We consider a model that provides insight into the well-known Folk theorem in economics that when the discount factor $\beta$ is sufficiently close to 1, expropriation will never occur. Although this Folk theorem is true in our model, our perspective is different. The discount factor $\beta$ often is described as a “deep structural parameter” that is difficult to alter at a point in time. In contrast, we analyze the determinants of two thresholds $\beta$ and $\beta^*$ that segment the unit interval on which $\beta$ is defined into three subintervals. These subintervals correspond to the three possible equilibria for investment flows: autarky, underinvestment, and unconstrained optimal investment. These thresholds are of interest because they can be altered by specific policy interventions. As a consequence, even if $\beta$ is small, some level of foreign investment can be supported. We construct measures of $\beta$ for 40 countries, characterize $\beta^*$ and $\beta^*$, and discuss recent trends in investment flows.

Keywords: Intertemporal Enforcement, Foreign Investment, Discount Factor

1. INTRODUCTION

In a seminal paper, Eaton and Gersovitz (1981) showed that underinvestment might occur when foreign capital is subject to endogenous expropriation risk. They focused on settings in which borrowers simply may choose not to repay their debt. Rational investors anticipate this, which imposes a repayment incentive constraint on the optimal contracting problem that causes investment to fall short of the unconstrained optimal level. We focus on the implications of this “pure enforceability problem,” where underinvestment occurs because of an unwillingness, rather than an inability, to pay. Because there is no supranational legal authority that can enforce contracts across borders, the “tightness” of the incentive constraint can be quite severe in an international setting. North (1990, p. 54) argues that “the
inability of societies to develop effective, low-cost enforcement of contracts is the most important source of both historical stagnation and contemporary under development in the Third World.”

We consider a model of investment when enforcement is imperfect. The paper\(^2\) (i) characterizes the determinants of a country’s discount factor \(\beta\); (ii) characterizes when unconstrained foreign investment, underinvestment, and no foreign investment will occur by constructing two thresholds on the unit interval; and (iii) characterizes the determinants of the incentive constraint.

The results show that a country’s investment problem can be reduced to a comparison between the country’s discount factor \(\beta\) and an interval with three regions that correspond to autarky, underinvestment, and unconstrained investment. These intervals are determined by critical thresholds \(\beta^0\) and \(\beta^*\) that we construct. Further, we show that, when enforcement is imperfect, country-specific factors are crucial determinants of investment flows. This differs from the case of perfect enforcement in which the interest rate alone is sufficient.

The model consists of a host country whose firms finance projects by borrowing from abroad at an interest rate determined by competitive world markets. Projects are operated in the host country (i.e., foreign investors do not control projects) and domestic firms are unable to credibly commit to honor investment agreements. As a consequence, the Pareto-efficient investment plan must be time consistent. When this constraint binds, the optimal investment plan need not equate the marginal product of capital to its return. Section 2 specifies the model and equilibria and analyzes the incentive constraint. Section 3 contains some facts about private market investment flows that motivate the analysis. Finally, Section 4 concludes.

2. THE MODEL AND EQUILIBRIA

Consider an economy with an infinite time horizon and agents of two types: domestic and foreign. Domestic agents are identical and have a common risk-neutral utility function.\(^3\) The domestic country has access to a constant returns to scale production function \(F(\cdot)\), which requires two inputs, labor and capital. Let \(f(\cdot)\) denote output per capita, which satisfies the Inada conditions. Labor is provided inelastically by domestic residents. Risk-neutral foreign agents provide capital to supplement domestic capital and earn a competitive return given by the gross world interest rate \(r\), which we assume is constant over time. Let \(k\), \(k^f\), and \(k^d\) be the aggregate, foreign, and domestic capital stock per capita, respectively. Assume that capital depreciates completely, that the domestic capital stock is constant, and that the foreign capital stock is elastic (i.e., we consider a small open economy). Assume further that the country’s domestic capital stock is less than the optimal capital stock. The domestic country does not invest abroad, and its capital stock neither depreciates nor can be augmented. As a consequence, the country’s current output net of interest payments is consumed in the current period.
The host country chooses an investment plan to maximize the present discounted utility of income denoted by

$$W_t = \sum_{s=t}^{\infty} \beta^{s-t} y_s,$$

with $0 < \beta < 1$ and $y_t = f(k_t) - r(k_t - k^d)$. As is standard [cf., Yaari (1965)], $\beta$ is the discount factor with $\beta = \theta \rho$, where $\theta$ is the probability of survival, an idiosyncratic factor that reflects the “patience” of decision makers in a particular country, and $\rho = 1/r$ is the common pure discount factor determined by the world market. As the world interest rate rises, $\beta$ falls, indicating that all countries value the future less highly. In contrast, as $\theta$ falls, the country-specific $\beta$ falls, indicating that the country behaves as if it has become more myopic. Following Barro and Sala-i-Martin (1995, p. 439), $\theta$ can be interpreted as a measure of country-specific risk.4

The host country faces a stationary problem; thus the optimal investment plan is time invariant. In any period $t$, the host has the option to renege on foreign-investment agreements. The consequence of this action is that the host loses access to international capital markets in subsequent periods. If default occurs in any period $t$, for all future periods $s > t$, foreign investors abstain from the market (i.e., $k_s = k^d$ and $k^f_s = 0$). Clearly, the need for international capital provides some incentive for borrowers to honor investment agreements in intertemporal problems, and the host will expropriate only if the gain exceeds the cost. Because countries that expropriate eventually return to world capital markets, the punishment of complete exclusion in the future may seem severe. We show that when agents discount the future highly, however, even this severe punishment is not strong enough to deter default.

We now analyze the incentive constraint. The discounted present value from reneging on an agreement in period $t$ and remaining autarkic thereafter is given by

$$B(k_t) = f(k_t) + \sum_{s=t+1}^{\infty} \beta^{s-t} y_s = f(k) + \frac{\beta}{1 - \beta} f(k^d).$$

The discounted present value of not reneging on an agreement in period $t$ and maintaining access to international capital markets is given by

$$G(k_t) = \sum_{s=t}^{\infty} \beta^{s-t} y_s = \sum_{s=t}^{\infty} \beta^{s-t} [f(k_s) - r(k_s - k^d)] = \frac{1}{1 - \beta} [f(k) - r(k - k^d)].$$

As a consequence of stationarity, $B(k)$ and $G(k)$ are time invariant.

The problem solved by a benevolent social planner now can be specified. The planner chooses a stationary level of aggregate investment $k$ to maximize $W(k)$ subject to a repayment incentive constraint that ensures that the host will honor investment agreements, where $k^d$ is given and $k = k^d + k^f$. 
Problem 1. Choose $k$ to maximize

$$W(k) = \frac{1}{1-\beta} [f(k) - r(k - k^d)]$$

subject to

$$B(k) \leq G(k), \quad \forall k.$$   \hspace{1cm} (1)

The host maximizes the discounted utility of output subject to incentive constraint (1). When (1) holds, contracts are self-enforcing. Let $\lambda$ be the Lagrangian multiplier on (1). The first-order conditions are $B(k) = G(k)$ and

$$f'(k) - r = \frac{r\lambda(1-\beta)}{1+\lambda \beta}.$$   \hspace{1cm} (2)

Define the right-hand side of (2) by $\Omega(\beta, \lambda, r) = \frac{r\lambda(1-\beta)}{1+\lambda \beta}$, where $\Omega$ is the wedge between the marginal product of capital and the return on capital, which is the efficiency loss due to inadequate contractual enforcement. When enforcement is imperfect and (1) binds, this causes $\lambda > 0$, and the well-known underinvestment result follows immediately. We focus on the determinants of this inefficiency when enforcement is imperfect. Clearly, it is desirable for $\Omega(\beta, \lambda, r) \rightarrow 0$, which occurs as

(i) $\beta \rightarrow 1$: There is less inefficiency when agents care more about the future.

(ii) $r \rightarrow 0$: Low interest rates reduce inefficiency.

(iii) $\lambda \rightarrow 0$: There is less inefficiency when the constraint is weak.

We begin by characterizing the solutions to Problem 1, which are described by one of three cases. Let $k^u_k$ denote the optimal capital sequence when (1) does not bind (the unconstrained optimal plan) and $k^c_k$ denote the plan when (1) binds (the constrained optimal plan). Then:

Case 1. When $B(k) > G(k)$ for all $k > k^d$, the constraint set is empty and no foreign investment occurs.

Case 2. When the constraint binds (i.e., $\lambda > 0$), $f'(k) > r$ and $k = k^c_k$ is the optimal investment plan.

Case 3. When the constraint does not bind (i.e., $\lambda = 0$), $f'(k) = r$ and $k = k^u_k$ is the optimal investment plan.

In Case 1, the benefit from expropriation exceeds the gain from not expropriating for all levels of investment. As a result, the host always will expropriate. Foreign investors realize the incentive for the host to expropriate; hence, in equilibrium, no foreign investment occurs. In Case 2, the constraint binds and this leads to underinvestment relative to the unconstrained optimal plan (i.e., Case 3). Underinvestment is optimal, given that (1) binds, as this is a version of Kydland and Prescott’s (1977) time-consistency result: The constrained equilibrium is Pareto inferior relative to the unconstrained equilibrium, where there is full commitment to the ex-ante optimal plan. The problem is that no “commitment technology”
exists that can induce agents to adhere to the “first best” intertemporal plan ex post [cf., Krasa and Villamil (2000)].

When there is no supranational legal authority (e.g., court) that can enforce contracts across borders, contracts must be self-enforcing. As in Atkeson (1991), Eaton (1993), and Thomas and Worrall (1994), a contract is self-enforcing if it is incentive compatible for the agent to honor prior commitments (i.e., to not expropriate). The set of self-enforcing investment plans in our model is characterized as follows: First note that $B'(k) = f'(k) > 0$ and $B''(k) < 0$. Further, $G'(k^*_u) = 0$ and $G''(k) < 0$. As a consequence, $k^*_u$ maximizes $G(k)$. Also note that $G(k)$ and $B(k)$ intersect at $k^d$. Figures 1, 2, and 3 illustrate the three possible cases of the solution to Problem 1.

Given the self-enforcing interval,

(i) if $k^*_u \in [k^d, \bar{k})$, then (1) does not bind and $k^*_u$ is optimal;
(ii) if $k^*_u \notin [k^d, \bar{k}]$, then (1) binds, $B(k) = G(k)$, and $k^*_u = \bar{k}$ is the maximum investment level for which $k$ is self-enforcing.

The upper bound of the interval, $\bar{k}$, can be interpreted as a debt ceiling. If $k^*_u > \bar{k}$, the country faces a borrowing constraint. If $k^*_u \leq \bar{k}$, the country can borrow as much as it desires.

Finally, it is evident in Problem 1 that the host country’s discounted utility (i.e., welfare) depends on its domestic capital stock, $k^d$, and its idiosyncratic discount factor $\beta = \theta / \mu$, regardless of whether enforcement is a problem. When enforcement is a problem, however, country-specific characteristics $k^d$ and $\beta(\theta, \mu)$ play
FIGURE 2. Case 2: Constrained optimal equilibrium.

an additional role: They determine, in conjunction with \( r \), a country’s ability to attract foreign capital, \( k_f \). In contrast, when enforcement is perfect, \( r \) is sufficient to determine capital flows, and each country receives the unconstrained optimal level of investment, \( k_u^* \). This is a crucial observation, which motivates the analysis that follows. We now show how the optimal investment plan is affected by these “fundamentals,” and construct \( \theta \) and \( \beta(\theta, r) \) for 40 countries.

### 2.1. Threshold Discount Factors

In this section we show that the solutions to Problem 1 can be characterized completely by restrictions on the discount factor that segment the unit interval into the three cases. The equilibrium then is determined completely by a comparison of \( \beta \) and the relevant case on the unit interval. It follows from \( B(k) \leq G(k) \) that an investment plan \( k \) is self-enforcing if and only if the following restriction on \( \beta \) is satisfied:

\[
0 < \frac{r(k - k^d)}{f(k) - f(k^d)} \leq \beta < 1.
\]  

(3)

Define two critical thresholds on the unit interval \((0, 1)\) by

\( \beta^* \), the minimum discount factor for which plan \( k_u^* \) is self-enforcing; and

\( \beta_\star \), the minimum discount factor required to attract foreign investment.

Proposition 1 shows that these thresholds \( \beta_\star \) and \( \beta^* \) segment the unit interval into three subintervals, which correspond to Cases 1, 2, and 3 such that \( 0 < \beta_\star < \beta^* < 1 \).

A foreign agent’s investment decision is thus effectively a comparison of the host country’s idiosyncratic \( \beta(\theta, r) \) and the relevant subinterval: Case 1 prevails when \( \beta \in (0, \beta_\star) \), Case 2 prevails when \( \beta \in [\beta_\star, \beta^*) \), and Case 3 prevails when \( \beta \in [\beta^*, 1) \).

**Proposition 1.** The threshold discount factors \( \beta_\star, \beta^* \in (0, 1) \) are critical determinants of the solution with \( \beta_\star < \beta^* \) such that

- **Case 1.** For \( \beta(\theta, r) \in (0, \beta_\star) \), there is no foreign investment (autarky).
- **Case 2.** For \( \beta(\theta, r) \in [\beta_\star, \beta^*) \) investment is constrained optimal \( (k^*_u) \).
- **Case 3.** For \( \beta(\theta, r) \in [\beta^*, 1) \), investment is unconstrained optimal \( (k^*_u) \).

We define a country’s discount factor \( \beta(\theta, r) \) as “low” if it is in the Case-1 interval, “moderate” if it is in the Case-2 interval, and “high” if it is in the Case-3 interval. The three possible outcomes in Proposition 1 are depicted clearly in Figure 4.

\[
\begin{array}{ccc}
\text{Case 1} & \text{Case 2} & \text{Case 3} \\
(\quad) & (\quad) & (\quad)
\end{array}
\]

\[
\begin{array}{c|c|c|c}
0 & \beta_\star & \beta^* & 1 \\
\hline
\end{array}
\]

**Figure 4.** Possible outcomes of Proposition 1.
Proposition 1 indicates that countries with low discount factors will have difficulty attracting foreign investment. Recall that $\beta = \theta / r$, where $1/r$ is the pure (market) discount factor and $\theta$ is the probability of survival. By linking the discount factor $\beta$ to the country-specific $\theta$, Proposition 1 provides a plausible explanation for the inability of some countries to attract foreign investment. When $\beta(\theta, r)$ is very low, the country is more likely to be in the low Case-1 equilibrium with $\beta(\theta, r) \in (0, \beta)$. Barro and Sala-i-Martin’s (1995, p. 440) and Mauro’s (1995) empirical results indicate that effective legal and political institutions are statistically significant for explaining growth. These results are consistent with the model. Effective institutions tend to raise $\beta(\theta, r)$, thus increasing the likelihood that a country is in Case 2 or 3. As a consequence, the country will be able to sustain some level of foreign investment. Similarly, it is well known that stability is correlated with successful development and that instability is associated with “growth disasters” [cf., Parente and Prescott (1986)]. This result is also consistent with the predictions of the model. Figure 5 illustrates clearly the impact that $\beta$ has on private foreign investment flows.

Proposition 1 and Figure 5 indicate that, when agents discount the future highly, the threat of permanent and complete exclusion from international capital markets is not sufficient to deter default. Rational investors anticipate this problem and, as a result, no foreign investment occurs. For moderate levels of the discount factor, the threat of permanent exclusion can sustain some amount of investment. Thus, Proposition 1 reconciles the apparently contradictory results of Cohen (1991) and Eaton (1993). Cohen (p. 94) argues that the threat of complete financial autarky is never enough to prevent default. Bulow and Rogoff (1989) obtain a similar result when the punishment is partial financial autarky. We arrive at a similar conclusion for Case 1, where $\beta \in (0, \beta)$. In contrast, Eaton (1993), on the basis of a model where agents borrow for consumption smoothing, concludes that maintaining access to credit markets is a sufficient reason to honor contracts. This is consistent with our results for Case 2, where $\beta \in [\beta, \beta^*)$. Thus both results are consistent with our model in theory. Given $\beta$ and $\beta^*$, the empirically relevant case will be determined by the magnitude of $\beta$. We consider this issue in the next section.

Finally, note that our model provides some insights into the well-known Folk theorem that, for $\beta$ sufficiently close to 1, expropriation will never occur [cf., Chari and Kehoe (1990)]. Although this Folk theorem is true in our model, our perspective is different. Specifically, $\beta(\theta, r)$ often is described as a “deep structural parameter” that is difficult to alter at a point in time. In contrast, in the next section, we analyze the determinants of the thresholds $\beta$ and $\beta^*$, and hence the length of the subintervals in Figure 4. We show that these thresholds can be altered by specific policy interventions. As a consequence, the Case 2 and 3 intervals in Figure 4 can be made bigger. When this type of policy intervention is possible, then even if $\beta(\theta, r)$ is small, some level of foreign investment may be supported. Further, because Figure 5 indicates that the level of private foreign investment increases exponentially in Case 2, changes in $\beta$ on this interval can lead to volatility in investment flows.
FIGURE 5. The discount factor and private foreign investment.
2.2. Calibration

In Table 1, we present estimates of $\theta$ and $\beta$ for 40 countries. The estimate for $\theta$ is constructed from five risk indicators, averaged over the period 1990–1995 using data from the *International Country Risk Guide*. The risk indicators evaluate contract repudiation, expropriation risk, government corruption, the rule of law, and the quality of the bureaucracy. We use the average of the five indicators to construct $\theta$, and hence $\beta$, for these countries. Indeed, $\beta$ varies from a high of 0.95 to a low of 0.23. In contrast, when $\beta = 1/r$, a frequent calibration in many real business-cycle models, $\beta = 0.95$ is the common discount factor for all countries.

Table 2 provides data on loans to 24 low- and medium-$\beta$ countries over the period 1989–1996. One of the striking features of the data is the strong association between the estimated discount factor and private foreign investment. This relationship is depicted clearly in Figure 6, the scatter plot of the share of foreign investment provided by private entities versus the discount factor, which follows Table 2. The correlation coefficient for the two variables is quite high (0.77) and is significant at 5%. Furthermore, five of the countries, namely, Burkina Faso, Congo, Haiti, Swaziland, and Togo, are in the autarky (Case 1) equilibrium, and did not receive any private foreign investment. Note that the average estimated discount factor for these countries is quite low, equaling 0.39. These results are an indication of the importance of the magnitude of $\beta$ for determining private investment flows.

2.3. Comparative Statics

We now consider how changes in “fundamentals” affect the length of each of the subintervals in Figure 4. We first ask how changes in a country’s domestic capital stock $k^d$ and the world interest rate $r$ affect $\beta$ and $\beta^\ast$. From the proof of Proposition 1 (in the Appendix), the threshold discount factors are determined by $k^d$ and $r$, and are given by

$$\beta(k^d, r) = \frac{r}{f'(k^d)},$$

$$\beta^\ast(k^d, r) = \frac{r [k^\ast(r) - k^d]}{f(k^\ast(r)) - f(k^d)}.$$  

Claim 1. $k^d$ and $r$ affect the threshold discount factors as follows:

(a) $\frac{d\beta}{dk^d} > 0$ and $\frac{d\beta^\ast}{dk^d} > 0$.

(b) $\frac{d\beta}{dr} > 0$ and $\frac{d\beta^\ast}{dr}$ is ambiguous.

Part (a) indicates that capital-poor countries have relatively lower threshold discount factors than countries with large domestic capital stocks. This result may seem odd at first glance. However, a low domestic capital stock implies that most of
### Table 1. Country risk indicators

<table>
<thead>
<tr>
<th>Country</th>
<th>Contract repudiation</th>
<th>Expropriation risk</th>
<th>Corruption</th>
<th>Rule of law</th>
<th>Bureaucratic quality</th>
<th>Average risk, $\theta^b$</th>
<th>Discount factor, $\beta$</th>
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<td>6.0</td>
<td>6.0</td>
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**Table 1.** (Continued.)

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<td>6.8</td>
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<td>2.5</td>
<td>4.0</td>
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<td>8.4</td>
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<td>0.60</td>
<td>0.57</td>
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<td>7.2</td>
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<td>2.4</td>
<td>2.0</td>
<td>0.45</td>
<td>0.42</td>
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<tr>
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<td>3.3</td>
<td>3.0</td>
<td>4.0</td>
<td>0.62</td>
<td>0.59</td>
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<td>2.7</td>
<td>2.0</td>
<td>0.47</td>
<td>0.44</td>
</tr>
<tr>
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<td>8.5</td>
<td>2.5</td>
<td>2.7</td>
<td>4.0</td>
<td>0.61</td>
<td>0.58</td>
</tr>
<tr>
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<td>2.5</td>
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<td>0.48</td>
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<td>1.5</td>
<td>0.34</td>
<td>0.32</td>
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<tr>
<td>Togo</td>
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<td>2.3</td>
<td>2.0</td>
<td>0.45</td>
<td>0.43</td>
</tr>
<tr>
<td>Zambia</td>
<td>2.6</td>
<td>7.2</td>
<td>3.1</td>
<td>2.4</td>
<td>2.0</td>
<td>0.45</td>
<td>0.42</td>
</tr>
</tbody>
</table>

*Contract repudiation and expropriation risk fall in the range of 1 to 10; other indicators range from 1 to 6.*

*Average risk $\theta$ is computed by averaging each risk indicator over the period 1990–1995, dividing each indicator by its maximum (6 or 10), and taking the average of all the risk indicators.*
## TABLE 2. Loan information for selected low- and medium-$\beta$ countries$^a$

<table>
<thead>
<tr>
<th>Country</th>
<th>Discount factor, $\beta$</th>
<th>Official</th>
<th>Private</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>% of Total loans$^b$</td>
<td>Interest rate</td>
</tr>
<tr>
<td>Argentina</td>
<td>0.62</td>
<td>43</td>
<td>6.99</td>
</tr>
<tr>
<td>Bangladesh</td>
<td>0.36</td>
<td>95</td>
<td>1.16</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.67</td>
<td>40</td>
<td>6.99</td>
</tr>
<tr>
<td>Burkina Faso</td>
<td>0.47</td>
<td>100</td>
<td>1.27</td>
</tr>
<tr>
<td>Cameroon</td>
<td>0.55</td>
<td>93</td>
<td>4.29</td>
</tr>
<tr>
<td>Chile</td>
<td>0.65</td>
<td>68</td>
<td>6.13</td>
</tr>
<tr>
<td>China</td>
<td>0.66</td>
<td>33</td>
<td>5.44</td>
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<tr>
<td>Colombia</td>
<td>0.57</td>
<td>39</td>
<td>7.03</td>
</tr>
<tr>
<td>Congo</td>
<td>0.42</td>
<td>100</td>
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<tr>
<td>Ghana</td>
<td>0.59</td>
<td>81</td>
<td>1.73</td>
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<tr>
<td>Haiti</td>
<td>0.23</td>
<td>100</td>
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</tr>
<tr>
<td>India</td>
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<td>67</td>
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<td>Indonesia</td>
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<td>Malaysia</td>
<td>0.61</td>
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<td>Nigeria</td>
<td>0.47</td>
<td>82</td>
<td>5.58</td>
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<tr>
<td>Peru</td>
<td>0.49</td>
<td>97</td>
<td>6.16</td>
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<tr>
<td>Philippines</td>
<td>0.46</td>
<td>64</td>
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<tr>
<td>Russia</td>
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</tr>
<tr>
<td>Sierra Leone</td>
<td>0.32</td>
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<tr>
<td>Zambia</td>
<td>0.42</td>
<td>89</td>
<td>1.84</td>
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</tbody>
</table>

$^a$ All data were obtained from the Global Development Finance [cf., World Bank (1998a)].

$^b$ Official loans include multilateral and bilateral loans.

$^c$ Private loans include bonds, commercial banks, and other private creditors.

The host country’s production is financed by foreign investment. As a consequence, foreign investment is relatively essential. Indeed, the plausible intuition for the result is that, when foreign investment is relatively essential, a forward-looking host will not expropriate even when it values the future in only a limited way. This intuition is reminiscent of Alexander Hamilton’s views on sovereign debt. He claimed that in order for governments to honor their debt they must have a “commonality of interest” with debt holders [cf., Smith and Villamil (1998)]. A similar argument applies in this case. If the host country and foreign investors have a commonality of interest because foreign investment is essential, the host will not expropriate even when it cares about the future only in a limited way. When it is not essential (i.e., $k_d$ is high), the threshold discount factors are high and this makes it more difficult to attract $k_f$. 

Figure 6. Private foreign loans and the discount factor.
An important question arises from this comparative static result: If poor countries require relatively small discount factors to attract foreign investment, why are most poor countries, such as countries in Sub-Saharan Africa, unable to attract foreign investment? The answer in our model is that the country-specific discount factors of these countries, $\beta(\theta, r)$, are lower than the critical threshold $\beta$, perhaps because their idiosyncratic survival probabilities $\theta$ are low. That is, these countries do not achieve the threshold required to attract foreign investment because $\beta(\theta, r) < \beta$. Our results suggest that a need for foreign capital in the future can have good incentive effects, but a minimum threshold must be attained before this effect is operative. Put differently, $\beta$ need not necessarily tend to 1 as in the Folk theorem, but it must at least attain $\beta$.

The result $d\beta/dr > 0$ in part (b) implies that, when the world interest rate rises, more countries will revert to the autarky equilibrium because higher world interest rates literally "raise the bar" $\beta$ for these countries to attract foreign investment. In contrast, the sign of $d\beta*/dr$ is ambiguous. When $d\beta*/dr > 0$, an increase in the world interest rate depresses international capital flows because it increases the Case 2 interval. This is undesirable because it shrinks the Case 3 interval where the unconstrained optimal level of investment occurs. However, $d\beta*/dr < 0$ is problematic as well. Although $\beta^*$ decreases, and this is a "good" effect because it lengthens the Case 3 interval, the country-specific discount factor $\beta(\theta, r)$ decreases, which impedes private investment. The net effect on private investment is unclear when both $\beta(\theta, r)$ and the lengths of the intervals change.

We now explore factors that affect the incentive constraint (i.e., factors that tend to tighten or loosen the constraint, ceteris paribus). We begin by noting that when (1) binds, then $B(k) = G(k)$, and the following equation holds:

$$\beta \left[ f \left( k^* \right) - f \left( k^d \right) \right] - r \left( k^*_c - k^d \right) = 0. \tag{6}$$

We use (6) to obtain a series of comparative static results. These results characterize how the constrained optimal investment plan $k^*_c$ changes when there are changes in the level of the domestic capital stock $k^d$, the country-specific discount factor $\beta(\theta, r)$, and the world interest rate $r$. These results are summarized in Claim 2.

Claim 2. When (1) binds, the constrained optimal level of investment has the following properties:

1. \( \frac{dk^*_c}{dk^d} < 0. \)
2. \( \frac{dk^*_c}{d\beta} > 0. \)
3. \( \frac{dk^*_c}{dr} < 0. \)

First suppose that the country can affect the domestic capital stock $k^d$, for example, via policies designed to increase savings. Claim 2(a) indicates that when equation (1) binds, an increase in the domestic capital stock decreases the constrained...
optimal level of investment by crowding out foreign investment more than one for one. This occurs because an increase in $k^d$ increases the host’s incentive to default (i.e., tightens the constraint) because foreign investment is less essential. Foreign investors, realizing this, reduce their investment further, causing a decrease in the total amount of investment.

Now suppose that the country can affect $\beta$, for example, via measures that improve one or more of the five risk factors in Table 1. Claim 2(b) indicates that when equation (1) binds, an increase in the country-specific discount factor $\beta(\theta, r)$ leads to an increase in the constrained optimal level of investment. If the host values the future more highly, then the host is less likely to default. This weakens (1) and allows for better outcomes because we effectively differentiate with respect to $\theta$ for a given $r$.

Now consider the effect of a change in the world interest rate on the constrained optimal level of investment, where $r$ is exogenous to all agents. Claim 2(c) shows that when equation (1) binds, an increase in the world interest rate leads to a decrease in the constrained level of investment. This result is not surprising, but the means by which it occurs differs from the standard argument. As the interest rate rises in our model, the net gain to the host from default increases. This tightens the incentive constraint and depresses foreign investment. Because domestic capital is fixed, total investment falls.

One explanation for the surge in capital flows to developing countries in this decade is that inflows are “pushed” by prevailing conditions in industrialized countries, especially low interest rates. Some predict that an increase in the world interest rate will result in a reversal of these massive capital flows, causing a “capital crunch” in developing countries. This explanation is based on two assumptions: (i) investment is driven solely by higher returns, and (ii) capital is scarce. In contrast, if increased investment in developing countries does not significantly affect the capital stock in the rest of the world, then our model provides an alternative explanation for the decrease in foreign investment when world interest rates are high. High interest rates reduce national income and therefore increase the net gain from expropriation when enforcement of contracts is inadequate. Using a willingness-to-pay Probit model, Lee (1991) finds that the probability that a country will default increases with the interest rate (based on data from 75 developing countries during 1970–1985). This finding is consistent with our model. Rational investors anticipate the increase in expropriation risk [i.e., the tightness of (1)] and therefore invest less.

The increase in expropriation risk that results from high interest rates also provides a plausible explanation for the differential default rates observed during the 1980s. During that period, tight monetary policies in industrialized countries caused interest rates to soar. The default rates were highest among countries whose external debt was owed to private investors such as commercial banks. Relatively lower default rates were observed for countries that relied on official sources for investment, notably the World Bank and IMF. Lee (1993) shows that during the period 1979–1987, the default probability was higher for countries from Latin America and the Caribbean who owed a substantial portion of their debt to
private investors than for poor African countries, whose external debt was owed
mainly to multilateral and bilateral institutions. Loans from official sources gener-
ally carry below-market rates; hence this provides a plausible explanation for the
lower default rates. See Table 2 for data on the interest rates charged on official
loans.

Finally, another implication of claim 2(c) is that, when private investment is
subject to expropriation risk, luring investors with high returns may be dangerous.
High returns increase the net gain from expropriation, which in turn results in a
decline in investment. Thus, as intuition would suggest, our model shows that high
returns alone are not enough to induce foreign investment in a country with a high
level of expropriation risk.

3. PRIVATE FOREIGN INVESTMENT: STYLIZED FACTS

In this section we summarize some stylized facts about private foreign investment
that motivate our analysis. Private foreign investment can be decomposed into two
components, foreign direct investment (FDI) and indirect foreign investment (IFI).
FDI is investment made to acquire a management interest (usually 10% of voting
stock) in a foreign enterprise. In contrast, IFI investors hold debt or own less than
10% equity in projects that are operated by the government or a private entity in
the host country. IFI includes commercial bank loans, bonds, and portfolio equity
flows. Thus, FDI involves “control rights” but IFI does not [cf., Aghion and Bolton
(1992)]. Our model focuses on IFI, and is consistent with the following stylized
facts.

1. **IFI is an important source of finance to developing countries:** IFI to developing
countries has increased from $18.2 billion in 1990 (18% of total foreign investment)
to $128 billion in 1996 (45% of total foreign investment).

2. **IFI is highly susceptible to expropriation:** FDI involves control rights and therefore
foreign investors can engage in “defensive” acts to minimize expropriation risk. See
Eaton and Gersovitz (1983) for specific strategies that firms can use to decrease FDI
expropriation risk (e.g., if the firm is a monopolist, it can locate production in different
countries to drive the firm’s present value to zero if expropriation occurs). In contrast,
it is difficult for IFI investors to engage in defensive actions. They could add restrictive
covenants to deter expropriation, but these contracts are difficult to enforce. It is this
underlying enforcement problem that creates the expropriation risk, and may explain
the inability of some countries to attract IFI despite their relative success in attracting
FDI. For example, Nigeria ranks among the top 12 developing countries in terms of
FDI, with FDI increasing from about $400 million each year over the period 1981–
1985, to $723 million over the period 1986–1990, and to $1.1 billion over the period
1990–1994. Despite this significant rise in FDI, it received no IFI until 1994, when it
received a meager $17 million.

3. **IFI is highly concentrated in geographic areas and countries:** In 1996, 48% of bond
issues and loans to developing countries went to Latin America compared to 13%
for Europe and Central Asia and 4% for Sub-Saharan Africa. This suggests that
country-specific factors are important (i.e., interest rates are not sufficient to determine
investment flows).
4. IFI investment is volatile: The World Bank documents in Global Economic Prospects that five East Asian countries shifted from a net inflow of $94 billion in 1996 to a net outflow of $6 billion in 1997 [cf., World Bank (1998b)]. This volatility is consistent with the exponential structure of $\beta$.

4. CONCLUSIONS

This paper examines factors that determine a country’s ability to attract foreign private investment when contractual enforcement is problematic. We find that when enforcement is imperfect, a country’s fundamental characteristics affect its ability to borrow from abroad. By analyzing the incentive constraint that the country faces, we derive two thresholds, $\beta(r, k^d)$ and $\beta^*(r, k^d)$, that segment the unit interval on which the “standard” discount factor $\beta(\theta, r)$ is defined into three disjoint subintervals. These subintervals determine the optimal private investment flows. The analysis effectively reduces the foreign investment problem to a comparison of the country-specific discount factor $\beta(\theta, r)$ with each of the subintervals: If $\beta(\theta, r) \in (0, \beta)$, autarky occurs; if $\beta(\theta, r) \in [\beta, \beta^*)$, investment is constrained; and if $\beta(\theta, r) \in [\beta^*, 1)$, investment is unconstrained optimal.

When private markets fail, some type of intervention is generally necessary and desirable to correct the problem. Our results indicate that when the source of the market failure is imperfect enforcement of contracts, there are at least two indirect ways to address enforcement problems:

(i) Policy makers may try to alter $\beta(\theta, r)$ via three types of policies. First, the country could take measures to raise $\theta$ by altering the risk factors in Table 1. Second, lower world interest rates will raise $\beta$ for all countries, although no country can achieve this unilaterally. Third, policies directed at altering the exponential structure of $\beta$ in Case 2 (see Figure 5) may reduce the volatility of private investment flows.

(ii) Policy makers may try to alter the critical thresholds $\beta$ and $\beta^*$ to enlarge the set of Case 2 and 3 equilibria. In our simple model, this involves policies designed to alter $k^d$ and $r$. However, Asiedu and Villamil (1999) build on this model to show how policies often used by multilateral agencies (i.e., loan subsidies and technical assistance) can be used to alter these thresholds.

The most direct way to solve problems that arise from imperfect contractual enforcement is obviously to improve the enforcement technology. Although some progress has been made recently on implementing multilateral institutions (e.g., the WTO and the European Central Bank), the absence of a supranational legal authority that can enforce contracts in an international setting remains an important and difficult problem.

The indirect policies that we have characterized to improve private investment flows appear to constitute a useful interim measure in the absence of a supranational authority. The simple structure of the model provides a useful theoretical framework for additional policy analysis. In addition, the model provides an interesting contrast to recent models of international capital flows based on information asymmetries [e.g., Gertler and Rogoff (1990); Boyd and Smith (1997)].
These alternative models have the feature that an ability to provide internal finance mitigates an information friction. Wealthier economies have a superior ability to finance investments internally and, as a consequence, international markets favor these “richer” countries. In contrast, in our model a wealthier country experiences less “pain” by being excluded from world capital markets than a poorer country. Rich countries therefore have a greater incentive to default on debt than poorer countries, ceteris paribus. These alternative predictions potentially present an interesting empirical test of the models.

NOTES
1. Expropriation is the violation of any condition of an investment agreement. It may involve direct government default on foreign investment contracts or guarantees, prevention of domestic residents from honoring obligations to foreign creditors, and so on.
3. We focus on the risk-neutral case to study the effect of “pure enforcement problems” on investment. Underinvestment would be even more severe if the host used international capital markets for consumption insurance (i.e., if agents were risk averse). The model is related closely to that of Eaton and Gersovitz (1983), although their focus is on taxation whereas ours is on the discount factor.
4. We provide measures of \( \theta \) and \( \beta \) and determine their quantitative significance for private investment flows.
5. See Knack and Keefer (1995) for measures of the quality of political and legal institutions for 111 countries, which they use to assess the riskiness of investment using the same data set.
6. In real business-cycle models, \( \beta \) is calibrated from a nonstochastic version of the growth model and data on the average experience of an economy over a particular period. The calibration typically uses the equations \( \Delta K_{t+1}/K_t \) and \( Y_{t+1}/Y_t \) from the growth model.
7. The stock of financial assets in industrial countries is estimated to be $20 trillion, whereas capital flows to developing countries are in the range of $120 billion [cf., Cline (1995)].

REFERENCES
APPENDIX

Proof of Proposition 1. Consider each case:

Case 1. When $B(k)$ cuts $G(k)$ at $k^d$ from below, $B'(k^d) < G'(k^d)$ and the autarky equilibrium occurs (see Figure 1). This implies that $\beta > r$. Define

$$\beta^*(k^d, r) = \frac{r}{f'(k^d)}.$$ 

Then $\beta^*$ is the minimum discount factor required to attract foreign investment. Hence for any $\beta \in (0, \beta^*)$, the inefficient autarky equilibrium occurs.

Case 2. Note that $\beta > \beta^*$ implies that $B(k)$ cuts $G(k)$ at $k^d$ from above. This implies that there exists an upper bound $\tilde{k} > k^d$ such that the interval $[k^d, \tilde{k}]$ is self-enforcing. To prove the result, it suffices to show that $k^*_u \notin [k^d, \tilde{k}]$ for $\beta \in [\beta^*, \beta^*]$. If $k^*_u \notin [k^d, \tilde{k}]$; then
equation (1) binds and the optimal self-enforcing investment plan is \( k^*_u = \bar{k} < k^*_u \). The result follows because \( \beta < \beta^* \) implies that \( k^*_u \) is not self-enforcing; i.e., \( k^*_u \notin [k^d, \bar{k}] \).

We also show that \( 0 < \beta < \beta^* < 1 \). Note that \( k^d < k^*_u \) implies that a straight line that passes through \( k^d \) and \( k^*_u \) cuts \( f(k) \) from above at \( k^d \) and from below at \( k^*_u \). This implies that
\[
f'(k^d) > \frac{f(k^*_u) - f(k^d)}{k^*_u - k^d} > f'(k^*_u) = r.
\]
The result follows from the inequalities.

**Case 3.** Substitute \( k = k^*_u \) and \( r = f'(k^*_u) \) in equation (3) and define \( \beta^* \) as
\[
\beta^*(k^d, r) = \frac{(k^*_u - k^d)f'(k^*_u)}{f(k^*_u) - f(k^d)}.
\]
Then \( k^*_u \) is self-enforcing if and only if \( \beta(\theta, r) \geq \beta^* \). Note that \( \beta^* \) is the minimum discount factor for which \( k^*_u \) is self-enforcing.

**Proof of Claim 1.** Part (a) follows from differentiation of equations (4) and (5) with respect to \( k^d \):
\[
\frac{d\beta}{dk^d} = \frac{\beta f''(k^d) - \beta f'(k^d) f'(k^d)}{f'(k^d)^2},
\]
\[
\frac{d\beta^*}{dk^d} = \frac{\beta^* f'(k^d) - r}{f(k^*_u) - f(k^d)}.
\]
Note that \( f''(k) < 0 \). Further, because \( \beta^* > \beta^* \), equation (4) implies that \( \beta^* f'(k^d) > r \). The result follows immediately.

Part (b) follows from differentiation of (4) and (5); \( \frac{d\beta}{dr} > 0 \) is obvious and
\[
\left[ f(k^*_u) - f(k^d) \right] \frac{d\beta^*}{dr} = r(1 - \beta) \frac{dk^*_u}{dr} + (k^*_u - k^d).
\]
The result follows from the fact that
\[
\frac{dk^*_u}{dr} = \frac{1}{f''(k^*_u)}.
\]

**Proof of Claim 2.** Differentiating (6) with respect to \( k^d, \beta, \) and \( r \) yields:
\[
\frac{dk^*_u}{dk^d} = \frac{\beta f'(k^d)-r}{\beta f'(k^*_u) - r},
\]
\[
\frac{dk^*_u}{d\beta} = \frac{f(k^*_u) - f(k^d)}{\beta f'(k^*_u) - r}.
\]
\[
\frac{dk^*_u}{dr} = \frac{1}{\beta f'(k^*_u) - r} \left\{ \left( k^*_u - k^d \right) - \left[ f(k^*_u) - f(k^d) \right] \frac{d\beta}{dr} \right\}.
\]
This is the Case-2 equilibrium where \( B(k) \) cuts \( G(k) \) from above at \( k^d \) and from below at \( k^*_u \) (see Figure 2). Hence, \( B'(k^d) < G'(k^d) \) and \( B'(k^*_u) > G'(k^*_u) \). This implies that \( [\beta f'(k^d) - r] > 0 \) and \( [\beta f'(k^*_u) - r] < 0 \), and the result follows.