The History of the Teaching of Arithmetic in the United States Prior to 1860

by Thornton Lynn Bouse

June, 1912

Submitted to the Department of Mathematics of the University of Kansas in partial fulfillment of the requirements for the Degree of Master of Arts

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The History of the teaching of Arithmetic in the United States, prior to the year I860, must be written principally from, the text-books used before that time, for, as Supt. Greenwood very aptly says; "In the Arithmetics of the past are mirrored the methods of instruction that prevailed at that time".(I) This study of texts, however, may be supplemented

(I) Greenwood p.

ocassionally by stray paragraphs found in the writings of some of our earlier authors.

The earliest~~s~~ texts used in what is now the United States were either imported directly from England, or close paterns of the English texts. The French and German texts, before reaching America usually passed through the hands of the English.

Little can be said concerning the prominence of Arithmetic before the year I750. At that time it had not been introduced in Halstead N.H., and as late as I800 there were country districts in which it was not taught.(2)

(2) Cajori p. 9.

Before the Revolution regular texts were ~~very~~ few.(3)

(3) Cajori p. II.

Cajori names the Hornbook as the first mathematical primer, because it contained the Roman ~~N~~numerals, and at that time it was considered gross ignorance not to know one's Hornbook.

George Fox's primer (4) was published in Philidalphia, I70I,

(4) Cajori p. II.

and the Koffer Konst in New York, I730.(5)

(5) Cajori p. I3

Hodder's Arithmetick, or that necessary art made most easy" published in Boston, 1719,(1) was the first purely Arithmetical book published in the United States. Cocker's Arithmetic is

(1) Cajori p. 13.

probably the second oldest arithmetic printed in America. Benjamin Franklin used it about 1722. This book, according to De Morgan (2) excluded all demonstration and reasoning and

(2) De Morgen, Augustus , Arithmetical Books

confined itself to commercial questions only.

In 1729, Prof. Isaac Greenwood, of Harvard, published the first Arithmetic, written by an American Author. This book contained 158 pages and covered the following topics: numeration, addition, subtraction, multiplication, division, reduction, vulgar fractions, decimal fractions, roots and powers, continued proportion, practice, and rules relating to trade and commerce. In the preface the author stated that his "design is to give a very concise Account of Such Rules, as are of the easiest Practice in all Parts of Vulgar and Decimal Fractions; and to illustrate each with such examples as may be sufficient to lead the Learner to the full use thereof in all other instances". (3)

(3) Greenwood p 804

He stated further that he thought it "improper to go into an elaborate Explanation of the Rules in the lower parts of Arithmetic, as most authors have done". A principal object was to enable the student to obtain a "comprehensive collection of all the best rules in the art of numbering". The words and phrases that the author wishes to emphasize are printed in italics. More than Half is italisized. (I)

(I) Greenwood p 805

From the preceding it appears that the work was entirely deductive. The rule is stated and applied with nothing showing its development.

In the secondary schools before the Revolution, arithmetic as taught, consisted largely in drills in the use of integral numbers. Few teachers had knowledge of fractions, or of the "rule of three". (2) Women were not supposed to know any-

(2) Cajori p II

thing about arithmetic and were not required to teach it. Texts not being in the hands of the pupils, the teacher dictated the "sums". There was no explanation of processes, or demonstrations of principals, and the work was considered complete~~d~~ when the problem was solved and the solution ~~was~~ copied. The Teacher was usually supplied with a "ciphering book" which contained exercises that were given to advanced pupils, who made their own ciphering books. A few teachers used printed arithmetics instead of ciphering books as guides. (3)

(3) Cajori p I6

Thomas Dilworth's "School-master's Assistant", according to Cajori (4) was the most popular arithmetic at the beginning

(4) Cajori p ~~I5~~ I4

of the Revolution. In this ~~book~~ work the order of the subjects is very different from what we find in the texts of today. Elementary rules, interest, fellowship, exchange, double rule of three, alligation, single and double position, geometrical progression, and permutations all precede interest and fractions follow interest. The rule of three is explained first with whole numbers, then with fractions and finally with decimals. In the end the author fails to generalize the rule. (I) The

(I) Cajori p I5

book used many perplexing terms, some of which follow: practice, conjoined proportion, alligation medial, alligation alternate, biquadratic roots, sursolids, square cubes, second and third sursolids, biquadrates squared, and square cubes squared. Rules are not always stated properly; for example, "Feet multiplied by feet give feet". (2) There is also a "Short Collection of

(2) Cajori p I6

Pleasant and Diverting Questions" among which may be mentioned the fox, goose, and peck of corn problem, the problem of the three jealous husbands, and the magic square of the first nine digits. (3)

(3) Cajori p I7

The author's style is shown by the following, which is found on page 44, of the London Edition of I784;

"Of the Single Rule of Three.

Q. How many parts are there in the Rule of Three?

A. Two; Single or Simple, and Double or Compound.

Q. By what is the Single Rule of Three known?

A. By three terms, which are always given in the question to find the fourth.

Q. Are any of the terms given to be reduced from one Denominator to another?

A. If any of the ~~terms~~ given terms be of several denominations, they must be reduced into the lowest Denomination mentioned.

Q. What do you observe concerning the first and third terms?

A. They must be of the same Name and Kind.

Q. What do you observe concerning the fourth term?

A. It must be of the same Name and Kind as the second.

Q. What do you observe of the three Terms taken together?

A. That the first two is a Supposition, the last a Demand.

Q. How is the third Term known?

A. It is known by these or the like words, What cost? How many, How much?

Q. How many Sorts of Proportion are there?

A. Two, Direct and Inverse. (I)

(I) Cajori p I5

Equally clear is the following: "A fraction is a broken number and signifies the part or parts of a whole number". (2)

(2) Cajori p I5

In I878, Nicholas Pike published his New and Complete System of Arithmetic. This was a very complete book forthe time. The first edition contained logarithms, and some trigonometry, some algebra, and some treatment of the conic sections.

It contained many rules, fourteen under simple multiplication, many of which were given without demonstration, and some proved proved algebraically.
Pike stated in the preface that he regarded the books then in use as generally deficient in the illustration and application of rules, and that that was the general coplaint of the School-masters.(I)

The following is from the preface of the first edition as given in the I797 edition. "I have given several methods of extracting the Cube Root, and am indebted to a learned friend, who declines having his name made public, for the investigation of two very concise Algebraic Theorems for the extraction of all Roots, and aparticular Theorem for the Sursolids.

Among the Miscelaneous Questions, I have given some of a philosophical nature, as well ~~as~~ with a view to inspire the pupil with a relish for philosophical studies, as to the usefulness of them in the common business of life".

The following illustrates the multiplicity of rules, all of which the student was required to memorize and apply (I779 edition). Nine kinds of currency were in use in commercial transactions and students were taught to express each in terms of the others, making 72 distinct rules. The student was required to commit a page of tables of aliquot parts of pounds,shillings,hundredweights,andtons,and also a table of per cents of the pound in shillings and pence. These tables contain more than a hundred relations,and their applications are in more than 34 cases,each with a rule. Many of the rules would tax the understanding of the pupil to grasp their meaning sufficiently to apply them,as,for instance, "When the price is shillings,pençe,and farthings,and not an even part of a pound,multiply the given quantity by the shillings in the price of one yard,etc. and take parts of parts from the quantity for the pence,etc. then add them together,and their sums will be their answer in shillings,etc. ". Pike's Arithmetic was long the standard in New England Schools.(I)

(I) Cajori p 46

"It gave tone to all arithmetical study of the district school period,and is responsible for that excessive devotion to arithmetic which has of late been the subject of such just complaint".(2)

(2) Martin p I64

Thus at the beginning of the I9 th century,arithmetic, as taught,had but little educational value. "Trade and the counting room" said Meriwether, "set the pace,and arithmetic was only a means of getting along in the world of bartering and dealing with your fellow men".

Facilities were very scanty. There being no blackboards or slates,pupils were obliged to use blank leaves in daybooks,backs of letters and birch bark.(I)

(I) Meriwether p I60

Chancey Lee was one of the first American Authors to see the defects of the old system,and to comment on the superficial mode of instruction in the common schools. In I797,he published the American Accomptant,which was an attempt to treat of and to simplify commercial arithmetic only. In a long introduction of 37 pages he stated that his object was to render rules more easy,and concise and better adapted to the instruction of young minds in the method and progress of common school education. He claimed to have om~~m~~itted rules,contained in former treatises, more calculated to amuse and puzzle than to profit the common school student. He said further: "By taking things for granted as already known and understood though needing at the same time as particular explanation as the more abtruse and complicated,the studeht,as he proceeds,is subjected to the arduous task of erecting a superstructure without a foundation. By his "teens" he goes to school,vulgarly speaking,raw,perhaps scarcely able to form an arithmetical figure. The master sets a sum in addition and tells how to carry one for every ten. There is no explanation why. When taught to commit rules to memory,he learns them like a parrot without any knowledge of their reasoning or application. He gropes along in this way from rule to rule till he ends his blind career with the rule of three. He is then deemed qualified to teach school himself the next winter".[I]

(I) Greenwood p 8II

Jos. T Buckingham at the age of I2,(about I790),received his first instruction in arithmetic,which he described thus: "He,the master,set me a sum in simple addition,five columns of figures and ~~six~~ six figures in each column. All the instruct~~XX~~ion he gave me was -- 'add the figures in the first column, carry one for every ten,and set the overplus down under the ~~co~~ column',I supposed supposed he meant by the first column,the left-hand column". An hour or so later the master added: "Add up the column on the right and set down the remainder".[2]

(2) Cajori p 52

Daboll's Schoolmaster's Assistant,published in I779,was one of the first adapted to our measure of length,weight,and currency. The general plan of the work was ~~the~~ to explain a subject,if necessary,and then give the rule. Federal money is introduced immediately after addition of whole numbers and anly a few pages precede~~s~~ square root. Decimal fractions follow compound numbers and precede vulgar fractions.[3]

(3) Greenwood p 8I3

In I80I,Daniel Adam*s' Scholar's Arithmetic appeared. this book contained four sections; (I),Fundamental rules;(2), rules essentially necessary for every person to fit and qualify himself for the transaction of business;(3),rules occasionally useful to men in particular employments of life;(4), miscellaneous questions. He defines multiplication,and illustrates it ba a problem,then follows the table,rule,and examples with methods of proof. (I)

(I) Greenwood p 8I5

Carleton's Compendium of Practical Arithmetic,published in I8I0,was compiled at the request of the associated instructors of youth in Boston. Demonstrations are omitted because "They swell the book to little advantage to young minds". There is an attempt to make the book interesting to little pupils and each problem is stated as a little poem or story.

Example:

"Nineteen and ten,good honest men,
With ~~10~~ sixteen more,and 40;
Spoke to a victualler to provide,
Meals for his little party;
He charged them ninety-five cents each,
Because they ate quite hearty;
Pray how much money was the bill
Paid by the little party?" (2)

(2) (Greenwood p 8I9

Thus up to the year I820 but little progress had been made in the teaching of Arithmetis. Pupils were taught to depend on rules and not to master principles. Mental ~~/~~ arithmetic received no emphasis,and fractions received very little treatment. Cancellation was then unknown. The system of numeration was the English in which the digits of the number are distributed in periods of six. The Rule of Three was taught merely as a rule. Arithmeticians were not accustomed to think proportion as an equality of ratios which is probably the reason four dots were ~~placed~~ used instead of the equality sign. Methods of "Single position" and"double position" were used,or the method of resolving questions by making one or two suppositions of false numbers,then correcting the resulting error. This was,in reality,a method of solving proportion. The word proof meant a test of correctness of a particular operation by reversing the process. Problems were,as a rule,poorly graded,and in some there is no connection between the opening statement and the conclusion, e.g., "When hens are 9 shillings a dozen,what will be the price of eggs at 2 cents for 3 eggs?"--- Willet's Scholar's Arithmetic.

No uniform text was found in the country school,besides not all pupils were supplied with texts.

To Warren Colburn,more than to any one else belongs the credit of the first great advancement in the teaching of arithmetic. In I82I,he published his First Lessons in Arithmetic. This is one of the epoch making books of the I9th century.

Colburn's idea was to begin with the concrete and known and proceed gradually,step by step, to more difficult subjects. Here the pupil is brought into the subject of addition by simple questions,such as: How many thumbs have you on your right hand? On your left? On both? etc.

This was a new idea in teaching . "It touched the Subject as with a wand of an enchantress,and it began to glow with interest and beauty. What was before was dull routine, now became animated with the spirit of logic." (I)

(I) ~~XXXX~~ Brooks p I06

In I826 Colburn published a sequel to his First Lessons, An Arithmetic upon the Inductive Method of Instruction.

Colburn's books at once began to gain ground. His ideas were adopted by different teachers,who began to write upon them. Old books were revised to embody the new methods, but in most casses only in ~~part~~ part. In fact Colburn's ideas have never been fully adopted and the later day arithmetics are combinations of the old and new methods. The better class of arithmetics,after Colburn's not only contained rules but gave demonstrations and sought to encourage the pupil to think.

Colburn's ideas belong to Pestalozzi and were first introduced into this country by F.J.N.Neff,who came to America in 1806. (2)

(2) Cajori p I06

About ~~XXXX~~ I825 or 30,the French notation began to replace the English. Cajori thinks that the oldest book to adopt the French notation was the I805 Edition of Dilworth'S School-master's Assistant, and that the latest to use the English was Gibson's I850 revised Edition of Abijah and Josiah Fowler's Youths Assistant. (I)

(I) Cajori p I08

Cancellation was first introduced by C.Tracy, and came to general use about I850. (2)

(2) Greenwood p

The arrangement of subjects now ,(about I825 or 30), began to assume a definite order. Fractions had been brough towards the front now precede Federal money and compound interest. Single and double position have almost entirely disappeared. Mental arithmetic now has a clearly defined place, but is rather loosely connected with written arithmetic. In fact they were often taught in such a manner that a pupil might grow proficient in one without knowing much about the other.

Teachers began to study how to make a subject attractive and how to aid the pupil to a clear understanding of ~~pr~~ princip~~l~~les. Benard's Arithmetic, I830, was probably the first to use pictures as aids. This book was "rendered entertaining to the pupil by a great variety of amusing problems". Some of these took the form of a continued story. (3)

(3) Johnson p 3I6

Emerson's North American Arithmetic, published in 1832, was a beginners book of of the modern type. (1) In 1834 it displaced

(1) Johnson p 316

Colburn's First Lessons and Sequel in the Boston School. (2)

(2) Greenwood p 833

In the preface the author said: "The practice of postponing arithmetic till the children arrive at the age of nine or ten years still prevails in many of our schools". (3)

(3) Johnson p 316

In 1854, Charles Davies wrote " The Intellectual Arithmetic". This was, to use his own words "an analysis of the science of numbers with special reference to mental training and development". (4) The work is based upon the principle that

(4) Quoted by Greenwood, p 837

every operation in arithmetic has reference to the unit 1.

Davies is the author of a series of mathematical texts. The influence of his writings was such that Greenwood regards it as the beginning of a revolution in ~~the~~ School book making. Greenwood adds: "Simplicity and extreme clearness became the leading ideas in the minds of authors, who studied how to be understood by children and young people".

Thus by the year 1860 arithmetic had become a definitely organized subject, with a clearly defined place in the school curriculum.

A study of the typical text of that time, reveals the fact, that the scope of topics covered, and their order of arrangement was verysimilar to the texts of the present time.

The general plan, if we may use the I856 Edition of Ray's Higher Arithmetic as a representative text of that period, was to introduce a subject with a few definitions, define the process to be used, ~~and~~ state the rule, give a practical ~~problem~~ application, and then a demonstration, which is to some extent a proof of the rule. --- This method is followed throughout the text.

Ray's Arithmetics have had a large circulation. They set the standard for about half a century.

A comparison of the treatment of the fundamental processes, by Pike, Ray, and Myers-Brooks.

Rules	Addition.	Subtraction.	Multiplication.	Division.
Pike	I	I	I6	5
Ray	I	I	7	6
Myers-Brooks	———	———	———	———
Pages				
Pike	20-22	22	23-34	34-4I
Ray	I9-26	22-26	26-35	35-47
Myers-Brooks	I5-26	26-35	35-50	50-76
Exercises				
Pike	I0	9	I05	38
Ray	I9	Both Add & 3I, Subt. II	65	83

Pike and Ray introduce a subject by definitions. Myers-Brooks leads up leads up to the definition by simple questions out of which the definitions naturally follow.

For example, Ray begins Addition with the definition, "Addition is the process of collecting two or more numbers into one sum". Myers-Brooks begins with the ~~q~~ question, "A bicyclist rides I9 miles Monday and IO mi. Tuesday; how far does he ride both days?" It is stated further that the answer to this question is the same as the answer to the question, "What single distance is just as long as the distance I9 mi. and IO mi. combined?" This combining numbers into a single number is called addition.

Following the definition Pike gives the rule and the method of proof. In addition and subtraction no illustrat-~~ing~~ ion of the application of the rules are given. In multiplication and division each rule is followed by the solution of an example, and in a few cases a complete explanation of the process.

Pike proceeds at once into long and difficult problems. His seventh example in Subtraction is:

"From IOO2OO3OO4OO5OO6OO7OO8OO9OO
Take 98O76O54O32OIIO23O45O67O89

Rem. "

Ray applies each rule to the solution of a problem. This solution is followed by a demonstration or an analysis. The following taken from page 29 clearly illustrates his method:

At the rate of 53 miles ~~per~~ an hour, how far will a railroad car run in four hours?

Solution. --- Here say, 4 times 3 (units) are I2 (units); write the 2 in units' place, and carry the I (ten); then, 4 times 5 are 20, and I carried makes 2I (tens), and the work is complete.

53	miles.
4	
2I2	miles.

Demonstration.---- The multiplier being written under the multiplicand for convenience, begins with units, so that if the product should contain tens, they may be carried to the tens; and so on for each successive order.

Since every figure of the multiplicand is multiplied, therefore, the <u>whole</u> multiplicand is multiplied.

Proof. ---- Separate the multiplier into any two parts; multiply by these separately. The sum of the products must be equal to the first product.

Myers-Brooks attempts to lead the pupil inductively to a grasp of the fundamental principles. The rule is not formal~~l~~-ly stated but is embodied in the solution of a practical problem, as may be seen by ~~the~~ a study of the following which is his introduction to written work in subtraction. ---- Myers-Brooks p 27.

I. A traveler has a journey of 437 mi. to make and he has already traveled I99 mi. of it; how much farther must he travel?

Solution.---- 437 means 4oo plus 30 plus 7, and ~~100~~ I99 means I00 plus 90 plus 9. We arrange the numbers conveniently, thus:

Minuend	437 mi.	equals	300 mi.	plus I20mi.	plus	I7mi.
Subtrahend	I99 mi.	equals	I00 mi.	plus 90mi.	plus	9mi.
Remainder	238 mi.	equals	200 mi.	plus 30mi.	plus	8mi.

Beginning on the right, 9 units cannot be taken from 7 units. We take one of the ~~three~~ 3 tens and add it to the 7 units, giving I7 units. Then I7 units minus 9 units equals 8 units. Write 8 in units column. Passing to tens column, we cannot take 9 tens from 2 tens remaining, so we take one of the ~~four~~ 4 hundreds and add it to 2 tens, making I2 tens. Then I2 tens minus 9 tens equals 3 tens. Write 3 in tens column. Finally, 3 hundreds minus I hundred equals 2 hundreds. Write the 2 in hundreds column. The remainder is 238 miles.

Pike gives no practical problems, Ray only a few, while Myers-Brooks gives more practical than abstract exercises. In neither Pike nor Ray are the exercises, as a rule, well graded. Both exclude oral work. Myers -Brooks begins each subject with simple oral questions and proceed gradually, step by step, to the more difficult exercises. Pike and Ray give the complete multiplication table at the beginning while Myers-Brooks builds it up gradually. All three give similar methods of proof, which, excepting the method of casting out nines,are tests of correctness by reversing the process.

Pike shows how to multiply feet by feet. Ray and Myers-Brooks holds that the multiplier must be an abstract number. The student of Pike or Ray must almost necessarily depend on rules,and hence on his memory,while the student of Myers-Brooks masters the fundamental principles and can make his rules as he needs them.

Myers-Brooks defines and uses the plus,minus,and equality signs. He introduces the use of letters to represent numbers ,and makes some use of the equation. This tends to make the transition from Arithmetic to Algebra easy.

The following chart is a comparison of the contents of seven Arithmetics which were prominent in their time, showing the arrangement of topics, and the space devoted to each topic.