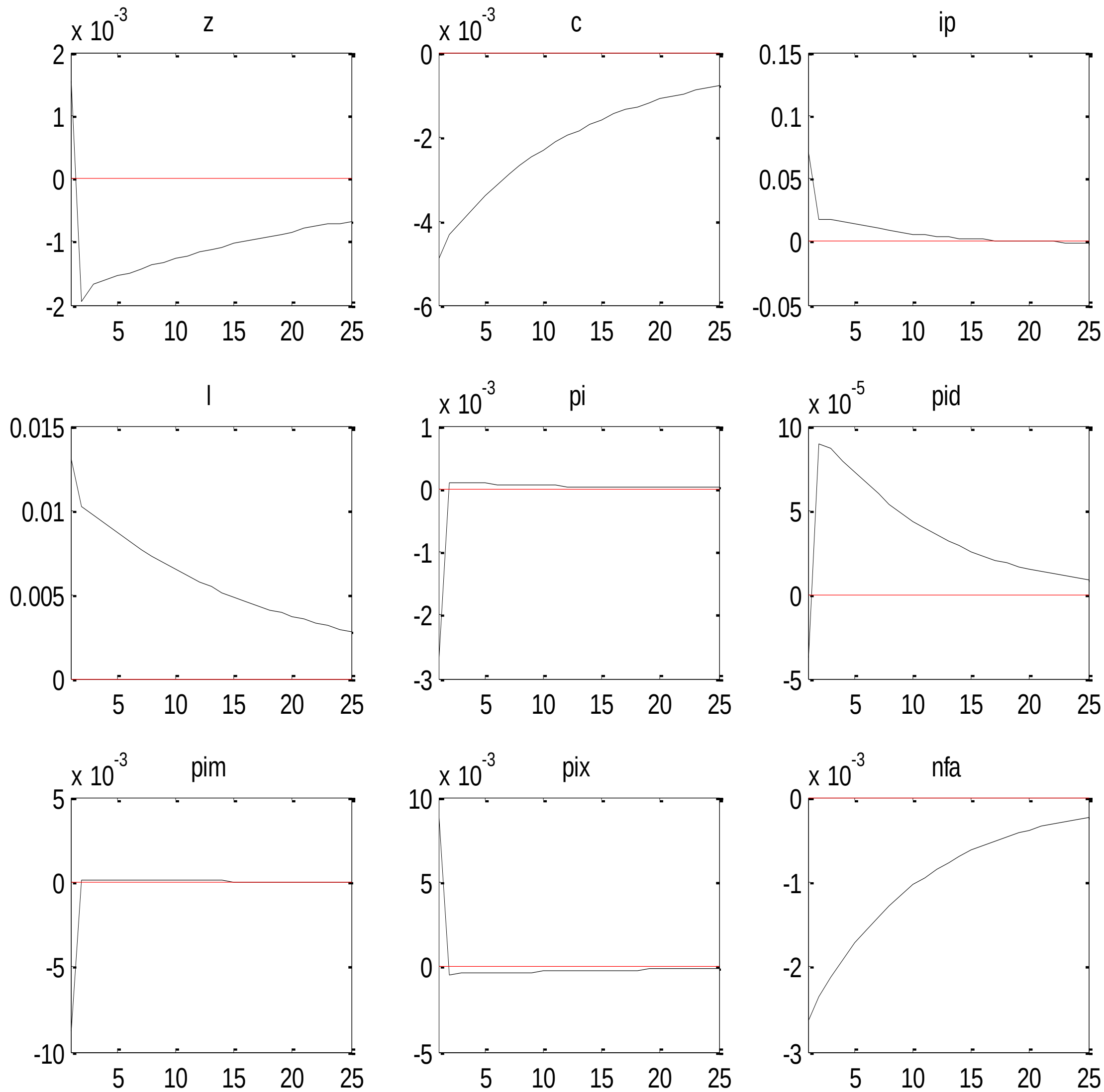

Figure 2: Impulse Response for 1% increase in World Output

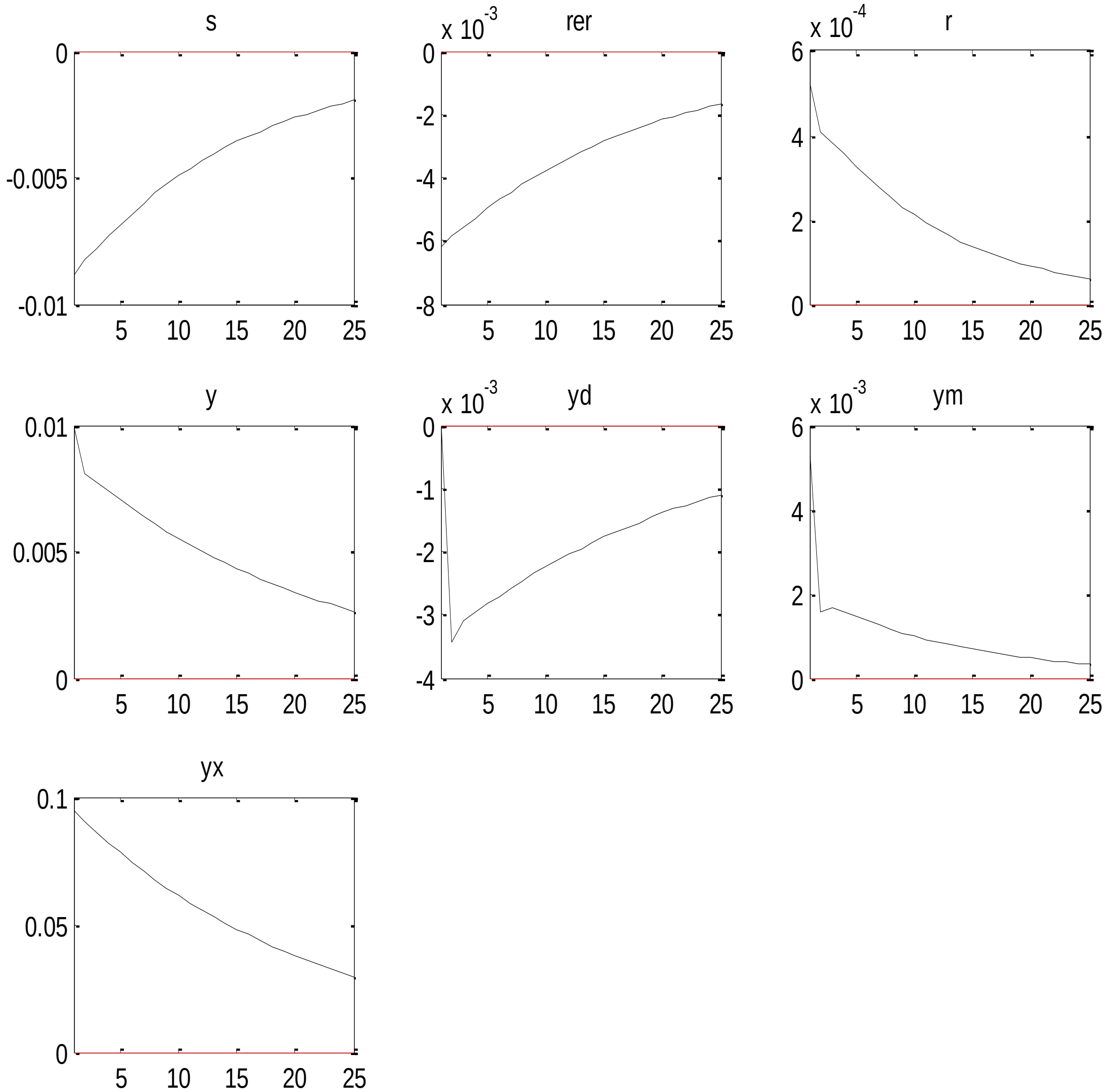

Figure 3: Impulse Response for 1% increase in World Inflation

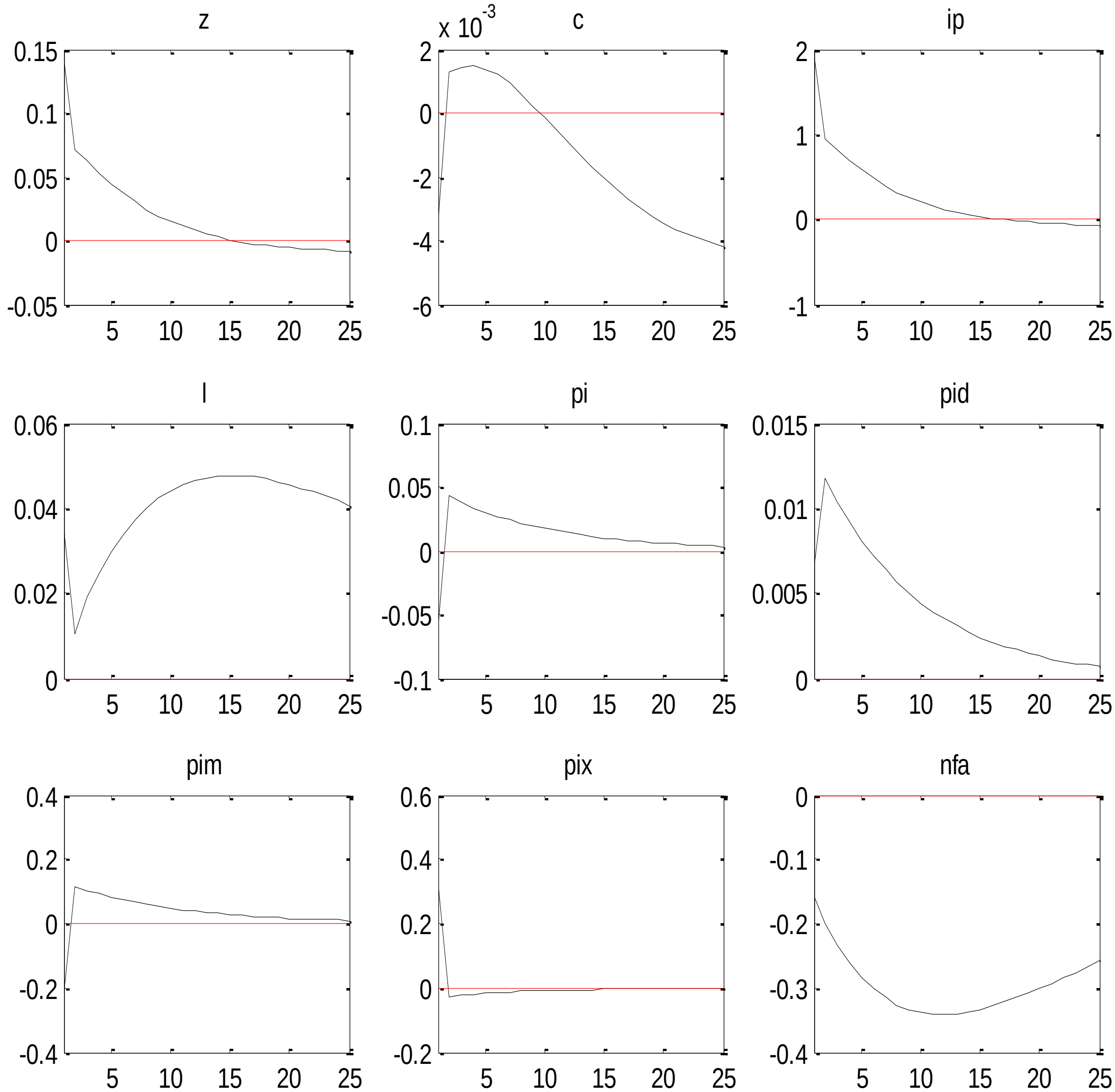

Figure 4: Impulse Response for 1% increase in World Inflation

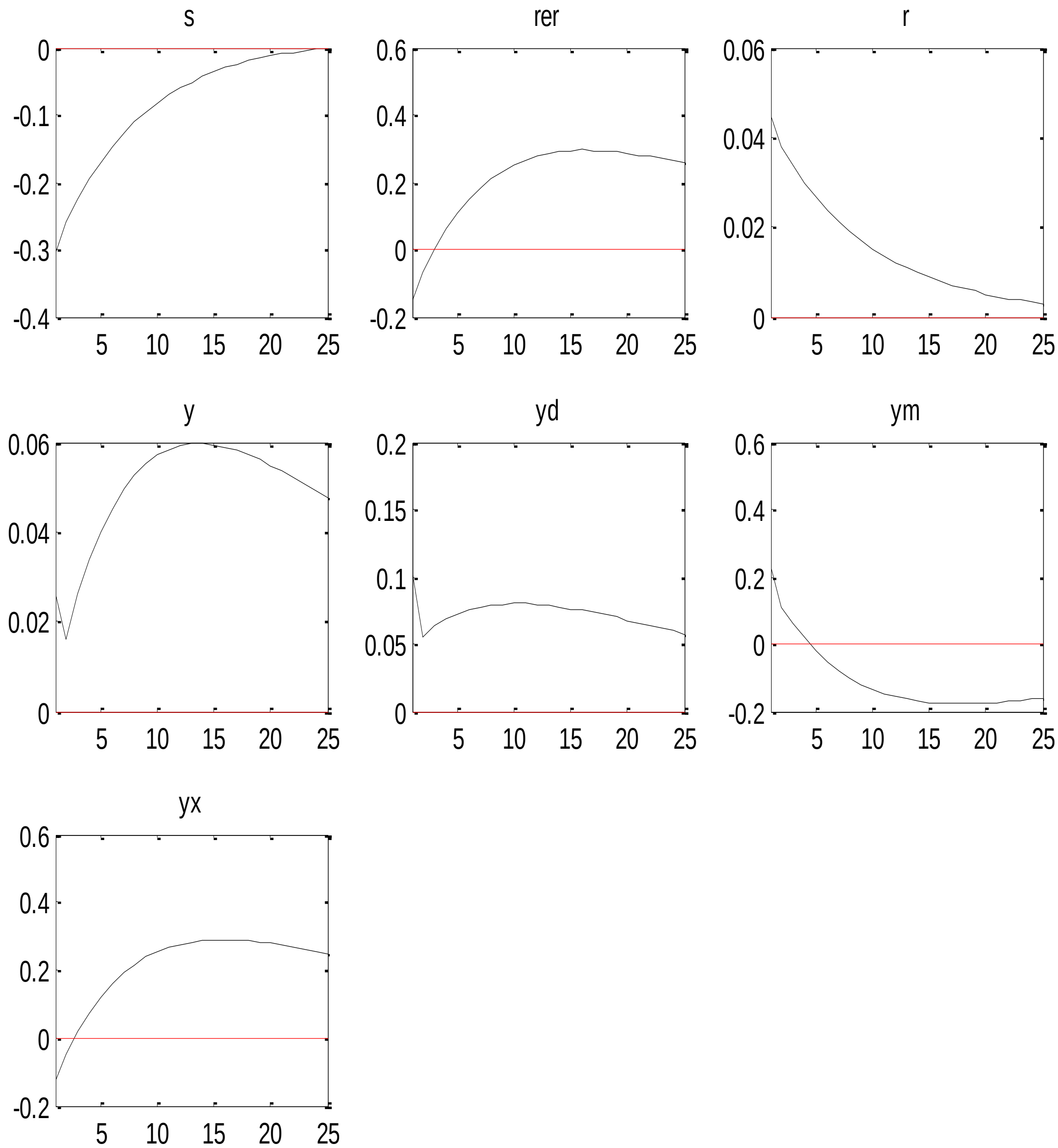

Figure 5: Impulse Response for 1% increase in Government Consumption

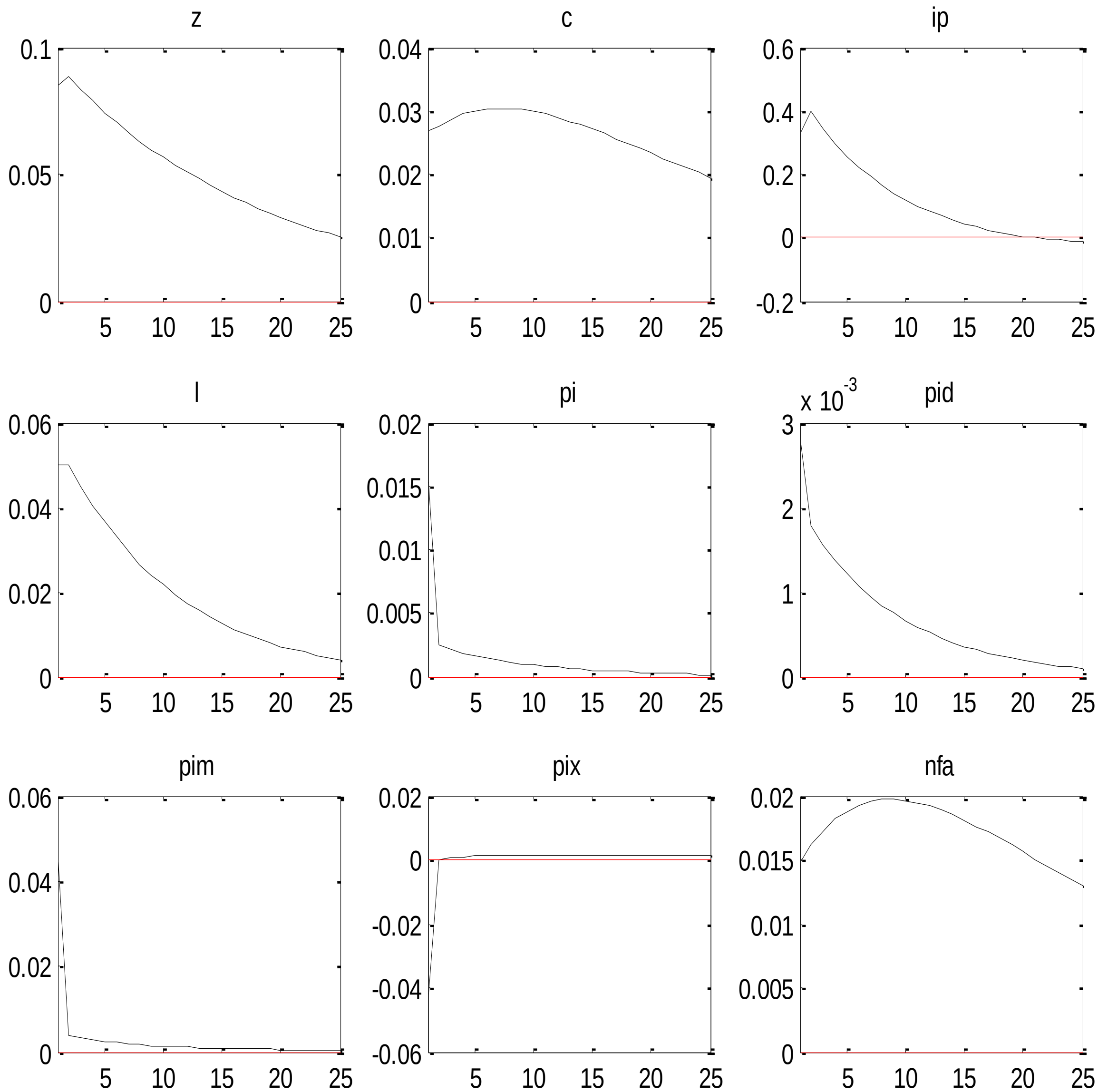

Figure 6: Impulse Response for 1% increase in Government Consumption

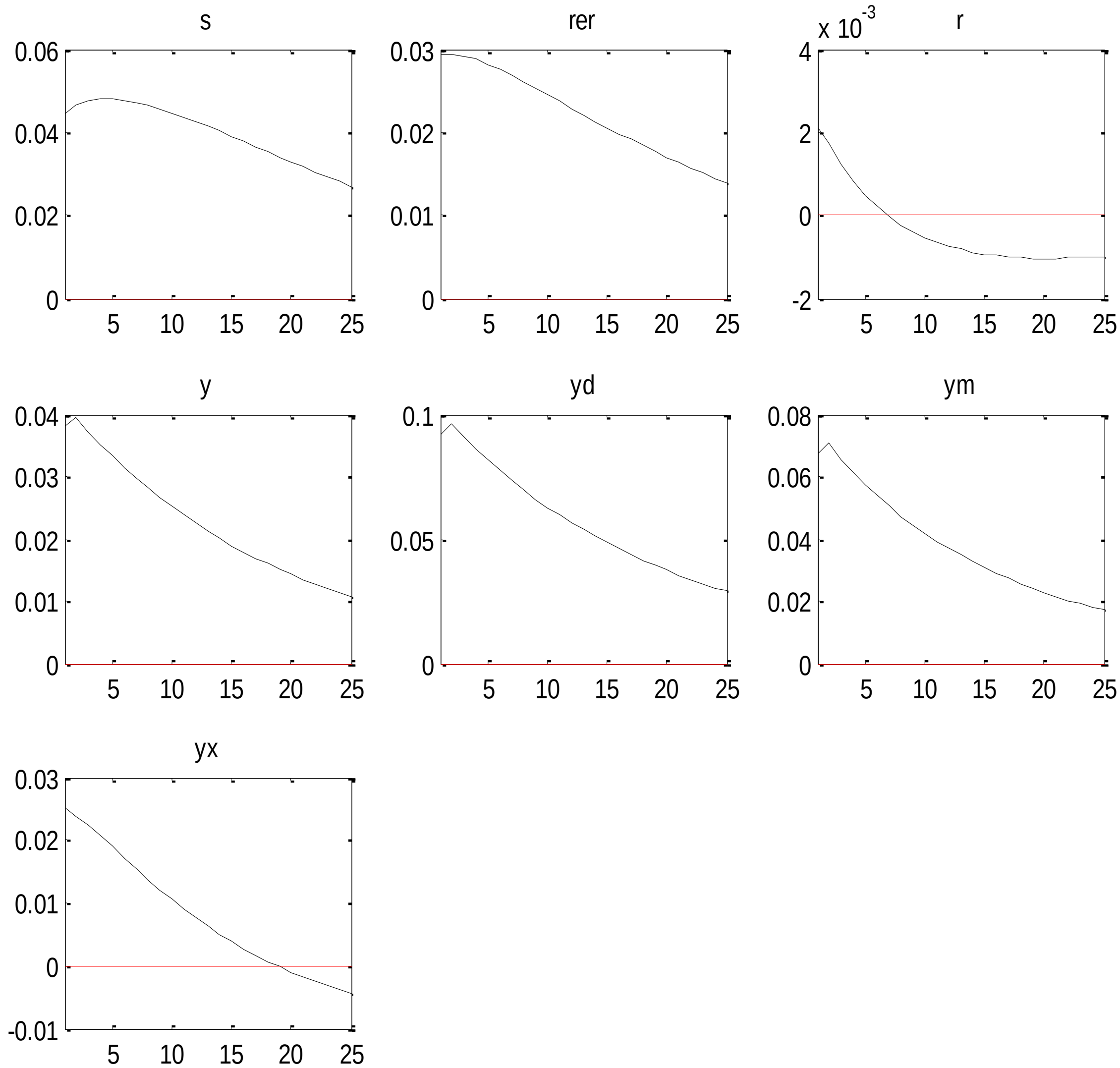

Figure 7: Impulse Response for 1% increase in Government Investment

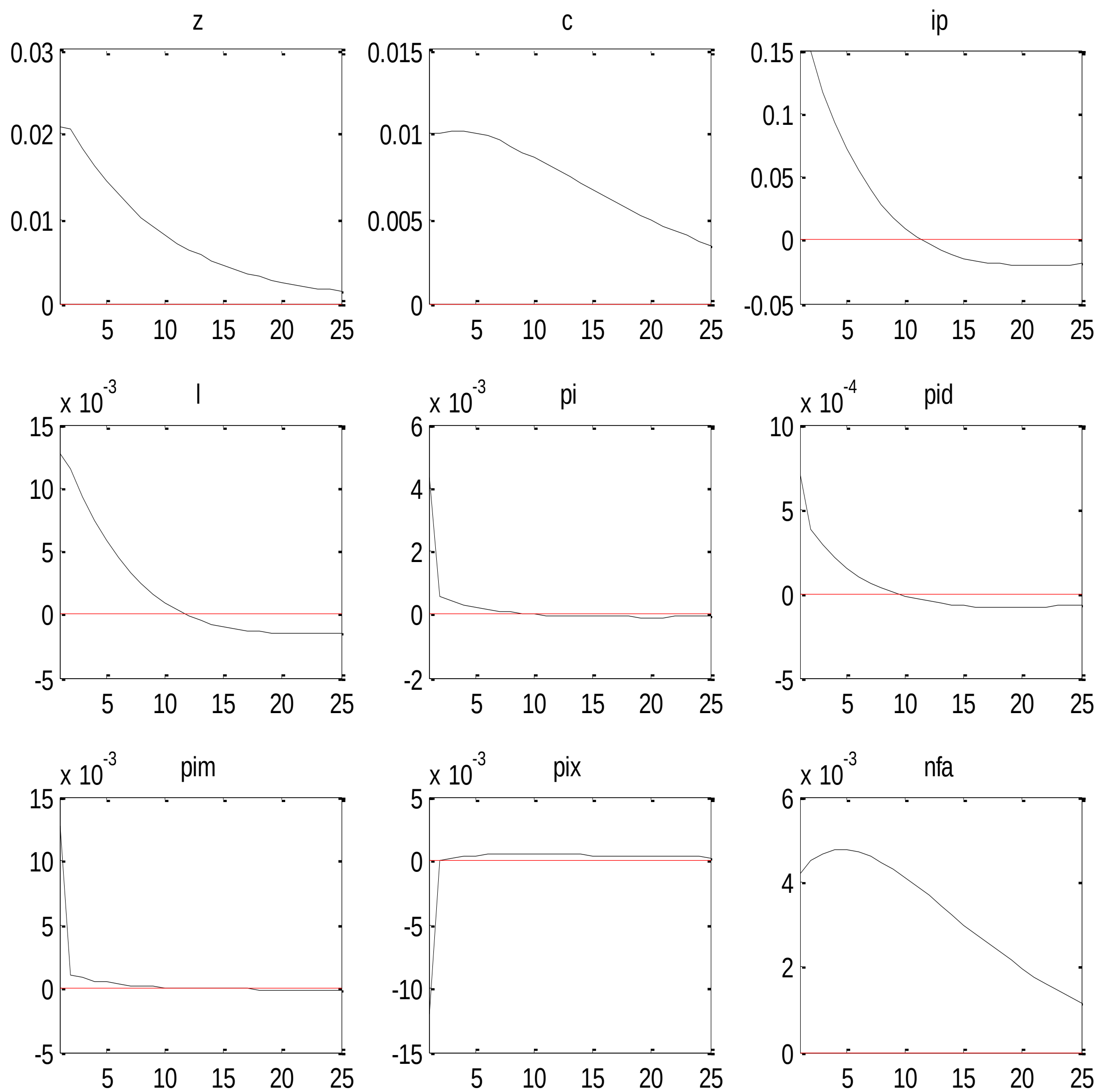

Figure 8: Impulse Response for 1% increase in Government Investment

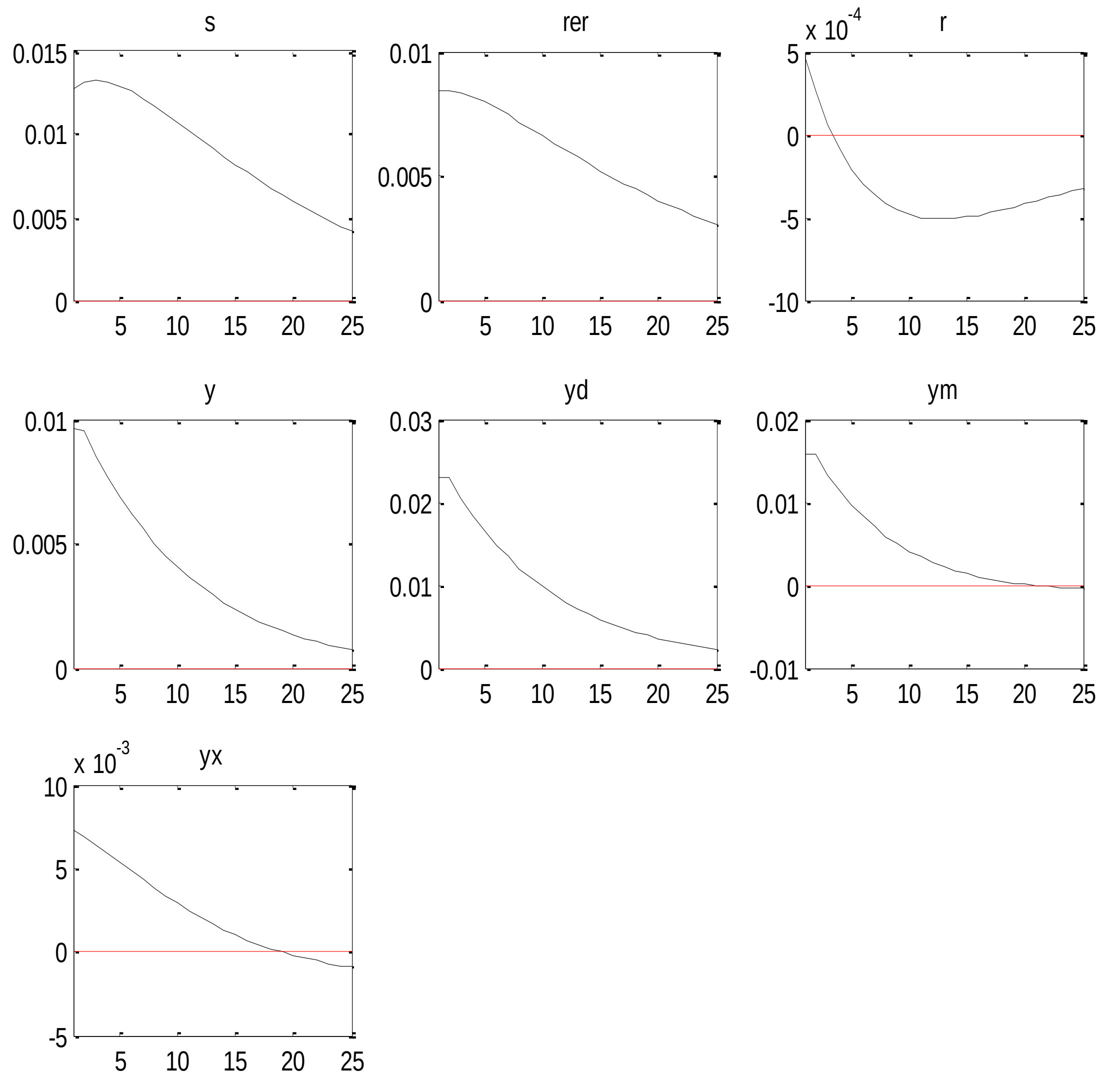

Figure 9: Impulse Response for 1% increase in Risk Premium

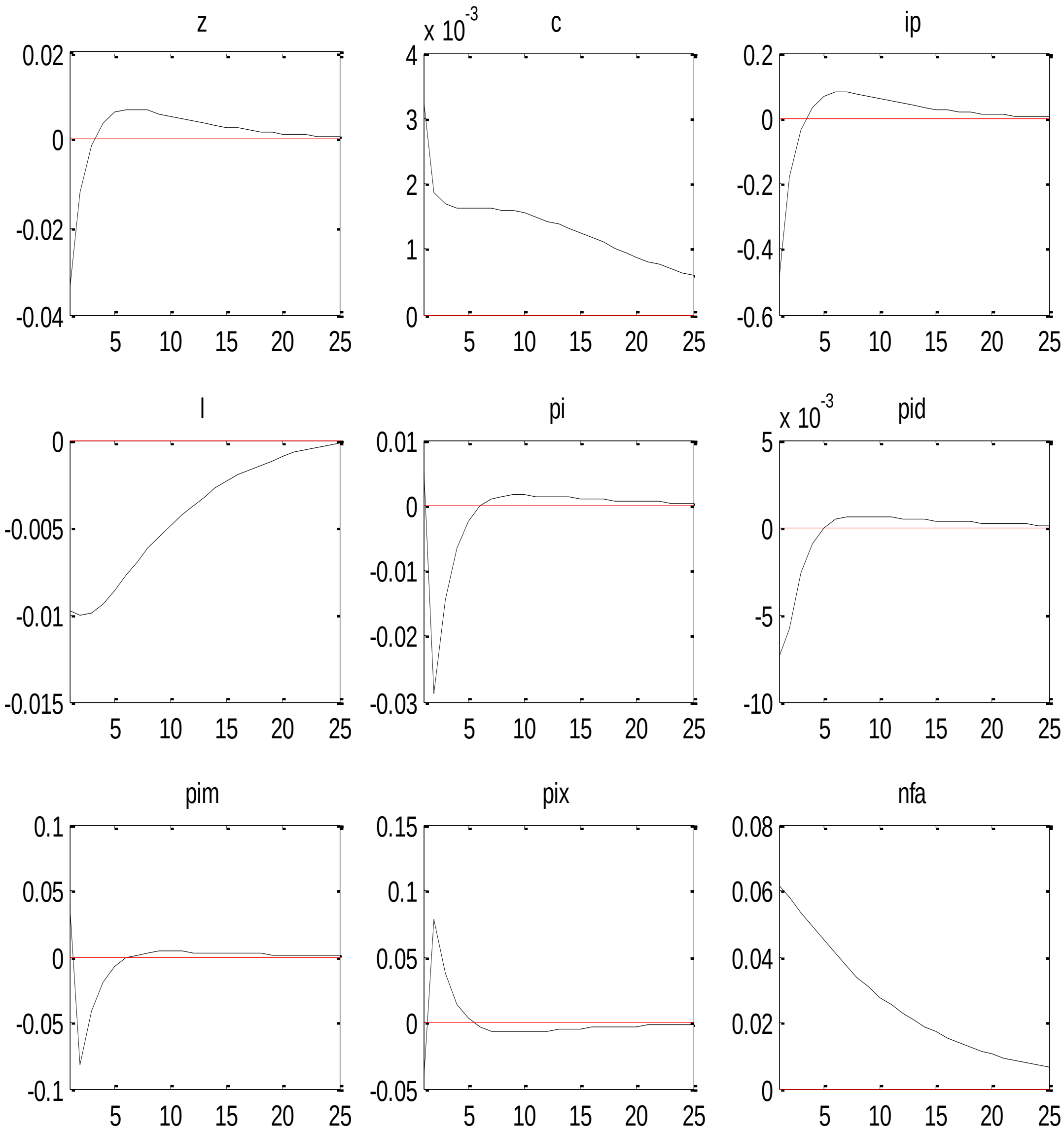

Figure 10: Impulse Response for 1% increase in Risk Premium

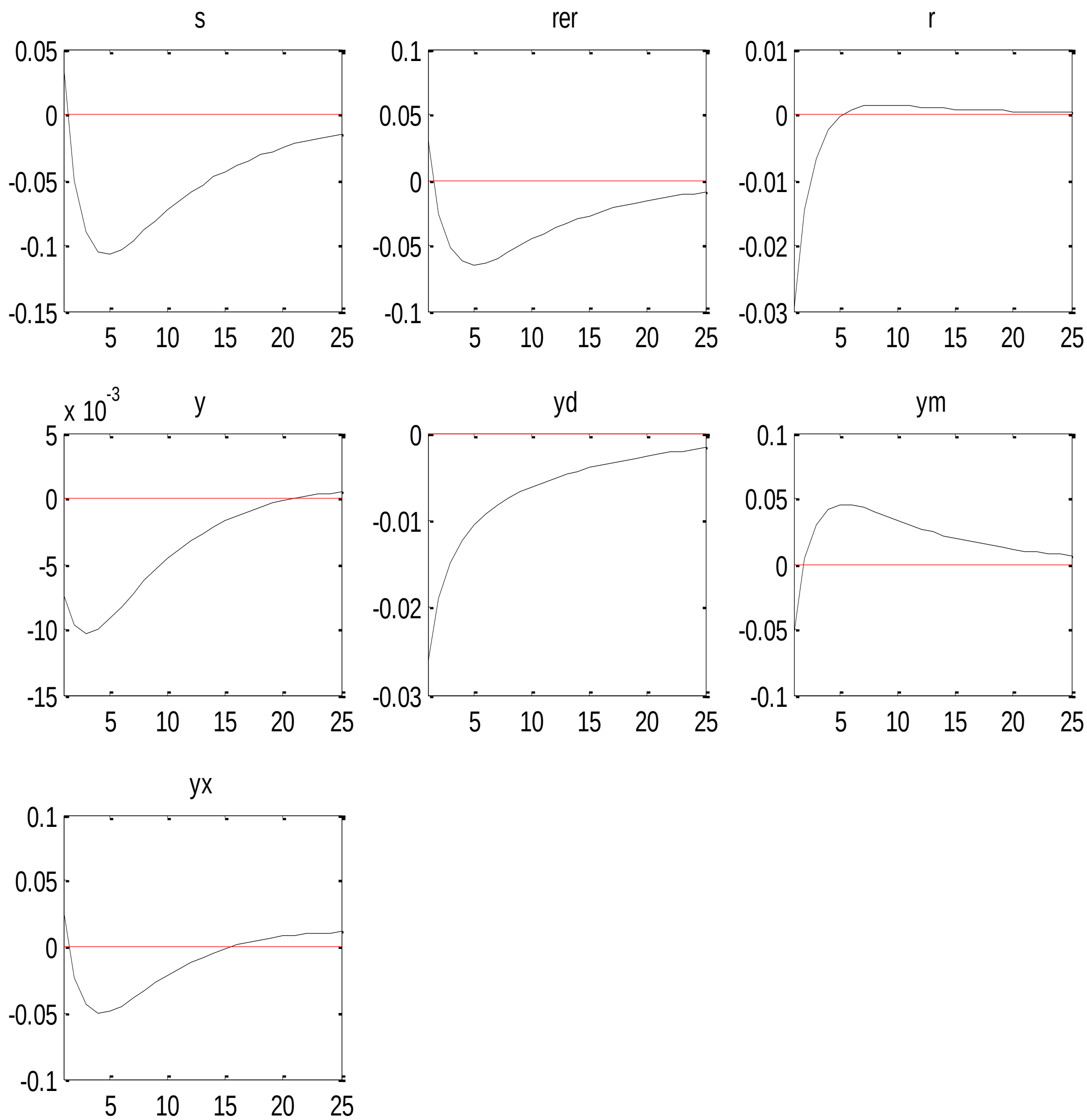

Figure 11: Impulse Response for 1% increase in World Interest Rate

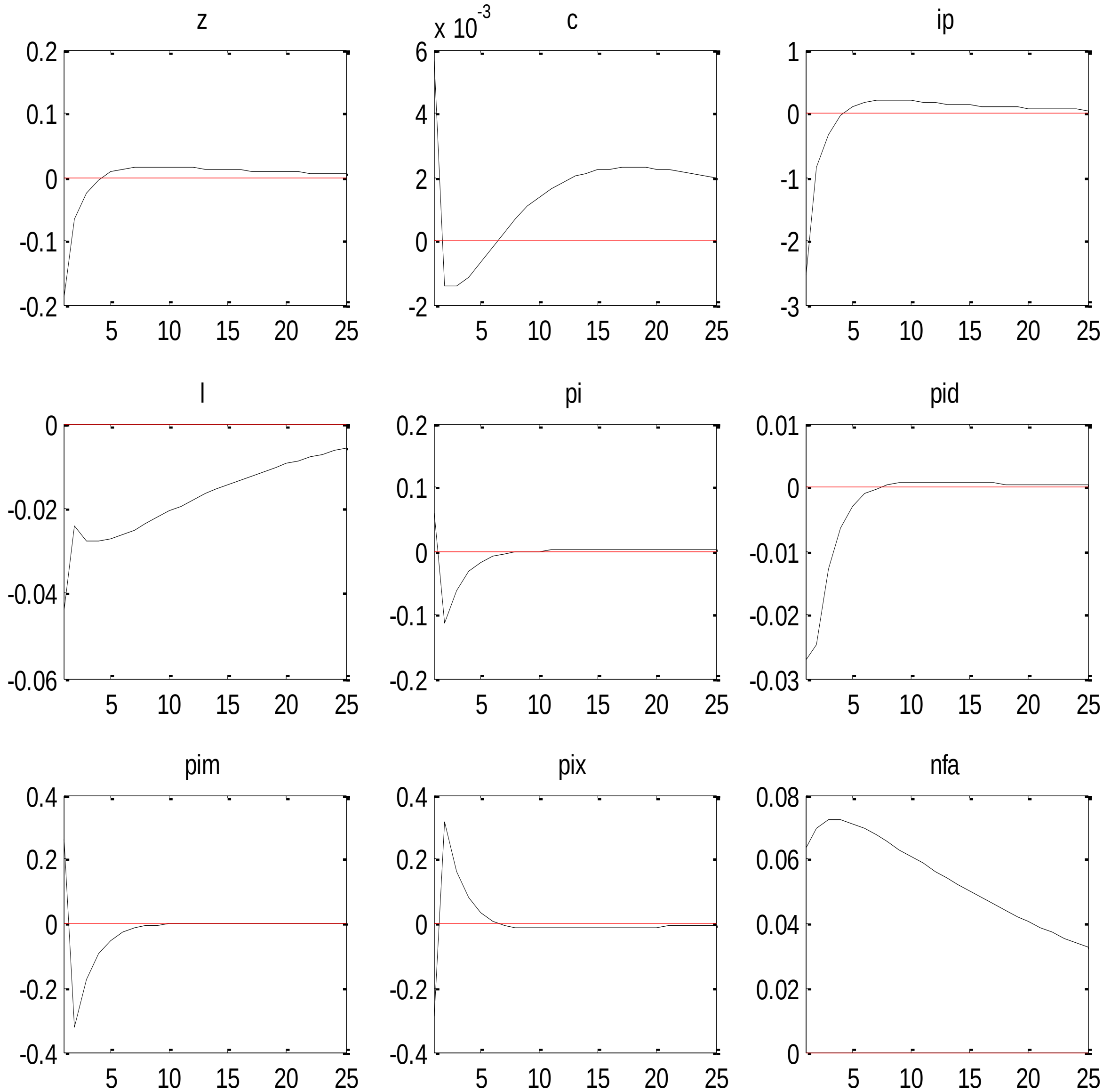

Figure 12: Impulse Response for 1% increase in World Interest Rate

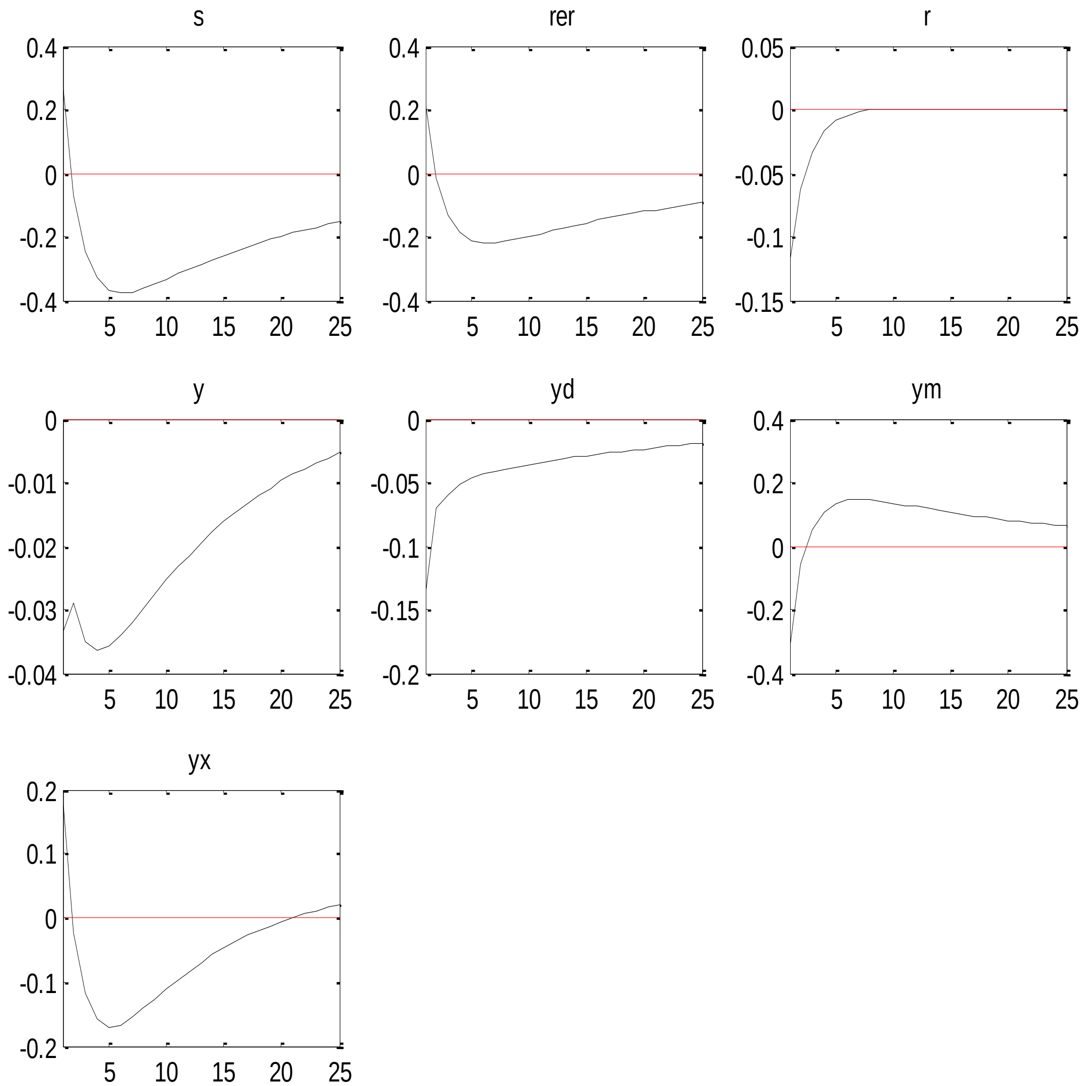

Figure 13: Impulse Response for 1% increase in Domestic Productivity

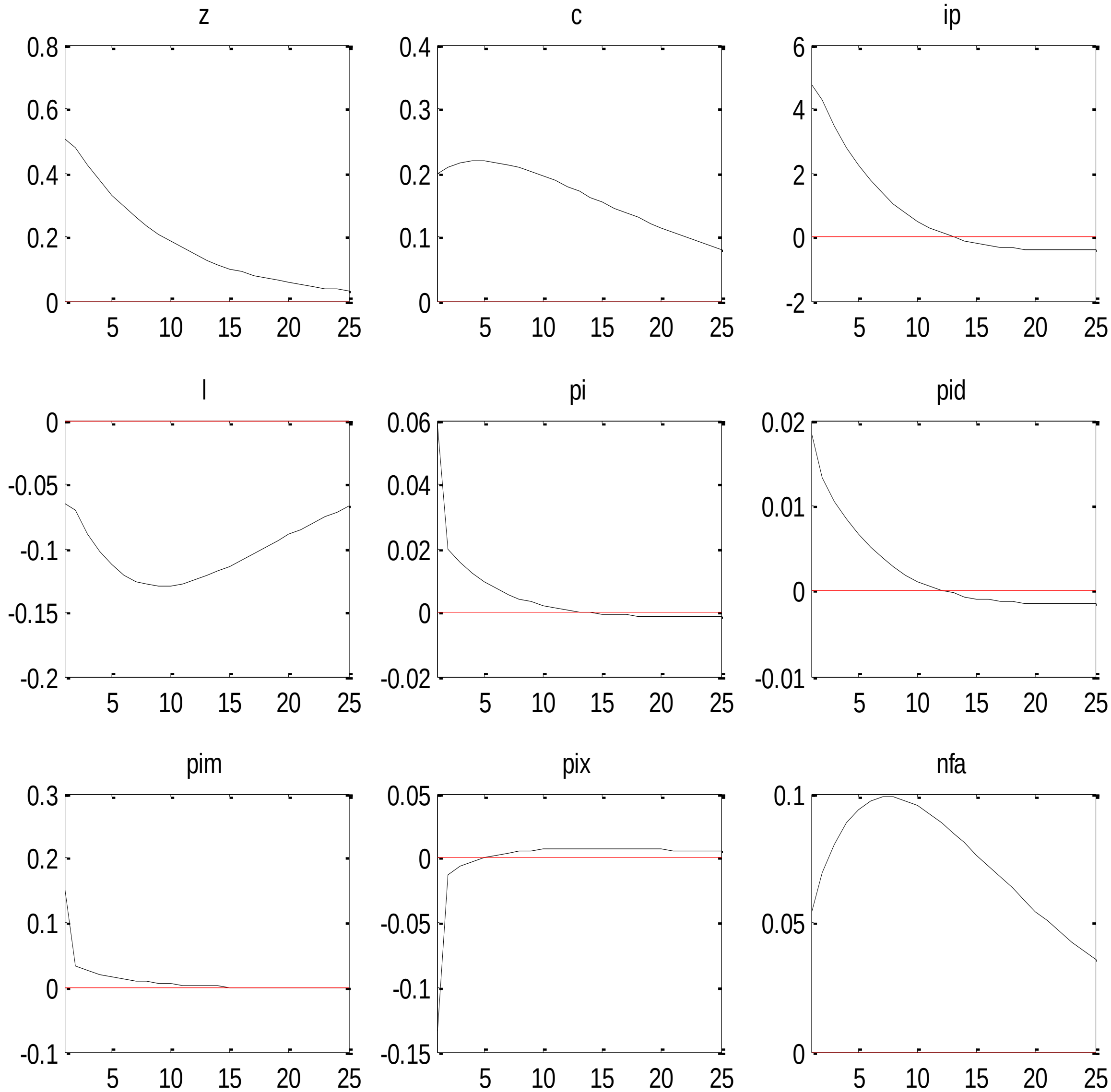

Figure 14: Impulse Response for 1% increase in Domestic Productivity

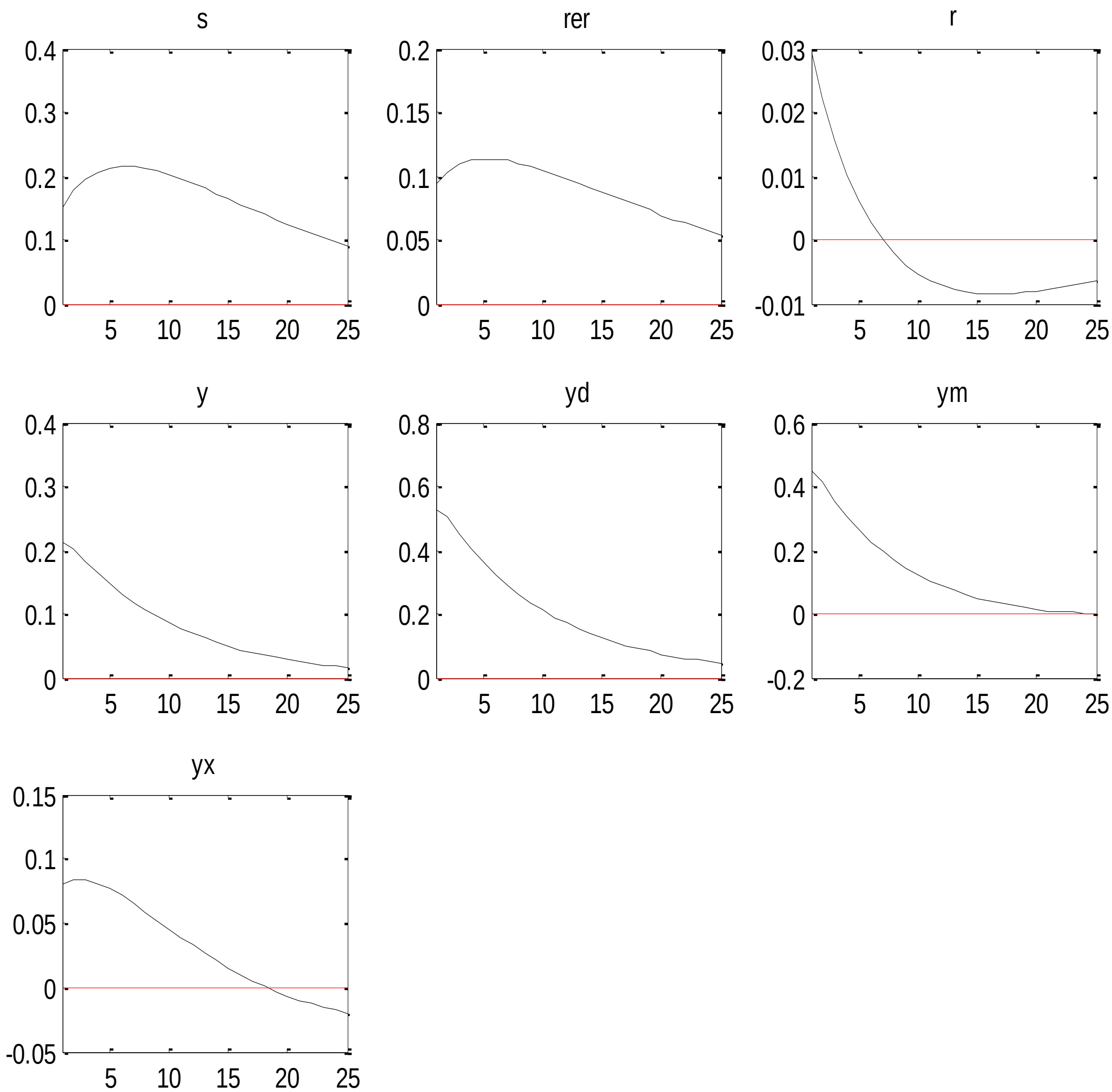

Table 2.1: **Baseline model. Optimized policy rule**

	SP	FP	PCP	FER
	(1)	(2)	(3)	(4)
Standard deviations (in %)				
Y	1.73	0.22	0.22	3.03
C	1.53	0.38	0.38	1.81
Ip	11.82	7.93	7.64	25.02
L	2.52	0.30	0.26	4.28
π	0.64	0.39	0.20	0.94
π^d	0.79	0.22	0.16	0.98
GC	0.91	0.37	0.37	1.85
IG	0.16	0.09	0.09	0.38
G	0.92	0.38	0.38	1.82
R	0.63	0.30	0.19	0.98
S	3.20	1.14	0.92	6.52
RER	2.09	0.72	0.73	4.26
NFA	3.55	0.74	0.54	8.23
μ^d	2.67	0.00	0.00	3.87
μ^x	2.51	0.00	0.00	2.81
μ^m	3.82	0.00	0.00	1.56
Means (in%)				
Y	-0.91	-0.13	-0.21	1.57
C	0.15	1.08	1.09	-2.76
Y^d	-0.01	-0.27	-0.24	0.63
Y^m	-0.10	3.77	3.68	-0.11
Y^x	-8.71	-0.30	-1.17	-12.48
L	-0.74	-0.55	-0.62	-2.64
K	-1.47	1.18	1.07	-0.83
PK	-2.07	1.68	1.51	-1.54
RER	0.10	-3.87	-4.57	-1.65
NFA	0.30	0.55	0.54	0.46
μ^d	0.00	1.2	1.2	-1.36
μ^x	-0.09	1.2	1.2	0.01
μ^m	0.12	1.2	1.2	0.23
Welfare (% equivalent variation in consumption)				
ζ	0.76	2.40	2.46	0.04
ζ^m	0.68	2.18	2.23	0.01
ζ^v	0.08	0.22	0.23	0.03

Chapter 3

Unconventional Monetary Policy in a Small Open Economy with Liquidity Constraint

Abstract

What are the effects of liquidity facilities (unconventional credit policy) on macroeconomic and financial variables when a small open economy faces a liquidity shock? To answer this question, we introduction the external sector with foreign private paper and bonds in a DSGE model with both nominal and real rigidities. The main result of this paper is that both unconventional credit policy has large quantitative effects on macroeconomic and financial variables. In fact, with the quantitative easing, output, consumption and investment stops to decrease after two quarters and then become positive. However, without the liquidity facilities, output, consumption and investment would have dropped continuously up to 10%, 15% 10%, respectively. This result is closely related to DEFK 's finding in terms of sign. Besides, the domestic inflation, the objective of the conventional monetary policy, falls and becomes negative after four quarters. Then, it raises and becomes positive up to two percent (2%) after eight quarters. Furthermore, a negative liquidity shock under the quantitative easing has positive impact on employment. Nominal and real exchange rates depreciate due to a negative impact of liquidity shock. Finally, the liquidity shock has positive impact on the financial variable (domestic and foreign spreads). The domestic and foreign spreads increase up to 100 and 120 basis points, respectively.

JEL classification : E44; E52; E62; F41

Keywords : Small Open Economy; Financial Frictions; Liquidity constraints; Unconventional Monetary Policy; Fiscal Policy.

3.1 Introduction

During the financial crisis of 2007-2009, most of central banks from advanced economies has implemented policies which departure from policy that is based on the management of the nominal interest rate well-known as Taylor rule. Those policies consist of loans and liquidity facilities, swap lines with other central banks, extending the list of collateral, asset purchases, and direct lending to financial firms. Many papers have focused on the impact of those policies on the macroeconomic and financial variables such especially the output, investment, consumption, inflation and interest rate. Del Negro, Eggertsson, Ferrero,and Kiyotaki(2011), DEFK, study the quantitative effects of liquidity facilities, that they called liquidity shock, on macroeconomic and financial variables such as output, nominal interest rate, investment... in the context of a closed economy. These falicities consist of non-standard open market operations in which the central bank (governement) exchanges highly liquid government paper (bonds) for less liquid private paper (equity). This paper expands DEFK (2011) to a small open economy which faces credit frictions not only from the domestic financial market, but also the foreign ones. In this context, the households' portfolio includes two types of bonds and private equity: domestic and foreign. Besides, in addition to the interventions in domestic credit market through the liquidity facilities, this paper introduces the international reserves as the form of interventions that the government implements on the objective of alleviating liquidity constraint in the foreign exchange and financial market by changing its holdings of international reserves. This type of interventions is more implemented by many emerging economies in form of foreign exchange liquidity injections (Ishi, Stone and Yehoue (2009)). The aim of this paper is to evaluate the effects of the central banks and/or government interventions (liquidity facilities and foreign reserves) on macroeconomic and financial variables.

DEFK's paper incorporates a particular form of credit frictions proposed by Kiyotaki and Moore (2008) to evaluate the quantitative effects of the liquidity policies on macroeconomic and financial variables. The credit frictions proposed by DEFK consist of "resaleability" and borrowing constraints. In fact, the latter constraint is more standard financial friction which means that the entrepreneur who faces the opportunity to invest can borrow only up to a fraction of the value of its current investment. However, the resaleability constraint that is less standard means that the entrepreneur who faces the opportunity to invest can sell only up to a certain fraction of his/her "illiquid" private paper in each period. The illiquid paper consists of all privately issued papers such as commercial papers, bank loans, mortgages... The results of their simulations suggest that the liquidity facilities' policy is strongly effective. In this sense, in the absence of this policy, the output and interest rate would have dropped by an additional fifty percent (50%) when the economy faces both borrowing and reseleability constraints.

In this paper, I incorporate the private foreign paper and foreign bonds in the DEFK's model. As for the foreign bonds, I assume that only the foreign issued private paper can be claimed by the domestic households and not the contrary. This assumption on foreign bonds is standard to small open economy's models. Therefore, the net equity of the households consist of the domestic and foreign equities that face the same reseleability and borrowing constraints. As for DEFK, the main credit friction in this model is the reseleability constraint which is considered as liquidity shock. Like DEFK (2011), I claim that the liquidity shock represents the source of financial crisis of 2008. In addition to the liquidity shock, the paper considers six more shocks which are standard for small open economy. These shocks are other sources of perturbation of small open economy due to (i) productivity, (ii) uncovered interest parity (UIP), (iii) international interest rate, (iv) international output, (v) international inflation, and (vi) government spending. Moreover, I assume that the domestic and foreign bonds do not face the reseleability constraint which plays a crucial role in my analysis [1]. When the economy faces a negative impact of liquidity shock, the level of investment decreases which lower the output.

Furthermore, the model includes both nominal and real rigidities that have strong policy's implications. The nominal rigidities consist of prices' stickiness for intermediate firms. This friction causes inflation. Therefore, the central bank must keep the price levels constant in order to eliminate the costs introduced by the inflation. However, the real rigidity is related to the types of households: (i) workers and (ii) entrepreneurs. This friction allows the model to explain not only the cause of crisis of 2008, but also the intervention of central banks to eliminate the crisis. In addition to the above rigidities, the model considers the monopolistic competition for intermediate firms that produce less than the perfectly competitive firms. Therefore, the central bank should deviate from its main objective of zero inflation by targeting the output gap as well. Based on the above frictions, the paper proposes two types of monetary policies that are available for the central bank: (i) conventional and (ii) unconventional. The conventional policy follows the Taylor's rule. However, the unconventional monetary policies are related to the credit market. This policy aims to eliminate the liquidity constraints that accounts for a drop in investment by swapping partly illiquid private paper for liquid bonds. Therefore, the policy makes the aggregate portfolio holdings of the private sector more liquid and the financial market more lubricated. Also, the policy reduces the fall in consumption and investment. Nevertheless, the unconventional policy does not affect the reseleability constraint of the private sector.

The main result of this paper suggests that both unconventional and conventional policies have a large qualitative effect on macroeconomic and financial variables. In fact, with the quantitative easing, the output, consumption and investment stops to decrease after two quarters and then becomes positive. However, without the liquidity facilities, output, consumption and investment would have dropped continuously up to 10%, 15% 10%, respectively. This main result is closely

[1] This assumption is the same as in DEFK

related to DEFK 's finding. Besides, the domestic inflation, the objective of the conventional monetary policy, falls and becomes negative after four quarters. Then, it raises and becomes positive up to two percent (2%) after eight quarters. Furthermore, a negative liquidity shock under the quantitative easing has positive impact on employment. Both nominal and real exchange rates depreciate due to a negative impact of liquidity shock. Finally, there is a positive impact on domestic and foreign spreads which attains 120 basis points.

The unconventional monetary policy introduces new branch of the literature into macroeconomic models with financial frictions using the New Keynesian DSGE framework. But, the majority of them focuses on closed economy and has neglected the case of small open economy. The seminal work of Bernanke et al. (1999) launched this literature. This paper has been inspired by other works such the work of Christiano et al. (2003, 2009, 2011), Goodfriend and McCallum (2007), Curdia and Woodford (2009), Gertler and Karadi (2011), Gertler and Kiyotaki (2010), Martins (2012), Kara and Sin (2013), and Garcia(2013). However, the introduction of the financial frictions in the models is exogenous and does not have any microeconomic foundations. This paper expands DEFK model to a small open economy by modeling the international illiquid equity and the policy related to the interventions of the central bank in the foreign exchange market. Even though the policy is proposed by Garcia (2013) and Martins (2012), they do not consider the reseleability constraint.

The rest paper will be organized as follow. I will present the model in details in section 2. The results are presented in section 3 and I will conclude the paper in section 4.

3.2 Model

The model is build on DEFK augmented with foreign sectors and retailers. There are six types of agents: households, distribution sector, intermediate good producers, foreign sector, the monetary authority, and the government.

3.2.1 Households

The economy is populated by a continuum of identical households of the measure one. Each household consists of a continuum of members indexed by $j \in [0,1]$. In every period , household members receive an identical independent distributed draw that determines whether they are entrepreneurs or workers. The probability of being an entrepreneur is $j \in [0,\chi)$, which, by the law of large numbers, is also the fraction of entrepreneurs in the household. Each entrepreneur χ has an opportunity to invest but does not work. Each worker member$j \in [\chi,1]$ supplies differentiated labor of type j but does not invest.

Let C_t be the aggregate consumption of goods that each member of the household purchases in the market at period t

$$C_t = \left[(1-\alpha_c)^{\frac{1}{\eta_c}}(C_t^d)^{\frac{\eta_c-1}{\eta_c}} + (\alpha_c)^{\frac{1}{\eta_c}}(C_t^m)^{\frac{\eta_c-1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}} \tag{3.1}$$

where C_t^d, C_t^m, η_c, and α_c are respectively the consumption of domestic goods, the consumption of import goods, the elasticity of substitution between import and domestic goods, and the share of import goods in consumption. The total expenditure on consumption is the sum of domestic and import good purchases by each member of the household.

$$P_tC_t = P_t^dC_t^d + P_t^mC_t^m \tag{3.2}$$

where P_t, P_t^d, and P_t^m denote respectively consumer price index(CPI), the domestic producer prices and import prices. The maximization of the equation (1) subject to (2) yields

$$C_t^d = (1-\alpha_c)\left(\frac{P_t^d}{P_t}\right)^{-\eta_c} C_t, \tag{3.3}$$

$$C_t^m = \alpha_c \left(\frac{P_t^m}{P_t}\right)^{-\eta_c} C_t. \tag{3.4}$$

Plugging (3) and (4) into (1) yields the CPI as

$$P_t^{1-\nu} = [(1-\alpha_c)(P_t^d)^{1-\nu} + \alpha_c(P_t^m)^{1-\nu}]. \tag{3.5}$$

Each household optimizes

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\sigma}}{1-\sigma} - \frac{L_s^{1+\gamma}(j)}{1+\gamma} \tag{3.6}$$

where β, σ, and γ denote the discount factor, the relative risk aversion, and the inverse elasticity of work efforts, respectively.

Now, let recall the main assumptions for the household members. First of all, the entrepreneurs do not work. Therefore, the hours of work or labor supply (L_t) is zero ($L_t(j) = 0$). Secondly, on each period, the members of household choose to consume non-durable goods; to save on domestic bonds or equity,and foreign bonds or equity; and to invest in capital. Besides, the foreign households do not buy the equity issued by the domestic household and bonds issued by the domestic government. Therefore, only the domestic households are allowed to buy both foreign bonds and equity. Furthermore, after the members have found out whether they are workers or entrepreneurs, the households cannot reallocate their assets. Thus, any additional funds must be obtained outside the household. Moreover, at the end of the each period, the household return all their earnings (assets and income) to the household's asset pool.

As each household member has a choice to save or invest depending on his type, their behaviors are not only different, but also complementary. The entrepreneurs have the opportunity to invest in new homogeneous capital, I_t, whose cost per unit is P_t^I and rental income per unit is R_t^k. Besides, the new capital depreciates each period at the rate of δ and has market value of Q_t. The entrepreneurs have also the opportunity to borrow in order to finance their investment by issuing equity, N_t^I, that is claimed by other households, N_t^O, in form of future returns. Therefore, the return on new capital is $\frac{R_t^k+(1-\delta)Q_t}{p_t^I}$. However, the workers can save by buying government bonds (B_t^d), foreign bonds (B_t), private domestic equity (N_t^O), and private foreign equity (N_t^F). The market price of private equity is Q_t. Therefore, the return on equity, domestic bonds, and foreign bonds are, respectively, $\frac{R_t^k+(1-\delta)Q_t}{Q_t}$, R_t, and R_t^F.

The household's net worth on equity is defined as following

$$N_t = N_t^O + K_t + S_t \varphi_t N_t^F - N_t^I. \tag{3.7}$$

In comparison with both DEFK and KS, this formulation of the equation (7) includes the nominal exchange rate (S_t), the risk premium (φ_t), and the private foreign equity (N_t^F)issued in the rest of the world.

$$\varphi_t = exp[(\frac{B_t}{B_{ss}} + \frac{N_t^F}{N_{ss}^F}) + \psi_t], \tag{3.8}$$

where ψ_t is the the uncovered interest parity shocks.

The households also owns firms which produce not only goods (intermediate and final), but also capital. At the end of each period, they earn profits, D_t, from those firms. For simplicity, I assume that the households pay non-distortive taxes, T_t, to the government. Thus, the inter-temporal budget constraint for the household is

$$C_t + p_t^I I_t + Q_t[N_t - I_t] + \frac{B_t^d}{P_t} + \frac{S_t B_t}{P_t} = [R_t^k + (1-\delta)Q_t]N_t + \frac{R_{t-1}B_{t-1}^d}{P_t} + \frac{S_t R_t^F \varphi_{t-1} B_{t-1}}{P_t} + \frac{W_t L_t}{P_t} + D_t - T_t \tag{3.9}$$

where $D_t = D_t^B + D_t^I$, D_t^B is the profit of firms producing goods, D_t^I is the profit of firms producing capital, W_t is the wage, and L_t is the hours work.

Following KM, DEFK and KS, the model has two financial frictions :

- Borrowing constraint which implies that any entrepreneur can only issue new equity up to a fraction θ of the investment, and
- resaleability constraint which implies that in any given perioda household member can sell only a fraction of ϕ_t of her existing equity holdings (both domestic and foreign).

The latter constraint is interpreted as liquidity shocks which capture changes in market liquidity (e.g, DEFK and KM, KS). However, both domestic and foreign bonds are not subject to any resaleability constraint. Therefore, they are liquid. Following the borrowing and resaleability constraints, the net equity can be written as

$$N_t = (1-\theta)I_t + (1+\phi_t)(1-\delta)N_{t-1}. \tag{3.10}$$

The equation (9) is valid only if the constraints are both binding. If θ is equal to one, the entrepreneur is free to finance his entire investment by selling equity in financial market. The desired θ must be strictly less than one which implies that the entrepreneur has to keep a fraction of $1-\theta$ of his investment as own equity whose cost must be financed partly by his own fund.

Similar to the consumption, the total investment goods, I_t, consists of domestically and import produced investment goods as

$$I_t = \left[(1-\alpha_I)^{\frac{1}{\eta_I}}(I_t^d)^{\frac{\eta_I-1}{\eta_I}} + (\alpha_I)^{\frac{1}{\eta_I}}(I_t^m)^{\frac{\eta_I-1}{\eta_I}}\right]^{\frac{\eta_I}{\eta_I-1}} \tag{3.11}$$

where I_t^d, I_t^m, η_I, and α_I denote respectively the domestically produced investment goods, the import investment goods, the elasticity of substitution between import and domestic investment goods, and the share of import investment in total investment goods. Let the price of the total investment goods be p_t^I, then the demand for domestic and import investment goods are

$$I_t^d = (1-\alpha_I)\left(\frac{P_t^d}{p_t^I}\right)^{-\eta_I} I_t, \tag{3.12}$$

$$I_t^m = \alpha_I \left(\frac{P_t^m}{p_t^I}\right)^{-\eta_I} C_t. \tag{3.13}$$

Entrepreneurs

Since the entrepreneurs seek to invest, the equity's price, Q_t, must be greater than the price of newly produced capital, p_t^I. This condition guaranties the equilibrium. In fact, when $Q_t > p_t^I$ the entrepreneurs prefer to sell all his holdings of government and foreign bonds. Besides, they will prefer to issue equity that other households will claim than buy other households' equity. Therefore, combining the condition (9),

$$L_t = 0, \tag{3.14}$$

$$B_t = 0, \tag{3.15}$$

and

$$C_t = 0 \tag{3.16}$$

with (8) help to obtain the aggregate investment in the economy :

$$I_t = \chi \frac{[R_t^k + (1-\delta)Q_t\phi_t]N_t + \frac{R_{t-1}B_t^d}{P_t} + \frac{S_t\varphi_t R_t^F B_{t-1}}{P_t} + D_t - T_t}{p_t^I - \theta Q_t}. \tag{3.17}$$

The numerator of (13) measures the amount of liquidity available to finance the investment. However, the denominator measures the liquidity needed for one unit of investment. θQ_t represents the amount that the entrepreneur finances his new capital by issuing equity. Therefore, any drop in ϕ_t reduces the amount of liquidity available to finance investment (DEFK, 2011).

Workers

The workers choose C_t, L_t, B_t^d, B_t, and N_t in order to maximize his utility subject to (8) and (13). The first order conditions with respect to hours work, domestic bonds, foreign bonds, and net equity are

$$L_t = \left[\frac{W_t C_t^{-\sigma}}{P_t}\right]^{-\frac{1}{\gamma}}, \tag{3.18}$$

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma}\left[\chi \frac{R_t}{\pi_{t+1}} \frac{(Q_{t+1-p_t^I})}{p_{t+1}^I - \theta Q_{t+1}} + \frac{R_t}{\pi_{t+1}}\right], \tag{3.19}$$

$$C_t^{-\sigma} = \beta E_t \frac{S_{t+1}}{S_t} \varphi_t C_{t+1}^{-\sigma}\left[\chi \frac{R_t^F}{Q_t} \frac{(Q_{t+1-p_t^I})}{p_{t+1}^I - \theta Q_{t+1}} + \frac{R_t^F}{Q_t}\right], \tag{3.20}$$

$$C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma}\left[\chi \frac{(R_{t+1}^k + (1-\delta) Q_{t+1}\phi_{t+1})}{Q_t} \frac{(Q_{t+1-p_t^I})}{p_{t+1}^I - \theta Q_{t+1}} + \frac{(R_{t+1}^k + (1-\delta) Q_{t+1})}{Q_t}\right]. \tag{3.21}$$

If $\chi = 0$, the equations (15) through (17) are reduced to the standard euler equations without financial frictions. According to DEFK for closed economy, there are two parts on the payoff from holding bonds or equity. The first part is the standard return related to paper holding: $\frac{R_t}{\pi_{t+1}}$ for domestic bonds and $\frac{R_{t+1}^k + (1-\delta)Q_{t+1}}{Q_t}$ for domestic equity. For an small open economy, there are two more standard returns : $S_t \varphi_t \frac{R_t^F}{\pi_{t+1}}$ for foreign bonds and $S_t \varphi_t \frac{(R_{t+1}^k + (1-\delta)Q_{t+1})}{Q_t}$ for foreign equity. The second part of the payoff is the premium $\chi\frac{(Q_t-p_t^I)}{p^I-\theta Q_t}$related to this paper (bonds or equity), when in the hand of the entrepreneurs, relaxes their investment constraint. Therefore, if the entrepreneur chooses to buy one extra unit of domestic government or foreign bonds at time t instead of consumption, he gains $\frac{R_t}{\pi_{t+1}}$ or $\frac{S_t\varphi_t R_t^F}{\pi_{t+1}}$, respectively, extra units of liquidity at time $t+1$. Besides, $\frac{\chi}{p_t^I-\theta Q_t}$ measures the increase in investment afforded by an extra dollar liquidity (DEFK). However, the magnitude of $(Q_t - p_t^I)$ measures the value to the household of relaxing the constraint. If the magnitude is larger, the more valuable for the household to acquire capital by investing and pay p_t^I per unit, rather than pay Q_t on the market. Therefore, the more liquid a paper is, the less it bears premium.

3.2.2 Distribution Sector

This sector consists of consumption good producers and investment good producers. Both goods are an aggregation of domestic and import goods that are produced by perfect competitive final good firms.

Final Consumption and Investment Good Producers

The equations (1), (3), (4), and (5) give the details about the technology of the consumption good's production, the demand for domestic consumption good, the demand for import consumption good, and the aggregate price level (CPI).

Similarly, the investment good is produced following the technology described by the equation (10). The demands for domestic and import investment are described by the equations (11) and (12). However, the investment goods are produced by transforming the consumption goods into investment goods. Therefore, the producers choose the amount of investment good to maximize their profits

$$D_t^I = [p_t^I - (1 + F\left(\frac{I_t}{I_{ss}}\right)]I_t, \tag{3.22}$$

by taking the price of new investment, p_t^I, as given. Following DEFK, the price of investment goods is different from the price of consumption goods due to the adjustment cost function of the investment with $F(1) = F'(1) = 0$ and $F''(1) > 0$.

Final Domestic Good Producer

The final good is produced by a single firm that operates in perfect competitive market according to the following technology by taking the price P_t as given

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}}, \tag{3.23}$$

where η is the elasticity of substitution across domestic intermediate goods and Y_{it} is the aggregate intermediate goods.

Let the price of intermediate goods be P_{it}, then the maximization problem is

$$max_{Y_{it}} P_t Y_t - \int_0^1 P_{it} Y_{it} di$$

. Thus, the demand for $i-th$ intermediate goods firm's output is

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\eta} Y_t, \tag{3.24}$$

where the aggregate price level is given as

$$P_t = \left(\int_0^1 P_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}}. \tag{3.25}$$

3.2.3 Intermediate Good Producers

Each intermediate good i is produced through the following technology

$$Y_{it} = A_t K_{it-1}^{\alpha} L_{it}^{1-\alpha}, \tag{3.26}$$

where K_t, L_t, A_t, and $0 < \alpha < 1$ denote, respectively, the capital stock, the hours work, the total factor productivity shocks and the share of capital stock in production. A_t is exogenous to all intermediate good producers.

As the firms operate in monopoly competitive market, they face two-stage problem : (i) minimization of the cost and (ii) maximization of the discounted real profit.

On the first stage, each firm minimizes the real cost of production by taking the real wage (W_t) and the real rent of capital stock (R_t^k) as given

$$\min_{L_{it}, K_{it}} W_t L_{it} + R_t^k K_{it}$$

subject to (25). The first order conditions are given as following with respect, respectively, to L_{it} and K_{it}

$$W_t = (1-\alpha) A_t \left(\frac{K_{it}}{L_{it}} \right)^{\alpha} = (1-\alpha) MC_t \frac{Y_{it}}{L_{it}}, \tag{3.27}$$

$$R_t^k = \alpha A_t \left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} = \alpha MC_t \frac{Y_{it}}{K_{it}}, \tag{3.28}$$

where MC_t is the real marginal cost given by

$$MC_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \frac{W_t^{1-\alpha}(R_t^k)^{\alpha}}{A_t}. \tag{3.29}$$

However, on the second stage, each firm maximizes its discounted real profits in in Calvo's setting. Therefore, in each period, a fraction of $(1-\lambda)$ of firms can change their prices. Other firms can only report their previous prices. Let $P_t^{\star d}$ and P_t^d be the average of the domestic price and the domestic price respectively then the firms solve the following problem

$$\max_{P_t^{\star d}} E_t \sum_{s=0}^{\infty} (\beta\lambda)^s \rho_{t+s} \left[Y_{it+s}\left(\frac{P^{\star d}}{P_{t+s}^d} - MC_{t+s}\right)\right]$$

subject to

$$Y_{it+s} = \left(\frac{P_t^{\star d}}{P_{t+s}^d}\right)^{-\eta_d} Y_{t+s}$$

the first order condition is

$$P_t^{\star d} = \frac{\eta}{\eta-1} \frac{E_t \sum_{s=0}^{\infty} (\beta\lambda)^s \rho_{t+s} P_{t+s}^d MC_{t+s} Y_{it+s}}{E_t \sum_{s=0}^{\infty} (\beta\lambda)^s \rho_{t+s} Y_{it+s}} \tag{3.30}$$

where

$$(P_t^d)^{1-\eta_d} = \lambda (P_{t-1}^d)^{1-\eta_d} + (1-\lambda)(P_t^{\star d})^{1-\eta_d}. \tag{3.31}$$

and

$$\rho_{t+s} = \left(\frac{C_{t+s}}{C_t}\right)^{-\sigma}. \tag{3.32}$$

3.2.4 Foreign Sector

This sector consists of import and export firms that operate in a monopolistic competition market and set their prices à la Calvo.

Importing Firms

There is a continuum of importing firms that purchase a homogeneous goods at world market prices $P_t^{\star}$ which are set in producers' currency and transformed them into a differentiated final import good, Y_t^m. The latter is sold to the household in form of consumption and investment goods. The technology used by $i-th$ producer is

$$Y_t^m = \left[\int_t^1 (Y_{it}^m)^{\frac{\eta_m - 1}{\eta_m}} di\right]^{\frac{\eta_m}{\eta_m - 1}}, \tag{3.33}$$

where $Y_t^m = C_t^m + I_t^m$. As for the final good producer, the following demands for import consumption and investment goods are derived from the maximization problem solved by the $i-th$ producer

$$C_{it} = \left(\frac{P_{it}^m}{P_t^m}\right)^{-\eta_m} C_t^m, \tag{3.34}$$

$$I_{it} = \left(\frac{P_{it}^m}{P_t^m}\right)^{-\eta_m} I_t^m, \tag{3.35}$$

where P_{it}^m is the price of import intermediate goods.

Similar to the intermediate Good Producers, the import good producers set their prices à Calvo. Therefore, only a fraction of $(1-\lambda)$ of producers can reset their prices. All other firms are not allowed to reset their prices. The firms that reset their prices can choose the same price $P_t^{\star m}$. Thus, the firm's problem is to solve the following

$$\max_{P_t^{\star d}} E_t \sum_{s=0}^{\infty} (\beta\lambda)^s \rho_{t+s} \left[Y_{it+s}^m \left(\frac{P^{\star m}}{P_{t+s}^m} - MC_{t+s}^m \right) \right]$$

subject to

$$Y_{it+s}^m = \left(\frac{P_t^{\star m}}{P_{t+s}^m} \right)^{-\eta_m} Y_{t+s}^m$$

.

where

$$MC_t^m = \frac{P_t^\star S_t}{P_t^m} \tag{3.36}$$

is the marginal cost for import good producer.

The first order condition is

$$P_t^{\star m} = \frac{\eta}{\eta - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\lambda)^s \rho_{t+s} P_{t+s}^m MC_{t+s}^m Y_{it+s}^m}{E_t \sum_{s=0}^{\infty} (\beta\lambda)^s \rho_{t+s} Y_{it+s}^m} \tag{3.37}$$

Finally, the aggregate import price is defined as

$$(P_t^m)^{1-\eta_m} = \lambda (P_{t-1}^m)^{1-\eta_m} + (1-\lambda)(P_t^{\star m})^{1-\eta_m}. \tag{3.38}$$

Exporting Firms

Similar to the import good producers, there is a continuum of export firms that purchase the homogeneous good at price P_t^d from the final good producer and differentiate it by brand naming. Then, the export producers sell their goods to foreign markets at aggregate price P_t^x in terms of consumption and investment. Therefore, the $i-th$ exporting good producer face the following demand for its product Y_{it}^x

$$Y_{it}^x = \left(\frac{P_{it}^x}{P_t^x}\right)^{-\eta_m} Y_t^x \tag{3.39}$$

where $Y_t^x = C_t^x + I_t^x$, P_{it}^x is the price for a single producer. Besides, the real marginal cost for the firm, MC_t^x, is given as

$$MC_t^x = \frac{P_t^d}{S_t P_t^x}. \tag{3.40}$$

Besides, as for importing good producers, the exporting good producers choose the price, $P_t^{\star x}$, that maximizes their discounted real profits à la Calvo. Then, only a fraction of $(1-\lambda)$ firms is allowed to change the price. The problem consists of

$$\max_{P_t^{\star x}} E_t \sum_{s=0}^{\infty} (\beta\lambda)^s [Y_{it+s}^x \left(\frac{P^{\star x}}{P_{t+s}^x} - MC_{t+s}^x\right)]$$

subject to

$$Y_{it+s}^x = \left(\frac{P_t^{\star x}}{P_{t+s}^x}\right)^{-\eta_x} Y_{t+s}^x.$$

The first order condition yields

$$P_t^{\star x} = \frac{\eta}{\eta-1} \frac{E_t \sum_{s=0}^{\infty} (\beta\lambda)^s \rho_{t+s} P_{t+s}^x MC_{t+s}^x Y_{it+s}^x}{E_t \sum_{s=0}^{\infty} (\beta\lambda)^s \rho_{t+s} Y_{it+s}^x}. \tag{3.41}$$

The aggregate import price has the following law of motion

$$(P_t^x)^{1-\eta_x} = \lambda (P_{t-1}^x)^{1-\eta_x} + (1-\lambda)(P_t^{\star x})^{1-\eta_x}. \tag{3.42}$$

Finally, the aggregate export demand maybe expressed in terms of foreign output or demand for domestic goods (investment and consumption) as following

$$Y_t^x = \left(\frac{P_t^x}{P_t^\star}\right)^{-\eta_x} Y_t^\star, \tag{3.43}$$

where $P_t^\star$ and $Y_t^\star$ are world price and output, respectively.

3.2.5 Relative Prices and Real Exchange Rate

Now let define some relative prices that will be helpful to solve the model in real terms.

$$\psi_t^m = \frac{P_t^m}{P_t} = \frac{\pi_t^m}{\pi_t}\psi_{t-1}^m, \tag{3.44}$$

$$\psi_t^d = \frac{P_t^d}{P_t} = \frac{\pi_t^d}{\pi_t}\psi_{t-1}^d, \tag{3.45}$$

$$\psi_t^x = \frac{P_t^x}{P_t^\star} = \frac{\pi_t^x}{\pi_t^\star}\psi_{t-1}^x, \tag{3.46}$$

$$\psi_t^\star = \frac{P_t^\star}{P_t} = \frac{\pi_t^\star}{\pi_t}\psi_{t-1}^\star, \tag{3.47}$$

$$\psi_t^{x\star} = \frac{P_t^x S_t}{P_t} = \frac{\pi_t^x e_t}{\pi_t}\psi_{t-1}^\star, \tag{3.48}$$

$$\psi_t^{I,d} = \frac{P_t^d}{p_t^I} = \frac{\pi_t^d}{\pi_t^I}\psi_{t-1}^{I,d}, \tag{3.49}$$

$$\psi_t^{I,m} = \frac{P_t^m}{p_t^I} = \frac{\pi_t^m}{\pi_t^I}\psi_{t-1}^{I,m}, \tag{3.50}$$

where $\pi_t^m = \frac{P_t^m}{P_{t-1}^m}$, $\pi_t^d = \frac{P_t^d}{P_{t-1}^d}$, $\pi_t^x = \frac{P_t^x}{P_{t-1}^x}$, $\pi_t^\star = \frac{P_t^\star}{P_{t-1}^\star}$, $\pi_t^I = \frac{p_t^I}{p_{t-1}^I}$, and $e_t = \frac{S_t}{S_{t-1}}$.

The real exchange rate is defined as

$$RER_t = \frac{S_t P_t^\star}{P_t} = \frac{e_t \pi_t^\star}{\pi_t} = S_t \psi_t^\star \tag{3.51}$$

3.2.6 Government

Since the entrepreneurs face the resaleability of private equity, the government executes the quantitative easing to solve the problem of credit crisis in the economy. Any unexpected negative deviation in the resaleability parameter ϕ_t from its steady state value represents the credit crisis. Therefore, the government buys private domestic equities from households by selling bonds (N_t^g). Besides, it buys the private foreign equities from households by using its foreign reserves (FA_t) in order to intervene in the foreign exchange market. Then, the total intervention of the government is given by

$$N_t^T = N_t^g + S_t FA_t. \tag{3.52}$$

The government conducts three types of policy : (i) the conventional monetary policy through the standard Taylor's feedback rule, (ii) the unconventional credit policy, and (iii) the unconventional fiscal policy.

The monetary policy is conducted by the central bank according to the Tarlor's feedback rule

$$R_t = R_{ss}(\frac{\pi_t^d}{\pi_{ss}^d})^{\gamma_\pi}(\frac{Y_t}{Y_{ss}})^{\gamma_Y}(\frac{e_t}{e_{ss}})^{\gamma_e} \tag{3.53}$$

where the subscript *ss* denotes the steady state value.

The unconventional credit policy consists of purchasing of private paper, N_t^T, as function of liquidity facilities used by the most of central banks like the Federal Reserve

$$\frac{N_t^T}{K_{ss}} = \gamma_k \left(\frac{\phi_t - \phi_{ss}}{\phi_{ss}} \right) \tag{3.54}$$

where the policy's parameter γ_k must be negative since the economy faces a unexpected drop in the resaleability parameter ϕ_t from its steady state value. According to DEFK, the above rule suggests that the government intervenes in the credit through the open market operations only when the liquidity of private paper is abnormally low. Therefore, when the liquidity becomes normal, the facilities are discontinued. Besides, the government intervention in the credit market does not relax the resaleability constraint. It affects only macroeconomic outcomes by changing the aggregate portfolio composition of the private sector, skewing it toward liquid assets.On impact, the intervention is effective only through its impact on expectations and prices.

The government budget constraint is given as following

$$G_t + Q_t N_t^T + \frac{R_{t-1} B_{t-1}^d}{P_t} = T_t + [R_t^k + [1-\delta) Q_t] N_t^T + \frac{B_t^d}{P_t}, \tag{3.55}$$

where G_t is the government purchases. The equation (54) shows that the government finances its purchases, its intervention in the credit market by purchasing private equity, and its debt repayment by lump sum tax, T_t, returns on equity holdings, and new debt issuance.

The following fiscal rule ensures the government intertemporal solvency in order to avoid the Ponzi's Scheme

$$T_t - T_{ss} = \gamma_T \left[\left(\frac{R_{t-1} B_{t-1}^d}{P_t} - \frac{R_{ss} B_{ss}^d}{P_{ss}} \right) - Q_t N_t^T \right], \tag{3.56}$$

where $\gamma_T > 0$, T_{ss}, and $\frac{R_{ss}B^d_{ss}}{P_{ss}}$ denotes, respectively, the policy parameter, the steady state of taxes and beginning of period government debt. However, the state state value of N_t^T is assumed to be zero. The value of γ_T is low because, by assumption, the adjustment of taxes to debt is slow to reflect the fact that the government has to finance its spending by issuing mostly bonds.

3.2.7 Equilibrium Conditions

The market clearing conditions for the composite labor and capital are

$$L_t = \int_0^1 L_{it} di$$

, and

$$K_t = \int_0^1 K_{it} di$$

. The law of motion of capital stock is

$$K_t = (1-\delta)K_{t-1} + I_t, \tag{3.57}$$

and the capital stock is owned by either household or government as

$$K_t = N_t + N_t^T \tag{3.58}$$

and the resource constraint is given as

$$Y_t = C_t^d + C_t^x + G_t + I_t^d + I_t^x + F\left(\frac{I_t}{I_{ss}}\right) I_t, \tag{3.59}$$

the balance of payments is

$$S_t P_t^x (C_t^x + I_t^x) - S_t P_t^\star (C_t^m + I_t^m) = S_t FA_t - S_t FA_{t-1} + S_t B_t - S_t R_{t-1}^F \varphi_{t-1} B_{t-1}. \tag{3.60}$$

Finally, let define two more variables the expected rate of return on equity as

$$R_t^q = E_t \frac{[R^k + (1-\delta)Q_{t+1}]}{Q_t}, \tag{3.61}$$

and the expected return to capital as

$$R_t^{rk} = E_t[R_t^k + (1-\delta)]. \tag{3.62}$$

3.2.8 Exogenous Variables

The model considers seven exogenous variables:

$$log(G_t) = \rho_G log(G_{t-1}) + \varepsilon_{Gt}, \tag{3.63}$$

$$log(\phi_t) = \rho_\phi log(\phi_{t-1}) + \varepsilon_{\phi t}, \tag{3.64}$$

$$log(R_t^F) = \rho_{R^F} log(R_{t-1}^F) + \varepsilon_{R^F t}, \tag{3.65}$$

$$log(\psi_t) = \rho_\psi log(\psi_{t-1}) + \varepsilon_{\psi t}, \tag{3.66}$$

$$log(Y_t^\star) = \rho_{Y^\star} log(Y_{t-1}^\star) + \varepsilon_{Y^\star t}, \tag{3.67}$$

$$log(\pi_t^\star) = \rho_{\pi^\star} log(\pi_{t-1}^\star) + \varepsilon_{\pi^\star t}, \tag{3.68}$$

and

$$log(A_t) = \rho_A log(A_{t-1}) + \varepsilon_{At}. \tag{3.69}$$

3.2.9 Solution Method and Welfare Measure

I solve my model using Sims' (2000) second-order accurate method. The welfare is evaluated through a second-order Taylor expansion of the utility function around the steady state which yields

$$E(U(C_t, L_t)) \cong U(C_t, L_t) + E(\hat{C}_t) - L_{ss}E(\hat{L}_t) \tag{3.70}$$

$$-Var(\hat{C}_t) \tag{3.71}$$

where $Var(\hat{C}_t)$ is the variance of $(\hat{C}_t)$.

By expressing the welfare as the permanent relative change in consumption (compared to the stady state), ξ, which gives

$$E(U(C_t, L_t)) : U((1+\xi)C_t, L_t) \cong U(C_t, L_t) + E(\hat{C}_t) - L_{ss}E(\hat{L}_t) \tag{3.72}$$

$$-Var(\hat{C}_t). \tag{3.73}$$

Hence, the welfare ξ can be decomposed into two components ξ^m and ξ^v as following:

$$U((1+\xi^m)C_t, L_t) = U(C_t, L_t) + E(\hat{C}_t) - L_{ss}E(\hat{L}_t) \tag{3.74}$$

$$U((1+\xi^v)C_t, L_t) = U(C_t, L_t) - Var(\hat{C}_t), \tag{3.75}$$

where ξ^m and ξ^v represent the mean of consumption and hours worked, and variance of consumption. By applying the equation (6) into the previous equations yields

$$\ln(1+\xi) = E(\hat{C}_t) - L_{ss}E(\hat{L}_t) - Var(\hat{C}_t) \tag{3.76}$$

$$\ln(1+\xi^m) = E(\hat{C}_t) - L_{ss}E(\hat{L}_t) \tag{3.77}$$

$$\ln(1+\xi^v) = -Var(\hat{C}_t) \tag{3.78}$$

Therefore,

$$(1+\xi) = (1+\xi^m)(1+\xi^v). \tag{3.79}$$

3.2.10 Parameters (non policy)

Most of non-policy parameters is drawn from the estimations of Garcia-Cicco (2011) and Kollmann (2002). However, the parameters related to credit frictions are drawn from DEFK (2011). The financial variables of model are calibrated to quarterly data for Chile and South Africa in average from 1982 to 2012.

In order to computer the steady state value of different variables, it is important to choose the steady state values of exogenous variables. As for DEFK, I set the steaty value of liquidity shock, ϕ_{ss}, equal to the value of borrowing constraint, θ. This value is $\phi_{ss} = \theta = 0.185$ which means that the entrepreneur can sell up to 18%of the equity holding within a quarter and $1-(0.815)^4 = 56\%$ within a year. This is consistent with the data in average from Chile and South Africa.

The indexation of prices ε is set to 6 which corresponds the price-marginal cost steady state markup

Table 3.1: **Parameters' values**

β	= 0.99	Discount factor
σ	= 1.39	Relative Risk Aversion
δ	= 0.025	Depreciation rate of capital stock
α	= 0.24	capital share
α_c	= 0.36	Import share on consumption
α_I	= 0.55	Import share on investment
θ	= 0.185	Borrowing constraint
χ	= 0.05	Probability of investment opportunity
γ	= 0.05	Probability of investment opportunity
ε	= 6	Indexation of prices
η_c	= 11.952	Elasticity of substitution between import and domestic consumption goods
η_I	= 2.056	Elasticity of substitution between import and domestic investment goods
η_f	= 3.809	Share of domestic export goods in foreign output
G_{ss}	= 0.1	Steady state value of Government spending
L_{ss}	= 1/3	Steady state value of hours worked
ψ_k	= -0.063	Central Bank intervention coefficient
ψ_T	= 0.1	Transfer rule coefficient
$\frac{B^d_{ss}}{Y_{ss}}$	= 0.40	Steady state liquidity/GDP
$\frac{C_{ss}}{Y_{ss}}$	= 0.60	Steady state consumption/GDP
$\frac{I_{ss}}{Y_{ss}}$	= 0.22	Steady state investment/GDP
$\frac{G_{ss}}{Y_{ss}}$	= 0.18	Steady state Government spending/GDP
$\frac{T_{ss}}{Y_{ss}}$	= 0.19	Steady state lum sum tax/GDP

factor for intermediate goods $\frac{\varepsilon}{(\varepsilon-1)} = 1.2$. The price elasticity of substitution between import and domestic consumption goods and domestic and import investment goods, are set $\eta_C = 11.9252$ and $\eta_I = 2.056$, respectively. The share of domestic export to foreign economy is set $\eta_f = 3.809$. Moreover, the elasticity of output with respect to capital, α, is set to 0.24 which is consistent also with Kollmann (2002). The depreciation rate of the capital stock, δ, is set to 0.025 which corresponds to many previous studies (e.g., Bouakez and Rebei, 2007; Kollmann, 2002; Garcia-Cicco, 2011). Following Kollmann (2002), $\lambda = 0.75$, the average interval that producers of intermediate goods change their prices à la Calvo. Table 1 shows the value of the remained parameters.

3.3 Results

The model is solved using Dynare (version 4.4.3). The appendices A and B shows the response of the main macroeconomic and financial variable to different shocks. The appendix A shows the response of the variables under the unconventional policy (liquidity facility). However, the appendix B reports the response of these variables in absence of the unconventional policy. All the responses depend on the optimal conventional policy based on nominal interest rate (R_t) which reacts to inflation gap and output gap. For the inflation, I use the producer price index(PPI). The optimized policy rule depends on whether or not the unconventional policy is used. When the latter is used concomitantly with the first one, the optimized policy rule has inflation stance

$$R_t = R_{ss} + 1.23\hat{\pi}_t^d + 3.58\hat{Y}_t. \tag{3.80}$$

But when the liquidity facilities policy is not used, the optimized policy rule has output stance

$$R_t = R_{ss} + 2.7\hat{\pi}_t^d + 3.9\hat{Y}_t. \tag{3.81}$$

From the appendices A and B, under the unconventional monetary, the negative liquidity shock has a negative impact on output for the first two quarters and a positive one after three quarters due to the rise on investment.In fact, a drop of 1 percent in the liquidity shock has immediate positive impact on investment after one quarter because the objective of the liquidity facilities is to alleviate the credit constraint that burdens the entrepreneurs who have the possibility to invest. However, in absence of the liquidity facilities of the central bank, when liquidity shock hits the small open economy, the output drops continuously up to nearly 90% at the end of 25th quarter due to a continuous negative impact of liquidity shock in investment drop. Even though the liquidity shock has negative impact in investment and output for the entire period of the simulations, the fall stops after two quarters for the investment and ten quarters for the output. Besides, when the central bank uses the liquidity facilities policy, a negative liquidity shock has positive impact on consumption for about thirteen quarters and negative after that due to an increase in nominal interest rate. Nevertheless, in absence of liquidity facilities, a negative liquidity shock has negative impact on consumption for the entire period of the simulations.

The negative liquidity shock also affects the real exchange rate. Under the unconventional monetary policy, the liquidity shock depreciates the real exchange rate for the first thirteen quarters and appreciates slightly after that. This depreciation of the real exchange rate boots the domestic export in comparison to the import of foreign goods (investment and consumption). Thus, the country accumulates more international reserves for its unconventional policy. In absence of the unconventional policy, the liquidity shock appreciates the real exchange rate which lowers the demands of the domestic good from the foreign consumers but increases the demand of the foreign goods from the domestic consumers. Consequently, the aggregate consumption and investment will not fall dramatically due to a decrease in both import and export.

Furthermore, the interventions of the central bank affects the inflation. The liquidity shock has a negative impact of inflation (CPI) which declines up to 5% after two quarters and then rises up to 0.5% at the end of the period of the simulation (25th quarter). The inflation, measured in terms of PPI and the target of the central bank, drops to 5% after 8 quarters and then rises up to 2% at the end of period of simulation. Since the central bank uses two different instruments for its monetary policy, the objective of the management of nominal interest through the inflation and output gap combined with facilities are effective. In absence of the central bank's interventions, the PPI inflation decreases continuously up to 2%. The CPI inflation, however, increases up to 2% and decreases continuously.

The impact of the liquidity shock on financial variable is positive. A negative shock affects positively the prices, especially p_t^I, R_t^k, and Q_t, when the central bank intervenes in the credit market by its liquidity. The impact is negative on the return of capital, R_t^k, if there are no central bank intervention in the credit market. As for DEFK, I define a financial variable called spread of equity versus liquid paper. The spread is defined as the difference in expected returns between illiquid and liquid paper. In the domestic credit market, the spread is defined as

$$E_t\left[\frac{R_{t+1}^k+(1-\delta)Q_t}{Q_t}-\frac{R_t}{\pi_{t+1}}\right]. \tag{3.82}$$

However, for the foreign papers, the households receive a premium φ_t for buying a foreign paper (bonds or equity)

$$E_tS_t\varphi_t\left[\frac{R_{t+1}^k+(1-\delta)Q_t}{Q_t}-\frac{R_t^F}{\pi_{t+1}}\right]. \tag{3.83}$$

The figures in appendices C.1 and C.2 show the impact of the liquidity shock on the spreads with and without the central bank's interventions. With the central bank's interventions in the credit market, the liquidity shock generates positive impact on both domestic and foreign spreads. In terms of magnitude, the impact on foreign spread is more pronounced than on domestic one. In fact, the liquidity shock causes the foreign spread to increase by 120 basis points against less than 100 basis points for the domestic spread. This maybe due to not only on the premium that households receive by buying foreign bonds and equity, φ_t, but also the combined external shocks (foreign inflation, output and nominal interest rate). Nevertheless, the impact is negative on both spreads if there are no interventions of the central bank.

This paper expand DEFK's model to a small open economy. When comparing the effects of the interventions on the key macroeconomic and financial variables, there is a difference on the magnitude of the impact of the intervention on key macroeconomic variables. The result is consistent with actual data for both Chile and South Africa. In fact, my simulations generate not only an immediate positive increase after a negative impact, but also a persistent expansion of the output. In period of 2010 to 2012, the growth rate of GDP for South Africa was 3.0%, 3.2%,and 2.2% respectively again -1.5% in 2009. For the same period, Chile realized 5.7%, 5.8%,and 5.5% respectively again -0.7% in 2009. DEFK's simulations for developed countries predict a negative impact on output when the liquidity shock hits the economy, but the drop has stopped and the output has initiated its expansion. The immediate impact for developing countries maybe due to the combined effects of the government spending and the international reserve which are not taking into account in DEFK's analysis. Similarly, the difference is observed on the impact of variable inflation targeted by the central bank. My model generates a positive increase on PPI inflation after nine quarters. The DEFK's model, however, predict only a immediate stop on inflation which is still negative for the entire period under simulations. Besides, the same difference is observed for consumption and investment. My model predicts a positive impact on investment when the liquidity shock hits the small open economy, but the DEFK's model predicts a negative one. Finally, for the financial variable, the spreads, there is no difference between the two models.

3.4 Conclusion

What are the effects of liquidity facilities (unconventional credit policy) on macroeconomic and financial variables when a small open economy faces a liquidity shock? To answer this question, we introduction the external sector with foreign private paper and bonds in a DSGE model with both nominal and real rigidities. The main result of this paper is that both unconventional credit policy has large quantitative effects on macroeconomic and financial variables. In fact, with the quantitative easing, output, consumption and investment stops to decrease after two quarters and then become positive. However, without the liquidity facilities, output, consumption and investment would have dropped continuously up to 10%, 15% 10%, respectively. This result is closely related to DEFK 's finding in terms of sign. Besides, the domestic inflation, the objective of the conventional monetary policy, falls and becomes negative after four quarters. Then, it raises and becomes positive up to two percent (2%) after eight quarters. Furthermore, a negative liquidity shock under the quantitative easing has positive impact on employment. Nominal and real exchange rates depreciate due to a negative impact of liquidity shock. Finally, the liquidity shock has positive impact on the financial variable (domestic and foreign spreads). The domestic and foreign spreads increase up to 100 and 120 basis points, respectively.

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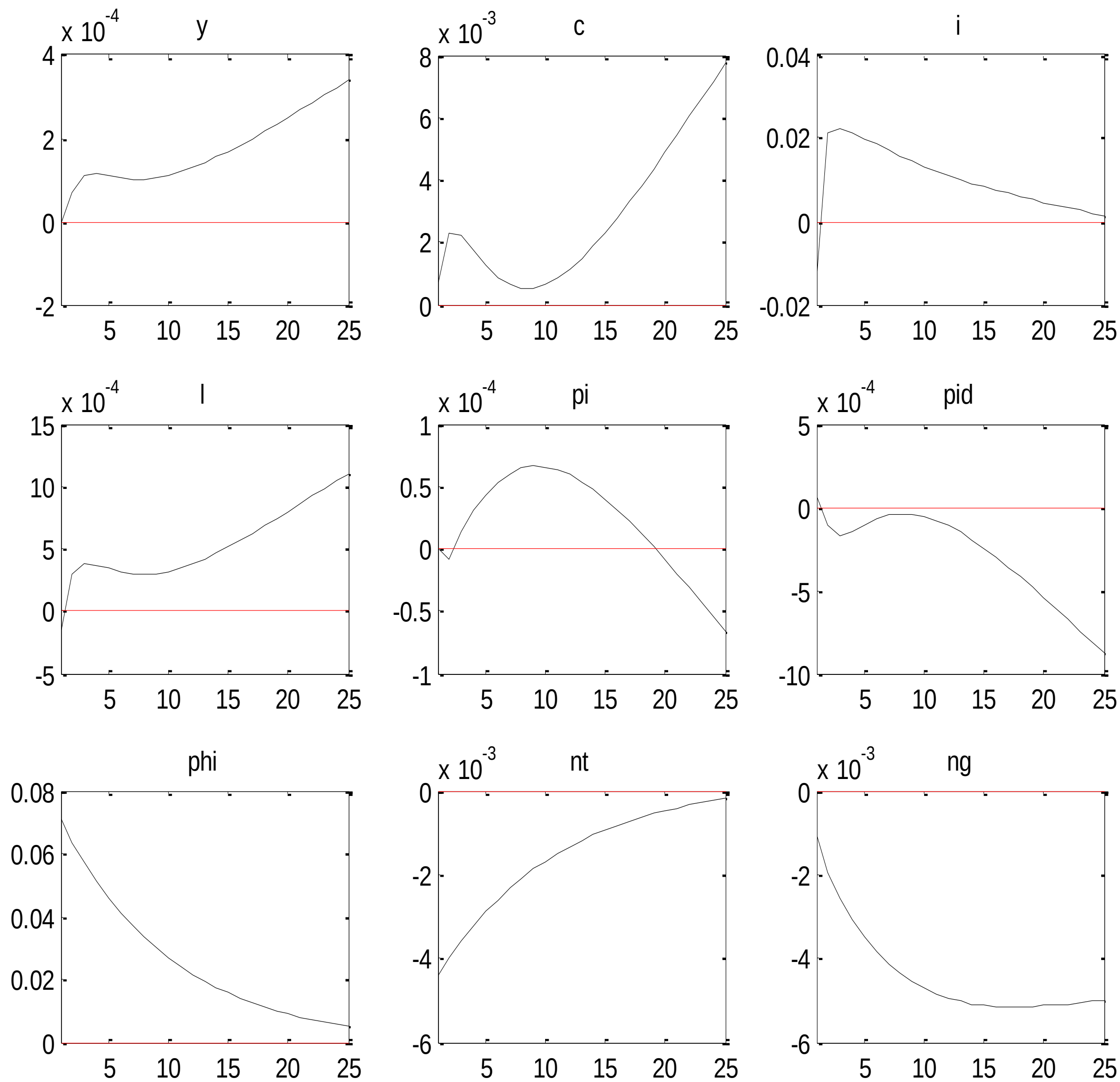
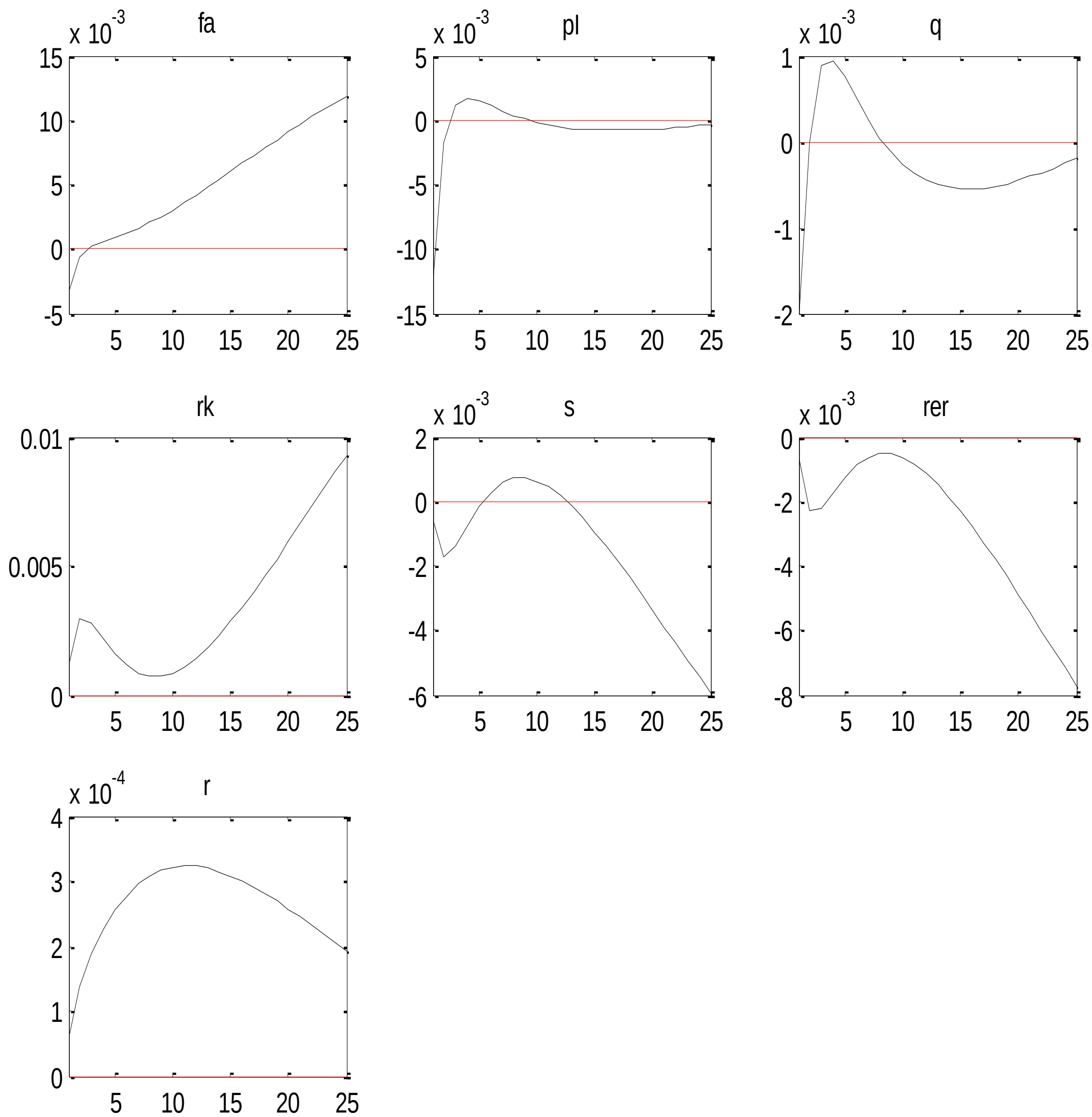
Appendices

A. IRF with Quantitative easing

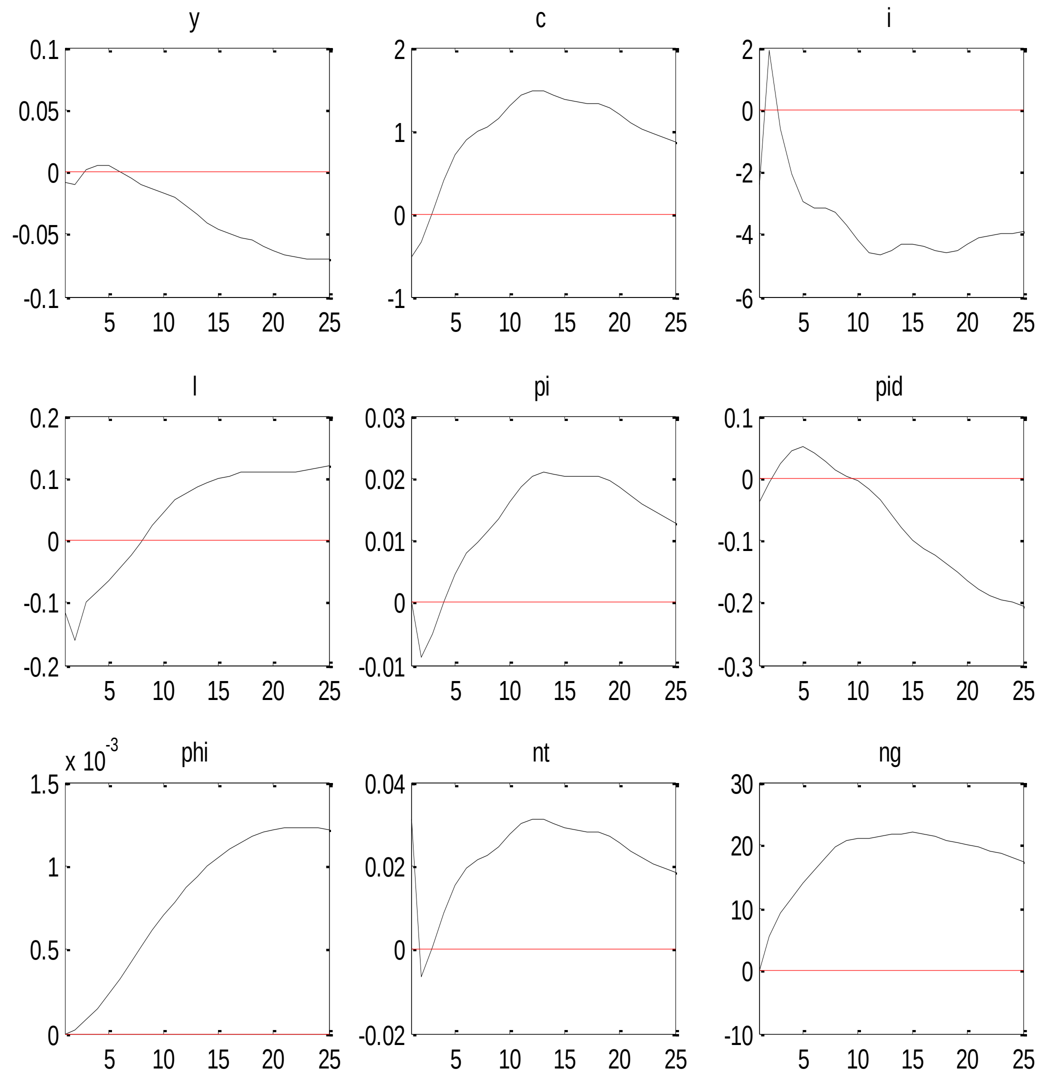
A.1. Impact of 1% drop in liquidity shock



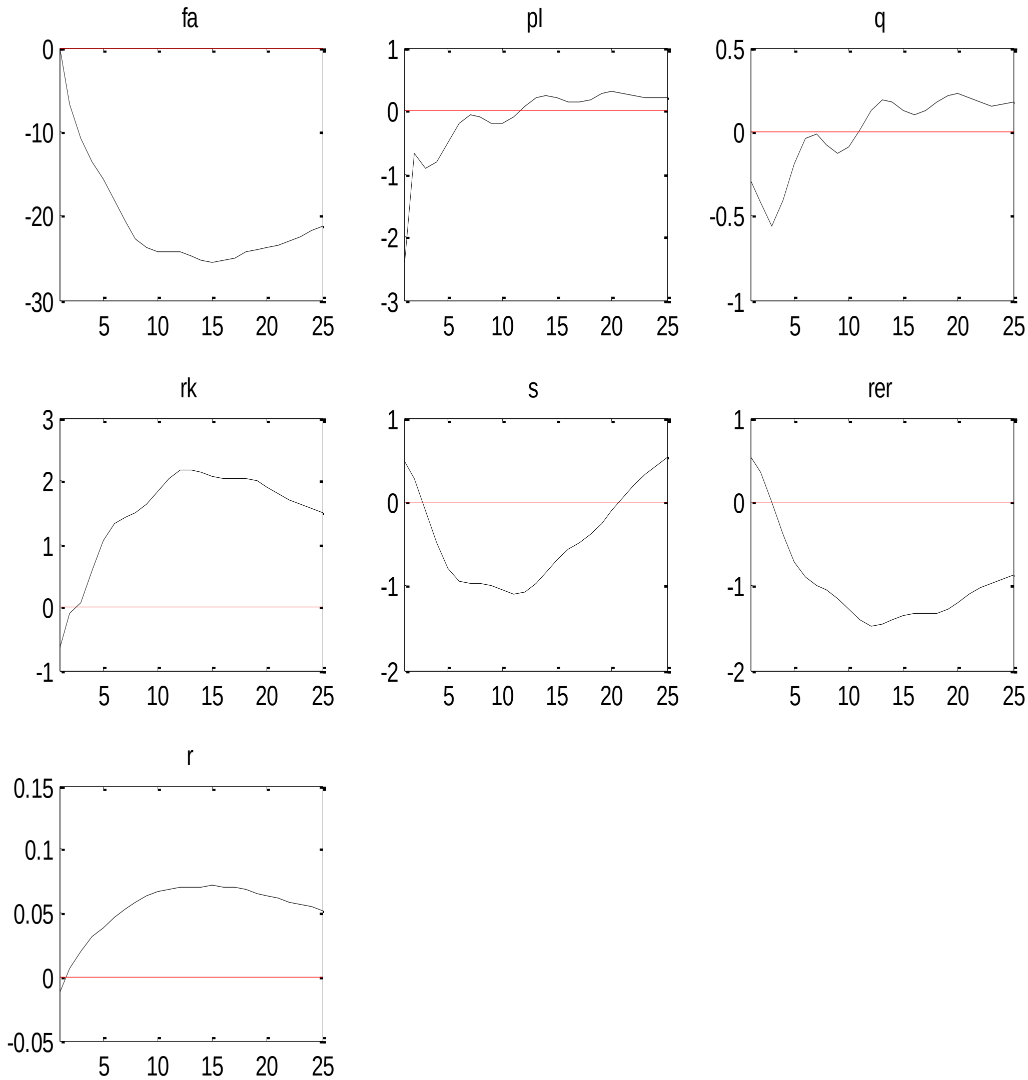
A.2. Impact of 1% drop in liquidity shock



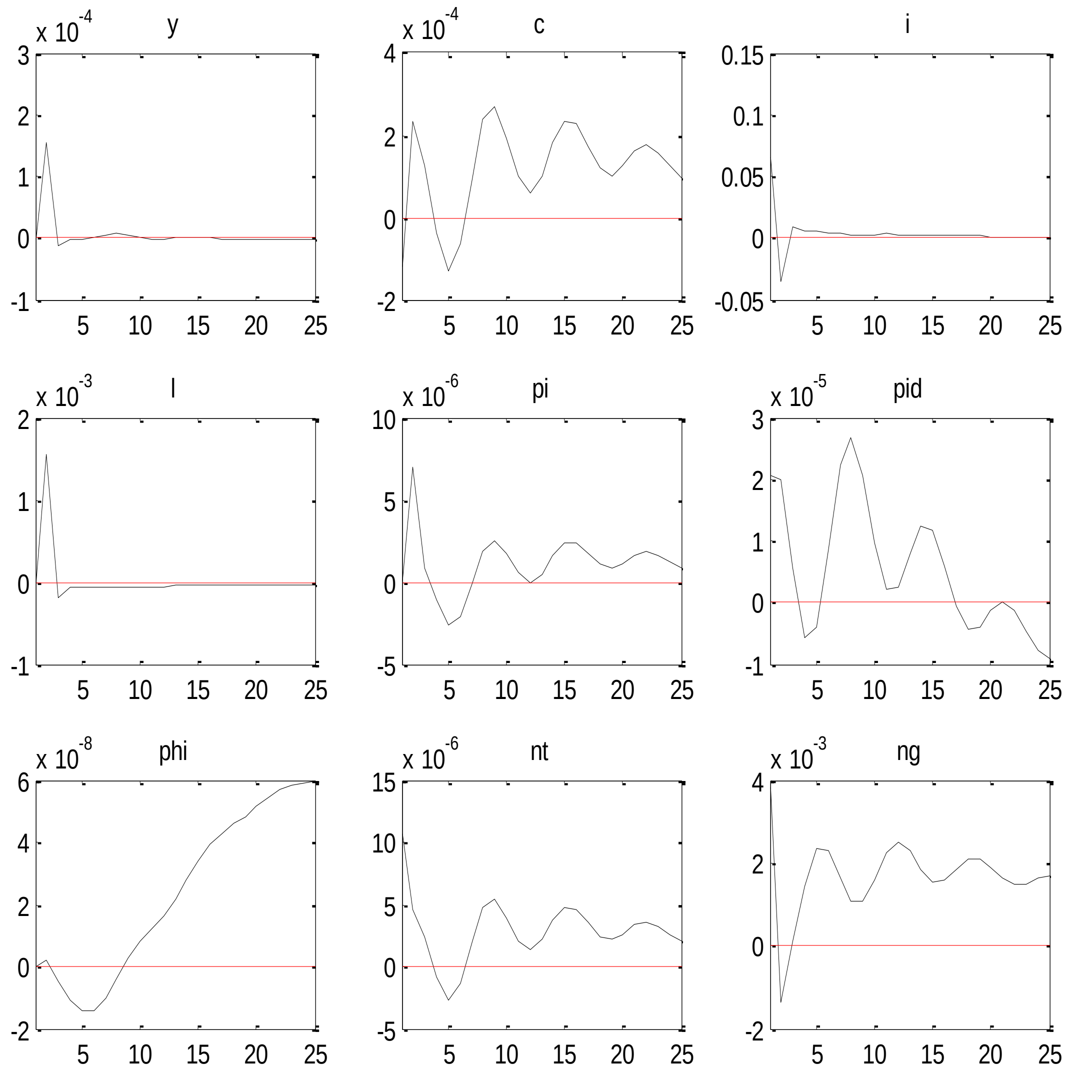
A.3. Impact of 1% increase in UIP shock



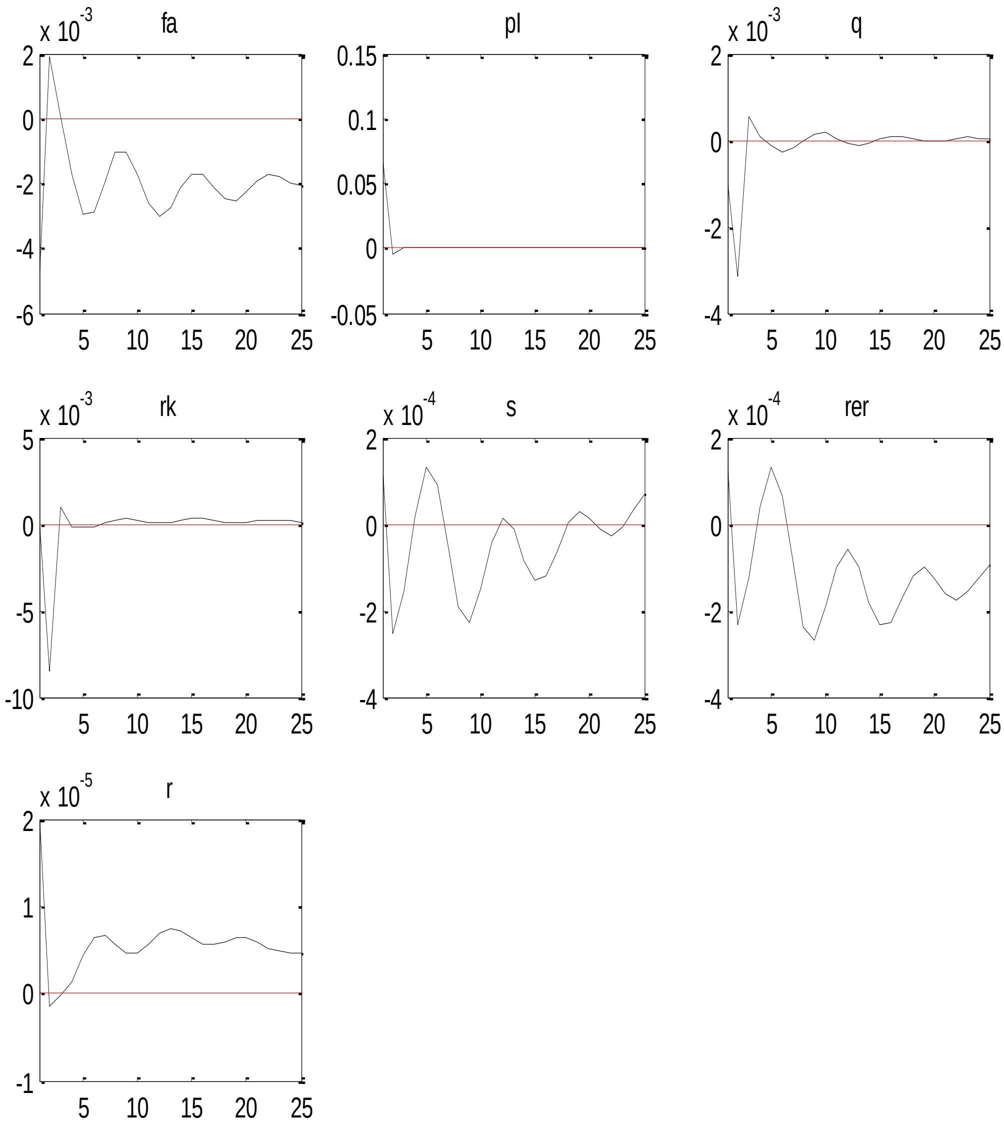
A.4. Impact of 1% increase in UIP shock



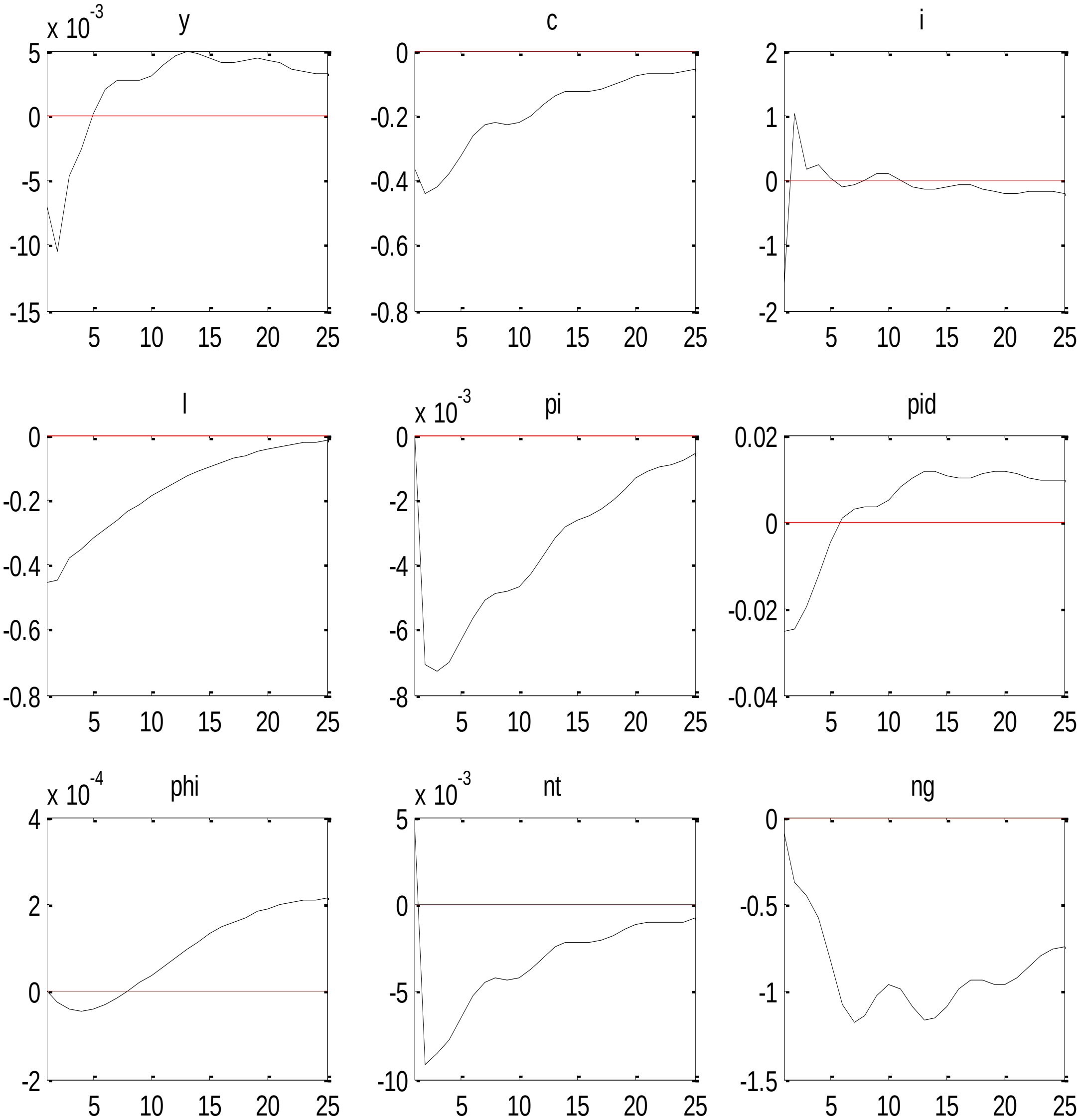
A.5. Impact of 1% increase in Government Spending shock



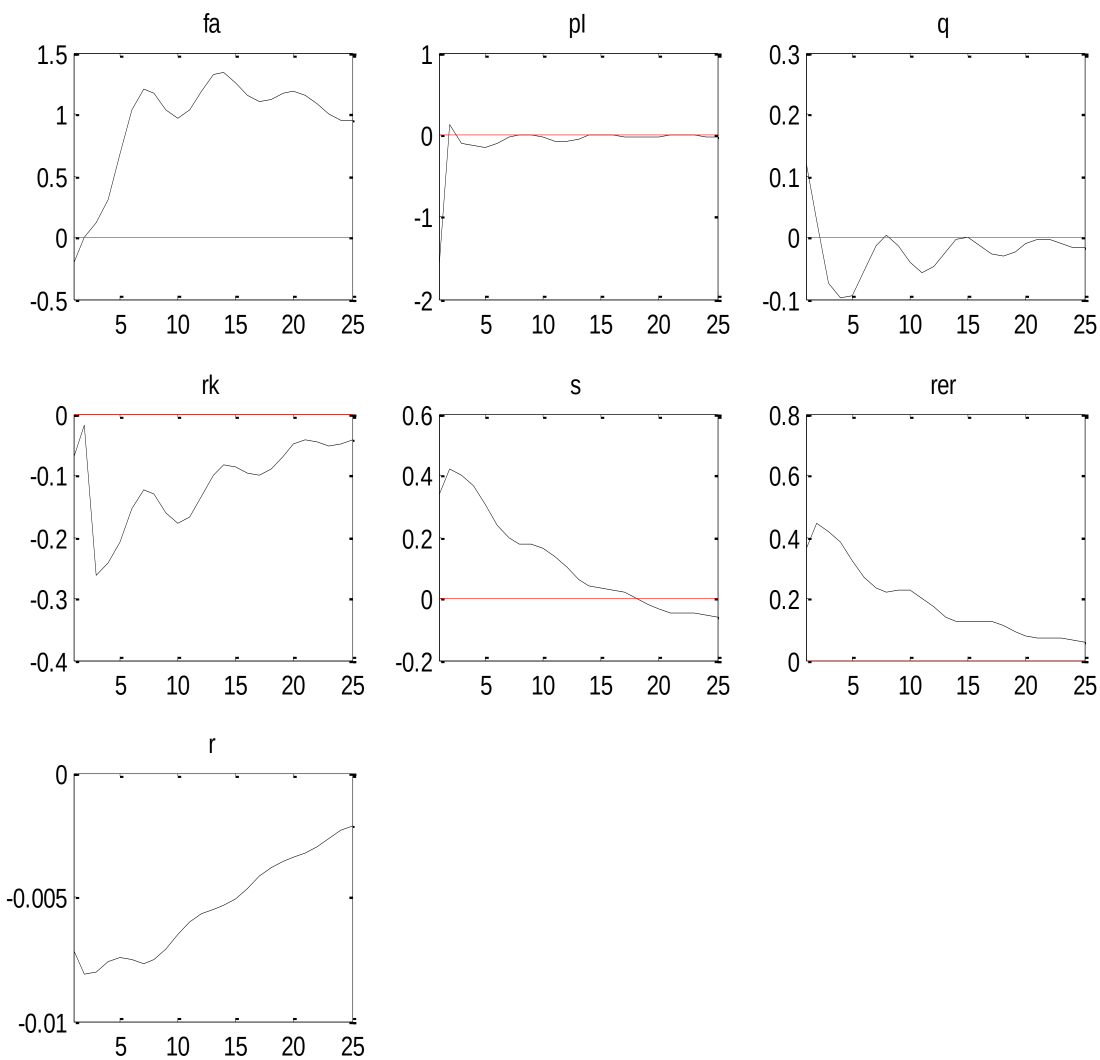
A.6. Impact of 1% increase in Government Spending shock



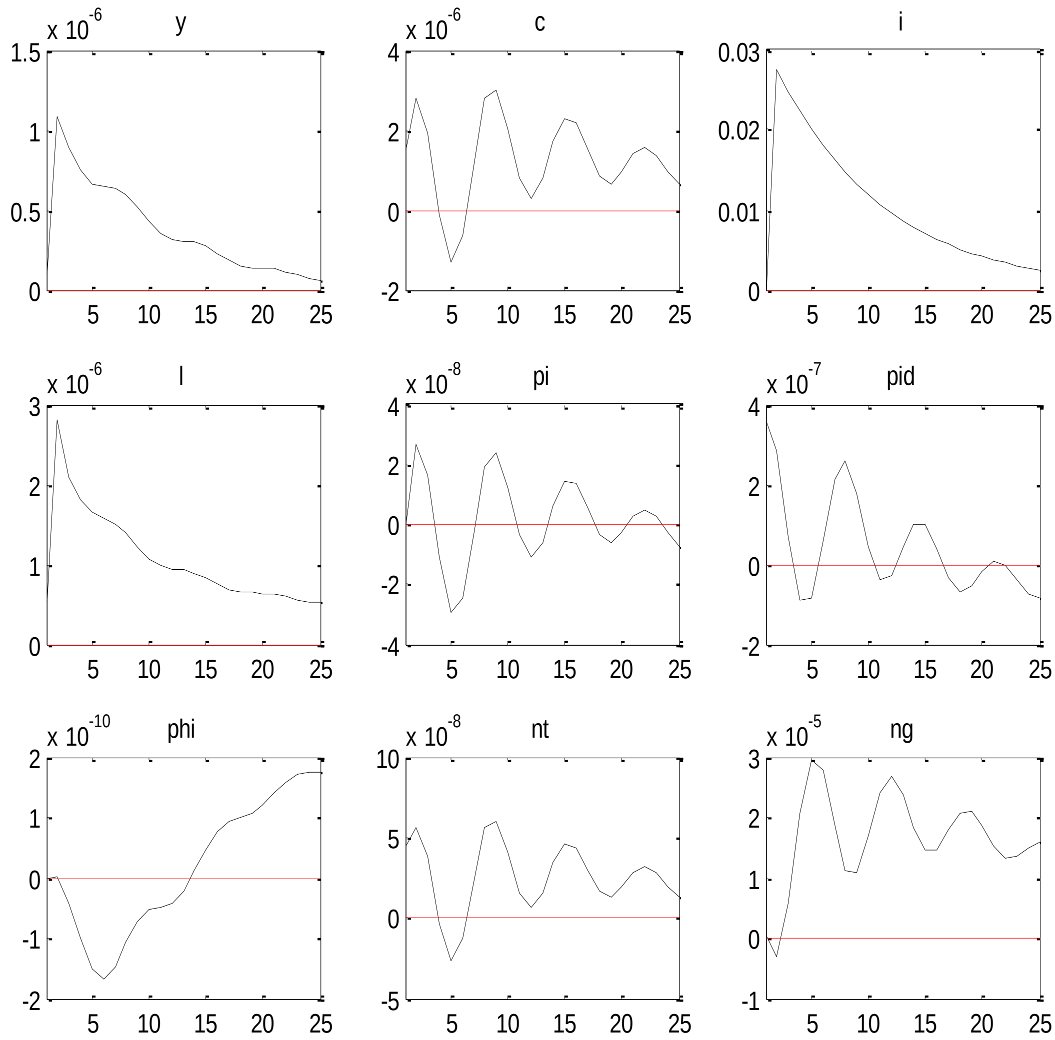
A.7. Impact of 1% increase in Productivity shock



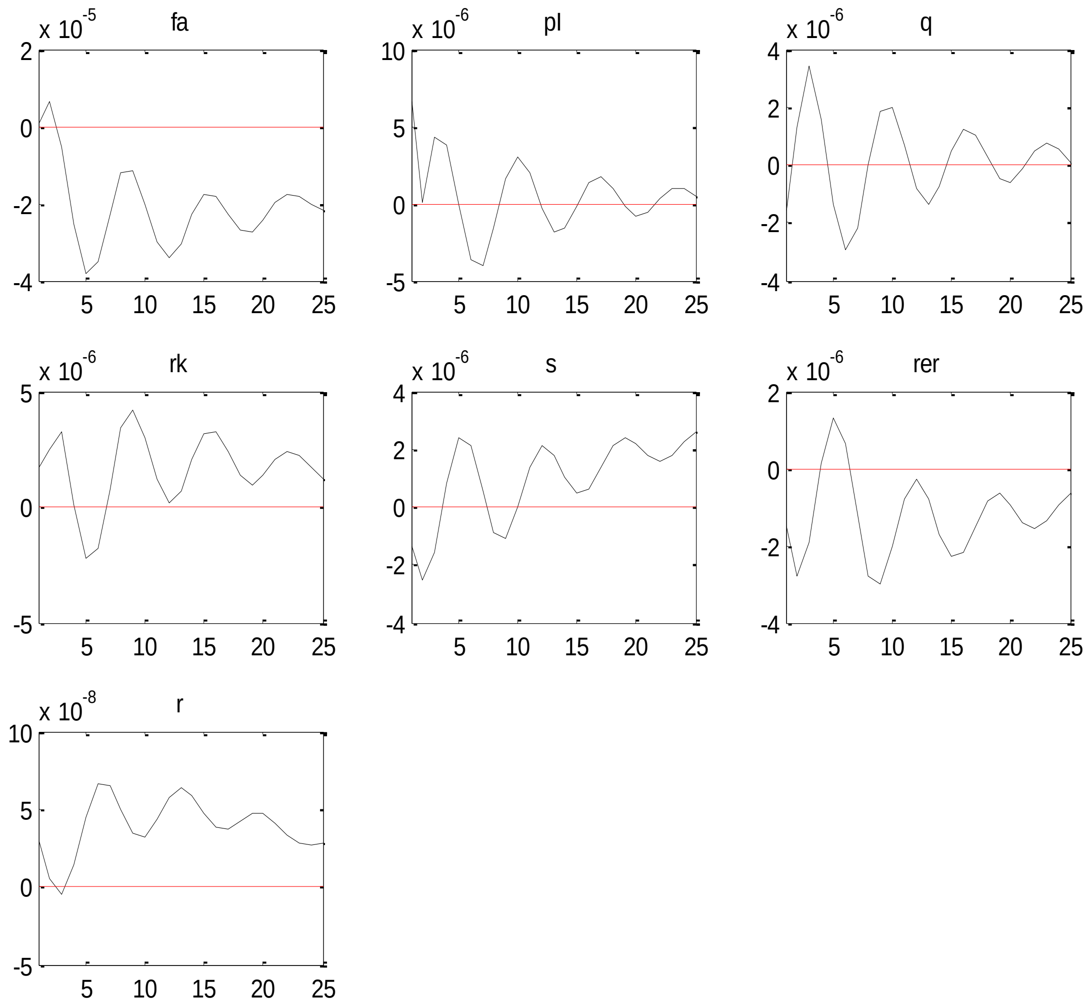
A.8. Impact of 1% increase in Productivity shock



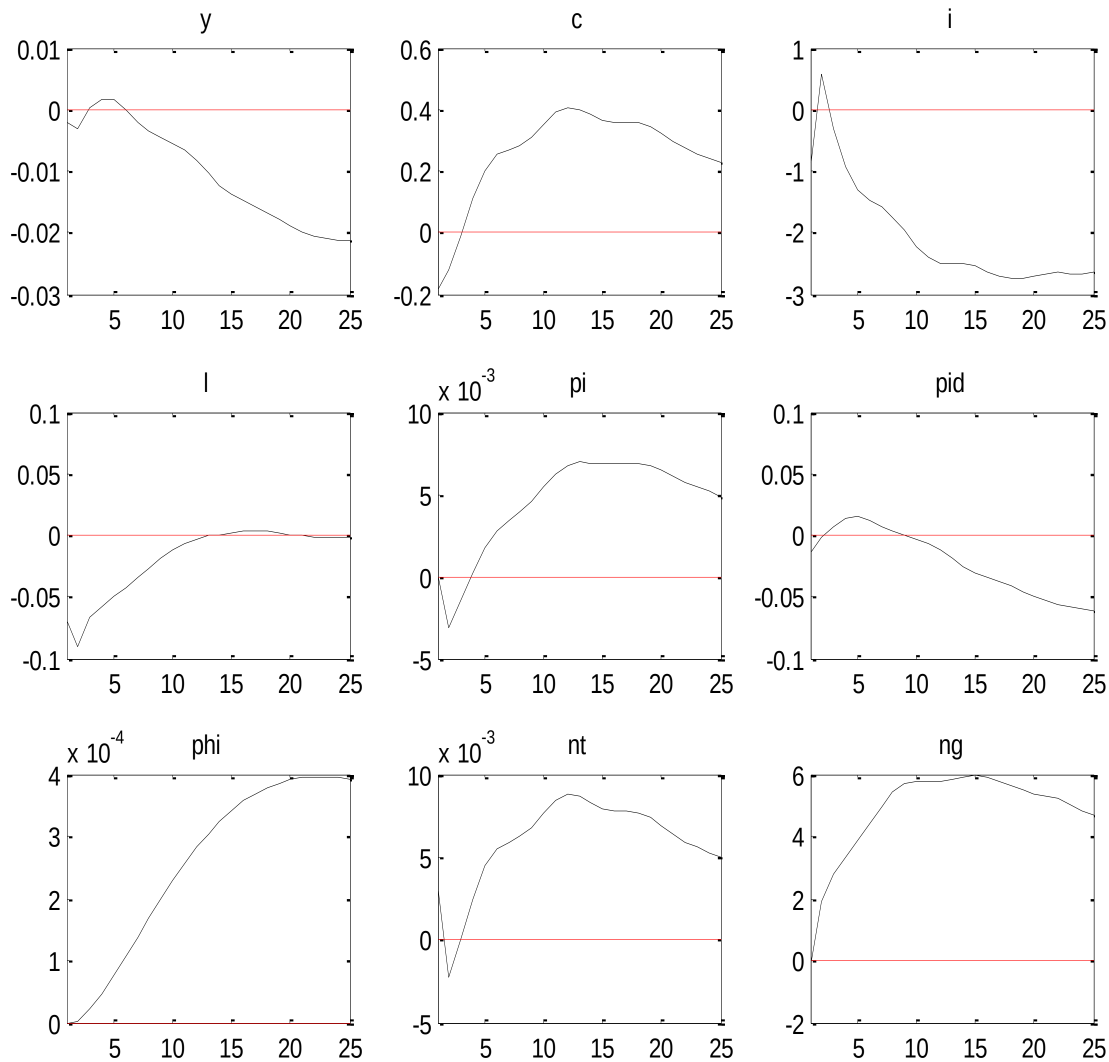
A.9. Impact of 1% increase in Foreign Output shock



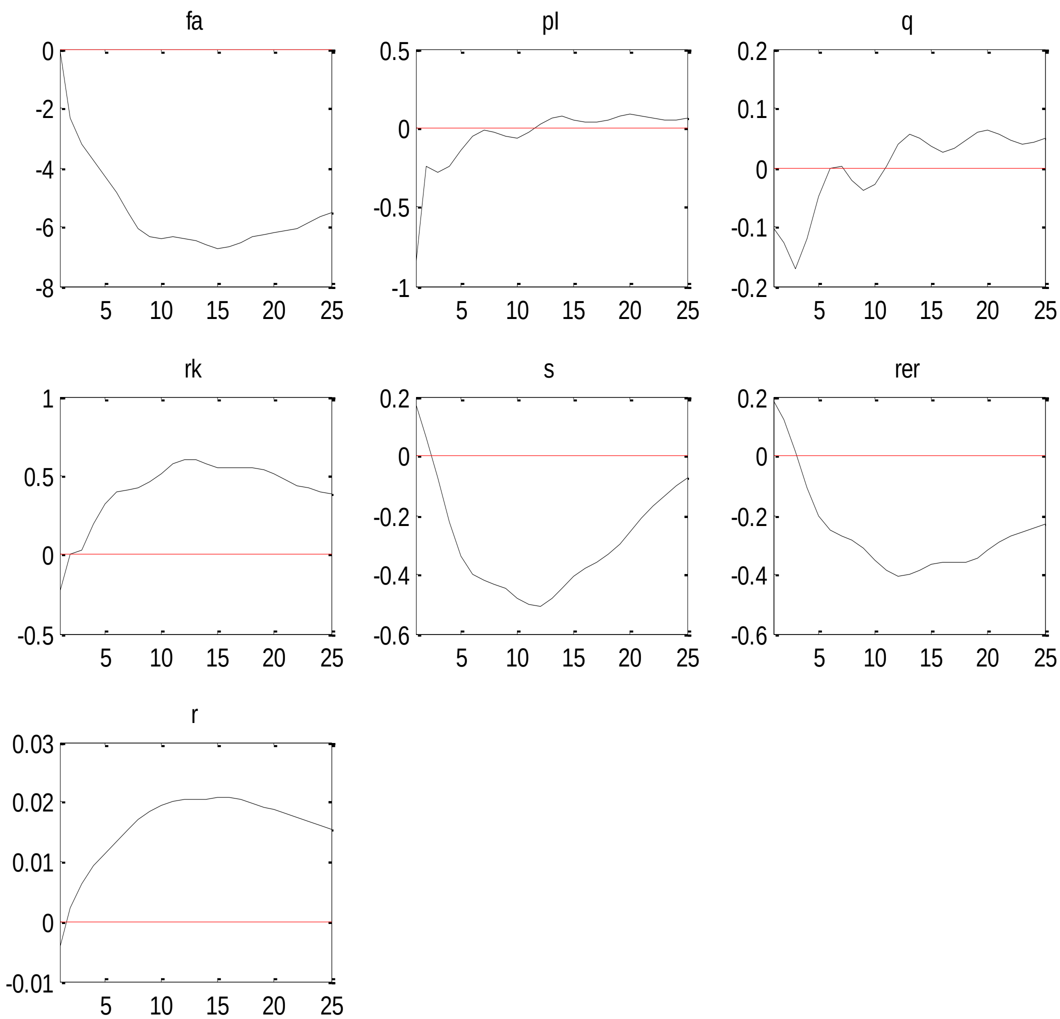
A.10. Impact of 1% increase in Foreign Output shock



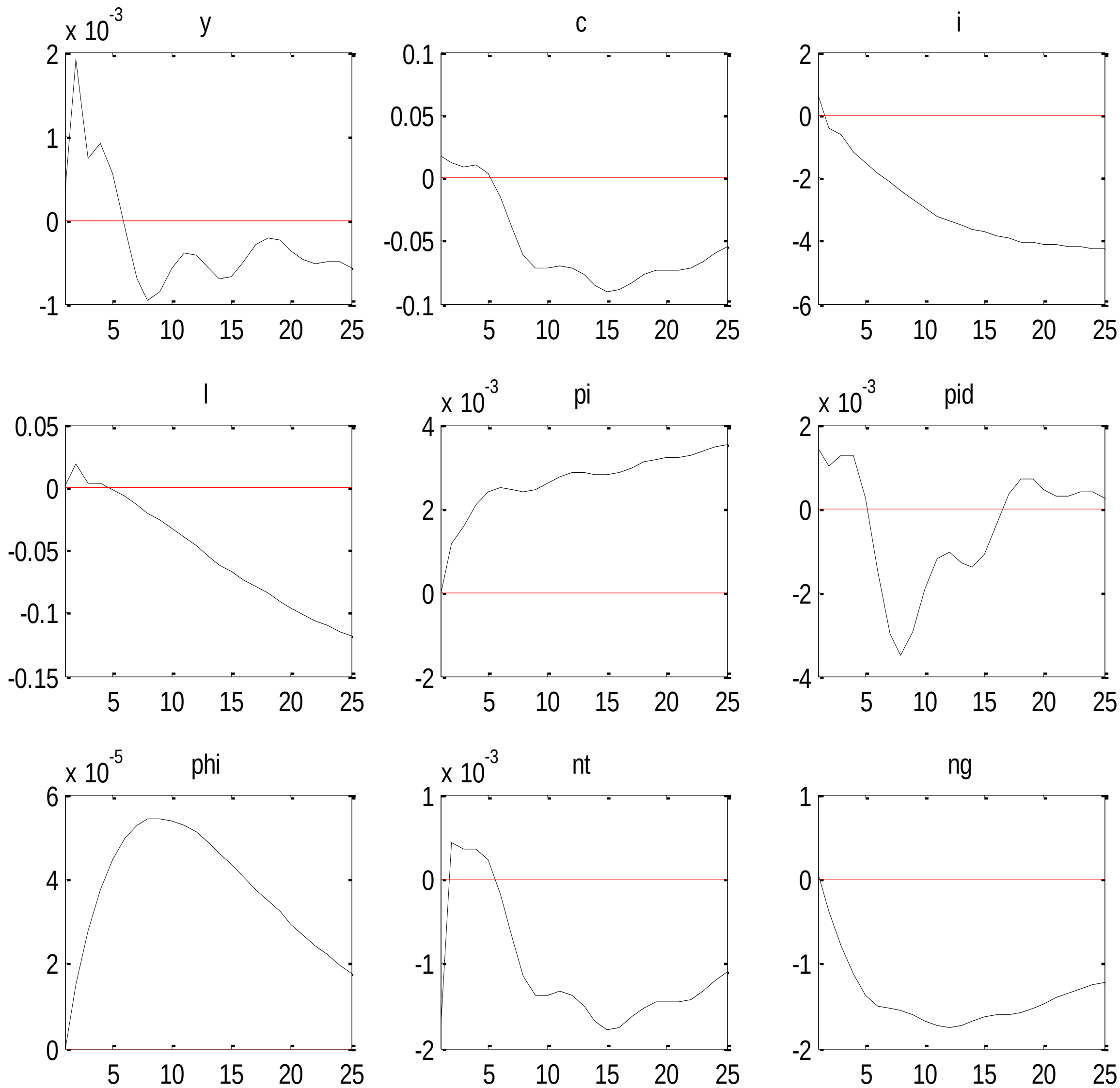
A.11. Impact of 1% increase in Foreign Nominal Interest shock



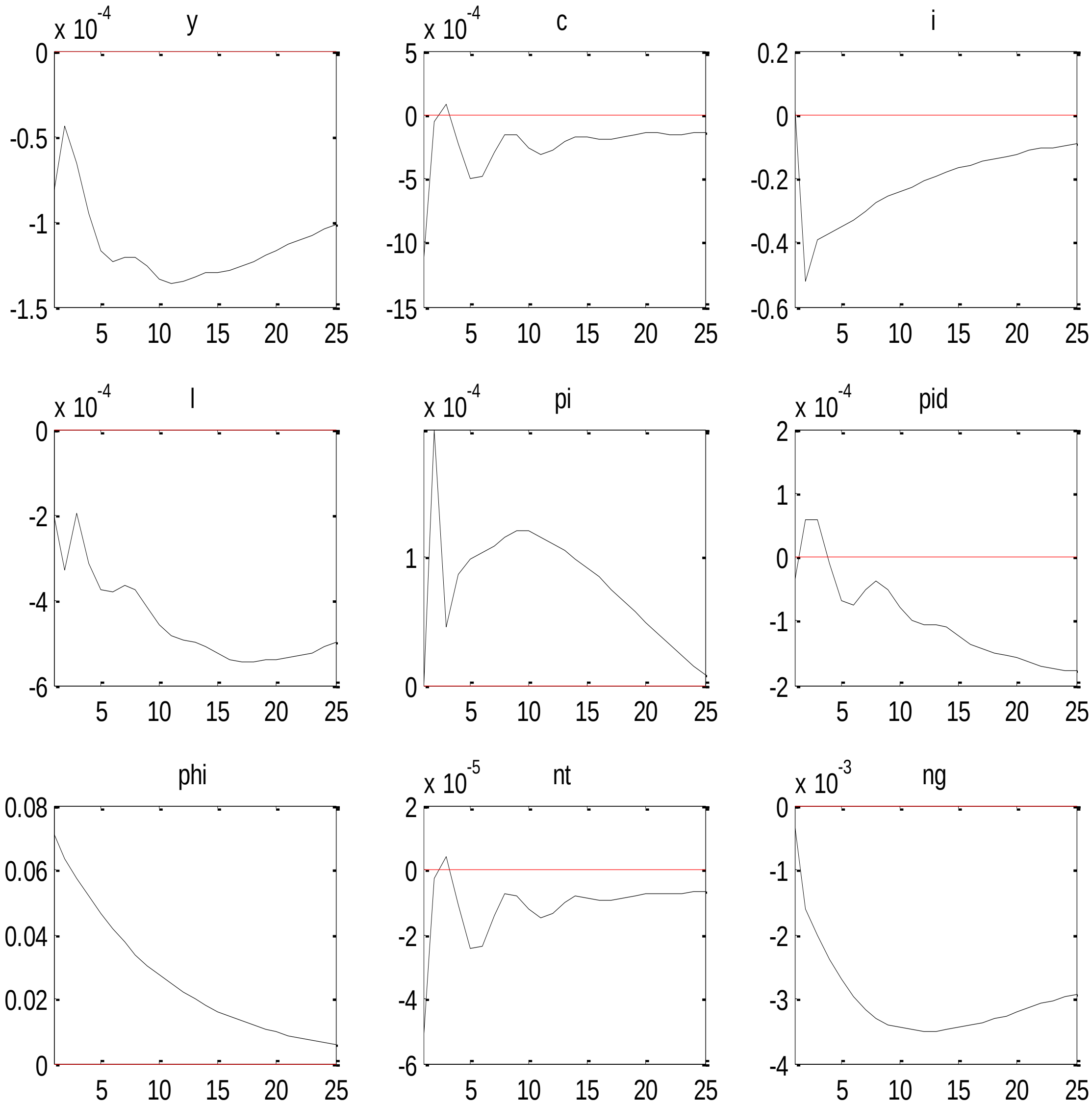
A.12. Impact of 1% increase in Foreign Nominal Interest shock



A.13. Impact of 1% increase in Foreign Inflation shock

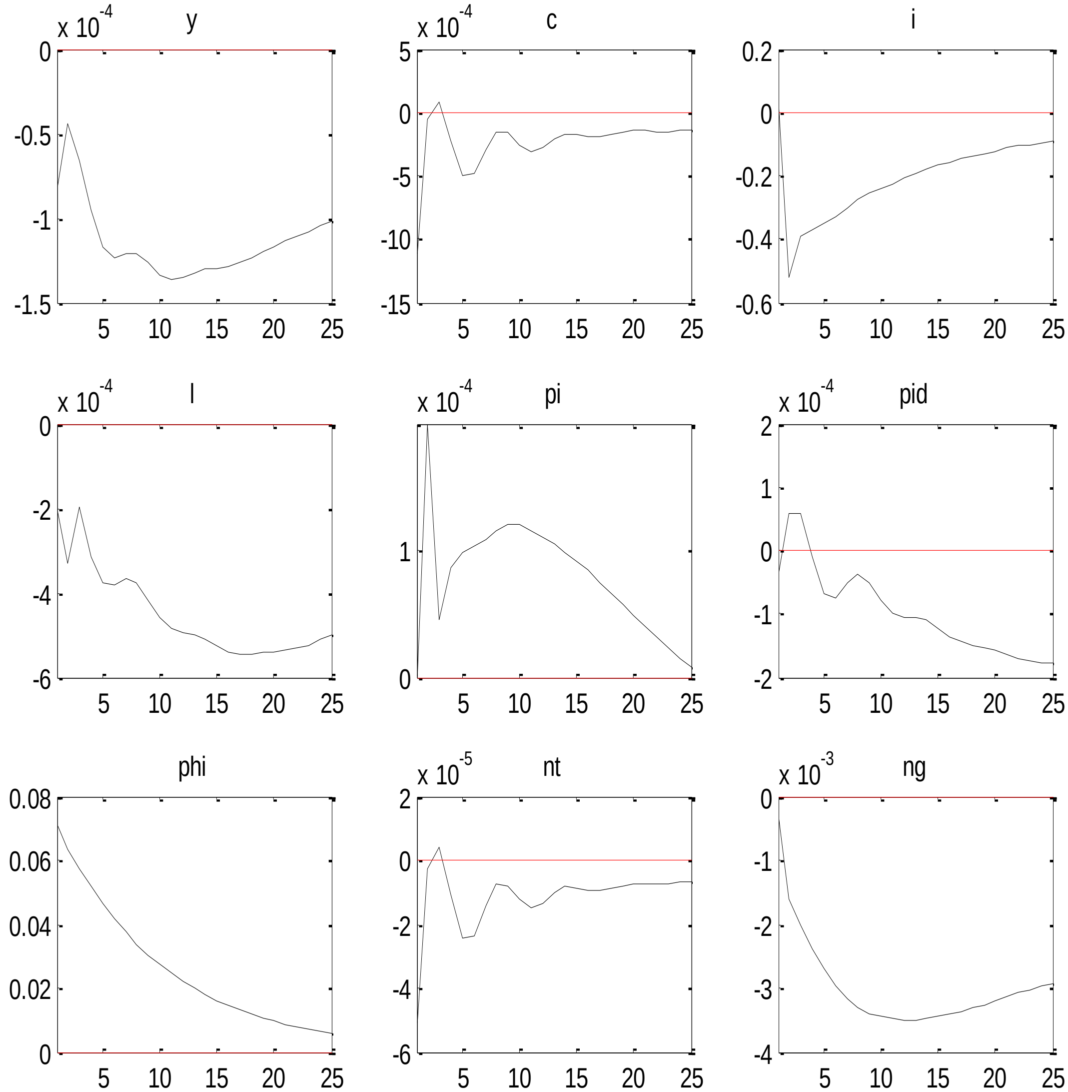


A.14. Impact of 1% increase in Foreign Inflation shock

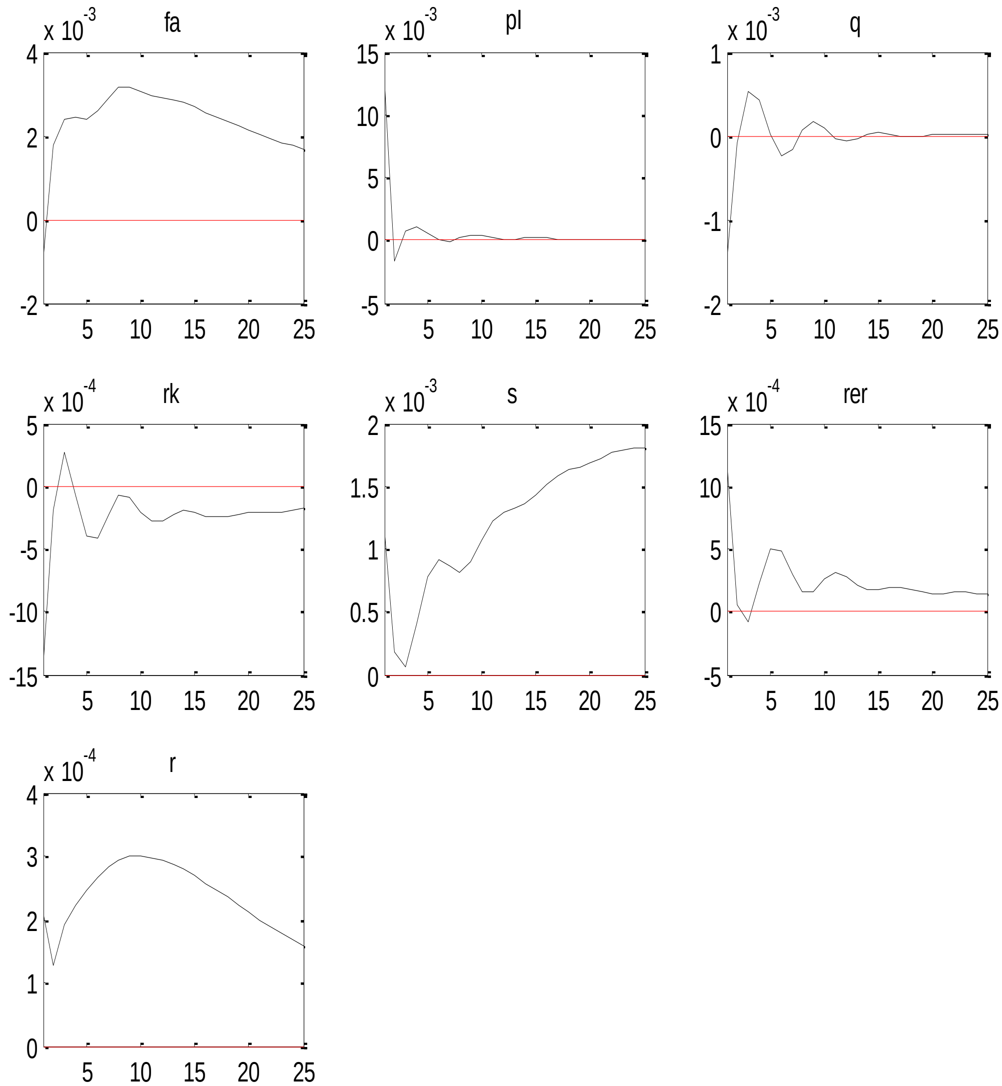


B. IRF without Quantitative easing

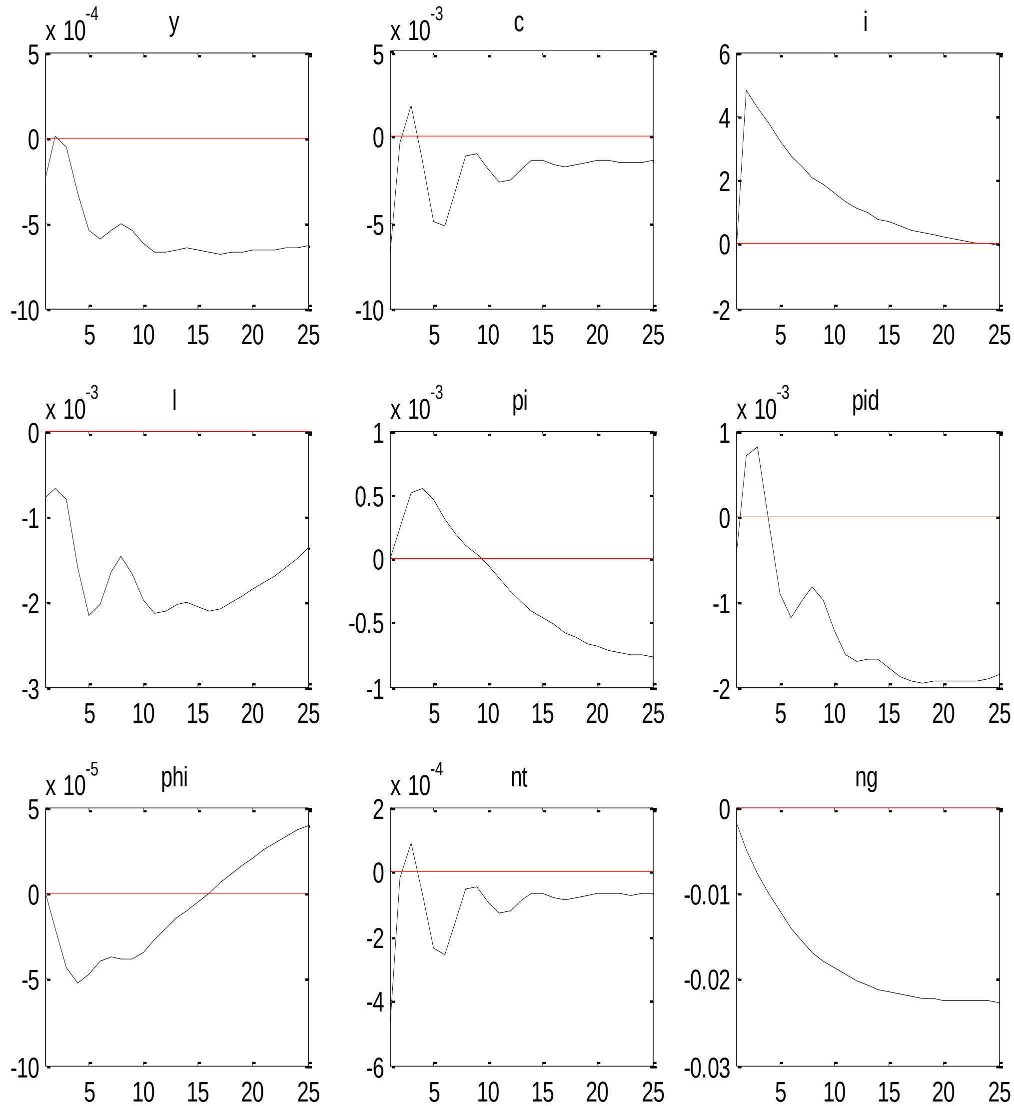
B.1. Impact of 1% drop in liquidity shock



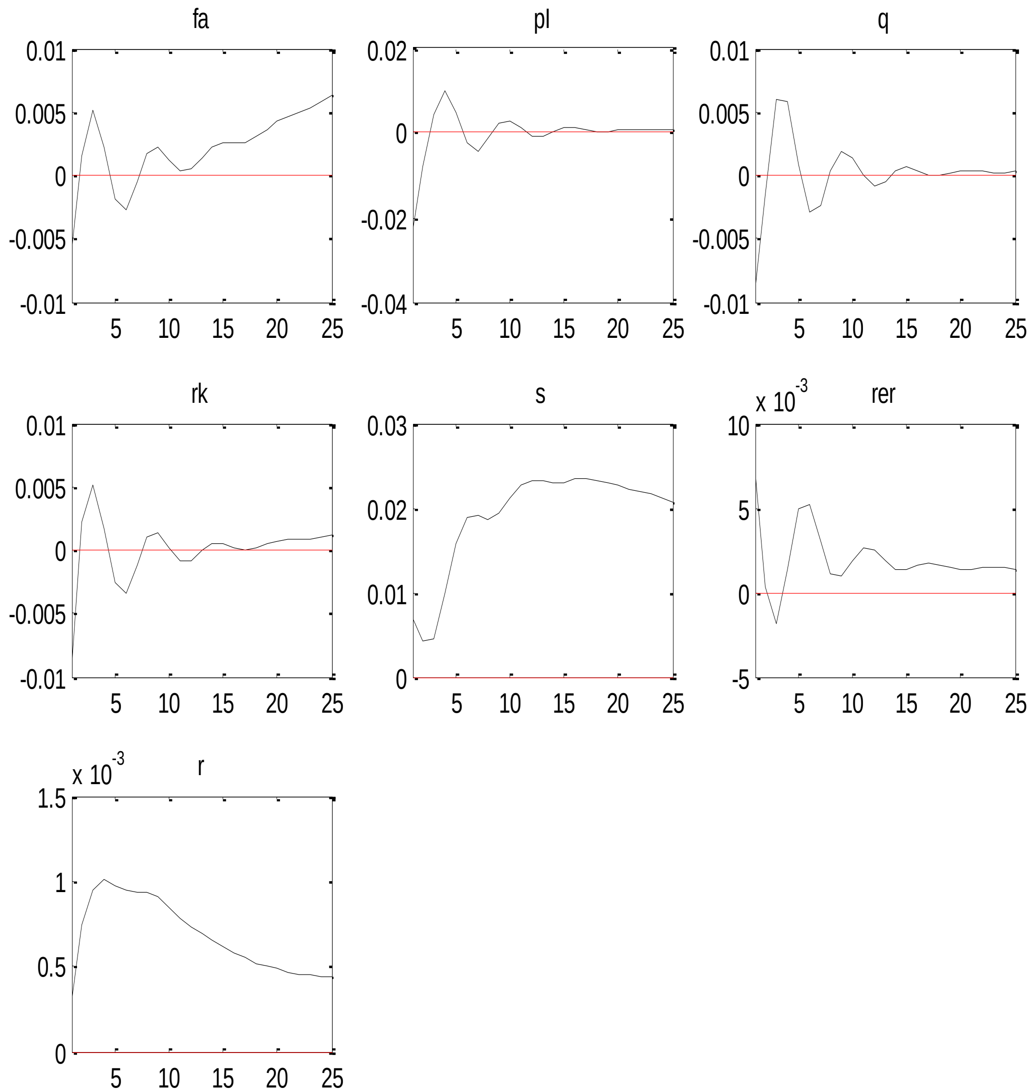
B.2. Impact of 1% drop in liquidity shock



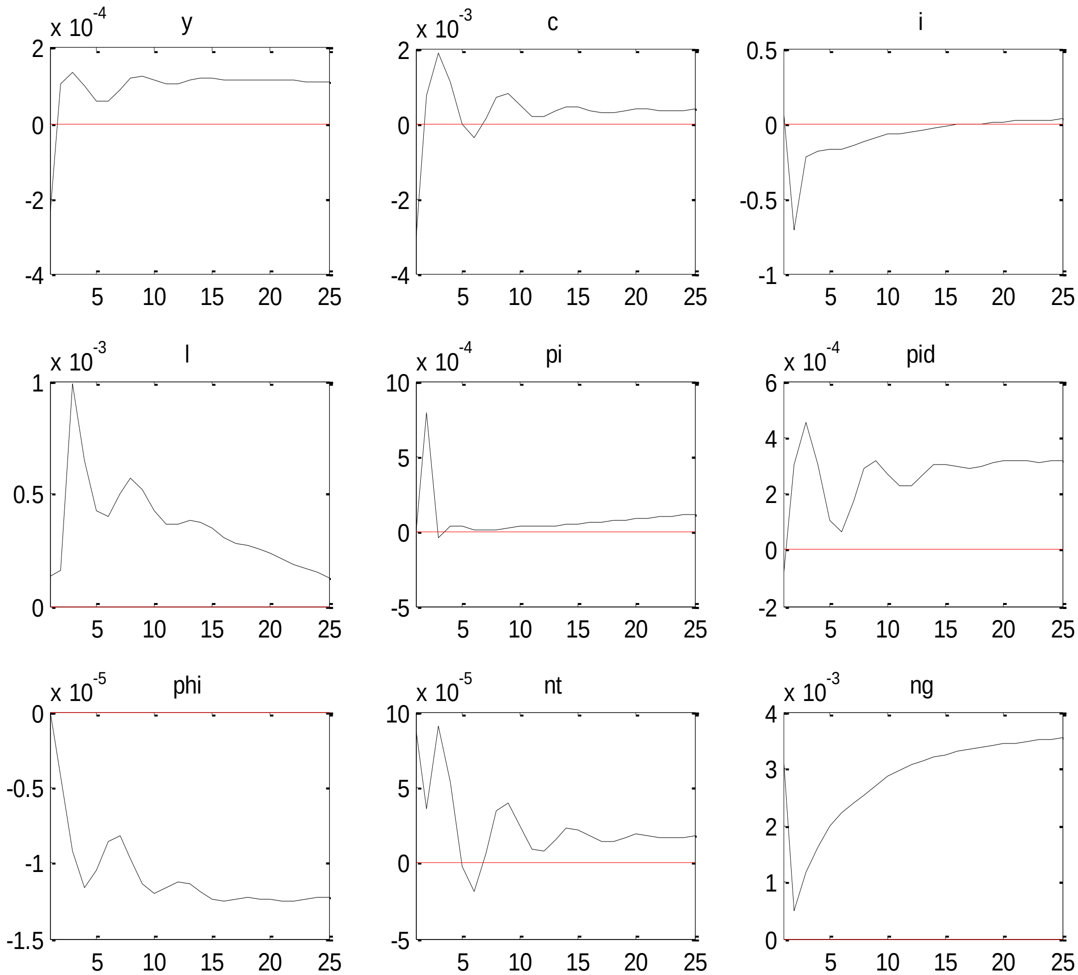
B.3. Impact of 1% increase in UIP shock



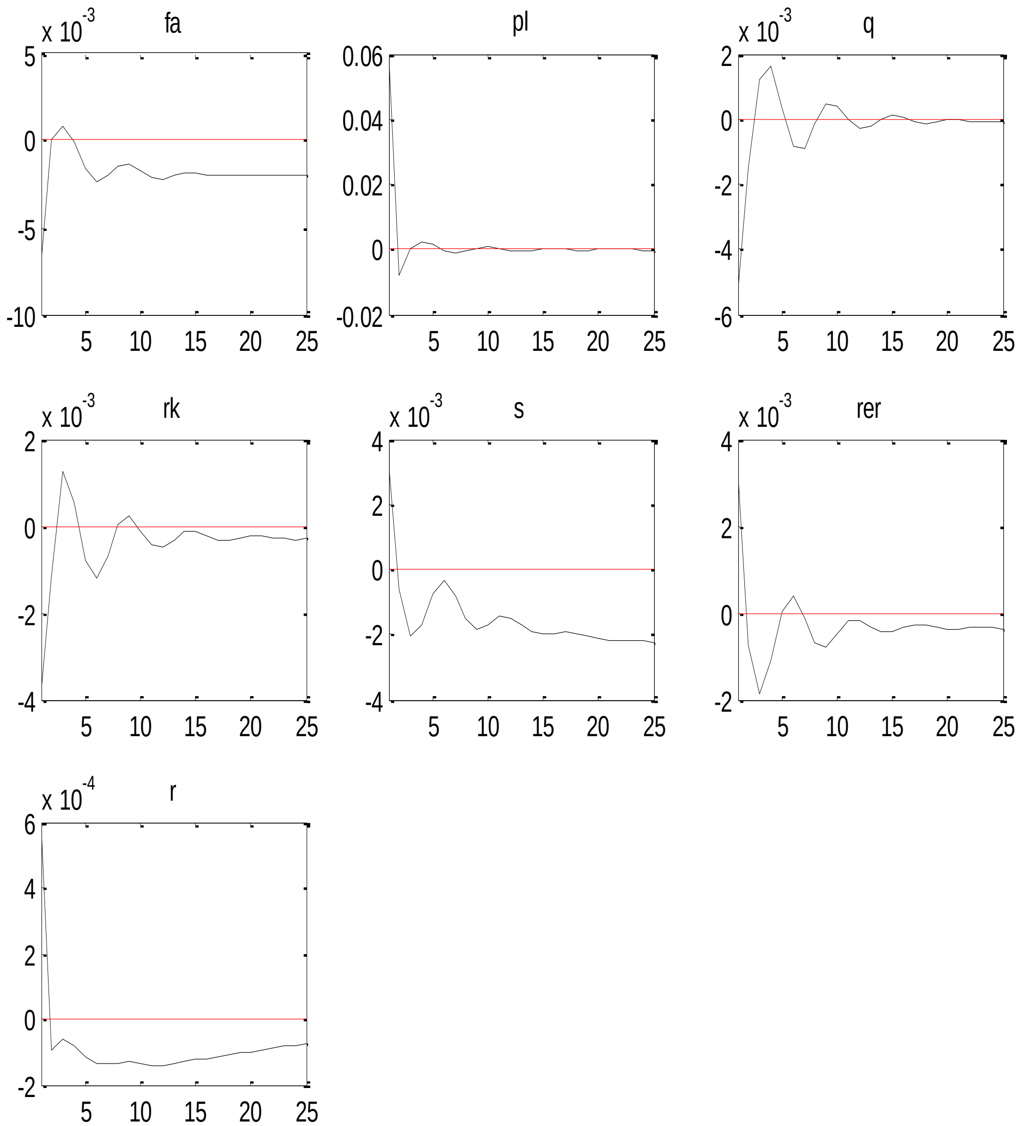
B.4. Impact of 1% increase in UIP shock



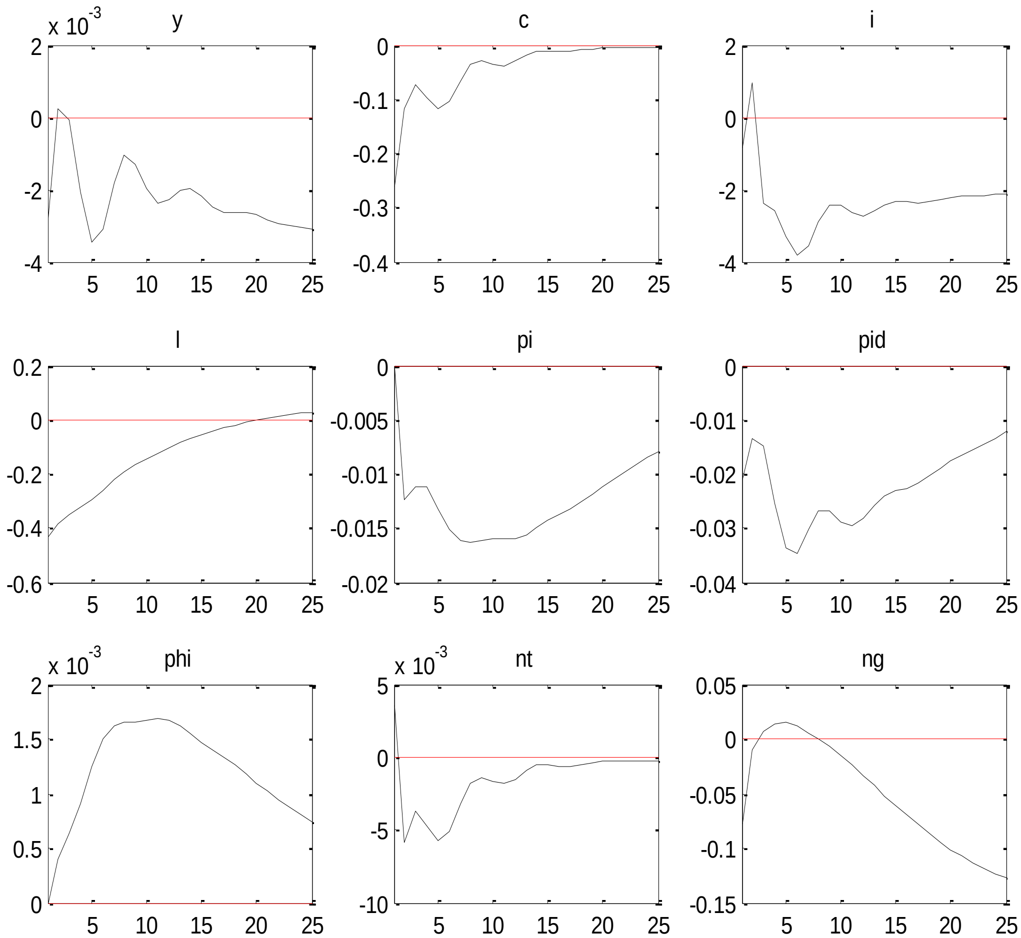
B.5. Impact of 1% increase in Government Spending shock



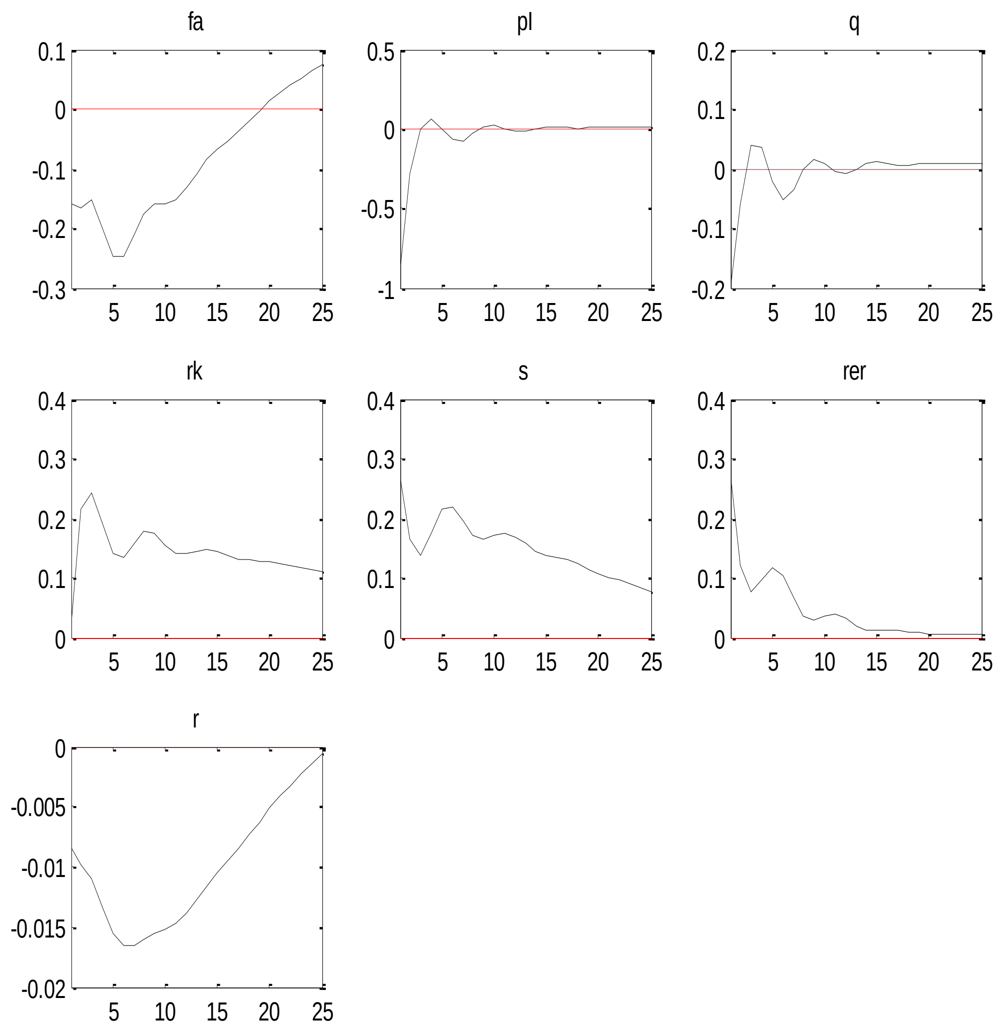
B.6. Impact of 1% increase in Government Spending shock



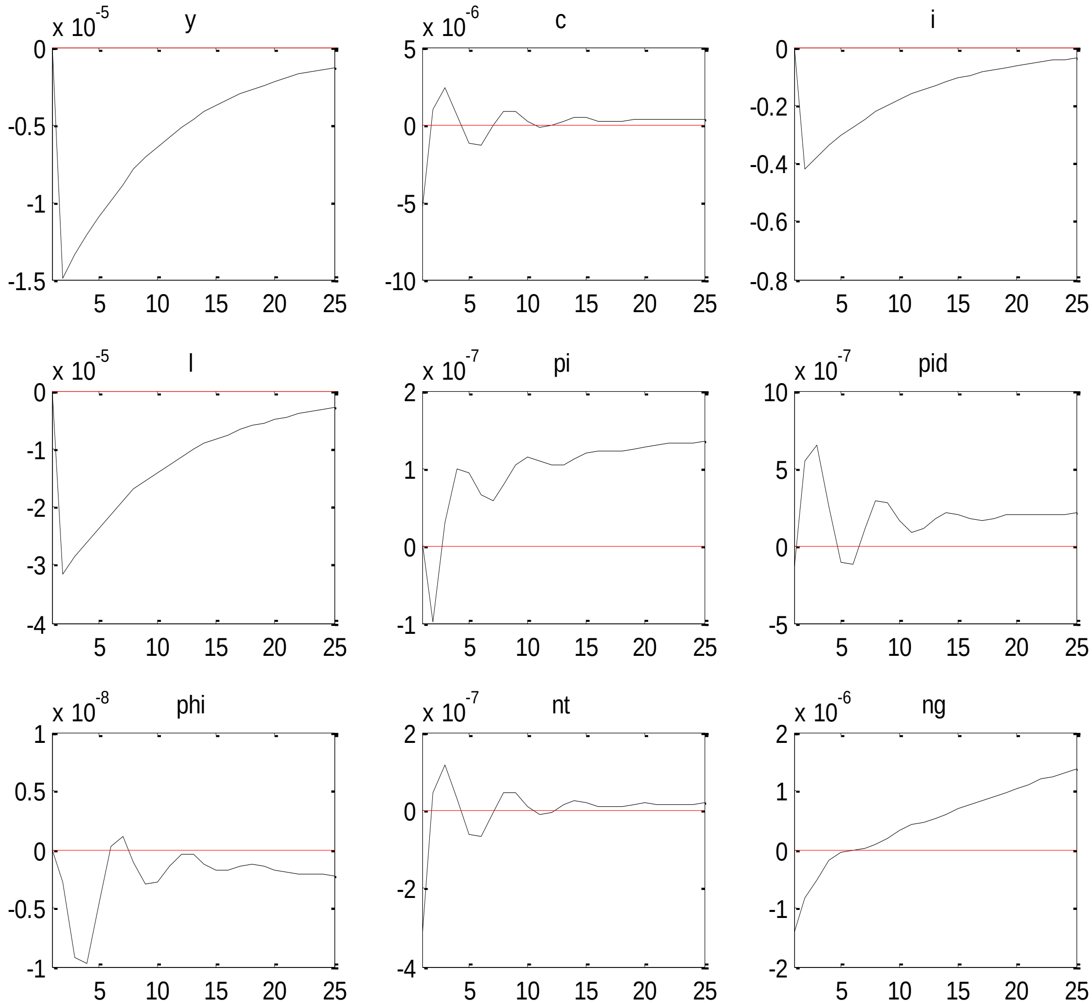
B.7. Impact of 1% increase in Productivity shock



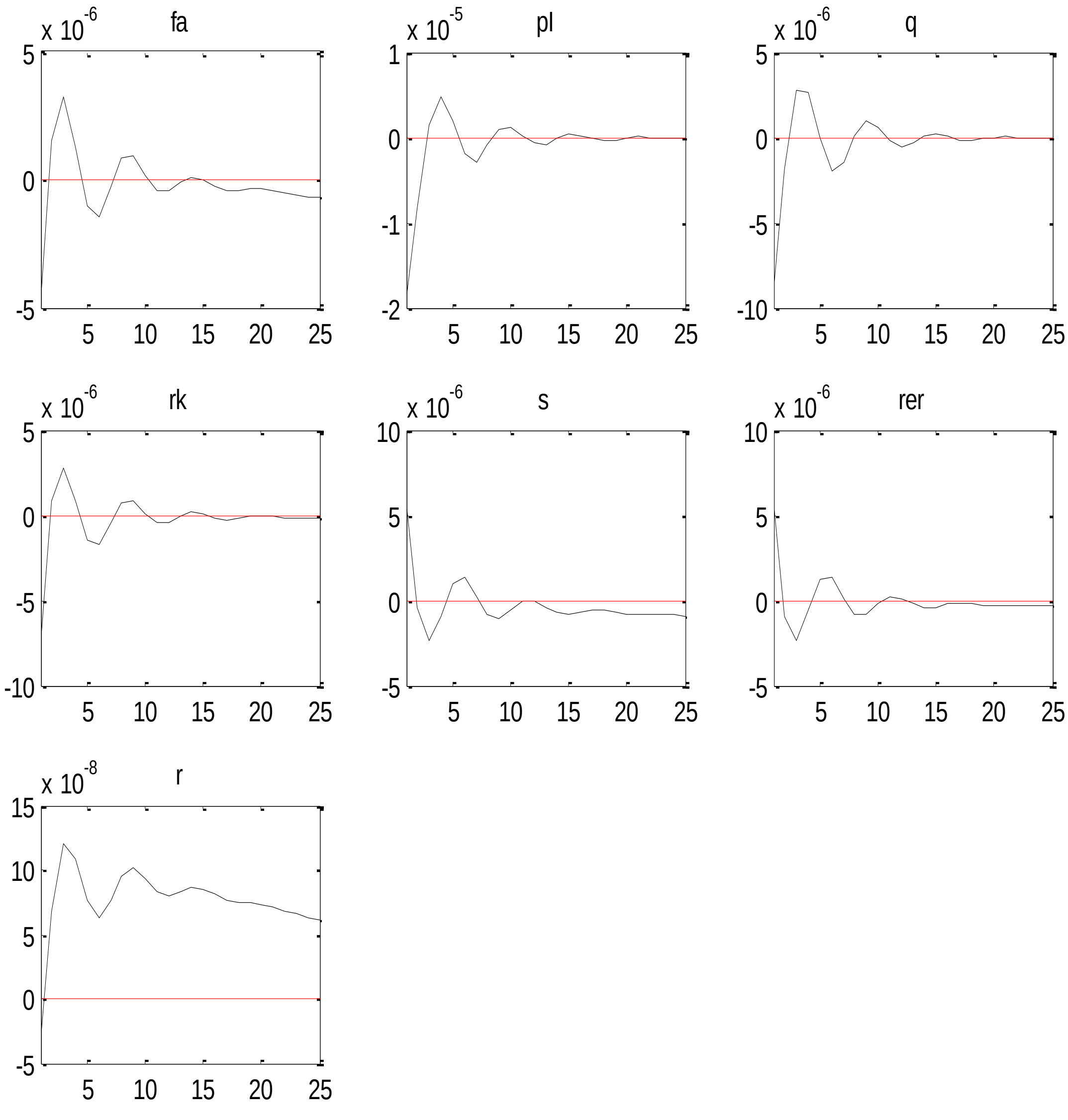
B.8. Impact of 1% increase in Productivity shock



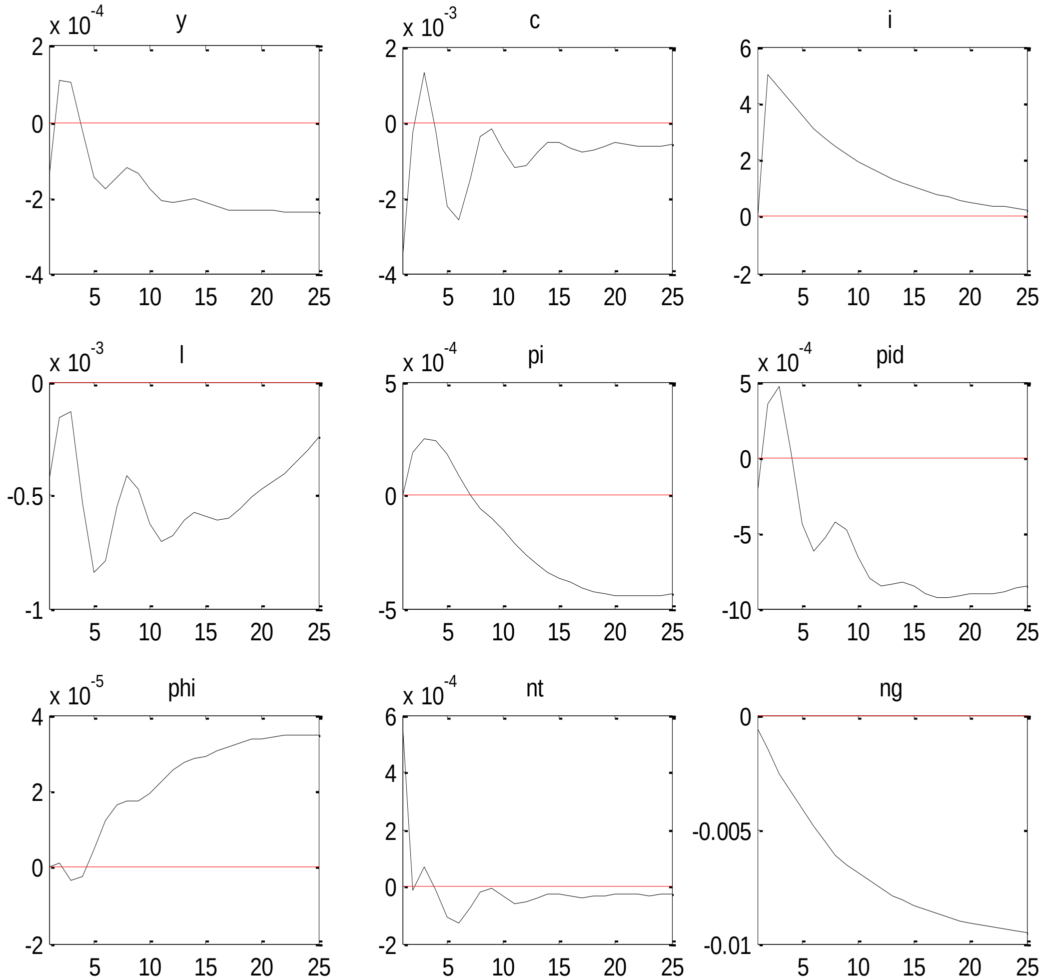
B.8. Impact of 1% increase in World Output shock



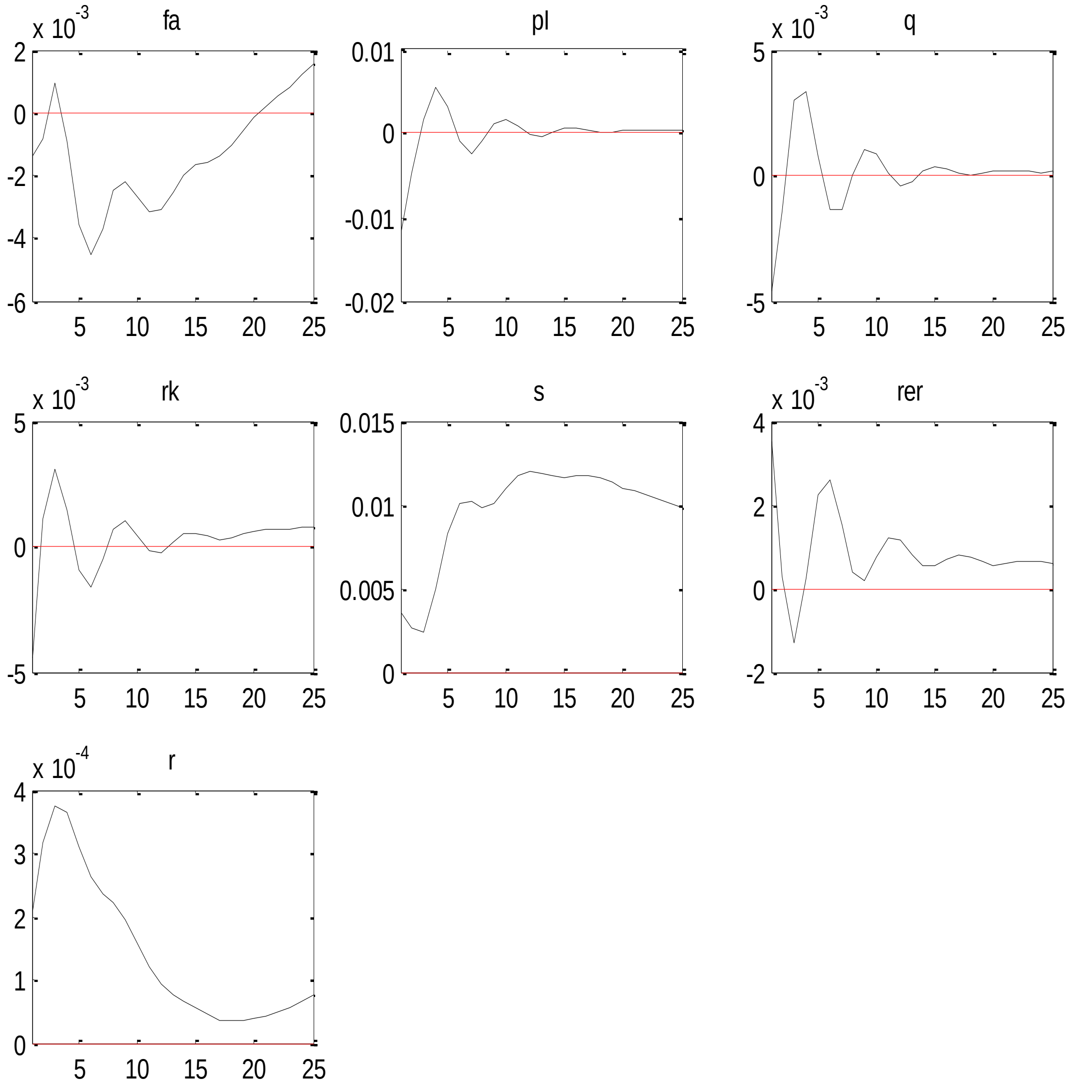
B.10. Impact of 1% increase in World Output shock



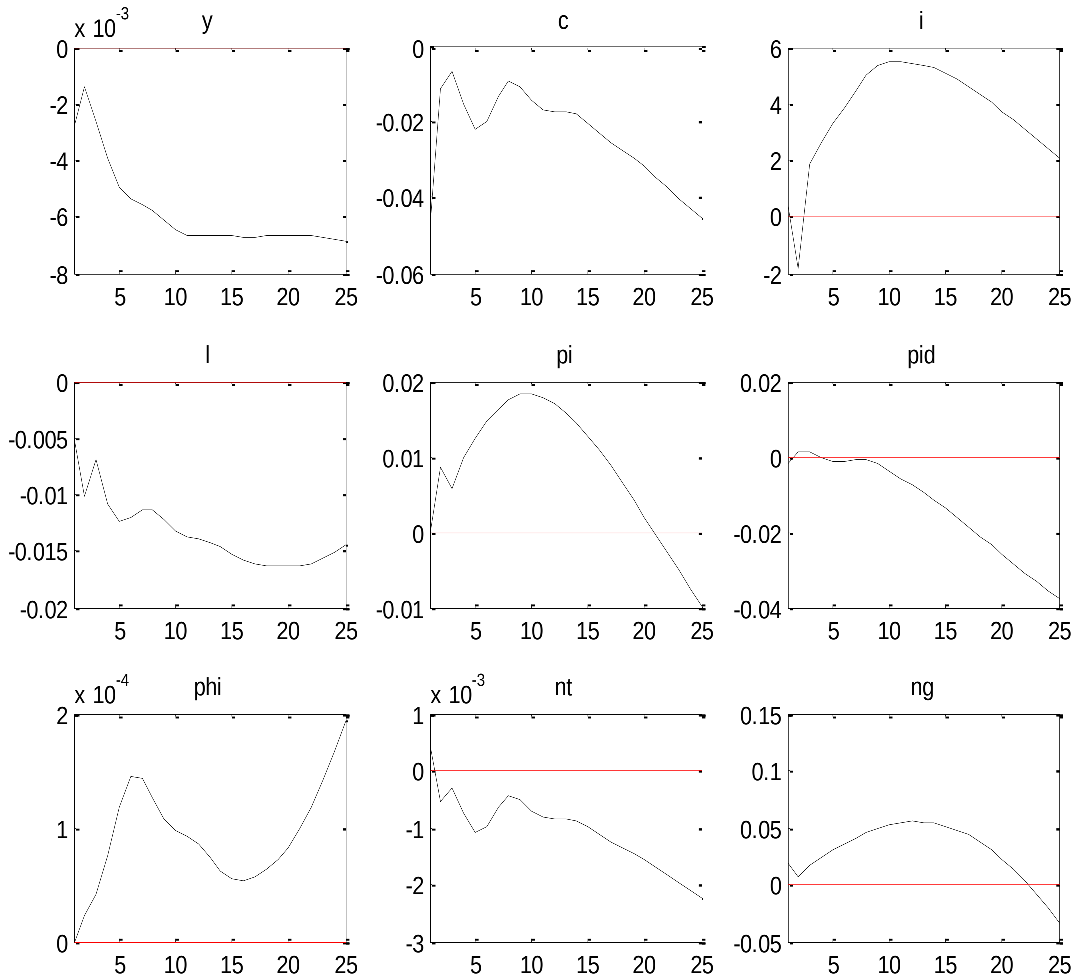
B.11. Impact of 1% increase in World Nominal Interest Rate shock



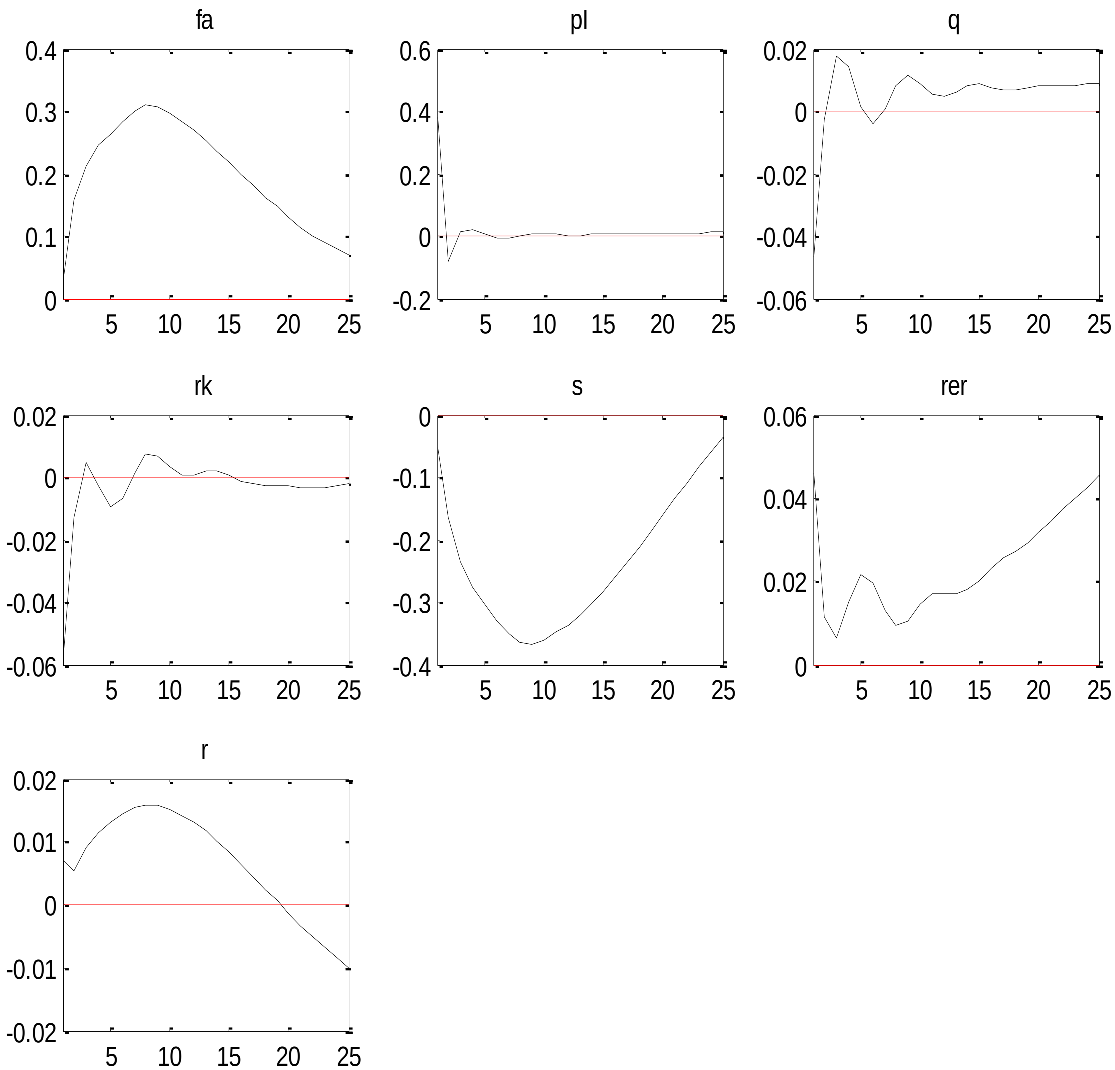
B.12. Impact of 1% increase in World Nominal Interest Rate shock



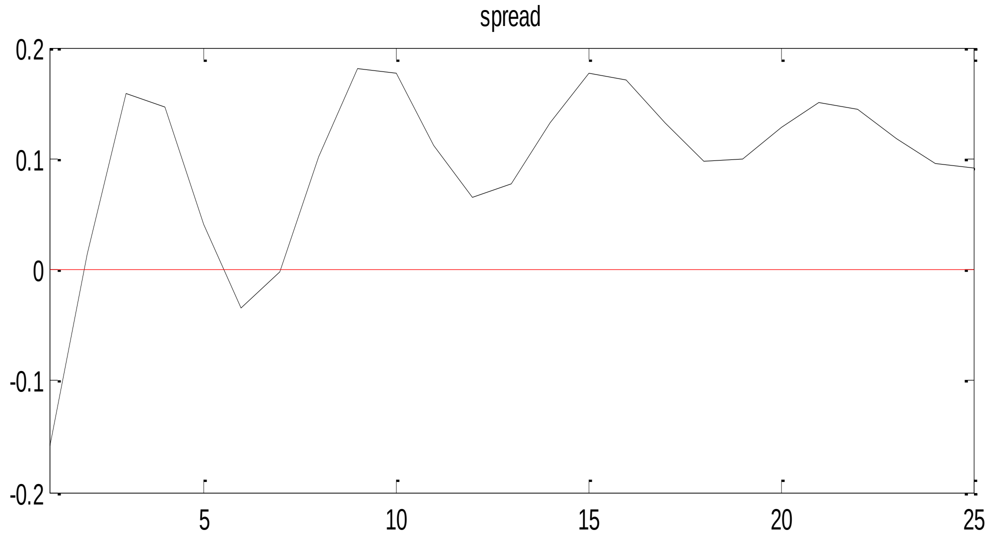
B.13. Impact of 1% increase in World Inflation shock

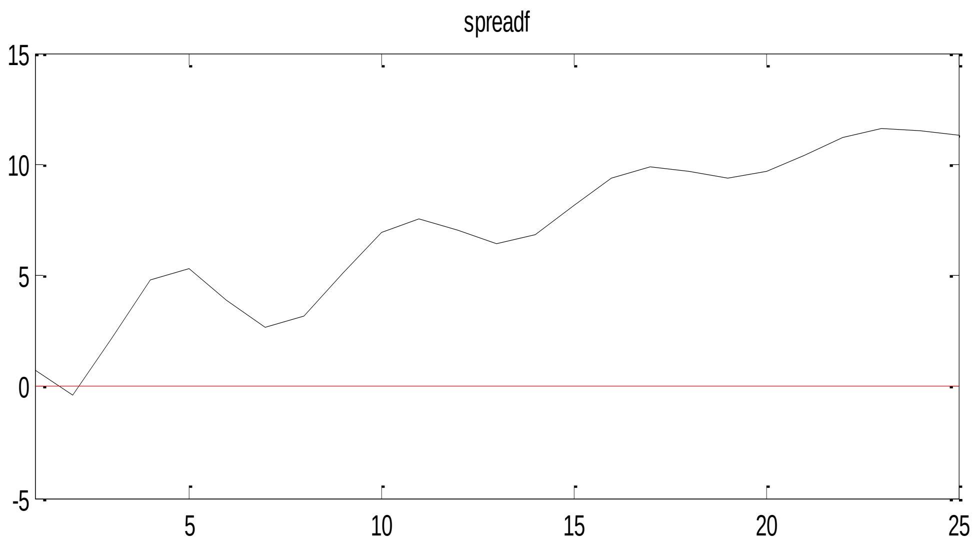


B.14. Impact of 1% increase in World Inflation shock



C.1 Impact on Domestic and Foreign Spreads with Quantitative Easing





C.2 Impact on Domestic and Foreign Spreads without Quantitative Easing