

ESSAYS ON INTERNATIONAL RISK SHARING

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## **Abstract**

International risk sharing is an intertemporal utility maximizing process in which countries of different economic prospects engage in cross-border trade and financial asset transactions to mitigate impacts of idiosyncratic income shocks on consumption. Measuring the extent of international risk sharing (IRS) remains an open empirical question. This dissertation provides a new approach to measuring the extent of IRS for countries and conditions for the measure of consumption correlation to hold and a possible cause to the consumption correlation puzzle.

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## Essay 1 : Literature Review

### 1. The concept of international risk sharing

Aggregate volatilities or risks refer to fluctuations in economic variables, e.g. consumption, gross domestic products, saving, investment, trade balance, exchange rate, and interest rates. Shocks originated from within and other countries are normally the cause of fluctuations of these variables in interconnected economies. Studying the extent of disconnection of these shocks and domestic aggregate fluctuations is an important branch in economic research and of practical meanings in making economic policies.

Consumption risk sharing refers to an intertemporal utility maximization process in which economic agents attempt to reduce impacts of income shocks on consumption via directly engaging in an insurance contract or by buying and selling a variety of financial assets or securities on capital and credit markets. The former arrangement separates risk hedgers from risk takers, while the latter lets both parties exchange risks with each other.

Risk sharing could be accomplished via within- or cross-country financial transactions. Domestic risk sharing describes situations in which residents of a country try to reduce volatilities of their consumption by carrying out financial transactions with each other. For a country as a whole, domestic risk sharing alone is not sufficient to avoid consumption fluctuations as the process is still limited to the country's own income. International risk sharing, on the other hand, describes processes in which countries of different income prospects would carry out international trade and financial transactions to share among them the income volatility risks affecting countries' consumption. The process would enable countries to delink countries' output shocks and their consumption.

The foundation of the concept lies in the observation that economic agents attempt to maintain stable consumption patterns over periods of turbulent economic activities. Leme (1984) and Scheinman (1984) observed that national consumption levels should move in a synchronized fashion when insurance against idiosyncratic national risk is possible, e.g. by using available financial instruments in international financial markets. Cochrane (1991) emphasized that consumption insurance is the cross-sectional counterpart of permanent income hypothesis in the sense that consumption of agents would not vary in response to idiosyncratic income shocks.

Employing dynamic general equilibrium models to study transmissions of aggregate fluctuations across countries, international business cycle literature has shown that in an Arrow-Debreu complete-markets world, consumption growth correlation should be higher than output growth correlation. In this ideal world, country-specific output risk would be pooled, and hence domestic per capita consumption growth should not depend on country-specific income shocks. A theoretical definition of perfect international risk sharing can be shown as the equality of marginal utility between any two identical countries on any date and state of nature. Put it differently, the insurance process theoretically results in much stronger comovement of any two countries' consumption growth than comovement of their income growth. Moreover, a country's consumption growth should be theoretically independent from its country-specific income fluctuations if perfect insurance were achieved.

## 2. A canonical model of international risk sharing

### 2.1 The model

The theoretical correlation result is derived based on the following model. Assume the world consists of a finite number of  $J$  countries,  $j = 1 \dots J$ , each of which is represented by an economic agent who wishes to maximize a time-separable CRRA utility function. All economic agents share



a common state-independent discount factor  $\beta$  and rate of relative risk aversion  $\sigma$ . There are a finite number of states  $S$  at time  $t$ ,  $s = 1 \dots S < \infty$ , each of which has an occurrence probability of  $\pi(s_t)$  and  $\sum_s \pi(s_t) = 1$ . Each country is endowed with an exogenous stochastic amount  $y_j(s_t)$  of good at each  $t$ . The lifetime utility maximization problem of each of the agents is<sup>1</sup>

$$U_j = \sum_{t=0}^{\infty} \beta^t \sum_s \pi_t u(c_{jt}) = \sum_{t=0}^{\infty} \beta^t \sum_s \pi_t \frac{c_{jt}^{1-\sigma}}{1-\sigma} \quad (0.1)$$

The agent's problem is solved using a social planner's method that assigns a social weight to each agent (or country).

$$\sum_{j=1}^J \omega_j U_j = \sum_{j=1}^J \omega_j \sum_{t=0}^{\infty} \beta^t \sum_s \pi_t u(c_{jt}) = \sum_{i=1}^J \omega_j \sum_{t=0}^{\infty} \beta^t \sum_s \pi_t \frac{c_{jt}^{1-\sigma}}{1-\sigma} \quad (0.2)$$

The aggregate constraint at each time and state is

$$\sum_{j=1}^J c_{jt} \leq \sum_{j=1}^J y_{jt} \quad (0.3)$$

The first-order condition for each country  $j$  at time  $t$  is

$$\frac{\lambda_t}{\beta \pi_t} = \omega_j c_{jt}^{\sigma} \quad (0.4)$$

where  $\lambda_t$  is the Lagrangian multiplier of the resource constraint at time  $t$ .

Equation (0.4) holds for any country  $j$  and any state and time. Hence, we will have

$$1 = \frac{\omega_j c_{jt}^{\sigma}}{\omega_k c_{kt}^{\sigma}} \quad (0.5)$$

Taking logarithm both sides of (0.5), we will have

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<sup>1</sup> We suppress the state notation for expositional convenience. All state-dependent variables are denoted with time  $t$  subscript.

$$\ln c_{jt} - \ln c_{kt} = \frac{\ln \omega_k - \ln \omega_j}{\sigma} \quad (0.6)$$

There are three more unknowns in equation (0.6), but taking first difference both sides will eliminate them

$$\ln c_{jt} - \ln c_{jt-1} = \ln c_{kt} - \ln c_{kt-1} \quad (0.7)$$

Aggregating equation (0.4) over countries and taking average of the result upon taking logarithm both sides of (0.4), we will get

$$\frac{1}{J} \sum_j \ln \lambda_t - \frac{1}{J} \sum_j \ln \beta^t \pi_t = \frac{1}{J} \sum_j \ln \omega_j + \frac{\sigma}{J} \sum_{j=1}^J \ln c_{jt} \quad (0.8)$$

Subtracting equation (0.8) from logarithm of equation (0.4)

$$\ln c_{jt} = \frac{1}{J} \sum_{j=1}^J \ln c_{jt} + \frac{1}{\sigma} \left( \ln \omega_j - \frac{1}{J} \sum_{j=1}^J \ln \omega_j \right) = c_t^a + \frac{1}{\sigma} \left( \ln \omega_j - \frac{1}{J} \sum_{j=1}^J \ln \omega_j \right) \quad (0.9)$$

where  $c_t^a = \frac{1}{J} \sum_{j=1}^J \ln c_{jt}$

Upon taking first difference of equation (0.9), we finally get

$$\ln c_{jt} - \ln c_{jt-1} = c_t^a - c_{t-1}^a \quad (0.10)$$

The important economic result learned from equations (0.7) is there exists a perfect consumption growth correlation between any two countries. Equation (0.10) also indicates a similar relationship between a country's consumption growth and the world's under perfect international risk sharing. In other words, a country's consumption only reacts to aggregate consumption shock when risk sharing is complete. The literature frequently tests equation (0.7) and (0.10) for full international risk sharing. In short, under these above assumptions, national consumption growth should move in a lockstep fashion to aggregate consumption growth.

## 2.2 Remarks

It is important to observe that the tight movements of countries' consumption growth and the world's depend on several ideal assumptions for all countries, including complete markets with a single good for trade, no default possibility, identical agents, and risk averse coefficients. Without these assumptions, the theoretical conclusions would not hold. One important implicit assumption in the model is that the income prospect of each country is exactly the same, hence, countries have no restrictions on cross-border financial assets transactions.

Another observation is that we cannot obtain the extent of risk sharing measured by pairwise correlations under imperfect risk sharing. Last but not least, international consumption risk sharing is different from domestic consumption smoothing, which could be done simply by saving one's own income or borrowing from relatives. In addition, the setup of the canonical model ignores possible reallocations of income across countries. In this perspective, receiving unilateral foreign aids cannot be considered as international consumption risk sharing (but consumption smoothing) as the aids does not represent contingent claims on the givers' income streams.

### 3. Current empirical approaches and their limitations

There are three main approaches used to measure international risk sharing in the literature. The first approach is based on general equilibrium dynamic economic models similar to one in Backus, Kehoe, and Kydland (1992). The result from a benchmark world economy with complete markets shows that there would be a very high consumption correlation compared with income correlation. Specifically, Backus et al. (1992) found that consumption correlation is 0.88, compared with -0.21 for income correlation from their model. Accounting for transport costs increases consumption correlation to 0.89 while decreasing income correlation to -0.5. The very high consumption correlation is interpreted as reflecting agent's high ability of sharing risks internationally (Backus et al., 1992). The paper also found completely opposite results in observed data, a situation often

referred to as consumption correlation puzzle in the literature. Despite several papers have tried to explain the puzzle, e.g. due to unobserved preference shocks, non-traded goods or capital controls (Backus & Smith, 1993; Baxter, 2011; Kollmann, 1995; Lewis, 1995, 1996, 1997, 1999; Obstfeld, 1994; Obstfeld & Rogoff, 2000; Stockman & Tesar, 1995; Tesar, 1993, 1995), a conclusive answer has not been found.

Another approach is based on testing equation (0.10) and the observation that consumption growth should be independent from idiosyncratic income shocks. The test could be done by running the following regression<sup>2</sup>  $\Delta c_{it} = \alpha_i + \beta_i \Delta C_t + \varepsilon_{it}$  and test if  $\beta_i = 1$ . The idea is similar to the correlation approach in the sense that under perfect risk sharing, idiosyncratic consumption fluctuations would be moving in a synchronized fashion with aggregate consumption fluctuation.

Therefore,  $\beta_i = \text{corr}(\Delta c_{it}, \Delta C_t) \frac{\sqrt{\text{var}(\Delta C_t)}}{\sqrt{\text{var}(\Delta c_{it})}} = \text{corr}(\Delta c_{it}, \Delta C_t)$  when  $\frac{\sqrt{\text{var}(\Delta C_t)}}{\sqrt{\text{var}(\Delta c_{it})}} = 1$ . This

approach relies on number of countries in the sample, the weighting scheme to obtain aggregate consumption shock, and does not tell the extent of risk sharing when null hypothesis is rejected.

A variant of the above regression form is the following  $\Delta c_{it} = \alpha_i + \eta \Delta C_t + \gamma \Delta y_{it} + \varepsilon_{it}$ , where the null hypotheses are  $\eta = 1$  and  $\gamma = 0$  in observance that country-specific income shock does not affect consumption growth. This form, however, is not entirely compatible with the correlation approach. Under perfect risk sharing, i.e.  $\text{corr}(\Delta c_{it}, \Delta C_t) = 1$ , we will have

$\eta = \frac{r_{\Delta c_{it}, \Delta C_t} - r_{\Delta c_{it}, \Delta y_{it}} r_{\Delta y_{it}, \Delta C_t}}{1 - r_{\Delta y_{it}, \Delta C_t}^2} = 1$  and  $\gamma = \frac{r_{\Delta c_{it}, \Delta y_{it}} - r_{\Delta c_{it}, \Delta C_t} r_{\Delta y_{it}, \Delta C_t}}{1 - r_{\Delta y_{it}, \Delta C_t}^2} = 0$  if and only if  $r_{\Delta y_{it}, \Delta C_t} = 0$ <sup>3</sup>, which

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<sup>2</sup>  $\Delta c_{it}(\Delta C_t)$  indicates growth rate of country  $i$ 's (the world's) consumption at time  $t$ .

<sup>3</sup>  $r_{\Delta y_{it}, \Delta C_t}$  is correlation between  $\Delta c_{it}$  and  $\Delta C_t$ .

has no supporting theoretical results.

The panel regression  $\Delta c_{it} = \alpha_i + \beta \Delta y_{it} + \varepsilon_{it}$  allows Asdrubali, Sorensen, and Yosha (1996) to the extent of unshared risks shown by the coefficient  $\beta$ . The country's degree of international risk sharing could be found by using the expression  $r_s = 1 - \beta$ , where the value of 1 comes from analytical results while deriving the regression form. The form could be used to measure the degree of risk sharing for a representative country in an economic union. The current economic situation in the European Union is a vivid example of how different macroeconomic fundamentals among countries are, even when they are in a monetary union. Failure to take into account some of the differences would yield uninterpretable results when using the approach to measure the extent of risk sharing for a specific country.

#### 4. Conclusions

The theory of international risk sharing is based on a simple pooling equilibrium in which countries have the same income prospects, hence, their consumption fluctuations would be affected by world aggregate only. The optimal condition, while it is useful for pedagogical purposes, is failed to be found in observed data. Current empirical approaches rely on such optimal condition to form the empirical forms, which are likely the reason for conflicting results and conclusions found in the empirical literature.

## Essay 2 : Measuring International Risk Sharing: The New Approach

### 1. Introduction

It is well known that an economic agent in one country can lessen the impact of a country-specific income shock on his consumption decision by engaging in asset transactions across countries. This so-called international risk sharing (IRS) is as fundamental as commodity transactions itself.

However, it is not an easy task to empirically measure the extent of risk sharing in a particular set of countries. Recently some economists propose a simple approach to regress the growth rate of consumption of a particular country (over the world average) on the growth rate of income similarly constructed and then take the figure of one minus the estimated coefficient as a measure of international risk sharing<sup>4</sup>. We call it the regression measure of IRS. The idea behind this intuitively attractive approach is a simple pedagogical model of a pooling equilibrium. By putting income of all members together, each participant can enjoy a more stable consumption level over time than one can do individually. In such a case, the consumption growth fluctuation over the world average should come from the source not related to income fluctuations. Therefore, in full international risk sharing, the coefficient of the above regression must be close to zero, implying this measure of IRS close to 1.

Unfortunately, the empirical results observed in many countless countries are not what is expected. The estimated measure is far from one even for a group of most advanced countries with massive international asset transactions. Two interpretations are possible. First, the degree of international risk sharing is actually low in reality even among advanced countries. Second, there is something

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<sup>4</sup> See Asdrubali et al. (1996), Arreaza, Sorensen, and Yosha (1998), A. M. Kose, Prasad, and Terrones (2009), Moser, Pointner, and Scharler (2004), Sørensen, Wu, Yosha, and Zhu (2007)

wrong with the proposed measure.

We do not know with certainty which is the case. However, we suspect the second is more likely a culprit. In figure 1, we summarize the regression measure of IRS estimated from time series data of a variety of countries. What we find is quite disturbing. The estimated measures are not only much smaller than one but also are relatively larger in developing countries than in advanced countries. Actually our average estimates are 0.12 for industrial economies, 0.22 for the emerging economies, and 0.3 for the developing economies. If we take these numbers literally, we have to say that the developing countries are achieving higher degrees of risk sharing on average than the industrial countries, which is simply absurd. We call this incident of anomaly the “order reversal puzzle” of the regression measure of IRS.

The purpose of this paper is to develop an empirical framework to obtain a more reasonable measure of IRS. To do so, we base our argument on the classical permanent income hypothesis (PIH) and develop an empirical model of the income and consumption processes in the form of bivariate unobserved component (UC) model.

In our model, the income and consumption processes share a common random walk component, which is interpreted as the permanent income. Transitory shocks to income, which is interpreted as country-specific business cycle shocks, has a direct impact on consumption, but this effect can be partially or totally eliminated by international asset transactions.

The extent of how independent the transitory component of consumption is from the transitory shock to income is, in our view, a reasonable measure of international risk sharing. In other words, our measure of IRS is nothing but an indicator of how much of the country specific business cycle shocks of an individual country are prevented from affecting the level of consumption contemporaneously.

It is found that the size of our measure of IRS estimated from data is close to one, and is larger in advanced countries than in developing countries. We also find that much smaller size of the regression measure of IRS is caused by the factors not directly related to the country's risk sharing behavior.

The organization of this paper is as follows: In the next section we develop an empirical framework to analyze risk sharing in the unobserved component model, and propose an alternative measure of IRS. Section 3 reports the empirical estimates of our UC measure of IRS and investigate the possible factors leading to the puzzling results of the regression measure of IRS. The last section provides a brief conclusion.

## 2. A model of international risk sharing

In this section, we first discuss the basic idea behind our empirical model, and then layout the mathematical detail of the model.

### 2.1 Basic ideas

To disentangle the empirical puzzle observed in the regression measure of IRS, we need to start our discussion with the permanent income hypothesis (PIH). Roughly speaking, each of the income and consumption processes is expressed as the sum of the permanent and transitory components. The two processes share the common random walk process as their permanent components. Their transitory components are separate stationary processes but can be correlated.

Given this rough structure, the regression of the growth rate of consumption on that of income would pickup the common random walk shock of two processes, which produces the positive lower bound on the slope coefficient, or the upper bound on the proposed measure of international risk sharing. This upper bound could potentially be quite low and could yield misleading conclusions. More seriously, the proposed measure is apparently measuring something other than



international risk sharing.

We now turn to an important aspect of the risk sharing across the national border in contrast with that within the border.

The permanent and transitory components of income and consumption of a representative agent in one country's economy are like the trend and cycle components of the country's income and consumption. In the absence of international financial transactions an individual can share individual specific idiosyncratic risks with his fellow citizens but cannot avoid the country-specific business cycle risks because they are systematic within the country's economy.

The representative agent model is more or less justifiable in one country's economy since at least the country specific shocks to the trend and cycle processes are likely to be shared by most members. This is partly because an economy is integrated to a large extent within a border and partly because the government can facilitate social programs including a variety forms of income transfers.

When we look at income and consumption of different countries and allow cross border financial transactions, we need to have heterogeneous agent models.

Financial markets integration alone cannot bring in the situation in which all different countries share a similar trend component of income and consumption unless the world is fiscally integrated and allows income transfers among countries. But the latter is politically quite difficult even within a small group of countries as witnessed by the current struggle of the EU nations. Given a current situation of the world, we assume that different countries have different long run income prospects or trend components and financial managements can only lessen an impact of the country specific business cycle on the transitory component of its consumption. Hence, what we expect from international risk sharing should not be an equalization of consumption growth rates among

countries, which is implied by a simple pooling equilibrium idea, but a consumption smoothing of each country along its specific trend line. A low income growth country cannot obtain a high growth rate of consumption through international financial arrangement but can lessen temporal fluctuations of consumption from its own (low) growth trend.

## 2.2 Empirical framework

We express income and consumption each as the sum of its permanent and transitory components in the form:

$$y_t = y_t^p + y_t^T$$

$$c_t = c_t^p + c_t^T$$

where  $y_t$  and  $c_t$  are log of per capita income and consumption, respectively, and each of the permanent components is an  $I(1)$  and each of transitory components is an  $I(0)$  process. Also, assume innovations to the permanent and transitory components are uncorrelated for income and consumption

In the absence of risk sharing, we set

$$y_t^T = \xi_t^y$$

$$c_t^T = \xi_t^c$$

where  $\xi_t^y$  and  $\xi_t^c$  are zero mean auto-regressive processes of degree  $p_y$  and  $p_c$ .

We now make more particular assumptions in order to set up our empirical framework as follows.

There are three assumptions

1. An income shock has one-for-one impact on consumption
2. There is a consumption shock with an exogenous origin
3. In an ideal environment all transitory income shocks can be eliminated by some by some financial arrangements

We elaborate these assumptions in turn. The first assumption is simply manifestation of the situation that in the absence of financial arrangement what you can consume is what you earn. It implies

$$\begin{bmatrix} \phi_y(L) & 0 \\ 0 & \phi_c(L) \end{bmatrix} \begin{bmatrix} \xi_t^y \\ \xi_t^c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^c \end{bmatrix} \quad (*)$$

where  $\varepsilon_t^y$  and  $\varepsilon_t^c$  are income and consumption shocks with  $\phi_y(z) = 1 - \phi_1^y z - \dots - \phi_{p_y}^y z^{p_y}$  and  $\phi_c(z) = 1 - \phi_1^c z - \dots - \phi_{p_c}^c z^{p_c}$ . The innovation for  $\xi_t^y$  and  $\xi_t^c$  are related in the form of the right-hand expression of equation (\*).

Second, we assume the consumption process is subject to the consumption shock  $\varepsilon_t^c$  in addition to the full-impact from income shock  $\varepsilon_t^y$ . The consumption shock is purely exogenous to the optimization process.

Third, we assume that the impact of the transitory shock to income of one country can be fully eliminated through financial arrangements internationally at least in an ideal condition. This is achieved by minimizing the conditional variance  $\text{var}_t(\Delta c_{t+1})$  through choosing the value of  $\beta$  equal to unity.

To see this, note that the above equations can be written as

$$\begin{bmatrix} y_t \\ c_t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tau_t + \begin{bmatrix} 1 & 0 \\ -\beta & 1 \end{bmatrix} \begin{bmatrix} 1/\phi_y(L) & 0 \\ 1/\phi_c(L) & 1/\phi_c(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^c \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tau_t + \begin{bmatrix} 1/\phi_y(L) & 0 \\ -\beta/\phi_c(L) + 1/\phi_c(L) & 1/\phi_c(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^c \end{bmatrix}$$

Then  $\text{var}_t(\Delta c_{t+1}) = \sigma_n^2 + (1 - \beta)^2 \sigma_y^2 + \sigma_c^2$ , which is minimized by choosing  $\beta = 1$ .

The actual value of  $\beta$  is of course influenced by the availability of financial arrangements, which in turn is constrained by the degree of development of financial markets. So, we think it appropriate to measure a country's degree of risk sharing by the magnitude of estimated value of  $\beta$ .

Comparison of the above model with those in the literature may give a further insight.

First, if we assume a simplified case in which  $\phi_y(z) = \phi_c(z) = 1$ , then our model reduces to

$$\begin{bmatrix} y_t \\ c_t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tau_t + \begin{bmatrix} 1 & 0 \\ 1-\beta & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^c \end{bmatrix}$$

So in the absence of the consumption shock, the full risk sharing situation  $\beta = 1$  is

$$\begin{bmatrix} y_t \\ c_t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tau_t + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \varepsilon_t^y$$

That is identical to the illustration of Hall's (1978) random walk model of consumption by Blanchard and Quah (1989).

Second, if we assume  $\sigma_\eta^2 = 0$  or the trend stationary model of the income process, we find

$$\begin{bmatrix} \Delta y_t \\ \Delta c_t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mu + \begin{bmatrix} 1 & 0 \\ 1-\beta & 1 \end{bmatrix} \begin{bmatrix} \Delta \varepsilon_t^y \\ \Delta \varepsilon_t^c \end{bmatrix}.$$

implying

$$\text{var} \begin{bmatrix} \Delta y_t \\ \Delta c_t \end{bmatrix} = 2 \begin{bmatrix} \sigma_y^2 & (1-\beta)\sigma_y^2 \\ \bullet & (1-\beta)^2 \sigma_y^2 + \sigma_c^2 \end{bmatrix}$$

In this case, the OLS coefficient of the regression of  $\Delta c_t$  on  $\Delta y_t$  is given by

$$\frac{\text{cov}(\Delta y_t, \Delta c_t)}{\text{var}(\Delta y_t)} = 1 - \beta$$

which is exactly the measure of IRS proposed by Asdrubali et al. (1996).

### 3. Empirical results

#### 3.1 The UC measure of IRS

We discuss both estimated regression and UC measures of IRS in this subsection. The results for these measures are presented in figures 3a to 3c.

The noticeable feature of results of regression measures is that they are not economically sensible. Looking at the results for advanced countries, they suggest that some most advanced countries, in particular the United States, UK, or Japan, share very small amount of income shocks internationally. In contrast, other least developing countries and emerging countries, e.g. Ghabon, Tunisia, Honduras, Trinidad & Tobago, Egypt, are able to internationally share much higher percentages of their income shocks. It is also seen from the results of regression measures that Greece and Switzerland appear to be able to share risks internationally better than other advanced counterparts. However, the figures for other highest-ranked emerging and developing countries also suggest that they have carried out IRS almost twice as well. The mean regression measures of IRS for advanced, emerging, and developing countries are 0.19, 0.28, and 0.40, respectively.

In contrast with the above situation of regression measure, the results of UC measure of IRS appear to be intuitive and economically sensible. As can be seen from these figures, the UC measure suggests that the extent of IRS turns out to be better than previously thought, especially among advanced countries. Specifically, the average value of UC measure of IRS for advanced countries is 0.86. This number indicates that existing IRS mechanisms in advanced countries are able to eliminate 86 percent of an impact of an income shock on consumption. In contrast, emerging and developing countries are able to eliminate 68 and 48 percent of an income shock on their consumption.

Looking at individual results for the advanced country group, the UC measure suggests that Switzerland and Germany are better at sharing risks internationally than other advanced countries. These two countries have similar extents of IRS and are individually able to reduce 94 percent of the impact of an income shock on their consumption. The figure 2a also indicates that the United States, Netherlands, and Norway are able to eliminate 87 percent of an impact of an income shock

on their consumption. Other advanced countries have also obtained relatively high extents of IRS, such as 80 percent for Italy, 77 percent for Spain, and 75 percent for Greece.

Relatively advanced emerging countries, such as South Korea, Israel, or Brazil are able to obtain the extents of IRS that are comparable to some lower-ranked advanced countries, for instance Greece's or Spain's. Lastly, the UC measure are available for only six developing countries in our study sample, whose magnitudes of UC measure appear to be relatively low.

To summary, the UC measure of IRS suggests that advanced and a few emerging countries can internationally share income risks much better than what have been found in the literature. We now turn to providing possible explanations for the situation in the next section.

### 3.2 Analyzing the order reversal puzzle

In this subsection we investigate the potential sources of the puzzle with the regression measure of IRS. As we show previously, there are two puzzles. First, the estimated regression measures are found too small. Second, when estimated separately for each country using time series data, the measures are found smaller on average for advanced countries than for developing countries, implying that the latter achieve higher degree of international risk sharing than the former.

We guess in the previous section that the first puzzle appears to be a result of the fact that  $\Delta c_t$  and  $\Delta y_t$  share the common random walk shock  $\eta_t$ , but we have not precisely pinned down the cause.

For simplicity, we assume that  $\phi_y(z) = \phi_c(z) = 1$ . From the form for

$$b = \frac{\text{cov}(\Delta y_t, \Delta c_t)}{\text{var}(\Delta y_t)} = \frac{\sigma_\eta^2 + 2(1-\beta)\sigma_y^2}{\sigma_\eta^2 + 2\sigma_y^2} = \frac{\sigma_\eta^2/2\sigma_y^2 + (1-\beta)}{\sigma_\eta^2/2\sigma_y^2 + 1},$$

we see that  $1-b = \frac{\beta}{\sigma_\eta^2/2\sigma_y^2 + 1}$ . When

the permanent-transitory variance ratio becomes bigger, the regression measure of IRS will be smaller, given a value of  $\beta$ . We report empirical values of the ratio in table 1. As can be seen, advanced countries on average have much larger permanent income shock than other groups. In

contrast, developing countries have the smallest ratio. These empirical results suggest that the ratio very likely plays an important role in the conclusion of low extent of risk sharing found in the literature.

The second possible explanation comes from the cointegration of consumption and income. Statistically, when income and consumption are not cointegrated,  $\text{cov}(\Delta c_t, \Delta y_t) \rightarrow 0$ . In this case, the regression measure of IRS tend to become larger due to smaller numerator. The empirical evidence of this theoretical result is also seen from table 1.

#### 4. Conclusions

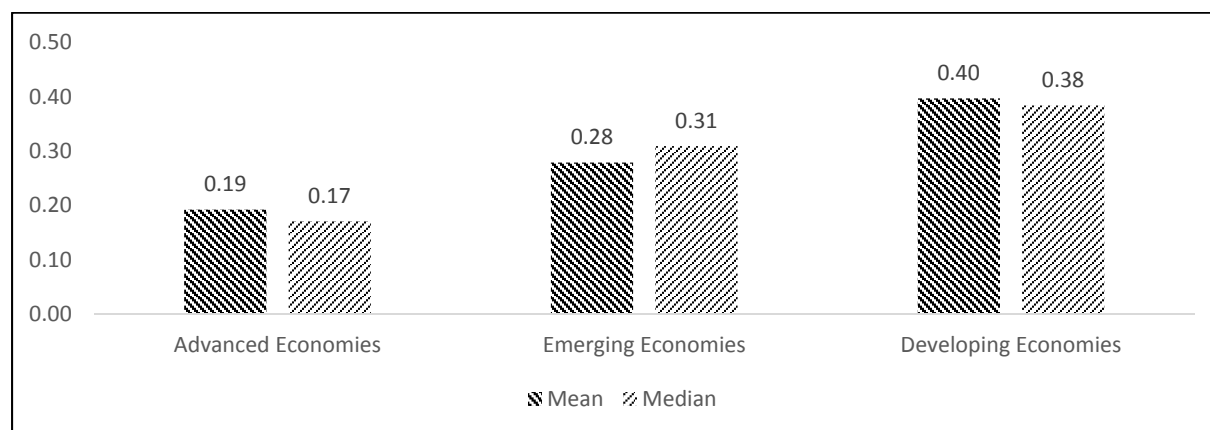
In this paper, we provide a new approach to measuring the extent of IRS for individual countries and pin down two possible causes for the low degree of risk sharing conclusion found in the literature. The new approach is based on an unobserved component model where an indicator for IRS is modeled to reduce a contemporaneous impact of income shock on consumption. The empirical results of the new measure suggest that the degrees of risk sharing for advanced countries and some emerging countries are relatively high. Finally, regression measure of IRS are shown to be influenced by permanent-transitory variance ratio and cointegration of income and consumption.

**Table 1: Variance ratio and regression measure of IRS**

|                  | Cointegration |              |    | Non-Cointegration |              |    | Total      |              |    |
|------------------|---------------|--------------|----|-------------------|--------------|----|------------|--------------|----|
|                  | Var. ratio    | Reg. measure | N  | Var. ratio        | Reg. measure | N  | Var. ratio | Reg. measure | N  |
| Advance Econ.    | 4.64          | 0.81         | 21 | -                 | -            | 0  | 4.64       | 0.81         | 21 |
| Emerging Econ.   | 1.93          | 0.75         | 12 | -                 | 0.67         | 9  | 1.93       | 0.72         | 21 |
| Developing Econ. | 0.45          | 0.62         | 6  | -                 | 0.59         | 21 | 0.45       | 0.60         | 27 |
| Total            | 2.20          | 0.73         | 39 | -                 | 0.63         | 30 | 2.20       | 0.68         | 69 |

Notes: The cointegration test for each country was run with and without time trend component in the specification (Søren Johansen, 1988; Soren Johansen & Juselius, 1990).

**Figure 1: Regression measure of IRS**

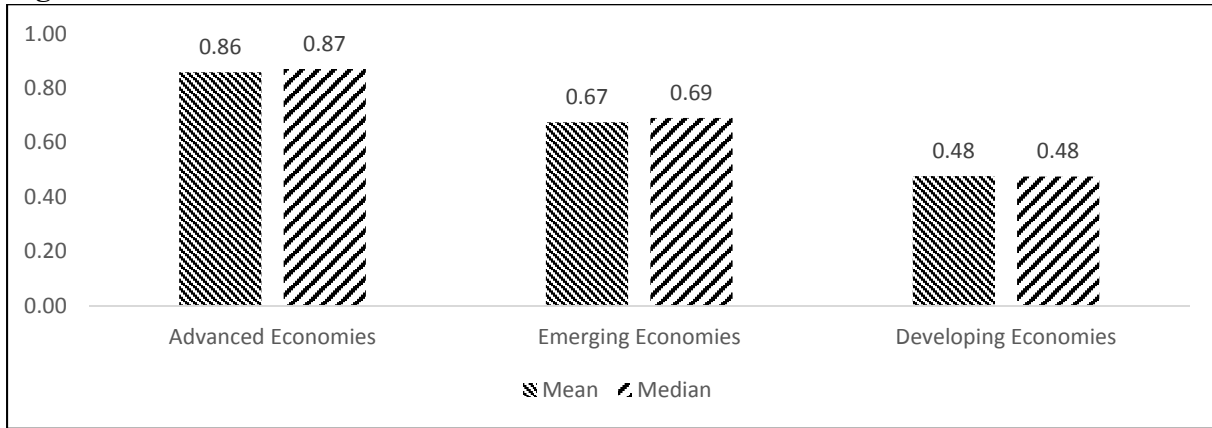


Notes:

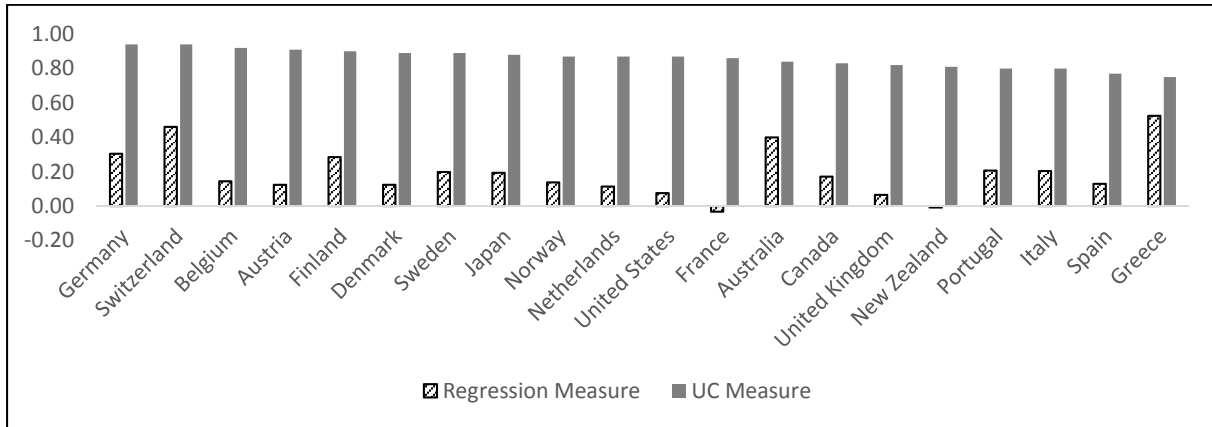
1. The regression measure of IRS for one country is obtained from the regression  $(\Delta y_{it} - \Delta Y_t) - (\Delta c_{it} - \Delta C_t) = a_i + b_i (\Delta y_{it} - \Delta Y_t) + \varepsilon_{it}$ .
2. The figure is obtained from calculating the mean and median of  $b_i$  by country groups.



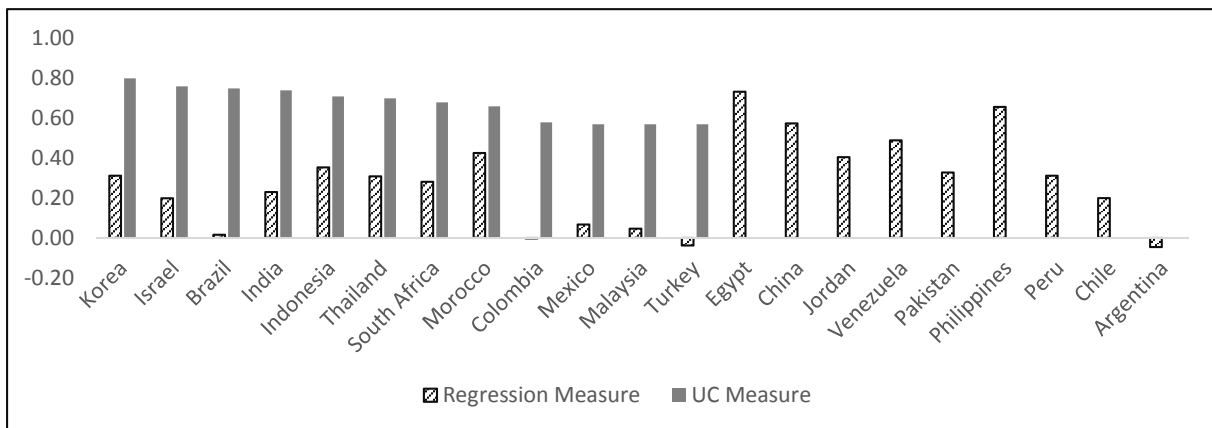
**Figure 2: UC measure of IRS**



**Figure 3a: Regression measure and UC measure of IRS for advanced countries**

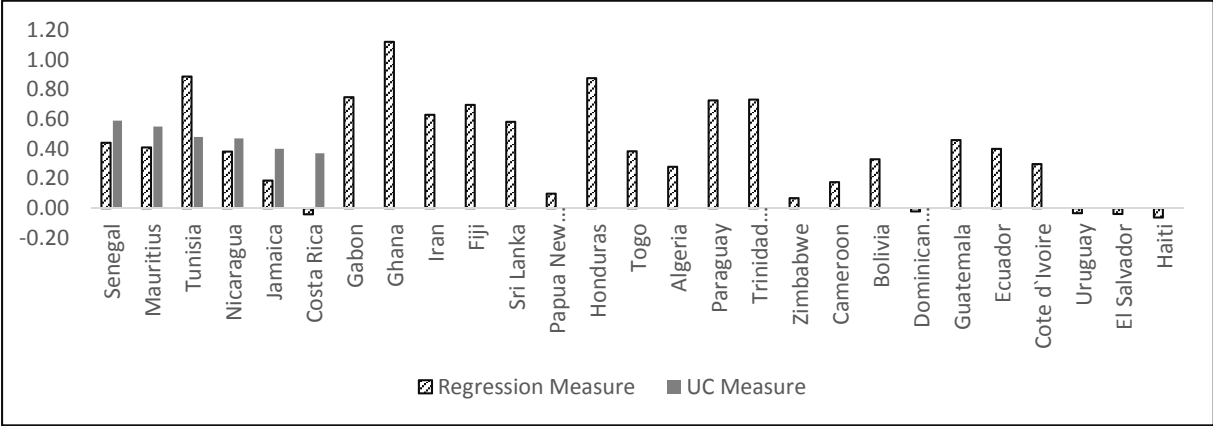


**Figure 3b: Regression measure and UC measure of IRS for emerging countries**



Notes: UC measures are available for countries with rank of 1 in a cointegration test.

**Figure 3c: Regression measure and UC measure of IRS for developing countries**



Notes: UC measures are available for countries with rank of 1 in a cointegration test.

## Essay 3 : Conditions for Consumption Correlation Puzzle

### 1. Introduction

It is well known that an economic agent in one country can lessen the impact of a country-specific income shock on his consumption decision by engaging in asset transactions across countries. This so-called international risk sharing (IRS) is as fundamental as commodity transactions itself. The knowledge of a country's temporal pattern of consumption not only has important welfare and policy implications to the country's government but also to the works of global institutions (e.g. IMF (2013)), and economic research.

However, it is very difficult to measure the extent of risk sharing for a country. Backus et al. (1992) propose a simple approach to compare the correlation of growth rate of consumption of two countries to correlation of income growth. The idea behind this simple approach is a simple pedagogical model of a pooling equilibrium. By putting income of all members together, each participant can enjoy a more stable consumption level over time than one can do individually. In such a case, the consumption growth fluctuation should come from the source not related to income fluctuations. Therefore, in full international risk sharing, the correlation of consumption growth must be 1 or much larger than correlation of income growth.

Unfortunately, the empirical results observed in many countless countries are not what is expected. The consumption correlation (CC) is far from one while income correlation (IC) is relatively higher than consumption correlation even for a group of most advanced countries with massive international asset transactions. This result is well known in the literature as consumption correlation puzzle.

Two interpretations are possible. First, the degree of international risk sharing is actually low in reality even among advanced countries. Second, there is something missing with the proposed measure. We do not know with certainty which is the case. However, we suspect the second is more likely a culprit. In table 2, we summarize the correlation measure of IRS estimated from time series data of a variety of countries. What we find is starkly surprising. The unsurprising part is the puzzle remains applicable to all countries on average. The average CC and IC are 0.12 and 0.18 for advanced economies, 0.05 and 0.08 for the emerging economies, and 0.05 and 0.07 for developing economies. However, the examining all possible pairwise CCs and ICs reveals that developing countries are more likely to observe a higher CC than IC among themselves while developed countries is the least likely. Moreover, an advanced country is more likely to have a higher CC than IC when pairing with a developing country than with an emerging counterpart. If we take these numbers literally, we have to say that the developing countries share income risks better on average than the advanced countries who also shares risks more often with developing countries, which is simply economically insensible.

The purpose of this paper is to explain the above situation and provide economic conditions for the CC to hold. To do so, we base our argument on the classical permanent income hypothesis (PIH) and develop a model of the income and consumption processes in the form of bivariate unobserved component (UC) model.

The extent of how much of the country-specific business cycle shocks of an individual country being prevented from affecting the level of consumption contemporaneously is, in our view, a reasonable measure of international risk sharing. The sufficient condition for risk sharing activities to happen is different business cycle conditions across countries. The fact is business cycles tend to be similar among countries within the same group while they are different between countries in

different economic groups likely contribute to the failure of the consumption correlation measure and the above empirical puzzle.

The organization of this paper is as follows: In the next section we develop a framework to analyze risk sharing in the unobserved component model, and provide conditions for the consumption correlation. Section 3 reports the data analysis for our conditions. The last section provides a brief conclusion.

## 2. An analytical framework for consumption correlation puzzle

In this section, we first discuss the basic idea behind our models, and then discuss the conditions for having a larger CC than IC.

### 2.1 Basic ideas

We start our discussion with the permanent income hypothesis (PIH). Roughly speaking, for a country each of its income and consumption processes is expressed as the sum of the permanent and transitory components. The two processes share the common random walk process as their permanent components. Their transitory components are separate stationary processes but can be correlated.

The permanent and transitory components of income and consumption of a representative agent in one country's economy are like the trend and cycle components of the country's income and consumption. In the absence of international financial transactions an individual can share individual specific idiosyncratic risks with his fellow citizens but cannot avoid the country-specific business cycle risks because they are systematic within the country's economy.

The representative agent model is more or less justifiable in one country's economy since at least the country specific shocks to the trend and cycle processes are likely to be shared by most

members. This is partly because an economy is integrated to a large extent within a border and partly because the government can facilitate social programs including a variety forms of income transfers.

When we look at income and consumption of different countries and allow cross border financial transactions, we need to have heterogeneous agent models.

Financial markets integration alone cannot bring in the situation in which all different countries share a similar trend component of income and consumption unless the world is fiscally integrated and allows income transfers among countries. But the latter is politically quite difficult even within a small group of countries as witnessed by the current struggle of the EU nations. Given a current situation of the world, we assume that different countries have different long run income prospects or trend components and financial managements can only lessen an impact of the country specific business cycle on the transitory component of its consumption. Hence, what we expect from international risk sharing should not be an equalization of consumption growth rates among countries, which is implied by a simple pooling equilibrium idea, but a consumption smoothing of each country along its specific trend line. A low income growth country cannot obtain a high growth rate of consumption through international financial arrangement but can lessen temporal fluctuations of consumption from its own (low) growth trend.

## 2.2 Intra-national risk sharing

We express income and consumption each as the sum of its permanent and transitory components in the form:

$$y_t = y_t^p + y_t^T$$
$$c_t = c_t^p + c_t^T$$

where  $y_t$  and  $c_t$  are log of per capita income and consumption, respectively, and each of the permanent components is an  $I(1)$  and each of transitory components is an  $I(0)$  process. Also, assume innovations to the permanent and transitory components are uncorrelated for income and consumption.

In the absence of IRS, only individual income shocks are diversifiable while country-specific business cycle and permanent shocks are not due to their systematic natures. We set for  $i$ -th individual in the  $I$ -th country as follows:

$$\begin{aligned} y_t^i &= \tau_t^y + \xi_t^y + \varepsilon_t^i \\ c_t^i &= \tau_t^y + \xi_t^y + (1 - \beta_i) \varepsilon_t^i \\ \tau_t &= \mu_i + \tau_{t-1} + \eta_t \end{aligned}$$

where  $\varepsilon_t^i$  is zero mean auto-regressive processes of degree  $p_y$ ,  $\xi_t^y$  indicates country-specific business cycle,  $\beta_i$  indicates the measure of risk sharing for the individual.

From this structure, we get

$$\begin{aligned} \text{corr}(\Delta y_t^i, \Delta y_t^j) &= \frac{\sigma_\eta^2 + 2\sigma_\xi^2}{\sigma_\eta^2 + 2\sigma_\xi^2 + 2\sigma_\varepsilon^2} < 1 \\ \text{corr}(\Delta c_t^i, \Delta c_t^j) &= \frac{\sigma_\eta^2 + 2\sigma_\xi^2}{\left(\sigma_\eta^2 + 2\sigma_\xi^2 + 2(1 - \beta_i)^2 \sigma_{\varepsilon^i}^2\right)^{\frac{1}{2}} \left(\sigma_\eta^2 + 2\sigma_\xi^2 + 2(1 - \beta_j)^2 \sigma_{\varepsilon^j}^2\right)^{\frac{1}{2}}} < 1 \end{aligned}$$

$$\text{Under full risk sharing or } \beta_i = \beta_j = 1: \text{corr}(\Delta c_t^i, \Delta c_t^j) = \frac{\sigma_\eta^2 + 2\sigma_\xi^2}{\left(\sigma_\eta^2 + 2\sigma_\xi^2\right)^{\frac{1}{2}} \left(\sigma_\eta^2 + 2\sigma_\xi^2\right)^{\frac{1}{2}}} = 1$$

### 2.3 International risk sharing

In the case of IRS, business cycle becomes diversifiable risks thanks to different business cycles across countries. We set, for any two countries  $i$  and  $j$ , income and consumption to share a similar structure of the following form:

$$\begin{aligned} y_i^I &= \tau_i^y + \xi_{it}^y \\ c_i^I &= \tau_i^y + (1 - \beta_i) \xi_{it}^y \\ \tau_i^y &= \mu_i + \tau_{i,t-1}^y + \eta_{it} \end{aligned}$$

$$\text{where } \begin{bmatrix} \eta_{it} \\ \eta_{jt} \end{bmatrix} \sim \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{\eta_i}^2 & \sigma_{\eta_{ij}} \\ \sigma_{\eta_{ij}} & \sigma_{\eta_j}^2 \end{bmatrix} \right), \begin{bmatrix} \xi_{it}^y \\ \xi_{jt}^y \end{bmatrix} \sim \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{\xi_i}^2 & \sigma_{\xi_{ij}} \\ \sigma_{\xi_{ij}} & \sigma_{\xi_j}^2 \end{bmatrix} \right)$$

We then get

$$\begin{aligned} \text{corr}(\Delta y_i^I, \Delta y_j^I) &= \frac{\sigma_{\eta_{ij}} + 2\sigma_{\xi_{ij}}}{\left(\sigma_{\eta_i}^2 + 2\sigma_{\xi_i}^2\right)^{1/2} \left(\sigma_{\eta_j}^2 + 2\sigma_{\xi_j}^2\right)^{1/2}} \\ \text{corr}(\Delta c_i^I, \Delta c_j^I) &= \frac{\sigma_{\eta_{ij}} + 2(1 - \beta_i)(1 - \beta_j)\sigma_{\xi_{ij}}}{\left(\sigma_{\eta_i}^2 + 2(1 - \beta_i)^2 \sigma_{\xi_i}^2\right)^{1/2} \left(\sigma_{\eta_j}^2 + 2(1 - \beta_j)^2 \sigma_{\xi_j}^2\right)^{1/2}} \end{aligned}$$

$$\text{Under no IRS or } \beta_i = \beta_j = 0: \text{corr}(\Delta y_i^I, \Delta y_j^I) = \frac{\sigma_{\eta_{ij}} + 2\sigma_{\xi_{ij}}}{\left(\sigma_{\eta_i}^2 + 2\sigma_{\xi_i}^2\right)^{1/2} \left(\sigma_{\eta_j}^2 + 2\sigma_{\xi_j}^2\right)^{1/2}} = \text{corr}(\Delta c_i^I, \Delta c_j^I)$$

On the other hand, under full risk sharing, i.e.  $\beta_i = \beta_j = 1$ :

$$\text{corr}(\Delta c_i^I, \Delta c_j^I) = \frac{\sigma_{\eta_{ij}}}{\sigma_{\eta_i} \sigma_{\eta_j}}$$

$$\text{corr}(\Delta y_i^I, \Delta y_j^I) = \frac{\sigma_{\eta_{ij}} + 2\sigma_{\xi_{ij}}}{\left(\sigma_{\eta_i}^2 + 2\sigma_{\xi_i}^2\right)^{1/2} \left(\sigma_{\eta_j}^2 + 2\sigma_{\xi_j}^2\right)^{1/2}}$$



We then get

$$\frac{corr(\Delta y_t^I, \Delta y_t^J)}{corr(\Delta c_t^I, \Delta c_t^J)} = \frac{\sigma_{\eta_I} \sigma_{\eta_J}}{\sigma_{\eta_{IJ}}} \frac{\sigma_{\eta_{IJ}} + 2\sigma_{\xi_{IJ}}}{(\sigma_{\eta_I}^2 + 2\sigma_{\xi_I}^2)^{1/2} (\sigma_{\eta_J}^2 + 2\sigma_{\xi_J}^2)^{1/2}} = \frac{1 + 2\frac{\sigma_{\xi_{IJ}}}{\sigma_{\eta_{IJ}}}}{\left(1 + 2\frac{\sigma_{\xi_I}^2}{\sigma_{\eta_I}^2}\right)^{1/2} \left(1 + 2\frac{\sigma_{\xi_J}^2}{\sigma_{\eta_I}^2}\right)^{1/2}}$$

which is observably larger than 1.

Let  $\rho_{IJ}^{\Delta y} = corr(\Delta y_t^I, \Delta y_t^J)$  and  $\rho_{IJ}^{\Delta c} = corr(\Delta c_t^I, \Delta c_t^J)$

We then have the following conditions

- A.  $\sigma_{\xi_{IJ}} < 0$  is sufficient for  $\rho_{IJ}^{\Delta y} < \rho_{IJ}^{\Delta c}$
- B. Suppose  $\sigma_{\xi_{IJ}} > 0$ 
  - a. If  $\sigma_{\eta_{IJ}} < 0$ , then  $\rho_{IJ}^{\Delta y} < \rho_{IJ}^{\Delta c}$
  - b. Suppose  $\sigma_{\eta_{IJ}} > 0$ ,  $\rho_{IJ}^{\xi} > \rho_{IJ}^{\eta}$  is sufficient for  $\rho_{IJ}^{\Delta y} > \rho_{IJ}^{\Delta c}$

The condition Bb is seen as follows: Assuming positive in both numerator and denominator, if

$$\frac{\sigma_{\xi_{IJ}}}{\sigma_{\eta_{IJ}}} \text{ is greater than both } \left(\frac{\sigma_{\xi_I}}{\sigma_{\eta_I}}\right)^2 \text{ and } \left(\frac{\sigma_{\xi_J}}{\sigma_{\eta_I}}\right)^2 \text{ or } \left(\frac{\sigma_{\xi_{IJ}}}{\sigma_{\eta_{IJ}}}\right)^2 > \left(\frac{\sigma_{\xi_I}}{\sigma_{\eta_I}}\right)^2 \left(\frac{\sigma_{\xi_J}}{\sigma_{\eta_I}}\right)^2 \text{ or}$$

$$\left(\frac{\sigma_{\xi_{IJ}}}{\sigma_{\xi_I} \sigma_{\xi_J}}\right)^2 > \left(\frac{\sigma_{\eta_{IJ}}}{\sigma_{\eta_I} \sigma_{\eta_I}}\right)^2 \text{ i.e. } corr(\xi_I, \xi_J) > corr(\eta_I, \eta_J).$$

## 2.4 Conclusions

If a given country's business cycle is more correlated to the world business cycle than its trend is correlated to the world trend, then this country's income growth tends to be more correlated to that

of the rest of the countries than its consumption growth is correlated to that of the rest of the countries.

Probably a most important factor that would make the correlation of income across countries larger than that of consumption despite the presence of high degrees of IRS is large positive correlation of business cycles across countries. In other words, it is a world business cycle.

### 3. Empirical evidence

We report in this section the empirical results of the above theoretical arguments. The results are presented in tables 2 to 5.

The noticeable result is that the conditions (A) and (Ba) do not hold well in the whole sample. However, they do hold relatively well within the group that meets the conditions. The economic meaning of condition (A) is that countries should have different business cycles to facilitate risk-sharing activities, leading to a larger consumption correlation than income correlation. In the real world, however, studies have shown that business cycles among advanced countries are relatively synchronized (see, for example M. A. Kose, Otrok, and Whiteman (2003)). This is likely the cause of the limited applicability of the correlation measure while provides a possible explanation for the limited occurrence of larger consumption correlation than income correlation among advanced countries seen in **Table 2**.

Condition (Bb) is a theoretically interesting result. As seen in table 4, the condition holds very well for country groups where the condition is met. However, it does not hold for a larger share of the sample. Results shown in table 5 indicates that country groups, where a larger income correlation than consumption correlation is observed, experience larger correlations of trend and cycles than the group without this feature. Therefore, direct empirical evidence for condition (Bb) is obviously

scarce, results in table 5 suggest certain role to play of the world business cycle in the consumption correlation puzzle.

#### 4. Conclusions

In this paper we provide sufficient conditions for the consumption correlation measure and the empirical fact in observed data. The sufficient condition is when countries experience different business cycles. Moreover, the world business cycle likely plays a role in the consumption correlation puzzle. Empirical results suggest that these conditions hold relatively well within the groups where these conditions are found valid.

While supports for these conditions are scarce, these conditions offer new directions from which they can be extended and gain further insights into the consumption correlation puzzle. Data analysis indicates that the first and second moments of income growth and consumption growth are markedly different between country groups where opposite relationship between income and consumption correlations are observed and where cointegration is found (see tables 7 and 8). Moreover, there are substantial differences in these moments across countries within the group where consumption correlation is larger than income correlation. The general conclusion from comparing first and second moments of income and consumption growth of country pairs with  $\rho^{\Delta c} > \rho^{\Delta y}$  and those with  $\rho^{\Delta c} < \rho^{\Delta y}$  is that within the former group the first moment of income and consumption growth are larger while the second moments of these variables are smaller. This observation suggests that the causes of consumption correlation puzzle could be found by removing identical countries assumption in the canonical model and allowing consumption and income shocks to be differences among countries.

**Table 2: Summary of consumption and income correlation**

|  | <b>AEs</b> | <b>EEs</b> | <b>DEs</b> | <b>Total</b> |
|--|------------|------------|------------|--------------|
| $\rho^{\Delta c}$                          |            |            |            |              |
| AEs  | 0.28       | 0.02       | 0.05       | 0.12         |
| EEs  | 0.02       | 0.09       | 0.04       | 0.05         |
| DEs  | 0.05       | 0.04       | 0.06       | 0.05         |
| $\rho^{\Delta y}$                          |            |            |            |              |
| AEs  | 0.38       | 0.08       | 0.09       | 0.18         |
| EEs  | 0.08       | 0.11       | 0.05       | 0.08         |
| DEs  | 0.09       | 0.05       | 0.08       | 0.07         |
| $N_1(\rho^{\Delta c} > \rho^{\Delta y})/N$ |            |            |            |              |
| AEs  | 0.18       | 0.33       | 0.42       | 0.31         |
| EEs  | 0.33       | 0.41       | 0.47       | 0.40         |
| DEs  | 0.42       | 0.47       | 0.47       | 0.45         |

**Table 3: Condition (A)**

|     | <b>AEs</b> |        | <b>EEs</b> |        | <b>DEs</b> |        | <b>World</b> |        |
|-----|------------|--------|------------|--------|------------|--------|--------------|--------|
|     | A1 (N)     | A2 (N) | A1 (N)     | A2 (N) | A1 (N)     | A2 (N) | A1 (N)       | A2 (N) |
| AEs | 2          | 1      | 2          | 2      | 2          | 2      |              |        |
| EEs |            |        | 3          | 2      |            |        |              | 1      |
| DEs |            |        |            |        | 2          | 1      |              | 1      |
|     | S1 (%)     | S2 (%) | S1 (%)     | S2 (%) | S1 (%)     | S2 (%) | S1 (%)       | S2 (%) |
| AEs | 67         | 33     | 50         | 50     | 50         | 50     |              |        |
| EEs |            |        | 60         | 40     |            |        |              | 100    |
| DEs |            |        |            |        | 67         | 33     |              | 100    |

Note:

1. A1: average number of country pairs satisfying  $\sigma_{\xi_{IJ}} < 0 \rightarrow \rho^{\Delta y} < \rho^{\Delta c}$
2. A2: average number of country pairs satisfying  $\sigma_{\xi_{IJ}} < 0 \rightarrow \rho^{\Delta y} > \rho^{\Delta c}$
3. S1: percentage of number of country pairs satisfying  $\sigma_{\xi_{IJ}} < 0 \rightarrow \rho^{\Delta y} < \rho^{\Delta c}$  in (A1+A2) for respective row and column country group.
4. S2: percentage of number of country pairs satisfying  $\sigma_{\xi_{IJ}} < 0 \rightarrow \rho^{\Delta y} > \rho^{\Delta c}$  in (A1+A2) for respective row and column country group.

**Table 4: Condition (Ba)**

|     | AEs    |        | EEs    |        | DEs    |        | World |
|-----|--------|--------|--------|--------|--------|--------|-------|
|     | A1 (N) | A2 (N) | A1 (N) | A2 (N) | A1 (N) | A2 (N) |       |
| AEs |        |        |        |        | 1      | 1      |       |
| EEs |        |        |        |        |        |        |       |
| DEs |        |        |        |        | 1      | 0      |       |
|     | S1 (%) | S2 (%) | S1 (%) | S2 (%) | S1 (%) | S2 (%) |       |
| AEs |        |        |        |        | 50     | 50     |       |
| EEs |        |        |        |        |        |        |       |
| DEs |        |        |        |        | 100    | 0      |       |

Note:

1. A1: average number of country pairs satisfying  $\begin{cases} \sigma_{\xi_{ij}} > 0 \\ \sigma_{\eta_{ij}} < 0 \end{cases} \rightarrow \rho^{\Delta y} < \rho^{\Delta c}$

2. A2: average number of country pairs satisfying  $\begin{cases} \sigma_{\xi_{ij}} > 0 \\ \sigma_{\eta_{ij}} < 0 \end{cases} \rightarrow \rho^{\Delta y} > \rho^{\Delta c}$

3. S1: percentage of number of country pairs satisfying  $\begin{cases} \sigma_{\xi_{ij}} > 0 \\ \sigma_{\eta_{ij}} < 0 \end{cases} \rightarrow \rho^{\Delta y} < \rho^{\Delta c}$  in (A1+A2) for respective row and column country group.

4. S2: percentage of number of country pairs satisfying  $\begin{cases} \sigma_{\xi_{ij}} > 0 \\ \sigma_{\eta_{ij}} < 0 \end{cases} \rightarrow \rho^{\Delta y} > \rho^{\Delta c}$  in (A1+A2) for respective row and column country group.

**Table 5: Condition (Bb)**

|     | AEs    |        | EEs    |        | DEs    |        | World |
|-----|--------|--------|--------|--------|--------|--------|-------|
|     | A1 (N) | A2 (N) | A1 (N) | A2 (N) | A1 (N) | A2 (N) |       |
| AEs | 1      |        |        |        |        | 1      |       |
| EEs |        |        | 1      |        |        |        |       |
| DEs |        |        |        |        | 1      |        |       |
|     | S1 (%) | S2 (%) | S1 (%) | S2 (%) | S1 (%) | S2 (%) |       |
| AEs | 100    |        |        |        |        | 100    |       |
| EEs |        |        | 100    |        |        |        |       |
| DEs |        |        |        |        | 100    | 0      |       |

Note:

1. A1: average number of country pairs satisfying  $\begin{cases} \sigma_{\xi_{IJ}} > 0 \\ \sigma_{\eta_{IJ}} > 0 \end{cases}$   
 $\rho_{IJ}^{\xi} > \rho_{IJ}^{\eta} \rightarrow \rho^{\Delta y} > \rho^{\Delta c}$
2. A2: average number of country pairs satisfying  $\begin{cases} \sigma_{\xi_{IJ}} > 0 \\ \sigma_{\eta_{IJ}} > 0 \end{cases}$   
 $\rho_{IJ}^{\xi} > \rho_{IJ}^{\eta} \rightarrow \rho^{\Delta y} < \rho^{\Delta c}$
3. S1: percentage of number of country pairs satisfying  $\begin{cases} \sigma_{\xi_{IJ}} > 0 \\ \sigma_{\eta_{IJ}} > 0 \end{cases}$  in (A1+A2) for respective  
 $\rho_{IJ}^{\xi} > \rho_{IJ}^{\eta} \rightarrow \rho^{\Delta y} > \rho^{\Delta c}$   
row and column country group.
4. S2: percentage of number of country pairs satisfying  $\begin{cases} \sigma_{\xi_{IJ}} > 0 \\ \sigma_{\eta_{IJ}} > 0 \end{cases}$  in (A1+A2) for respective  
 $\rho_{IJ}^{\xi} > \rho_{IJ}^{\eta} \rightarrow \rho^{\Delta y} < \rho^{\Delta c}$   
row and column country group.

**Table 6: Comparison of correlation of cycles and trends for groups of  $\rho^{\Delta y} < \rho^{\Delta c}$  and  $\rho^{\Delta y} > \rho^{\Delta c}$**

|                               | $\rho^{\Delta y} < \rho^{\Delta c}$ |      |       | $\rho^{\Delta y} > \rho^{\Delta c}$ |      |      |       |
|-------------------------------|-------------------------------------|------|-------|-------------------------------------|------|------|-------|
|                               | AEs                                 | EEs  | DEs   | AEs                                 | EEs  | DEs  |       |
| $\text{corr}(\xi_I, \xi_J)$   |                                     |      |       | $\text{corr}(\xi_I, \xi_J)$         |      |      |       |
| AEs                           | 0.20                                | 0.06 | -0.03 | AEs                                 | 0.39 | 0.17 | 0.09  |
| EEs                           |                                     | 0.02 | -0.11 | EEs                                 |      | 0.14 | 0.08  |
| DEs                           |                                     |      | 0.02  | DEs                                 |      |      | -0.06 |
| Total                         | 0.10                                | 0.00 | -0.04 | Total                               | 0.21 | 0.13 | 0.04  |
| $\text{corr}(\eta_I, \eta_J)$ |                                     |      |       | $\text{corr}(\eta_I, \eta_J)$       |      |      |       |
| AEs                           | 0.98                                | 0.97 | 0.32  | AEs                                 | 0.98 | 0.96 | 0.71  |
| EEs                           |                                     | 0.94 | 0.48  | EEs                                 |      | 0.96 | 0.56  |
| DEs                           |                                     |      | 0.05  | DEs                                 |      |      | 0.67  |
| Total                         | 0.82                                | 0.81 | 0.28  | Total                               | 0.83 | 0.82 | 0.65  |

**Table 7: Mean and variance ratios of income and consumption growth**

|   | $\Delta y_{it}/\Delta y_{jt}$ | $V(\Delta y_{it})/V(\Delta y_{jt})$ | $\Delta c_{it}/\Delta c_{jt}$ | $V(\Delta c_{it})/V(\Delta c_{jt})$ | $\rho^{\Delta y}$ | $\rho^{\Delta c}$ |
|---|-------------------------------|-------------------------------------|-------------------------------|-------------------------------------|-------------------|-------------------|
| <i>corr</i> ( $\Delta c_{it}, \Delta c_{jt}$ ) > <i>corr</i> ( $\Delta y_{it}, \Delta y_{jt}$ ) |                               |                                     |                               |                                     |                   |                   |
| AEs   | 1.34                          | 0.20                                | 1.30                          | 0.09                                | 0.00              | 0.12              |
| EEs   | 1.37                          | 0.43                                | 1.31                          | 0.31                                | -0.01             | 0.10              |
| Des   | 0.96                          | 1.18                                | 0.95                          | 1.26                                | -0.02             | 0.13              |
| <b>Total</b>  | <b>1.05</b>                   | <b>0.98</b>                         | <b>1.04</b>                   | <b>1.01</b>                         | <b>-0.02</b>      | <b>0.12</b>       |
| <i>corr</i> ( $\Delta c_{it}, \Delta c_{jt}$ ) < <i>corr</i> ( $\Delta y_{it}, \Delta y_{jt}$ ) |                               |                                     |                               |                                     |                   |                   |
| AEs   | 1.22                          | 0.27                                | 1.20                          | 0.13                                | 0.20              | 0.05              |
| EEs   | 1.28                          | 0.50                                | 1.24                          | 0.36                                | 0.12              | -0.01             |
| Des   | 0.89                          | 1.24                                | 0.91                          | 1.26                                | 0.15              | 0.00              |
| <b>Total</b>  | <b>0.97</b>                   | <b>1.04</b>                         | <b>0.99</b>                   | <b>1.02</b>                         | <b>0.16</b>       | <b>0.00</b>       |

**Table 8: Mean and variances of consumption and income across countries with and without cointegration**

|   | $\Delta y_{it}/\Delta y_{jt}$ |             | $V(\Delta y_{it})/V(\Delta y_{jt})$ |             | $\Delta c_{it}/\Delta c_{jt}$ |             | $V(\Delta c_{it})/V(\Delta c_{jt})$ |             |
|---|-------------------------------|-------------|-------------------------------------|-------------|-------------------------------|-------------|-------------------------------------|-------------|
|   | NC                            | C           | NC                                  | C           | NC                            | C           | NC                                  | C           |
| <i>corr</i> ( $\Delta c_{it}, \Delta c_{jt}$ ) > <i>corr</i> ( $\Delta y_{it}, \Delta y_{jt}$ ) |                               |             |                                     |             |                               |             |                                     |             |
| AEs   | 2.08                          | 1.00        | 0.14                                | 0.32        | 1.96                          | 0.98        | 0.06                                | 0.18        |
| EEs   | 2.12                          | 1.01        | 0.31                                | 0.74        | 1.96                          | 0.97        | 0.21                                | 0.62        |
| Des   | 1.53                          | 0.72        | 0.83                                | 1.92        | 1.46                          | 0.73        | 0.82                                | 2.43        |
| <b>Total</b>  | <b>1.67</b>                   | <b>0.79</b> | <b>0.69</b>                         | <b>1.59</b> | <b>1.58</b>                   | <b>0.79</b> | <b>0.66</b>                         | <b>1.95</b> |
| <i>corr</i> ( $\Delta c_{it}, \Delta c_{jt}$ ) < <i>corr</i> ( $\Delta y_{it}, \Delta y_{jt}$ ) |                               |             |                                     |             |                               |             |                                     |             |
| AEs   | 1.94                          | 0.93        | 0.18                                | 0.47        | 1.88                          | 0.93        | 0.08                                | 0.28        |
| EEs   | 2.07                          | 0.99        | 0.34                                | 0.82        | 1.94                          | 0.97        | 0.22                                | 0.75        |
| Des   | 1.44                          | 0.67        | 0.87                                | 2.12        | 1.43                          | 0.69        | 0.81                                | 2.64        |
| <b>Total</b>  | <b>1.57</b>                   | <b>0.74</b> | <b>0.72</b>                         | <b>1.77</b> | <b>1.54</b>                   | <b>0.75</b> | <b>0.65</b>                         | <b>2.13</b> |

Note: NC indicates non-cointegration, C cointegration

## References

- Arreaza, A., Sorensen, B. E., & Yosha, O. (1998). Consumption Smoothing through Fiscal Policy in OECD and EU Countries. *National Bureau of Economic Research Working Paper Series, No. 6372*(published as Adriana Arreaza, Bent E. Sgrensen, Oved Yosha. "Consumption Smoothing through Fiscal Policy in OECD and EU Countries," in James M. Poterba and Jürgen von Hagen, editors, "Fiscal Institutions and Fiscal Performance" University of Chicago Press (1999)).
- Asdrubali, P., Sorensen, B. E., & Yosha, O. (1996). Channels of Interstate Risk Sharing: United States 1963-1990. *The Quarterly Journal of Economics, 111*(4), 1081-1110.
- Backus, D. K., Kehoe, P. J., & Kydland, F. E. (1992). International Real Business Cycles. *The Journal of Political Economy, 100*(4), 745-775.
- Backus, D. K., & Smith, G. W. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics, 35*(3-4), 297-316.
- Baxter, M. (2011). International Risk Sharing in the Short Run and in the Long Run. *National Bureau of Economic Research Working Paper Series, No. 16789*.
- Blanchard, O. J., & Quah, D. (1989). The Dynamic Effects of Aggregate Demand and Supply Disturbances. *The American Economic Review, 79*(4), 655-673. doi: 10.2307/1827924
- Hamilton, J. D., & Flavin, M. A. (1986). On the Limitations of Government Borrowing: A Framework for Empirical Testing. *The American Economic Review, 76*(4), 808-819.
- IMF. (2013). West African Economic and Monetary Union (WAEMU): International Monetary Fund.
- Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control, 12*(2-3), 231-254. doi: 10.1016/0165-1889(88)90041-3
- Johansen, S., & Juselius, K. (1990). Maximum Likelihood Estimation and Inference on Cointegration--With Applications to the Demand for Money. *Oxford Bulletin of Economics and Statistics, 52*(2), 169-210.
- Kollmann, R. (1995). Consumption, real exchange rates and the structure of international asset markets. *Journal of International Money and Finance, 14*(2), 191-211.
- Kose, A. M., Prasad, E. S., & Terrones, M. E. (2009). Does financial globalization promote risk sharing? *Journal of Development Economics, 89*(2), 258-270.
- Kose, M. A., Otrok, C., & Whiteman, C. H. (2003). International Business Cycles: World, Region, and Country-Specific Factors. *The American Economic Review, 93*(4), 1216-1239.
- Lewis, K. K. (1995). What Can Explain the Apparent Lack of International Consumption Risk Sharing? *National Bureau of Economic Research Working Paper Series, No. 5203*(published as Journal of Political Economy, April 1996, vol.104, pp.267-297.).
- Lewis, K. K. (1996). Consumption, Stock Returns, and the Gains from International Risk-Sharing. *National Bureau of Economic Research Working Paper Series, No. 5410*(published as Lewis, Karen K. "Why Do Stocks And Consumption Imply Such Different Gains From International Risk Sharing?," *Journal of International Economics*, 2000, v52(1,Oct), 1-35.).
- Lewis, K. K. (1997). Are countries with official international restrictions liquidity constrained? *European Economic Review, 41*(6), 1079-1109. doi: Doi: 10.1016/s0014-2921(97)00056-1
- Lewis, K. K. (1999). Trying to Explain Home Bias in Equities and Consumption. *Journal of Economic Literature, 37*(2), 571-608.



- Moser, G., Pointner, W., & Scharler, J. (2004). International risk sharing in Europe: has anything changed? *The Economic Potential of a Larger Europe* (pp. 260-275). Northampton: Edward Elgar Publishing.
- Obstfeld, M. (1994). Are Industrial-Country Consumption Risks Globally Diversified? *National Bureau of Economic Research Working Paper Series, No. 4308*(published as forthcoming in Leonardo Leiderman and Assaf Razin, eds., *Capital Mobility* Cambridge, UK, Cambridge University Press, 1994).
- Obstfeld, M., & Rogoff, K. (2000). The Six Major Puzzles in International Macroeconomics: Is There a Common Cause? *NBER Macroeconomics Annual, 15*(ArticleType: primary\_article / Full publication date: 2000 / Copyright © 2000 The University of Chicago Press), 339-390.
- Sørensen, B. E., Wu, Y.-T., Yosha, O., & Zhu, Y. (2007). Home bias and international risk sharing: Twin puzzles separated at birth. *Journal of International Money and Finance, 26*(4), 587-605. doi: <http://dx.doi.org/10.1016/j.jimonfin.2007.03.005>
- Stockman, A. C., & Tesar, L. L. (1995). Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements. *National Bureau of Economic Research Working Paper Series, No. 3566*(published as *American Economic Review*, vol 85, no. 1, pp. 168-185, (March 1995)).
- Tesar, L. L. (1993). International risk-sharing and non-traded goods. *Journal of International Economics, 35*(1-2), 69-89.
- Tesar, L. L. (1995). Evaluating the gains from international risksharing. *Carnegie-Rochester Conference Series on Public Policy, 42*, 95-143.