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Field Project

# Staffing Optimization Tools for a 24 x 7 Telecommunications Control Center Environment

By

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## Executive Summary

This research explored optimization techniques to assist in personnel scheduling in a telecommunications environment. A survey of personnel was conducted to help determine basic employee priorities and two models were implemented in Microsoft Excel Solver to assist in providing data to support employee scheduling.

The first model is linear and, given the number of people required during each hour of the week, determines the optimal number of personnel that should be working each of the available shift types, with an objective of minimizing cost. This tool includes the option to include or exclude particular shifts depending on employee and management preference. This model is found to be a useful tool in learning about the cost implications of various shifts and is suitable for assisting in shift scheduling.

The second model designed is nonlinear and minimizes variance in the hourly utilization rate of personnel. Given a fixed number of people, the model seeks to determine the optimal number of personnel that should be working each of the available shift types, with an objective of balancing capacity to demand across each hour of the week. Microsoft Excel Solver is found to be insufficient in handling the number of constraints in this nonlinear model. Implementation of this methodology within a more powerful software package is suggested for future research.

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## Chapter 1 – Introduction

This author has worked in a variety of telecommunications positions, many of which included managing centralized groups of people who monitor, control or otherwise support networks. Due to the continuous-operating nature of telecommunications, it became clear that human resource capacity planning and scheduling were key skills that can directly impact the success of departments. This research stems from the need to better understand scheduling in a control center environment, to stay cost competitive.

As a matter of introduction, this section describes the environment that was researched, offers insight into existing practices for shift scheduling, the challenges with objectively measuring staff needs and explains the purpose for the current research.

Description of Environment

Within telecommunications companies, a common fixture is the network operations control center (NOCC). At this author's employer, the NOCC evolved significantly. Because the company provides multiple technologies to its customers, each technology was originally supported by its own business segment. Internal IT applications and networks, classic wireline (dialtone, phone operators, etc.) networks, IP or other packet-based data networks, optical fiber networks, wireless networks, and more. Due to technology evolution, consolidation within the marketplace and ever-decreasing profit margins, individual business segments have been forced to merge to some degree.

While there were multiple NOCCs with different locations, distinct software applications, disparate management teams and physical layouts, there is now predominantly one control center. Multiple technologies and networks supported from one large center. The center is a collection of pods or small groupings of PC-based workstations. Each worker has multiple monitors to help assist with managing the multiple software applications and simultaneous moving parts. Within the one center, there are different groups that focus on different networks, but all of the groups have similar furniture, software tools, management practices and compensation structure.

Existing Shift Scheduling Practices

One fundamental difference that can persist between control center groups and even between managers within a specific network segment is how personnel shift scheduling is handled. In many cases, differences in processes will drive inefficiencies or cost and may be perceived as negatives as they appear as inconsistencies to customers. Differences in shift planning practices persist because each manager has their own opinion about what is the most cost-efficient method and the differences are not apparent to internal and external customers.

Without thorough analysis of scheduling and staffing methods, crude guidelines develop amongst workgroups. For example, the table below was offered as a static rule of thumb for determining how many total staff are required if a person needed a certain amount of people on any given shift:

| **# of staff required per shift** | **# of staff required for 24x7 coverage** |
|---|---|
| 1 | 6 |
| 2 | 9 |
| 3 | 15 |
| • | |
| • | |
| • | |
| 26 | 117 |
| 27 | 123 |
| 28 | 126 |

Many state labor laws and much of the American social culture are built around a five day work week, Monday through Friday. In a continuous staffing environment, there is considerable debate about how to best schedule personnel so there is adequate weekend coverage. If personnel are scheduled as in the five continuous days per week example shown in Figure 1, the end result is excess resources in the middle of the week and a staffing gap on Friday and Monday.

| | Su | Mo | Tu | We | Th | Fr | Sa |
|---|---|---|---|---|---|---|---|
| Analyst 1 | X | X | X | X | X | | |
| Analyst 2 | X | X | X | X | X | | |
| Analyst 3 | X | X | X | X | X | | |
| Analyst 4 | | X | X | X | X | X | |
| Analyst 5 | | X | X | X | X | X | |
| Analyst 6 | | X | X | X | X | X | |
| Analyst 7 | | X | X | X | X | X | |
| Analyst 8 | | | X | X | X | X | X |
| Analyst 9 | | | X | X | X | X | X |
| Analyst 10 | | | X | X | X | X | X |
| **Total Staff:** | **3** | **7** | **10** | **10** | **10** | **7** | **3** |

Figure 1: Example of simplified schedule

Management will often resort to smoothing and filling gaps across the weekends with personnel working four 10-hour shifts from Friday to Monday, as the undesirability of working weekends is perceived to be mitigated by only having to commute and work four days a week.

Management decisions around shift scheduling are largely made with the intent to maintain employee satisfaction while maintaining adequate staffing. To ensure that management better understood what some of the control center staff preferences were, this author deemed it necessary to perform a survey to gather feedback from employees. Detail about the questions asked and the responses received are included in chapter 4.

Challenges with Objectively Measuring Staff Needs

One approach to staffing and scheduling seeks a quantitative method to look at various work drivers to determine how many total staff are needed within a workgroup and at any particular time. This avenue is problematic for multiple reasons.

Managers find ways to perform work with less staff as cost reductions are forced upon organizations. While various spreadsheets and models may indicate that a certain staffing level is required to effectively perform a function, one can observe that over the course of years, the job can be done with less staff. Due to necessity, when faced with market-driven budget cuts, telecommunications groups will manage the same number of network elements with less people. As staffing is reduced per network element managed, various process quality levels may change, but are difficult to quantify. Items such as the quality of ticket notes, the length of time that a questionable circuit is tested before calling it "restored" or the quality of handoff information between shifts or groups will vary without easy means to measure them. With this variation, the impact upon the end product can be minimal or at least difficult to determine in an objective fashion.

One example of this subjectivity in staffing levels is using the number of trouble tickets (instances of problems) as a driver for staff levels. The use of trouble tickets as work tasks is common for IT help desks and network control centers. If the number of network elements being managed by a control center increase, logically the number of trouble tickets will increase at some rate that can be correlated to the quantity of elements. However, control center employees may change their criteria for when tickets are opened, or criteria for when the tickets are allowed to be closed or "parked". This can happen informally- it can start with one or two personnel on a less visible shift and spread to other employees as they socialize about work volumes and methods for managing time. It can be started during a disaster event when the volume is unmanageable and the differences in behavior then inadvertently carry over into subsequent everyday practices.

Another factor that detracts from the ability to directly measure the amount of needed staff is that assumptions about the amount of time to perform functions such as open or close a ticket or restore an outage are ever-changing as people develop shortcuts and IT systems are improved.

Purpose of Current Research

Given the difficulties described above with directly measuring the required number of people on any given shift, this research focuses instead on methods for knowing how to best schedule resources when either the number needed at a particular time is given or if the total staff available to apply to the problem is given.

When employee preferences are known, there is a desire to understand the cost of accommodating employee preferences such as consecutive days off and 10-hour work shifts. For example, when three 10-hour shifts are used in a day, the result can include extra personnel for two or more hours, two to three times a day. An objective is to determine whether or not this overlap in employee coverage is financially significant.

This research explores methods of optimizing employee scheduling to reduce inefficiencies and validate if reducing those inefficiencies has significant financial benefit as compared to accommodating employee preferences. This author researches shift planning methods and seeks to introduce empirical data and optimization tools to learn more about the most efficient ways to schedule employee shifts.

This research on telecommunications scheduling is unique as it: (1) assumes that the correct/appropriate ratio of servers to demand is unknown or at least highly dynamic and, therefore, efforts are largely focused on how to optimally staff given that a certain amount of workers are available, (2) seeks to understand whether or not there is a significant financial impact that can be realized by using more formal scheduling methods as opposed to traditional methods, and (3) endeavors to explore which aspects of the scheduling problem can and cannot be adequately optimized using Excel Solver.

## Chapter 2 – Literature Review

In performing a review of prior research, this author focused on three major categories:

- Overall methods and guidance for modeling a continuous operation.
- Modeling of environments similar to telecommunications NOCCs, specifically healthcare.
- The prospects for using a spreadsheet based tool for modeling.

The findings below helped shape the course of the current research.

Modeling Continuous Operations Environments

In 1993, Brusco and Jacobs performed a study of personnel scheduling in continuously operating organizations. Specifically, an environment where the staff are employed 24 x 7 is referred to as a continuous tour-scheduling problem and involves personnel shifts overlapping 24 hour periods. They point to continuous operating environments often employing full-time and part-time employees and assert that "continuous models are extremely difficult to solve optimally" and are "typically impervious to optimal solution approaches." (Brusco and Jacobs, 1993).

However, the Brusco and Jacobs study assumes that you have both full-time and part-time staff, which further increases complexity. They note that one method is to reduce the number of variables by assuming there is no overlap between 24 hour periods and refer to the resulting problem as a "discontinuous tour-scheduling" problem. (Brusco and Jacobs, 1993).

Their study found that when you model a continuous operating environment as a discontinuous operating environment, it is often not a good fit and "the potential exists for significant excess schedule costs." (Brusco and Jacobs, 1993). This is significant and directly relevant to this current research: a continuous telecommunications environment needs to be modeled as such. Brusco and Jacobs used a heuristic approach and found that it produced "superior schedules at far less computational effort" than optimization techniques. (Brusco and Jacobs, 1993). However, because the Brusco and Jacobs study was performed in 1993, the computational methods are considerably different than those used today. For their linear programming, they used SAS/OR run on an HP9000-835s minicomputer, with a maximum of 1000 CPU seconds (approximately 17 minutes). Their heuristic approach used an application coded in Fortran and run on the same minicomputer platform. Their platform used in their study was capable of approximately 3 Million Instructions Per Second, whereas the common personal computer today is capable of significantly more computational power.

Gartner, Musliu and Slany developed a method of creating rotational schedules using a semi-automatic approach which uses some degree of user response/input in conjunction with the software routines. They compare their approach to a benchmark which is fully automated and seeks the optimal solution. Their finding is that the method that uses some human interaction is faster than going straight for an automatic, optimal solution. Further, they point to the advantages of "interaction with human decision makers" as a way to account for "soft constraints, which are otherwise more difficult to assess and model." Their solution is now contained in a commercial package from Ximes, with a non-trivial cost. (Gartner et al., 2001).

Healthcare Modeling

In a review of literature, it becomes clear that nurse-related scheduling is one of the most closely studied, using a variety of methods. Since the early 1970's, there have been numerous studies surrounding scheduling nurses in a hospital environment. Hospitals, in some respects, are analogous to a telecommunications control center environment due to their continuous 24 hours a day, seven days a week staffing and the tendency for all resources to be employed in the same physical building or cluster of buildings.

Nursing-related studies largely focus around how many nurses are needed at any time to satisfy nurse to patient ratios and assume that nurse schedules rotate. Past studies point to more complex problems that cannot be easily solved through optimization and therefore alternate methods are created, however evidence also exists that optimization is a realistic tool for this environment.

In 2006, Wright, Bretthauer and Cote performed an extensive study on the nurse scheduling challenge. In their model, shifts were either eight hours or 12 hours. They designed the problem with two objectives and as a nonlinear set of relations. The first objective was to minimize nurse wages and second was to minimize undesirable shifts. When all constraints are in place, the problem is described as "difficult to solve." To tackle the entire problem, they resort to a heuristic approach. They are also able to use optimization to solve a portion of the problem by removing one constraint and making the problem linear. (Wright et al., 2006).

A 2007 study of nurse scheduling was performed by Parr and Thompson. They point to the difficulty of satisfying all the constraints in the nursing problem due to multiple objectives, however they proceed to examine two methods of linear optimization: stepwise allocation of weights (SAWing) and the noising method. Noising consists of "varying the cost function during a search by adding random error (noise) to the data." They found that using their datasets and methods, particularly the noising method, "produced better solutions than those produced manually," pointing to the value of optimization techniques for scheduling problems. (Parr and Thompson, 2007).

Spreadsheet-based Tools for Optimization

Another area of interest in performing a review of prior studies was related to using Microsoft Excel and Solver to approach the current research.

In 2003, Kuo, Schroeder, Mahaffey and Bollinger performed a study that optimized the scheduling of hospital operating rooms using constraint-based optimization methods, specifically using Excel Solver. The authors use methods and tools that "commonly used in the field of management science and are taught to first-year students in MBA courses." The objective function they used was financial profit. Their analysis illustrated that Solver could yield real business solutions to significantly increase profit through optimized scheduling, however their variables and constraints were limited and the problem was strictly linear. (Kuo et al, 2003).

A 2008 study by McCullough and Heiser point to ongoing deficiencies related to statistical functions within Excel 2007. Among the weaknesses, they point to Solver difficulties with non-linear optimization. In their tests, they compare Solver solutions to known solutions and find that the tool is accurate for "16 of the 27 nonlinear problems." (McCullough and Heiser, 2008). Their research clearly pointed at the need for caution with Excel based non-linear models.

Herve Thiriez examined the use of Microsoft Excel as a tool for teaching Operations Research concepts. He points to many advantages of using this ubiquitous tool, while at the same time he warns the reader about Excel's solver accuracy on non-linear problems. Solver uses a "gradient method and therefore finds local optima" for non-linear problems and for linear problems, results can sometimes be inaccurate. Thiriez notes however that "such problems have also been observed with some specialized LP packages", pointing out that problems are not unique to Excel. (Thiriez, 2001).

In 1994, David Ashley developed a librarian staffing model using a spreadsheet based solver- in this case it was Lotus 123 with the add-in "What's Best!" The solution was scalable to a larger number of staff, with the only limitation being the maximum number of variables supported by the software package. While this was not a 24 x 7 continuous operating environment, it serves as a clear example of linear programming to create schedules, all while using common business tools. (Ashley, 1994). What's Best! is also available for Microsoft Excel, but at significant cost.

# Chapter 3 – Procedure and Methodology

This chapter outlines the process used in the research, including the six fundamental steps of modeling: identifying the purpose, collecting information, formulating the model, validating the model, exercising the model, and reporting results.

## Identifying the Purpose

The purpose of this research and modeling was identified by the author through the course of managing a team of technicians in a NOCC environment. It was observed that a variety of shift scheduling theories and practices were used by peer managers, however there was no definitive answer about the best approach and there were not any quantitative methods to help make decisions about staff scheduling. Therefore, the need for quantitative methods and data to support scheduling decisions drove this research.

## Collecting Information

A variety of information was then collected. This author gathered information from peer groups about what shifts were used for their operation and how well the shifts were working. Information was gathered about why managers chose particular shifts and how they determined the right number of people at any given time. This author also performed a survey to collect hard data from employees concerning their shift preferences.

A literature review was performed to learn what research was performed previously around quantitative tools for scheduling in telecommunications or other similar environments.

An interview with a financial analyst was performed to learn how headcount budgets were being determined and how cost drivers were being tracked. Financial cost information was gathered, to include items such as shift pay differentials and fully loaded technician costs.

Formulating the Model

From information gathered about current scheduling practices, it became clear that finding objective and reliable drivers for NOCC staffing models is particularly challenging and dynamic. Instead of using drivers to calculate the desired staffing levels for a control center, this author arrived at two approaches for modeling that would offer insights for how to allocate staff amongst different shifts:

1. An optimization problem with low total labor cost being the objective. This allows the user to specify the number of people needed on each of the various hours, then observe how cost varies as different shifts are used. This helps to quantify the cost of accommodating employee preferences.
2. An optimization model seeking to minimize the variance (nonlinear objective) of human resource utilization between the 168 hours of a week. This tool allows the user to input work drivers and the total staff available, with the output being the optimal allocation of particular shifts that would best serve the work demand that is offered.

For both approaches, detailed algebraic formulations were created to design the constraints and parameters involved. These algebraic formulations were then implemented as Microsoft Excel Solver models. Excel Solver was selected due to its ubiquitous availability and low cost.

Validating the Model

The validation process included two aspects: checking formulas and gauging model outputs against reality.

This author manually verified that totals were being calculated correctly and that formulas were correct. As problems appeared, debugging was performed to correct the models.

Model outputs were examined and compared to the constraints and were also considered against actual industry experience. The model rendered realistic solutions.

Exercising the Model

The two models were exercised through repeated runs using Excel Solver. The model with the low labor cost objective was the predominant focus with repeated runs to observe the effects of turning shifts on and off and how cost was affected by introducing different staffing needs. Run durations were also observed and recorded.

Reporting Results

Chapter 4 includes a full report of results from the modeling research. Results discussed include the following sections: findings from information gathering on employee preferences, an examination of each of the two models, and a discussion on the suitability of the hardware and software platforms.

For each of the models, the results include detail about the user interface, algebraic formulations and what was learned from modeling iterations.

# Chapter 4 – Results

This research produced results in four categories and each is discussed in this chapter. First, a section on employee preferences for shift scheduling includes survey results and a brief analysis of the responses. Second, there is a significant discussion of a model that performs linear optimization while minimizing total labor cost and how that was implemented in Excel Solver. Included third is discussion of a model for nonlinear optimization while minimizing variance of resource utilization. Finally, this chapter briefly addresses the suitability of the software application and hardware platform.

## Employee Preferences for Shift Scheduling

It was important to answer some questions about employee preferences to help shape the shift options in the optimization models. A brief survey was conducted for NOCC employees. The survey tool used was an internal company survey engine that allows for anonymous responses. Figure 2 shows the exact questions that the employees saw and the possible responses that the employees could select.

| Survey question | Possible answers | Qty of responses |
|---|---|---|
| What shift do you work? | | |
| | Midshift | 13 |
| | Dayshift | 11 |
| | Nightshift | 4 |
| Which one would you prefer? | | |
| | Work five 8-hour shifts from Thurs to Mon | 10 |
| | Work four 10-hour shifts from Fri to Mon | 18 |
| Which days off would you prefer? | | |
| | Tues and Wed consecutive days | 23 |
| | Tues and Saturday off | 5 |
| Which days off would you prefer? | | |
| | Tues and Sundays off | 6 |
| | Tues and Wed consecutive days | 22 |
| Which one is more important to you? | | |
| | To have Sundays off | 19 |
| | To have Saturdays off | 6 |
| | To have Fridays off | 3 |

Figure 2: Shift preference survey questions and responses

Out of 28 responses received, some preferences emerge:

- When the choice is between consecutive days off in the middle of the week and non-consecutive days off including one weekend day, approximately 80% prefer to have consecutive days off.
- When having to work through the weekend, 64% prefer four days of 10-hour shifts as opposed to five days of eight-hour shifts.
- Approximately 70% of respondents want Sundays off as opposed to Saturdays or Fridays.

Some of the survey results reinforce the experienced-based knowledge of the control center managers, some are not as evident. Clearly consecutive days off are important, even if they occur in the middle of a week. The preference for working four days of 10-hours shifts across the weekend instead of five days of eight-hour shifts across the weekend is clearly the majority but was less of a majority than this author would have previously believed. The strong preference towards having Sundays versus Saturdays off was also somewhat unexpected and offers insight to help when assigning shifts.

## Approach #1 Linear Optimization Minimizing Total Labor Cost

The first approach researched starts with the assumption that the minimum number of people required at any given time is known, but determines how many total employees are required and what shifts they should work. Typically, a supervisor will have recent experience that tells them the minimum number of staff members that are required on a given shift. Here is an example of what a supervisor might specify:

Group minimum staffing at any given time, M-F day shift = 11:

| Technology A | Technology B | Technology | Technology D |
|---|---|---|---|
| 2 | 1.5 (need total 3 between technology B and C) | 1.5 (need total 3 between technology B and C) | 6 |

This approach is performed as an optimization problem using the scenario set up in the Excel Solver tool, with low total labor cost being the objective. Total labor is based on a calculation that considers the loaded hourly cost per hour per employee as well as the additional shift differentials which are traditionally present as an incentive for employees to work non-traditional hours.

The variables used are defined below:

hrlylabor ≡ Loaded labor cost per hour for NOCC technicians

swingdiff ≡ Hourly pay differential for swing shift = 10%

midsdiff ≡ Hourly pay differential for midshift (graveyard) = 15%

$hc_h$ ≡ Total headcount working during hour h

$hcmin_h$ ≡ The minimum number of people required for staffing at hour h

days1, days2, days3, etc. ≡ Number of personnel working that particular day shift, with shifts defined in table below.

swings1, swings2, swings3, etc. ≡ Number of personnel working that particular swing shift, with shifts defined in table below.

mids1, mids2, mids3, etc. ≡ Number of personnel working that particular mid shift, with shifts defined in the table below

A large number of shift options were considered and are defined in Figure 3 below:

| | Weekdays | Midshift Hours/Shiftname | Dayshift Hours/Shiftname | Swingshift Hours/Shiftname |
|---|---|---|---|---|
| 5 X 8's | Mon – Fri | 11pm to 7am-Mids1 | 7am to 3pm-Days1 | 3pm to 11pm-Swings1 |
| 5 X 8's | Tues – Sat | Mids2 | Days2 | Swings2 |
| 5 X 8's | Wed – Sun | Mids3 | Days3 | Swings3 |
| 5 X 8's | Th – Mon | Mids4 | Days4 | Swings4 |
| 5 X 8's | Fri – Tues | Mids5 | Days5 | Swings5 |
| 5 X 8's | Sat – Wed | Mids6 | Days6 | Swings6 |
| 5 X 8's | Sun – Th | Mids7 | Days7 | Swings7 |
| | | | | |
| 4 X 10's | Mon – Th | 11pm to 9am-Mids8 | 7am to 5pm-Days8 | 3pm to 1am-Swings8 |
| 4 X 10's | Tues – Fri | Mids9 | Days9 | Swings9 |
| 4 X 10's | Wed – Sat | Mids10 | Days10 | Swings10 |
| 4 X 10's | Th – Sun | Mids11 | Days11 | Swings11 |
| 4 X 10's | Fri – Mon | Mids12 | Days12 | Swings12 |
| 4 X 10's | Sat – Tues | Mids13 | Days13 | Swings13 |
| 4 X 10's | Sun – Wed | Mids14 | Days14 | Swings14 |
| | | | | |
| 5 X 8's non-consecutive | Tues & Sat off | 11pm to 7am-Mids15 | 7am to 3pm-Days15 | 3pm to 11pm-Swings15 |
| 5 X 8's non-consecutive | Wed & Sat off | Mids16 | Days16 | Swings16 |
| 5 X 8's non-consecutive | Thurs & Sat off | Mids17 | Days17 | Swings17 |
| 5 X 8's non-consecutive | Tues & Sun off | Mids18 | Days18 | Swings18 |
| 5 X 8's non-consecutive | Wed & Sun off | Mids19 | Days19 | Swings19 |
| 5 X 8's non-consecutive | Thurs & Sun off | Mids20 | Days20 | Swings20 |
| | | | | |
| 3x13 1/3's | | 6PM to 7:20 AM-Mids21 Fri morn, Sat morn, Sun morn | 6AM to 7:20 PM-Days21 Thurs, Fri, Sat | |

Figure 3: Shift options used in linear optimization model

The objective function used was to minimize total cost:

dayscost + swingscost + midscost
= (HCh values for days) x hrlylabor
+ (HCh values for swings) x hrlylabor x (swingdiff + 1.00)
+ (HCh values for mids) x hrlylabor x (midsdiff+1.00)

Constraints include the following:

swingdiff, midsdiff, hrlylabor $\geq 0$ constants

$hc_h$, $hcmin_h \geq 0 \quad \forall_h$

mids1, mids2 … mids20, mids21 $\geq 0$

days1, days2 … days20, days21 $\geq 0$

swings1, swings2 … swings19, swings20 $\geq 0$

$hc_h \geq hcmin_h \quad \forall h$ (must have at least minimum personnel each hour)

h has a range of 0 to 167 (168 hours in one week)

The full algebraic formulation is contained in Appendix A, including a table which shows which shifts span which hours of the week.

Implementation of Approach #1 in Excel and Solver

The actual implementation of the optimization tool is shown below. Parameters of loaded labor rate, swing shift pay differential and midshift pay differential can be modified by the user and are shown in gray in Figure 4. Throughout this research, values of 10% differential for swing shift and 15% for midshift were used. A loaded labor rate of $55.00 is intended to reflect the technician hourly pay, benefits, information technology cost allocations, and other incidental costs of employment.

The tool displays the calculations for the labor costs per shift and seeks to minimize the total labor cost (the objective function) as shown in the oval.

For each hour of the week, the user inputs values for the minimum required staff. This is outlined in red in Figure 4. As the tool calculates the optimal shift schedules, the tool auto sums and displays how many people will actually be working during each hour to compare to the minimum required staffing, as shown in the "Computed Staff HC" column.

| | |
|---|---|
| Loaded labor per hour: | $55.00 |
| Swing shift differential: | 0.10 |
| Midshift differential: | 0.15 |

| | |
|---|---|
| Midshift Labor Cost: | $5,313.00 |
| Dayshift Labor Cost: | $17,160.00 |
| Swingshift Labor Cost: | $7,260.00 |
| **Minimized Total Labor Cost:** | **$29,733.00** |

| | Sunday | | | Monday | | | Tuesday | | |
|---|---|---|---|---|---|---|---|---|---|
| **Hour of day, starting at:** | Hour of Week | Min Staff | Computed Staff $HC_h$ | Hour of Week | Min Staff | Computed Staff $HC_h$ | Hour of Week | Min Staff | Computed Staff $HC_h$ |
| 12 AM Midnight | 0 | 2 | 2.00 | 24 | 2 | 2.00 | 48 | 2 | 2.00 |
| 1:00 AM | 1 | 2 | 2.00 | 25 | 2 | 2.00 | 49 | 2 | 2.00 |
| 2:00 AM | 2 | 2 | 2.00 | 26 | 2 | 2.00 | 50 | 2 | 2.00 |
| 3:00 AM | 3 | 2 | 2.00 | 27 | 2 | 2.00 | 51 | 2 | 2.00 |
| 4:00 AM | 4 | 2 | 2.00 | 28 | 2 | 2.00 | 52 | 2 | 2.00 |
| 5:00 AM | 5 | 2 | 2.00 | 29 | 2 | 2.00 | 53 | 2 | 2.00 |

Figure 4: Extract of input and output fields of cost minimization model

The tool also includes the option to exclude or include a particular shift. This is shown in gray/blue in Figure 5. The extract shown only displays midshift, but the same fields also exist for the dayshifts and swingshifts. If a shift is particularly unpopular or if the user wants to see the cost impact of including or excluding a set of shift options, one or more shifts can be excluded from the optimization to measure the resulting total cost in dollars and total headcount. For example, if one considers the employee preference survey discussed earlier in this chapter, it indicates a strong preference for consecutive days off, therefore shifts that include non-consecutive days off might be excluded by the user. If a shift should be excluded as a possibility, the value can be manually set to "0."

In the example in Figure 5, in gray/blue, all of the midshift shift options have a value of "1" and therefore are considered possibilities for the optimization.

| | Midshift | | | |
|---|---|---|---|---|
| **Shift Description** | **Midshift Hours/Shiftname** | **Use shift in calculation?** | **Solver Parameter** | **Optimal # of ppl working this shift** |
| Mon – Fri 5x8's | 11pm to 7am- Mids1 | 1 | 0.0000 | 0.0000 |
| Tues – Sat 5x8's | Mids2 | 1 | 0.0000 | 0.0000 |
| Wed – Sun 5x8's | Mids3 | 1 | 1.0000 | 1.0000 |
| Th – Mon 5x8's | Mids4 | 1 | 0.0000 | 0.0000 |
| Fri – Tues 5x8's | Mids5 | 1 | 1.0000 | 1.0000 |
| Sat – Wed 5x8's | Mids6 | 1 | 0.0000 | 0.0000 |
| Sun – Th 5x8's | Mids7 | 1 | 0.0000 | 0.0000 |
| Mon – Th 4x10's | 11pm to 9am- Mids8 | 1 | 1.0000 | 1.0000 |
| Tues – Fri 4x10's | Mids9 | 1 | 0.0000 | 0.0000 |
| Wed – Sat 4x10's | Mids10 | 1 | 0.0000 | 0.0000 |
| Th – Sun 4x10's | Mids11 | 1 | 0.0000 | 0.0000 |
| Fri – Mon 4x10's | Mids12 | 1 | 0.0000 | 0.0000 |
| Sat – Tues 4x10's | Mids13 | 1 | 0.0000 | 0.0000 |
| Sun – Wed 4x10's | Mids14 | 1 | 0.0000 | 0.0000 |
| Tues & Sat off 5x8's non-consec | 11pm to 7am- Mids15 | 1 | 0.0000 | 0.0000 |
| Wed & Sat off 5x8's non-consec | Mids16 | 1 | 0.0000 | 0.0000 |
| Thurs & Sat off 5x8's non-consec | Mids17 | 1 | 0.0000 | 0.0000 |
| Tues & Sun off 5x8's non-consec | Mids18 | 1 | 0.0000 | 0.0000 |
| Wed & Sun off 5x8's non-consec | Mids19 | 1 | 0.0000 | 0.0000 |
| Thurs & Sun off 5x8's non-consec | Mids20 | 1 | 0.0000 | 0.0000 |
| 3x13 1/3's | 6PM to 7:20 AM Fri, Sat & Sun morn- Mids21 | 1 | 0.0000 | 0.0000 |
| | | | | 3.0000 |

Figure 5: Fields used in shift toggling and displaying shift counts

Exercising the Model

After all input variables have been set by the user, the user executes the model using Solver. Figures 6 is a screenshot of the Solver Parameters dialog box where the user sets the objective function, defines the adjustable parameters and defines the constraints that match the algebraic formulation. Figure 7 is a screenshot of a secondary dialog box, the Solver Options, where the user specifies non-negative (part of the constraints in the algebraic formula) and that the model is linear.

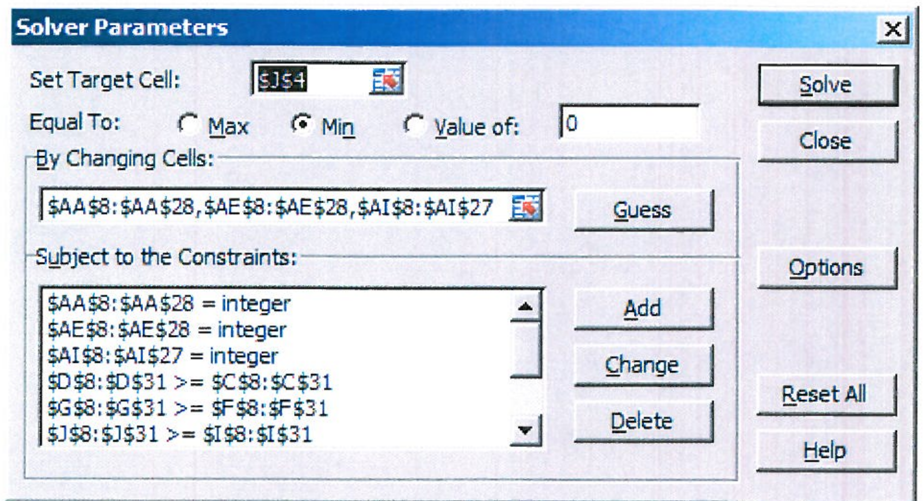


Figure 6: Solver parameters dialog box

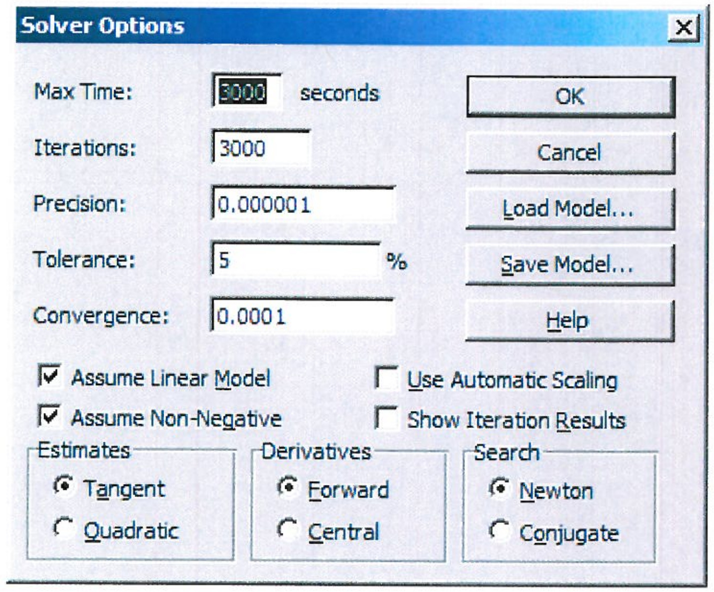


Figure 7: Solver options dialog box

Quantifying the Cost of Accommodating Employee Preferences

The Figure 5 also shows "Optimal # of ppl working this shift" fields which are the results of the optimization and are the primary outputs of the tool. These fields contain the data that a manager would utilize in helping to form an employee schedule.
In this particular example, the tool output indicates that, for the midshift, the optimal solution to minimize cost and yet still have the required minimum headcount for each hour would be to use:

- One person working five shifts of eight hours each on Wednesday through Sunday.
- One person working five shifts of eight hours each on Friday through Tuesday.
- One person working four shifts of 10 hours each on Monday through Thursday.

This example includes two unpopular shifts that force the technician to have days off mid week instead of on the weekend. It is then useful to do comparisons in order to understand the cost of accommodating employee preference in the interest of helping NOCC management in the decision-making process.

In this example, all shifts are considered "fair game" and the total labor cost for all day, mid and swing shifts is $29,733 for 13 headcount as calculated by the optimization tool. If non-consecutive day off shifts are excluded from consideration, there is no change in cost, it remains at $29,733 for 13 headcount. If all 10-hour shifts and non-consecutive day off shifts are excluded, 13 headcount are still required but the total cost is slightly higher at $29,788 due to differences in shift pay differential. If the user excludes the most unpopular shifts: non-consecutive days off and shifts that include working both Saturday and Sunday, then 16 headcount are required at a cost of $36,905.

This is significant as it starts to quantify the cost of accommodating employee preferences. There is no cost to excluding non-consecutive day off shifts; that employee preference is "free" in this particular scenario. However, the cost difference between "all shifts are fair game" and excluding shifts that include working both Saturday and Sunday is three headcount, and $36,905 - $29,733 = $7172 per week. It is then expedient for NOCC management to determine if $7172 per week is money well spent to boost morale (and possibly productivity) and to decrease employee turnover costs. Quantifying morale impacts and turnover costs are not within the scope of this research and would vary by workgroup, local culture and current economic conditions.

Each of the calculations discussed above required less than one minute for Solver to obtain the solution.

The example used here assumed that a minimum staffing of four people was needed at all times Monday through Friday from 7AM to 7PM and a minimum of two people was needed at all other hours of the week. This is but one scenario and the cost of accommodating employee shift preference would vary greatly depending on how the minimum staffing per hour of the week is specified. The tool can be readily modified to match the actual resource needs of the NOCC environment.

Validating Particular Shift Options

Within the author's workplace, there were two shift types that caused debate as to the cost efficiency of the shifts. One is the practice of utilizing 10-hour shifts. Simple consideration would cause concern about inefficiency stemming from employee redundancies, as compared to eight hour shifts, because three shifts of eight hours each have no overlap during a given day yet three shifts of 10 hours each have significant overlap. In the example previously discussed, if 10-hour shifts are eliminated as an option, the labor cost actually rises by $55 a week, pointing to the validity of 10 hour shifts.

The other practice that was debated is the use of a shift that uses three shifts of 13.33 hours each to cover weekend work. If one technician's worth of continual coverage capacity is needed on the weekend, one person works the daytime equivalent of this shift and one person works the nighttime equivalent of this shift and they only work the weekend. Only two shifts provide coverage as opposed to three shifts and while the worker has to work the whole weekend, their entire "workweek" is completed in approximately three days. Using the optimization tool, if one excludes all of the shifts that work weekend days except the 13.33 hour special shift, then 14 total headcount are required at a total labor cost of $32,208. While this represents a $2475 increase per week over the "all shifts are fair game" option, it offers an opportunity to accommodate employee preferences for not working weekends (except for four employees on the special shift) at what may be considered a reasonable cost.

**Approach #2- Nonlinear Optimization Minimizing Variance of Resource Utilization**

The second approach that was researched assumes that the total number of staff is known and constant and that full utilization of the personnel is desired. Using hourly incoming alarm volume as an indicator of relative work volume between the hours of the day, this model seeks to even out the utilization of personnel across the week. Hourly alarm data is an objective indicator of network faults and it is easily derived from software queries of NOCC operational support systems. This approach is also set up as an optimization model within the Excel Solver tool and it seeks to minimize the variance (nonlinear objective) of human resource utilization between the 168 hours of a week.

The variables for this model are defined below:

maxhd ≡ Maximum headcount available

$avol_h$ ≡ Alarm volume at hour h of the week
(h = 0, 1, 2, …165, 166, 167)

$hc_h$ ≡ Total headcount working during hour h

$util_h$ ≡ Alarms per person at hour h of the week, utilization
= $avol_h / hc_h$ for (h = 0, 1, 2, …165, 166, 167)

meanutil ≡ Mean utilization ≡ $\overline{util} \equiv \frac{1}{168} \sum_{h=0}^{167} util_h$

$mincover_h$ ≡ The minimum number of people required to at hour h

As compared to the model in Approach #1, a reduced subset of shift options was used and these are defined in Figure 8. Shift options were decreased to make the optimization problem more computationally feasible.

| | Weekdays | Midshift Hours/Shiftname | Dayshift Hours/Shiftname | Swingshift Hours/Shiftname |
|---|---|---|---|---|
| 5 X 8's | Mon – Fri | 11pm to 7am- Mids1 | 7am to 3pm- Days1 | 3pm to 11pm- Swings1 |
| 5 X 8's | Tues – Sat | Mids2 | Days2 | Swings2 |
| 5 X 8's | Wed – Sun | Mids3 | Days3 | Swings3 |
| 5 X 8's | Th – Mon | Mids4 | Days4 | Swings4 |
| 5 X 8's | Fri – Tues | Mids5 | Days5 | Swings5 |
| 5 X 8's | Sat – Wed | Mids6 | Days6 | Swings6 |
| 5 X 8's | Sun – Th | Mids7 | Days7 | Swings7 |
| 4 X 10's | Mon – Th | 11pm to 9am- Mids8 | 7am to 5pm- Days8 | 3pm to 1am- Swings8 |
| 4 X 10's | Tues – Fri | Mids9 | Days9 | Swings9 |
| 4 X 10's | Wed – Sat | Mids10 | Days10 | Swings10 |
| 4 X 10's | Th – Sun | Mids11 | Days11 | Swings11 |
| 4 X 10's | Fri – Mon | Mids12 | Days12 | Swings12 |
| 4 X 10's | Sat – Tues | Mids13 | Days13 | Swings13 |
| 4 X 10's | Sun – Wed | Mids14 | Days14 | Swings14 |

Figure 8: Shift options used for nonlinear optimization model

The objective function is to minimize variance in headcount utilization from hour to hour, in essence maximizing efficiency by reducing wasted resources

$$\text{Variance} = \sigma^2 = \frac{1}{168} \cdot \sum_{h=0}^{167} \left(\text{util}_h - \text{meanutil}\right)^2$$

The constraints include the following:

maxhd, mincover $\geq 0$

$hc_h$, $avol_h \geq 0 \quad \forall_h$

mids1, mids2 … mids13, mids14 $\geq 0$

days1, days2 … days13, days14 $\geq 0$

swings1, swings2 … swings13, swings14 $\geq 0$

$\sum_{i=1}^{14} \left( \text{mids}(i) + \text{days}(i) + \text{swings}(i) \right) = \text{maxhd}$ (all headcount are used)

$hc_h \geq \text{mincover} \quad \forall_h$ (must have at least minimum personnel each hour)

The full algebraic formulation, including a table of what shifts span which hours of the week, is contained in Appendix B.

Implementation of Approach #2 in Excel and Solver

This author created the optimization model in Excel with all constraints as designed; however, Solver was insufficient to solve the optimization problem. Solver is not able to handle all of the constraints in the algebraic formula. Specifically, Solver returned errors as a result of the constraints which ensure that the computed headcount is greater than or equal to the minimum coverage:

$hc_h \geq \mathbf{mincover} \quad \forall_h$

The error encountered is shown in Figure 9.

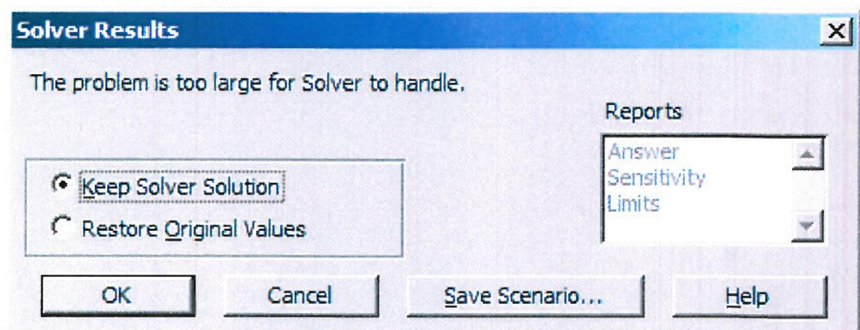


Figure 9: Solver results window for nonlinear model.

By deleting the minimum coverage constraints, as shown in Figure 10, the model can be made to start running in Solver, but further errors occur and the output is then not valid as the model divides by zero as it iterates through possible solutions.

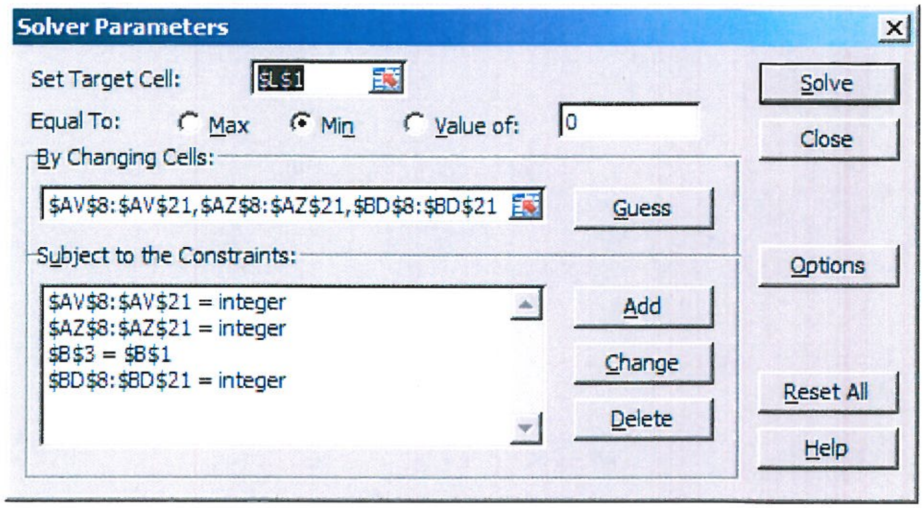


Figure 10: Solver parameters window for non linear model with reduced contraints.

## Suitability of the Software Application and Hardware Platform

The platform used in this research was Microsoft Excel 2007 with the standard Solver Add-in tool. This application ran on a Dell laptop using dual core processors running at 1.83 GHz. The adequacy of this combination varied widely, based on multiple factors.

For the linear cost optimization model, when the hourly staffing had significant peaks or valleys throughout the week, a solution was typically obtained within less than 1 minute and the combination was adequate and practical for use. However, for the same model, if the desired staffing levels were flat (roughly equal across all hours of the week), Solver had extensive troubles arriving at an optimal solution, requiring 45 minutes or more to arrive at an optimal solution or not arriving at an optimal solution at all.

For the non-linear variance optimization model, Excel solver proved to be inadequate as it was not able to handle the large number of constraints and even when these were decreased, the tool struggled to find an optimal solution.

The weaknesses described above appear to be functions of the software package and not the computer itself.

## Chapter 5 – Suggestions for Additional Work

This chapter discusses areas for further research. The wide body of research that exists on the modeling of employee schedules points to vast additional research opportunities, however this author sees particular benefit in exploring four specific areas: accounting for morale/turnover costs, further development of the utilization variance minimization model, the use of alternate software packages, and educational approaches for increasing awareness of optimization and modeling possibilities.

### Accounting for Morale and Turnover Costs

In the cost optimization model used in this research, the algebraic formulation and the model itself can be readily adapted to include a factor for morale and employee turnover in the total cost calculation. If a user can assign a cost to those "soft factors" and create a linkage between unpopular shifts and morale or turnover problems, the true costs of employee preferences can be seen.

### Minimizing Utilization Variance

Due to tool limitations explained in the previous chapter, this research was not able to fully explore the value of the utilization variance minimization model (also referred to as Approach #2). This model uses a principle and method that may provide value to a variety of industries. Given a fixed amount of resources currently available and the incoming demand, this technique offers a method to even out resource capacity vs. demand ratio. The principle of minimizing variance assumes what is often the case in business: that capacity cannot be increased or decreased easily and that while the current capacity exists, a primary responsibility for managers is to use the capacity efficiently.

This approach can be used beyond telecommunications and would be directly beneficial to industries where the cost or lead time of increasing or decreasing capacity is high, such as, healthcare staffing, fleets, and warehousing.

Alternate Modeling Tools

While Microsoft Excel Solver was the tool used in this research due to Solver's low cost and ready availability, there exists a large variety of modeling tools that may be more appropriate for both the linear and nonlinear models used in this research.

Through this author's review of past research and literature, it is apparent that multiple optimization and modeling packages exist, however the cost can be significant and were not appropriate to the economics of this research. By translating the nonlinear model from this current research into other software packages, the true usefulness of the model design can be determined.

Educational Approaches for Increasing Familiarity with Optimization

Quantitative methods for modeling and optimization are typically taught in graduate level courses due to the need to thoroughly understand the mathematical or statistical methods at their foundation. As a result, there is often little awareness of the methods or their value by many industry personnel. This author has observed groups making significant decisions that would have been eased and would have benefited from quantitative modeling (most likely by a consulting subject matter expert), however it was not a consideration due to lack of awareness. There may be significant value in researching methods to increase awareness about decision science capabilities at the undergraduate course level.

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# Appendix A

Algebraic Formulation of Cost Objective Model

## Algebraic Formulation- Cost Objective Model

Let:

hrlylabor ≡ Loaded labor cost per hour for NOCC technicians
swingdiff ≡ hourly pay differential for swing shift = 10%
midsdiff ≡ hourly pay differential for midshift (graveyard) = 15%
$hc_h$ ≡ Total headcount working during hour h
$hcmin_h$ ≡ The minimum number of people required for staffing at hour h
days1, days2, days3, etc. ≡ Number of personnel working that particular day shift. Shifts defined in table below.
swings1, swings2, swings3, etc. ≡ Number of personnel working that particular swing shift. Shifts defined in table below.
mids1, mids2, mids3, etc. ≡ Number of personnel working that particular mid shift. Shifts defined in table below.

Shift designations are defined below:

| | Weekdays | Midshift Hours/Shiftname | Dayshift Hours/Shiftname | Swingshift Hours/Shiftname |
|---|---|---|---|---|
| 5 X 8's | Mon – Fri | 11pm to 7am- Mids1 | 7am to 3pm- Days1 | 3pm to 11pm- Swings1 |
| 5 X 8's | Tues – Sat | Mids2 | Days2 | Swings2 |
| 5 X 8's | Wed – Sun | Mids3 | Days3 | Swings3 |
| 5 X 8's | Th – Mon | Mids4 | Days4 | Swings4 |
| 5 X 8's | Fri – Tues | Mids5 | Days5 | Swings5 |
| 5 X 8's | Sat – Wed | Mids6 | Days6 | Swings6 |
| 5 X 8's | Sun – Th | Mids7 | Days7 | Swings7 |
| | | | | |
| 4 X 10's | Mon – Th | 11pm to 9am- Mids8 | 7am to 5pm- Days8 | 3pm to 1am- Swings8 |
| 4 X 10's | Tues – Fri | Mids9 | Days9 | Swings9 |
| 4 X 10's | Wed – Sat | Mids10 | Days10 | Swings10 |
| 4 X 10's | Th – Sun | Mids11 | Days11 | Swings11 |
| 4 X 10's | Fri – Mon | Mids12 | Days12 | Swings12 |
| 4 X 10's | Sat – Tues | Mids13 | Days13 | Swings13 |
| 4 X 10's | Sun – Wed | Mids14 | Days14 | Swings14 |
| | | | | |
| 5 X 8's non-consecutive | Tues & Sat off | 11pm to 7am- Mids15 | 7am tc 3pm- Days15 | 3pm to 11pm- Swings15 |
| 5 X 8's non- | Wed & Sat off | Mids16 | Days16 | Swings16 |

| | | | | |
|---|---|---|---|---|
| consecutive | | | | |
| 5 X 8's non-consecutive | Thurs & Sat off | Mids17 | Days17 | Swings17 |
| 5 X 8's non-consecutive | Tues & Sun off | Mids18 | Days18 | Swings18 |
| 5 X 8's non-consecutive | Wed & Sun off | Mids19 | Days19 | Swings19 |
| 5 X 8's non-consecutive | Thurs & Sun off | Mids20 | Days20 | Swings20 |
| | | | | |
| 3x13 1/3's | | 6PM to 7:20 AM- Mids21<br>Fri morn, Sat morn, Sun morn | 6AM to 7:20 PM- Days21<br>Thurs, Fri, Sat | |

Let (continued):

Hours of the week (h) are defined below:

| | **Sunday** | **Monday** | **Tuesday** | **Wednesday** | **Thursday** | **Friday** | **Saturday** | |
|---|---|---|---|---|---|---|---|---|
| **Hour of day** | Hour of Week | Hour of Week | Hour of Week | Hour of Week | Hour of Week | Hour of Week | Hour of Week | |
| 12 AM Midnight | 0 | 24 | 48 | 72 | 96 | 120 | 144 | Midshift Pay Differential |
| 1 | 1 | 25 | 49 | 73 | 97 | 121 | 145 | |
| 2 | 2 | 26 | 50 | 74 | 98 | 122 | 146 | |
| 3 | 3 | 27 | 51 | 75 | 99 | 123 | 147 | |
| 4 | 4 | 28 | 52 | 76 | 100 | 124 | 148 | |
| 5 | 5 | 29 | 53 | 77 | 101 | 125 | 149 | |
| 6 | 6 | 30 | 54 | 78 | 102 | 126 | 150 | |
| 7 | 7 | 31 | 55 | 79 | 103 | 127 | 151 | |
| 8 | 8 | 32 | 56 | 80 | 104 | 128 | 152 | |
| 9 | 9 | 33 | 57 | 81 | 105 | 129 | 153 | |
| 10 | 10 | 34 | 58 | 82 | 106 | 130 | 154 | |
| 11:00 AM | 11 | 35 | 59 | 83 | 107 | 131 | 155 | |
| 12:00 PM | 12 | 36 | 60 | 84 | 108 | 132 | 156 | |
| 1:00 PM | 13 | 37 | 61 | 85 | 109 | 133 | 157 | |
| 2 | 14 | 38 | 62 | 86 | 110 | 134 | 158 | |
| 3 | 15 | 39 | 63 | 87 | 111 | 135 | 159 | |
| 4 | 16 | 40 | 64 | 88 | 112 | 136 | 160 | |
| 5 | 17 | 41 | 65 | 89 | 113 | 137 | 161 | |
| 6 | 18 | 42 | 66 | 90 | 114 | 138 | 162 | Swingshift Pay Differential |
| 7 | 19 | 43 | 67 | 91 | 115 | 139 | 163 | |
| 8 | 20 | 44 | 68 | 92 | 116 | 140 | 164 | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | 21 | 45 | 69 | 93 | 117 | 141 | 165 | |
| 10 | 22 | 46 | 70 | 94 | 118 | 142 | 166 | |
| 11:00 PM | 23 | 47 | 71 | 95 | 119 | 143 | 167 | |

O.F.:

Min dayscost + swingscost + midscost Total labor cost

= (HCh values for days) x hrlylabor
+ (HCh values for swings) x hrlylabor x (swingdiff + 1.00)
+ (HCh values for mids) x hrlylabor x (midsdiff+1.00)

S.T.:

swingdiff, midsdiff, hrlylabor $\geq 0$ constants

$hc_h$, $hcmin_h \geq 0 \ \forall_h$

mids1, mids2 … mids20, mids21 $\geq 0$

days1, days2 … days20, days21 $\geq 0$

swings1, swings2 … swings19, swings20 $\geq 0$

$hc_h \geq hcmin_h \ \forall_h$ (must have at least minimum personnel each hour)

Values for $hc_h$ are defined in the table below:

| h | $HC_h$ equals summation of the following: | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | mids 3 | mids 4 | mids 5 | mids 6 | mids 7 | swin gs10 | swin gs11 | mids 11 | swin gs12 | mids 12 | swin gs13 | mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 21 | | | |
| 1 | mids 3 | mids 4 | mids 5 | mids 6 | mids 7 | | | mids 11 | | mids 12 | | mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 21 | | | |
| 2 | mids 3 | mids 4 | mids 5 | mids 6 | mids 7 | | | mids 11 | | mids 12 | | mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 21 | | | |
| 3 | mids 3 | mids 4 | mids 5 | mids 6 | mids 7 | | | mids 11 | | mids 12 | | mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 21 | | | |
| 4 | mids 3 | mids 4 | mids 5 | mids 6 | mids 7 | | | mids 11 | | mids 12 | | mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 21 | | | |
| 5 | mids 3 | mids 4 | mids 5 | mids 6 | mids 7 | | | mids 11 | | mids 12 | | mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 21 | | | |
| 6 | mids 3 | mids 4 | mids 5 | mids 6 | mids 7 | | | mids 11 | | mids 12 | | mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 21 | | | |
| 7 | days 3 | days 4 | days 5 | days 6 | days 7 | days 11 | | mids 11 | days 12 | mids 12 | Days 13 | mids 13 | days 14 | mids 14 | days 15 | days 16 | days 17 | | | | |
| 8 | days 3 | days 4 | days 5 | days 6 | days 7 | days 11 | | mids 11 | days 12 | mids 12 | Days 13 | mids 13 | days 14 | mids 14 | days 15 | days 16 | days 17 | | | | |
| 9 | days 3 | days 4 | days 5 | days 6 | days 7 | days 11 | | | days 12 | | Days 13 | | days 14 | | days 15 | days 16 | days 17 | | | | |
| 10 | days 3 | days 4 | days 5 | days 6 | days 7 | days 11 | | | days 12 | | Days 13 | | days 14 | | days 15 | days 16 | days 17 | | | | |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | days 3 | days 4 | days 5 | days 6 | days 7 | days 11 | | | days 12 | | Days 13 | | days 14 | | days 15 | days 16 | days 17 | | | | |
| 12 | days 3 | days 4 | days 5 | days 6 | days 7 | days 11 | | | days 12 | | Days 13 | | days 14 | | days 15 | days 16 | days 17 | | | | |
| 13 | days 3 | days 4 | days 5 | days 6 | days 7 | days 11 | | | days 12 | | Days 13 | | days 14 | | days 15 | days 16 | days 17 | | | | |
| 14 | days 3 | days 4 | days 5 | days 6 | days 7 | days 11 | | | days 12 | | Days 13 | | days 14 | | days 15 | days 16 | days 17 | | | | |
| 15 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | days 11 | swin gs11 | swin gs12 | days 12 | swin gs13 | Days 13 | swin gs14 | days 14 | swin gs15 | swin gs16 | swin gs17 | | | | | |
| 16 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | days 11 | swin gs11 | swin gs12 | days 12 | swin gs13 | Days 13 | swin gs14 | days 14 | swin gs15 | swin gs16 | swin gs17 | | | | | |
| 17 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs11 | swin gs12 | | swin gs13 | | swin gs14 | | swin gs15 | swin gs16 | swin gs17 | | | | | |
| 18 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs11 | swin gs12 | | swin gs13 | | swin gs14 | | swin gs15 | swin gs16 | swin gs17 | | | | | |
| 19 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs11 | swin gs12 | | swin gs13 | | swin gs14 | | swin gs15 | swin gs16 | swin gs17 | | | | | |
| 20 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs11 | swin gs12 | | swin gs13 | | swin gs14 | | swin gs15 | swin gs16 | swin gs17 | | | | | |
| 21 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs11 | swin gs12 | | swin gs13 | | swin gs14 | | swin gs15 | swin gs16 | swin gs17 | | | | | |
| 22 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs11 | swin gs12 | | swin gs13 | | swin gs14 | | swin gs15 | swin gs16 | swin gs17 | | | | | |
| 23 | mids 1 | mids 4 | mids 5 | mids 6 | mids 7 | mids 8 | swin gs11 | swin gs12 | mids 12 | swin gs13 | Mids 13 | swin gs14 | mids 14 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | |
| 24 | mids 1 | mids 4 | mids 5 | mids 6 | mids 7 | swin gs11 | mids 8 | swin gs12 | mids 12 | swin gs13 | Mids 13 | swin gs14 | mids 14 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | |
| 25 | mids 1 | mids 4 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 12 | | Mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | |
| 26 | mids 1 | mids 4 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 12 | | Mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | |
| 27 | mids 1 | mids 4 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 12 | | Mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | |
| 28 | mids 1 | mids 4 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 12 | | Mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 29 | mids 1 | mids 4 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 12 | | Mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | |
| 30 | mids 1 | mids 4 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 12 | | Mids 13 | | mids 14 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | |
| 31 | days 1 | days 4 | days 5 | days 6 | days 7 | days 8 | mids 8 | days 12 | mids 12 | days 13 | Mids 13 | days 14 | mids 14 | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | | |
| 32 | days 1 | days 4 | days 5 | days 6 | days 7 | days 8 | mids 8 | days 12 | mids 12 | days 13 | Mids 13 | days 14 | mids 14 | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | | |
| 33 | days 1 | days 4 | days 5 | days 6 | days 7 | days 8 | | days 12 | | days 13 | | days 14 | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | | |
| 34 | days 1 | days 4 | days 5 | days 6 | days 7 | days 8 | | days 12 | | days 13 | | days 14 | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | | |
| 35 | days 1 | days 4 | days 5 | days 6 | days 7 | days 8 | | days 12 | | days 13 | | days 14 | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | | |
| 36 | days 1 | days 4 | days 5 | days 6 | days 7 | days 8 | | days 12 | | days 13 | | days 14 | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | | |
| 37 | days 1 | days 4 | days 5 | days 6 | days 7 | days 8 | | days 12 | | days 13 | | days 14 | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | | |
| 38 | days 1 | days 4 | days 5 | days 6 | days 7 | days 8 | | days 12 | | days 13 | | days 14 | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | | |
| 39 | swin gs1 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | days 8 | swin gs8 | days 12 | swin gs12 | days 13 | swin gs13 | days 14 | swin gs14 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | | |
| 40 | swin gs1 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | days 8 | swin gs8 | days 12 | swin gs12 | days 13 | swin gs13 | days 14 | swin gs14 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | | |
| 41 | swin gs1 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs12 | | swin gs13 | | swin gs14 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | | |
| 42 | swin gs1 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs12 | | swin gs13 | | swin gs14 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | | |
| 43 | swin gs1 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs12 | | swin gs13 | | swin gs14 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | | |
| 44 | swin gs1 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs12 | | swin gs13 | | swin gs14 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | | |
| 45 | swin gs1 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs12 | | swin gs13 | | swin gs14 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | | |
| 46 | swin gs1 | swin gs4 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs12 | | swin gs13 | | swin gs14 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | | |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 47 | mids 1 | mids 2 | mids 5 | mids 6 | mids 7 | mids 8 | swin gs8 | mids 9 | swin gs12 | mids 13 | swin gs13 | mids 14 | swin gs14 | mids 16 | mids 17 | mids 19 | mids 20 | | | | |
| 48 | mids 1 | mids 2 | mids 5 | mids 6 | mids 7 | swin gs8 | mids 8 | swin gs12 | mids 9 | swin gs13 | Mids 13 | swin gs14 | mids 14 | mids 16 | mids 17 | mids 19 | mids 20 | | | | |
| 49 | mids 1 | mids 2 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 13 | | mids 14 | mids 16 | mids 17 | mids 19 | mids 20 | | | | |
| 50 | mids 1 | mids 2 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 13 | | mids 14 | mids 16 | mids 17 | mids 19 | mids 20 | | | | |
| 51 | mids 1 | mids 2 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 13 | | mids 14 | mids 16 | mids 17 | mids 19 | mids 20 | | | | |
| 52 | mids 1 | mids 2 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 13 | | mids 14 | mids 16 | mids 17 | mids 19 | mids 20 | | | | |
| 53 | mids 1 | mids 2 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 13 | | mids 14 | mids 16 | mids 17 | mids 19 | mids 20 | | | | |
| 54 | mids 1 | mids 2 | mids 5 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 13 | | mids 14 | mids 16 | mids 17 | mids 19 | mids 20 | | | | |
| 55 | days 1 | days 2 | days 5 | days 6 | days 7 | days 8 | mids 8 | days 9 | mids 9 | days 13 | Mids 13 | days 14 | mids 14 | days 16 | days 17 | days 19 | days 20 | | | | |
| 56 | days 1 | days 2 | days 5 | days 6 | days 7 | days 8 | mids 8 | days 9 | mids 9 | days 13 | Mids 13 | days 14 | mids 14 | days 16 | days 17 | days 19 | days 20 | | | | |
| 57 | days 1 | days 2 | days 5 | days 6 | days 7 | days 8 | | days 9 | | days 13 | | days 14 | | days 16 | days 17 | days 19 | days 20 | | | | |
| 58 | days 1 | days 2 | days 5 | days 6 | days 7 | days 8 | | days 9 | | days 13 | | days 14 | | days 16 | days 17 | days 19 | days 20 | | | | |
| 59 | days 1 | days 2 | days 5 | days 6 | days 7 | days 8 | | days 9 | | days 13 | | days 14 | | days 16 | days 17 | days 19 | days 20 | | | | |
| 60 | days 1 | days 2 | days 5 | days 6 | days 7 | days 8 | | days 9 | | days 13 | | days 14 | | days 16 | days 17 | days 19 | days 20 | | | | |
| 61 | days 1 | days 2 | days 5 | days 6 | days 7 | days 8 | | days 9 | | days 13 | | days 14 | | days 16 | days 17 | days 19 | days 20 | | | | |
| 62 | days 1 | days 2 | days 5 | days 6 | days 7 | days 8 | | days 9 | | days 13 | | days 14 | | days 16 | days 17 | days 19 | days 20 | | | | |
| 63 | swin gs1 | swin gs2 | swin gs5 | swin gs6 | swin gs7 | days 8 | swin gs8 | days 9 | swin gs9 | days 13 | swin gs13 | days 14 | swin gs14 | swin gs16 | swin gs17 | swin gs19 | swin gs20 | | | | |
| 64 | swin gs1 | swin gs2 | swin gs5 | swin gs6 | swin gs7 | days 8 | swin gs8 | days 9 | swin gs9 | days 13 | swin gs13 | days 14 | swin gs14 | swin gs16 | swin gs17 | swin gs19 | swin gs20 | | | | |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 65 | swin gs1 | swin gs2 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs13 | | swin gs14 | swin gs16 | swin gs17 | swin gs19 | swin gs20 | | | | |
| 66 | swin gs1 | swin gs2 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs13 | | swin gs14 | swin gs16 | swin gs17 | swin gs19 | swin gs20 | | | | |
| 67 | swin gs1 | swin gs2 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs13 | | swin gs14 | swin gs16 | swin gs17 | swin gs19 | swin gs20 | | | | |
| 68 | swin gs1 | swin gs2 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs13 | | swin gs14 | swin gs16 | swin gs17 | swin gs19 | swin gs20 | | | | |
| 69 | swin gs1 | swin gs2 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs13 | | swin gs14 | swin gs16 | swin gs17 | swin gs19 | swin gs20 | | | | |
| 70 | swin gs1 | swin gs2 | swin gs5 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs13 | | swin gs14 | swin gs16 | swin gs17 | swin gs19 | swin gs20 | | | | |
| 71 | mids 1 | mids 2 | mids 3 | mids 6 | mids 7 | mids 8 | swin gs8 | mids 9 | swin gs9 | mids 10 | swin gs13 | mids 14 | swin gs14 | mids 15 | mids 17 | mids 18 | mids 20 | | | | |
| 72 | mids 1 | mids 2 | mids 3 | mids 6 | mids 7 | swin gs8 | mids 8 | swin gs9 | mids 9 | swin gs13 | Mids 10 | swin gs14 | mids 14 | mids 15 | mids 17 | mids 18 | mids 20 | | | | |
| 73 | mids 1 | mids 2 | mids 3 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 10 | | mids 14 | mids 15 | mids 17 | mids 18 | mids 20 | | | | |
| 74 | mids 1 | mids 2 | mids 3 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 10 | | mids 14 | mids 15 | mids 17 | mids 18 | mids 20 | | | | |
| 75 | mids 1 | mids 2 | mids 3 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 10 | | mids 14 | mids 15 | mids 17 | mids 18 | mids 20 | | | | |
| 76 | mids 1 | mids 2 | mids 3 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 10 | | mids 14 | mids 15 | mids 17 | mids 18 | mids 20 | | | | |
| 77 | mids 1 | mids 2 | mids 3 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 10 | | mids 14 | mids 15 | mids 17 | mids 18 | mids 20 | | | | |
| 78 | mids 1 | mids 2 | mids 3 | mids 6 | mids 7 | | mids 8 | | mids 9 | | Mids 10 | | mids 14 | mids 15 | mids 17 | mids 18 | mids 20 | | | | |
| 79 | days 1 | days 2 | days 3 | days 6 | days 7 | days 8 | mids 8 | days 9 | mids 9 | days 10 | Mids 10 | days 14 | mids 14 | days 15 | days 17 | days 18 | days 20 | | | | |
| 80 | days 1 | days 2 | days 3 | days 6 | days 7 | days 8 | mids 8 | days 9 | mids 9 | days 10 | Mids 10 | days 14 | mids 14 | days 15 | days 17 | days 18 | days 20 | | | | |
| 81 | days 1 | days 2 | days 3 | days 6 | days 7 | days 8 | | days 9 | | days 10 | | days 14 | | days 15 | days 17 | days 18 | days 20 | | | | |
| 82 | days 1 | days 2 | days 3 | days 6 | days 7 | days 8 | | days 9 | | days 10 | | days 14 | | days 15 | days 17 | days 18 | days 20 | | | | |

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| 83 | days 1 | days 2 | days 3 | days 6 | days 7 | days 8 | | days 9 | | days 10 | | days 14 | | days 15 | days 17 | days 18 | days 20 | | | | |
| 84 | days 1 | days 2 | days 3 | days 6 | days 7 | days 8 | | days 9 | | days 10 | | days 14 | | days 15 | days 17 | days 18 | days 20 | | | | |
| 85 | days 1 | days 2 | days 3 | days 6 | days 7 | days 8 | | days 9 | | days 10 | | days 14 | | days 15 | days 17 | days 18 | days 20 | | | | |
| 86 | days 1 | days 2 | days 3 | days 6 | days 7 | days 8 | | days 9 | | days 10 | | days 14 | | days 15 | days 17 | days 18 | days 20 | | | | |
| 87 | swin gs1 | swin gs2 | swin gs3 | swin gs6 | swin gs7 | days 8 | swin gs8 | days 9 | swin gs9 | days 10 | swin gs10 | days 14 | swin gs14 | swin gs15 | swin gs17 | swin gs18 | swin gs20 | | | | |
| 88 | swin gs1 | swin gs2 | swin gs3 | swin gs6 | swin gs7 | days 8 | swin gs8 | days 9 | swin gs9 | days 10 | swin gs10 | days 14 | swin gs14 | swin gs15 | swin gs17 | swin gs18 | swin gs20 | | | | |
| 89 | swin gs1 | swin gs2 | swin gs3 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs10 | | swin gs14 | swin gs15 | swin gs17 | swin gs18 | swin gs20 | | | | |
| 90 | swin gs1 | swin gs2 | swin gs3 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs10 | | swin gs14 | swin gs15 | swin gs17 | swin gs18 | swin gs20 | | | | |
| 91 | swin gs1 | swin gs2 | swin gs3 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs10 | | swin gs14 | swin gs15 | swin gs17 | swin gs18 | swin gs20 | | | | |
| 92 | swin gs1 | swin gs2 | swin gs3 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs10 | | swin gs14 | swin gs15 | swin gs17 | swin gs18 | swin gs20 | | | | |
| 93 | swin gs1 | swin gs2 | swin gs3 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs10 | | swin gs14 | swin gs15 | swin gs17 | swin gs18 | swin gs20 | | | | |
| 94 | swin gs1 | swin gs2 | swin gs3 | swin gs6 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs10 | | swin gs14 | swin gs15 | swin gs17 | swin gs18 | swin gs20 | | | | |
| 95 | mids 1 | mids 2 | mids 3 | mids 4 | mids 7 | mids 8 | swin gs8 | mids 9 | swin gs9 | mids 10 | swin gs10 | mids 11 | swin gs14 | mids 15 | mids 16 | mids 18 | mids 19 | | | | |
| 96 | mids 1 | mids 2 | mids 3 | mids 4 | mids 7 | swin gs8 | mids 8 | swin gs9 | mids 9 | swin gs10 | mids 10 | swin gs14 | mids 11 | mids 15 | mids 16 | mids 18 | mids 19 | | | | |
| 97 | mids 1 | mids 2 | mids 3 | mids 4 | mids 7 | | mids 8 | | mids 9 | | mids 10 | | mids 11 | mids 15 | mids 16 | mids 18 | mids 19 | | | | |
| 98 | mids 1 | mids 2 | mids 3 | mids 4 | mids 7 | | mids 8 | | mids 9 | | mids 10 | | mids 11 | mids 15 | mids 16 | mids 18 | mids 19 | | | | |
| 99 | mids 1 | mids 2 | mids 3 | mids 4 | mids 7 | | mids 8 | | mids 9 | | mids 10 | | mids 11 | mids 15 | mids 16 | mids 18 | mids 19 | | | | |
| 100 | mids 1 | mids 2 | mids 3 | mids 4 | mids 7 | | mids 8 | | mids 9 | | mids 10 | | mids 11 | mids 15 | mids 16 | mids 18 | mids 19 | | | | |

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| 101 | mids 1 | mids 2 | mids 3 | mids 4 | mids 7 | | mids 8 | | mids 9 | | mids 10 | | mids 11 | mids 15 | mids 16 | mids 18 | mids 19 | | | | |
| 102 | mids 1 | mids 2 | mids 3 | mids 4 | mids 7 | | mids 8 | | mids 9 | | mids 10 | | mids 11 | mids 15 | mids 16 | mids 18 | mids 19 | days 21 | | | |
| 103 | days 1 | days 2 | days 3 | days 4 | days 7 | days 8 | mids 8 | days 9 | mids 9 | days 10 | mids 10 | days 11 | mids 11 | days 15 | days 16 | days 18 | days 19 | days 21 | | | |
| 104 | days 1 | days 2 | days 3 | days 4 | days 7 | days 8 | mids 8 | days 9 | mids 9 | days 10 | mids 10 | days 11 | mids 11 | days 15 | days 16 | days 18 | days 19 | days 21 | | | |
| 105 | days 1 | days 2 | days 3 | days 4 | days 7 | days 8 | | days 9 | | days 10 | | days 11 | | days 15 | days 16 | days 18 | days 19 | days 21 | | | |
| 106 | days 1 | days 2 | days 3 | days 4 | days 7 | days 8 | | days 9 | | days 10 | | days 11 | | days 15 | days 16 | days 18 | days 19 | days 21 | | | |
| 107 | days 1 | days 2 | days 3 | days 4 | days 7 | days 8 | | days 9 | | days 10 | | days 11 | | days 15 | days 16 | days 18 | days 19 | days 21 | | | |
| 108 | days 1 | days 2 | days 3 | days 4 | days 7 | days 8 | | days 9 | | days 10 | | days 11 | | days 15 | days 16 | days 18 | days 19 | days 21 | | | |
| 109 | days 1 | days 2 | days 3 | days 4 | days 7 | days 8 | | days 9 | | days 10 | | days 11 | | days 15 | days 16 | days 18 | days 19 | days 21 | | | |
| 110 | days 1 | days 2 | days 3 | days 4 | days 7 | days 8 | | days 9 | | days 10 | | days 11 | | days 15 | days 16 | days 18 | days 19 | days 21 | | | |
| 111 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs7 | days 8 | swin gs8 | days 9 | swin gs9 | days 10 | swin gs10 | days 11 | swin gs11 | swin gs15 | swin gs16 | swin gs18 | swin gs19 | days 21 | | | |
| 112 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs7 | days 8 | swin gs8 | days 9 | swin gs9 | days 10 | swin gs10 | days 11 | swin gs11 | swin gs15 | swin gs16 | swin gs18 | swin gs19 | days 21 | | | |
| 113 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs10 | | swin gs11 | swin gs15 | swin gs16 | swin gs18 | swin gs19 | days 21 | | | |
| 11 | swin | swin | swin | swin | swin | | swin | | swin | | swin | | swin | swin | swin | swin | swin | days | | mid | |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 4 | gs1 | gs2 | gs3 | gs4 | gs7 | | gs8 | | gs9 | | gs10 | | gs11 | gs15 | gs16 | gs18 | gs19 | 21 | | s21 | |
| 115 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs10 | | swin gs11 | swin gs15 | swin gs16 | swin gs18 | swin gs19 | | | mid s21 | |
| 116 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs10 | | swin gs11 | swin gs15 | swin gs16 | swin gs18 | swin gs19 | | | mid s21 | |
| 117 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs10 | | swin gs11 | swin gs15 | swin gs16 | swin gs18 | swin gs19 | | | mid s21 | |
| 118 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs7 | | swin gs8 | | swin gs9 | | swin gs10 | | swin gs11 | swin gs15 | swin gs16 | swin gs18 | swin gs19 | | | mid s21 | |
| 119 | mids 1 | mids 2 | mids 3 | mids 4 | mids 5 | mids 9 | swin gs8 | mids 10 | swin gs9 | mids 11 | swin gs10 | mids 12 | swin gs11 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | mid s21 | |
| 120 | mids 1 | mids 2 | mids 3 | mids 4 | mids 5 | swin gs8 | swin gs9 | mids 9 | swin gs10 | mids 10 | swin gs11 | mids 11 | mids 12 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | mid s21 |
| 121 | mids 1 | mids 2 | mids 3 | mids 4 | mids 5 | | | mids 9 | | mids 10 | | mids 11 | mids 12 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | mid s21 |
| 122 | mids 1 | mids 2 | mids 3 | mids 4 | mids 5 | | | mids 9 | | mids 10 | | mids 11 | mids 12 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | mid s21 |
| 123 | mids 1 | mids 2 | mids 3 | mids 4 | mids 5 | | | mids 9 | | mids 10 | | mids 11 | mids 12 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | mid s21 |
| 124 | mids 1 | mids 2 | mids 3 | mids 4 | mids 5 | | | mids 9 | | mids 10 | | mids 11 | mids 12 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | mid s21 |
| 125 | mids 1 | mids 2 | mids 3 | mids 4 | mids 5 | | | mids 9 | | mids 10 | | mids 11 | mids 12 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | | mid s21 |
| 126 | mids 1 | mids 2 | mids 3 | mids 4 | mids 5 | | | mids 9 | | mids 10 | | mids 11 | mids 12 | mids 15 | mids 16 | mids 17 | mids 18 | mids 19 | mids 20 | day s21 | mid s21 |
| 127 | days 1 | days 2 | days 3 | days 4 | days 5 | days 9 | days 10 | mids 9 | days 11 | mids 10 | days 12 | mids 11 | mids 12 | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | day s21 | |

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 128 | days 1 | days 2 | days 3 | days 4 | days 5 | days 9 | days 10 | mids 9 | days 11 | mids 10 | days 12 | mids 11 | mids 12 | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | day s21 | |
| 129 | days 1 | days 2 | days 3 | days 4 | days 5 | days 9 | days 10 | | days 11 | | days 12 | | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | day s21 | |
| 130 | days 1 | days 2 | days 3 | days 4 | days 5 | days 9 | days 10 | | days 11 | | days 12 | | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | day s21 | |
| 131 | days 1 | days 2 | days 3 | days 4 | days 5 | days 9 | days 10 | | days 11 | | days 12 | | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | day s21 | |
| 132 | days 1 | days 2 | days 3 | days 4 | days 5 | days 9 | days 10 | | days 11 | | days 12 | | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | day s21 | |
| 133 | days 1 | days 2 | days 3 | days 4 | days 5 | days 9 | days 10 | | days 11 | | days 12 | | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | day s21 | |
| 134 | days 1 | days 2 | days 3 | days 4 | days 5 | days 9 | days 10 | | days 11 | | days 12 | | | days 15 | days 16 | days 17 | days 18 | days 19 | days 20 | day s21 | |
| 135 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | days 9 | days 10 | swin gs9 | days 11 | swin gs10 | days 12 | swin gs11 | swin gs12 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | day s21 | |
| 136 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | days 9 | days 10 | swin gs9 | days 11 | swin gs10 | days 12 | swin gs11 | swin gs12 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | day s21 | |
| 137 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | | | swin gs9 | | swin gs10 | | swin gs11 | swin gs12 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | day s21 | |
| 138 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | | | swin gs9 | | swin gs10 | | swin gs11 | swin gs12 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | day s21 | mid s21 |
| 139 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | | | swin gs9 | | swin gs10 | | swin gs11 | swin gs12 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | | mid s21 |
| 140 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | | | swin gs9 | | swin gs10 | | swin gs11 | swin gs12 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | | mid s21 |
| 14 | swin | swin | swin | swin | swin | | | swin | | swin | | swin | swin | swin | swin | swin | swin | swin | swin | | mid |

| 1 | gs1 | gs2 | gs3 | gs4 | gs5 | | | gs9 | | gs10 | | gs11 | gs12 | gs15 | gs16 | gs17 | gs18 | gs19 | gs20 | | s21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 142 | swin gs1 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | | | swin gs9 | | swin gs10 | | swin gs11 | swin gs12 | swin gs15 | swin gs16 | swin gs17 | swin gs18 | swin gs19 | swin gs20 | | mid s21 |
| 143 | | mids 2 | mids 3 | mids 4 | mids 5 | mids 6 | mids 10 | swin gs9 | mids 11 | swin gs10 | mids 12 | swin gs11 | swin gs12 | mids 13 | mids 18 | mids 19 | mids 20 | | | | mid s21 |
| 144 | mids 2 | mids 3 | mids 4 | mids 5 | mids 6 | swin gs9 | swin gs10 | mids 10 | swin gs11 | mids 11 | swin gs12 | mids 12 | | mids 13 | mids 18 | mids 19 | mids 20 | | mids 21 | | |
| 145 | mids 2 | mids 3 | mids 4 | mids 5 | mids 6 | | | mids 10 | | mids 11 | | mids 12 | | mids 13 | mids 18 | mids 19 | mids 20 | | mids 21 | | |
| 146 | mids 2 | mids 3 | mids 4 | mids 5 | mids 6 | | | mids 10 | | mids 11 | | mids 12 | | mids 13 | mids 18 | mids 19 | mids 20 | | mids 21 | | |
| 147 | mids 2 | mids 3 | mids 4 | mids 5 | mids 6 | | | mids 10 | | mids 11 | | mids 12 | | mids 13 | mids 18 | mids 19 | mids 20 | | mids 21 | | |
| 148 | mids 2 | mids 3 | mids 4 | mids 5 | mids 6 | | | mids 10 | | mids 11 | | mids 12 | | mids 13 | mids 18 | mids 19 | mids 20 | | mids 21 | | |
| 149 | mids 2 | mids 3 | mids 4 | mids 5 | mids 6 | | | mids 10 | | mids 11 | | mids 12 | | mids 13 | mids 18 | mids 19 | mids 20 | | mids 21 | | |
| 150 | mids 2 | mids 3 | mids 4 | mids 5 | mids 6 | | | mids 10 | | mids 11 | | mids 12 | | mids 13 | mids 18 | mids 19 | mids 20 | days 21 | mids 21 | | |
| 151 | days 2 | days 3 | days 4 | days 5 | days 6 | days 10 | | mids 10 | days 11 | mids 11 | days 12 | mids 12 | days 13 | mids 13 | days 18 | days 19 | days 20 | days 21 | | | |
| 152 | days 2 | days 3 | days 4 | days 5 | days 6 | days 10 | | mids 10 | days 11 | mids 11 | days 12 | mids 12 | days 13 | mids 13 | days 18 | days 19 | days 20 | days 21 | | | |
| 153 | days 2 | days 3 | days 4 | days 5 | days 6 | days 10 | | | days 11 | | days 12 | | days 13 | | days 18 | days 19 | days 20 | days 21 | | | |
| 154 | days 2 | days 3 | days 4 | days 5 | days 6 | days 10 | | | days 11 | | days 12 | | days 13 | | days 18 | days 19 | days 20 | days 21 | | | |

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 155 | days 2 | days 3 | days 4 | days 5 | days 6 | days 10 | | | days 11 | | days 12 | | days 13 | | days 18 | days 19 | days 20 | days 21 | | | |
| 156 | days 2 | days 3 | days 4 | days 5 | days 6 | days 10 | | | days 11 | | days 12 | | days 13 | | days 18 | days 19 | days 20 | days 21 | | | |
| 157 | days 2 | days 3 | days 4 | days 5 | days 6 | days 10 | | | days 11 | | days 12 | | days 13 | | days 18 | days 19 | days 20 | days 21 | | | |
| 158 | days 2 | days 3 | days 4 | days 5 | days 6 | days 10 | | | days 11 | | days 12 | | days 13 | | days 18 | days 19 | days 20 | days 21 | | | |
| 159 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | days 10 | swin gs10 | swin gs11 | days 11 | swin gs12 | days 12 | swin gs13 | days 13 | | swin gs18 | swin gs19 | swin gs20 | days 21 | | | |
| 160 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | days 10 | swin gs10 | swin gs11 | days 11 | swin gs12 | days 12 | swin gs13 | days 13 | | swin gs18 | swin gs19 | swin gs20 | days 21 | | | |
| 161 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | | swin gs10 | swin gs11 | | swin gs12 | | swin gs13 | | | swin gs18 | swin gs19 | swin gs20 | days 21 | | | |
| 162 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | | swin gs10 | swin gs11 | | swin gs12 | | swin gs13 | | | swin gs18 | swin gs19 | swin gs20 | days 21 | mids 21 | | |
| 163 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | | swin gs10 | swin gs11 | | swin gs12 | | swin gs13 | | | swin gs18 | swin gs19 | swin gs20 | | mids 21 | | |
| 164 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | | swin gs10 | swin gs11 | | swin gs12 | | swin gs13 | | | swin gs18 | swin gs19 | swin gs20 | | mids 21 | | |
| 165 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | | swin gs10 | swin gs11 | | swin gs12 | | swin gs13 | | | swin gs18 | swin gs19 | swin gs20 | | mids 21 | | |
| 166 | swin gs2 | swin gs3 | swin gs4 | swin gs5 | swin gs6 | | swin gs10 | swin gs11 | | swin gs12 | | swin gs13 | | | swin gs18 | swin gs19 | swin gs20 | | mids 21 | | |
| 167 | | mids 3 | mids 4 | mids 5 | mids 6 | mids 7 | swin gs10 | swin gs11 | mids 11 | swin gs12 | mids 12 | swin gs13 | mids 13 | mids 14 | mids 15 | mids 16 | mids 17 | | mids 21 | | |

# Appendix B

Algebraic Formulation of Variance Objective Model

## Algebraic Formulation- Variance Objective Model

Let:

$maxhd \equiv$ Maximum headcount available

$avol_h \equiv$ Alarm volume at hour h of the week
(h = 0, 1, 2, …165, 166, 167)

$hc_h \equiv$ Total headcount working during hour h

$util_h \equiv$ Alarms per person at hour h of the week, utilization
$= avol_h / hc_h$ for (h = 0, 1, 2, …165, 166, 167)

$meanutil \equiv$ Mean utilization $\equiv \overline{util} \equiv \frac{1}{168} \sum_{h=0}^{167} util_h$

$mincover_h \equiv$ The minimum number of people required to at hour h

Shift designations are defined below:

| | Weekdays | Midshift Hours/Shiftname | Dayshift Hours/Shiftname | Swingshift Hours/Shiftname |
|---|---|---|---|---|
| 5 X 8's | Mon – Fri | 11pm to 7am- Mids1 | 7am to 3pm- Days1 | 3pm to 11pm- Swings1 |
| 5 X 8's | Tues – Sat | Mids2 | Days2 | Swings2 |
| 5 X 8's | Wed – Sun | Mids3 | Days3 | Swings3 |
| 5 X 8's | Th – Mon | Mids4 | Days4 | Swings4 |
| 5 X 8's | Fri – Tues | Mids5 | Days5 | Swings5 |
| 5 X 8's | Sat – Wed | Mids6 | Days6 | Swings6 |
| 5 X 8's | Sun – Th | Mids7 | Days7 | Swings7 |
| 4 X 10's | Mon – Th | 11pm to 9am- Mids8 | 7am to 5pm- Days8 | 3pm to 1am- Swings8 |
| 4 X 10's | Tues – Fri | Mids9 | Days9 | Swings9 |
| 4 X 10's | Wed – Sat | Mids10 | Days10 | Swings10 |
| 4 X 10's | Th – Sun | Mids11 | Days11 | Swings11 |
| 4 X 10's | Fri – Mon | Mids12 | Days12 | Swings12 |
| 4 X 10's | Sat – Tues | Mids13 | Days13 | Swings13 |
| 4 X 10's | Sun – Wed | Mids14 | Days14 | Swings14 |

Let (continued):

Hours of the week (h) are defined below:

| | **Sunday** | **Monday** | **Tuesday** | **Wednesday** | **Thursday** | **Friday** | **Saturday** |
|---|---|---|---|---|---|---|---|
| **Hour of day** | Hour of Week | Hour of Week | Hour of Week | Hour of Week | Hour of Week | Hour of Week | Hour of Week |
| 12 AM Midnight | 0 | 24 | 48 | 72 | 96 | 120 | 144 |
| 1 | 1 | 25 | 49 | 73 | 97 | 121 | 145 |
| 2 | 2 | 26 | 50 | 74 | 98 | 122 | 146 |
| 3 | 3 | 27 | 51 | 75 | 99 | 123 | 147 |
| 4 | 4 | 28 | 52 | 76 | 100 | 124 | 148 |
| 5 | 5 | 29 | 53 | 77 | 101 | 125 | 149 |
| 6 | 6 | 30 | 54 | 78 | 102 | 126 | 150 |
| 7 | 7 | 31 | 55 | 79 | 103 | 127 | 151 |
| 8 | 8 | 32 | 56 | 80 | 104 | 128 | 152 |
| 9 | 9 | 33 | 57 | 81 | 105 | 129 | 153 |
| 10 | 10 | 34 | 58 | 82 | 106 | 130 | 154 |
| 11:00 AM | 11 | 35 | 59 | 83 | 107 | 131 | 155 |
| 12:00 PM | 12 | 36 | 60 | 84 | 108 | 132 | 156 |
| 1:00 PM | 13 | 37 | 61 | 85 | 109 | 133 | 157 |
| 2 | 14 | 38 | 62 | 86 | 110 | 134 | 158 |
| 3 | 15 | 39 | 63 | 87 | 111 | 135 | 159 |
| 4 | 16 | 40 | 64 | 88 | 112 | 136 | 160 |
| 5 | 17 | 41 | 65 | 89 | 113 | 137 | 161 |
| 6 | 18 | 42 | 66 | 90 | 114 | 138 | 162 |
| 7 | 19 | 43 | 67 | 91 | 115 | 139 | 163 |
| 8 | 20 | 44 | 68 | 92 | 116 | 140 | 164 |
| 9 | 21 | 45 | 69 | 93 | 117 | 141 | 165 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 10 | 22 | 46 | 70 | 94 | 118 | 142 | 166 |
| 11:00 PM | 23 | 47 | 71 | 95 | 119 | 143 | 167 |

O.F.:

Min

$$\mathbf{Variance} = \sigma^2 = \frac{1}{168} \cdot \sum_{h=0}^{167} \left(\mathbf{util}_h - \mathbf{meanutil}\right)^2$$

Minimizing variance in headcount utilization from hour to hour, in essence maximizing efficiency by reducing wasted resources

S.T.:

maxhd, mincover $\geq 0$
$hc_h$, $avol_h \geq 0 \quad \forall_h$
mids1, mids2 … mids13, mids14 $\geq 0$
days1, days2 … days13, days14 $\geq 0$
swings1, swings2 … swings13, swings14 $\geq 0$

$$\sum_{i=1}^{14}\left(\mathbf{mids(i)} + \mathbf{days(i)} + \mathbf{swings(i)}\right) = \mathbf{maxhd}$$
(all headcount are used)

$hc_h \geq \mathbf{mincover} \quad \forall_h$ (must have at least minimum personnel each hour)

Values for $hc_h$ are defined in the table below:

| h | HCh equals summation of the following: | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | mids3 | mids4 | mids5 | mids6 | mids7 | swings10 | swings11 | mids11 | swings12 | mids12 | swings13 | mids13 | | mids14 |
| 1 | mids3 | mids4 | mids5 | mids6 | mids7 | | | mids11 | | mids12 | | mids13 | | mids14 |
| 2 | mids3 | mids4 | mids5 | mids6 | mids7 | | | mids11 | | mids12 | | mids13 | | mids14 |
| 3 | mids3 | mids4 | mids5 | mids6 | mids7 | | | mids11 | | mids12 | | mids13 | | mids14 |
| 4 | mids3 | mids4 | mids5 | mids6 | mids7 | | | mids11 | | mids12 | | mids13 | | mids14 |
| 5 | mids3 | mids4 | mids5 | mids6 | mids7 | | | mids11 | | mids12 | | mids13 | | mids14 |
| 6 | mids3 | mids4 | mids5 | mids6 | mids7 | | | mids11 | | mids12 | | mids13 | | mids14 |
| 7 | days3 | days4 | days5 | days6 | days7 | days11 | | mids11 | days12 | mids12 | days13 | mids13 | days14 | mids14 |
| 8 | days3 | days4 | days5 | days6 | days7 | days11 | | mids11 | days12 | mids12 | days13 | mids13 | days14 | mids14 |
| 9 | days3 | days4 | days5 | days6 | days7 | days11 | | | days12 | | days13 | | days14 | |
| 10 | days3 | days4 | days5 | days6 | days7 | days11 | | | days12 | | days13 | | days14 | |
| 11 | days3 | days4 | days5 | days6 | days7 | days11 | | | days12 | | days13 | | days14 | |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | days3 | days4 | days5 | days6 | days7 | days11 | | | days12 | | days13 | | days14 | |
| 13 | days3 | days4 | days5 | days6 | days7 | days11 | | | days12 | | days13 | | days14 | |
| 14 | days3 | days4 | days5 | days6 | days7 | days11 | | | days12 | | days13 | | days14 | |
| 15 | swings3 | swings4 | swings5 | swings6 | swings7 | days11 | swings11 | swings12 | days12 | swings13 | days13 | swings14 | days14 | |
| 16 | swings3 | swings4 | swings5 | swings6 | swings7 | days11 | swings11 | swings12 | days12 | swings13 | days13 | swings14 | days14 | |
| 17 | swings3 | swings4 | swings5 | swings6 | swings7 | | swings11 | swings12 | | swings13 | | swings14 | | |
| 18 | swings3 | swings4 | swings5 | swings6 | swings7 | | swings11 | swings12 | | swings13 | | swings14 | | |
| 19 | swings3 | swings4 | swings5 | swings6 | swings7 | | swings11 | swings12 | | swings13 | | swings14 | | |
| 20 | swings3 | swings4 | swings5 | swings6 | swings7 | | swings11 | swings12 | | swings13 | | swings14 | | |
| 21 | swings3 | swings4 | swings5 | swings6 | swings7 | | swings11 | swings12 | | swings13 | | swings14 | | |
| 22 | swings3 | swings4 | swings5 | swings6 | swings7 | | swings11 | swings12 | | swings13 | | swings14 | | |
| 23 | mids1 | mids4 | mids5 | mids6 | mids7 | mids8 | swings11 | swings12 | mids12 | swings13 | mids13 | swings14 | mids14 | |
| 24 | mids1 | mids4 | mids5 | mids6 | mids7 | swings11 | mids8 | swings12 | mids12 | swings13 | mids13 | swings14 | mids14 | |
| 25 | mids1 | mids4 | mids5 | mids6 | mids7 | | mids8 | | mids12 | | mids13 | | mids14 | |
| 26 | mids1 | mids4 | mids5 | mids6 | mids7 | | mids8 | | mids12 | | mids13 | | mids14 | |
| 27 | mids1 | mids4 | mids5 | mids6 | mids7 | | mids8 | | mids12 | | mids13 | | mids14 | |
| 28 | mids1 | mids4 | mids5 | mids6 | mids7 | | mids8 | | mids12 | | mids13 | | mids14 | |
| 29 | mids1 | mids4 | mids5 | mids6 | mids7 | | mids8 | | mids12 | | mids13 | | mids14 | |
| 30 | mids1 | mids4 | mids5 | mids6 | mids7 | | mids8 | | mids12 | | mids13 | | mids14 | |
| 31 | days1 | days4 | days5 | days6 | days7 | days8 | mids8 | days12 | mids12 | days13 | mids13 | days14 | mids14 | |
| 32 | days1 | days4 | days5 | days6 | days7 | days8 | mids8 | days12 | mids12 | days13 | mids13 | days14 | mids14 | |
| 33 | days1 | days4 | days5 | days6 | days7 | days8 | | days12 | | days13 | | days14 | | |
| 34 | days1 | days4 | days5 | days6 | days7 | days8 | | days12 | | days13 | | days14 | | |
| 35 | days1 | days4 | days5 | days6 | days7 | days8 | | days12 | | days13 | | days14 | | |
| 36 | days1 | days4 | days5 | days6 | days7 | days8 | | days12 | | days13 | | days14 | | |
| 37 | days1 | days4 | days5 | days6 | days7 | days8 | | days12 | | days13 | | days14 | | |
| 38 | days1 | days4 | days5 | days6 | days7 | days8 | | days12 | | days13 | | days14 | | |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 39 | swings1 | swings4 | swings5 | swings6 | swings7 | days8 | swings8 | days12 | swings12 | days13 | swings13 | days14 | swings14 | |
| 40 | swings1 | swings4 | swings5 | swings6 | swings7 | days8 | swings8 | days12 | swings12 | days13 | swings13 | days14 | swings14 | |
| 41 | swings1 | swings4 | swings5 | swings6 | swings7 | | swings8 | | swings12 | | swings13 | | swings14 | |
| 42 | swings1 | swings4 | swings5 | swings6 | swings7 | | swings8 | | swings12 | | swings13 | | swings14 | |
| 43 | swings1 | swings4 | swings5 | swings6 | swings7 | | swings8 | | swings12 | | swings13 | | swings14 | |
| 44 | swings1 | swings4 | swings5 | swings6 | swings7 | | swings8 | | swings12 | | swings13 | | swings14 | |
| 45 | swings1 | swings4 | swings5 | swings6 | swings7 | | swings8 | | swings12 | | swings13 | | swings14 | |
| 46 | swings1 | swings4 | swings5 | swings6 | swings7 | | swings8 | | swings12 | | swings13 | | swings14 | |
| 47 | mids1 | mids2 | mids5 | mids6 | mids7 | mids8 | swings8 | mids9 | swings12 | mids13 | swings13 | mids14 | swings14 | |
| 48 | mids1 | mids2 | mids5 | mids6 | mids7 | swings8 | mids8 | swings12 | mids9 | swings13 | mids13 | swings14 | mids14 | |
| 49 | mids1 | mids2 | mids5 | mids6 | mids7 | | mids8 | | mids9 | | mids13 | | mids14 | |
| 50 | mids1 | mids2 | mids5 | mids6 | mids7 | | mids8 | | mids9 | | mids13 | | mids14 | |
| 51 | mids1 | mids2 | mids5 | mids6 | mids7 | | mids8 | | mids9 | | mids13 | | mids14 | |
| 52 | mids1 | mids2 | mids5 | mids6 | mids7 | | mids8 | | mids9 | | mids13 | | mids14 | |
| 53 | mids1 | mids2 | mids5 | mids6 | mids7 | | mids8 | | mids9 | | mids13 | | mids14 | |
| 54 | mids1 | mids2 | mids5 | mids6 | mids7 | | mids8 | | mids9 | | mids13 | | mids14 | |
| 55 | days1 | days2 | days5 | days6 | days7 | days8 | mids8 | days9 | mids9 | days13 | mids13 | days14 | mids14 | |
| 56 | days1 | days2 | days5 | days6 | days7 | days8 | mids8 | days9 | mids9 | days13 | mids13 | days14 | mids14 | |
| 57 | days1 | days2 | days5 | days6 | days7 | days8 | | days9 | | days13 | | days14 | | |
| 58 | days1 | days2 | days5 | days6 | days7 | days8 | | days9 | | days13 | | days14 | | |
| 59 | days1 | days2 | days5 | days6 | days7 | days8 | | days9 | | days13 | | days14 | | |
| 60 | days1 | days2 | days5 | days6 | days7 | days8 | | days9 | | days13 | | days14 | | |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 61 | days1 | days2 | days5 | days6 | days7 | days8 | | days9 | | days13 | | days14 | | |
| 62 | days1 | days2 | days5 | days6 | days7 | days8 | | days9 | | days13 | | days14 | | |
| 63 | swings1 | swings2 | swings5 | swings6 | swings7 | days8 | swings8 | days9 | swings9 | days13 | swings13 | days14 | swings14 | |
| 64 | swings1 | swings2 | swings5 | swings6 | swings7 | days8 | swings8 | days9 | swings9 | days13 | swings13 | days14 | swings14 | |
| 65 | swings1 | swings2 | swings5 | swings6 | swings7 | | swings8 | | swings9 | | swings13 | | swings14 | |
| 66 | swings1 | swings2 | swings5 | swings6 | swings7 | | swings8 | | swings9 | | swings13 | | swings14 | |
| 67 | swings1 | swings2 | swings5 | swings6 | swings7 | | swings8 | | swings9 | | swings13 | | swings14 | |
| 68 | swings1 | swings2 | swings5 | swings6 | swings7 | | swings8 | | swings9 | | swings13 | | swings14 | |
| 69 | swings1 | swings2 | swings5 | swings6 | swings7 | | swings8 | | swings9 | | swings13 | | swings14 | |
| 70 | swings1 | swings2 | swings5 | swings6 | swings7 | | swings8 | | swings9 | | swings13 | | swings14 | |
| 71 | mids1 | mids2 | mids3 | mids6 | mids7 | mids8 | swings8 | mids9 | swings9 | mids10 | swings13 | mids14 | swings14 | |
| 72 | mids1 | mids2 | mids3 | mids6 | mids7 | swings8 | mids8 | swings9 | mids9 | swings13 | mids10 | swings14 | mids14 | |
| 73 | mids1 | mids2 | mids3 | mids6 | mids7 | | mids8 | | mids9 | | mids10 | | mids14 | |
| 74 | mids1 | mids2 | mids3 | mids6 | mids7 | | mids8 | | mids9 | | mids10 | | mids14 | |
| 75 | mids1 | mids2 | mids3 | mids6 | mids7 | | mids8 | | mids9 | | mids10 | | mids14 | |
| 76 | mids1 | mids2 | mids3 | mids6 | mids7 | | mids8 | | mids9 | | mids10 | | mids14 | |
| 77 | mids1 | mids2 | mids3 | mids6 | mids7 | | mids8 | | mids9 | | mids10 | | mids14 | |
| 78 | mids1 | mids2 | mids3 | mids6 | mids7 | | mids8 | | mids9 | | mids10 | | mids14 | |
| 79 | days1 | days2 | days3 | days6 | days7 | days8 | mids8 | days9 | mids9 | days10 | mids10 | days14 | mids14 | |
| 80 | days1 | days2 | days3 | days6 | days7 | days8 | mids8 | days9 | mids9 | days10 | mids10 | days14 | mids14 | |
| 81 | days1 | days2 | days3 | days6 | days7 | days8 | | days9 | | days10 | | days14 | | |
| 82 | days1 | days2 | days3 | days6 | days7 | days8 | | days9 | | days10 | | days14 | | |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 83 | days1 | days2 | days3 | days6 | days7 | days8 | | days9 | | days10 | | days14 | | |
| 84 | days1 | days2 | days3 | days6 | days7 | days8 | | days9 | | days10 | | days14 | | |
| 85 | days1 | days2 | days3 | days6 | days7 | days8 | | days9 | | days10 | | days14 | | |
| 86 | days1 | days2 | days3 | days6 | days7 | days8 | | days9 | | days10 | | days14 | | |
| 87 | swings1 | swings2 | swings3 | swings6 | swings7 | days8 | swings8 | days9 | swings9 | days10 | swings10 | days14 | swings14 | |
| 88 | swings1 | swings2 | swings3 | swings6 | swings7 | days8 | swings8 | days9 | swings9 | days10 | swings10 | days14 | swings14 | |
| 89 | swings1 | swings2 | swings3 | swings6 | swings7 | | swings8 | | swings9 | | swings10 | | swings14 | |
| 90 | swings1 | swings2 | swings3 | swings6 | swings7 | | swings8 | | swings9 | | swings10 | | swings14 | |
| 91 | swings1 | swings2 | swings3 | swings6 | swings7 | | swings8 | | swings9 | | swings10 | | swings14 | |
| 92 | swings1 | swings2 | swings3 | swings6 | swings7 | | swings8 | | swings9 | | swings10 | | swings14 | |
| 93 | swings1 | swings2 | swings3 | swings6 | swings7 | | swings8 | | swings9 | | swings10 | | swings14 | |
| 94 | swings1 | swings2 | swings3 | swings6 | swings7 | | swings8 | | swings9 | | swings10 | | swings14 | |
| 95 | mids1 | mids2 | mids3 | mids4 | mids7 | mids8 | swings8 | mids9 | swings9 | mids10 | swings10 | mids11 | swings14 | |
| 96 | mids1 | mids2 | mids3 | mids4 | mids7 | swings8 | mids8 | swings9 | mids9 | swings10 | mids10 | swings14 | mids11 | |
| 97 | mids1 | mids2 | mids3 | mids4 | mids7 | | mids8 | | mids9 | | mids10 | | mids11 | |
| 98 | mids1 | mids2 | mids3 | mids4 | mids7 | | mids8 | | mids9 | | mids10 | | mids11 | |
| 99 | mids1 | mids2 | mids3 | mids4 | mids7 | | mids8 | | mids9 | | mids10 | | mids11 | |
| 100 | mids1 | mids2 | mids3 | mids4 | mids7 | | mids8 | | mids9 | | mids10 | | mids11 | |
| 101 | mids1 | mids2 | mids3 | mids4 | mids7 | | mids8 | | mids9 | | mids10 | | mids11 | |
| 102 | mids1 | mids2 | mids3 | mids4 | mids7 | | mids8 | | mids9 | | mids10 | | mids11 | |
| 103 | days1 | days2 | days3 | days4 | days7 | days8 | mids8 | days9 | mids9 | days10 | mids10 | days11 | mids11 | |
| 104 | days1 | days2 | days3 | days4 | days7 | days8 | mids8 | days9 | mids9 | days10 | mids10 | days11 | mids11 | |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 105 | days1 | days2 | days3 | days4 | days7 | days8 | | days9 | | days10 | | days11 | | |
| 106 | days1 | days2 | days3 | days4 | days7 | days8 | | days9 | | days10 | | days11 | | |
| 107 | days1 | days2 | days3 | days4 | days7 | days8 | | days9 | | days10 | | days11 | | |
| 108 | days1 | days2 | days3 | days4 | days7 | days8 | | days9 | | days10 | | days11 | | |
| 109 | days1 | days2 | days3 | days4 | days7 | days8 | | days9 | | days10 | | days11 | | |
| 110 | days1 | days2 | days3 | days4 | days7 | days8 | | days9 | | days10 | | days11 | | |
| 111 | swings1 | swings2 | swings3 | swings4 | swings7 | days8 | swings8 | days9 | swings9 | days10 | swings10 | days11 | swings11 | |
| 112 | swings1 | swings2 | swings3 | swings4 | swings7 | days8 | swings8 | days9 | swings9 | days10 | swings10 | days11 | swings11 | |
| 113 | swings1 | swings2 | swings3 | swings4 | swings7 | | swings8 | | swings9 | | swings10 | | swings11 | |
| 114 | swings1 | swings2 | swings3 | swings4 | swings7 | | swings8 | | swings9 | | swings10 | | swings11 | |
| 115 | swings1 | swings2 | swings3 | swings4 | swings7 | | swings8 | | swings9 | | swings10 | | swings11 | |
| 116 | swings1 | swings2 | swings3 | swings4 | swings7 | | swings8 | | swings9 | | swings10 | | swings11 | |
| 117 | swings1 | swings2 | swings3 | swings4 | swings7 | | swings8 | | swings9 | | swings10 | | swings11 | |
| 118 | swings1 | swings2 | swings3 | swings4 | swings7 | | swings8 | | swings9 | | swings10 | | swings11 | |
| 119 | mids1 | mids2 | mids3 | mids4 | mids5 | mids9 | swings8 | mids10 | swings9 | mids11 | swings10 | mids12 | swings11 | |
| 120 | mids1 | mids2 | mids3 | mids4 | mids5 | swings8 | swings9 | mids9 | swings10 | mids10 | swings11 | mids11 | mids12 | |
| 121 | mids1 | mids2 | mids3 | mids4 | mids5 | | | mids9 | | mids10 | | mids11 | mids12 | |
| 122 | mids1 | mids2 | mids3 | mids4 | mids5 | | | mids9 | | mids10 | | mids11 | mids12 | |
| 123 | mids1 | mids2 | mids3 | mids4 | mids5 | | | mids9 | | mids10 | | mids11 | mids12 | |
| 124 | mids1 | mids2 | mids3 | mids4 | mids5 | | | mids9 | | mids10 | | mids11 | mids12 | |
| 125 | mids1 | mids2 | mids3 | mids4 | mids5 | | | mids9 | | mids10 | | mids11 | mids12 | |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 126 | mids1 | mids2 | mids3 | mids4 | mids5 | | | mids9 | | mids10 | | mids11 | mids12 | |
| 127 | days1 | days2 | days3 | days4 | days5 | days9 | days10 | mids9 | days11 | mids10 | days12 | mids11 | mids12 | |
| 128 | days1 | days2 | days3 | days4 | days5 | days9 | days10 | mids9 | days11 | mids10 | days12 | mids11 | mids12 | |
| 129 | days1 | days2 | days3 | days4 | days5 | days9 | days10 | | days11 | | days12 | | | |
| 130 | days1 | days2 | days3 | days4 | days5 | days9 | days10 | | days11 | | days12 | | | |
| 131 | days1 | days2 | days3 | days4 | days5 | days9 | days10 | | days11 | | days12 | | | |
| 132 | days1 | days2 | days3 | days4 | days5 | days9 | days10 | | days11 | | days12 | | | |
| 133 | days1 | days2 | days3 | days4 | days5 | days9 | days10 | | days11 | | days12 | | | |
| 134 | days1 | days2 | days3 | days4 | days5 | days9 | days10 | | days11 | | days12 | | | |
| 135 | swings1 | swings2 | swings3 | swings4 | swings5 | days9 | days10 | swings9 | days11 | | days12 | | | |
| 136 | swings1 | swings2 | swings3 | swings4 | swings5 | days9 | days10 | swings9 | days11 | swings10 | days12 | swings11 | swings12 | |
| 137 | swings1 | swings2 | swings3 | swings4 | swings5 | | | swings9 | | swings10 | | swings11 | swings12 | |
| 138 | swings1 | swings2 | swings3 | swings4 | swings5 | | | swings9 | | swings10 | | swings11 | swings12 | |
| 139 | swings1 | swings2 | swings3 | swings4 | swings5 | | | swings9 | | swings10 | | swings11 | swings12 | |
| 140 | swings1 | swings2 | swings3 | swings4 | swings5 | | | swings9 | | swings10 | | swings11 | swings12 | |
| 141 | swings1 | swings2 | swings3 | swings4 | swings5 | | | swings9 | | swings10 | | swings11 | swings12 | |
| 142 | swings1 | swings2 | swings3 | swings4 | swings5 | | | swings9 | | swings10 | | swings11 | swings12 | |
| 143 | | mids2 | mids3 | mids4 | mids5 | mids6 | mids10 | swings9 | mids11 | swings10 | mids12 | swings11 | swings12 | mids13 |
| 144 | mids2 | mids3 | mids4 | mids5 | mids6 | swings9 | swings10 | mids10 | swings11 | mids11 | swings12 | mids12 | | mids13 |
| 145 | mids2 | mids3 | mids4 | mids5 | mids6 | | | mids10 | | mids11 | | mids12 | | mids13 |
| 146 | mids2 | mids3 | mids4 | mids5 | mids6 | | | mids10 | | mids11 | | mids12 | | mids13 |
| 147 | mids2 | mids3 | mids4 | mids5 | mids6 | | | mids10 | | mids11 | | mids12 | | mids13 |

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| 148 | mids2 | mids3 | mids4 | mids5 | mids6 | | | mids10 | | mids11 | | mids12 | | mids13 |
| 149 | mids2 | mids3 | mids4 | mids5 | mids6 | | | mids10 | | mids11 | | mids12 | | mids13 |
| 150 | mids2 | mids3 | mids4 | mids5 | mids6 | | | mids10 | | mids11 | | mids12 | | mids13 |
| 151 | days2 | days3 | days4 | days5 | days6 | days10 | | mids10 | days11 | mids11 | days12 | mids12 | days13 | mids13 |
| 152 | days2 | days3 | days4 | days5 | days6 | days10 | | mids10 | days11 | mids11 | days12 | mids12 | days13 | mids13 |
| 153 | days2 | days3 | days4 | days5 | days6 | days10 | | | days11 | | days12 | | days13 | |
| 154 | days2 | days3 | days4 | days5 | days6 | days10 | | | days11 | | days12 | | days13 | |
| 155 | days2 | days3 | days4 | days5 | days6 | days10 | | | days11 | | days12 | | days13 | |
| 156 | days2 | days3 | days4 | days5 | days6 | days10 | | | days11 | | days12 | | days13 | |
| 157 | days2 | days3 | days4 | days5 | days6 | days10 | | | days11 | | days12 | | days13 | |
| 158 | days2 | days3 | days4 | days5 | days6 | days10 | | | days11 | | days12 | | days13 | |
| 159 | swings2 | swings3 | swings4 | swings5 | swings6 | days10 | swings10 | swings11 | days11 | swings12 | days12 | swings13 | days13 | |
| 160 | swings2 | swings3 | swings4 | swings5 | swings6 | days10 | swings10 | swings11 | days11 | swings12 | days12 | swings13 | days13 | |
| 161 | swings2 | swings3 | swings4 | swings5 | swings6 | | swings10 | swings11 | | swings12 | | swings13 | | |
| 162 | swings2 | swings3 | swings4 | swings5 | swings6 | | swings10 | swings11 | | swings12 | | swings13 | | |
| 163 | swings2 | swings3 | swings4 | swings5 | swings6 | | swings10 | swings11 | | swings12 | | swings13 | | |
| 164 | swings2 | swings3 | swings4 | swings5 | swings6 | | swings10 | swings11 | | swings12 | | swings13 | | |
| 165 | swings2 | swings3 | swings4 | swings5 | swings6 | | swings10 | swings11 | | swings12 | | swings13 | | |
| 166 | swings2 | swings3 | swings4 | swings5 | swings6 | | swings10 | swings11 | | swings12 | | swings13 | | |
| 167 | | mids3 | mids4 | mids5 | mids6 | mids7 | swings10 | swings11 | mids11 | swings12 | mids12 | swings13 | mids13 | mids14 |