Essays on Economic Signals with Credit-Card-Augmented Divisia Monetary Aggregates

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Abstract

When the economy encounters unforeseen economic events such as the pandemic or the Great Recession, volatile macroeconomic variables are the issue. Especially during the pandemic, the economy has faced high inflation pressure. Therefore, the prediction ability is getting more crucial at this time and I predict macroeconomic variables in many different ways. Moreover, the share of payment instrument usage of the credit card is getting bigger. There is a new idea, credit-card-augmented Divisia monetary aggregates which include the amount of credit card transactions volume in money measure. The purpose of my dissertation is to examine the performance of the prediction ability of the credit-card-augmented Divisia for various macroeconomic variables. I find the better research method to make more accurate predictions of macroeconomic variables such as inflation and GDP. My dissertation consists of four chapters:

The study of the new monetary aggregates, Divisia monetary aggregates, is continually being developed. While simple sum measures have the same weight for each monetary asset, Divisia monetary aggregates impute different component expenditure shares with user cost pricing and reflect the real world. First chapter in this dissertation **"Development of Divisia Monetary Aggregates and Its Applications"** is the literature survey of the Divisia monetary aggregates that are prefered to the simple sum in much economic research since monetary components are not perfect substitutes. We introduce and organize various literature about the traditional Divisia, credit-card-augmented Divisia, and credit-card augmented Divisia inside money.

Second chapter **"Forecasting Inflation and Output Growth with Credit-Card-Augmented Divisia Monetary Aggregates"** investigates the performance of the Credit-Card-Augmented Divisia monetary aggregates in forecasting U.S. inflation and output growth at the 12-month horizon. We compute recursive and rolling out-of-sample forecasts using an Autoregressive Distributed Lag (ADL) model based on Divisia monetary aggregates. We use the three available versions of those

monetary aggregate indices, including the original Divisia aggregates, the credit-card-augmented Divisia, and the credit-card-augmented Divisia inside money aggregates. The source of each is the Center for Financial Stability (CFS). We find that the smallest Root Mean Square Forecast Errors (RMSFE) are attained with the credit-card-augmented Divisia indices used as the forecast indicators. We also consider Bayesian vector autoregression (BVAR) for forecasting annual inflation and output growth.

Third chapter **“Welfare Cost of Inflation with Credit Card Transactions in Money Measures"** investigates the welfare costs that occurred by anticipated inflation when we add the volume of credit card transactions to the measurement of money. First, we use the concept of credit card-augmented Divisia in the dynamic stochastic general equilibrium (DSGE) model and calculate the welfare costs of inflation. This paper assumes money yields utility in the money-in-the-utility function of Sidrauski (1967). This paper also empirically examines the welfare costs of inflation in the U.S. by deriving the inverse money demand functions with the consumer surplus approach using the Divisia indices from the Center for Financial Stability (CFS). The welfare costs of inflation with credit card services are lower than those with no credit card services in the New Keynesian model. With the empirical method, we see more sensitive changes in the welfare cost of inflation with broad money and the monetary aggregation containing the credit card transactions volume when the inflation target changes.

Last chapter is **“The Role of Broad Money: Tracking Economic Signals."** Fed discontinued announcing broader monetary aggregate. For example, M3 was discontinued to publish in 2006 and only M3 data set borrowed from OECD has been displayed. Moreover, we cannot track M4 data series in FRED anymore. Although the importance of monetary aggregate is increasing (as a tool of understanding monetary transmission mechanism), researchers cannot approach a reliable data set. As we see in the Divisia data provided by the Center for Financial Stability (CFS), many economic events can be tracked by the broader money rather than the narrow money. Clearly, there would be an economic implication provided by the broader monetary aggregate. The purpose of this paper is to examine the availability of Divisia type monetary aggregates for all ranges by conducting several tests. Following the philosophy of Bernanke and Blinder (1992) and Belongia and Ireland (2015), we test the causal relationship between real economic variables and monetary aggregates. We also set up a basic recursive VAR and Leeper-Roush type (2003) non-recursive VAR model, and study the economic implications given by the results.

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Chapter 1

Development of Divisia Monetary Aggregates and Its Applications

Sohee Park

Abstract

The study of the new monetary aggregates, Divisia monetary aggregates, is continually being developed. While simple sum measures have the same weight for each monetary asset, Divisia monetary aggregates impute different component expenditure shares with user cost pricing and reflect the real world. This paper is the literature survey of the Divisia monetary aggregates that are preferred to the simple sum in much economic research since monetary components are not perfect substitutes. We introduce and organize various literature about the traditional Divisia, credit-card-augmented Divisia, and credit-card-augmented Divisia inside money.

1.1 Introduction

The purpose of this paper is to investigate the history of the Divisia monetary aggregates with index number theory. Since Barnett (1980) proposed the Divisia monetary aggregates that measure demand side monetary services using the economic aggregation and index number theory, Divisia monetary aggregates have been supported by many researchers in various different ways. Divisia is the growth rate with a weighted average and it reflects the reality.

Simple sum money has the same weight on each monetary asset. Unlike the simple sum measure, the Divisia reflects the weight of each monetary asset. Since the component monetary assets are not perfect substitutes, Divisia monetary aggregates are strictly preferable to the simple sum monetary aggregates. The considerable literature shows that the Divisia is strictly preferable to a simple sum. Thornton and Yue (1992) also point out that the convention method of monetary aggregation has been criticized because equally weighting each component financial asset is relevant only under special circumstances.

Central banks of many countries agree with the importance of the use of measures of Divisia and publish crucial research. In Federal Reserve Bank of St. Louis Review, Anderson et al. (1997) study microeconomic theory of monetary aggregation which is the base of Divisia monetary aggregates. Hancock (2005) from the Bank of England reviews the measure of Divisia money that assigns the greatest weight to those components most used in transactions and explains its calculation.

Recently, since credit card transactions increase as time goes by, there is a new idea for the monetary aggregates including the amount of credit card transactions which is so-called credit-card-augmented Divisia monetary aggregates. Barnett and Su (2016) generalize the theory to contain credit card transaction services on the demand side and derive the theory needed to measure the joint services of credit cards and monetary assets. Based on the index number theory, their results are derived from economic aggregation theory. Extending Barnett and Su (2016), Barnett and Su (2020) derive credit-card-augmented Divisia inside money which is about supply side money and the estimation of the output supply function of banks and measure of value-added in banking. They derive the theory needed to measure the production of the joint services of credit cards and inside money. Financial intermediaries' monetary production theory is augmented to include credit card transaction services. This paper also introduce these new monetary aggregates.

This paper is organized as follows: Section 1.2 discusses the paper explaining the theory and the application of the traditional Divisia, and Section 1.3 contains the research of credit-card-augmented Divisia. Section 1.4 describes the credit-card-augmented Divisia inside money proposed recently and we conclude in Section 1.5.

1.2 Traditional Divisia

In this section, we introduce theoretical literature and application literature of the conventional Divisia monetary aggregates. The Barnett Critique indicated an internal inconsistency between the theory that is implied by simple sum monetary aggregation and the economic theory that produces the models within which those aggregates are used. This inconsistency leads to the emergence of unstable money demand and supply and this unstable money demand is harmful to the field of monetary economics. Barnett et al. (2021) explain the origin and definition of the Barnett critique again and also organize the development of the Divisia monetary aggregates.

1.2.1 Theory

Barnett (1978) and Barnett (1980) proposed the use of either the Divisia or Fisher ideal index for monetary quantity aggregation and derived the user cost of money, using economic aggregation theory and index number theory. Those papers assumed that the consumer's current intertemporal utility function is weakly separable in each period's consumption of goods and monetary assets in the consumer decision model and the aggregator function is linearly homogeneous. While simple sum money has the same weight on all monetary assets, Divisia imputes different share weights to each monetary component depending on the expenditure share of each component. Barnett et al. (1984) say that Barnett's Divisia monetary aggregates were derived to be elements of Diewert's class of superlative quantity index numbers and the Divisia monetary aggregates are strictly preferable to the simple sum monetary aggregates since the component monetary assets are not perfect substitutes. Belongia (1996) also supports the excellence of Divisia monetary aggregates constructed by the economic aggregation theory and index number theory compared to the simple sum. Barnett et al. (1992) propose the use of the method of Divisia monetary aggregates again and study the consumer theory.

The decision maker maximizes monotonically increasing and strictly concave utility function for the optimal portfolio allocation decision:

$$\begin{aligned} &\max \quad u(m_t) \\ &\text{subject to} \quad \pi_t' m_t = y_t \end{aligned} \tag{1.1}$$

where m_t' is the vector of real balances of monetary assets during period t, y_t is the real value of the total budgeted expenditure on monetary services, and π_t' is the vector of monetary asset real user costs. The real user cost (equivalent rental price) π_{it} of monetary asset i at period t is given by

$$\pi_{it} = \frac{R_t - r_{it}}{1 + R_t} \tag{1.2}$$

where r_{it} is the own rate of return on asset i during period t, and R_t is the risk-free rate of return on a completely illiquid asset (benchmark rate) during period t.

The exact monetary aggregate M_t of economic theory is the utility level associated with holding the portfolio that solves the optimal portfolio allocation decision, equation (1.1).

$$M_t = u(m_t^*) \tag{1.3}$$

where m_t^* is the optimized value of the decision's objective function.

The Divisia index in growth rate form in continuous time is

$$d\log M(m_t^*) = \sum_i s_{it} d\log m_{it}^* \tag{1.4}$$

where $s_{it} = \pi_{it} m_{it}^* / \pi_t' m_t$ is the expenditure share on monetary asset i and m_{it}^* is the real balances of monetary asset i during period t and must continually solve equation (1.1) for equation (1.4) to hold.

The discrete time Divisia quantity index[1] is

$$\log M_t - \log M_{t-1} = \sum_i \bar{s}_{it} (\log m^*_{it} - \log m^*_{i,t-1}) \tag{1.5}$$

where $\bar{s}_{it} = \frac{1}{2}(s_{it} + s_{i,t-1})$ is the average expenditure share between the beginning and end of the period. User cost aggregates are duals to monetary quantity aggregates. The price aggregator function dual to quantity aggregator function is unique. The Divisia price index in the continuous time and the Törnqvist-Theil discrete time are following:

$$d \log \Pi_t = \sum_i s_{it} d \log \pi_{it} \tag{1.6}$$

$$\log \Pi_t - \log \Pi_{t-1} = \sum_i \bar{s}_{it} (\log \pi_{it} - \log \pi_{i,t-1}) \tag{1.7}$$

Barnett et al. (2013) introduce a new Divisia monetary aggregates database offered by the Center for Financial Stability (CFS) and their Advances in Monetary and Financial Measurement (AMFM) program. That paper documents the sources of components monetary assets of Divisia monetary aggregates database and introduces all types of Divisia monetary aggregates that are Divisia M1, M2, M3, M4-, and M4 while the Federal Reserve discontinued publishing broad money from 2006. Anderson and Jones (2011) introduce a revision of the Divisia monetary aggregates published by the Federal Reserve Bank of St. Louis, Monetary Services Indexes (MSI). They deal with five levels of aggregations and find that the growth rates of MSI-M2 and MSI-ALL are highly correlated with the growth rates of MSI-M3. Like other literature, that paper shows the evidence of the importance of broad money (M3).

[1] This Divisia quantity index is also called the Törnqvist-Theil approximation (Tönrnqvist index).

1.2.2 Applications

There are many pieces of literature containing the application of the Divisia monetary aggregates and they have proven more useful than the simple sum money in predicting macroeconomic signals.

1.2.2.1 Prediction

Much literature show the evidence of the better prediction ability for inflation and output of Divisia monetary aggregates compared to the simple sum in various methods. Swofford and Whitney (1991) construct Divisia monetary aggregates and examine the monetary aggregates in forecasting inflation rate. They demonstrate that their economic monetary aggregates are superior to forecasts based on the simple sum M1 or M2. Schunk (2001) explores the forecasting performance of the Divisia monetary aggregates compared to the simple sum monetary aggregates. That paper uses a four-variable[2] vector autoregression to forecast U.S. real GDP and finds that forecasting by using broad Divisia aggregates that contain valuable information for forecasting future real economic variables rather than the simple sum. In forecasting future prices, a narrow Divisia aggregate is most useful. Drake and Mills (2005) also compare the forecasting performance of the Divisia and the simple sum and demonstrate the advantages of an empirically weighted broad monetary aggregate from a monetary policy perspective in the U.S. This new empirically weighted aggregate works well in out-of-sample forecasting of nominal income and inflation. Elger et al. (2006) consider the situation in the United States. They use vector autoregressive (VAR) and regime-switching (RS) VAR models to investigate the out-of-sample forecasting performance of Divisia and simple sum monetary aggregates. They confirm that aggregation matter has a big impact on the forecasting performance with respect to inflation and real output growth.

Many researchers have tried to conduct new method to investigate the monetary aggregation related research. Findings and results depend on the research methods. Binner et al. (2010) consider a wide range of different definitions of money, various monetary aggregates (simple sum and Divisia) by different methods of aggregation and different components of included monetary assets,

[2]Four variables are real GDP, GDP deflator, nominal 6-month treasury bill rate, and monetary aggregates.

for forecasting US inflation in early to mid 2000s. They use two new macroeconomics nonlinear techniques, recurrent neural networks and kernel recursive least squares regression, and their results do not support much for the role of monetary aggregates in forecasting inflation. Barnett et al. (2016a) investigate the assessments of current U.S. nominal GDP growth and the performance of various econometric models (univariate and multivariate, linear and nonlinear) in nowcasting nominal GDP growth in real-time with the traditional Divisia monetary aggregates. They also confirm that the model containing information on real economic activity, inflation, interest rates, and Divisia monetary aggregates, makes the most precise real-time nowcasts of nominal GDP growth.

Barnett and Gaekwad (2018) explore the circumstances in Euro area and analyze aggregation over monetary assets for the European Monetary Union (EMU). They consider multilateral Divisia monetary aggregates for the EMU-11[3] and find that those aggregates are more informative and the better signal of economic trends than simple sum money. That paper analyzes substitutability among those EMU-11's monetary assets considering the representative consumer's utility function, the minflex Laurent indirect utility function. From the analysis of elasticities with respect to asset's user cost prices, they find that transaction balances and deposits are income elastic and EMU-11's monetary assets are not good substitutes for each other.

Ellington (2018) adopts time-varying coefficient VAR models and assesses the empirical benefits of Divisia monetary aggregates. That paper finds a strong linkage between Divisia money and economic activity over the business cycle, while the relationship between simple sum aggregates and economic activity is substantially less prominent. Also, out-of-sample forecasts of economic activity from the models using Divisia monetary aggregates outdo those using simple sum measures.

Belongia and Ireland (2015) explore the superiority of superlative (Divisia) measures of money in forecasting movements in key macroeconomic variables. They find that the Divisia monetary aggregates play a significant role as an intermediate target or indicator variable and make no striking deterioration in the information content in Granger-causality tests.

[3]The eleven EMU countries are Estonia, Finland, France, Germany, Ireland, Italy, Luxembourg, Malta, Netherlands, Slovakia, and Slovenia.

1.2.2.2 In Dynamic Stochastic General Equilibrium (DSGE) Model

Divisia monetary aggregates are utilized in the New Keynesian model and Real Business Cycle model to investigate more sophisticated models reflecting reality. Belongia and Ireland (2006) study both related empirical models using a small VAR framework and theoretical model using the Real Business Cycle model to show the effects of price changes in the monetary transmission mechanism. This paper introduces the quantity of money (Divisia index) and the own-price of aggregate money which is the share-weighted sum of each asset's user cost and found that movements of the own-price of money and output are strongly related. An empirical VAR framework shows that increases in the own-price money are linked with declines in output and the results of the RBC model are consistent with an empirical approach.

Revisiting the Barnett critique, Belongia and Ireland (2014) adopt the Divisia concept of monetary aggregations in the New Keynesian model and include currency and deposits in monetary aggregation. They measure the true aggregate of monetary or liquidity services demanded by the representative household by using CES function and compare the impulse responses of the growth rates of the true aggregate, Divisia approximation, and the simple sum aggregate to each of the model's six exogenous shocks.[4] In the impulse response functions, they see that the Divisia monetary aggregate tracks the true aggregate almost perfectly following various macroeconomic shocks while the simple sum measure behaves differently. In other words, Divisia monetary aggregates are convinced to reflect reality in the systematic model. They point out that "measurement matters" because the results of their paper support the "Barnett critique". Ireland (2014) sets up a New Keynesian model including banks and deposits and studies the macroeconomic effects of policies that pay interest on reserves. Their results display that there are small macroeconomic effects, specifically on output and inflation, of paying interest on reserves, however, the monetary authority needs to adjust the way of management of the supply of reserves because liquidity effects would be disappeared in the short run.

[4] Belongia and Ireland (2014) investigate money demand shock, preference shock, technology shock, reserves demand shock, deposit cost shock, and monetary policy shock.

Belongia and Ireland (2019) set a money-in-the-utility function model and include currency and deposits in the monetary aggregates to capture the noticeable roles of currency (noninterest-bearing) and deposits (interest-bearing) in providing liquidity services to households. Their paper suggests that if stable long-run money demand relationships exist, they may serve to the connection between the demand for a Divisia monetary aggregates and its user cost dual. They also use the U.S. Divisia M2 and MZM data in empirical works and estimate the stable demand relationships that link a Divisia monetary aggregate to consumption including the periods before, during, and after the Great Recession (2007-2009). They confirm that a measurement matters because properly measured money plays a role in the conduct of monetary policy.

Keating and Smith (2019) investigate the optimal monetary instrument in a New Keynesian framework with the simple sum measure of money, the Divisia measure, and the monetary base. They consider a Taylor interest rate fule, a fixed growth rate for the monetary base, a fixed growth rate for simple-sum money, and a fixed growth rate for the Divisia monetary aggregate to evaluate welfare for the representative household in the calibrated model. That paper finds that the constant Divisia growth rule outperforms all other rules while the constant simple-sum growth rule causes an infinite welfare loss. They also conduct both standard Granger causality tests and Sims lagged dependent variable tests on simulated data from their DSGE model. In the Granger causality test, they confirm that the interest rate Granger causes both output and prices, and each measure of money causes real GDP. The Divisia fails to cause the price level. In the Sims lagged dependent variable tests, the results also show that the Divisia measure of money is the only variable that fails to cause the price level, therefore, the variable offering the most welfare enhancement to a central bank has the weakest causality with the real GDP and price level.

1.2.2.3 Welfare Cost of Inflation

Divisia monetary aggregates have been compared to the other monetary aggregations in calculation of the welfare cost of inflation which is the change in social welfare caused by various inflation. Serletis and Virk (2006) use the simple sum, Divisia monetary aggregation, and currency equivalent monetary aggregates to measure the welfare cost of inflation. They mention that welfare cost calculations are sensitive to the specification of the money demand function. Welfare cost of inflation calculated by using the Divisia monetary aggregates is smaller than the welfare cost by using simple sum and currency equivalent aggregates and the choice of monetary aggregation procedure is important in evaluating the welfare cost of inflation.

Cysne and Turchick (2010) think the consideration of the liquidity of interest-bearing deposits increased by the technological innovations from 1980s and new regulations on the research of the welfare cost of inflation. They use bidimensional bilogarithmic money demands and show the evidence that ingoring interest-bearing money may cause a non-negligible overestimation of the welfare costs of inflation.

Dai and Serletis (2019) investigate instabilities in the long-run money demand function and observe the welfare cost of inflation in the U.S using the Markov regime switching approach. Their paper focuses on narrow monetary aggregates rather than broad monetary aggregates because households hold non-interest bearing money in the welfare cost of inflation in the money demand framework. Especially, they adopt simple sum M1 and Divisia M1 for their measure of the welfare cost of inflation and estimate the log-log and semi-log money demand functions. They display the results that the semi-log money demand function measures larger changes than the log-log estimated with both simple sum M1 and Divisia M1.

1.2.2.4 Others

There are a variety of other topics of literature using Divisia monetary aggregates. Barnett et al. (1984) say Barnett's Divisia monetary aggregates were derived to be element of Diewert's class of superlative quantity index numbers. They also showed the performance of the Divisia in the empirical tests. Belongia and Chalfant (1989) examined whether interest-bearing checkable deposits should be included in measures of the U.S. money stock comparing Divisia M1 and simple sum M1. They focus on the components asset of M1 and test weak separability in a model of the demand for financial assets. They test Divisia M1 and simple sum M1 based on a St. Louis equation and in terms of controllability and find that the performance of Divisia is better than it of the simple sum measure. While the CFS provides a traditional Divisia dataset for the period from 1967, Anderson et al. (2019) construct Divisia indexes for the period from 1947 to 1967 at the M2, M3, M4- and M4 levels employing calculation methods designed the corresponding series published by the CFS. By using this constructed data, they estimate a Fourier flexible form model to see the money demand system for the M3 components' properties.

We see a lot of research about Divisia of foreign countries other than the U.S. Belongia and Chrystal (1991) utilize the data of the United Kingdom and evaluate the performance of Divisia monetary aggregate. They test weak separability and find that wholesale deposits are not suggested to aggregate with other U.K. financial assets. Also, Divisia aggregates were more deeply connected to the nominal GDP growth and had stable money demand functions. Bissoondeeal et al. (2019) also use UK data to investigate the importance of monetary aggregates in determining output in IS curve specification. They study whether or not the impact of money on output is time varying data-driven procedures to see breaks in the data. That paper concludes that simple sum seems to be more affected by the breaks than Divisia monetary aggregates and Divisia has an effect on output though the role played in monetary policy is diminished. Yue and Fluri (1991) investigate the case of Switzerland and derive Divisia M1 and Divisia M2 to compare with simple sum M1 and M2. They show evidence of the better performance of Divisia M2 compared to simple sum M2. However, since simple sum M2 cannot be controlled and is an inadequate substitute for the monetary base for policy purposes, they conclude that Divisia monetary aggregates are worth to investigate. Darvas (2015) constructs the dataset of euro-area Divisia monetary aggregates and uses structural vector autoregressions to show the responses to money, user cost, and interest rate shocks. Especially, a Divisia-shock affects significantly output and prices. That paper finds that money matters for output, prices, and interest rates, while the European Central Bank (ECB) can affect monetary developments.

Some researchers try to adopt sophisticated econometrics methods to investigate the various effects of Divisia monetary aggregates. Serletis and Xu (2020) examine the effect of money growth volatility on real output growth in the U.S. by using Divisia monetary data from CFS and they show that money growth volatility causes asymmetric effects on output growth the business cycle. They extend the research methods and adopt the recent state-of-the-art advance in macroeconometrics and financial econometrics such as a bivariate, Markov switching, and identified structural vector error correction (VEC) model with generalized autoregressive conditional heteroscedasticity (GARCH)-in-mean errors. Their results show that money growth volatility causes asymmetric effects on output in the business cycle.

1.3 Credit-Card-Augmented Divisia and Inside Money

1.3.1 Theory

Barnett and Su (2016) and Barnett et al. (2016b) introduce the new idea that the monetary aggregates include the credit card transaction volume for the first. Credit-card-augmented Divisia is the measure of demand-side monetary services including credit card transactions. Credit card balances have never been included in measures of the money although credit cards provide transaction services since accounting conventions do not allow adding liabilities to assets. The new monetary aggregates containing credit card transactions are derived from the consumer decision with credit card transaction volumes, not credit card balances, along with monetary balances entered into the utility function. The theory in the paper is to measure the joint services of credit cards and money by index number theory which measures service flow and is based on aggregation theory, not accounting. Monetary assets and credit card transaction volumes provide services, such as liquidity and transaction services.

Now, the credit-card-augmented Divisia is a function of monetary assets and credit card services. The aggregation-theoretic exact approach provides credit-card-augmented structural aggregate,

$$M_t = M(\mathbf{m}_t, \mathbf{c}_t) = v(\mathbf{m}_t, \mathbf{c}_t) \tag{1.8}$$

derived under the assumption of risk neutrality. The growth rate of the Divisia index $M(\mathbf{m}_t, \mathbf{c}_t)$ is the share weighted average of the growth rates of the components.

The real user cost of credit card services was derived in Barnett et al. (2016b) as follow:

$$\tilde{\pi}_{jt} = \frac{e_{jt} - R_t}{1 + R_t} \tag{1.9}$$

where e_{jt} is credit card interest rate, and R_t is the risk-free rate of return on a completely illiquid asset (benchmark rate) during period t. The j is the credit card type and CFS considers four credit card companies, Visa, MasterCard, Discover, and American Express as primary sources. Barnett et al. (2016b) consider two categories of consumers. First, they consider 80% of consumers who do pay interest to the credit card issuing banks and another is 20% of consumers who do not pay interest. e_{jt} is averaged over both those consumers who maintain such rotating balances, and pay interest on contemporaneous credit card transactions and also those consumers who pay off such credit card transactions before the end of the period, and do not pay explicit interest on the credit card transactions.[5] This theory does not include rotating balances used for transactions in prior periods to avoid double counting of transactions services.

The credit card quantities to include in the augmented Divisia index formula are the monthly credit card transactions volumes, not the credit card balances. The credit-card-augmented Divisia index is as follow:

$$d \log M(\mathbf{m}_t, \mathbf{c}_t) = \sum_i s_{it} d \log m_{it} + \sum_j \tilde{s}_{jt} d \log c_{jt} \tag{1.10}$$

where the share $s_{it} = \pi_{it} m_{it} / (\pi_t' \mathbf{m}_t + \tilde{\pi}_t' \mathbf{c}_t)$ is the expenditure share of monetary asset i in the total service of monetary assets and credit cards while the share $\tilde{s}_{jt} = \tilde{\pi}_{jt} c_{jt} / (\pi_t' \mathbf{m}_t + \tilde{\pi}_t' \mathbf{c}_t)$ is the expenditure share of credit card services, j, in the total services of monetary assets and credit cards. They also examined the use of the data as indicators, rather than as policy targets or as structural variables in the macroeconomy. They found that the new aggregator function containing credit card transaction volumes uniquely captures the contributions of monetary services and credit card services as indicators of nowcasting nominal GDP. The indicator-optimized augmented aggregates,

[5] Barnet and Su (2016) explains that the Federal Reserve provides two interest rate series for credit card debt.

$M_t^* = M_t^*(\mathbf{m}_t, \mathbf{c}_t)$, will be provided by the Center for Financial Stability (CFS).

While Barnett et al. (2016b) assume perfect certainty or risk neutrality, Barnett and Su (2019) extend the aggregation theory removing the assumption of risk neutrality. They mention that risk adjustment of the credit-card-augmented Divisia monetary aggregates is more important than risk adjustment of the original Divisia monetary aggregates because credit card interest rates are high and volatile. They also argue that a more demanding approach would remove the CCAPM assumption of intertemporal separability, in accordance with Barnett and Wu (2005). Barnett and Su (2016) include the services of credit card transactions volumes under risk and emphasize the importance of the adjustment for risk using their results for for the new augmented Divisia monetary aggregates rather than the existing Divisia monetary aggregates (traditional Divisia).

Now, let's think about the supply side of money by financial intermediaries which explains value-added banking. Credit-card-augmented Divisia inside money measures service flows produced by financial intermediaries. Banks make more and more inside money by banking services. In a credit-card-augmented Divisia inside money point of view, monetary services and credit card transactions services are outputs of the financial firms. Barnett and Su (2020) derived theory needed to measure the supply of the joint services of credit cards and inside money[6], needed to estimate the output supply function and to compute value-added since financial firms' monetary production model is employed to investigate supply-side inside money aggregation, augmented to include credit card transaction services when financial firms produce both monetary services and credit card transaction services. The main assumption is the risk neutrality of bank managers. They expect the importance of the theory produced in this paper to grow because a major factor affecting monetary policy is inside money produced by banking and shadow banking. The conditions under which a joint supply-side aggregate over monetary and credit card transactions service exists. The existence condition for a supply-side inside-money aggregator function is blockwise weak separability within tastes and for a demand-side inside-money aggregator function is blockwise weak separability in bank technology. They augmented the monetary production theory of the financial

[6]Outside money is money base.

firms to include credit card transaction services.

The user cost formula for supplied monetary assets differs from the demand-side user cost formula because of the regulatory wedge produced by the existence of reserve requirements, imposing an implicit tax on banks. The difference between credit-card-augmented Divisia and credit-card-augmented Divisia inside money is adding the required reserve ratio on monetary asset in user cost equation. Barnett (1987) showed the user cost of monetary asset with the implicit tax as follows:

$$\pi_{it} = \frac{(1-k_i)R_t - r_{it}}{1+R_t} \tag{1.11}$$

where k_i is the required reserve ratio on monetary asset i and it is a kind of an implicit tax on the financial firms. In other words, this implicit tax is foregone interest on uninvested required reserves. Then the credit-card-augmented Divisia inside money index is calculated in equation (1.10) with the monetary asset user cost of equation (1.11).

Barnett and Su (2020) concluded that financial firm outputs of demand deposit services, time deposit services, and credit card transactions services can be aggregated to produce an inside money output aggregate. Then an inside money output aggregate enters an aggregate services supply function for the financial firm. The demand-side Divisia monetary aggregates have no currency because excess reserves are not an output of the financial firms, but an input to the technologies of financial firms. Currency is not blockwise weakly separable within utility functions from other monetary assets in their empirical works. Thus, a demand-side inside money aggregate without currency fails the existence condition. They also pointed out that this theory can be applied to figure out the role of financial intermediaries for the inside money production. Inside money plays a role in the transmission mechanism of monetary policy. They also mentioned that this theory can also be applied to produce a monthly economic indicator. Since the GDP data is provided quarterly, the inside-money aggregate would be a good substitution for the monthly economic data.

The credit-card-augmented Divisa monetary aggregates data sources are introduced in Barnett and Su (2017). They explain detailed information on the data sources used in producing the new augmented Divisia monetary aggregates, especially displaying the information of the credit card companies[7] whose transaction volumes are used in the new monetary aggregates, and how the public can approach the data set.

1.3.2 Applications

There is not much literature on credit-card-augmented Divisia monetary aggregates yet compared to the traditional Divisia monetary aggregates. Liu et al. (2020) consider the conventional Divisia, credit-card-augmented Divisia, and credit-card-augmented Divisia inside monetary aggregates and. They use cyclical correlations analysis and Granger causality test to find the inference ability of each monetary aggregation. They find that both credit-card-augmented Divisia and credit-card-augmented Divisia inside money measures behave better in predicting real economic activity than the conventional Divisia during and in the aftermath of the financial crisis and broad Divisia monetary aggregates show better performance measuring the flow of monetary services.

Recently, Liu and Serletis (2020) utilize the credit-card-augmented Divisia monetary aggregates to examine the effects of the variability of money growth on output, extending Serletis and Xu (2020)[8]. They use advanced research methods, bivariate VARMA, GARCH-in-Mean, and asymmetric BEKK model. That paper find that credit-card-augmented Divisia M4 has a negative effect on output, but traditional Divisia M4 growth volatility has no effect on real economic activity. They conclude that the increased uncertainty of credit-card-augmented Divisia M4 growth rate is associated with a lower average growth rate of output in the U.S.

1.4 Conclusion

Since Barnett (1980) proposed and introduced the use of Divisia monetary aggregates using economic aggregation theory and index number theory, there has been a lot of research on this new monetary aggregation and its applications using Divisia data produced by the CFS. Divisia mon-

[7]See section 1.3.1.
[8]See section 1.2.2.4.

etary aggregates which impute different expenditure share depending on each monetary asset's usage to all monetary components are preferable to the simple sum that has the same weight on each component in aggregation because monetary assets are not perfect substitute and it is only possible in specific situation.

This paper looks into the flow of theory of the Divisia monetary aggregates and also looks through a variety of literature on the Divisia, credit-card-augmented Divisia, and credit-card-augmented Divisia inside money. We organize much previous research to investigate and introduce development of Divisia monetary aggregates, as well as the new monetary aggregates, so-called credit-card-augmented Divisia monetary aggregates.

We look forward to seeing a variety of applications of Divisia monetary aggregate in various fields and it would be useful research to investigate the new money measure.

Chapter 2

Forecasting Inflation and Output Growth with Credit-Card-Augmented Divisia Monetary Aggregates

William A. Barnett, Sohee Park

Abstract

This paper investigates the performance of the credit-card-augmented Divisia monetary aggregates in forecasting U.S. inflation and output growth at the 12-month horizon. We compute recursive and rolling out-of-sample forecasts using an Autoregressive Distributed Lag (ADL) model based on Divisia monetary aggregates. We use the three available versions of those monetary aggregate indices, including the original Divisia aggregates, the credit card-augmented Divisia, and the credit-card-augmented Divisia inside money aggregates. The source of each is the Center for Financial Stability (CFS). We find that the smallest Root Mean Square Forecast Errors (RMSFE) are attained with the credit-card-augmented Divisia indices used as the forecast indicators. We also consider Bayesian vector autoregression (BVAR) for forecasting annual inflation and output growth.

2.1 Introduction

Credit card transactions have never been included in central bank measures of the money supply, since accounting conventions do not permit adding liabilities, such as credit card balances, to assets, such as money. But as credit card transactions have increased, the need to measure the contributions of credit cards to liquidity has grown. Economic aggregation theory permits aggregating over service flows, such as monetary services and credit card transaction volumes, regardless of whether the source of the services are assets or liabilities. The result, as maintained by the Center for Financial Stability (CFS) in New York City, is called the credit-card-augmented Divisia monetary aggregates.

Simple sum measures, which are not based on economic aggregation or index number theory, impute the same weight to each monetary asset in the aggregate, despite the fact that the services of different monetary assets are not perfect substitutes. Unlike the simple sum measure, Divisia monetary aggregates are directly derived from economic theory and impute user cost prices to the marginal utilities of component assets. The resulting component growth rate weights in measuring the growth rate of the Divisia index are the component expenditure shares with user cost pricing. As displayed in Figure 2.1[1] and Table 2.1, the share of cash and checks as payment instruments has been declining. The role of credit cards in transactions can no longer be ignored.

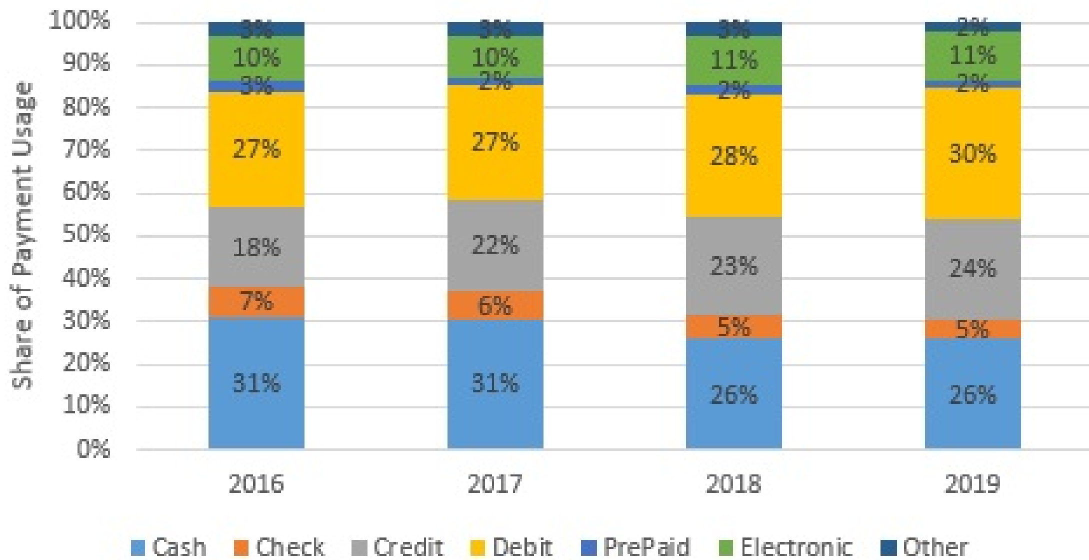


Figure 2.1: Shares of Payment Instruments

Barnett and Su (2016), extending the well-known Divisia monetary aggregates originated by Barnett (1980), generalized the theory to include credit card transaction services on the demand side. They derive the theory needed to measure the joint services of credit cards and money.

[1]Source: The 2019 Federal Reserve Payment Study

	2012	2015	2018	2012-15	2015-18
	Value ($ trillions)	Value ($ trillions)	Value ($ trillions)	% change in value	% change in value
Total	78.01	86.78	97.04	3.6	3.8
Credit Cards	2.55	3.05	3.98	**6.2**	**9.3**
Debit Cards	2.10	2.47	3.10	5.5	7.8

Table 2.1: Noncash Payment

Source: The 2019 Federal Reserve Payments Study, https://www.federalreserve.gov/paymentsystems/2019-December-The-Federal-Reserve-Payments-Study.htm

Their results are based on index number theory, which measures service flows, and are derived from economic aggregation theory. Barnett and Su (2020) derived the corresponding supply side theory needed to measure the production of the joint services of credit cards and inside money. The resulting supply side index is needed in estimation of the output supply function of banks and to measure value-added in banking. Financial firms' monetary production theory is thereby augmented to include credit card transaction services.

Using the conventional Divisia monetary aggregates, Barnett et al. (1984) explored the forecast properties of the well-known, linear, fixed-coefficient money demand functions. Barnett et al. (2016a) investigate the performance of univariate and multivariate, linear and nonlinear econometric models in nowcasting nominal GDP growth in real-time with conventional Divisia monetary aggregates includes as possible indictors. Their model, containing information on real economic activity, inflation, interest rates, and Divisia monetary aggregates, is found to be the most accurate in producing real-time nowcasts of nominal GDP growth. Barnett et al. (2016b) consider the use of credit-card-augmented Divisia monetary aggregates as indicators in nowcasting nominal GDP growth. Nowcasting is the prediction of present values, not yet directly measured, such as monthly GDP, which is directly measured only quarterly. The purpose of our paper is to evaluate the performance of credit-card-augmented Divisia monetary aggregates as indicators in predicting future economic variables, rather than current variables. We use the methods of forecasting rather than nowcasting to investigate prediction.

Schunk (2001) compares the forecasting performance of the conventional Divisia monetary aggregates versus the simple sum monetary aggregates and shows that forecasts of U.S. real GDP with a four-variable vector autoregression are most accurate, when including Divisia monetary aggregates.[2] Ellington (2018) assesses the relative empirical benefits of Divisia monetary aggregates by fitting time-varying coefficient VAR models. That paper finds that there is a strong link between Divisia money and economic activity over the business cycle, but the relationship is substantially less prominent with simple sum aggregates. That paper also finds that out of sample forecasts of economic activity from models using Divisia aggregates surpass those using simple sum measures.

We investigate credit-card-augmented Divisia monetary aggregates as predictors of inflation and output growth. We compare the results of forecasting inflation and output growth using these three monetary aggregates: the original Divisia monetary aggregates, the demand-side credit-card-augmented Divisia monetary aggregates, and the supply-side credit-card-augmented Divisia inside money aggregates. All three types of Divisia monetary aggregates are available from the Center for Financial Stability (CFS).

Stock and Watson (1999) investigated forecasts of U.S. inflation at the 12-month horizon, and examined the conventional unemployment rate Phillips curve in a simulated out-of-sample framework. They found that forecasting inflation by the Phillips curve was more accurate than with other macroeconomic variables as indicators, such as interest rates, money, and commodity prices. Stock and Watson (2003) forecasted output and inflation using asset prices and other leading indicators for seven OECD countries. They calculated out-of-sample mean square forecast errors. Rossi and Sekhposyan (2010) empirically analyzed the ability of various economic models in predicting both future industrial production growth and inflation.

We also consider Bayesian vector autoregression. Chin and Li (2019) explored BVAR and evaluated the performance in individual and combination forecasts. They studied whether the inclusion of prior economic and/or non-economic information can improve the forecasting performance of VAR and found that using prior information produces more accurate forecasts.

[2]The four variables in Schunk (2001) are real GDP, GDP deflator, nominal 6-month treasury bill rate, and monetary aggregates.

Our paper is organized as follows. Section 2.2 discusses the regular Divisia monetary aggregates, the credit-card-augmented Divisia, and credit-card-augmented Divisia inside money aggregates. Section 2.3 explains the out-of-sample method used in this paper for forecasting inflation and output growth. Section 2.4 describes the data used for forecasting, and the results of the model are investigated in Section 2.5. Section 2.6 explores forecasting by Bayesian vector autoregression. We conclude in Section 2.7.

2.2 Divisia Monetary Aggregates

Divisia monetary aggregate growth rates are the share weighted average of component growth rates, with user cost pricing used in computing the expenditure shares. This paper considers three types of Divisia monetary aggregates: the original Divisia, the credit-card-augmented Divisia, and the credit card-augmented Divisia inside money aggregates. We define each in this section.

2.2.1 Original Divisia Monetary Aggregates

The original Divisia monetary aggregates measure demand-side monetary services, using the economic aggregation and index number theory developed by Barnett (1980). Barnett et al. (1984) describe the details of the production and use of those aggregates and provide initial empirical results. Barnett (1980) proposes the use of either the Divisia or Fisher ideal index for monetary quantity aggregation with user cost pricing of components. The difference between the two indexes is negligible, being less than the roundoff error in the components, but the Divisia index is easier to explain and interpret than the Fisher ideal index. The resulting quantity index numbers, by either formula, are elements of Diewert's class of superlative quantity index numbers. Barnett's resulting monetary aggregates are strictly preferable to the simple sum monetary aggregates, since the component monetary assets are not perfect substitutes. Relative to aggregation and index number theory, simple sum aggregation over imperfect substitutes is inadmissible.

Barnett proved that the real user cost (equivalent rental price) of monetary asset i is

$$\pi_{it} = \frac{R_t - r_{it}}{1 + R_t}, \tag{2.1}$$

where r_{it} is the own rate of return on asset i during period t, and R_t is the risk-free rate of return on pure capital, a completely illiquid asset (benchmark rate), during period t.

The Divisia index in growth rate form in continuous time is

$$d \log M(\mathbf{m}_t) = \sum_i s_{it} d \log m_{it} \tag{2.2}$$

where $s_{it} = \pi_{it} m_{it} / \pi_t' \mathbf{m}_t$ is the expenditure share on monetary asset i and m_{it} is real balances of monetary asset i during period t.

2.2.2 Credit-Card-Augmented Divisia and Inside Money Divisia

Credit-card-augmented Divisia money is the measure of demand-side monetary services, including credit card transaction volumes. Barnett and Su (2016) and Barnett et al. (2016b) introduced the credit card services augmented Divisia monetary aggregates. Conventional simple sum monetary aggregates have never been augmented to include credit card balances, since accounting conventions do not allow adding liabilities to assets. Credit-card-augmented Divisia monetary aggregates are derived from the consumer decision with credit card transaction volumes along with monetary balances entered into the utility function, reflecting the fact that money and credit card transaction volumes provide services, such as liquidity and transactions services. Economic aggregation and index number theory measure service flow, independently of whether from assets or liabilities.

The real user cost of credit card services, derived in Barnett and Su (2016), is:

$$\tilde{\pi}_{jt} = \frac{e_{jt} - R_t}{1 + R_t} \tag{2.3}$$

where R_t is the risk-free rate of return on a completely illiquid asset (pure capital) during period

t, and e_{jt} is the interest rate on credit card type j. There are two categories of consumers using credit cards. Some consumers pay interest to the credit card issuing banks, and the others do not pay interest. The "representative consumer" pays the interest rate e_{jt}, averaged over both categories of consumers, including those who maintain rotating balances, and thereby pay interest on contemporaneous credit card transactions, and those consumers who pay off such credit card transactions before the end of the period, and thereby do not pay explicit interest on credit card transactions. The CFS considers four credit card types, j , including Visa, MasterCard, Discover, and American Express.

The aggregation-theoretic exact approach defines the credit-card-augmented structural aggregator function, M, to be the utility function, v,

$$M_t = M(\mathbf{m}_t, \mathbf{c}_t) = v(\mathbf{m}_t, \mathbf{c}_t) \tag{2.4}$$

nested as a weakly separable category utility function within the full utility function, which also contains consumer goods. The growth rate of the Divisia index tracks the growth rate of the exact aggregate $M(\mathbf{m}_t, \mathbf{c}_t)$, where $\mathbf{m}_t$ is the vector of monetary asset quantities held by the representative consumer during period t, and $\mathbf{c}_t$ is the vector of the consumer's four credit card transaction volumes during period t.

The credit card quantities to include in the augmented Divisia index formula are the monthly credit card transactions volumes, not the credit card balances. Including credit card balances would produce double counting, since they include carried forward rotating balances used for transactions in prior periods. The growth rate of the credit-card-augmented Divisia index is

$$d \log M(\mathbf{m}_t, \mathbf{c}_t) = \sum_i s_{it} d \log m_{it} + \sum_j \tilde{s}_{jt} d \log c_{jt}, \tag{2.5}$$

where $s_{it} = \pi_{it} m_{it} / (\pi_t' \mathbf{m}_t + \tilde{\pi}_t' \mathbf{c}_t)$ is the expenditure share of monetary asset i in the total service of monetary assets and credit cards, while $\tilde{s}_{jt} = \tilde{\pi}_{jt} c_{jt} / (\pi_t' \mathbf{m}_t + \tilde{\pi}_t' \mathbf{c}_t)$ is the expenditure share of the services of credit card type j in the total services of monetary assets and credit cards.

Barnett and Su (2020) employ a production model of financial firms and investigate supply-side aggregation, when financial firms produce both monetary services and credit card transactions services. Inside money is a supply-side concept. They derive the conditions under which a joint supply-side aggregate over monetary and credit card transactions services exists. The existence condition for a supply-side inside-money aggregator function is blockwise weak separability of outputs within financial firm technology, while for a demand-side aggregator function the existence condition is blockwise weak separability within consumer tastes.

The user cost formula for supplied monetary assets differs from the demand-side user cost formula, when a regulatory wedge results from the existence of required reserve, imposing an implicit tax on banks. Until recently, reserve requirements existed and produced an especially large implicit tax during the late 1970s, when interest rates on bank loans were high. Barnett (1987) and Liu et al. (2020) show that the user cost of supplied monetary assets subject to reserve requirements is

$$\pi_{it} = \frac{(1-k_i)R_t - r_{it}}{1+R_t} \tag{2.6}$$

where k_i is the required reserve ratio on monetary asset i, r_{it} is the interest rate paid by the bank on those deposits, and R_t is the bank's rate of return on invested loans. The implicit tax is the foregone interest on uninvested required reserves. The credit-card-augmented Divisia inside money index is calculated from equation (2.5), using the monetary asset user cost from equation (2.6).

2.3 Out-of-sample Forecasting Method

This section provides our forecasting models. This paper forecasts inflation and real GDP growth at the h-period horizon and focuses on the forecasting model used in Stock and Watson (1999)and Stock and Watson (2003) and Rossi and Sekhposyan (2014). We test the estimated model's ability to forecast in out-of-sample data. That is, some data are dropped from the estimation sample to see how the model forecasts in the region of the data omitted from the estimation. The model is defined as

$$Y_{t+h}^{h} = \beta_0 + \beta_1(L)X_t + \beta_2(L)Y_t + e_{t+h}^{h} \tag{2.7}$$

where Y_{t+h}^{h} is either the h-period inflation at time t, defined by $Y_{t+h}^{h} = (1200/h)\ln(CPI_{t+h}/CPI_t) - 1200\ln(CPI_t/CPI_{t-1})$ or the cumulative growth of output over the h-periods at an annual percentage rate, defined by $Y_{t+h}^{h} = (1200/h)\ln(Q_{t+h}/Q_t)$, where X_t is a possible explanatory variable. Output during period t is Q_t, and the consumer price index during period t is CPI_t. The variable Y_t on the right hand side of (7) is either the first difference of the period t rate of inflation, $Y_t = 1200\ln(CPI_t/CPI_{t-1}) - 1200\ln(CPI_{t-1}/CPI_{t-2})$, or the period t output growth, $Y_t = 1200\ln(Q_t/Q_{t-1})$, while e_{t+h}^{h} is an error term. The lag polynomials in the lag operator L are $\beta_1(L)$ and $\beta_2(L)$.

We use Autoregressive Distributed Lag (ADL) time series regression in accordance with equation (2.7). In particular, we estimate the following model

$$Y_{t+h}^{h} = \alpha + \sum_{i=0}^{p} \beta_i Y_{t-i} + \sum_{j=0}^{r} \gamma_j D_{t-j} + e_{t+h} \tag{2.8}$$

where Y_{t+h}^{h} is the same as in equation (2.7) and Y_{t-i} for $i = 0, \cdots, p$ is either p lags of the change in monthly inflation or p lags of the monthly output growth, and D_{t-j}, $i = 0, \cdots, r$ is r lags of the change in log difference of Divisia monetary aggregates at time t. We set $h = 12$ to model one year ahead inflation change and output growth.

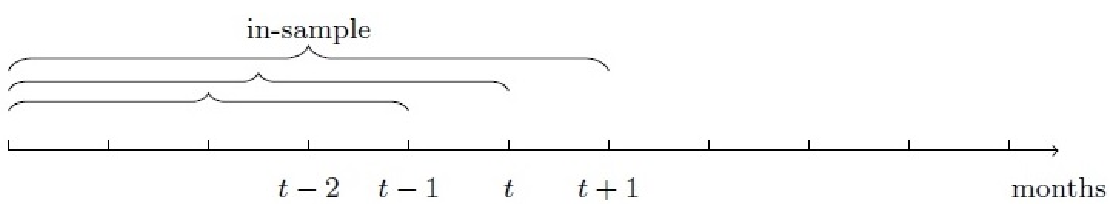


Figure 2.2: Recursive regression (expanding forecasting window)

We estimate equation (2.8) using in-sample data and recursively forecast inflation and output growth in the out-of-sample period. We forecast using only data before the forecast period in

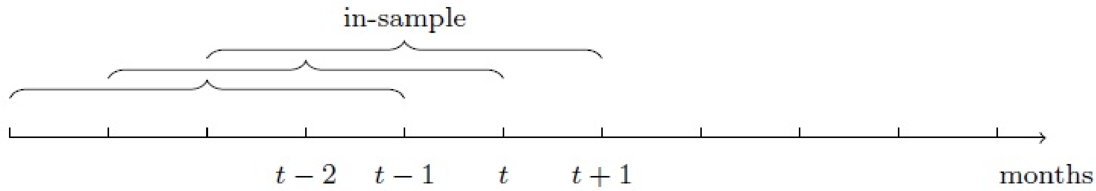


Figure 2.3: Rolling regression (fixed forecasting window)

the parameter estimation. The model is estimated by two different methods for out-of-sample forecasting. Figures 2.2 and 2.3 show these two forecasting methods. By the first method, we use recursive regression, setting the forecasting window three months ahead to forecast the observation at $t = 4$. We sequentially expand the forecasting window by one month until we have a forecast for all out-of-sample observations. For example, in 2006:09 we consider the forecast of the 12-month inflation rate and output growth from 2006:09 to 2007:09. Then moving forward one month, all the models are re-estimated using all prior data through 2006:10, and the forecast of inflation and output growth is computed from 2006:10 to 2007:10. The data period used in this paper is from 2006:07 to 2020:07, as is discussed in section 2.4. By using this method recursively, the period over which out-of-sample forecasts are computed is 2007:12 through 2020:07.

The second method to estimate the model is rolling regression using fixed windows of data to re-estimate the model over the periods. We fix the forecasting window at 60, 84, and 120 months. By using those three fixed windows, the periods of out-of-sample forecasts are 2012:09 through 2020:07, 2014:09 through 2020:07, and 2017:09 through 2020:07, respectively.

We calculate the forecast error and the root mean square forecast error (RMSFE) for each estimation to measure forecast performance. The forecast error is defined to be

$$Forecast\ Error = Y_{t+1} - \widehat{Y}_{t+1|t} \tag{2.9}$$

where Y_{t+1} is the value from the actual data not included in the estimation of coefficients in the regression and $\widehat{Y}_{t+1|t}$ is a forecast of Y_{t+1} based on the data in the past with the coefficients estimated using data through period T. The RMSFE of the recursive regression is

$$RMSFE_{recursive} = \sqrt{\frac{1}{T-3}\sum_{j=3}^{T} E[(Y_{j+1} - \widehat{Y}_{j+1|j})^2]}, \tag{2.10}$$

and the RMSFE of the rolling regression is

$$RMSFE_{rolling} = \sqrt{\frac{1}{T-N}\sum_{j=N}^{T} E[(Y_{j+1} - \widehat{Y}_{j+1|j})^2]}, \tag{2.11}$$

where N is the size of the forecasting window, set to be 60, 84, or 120, respectively. To measure the spread of the forecast error distribution and magnitude of a typical forecasting error, we compute RMSFE for all measures of aggregate monetary services. Comparing RMSFE, we can determine whether credit-card-augmented Divisia monetary aggregates perform better as indicators in forecasting than the original Divisia monetary aggregates.

2.4 Data

To evaluate the credit-card-augmented Divisia monetary aggregates as indicators in forecasting, we forecast inflation and output growth using three different Divisia monetary aggregates: (1) the original Divisia monetary aggregates, (2) the credit-card-augmented Divisia monetary aggregates, and (3) the credit-card-augmented Divisia inside money aggregates.

The three types of Divisia monetary aggregates are available on a monthly basis from the Center for Financial Stability (CFS). The CFS provides Divisia monetary aggregates to the public from 1967:01 along with credit-card-augmented Divisia monetary services and credit-card-augmented Divisia inside-money services from 2006:07. The CFS original Divisia data are level normalized to equal 100 in 1967:01. The credit-card-augmented Divisia money and credit card-augmented Divisia inside money data are level normalized to equal 100 in 2006:7.

Inflation is measured using the seasonally adjusted Consumer Price Index (CPI) with base year of 1984, as reported monthly by the Bureau of Labor Statistics (BLS). For output growth, we use

Variable	Description	Source
CPI	Consumer price index (1982-1984=100, Seasonally Adjusted)	BLS
GDP	Gross Domestic Product (Real)	IHS Markit
DivisiaM1	Divisia monetary aggregates: M1	CFS
DivisiaM1A	Credit card-augmented Divisia: M1	CFS
DivisiaM1AI	Credit card-augmented Divisia inside money: M1	CFS
DivisiaM2	Divisia monetary aggregates: M2	CFS
DivisiaM2A	Credit card-augmented Divisia: M2	CFS
DivisiaM2AI	Credit card-augmented Divisia inside money: M2	CFS
DivisiaMZM	Divisia monetary aggregates: MZM	CFS
DivisiaMZMA	Credit card-augmented Divisia: MZM	CFS
DivisiaMZMAI	Credit card-augmented Divisia inside money: MZM	CFS
DivisiaALL	Divisia monetary aggregates: ALL	CFS
DivisiaALLA	Credit card-augmented Divisia: ALL	CFS
DivisiaALLAI	Credit card-augmented Divisia inside money: ALL	CFS
DivisiaM3	Divisia monetary aggregates: M3	CFS
DivisiaM3A	Credit card-augmented Divisia: M3	CFS
DivisiaM3AI	Credit card-augmented Divisia inside money: M3	CFS
DivisiaM4-	Divisia monetary aggregates: M4-	CFS
DivisiaM4A-	Credit card-augmented Divisia: M4-	CFS
DivisiaM4AI-	Credit card-augmented Divisia inside money: M4-	CFS
DivisiaM4	Divisia monetary aggregates: M4	CFS
DivisiaM4A	Credit card-augmented Divisia: M4	CFS

Table 2.2: Description of Data Set

Note. BLS is Bureau of Labor Statistics. CFS is Center for Financial Stability.

monthly real GDP, as provided by IHS Markit.[3] The period of our data is 2006:07-2020:07 for all variables, since the credit-card-augmented Divisia data are provided from 2006.7. Table 2.2 contains a description of our data.

Stock and Watson (1999) include M1, M2, M3, and L (now M4) as explanatory variables. Rossi and Sekhposyan (2010), who include M2 and M3 as indicators in forecasting US output

[3]See https://ihsmarkit.com/products/us-monthly-gdp-index.html. First, IHS Markit derives a raw index from various monthly data, mostly source data of the official quarterly GDP of the Bureau of Economic Analysis (BEA). Then they calculate a monthly residual that reconciles the raw index with official GDP at the quarterly frequency. While nowcasting methods provide alternative measures of monthly GDP, the IHS Market measure is the closest to a direct measure. The BEA quarterly GDP data are currently the only available fully directly measured GDP data.

growth and inflation, use an identical model to Stock and Watson (2003), but with different data. We use the M1, MZM, M2, and ALL clusterings of components as narrow money measures and the M3, M4-, and M4 component groupings in the broad money measures for all three Divisia indices. The groupings are as defined by the CFS.

M1 includes currency, demand deposits, travelers checks, and other checkable deposits. M2 includes the components of M1 along with savings deposits, money market accounts, retail money market mutual funds, and small time deposits. MZM stands for Money Zero Maturity. It includes the components of M1 along with the components of M2, other than time deposits, and also money market funds. ALL adds institutional money market funds to the components of M2. M3 is a broad concept of the money, including the components of M2 along with large time deposits, institutional money market funds, and term Eurodollars.[4] M4- adds commercial paper to the components of M3, while M4 adds Treasury Bills to the components of M4-. Table 2.3 fully defines the component groupings.

[4]See Barnett et al. (1992) and Barnett et al. (2013).

Asset	Divisia M1	Divisia M2M	Divisia MZM	Divisia M2	Divisia M2 ALL	Divisia M3	Divisia M4-	Divisia M4
Currency	∨	∨	∨	∨	∨	∨	∨	∨
Travelers Checks	∨	∨	∨	∨	∨	∨	∨	∨
Demand Deposits	∨	∨	∨	∨	∨	∨	∨	∨
OCD Commercial	∨	∨	∨	∨	∨	∨	∨	∨
OCD Thrift	∨	∨	∨	∨	∨	∨	∨	∨
Savings Deposits Commercial		∨	∨	∨	∨	∨	∨	∨
Savings Deposits Thrift		∨	∨	∨	∨	∨	∨	∨
Retail Money Market Funds		∨	∨	∨	∨	∨	∨	∨
Small Time Deposits Commercial				∨	∨	∨	∨	∨
Small Time Deposits Thrift				∨	∨	∨	∨	∨
Institutional Money Market Funds			∨		∨	∨	∨	∨
Large Time Deposits						∨	∨	∨
Overnight and Term Repos						∨	∨	∨
Commercial Paper							∨	∨
T-Bills								∨

Table 2.3: Components of Aggregates

Source: Barnett et al. (2013).

2.5 Empirical Analysis

2.5.1 Stationarity

Before forecasting, we conduct a unit root test to check whether all variables are stationary. We use the Augmented Dickey-Fuller (ADF) test, which allows for higher order autoregressive processes, from the following equation:

$$\Delta y_t = \mu + \delta t + \phi y_{t-1} + \sum_{i=1}^{k} \theta_i \Delta y_{t-k} + \varepsilon_t \tag{2.12}$$

The null hypothesis is that variables are non-stationary, so that $\phi = 1$. We reject the null hypothesis at 5% significance level, if a a p-value is less than 0.05. Then the data have no unit root and are stationary. The test statistic of this test is

$$\tau = \frac{\phi - 1}{SE(\phi)} \tag{2.13}$$

where $SE(\phi)$ is standard error of ϕ in equation (2.12). Table 2.2 shows the results of the Augmented Dickey-Fuller test. All variables are found to be stationary in the forecasting model.

2.5.2 Bayesian Information Criterion (BIC)

Before we can forecast inflation and output growth, we must choose the number of lags, p and r, in equation (2.8). We determine the number of lags by the Bayesian Information Criterion (BIC). BIC is a criterion for model selection among a finite set of models and widely used for model identification in time series and linear regression.

BIC is defined as

$$BIC = k\ln(n) - 2\ln(L) \tag{2.14}$$

where k is the number of parameters in the model, n is the number of observations, and L is the maximized value of the model's likelihood function. Using that criterion, we find $p = 0$ and $r = 0$

Variable	ADF test statistic (ϕ)	p-Value	Decision
CPI (Log difference)	-7.3912	0.0000	Stationary
GDP(Log difference)	-11.2480	0.0000	Stationary
DivisiaM1 (Log difference)	-14.1586	0.0000	Stationary
DivisiaM1A (Log difference)	-14.5593	0.0000	Stationary
DivisiaM1AI (Log difference)	-15.3340	0.0000	Stationary
DivisiaM2 (Log difference)	-9.1965	0.0000	Stationary
DivisiaM2A (Log difference)	-10.0913	0.0000	Stationary
DivisiaM2AI (Log difference)	-10.5895	0.0000	Stationary
DivisiaMZM (Log difference)	-6.6509	0.0000	Stationary
DivisiaMZMA (Log difference)	-7.2618	0.0000	Stationary
DivisiaMZMAI (Log difference)	-7.5137	0.0000	Stationary
DivisiaALL (Log difference)	-6.0977	0.0000	Stationary
DivisiaALLA (Log difference)	-6.6952	0.0000	Stationary
DivisiaALLAI (Log difference)	-6.8814	0.0000	Stationary
DivisiaM3 (Log difference)	-10.7024	0.0000	Stationary
DivisiaM3A (Log difference)	-10.6399	0.0000	Stationary
DivisiaM3AI (Log difference)	-10.4619	0.0000	Stationary
DivisiaM4- (Log difference)	-10.0059	0.0000	Stationary
DivisiaM4A- (Log difference)	-9.9153	0.0000	Stationary
DivisiaM4AI- (Log difference)	-9.7785	0.0000	Stationary
DivisiaM4 (Log difference)	-9.8832	0.0000	Stationary
DivisiaM4A (Log difference)	-10.0719	0.0000	Stationary

Table 2.4: Augmented Dickey-Fuller (ADF) Test Results

in the model, which thereby is ADL(0,0) for both inflation and output growth. Then the right-hand side of the equation (2.8) has Y_t and D_t with no lags. The model which includes only the information immediately before time t is best at 12-month-ahead forecasting of both inflation and output growth.

2.5.3 Forecasting Results

We forecast annual US inflation and output growth using the original Divisia, the credit-card-augmented Divisia, and the credit-card-augmented Divisia inside money aggregates, as provided by the Center for Financial Stability (CFS) at M1, M2, MZM, ALL, M3, M4-, and M4 levels of aggregation. We compute Root Mean Square Forecast Error (RMSFE) to evaluate the forecasts. We now provide the results of the forecasts.

2.5.3.1 Inflation Forecasting

Using 3 months in-sample data at first and expanding the forecasting window by 1 month for recursive regression, we estimate equation (2.8) by multivariate linear regression in samples of 155 observations (T=155). We then recursively forecast the change in inflation and output growth in the out-of-sample periods. Hence, our first 12-months-ahead out-of-sample forecast is computed at 2006:12. The data set starts at 2006:07. We lose the first two observations, because we generate new variables by taking differences and using lags. Also, since the first regression uses 3 months of observations, forecasts of those 3 months will not be calculated. Table 2.5 shows the results of RMSFE in the case of the recursive regression.

When we consider M1 as a monetary indicator, RMSFE of the original Divisia is 3.8607, of the credit-card-augmented Divisia is 3.8169, and of the credit-card-augmented Divisia inside money aggregate is 3.6567. The RMSFE for M2 of the original Divisia is 3.8330, of the credit-card-augmented Divisia is 3.8070, and of the credit-card-augmented Divisia inside money aggregate is 3.7834. The RMSFE for MZM of original Divisia is 3.7828, of the credit-card-augmented Divisia is 3.7598, and of the credit-card-augmented Divisia inside money aggregate is 3.7304. The

		full sample	Great Recession	After Great Recession
	Divisia	3.8607	5.0661	3.7047
M1	CA-Divisia	3.8169	5.2434	3.6140
	CA-Divisia Inside	3.6567	4.9477	3.4773
	Divisia	3.8330	5.4783	3.5741
M2	CA-Divisia	3.8070	5.5062	3.5367
	CA-Divisia Inside	3.7834	5.4839	3.5119
	Divisia	3.7828	5.3862	3.5310
MZM	CA-Divisia	3.7598	5.4196	3.4960
	CA-Divisia Inside	3.7304	5.3902	3.4652
	Divisia	3.8243	5.4984	3.5538
ALL	CA-Divisia	3.8065	5.5207	3.5284
	CA-Divisia Inside	3.7853	5.5034	3.5050
	Divisia	3.6903	5.8382	3.3143
M3	CA-Divisia	3.6902	5.8328	3.3172
	CA-Divisia Inside	3.6920	5.8495	3.3160
	Divisia	3.6491	5.6831	3.2991
M4-	CA-Divisia	3.6482	5.6806	3.3001
	CA-Divisia Inside	3.6492	5.6916	3.2993
M4	Divisia	3.7002	5.7348	3.3514
	CA-Divisia	3.7017	5.7326	3.3550

Table 2.5: Forecasting Inflation (recursive regression): Root Mean Square Forecast Error

Note. Full sample: 2007.12-2020.2. Great Recession: 2007.12-2009.6. After Great Recession: 2009.7-2020.2.

RMSFE for the ALL level of aggregation of the original Divisia is 3.8243, of the credit-card-augmented Divisia is 3.8065, and of the credit-card-augmented Divisia inside money aggregate is 3.7853. For each of the narrow money levels of aggregation, M1, M2, MZM, and ALL, the credit-card-augmented Divisia aggregate produces smaller forecast errors than the original Divisia, while the credit-card-augmented Divisia inside money aggregate performs best in forecasting inflation.

The following are the result when the model uses M3 as a monetary indicator for forecasting inflation. The RMSFE of the original Divisia is 3.6903, of the credit-card-augmented Divisia is 3.6902, and of the credit-card-augmented Divisia inside money aggregate is 3.6920. For M4-, the RMSFE of the original Divisia is 3.6491, of the credit-card-augmented Divisia is 3.6482, and of the credit-card-augmented Divisia inside money aggregate is 3.6492. The RMSFE of M4 for original Divisia is 3.7002 and of the credit-card-augmented Divisia is 3.7017. Credit-card-augmented Divisia inside money does not exist for M4 separately from M4-, since M4 is defined to be M4- plus T-Bills. But credit-card-augmented Divisia inside money must exclude T-Bills, which are not produced outputs of bank services. Hence, credit-card-augmented Divisia inside money for M4 is identical to credit-card-augmented Divisia for M4-.

For broad money, we find little difference in RMSFEs among the three Divisia indices. Unlike narrow money, RMSFEs of credit-card-augmented Divisia inside money for M3, and M4- are not the smallest among the three Divisia indices. The demand side credit-card-augmented Divisia aggregates tend to perform best with the broad aggregates. Also, we find that RMSFEs of the broad aggregates are smaller than those of the narrow aggregates. Overall, the M4- level of aggregation performs best for forecasting inflation, especially when the monetary aggregate is credit-card-augmented. Similarly, Barnett et al. (1984), with much earlier data, found that the most stable demand-for-money functions were acquired with the broadest Divisia monetary aggregates, although credit-card-augmented aggregates were not known at that time.

Our other forecasting method is rolling regression with a fixed forecasting window. Table 2.6 reports the rolling regression results for forecasting annual inflation. We find that RMSFEs with all types of money are smaller than with recursive regression. When the forecasting window is

Forecasting window (months)		60	84	120
	Divisia	2.4742	2.3209	2.0679
M1	CA-Divisia	2.4282	2.3120	2.0404
	CA-Divisia Inside	2.4066	2.3051	2.0348
	Divisia	2.4815	2.3333	2.0565
M2	CA-Divisia	2.4548	2.3258	2.0438
	CA-Divisia Inside	2.4488	2.3229	2.0414
	Divisia	2.5006	2.3195	2.0688
MZM	CA-Divisia	2.4795	2.3165	2.0558
	CA-Divisia Inside	2.4751	2.3127	2.0541
	Divisia	2.5245	2.3180	2.0917
ALL	CA-Divisia	2.5066	2.3169	2.0774
	CA-Divisia Inside	2.5033	2.3116	2.0755
	Divisia	2.3943	2.3820	2.0188
M3	CA-Divisia	2.4089	2.4003	2.0152
	CA-Divisia Inside	2.4069	2.4115	2.0160
	Divisia	2.3824	2.3629	2.0127
M4-	CA-Divisia	2.3909	2.3747	2.0057
	CA-Divisia Inside	2.3877	2.3831	2.0049
M4	Divisia	2.3672	2.3374	2.0299
	CA-Divisia	2.3743	2.3452	2.0270

Table 2.6: Forecasting Inflation (rolling regression): Root Mean Square Forecast Error

fixed, credit-card-augmented Divisia inside money works better than the other two Divisia indices for narrow money, but not for the broad money. The larger the forecasting window, the smaller the forecasting error. To be specific, when the forecasting window is 120 months (N=120), the M4-component grouping is the best for forecasting inflation with both rolling regression and recursive regression.

In particular, we find that M4- and M4 work best for forecasting inflation by the recursive regression and rolling regression, respectively. Although the results in this paper show the importance of broad money in the prediction of inflation, the Federal Reserve no longer publishes M3, M4-, and M4 data, but the Center for Financial Stability does.

2.5.3.2 Output Growth Forecasting

Table 2.7 contains the RMSFE results with recursive regression for forecasting output growth. The RMSFE of M1 is 2.5595 for the original Divisia, 2.5039 for the credit-card-augmented Divisia, and 2.5519 for credit-card-augmented Divisia inside money aggregate. For M2, the RMSFE of the original Divisia is 2.5393, of the credit-card-augmented Divisia is 2.5028, and of the credit-card-augmented Divisia inside money aggregate is 2.5299. The RMSFE of the original Divisia for MZM is 2.6219, of the credit-card-augmented Divisia is 2.5907, and of the credit-card-augmented Divisia inside money aggregate is 2.6194. The RMSFE of the ALL component grouping for the original Divisia is 2.6771, for the credit-card-augmented Divisia is 2.6461, and for the credit-card-augmented Divisia inside money aggregate is 2.6730. When we consider the broad M3 as a monetary indicator for forecasting output growth, the RMSFE of the original Divisia is 2.7523, for the credit-card-augmented Divisia is 2.7357, and for the credit-card-augmented Divisia inside money aggregate is 2.7510. For M4-, the RMSFE of the original Divisia is 2.7576, for the credit-card-augmented Divisia is 2.7448, and for the credit-card-augmented inside money aggregate is 2.7563. The RMSFE of M4 for the original Divisia is 2.7761 and for the credit-card-augmented Divisia is 2.7682.

The RMSFEs of each of the Divisia indices for M2 are the lowest among the seven types of money, and the narrow money performs better than the broad money. Also, the results show that for each of the component groupings, credit-card-augmented Divisia works best for forecasting output growth among all three Divisia indices.

When we compare the results of forecasting inflation and output growth during the Great Recession, the RMSFEs of inflation forecasting during the Great Recession are greater than during other periods. The Great Recession between 2007-2009 was associated with the financial crisis, which was deeply related to money.

Table 2.8 focuses on forecasting output growth using rolling regression. Contrary to the results when forecasting inflation, the smaller the forecasting window the smaller the forecasting errors. Consistent with the results from recursive regression, we see that the M2 component grouping

		full sample	Great Recession	After Great Recession
M1	Divisia	2.5595	2.4544	2.0181
	CA-Divisia	2.5039	2.5270	1.8894
	CA-Divisia Inside	2.5519	2.4096	2.0050
M2	Divisia	2.5393	2.5888	1.8876
	CA-Divisia	2.5028	2.6029	1.8048
	CA-Divisia Inside	2.5299	2.5994	1.8571
MZM	Divisia	2.6219	2.5190	2.0604
	CA-Divisia	2.5907	2.5353	1.9931
	CA-Divisia Inside	2.6194	2.5283	2.0474
ALL	Divisia	2.6771	2.5691	2.1254
	CA-Divisia	2.6461	2.5817	2.0583
	CA-Divisia Inside	2.6730	2.5770	2.1093
M3	Divisia	2.7523	2.6267	2.2512
	CA-Divisia	2.7357	2.6306	2.2106
	CA-Divisia Inside	2.7510	2.6305	2.2447
M4-	Divisia	2.7576	2.6356	2.2662
	CA-Divisia	2.7448	2.6390	2.2319
	CA-Divisia Inside	2.7563	2.6392	2.2597
M4	Divisia	2.7761	2.6177	2.3092
	CA-Divisia	2.7682	2.6221	2.2861

Table 2.7: Forecasting Output Growth (recursive regression): Root Mean Square Forecast Error

Note. Full sample: 2007.12-2020.2. Great Recession: 2007.12-2009.6. After Great Recession: 2009.7-2020.2.

Forecasting window (months)		60	84	120
	Divisia	2.6484	2.9967	3.7988
M1	CA-Divisia	2.5683	2.9278	3.7748
	CA-Divisia Inside	2.6377	2.9904	3.8031
	Divisia	2.5703	2.9767	3.8593
M2	CA-Divisia	2.5237	2.9336	3.8408
	CA-Divisia Inside	2.5581	2.9708	3.8617
	Divisia	2.6212	3.0134	3.8916
MZM	CA-Divisia	2.5796	2.9728	3.8743
	CA-Divisia Inside	2.6161	3.0117	3.8980
	Divisia	2.7072	3.0894	3.9364
ALL	CA-Divisia	2.6599	3.0420	3.9240
	CA-Divisia Inside	2.7017	3.0859	3.9439
	Divisia	2.9046	3.2254	3.9306
M3	CA-Divisia	2.8653	3.1955	3.9363
	CA-Divisia Inside	2.8990	3.2287	3.9487
	Divisia	2.9499	3.2641	3.9165
M4-	CA-Divisia	2.9160	3.2384	3.9205
	CA-Divisia Inside	2.9445	3.2684	3.9297
M4	Divisia	2.9418	3.2541	3.9261
	CA-Divisia	2.9160	3.2358	3.9332

Table 2.8: Forecasting Output Growth (rolling regression): Root Mean Square Forecast Error

works best for forecasting output growth with rolling regression.

2.6 Bayesian Vector Autoregression (BVAR) Approach

In this section, we examine the forecasting performance of credit-card-augmented Divisia money in another way. We estimate a Bayesian Vector Autoregression (BVAR) model for forecasting U.S. annual inflation and output growth.

2.6.1 Model Specification

Let Y_t be the vector containing the M variables at time t with the model having p lags of the variables as follows:

$$\mathbf{Y}_t = \mathbf{B}_0 + \mathbf{B}_1\mathbf{Y}_{t-1} + \mathbf{B}_2\mathbf{Y}_{t-2} + \cdots + \mathbf{B}_p\mathbf{Y}_{t-p} + \varepsilon_t \tag{2.15}$$

where $\varepsilon_t \sim N(\mathbf{0}, \Psi)$, $\mathbf{B}_0$ is an $M \times 1$ vector of intercepts, $\mathbf{B}_1, \cdots, \mathbf{B}_p$ are $M \times M$ autoregressive matrices of coefficients, and Ψ is a $M \times M$ variance-covariance matrix. In this application, we set $M = 3$. The three variables for forecasting inflation are the h-period inflation, the change in monthly inflation, and the change in the growth rate of Divisia monetary aggregates, or alternatively for forecasting output growth the three variables are the h-period output growth, the monthly output growth, and the change in the growth rate of a Divisia monetary aggregate. We still set $h = 12$ in estimating the annual inflation and the annual output growth.

Equation (2.15) can equivalently be written as

$$\mathbf{y}_t = \mathbf{x}_t\beta + \varepsilon_t \tag{2.16}$$

where $\mathbf{y}_t = \mathbf{Y}_t$, $\mathbf{x}_t = \mathbf{I}_M \otimes [\mathbf{1}, \mathbf{Y}'_{t-1}, \cdots, \mathbf{Y}'_{t-p}]$, $\beta = vec([\mathbf{B}_0, \mathbf{B}_1, \cdots, \mathbf{B}_p])$, and $\varepsilon_t \sim N(0, \Psi)$.

In BVAR estimation, the choice of prior is central to the procedure. We adopt the Minnesota prior. Litterman (1980) introduced the Minnesota prior, which assumes that each variable follows a random walk process. The Minnesota prior then assumes that the covariance matrix is known and diagonal, $\hat{\Psi} = diag(\sigma_1^2, \cdots, \sigma_M^2)$, and its hyperparameters are fixed. Kadiyala and Karlsson (1997) investigate BVAR forecasting. They observe that the Minnesota prior is frequently used in forecasting applications under this prior distribution specification:

$$\beta | \mathbf{y}, \Psi \sim N(\bar{\beta}, \bar{\Omega}) \tag{2.17}$$

so that the prior distribution of β is normal with the prior mean $\bar{\beta}$ and covariance matrix $\bar{\Omega}$.

Given the prior distribution and the observations, the posterior distribution was produced numerically using the Gibbs sampling algorithm with 1000 iterations of the Gibbs sampler. As in the previous forecasting methods, we use the same period of the data, from 2006:07 to 2020:02, for all variables, since the credit card-augmented Divisia indices are provided from 2006:07. In our BVAR estimation, we set 120 months as the in-sample period and forecast annual inflation and output growth for 35 months. Unlike in the previous forecasting methods, the optimal number of lags result in our estimation of the VAR model with 3 lags for forecasting inflation and 2 lags for forecasting output growth.

2.6.2 Results

Table 2.9 displays Root Mean Squared Forecast Errors (RMSFEs) from the results of BVAR estimation in forecasting annual inflation and output growth. Conditionally on the prior specification over parameters, the credit-card-augmented Divisia performs better than the original Divisia in forecasting inflation with narrow money.

In the BVAR estimation, credit-card-augmented Divisia forecast output growth better than original Divisia with broad money, M3, M4-, and M4. However, all Divisia indices show very small differences in RMSFEs as indicators for output growth.

2.7 Conclusion

Credit-card-augmented Divisia monetary aggregates provide valuable measures of monetary service flows in the economy. We assess the credit-card-augmented Divisia monetary aggregates, provided by the CFS, as indicators in forecasting annual US inflation and output growth. We estimate an Autoregressive Distributed Lag (ADL) model using in-sample data with an expanding forecast window and recursively predict the change in annual inflation and output growth in the out-of-sample period. The model is also estimated with a fixed forecast window as a robustness check. In each case, we evaluate the forecasts by calculating Root Mean Square Forecast Error

		Inflation	Output Growth
M1	Divisia	2.1351	3.7814
	CA-Divisia	2.1163	3.7887
	CA-Divisia Inside	2.1363	3.7833
M2	Divisia	2.1690	3.7654
	CA-Divisia	2.1616	3.7674
	CA-Divisia Inside	2.1710	3.7688
MZM	Divisia	2.1166	3.7671
	CA-Divisia	2.1157	3.7708
	CA-Divisia Inside	2.1269	3.7713
ALL	Divisia	2.1333	3.7642
	CA-Divisia	2.1303	3.7671
	CA-Divisia Inside	2.1390	3.7689
M3	Divisia	2.2358	3.7776
	CA-Divisia	2.2361	3.7761
	CA-Divisia Inside	2.2358	3.7767
M4-	Divisia	2.2286	3.7788
	CA-Divisia	2.2290	3.7772
	CA-Divisia Inside	2.2264	3.7778
M4	Divisia	2.2528	3.7746
	CA-Divisia	2.2571	3.7743

Table 2.9: BVAR Forecasting Inflation and Output Growth: Root Mean Square Forecast Error

Note. Full sample: 2007.12-2020.2. Great Recession: 2007.12-2009.6. After Great Recession: 2009.7-2020.2.

(RMSFE).

We find that credit-card-augmented Divisia is a valuable indicator in predicting annual inflation. Moreover, the credit-card-augmented Divisia inside money aggregates are even better than the credit-card-augmented Divisia monetary aggregates for forecasting inflation, when the monetary aggregates use narrow money component clusters. The broad money aggregates are very effective indicators in predicting annual inflation with all seven component groupings, with the M4- component cluster being especially successful.

Also, we investigate the ability of credit-card-augmented Divisia as an indicator in forecasting annual output growth. For all types of money this paper examines, we see that credit-card-augmented Divisia is consistently the best in forecasting output growth among the original Divisia, the demand-side credit-card-augmented Divisia, and the supply-side credit-card-augmented Divisia inside money aggregate. In addition, narrow money component groupings are more effective than the broad money component clusterings as indicators in forecasting output growth, with the M2 being the best.

We also examine the forecasting performance of credit-card-augmented Divisia in an alternative approach. We use Bayesian vector autoregression and find that the narrow credit-card-augmented Divisia measures forecast U.S. annual inflation well, while the broad credit-card-augmented Divisia are best in forecasting output growth. But in all cases, the forecasting errors are very small.

Barnett et al. (2016b) are working on an indicator-optimized variant of the credit-card-augmented monetary aggregates, but that variant is not currently available to the public. As a result, we are not yet able to determine its performance in forecasting.

Chapter 3

Welfare Cost of Inflation with Credit Card Transactions in Money Measures

Sohee Park

Abstract

This paper investigates the welfare costs that occurred by anticipated inflation when we add the volume of credit card transactions to the measurement of money. First, we use the concept of credit card-augmented Divisia in the dynamic stochastic general equilibrium (DSGE) model and calculate the welfare costs of inflation. This paper assumes money yields utility in the money-in-the-utility function of Sidrauski (1967). This paper also empirically examines the welfare costs of inflation in the U.S. by deriving the inverse money demand functions with the consumer surplus approach using the Divisia indices from the Center for Financial Stability (CFS). The welfare costs of inflation with the credit card services are lower than those with no credit card services in the New Keynesian model. With the empirical method, we see more sensitive changes in the welfare cost of inflation with broad money and the monetary aggregation containing the credit card transactions volume when the inflation target changes.

3.1 Introduction

When the economy encounters unforeseen economic events such as the Great Recession and the pandemic, soaring inflation causes the matter that makes the economy volatile and reduces the purchasing power of economic agents. Especially during the pandemic, the economy has faced high inflation pressure since federal government spending has dramatically increased and there is a considerable amount of money injected into the economy. This government spending has been financed by issuing Treasury bond which is a debt of government with a long maturity, while the Federal Reserve has been monetizing the new debt by buying a comparable amount of Treasury securities financed by “printing" money. The money supply has been growing at alarmingly rapid rates. The Federal Reserve’s “monetization" of the new debt will increase in inflationary pressures as the pandemic ends and the economy recovers. The Federal Reserve will likely need to increase interest rates to control the inflationary pressure, thereby increasing the Treasury’s interest to pay on its debt. The percentage of tax revenue needed to be paid on federal debt service will grow. With these worrisome risks approaching, there will be a growing need for accurate information on federal debt monetization and the resulting growth in monetary services, including credit card transaction services.

This paper investigates the welfare cost of inflation which is the change in social welfare caused by various inflation. This paper sheds light on the prediction of the welfare cost of inflation with monetary aggregation including credit card services by investigating quantitative and empirical answers to the following questions: How much of the reduction of output or consumption does the economy have to endure when we encounter certain inflation in the economy? How different is the welfare cost of inflation in the economy with credit card services compared to the economy without credit card services?

At the present time, the matter of credit card payment is increasingly notable because credit card services are growing in importance and the usage of the credit card deserves much consideration as credit card transactions have increased. Figure 3.1[1] displays the share of consumer payments instruments. While the share of cash is decreasing, the share of credit cards as payment instruments has been increasing and surpassed cash usage. It is 26.9% in 2020 and worth thinking about. Credit card transactions have never been included in central bank measures of the money

[1]Source: 2020 SCPC Tables, Federal Reserve Bank of Atlanta

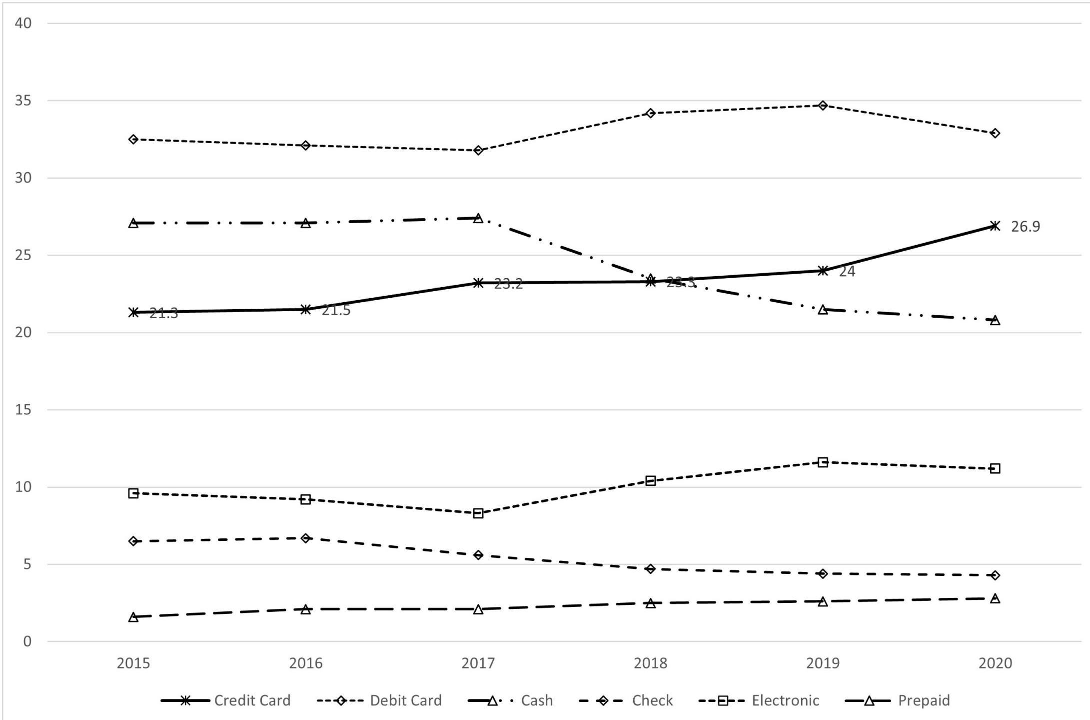


Figure 3.1: Share of Consumer Payments Instrument (%)

supply[2], since accounting conventions do not permit adding liabilities, such as credit card balances, to assets, such as money.[3] However, economic aggregation theory can be used to aggregate over service flows, regardless of whether the monetary services are from assets or liabilities. This paper adopts the concept of credit-card-augmented Divisia monetary aggregation including the amount of credit card transactions volume and introduces it to the model. In addition to the Divisia introduced by Barnett (1980)[4], Barnett and Su (2016) introduced the model to include the production of credit card transaction services using the approach developed initially for the demand side. They derive a theory needed to measure the joint services of credit cards and money using index number theory, which measures service flow and is based on aggregation theory.

This paper estimates the welfare cost which is occurred by anticipated inflation when the credit

[2]The money supply is the traditional simple sum monetary aggregates, which are based upon accounting conventions.

[3]See Barnett and Su (2016).

[4]Barnett (1980) mentions that monetary policy is related to the behavior of indices of the quantity, price, and velocity of money. On behalf of the simple sum, Barnett (1980) explains that Divisia reflects the expenditure share of each monetary asset because monetary assets are not a perfect substitute and they also demonstrate that the monetary aggregates need user cost price which is used for calculation on Divisia monetary aggregates.

card transactions volume is added to the money measure. We separately introduce the quantitative model and empirical approach for the welfare cost of inflation. First, we set up a New Keynesian model and include money in the utility function. It means that credit card transactions of the household are included in their utility and affect their welfare. This is a very new attempt to use the concept of the credit-card-augmented Divisia in the dynamic stochastic general equilibrium (DSGE) model and calculate welfare cost. Second, we estimate the money demand functions and the welfare cost of inflation by the consumer surplus approach. It would make another attempt to investigate the welfare cost of inflation in another different way. In other words, the research question of this paper is that how do anticipated inflation affect the long-run values of macroeconomic variables when we consider credit-card-augmented Divisia index as money. This paper uses Divisia indices as money to see how monetary aggregates would be revised over time as interest rates change and see the effects from them.

The key results of this paper are the following. First, we find that the welfare costs of inflation with the credit card services in the model are lower than those with no credit card services in the model. Also, the money aggregation including credit card transactions makes a more sensitive welfare cost of inflation depending on the inflation targets compared to the monetary aggregation with no credit card transactions volume. When we estimate two different money demand functions, log-log and semi-log money demand functions, the welfare costs of inflation measured in the semi-log function are changed more than the log-log function. Another finding is that the estimation with broad money (M4) shows larger changes in welfare costs than the narrow money (M1).

We consider not only Divisia but also the concept of credit-card-augmented Divisia that considers credit card transactions volume in the economic model. In that sense, this paper penetrates money specification precisely in the model. It is new research of application of the new monetary aggregation and related to notable macroeconomic variables, especially inflation, in volatile economic circumstances. Moreover, we explore both quantitative model and empirical approach to investigate in many different ways. Especially, Lucas (2000) and Ireland (2009) mentioned another remaining critical task replacing money with Divisia for future research of welfare cost of inflation.

Related Literature A variety of research studies the welfare cost of inflation with DSGE framework and empirical method with several types of monetary aggregates. Much of the previous literature used the simple sum monetary aggregation in the model for the calculation of welfare costs. Bailey (1956) treated real money balances as a consumption good and inflation as a tax on real balances. Cooley and Hansen (1989) used a cash-in-advance model with production, stochastic optimal growth model and an endogenous labor-leisure decision. They studied the quantitative importance of money in a Real Business Cycle model where money, M1, is introduced in a way that emphasizes the influence on real variables of anticipated inflation operating through the inflation tax. Cooley and Hansen (1991) pointed out a policy issue of zero inflation and found conditions which the costs of achieving zero inflation exceeded the benefits. Dotsey and Ireland (1996) calculated the amount of output which had to be given up for a certain inflation level, using a general equilibrium monetary model in which an inflationary tax distorted a variety of marginal decisions. They say that a sustained 4 percent inflation costs the economy the equivalent of 0.41 percent of output per year when currency is identified as the relevant definition of money and over 1 percent of output per year when simple sum M1 is defined as money. In an approach adopted here, Teo and Yang (2011) studied the welfare cost of inflation in a New Keynesian dynamic stochastic general equilibrium model where money was introduced into the model by adopting cash-in-advance (CIA) constraint for the consumption of the representative household. They argued that the welfare cost of inflation in a New Keynesian framework was much higher than its counterpart in a Real Business Cycle (RBC) model. The method of this paper is a New Keynesian for this reason.

The welfare cost of inflation was also approached by analyzing money demand. Phylaktis and Taylor (1993) demonstrated that the Cagan model of money demand under hyperinflation provided an adequate characterization of the monetary and inflationary experiences of five Latin American countries during the 1970s and 1980s. Lucas (2000) estimated the welfare cost of inflation in the Sidrauski model and McCallum-Goodfriend model that used M1 as money. That paper found that the gain from reducing the annual inflation rate from 10 percent to zero is equivalent to an increase in real income of slightly less than 1 percent. Ireland (2009) used post-1980 US data tracing out a stable long-run money demand relationship of Cagan's semi-log form between the M1-income ratio and the nominal interest rate. Integrating under this money demand curve yielded estimates of the welfare costs of modest departures from Friedman's zero nominal interest rate rule for the optimum quantity of money that was quite small.

There is also a couple of pieces of literature that estimates the welfare cost of inflation using Divisia monetary aggregates. Cysne (2003) considered economies where interest-bearing and non-interest-bearing monetary assets are used for transacting purposes and calculated the welfare costs of inflation using the Divisia methodology. Serletis and Virk (2006) investigated that the welfare cost of inflation using simple sum, Divisia monetary aggregation, and currency equivalent monetary aggregates. They set up the welfare cost functions and found that the Divisia monetary aggregates suggest a smaller welfare cost than the simple sum and currency equivalent aggregates. Serletis and Xu (2021) calculated the welfare cost of inflation by the Bailey (1956) approach in neoclassical demand theory and applied consumption analysis. They demonstrated that raising the rate of inflation from 2% to 4% in the U.S. would impose the welfare cost of 0.30 percent of output when money is measured by Divisia M4 monetary aggregates.

Layout This paper is organized as follows: Section 3.2 discusses the development of the quantitative model. Section 3.3 explains the calibration of the model and the results are in Section 3.4. Section 3.5 describes the empirical approach with the Cagan model and Section 3.6 concludes the paper.

3.2 Quantitative Model

In this section, we set up the environment to solve the dynamic stochastic general equilibrium model with credit card transaction volume in the money measure to estimate the welfare cost of inflation and monetary policy shock. Belongia and Ireland (2014) considered Divisia containing the noninterest-earning currency and interest-earning deposits in the money for a closed economy, New Keynesian model.

This paper also adopts a closed economy New Keynesian model and considers Divisia containing the currency, deposits, and the volume of credit card transactions in the money. We consider the credit card transaction volume, not the credit card balances. If we include credit card balances, they would produce double-counting problem, since they include carried forward rotating balances used for transactions in prior periods.

3.2.1 The Representative Household

This paper assumes money yields utility. The existence of money makes the easier exchange and increasing utility. Inflation makes a welfare loss because money holdings yield direct utility and higher inflation reduces real money balances. I adopt the money-in-the-utility function of Sidrauski (1967) for the representative household. Lucas (2000) also employs the general equilibrium model of Sidrauski which contains the consumption of goods and real balances. In the money-in-the-utility function, money is a commodity, and holding it creates a household's utility. Assume that the representative household considers the consumption, labor, and the monetary aggregation formed from currency, deposits, and credit card transactions volume. By including money in the utility of the representative household, we expect the calibration result that Divisia monetary aggregates including credit card services explain the economic circumstances well compared to the result of the model with the original Divisia money.

The representative household preferences at time t are

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U\left(C_t, \ell_t, \frac{M_t^{cc}}{P_t}\right) \tag{3.1}$$

where $\beta \in (0,1)$ is the subjective discount factor, C_t is consumption, ℓ_t is the leisure of the household, and $\frac{M_t^{cc}}{P_t}$ is real money holdings including credit card transactions volume. This paper assumes that the current period utility function of the representative agent is given by

$$U\left(C_t,\ell_t,\frac{M_t^{cc}}{P_t}\right)=\zeta\ln C_t+\eta\ln\ell_t+\theta\ln\left(\frac{M_t^{cc}}{P_t}\right) \tag{3.2}$$

where $\zeta>0$, $\eta>0$ and $\theta>0$ are utility weights on consumption, leisure and real money balances, respectively. The utility function is the form of logarithm of Cobb-Douglas and we assume $\zeta+\eta+\theta=1$.

This paper employs new money measure M_t^{cc} including currency N_t, the total nominal value of the household's deposits D_t, and the credit card transactions volume CC_t. The true monetary aggregate M_t^{cc} including credit card transactions volume CC_t in this model is collected by CES specification[5] as follow:

$$M_t^{cc}=\left[\nu^{\frac{1}{\omega}}N_t^{\frac{\omega-1}{\omega}}+\phi^{\frac{1}{\omega}}D_t^{\frac{\omega-1}{\omega}}+(1-\nu-\phi)^{\frac{1}{\omega}}CC_t^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \tag{3.3}$$

where $\omega>0$ is the elasticity of substitution[6] among currency, deposits, and real balance in producing the monetary aggregate M_t^{cc} and $\nu,\phi\in(0,1)$ denote the weights placed on the currency and deposits. This equation would be used in real terms by dividing by the nominal price level P_t. Substituting equation (3.3) into (3.2), an aggregator function over monetary and credit card services exists in this decision problem, since it can be factored out of the decision problem as a subfunction. Furthermore, this monetary aggregator function exists as a blockwise strongly separable subfunction of economic structure.[7]

Each period $t=0,1,2,\cdots$, the resources of the household are M_{t-1} units of currency, B_{t-1} units of bonds, τ_t of the lump-sum transfer from the monetary authority, L_t dollars of loan from

[5]See Barnett (1980), Belongia and Ireland (2014) and Belongia and Ireland (2019). Belongia and Ireland (2019) assume that monetary aggregator function takes the CES form because Lucas (2000) explains the specific form of the utility function makes the model consistent with balanced growth and CES function satisfies the homogeneity assumption.

[6]Since this is the early part of the attempt of calculation of true money aggregation with credit card transactions volume, for simplicity, we assume the elasticity of substitution between each pair of monetary assets is constant.

[7]For an index number theory to be able to track the aggregator function, the aggregator function must exist in the structure of the economy. That existence condition is blockwise weak separability and the aggregator function, equation (3.3), satisfies this existence condition.

the representative bank, and CC_t dollars worth of the capacity of credit card transactions volume. Household purchases B_t units of new bonds at the price of $1/r_t$ dollars per bond, where r_t is the gross nominal interest rate and remaining currency goes into an amount N_t. The share of the intermediate goods-producing firm $i \in (0,1)$ is $s_{i,t}$ at the price of $Q_{i,t}$ dollars per share[8]. Household's deposit D_t satisfies following constraint equation (3.4) and it can be written in real terms in equation (3.5).

$$M_{t-1} + B_{t-1} + \tau_t + \int_0^1 Q_{i,t} s_{i,t-1} \, di - \frac{B_t}{r_t} - \int_0^1 Q_{i,t} s_{i,t} \, di - N_t + L_t + CC_t \geq D_t \tag{3.4}$$

$$\frac{M_{t-1} + B_{t-1} + \tau_t - B_t/r_t - N_t + L_t + CC_t}{P_t} + \int_0^1 \frac{Q_{i,t}}{P_t}(s_{i,t-1} - s_{i,t}) \, di \geq \frac{D_t}{P_t} \tag{3.5}$$

Also, the household holds M_t units of currency into the next period at the end of each period. The household can also make CC_t amount of the credit card transactions at the start of the period and repay the credit card company $e_t CC_t$ dollars at the end of the period where e_t is the credit card gross interest rate.[9] Since CC_t is the volume of credit card transactions, not a level, the household does not face the double-counting problem. M_t satisfies following constraint

$$N_t + r_t^D D_t + \int_0^1 F_{i,t} s_{i,t} \, di + W_t(1 - \ell_t) - P_t C_t - r_t^L L_t - e_t CC_t \geq M_t \tag{3.6}$$

where r_t^D denotes the gross own yield on the deposits, C_t is the consumption excluding the amount of credit card use, $F_{i,t}$ is a nominal dividend payment the household receives for each share in each intermediate goods-producing firm $i \in (0,1)$, and r_t^L is the gross nominal interest rate on loans. W_t is the nominal wage rate. Since ℓ_t is the leisure of the household, $(1 - \ell_t)$ is the total units of labor

[8] See Belongia and Ireland (2014) and Belongia and Ireland (2019).

[9] See Barnett et al. (2016b). There are two categories of consumers using credit cards. First, they consider consumers who do pay interest to the credit card issuing banks, and the others do not pay interest. The "representative consumer" pays the interest rate e_{jt}, averaged over both categories of consumers who maintain such rotating balances, and thereby pay interest on contemporaneous credit card transactions, and also those consumers who pay off such credit card transactions before the end of the period, and do not pay explicit interest on the credit card transactions. This paper adopt this averaged credit card interest rate e_{jt} as e_t.

which consist of the household's labor to each intermediate goods-producing firm i as following and the household earns $W_t(1-\ell_t)$ of labor income. Equation (3.6) can also be written in real terms.

$$\frac{N_t + r_t^D D_t + W_t(1-\ell_t)}{P_t} + \int_0^1 \frac{F_{i,t}}{P_t} s_{i,t}\, di \geq C_t + \frac{r_t^L L_t + e_t CC_t + M_t}{P_t} \tag{3.7}$$

Finally, the representative household maximizes the utility function (3.2) subject to the constraints (3.3), (3.5), (3.7), and the Lagrangian for the problem and the first-order conditions are in Appendix B.

3.2.2 Production Sector

This paper follows the production sector in Belongia and Ireland (2014). In this model, the production sector consists of the representative firm producing finished goods and the representative firm producing intermediate goods, respectively.

3.2.2.1 The Representative Finished Goods-Producing Firm

The representative finished goods-producing firm uses $Y_{i,t}$ units of intermediate good produced by firm i to manufacture Y_t units of the finished good. The production function is the constant returns to scale and CES form given by

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\sigma-1}{\sigma}}\, di \right]^{\frac{\sigma}{\sigma-1}} \tag{3.8}$$

where $\sigma > 1$ is the elasticity of substitution between the various intermediate goods in producing the finished good. The finished goods-producing firm chooses $Y_{i,t}$ for all $i \in (0,1)$ to maximize profits:

$$P_t \left[\int_0^1 Y_{i,t}^{\frac{\sigma-1}{\sigma}}\, di \right]^{\frac{\sigma}{\sigma-1}} - \int_0^1 P_{i,t} Y_{i,t}\, di$$

where $P_{i,t}$ is the nominal price of the intermediate good i for all $t = 0, 1, 2, \cdots$. The first-order condition is

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} Y_t \tag{3.9}$$

for all $i \in (0,1)$ at $t = 0, 1, 2, \cdots$.

This paper assumes the market is competitive and the profit of the representative finished goods-producing firm in a competitive market is zero. Then the following condition satisfies in equilibrium for all $t = 0, 1, 2, \cdots$.

$$P_t = \left[\int_0^1 P_{i,t}^{1-\sigma}\, di\right]^{\frac{1}{1-\sigma}}$$

3.2.2.2 The Representative Intermediate Goods-Producing Firm

In this section, we define h_t as a unit of labor of household and $\ell_t + h_t = 1$. The representative intermediate goods-producing firm produces $Y_{i,t}$ units of intermediate good i by investing $h_{i,t}$ units of labor from the representative household.

$$h_t = \int_0^1 h_{i,t}\, di$$

The constant returns to scale technology is given by

$$Y_{i,t} = Z_t h_t \tag{3.10}$$

where Z_t is the aggregate technology shock. It follows a random walk process

$$\ln Z_t = \ln z + \ln Z_{t-1} + \varepsilon_{z,t} \tag{3.11}$$

where $z > 1$ and $\varepsilon_{z,t} \sim (0, \sigma_z^2)$.

The representative intermediate goods-producing firm sells output in a monopolistically competitive market and the firm sets the nominal price $P_{i,t}$ for its output $Y_{i,t}$ subject to the representative finished goods-producing firm's demand, equation (3.9). Furthermore, as Belongia and Ireland (2014), the intermediate goods-producing firm has a quadratic cost of adjusting its nominal price measured in units of the finished good as following:

$$\frac{\gamma}{2}\left[\frac{P_{i,t}}{\pi P_{i,t-1}}-1\right]^2 Y_t$$

where $\gamma \geq 0$ is the magnitude of the price adjustment costs and $\pi > 1$ is the gross steady-state inflation rate.

As in Belongia and Ireland (2014), we solve the intermediate goods-producing firm problem. From the Euler equation (3.53) in Appendix A, the intermediate goods-producing firm maximizes its real market value as follows:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Lambda_{3,t}\left[\frac{F_{i,t}}{P_t}\right]$$

subject to

$$\frac{F_{i,t}}{P_t} = \left[\frac{P_{i,t}}{P_t}\right]^{1-\sigma} Y_t - \left[\frac{P_{i,t}}{P_t}\right]^{-\sigma}\left(\frac{W_t Y_t}{P_t Z_t}\right) - \frac{\gamma}{2}\left[\frac{P_{i,t}}{\pi P_{i,t-1}}-1\right]^2 Y_t. \tag{3.12}$$

Then the firm's first-order condition is

$$\begin{aligned}&(1-\sigma)\Lambda_{3,t}\left[\frac{P_{i,t}}{P_t}\right]^{-\sigma} Y_t + \sigma\Lambda_{3,t}\left[\frac{P_{i,t}}{P_t}\right]^{-\sigma-1}\left(\frac{W_t Y_t}{P_t Z_t}\right) - \gamma\Lambda_{3,t}\left[\frac{P_{i,t}}{\pi P_{i,t-1}}-1\right]\left[\frac{Y_t P_t}{\pi P_{i,t-1}}\right] \\ &+ \beta\gamma\mathbb{E}_t\left\{\Lambda_{3,t+1}\left[\frac{P_{i,t+1}}{\pi P_{i,t}}-1\right]\left[\frac{Y_{t+1}P_{i,t+1}P_t}{\pi P_{i,t}^2}\right]\right\} = 0\end{aligned} \tag{3.13}$$

for all $t = 0, 1, 2, \cdots$.

3.2.3 Social Planner's Allocations

Social planner allocates the representative household's labor $h^*_{i,t}$ to produce each intermediate good i, $Y^*_{i,t}$, and finally to attain the most efficient level of finished good Y^*_t by maximizing following preferences at time t.

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\zeta\ln Y_t^* + \eta\ln\left(1-\int_0^1 h_{i,t}^*\,di\right)+\theta\ln\left(\frac{M_t^{cc}}{P_t}\right)\right]$$

The most efficient level of finished good Y_t^* is defined by

$$Y_t^* = \frac{\zeta}{\eta}Z_t(1-h_t) \tag{3.14}$$

and the output gap g_t^* is defined by

$$g_t^* = \frac{Y_t}{Y_t^*} = \frac{\eta Y_t}{\zeta Z_t(1-h_t)}. \tag{3.15}$$

3.2.4 The Representative Bank

The representative bank not only makes loans worth L_t to the representative household and accepts deposits worth D_t dollars but also offers credit card services. The bank's loans and deposits have following relationship.

$$L_t = (1-k_t)D_t \tag{3.16}$$

k_t is the required reserve ratio and it follows the AR(1) process:

$$\ln k_t = (1-\rho_k)\ln k + \rho_k \ln k_{t-1} + \varepsilon_{k,t} \tag{3.17}$$

where $k_t \in (0,1)$, $\rho_k \in [0,1)$, and $\varepsilon_{k,t}{\sim}(0,\sigma_k^2)$. Also, the credit card services that the bank offers allows the household to make transactions within the certain credit limit. For the simplicity, we assume the credit card transactions volume of the household is some part of its deposits as following.

$$CC_t = a_t D_t \tag{3.18}$$

Credit-deposit ratio a_t follows the AR(1) process:

$$\ln a_t = (1-\rho_a)\ln a + \rho_a \ln a_{t-1} + \varepsilon_{a,t} \tag{3.19}$$

$a_t \in (0,1)$, $\rho_a > 0$, and $\varepsilon_{a,t} \sim (0, \sigma_a^2)$.

The bank creates $x_t \frac{D_t}{P_t}$ units of output with total real value of deposit $\frac{D_t}{P_t}$ and $x_t \frac{CC_t}{P_t}$ units of output with total real value of credit card transaction volume $\frac{CC_t}{P_t}$ using a technology. x_t is the financial-sector cost shock.

$$\ln x_t = (1-\rho_x)\ln x + \rho_x \ln x_{t-1} + \varepsilon_{x,t} \tag{3.20}$$

where $x > 0$, $\rho_x \in (0,1)$, and $\varepsilon_{x,t} \sim (0, \sigma_x^2)$. Then the representative bank maximizes its profits.

$$\Pi_t = (r_t^L - 1)L_t + (e_t - 1)CC_t - (r_t^D - 1)D_t - P_t x_t \frac{D_t}{P_t} - P_t x_t \frac{CC_t}{P_t}$$

The representative bank is in a competitive market and its economic profits are zero. Setting $\Pi_t = 0$, we get the following equation.

$$r_t^D = (r_t^L - 1)(1 - k_t) + (e_t - x_t - 1)a_t - x_t + 1 \tag{3.21}$$

Equation (3.21) implies that the financial-sector cost shock x_t, the required reserve ratio k_t, and credit card interest rate e_t impact directly on the gross rate of deposits r_t^D.

3.2.5 The Monetary Authority

The monetary base is defined as the total amount of currency and commercial banks' deposits in the reserves in the central bank. M_t is sum of currency N_t and the amount of reserve N_t^{υ} which equals to $k_t D_t$, where k_t is the required reserve ratio and D_t is the amount of deposits.

$$M_t = N_t + N_t^{\upsilon} \tag{3.22}$$

$$N_t^{\upsilon} = k_t D_t \tag{3.23}$$

This paper assumes monetary authority in this model follows a simple Taylor-type class of rules similar in form to Annicchiarico and Rossi (2013) and Belongia and Ireland (2014). Let π_t and g_t^y be the gross rate of inflation and output growth.

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{3.24}$$

$$g_t^y = \frac{Y_t}{Y_{t-1}} \tag{3.25}$$

Then the gross nominal interest rate r_t is determined by the following equation.

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_r} \left(\frac{\pi_t}{\pi}\right)^{\rho_\pi} \left(\frac{g_t^*}{g^*}\right)^{\rho_{g^*}} \left(\frac{g_t^y}{g^y}\right)^{\rho_{g^y}} \exp(\varepsilon_{r,t}) \tag{3.26}$$

where ρ_r, ρ_π, ρ_{g^*}, and ρ_{g^y} are policy parameters and $\varepsilon_{r,t} \sim (0, \sigma_r^2)$.

3.2.6 Monetary Aggregation

Belongia and Ireland (2014) consider the true aggregate of monetary or liquidity services, the simple sum aggregate, and Divisia aggregate as the monetary aggregation. Keating and Smith (2019) take into account not only the true aggregate but also three other monetary aggregates - a weighted non-parametric Divisia aggregate, an unweighted non-parametric aggregate simple sum, and the monetary base. In this paper, the monetary aggregation is considered in two different ways other than the true aggregate that are traditional Divisia and credit-card-augmented Divisia to compare them.

First of all, the simple sum measure is sum of currency and deposits. Due to the accounting conventions which do not permit adding liabilities to asset, credit cards have never been included in measures of the money supply[10]. Therefore, we define the simple sum without the volume of the credit card transactions.

$$M_t^{sim} = N_t + D_t \tag{3.27}$$

While the simple sum measures impute the same weight to each asset and it is not appropriate and it may differ from the amount of money that people actually consider in certain circumstances, Barnett et al. (1984) show the Divisia monetary aggregates are strictly preferable to the simple sum monetary aggregates since the component monetary assets are not perfect substitutes. Moreover, for the money definition, considering user costs reflects reality since we always consider the opportunity costs for holding a certain monetary asset. Divisia is the money growth rate with a weighted average and it collects all of the flavor of monetary assets except investment yield.

The credit card quantities to include in the augmented Divisia index formula are credit card transactions volumes, not the credit card balances. The credit-card-augmented Divisia monetary aggregates in this model are formed of currency, interest-bearing deposits, and the credit card transactions volume in money with different user costs price each. The real user cost (equivalent rental price) of currency is defined as

$$u_t^N = \frac{r_t - 1}{r_t} \tag{3.28}$$

where r_t is the maximum expected holding-period yield available in the economy. In other words, r_t is the risk-free gross rate of return on a completely illiquid benchmark asset (pure capital). Since the nominal interest rate is the opportunity cost of holding non-interest-bearing money, the user costs of currency is the price of holding currency. The user cost of deposits is

$$u_t^D = \frac{r_t - r_t^D}{r_t} \tag{3.29}$$

[10]See Barnett et al. (2016b).

where r_t^D is the gross own yield on the deposits. Based on the real user costs of credit card services derived in Barnett and Su (2016), the real user cost of credit card services in this model is given by

$$u_t^{cc} = \frac{e_t - r_t}{r_t} \tag{3.30}$$

where e_t[11] is the credit card gross interest rate. Then we calculate the expenditure share of currency, deposits, and credit card services, respectively.

$$s_t^N = \frac{u_t^N N_t}{u_t^N N_t + u_t^D D_t + u_t^{cc} CC_t} \tag{3.31}$$

$$s_t^D = \frac{u_t^D D_t}{u_t^N N_t + u_t^D D_t + u_t^{cc} CC_t} \tag{3.32}$$

$$s_t^{cc} = \frac{u_t^{cc} CC_t}{u_t^N N_t + u_t^D D_t + u_t^{cc} CC_t} \tag{3.33}$$

Now, the growth rates of the credit-card-augmented Divisia quantity index for monetary services μ_t^{CDiv} is a function of monetary assets, currency and deposits, and credit card services and is as follow.

$$\mu_t^{CDiv} = \mu_t^{N^{(s_t^N + s_{t-1}^N)/2}} \mu_t^{D^{(s_t^D + s_{t-1}^D)/2}} \mu_t^{cc^{(s_t^{cc} + s_{t-1}^{cc})/2}} \tag{3.34}$$

where μ_t^N, μ_t^D, and μ_t^{cc} are the growth rate of currency, deposits, and the credit card transactions volume at time t, respectively.

$$\mu_t^N = \frac{N_t}{N_{t-1}} \tag{3.35}$$

[11]Barnett et al. (2016b) introduce the credit card interest rate e_{jt}, and consider two categories of consumers using their credit cards. First, they consider consumers who do pay interest to the credit card issuing banks and another category is consumers who do not pay interest. e_{jt} is averaged over both those consumers who maintain such rotating balances and pay interest on contemporaneous credit card transactions and also those consumers who pay off such credit card transactions before the end of the period, and do not pay explicit interest on the credit card transactions. The j is the credit card type and CFS considers four credit card companies, Visa, MasterCard, Discover, and American Express as primary sources. This paper assumes that there is only one credit card company (bank).

$$\mu_t^D = \frac{D_t}{D_{t-1}} \tag{3.36}$$

$$\mu_t^{cc} = \frac{CC_t}{CC_{t-1}} \tag{3.37}$$

3.2.7 Equilibrium Condition

In equilibrium, by Calvo (1983), $P_{i,t} = P_t$, $h_{i,t} = h_t$, $Y_{i,t} = Y_t$, $F_{i,t} = F_t$, and $Q_{i,t} = Q_t$ for all i. Market clearing conditions are $B_t = B_{t-1} = 0$, $M_t = M_{t-1} + \tau_t$, and $s_{i,t} = s_{i,t-1} = 1$ for all time t.

Applying these equilibrium conditions and after solving the optimization problem and stationary process[12], we collect (3.3), (3.5), (3.7), (3.10)-(3.23), (3.25)-(3.37), (A.1)-(A.10) and (B.1) in Appendix B which explains the process of the stationary system of those equations.

3.2.8 Welfare Cost Calculation

This section investigates the consumption equivalent variation approach to calculate the welfare costs. We approach the welfare cost of steady-state inflation through the change in the steady-state of the utility of the representative household as in Cooley and Hansen (1989) and Cooley and Hansen (1991). We calculate the change in consumption and credit card transactions volume to attain the target rate of inflation by the conventional approach which is an equivalent variation in consumption and the amount of credit card transactions. In other words, welfare cost in this model means that how much amount of consumption and credit card transactions volume the household has to give up to achieve indifferent utility as the steady-state.

Let ψ be the welfare cost and ψ satisfies the following equation after applying the stationary system on equations by Appendix B:

$$U\Big[c_\pi, \ell_\pi, m_\pi^{cc}(n_\pi, d_\pi, cc_\pi)\Big] = U\Big[(1+\psi)c_*, \ell_*, m_*^{cc}(n_*, d_*, (1+\psi)cc_*)\Big] \tag{3.38}$$

[12] See Appendix A and Appendix B.

where c_π, ℓ_π, $m_\pi^{cc} = M_\pi^{cc}/P_\pi$, n_π, d_π, and cc_π are the consumption, leisure, real money balance, currency, deposits, and credit card transactions associated with certain inflation rate of π, respectively. c_*, ℓ_*, $m_*^{cc} = M_*^{cc}/P_*$, n_*, d_*, and cc_* are the steady-state consumption, leisure, real money balance, currency, deposits, and credit card transactions volume associated with a steady-state net annual inflation rate (benchmark inflation rate). That is, we compute the welfare costs when the inflation rate π changes from the certain constant rate to the steady-state inflation rate. In addition, real money balance is a function of the currency, deposits, and credit card transactions volume as equation (3.3), we measure not only the change in consumption c_t but also the credit card transaction volume cc_t.

Applying the utility function of the representative household, the welfare cost ψ satisfies equation (3.38) and it can be written as follows:

$$\begin{aligned} &\zeta \ln c_\pi + \eta \ln \ell_\pi + \theta \ln \left\{ \left[\nu^{\frac{1}{\omega}} n_\pi^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}} d_\pi^{\frac{\omega-1}{\omega}} + (1-\nu-\phi)^{\frac{1}{\omega}} cc_\pi^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \right\} \\ &= \zeta \ln[(1+\psi)c_*] + \eta \ln \ell_* + \theta \ln \left\{ \left[\nu^{\frac{1}{\omega}} n_*^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}} d_*^{\frac{\omega-1}{\omega}} + (1-\nu-\phi)^{\frac{1}{\omega}} [(1+\psi)cc_*]^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} \right\} \end{aligned} \tag{3.39}$$

3.3 Calibration

This paper sets discount factor β is set equal to 0.99 and it means that the annual interest rate equals to 4%. In the utility function of the representative household, we follow the method of the estimation for utility weight on labor and real money balances in Chari et al. (2000) and Liu and Spiegel (2015). ζ, η and θ are calibrated 0.944, 0.0.028 and 0.028 as we estimate them by using U.S. data from July 2006 through May 2021. In the benchmark model without credit card services in the model, we get $\zeta = 0.669$, $\eta = 0.31$, and $\theta = 0.021$ in the same way.

Since Belongia and Ireland (2014) and Keating and Smith (2019) have only currency and deposits in their true monetary aggregation, they set the weight for the currency ν and for the deposits ϕ as 0.225 and 0.275 each for the true monetary aggregation. We set the weight for the currency ν as 0.198, and the weight for the deposits ϕ as 0.480, then the weight for the credit card transactions volume $(1-\nu-\phi)$ is equal to 0.322. These parameters are estimated to make the proportion of steady-state currency, deposits, and credit card transactions volume to the total amount of three components in the true money aggregation, 0.103, 0.877, and 0.019, respectively, by using the U.S. Divisia components data running from July 2006 through May 2021.[13] It reflects a more recent monetary asset share while previous literature calculated ν by using U.S. data from 1959 through 2009. Meanwhile, we set $\nu = 0.224$ in the benchmark model that the true money aggregation consists of only currency and deposits. It means that the ratio of steady-state currency to simple sum equals 0.1012 and deposits to simple sum is 0.8988. We follow the elasticity of substitution among currency, deposits, and real balance $\omega = 1.5$ as in Belongia and Ireland (2014).

σ is calibrated 6 as in Annicchiarico and Rossi (2013) and Belongia and Ireland (2014). The setting $z = 1.005$ implies 2% growth per year on average for most of the real variables and $\gamma = 50$ means a speed of price adjustment in the model is every 3.75 quarters on average as in Belongia and Ireland (2014).

Since the initial steady-state rate of inflation is 2% at the annual level in this paper, the inflation target $\pi = 1.005$. Based on the method in Belongia and Ireland (2014), the ratio of total reserves to the deposit of M2 (k) is calibrated 0.04. We regress the U.S. Divisia components data from July 2006 through May 2021 to get $a = 0.0217$ and $\rho_a = 0.80$. Following Belongia and Ireland (2014), $x = 0.01$, $\rho_k = 0.50$, and $\rho_x = 0.50$.

In monetary policy rules, the policy parameters ρ_r, and ρ_π are calibrated 0.75, and 0.3. Following Belongia and Ireland (2014), we set the policy coefficients on the output gap ρ_{g^*} and output growth ρ_{g^y} as zero. The standard deviations of exogenous shocks are $\sigma_z = 0.01$, $\sigma_k = 1$, $\sigma_x = 0.25$, $\sigma_r = 0.0025$, and $\sigma_a = 0.07$.

[13]The Center for Financial Stability (CFS) offers Divisia indices data, and we use the Divisia M2 components data. See http://www.centerforfinancialstability.org/.

3.4 Numerical Results

As we set, we have two different model settings. For comparison purposes, one is the benchmark model with only the currency and the deposits in the true monetary aggregation (Case 1), and this model considers no credit card services. We also consider the new true monetary aggregation (Case 2) including not only currency and the deposit but also the credit card transactions volume as equation (3.3).

Table 3.1 represents the welfare costs of inflation and real variables in a New Keynesian model. We experiment with the target inflation rates as 0%, 2%, 5%, and 10%. The welfare cost in this paper means the burden of the household when the economy goes toward the steady-state inflation from each experiment inflation rate. Since we assume that 2% inflation is steady-state, the welfare cost of inflation at 2% inflation is zero. As we know, zero inflation is not the optimal rate of inflation and the Fed usually targets the rate of inflation of 2%. Poole (1999) states that the target of the Federal Reserve should be zero inflation and it would maximize the credibility of monetary policy and minimize distortions in the economy from the experience of the U.S. He also mentions that falling inflation will cause slower real growth or a higher unemployment rate. Table 1 indicates the smaller welfare costs of inflation of the model with credit card services than the model with no credit card transactions to reach a steady-state inflation rate. When we have no credit card services in the model (upper panel), the welfare costs of 0%, 5%, and 10% inflation are 1.0041, 0.9951, and 0.9837. When we consider the credit card transactions of the representative household (lower panel), the welfare costs of 0%, 5%, and 10% inflation are 0.9264, 0.9528, and 0.9787, respectively. The differences among results seem small, however, if we consider the actual amount of personal consumption expenditures (PCE) and the credit card transactions volume in the U.S., for example, the costs of actual amounts would be huge[14].

We conjecture the reason for the lower welfare costs of the model with credit card services is that when people face high inflation, they not only have cash but also have credit cards like their

[14] According to the data from FRED, quarterly personal consumption expenditure (PCE) is around 15 trillion USD in 2021:Q2 (seasonally adjusted). https://fred.stlouisfed.org/series/PCEC

	0% inflation	2% inflation (Steady-state)	5% inflation	10% inflation
Case 1: Benchmark Model				
Consumption (c)	1.2676	0.6361	1.2813	1.2950
Leisure (ℓ)	0.3585	0.3519	0.3446	0.3306
Currency (n)	0.4170	0.1720	0.5175	0.6179
Deposits (d)	0.5536	1.5253	0.5830	0.6123
Welfare Cost (ψ)	1.0041	0	0.9951	0.9837
Case 2: Including credit card services				
Consumption (c)	0.9859	0.4937	0.9980	1.0102
Leisure (ℓ)	0.3704	0.3537	0.3583	0.3462
Currency (n)	4.3702	1.7818	5.5598	6.7493
Deposits (d)	5.5518	15.2493	5.5534	5.5551
Credit card transactions volume (cc)	0.7161	0.3309	0.7161	0.7162
Welfare Cost (ψ)	0.9264	0	0.9528	0.9787

Table 3.1: Welfare Cost of Inflation Estimated in New Keynesian Model

“bumper". Credit card use can be the method that relieves the welfare cost they face.

In both panels, consumption of the household in 10% inflation is the highest level among experiments and the level of deposits is significantly low compared to the steady-state level inflation (2%). The reason for this seems obvious. The household encounters a very high inflation rate and low interest rate simultaneously by the Fisher equation.

Furthermore, we see the reactions of true money aggregation, simple sum, and credit-card-augmented Divisia to the various shocks when the model includes the credit card transactions volume in Figure C.1 in Appendix C. For monetary policy shock, reserve demand shock, deposit cost shock, and credit-deposit ratio shock, the responses of the true money aggregation and credit-card-augmented Divisia show nearly the same patterns while simple sum shows a bit different peak points. Meanwhile, we see similar responses of true money aggregation and simple sum to a technology shock. Therefore, credit-card-augmented Divisia tracks the true money aggregation well and we believe the welfare cost of inflation with credit card services in the model accounts for the real welfare costs of a certain inflation in the economy.

Figure C.6 in Appendix C plots the impulse responses for the true money aggregation, simple sum, and traditional Divisia when the model doesn't consider the credit card services (benchmark model). We also see a similar movement of impulse responses between true monetary aggregation and Divisia index to monetary policy shock, reserve demand shock, and deposit cost shock.

Additionally, Figure C.2 - C.5 and Figure C.7 - C.9 draw the impulse response for the macroeconomic variables to monetary policy shock with credit card services in the model and without credit card services, respectively. Credit card transactions volume (cc) and consumption (c) of the representative household drop initially, and recover slowly. Especially, inflation (π), interest rate (r), and output (y) show similar reactions to those in Belongia and Ireland (2014).

3.5 Empirical Approach

In addition to the quantitative model estimation, this paper investigates the empirical approach for the welfare cost of inflation as well. This section adds empirical evidence of the different patterns between two different monetary aggregates by using Divisia data and it makes another attempt to investigate the welfare cost of inflation in another different way. Lucas (2000) and Ireland (2009) use simple sum M1 as the measure of money in their research of the welfare cost of inflation by deriving the money demand and they mention that the estimates of the welfare cost of inflation through the Divisia approach to monetary aggregation is another critical task for future research. Especially, Ireland (2009) mentions it would be a critical task to examine how the demand for other liquid assets reacts under very low nominal interest rates and estimate the welfare cost of inflation with this behavior through the Divisia approach. In addition, this paper examines not only M1 but also the broad money M4. Although the Fed discontinued announcing broader monetary aggregates, many economic events can be tracked by the broad money rather than the narrow money.

3.5.1 Cagan Money Demand

Extending Lucas (2000) and Ireland (2009), we estimate the Cagan money demand function. Cagan (1956) developed a simple model of the behavior of inflation and the demand for money during the hyperinflation period. Cagan explains that hyperinflation is an increase in the price level of goods in terms of money at a rate averaging at least 50% per month. In the Cagan model, the velocity of money depends on the nominal interest rate. The velocity of money rises if the nominal interest rate increases because the opportunity cost of money is the foregone nominal interest.

The demand of real money balances is log-linear form as following:

$$m_t - p_t = -\alpha E_t(p_{t+1} - p_t) \tag{3.40}$$

where m_t is log of money at the end of period t, p_t is log of price level P at time t. Then $(m_t - p_t)$ is the log of real money and α is the semi-elasticity of real money demand for real balances with respect to expected inflation. Then the elasticity of real money demand with respect to expected inflation is $|\alpha E_t(p_{t+1} - p_t)|$. We can replace expected with actual inflation in equation (3.50).

$$m_t - p_t = -\alpha(p_{t+1} - p_t) \tag{3.41}$$

Based on the Cagan money demand function, this paper considers two forms of money demand function as in Lucas (2000) and Ireland (2009). First, the log-log function by Meltzer (1963), and another is the semi-log function by Bailey (1956). Meltzer (1963) uses the log-log function that shows the relationship between the natural logarithm of money and the logarithm of nominal interest rate. The log-log function is given by

$$\ln m = \ln A - \eta \ln r \tag{3.42}$$

where m is the ratio of money to nominal income, $A > 0$ is a constant, $\eta > 0$ represents the interest elasticity of money demand, and r is the nominal interest rate. The semi-log function in Bailey

(1956) is

$$\ln m = \ln B - \xi r \tag{3.43}$$

where $B > 0$ is a constant and $\xi > 0$ is the interest semi-elasticity of money demand.

Bailey (1956) and Lucas (2000) measure the welfare cost of inflation by the welfare triangle which is the area under the inverse demand function - the consumers' surplus. We could gain the welfare triangle by reducing the interest rate from r to zero. Let $w(r)$ be the welfare cost function and $w(r)$ is defined by

$$w(r) = \int_{m(r)}^{m(0)} \psi(x)\, dx = \int_0^r m(x)\, dx - rm(r) \tag{3.44}$$

where $m(r)$ is the estimated function and $\psi(m)$ is the inverse function.

The welfare cost function $w(r)$ for the log-log function (3.42) and the semi-log function (3.43) can be written by following, respectively.

$$w(r) = A\left(\frac{\eta}{1-\eta}\right) r^{1-\eta} \tag{3.45}$$

$$w(r) = \frac{B}{\xi}[1 - (1 + \xi r)e^{-\xi r}] \tag{3.46}$$

3.5.2 Data

In this study, we estimate the money demand and the welfare cost of inflation using U.S. data. To see the effect of credit-card-augmented Divisia on the money demand, this paper adopts two different Divisia indices, traditional Divisia and credit-card-augmented Divisia monetary aggregates, as economic indicators to compare each other. Divisia indices are available on a monthly basis offering from the Center for Financial Stability (CFS). CFS provides Divisia monetary aggregates to the public from 1967:01 and credit-card-augmented Divisia from 2006:07. Divisia data is level normalized to equal 100 in 1967:01 and credit-card-augmented Divisia data is level normalized to equal 100 in 2006:07. This monthly Divisia data is converted into quarterly in this paper.

This paper uses the rate of return on the 3-month Treasury bill (in percent) for the nominal interest rates. We use quarterly nominal GDP (seasonally adjusted) for computing the money income ratio and Consumer Price Index (CPI) for the flexible money demand functions, published by Federal Reserve Economic Data (FRED).

Considering the periods of all affordable data, the money demand with traditional Divisia is estimated using data period from 1992:Q1 to 2020:Q4 and the money demand with credit-card-augmented Divisia is estimated using data set running from 2006:Q3 through 2020:Q4.

3.5.3 Results

3.5.3.1 Money Demand

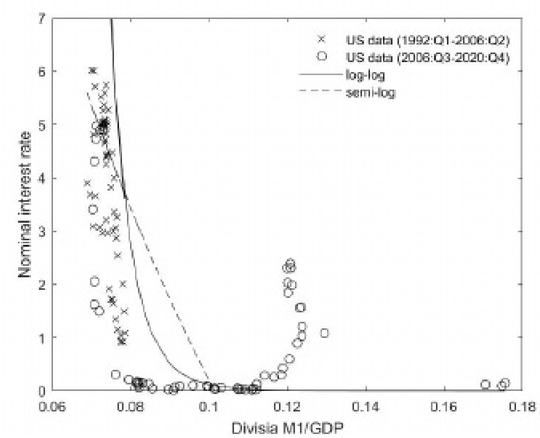
First of all, we run the simple regression to estimate η and ξ in equations (3.42) and (3.43). Table 3.2 shows the estimation of log-log and semi-log money demand functions with four money specifications. Whereas the setting elasticity of the log-log money demand in Lucas (2000) is 3, 5, and 7, the elasticity η in this paper is 0.0705, 0.0140, 0.0206, and 0.0010 for Divisia M1, credit-card-augmented Divisia M1, Divisia M4, and credit-card-augmented Divisia M4. The elasticity ξ is measured 0.0689, 0.0466, 0.0240, and 0.0081 for Divisia M1, credit-card-augmented Divisia M1, Divisia M4, and credit-card-augmented Divisia M4 for the semi-log money demand specification, while ξ in Lucas (2000) is 5, 7, and 9. Since the Divisia index is the index level normalized to equal 100 in 1967:01 and credit-card-augmented Divisia index is level normalized to equal 100 in 2006:07, the unit of m in equation (3.42) and (3.43) and the elasticities in both log-log and semi-log functions are different from Lucas (2000) and Ireland (2009). However, we would be able to observe their patterns and compare the results with each other according to the inflation rate change.

Figure 3.2 displays the U.S. money demand using Divisia M1 and M4 from 1992:Q1 to 2020:Q4 and the U.S. money demand with credit-card Divisia M1 and M4 from 2006:Q3 to 2020:Q4. The x-axis is the ratio of Divisia to nominal GDP or the ratio of credit-card-augmented Divisia to nom-

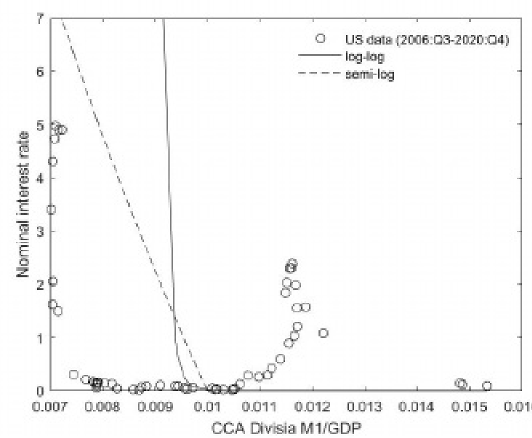
	LOG-LOG		SEMI-LOG	
	$\ln A$	η	$\ln B$	ξ
With Divisia M1	-2.4548^{***}	0.0705^{***}	-2.2891^{***}	0.0689^{***}
	(0.0173)	(0.0620)	(0.0253)	(0.0081)
With CCA-Divisia M1	-4.6698^{***}	0.0140^{***}	-4.6057^{***}	0.0466^{***}
	(0.0343)	(0.0159)	(0.0322)	(0.0186)
With Divisia M4	-2.5940^{***}	0.0206^{***}	-2.5364^{***}	0.0240^{***}
	(0.0061)	(0.0034)	(0.0082)	(0.0026)
With CCA-Divisia M4	-4.8862^{***}	0.0010^{***}	-4.8769^{***}	0.0081^{***}
	(0.0113)	(0.0052)	(0.0109)	(0.0063)

Table 3.2: Money Demand Estimates

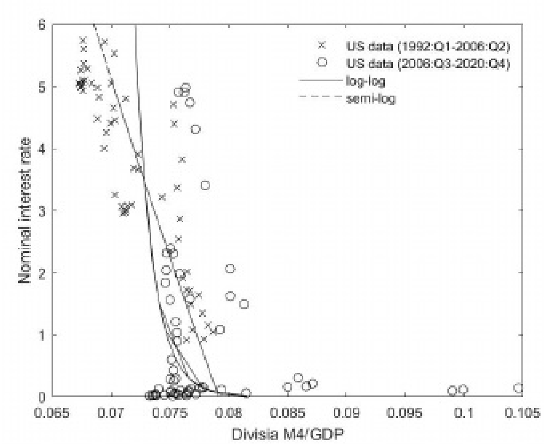
Note: Standard errors in parenthesis. *, **, *** denote statistical significance at the 10%, 5%, and 1% critical levels, respectively.



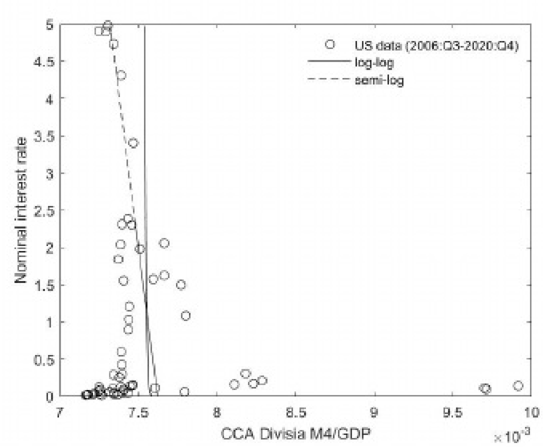
(a) With Divisia M1 (1992:Q1-2020:Q4)



(b) With CCA-Divisia M1 (2006:Q3-2020:Q4)



(c) With Divisia M4 (1992:Q1-2020:Q4)



(d) With CCA-Divisia M4 (2006:Q3-2020:Q4)

Figure 3.2: U.S. Money Demand with Divisia and Credit-Card-Augmented Divisia

inal GDP, respectively, and the y-axis is the nominal interest rate (in percent) for all cases. Despite using different data that is Divisia indices, we could find consistent patterns and implications with those in Ireland (2009). As Lucas (2000) mentioned, the log-log demand and the semi-log demand show very different results and implications for each other for the welfare cost. As the nominal interest rate approaches zero, the real balances come to be large in most cases.

3.5.3.2 Empirical Tests

This paper examines several tests with two types of the money demand function, equation (3.42) and (3.43). Following Hafer and Jansen (1991), Hoffman and Rasche (1991), and Ireland (2009), we test the order of integration for all variables in the log-log and semi-log money demand functions by unit root test.

First of all, we test the stationarity by Phillips-Perron unit root test, and Table 3.3 displays the results of the test. Ireland (2009) also shows the result of the Phillips-Perron unit root test that is adjusted augmented Dickey-Fuller test. Phillips-Perron unit root test is for the following equation:

$$y_t = c + ay_{t-1} + e_t$$

where y_t is the time series variable at time t, c is constant, a is AR(1) coefficient, and e_t is the error term. The setting for the character vector is autoregressive and the null hypothesis is AR(1) coefficient $a = 1$. There is a statistic Z calculated as

$$Z = n(\hat{a} - 1) - \frac{1}{2}\frac{n^2\hat{\sigma}^2}{s_n^2}(\hat{\lambda}_n^2 - \hat{\gamma}_{0,n})$$

We estimate the log of Divisia M1, log of credit-card Divisia M1, log of Divisia M4, log of credit-card Divisia M4, log of interest rates, and the level of interest rates by q lags, respectively. $\hat{c}$ and $\hat{a}$ denote the intercept and the slope coefficients from an OLS regression of each variable. The results display that the null hypothesis of a unit root cannot be rejected.

Second, we test Engle-Granger cointegration test using residual after Phillips-Perron test. The

$\ln(DM1)$	$\hat{c}$	$\hat{a}$	q	Z_t
	0.0874	1.0323	0	2.4122
			1	2.3356
			2	2.2638
			3	2.2105
			4	2.1998
			5	2.2062
			6	2.2201
$\ln(DMA1)$	$\hat{c}$	$\hat{a}$	q	Z_t
	0.1234	1.0236	0	1.3148
			1	1.2323
			2	1.1603
			3	1.1188
			4	1.1179
			5	1.1342
			6	1.1551
$\ln(DM4)$	$\hat{c}$	$\hat{a}$	q	Z_t
	−0.0309	0.9870	0	−0.3408
			1	−0.4356
			2	−0.5103
			3	−0.5746
			4	−0.6114
			5	−0.6363
			6	−0.6477
$\ln(DMA4)$	$\hat{c}$	$\hat{a}$	q	Z_t
	−0.0369	0.9914	0	−0.1126
			1	−0.2586
			2	−0.3685
			3	−0.4511
			4	−0.4920
			5	−0.5162
			6	−0.5253
$\ln(r)$	$\hat{c}$	$\hat{a}$	q	Z_t
	−0.0329	0.9794	0	−0.9734
			1	−1.1671
			2	−1.2834
			3	−1.3466
			4	−1.4091
			5	−1.4346
			6	−1.4518
r	$\hat{c}$	$\hat{a}$	q	Z_t
	0.0011	0.9857	0	−0.8196
			1	−1.1305
			2	−1.3335
			3	−1.4849
			4	−1.5816
			5	−1.6361
			6	−1.6658

Table 3.3: Phillips-Perron Unit Root Test Results

Note: DM1, DMA1, DM4, and DMA4 denote Divisia M1, credit-card-augmented Divisia M1, Divisia M4, and credit-card-augmented Divisia M4, respectively. Each panel reports the intercept $\hat{c}$ and the slope coefficient $\hat{a}$ from an ordinary least squares regression of the variable. q is the own lag of each variable and Z_t is the Phillips-Perron statistic corrected for autocorrelation in the regression error, computed using the Newey-West estimate of the error variance for various values of the lag truncation parameter q. The critical values for Z_t are -2.5798 (10%), -2.8871 (5%), -3.4918 (1%) in first, third, fifth and sixth panel, -2.5958 (10%), -2.9158 (5%), -3.5557 (1%) in second and fourth panel, respectively. *, **, *** denote statistical significance at the 10%, 5%, and 1% critical levels, respectively.

Engel-Granger test is for examination of cointegrating relationships and the null hypothesis is that no cointegration exists. This test uses the residuals to observe unit roots using the Phillips-Perron test and if the time series are cointegrated, the residuals are stationary. We test cointegration between the log of money and log of interest rate in equation (3.42), and log of money and interest rate in equation (3.43). Table 3.4 and Table 3.5 display the results of the cointegration test of the money demand functions including M1 and M4, respectively. We see that all series are not cointegrated.

Furthermore, we examine different types of money demand that are more flexible than the traditional money demand. Now, we consider the GDP divided by the price level. Table 3.6 and Table 3.7 represent the results of the Engle-Granger cointegration test with M1 and M4 in the flexible money demand functions. Interestingly, we confirm that the flexible money demand functions including Divisia M1, credit-card-augmented Divisia M1, and Divisia M4 have cointegration for all settings of lags. Exploration of these flexible money demand functions remains worthy of study for future research.

3.5.3.3 Welfare Cost

Figure 3.3 shows the welfare cost functions using Divisia M1, credit-card-augmented Divisia M1, Divisia M4, and credit-card-augmented Divisia M4. These figures are very similar in shape to each other and the only difference is the level of variables. These figures also show consistent shape with Lucas (2000).

This paper assumes the steady-state real interest rate is 3% as in Calza and Zaghini (2011) and Dai and Serletis (2019). In accordance with the Fisher equation, we assume the nominal interest rate is 5% to set 2% of the steady-state inflation rate as in the previous computational section. This paper examines the effects of three different inflation experiments, 2%, 5%, and 10% inflation. Therefore, we estimate $w(0.05)-w(0.03)$, $w(0.08)-w(0.03)$, and $w(0.13)-w(0.03)$. Since this paper uses the Divisia index and the credit-card-augmented Divisia index for the money measurement and they are level normalized to equal 100 in 1967:01 and 2006:07 each, we focus

	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	q	Test Statistic
$\ln(DM1) = \alpha - \beta \ln(r)$					
	-2.4548	0.0705	1.0244	0	1.6053
				1	1.4745
				2	1.3309
				3	1.2693
				4	1.1688
				5	1.1132
				6	1.0490
$\ln(DM1) = \alpha - \beta r$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	q	Test Statistic
	-2.2891	0.0689	1.0251	0	1.3766
				1	1.0489
				2	0.8037
				3	0.6220
				4	0.5109
				5	0.4464
				6	0.3885
$\ln(DMA1) = \alpha - \beta \ln(r)$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	q	Test Statistic
	-4.6698	0.0140	1.0248	0	1.3989
				1	1.1546
				2	0.9415
				3	0.7967
				4	0.6891
				5	0.6009
				6	0.5206
$\ln(DMA1) = \alpha - \beta r$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	q	Test Statistic
	-4.6057	0.0466	1.0326	0	1.7545
				1	1.4354
				2	1.2031
				3	1.0489
				4	0.9379
				5	0.8488
				6	0.7633

Table 3.4: Engle-Granger Cointegration Test Results (M1)

Note: DM1 and DMA1 denote Divisia M1 and credit-card-augmented Divisia M1, respectively. Each panel reports $\hat{\alpha}$ is the intercept, $\hat{\beta}$ is the slope coefficient from an ordinary least squares regression of the variables, $\hat{\rho}$ is the slope coefficient from the residual regression. q is the own lag of each variable and test statistic is the Phillips-Perron statistic corrected for autocorrelation in the regression error, computed using the Newey-West estimate of the error variance for various values of the lag truncation parameter q. The critical values for the test statistic are -3.0828 (10%), -3.3913 (5%), -3.9962 (1%) in first and second panel, -3.1214 (10%), -3.4476 (5%), -4.0996 (1%) in third and fourth panel, respectively. *, **, *** denote statistical significance at the 10%, 5%, and 1% critical levels, respectively.

$\ln(DM4) = \alpha - \beta \ln(r)$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	q	Test Statistic
	−2.5940	0.0206	0.9763	0	−0.6131
				1	−0.6888
				2	−0.7588
				3	−0.8051
				4	−0.8385
				5	−0.8689
				6	−0.8993
$\ln(DM4) = \alpha - \beta r$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	q	Test Statistic
	−2.5364	0.0240	0.9583	0	−0.9120
				1	−0.9558
				2	−1.0087
				3	−1.0354
				4	−1.0492
				5	−1.0645
				6	−1.0940
$\ln(DMA4) = \alpha - \beta \ln(r)$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	q	Test Statistic
	−4.8862	0.0010	0.9870	0	−0.1716
				1	−0.3581
				2	−0.5085
				3	−0.6116
				4	−0.6667
				5	−0.7018
				6	−0.7199
$\ln(DMA4) = \alpha - \beta r$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	q	Test Statistic
	−4.8769	0.0081	0.9933	0	−0.0911
				1	−0.2274
				2	−0.3400
				3	−0.4106
				4	−0.4437
				5	−0.4670
				6	−0.4834

Table 3.5: Engle-Granger Cointegration Test Results (M4)

Note: DM4 and DMA4 denote Divisia M4 and credit-card-augmented Divisia M4, respectively. Each panel reports $\hat{\alpha}$ is the intercept, $\hat{\beta}$ is the slope coefficient from an ordinary least squares regression of the variables, $\hat{\rho}$ is the slope coefficient from the residual regression. q is the own lag of each variable and test statistic is the Phillips-Perron statistic corrected for autocorrelation in the regression error, computed using the Newey-West estimate of the error variance for various values of the lag truncation parameter q. The critical values for the test statistic are -3.0828 (10%), -3.3913 (5%), -3.9962 (1%) in first and second panel, -3.1214 (10%), -3.4476 (5%), -4.0996 (1%) in third and fourth panel, respectively. *, **, *** denote statistical significance at the 10%, 5%, and 1% critical levels, respectively.

$\ln(DM1/P) = \alpha + \beta_y \ln(Y/P) - \beta_r \ln(r)$	$\hat{\alpha}$	$\hat{\beta}_y$	$\hat{\beta}_r$	$\hat{\rho}$	q	Test Statistic
	-105.8560	0.0949	159.6760	-0.0072	0	-9.9747^{***}
					1	-9.9748^{***}
					2	-9.9776^{***}
					3	-10.0017^{***}
					4	-10.0245^{***}
					5	-10.0452^{***}
					6	-10.1023^{***}
$\ln(DM1/P) = \alpha + \beta_y \ln(Y/P) - \beta_r r$	$\hat{\alpha}$	$\hat{\beta}_y$	$\hat{\beta}_r$	$\hat{\rho}$	q	Test Statistic
	209.5940	0.0947	129.0275	-0.0003	0	-9.8902^{***}
					1	-9.8902^{***}
					2	-9.8979^{***}
					3	-9.9301^{***}
					4	-9.9622^{***}
					5	-9.9927^{***}
					6	-10.0618^{***}
$\ln(DMA1/P) = \alpha + \beta_y \ln(Y/P) - \beta_r \ln(r)$	$\hat{\alpha}$	$\hat{\beta}_y$	$\hat{\beta}_r$	$\hat{\rho}$	q	Test Statistic
	-15.2509	0.0103	0.5810	0.1716	0	-5.5439^{***}
					1	-5.5462^{***}
					2	-5.5478^{***}
					3	-5.6153^{***}
					4	-5.6284^{***}
					5	-5.6258^{***}
					6	-5.6578^{***}
$\ln(DMA1/P) = \alpha + \beta_y \ln(Y/P) - \beta_r r$	$\hat{\alpha}$	$\hat{\beta}_y$	$\hat{\beta}_r$	$\hat{\rho}$	q	Test Statistic
	-10.6722	0.0103	4.9635	0.1700	0	-5.5599^{***}
					1	-5.5608^{***}
					2	-5.5635^{***}
					3	-5.6337^{***}
					4	-5.6549^{***}
					5	-5.6629^{***}
					6	-5.7088^{***}

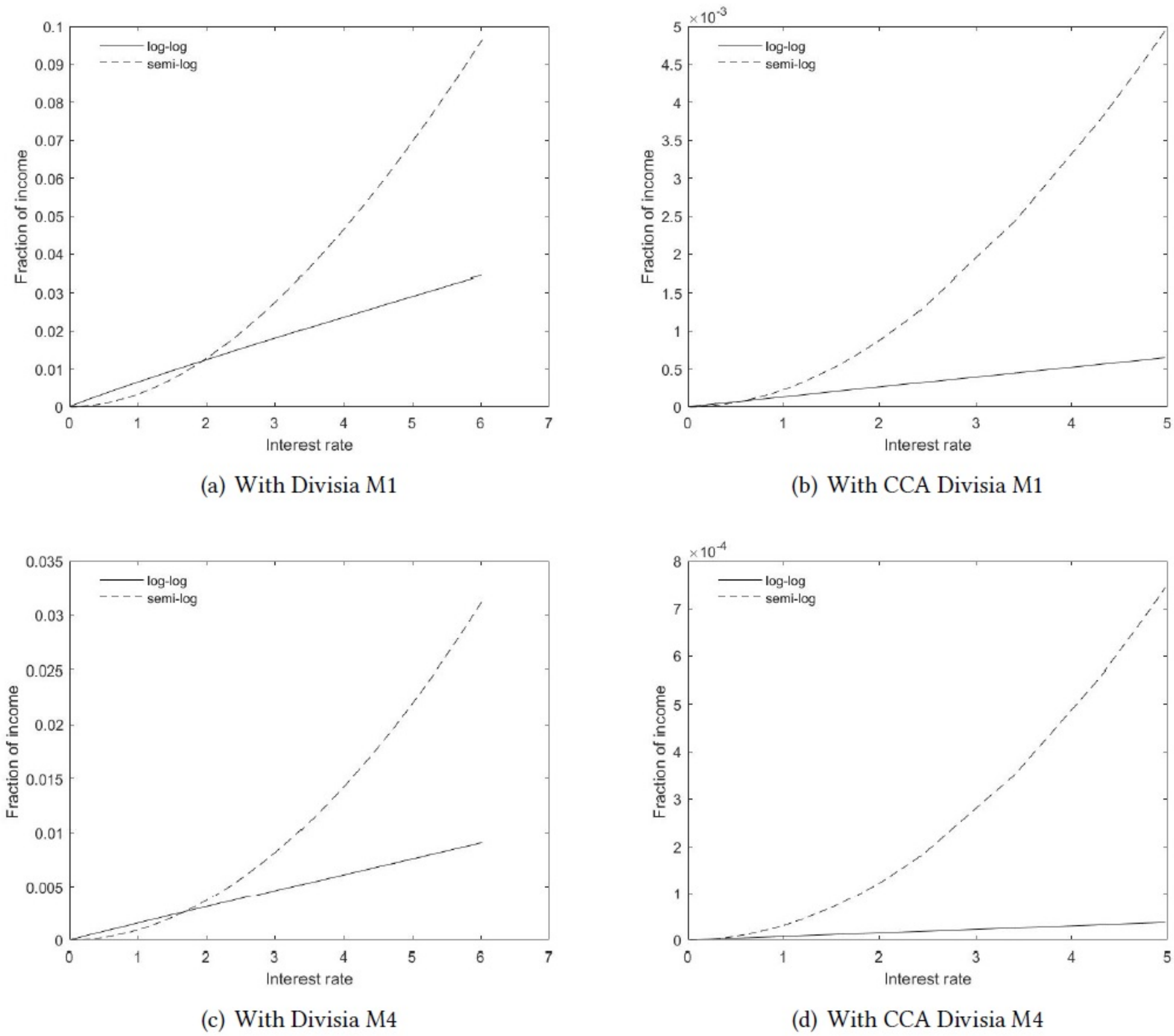
Table 3.6: Engle-Granger Cointegration Test Results with the Flexible Money Demand (M1)

Note: DM1 and DMA1 are Divisia M1 and credit-card-augmented Divisia M1, respectively. Each panel reports $\hat{\alpha}$ is the intercept, $\hat{\beta}_y$, $\hat{\beta}_r$ are slope coefficients from an ordinary least squares regression of the variables, and $\hat{\rho}$ is the slope coefficient from the residual regression. q is the own lag of each variable and test statistic is the Phillips-Perron statistic corrected for autocorrelation in the regression error, computed using the Newey-West estimate of the error variance for various values of the lag truncation parameter q. The critical values for the test statistic are -3.5076 (10%), -3.8178 (5%), -4.4250 (1%) in first and second panel, -3.5642 (10%), -3.8968 (5%), -4.5619 (1%) in third and fourth panel, respectively. *, **, *** denote statistical significance at the 10%, 5%, and 1% critical levels, respectively.

$\ln(DM4/P) = \alpha + \beta_y \ln(Y/P) - \beta_r \ln(r)$	$\hat{\alpha}$	$\hat{\beta}_y$	$\hat{\beta}_r$	$\hat{\rho}$	q	Test Statistic
	−12.0346	0.0762	28.3939	0.2090	0	-5.3742^{***}
					1	-5.6771^{***}
					2	-4.6961^{***}
					3	-4.4086^{**}
					4	-4.3924^{**}
					5	-4.5040^{***}
					6	-4.4505^{***}
$\ln(DM4/P) = \alpha + \beta_y \ln(Y/P) - \beta_r r$	$\hat{\alpha}$	$\hat{\beta}_y$	$\hat{\beta}_r$	$\hat{\rho}$	q	Test Statistic
	61.7168	0.0762	30.0473	0.1813	0	-5.4711^{***}
					1	-5.7635^{***}
					2	-4.6869^{***}
					3	-4.2789^{**}
					4	-4.1394^{**}
					5	-4.1259^{**}
					6	-3.9447^{**}
$\ln(DMA4/P) = \alpha + \beta_y \ln(Y/P) - \beta_r \ln(r)$	$\hat{\alpha}$	$\hat{\beta}_y$	$\hat{\beta}_r$	$\hat{\rho}$	q	Test Statistic
	2.4543	0.0074	0.6149	0.2605	0	−2.9114
					1	−3.3642
					2	−2.2369
					3	−1.7539
					4	−1.6502
					5	−1.6400
					6	−1.6260
$\ln(DMA4/P) = \alpha + \beta_y \ln(Y/P) - \beta_r r$	$\hat{\alpha}$	$\hat{\beta}_y$	$\hat{\beta}_r$	$\hat{\rho}$	q	Test Statistic
	4.2431	0.0074	0.9749	0.2597	0	−2.9157
					1	−3.3701
					2	−2.2403
					3	−1.7457
					4	−1.6294
					5	−1.6077
					6	−1.5863

Table 3.7: Engle-Granger Cointegration Test Results with the Flexible Money Demand (M4)

Note: DM4 and DMA4 are Divisia M4 and credit-card-augmented Divisia M4, respectively. Each panel reports $\hat{\alpha}$ is the intercept, $\hat{\beta}_y$, $\hat{\beta}_r$ are slope coefficients from an ordinary least squares regression of the variables, and $\hat{\rho}$ is the slope coefficient from the residual regression. q is the own lag of each variable and test statistic is the Phillips-Perron statistic corrected for autocorrelation in the regression error, computed using the Newey-West estimate of the error variance for various values of the lag truncation parameter q. The critical values for the test statistic are -3.5076 (10%), -3.8178 (5%), -4.4250 (1%) in first and second panel, -3.5642 (10%), -3.8968 (5%), -4.5619 (1%) in third and fourth panel, respectively. *, **, *** denote statistical significance at the 10%, 5%, and 1% critical levels, respectively.



(a) With Divisia M1

(b) With CCA Divisia M1

(c) With Divisia M4

(d) With CCA Divisia M4

Figure 3.3: Welfare Cost Functions with Divisia and Credit-Card-Augmented Divisia

on the change in the welfare cost when the inflation target changes rather than the number itself. Divisia is the index calculated by index number theory, not the level.

Table 3.8 represents the welfare cost of inflation for three different targets, 2% inflation, 5% inflation, and 10% inflation in both log-log and semi-log money demand functions. We have four money specifications: 1) Divisia M1, 2) credit-card-augmented Divisia M1, 3) Divisia M4, and 4) credit-card-augmented Divisia M4. The first panel in Table 3.8 shows the welfare cost of inflation using the log-log money demand functions using Divisia M1 and credit-card-augmented Divisia M1 and we see that the welfare costs change by 144.5% and 378.2% from the target inflation rate of 2% to 5% and 10% when we consider Divisia M1 and 100.0% and 333.3% when we consider credit-card-augmented Divisia M1 specification. The second panel is using the semi-log money demand functions with M1. The welfare costs increase by 205.0% and 622.7% using Divisia M1, 216.1% and 696.8% with credit-card-augmented Divisia M1. In the third panel, when we calculate the welfare costs using Divisia M4 using the log-log function, these changes by 148.8% and 394.0% using Divisia M4, 300% and 600% when the credit-card-augmented Divisia M4 is included in the log-log demand function. In the last panel, the welfare costs using Divisia M4 increase by 229.4% and 789.6%. With credit-card-augmented Divisia M4, the welfare costs change by 237.5% and 856.3%.

These results implicate that the monetary aggregation including credit card transactions tends to make more sensitive changes in the welfare cost of inflation when the inflation target changes. Moreover, these results are in line with the results in quantitative model that we confirm in Section 3.4. These sensitive changes make sense because credit-card-augmented Divisia includes not only currency and Treasury bills but also credit card transactions volume that is not a negligible amount and might be more intimate with economic agents than any other components. Also, the welfare costs of inflation measured in the semi-log money demand function show greater changes than the log-log demand function. Dai and Serletis (2019) also display similar results that the semi-log money demand function measures larger changes than the log-log estimated with both simple sum M1 and Divisia M1. Serletis and Virk (2006) also mention that welfare cost calculations

LOG-LOG	$A = \exp(\hat{\alpha})$	$\eta = \hat{\beta}$	Welfare Cost 2% inflation	5% inflation (Δ%)	10% inflation (Δ%)
DM1 specification	0.0859	0.0705	0.0110	0.0269 (144.5%)	0.0526 (378.2%)
DMA1 specification	0.0094	0.0140	0.0003	0.0006 (100.0%)	0.0013 (333.3%)

SEMI-LOG	$B = \exp(\hat{\alpha})$	$\xi = \hat{\beta}$	Welfare Cost 2% inflation	5% inflation (Δ%)	10% inflation (Δ%)
DM1 specification	0.1014	0.0689	0.0422	0.1287 (205.0%)	0.3050 (622.7%)
DMA1 specification	0.0100	0.0466	0.0031	0.0098 (216.1%)	0.0247 (696.8%)

LOG-LOG	$A = \exp(\hat{\alpha})$	$\eta = \hat{\beta}$	Welfare Cost 2% inflation	5% inflation (Δ%)	10% inflation (Δ%)
DM4 specification	0.0747	0.0206	0.00299	0.00744 (148.8%)	0.01477 (394.0%)
DMA4 specification	0.0076	0.0010	0.00001	0.00004 (300.0%)	0.00007 (600.0%)

SEMI-LOG	$B = \exp(\hat{\alpha})$	$\xi = \hat{\beta}$	Welfare Cost 2% inflation	5% inflation (Δ%)	10% inflation (Δ%)
DM4 specification	0.0792	0.0240	0.01378	0.04539 (229.4%)	0.12258 (789.6%)
DMA4 specification	0.0076	0.0081	0.00048	0.00162 (237.5%)	0.00459 (856.3%)

Table 3.8: Welfare Cost Estimates

Note: DM1, DMA1, DM4, and DMA4 denote Divisia M1, credit-card-augmented Divisia M1, Divisia M4, and credit-card-augmented Divisia M4, respectively. Percentage change compared to the result of 2% inflation in parenthesis.

are sensitive to the specification of the money demand function. Moreover, the broad money M4 makes larger changes in welfare costs of inflation in both log-log and semi-log demand functions compared to the narrow money M1.

We also see that the welfare costs of 2% inflation in each estimation are lowest among three experiments. Following the results here, the 2% inflation target is persuaded to implement ideal targeting of the Fed's policy.

3.6 Conclusion

To predict a myriad of macroeconomic variables, it is meaningful to use many different ways with the new attempt. In that sense, this paper is a new try of application of the new monetary aggregation, credit-card-augmented Divisia. This paper explores the welfare cost which is occurred by anticipated inflation considering credit card transactions volume in the measurement of money in both quantitative and empirical methods.

First, we adopt the concept of credit-card-augmented Divisia in the New Keynesian model and assume that money yields utility in the money-in-the-utility function. We could measure the lower welfare cost of inflation with credit card services in the model than without credit card services in the model. Extending Lucas (2000), this paper also looks into the empirical evidence of the U.S. welfare cost of inflation by deriving the inverse money demand functions with the consumer surplus approach using Divisia indices data from CFS. In the empirical estimation, when the inflation target changes, we see more sensitive changes in the welfare cost of inflation with credit-card-augmented Divisia as a money measure. Broad money specification and money demand specification of semi-log make greater changes in welfare cost.

One of the possible future works is to investigate the welfare cost of other shocks. Evans and Kenc (2003) explore the welfare cost of volatility due to monetary and fiscal policy shocks and Mao et al. (2019) observe the welfare costs of monetary policies regarding the money growth rate, required reserve ratio, and leverage ratio. Furthermore, another future plan is to include the required reserve ratio, k_t, in the user costs equation in the model as introduced in Barnett (1987). Then credit-card-augmented Divisia inside money would be another type of money in the model considering the supply side. Since the required reserve ratio is a kind of implicit tax for the banks, there would be foregone interest on uninvested required reserve. Barnett and Su (2020) employ a production model of financial firms, which produce services through financial intermediation, and investigate supply-side aggregation when financial firms produce both monetary services and credit card transactions services. They derive a theory for measuring the supply of the joint services of credit cards and inside money for estimating the output supply function and computing value-added.

Chapter 4

The Role of Broad Money: Tracking Economic Signals

Hyun Park, Sohee Park

Abstract

Fed discontinued announcing broader monetary aggregate. For example, M3 was discontinued to publish in 2006 and only M3 data set borrowed from OECD has been displayed. Moreover, we cannot track M4 data series in FRED anymore. Although the importance of monetary aggregate is increasing (as a tool of understanding monetary transmission mechanism), researchers cannot approach a reliable data set. As we see in the Divisia data provided by the Center for Financial Stability (CFS), many economic events can be tracked by the broader money rather than the narrow money. Clearly, there would be an economic implication provided by the broader monetary aggregate.

The purpose of this paper is to examine the availability of Divisia type monetary aggregates for all ranges by conducting several tests. Following the philosophy of Bernanke and Blinder (1992) and Belongia and Ireland (2015), we test the causal relationship between real economic variables and monetary aggregates. We also set up a basic recursive VAR and Leeper and Roush (2003) type non-recursive VAR model, and study the economic implications given by the results.

4.1 Introduction

In these circumstances of the huge amount of money injected into the economy, alarming is crucial for the economic outlook. How can we track more accurate monetary fluctuations? In an effort to make a precise prediction, this paper shed light on the broad money which includes more components. Thomas (1996) mentions that broad money has played a significant role in the formulation of monetary policy in the United Kingdom over the past 25 years. He also states that the primary role of broad money is to provide information about future fluctuations in nominal demand and inflation along with a wide range of other indicators. However, the Federal Reserve has discontinued publishing broad money data since March 2006. They have borrowed a M3 data from OECD and displayed it on the FRED website. Furthermore, we could no longer get the M4 data series from FRED and researchers have no way to approach reliable broad monetary data set.

In the absence of the official broad money measurement, this paper focuses on the characteristics of the traditional Divisia monetary aggregates and credit-card-augmented Divisia monetary aggregates, which are suggested as a new alternative, and analyzes the role of these new monetary variables. First, we investigate the relationship between a set of real variables and monetary aggregates, and between a set of real variables and the federal funds rate by conducting Granger-causality tests. Second, we adopt the basic recursive VAR and non-recursive VAR to set a variety of economic environment. By introducing non-recursive VAR estimation, we can be freed from the conventional premise that 'The less exogenous variable is affected by the exogenous variables'. And the equation of structural shock represented by the endogenous variables can be converted into the desired form through the free adjustment of the elements of the lower triangular matrix. To be specific, by introducing the non-recursive form, we introduce the Taylor-rule to the monetary policy shock equation. The interpretation of the impulse responses will be followed. By conducting these works, we expect to be able to grasp the unique characteristics of the broad money that is distinguished from the narrow money.

At its simplest, displaying raw time-series data itself allows us to evaluate the ability of two data sets to track the economic unusual events. Figure 4.1 displays the log-level of narrow money

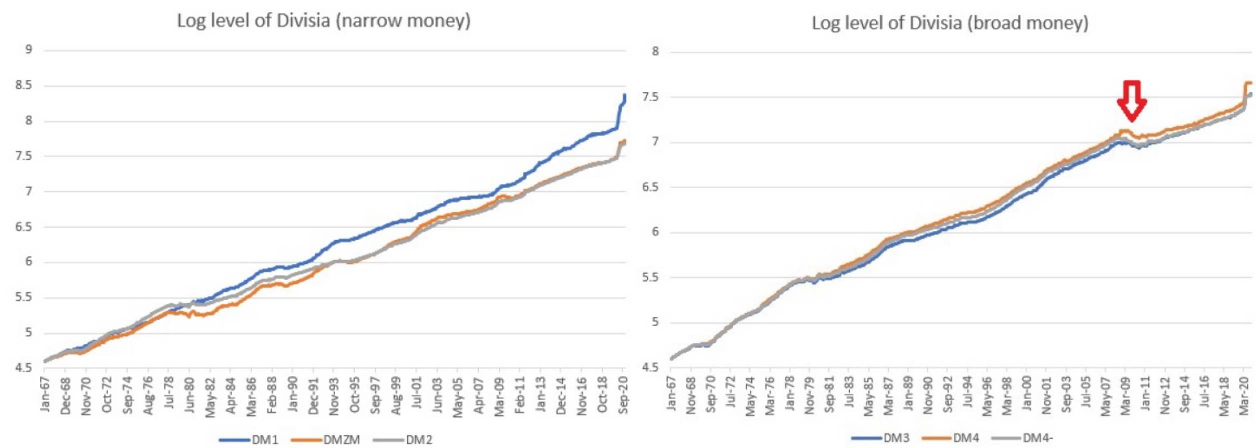


Figure 4.1: Log Level of U.S. Divisia Monetary Aggregates

and that of broad money of U.S. Divisia monetary aggregates, respectively. We observe that broad money (in the second figure) indicates the lowered paths after the Great Recession that are relevant to the low inflation for nearly a decade, but not in narrow money. Therefore, we can secure the broad money as one of several pieces of evidence that can be used as an indicator of a sensitive economic event.

Since the Fed started targeting the federal funds rate instead of controlling the amount of money supply in 1982, many empirical research papers examine the availability of the policy rate. Bernanke and Blinder (1992) explore the role of the federal funds rate in the monetary transmission mechanism and find that the interest rate on federal funds is extremely informative about future movements of real macroeconomic variables. They especially focus on the federal funds rates that work through bank loans and deposits. This policy shift makes money appear to have lost its status, however, in reality, it still plays an essential role in understanding the economic environment. The simple sum narrow money is still being published by the Fed, and the simple sum broad money is also published by the OECD. The state-of-the-art Divisia monetary aggregate, which is characterized by a share weighted average of the growth rate of monetary components, is published by the Center for Financial Stability (CFS), and many empirical works support the importance of monetary measurement.

Belongia and Ireland (2015) emphasize the superiority of superlative (Divisia) measures of money in forecasting movements in key macroeconomic variables. They revisit Bernanke and Blinder (1992) and find that the Divisia monetary aggregates play a noteworthy role as an intermediate target or indicator variable and make no noticeable deterioration in the information content.

To investigate the importance of broad money and examine the interest-bearing monetary aggregates, we revisit notable previous literature and extend the period of the data set. While Bernanke and Blinder (1992) and Belongia and Ireland (2015) examine simple sum measures which impute the same weight to all monetary assets in the aggregate, this paper examines Divisia monetary aggregates which are directly derived from economic aggregation theory and impute user cost prices to the marginal utilities of component assets. We explore the availability of Divisia type monetary aggregates for all ranges by conducting several tests.

We also examine credit-card-augmented Divisia monetary aggregates which include monetary services and credit card transaction volumes by the economic aggregation theory. The importance of the credit is already emphasized by Bernanke and Blinder (1988), more than 30 years ago. They state their critical view on the dichotomy between “money" and “credit". Their argument was supported by Liu et al. (2020), who criticize the existing New Keynesian models which are ignoring the importance of liquidity services provided by credit card transactions. Barnett and Su (2016) extend Divisia monetary aggregates originated by Barnett (1980) and they generalize the theory to include credit card transaction services on the demand side. Although credit card transactions have never been included in central bank measures of the money supply due to the accounting conventions, economic aggregation theory allows to aggregate over service flows, such as monetary assets and credit card transaction volumes, regardless of whether the source of the services is assets or liabilities.

Layout This paper is organized as follows: Section 4.2 explains Divisia-type monetary aggregations and Section 4.3 introduces the narrow money and broad money. Section 4.4 displays the causal relationship between a set of real variables and various ranges of monetary aggregates. Section 4.5 reports the recursive VAR and non-recursive VAR estimation for the comparison of the narrow- and broad-money. Section 4.6 concludes.

4.2 Divisia Monetary Aggregates

Divisia monetary aggregate growth rates are the share weighted average of component growth rates, with user cost pricing used in computing the expenditure shares. This paper considers the original Divisia and the credit-card-augmented Divisia monetary aggregates.

4.2.1 Original Divisia Monetary Aggregates

The original Divisia monetary aggregates measure demand-side monetary services, using the economic aggregation theory and index number theory developed by Barnett (1980). Barnett (1980) proposes the use of either the Divisia or Fisher ideal index for monetary quantity aggregation with user cost pricing of components. The difference between the two indexes is negligible, being less than the roundoff error in the components, but the Divisia index is easier to explain and interpret than the Fisher ideal index. The resulting quantity index numbers, by either formula, are elements of Diewert's class of superlative quantity index numbers. Barnett's resulting monetary aggregates are strictly preferable to the simple sum monetary aggregates, since the monetary assets' components are not perfect substitute. Relative to aggregation and index number theory, simple sum aggregation over imperfect substitutes is inadmissible.

Barnett (1980) proved that the real user cost (equivalent rental price) of monetary asset i is

$$\pi_{it} = \frac{R_t - r_{it}}{1 + R_t}, \tag{4.1}$$

where r_{it} is the own rate of return on asset i during period t, and R_t is the risk-free rate of return on pure capital, a completely illiquid asset (benchmark rate), during period t.

The Divisia index in growth rate form in continuous time is

$$d \log M(\mathbf{m}_t) = \sum_i s_{it} d \log m_{it} \tag{4.2}$$

where $s_{it} = \pi_{it} m_{it} / \pi_t' \mathbf{m}_t$ is the expenditure share on monetary asset i and m_{it} is real balances of monetary asset i during period t.

4.2.2 Credit-Card-Augmented Divisia Monetary Aggregates

Credit-card-augmented Divisia is the measure of demand-side monetary services, including credit card transaction volumes, not credit card balances. Barnett and Su (2016) and Barnett et al. (2016b) introduce this new monetary aggregates. Conventional simple sum monetary aggregates have never been augmented to include credit card balances, since accounting conventions do not permit adding liabilities to assets. Credit-card-augmented Divisia can be derived from the consumer decision with credit card transaction volumes along with monetary balances entered into the utility function, reflecting the fact that money and credit card transaction volumes provide services, such as liquidity and transactions services. Economic aggregation theory and index number theory measure service flow, independently of whether from assets or liabilities.

The real user cost of credit card services, derived in Barnett and Su (2016), is:

$$\tilde{\pi}_{jt} = \frac{e_{jt} - R_t}{1 + R_t} \tag{4.3}$$

where R_t is the risk-free rate of return on a completely illiquid asset (pure capital) during period t, and e_{jt} is the interest rate on credit card type j. There are two categories of consumers using credit cards. Some consumers pay interest to the credit card issuing banks, and the others do not pay interest. The "representative consumer" pays the interest rate e_{jt}, averaged over both categories of consumers, including those who maintain rotating balances, and thereby pay interest on contemporaneous credit card transactions, and those consumers who pay off such credit card transactions before the end of the period, and thereby do not pay explicit interest on credit card transactions. The CFS considers four credit card types, j , including Visa, MasterCard, Discover, and American Express.

The aggregation-theoretic exact approach defines the credit-card-augmented structural aggregator function, M, to be the utility function, v,

$$M_t = M(\mathbf{m}_t, \mathbf{c}_t) = v(\mathbf{m}_t, \mathbf{c}_t) \tag{4.4}$$

nested as a weakly separable category utility function within the full utility function, which also contains consumer goods. The growth rate of the Divisia index tracks the growth rate of the exact aggregate $M(\mathbf{m}_t, \mathbf{c}_t)$, where $\mathbf{m}_t$ is the vector of monetary asset quantities held by the representative consumer during period t, and $\mathbf{c}_t$ is the vector of the consumer's four credit card transaction volumes during period t.

The credit card quantities to include in the augmented Divisia index formula are the monthly credit card transactions volumes, not the credit card balances. Including credit card balances would produce double counting problem, since they include carried forward rotating balances used for transactions in prior periods. The growth rate of the credit-card-augmented Divisia index is

$$d\log M(\mathbf{m}_t, \mathbf{c}_t) = \sum_i s_{it} d\log m_{it} + \sum_j \tilde{s}_{jt} d\log c_{jt}, \tag{4.5}$$

where $s_{it} = \pi_{it} m_{it} / (\pi_t' \mathbf{m}_t + \tilde{\pi}_t' \mathbf{c}_t)$ is the expenditure share of monetary asset i in the total service of monetary assets and credit cards, while $\tilde{s}_{jt} = \tilde{\pi}_{jt} c_{jt} / (\pi_t' \mathbf{m}_t + \tilde{\pi}_t' \mathbf{c}_t)$ is the expenditure share of the services of credit card type j in the total services of monetary assets and credit cards.

4.3 Narrow Money and Broad Money

This paper focuses on the broad money, especially M3 and M4, since the benchmark literature gives evidence of the superior of broad money for tracking economic signals. The definition of broad money of OECD is M3, including currency, deposits with an agreed maturity of up to two years, deposits redeemable at notice of up to three months and repurchase agreements, money market fund shares/units and debt securities up to two years. Bernanke and Blinder (1992) deal with simple sum M1 and M2, and Belongia and Ireland (2015) investigate simple sum M1, M2, Divisia M1, M2, and MZM (Money Zero Maturity) in their models.

Extending the benchmark literature, this paper considers simple sum M1, M2, and M3, as well as whole types of money of the original Divisia and credit-card-augmented Divisia that are affordable, both narrow and broad money. We define the narrow money as M1, M2M, MZM, M2, M2ALL, and the broad money as M3, M4-, and M4. Compared to the narrow money, M3 includes large time deposits, overnight and term repos, M4- adds the commercial paper, and M4 contains treasury bills.

4.4 The Information Content of the Various Monetary Aggregations and Federal Funds Rates

Bernanke and Blinder (1992) and Belongia and Ireland (2015) explore the relationship between the federal funds rate and the real variables, and between several monetary aggregations and the real variables. We examine the following equation to explore the information content of money and federal funds rate in explaining the past real activity:

$$Y_t = \alpha + \sum_{i=1}^{6} \lambda_i Y_{t-i} + \sum_{i=1}^{6} \beta_i X_{t-i} + \sum_{i=1}^{6} \gamma_i P_{t-i} + e_t \tag{4.6}$$

where Y_t is one of several measures of real variables, X_t is a measure of monetary policy such as various monetary aggregates and the federal funds rate. P_t is the Consumer Price Index (CPI) that adjusts every estimation for the effects of the price level changes, α, λ_i, β_i, and γ_i $(i = 1, 2, \cdots, 6)$ are regression coefficients. We set six lags of each variable $(i = 1, 2, \cdots, 6)$ and we take a log for all variables. The null hypothesis here is that the monetary aggregations and the Federal funds rate don't Granger-cause the real variables.

Exploring the role of broad money in the economy, we examine various monetary aggregates as measures of X_t in equation (4.6). As an extension of the benchmark works, we adopt the whole ranges of the traditional simple sum, the original Divisia and credit-card-augmented Divisia for the monetary aggregations to test causality. The measures of the simple sum M1 and M2 are published by the FRED and the unit of them is billions of dollars (monthly, not seasonally adjusted). We borrow M3 data from the OECD data set (monthly, not seasonally adjusted). Divisia data is available on a monthly basis offering from the Center for Financial Stability (CFS). The CFS produces Divisia monetary aggregates to the public from January 1967 along with credit-card-augmented Divisia monetary services from July 2006. Traditional Divisia is level normalized to equal 100 in January 1967 and credit-card-augmented Divisia data is level normalized to equal 100 in July 2006.

We explore the Granger-causality test 1) between the real variables and the various monetary aggregations, 2) between the real variables and the Federal funds rate. A set of real variables in this paper includes industrial production, capacity utilization, employment, unemployment rate, housing starts, personal income, retail sales, consumption, and durable-goods orders. The FRED provides the monthly frequency of the all real variables. The industrial production is a seasonally adjusted total index, a level normalized to equal 100 in 2017. Capacity utilization is a total index and seasonally adjusted percent of capacity. The unit of employment is thousands of persons and it is a seasonally adjusted data. Unemployment rate is a seasonally adjusted percent. Housing starts is a seasonally adjusted annual rate expressed in thousands of units. The unit of personal income and consumption are billions of dollars and those are seasonally adjusted annual rates. Retail sales and durable-goods orders are measured in millions of dollars and seasonally adjusted.

Table 4.1 - 4.5 display the Granger-causality test results 1) between the set of real variables and several monetary aggregates, 2) between the set of real variables and the policy rate. Values in the tables show the marginal significance levels for the hypothesis that all lags of monetary aggregates (or the federal funds rate) can be excluded from the equation (4.6). That is, the smaller number indicates the stronger role for the monetary measures toward real variables. Since the credit-card-augmented Divisia data is available from July 2006 and other data is available from January 1992, we divide the periods of the data from January 1992 to December 2019 and from July 2006 to December 2019.

Table 4.1 reports the results of Granger-causality test between a set of real variables and simple

Forecasted variable	Simple Sum M1	Simple Sum M2	Simple Sum M3	FFR	Shadow FFR
Sample Period 1992:01 - 2019:12					
Industrial production	0.2756	0.6077	0.8152	0.4338	0.0892
Capacity utilization	0.1526	0.7134	0.9924	0.0508	**0.0321**
Employment	0.7439	0.8548	**0.0477**	0.5170	0.7714
Unemployment rate	0.1502	0.8019	0.0636	**0.0174**	**0.0463**
Housing starts	0.7661	0.0749	**0.0049**	**0.0293**	0.2604
Personal income	**0.0422**	0.6488	0.4128	**0.0047**	0.1447
Retail sales	0.3909	0.9555	0.1294	0.4634	0.2469
Consumption	0.9056	0.9129	0.3626	0.1912	0.1918
Durable-goods orders	0.3886	0.5225	0.4453	0.1098	0.2012
Sample Period 2006:07 - 2019:12					
Industrial production	0.6938	0.6761	0.7070	**0.0007**	0.1073
Capacity utilization	0.6900	0.7648	0.8283	**0.0003**	0.0970
Employment	0.5439	0.8023	0.0511	**0.0051**	0.5266
Unemployment rate	0.4595	0.8919	0.5324	**0.0010**	0.1020
Housing starts	0.8583	0.3835	0.1204	0.0885	0.3685
Personal income	**0.0452**	0.4960	0.4073	**0.0000**	0.2764
Retail sales	0.5588	0.1874	0.1153	**0.0003**	0.3323
Consumption	0.9484	0.5901	0.0852	**0.0002**	0.1156
Durable-goods orders	0.6158	0.7133	0.8749	**0.0138**	0.4294

Table 4.1: Causality Test Results: Simple Sum and Federal Funds

Note. Simple sum M3 is from OECD. Values in the table are marginal significance levels for the hypothesis that all lags of the monetary aggregates or the federal funds rate can be excluded from the equation (4.6).

Forecasted variable	Divisia M1	Divisia M2M	Divisia MZM	Divisia M2	Divisia M2 ALL
Sample Period 1992:01 - 2019:12					
Industrial production	0.3579	0.8692	0.9902	0.8524	0.9643
Capacity utilization	0.4382	0.9978	0.9773	0.9971	0.9339
Employment	0.1463	0.1351	0.2214	0.0511	**0.0445**
Unemployment rate	**0.0028**	**0.0428**	0.0509	**0.0301**	**0.0342**
Housing starts	0.0679	**0.0044**	**0.0137**	**0.0092**	**0.0386**
Personal income	0.3641	0.2934	0.0820	0.1304	**0.0147**
Retail sales	**0.0033**	**0.0097**	**0.0005**	**0.0479**	**0.0018**
Consumption	0.1684	0.1025	**0.0073**	0.1680	**0.0132**
Durable-goods orders	0.5908	0.3947	0.2733	0.3179	0.1692
Sample Period 2006:07 - 2019:12					
Industrial production	0.3688	0.6329	0.5933	0.5795	0.6787
Capacity utilization	0.4116	0.6131	0.6873	0.6863	0.8192
Employment	0.3023	0.0844	0.4048	**0.0405**	0.1908
Unemployment rate	**0.0394**	0.2874	0.2743	0.3400	0.2167
Housing starts	0.3949	0.1542	0.1722	0.1917	0.2053
Personal income	0.6750	0.1875	**0.0350**	0.2391	**0.0119**
Retail sales	0.0726	0.1498	**0.0046**	0.2296	**0.0211**
Consumption	0.2183	**0.0312**	**0.0025**	0.0711	**0.0075**
Durable-goods orders	0.6714	0.7450	0.9575	0.7908	0.6861

Table 4.2: Causality Test Results: Traditional Divisia, Narrow Money

Note. Values in the table are marginal significance levels for the hypothesis that all lags of the monetary aggregates or the federal funds rate can be excluded from the equation (4.6).

sum money, between a set of real variables and federal funds rate as in Bernanke and Blinder (1992) and Belongia and Ireland (2015). We observe that the federal funds rate has more significant effect on the real activities compared to other measures in X and these results are consistent with the benchmark literature. However, unlike previous research, simple sum M1 affects personal income on both sample periods and simple sum M2 has no effect on all real variables.

In Table 4.2 - 4.5, we find that the original Divisia and credit-card-augmented Divisia tend to have a stronger linkage with real activities compared to the simple sum measure in Table 4.1. While simple sum M1 has an effect on personal income, Divisia M1 and credit-card-augmented Divisia M1 are connected to the unemployment rate and retail sales. Simple sum M2 has no effect

Forecasted variable	Divisia M3	Divisia M4-	Divisia M4
Sample Period 1992:01 - 2019:12			
Industrial production	0.5296	0.8017	0.1219
Capacity utilization	0.4772	0.6942	0.1095
Employment	0.8320	0.9886	0.0517
Unemployment rate	0.4106	0.5887	0.3055
Housing starts	0.5829	0.6986	0.8865
Personal income	**0.0297**	0.0616	**0.0007**
Retail sales	0.1109	0.0881	0.4930
Consumption	0.3616	0.2609	0.0790
Durable-goods orders	0.5342	0.3113	0.4745
Sample Period 2006:07 - 2019:12			
Industrial production	0.1336	0.2906	**0.0466**
Capacity utilization	0.0792	0.2257	**0.0309**
Employment	0.6522	0.9074	0.1339
Unemployment rate	0.4752	0.3708	0.0711
Housing starts	0.9339	0.9702	0.6317
Personal income	0.0740	0.0590	**0.0004**
Retail sales	0.4269	0.2853	**0.0249**
Consumption	0.4998	0.1799	**0.0006**
Durable-goods orders	0.7529	0.7623	0.3527

Table 4.3: Causality Test Results: Traditional Divisia, Broad Money

Note.Values in the table are marginal significance levels for the hypothesis that all lags of the monetary aggregates or the federal funds rate can be excluded from the equation (4.6).

Forecasted variable	Divisia M1	Divisia M2M	Divisia MZM	Divisia M2	Divisia M2 ALL
Sample Period 2006:07 - 2019:12					
Industrial production	0.3619	0.4408	0.3710	0.4182	0.4413
Capacity utilization	0.3885	0.4286	0.4415	0.4964	0.5541
Employment	0.4183	0.1954	0.5689	0.0783	0.2665
Unemployment rate	**0.0090**	0.1999	0.2146	0.2069	0.1622
Housing starts	0.6410	0.3237	0.4013	0.4089	0.5389
Personal income	0.5807	0.3223	0.0982	0.4225	**0.0500**
Retail sales	**0.0333**	0.1166	**0.0048**	0.1957	**0.0219**
Consumption	0.2407	**0.0112**	**0.0007**	**0.0274**	**0.0017**
Durable-goods orders	0.4523	0.6613	0.9814	0.7998	0.8713

Table 4.4: Causality Test Results: credit-Divisia, Narrow Money

Note.Values in the table are marginal significance levels for the hypothesis that all lags of the monetary aggregates or the federal funds rate can be excluded from the equation (4.6).

Forecasted variable	Divisia M3	Divisia M4-	Divisia M4
Sample Period 2006:07 - 2019:12			
Industrial production	0.0717	0.2028	**0.0346**
Capacity utilization	**0.0397**	0.1520	**0.0235**
Employment	0.4934	0.8808	0.2038
Unemployment rate	0.5006	0.4824	0.1029
Housing starts	0.9079	0.9554	0.6657
Personal income	0.2081	0.1418	**0.0013**
Retail sales	0.3023	0.1988	**0.0219**
Consumption	0.3976	0.1452	**0.0003**
Durable-goods orders	0.5657	0.6336	0.4694

Table 4.5: Causality Test Results: credit-Divisia, Broad Money

Note.Values in the table are marginal significance levels for the hypothesis that all lags of the monetary aggregates or the federal funds rate can be excluded from the equation (4.6).

on any real variable, however, Divisia M2 affects unemployment rate, housing starts, retail sales (upper panel in Table 4.2, whole period), employment (lower panel in Table 4.2, second half of the period) and credit-card-augmented Divisia M2 affects consumption (Table 4.4), respectively. The original Divisia M2 ALL and M4 are significantly associated with many economic variables. Especially, the lower panel of Table 4.3 and Table 4.5 show that both original Divisia M4 and credit-card-augmented Divisia M4 are linked to five out of nine real variables. As we have seen in the result of Figure 4.1, the CFS argues that Divisia M4 is the broadest and the most important measure of money. In line with the argument of the CFS, we also find that both original Divisia M4 and credit-card-augmented Divisia M4 are the most significantly associated with many real activities. Unfortunately, since simple sum M4 is not produced, we cannot compare the effects of simple sum M4 on real activities.

4.5 Vector Autoregressive (VAR) estimation

To capture the economic implications of the monetary variables of different ranges, we construct the VAR model. The theoretical reduced form VAR model is (Sims (1980)):

$$\mathbf{X}_t = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{X}_{t-1} + \cdots + \mathbf{A}_p \mathbf{X}_{t-p} + \mathbf{u}_t, \tag{4.7}$$

where p is the number of lags, and assume $\mathbf{u}_t$ to be serially uncorrelated and to have covariance matrix for residuals, $\mathbb{E}\mathbf{u}_t\mathbf{u}_t' = \mathbf{V}$. And the reduced form VAR disturbances are related to the underlying economic shock (Structural VAR), ε_t by

$$\mathbf{u}_t = \mathbf{B}_0 \varepsilon_t, \tag{4.8}$$

where ε_t is serially uncorrelated with a diagonal covariance matrix Σ_ε of full rank (white noise), and $\mathbf{B}_0$ is lower triangular matrix. Put equation (4.8) into the VAR equation (4.7) and multiply $\mathbf{B}_0^{-1}$

to both hand side then we can get

$$\mathbf{B}_0^{-1}\mathbf{X}_t = \mathbf{B}_0^{-1}\mathbf{A}_0 + \mathbf{B}_0^{-1}\mathbf{A}_1\mathbf{X}_{t-1} + \cdots + \mathbf{B}_0^{-1}\mathbf{A}_p\mathbf{X}_{t-p} + \varepsilon_t. \tag{4.9}$$

We borrow 6-endogenous variables from Leeper and Roush (2003) as following. The data starts from July 2006 to December 2019 which includes the Great Recession era and we focus on the behavior of economic variables during economic downturn. First, real GDP and GDP deflator are considered as a measure of aggregate output ($\mathbf{Y}_t$) and price level ($\mathbf{P}_t$), respectively. These data are collected from IHS-markit[1]. We choose shadow federal funds rate ($\mathbf{R}_t$) as a measurement of short-term nominal interest rate, since traditional federal funds rate has limited role in post-Great Recession period. As a measure of flow of monetary service, we choose credit-card-augmented Divisia ($\mathbf{CM}_t$) and its user cost ($\mathbf{UCM}_t$). Use of quantity and price index allows us to distinguish the shock of money demand and that of money supply (Belongia and Ireland (2015)). Finally, we adopt commodity price ($\mathbf{CP}_t$) as a potential solution of price puzzle problem, which is suggested by Sims (1992). All of the data are provided as monthly frequency. Price, output, commodity price, and credit-card-augmented Divisia are converted into stationary, while shadow federal funds rate and user cost keep original unit, not a decimal form, since all of the data are represented by the percentage expression. Following conventional lag-determination rule, the statistical criteria AIC allows us to use 13 lags.

In short, the vector of endogenous variables are given by;

$$\mathbf{X}_t = [\mathbf{P}_t \;\; \mathbf{Y}_t \;\; \mathbf{CP}_t \;\; \mathbf{R}_t \;\; \mathbf{CM}_t \;\; \mathbf{UCM}_t]'. \tag{4.10}$$

Now we have structural VAR model with 6-endogenous variables and determine the relationship $\mathbf{V} = \mathbf{B}_0\mathbf{B}_0'$ uniquely by using Cholesky decomposition. The order of variables becomes important when we are imposing zero restriction with Cholesky decomposition. There is no fixed rule for the variable ordering, but conventionally the most exogenous variable is located in the first row of the

[1] See https://ihsmarkit.com/products/us-monthly-gdp-index.html.

vector $\mathbf{X}_t$, and the least exogenous variable is located in the last row of the vector $\mathbf{X}_t$. Following the identification scheme of Cholesky decomposition, the fourth element of the vector ε_t, or ε_t^{mp}, can be expressed by

$$b_{41}p_t + b_{42}y_t + b_{43}cp_t + b_{44}r_t = \varepsilon_t^{mp}, \tag{4.11}$$

which explains the first three variables; price, output, and commodity price that behave sluggishly. The last variable, monetary policy instrument, is adjusted by Fed instantly in response to monetary policy shock. In the same manner, the fifth and sixth elements of the vector ε_t, ε_t^{md} and ε_t^{ms} can be expressed respectively, by

$$b_{51}p_t + b_{52}y_t + b_{53}cp_t + b_{54}r_t + b_{55}m_t = \varepsilon_t^{md}, \tag{4.12}$$

$$b_{61}p_t + b_{62}y_t + b_{63}cp_t + b_{64}r_t + b_{65}m_t + b_{66}u_t = \varepsilon_t^{ms}. \tag{4.13}$$

Equation (4.12) represents the money demand equation. It explains that the first four variables are determinants of money demand which respond not immediately to the money demand shock. The last variable is the only factor that response instantly toward the shock. Equation (4.13) demonstrates the behavior monetary service which is considered to be the least exogenous.

Use of Cholesky decomposition as an identification method is powerful and easy to understand, however, it is strict rule, since the order of the variable is the only factor for determining the composition of the shock and does not allow exception. To loose the restriction and give the variation, we introduce non-recursive VAR. Keating (1992) introduces non-recursive VAR estimation in the long-run and contemporaneous model. By allowing variation into the contemporaneous model, he induces flexible form of $\mathbf{B}_0^{-1}$ and shows 4-meaningful economic representations (AS curve, IS curve, money supply curve, and money demand curve). Inspired by this work, we follow the algorithm for solving non-linear equations system provided by MATLAB[2]. The use of MATLAB fsolve function set provides the solution of non-linear equation systems.

Following Leeper and Roush (2003) and Belongia and Ireland (2015), the benchmark non-

[2] See Kilian and Lütkepohl (2017) Chapter 9, p. 241 - 265.

recursive model specifies the matrix $\mathbf{B}_0^{-1}$ as

$$\mathbf{B}_0^{-1} = \begin{pmatrix} b_{11} & 0 & 0 & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \\ b_{41} & b_{42} & 0 & b_{44} & b_{45} & 0 \\ -b_{55} & b_{52} & 0 & 0 & b_{55} & b_{56} \\ -b_{65} & 0 & 0 & b_{64} & b_{65} & b_{66} \end{pmatrix} \tag{4.14}$$

which is called as a model of Taylor rule with money, where,

$$\mathbf{B}_0^{-1}\mathbf{u}_t = \varepsilon_t. \tag{4.15}$$

Since the restrictions are imposed to $\mathbf{B}_0^{-1}$, and the matrix is over-identified (number of the restrictions are greater than 15), we apply GMM estimation. By focusing on the fourth, fifth, and sixth rows of matrix $\mathbf{B}_0^{-1}$, the Equation (4.11), (4.12), and (4.13) can be restated as

$$b_{41}p_t + b_{42}y_t + b_{44}r_t + b_{45}m_t = \varepsilon_t^{mp}, \tag{4.16}$$

$$b_{52}y_t + b_{55}(m_t - p_t) + b_{56}u_t = \varepsilon_t^{md}, \tag{4.17}$$

$$b_{64}r_t + b_{65}(m_t - p_t) + b_{66}u_t = \varepsilon_t^{ms}, \tag{4.18}$$

respectively. Equation (4.16) explains the extended version of Taylor rule with monetary variable. To determine the federal funds rate, Federal Reserve considers the changes in the aggregate output, price, and additionally, change in money supply. The last two equations allow us to fully concentrate on the monetary variable. The use of Divisia-types of monetary variables in money demand Equation (4.17) offers a more precise quantity and price of monetary service and it leads the measurement more accurately (Belongia (2006)). Equation (4.18) describes the behavior of the financial institutions that create the liquid assets that provide households and firms with monetary services. This model illustrates how the increase in the cost of bank resulting from an increase in the federal funds rate is directly passed on to consumers in the form of an increase in the user costs.

The real money service is also considered to allow for the possibility of short-term cost increases due to the expansion of the bank's operation scale. Belongia and Ireland (2015) emphasize this non-recursive modeling offers a more detailed and theoretically motivated description of the banks that supply monetary assets and the non-bank public that demands those same assets.

Meanwhile we impose additional specification on the elements of $\mathbf{B}_0^{-1}$. If we apply $b_{45} = 0$ to $\mathbf{B}_0^{-1}$ additionally, then we can introduce a typical Taylor rule that does not take into account the money supply (called Taylor rule without money model). Instead, if we apply $b_{41} = 0$, $b_{42} = 0$ to $\mathbf{B}_0^{-1}$, then we can introduce Leeper and Roush (2003) specification (called Money-interest rate rule model). This model shifts the money supply toward more center as a (sole) determinant of the short-term interest rate. For the inference of the model, we investigate the impulse responses with the 2-standard deviation confidence interval calculated by the delta-method bootstrapping[3].

For extension of the work, the original Divisia monetary aggregates and the corresponding user costs visit the model. Since the original Divisia has a long history compared to the credit-card-augmented Divisia, we can extend the sample period from January 1992 to December 2019. After, we split the sample period by two, January 1992 - June 2006 and July 2006 - December 2019, and investigate the model of each sample periods. Then the work of the first sub-sample period would be the direct comparison to the benchmark work (Belongia and Ireland (2015)), and that of the second sub-sample period would be the comparison to the main discussion in this paper. These data set also report the optimal length of lag 13. Finally, we repeat the whole process by switching the policy rate to the effective federal funds rate.

Table D.1 - D.16 in Appendix D show the estimates of the fourth-row (monetary policy) and fifth-row (money demand) equations for the typical recursive model calculated by Cholesky decomposition, and the estimates of the fourth-row (monetary policy), fifth-row (money demand), and sixth-row (monetary service) equations for three non-recursive models. The diagonal elements of $\mathbf{B}_0^{-1}$ are normalized to 1 which allow the model to isolate interest rate, real money service, and the user cost to the left-hand side of monetary policy, money demand and monetary service equa-

[3]See Kilian and Lütkepohl (2017) Chapter 12, p. 416 - 420.

tions, respectively. The benchmark paper confirms that the policy rate has a positive relationship with the output, price, and real money has a positive relationship with output and negative relationship with the user costs. And it demonstrate the user costs have a positive relationship with the policy rate and real money service. The results of the Table D.11 and D.12, use of original Divisia and effective shadow federal funds rate, first sub-sample period, show the sign of the coefficient is consistent with the benchmark results.

Tables D.1 and D.2 describe the result of equations using credit-card-augmented Divisia and shadow federal funds rate, July 2006 - December 2019. In Table 4.6 of the broader money case, the Taylor-type monetary policy rules show that the increase in federal funds rate can be explained by the increase in output and decrease in price. Meanwhile, in the case of the non-Taylor-type monetary policy rule and the money-interest rate rule, it shows a roughly negative relationship between the money supply and the policy rate. In other words, we can interpret that a positive monetary policy shock is associated with an increase in the policy rate and money supply. And the money demand equations show the positive relationship between real money and output, and the negative relationship between real money and user cost variables. Equation (4.18) explains how the monetary system delivers an increase in policy rate along to consumers of monetary services in the form of higher user costs. We can track a positive relationship between real money services and the user cost. This relationship can be re-evaluated as a supply curve for the monetary service. In Table D.2, the narrow money case, the results are somewhat different from that of Table D.1. Most of the monetary policy rule shows the negative relationship between the policy rate and explanatory variables on the right-hand side. The results of money demand and money service equations show a consistency with that of the broad money case.

Tables D.3 and D.4 show the results when we switch only the policy rate to the traditional, conventional effective federal funds rate. In Taylor-type monetary policy, both broad and narrow money show the increase in the policy rate in response to increases in output and price. In the money-interest rate rule case, we can distinguish the behaviors between broad and narrow money in the monetary policy equation. In broad money case, we can check the negative relationship between the policy rate and money supply, however, in narrow money case, we can capture the opposite relationship. In the money demand equations, narrow money shows a relatively strong tendency for a positive relationship between real-money service and output, and a negative relationship between real-money service and user costs. For the monetary system equation, we cannot track the common signs of the variables, but the narrow money-interest rate rule shows the positive relationship between user cost and the explanatory variables. We find that the use of the effective federal funds rate gives us a more clear understanding of the monetary policy equations, meanwhile the demand behavior of money and the monetary system can be explained clearly with the shadow federal funds rate.

Tables D.5 and D.6 describe the result of equations using original Divisia and shadow federal funds rate, July 2006 - December 2019. In this case, we cannot capture the differences between broad and narrow money. In the Taylor-type monetary policy rule, we observe that the increase in the policy rate is attributed to a decrease in output and price. In the non-Taylor-type monetary policy rule, we check the negative relationship between the money supply and the policy rate. In the money demand equations, the user cost affects the dominant effect on money demand. In detail, the user cost has a positive effect in the Taylor rule with money and the money-interest rate rule. However, it has a negative effect in the Taylor rule without money. For the monetary service equation, the user cost has a positive relationship with the policy rate and the real-money service.

Following Table D.7 and D.8 illustrate the equations using original Divisia and effective federal funds rate, July 2007 - December 2019. The Taylor-type monetary policy shows the increase in the policy rate can be explained by the increase in output. For the money demand equation, the effect of the user costs dominates the effect of the output. We cannot observe the negative relationship between the real money and the user costs, and the positive relationship between the real money and the output. For the monetary system equation, the Taylor-type equations show the positive relationship between the user cost and policy rate, and real-money service.

At a glance, a switch of the monetary variables, from credit-card-augmented Divisia to original Divisia, and a switch of the policy rate from shadow federal funds rate to effective federal funds rate does not make noticeable differences for both broad and narrow money. However, the sample periods matter. We confirm that by applying the different sample periods for the case of the money-interest rate rule which is using the effective federal funds rate, meaningful results are obtained for distinguishing between broad money and narrow money. First of all, we observe the conventional results in the case of the whole sample period (January 1992 - December 2019) and the first sample period (January 1992 - June 2006). Positive monetary policy shock is associated with the increase in the policy rate and decrease in the original Divisia money supply. Meanwhile, different behaviors of the money supply are captured in the second sample period which includes the Great recession (July 2007 - December 2019). In the original Divisia case, the results show the negative relationship between money supply (both broad and narrow money) and policy rate. Whereas, the use of credit-card-augmented Divisia as a monetary variable displays slightly contrasting results. To be specific, the use of broad money shows the negative relationship between the policy rate and broad money supply, and the use of narrow money shows the positive relationship between the policy rate and narrow money supply. In this sub-sample period, the economy experienced a compulsory-zero-lower bound, several massive amounts of money supply, and a negative real interest rate. These unusual economic incidents are expected to have contributed to undermining the negative relationship between policy rates and the money supply.

The result of the impulse response reinforces the argument that monetary variables should be at the center of economic analysis. Compared to the whole cases of equations (recursive and non-recursive cases), the use of the Money-interest rate rule reports a relatively small amount of liquidity puzzle and its duration is also relatively short (Figure 4.2[4]). We choose the case of using the shadow federal funds rate, credit-card-augmented Divisia M4, and its corresponding price index as a policy rate and monetary variables, respectively.

Following the Money-interest rate rule equations, Figure 4.3[5] displays the impulse responses of

[4] 1st row: Recursive, 2nd row: Taylor rule with money, 3rd row: Taylor rule without money, 4th row: Money-interest rate rule. Dotted red line stands for the 2-standard deviation confidence interval calculated by the delta-method bootstrapping.

[5] Dotted red line stands for the 2-standard deviation confidence interval calculated by the delta-method bootstrapping.

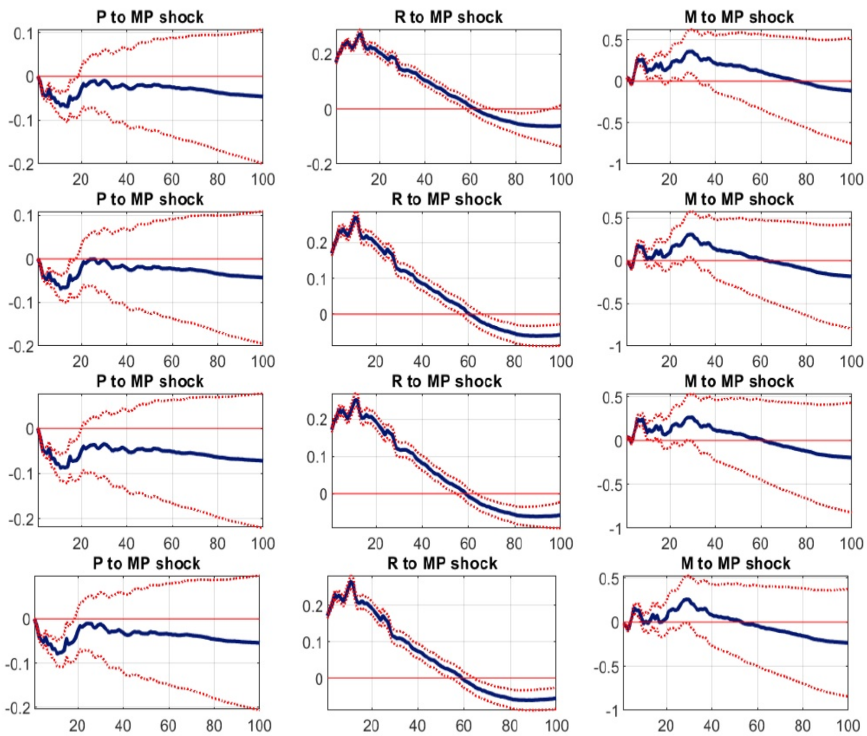


Figure 4.2: Impulse Responses to monetary policy shock

endogenous variables. Each column describes the responses to monetary policy, money demand, and monetary system shock, respectively. In the first column, we observe the decreases in price, output, and the quantity of money. An increase in the policy rate affects to increase in the user cost price index. This can be explained by the monetary transmission mechanism that the inflation tax effect caused by the monetary policy shock is transmitted to the output through the increase in the user cost price.

The second column describes the impulse responses to the money demand shock. We observe the decrease in output and the increase in the policy rate; these results are consistent with the logic of the conventional economic theory. These results reconfirm the argument of Belongia and Ireland (2015). Meanwhile, the decreasing behavior of price to money demand shock is also well matched with the theory, but this result could not be captured by the benchmark research. As we have seen, the positive money demand shock has a positive effect on the policy rate. Due to the effect of this monetary policy channel, we observe a negative behavior on the price level.

The last column describes the impulse responses to monetary system shock. The behavior of

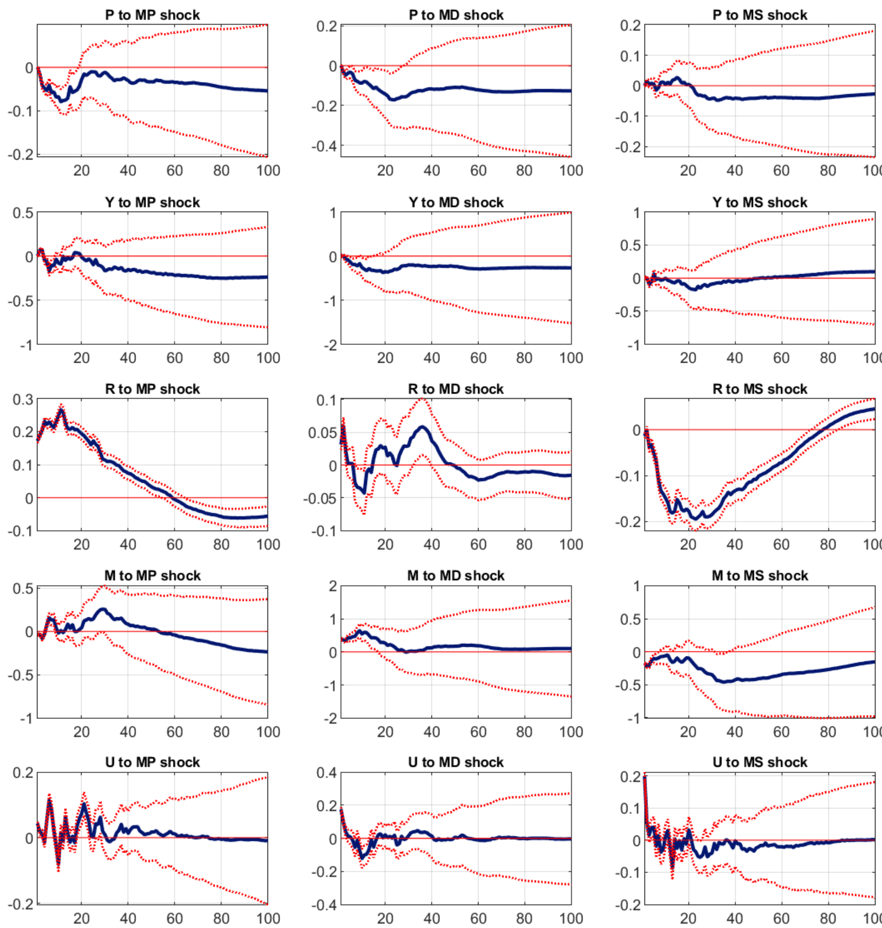


Figure 4.3: Impulse Responses to monetary policy shock, money demand shock, and monetary service shock

	R: polr	R: lrm	SAS	Stata
$\hat{b}_1$	-0.28	-0.28	0.28	-0.28
$\hat{\zeta}_1$	-4.24	4.24	-4.24	-4.24
$\hat{\zeta}_2$	-2.32	2.32	-2.32	-2.32

Table 4.6: Table for the listoftables in the contents page.

the private financial sector induces the temporary decrease in output and the relatively persistent decrease in price. It can be confirmed that the continuous decline in the nominal policy rate to respond to the disturbances in the financial sector causes an instant decrease in the user cost price. However, at the same time, we capture that it takes a long haul for the response of the quantity of money to return to a positive level.

4.6 Conclusion

The economic significance of the money supply seems to have lost its status since the Fed has been focusing on the federal funds rate instead of controlling the amount of money supply for monetary policy. However, it still plays an essential role in economic analysis. In particular, the monetary aggregate variables play a key role in explaining the monetary policy mechanism in the recent era of zero-lower bound, where the traditional federal funds rate conducts only a limited role.

Many previous research papers proved that Divisia-type monetary aggregates published by the CFS are superior to the simple sum in tracking true liquidity service flow and in forecasting real economic variables. In this paper, we mainly focus on the broad Divisia monetary aggregates to track the economic role. We found that the broader Divisia have a strong causality with various section of real economic variables. This result can be a case that justifies the use of broad Divisia in economic analysis. In a value-neutral point of view, we confirmed that the broad Divisia can be used as a detectable instrument for economic abnormality regardless of whether the direction of the signal is correct.

References

Anderson, R. G., Duca, J. V., Fleissig, A. R., and Jones, B. E. (2019). New monetary services (Divisia) indexes for the post-war U.S. *Journal of Financial Stability*, 42:3–17.

Anderson, R. G. and Jones, B. E. (2011). A comprehensive revision of the U.S. monetary services (Divisia) indexes. *Federal Reserve Bank of St. Louis Review*, 93(5):235–259.

Anderson, R. G., Jones, B. E., and Nesmith, T. D. (1997). Monetary aggregation theory and statistical index numbers. *Federal Reserve Bank of St. Louis Review*, 79(1):31–51.

Annicchiarico, B. and Rossi, L. (2013). Optimal monetary policy in a New Keynesian model with endogenous growth. *Journal of Macroeconomics*, 38:274–285.

Bailey, M. J. (1956). The welfare cost of inflationary finance. *Journal of Political Economy*, 64(2):93–110.

Barnett, W. A. (1978). The user cost of money. *Economics Letters*, 1:145–149. Reprinted in Barnett, William A. and Serletis, A. (eds.) (2000) *The Theory of Monetary Aggregation*, North Holland, Amsterdam, 6–10.

Barnett, W. A. (1980). Economic monetary aggregates: an application of aggregation and index number theory. *Journal of Econometrics*, 14(1):11–48. Reprinted in Barnett, William A. and Serletis, A. (eds.) (2000) *The Theory of Monetary Aggregation*, North Holland, Amsterdam, 11–48.

Barnett, W. A. (1987). The microeconomic theory of monetary aggregation. in W. A. Barnett and K. J. Singleton (eds.). *New Approaches to Monetary Economics*, Cambridge University Press, Cambridge:115–168.

Barnett, W. A., Chauvet, M., and Leiva-Leon, D. (2016a). Real-time nowcasting of nominal GDP with structural breaks. *Journal of Econometrics*, 191(2):312–324.

Barnett, W. A., Chauvet, M., Leiva-Leon, D., and Su, L. (2016b). The credit-card-services augmented Divisia monetary aggregates. *Journal of Econometrics*, MPRA Paper 73245:University Library of Munich, Germany.

Barnett, W. A., Fisher, D., and Serletis, A. (1992). Consumer theory and the demand for money. *Journal of Economic Literature*, 30(4):2086–2119.

Barnett, W. A. and Gaekwad, N. B. (2018). The demand for money for EMU: A flexible functional form approach. *Open Economies Review*, 29:353–371.

Barnett, W. A., Liu, J., Mattson, R. S., and van den Noort, J. (2013). The new CFS Divisia monetary aggregates: Design, construction, and data sources. *Open Economies Review*, 24(1):101–124.

Barnett, W. A., Offenbacher, E. K., and Spindt, P. A. (1984). The new Divisia monetary aggregates. *Journal of Political Economy*, 92(6):1049–1085.

Barnett, W. A., Park, H., and Park, S. (2021). The Barnett Critique. *Encyclopedia*, 1(3):964–973. In "Encyclopedia of Social Sciences," edited by Chia–Lin Chang, Michael McAleer, and Philip Hans Franses.

Barnett, W. A. and Su, L. (2016). Joint aggregation over money and credit card services under risk. *Economics Bulletin*, 36(4):2301–2310.

Barnett, W. A. and Su, L. (2017). Data sources for the credit-card-augmented Divisia monetary aggregates. *Research in International Business and Finance*, 39(B):899–910.

Barnett, W. A. and Su, L. (2019). Risk adjustment of the credit-card-augmented Divisia monetary aggregates. *Macroeconomic Dynamics*, 23(S1):90–114.

Barnett, W. A. and Su, L. (2020). Financial firm production of inside monetary and credit card services: an aggregation theoretic approach. *Macroeconomic Dynamics*, 24(S1):1–31.

Barnett, W. A. and Wu, S. (2005). On user costs of risky monetary assets. *Annals of Finance*, 1:35–50. Reprinted in Barnett, W. A. and Chauvet, M. (eds.) (2010) *Financial Aggregation and Index Number Theory*, World Scientific: Singapore, 85–106.

Belongia, M. T. (1996). Measurement matters: Recent results from monetary economics reexamined. *Journal of Political Economy*, 104(5):1065–1083.

Belongia, M. T. and Chalfant, J. A. (1989). The changing empirical definition of money: Some estimates from a model of the demand for money substitutes. *Journal of Political Economy*, 97(2):387–397.

Belongia, M. T. and Chrystal, K. A. (1991). An admissible monetary aggregate for the United Kingdom. *Review of Economics and Statistics*, 73(3):491–502.

Belongia, M. T. and Ireland, P. N. (2006). The own-price of money and the channels of monetary transmission. *Journal of Money, Credit, and Banking*, 38(2):429–445.

Belongia, M. T. and Ireland, P. N. (2014). The Barnett critique after three decades: A New Keynesian analysis. *Journal of Econometrics*, 183:5–21.

Belongia, M. T. and Ireland, P. N. (2015). Interest rates and money in the measurement of monetary policy. *Journal of Business and Economic Statistics*, 33(2):255–269.

Belongia, M. T. and Ireland, P. N. (2019). The demand for Divisia money: Theory and evidence. *Journal of Macroeconomics*, 61:103128.

Bernanke, B. S. and Blinder, A. S. (1988). Credit, money, and aggregate demand. *American Economic Review*, 78(2):435–439.

Bernanke, B. S. and Blinder, A. S. (1992). The federal funds rate and the channels of monetary transmission. *American Economic Review*, 82(4):901–921.

Binner, J. M., Tino, P., Tepper, J., Anderson, R. G., Jones, B., and Kendall, G. (2010). Does money matter in inflation forecasting? *Phisica A*, 389:4793–4808.

Bissoondeeal, R. K., Karoglou, M., and Binner, J. M. (2019). Structural changes and the role of monetary aggregates in the UK. *Journal of Financial Stability*, 42:100–107.

Cagan, P. (1956). The monetary dynamics of hyperinflation. *Studies in the Quantity Theory of Money*, ed. M. Friedman:25–117. Chicago: University of Chicago Press.

Calvo, G. A. (1983). Staggered prices in a Utility-Maximizing framework. *Journal of Monetary Economics*, 12(3):383–398.

Calza, A. and Zaghini, A. (2011). Welfare costs of inflation and the circulation of US currency abroad. *ECB Working Paper*, No.1326.

Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2000). Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem? *Econometrica*, 68(5):1151–1179.

Chin, K. H. and Li, X. (2019). Bayesian forecast combination in VAR-DSGE models. *Journal of Macroeconomics*, 59:278–298.

Cooley, T. F. and Hansen, G. D. (1989). The inflation tax in a Real Business Cycle model. *The American Economic Review*, 79(4):733–748.

Cooley, T. F. and Hansen, G. D. (1991). The welfare costs of moderate inflations. *Journal of Money, Credit, and Banking*, 23(3):483–503.

Cysne, R. P. (2003). Divisia index, inflation, and welfare. *Journal of Money, Credit, and Banking*, 35(2):221–238.

Cysne, R. P. and Turchick, D. (2010). Welfare costs of inflation when interest-bearing deposits are disregarded: A calculation of the bias. *Journal of Economic Dynamics and Control*, 34(6):1015–1030.

Dai, W. and Serletis, A. (2019). On the Markov Switching welfare cost of inflation. *Journal of Economic Dynamics and Control*, 108:103748.

Darvas, Z. (2015). Does money matter in the euro area? Evidence from a new Divisia index. *Economics Letters*, 133:123–126.

Dotsey, M. and Ireland, P. (1996). The welfare cost of inflation in general equilibrium. *Journal of Monetary Economics*, 37:29–47.

Drake, L. and Mills, T. C. (2005). A new empirically weighted monetary aggregate for the United States. *Economic Inquiry*, 43:138–157.

Elger, T., Jones, B. E., and Nilssons, B. (2006). Forecasting with monetary aggregates: Recent evidence for the United States. *Journal of Economics and Business*, 58:428–446.

Ellington, M. (2018). The case for Divisia monetary statistics: A Bayesian time-varying approach. *Journal of Economic Dynamics and Control*, 96:26–41.

Evans, L. and Kenc, T. (2003). Welfare cost of monetary and fiscal policy shocks. *Macroeconomic Dynamics*, 7:212–238.

Hafer, R. W. and Jansen, D. W. (1991). The demand for money in the United States: Evidence from cointegration tests. *Journal of Money, Credit, and Banking*, 23(2):155–168.

Hancock, M. (2005). Divisia money. *Bank of England Quarterly Bulletin*, 45(1):39–46.

Hoffman, D. L. and Rasche, R. H. (1991). Long-run income and interest elasticities of money demand in the United States. *The Review of Economics and Statistics*, 73(4):665–674.

Ireland, P. N. (2009). On the welfare cost of inflation and the recent behavior of money demand. *The American Economic Reviews*, 99(3):1040–1052.

Ireland, P. N. (2014). The macroeconomic effects of interest on reserves. *Macroeconomic Dynamics*, 18(6):1271–1312.

Kadiyala, K. R. and Karlsson, S. (1997). Numerical methods for estimation and inference in Bayesian VAR-Models. *Journal of Applied Econometrics*, 12(2):99–132.

Keating, J. W. (1992). Structural approaches to Vector Autoregressions. *Federal Reserve Bank of St. Louis Review*, September/October 1992:37–57.

Keating, J. W. and Smith, A. L. (2019). The optimal monetary instrument and the (mis)use of causality tests. *Journal of Financial Stability*, 42:90–99.

Kilian, L. and Lütkepohl, H. (2017). *Structural Vector Autoregressive Analysis*. Cambridge University Press.

Leeper, E. M. and Roush, J. E. (2003). Putting 'M' back in monetary policy. *Journal of Money, Credit, and Banking*, 35:1217–1256.

Litterman, R. B. (1980). A Bayesian procedure for rorecasting with Vector Autoregression. Working Paper, Massachusetts Institute of Technology, Department of Economics, Cambridge, MA.

Liu, J., Dery, C., and Serletis, A. (2020). Recent monetary policy and the credit-card-augmented Divisia monetary aggregates. *Journal of Macroeconomics*, 64:103203.

Liu, J. and Serletis, A. (2020). Money growth variability and output: Evidence with credit-card-augmented Divisia monetary aggregates. *Studies in Nonlinear Dynamics and Econometrics*, 24(5):1–11.

Liu, Z. and Spiegel, M. M. (2015). Optimal monetary policy and capital account restrictions in a small open economy. *IMF Economic Review*, 63(2):298–324.

Lucas, R. E. J. (2000). Inflation and welfare. *Econometrica*, 68(2):247–274.

Mao, S. Z., Huang, C. Y., and Chang, J. J. (2019). Growth effects and welfare costs in an Innovation-Driven Growth model of money and banking. *Journal of Macroeconomics*, 62:103049.

Meltzer, A. H. (1963). The demand for money: The evidence from the time series. *Journal of Political Economy*, 71(3):219–246.

Phylaktis, K. and Taylor, M. P. (1993). Money demand, the Cagan model and the inflation tax: Some Latin American evidence. *The Review of Economics and Statistics*, 75(1):32–37.

Poole, W. (1999). Is inflation too low. *Federal Reserve Bank of St. Louis Review*, July/August 1999:3–10.

Rossi, B. and Sekhposyan, T. (2010). Have economic models' forecasting performance for US output growth and inflation changed over time, and when? *International Journal of Forecasting*, 26(4):808–835.

Rossi, B. and Sekhposyan, T. (2014). Evaluating predictive densities of US output growth and inflation in a large macroeconomic data set. *International Journal of Forecasting*, 30(3):662–682.

Schunk, D. L. (2001). The relative forecasting performance of the Divisia and simple sum monetary aggregates. *Journal of Money, Credit and Banking*, 33(2):272–283.

Serletis, A. and Virk, J. J. (2006). Monetary aggregation, inflation, and welfare. *Applied Financial Economics*, 16:499–512.

Serletis, A. and Xu, L. (2020). Money supply volatility and the macroeconomy. *Macroeconomic Dynamics*, 24(6):1392–1402.

Serletis, A. and Xu, L. (2021). The welfare cost of inflation. *Journal of Economic Dynamics and Control*, 128:104144.

Sidrauski, M. (1967). Rational choice and patterns of growth in a monetary economy. *American Economic Review*, 57(2):534–544.

Sims, C. (1980). Macroeconomics and reality. *Econometrica*, 48(1):1–49.

Sims, C. (1992). Interpreting the macroeconomic time series facts: The effects of monetary policy. *European Economic Review*, 36(5):1–49.

Stock, J. H. and Watson, M. W. (1999). Forecasting inflation. *Journal of Monetary Economics*, 44(2):293–335.

Stock, J. H. and Watson, M. W. (2003). Forecasting output and inflation: The role of asset prices. *Journal of Economic Literature*, 41(3):788–829.

Swofford, J. L. and Whitney, G. A. (1991). The composition and construction on monetary aggregates. *Economic Inquiry*, 29(4):752–761.

Teo, W. L. and Yang, P. C. (2011). Welfare cost of inflation in a New Keynesian model. *Pacific Economic Review*, 16(3):313–338.

Thomas, R. (1996). Understanding broad money. *Bank of England Quarterly Bulletin*, June:163–179.

Thornton, D. L. and Yue, P. (1992). An extended series of Divisia monetary aggregates. *Federal Reserve Bank of St. Louis Review*, 74(6):35–52.

Yue, P. and Fluri, R. (1991). Divisia monetary services indexes for Switzerland: Are they useful for monetary targeting? *Federal Reserve Bank of St. Louis Review*, 73(5):19–33.

Appendix A

Household Optimization

In Section 3.2.1, the representative household maximizes the utility function (3.2) subject to the constraints (3.3), (3.5), (3.7), and its Lagrangian for the problem can be written:

$$
\begin{aligned}
\mathscr{L}_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ & \zeta \ln C_t + \eta \ln \ell_t + \theta \ln \left(\frac{M_t^{cc}}{P_t} \right) \\
& + \Lambda_{1,t} \Big[\left(\nu^{\frac{1}{\omega}} \left(\frac{N_t}{P_t} \right)^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}} \left(\frac{D_t}{P_t} \right)^{\frac{\omega-1}{\omega}} + (1-\nu-\phi)^{\frac{1}{\omega}} \left(\frac{CC_t}{P_t} \right)^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} - \frac{M_t^{cc}}{P_t} \Big] \\
& + \Lambda_{2,t} \Big[\frac{M_{t-1} + B_{t-1} + \tau_t - B_t/r_t - N_t + L_t + CC_t}{P_t} + \int_0^1 \frac{Q_{i,t}}{P_t} (s_{i,t-1} - s_{i,t})\, di - \frac{D_t}{P_t} \Big] \\
& + \Lambda_{3,t} \Big[\frac{N_t + r_t^D D_t + W_t(1-\ell_t)}{P_t} + \int_0^1 \frac{F_{i,t}}{P_t} s_{i,t}\, di - C_t - \frac{r_t^L L_t + e_t CC_t + M_t}{P_t} \Big] \Big\}
\end{aligned}
$$

The first-order conditions with respect to B_t, C_t, L_t, M_t, h_t, M_t^{cc}, $s_{i,t}$, N_t, D_t, and CC_t are as follow and $\Lambda_{1,t}$, $\Lambda_{2,t}$, and $\Lambda_{3,t}$ denote the Lagrange multipliers on the constraints (3.3) in real terms, (3.5), and (3.7).

$$\frac{\Lambda_{2,t}}{P_t r_t} = \beta \mathbb{E}_t \Big[\frac{\Lambda_{2,t+1}}{P_{t+1}} \Big] \tag{A.1}$$

$$\Lambda_{3,t} = \frac{\zeta}{C_t} \tag{A.2}$$

$$\Lambda_{2,t} = r_t^L \Lambda_{3,t} \tag{A.3}$$

$$\frac{\Lambda_{3,t}}{P_t} = \beta \mathbb{E}_t \Big[\frac{\Lambda_{2,t+1}}{P_{t+1}} \Big] \tag{A.4}$$

$$\Lambda_{3,t}\frac{W_t}{P_t}=\frac{\eta}{\ell_t} \tag{A.5}$$

$$\Lambda_{1,t}\frac{M_t^{cc}}{P_t}=\theta \tag{A.6}$$

$$\Lambda_{2,t}\frac{Q_{i,t}}{P_t}-\Lambda_{3,t}\frac{F_{i,t}}{P_t}=\beta\mathbb{E}_t\left[\Lambda_{2,t+1}\frac{Q_{i,t+1}}{P_{t+1}}\right] \tag{A.7}$$

$$\Lambda_{2,t}-\Lambda_{3,t}=\Lambda_{1,t}\left[\nu^{\frac{1}{\omega}}\left(\frac{N_t}{P_t}\right)^{\frac{\omega-1}{\omega}}+\phi^{\frac{1}{\omega}}\left(\frac{D_t}{P_t}\right)^{\frac{\omega-1}{\omega}}+(1-\nu-\phi)^{\frac{1}{\omega}}\left(\frac{CC_t}{P_t}\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}}\nu^{\frac{1}{\omega}}\left(\frac{N_t}{P_t}\right)^{-\frac{1}{\omega}} \tag{A.8}$$

$$\Lambda_{2,t}-r_t^D\Lambda_{3,t}=\Lambda_{1,t}\left[\nu^{\frac{1}{\omega}}\left(\frac{N_t}{P_t}\right)^{\frac{\omega-1}{\omega}}+\phi^{\frac{1}{\omega}}\left(\frac{D_t}{P_t}\right)^{\frac{\omega-1}{\omega}}+(1-\nu-\phi)^{\frac{1}{\omega}}\left(\frac{CC_t}{P_t}\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}}\phi^{\frac{1}{\omega}}\left(\frac{D_t}{P_t}\right)^{-\frac{1}{\omega}} \tag{A.9}$$

$$e_t\Lambda_{3,t}-\Lambda_{2,t}=\Lambda_{1,t}\left[\nu^{\frac{1}{\omega}}\left(\frac{N_t}{P_t}\right)^{\frac{\omega-1}{\omega}}+\phi^{\frac{1}{\omega}}\left(\frac{D_t}{P_t}\right)^{\frac{\omega-1}{\omega}}+(1-\nu-\phi)^{\frac{1}{\omega}}\left(\frac{CC_t}{P_t}\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}}(1-\nu-\phi)^{\frac{1}{\omega}}\left(\frac{CC_t}{P_t}\right)^{-\frac{1}{\omega}} \tag{A.10}$$

Appendix B

The Stationary System

Since most variables will be nonstationary in Section 3.2, we make the system to be stationary by defining new set of variables as $c_t = C_t/Z_{t-1}$, $y_t = Y_t/Z_{t-1}$, $y_t^* = Y_t^*/Z_{t-1}$, $f_t = (F_t/P_t)/Z_{t-1}$, $\lambda_{1,t} = Z_{t-1}\Lambda_{1,t}$, $\lambda_{2,t} = Z_{t-1}\Lambda_{2,t}$, $\lambda_{3,t} = Z_{t-1}\Lambda_{3,t}$, $m_t = (M_t/P_t)/Z_{t-1}$, $m_t^{cc} = (M_t^{cc}/P_t)/Z_{t-1}$, $n_t = (N_t/P_t)/Z_{t-1}$, $n_t^{\nu} = (N_t^{\nu}/P_t)/Z_{t-1}$, $l_t = (L_t/P_t)/Z_{t-1}$, $d_t = (D_t/P_t)/Z_{t-1}$, $cc_t = (CC_t/P_t)/Z_{t-1}$, $w_t = (W_t/P_t)/Z_{t-1}$, $z_t = Z_t/Z_{t-1}$, $\pi_t = P_t/P_{t-1}$, $q_t = (Q_t/P_t)/Z_{t-1}$.

We get equations (3.3), (3.5), (3.7), (3.10)-(3.23), (3.25)-(3.37), and (A.1)-(A.10) in terms of transformed new variables.

$$m_t^{cc} = \left[\nu^{\frac{1}{\omega}} n_t^{\frac{\omega-1}{\omega}} + \phi^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} + (1-\nu-\phi)^{\frac{1}{\omega}} cc_t^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \tag{3.3}$$

$$m_t - n_t + l_t + cc_t = d_t \tag{3.5}$$

$$n_t + r_t^D d_t + w_t h_t + f_t = c_t + r_t^L l_t + e_t cc_t + m_t \tag{3.7}$$

$$y_t = z_t h_t \tag{3.10}$$

$$\ln z_t = \ln z + \varepsilon_{z,t} \tag{3.11}$$

$$f_t = y_t - \frac{w_t h_t}{z_t} - \frac{\gamma}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 y_t \tag{3.12}$$

$$(1-\sigma)\lambda_{3,t} y_t + \sigma\lambda_{3,t}\left(\frac{w_t y_t}{z_t}\right) - \gamma\lambda_{3,t}\left(\frac{\pi_t}{\pi} - 1\right)\left(\frac{y_t \pi_t}{\pi}\right) + \beta\gamma\mathbb{E}_t\left[\lambda_{3,t+1}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\left(\frac{y_{t+1}\pi_{t+1}}{\pi}\right)\right] = 0 \tag{3.13}$$

$$y_t^* = \frac{\zeta}{\eta} z_t (1 - h_t) \tag{3.14}$$

$$g_t^* = \frac{\eta y_t}{\zeta z_t (1 - h_t)} \tag{3.15}$$

$$l_t = (1 - k_t) d_t \tag{3.16}$$

$$\ln k_t = (1 - \rho_k)\ln k + \rho_k \ln k_{t-1} + \varepsilon_{k,t} \tag{3.17}$$

$$cc_t = a_t d_t \tag{3.18}$$

$$\ln a_t = (1 - \rho_a)\ln a + \rho_a \ln a_{t-1} + \varepsilon_{a,t} \tag{3.19}$$

$$\ln x_t = (1 - \rho_x)\ln x + \rho_x \ln x_{t-1} + \varepsilon_{x,t} \tag{3.20}$$

$$r_t^D = (r_t^L - 1)(1 - k_t) + (e_t - x_t - 1)a_t - x_t + 1 \tag{3.21}$$

$$m_t = n_t + n_t^{\nu} \tag{3.22}$$

$$n_t^{\nu} = k_t d_t \tag{3.23}$$

$$g_t^y y_{t-1} = y_t z_{t-1} \tag{3.25}$$

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_r} \left(\frac{\pi_t}{\pi}\right)^{\rho_\pi} \left(\frac{g_t^*}{g^*}\right)^{\rho_{g^*}} \left(\frac{g_t^y}{g^y}\right)^{\rho_{g^y}} \exp(\varepsilon_{r,t}) \tag{3.26}$$

$$m_t^{sim} = n_t + d_t \tag{3.27}$$

$$u_t^N = 1 - \frac{1}{r_t} \tag{3.28}$$

$$u_t^D = 1 - \frac{r_t^D}{r_t} \tag{3.29}$$

$$u_t^{cc} = \frac{e_t}{r_t} - 1 \tag{3.30}$$

$$s_t^N = \frac{u_t^N n_t}{u_t^N n_t + u_t^D d_t + u_t^{cc} cc_t} \tag{3.31}$$

$$s_t^D = \frac{u_t^D d_t}{u_t^N n_t + u_t^D d_t + u_t^{cc} cc_t} \tag{3.32}$$

$$s_t^{cc} = \frac{u_t^{cc} cc_t}{u_t^N n_t + u_t^D d_t + u_t^{cc} cc_t} \tag{3.33}$$

$$\mu_t^{CDiv} = \mu_t^{N^{(s_t^N + s_{t-1}^N)/2}} \mu_t^{D^{(s_t^D + s_{t-1}^D)/2}} \mu_t^{cc^{(s_t^{cc} + s_{t-1}^{cc})/2}} \tag{3.34}$$

$$n_{t-1}\mu_t^N = n_t z_{t-1}\pi_t \tag{3.35}$$

$$d_{t-1}\mu_t^D = d_t z_{t-1}\pi_t \tag{3.36}$$

$$cc_{t-1}\mu_t^{cc} = cc_t z_{t-1}\pi_t \tag{3.37}$$

$$z_t\lambda_{2,t} = \beta r_t \mathbb{E}_t\left[\frac{\lambda_{2,t+1}}{\pi_{t+1}}\right] \tag{A.1}$$

$$\lambda_{3,t} = \frac{\zeta}{c_t} \tag{A.2}$$

$$\lambda_{2,t} = r_t^L \lambda_{3,t} \tag{A.3}$$

$$z_t\lambda_{3,t} = \beta \mathbb{E}_t\left[\frac{\lambda_{2,t+1}}{\pi_{t+1}}\right] \tag{A.4}$$

$$\lambda_{3,t} w_t = \frac{\eta}{\ell_t} \tag{A.5}$$

$$\lambda_{1,t} m_t^{cc} = \theta \tag{A.6}$$

$$\lambda_{2,t} q_t - \lambda_{3,t} f_t = \beta \mathbb{E}_t[\lambda_{2,t+1} q_{t+1}] \tag{A.7}$$

$$\lambda_{2,t} - \lambda_{3,t} = \lambda_{1,t} m_t^{cc\frac{1}{\omega}} \nu^{\frac{1}{\omega}} n_t^{-\frac{1}{\omega}} \tag{A.8}$$

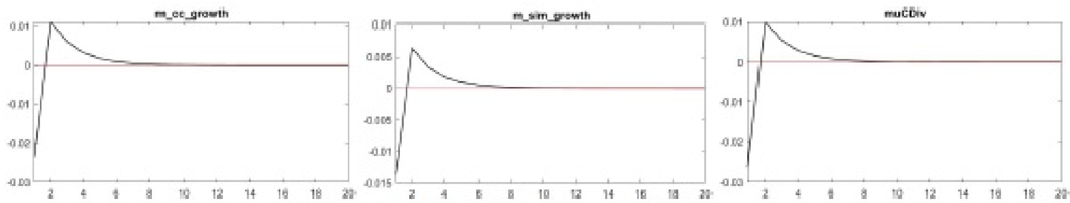
$$\lambda_{2,t} - r_t^D \lambda_{3,t} = \lambda_{1,t} m_t^{cc\frac{1}{\omega}} \phi^{\frac{1}{\omega}} d_t^{-\frac{1}{\omega}} \tag{A.9}$$

$$e_t \lambda_{3,t} - \lambda_{2,t} = \lambda_{1,t} m_t^{cc\frac{1}{\omega}} (1 - \nu - \phi)^{\frac{1}{\omega}} cc_t^{-\frac{1}{\omega}} \tag{A.10}$$

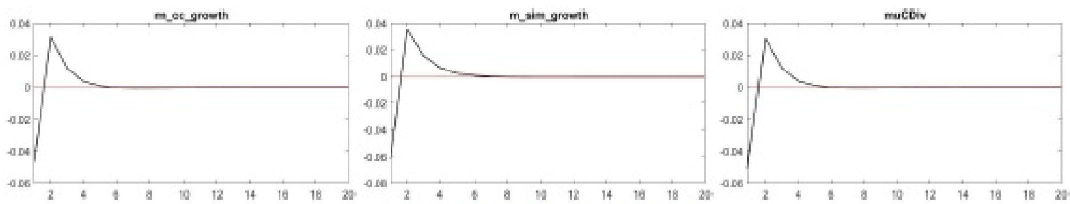
$$\ell_t + h_t = 1 \tag{B.1}$$

Appendix C

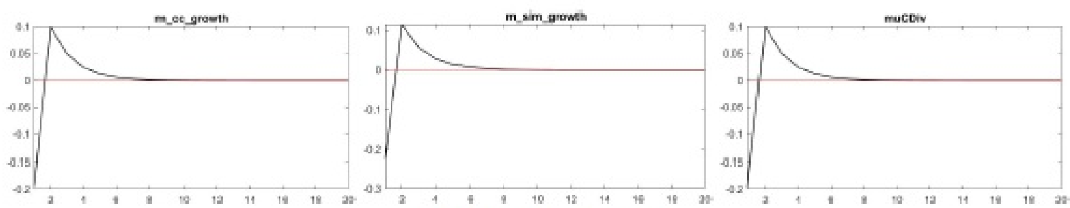
Figures



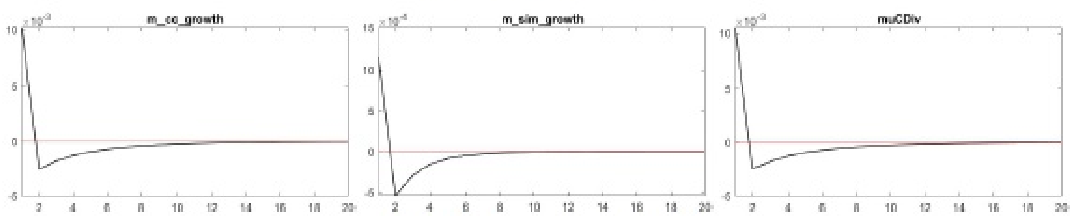
(a) Monetary Policy Shock



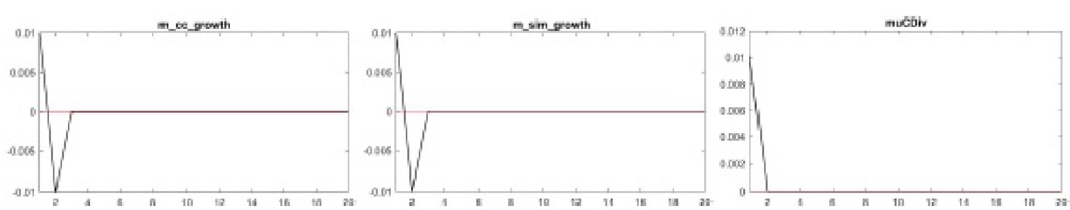
(b) Reserve Demand Shock



(c) Deposit Cost Shock



(d) Credit-Deposit Ratio Shock



(e) Technology Shock

Figure C.1: Impulse responses for nominal growth of the true money aggregation, simple sum, and credit-card-augmented Divisia

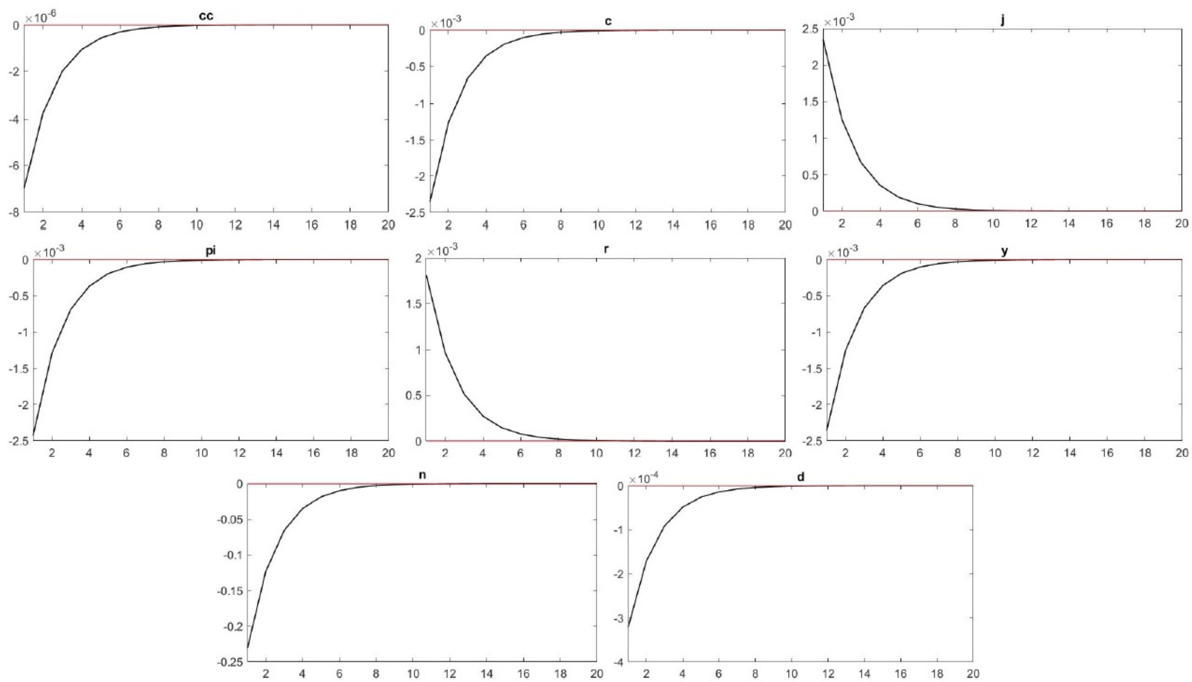


Figure C.2: Impulse responses for macroeconomic variables (credit card transactions volume, consumption, leisure, rate of inflation, nominal interest rate, output, currency, and deposits) to monetary policy shock in the model with credit card services

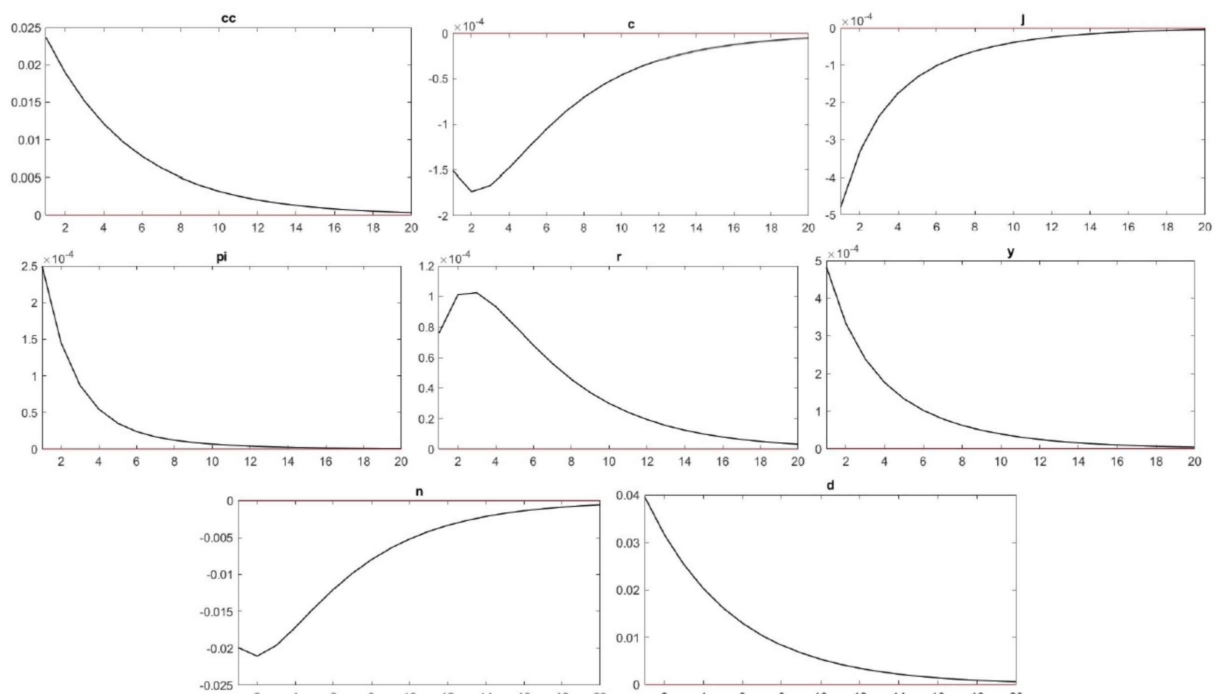


Figure C.3: Impulse responses for macroeconomic variables (credit card transactions volume, consumption, leisure, rate of inflation, nominal interest rate, output, currency, and deposits) to Credit-Deposit ratio shock in the model with credit card services

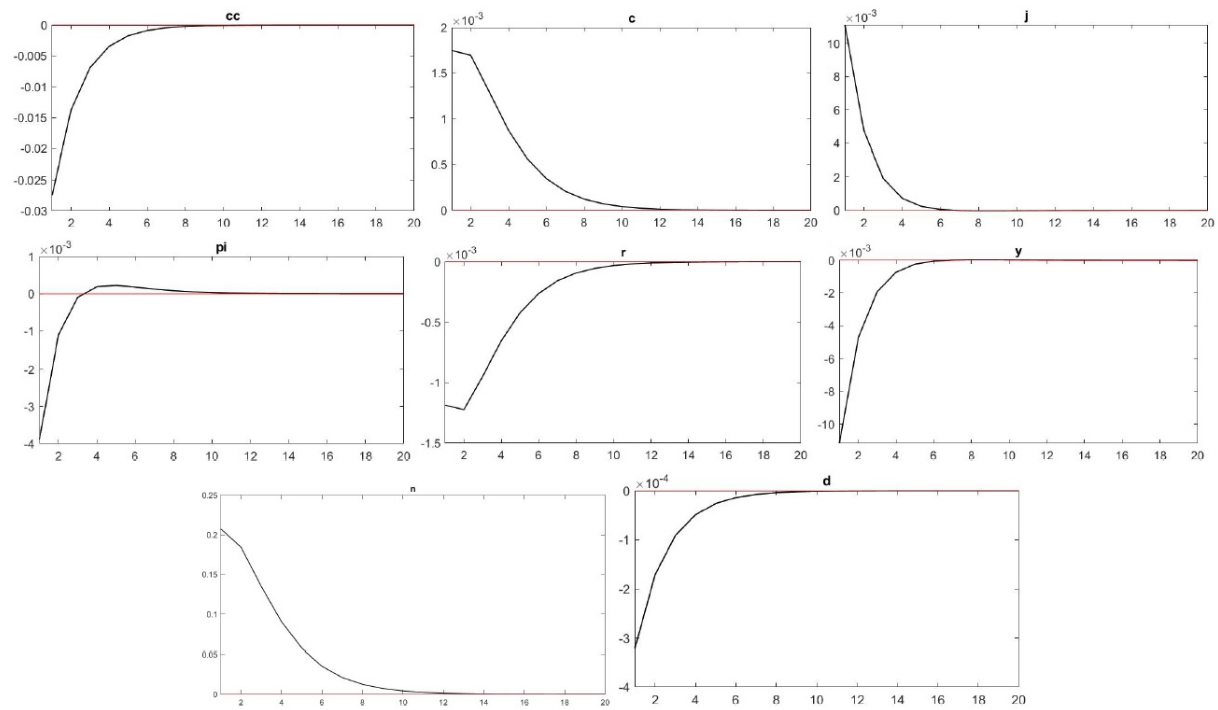


Figure C.4: Impulse responses for macroeconomic variables (credit card transactions volume, consumption, leisure, rate of inflation, nominal interest rate, output, currency, and deposits) to reserve demand shock in the model with credit card services

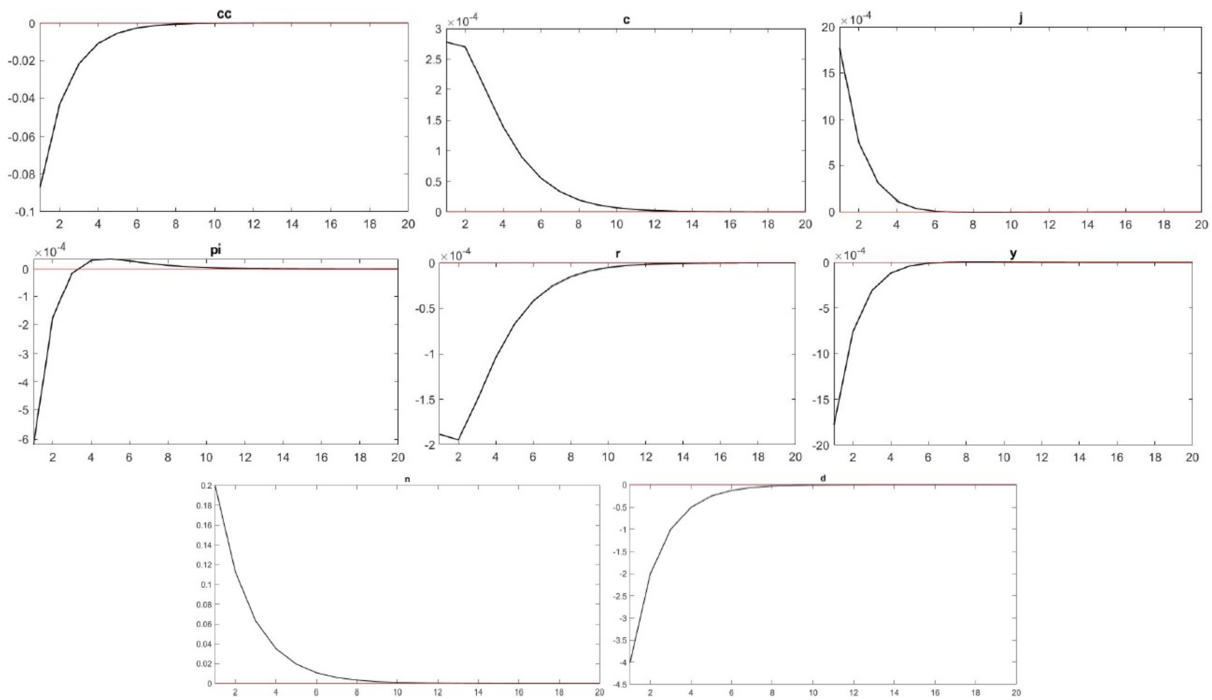
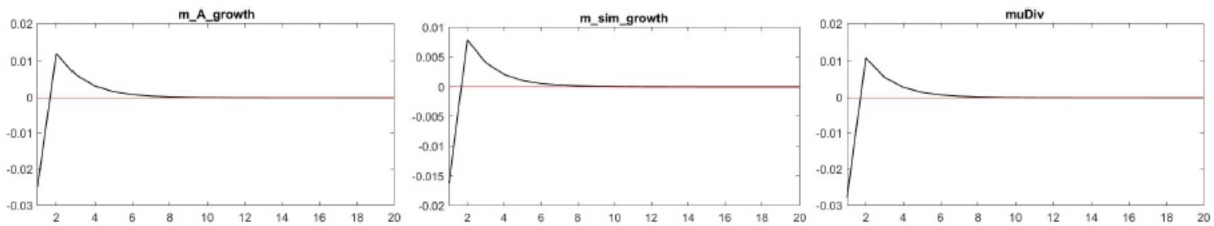
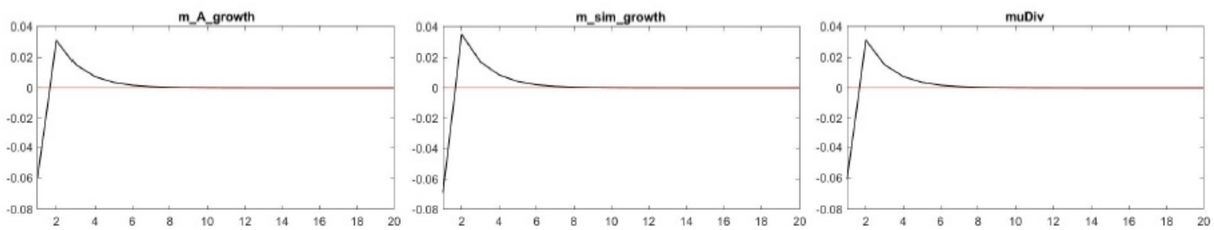


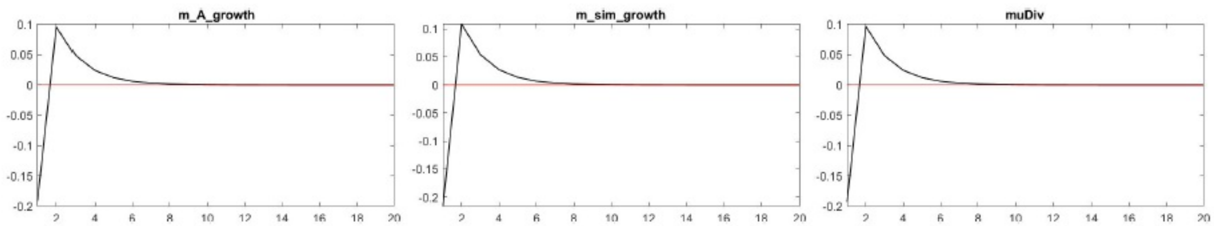
Figure C.5: Impulse responses for macroeconomic variables (credit card transactions volume, consumption, leisure, rate of inflation, nominal interest rate, output, currency, and deposits) to demand cost shock in the model with credit card services



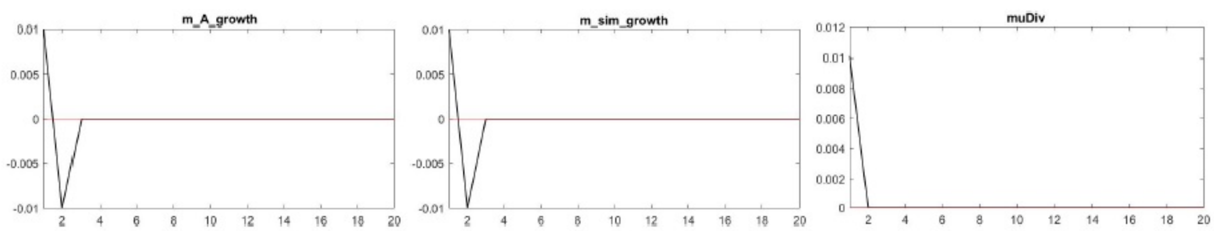
(a) Monetary Policy Shock



(b) Reserve Demand Shock



(c) Deposit Cost Shock



(d) Technology Shock

Figure C.6: Impulse responses for nominal growth of the true money aggregation, simple sum, and traditional Divisia

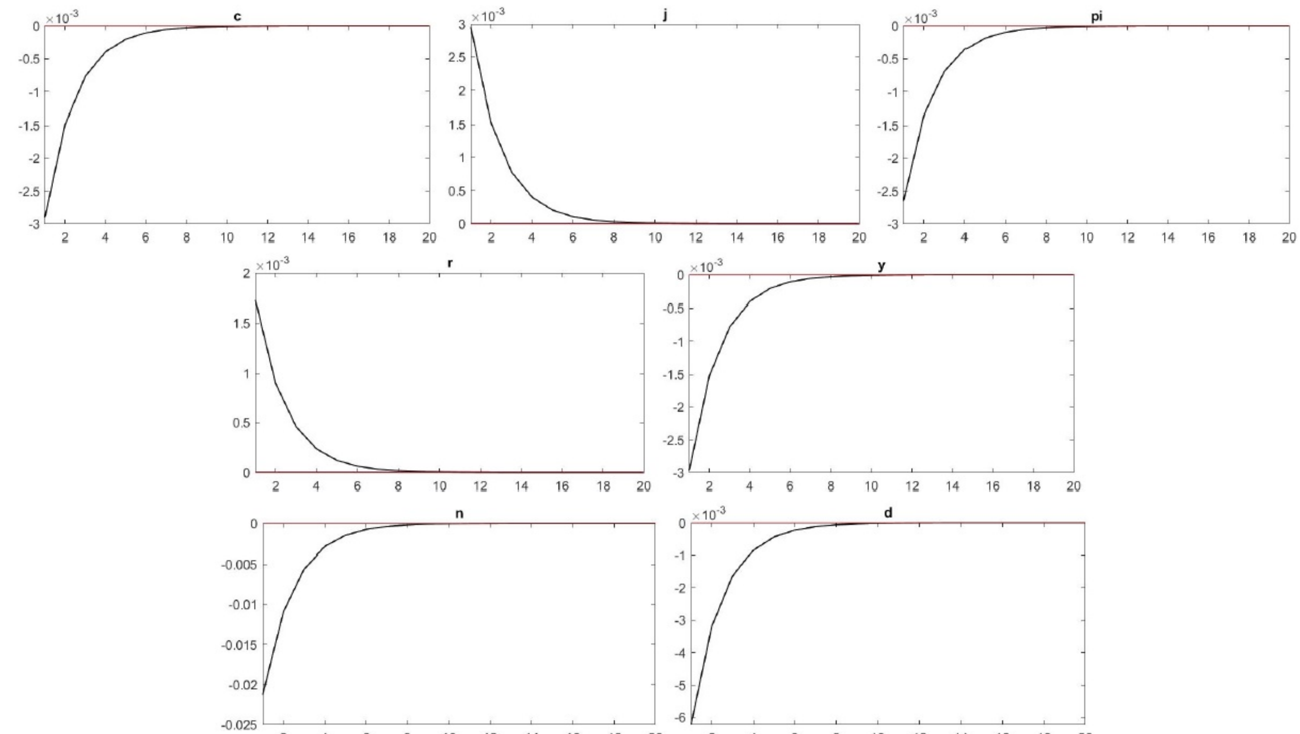


Figure C.7: Impulse responses for macroeconomic variables (consumption, leisure, rate of inflation, nominal interest rate, output, currency, and deposits) to monetary policy shock in the model with no credit card services

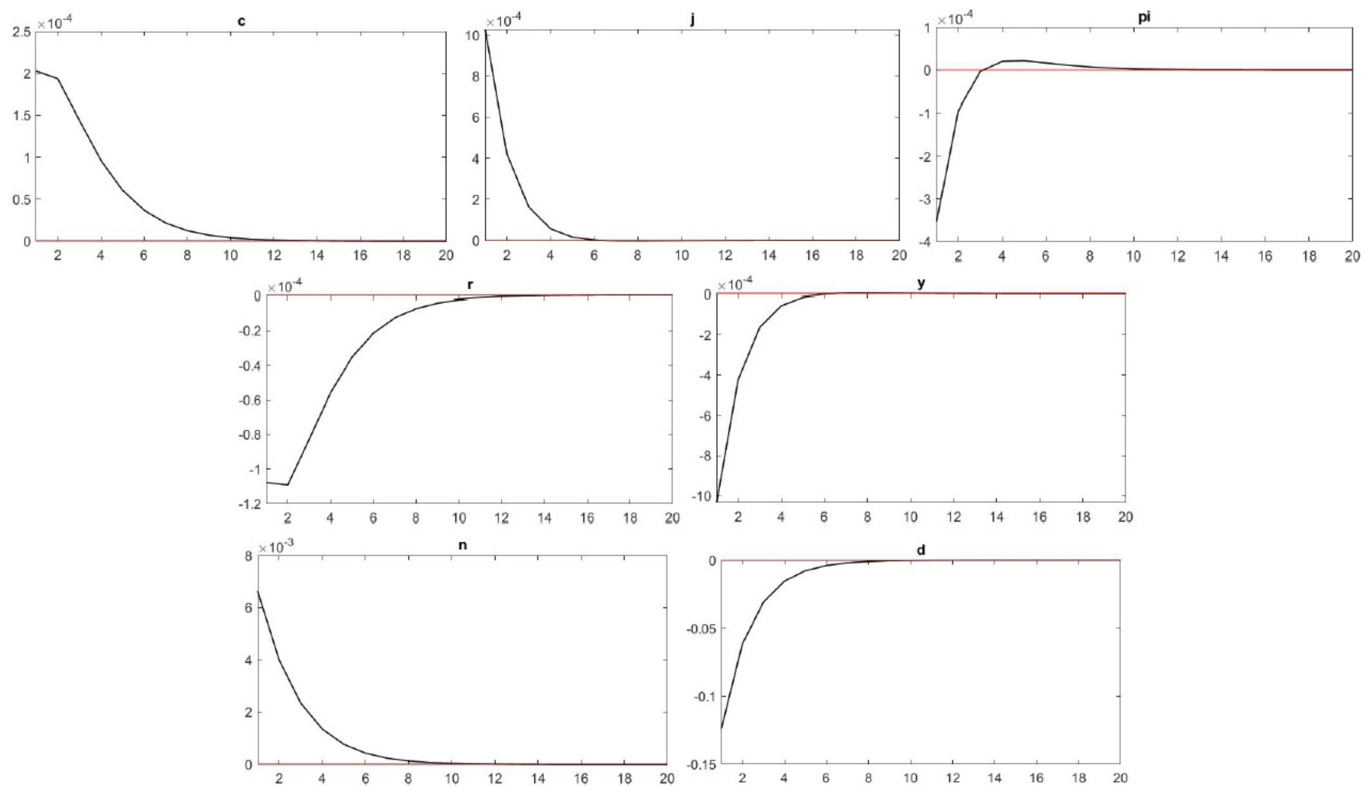


Figure C.8: Impulse responses for macroeconomic variables (consumption, leisure, rate of inflation, nominal interest rate, output, currency, and deposits) to reserve demand shock in the model with no credit card services

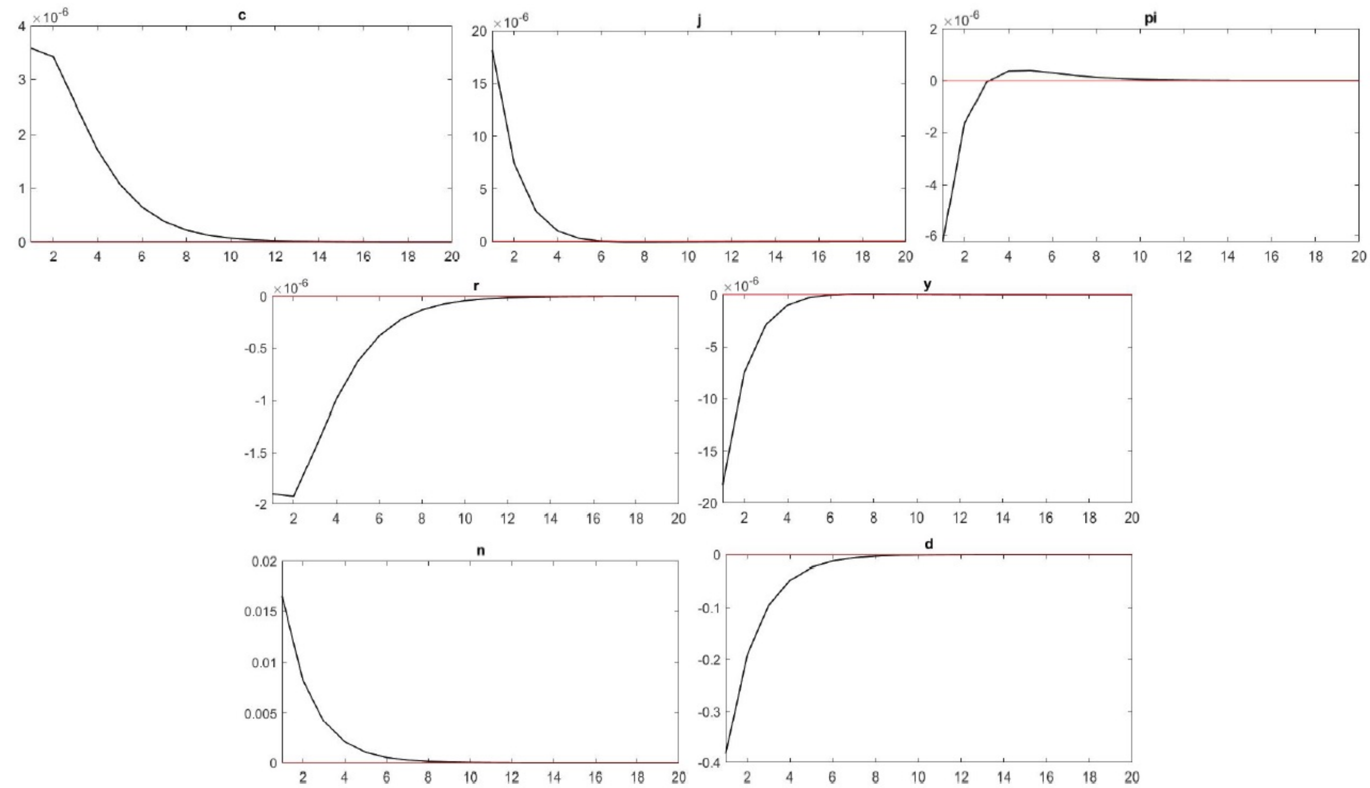


Figure C.9: Impulse responses for macroeconomic variables (consumption, leisure, rate of inflation, nominal interest rate, output, currency, and deposits) to demand cost shock in the model with no credit card services

Appendix D

Tables

A. Credit Divisia M4	Equations
Recursive	$r = 0.017y - 0.093p - 0.050cp$ $m = 0.071y + 0.920p + 0.269r - 0.107cp$
Taylor rule with money	$r = 0.005y - 0.378p + 0.085m$ $m - p = 0.110y - 0.593u$ $u = 0.279r + 0.354(m - p)$
Taylor rule without money	$r = 0.031y - 0.069p$ $m - p = 0.079y + 0.507u$ $u = 0.425r - 0.023(m - p)$
Money-interest rate rule	$r = 0.081m$ $m - p = 0.122y - 0.875u$ $u = 0.349r + 0.426(m - p)$
B. Credit Divisia M4-	**Equations**
Recursive	$r = 0.004y - 0.064p - 0.048cp$ $m = 0.003y + 0.544p - 0.207r + 0.010cp$
Taylor rule with money	$r = 0.009y + 0.358p - 0.786m$ $m - p = -0.814y + 17.452u$ $u = 5.515r - 1.278(m - p)$
Taylor rule without money	$r = 0.012y - 0.033p$ $m - p = 0.098y - 1.629u$ $u = 0.441r + 0.752(m - p)$
Money-interest rate rule	$r = -0.463m$ $m - p = -0.438y + 9.752u$ $u = 3.980r - 1.726(m - p)$
C. Credit Divisia M3	**Equations**
Recursive	$r = -0.003y - 0.167p - 0.038cp$ $m = -0.037y + 0.352p - 0.108r - 0.021cp$
Taylor rule with money	$r = -0.029y - 0.012p - 0.680m$ $m - p = -0.956y + 16.807u$ $u = 4.563r - 1.361(m - p)$
Taylor rule without money	$r = -0.017y - 0.802p$ $m - p = -0.237y + 4.022u$ $u = 0.341r - 1.578(m - p)$
Money-interest rate rule	$r = -0.441m$ $m - p = -0.487y + 8.413u$ $u = 3.078r - 1.420(m - p)$
D. Credit Divisia ALL	**Equations**
Recursive	$r = -0.008y - 0.219p - 0.025cp$ $m = 0.046y + 0.442p - 0.187r - 0.057cp$
Taylor rule with money	$r = -0.020y - 0.480p + 0.091m$ $m - p = 0.168y - 1.385u$ $u = 0.422r + 0.923(m - p)$
Taylor rule without money	$r = -0.009y - 0.299p$ $m - p = 0.163y - 1.312u$ $u = 0.274r + 0.888(m - p)$
Money-interest rate rule	$r = 0.022m$ $m - p = 0.168y - 1.389u$ $u = 0.410r + 0.937(m - p)$

Table D.1: GMM estimates of SVAR, Credit-card-augmented Divisia Broad Money, Shadow federal funds rate, Sample period: Jul.06 - Dec.19

E. Credit Divisia M2	Equations
Recursive	$r = -0.016y - 0.290p - 0.028cp$ $m = 0.067y + 0.478p - 0.365r - 0.062cp$
Taylor rule with money	$r = -0.025y - 0.541p + 0.015m$ $m - p = 0.197y - 1.873u$ $u = 0.345r + 0.653(m - p)$
Taylor rule without money	$r = -0.023y - 0.511p$ $m - p = 0.195y - 1.845u$ $u = 0.312r + 0.644(m - p)$
Money-interest rate rule	$r = -0.048m$ $m - p = 0.195y - 1.836u$ $u = 0.322r + 0.643(m - p)$

F. Credit Divisia MZM	Equations
Recursive	$r = -0.008y - 0.224p - 0.025cp$ $m = 0.062y + 0.444p - 0.191r - 0.041cp$
Taylor rule with money	$r = -0.019y - 0.461p + 0.053m$ $m - p = 0.171y - 1.507u$ $u = 0.292r + 0.655(m - p)$
Taylor rule without money	$r = -0.012y - 0.357p$ $m - p = 0.169y - 1.471u$ $u = 0.205r + 0.644(m - p)$
Money-interest rate rule	$r = -0.009m$ $m - p = 0.172y - 1.515u$ $u = 0.277r + 0.666(m - p)$

G. Credit Divisia M2M	Equations
Recursive	$r = -0.008y - 0.291p - 0.031cp$ $m = 0.078y + 0.372p - 0.433r - 0.031cp$
Taylor rule with money	$r = -0.014y - 0.528p - 0.038m$ $m - p = 0.219y - 2.434u$ $u = 0.190r + 0.506(m - p)$
Taylor rule without money	$r = -0.018y - 0.588p$ $m - p = 0.223y - 2.498u$ $u = 0.283r + 0.517(m - p)$
Money-interest rate rule	$r = -0.064m$ $m - p = 0.222y - 2.482u$ $u = 0.242r + 0.522(m - p)$

H. Credit Divisia M1	Equations
Recursive	$r = 0.026y - 0.152p - 0.045cp$ $m = 0.071y + 1.187p + 0.147r - 0.118cp$
Taylor rule with money	$r = 0.027y - 0.268p - 0.066m$ $m - p = 0.016y + 7.905u$ $u = 0.039r + 0.070(m - p)$
Taylor rule without money	$r = 0.021y - 0.369p$ $m - p = 0.061y + 3.016u$ $u = 0.083r + 0.018(m - p)$
Money-interest rate rule	$r = 0.262m$ $m - p = 1.181y - 54.846u$ $u = 0.354r + 0.181(m - p)$

Table D.2: GMM estimates of SVAR, Credit-card-augmented Divisia Narrow Money, Shadow federal funds rate, Sample period: Jul.06 - Dec.19

A. Credit Divisia M4	Equations
Recursive	$r = 0.059y - 0.115p + 0.050cp$ $m = 0.018y + 0.913p - 0.130r - 0.088cp$
Taylor rule with money	$r = 0.069y + 0.104p - 0.058m$ $m - p = -0.270y + 7.246u$ $u = 1.328r - 5.641(m - p)$
Taylor rule without money	$r = 0.070y + 0.089p$ $m - p = 0.393y - 9.833u$ $u = 0.402r + 0.1.541(m - p)$
Money-interest rate rule	$r = -0.231m$ $m - p = -0.320y + 8.428u$ $u = 11.110r - 6.355(m - p)$

B. Credit Divisia M4-	Equations
Recursive	$r = 0.071y - 0.093p + 0.050cp$ $m = -0.058y + 0.192p + 0.629r + 0.012cp$
Taylor rule with money	$r = 0.078y + 0.036p + 0.084m$ $m - p = 0.162y - 3.711u$ $u = -0.098r + 1.035(m - p)$
Taylor rule without money	$r = 0.067y - 0.076p$ $m - p = -0.853y + 26.535u$ $u = 5.330r - 8.066(m - p)$
Money-interest rate rule	$r = -0.567m$ $m - p = 3.696y - 110.322u$ $u = -40.259r + 13.058(m - p)$

C. Credit Divisia M3	Equations
Recursive	$r = 0.093y - 0.116p + 0.059cp$ $m = -0.114y + 0.145p + 0.505r - 0.029cp$
Taylor rule with money	$r = 0.127y + 0.418p + 0.136m$ $m - p = 0.237y - 5.646u$ $u = 0.880r + 1.381(m - p)$
Taylor rule without money	$r = 0.111y + 0.293p$ $m - p = -0.200y + 3.933u$ $u = 0.363r - 0.927(m - p)$
Money-interest rate rule	$r = -0.410m$ $m - p = -0.944y + 16.075u$ $u = 3.369r - 1.413(m - p)$

D. Credit Divisia ALL	Equations
Recursive	$r = 0.063y - 0.110p + 0.076cp$ $m = 0.023y + 0.285p + 0.124r - 0.070cp$
Taylor rule with money	$r = 0.081y + 0.285p - 0.109m$ $m - p = 0.209y - 2.708u$ $u = -0.289r + 0.979(m - p)$
Taylor rule without money	$r = 0.076y + 0.239p$ $m - p = 0.181y - 2.188u$ $u = -0.002r + 0.742(m - p)$
Money-interest rate rule	$r = 0.134m$ $m - p = 0.203y - 2.318u$ $u = 0.249r + 0.725(m - p)$

Table D.3: GMM estimates of SVAR, Credit-card-augmented Divisia Broad Money, Effective federal funds rate, Sample period: Jul.06 - Dec.19

E. Credit Divisia M2	Equations
Recursive	$r = 0.042y - 0.211p + 0.076cp$ $m = 0.028y + 0.559p - 0.111r - 0.104cp$
Taylor rule with money	$r = 0.060y + 0.213p - 0.006m$ $m - p = 0.251y - 4.298u$ $u = 0.081r + 0.695(m - p)$
Taylor rule without money	$r = 0.060y + 0.219p$ $m - p = 0.249y - 4.255u$ $u = 0.102r + 0.682(m - p)$
Money-interest rate rule	$r = 3.051m$ $m - p = 1.033y - 17.903u$ $u = 1.432r + 0.079(m - p)$

F. Credit Divisia MZM	Equations
Recursive	$r = 0.061y - 0.132p + 0.081cp$ $m = 0.055y + 0.318p + 0.162r - 0.071cp$
Taylor rule with money	$r = 0.081y + 0.236p - 0.087m$ $m - p = 0.217y - 2.404u$ $u = -0.217r + 0.579(m - p)$
Taylor rule without money	$r = 0.074y + 0.195p$ $m - p = 0.199y - 2.028u$ $u = -0.037r + 0.469(m - p)$
Money-interest rate rule	$r = 1.343m$ $m - p = 0.540y - 8.312u$ $u = 0.734r + 0.066(m - p)$

G. Credit Divisia M2M	Equations
Recursive	$r = 0.048y - 0.256p + 0.084cp$ $m = 0.065y + 0.630p + 0.226r - 0.108cp$
Taylor rule with money	$r = 0.064y + 0.088p - 0.016m$ $m - p = 0.267y - 4.422u$ $u = -0.060r + 0.425(m - p)$
Taylor rule without money	$r = 0.063y + 0.103p$ $m - p = 0.258y - 4.188u$ $u = -0.013r + 0.397(m - p)$
Money-interest rate rule	$r = 1.065m$ $m - p = 1.062y - 22.735u$ $u = 0.618r + 0.086(m - p)$

H. Credit Divisia M1	Equations
Recursive	$r = 0.049y - 0.153p + 0.063cp$ $m = 0.006y + 1.041p - 0.111r - 0.111cp$
Taylor rule with money	$r = 0.077y - 0.326p + 0.601m$ $m - p = 44.206y - 4341.523u$ $u = 0.177r + 0.040(m - p)$
Taylor rule without money	$r = 0.066y + 0.124p$ $m - p = 0.029y - 9.511u$ $u = 0.048r + 0.056(m - p)$
Money-interest rate rule	$r = -0.470m$ $m - p = -0.395y + 58.614u$ $u = 0.159r - 0.024(m - p)$

Table D.4: GMM estimates of SVAR, Credit-card-augmented Divisia Narrow Money, Effective federal funds rate, Sample period: Jul.06 - Dec.19

A. Divisia M4	Equations
Recursive	$r = 0.018y - 0.102p - 0.049cp$ $m = 0.047y + 1.043p + 0.274r - 0.126cp$
Taylor rule with money	$r = 0.021y - 0.154p - 0.202m$ $m - p = 0.004y + 31.774u$ $u = 0.036r + 0.010(m - p)$
Taylor rule without money	$r = 0.010y - 0.330p$ $m - p = 0.019y + 19.562u$ $u = 0.010r + 0.025(m - p)$
Money-interest rate rule	$r = -0.337m$ $m - p = 0.011y + 34.461u$ $u = 0.029r + 0.018(m - p)$
B. Divisia M4-	**Equations**
Recursive	$r = 0.004y - 0.073p - 0.047cp$ $m = -0.051y + 0.637p - 0.197r - 0.024cp$
Taylor rule with money	$r = -0.021y - 0.025p - 0.343m$ $m - p = -0.363y + 53.307u$ $u = 0.237r - 0.119(m - p)$
Taylor rule without money	$r = -0.003y - 0.238p$ $m - p = -0.014y - 125.693u$ $u = 0.004r + 0.008(m - p)$
Money-interest rate rule	$r = -0.231m$ $m - p = -0.222y + 35.591u$ $u = 0.120r - 0.087(m - p)$
C. Divisia M3	**Equations**
Recursive	$r = -0.001y - 0.175p - 0.035cp$ $m = -0.072y + 0.453p - 0.115r - 0.060cp$
Taylor rule with money	$r = -0.042y - 0.167p - 0.438m$ $m - p = -0.563y + 73.513u$ $u = 0.288r - 0.103(m - p)$
Taylor rule without money	$r = -0.018y - 0.572p$ $m - p = -0.018y + 11.462u$ $u = -0.001r + 0.070(m - p)$
Money-interest rate rule	$r = -0.125m$ $m - p = -0.032y + 17.606u$ $u = 0.011r + 0.036(m - p)$
D. Divisia ALL	**Equations**
Recursive	$r = -0.007y - 0.267p - 0.021cp$ $m = -0.011y + 0.438p - 0.214r - 0.059cp$
Taylor rule with money	$r = -0.025y - 0.509p - 0.245m$ $m - p = 0.003y + 8.483u$ $u = 0.012r + 0.087(m - p)$
Taylor rule without money	$r = -0.026y - 0.717p$ $m - p = 0.294y - 30.145u$ $u = 0.040r + 0.749(m - p)$
Money-interest rate rule	$r = -0.474m$ $m - p = -0.004y + 9.758u$ $u = 0.024r + 0.069(m - p)$

Table D.5: GMM estimates of SVAR, Divisia Broad Money, Shadow federal funds rate, Sample period: Jul.06 - Dec.19

E. Divisia M2	Equations
Recursive	$r = -0.021y - 0.368p - 0.021cp$ $m = 0.021y + 0.458p - 0.383r - 0.070cp$
Taylor rule with money	$r = -0.027y - 0.540p - 0.198m$ $m - p = 0.006y + 6.380u$ $u = 0.010r + 0.133(m - p)$
Taylor rule without money	$r = -0.039y - 0.819p$ $m - p = 0.195y - 27.764u$ $u = 0.030r + 0.121(m - p)$
Money-interest rate rule	$r = -0.318m$ $m - p = 0.004y + 6.259u$ $u = 0.013r + 0.140(m - p)$
F. Divisia MZM	**Equations**
Recursive	$r = -0.008y - 0.273p - 0.021cp$ $m = -0.001y + 0.444p - 0.230r - 0.045cp$
Taylor rule with money	$r = -0.020y - 0.457p - 0.195m$ $m - p = 0.001y + 5.949u$ $u = 0.010r + 0.135(m - p)$
Taylor rule without money	$r = -0.024y - 0.659p$ $m - p = 0.211y - 24.870u$ $u = 0.035r + 0.372(m - p)$
Money-interest rate rule	$r = -0.314m$ $m - p = 0.001y + 5.993u$ $u = 0.013r + 0.136(m - p)$
G. Divisia M2M	**Equations**
Recursive	$r = -0.017y - 0.372p - 0.026cp$ $m = 0.033y + 0.295p - 0.499r - 0.038cp$
Taylor rule with money	$r = -0.019y - 0.519p - 0.189m$ $m - p = 0.008y + 7.023u$ $u = 0.008r + 0.122(m - p)$
Taylor rule without money	$r = -0.032y - 0.767p$ $m - p = 0.031y - 17.086u$ $u = 0.007r - 0.039(m - p)$
Money-interest rate rule	$r = -0.289m$ $m - p = 0.007y + 7.748u$ $u = 0.013r + 0.112(m - p)$
H. Divisia M1	**Equations**
Recursive	$r = 0.00014y - 0.213p - 0.028cp$ $m = -0.357y + 0.844p + 0.109r - 0.245cp$
Taylor rule with money	$r = -0.065y - 0.346p - 0.172m$ $m - p = -1.616y + 43.411u$ $u = 0.367r - 0.079(m - p)$
Taylor rule without money	$r = -0.011y - 0.888p$ $m - p = -0.838y + 17.478u$ $u = 0.055r - 0.078(m - p)$
Money-interest rate rule	$r = -0.033m$ $m - p = -0.804y + 16.445u$ $u = 0.117r - 0.073(m - p)$

Table D.6: GMM estimates of SVAR, Divisia Narrow Money, Shadow federal funds rate, Sample period: Jul.06 - Dec.19

A. Divisia M4	Equations
Recursive	$r = 0.050y - 0.130p + 0.045cp$ $m = -0.015y + 1.024p - 0.106r - 0.111cp$
Taylor rule with money	$r = 0.069y - 0.066p + 0.164m$ $m - p = 0.207y - 125.223u$ $u = 0.034r + 0.013(m - p)$
Taylor rule without money	$r = 0.060y + 0.024p$ $m - p = -0.001y - 77.807u$ $u = 0.003r + 0.006(m - p)$
Money-interest rate rule	$r = -0.480m$ $m - p = -0.050y + 30.392u$ $u = 0.025r + 0.028(m - p)$

B. Divisia M4-	Equations
Recursive	$r = 0.059y - 0.097p + 0.045cp$ $m = -0.105y + 0.286p + 0.644r - 0.024cp$
Taylor rule with money	$r = 0.072y + 0.038p + 0.067m$ $m - p = -0.011y + 6.139u$ $u = 0.007r + 0.120(m - p)$
Taylor rule without money	$r = 0.063y - 0.027p$ $m - p = -0.010y + 6.561u$ $u = 0.011r - 0.137(m - p)$
Money-interest rate rule	$r = -0.399m$ $m - p = 0.002y - 10.757u$ $u = -0.0003r - 0.093(m - p)$

C. Divisia M3	Equations
Recursive	$r = 0.081y - 0.118p + 0.053cp$ $m = -0.134y + 0.252p + 0.500r - 0.066cp$
Taylor rule with money	$r = 0.086y - 0.021p - 0.007m$ $m - p = -0.014y + 7.696u$ $u = 0.005r + 0.121(m - p)$
Taylor rule without money	$r = 0.085y - 0.059p$ $m - p = -0.013y + 7.336u$ $u = 0.005r + 0.127(m - p)$
Money-interest rate rule	$r = -0.176m$ $m - p = -0.045y - 4.651u$ $u = -0.137r - 0.122(m - p)$

D. Divisia ALL	Equations
Recursive	$r = 0.052y - 0.109p + 0.073cp$ $m = -0.028y + 0.353p + 0.171r - 0.075cp$
Taylor rule with money	$r = 0.129y - 0.118p + 2.177m$ $m - p = 0.990y - 141.295u$ $u = 0.117r + 0.012(m - p)$
Taylor rule without money	$r = 0.075y + 0.341p$ $m - p = -0.001y + 4.408u$ $u = 0.001r + 0.213(m - p)$
Money-interest rate rule	$r = -0.494m$ $m - p = 0.002y - 3.985u$ $u = -0.003r - 0.248(m - p)$

Table D.7: GMM estimates of SVAR, Divisia Broad Money, Effective federal funds rate, Sample period: Jul.06 - Dec.19

E. Divisia M2	Equations
Recursive	$r = 0.035y - 0.228p + 0.077cp$ $m = -0.008y + 0.625p + 0.099r - 0.120cp$
Taylor rule with money	$r = 0.049y + 0.085p - 0.049m$ $m - p = -0.001y + 5.061u$ $u = 0.002r + 0.188(m - p)$
Taylor rule without money	$r = 0.050y + 0.079p$ $m - p = 0.030y - 28.934u$ $u = 0.001r + 0.001(m - p)$
Money-interest rate rule	$r = -0.298m$ $m - p = 0.003y - 6.080u$ $u = -0.009r - 0.157(m - p)$

F. Divisia MZM	Equations
Recursive	$r = 0.050y - 0.131p + 0.078cp$ $m = 0.002y + 0.406p + 0.211r - 0.080cp$
Taylor rule with money	$r = 0.060y - 0.182p + 1.938m$ $m - p = 0.966y - 143.797u$ $u = 0.106r + 0.010(m - p)$
Taylor rule without money	$r = 0.071y + 0.286p$ $m - p = 0.004y + 4.424u$ $u = 0.001r + 0.210(m - p)$
Money-interest rate rule	$r = -0.098m$ $m - p = 0.002y + 4.713u$ $u = 0.003r + 0.204(m - p)$

G. Divisia M2M	Equations
Recursive	$r = 0.040y - 0.281p + 0.085cp$ $m = 0.028y + 0.692p + 0.204r - 0.118cp$
Taylor rule with money	$r = 0.054y + 0.005p - 0.027m$ $m - p = 0.004y + 5.715u$ $u = 0.002r + 0.165(m - p)$
Taylor rule without money	$r = 0.054y - 0.006p$ $m - p = 0.005y + 5.680u$ $u = 0.001r + 0.161(m - p)$
Money-interest rate rule	$r = 0.008m$ $m - p = 0.033y + 4.802u$ $u = 0.001r + 0.087(m - p)$

H. Divisia M1	Equations
Recursive	$r = 0.072y - 0.144p + 0.088cp$ $m = -0.331y + 0.946p - 0.172r - 0.287cp$
Taylor rule with money	$r = 0.123y + 0.230p + 0.085m$ $m - p = -0.331y + 572.166u$ $u = -0.005r - 0.001(m - p)$
Taylor rule without money	$r = 0.089y + 0.214p$ $m - p = -0.237y + 133.732u$ $u = -0.001r - 0.003(m - p)$
Money-interest rate rule	$r = -0.200m$ $m - p = -0.088y - 41.391u$ $u = -0.011r - 0.022(m - p)$

Table D.8: GMM estimates of SVAR, Divisia Narrow Money, Effective federal funds rate, Sample period: Jul.06 - Dec.19

A. Divisia M4	Equations
Recursive	$r = 0.055y + 0.071p - 0.019cp$ $m = -0.070y - 0.309p - 0.431r + 0.029cp$
Taylor rule with money	$r = 0.321y + 1.125p + 2.319m$ $m - p = 0.022y - 13.993u$ $u = 0.019r - 0.064(m - p)$
Taylor rule without money	$r = 0.182y + 1.351p$ $m - p = 0.090y - 33.319u$ $u = 0.037r + 0.028(m - p)$
Money-interest rate rule	$r = -1.381m$ $m - p = -0.021y + 10.433u$ $u = 0.021r + 0.093(m - p)$
B. Divisia M4-	**Equations**
Recursive	$r = 0.061y + 0.078p - 0.025cp$ $m = -0.083y - 0.279p - 0.272r + 0.039cp$
Taylor rule with money	$r = 0.366y + 1.014p + 2.487m$ $m - p = 0.021y - 12.826u$ $u = 0.017r - 0.072(m - p)$
Taylor rule without money	$r = 0.205y + 1.363p$ $m - p = 0.077y - 18.363u$ $u = 0.037r + 0.031(m - p)$
Money-interest rate rule	$r = -1.297m$ $m - p = -0.042y + 15.684u$ $u = 0.016r + 0.059(m - p)$
C. Divisia M3	**Equations**
Recursive	$r = 0.060y + 0.090p - 0.026cp$ $m = -0.079y - 0.388p - 0.361r + 0.030cp$
Taylor rule with money	$r = 0.003y - 0.384p - 0.210m$ $m - p = 0.059y - 1.581u$ $u = 0.025r + 0.013(m - p)$
Taylor rule without money	$r = 0.224y + 1.599p$ $m - p = 0.010y - 6.048u$ $u = 0.014r - 0.137(m - p)$
Money-interest rate rule	$r = -1.148m$ $m - p = -0.014y + 11.270u$ $u = 0.021r + 0.082(m - p)$
D. Divisia ALL	**Equations**
Recursive	$r = 0.059y + 0.082p - 0.026cp$ $m = 0.034y - 0.283p - 0.500r - 0.040cp$
Taylor rule with money	$r = 0.121y + 0.884p + 0.518m$ $m - p = 0.045y - 11.096u$ $u = 0.012r - 0.080(m - p)$
Taylor rule without money	$r = 0.171y + 1.226p$ $m - p = 0.048y - 11.034u$ $u = 0.014r - 0.069(m - p)$
Money-interest rate rule	$r = 1.563m$ $m - p = 0.044y - 14.783u$ $u = 0.007r - 0.064(m - p)$

Table D.9: GMM estimates of SVAR, Divisia Broad Money, Shadow federal funds rate, Sample period: Feb.92 - Jul.06

E. Divisia M2	Equations
Recursive	$r = 0.054y + 0.098p - 0.030cp$ $m = 0.046y - 0.276p - 0.404r - 0.023cp$
Taylor rule with money	$r = 0.098y + 0.882p + 0.632m$ $m - p = 0.046y - 11.090u$ $u = 0.014r - 0.080(m - p)$
Taylor rule without money	$r = 0.168y + 1.293p$ $m - p = 0.055y - 12.904u$ $u = 0.011r - 0.058(m - p)$
Money-interest rate rule	$r = 1.028m$ $m - p = 0.055y - 12.364u$ $u = 0.013r - 0.072(m - p)$

F. Divisia MZM	Equations
Recursive	$r = 0.059y + 0.084p - 0.026cp$ $m = 0.029y - 0.305p - 0.555r - 0.038cp$
Taylor rule with money	$r = 0.138y + 0.958p + 0.303m$ $m - p = 0.041y - 10.692u$ $u = 0.013r - 0.083(m - p)$
Taylor rule without money	$r = 0.175y + 1.258p$ $m - p = 0.048y - 10.687u$ $u = 0.017r - 0.070(m - p)$
Money-interest rate rule	$r = -0.087m$ $m - p = 0.317y - 20.864u$ $u = 0.019r - 0.003(m - p)$

G. Divisia M2M	Equations
Recursive	$r = 0.052y + 0.096p - 0.030cp$ $m = 0.055y - 0.277p - 0.456r - 0.017cp$
Taylor rule with money	$r = 0.097y + 0.828p + 0.448m$ $m - p = 0.048y - 11.818u$ $u = 0.011r - 0.077(m - p)$
Taylor rule without money	$r = 0.162y + 1.244p$ $m - p = 0.081y - 12.115u$ $u = 0.023r - 0.047(m - p)$
Money-interest rate rule	$r = 0.769m$ $m - p = 0.059y - 13.993u$ $u = 0.014r - 0.063(m - p)$

H. Divisia M1	Equations
Recursive	$r = 0.053y + 0.072p - 0.021cp$ $m = -0.026y - 0.505p - 0.457r - 0.039cp$
Taylor rule with money	$r = 0.340y + 3.479p + 0.227m$ $m - p = 0.021y - 13.524u$ $u = 0.013r - 0.065(m - p)$
Taylor rule without money	$r = 0.220y + 2.028p$ $m - p = 0.050y - 8.973u$ $u = 0.035r - 0.061(m - p)$
Money-interest rate rule	$r = -1.409m$ $m - p = -0.114y + 49.529u$ $u = 0.054r + 0.016(m - p)$

Table D.10: GMM estimates of SVAR, Divisia Narrow Money, Shadow federal funds rate, Sample period: Feb.92 - Jul.06

A. Divisia M4	Equations
Recursive	$r = -0.018y + 0.216p + 0.042cp$ $m = -0.067y - 0.259p - 0.277r + 0.058cp$
Taylor rule with money	$r = 0.147y + 2.308p - 0.661m$ $m - p = 0.052y + 0.904u$ $u = -0.001r - 0.011(m-p)$
Taylor rule without money	$r = 0.111y + 1.665p$ $m - p = 0.111y - 71.696u$ $u = 0.013r + 0.027(m-p)$
Money-interest rate rule	$r = 30.471m$ $m - p = 0.255y + 140.986u$ $u = -0.173r - 0.108(m-p)$

B. Divisia M4-	Equations
Recursive	$r = -0.035y + 0.239p + 0.026cp$ $m = -0.087y - 0.194p - 0.227r + 0.078cp$
Taylor rule with money	$r = 1.228y + 3.516p + 14.993m$ $m - p = -0.208y - 45.124u$ $u = 0.125r + 0.058(m-p)$
Taylor rule without money	$r = 0.092y + 1.578p$ $m - p = 0.167y - 60.672u$ $u = 0.041r + 0.146(m-p)$
Money-interest rate rule	$r = 1.446m$ $m - p = 0.200y - 69.991u$ $u = 2.803r + 4.416(m-p)$

C. Divisia M3	Equations
Recursive	$r = -0.018y + 0.220p + 0.007cp$ $m = -0.089y - 0.307p - 0.256r + 0.082cp$
Taylor rule with money	$r = 3.008y + 12.060p + 32.034m$ $m - p = -0.082y - 111.144u$ $u = 3.552r + 2.007(m-p)$
Taylor rule without money	$r = 0.239y - 1.582p$ $m - p = 0.327y - 46.434u$ $u = 0.020r + 0.045(m-p)$
Money-interest rate rule	$r = 1.216m$ $m - p = 0.238y - 105.924u$ $u = 0.094r + 0.138(m-p)$

D. Divisia ALL	Equations
Recursive	$r = 0.010y + 0.357p + 0.039cp$ $m = 0.007y - 0.174p - 0.252r + 0.022cp$
Taylor rule with money	$r = -0.021y + 1.607p + 5.204m$ $m - p = 0.116y - 24.273u$ $u = 0.061r + 0.010(m-p)$
Taylor rule without money	$r = 0.134y + 1.677p$ $m - p = 0.211y - 57.564u$ $u = 0.009r + 0.013(m-p)$
Money-interest rate rule	$r = -0.507m$ $m - p = 0.139y - 5.774u$ $u = 0.004r + 0.025(m-p)$

Table D.11: GMM estimates of SVAR, Divisia Broad Money, Effective federal funds rate, Sample period: Feb.92 - Jul.06

E. Divisia M2	Equations
Recursive	$r = -0.003y + 0.267p + 0.022cp$ $m = 0.025y - 0.121p - 0.228r + 0.023cp$
Taylor rule with money	$r = -0.006y + 0.852p + 1.328m$ $m - p = 0.095y - 22.012u$ $u = 0.025r - 0.009(m - p)$
Taylor rule without money	$r = 0.085y + 1.269p$ $m - p = 0.104y - 18.199u$ $u = 0.012r - 0.008(m - p)$
Money-interest rate rule	$r = 0.275m$ $m - p = 0.176y - 68.940u$ $u = 0.007r + 0.007(m - p)$

F. Divisia MZM	Equations
Recursive	$r = 0.022y + 0.337p + 0.034cp$ $m = -0.009y - 0.224p - 0.270r + 0.033cp$
Taylor rule with money	$r = 0.108y + 1.843p + 4.450m$ $m - p = 0.105y - 25.095u$ $u = 0.065r + 0.006(m - p)$
Taylor rule without money	$r = 0.174y + 1.874p$ $m - p = 0.174y - 53.080u$ $u = 0.009r + 0.011(m - p)$
Money-interest rate rule	$r = 26.079m$ $m - p = 0.270y - 117.631u$ $u = 0.068r + 0.024(m - p)$

G. Divisia M2M	Equations
Recursive	$r = 0.007y + 0.243p + 0.015cp$ $m = 0.029y - 0.143p - 0.249r + 0.039cp$
Taylor rule with money	$r = 0.060y + 1.015p + 0.429m$ $m - p = 0.100y - 22.460u$ $u = 0.017r - 0.011(m - p)$
Taylor rule without money	$r = 0.101y + 1.262p$ $m - p = 0.113y - 17.782u$ $u = 0.017r - 0.007(m - p)$
Money-interest rate rule	$r = -0.309m$ $m - p = 0.142y - 6.115u$ $u = 0.008r + 0.017(m - p)$

H. Divisia M1	Equations
Recursive	$r = -0.049y + 0.222p + 0.042cp$ $m = -0.032y - 0.268p - 0.302r + 0.018cp$
Taylor rule with money	$r = 0.012y + 1.266p + 2.166m$ $m - p = -0.017y - 26.116u$ $u = 0.089r + 0.017(m - p)$
Taylor rule without money	$r = 0.079y + 1.732p$ $m - p = 0.103y - 27.032u$ $u = 0.019r + 0.018(m - p)$
Money-interest rate rule	$r = -0.780m$ $m - p = 0.317y + 73.941u$ $u = 0.005r + 0.007(m - p)$

Table D.12: GMM estimates of SVAR, Divisia Narrow Money, Effective federal funds rate, Sample period: Feb.92 - Jul.06

A. Divisia M4	Equations
Recursive	$r = 0.014y - 0.039p - 0.030cp$ $m = -0.064y + 0.044p - 0.145r - 0.003cp$
Taylor rule with money	$r = -0.083y - 0.027p - 1.387m$ $m - p = -0.011y + 34.988u$ $u = 0.041r + 0.023(m - p)$
Taylor rule without money	$r = 0.012y - 0.058p$ $m - p = -0.004y - 39.505u$ $u = 0.006r + 0.003(m - p)$
Money-interest rate rule	$r = -0.522m$ $m - p = -0.023y + 18.550u$ $u = 0.069r + 0.025(m - p)$

B. Divisia M4-	Equations
Recursive	$r = 0.024y - 0.028p - 0.027cp$ $m = -0.001y - 0.123p - 0.217r + 0.039cp$
Taylor rule with money	$r = 0.015y - 0.193p - 1.196m$ $m - p = -0.009y + 29.406u$ $u = 0.063r + 0.024(m - p)$
Taylor rule without money	$r = 0.036y + 0.103p$ $m - p = 0.095y - 23.037u$ $u = 0.012r - 0.004(m - p)$
Money-interest rate rule	$r = -2.251m$ $m - p = -0.006y + 26.341u$ $u = 0.038r - 0.036(m - p)$

C. Divisia M3	Equations
Recursive	$r = 0.026y - 0.041p - 0.022cp$ $m = -0.007y - 0.147p - 0.208r + 0.034cp$
Taylor rule with money	$r = 0.008y - 0.234p - 1.168m$ $m - p = -0.019y + 28.646u$ $u = 0.063r + 0.025(m - p)$
Taylor rule without money	$r = 0.036y + 0.067p$ $m - p = 0.074y - 24.705u$ $u = 0.007r - 0.017(m - p)$
Money-interest rate rule	$r = -2.259m$ $m - p = -0.001y + 25.925u$ $u = 0.013r + 0.038(m - p)$

D. Divisia ALL	Equations
Recursive	$r = 0.019y - 0.054p - 0.019cp$ $m = -0.033y - 0.137p - 0.297r - 0.011cp$
Taylor rule with money	$r = -0.073y - 0.369p - 2.263m$ $m - p = -0.078y + 75.427u$ $u = 0.052r + 0.009(m - p)$
Taylor rule without money	$r = 0.039y + 0.156p$ $m - p = 0.067y - 23.317u$ $u = 0.010r - 0.014(m - p)$
Money-interest rate rule	$r = -1.884m$ $m - p = -0.038y + 29.320u$ $u = 0.063r + 0.027(m - p)$

Table D.13: GMM estimates of SVAR, Divisia Broad Money, Shadow federal funds rate, Sample period: Feb.92 - Dec.19

E. Divisia M2	Equations
Recursive	$r = 0.015y - 0.038p - 0.017cp$ $m = -0.013y - 0.034p - 0.270r - 0.007cp$
Taylor rule with money	$r = -0.030y - 0.126p - 2.526m$ $m - p = -0.042y + 101.585u$ $u = 0.027r + 0.008(m - p)$
Taylor rule without money	$r = 0.028y + 0.119p$ $m - p = 0.070y - 24.296u$ $u = 0.008r - 0.015(m - p)$
Money-interest rate rule	$r = -2.172m$ $m - p = -0.029y + 48.509u$ $u = 0.046r + 0.017(m - p)$

F. Divisia MZM	Equations
Recursive	$r = 0.019y - 0.052p - 0.019cp$ $m = -0.023y - 0.174p - 0.334r - 0.001cp$
Taylor rule with money	$r = -0.041y - 0.375p - 1.980m$ $m - p = -0.370y + 174.391u$ $u = 0.115r - 0.002(m - p)$
Taylor rule without money	$r = 0.046y + 0.235p$ $m - p = 0.063y - 21.247u$ $u = 0.010r - 0.022(m - p)$
Money-interest rate rule	$r = -2.183m$ $m - p = -0.063y + 67.298u$ $u = 0.040r + 0.013(m - p)$

G. Divisia M2M	Equations
Recursive	$r = 0.016y - 0.040p - 0.016cp$ $m = -0.002y - 0.068p - 0.296r + 0.008cp$
Taylor rule with money	$r = 0.003y - 0.143p - 2.120m$ $m - p = -0.206y + 157.648u$ $u = 0.092r + 0.001(m - p)$
Taylor rule without money	$r = 0.033y + 0.153p$ $m - p = 0.081y - 20.952u$ $u = 0.014r - 0.007(m - p)$
Money-interest rate rule	$r = -2.670m$ $m - p = -0.033y + 46.786u$ $u = 0.050r + 0.020(m - p)$

H. Divisia M1	Equations
Recursive	$r = 0.025y - 0.018p - 0.018cp$ $m = -0.144y - 0.461p - 0.254r - 0.003cp$
Taylor rule with money	$r = -0.216y - 0.770p - 1.605m$ $m - p = -0.209y + 1024.851u$ $u = 0.006r + 0.001(m - p)$
Taylor rule without money	$r = 0.086y + 0.644p$ $m - p = -0.011y - 7.747u$ $u = 0.007r - 0.107(m - p)$
Money-interest rate rule	$r = -0.107m$ $m - p = -0.032y + 11.398u$ $u = 0.026r + 0.056(m - p)$

Table D.14: GMM estimates of SVAR, Divisia Narrow Money, Shadow federal funds rate, Sample period: Feb.92 - Dec.19

A. Divisia M4	Equations
Recursive	$r = -0.004y + 0.156p + 0.025cp$ $m = -0.072y + 0.078p - 0.341r + 0.011cp$
Taylor rule with money	$r = 0.131y + 0.163p + 1.812m$ $m - p = -0.036y - 39.098u$ $u = 0.039r - 0.004(m - p)$
Taylor rule without money	$r = 0.038y + 0.630p$ $m - p = 0.005y + 10.911u$ $u = -0.020r + 0.038(m - p)$
Money-interest rate rule	$r = 0.296m$ $m - p = -0.023y - 39.219u$ $u = 0.018r + 0.001(m - p)$

B. Divisia M4-	Equations
Recursive	$r = 0.004y + 0.149p + 0.041cp$ $m = -0.003y - 0.093p - 0.225r + 0.048cp$
Taylor rule with money	$r = 0.047y + 0.607p + 0.381m$ $m - p = 0.078y + 18.964u$ $u = -0.024r + 0.016(m - p)$
Taylor rule without money	$r = 0.064y + 0.745p$ $m - p = 0.065y + 15.479u$ $u = -0.009r + 0.027(m - p)$
Money-interest rate rule	$r = 0.040m$ $m - p = 0.064y + 16.596u$ $u = -0.008r + 0.027(m - p)$

C. Divisia M3	Equations
Recursive	$r = 0.023y + 0.152p + 0.041cp$ $m = -0.002y - 0.122p - 0.257r + 0.046cp$
Taylor rule with money	$r = 0.085y + 0.781p + 0.169m$ $m - p = 0.073y + 16.902u$ $u = -0.018r + 0.023(m - p)$
Taylor rule without money	$r = 0.095y + 0.876p$ $m - p = 0.063y + 12.628u$ $u = -0.012r + 0.036(m - p)$
Money-interest rate rule	$r = -0.119m$ $m - p = 0.053y + 13.912u$ $u = -0.003r + 0.038(m - p)$

D. Divisia ALL	Equations
Recursive	$r = -0.014y + 0.165p + 0.044cp$ $m = -0.035y - 0.121p - 0.219r + 0.001cp$
Taylor rule with money	$r = 1.967y + 9.556p + 60.667m$ $m - p = -0.142y - 251.580u$ $u = 0.714r + 0.234(m - p)$
Taylor rule without money	$r = 0.064y + 1.006p$ $m - p = 0.043y - 22.565u$ $u = 0.006r - 0.021(m - p)$
Money-interest rate rule	$r = 19.935m$ $m - p = 0.089y - 63.695u$ $u = 0.174r + 0.052(m - p)$

Table D.15: GMM estimates of SVAR, Divisia Broad Money, Effective federal funds rate, Sample period: Feb.92 - Dec.19

E. Divisia M2	Equations
Recursive	$r = -0.029y + 0.139p + 0.049cp$ $m = -0.026y - 0.031p - 0.198r - 0.002cp$
Taylor rule with money	$r = 0.217y + 0.744p + 12.120m$ $m - p = -0.047y - 38.696u$ $u = 0.126r + 0.021(m - p)$
Taylor rule without money	$r = 0.027y + 0.774p$ $m - p = 0.046y - 19.535u$ $u = 0.010r - 0.009(m - p)$
Money-interest rate rule	$r = 13.116m$ $m - p = -0.046y - 47.164u$ $u = 0.152r + 0.034(m - p)$

F. Divisia MZM	Equations
Recursive	$r = -0.008y + 0.157p + 0.045cp$ $m = -0.021y - 0.166p - 0.237r + 0.015cp$
Taylor rule with money	$r = 0.233y + 2.732p + 13.472m$ $m - p = 0.100y - 47.957u$ $u = 0.162r + 0.034(m - p)$
Taylor rule without money	$r = 0.082y + 1.124p$ $m - p = 0.052y - 18.030u$ $u = 0.009r - 0.021(m - p)$
Money-interest rate rule	$r = -0.275m$ $m - p = 0.084y - 0.361u$ $u = 0.004r + 0.002(m - p)$

G. Divisia M2M	Equations
Recursive	$r = -0.020y + 0.130p + 0.051cp$ $m = -0.012y - 0.073p - 0.205r + 0.018cp$
Taylor rule with money	$r = 0.008y + 0.429p + 2.800m$ $m - p = 0.032y - 19.128u$ $u = 0.081r - 0.009(m - p)$
Taylor rule without money	$r = 0.043y + 0.842p$ $m - p = 0.048y - 12.233u$ $u = 0.012r - 0.030(m - p)$
Money-interest rate rule	$r = 13.573m$ $m - p = -0.011y - 58.486u$ $u = 0.117r + 0.016(m - p)$

H. Divisia M1	Equations
Recursive	$r = -0.011y + 0.124p + 0.052cp$ $m = -0.156y - 0.453p - 0.189r + 0.010cp$
Taylor rule with money	$r = 0.647y + 2.206p + 4.226m$ $m - p = -1.019y - 4764.450u$ $u = 0.010r + 0.001(m - p)$
Taylor rule without money	$r = 0.096y + 1.396p$ $m - p = -0.019y - 9.052u$ $u = 0.005r - 0.087(m - p)$
Money-interest rate rule	$r = 0.048m$ $m - p = -0.042y - 30.991u$ $u = 0.005r - 0.013(m - p)$

Table D.16: GMM estimates of SVAR, Divisia Narrow Money, Effective federal funds rate, Sample period: Feb.92 - Dec.19