

Pricing Kernel Specification for User Cost of Monetary Assets

By

Copyright 2010

Wei Zhou

Submitted to the graduate degree program in Economics
and the Graduate Faculty of the University of Kansas
in partial fulfillment of the requirements for the degree of
Doctoral of Philosophy

(Chairperson)

Committee members*

Date Defended: _____

The Dissertation Committee for Wei Zhou certifies that this is the approved version of the following dissertation.

Pricing Kernel Specification for User Cost of Monetary Assets

(Chairperson)

Committee members*

Date Defended: _____

Abstract

Wei Zhou
Department of Economics,
University of Kansas

This paper studies the nonlinear asset pricing kernel approximation by using orthonormal polynomials of state variables in which the pricing kernel specification is restricted by preference theory. We approximate the true asset pricing kernel for monetary assets by considering consumption-based and Fama-French asset pricing models in which the consumer is assumed to have inter-temporally non-separable preference. We study the classical consumption-based kernels and multifactor (Fama-French) kernels in our asset pricing models. Our results suggest that the multi-factor pricing kernels with nonlinearity and non-separable utility specifications have significantly improved performance.

Acknowledgements

I would like to express my sincere gratitude to my advisor, Professor William Barnett for his guidance and support. I would also like to thank Professor Shu Wu for his constructive comments and his important support throughout this work. Their wide knowledge, understanding, encouraging and personal guidance have provided a significant basis for this dissertation.

I wish to express my warm and sincere thanks to Professor John Keating, Professor Shigeru Iwata, and Professor Prakash P. Shenoy for their contributions to this work.

I owe my loving thanks to my wife Ying Huang and my mother Liyu Su. Without their encouragement and understanding it would have been impossible for me to finish this work. My special gratitude is due to my brothers, my sisters and their families for their loving support.

Table of Contents

	Pages
Abstract _____	ii
Acknowledgements _____	iii
Table of Contents _____	iv
Chapter 1 Introduction _____	1
1.1. Asset Pricing Kernel Approximation _____	1
1.2. The User Cost of Monetary Asset and Pricing Kernel _____	3
Chapter 2 Asset Pricing Models and Kernel Specifications _____	6
2.1. Dynamic Economy Models and Implications _____	6
2.2. Linear Pricing Kernel Specification and Limitations _____	7
2.3. Alternative Kernel Specification Approaches _____	7
Chapter 3 Pricing Kernel for User Cost of Monetary Assets _____	14
3.1. The Economy Framework and the User Cost of Monetary Assets _____	14
3.2. IMRS of Representative Consumers _____	17
3.3. The Derived User Cost of Monetary Asset _____	23

Chapter 4.	An Empirical Study of Nonlinear Pricing Kernels	28
4.1.	Background	28
4.2.	Euler Equation and Assumptions	31
4.3.	Taylor Series Expansion and Pricing Kernel Specification	34
4.4.	Estimation of the Approximated Pricing Kernel	48
4.5.	Data Description and Estimation Process	55
4.6.	Estimation Results	59
4.7.	Conclusion	69
Reference		73
Table I		79
Table II		80
Table III		81
Table IV		82
Table V		83
Table VI		84
Table VII		86

Table VIII	89
Table IX	90
Table X	92
Table XI	96
Table XII	98
Plot.1 a-b	100
Plot.2 a-b	101
Plot.3 a-b	102
Plot.4 a-b	103
Plot.5 a-b	104
Plot.6 a-b	105
Plot.7	106
Plot.8	107
Plot.9	107
Plot.10	108
Plot.11	109

Plot.12 _____109

Plot.13 _____110

Chapter 1 Introduction

1.1. Asset Pricing Kernel Approximation

In the traditional approach to investigate the implications of dynamic asset pricing models from the framework set forth by Lucas (1978), the underpin economic model imposed the following assumptions that markets are complete, the economy is in equilibrium, and that the unique state-price discount factor can be interpreted as the intertemporal marginal rate of substitution of a representative consumer.

By observing the conditional or unconditional covariation of estimated intertemporal marginal rates of substitution with measured asset returns with assumptions imposed on the utility function of consumers and on the observability of aggregate consumption, the dynamic asset pricing model could be tested for consistency of predictions.

The conventional implication of Capital Asset Pricing Model (CAPM) is that the pricing kernel is linear in a single factor, the portfolio of aggregate wealth. Numerous studies over the past two decades have documented violations of this restriction, and the fact that these attempts have been mostly inconsistent with the data on asset

returns and consumption. For instance, some economic frameworks assume utility function to be intertemporal separable (see, for example, Mehra and Prescott (1985) and Weil (1989)), then the absolute and relative levels and variability of the real returns on risk-free and risky monetary assets won't be able to match with the model's predictions.

A number of works have examined the performance of alternative asset pricing models. These models could be categorized into two classes: (1) multifactor models such as Ross' APT or Merton's ICAPM, in which factors in addition to the market return determine asset prices; or (2) nonparametric models, such as Bansal et al. (1993), Bansal and Viswanathan (1993), and Chapman (1997), in which the pricing kernel is nonlinear in market returns. Empirical applications of these models suggest that they are much better at explaining cross-sectional variation in expected returns than the CAPM.

In this work we conduct an alternative approach to investigate the pricing kernels of monetary assets in nonparametric model analysis. We study the asset pricing kernel approximation with consumption-based utility function and multifactor (Fama-French) kernel specifications. We also investigate the specification of state-price deflator or

the “asset pricing kernel” by comparing different polynomial functions of a number of state variables.

1.2. The User Cost of Monetary Asset and Pricing Kernel

Demonstrated under similar economic framework, , Barnett, Liu, and Jensen (1997) and Barnett and Liu (2000) showed that a risk adjustment term should be added to the certainty-equivalent user cost in a consumption-based capital asset pricing model (C-CAPM) in producing the Divisia index approximations to the theory’s aggregator functions under risk. And an important covariance term between the rates of return on monetary assets and the growth rate of aggregate consumption will determine the magnitude of risk adjustment to the certainty-equivalent user cost. However, the consumption CAPM based risk adjustment to certainty equivalent user cost of monetary asset is slight and the gain from replacing the unadjusted Divisia index with the extended index is too trivial to match the observed risk premium consistently (Barnett, Liu, and Jensen (1997)).

The primary finding from a large literature testing consumption-based models is that the measured consumption is too “smooth” to rationalize the observed level and variability of asset returns for “reasonable” parameterizations of time-separable utility

functions.

The small adjustments been questioned are mainly due to the very low contemporaneous covariance between asset returns and the growth rate of consumption. Under the standard power utility function and a reasonable value of the risk-aversion coefficient, the low contemporaneous covariance between asset returns and consumption growth implies that the impact of risk on the user cost of monetary assets is very small. In other words, the standard power utility function is incapable to reconcile the observed large equity premium with the low covariance between the equity return and consumption growth.

The consumption CAPM adjustment to the certainty-equivalent monetary-asset user costs can similarly be larger under a more general utility function (see, Barnett and Wu (2004)) than those used in Barnett, Liu, and Jensen (1997), where a standard intertemporal separable power utility function was assumed. Besides, it was found that the basic results from Barnett, Liu, and Jensen (1997) still holds under a more general utility function. And by introducing intertemporal nonseparable utility function, the model can lead to substantial and more accurate consumption CAPM risk adjustment, even when a reasonable setting of the risk-aversion coefficient is

present.

The risk premium term in CAPM models can be similarly constructed and tracked in computing the risk premium adjustment in user cost of individual monetary assets to the user cost of consumers' wealth portfolio (see, Barnett and Wu (2004)). By introducing a basic form of pricing kernel, the risk premium adjustment term can be presented as the asset's risk premium exposure to the market portfolio.

In our work, we show that the monetary user cost referenced from pricing kernel can be represented as a risk free rate plus a market exposure adjusted risk premium term.

Chapter 2 Asset Pricing Models and Kernel Specifications

2.1. Dynamic Economy Models and Implications

A series of dynamic economy models has been investigated for their implication of the asset market data. The differences among the models range from payoffs' span on the tradable securities, heterogeneity of consumers' preferences and the role of money within the consumption goods acquisitions are also studied for their empirical implications (Hansen and Jagannathan 1991).

Regardless of these differences, there exist common implications from all these models. Those are the expectations of the product of payoffs which represents the equilibrium price of a future payoff from any tradable financial security and an appropriately rendered intertemporal marginal rate of substitution (IMRS) of any consumer (Lucas 1978, Breeden 1979, Harrison and Kreps 1979, Hansen and Richard 1987). Given these implications, numerous works over the past few decades have been carried out to examine the conditional and unconditional covariation of the IMRS with measures of financial assets' payoffs. The very common assumption of the intertemporal marginal rate of substitution of a representative consumer, or the "asset pricing kernel" of the capital asset pricing model (CAPM), is a linear function of a single factor from an aggregate wealth portfolio's return.

2.2. Linear Pricing Kernel Specification and Limitations

The linearity restriction in the asset pricing kernel has been documented for its inferior performance from numerous past studies. Similar cases can be seen in the poorly fitted data in the consumption-based asset pricing models derived from the framework set forth by Lucas (1978). Within the discrete state-space version of Lucas model, although it has been augmented by the riskless assets' rate of return, the generated equity risk premium will not be able to match the average premium observed in historical U.S data.

2.3. Alternative Kernel Specification Approaches

In order to capture more variation in expected returns, numerous approaches have been attempted. These approaches can be categorized into two directions, multifactor versions of CAPM and nonparametric asset pricing models with nonlinear pricing kernel specifications. Both multifactor versions of CAPM and nonparametric models are approved by empirical studies for their better performance in explaining expected return variations in CAPM. A multifactor alternative of CAPM will capture more variation in expected returns of financial assets than CAPM (Fama and French, 1995).

Various nonlinear pricing kernel specifications in monetary asset pricing models significantly outperform linear specifications (Bansal and Viswanathan 1993, Chapman 1997).

In addition, the choice of factors in multifactor model and the alternative nonlinear specifications of pricing kernel are not straightforward. Researchers would need considerable discretion over the form of models to be investigated. Either the multifactor model or the form of nonlinearity would require ad hoc specifications. As pointed out by Dittmar (2002) and Chapman (1997), given a specific assumption on investors' preferences or return distributions, the form of the pricing kernel investigated in non-parametric approaches would not follow endogenously. For example, in the large literature of consumption based asset pricing models, the primary finding is that the consumption based pricing kernel is too smooth to rationalize the observed variation in asset returns when the utility function of representative consumers are intertemporally separable.

Therefore the absolute and relative levels of real returns from financial assets are inconsistent with model predictions (Mehra and Prescott 1985). These nonparametric and multifactor approaches are problematic from their ad hoc assumptions and may

suffer from over fitting problems, factor dredging and low power (Dittmar 2002, Lo and MacKinlay 1990).

In Campbell and Cochrane (1999), a successful approach was attempted. They introduced the more general utility function to representative consumers with habit persistence assumed, which in other words differs from previous framework in that inter-temporally non-separable utility functions should be implemented. A large time-varying risk premium similar in magnitude to the data was observed by doing so.

In this paper we investigate the approximation of the state-price deflator or the “asset pricing kernel” by comparing different polynomial functions of a number of state variables. These state variables should be implied by the underlying economic models, in which we elect state variables based on a stochastic version of the neoclassical growth model, which suggests that aggregate consumption as a necessary state variable to be used in approximation of asset pricing kernels. In the same manner, the bedrock model implies that consumption is a time-invariant function of the level of capital stock and transitory shocks to total factor productivity (i.e., technology shocks), which suggests that technology shock growth rates might aid in the construction of an approximated kernel. The pricing kernels with polynomial

functions assume nonlinearity among the underlying variables and are capable of explaining nonlinear asset payoffs, therefore are distinctively advantageous over other approximation scenarios.

In our work, we derived statistical tests based on Hansen-Jagannathan bounds as many contemporaneous works did. These tests are applied to construct the confidence regions for the parameters of dynamic asset pricing models. The approximated pricing kernels are examined using an asymptotically chi-square statistic based on the mean square distance of the estimated kernel to the H-J bounds introduced in Hansen and Jagannathan (1991). By using both graphical evidence and Wald type tests, we evaluated the polynomial function approximations through testing the statistical significance of marginal polynomial orders and by examining the pricing errors on individual assets and groups of related assets.

The approximated asset pricing kernels based on consumption growth from one-period ahead of schedule are not mostly rejected by overall measures of model fit, however, they produce large pricing errors on individual assets both statistically and economically, see Chapman (1997). In our approximate kernel approach, the inclusion of additional future value of consumption growth and/or technology shock

growth motivated by an appeal to time nonseparable preferences or the durability of consumption goods (Barnett and Wu (2005)) induced the largest improvement in the performance of approximate kernels. The marginal power of technology shocks is difficult to detect directly in these cases, but the inclusion of technology shocks under intertemporal nonseparable utility framework generally improves the fit of approximated kernels. Especially, the approximated kernel under intertemporal nonseparable utility framework is the only specification that is capable of consistently reducing the average pricing errors on small market capitalization stocks.

Particularly, as motivated by the analysis in Hansen and Jagannathan (1997), our results show that a predetermined weighting matrix equal to the inverse of the second moment matrix of returns generally produces smaller pricing errors and the approximated kernels that are more consistent with the Hansen-Jagannathan bounds. It also shows that the parameters of approximated kernels estimated using this “optimal” weighting matrix will minimize the mean-square distance between the estimated kernel and the Hansen-Jagannathan bounds. The strong connection of using the optimal covariance matrix for pricing errors was also found with strong evidence in our results with, as been emphasized in Cochrane (1996), potential problems in estimating the polynomial parameters. And of the most importance, if the covariance

matrix is misspecified or poorly measured, the resulting kernel can have very mediocre performance in small size samples.

Results in our study can be regarded as a similar endeavor in asset pricing kernel approximation investigation for user cost of monetary assets based on macroeconomic aggregates by comparing different polynomial function scenarios. Bansal, Hsieh, and Viswanathan (1993) examine international pricing models using market returns based approximate kernels.

The parameters of polynomial functions in approximated kernels are estimated with the model by using GMM, as did in Hansen and Singleton (1982), Bansal and Viswanathan (1993) and Bansal, Hsieh, and Viswanathan (1993). The similarity among our work and the works done in Bansal and Viswanathan (1993) and Bansal, Hsieh, and Viswanathan (1993) is that they also approximated nonlinearity in asset pricing kernels. However, their approximated kernels are based on asset returns instead of on state variables of the economy which can not avoid potential numerical problems associated among the asset returns, in other words, the colinearity problems.

In our work, we also derive statistical tests based on Hansen-Jagannathan statistics.

And we construct the confidence regions for parameters of approximated pricing kernels. The principal implication from our results is that the treatment of model sampling error should be carefully managed.

Chapter 3 Pricing Kernel for User Cost of Monetary Assets

3.1. The Economy Framework and the User Cost of Monetary Assets

To develop the specific nonlinear pricing kernel desired, we start with the intertemporal consumption and portfolio choice problem for a long lived investor. Under a standard set of assumptions, we describe the competitive equilibrium of this economic framework as the solution formulated in Stokey and Lucas (1989), and that the consumption, labor supply and capital investments will all be represented as continuous, time-invariant functions of the model's state variables. And we setup the similar underlying economic framework as in Baret, Liu and Jensen (1997) that the utility function is time nonseparable, caused by either direct utility effects from past consumption or from the durability in past consumption.

We define $U(m_t, c_t, c_{t-1}, c_{t-2}, \dots, c_{t-n})$ over current and past consumption and L number of current period monetary assets $m_t = (m_{1,t}, m_{2,t}, m_{3,t}, m_{4,t}, \dots, m_{L,t})$. The economic agent also holds non-monetary assets, in other words the "investment", $k_t = (k_{1,t}, k_{2,t}, k_{3,t}, \dots, k_{J,t})$ which provide no monetary service other than investment returns and therefore don't enter the utility function.

Within this economic framework, there exists a presumed complete set of contingent claims market. Besides, including financial asset market does not change the economic agents' equilibrium decision rules and quantity allocations.

With the monetary assets specified and the assumption that there exists a linearly

homogeneous aggregator function $M(m_t)$, we have the utility function in the following,

$$U(m_t, c_t, c_{t-1}, c_{t-2}, \dots, c_{t-n}) = V(M(m_t), c_t, c_{t-1}, c_{t-2}, \dots, c_{t-n}) \quad (3.1)$$

Given the investment portfolio W_t , the utility maximization problem follows,

$$E_t \sum_{s=0}^{\infty} \beta^s U(m_{t+s}, c_{t+s}, c_{t+s-1}, c_{t+s-2}, \dots, c_{t+s-n}) \quad (3.2)$$

which is subject to the following budget constraints,

$$\begin{aligned} W_t &= p_t^* c_t + \sum_{i=1}^L p_t^* m_{i,t} + \sum_{j=1}^K p_t^* k_{j,t} \\ &= p_t^* c_t + p_t^* A_t \end{aligned} \quad (3.3)$$

and

$$W_{t+1} = \sum_{i=1}^L R_{i,t+1} p_t^* m_{i,t} + \sum_{j=1}^K \tilde{R}_{j,t+1} p_t^* k_{j,t} + Y_{t+1} \quad (3.4)$$

β is the subjective discount factor,

c_t is the consumption at period t ,

$k_{j,t}$ is the investment in non-monetary asset j at period t ,

p_t^* is the true cost-of-living index,

$A_t = \sum_{i=1}^L p_t^* m_{i,t} + \sum_{j=1}^K p_t^* k_{j,t}$ is the real value of the asset portfolio,

$\tilde{R}_{j,t+1}$ is the gross rate of return from holding non-monetary assets, “investments”,
from period t to $t+1$,

$R_{i,t+1}$ is the gross rate of return from holding monetary assets, which provide
monetary services to the representative consumers,

In this case, the chosen state variables for pricing kernel approximation should
include the current information of technology shocks and aggregate capital stock
growth rates with lagged values.

3.2. IMRS of Representative Consumers

Follow the same procedure in Barnett and Wu (2005), we first deduce the interpreted intertemporal marginal rate of substitution (IMRS) of the representative consumer (see, e.g., LeRoy (1973); Rubinstein (1976); Lucas (1978); Breeden (1979); Harrison and Kreps (1979); Hansen and Richard (1987); Hansen and Jagannathan (1991)).

We initially consider the following simplified consumer decision problem with a standard set of assumption. Given the wealth portfolio and consumption level at each period, the representative investor maximizes his expected utility for an intertemporal separable infinite horizon utility function.

The representative consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \text{ and } 0 < \beta < 1, \quad (3.5)$$

Subject to:

$$W_{t+1} = R_{t+1}(W_t - c_t), R_{t+1} = \sum_{i=1}^n s_t^i R_{t+1}^i, \text{ and } \sum_{i=0}^n s_t^i = 1. \quad (3.6)$$

Here W_t indicates the wealth portfolio held by representative investor at period t , R_t indicates the gross portfolio return at period t . The individual asset returns are indicated by superscripts with superscript 0 indicating the risk free asset. s_t^i indicates the portfolio share of individual asset and portfolio shares add to 1 in each period.

By following the dynamic programming approach, we reformulate the utility maximization problem into the following equation:

$$V(W_t) = \max_{c_t, \{s_t^i\}_{i=1}^n} [u(c_t) + \beta E_t V(W_{t+1})] \quad (3.7)$$

Subject to the following wealth constraints:

$$W_{t+1} = R_{t+1}(W_t - c_t), \text{ and } R_{t+1} = R_{t+1}^f + \sum_{i=1}^n s_t^i (R_{t+1}^i - R_{t+1}^f). \quad (3.8)$$

The first-order conditions of this decision problem at each period are:

$$u_c(c_t) = \beta E_t R_{t+1} V_W(W_{t+1}) \quad (3.9)$$

and

$$E_t (R_{t+1}^i - R_{t+1}^f) V_W(W_{t+1}) = 0 \quad (3.10)$$

Besides, we have the envelope condition:

$$V_W(W_t) = \beta E_t R_t V_W(W_{t+1}) \quad (3.11)$$

It implies that

$$V_W(W_{t+1}) = \beta E_t R_{t+1} V_W(W_{t+2}) \quad (3.12)$$

Or,

$$u_c(c_t) = V_W(W_t), \quad (3.13)$$

Such that, we update the equations in (3.13) with one period and substitute it into equation (3.9), it yields the following equation:

$$u_c(c_{t+1}) = \beta E_t R_t u_c(c_{t+1}) \quad (3.14)$$

It implies that

$$\beta E_t R_t [u_c(c_{t+1}) / u_c(c_t)] = 1 \quad (3.15)$$

And we deduced the following:

$$E_t Q_t = \beta E_t [u_c(c_{t+1}) / u_c(c_t)] \quad (3.16)$$

Equation (3.12) also demonstrates that,

$$E_t Q_t = \beta E_t [V_w(W_{t+1}) / V_w(W_t)] \quad (3.17)$$

Here the stochastic discount factor that prices all assets is equal to the marginal rate of intertemporal substitution. In equation (3.17), we show that it is also equal to the marginal rate of intertemporal substitution in terms of wealth.

In finance literature, a common implication of various asset pricing models is that the equilibrium price of a future payoff on any tradable security can be represented as the expectation (conditioned on current information) of the product of the payoff and IMRS. The equilibrium price is a generalization of the familiar tenet from price theory and it states that price should equal marginal rates of substitution. And within

our economic framework, this principle implies that the monetary assets are viewed as the claims to the numeraire good indexed by future states of the world. Since we further defined the utility function to be intertemporal nonseparable, we consider the decision problem for the representative investor in the following:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, \beta E_t u(c_{t+1})), \text{ and } 0 < \beta < 1 \quad (3.18)$$

Subject to:

$$W_{t+1} = R_{t+1}(W_t - c_t), R_{t+1} = \sum_{i=1}^n s_t^i R_{t+1}^i, \text{ and } \sum_{i=0}^n s_t^i = 1. \quad (3.19)$$

Then the utility maximization problem is reformulated into:

$$V(W_t) = \max_{c_t, \{s_t^i\}_{i=1}^n} [u(c_t, E_t u(c_{t+1})) + \beta E_t V(W_{t+1})] \quad (3.20)$$

which is subject to the same wealth constraints shown in equation (3.8).

And we have one of the first-order conditions of this decision problem at each period in the following:

$$\frac{\partial u(c_t)}{\partial c_t} + \beta \frac{\partial E_t u(c_{t+1})}{\partial c_t} + \beta^2 \frac{\partial E_t u(c_{t+2})}{\partial c_t} + \dots + \beta^n \frac{\partial E_t u(c_{t+n})}{\partial c_t} = \beta E_t R_{t+1} V(W_{t+1}) \quad (3.21)$$

Given the envelope condition:

$$V_w(W_t) = \beta E_t R_{t+1} V(W_{t+1}) \quad (3.22)$$

We have:

$$V_w(W_t) = \frac{\partial u(c_t)}{\partial c_t} + \beta \frac{\partial E_t u(c_{t+1})}{\partial c_t} + \beta^2 \frac{\partial E_t u(c_{t+2})}{\partial c_t} + \dots + \beta^n \frac{\partial E_t u(c_{t+n})}{\partial c_t} \quad (3.23)$$

And it follows that:

$$V_w(W_{t+1}) = \frac{\partial u(c_{t+1})}{\partial c_{t+1}} + \beta \frac{\partial E_t u(c_{t+2})}{\partial c_{t+1}} + \beta^2 \frac{\partial E_t u(c_{t+3})}{\partial c_{t+1}} + \dots + \beta^n \frac{\partial E_t u(c_{t+n+1})}{\partial c_{t+1}} \quad (3.24)$$

And we further deduced the following:

$$E_t Q_{t+1} = \beta E_t \left(\frac{\frac{\partial u(c_{t+1})}{\partial c_{t+1}} + \beta \frac{\partial E_t u(c_{t+2})}{\partial c_{t+1}} + \dots + \beta^n \frac{\partial E_t u(c_{t+n+1})}{\partial c_{t+1}}}{\frac{\partial u(c_t)}{\partial c_t} + \beta \frac{\partial E_t u(c_{t+1})}{\partial c_t} + \dots + \beta^n \frac{\partial E_t u(c_{t+n})}{\partial c_t}} \right) \quad (3.25)$$

Note that when the instantaneous utility function is time separable, the pricing kernel boils down to equation (3.16)

$$E_t Q_{t+1} = \beta E_t \left(\frac{\partial U_{t+1} / \partial c_{t+1}}{\partial U_t / \partial c_t} \right)$$

The pricing kernel was defined as a linear function based on asset returns

$Q_{t+1} = a_t - b_t r_{A,t+1}$, in which $r_{A,t+1}$ was defined as the gross real rate of return on the consumer's wealth portfolio in Barnett and Wu (2005). Comparable setup and results can be found in scenarios applied in related works done by Bansal and Viswanathan (1993) and Bansal, Hsieh, and Viswanathan (1993), in which, their approximated pricing kernels are nonlinear factor functions based on asset returns.

However, the approximated pricing kernels based on asset return data nest the market-based capital asset pricing models (CAPM), their results are still subject to the numerical problems associated within different asset returns data, which hasn't been scaled as pointed out in Chapman (1997).

3.3. The Derived User Cost of Monetary Asset

In our work, we consider the pricing kernel to be a linear factor function with the nonlinear structure of polynomials from various economic state variables and shocks.

We define the pricing kernel to be

$$Q(x_t, \theta) = \sum_{k=0}^K \theta_k n_k(x_t) \quad (3.26)$$

The linearity of the pricing kernel can be reflected in the following equation

$$Q_{t+1} = \theta_0 + n'_{t+1} \theta \quad (3.27)$$

where n_{t+1} is defined as a $(k+1) \times 1$ vector of orthonormal polynomials with order from 0 to k and the pricing kernel $Q(x_t, \theta)$ is evaluated at x_t .

Recall the *Proposition 2* in Barnett and Wu (2005), the user cost of asset i given the defined pricing kernel in (3.27) can be obtained in the following,

$$\begin{aligned} \pi_{i,t} &= \frac{(1 + \omega_{i,t}) E_t \tilde{r}_{j,t+1} - (1 + \omega_{i,t}) E_t r_{i,t+1}}{E_t \tilde{r}_{j,t+1}} \\ &= \frac{(1 + \theta_j \text{cov}_t(n_{t+1}, r_{i,t+1})) E_t \tilde{r}_{j,t+1} - (1 + \theta_j \text{cov}_t(n_{t+1}, \tilde{r}_{j,t+1})) E_t r_{i,t+1}}{E_t \tilde{r}_{j,t+1}} \end{aligned} \quad (3.28)$$

where $\omega_{i,t} = -Cov_t(Q_{t+1}, r_{i,t+1})$ and $\omega_{j,t} = -Cov_t(Q_{t+1}, \tilde{r}_{j,t+1})$. $r_{i,t+1}$ is the real rate of return on a monetary asset and $\tilde{r}_{j,t+1}$ is the real rate of return on an arbitrary non-monetary asset.

The user cost of asset wealth portfolio can also be obtained in the following:

$$\begin{aligned}\Pi_{A,t} &= 1 - E_t Q_{t+1} E_t r_{A,t+1} - Cov_t(Q_{t+1}, r_{A,t+1}) \\ &= 1 - \theta_0 E_t r_{A,t+1} - \theta_j Cov_t(n_{t+1}, r_{A,t+1})\end{aligned}\quad (3.29)$$

where $r_{A,t+1}$ is the gross real rate of return on consumer's wealth portfolio.

Then with the certainty-equivalent user cost of monetary assets and wealth portfolios shown in Barnett (1978)

$$\pi_{i,t}^e = \frac{r_t^f - E_t r_{i,t+1}}{r_t^f} \quad \text{and} \quad \Pi_{A,t}^e = \frac{r_t^f - E_t r_{A,t+1}}{r_t^f} \quad (3.30)^*$$

We have the following equations

$$\begin{aligned}\frac{\pi_{i,t} - \pi_{i,t}^e}{\Pi_{A,t} - \Pi_{A,t}^e} &= \frac{Cov_t(n_{t+1}, r_{i,t+1})}{Cov_t(n_{t+1}, r_{A,t+1})} \\ \pi_{i,t} - \pi_{i,t}^e &= \frac{Cov_t(n_{t+1}, r_{i,t+1})}{Cov_t(n_{t+1}, r_{A,t+1})} \bullet (\Pi_{A,t} - \Pi_{A,t}^e)\end{aligned}\quad (3.31)$$

Or,

$$\pi_{i,t} = \pi_{i,t}^e + \frac{Cov_t(n_{t+1}, r_{i,t+1})}{Cov_t(n_{t+1}, r_{A,t+1})} \bullet (\Pi_{A,t} - \Pi_{A,t}^e) \quad (3.32)$$

On the other hand, for any monetary asset i we have from (3.29) and (3.30) that

$$\pi_{i,t} = \pi_{i,t}^e + \frac{\text{Cov}_t(n_{t+1}, r_{i,t+1})}{\text{Cov}_t(n_{t+1}, r_{A,t+1})} \bullet (E_t r_{A,t+1} (\frac{1-r_t^f \theta_0}{r_t^f}) - \theta_j \text{Cov}_t(n_{t+1}, r_{A,t+1})) \quad (3.33)$$

Using a simple transformation, it follows that

$$\pi_{i,t} = \pi_{i,t}^e + \frac{\text{cov}_t(n_{t+1}, r_{i,t+1})}{\text{Var}_t(n_{t+1})} \bullet (\frac{\text{Var}_t(n_{t+1})}{\text{Cov}_t(n_{t+1}, r_{A,t+1})} \cdot E_t r_{A,t+1} (\frac{1-r_t^f \theta_0}{r_t^f}) - \theta_j \text{Var}_t(n_{t+1})) \quad (3.34)$$

We recall *Corollary 1* from Barnett and Wu (2004) that under uncertainty we can choose any non-monetary asset as the “benchmark” asset, when computing the risk-adjusted user-cost prices of the services of monetary assets.

Therefore we have $E_t Q_{t+1} = \frac{1}{r_t^f}$, hence from equation (3.34), we can conclude that

$$\pi_{i,t} = \pi_{i,t}^e - \frac{\text{cov}_t(n_{t+1}, r_{i,t+1})}{\text{Var}_t(n_{t+1})} \bullet (\theta_j \text{Var}_t(n_{t+1})) \quad (3.35)$$

Equation (3.35) demonstrates that the user cost of monetary asset can be very similarly constructed as the standard CAPM function for asset returns. We also consider a special case of a j -th order polynomial with only one state variable from money market return. The market return will have very similar volatility to the consumer’s wealth portfolio, then our risk adjusted user cost of monetary asset i in (3.32) will boil down to:

$$\pi_{i,t} - \pi_{i,t}^e = \frac{\text{cov}_t(n_{t+1}, r_{i,t+1})}{\text{Var}_t(r_{A,t+1})} \bullet (\Pi_{A,t} - \Pi_{A,t}^e) \quad (3.36)$$

Comparable result to (3.36) could be found in Barnett and Wu (2005), in which they defined the pricing kernel as a linear function based on asset returns.

Correspondingly, we consider the Euler equation from the consumer problem, which characterizes the equilibrium financial asset prices:

$$\begin{aligned} E_t \left[\beta \frac{E_{t+1}(\partial U_{t+1} / \partial c_{t+1})}{E(\partial U_t / \partial c_t)} R_{t+1} | x_t \right] &= 1 \\ E_t(Q_{t+1} R_{t+1}) &= 1 \\ E_t[R_{t+1}] &= \frac{1}{E_t(Q_{t+1})} - \frac{\text{Cov}_t(Q_{t+1}, R_{t+1})}{E_t(Q_{t+1})} \\ &= \frac{1}{\theta_0} - \frac{E_t(n'_{t+1} \theta_j R_{t+1})}{E_t(Q_{t+1})} \\ &= \frac{1}{\theta_0} - \theta_j E_t(n_{t+1} n'_{t+1}) \frac{E_t(n_{t+1} n'_{t+1})^{-1} E_t(n'_{t+1} R_{t+1})}{\theta_0} \end{aligned} \quad (3.37)$$

Here, for individual financial asset, its rate of return could also be represented as a risk free rate plus a market exposure adjusted risk premium.

$$E_t[R_{t+1}] = \frac{1}{\theta_0} - \theta_j E_t(n_{t+1} n'_{t+1}) \frac{E_t(n_{t+1} n'_{t+1})^{-1} E_t(n'_{t+1} R_{t+1})}{\theta_0} \quad (3.38)$$

And the market exposure adjusted risk premium can be similarly interpreted by the

instantaneous covariance of the aggregate portfolio's return with the instantaneous growth rate in the individual's consumption as demonstrated by Breeden (1979), Duffie and Zame (1989), and Chapman (1997).

In other words, the pricing kernel of the user cost of monetary assets can be similarly constructed as that of financial assets.

Chapter 4. An Empirical Study of Nonlinear Pricing Kernels

4.1. Background

We conduct an alternative approach to investigate the pricing kernels of monetary assets in nonparametric model analysis. We study the asset pricing kernel approximation with consumption-based utility function and multifactor (Fama-French) kernel specifications. Both approaches were explored extensively to identify IMRSs. Consumption-based asset pricing models were explored by approximating the true pricing kernel with various ordered polynomials based on aggregate consumption under different utility function specifications. Particularly, the nonlinearity and time nonseparability of these kernel specifications exhibit substantially improved model overfit comparing to the principal implication of CAPM with linear function of single factor. Various nonlinear pricing kernel specifications were also tested for their performance (Bansal and Viswanathan 1993, Bansal et al. 1993, Chapman 1997). These approaches have limitations in many ways such as ad hoc assumptions and specification errors.

Our approach avoids the limitations in previous studies in both model assumptions and polynomial specifications by utilizing an unknown marginal utility function, which is augmented with Taylor series expansion in a static setting. The resulted state

variable polynomials are transformed into orthonormalized polynomial functions with respect to state variables to avoid strong linear relationships over relevant portions of the polynomials' domain. Within this structure, our pricing kernels fall into 2 groups: a polynomial function in aggregate wealth and a polynomial function in aggregate consumption growth. The marginal utility function augmented by Taylor series expansion is restricted by imposing decreasing absolute prudence on representative consumers' preferences (Kimball 1993, Dittmar 2002).

Therefore, our pricing kernels would be free from ad hoc misspecification problems and would guarantee the kernel to be an exogenously obtained risky factor function with aggregate wealth portfolio or aggregate consumption. Chapman (1997) suggests that the inclusion of a temporary technology shock would substantially improve the performance of the model analyzed. We also incorporate a temporary technology shock into the pricing kernel since many recent works have shown that the pricing kernel specification of aggregate wealth would impact the conclusions of empirical asset pricing studies.

A multifactor version of Merton's (1973) intertemporal CAPM model is investigated in Fama and French (1993), in which the size and (BE/ME) proxy for sensitivity to

risk factors are consistent with stock returns and profitability. This model captures the common variation in stock returns and explains the cross-section of average returns. We further construct the pricing kernel with an aggregate portfolio as the state variable by implementing the multifactor model in Fama and French (1993) with the augmentation of a momentum factor which has been applied in a few studies. We also include the Fama-French 6 portfolio and Fama-French 25 portfolio returns as state variables into the pricing kernel. Fama-French portfolio returns will nest a significant part of the market risk in asset returns and the augmented Fama-French 3 factors model is only capable of explaining excess returns of financial assets.

The remainder of this chapter is organized as the following:

In section I we describe representative agent's preference restrictions, and in Section II we discuss the specification of the pricing kernel approximation. The empirical methods and tests are described in Section III, and in Section IV we describe the nature of the data we chose and the estimation process. In section V, we conduct the empirical analysis, and in Section VI we conclude the paper.

4.2. Euler Equation and Framework Assumptions

Under a standard set of assumptions, we develop a specific nonlinear pricing kernel with an intertemporal consumption and portfolio choice problem for a long-lived agent. We also suppose there are n long-lived financial assets. We first assume the representative agent's utility function is additively time separable, then the familiar Euler equation as the solution to an investor's portfolio choice problem that first presented in Lucas (1978) and also discussed in Hansen and Jagannathan (1991) will characterize the equilibrium asset prices as the following equation:

$$E_t[\beta \frac{E_{t+1}(\frac{\partial U_{t+1}}{\partial c_{t+1}})}{E(\frac{\partial U_t}{\partial c_t})} R_{t,t+1} | \zeta_t] = 1 \quad (4.1)$$

Or, sometimes represented as

$$E_t[(1 + R_{t,t+1})Q_{t+1} | \zeta_t] = 1 \quad (4.2)$$

$$E_t Q_{t+1} = \beta E_t[\frac{u_t(c_{t+1})}{u_t(c_t)}] \quad (4.3)$$

Where $1 + R_{i,t+1}$ and $R_{i,t+1}$ are a vector of gross returns on assets, β is the discount rate and ζ_t represents information set available at time t , Q_{t+1} is the investors' intertemporal marginal rate of substitution (IMRS). IMRS is represented in equation (3) under the assumption of time separable utility function of representative agent, and Q_{t+1} is a strictly positive stochastic process used to price financial assets.

We choose to represent the pricing kernel as a nonlinear polynomial function of the chosen state variables, since a suitable representation for the representative agent's utility function is unknown. In addition, numerous research works investigating the ideal utility function find that investors' risk tolerance and risk-free rate were not properly supported by data.

The same basic result would also hold when the representative agent's utility function is intertemporally non-separable over time. Under the same framework in Hansen and Jagannathan (1991), we extend the time non-separable IMRS that characterizes the equilibrium asset prices in the following Euler equation:

$$E_t[(1 + R_{i,t+1})Q_{t+1} | \zeta_t] = 1 \quad (4.4)$$

Such that,

$$E_t Q_{t+1} = \beta E_t \left(\frac{\frac{\partial u(c_{t+1})}{\partial c_{t+1}} + \beta \frac{\partial E_t u(c_{t+2})}{\partial c_{t+1}} + \dots + \beta^n \frac{\partial E_t u(c_{t+n+1})}{\partial c_{t+1}}}{\frac{\partial u(c_t)}{\partial c_t} + \beta \frac{\partial E_t u(c_{t+1})}{\partial c_t} + \dots + \beta^n \frac{\partial E_t u(c_{t+n})}{\partial c_t}} \right) \quad (4.5)$$

Or, the Euler equation in Ferson and Constantinides (1991) can be expressed as the following:

$$E_t \left[\sum_{i=1}^{\infty} \left(\frac{s_{t+i}}{s_t} \right)^{-\phi} (b_{i-1} R_{j,t,t+1} - b_i) \mid \zeta_t \right] = 1 \quad (4.6)$$

Where $s_t = \sum_{i=1}^{\infty} b_i c_{t-i}$ is the accumulation of all past consumption expenditure effects on current utility, b_t is the habit formation effect coefficient and also reflects the durability of prior consumption purchases, ϕ is the representative agent's relative risk aversion coefficient. With these general and specific utility function forms and assumptions, we can estimate the preference parameters and evaluate the models' performance and overfit by using the Generalized Method of Moments (GMM) estimation method (Chapman 1997).

4.3. Taylor Series Expansion and Pricing Kernel approximation

The next step in our analysis as a *priori* is to define the pricing kernel as a nonlinear polynomial function of state variables since the nonlinearity specification of pricing kernels was suggested from past studies to show exceptional improvement in models' overfit. In order to avoid the data overfitting problem, and low power from ad hoc specification of pricing kernel rising from past nonparametric and multifactor approaches, we choose to implement a Taylor series expansion in pricing kernel specification.

A viable representation for the pricing kernel function with the implementation of a Taylor series expansion is proposed in the following form. The pricing kernel is represented as a nonlinear function of state variable S_{t+1} , which can be equivalently represented by the aggregate consumption or by the end of period return on representative agent's aggregate wealth under the assumption of static setting. Brown and Gibbons (1985) address the assumption of static setting that will allow the equivalent implementation of wealth as the aggregate consumption conditionally to proxy for the function of intertemporal marginal rate of substitution .

$$Q_{t+1} = \theta_0 + \theta_1 \frac{U''}{U'} x_{t+1} + \theta_2 \frac{U'''}{U'} x_{t+1}^2 + \dots \quad (4.7)$$

From the perspective and standpoint of economic theory, this pricing kernel specification appears to be attractive. Besides, it may also solve the concerns related to the measurement of tentative aggregate consumption proxies as pointed out by Breeden, Gibbons, and Litzenberger (1989). Nonetheless, a potential problem is that although the pricing kernel follows the nonlinear functional form and may have improvement in model fitting, it raises another question of having strong linear relationships among certain polynomial domains. The second problem falls on the proper determination of the maximum polynomial order that the model should include, which is a balance to keep between losing power and improve models' overfitting (Dittmar 2002, Chapman 1997, Bansal and Viswanathan 1993, Bansal et, al 1993).

Therefore, we propose the following functional form for pricing kernel. We implement a Taylor series expansion with coefficients driven by derivatives of consumers' utility function and a set of orthonormal polynomials of aggregate consumption proxies.

$$Q_{t+1} = \theta_0 P_0(x_{t+1}) + \theta_1 \frac{U'}{U} P_1(x_{t+1}) + \theta_2 \frac{U''}{U} P_2(x_{t+1}) + \dots \quad (4.8)$$

In equation (4.8), x_{t+1} represents the state variable which is defined as the aggregate consumption proxy. $\{P_n(x_{t+1})\}_n^\infty$ is defined as the set of orthonormal polynomials of the state variable with order n . We denote the inner product of orthonormal polynomials $\{P_n(x_{t+1})\}_n^\infty$ as $\langle P_n, P_m \rangle$ and the orthonormal polynomials satisfy the following conditions:

$$\begin{aligned} \langle P_n, P_m \rangle &= 0, \text{ for all } n < m \\ \langle P_n, P_n \rangle &= 1 \end{aligned} \quad (4.9)$$

The inner products of the orthonormal polynomials $\{P_n\}_{n=0}^\infty$ are defined on the space of continuous functions which are defined on some closed and bounded domain D .

The chebyshev polynomials defined over $chv \in [-1, 1]$ were used extensively in Judd (1992) and has the form shown in equation (4.10), in which the polynomials are orthogonal with respected to the weighting function $w(chv) = 1/\sqrt{1-chv^2}$.

$$T_n(chv) = \frac{n}{2} \sum_{r=0}^{n/2} \frac{(-1)^r}{n-r} \binom{n-r}{r} (2chv)^{n-2r} \quad (4.10)$$

A more efficient approximation algorithm can be constructed by using a set of orthonormal polynomials for asset pricing kernels (Chapman 1997). Therefore, we propose using Legendre polynomial functions to generate orthonormalized polynomials for the state variables.

The Legendre polynomial function is defined as:

$$P_l^n(x) = \frac{(-1)^m}{2^l l!} (1-x^2)^{\frac{m}{2}} \frac{d^m}{dx^m} P_l(x) \quad (4.11)$$

In which, the sets of orthonormal polynomials are formed by the legendre polynomial with weighting function $w(x) = 1$,

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l \quad (4.12)$$

In equation (4.12), l is the legendre degree with order $l=0,1,2,3,\dots,L$. $P_l(x)$ is a orthonormal polynomial vector with dimension $(L+1) \times 1$, with order 0 to l and evaluated at state variable x_t .

Hence the pricing kernel can also be expressed as:

$$\begin{aligned}
 Q(x_t, \theta) &= \sum_{k=0}^K \theta_k \frac{d^{k+1}U}{dU} P_k(x_t) \\
 &= \frac{d^1U}{dU} P_t^1 \theta
 \end{aligned} \tag{4.13}$$

Typically, given all these assumption and specifications, we can safely and intuitively allow the asset pricing kernel to choose aggregate consumption as the state variable which incorporates all market information. A large literature investigating CCAPM (consumption based asset pricing model) suggest that aggregate consumption is incapable of rationalizing the variability of asset returns. Considerable endeavor is given to the application of consumption proxies. An alternative approach is to hold Euler equation conditionally in a static setting with aggregate wealth, and then aggregate portfolio returns can be used equivalently as the proxy for aggregate consumption in pricing kernel approximations.

Therefore the pricing kernels under consideration are in the following equations.

Equation (4.14) and (4.15) are the tentative forms of pricing kernel with orthonormalized polynomials of aggregate consumption and aggregate portfolio

returns respectively.

$$\begin{aligned}
Q(x_t, \theta) &= \sum_{k=0}^K \theta_k \frac{d^{k+1}U}{dU} P_k(c_{A,t}) \\
&= \theta_0 P_0(c_{A,t}) + \theta_1 \frac{U''}{U'} P_1(c_{A,t}) + \theta_2 \frac{U'''}{U''} P_2(c_{A,t}) + \dots
\end{aligned} \tag{4.14}$$

$$\begin{aligned}
Q(x_t, \theta) &= \sum_{k=0}^K \theta_k \frac{d^{k+1}U}{dU} P_k(R_{w,t}) \\
&= \theta_0 P_0(R_{w,t}) + \theta_1 \frac{U''}{U'} P_1(R_{w,t}) + \theta_2 \frac{U'''}{U''} P_2(R_{w,t}) + \dots
\end{aligned} \tag{4.15}$$

A multifactor version of Merton's (1973) intertemporal asset pricing model was investigated in Fama and French (1993), in which the firms' market capitalization and (BE/ME) proxy for sensitivity to risk factors are consistent with stock returns and profitability. These factors outperform the CAPM beta in capturing the cross sectional variation in asset returns and help explain the cross-section of average returns. We construct the pricing kernel with aggregate portfolio returns as the state variable by implementing the three-factor model in Fama and French (1993). We also augment the model with a momentum factor (UMD) which has been applied in a few studies. Therefore, in contrast to Fama and French (1993), we propose the following model for asset returns,

$$\begin{aligned}
E(R_{it}) - R_{ft} = & \theta_0 + \theta_1 * E(Mktrf_t) + \theta_2 * E(Smb_t) \\
& + \theta_3 * E(Hml_t) + \theta_4 * E(Umd_t)
\end{aligned} \tag{4.16}$$

In this model, $E(R_{it}) - R_{ft}$ represents the excess return on representative agent's market portfolio, $Mktrf_t$ is the excess return on the market which is calculated as the value weighted return on all NYSE, AMEX, and NASDAQ stocks minus the one-month Treasury bill rate. Smb_t (Small Minus Big) is the average return on the three small portfolios (including small value, small neutral and small growth) minus the average return on the three big portfolios (including small value, small neutral and small growth). Hml_t is the average return on the two *value* portfolios (that is, with high BE/ME ratios) minus the average return on the two *growth* portfolios (low BE/ME ratios) and Umd_t is the equally weighted average of the returns on the winner¹ stock portfolios minus the returns on the loser² stock portfolios. Therefore, the implied linear pricing kernel with the four-factor asset pricing model can be expressed as the following,

$$\begin{aligned}
Q(x_t, \theta) = & \theta_0 + \theta_1 * E(Mktrf_t) + \theta_2 * E(Smb_t) \\
& + \theta_3 * E(Hml_t) + \theta_4 * E(Umd_t)
\end{aligned} \tag{4.17}$$

¹ Winner: The winner stock portfolio consists of the top 30% of the total stocks with the highest average prior performance.

² Loser: The loser stock portfolio consists of the bottom 30% of the total stocks with the lowest average prior performance.

A similar pricing kernel specification is defined in Dittmar (2002) and Jagannathan and Wang (1996). In equation (4.17), coefficients h_i are intended to capture the prices of corresponding risk factors. In a more fledged pricing kernel specification, the nonlinearity of the pricing kernel is captured by higher order terms of the polynomial function.

We propose a more parsimonious approach to implement F.F four factor model in pricing kernel approximation to capture the nonlinearity in pricing kernels. Instead of directly including higher order polynomials of asset portfolio returns or factors in asset pricing model, we investigate the pricing kernel in a Taylor series expansion framework. The higher order polynomials are created by the inner product of orthonormalized polynomials to avoid the co-linearity problem.

$$Q(x_t, \theta) = \theta_0 P_0(R_{w,t}^{FF}) + \theta_1 \frac{U^*}{U} P_1(R_{w,t}^{FF}) + \theta_2 \frac{U^{**}}{U^*} P_2(R_{w,t}^{FF}) + \dots \quad (4.18)$$

In which, it follows that

$$\begin{aligned}
P_0(R_{w,t}^{FF}) &= P_0(Mktrf_t) + P_0(Smb_t) + P_0(Hml_t) + P_0(Umd_t) \\
P_1(R_{w,t}^{FF}) &= P_1(Mktrf_t) + P_1(Smb_t) + P_1(Hml_t) + P_1(Umd_t) \\
&\dots
\end{aligned}$$

Based on this kernel specification, we choose aggregate consumption and aggregate wealth portfolio returns as the state variables to summarize the state of the economy. If the utility function of the representative agent is timely non-separable, we should also consider modeling the effects of the intertemporal nonseparability of utility function on asset pricing kernels. Enlightened by equation (4.5), we model this effect by incorporating future aggregate consumption into the pricing kernel specification. A 2-fold tensor product of the one dimensional polynomials was proposed in (Judd (1992)) and the extension to more folds tensor products were considered and tested in (Chapman (1997)). The cross terms of the tensor products were found to provide insignificant contribution in models' overfit, and incorporating cross terms of different period also bring additional noise to the models' sampling error. In this case, we define the pricing kernel function by using the two fold tensor product of orthonormalized factor polynomials. Therefore, we consider the following form for the pricing kernel.

$$\begin{aligned}
Q(R_w^{FF}, \theta) &= \sum_{k=0}^K \theta_k \frac{d^{k+1}U}{dU} [P_i(Mktrf_t)P_i(Mktrf_{t+1}) + P_i(Smb_t)P_i(Smb_{t+1}) \\
&\quad + P_i(Hml_t)P_i(Hml_{t+1}) + P_i(Umd_t)P_i(Umd_{t+1})] \\
&= \theta_0 [P_0(Mktrf_t)P_0(Mktrf_{t+1}) + P_0(Umd_t)P_0(Umd_{t+1}) \\
&\quad + P_0(Smb_t)P_0(Smb_{t+1}) + P_0(Hml_t)P_0(Hml_{t+1})] \\
&\quad + \theta_1 \frac{U''}{U'} [P_1(Mktrf_t)P_1(Mktrf_{t+1}) + P_1(Umd_t)P_1(Umd_{t+1}) \\
&\quad + P_1(Smb_t)P_1(Smb_{t+1}) + P_1(Hml_t)P_1(Hml_{t+1})] \\
&\quad + \theta_2 \frac{U'''}{U'} [P_2(Mktrf_t)P_2(Mktrf_{t+1}) + P_2(Umd_t)P_2(Umd_{t+1}) \\
&\quad + P_2(Smb_t)P_2(Smb_{t+1}) + P_2(Hml_t)P_2(Hml_{t+1})] \\
&\quad + \dots
\end{aligned} \tag{4.19}$$

However, to further investigate the ability of the augmented Fama-French 3 factors model in approximating the nonlinear pricing kernel, we should consider including the Fama-French 6 portfolio and Fama-French 25 portfolio returns as state variable components into the pricing kernel in equation (4.19). Since Fama-French portfolio returns will nest a significant part of the market risk of asset returns and the augmented Fama-French 3 factors model is only capable of explaining the most part of excess returns of financial assets. Then, the market based CAPM pricing kernel under the intertemporally separable utility framework will follow the following form:

$$\begin{aligned}
Q(x_t, \theta) &= \theta_0 P_0(R_{pfs}^{FF}) + \theta_1 P_0(R_{w,t}^{FF}) \\
&\quad + \theta_2 \frac{U''}{U'} P_1(R_{w,t}^{FF}) + \theta_3 \frac{U'''}{U''} P_2(R_{w,t}^{FF}) + \dots
\end{aligned} \tag{4.20}$$

However, in the case of intertemporally nonseparable utility framework, the corresponding pricing kernel will be tested in the following equation:

$$\begin{aligned}
Q(x_t, \theta) = & \theta_0 P_0(R_{pfs}^{FF}) + \theta_1 P_0(R_{w,t}^{FF}) P_0(R_{w,t+1}^{FF}) \\
& + \theta_2 \frac{U''}{U'} P_1(R_{w,t}^{FF}) P_1(R_{w,t+1}^{FF}) + \theta_3 \frac{U'''}{U''} P_2(R_{w,t}^{FF}) P_2(R_{w,t+1}^{FF}) + \dots
\end{aligned} \tag{4.21}$$

As for taking aggregate consumption as the state variable, we can express the timely non-separable pricing kernel in the following equation:

$$\begin{aligned}
Q(C_t, \theta) = & \theta_0 P_0(C_t) P_0(C_{t+1}) + \theta_1 \frac{U''}{U'} P_1(C_t) P_1(C_{t+1}) + \theta_2 \frac{U'''}{U''} P_2(C_t) P_2(C_{t+1}) \\
& + \dots
\end{aligned} \tag{4.22}$$

The pricing kernels with more periods and state variables can be nested similarly. For instance, an intertemporally nonseparable specification for pricing kernel with higher order polynomials can be shown as the following.

$$\begin{aligned}
Q(C_t, \theta) = & (\theta_0^2) P_0(C_t) P_0(C_{t+1}) - (\theta_1^2) P_1(C_t) P_1(C_{t+1}) + (\theta_2^2) P_2(C_t) P_2(C_{t+1}) \\
& - (\theta_3^2) P_3(C_t) P_3(C_{t+1})
\end{aligned} \tag{4.23}$$

Another question in approximating pricing kernels is how to determine the maximum order in polynomial functions. As noted in Chapman(1997), choosing the appropriate maximum order for the pricing kernel polynomial is equivalent to measuring the magnitude of approximation error in any particular application. Nevertheless as noted in Judd(1992), the maximum order in the optimal pricing kernel function can not be determined at priori and the order should be infinity if it should have been determined (Judd 1992). Bansal et al, (1993) use the model to guide the orders truncation, however, allowing the data to determine the pricing kernel specification may have the risk of potentially overfitting models.

An alternative approach that allows preference theory to determine the maximum order is to impose restrictions on representative agent's utility function with decreasing absolute prudence (Dittmar 2002, Bansal et al, 1993). Through imposing this restriction in representative agent's utility function, it will not rule out certain counterintuitive risk-taking actions of the agent (Kimball 1993, Pratt and Zeckhauser 1987). When the agent's preference is restricted only to have decreasing absolute risk aversion, he maybe still willing to take the sequential gambles with negative mean even if this agent had already accepted a bet with negative outcome. Then by imposing standard risk aversion on agent's preferences, we will have the following

equation,

$$-\frac{d\frac{U'''}{U''}}{dW} = \frac{(U''')^2 - U''''U''}{(U''')^2} < 0 \quad (4.24)$$

$$(U''')^2 - U''''U'' < 0$$

In equation (4.24), it implies that $U'''' < 0$, and correspondingly we are able to determine the signs of the first three polynomial coefficients in the Taylor series expansion. Implementing standard risk aversion restriction on representative agent's utility function implicitly assumes that the covariance between asset returns and polynomial terms of chosen state variables with order greater than three is zero. Besides, the lost of power from omitting the higher order polynomial terms will be reimbursed by the increased power from following preference theory.

Therefore, the asset pricing kernels we proposed above are flexible and parsimonious in capturing the nonlinearity of pricing kernels. In contrast to nonparametric modeling in prior works, the functional forms are guided by preference theory to determine the signs of the coefficients and therefore are free from ad hoc specification problems. Furthermore, the effects of the intertemporal non-separability of utility function on

asset pricing kernels were also incorporated within the functional form. These specifications and restrictions will deliver more statistical power to the model testing.

4.4. Estimation of the Approximated Pricing Kernel

As discussed in section II, the imposition of decreasing absolute prudence on representative agent's utility function implies that the proposed pricing kernels are decreasing in the linear terms, increasing in the quadratic polynomial terms and decreasing in the cubic polynomial terms. With the guidance of these restrictions, we should investigate equation (4.14) and (4.15) in the following form:

$$\begin{aligned}
 Q(R_{w,t}, \theta) &= \sum_{k=0}^K \theta_k \frac{d^{k+1}U}{dU} P_k(R_{w,t}) \\
 &= \theta_0 P_0(R_{w,t}) + \theta_1 \frac{U''}{U'} P_1(R_{w,t}) + \theta_2 \frac{U'''}{U'} P_2(R_{w,t}) + \theta_3 \frac{U''''}{U'} P_3(R_{w,t}) \\
 &= (\hat{\theta}_0^2) P_0(R_{w,t}) - (\hat{\theta}_1^2) P_1(R_{w,t}) + (\hat{\theta}_1^2) P_2(R_{w,t}) - (\hat{\theta}_1^2) P_3(R_{w,t})
 \end{aligned} \tag{4.25}$$

$$\begin{aligned}
 Q(C_t, \theta) &= \sum_{k=0}^K \theta_k \frac{d^{k+1}U}{dU} P_k(c_{A,t}) \\
 &= \theta_0 P_0(c_{A,t}) + \theta_1 \frac{U''}{U'} P_1(c_{A,t}) + \theta_2 \frac{U'''}{U'} P_2(c_{A,t}) \\
 &= (\hat{\theta}_0^2) P_0(c_{A,t}) - (\hat{\theta}_1^2) P_1(c_{A,t}) + (\hat{\theta}_2^2) P_2(c_{A,t}) - (\hat{\theta}_3^2) P_3(c_{A,t})
 \end{aligned} \tag{4.26}$$

We will estimate the parameters of approximated pricing kernels by using the generalized method of moments (GMM).

Then the Euler equation (4.1) can be expressed as the following:

$$E_t\{[(1 + R_{i,t+1})^* Q_{t+1}] \otimes \zeta_t\} = 1_N \otimes \zeta_t \quad (4.27)$$

In which, ζ_t is the available information set to agents at period t .

Using aggregate consumption as the state variable and orthonormalized polynomials, equation (4.26) and equation (4.27) can be expressed as the following,

$$E_t\{[(1 + R_{i,t+1})^* ((\theta_0^2)P_0(c_{A,t}) - (\theta_1^2)P_1(c_{A,t}) + (\theta_1^2)P_2(c_{A,t}) - (\theta_1^2)P_3(c_{A,t}))] \otimes \zeta_t\} = 1_N \otimes \zeta_t \quad (4.28)$$

We approximate the pricing kernel as a static function of risk factors. By assuming static kernel settings we implicitly ignore the time variability of function parameters. We assume the state variables are time-invariant functions and will encompass all available homochronous information. As noted in Campbell (1996), the pricing of the time variability of risk factors are found evidently necessary in model specification and are proportional to the pricing of their market risk. Therefore, a more parsimonious kernel structure that is capable of incorporating the intertemporal variability of asset returns should be considered.

However, in this analysis we focus on analyzing nonlinearity effect and non-separability of utility function effect on models' performance. Hence, a conditionally restricted function will be more sensible comparing to modeling the coefficients as intertemporally varying functions of informative or instrumental variables. The intuitive configuration for the time-variant parameters would be directly modeling the coefficients as linear functions of instrumental variables or a specified set of informative variables. Pertinent coefficient structures can be found in Ferson and Harvey (1989), Dumas and Solnik (1995) and Dittmar (2002).

Therefore, modeling the intertemporally varying coefficient in pricing kernels will remain unanswered as an open question for future studies and we will pursue a different approach in this analysis. It was suggested that controlling the state variable directly by a set of instrumental variables is equivalently capable of driving the model without specifically estimating the time variability of coefficients. On the other hand it will restrict our models conditionally on the time risk (Shanken (1991), Cochrane (1996)).

The orthogonality condition of the Euler equation augmented by the set of instrumental variables ζ_t can be transformed into the following:

$$\begin{aligned}
& E_t \{ [(1 + R_{n,t+1}) \otimes \zeta_t]^* ((\theta_0^2) P_0(c_{A,t}) - (\theta_1^2) \frac{U^m}{U} P_1(c_{A,t}) + (\theta_2^2) \frac{U^m}{U} P_2(c_{A,t}) \\
& \quad - (\theta_3^2) \frac{U^m}{U} P_3(c_{A,t})) \} \\
& = 1_N \otimes \zeta_t
\end{aligned} \tag{4.29}$$

Therefore, the moment conditions for individual assets can be shown as the following:

$$\begin{aligned}
g_T(\theta) &= T^{-1} \sum_{t=1}^T \{ [(1 + R_{n,t+1}) \otimes \zeta_t]^* ((\theta_0^2) P_0(c_{A,t}) - (\theta_1^2) \frac{U^m}{U} P_1(c_{A,t}) + \\
& \quad (\theta_2^2) \frac{U^m}{U} P_2(c_{A,t}) - (\theta_3^2) \frac{U^m}{U} P_3(c_{A,t})) - 1_N \otimes \zeta_t \} \\
& = 0_{NL}
\end{aligned} \tag{4.30}$$

The gross asset return scaled by instrumental variable set $(1 + R_{n,t+1}) \otimes \zeta_t$ can be comprehended as the total return of a well diversified investment portfolio. In other words, as noted in Chapman (1997) that investors make their investment decisions or choices under the guidance of selectively observed information from a specific instrumental information set. Equation (4.30) is a system of $NL \times 1$ sample orthogonality conditions, in which, T represents the total number of time series observations, N refers to the total number of financial assets for analysis and L refer to the number of instrumental variables.

Furthermore, we obtain the objective function for GMM estimation of the model specification in the following form,

$$J_{GMM}(\theta) = \min_{\theta} g_T(\theta)' W_T g_T(\theta) \quad (4.31)$$

In the objective function, W_T is the optimal weighting matrix of the GMM estimators and is defined as the inversed long-run covariance matrix of the moment conditions' sampling errors $W_T^* = [g_T(\theta)g_T'(\theta)]^{-1}$. In later works Ferson & Foerster (1994) and Chapman (1997), studies suggest that this weighting matrix exhibits poor finite sample properties and its large pricing errors in estimation will produce very small J-statistics. A large literature has proven that this optimal weighting matrix can be consistently estimated with HAC (heteroskedasticity and autocorrelation consistent) estimators (Hansen (1982), Ogaki (1993)). In our case, we define the HAC with Bartlett kernel weighting matrix and a specified bandwidth. The moment condition sampling errors from the approximated pricing kernel will approach zero if the pricing kernels we proposed are well defined and the objective function will be minimized. With the test statistic defined in Jagannathan and Wang (1991), we test the model's overidentifying restrictions by minimizing the following function:

$$J_T = T \bullet J(\hat{\delta}_{GMM}) \quad (4.32)$$

The test statistic is a Chi-square distribution with NL-K degrees of freedom, NL is the total number of moment conditions implied in the GMM test and K is the number of parameters under estimation.

An alternative approach to evaluate the proposed pricing kernels is to replace the efficient estimator weighting matrix with the inversed second moment matrix of the asset returns being scaled by instrumental variables. As noted in Hansen and Jagannathan (1997), the instrumental variable scaled return weighting matrix can be shown as the following:

$$W_{HJ} = E\{[(1 + R_{n,t+1}) \otimes \zeta_t][(1 + R_{n,t+1}) \otimes \zeta_t]'\} \quad (4.33)$$

Then the minimum mean-square distance from any pricing kernel to the optimal bound given by the mean and the standard deviation of a given set of asset returns is the square root of the Hansen Jagannathan J-statistics, which is developed by Hansen and Jagannathan (1991) and can be expressed as the following:

$$J^{HI}(\theta) = \min_{\theta} g_T(\theta)' W_{HI} g_T(\theta) \quad (4.34)$$

As noted in Jagannathan and Wang (1996) and Dittmar (2002), replacing the inversed sampling error covariance weighting matrix with the instrument-scaled returns' weighting matrix will allow direct comparison among nested and non-nested models since the weighting matrix is invariant across all models tested. In addition, Dittmar (2002) and Cochrane (2001) suggest that parameter estimates using instrument-scaled weighting matrix may be more stable and more robust to heteroskedasticity and autocorrelation problems than in standard GMM estimation. Nonetheless, it is also argued that using the Hansen-Jagannathan estimator rather than standard GMM estimators may trade size for power. And using the iterated GMM estimators exhibit superior performance in finite samples.

To measure the tradeoff of Hansen-Jagannathan estimators in finite sample and compare its robustness and stability with standard GMM estimators, we also estimate the models using Hansen-Jagannathan, iterated and standard estimators in our analysis.

4.5. Data Description and Estimation Process

Our data set consists of observations on the personal nondurable consumption, industry asset portfolio returns from 20 SIC industries, Fama-French 6 and 25 portfolio returns and the 4 factors used in Fama-French models, and a number of instruments used as the conditioning information. Also, as motivated in Chapman (1997), we include a temporary technology shock defined as the growth rate of solow residuals from an aggregate Cobb-Douglas production function. The function specification will be detailed in Appendix.A. All data series cover the period from January 1970 to December 1999 in monthly frequency.

We obtain the 20 SIC stock return series from the Center for Research in Security Prices (CRSP). The portfolio returns and 4 factors included in Fama-French data series were obtained from Kenneth French's web site at Dartmouth and Wharton Research Data Services (WRDS). The per capita personal consumption data contains real "personal consumption expenditures" on nondurable goods scaled by residential population and is obtained from Fed St.louis. The instrumental variables s & $pdivyield_t$, $Crspexr_t$ and $Texr_t$ are obtained from Standard & Poor's COMPUSTAT North America monthly database.

We construct $\zeta_t = \{1_{NL}, s \& pdivyield_t, Crspexr_t, Texr_t, Tbill30_t\}$ as the instrumental variable set, where 1_{NL} denotes a vector of ones, $s \& pdivyield_t$ is the dividend yield on the S&P 500 composite index, $Crspexr_t$ is the excess return on the CRSP value-weighted index at time t , $Texr_t$ is the excess yield on the 10-year Treasury bill in excess of the yield on 30-day Treasury bill, and $Tbill30_t$ is the Treasury bill yield with maturity of 30 days.

All these instrumental variables in our instrumental variable set are investigated to be able to predict the asset returns. The 6 Fama-French Portfolios are formed by the intersections of two portfolios grouped by size (market equity, ME) and three portfolios formed on the ratio of book equity to market equity (BE/ME). The market equity or firm size (ME) is measured by a firm's market capitalization or market value of equity. It is in turn defined as the product of stock price and the number of outstanding shares at the end of the fiscal year t . The book-to-market equity (BE/ME) is measured as the ratio between a firm's book equity (BE) at the fiscal year-end in calendar year $t - 1$ and its market equity (ME) at the end of December of year $t - 1$. The 25 Fama-French Portfolios are formed by the intersections of five portfolios formed on size (market equity, ME) and five portfolios formed on the ratio of book equity to market equity (BE/ME). And all the portfolio data were created by

combining CRSP market equity data and COMPUSTAT book equity data. The size breakpoint for year t is the median NYSE market equity at the end of June of year t . (BE/ME) for June of year t is the book equity for the last fiscal year end in $t-1$ divided by ME for December of year $t-1$. The (BE/ME) breakpoint are the 30th and 70th NYSE percentiles.

As noted in Fama and French (1993), the Fama-French factors are the returns on Fama-French portfolios constructed from the intersections of two portfolios formed on size, as measured by market equity (ME), and three portfolios using, as proxy for value, the ratio of book equity to market equity (BE/ME) .

We choose these series to reflect the variations in financial markets and the real economy. The instrumental variables have been used in numerous empirical studies to investigate the time series properties of asset returns. Summary statistics for the instrumental variables are demonstrated in Table I.

In table II, we present a Wald type of test and a F-test on the predictive power of instrumental variable set $\zeta_t = \{1_{NL}, s \& pdivyield_t, Crspexr_t, Texr_t, Tbill30_t\}$ for the asset returns. The null hypothesis of the Wald test is that the instrumental variables have no

predictive power over the asset returns. And the results show that the instrumental variable set is well selected and the instrumental variables should be capable of predicting asset returns.

4.6. Estimation Results

In this section, we discuss the financial assets used in model specifications and the corresponding tests of Euler equations under different pricing kernel settings. The 20 industry-sorted asset portfolios follow the definitions of the four-digit SIC codes used initially in Moskowitz and Grinblatt (1999) and the portfolio returns are widely used by U.S. Securities and Exchange Commission (SEC).

In Table I we present the descriptive statistics for the set of state variables. In panel A through C in Table I, we show the statistics for the 20 SIC industry portfolio returns as in Moskowitz and Grinblatt (1999), the aggregate consumption growth rate, technology shock growth rates, the four factors used in Fama-French 3-factor models and the 6 and 25 Fama-French selected large portfolio returns. The summary statistics of instrumental variable set $\zeta_t = \{1_{NL}, s \& pdivyield_t, Crspexr_t, Texr_t, Tbill30_t\}$ are listed in Panel D of Table I.

In Table II, we present a wald test on the predictive power of the instrumental variables for the 20 SIC industry portfolio asset returns used in our study. We evaluate the predictive power of instrumental variables by following a similar projection of the portfolio asset returns onto the instrumental variables as shown in Dittmar (2002):

$$R_{i,t+1} = \beta' \zeta_t + u_{t+1} \quad (4.31)$$

The null hypothesis of the Wald test and F test are that the instrumental variables have no predictive power over asset portfolio returns. And the results show that the instrumental variables are capable of predicting asset returns and the list of instrumental variables is conceivably selected.

The results in Table III through Table V discuss the kernel specifications based on aggregate consumption growth rates. In Table III, we show the test results for kernel specifications with intertemporally separable utility function. The moment conditions are scaled by Hansen Jagannathan return-scaled weighting matrix and the state variables contains only the growth rates of aggregate consumption. We also present the Hansen Jagannathan distance measure and model specification test results. The first row in every Table shows the values of the estimated coefficients, F-value, H-J statistics with P-value and H-J distance Measure.

As shown in Table III, the linearly approximated pricing kernel, quadratic and cubic pricing kernels in Panel A, B and C are not rejected at 1% level of significance.

Besides, the coefficient estimates are significant and marginally significant for linear and quadratic pricing kernels respectively. The quadratic term in the quadratic pricing kernel is marginally significant at 10% level of significance and the quadratic pricing kernel exhibits marginal improvement in model's fit and the distance measure from linear pricing kernels. The distance measurement decreases from 0.8405 to 0.8360, dropped by 0.53%. However, in the case of cubic pricing kernel, the pricing kernel does not improve the fit of the model and none of the coefficient estimates is significant. The distance measure for cubic pricing kernel also shows zero improvement from quadratic pricing kernel specification. Therefore under the intertemporally separable utility function, incorporating a cubic term of the state variable will not improve the kernel specification and will invalidate lower order terms' significance.

Table IV demonstrates the effects of incorporating cross terms into the pricing kernel specifications under intertemporally non-separable utility function framework. The cross terms of current period consumption growth and the growth of one period ahead consumption do not improve the distance measurement, although the coefficient estimates in linear and cubic pricing kernel specifications are mostly significant. These results are consistent with the findings in Chapman (1997). Therefore, our

following analysis will focus on the pricing kernels with orthonormalized state variables at different orders without cross terms. We analyze the pricing kernels with higher order terms under the intertemporally non-separable utility framework in Table V. The outcomes of the specifications improve significantly comparing to the results shown in Table III. In the linear and cubic kernel specifications, all coefficient estimates are significant at 1% level and the distance measurement is significantly improved from 0.8405 to 0.8128. The improvement represents a decrease of 3.3% in the distance measurement relative to the linear pricing kernel without incorporating the one period ahead consumption growth. In cubic kernel specification, none of the coefficient estimates is significant. All these tests demonstrate that in consumption based asset pricing models, incorporating cubic terms into pricing kernel specifications does not improve the models' fit.

Thus far, we have discussed the specification tests of the Euler equation under the circumstances of consumption based pricing kernels with linear, quadratic and cubic time-varying coefficients. And we have observed considerable improvement in model's fit through moving from a linear specification to nonlinear specifications. In order to gain more insight of the performance of different kernel specifications, we will compare the performance of consumption polynomial pricing kernels to

multifactor models in pricing the cross section of financial portfolios. As noted earlier, multifactor asset pricing models have been more successful than single factor model in pricing the cross section of equities. In the following we will measure the performances of the polynomial pricing kernels to price the cross section of financial portfolios relative to a popular multifactor model, the Fama and French (1993) three-factor model and large portfolio returns.

Although multifactor models will allow researchers with considerable flexibility when the models give very little direction for the choice of factors, we explicitly define the portfolio of aggregate wealth as the relevant factor for pricing. And we impose restrictions on properties of the coefficients on each of pricing kernel polynomials by following the preference theory.

In Table VI we show the results for the estimation of the Fama-French three-factor model. The three factor model is augmented with the equally weighted average of returns on the winner stock portfolios minus the returns on the loser stock portfolios. The results suggest that the pricing kernels implied by the model perform poorly in describing the cross section of industry returns under the intertemporally separable utility framework. In the case of estimating orthonormalized linear pricing kernel, we

observe the H-J distance measure improves marginally from 0.831 to 0.828 compare to linear pricing kernel settings. In the case of estimating quadratic and cubic pricing kernels, although the distance measure improve marginally, the p-value of the specification tests are largely insignificant. This indicates that incorporating the quadratic and cubic polynomial terms may bring marginal improvement in fitting the pricing kernel, however it brings higher risk of losing degree of freedom from adding additional terms. More interestingly, the $Mktrf_t$ and Smb_t terms continue to be significant with a p-value less than 0.005.

Table VII presents the estimation results for pricing kernels based on portfolio returns with cross terms of one period ahead asset returns as expressed in equation (19). The large portfolio returns are not included in the estimation. The results are consistent with our observation in Table IV, The cross terms of current period factors and the factors of one period ahead do not significantly improve the distance measurement, however, almost none of the coefficient estimates in linear, quadratic and cubic pricing kernel specifications is significant. These further prove that adding the cross terms in between the state variables will provide insignificant contribution in improving models' overfit, and bring additional noise to the model's sampling error.

In Table VIII and IX we estimate pricing kernels motivated by adding the large portfolio asset returns into the four-factor model are examined. In panel A through c in Table VIII we show the estimation results for kernels based on one period state variable. In this case, incorporating the quadratic and cubic terms of large portfolio asset returns barely improves the distance measure and the coefficient estimates for the quadratic and cubic terms are mostly insignificant. It is noteworthy that incorporating the linear terms of the Fama-French three-factor model augmented with the momentum factor exhibit considerable stability. The Smb_t and Hml_t terms are mostly significant and are not impacted by incorporating the quadratic or cubic terms of large portfolio asset returns. In Table IX We consider incorporating the technology shock growth into kernel estimation. Adding the technology shock growth does improve the explanatory power of large portfolio asset returns in the linear and quadratic pricing kernels. However, adding higher terms barely improve the distance measure and none of the coefficient estimates is significant. It further prove that adding the higher terms of large portfolio asset returns and technology shock growth would dramatically offsets the degrees of freedom by adding unnecessary noise to the model. Consistent with the findings in Table VIII, the terms of the Fama-French three-factor model augmented with the momentum factor exhibit considerable stability in the linear and quadratic pricing kernel settings. The Smb_t and Hml_t

terms are significant and not impacted by adding the quadratic and cubic terms of large portfolio returns and technology shock growth. In the case of adding cubic terms of large portfolio returns and technology shock growth, none of the estimates is significant.

In Panel A through D in Table X we examine the pricing kernels motivated by the time nonseparable utility specification of equation (20) augmented by the technology shock growth. In the linear pricing kernel with one period ahead state variables augmented by the four factors, contemporary and one period ahead state variables are mostly significant except for the one period ahead technology shock growth. In the case of Legendre polynomials of the linear pricing kernel, the distance measure is significantly improved from 0.822 to 0.818 and the estimates for one period ahead technology shock growth are all insignificant, which are consistent with the results shown in the linear pricing kernel. The combined effects of large portfolio asset returns and technology shock growth are insignificant with higher order pricing kernel terms added to the model. As observed in Table VII and Table IX, the parameter estimates for Smb_t and Hml_t terms are significant and intact by adding the higher order polynomial terms of large portfolio returns and technology shock growth, even in the case of the existence of cubic pricing kernel terms of one period

ahead state variables.

We extend our analysis of pricing kernel specification from technology shock augmented two-period portfolio returns and Fama-French three factor models to include aggregate consumption growth rates in Table XI. Panel A shows estimation results for kernel based on two-period portfolio returns and cross terms of two period consumption growth. The results are consistent with Panel A in Table X that coefficient estimates for two period portfolio returns and first period technology shock are significant except for the one period ahead technology shock growth. The coefficient estimates for linear and cubic orthonormalized consumption cross terms are insignificant. By adding the two-period consumption polynomial cross terms, the H-J distance measure improved from 0.822 to 0.817 and the coefficient estimates for Smb_t , Hml_t , and Umd_t terms are still significant.

In Panel B, the coefficient estimates for most terms are insignificant except for current period large portfolio returns and technology shock when two-period orthonormalized consumption cross terms are added into the quadratic large portfolio return kernel specifications. The improvement in the H-J distance measure is barely noticeable however the point estimates for Smb_t , Hml_t , and Umd_t terms are still significant.

In Table XII, we conduct the similar analysis in Table XI that we include additive terms of orthonormalized consumption growth rates to the solow residual augmented two-period portfolio return pricing kernel. The coefficient estimates for two-period portfolio returns and first period technology shock are significant except for the one period ahead technology shock growth in Panel A when only linear terms are present. The H-J distance measure is significantly improved from 0.817 to 0.812 and most of the orthonormalized consumption terms are insignificant. In Panel B, the H-J distance measure is slightly improved from 0.812 to 0.810 and estimates for the polynomial terms are very consistent to the results shown in Panel B of Table XI. The linear terms of current period large portfolio returns and technology shock are significant when the additive terms of orthonormalized consumption growth rates are included in the kernel specification. We found that only the linear term of one period ahead consumption growth rate remains significant after been included. Beside, the point estimates for Smb_t , Hml_t , and Umd_t terms remain highly significant.

4.7. Conclusion

In this paper, we investigate pricing kernels in dynamic asset pricing models and the variation in financial asset returns. We implement nonparametric approach in parameter approximations and consider for nonlinear specifications in continuous pricing kernel functions of the state variables.

We also extend the tests of representative consumers' asset pricing model by approximating the pricing kernel for monetary assets to consumption-based and market-based environments in which the consumer is assumed to have intertemporal nonseparable utility functions. The classical consumption-based kernels and market-based kernels are investigated in our asset pricing models for their empirical performance and statistical significance with both kernel functions being guided by preference theory.

We propose a Taylor series expansion with coefficients being driven by the derivatives of consumers' utility function and a set of orthonormal polynomials of the aggregate state variables to approximate the pricing kernels. This approximation approach has the advantage of eliminating the linear relationships over certain portion of the state variables' domain, especially occur in between the higher order terms. In

addition, following the preference theory will help us restrict the properties of certain terms in the model to avoid ad hoc assumptions and data over fitting problems.

We conducted pricing kernel approximation with industry asset portfolio returns from 20 SIC industries, Fama-French 6 and 25 portfolio returns. The Fama-French three-factor model has been augmented with a momentum term and includes a number of instrumental variables as the conditioning information. The specification tests and approximations yield results that pricing kernels with intertemporal nonseparable utility functions will improve the pricing kernels' performance in measuring large pricing errors observed in asset returns in both cases of consumption based and market-based kernel specifications. It will further substantially improve pricing kernels' performance when future value of the state variables and nonlinear functions of orthonormalized state variables are incorporated. Particularly, incorporating orthonormalized aggregate consumption growth under the specification of intertemporal nonseparable utility function in pricing kernels demonstrates the most stable and significant performance. Results show that the approximated pricing kernels motivated by intertemporal nonseparable utility functions over the durable consumption goods and portfolio investments under static setting will outperform the pricing kernels with inter-temporally separable preferences in general. More

importantly, the marginal contribution of incorporating large portfolio asset returns into the Fama and French (1993) three factor model is substantial in improving the kernels' explanatory power over the pricing errors. Furthermore, adding higher order terms of the state variables in pricing kernels, the cubic terms for instance will offset the degrees of freedom through trading less improvement with losing more power by adding noise to the system. This result further necessitates parsimonious nonlinearity specifications in pricing kernels, as emphasized in Dittmar (2002) that ad hoc nonlinearity specifications will bring disastrous results and should be avoided.

A notable observation from incorporating the Fama and French (1993) three factor model in the pricing kernel is that the parameter estimates of the Smb_t and Hml_t terms are statistically significant in general. And the results suggest that linear functions of the three factors in the pricing kernels are significantly sufficient to account for the admissible pricing errors without restrictions in preferences and functional forms.

The results in this paper infer a few interesting questions and directions for future research. Since we allow the parameters of pricing kernel vary over time, what ex ante specification over the state variables should be imposed in order to track the

observed large pricing errors is still an unanswered question, particularly when nonlinearity is imposed. Alternative approaches may fall on the approximation of coefficient estimates as time-varying functions of the information set, or the time variability of pricing kernel parameters.

Reference

- Bansal, Ravi and S. Viswanathan, 1993, "No Arbitrage and Arbitrage Pricing: A New Approach", *Journal of Finance*, 48, 1231-62.
- Bansal, Ravi, David A. Hsieh, and S. Viswanathan, 1993, "A New Approach to International Arbitrage Pricing", *Journal of Finance*, 48, 1719-47.
- Barnett, William A. 1978, "The user cost of money", *Economic Letters*, 1, 145-149.
Reprinted in W. A. Barnett and A. Serletis (eds.) (2000), *The Theory of Monetary Aggregation*, North-Holland, Amsterdam, 1, 6-10.
- Barnett, William A., Yi Liu, and Mark J. Jensen, 1997, "CAPM risk adjustment for exact aggregation over financial assets", *Macroeconomic Dynamics*, 1, 485-512. Reprinted in W. A. Barnett and A. Serletis (eds) (2000), *The Theory of Monetary Aggregation*, North-Holland, Amsterdam, 12, 245-273.
- Barnett, William A. and Shu Wu, 2005. "On user costs of risky monetary assets." *Annals of Finance*, 1, 35-50.
- Breeden, Douglas T., 1979, "An inter-temporal asset pricing model with stochastic consumption and investment opportunities", *Journal of Financial Economics*, 7(3), 265-296.
- Breeden, Douglas T., 1979, "An intertemporal asset pricing model with stochastic consumption and investment opportunities", *Journal of Financial Economics*, 7,

265-296

- Breeden, Douglas T., and Robert H. Litzenberger, "Prices of State Contingent Claims Implicit in Option Prices." *Journal of Business*, 51, 621-51.
- Brown, David P., and Michael R. Gibbons, "A Simple Econometric Approach for Utility-based Asset Pricing Models.", *Journal of Finance*, 40, 359-81.
- Campbell, John, 1987, "Stock Returns and the Term Structure", *Journal of Financial Economics*, 18, 373-399
- Campbell, John, 1996, "Understanding Risk and Return", *Journal of Political Economy*, 104, 298-345.
- Campbell, John, Andrew Lo, and Craig MacKinlay, 1995, "The Econometrics of Financial Markets", Princeton University Press, Princeton, NJ.
- Chapman, David A., 1997, "Approximating the Asset Pricing Kernel", *Journal of Finance*, 52-4, 1383-1410.
- Cochrane, John, 1996, "A Cross-sectional Test of an Investment-based Asset Pricing Model", *Journal of Political Economy*, 104, 572-621.
- Dittmar, Robert F., 2002. "Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns", *Journal of Finance*, American Finance Association, 57(1), 369-403.
- Duffie, Darrell, and William Zame, 1989, "The Consumption-Based Capital Asset Pricing

- Model", *Econometrica*, Econometric Society, 57(6), 1279-97.
- Dumas, Bernard, and Bruno Solnik, 1995, "The World Price of Foreign Exchange Risk", *Journal of Finance*, 50, 445-479.
- Fama, Eugene F., and Keneth R. French, 1992, "The Cross Section of Expected Stock Returns", *Journal of Finance*, 47, 427-465.
- Fama, Eugene F., and Keneth R. French, 1993, "Common risk factors in the returns on stocks and bonds", *Journal of Financial Economics*, 33, 3-56.
- Fama, Eugene F., and Keneth R. French, 1995, "Size and Book-to-Market Factors in Earnings and Returns", *Journal of Finance*, 50, 131-155.
- Fama, Eugene F., and Keneth R. French, 1995, "Size and Book-to-Market Factors in Earnings and Returns", *Journal of Finance*, 50, 131-155.
- Fama, Eugene F., and Keneth R. French, 1996, "Multifactor Explanations of Asset Pricing Anomalies", *Journal of Finance*, 51(1), 55-84.
- Fama, Eugene F., and Keneth R. French, 1998, "Value versus growth: the international evidence", *Journal of Finance*, 53, 1975-1979.
- Ferson, Wayne E, and George M. Constantinides, 1991, "Habit Persistence and Durability in Aggregate Consumption: Empirical Tests", *Journal of Financial Economics*, 29, 199-240.

- Hansen, Lars Peter, and Ravi Jagannathan, 1991, "Implications of security market data for models of dynamic economies", *Journal of Political Economy*, 99, 225-262.
- Hansen, Lars Peter, 1982, "Large Sample Properties of Generalized Method of Moments Estimators", *Econometrica*, 50, 1029-1054.
- Hansen, Lars Peter, and Ravi Jagannathan, 1997, "Assessing Specification Errors in Stochastic Discount Factor Models", *Journal of Finance*, 52, 557-590.
- Hansen, Lars Peter, and Scott F. Richard, "The Role of Conditioning Information in Deducing Testable Restrictions implied by Dynamic Asset Pricing Models", *Econometrica*, 55, 587-613.
- Harrison, J. Michael, and David M. Kreps, "Martingales and Arbitrage in Multiperiod Securities Markets", *Journal of Economic Theory*, 20, 381-408.
- Jagannathan, Ravi, and Zhenyu Wang, 1996. "The Conditional CAPM and the Cross-Section of Expected Returns", *Journal of Finance*, American Finance Association, 51(1), 3-53.
- Judd, Kenneth L., 1992, "Projection Methods for Solving Aggregate Growth Models", *Journal of Economic Theory*, 58, 410-452.
- Kimball, Miles S., 1993, "Standard Risk Aversion", *Econometrica*, 61, 589-611.
- Leroy, Stephen F., 1973, "Risk aversion and the martingale property of stock returns". *International Economic Review*, 14, 436-46.

- Lo, Andrew W., and Andrew Archie C. MacKinlay, 1990, "Data Snooping and Biases in Tests of Financial Models", *Review of Financial Studies*, 3, 431-468.
- Lucas, Robert E., 1978. "Asset prices in an exchange economy", *Econometrica*, 46, 1429-46.
- Mehra, Rajnish, and Edward C. Prescott, 1985, "The Equity Premium: A Puzzle", *Journal of Monetary Economics*, 15, 145-161.
- Merton, Robert C., "Theory of Rational Option Pricing", *Bell Journal of Economics and Management Science*. 4, 141-83.
- Moskowitz, Tobias, and Mark Grinblatt, 1999, "Do Industries Explain Momentum?", *Journal of Finance*, 54, 1249-1290.
- Ogaki, M., 1993, "Generalized Method of Moments : Econometric Applications". In G.S. Maddala, C.R. Rao and H.D. Vinod, (eds), *Handbook of Statistics*, 11, 455-488.
- Pratt, John W., and Richard J. Zeckhauser, 1987, "Proper Risk Aversion", *Econometrica*, 55, 143-54.
- Rubinstein, Mark, 1976, "The valuation of uncertain income streams and the pricing of options", *Bell Journal of Economics*, 7, 407-25.
- Shanken, Jay, 1990, "Intertemporal Asset Pricing: An Empirical Investigation", *Journal of Econometrics*, 45, 99-120.
- Stokey, Nancy L. and Robert E. Lucas, 1989, "Recursive Methods in Economic

Dynamics”, Harvard University Press.

Table I
Descriptive Statistics for Industry Returns, State Variables
and Portfolio Returns Statistics Summary: 1970:01 – 1999:12

Panel A: Industry Return Statistics							
Sic Industry Series	Return Mean	Std.Dev.	Autocorrelation				
			1	2	3	4	5
Coal & Lignite Mining	0.014	0.178	0.016	-0.097	-0.001	0.107	-0.028
Food & Kindred Products	0.012	0.110	-0.008	-0.007	-0.001	0.026	-0.006
Papers & Allied Products	0.014	0.108	0.102	-0.012	-0.160	-0.002	-0.069
Plastic Materials	0.013	0.101	-0.060	-0.024	0.048	-0.010	0.020
Crude & Natural Gas	0.038	0.108	0.091	0.093	0.093	0.005	0.041
Water, Sewer, Pipeline	0.004	0.142	0.050	-0.094	0.102	0.001	-0.014
Primary Metals	0.019	0.167	0.017	-0.068	-0.033	0.018	0.053
Fabricated Plate	0.028	0.081	0.094	0.118	0.066	0.147	0.048
Construction Machinery	0.019	0.152	-0.127	0.022	-0.016	-0.070	-0.021
Electric Equipment	0.038	0.103	0.094	0.053	0.049	0.132	0.068
Transportation Equipment	0.007	0.145	-0.015	0.003	-0.017	-0.023	-0.067
Manufacturing Industries	0.017	0.123	0.006	-0.033	0.041	-0.122	0.107
Railroads, Line-Haul	0.012	0.153	-0.053	0.082	0.088	-0.022	0.004
Air Courier Services	0.027	0.133	0.030	0.057	0.070	0.072	0.053
Natural Gas Distrib.	0.022	0.111	0.096	0.015	-0.046	0.075	0.085
Retail-Department Stores	0.025	0.080	0.133	-0.057	0.019	0.057	0.097
Other Retail	0.051	0.132	0.293	0.316	0.269	0.124	0.232
Savings Insts., Finance	0.016	0.069	0.104	-0.006	0.023	-0.008	0.022
Real Estate	0.016	0.084	0.031	0.127	0.094	-0.126	-0.067
Other	0.011	0.187	-0.058	-0.037	0.070	-0.156	-0.030
Panel B: C-CAPM State Variables							
Consumption	0.001	0.007	-0.328	0.023	0.121	-0.047	0.034
Technology Shocks	0.002	0.007	0.284	0.192	0.210	0.090	0.065
Panel C: Inflation and M-CAPM State Variables and Fama-French 6&25 Portfolio Returns							
Inflation Rate	0.004	0.003	0.655	0.555	0.500	0.457	0.468
Mktrf	0.006	0.046	0.057	-0.036	-0.004	-0.046	-0.047
Smb	0.001	0.029	0.147	0.039	-0.042	-0.015	0.061
Hml	0.004	0.028	0.203	0.073	0.034	-0.008	0.013
Umd	0.009	0.035	0.089	-0.02	-0.049	-0.056	-0.062
F.F Small-Low P-6	0.009	0.069	0.189	-0.026	-0.033	-0.069	-0.057
F.F Small-High P-6	0.015	0.054	0.198	-0.067	-0.075	-0.075	-0.078
F.F Big-Low P.6	0.011	0.049	0.055	-0.024	0.005	-0.031	-0.023
F.F. Big-High P-6	0.013	0.044	0.004	-0.034	-0.024	-0.089	-0.08
F.F. Large P-25	0.011	0.049	0.048	-0.006	0.022	-0.017	0.004
Panel D: Instrumental Variables							
S&PDivdYield	0.086	0.043	0.991	0.979	0.968	0.959	0.928
CRSPEXR	0.006	0.046	0.057	-0.036	-0.004	-0.046	-0.047
TEXR	0.001	0.001	0.229	0.032	-0.044	0.014	0.126
Tbill30	0.005	0.002	0.914	0.869	0.851	0.81	0.749

Table II
Wald Test for Instrumental Variables

Table II contains a Wald type of test and an F-test on the predictive power of the instrumental variable set for asset returns.

The instrumental variable set is $\zeta_t = \{1_{NL}, s \& pdivyield_t, Crspexr_t, Texr_t, Tbill30_t\}$.

The null hypothesis of the Wald test is that the instrumental variables have no predictive power over asset returns. And the results show that the selection of instrumental variables that instrumental variable should be capable of predicting asset returns is well satisfied.

Summary Statistics				
Industry	χ^2	P-value	F	P-value
Coal & Lignite Mining	36.815***	0.000	9.204***	0.000
Food & Kindred Products	72.040***	0.000	18.010***	0.000
Papers & Allied Products	91.986***	0.000	22.997***	0.000
Plastic Materials	32.533***	0.000	8.138***	0.000
Crude & Natural Gas	101.461***	0.000	25.365***	0.000
Water, Sewer, Pipeline	46.152***	0.000	11.538***	0.000
Primary Metals	184.179***	0.000	46.045***	0.000
Fabricated Plate	121.899***	0.000	30.475***	0.000
Construction Machinery	30.404***	0.000	7.601***	0.000
Electric Equipment	239.919***	0.000	59.980***	0.000
Transportation Equipment	91.200***	0.000	22.800***	0.000
Manufacturing Industries	115.697***	0.000	28.924***	0.000
Railroads, Line-Haul	54.868***	0.000	13.717***	0.000
Air Courier Services	171.340***	0.000	42.835***	0.000
Natural Gas Distrib.	48.107***	0.000	12.027***	0.000
Retail-Department Stores	253.005***	0.000	63.251***	0.000
Other Retail	59.741***	0.000	14.935***	0.000
Savings Insts., Finance	77.924***	0.000	19.481***	0.000
Real Estate	112.204***	0.000	28.051***	0.000
Other	39.875***	0.000	9.969***	0.000

Table III
Kernel Specification Test:
Polynomial Pricing Kernels with Intertemporally Separable Utility Specification and
H-J Return Weighting Matrix

1. $\hat{m}(C_t, \theta) = \delta_0^2 P_0(c_{A,t}) - \delta_1^2 P_1(c_{A,t})$
2. $\hat{m}(C_t, \theta) = \delta_0^2 P_0(c_{A,t}) - \delta_1^2 P_1(c_{A,t}) + \delta_2^2 P_2(c_{A,t})$
3. $\hat{m}(C_t, \theta) = \delta_0^2 P_0(c_{A,t}) - \delta_1^2 P_1(c_{A,t}) + \delta_2^2 P_2(c_{A,t}) - \delta_3^2 P_3(c_{A,t})$

	δ_0^2	δ_1^2	δ_2^2	δ_3^2	<i>F</i> val	J_T^{HU}	J_{Dist}^{HU}
Panel A: Order 1, Linear Kernel							
Coefficient	-24.304***	-1.094***			0.7298	262.741***	0.854
P-value	(0.000)	(0.000)				(0.000)	
Panel B: Order 2, Quadratic Kernel							
Coefficient	968.110*	11.448***	161.710*		0.7266	261.563***	0.852
P-value	(0.070)	(0.001)	(0.069)			(0.000)	
Panel C: Order 3, Cubic Kernel							
Coefficient	52493.000	-53.265	5246.300	-5.401	0.724	260.721***	0.851
P-value	(0.114)	(0.722)	(0.114)	(0.718)		(0.000)	

Table IV
Kernel Specification Test:
Polynomial Pricing Kernels with Intertemporally Non-Separable Utility Specification and
H-J Return Weighting Matrix

1. $\hat{m}(C_t, \theta) = \delta_0^2 P_0(C_t) P_0(C_{t+1}) - \delta_1^2 P_1(C_t) P_1(C_{t+1})$
2. $\hat{m}(C_t, \theta) = \delta_0^2 P_0(C_t) P_0(C_{t+1}) - \delta_1^2 P_1(C_t) P_1(C_{t+1}) + \delta_2^2 P_2(C_t) P_2(C_{t+1})$
3. $\hat{m}(C_t, \theta) = \delta_0^2 P_0(C_t) P_0(C_{t+1}) - \delta_1^2 P_1(C_t) P_1(C_{t+1}) + \delta_2^2 P_2(C_t) P_2(C_{t+1}) - \delta_3^2 P_3(C_t) P_3(C_{t+1})$

	δ_0^2	δ_1^2	δ_2^2	δ_3^2	<i>F</i> val	J_T^{HJ}	J_{Dist}^{HJ}
Panel A: Order 1, Linear Kernel							
Coefficient	354.260***	0.979***			0.740	266.476***	0.860
P-value	(0.002)	(0.000)				(0.000)	
Panel B: Order 2, Quadratic Kernel							
Coefficient	-639.430***	41.614	17.866***		0.739	266.038***	0.859
P-value	(0.000)	(0.111)	(0.000)			(0.000)	
Panel C: Order 3, Cubic Kernel							
Coefficient	-3504800***	-80.968***	35033***	0.814***	0.7298	262.742***	0.854
P-value	(0.000)	(0.000)	(0.000)	(0.000)		(0.000)	

Table V
Kernel Specification Test:
Polynomial Pricing Kernels with Intertemporally Non-Separable Utility Specification and
H-J Return Weighing Matrix

1. $\hat{m}(C_t, \theta) = \delta_0^2 P_0(C_t) + \delta_1^2 P_0(C_{t+1}) + \delta_2^2 P_1(C_t) + \delta_3^2 P_1(C_{t+1})$
2. $\hat{m}(C_t, \theta) = \delta_0^2 P_0(C_t) + \delta_1^2 P_0(C_{t+1}) + \delta_2^2 P_1(C_t) + \delta_3^2 P_1(C_{t+1}) + \delta_4^2 P_2(C_t) + \delta_5^2 P_2(C_{t+1})$
3. $\hat{m}(C_t, \theta) = \delta_0^2 P_0(C_t) + \delta_1^2 P_0(C_{t+1}) + \delta_2^2 P_1(C_t) + \delta_3^2 P_1(C_{t+1}) + \delta_4^2 P_2(C_t) + \delta_5^2 P_2(C_{t+1}) + \delta_6^2 P_3(C_t) + \delta_7^2 P_3(C_{t+1})$

	δ_0^2	δ_1^2	δ_2^2	δ_3^2	δ_4^2	δ_5^2	δ_6^2	δ_7^2	F_{val}	J_T^{HJ}	J_{Dist}^{HJ}
Panel A: Order 1, Linear Kernel											
Coefficient	-41.030***	67.190***	3225.900***	-3226.800***					0.689	248.288***	0.831
P-value	(0.000)	(0.000)	(0.000)	(0.000)						(0.000)	
Panel B: Order 2, Quadratic Kernel											
Coefficient	754.500***	-574.870***	13.970***	-21.982***	-182.960*	213.190**			0.689	248.266***	0.830
P-value	(0.000)	(0.001)	(0.000)	(0.000)	(0.065)	(0.042)				(0.000)	
Panel C: Order 3, Cubic Kernel											
Coefficient	69275**	-862.230	-105.360	212.020	6923.800**	-81.738	-1.982	12.587	0.685	246.816***	0.828
P-value	(0.050)	(0.972)	(0.989)	(0.977)	(0.048)	(0.974)	(0.998)	(0.996)		(0.000)	

Table VI

Kernel Specification Test:												
Polynomial Pricing Kernels with Fama French 3 Factor Model and												
H-J Return Weighting Matrix												
Equation $\hat{m}(x, \theta) = h_0 + h_1 * Mkrf_t + h_2 * E(Smb_t) + h_3 * E(Hml_t) + h_4 * E(Umd_t)$												
Augmented by a constant vector												
1. $\hat{m}(x, \theta) = h_0 + h_1 P_0(Mkrf_t) + h_2 P_0(Smb_t) + h_3 P_0(Hml_t) + h_4 P_0(Umd_t) + h_5 P_1(Mkrf_t) + h_6 P_1(Smb_t) + h_7 P_1(Hml_t) + h_8 P_1(Umd_t)$												
2. $\hat{m}(x, \theta) = h_0 + h_1 P_0(Mkrf_t) + h_2 P_0(Smb_t) + h_3 P_0(Hml_t) + h_4 P_0(Umd_t) + h_5 P_1(Mkrf_t) + h_6 P_1(Smb_t) + h_7 P_1(Hml_t) + h_8 P_1(Umd_t) + h_9 P_2(Mkrf_t) + h_{10} P_2(Smb_t) + h_{11} P_2(Hml_t) + h_{12} P_2(Umd_t)$												
	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	Fval	J_T^{HJ}	J_{Dist}^{HJ}
Panel A: Original Equation F-3 Factor Model												
Coefficient	0.915***	3.964***	4.648***	5.578***	1.931					0.690	248.668***	0.831
P-value	(0.000)	(0.000)	(0.000)	(0.002)	(0.283)						(0.000)	
Panel B: Order 1, Linear Kernel												
Coefficient	-223.37	4.276***	5.492***	6.188***	1.729	-16.871	-112.950	-77.479	-17.092	0.685	246.495***	0.828
P-value	(0.108)	(0.002)	(0.002)	(0.008)	(0.416)	(0.605)	(0.322)	(0.496)	(0.724)		(0.000)	

Table VI (Continue.1)

	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	F_{val}	J_T^{HJ}	J_{Dist}^{HJ}
$\hat{m}(x, \theta) = h_0 + h_1 P_0(Mktrf_t) + h_2 P_0(Smb_t) + h_3 P_0(Hml_t) + h_4 P_0(Umd_t) + h_5 P_0(Mktrf_t) + h_6 P_0(Smb_t) + h_7 P_0(Hml_t) + h_8 P_0(Umd_t)$ $+ h_9 P_2(Mktrf_t) + h_{10} P_2(Smb_t) + h_{11} P_2(Hml_t) + h_{12} P_2(Umd_t)$												
2.	$\hat{m}(x, \theta) = h_0 + h_1 P_0(Mktrf_t) + h_2 P_0(Smb_t) + h_3 P_0(Mktrf_t) + h_4 P_0(Smb_t) + h_5 P_0(Smb_t) + h_6 P_0(Smb_t) + h_7 P_0(Smb_t) + h_8 P_0(Smb_t)$ $+ h_9 P_0(Hml_t) + h_{10} P_1(Hml_t) + h_{11} P_2(Hml_t) + h_{12} P_3(Hml_t) + h_{13} P_0(Umd_t) + h_{14} P_1(Umd_t) + h_{15} P_2(Umd_t) + h_{16} P_3(Umd_t)$											
3												
Panel C: Order 1, Quadratic Kernel												
Coefficient	-11.824	-16.099	-31.868	-35.391	-15.998	-1.436	-1.827	-2.067	-11.824	0.685	246.463***	0.827
P-value	(0.987)	(0.924)	(0.959)	(0.931)	(0.982)	(0.524)	(0.702)	(0.538)	(0.987)		(0.000)	
Panel D: Order 2, Cubic Kernel												
Coefficient	-0.574	-5.131	2.819	-4.897	-5.068							
P-value	(0.846)	(0.970)	(0.989)	(0.983)	(0.988)							
Panel E: Order 3, Cubic Kernel												
Coefficient	-31.377	-260.988	741.886	-1,478.806	-0.892	1.597	10.074	8.684	-31.377	0.676	243.428***	0.822
P-value	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)		(0.000)	
Panel F: Order 4, Cubic Kernel												
Coefficient	1.267	-25.666	74.446	-147.199	1.286e+39	2.46e+40	5.88e+41	-2.17e+42	2.205e+42			
P-value	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)			

Table VII

Kernel Specification Test: Polynomial Pricing Kernels with Fama French 3 Factor Model and H-J Return Weighting Matrix												
	$\hat{m}(R_W^{FF}, \theta) = \sum_j^L h_j \frac{d^{j+1}U}{dU} [P_1(Mktrf_t)P_1(Mktrf_{t+1}) + P_1(Smb_t)P_1(Smb_{t+1}) + P_1(Hml_t)P_1(Hml_{t+1}) + P_1(Umd_t)P_1(Umd_{t+1})]$ $= h_0[P_0(Mktrf_t)P_0(Mktrf_{t+1}) + P_0(Smb_t)P_0(Smb_{t+1}) + P_0(Hml_t)P_0(Hml_{t+1}) + P_0(Umd_t)P_0(Umd_{t+1})]$ $+ h_1 \frac{U^*}{U} [P_1(Mktrf_t)P_1(Mktrf_{t+1}) + P_1(Smb_t)P_1(Smb_{t+1}) + P_1(Hml_t)P_1(Hml_{t+1}) + P_1(Umd_t)P_1(Umd_{t+1})]$ $+ h_2 \frac{U^{**}}{U} [P_2(Mktrf_t)P_2(Mktrf_{t+1}) + P_2(Smb_t)P_2(Smb_{t+1}) + P_2(Hml_t)P_2(Hml_{t+1}) + P_2(Umd_t)P_2(Umd_{t+1})]$ $+ \dots$ $\hat{m}(R_W^{FF}, \theta) = h_0 P_0(Mktrf_t)P_0(Mktrf_{t+1}) + h_1 P_0(Smb_t)P_0(Smb_{t+1}) + h_2 P_0(Hml_t)P_0(Hml_{t+1}) + h_3 P_0(Umd_t)P_0(Umd_{t+1})$ $+ h_4 \frac{U^*}{U} P_1(Mktrf_t)P_1(Mktrf_{t+1}) + h_5 \frac{U^*}{U} P_1(Smb_t)P_1(Smb_{t+1}) + h_6 \frac{U^*}{U} P_1(Hml_t)P_1(Hml_{t+1}) + h_7 \frac{U^*}{U} P_1(Umd_t)P_1(Umd_{t+1})$											
Equation	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	Fval	J_T^{HJ}	J_{DM}^{HJ}	
Panel A: Order 1, Linear Kernel	25.950 (0.982)	105.783 (0.972)	-148.825 (0.964)	20.889 (0.988)	75.064 (0.950)	43.428 (0.988)	-156.611 (0.959)	39.228 (0.985)	0.692	248.966*** (0.000)	0.832	

Table VII (Continue.1)

$\hat{\pi}(R_W^{FF}, \theta) = h_0 P_0(Mktrf_t)P_0(Mktrf_{t+1}) + h_1 P_0(Smb_t)P_0(Smb_{t+1}) + h_2 P_0(Hml_t)P_0(Hml_{t+1}) + h_3 P_0(Umd_t)P_0(Umd_{t+1})$												
2.	$+ h_4 \frac{U''}{U'} P_1(Mktrf_t)P_1(Mktrf_{t+1}) + h_5 \frac{U''}{U'} P_1(Smb_t)P_1(Smb_{t+1}) + h_6 \frac{U''}{U'} P_1(Hml_t)P_1(Hml_{t+1}) + h_7 \frac{U''}{U'} P_1(Umd_t)P_1(Umd_{t+1})$											
	$+ h_8 \frac{U''}{U'} P_2(Mktrf_t)P_2(Mktrf_{t+1}) + h_9 \frac{U''}{U'} P_2(Smb_t)P_2(Smb_{t+1}) + h_{10} \frac{U''}{U'} P_2(Hml_t)P_2(Hml_{t+1}) + h_{11} \frac{U''}{U'} P_2(Umd_t)P_2(Umd_{t+1})$											
	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	F_{val}	J_T^{HU}	J_{Disk}^{HU}
Panel B: Order 2, Quadratic Kernel												
Coefficient	50.101	112.840	85.617	44.412	0.682	11.108*	-6.608	2.142	-1.202	0.677	243.67***	0.823
P-value	(0.975)	(0.992)	(0.993)	(0.992)	(0.689)	(0.070)	(0.319)	(0.419)	(0.993)		(0.000)	
	h_9	h_{10}	h_{11}									
Coefficient	-0.122	-2.946	-3.725									
P-value	(0.999)	(0.997)	(0.992)									

Table VII (Continue.2)

$\hat{m}(R_W^{FF}, \theta) = h_0 P_0(Mktrf_t)P_0(Mktrf_{t+1}) + h_1 P_0(Smb_t)P_0(Smb_{t+1}) + h_2 P_0(Hml_t)P_0(Hml_{t+1}) + h_3 P_0(Umd_t)P_0(Umd_{t+1})$ $+ h_4 \frac{U^*}{U} P_1(Mktrf_t)P_1(Mktrf_{t+1}) + h_5 \frac{U^*}{U} P_1(Smb_t)P_1(Smb_{t+1}) + h_6 \frac{U^*}{U} P_1(Hml_t)P_1(Hml_{t+1}) + h_7 \frac{U^*}{U} P_1(Umd_t)P_1(Umd_{t+1})$											
<p>3.</p> $+ h_8 \frac{U^*}{U} P_2(Mktrf_t)P_2(Mktrf_{t+1}) + h_9 \frac{U^*}{U} P_2(Smb_t)P_2(Smb_{t+1}) + h_{10} \frac{U^*}{U} P_2(Hml_t)P_2(Hml_{t+1}) + h_{11} \frac{U^*}{U} P_2(Umd_t)P_2(Umd_{t+1})$ $+ h_{12} \frac{U^{**}}{U} P_3(Mktrf_t)P_3(Mktrf_{t+1}) + h_{13} \frac{U^{**}}{U} P_3(Smb_t)P_3(Smb_{t+1}) + h_{14} \frac{U^{**}}{U} P_3(Hml_t)P_3(Hml_{t+1}) + h_{15} \frac{U^{**}}{U} P_3(Umd_t)P_3(Umd_{t+1})$											
h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	F_{val}	J^H_T	J^H_{Dist}
Panel C: Order 3, Cubic Kernel											
Coefficient	-5494.700	39067.000	4499.500	9.685	-45.918	41.436	-4.230	54.296	0.617	221.951***	0.785
P-value	(0.771)	(0.834)	(0.913)	(0.995)	(0.996)	(0.997)	(0.999)	(0.770)		(0.000)	
h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}					
Coefficient	498.580	-389.380	-44.571	-0.226	1.659	-1.633	0.195				
P-value	(0.839)	(0.834)	(0.913)	(0.997)	(0.996)	(0.996)	(0.998)				

Table VIII

**Kernel Specification Test:
Polynomial Pricing Kernels with 25 Large Portfolio Asset Returns and
H-J Return Weighting Matrix**

Equation:

$$\hat{m}(x_t, \theta) = h_0 P_0(R_{pifs}^{FF}) + h_1 P_1(R_{w,t}^{FF}) + h_2 \frac{U''}{U'} P_1(R_{w,t}^{FF}) + h_3 \frac{U''}{U'} P_2(R_{w,t}^{FF}) + \dots \quad (20)$$

$$\hat{m}(x_t, \theta) = h_0 + h_1 * P_0(R_{pifs}^{FF}) + h_2 * Mkt_{r,t} + h_3 * E(Smb_t) + h_4 * E(Hml_t) + h_5 * E(Umd_t) + \dots \quad (21)$$

$$1. \hat{m}(x_t, \theta) = h_0 P_0(R_{pifs}^{FF}) + h_1 * P_1(R_{pifs}^{FF}) + h_2 * P_2(R_{pifs}^{FF}) + h_3 * Mkt_{r,t} + h_4 * E(Smb_t) + h_5 * E(Hml_t) + \dots$$

$$2. \hat{m}(x_t, \theta) = h_0 P_0(R_{pifs}^{FF}) + h_1 * P_1(R_{pifs}^{FF}) + h_2 * P_2(R_{pifs}^{FF}) + h_3 * Mkt_{r,t} + h_4 * E(Smb_t) + h_5 * E(Hml_t) + h_6 * E(Umd_t) + \dots$$

$$3. \hat{m}(x_t, \theta) = h_0 P_0(R_{pifs}^{FF}) + h_1 * P_1(R_{pifs}^{FF}) + h_2 * P_2(R_{pifs}^{FF}) + h_3 * P_3(R_{pifs}^{FF}) + h_4 * Mkt_{r,t} + h_5 * E(Smb_t) + h_6 * E(Hml_t) + h_7 * E(Umd_t) + \dots$$

	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	Fval	J_T^{HU}	J_{Dist}^{HU}
Panel A: Order 1, Linear Kernel Augmented by Fama French Three Factor Model											
Coefficient	2.926***	-0.891***	1.338	5.253***	6.899**	2.239			0.690	248.448***	0.831
P-value	(0.004)	(0.000)	(0.742)	(0.000)	(0.031)	(0.292)				(0.000)	
Panel B: Order 2, Quadratic Kernel Augmented by Fama French Three Factor Model											
Coefficient	-10.985	-1.339	-1.527	0.706	4.868**	7.111**	2.121		0.689	248.105***	0.830
P-value	(0.292)	(0.378)	(0.379)	(0.864)	(0.011)	(0.039)	(0.326)			(0.000)	
Panel C: Order 3, Cubic Kernel Augmented by Fama French Three Factor Model											
Coefficient	-73.922	0.736	-7.104	0.014	1.021	4.825***	6.836**	2.125	0.688	247.775***	0.8296
P-value	(0.384)	(0.771)	(0.404)	(0.956)	(0.763)	(0.004)	(0.024)	(0.271)		(0.000)	

Table IX

Kernel Specification Test:												
Polynomial Pricing Kernels with 6 and 25 Large Portfolio Asset Returns												
Augmented by Solow Residual Growth Rates with H-J Return Weighting Matrix												
Equation,												
$\hat{m}(x_t, \theta) = h_0 \sum_{j=0}^L P_j(R_{pfs,t}^{FF}) + h_{L+1} * Solow_t + h_{L+2} * Mkt_{t-1} + h_{L+3} * E(Smb_t) + h_{L+4} * E(Hml_t) + h_{L+5} * E(Umd_t) \dots \quad (22)$												
$1. \hat{m}(x_t, \theta) = h_0 P_0(R_{pfs,t}^{FF}) + h_1 * P_1(R_{pfs,t}^{FF}) + h_2 P_2(Solow_t) + h_3 P_3(Solow_t) + h_4 * Mkt_{t-1} + h_5 * E(Smb_t) + h_6 * E(Hml_t) + h_7 * E(Umd_t) \dots$												
$2. \hat{m}(x_t, \theta) = h_0 P_0(R_{pfs,t}^{FF}) + h_1 * P_1(R_{pfs,t}^{FF}) + h_2 P_2(R_{pfs,t}^{FF}) + h_3 P_3(R_{pfs,t}^{FF}) + h_4 P_4(Solow_t) + h_5 P_5(Solow_t) + h_6 P_6(Solow_t) + h_7 * Mkt_{t-1} + h_8 * E(Smb_t) + h_9 * E(Hml_t) + h_{10} * E(Umd_t) \dots$												
h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	$Fval$	J_T^{HJ}	J_{Dist}^{HJ}		
Panel A: Order 1, Linear Kernel Augmented by Solow Residual Growth Rate and Fama French Three Factor Model												
Coefficient	7.055**	-24.680*	24.291***	23.834*	-1.769	5.651***	8.584***	2.553**	0.678	244.188***	0.824	
P-value	(0.012)	(0.071)	(0.000)	(0.081)	(0.462)	(0.000)	(0.000)	(0.050)		(0.000)		
h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	$Fval$	J_T^{HJ}	J_{Dist}^{HJ}		
Panel B: Order 2, Quadratic Kernel Augmented by Solow Residual Growth Rate and Fama French Three Factor Model												
Coefficient	-197.900	-2.330**	-95.507	-237.410	-7.425***	23.245	-1.814	5.862***	0.677	243.553***	0.823	
P-value	(0.999)	(0.020)	(0.999)	(0.999)	(0.000)	(0.999)	(0.509)	(0.000)		(0.000)		
h_8	h_9											
Coefficient	8.633***	2.695**										
P-value	(0.000)	(0.036)										

Table IX (Continue.1)

Kernel Specification Test:											
Polynomial Pricing Kernels with 6 and 25 Large Portfolio Asset Returns											
Augmented by Solow Residual Growth Rates with H-J Return Weighting Matrix											
Equation,											
$\hat{m}(x_t, \theta) = h_j \sum_{j=0}^L P_j(R_{pfs,t}^{FF}) + h_{L+1} * Solow_t + h_{L+2} * Mkt_{rf,t} + h_{L+3} * E(Smb_t) + h_{L+4} * E(Hml_t) + h_{L+5} * E(Umd_t) \dots \quad (22)$											
3. $\hat{m}(x_t, \theta) = h_0 P_0(R_{pfs,t}^{FF}) + h_1 * P_1(R_{pfs,t}^{FF}) + h_2 P_2(R_{pfs,t}^{FF}) + h_3 P_3(R_{pfs,t}^{FF}) + h_4 P_4(Solow_t) + h_5 P_5(Solow_t) + h_6 P_6(Solow_t) + h_7 P_7(Solow_t) + h_8 Mkt_{rf,t} + h_9 * E(Smb_t) + h_{10} * E(Hml_t) + h_{11} * E(Umd_t) \dots$											
	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	$Fval$	J_T^{HI}	J_{Dist}^{HI}
Panel C: Order 3, Cubic Kernel Augmented by Solow Residual Growth Rate and Fama French Three Factor Model											
Coefficient	-64.004	-3.934	-5.907	-1.678	-4350.200	31.139	-433.390	4.342	0.676	243.440***	0.822
P-value	(0.978)	(0.999)	(0.979)	(0.999)	(0.998)	(0.998)	(0.999)	(0.999)	(0.000)		
	h_8	h_9	h_{10}	h_{11}							
Coefficient	-1.564	5.756	8.444	2.602							
P-value	(0.998)	(0.907)	(0.910)	(0.938)							

Table X

Kernel Specification Test:											
Polynomial Pricing Kernels with Two-Period 6 and 25 Large Portfolio Asset Returns Augmented by Solow Residual Growth Rates with H-J Return Weighting Matrix											
Equation,	$\hat{m}(x_t, \theta) = h * \sum_{j=0}^L P_j(R_{pfs,t}^{FF}) + h * \sum_{j=0}^L P_j(R_{pfs,t+1}^{FF}) + h * \sum_{j=0}^L P_j(Solow_t) + h * \sum_{j=0}^L P_j(Solow_{t+1}) + h * Mktrf_t + h * E(Smb_t) + h * E(Hml_t) + h * E(Umd_t) \dots \quad (21.B)$										
1.	$\hat{m}(x_t, \theta) = h_0 P_0(R_{pfs,t}^{FF}) + h_1 P_1(R_{pfs,t}^{FF}) + h_2 P_2(R_{pfs,t+1}^{FF}) + h_3 P_3(R_{pfs,t+1}^{FF}) + h_4 P_4(Solow_t) + h_5 P_5(Solow_t) + h_6 P_6(Solow_{t+1}) + h_7 P_7(Solow_{t+1}) + h_8 * Mktrf_t + h_9 * E(Smb_t) + h_{10} * E(Hml_t) + h_{11} E(Umd_t) \dots$										
	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	$Fval$	J_T^{HU}	J_{Dist}^{HU}
Panel A: Linear Kernel, Two-Period Large Portfolio Returns augmented with Solow Residual and Fama French Three Factor Model											
Coefficient	0.843***	6.852*	-2.438***	24.916***	0.322	-2.105	6.570***	8.301***	0.676	243.220***	0.822
P-value	(0.000)	(0.055)	(0.009)	(0.001)	(0.966)	(0.515)	(0.000)	(0.002)		(0.000)	
	h_8										
Coefficient	2.877*										
P-value	(0.071)										

Table X (Continue.1)

Kernel Specification Test:											
Polynomial Pricing Kernels with 6 and 25 Large Portfolio Asset Returns											
Augmented by Solow Residual Growth Rates with H-J Return Weighting Matrix											
Equation,											
$\hat{m}(x_t, \theta) = h^* \sum_{j=0}^L P_j(R_{pfs,t}^{FF}) + h^* \sum_{j=0}^L P_j(R_{pfs,t+1}^{FF}) + h^* \sum_{j=0}^L P_j(Solow_t) + h^* \sum_{j=0}^L P_j(Solow_{t+1}) \quad (21.B)$ $+ h^* Mkt_{r,t} + h^* E(Smb_t) + h^* E(Hml_t) + h^* E(Umd_t) \dots$											
$1. \quad \hat{m}(x_t, \theta) = h_0 P_0(R_{pfs,t}^{FF}) + h_1 P_1(R_{pfs,t}^{FF}) + h_2 P_2(R_{pfs,t}^{FF}) + h_3 P_3(R_{pfs,t}^{FF}) + h_4 P_4(R_{pfs,t}^{FF}) + h_5 P_5(R_{pfs,t}^{FF}) + h_6 P_6(R_{pfs,t}^{FF}) + h_7 P_7(Solow_t) + h_8 P_8(Solow_{t+1}) + h_9 * E(Smb_t) + h_{10} * E(Hml_t) + h_{11} E(Umd_t) \dots$											
	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	$Fval$	J_T^{HJ}	J_{Dist}^{HJ}
Panel B: Order 1, Linear Kernel Augmented by Solow Residual Growth Rate and Fama French Three Factor Model											
Coefficient	6.938**	1.683	-1.086	-75.579***	25.052***	14.286	-0.547	58.689	0.669	240.800***	0.8179
P-value	(0.042)	(0.921)	(0.245)	(0.001)	(0.000)	(0.989)	(0.930)	(0.941)		(0.000)	
	h_8	h_9	h_{10}	h_{11}							
Coefficient	-3.036	6.906***	8.693***	3.410***							
P-value	(0.316)	(0.000)	(0.000)	(0.010)							

Table X (Continue.2)

Kernel Specification Test:												
Polynomial Pricing Kernels with 6 and 25 Large Portfolio Asset Returns												
Augmented by Solow Residual Growth Rates with H-J Return Weighting Matrix												
Equation,												
$\hat{m}(x_t, \theta) = h^* \sum_{j=0}^L P_j(R_{pfs,t}^{FF}) + h^* \sum_{j=0}^L P_j(R_{pfs,t+1}^{FF}) + h^* \sum_{j=0}^L P_j(Solow_t) + h^* \sum_{j=0}^L P_j(Solow_{t+1}) \quad (21.B)$												
$+ h^* Mkt_{rf,t} + h^* E(Smb_t) + h^* E(Hml_t) + h^* E(Umd_t) \dots$												
$\hat{m}(x_t, \theta) = h_0 P_0(R_{pfs,t}^{FF}) + h_1 P_1(R_{pfs,t}^{FF}) + h_2 P_2(R_{pfs,t}^{FF}) + h_3 P_3(R_{pfs,t}^{FF}) + h_4 P_4(R_{pfs,t}^{FF}) + h_5 P_5(R_{pfs,t}^{FF}) + h_6 P_6(R_{pfs,t}^{FF}) + h_7 P_7(R_{pfs,t}^{FF}) + h_8 P_8(Solow_t) + h_9 P_9(Solow_t) + h_{10} P_{10}(Solow_t) + h_{11} P_{11}(Solow_t) + h_{12} P_{12}(Solow_t) + h_{13} * E(Smb_t) + h_{14} * E(Hml_t) + h_{15} * E(Umd_t) \dots$												
2.	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}	J_{Dist}^{HU}
Panel B: Order 2, Quadratic Kernel Augmented by Solow Residual Growth Rate and Fama French Three Factor Model												
Coefficient	5.719	-2.424**	2.482	-25.681	0.390	0.002	38.175	-8.368***	0.667	240.276***	0.817	
P-value	(0.999)	(0.028)	(0.999)	(0.999)	(0.307)	(0.999)	(0.999)	(0.000)	(0.000)	(0.000)	(0.000)	
	h_8	h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}				
Coefficient	-7.191	-52.808	0.300	-0.750	-3.310	6.967***	8.763***	3.422***				
P-value	(0.998)	(0.989)	(0.888)	(0.999)	(0.266)	(0.000)	(0.000)	(0.017)				

Table X (Continue.3)

Kernel Specification Test:												
Polynomial Pricing Kernels with 6 and 25 Large Portfolio Asset Returns												
Augmented by Solow Residual Growth Rates with H-J Return Weighting Matrix												
Equation,	$\hat{m}(x_t, \theta) = h^* \sum_{j=0}^L P_j(R_{pfs,t}^{FF}) + h^* \sum_{j=0}^L P_j(R_{pfs,t+1}^{FF}) + h^* \sum_{j=0}^L P_j(Solow_t) + h^* \sum_{j=0}^L P_j(Solow_{t+1}) \quad (21.B)$ $+ h^* Mkt_{r,t} + h^* E(Smb_t) + h^* E(Hml_t) + h^* E(Umd_t) \dots$											
3.	$\hat{m}(x_t, \theta) = h_0 P_0(R_{pfs,t}^{FF}) + h_1 P_1(R_{pfs,t}^{FF}) + h_2 P_2(R_{pfs,t}^{FF}) + h_3 P_3(R_{pfs,t}^{FF}) + h_4 P_4(R_{pfs,t}^{FF}) + h_5 P_5(R_{pfs,t}^{FF}) + h_6 P_6(R_{pfs,t+1}^{FF}) + h_7 P_7(R_{pfs,t+1}^{FF})$ $+ h_8 P_8(Solow_t) + h_9 P_9(Solow_t) + h_{10} P_{10}(Solow_t) + h_{11} P_{11}(Solow_t) + h_{12} P_{12}(Solow_t) + h_{13} P_{13}(Solow_{t+1}) + h_{14} P_{14}(Solow_{t+1}) + h_{15} P_{15}(Solow_{t+1})$ $+ h_{16} * Mkt_{r,t} + h_{17} * E(Smb_t) + h_{18} * E(Hml_t) + h_{19} * E(Umd_t) \dots$											
	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	$Fval$	J_T^{HJ}	J_{Dkt}^{HJ}	
Panel C: Order 3, Cubic Kernel Augmented by Solow Residual Growth Rate and Fama French Three Factor Model												
Coefficient	-161.310	-0.163	-15.615	0.664	-21.931	8.023	-2.295	1.020	0.664	239.027***	0.815	
P-value	(0.370)	(0.999)	(0.385)	(0.998)	(0.943)	(0.995)	(0.941)	(0.998)		(0.000)		
	h_8	h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}				
Coefficient	65.735	5.369	8.261	1.163	-24.133	-3.203	-2.662	-1.905				
P-value	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)	(0.999)				
	h_{16}	h_{17}	h_{18}	h_{19}								
Coefficient	-2.696	7.094**	7.921*	3.644								
P-value	(0.627)	(0.047)	(0.059)	(0.107)								

Table XI

**Kernel Specification Test:
Polynomial Pricing Kernels with 6 and 25 Large Portfolio Asset Returns and Intertemporally Non-separable Utility
Augmented by Solow Residual Growth Rates with H-J Return Weighting Matrix**

Equation,

$$\begin{aligned} \hat{m}(x_t, \theta) = & h^* \sum_{j=0}^L P_j(R_{pfs,t}^{FF}) + h^* \sum_{j=0}^L P_j(R_{pfs,t+1}^{FF}) + h^* \sum_{j=0}^L P_j(\text{Solow}_t) + h^* \sum_{j=0}^L P_j(\text{Solow}_{t+1}) \\ & + h^* P_0(C_t)P_0(C_{t+1}) - h^* P_1(C_t)P_1(C_{t+1}) + h^* P_2(C_t)P_2(C_{t+1}) - h^* P_3(C_t)P_3(C_{t+1}) \\ & + h^* \text{Mktrf}_t + h^* E(\text{Smb}_t) + h^* E(\text{Hml}_t) + h^* E(\text{Umd}_t), \dots \end{aligned}$$

$$\begin{aligned} \mathbf{I.} \quad \hat{m}(x_t, \theta) = & h_0 + h_1 R_{pfs,t}^{FF} + h_2 R_{pfs,t+1}^{FF} + h_3 \text{Solow}_t + h_4 \text{Solow}_{t+1} + h_5 P_0(C_t)P_0(C_{t+1}) - h_6 P_1(C_t)P_1(C_{t+1}) + h_7 P_2(C_t)P_2(C_{t+1}) - h_8 P_3(C_t)P_3(C_{t+1}) \\ & + h_9^* \text{Mktrf}_t + h_{10}^* E(\text{Smb}_t) + h_{11}^* E(\text{Hml}_t) + h_{12}^* E(\text{Umd}_t) \end{aligned}$$

	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	F_{val}	J_T^{HJ}	$J_{\text{Dist}}^{\text{HJ}}$
Panel A: Linear Kernel Augmented by Solow Residual Growth Rate and Fama French Three Factor Model											
Coefficient	131.770	6.488*	-2.101**	26.731***	0.513	-2716600**	-1.419	27157**	0.668	240.439***	0.817
P-value	(0.998)	(0.066)	(0.030)	(0.000)	(0.944)	(0.035)	(0.999)	(0.035)		(0.000)	
	h_8	h_9	h_{10}	h_{11}	h_{12}						
Coefficient	-0.568	-1.974	6.629***	7.502***	3.007**						
P-value	(0.999)	(0.537)	(0.000)	(0.002)	(0.038)						

Table XI (Continue.1)

Equation,	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	F_{val}	J_T^{H0}	J_{Dist}^{H0}
$\hat{m}(x_t, \theta) = h^* \sum_{j=0}^L P_j(R_{pfs,t}^{FF}) + h^* \sum_{j=0}^L P_j(R_{pfs,t+1}^{FF}) + h^* \sum_{j=0}^L P_j(Solow_t) + h^* \sum_{j=0}^L P_j(Solow_{t+1})$ $+ h^* P_0(C_t)P_0(C_{t+1}) - h^* P_1(C_t)P_1(C_{t+1}) + h^* P_2(C_t)P_2(C_{t+1}) - h^* P_3(C_t)P_3(C_{t+1})$ $+ h^* Mktrf_t + h^* E(Smb_t) + h^* E(Hml_t) + h^* E(Umdl_t) \dots$											
2.											
$\hat{m}(x_t, \theta) = h_0 P_0(R_{pfs,t}^{FF}) + h_1 P_1(R_{pfs,t}^{FF}) + h_2 P_2(R_{pfs,t}^{FF}) + h_3 P_3(R_{pfs,t}^{FF}) + h_4 P_4(R_{pfs,t+1}^{FF}) + h_5 P_5(R_{pfs,t+1}^{FF}) + h_6 P_6(Solow_t) + h_7 P_7(Solow_t)$ $+ h_8 P_8(Solow_t) + h_9 P_9(Solow_{t+1}) + h_{10} P_{10}(Solow_{t+1}) + h_{11} P_{11}(Solow_{t+1}) + h_{12} P_{12}(C_t)P_0(C_{t+1}) - h_{13} P_{13}(C_t)P_1(C_{t+1})$ $+ h_{14} P_{14}(C_t)P_2(C_{t+1}) - h_{15} P_{15}(C_t)P_3(C_{t+1}) + h_{16} * Mktrf_t + h_{17} * E(Smb_t) + h_{18} * E(Hml_t) + h_{19} * E(Umdl_t)$											
Panel B: Quadratic Kernel Augmented by Solow Residual Growth Rate and Fama French Three Factor Model											
Coefficient	0.070	-2.559**	-0.011	-22.907	0.352	1.126	1.245	-8.378***	0.667	240.166***	0.817
P-value	(0.999)	(0.025)	(0.999)	(0.997)	(0.373)	(0.998)	(0.999)	(0.000)		(0.000)	
	h_8	h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}			
Coefficient	0.093	0.820	0.352	0.141	-7143.200	-0.054	74.510	-0.060			
P-value	(0.999)	(0.999)	(0.875)	(0.999)	(0.995)	(0.999)	(0.994)	(0.999)			
	h_{16}	h_{17}	h_{18}	h_{19}							
Coefficient	-3.659	7.246***	8.725***	3.570**							
P-value	(0.236)	(0.000)	(0.000)	(0.016)							

Table XII

Kernel Specification Test:

Polynomial Pricing Kernels with 6 and 25 Large Portfolio Asset Returns and Intertemporally Non-separable Utility Augmented by Solow Residual Growth Rates with H-J Return Weighting Matrix

Equation,

$$\begin{aligned} \hat{m}(x_t, \theta) = & h^* \sum_{j=0}^L P_j(R_{pfs,t}^{FF}) + h^* \sum_{j=0}^L P_j(R_{pfs,t+1}^{FF}) + h^* \sum_{j=0}^L P_j(\text{Solow}_t) + h^* \sum_{j=0}^L P_j(\text{Solow}_{t+1}) \\ & + h^* P_0(C_t)P_0(C_{t+1}) - h^* P_1(C_t)P_1(C_{t+1}) + h^* P_2(C_t)P_2(C_{t+1}) - h^* P_3(C_t)P_3(C_{t+1}) \\ & + h^* \text{Mktrf}_t + h^* E(\text{Smb}_t) + h^* E(\text{Hml}_t) + h^* E(\text{Umd}_t), \dots \end{aligned}$$

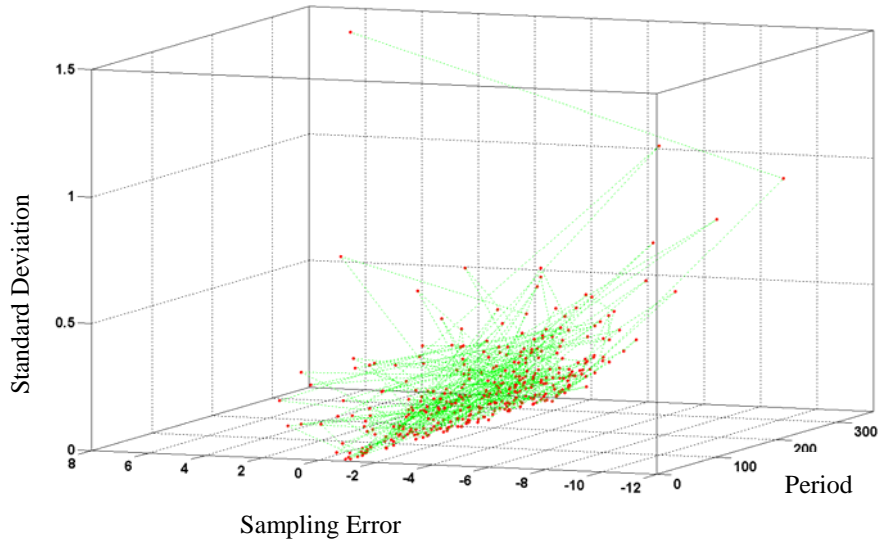
$$\begin{aligned} \mathbf{1.} \quad \hat{m}(x_t, \theta) = & h_0 R_{pfs,t}^{FF} + h_1 R_{pfs,t}^{FF} + h_2 R_{pfs,t+1}^{FF} + h_3 \text{Solow}_t + h_4 \text{Solow}_{t+1} + h_5 P_0(C_t) + h_6 P_0(C_{t+1}) + h_7 P_1(C_t) + h_8 P_1(C_{t+1}) + h_9 P_2(C_t) + h_{10} P_2(C_{t+1}) \\ & + h_{11} * \text{Mktrf}_t + h_{12} * E(\text{Smb}_t) + h_{13} * E(\text{Hml}_t) + h_{14} * E(\text{Umd}_t) \end{aligned}$$

	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	$Fval$	J_T^{HJ}	J_{Dist}^{HJ}
Panel B: Order 1, Linear Kernel Augmented by Solow Residual Growth Rate and Fama French Three Factor Model											
Coefficient	-38.000	8.405*	-2.029*	22.16***	1.813	43.811	5.926	-5.130	0.660	237.585***	0.812
P-value	(0.826)	(0.054)	(0.079)	(0.004)	(0.829)	(0.958)	(0.240)	(0.964)		(0.000)	
	h_8	h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}				
Coefficient	-102.440	-8.925*	8.293	-3.981	5.881***	7.639**	2.780				
P-value	(0.805)	(0.061)	(0.869)	(0.320)	(0.000)	(0.017)	(0.158)				

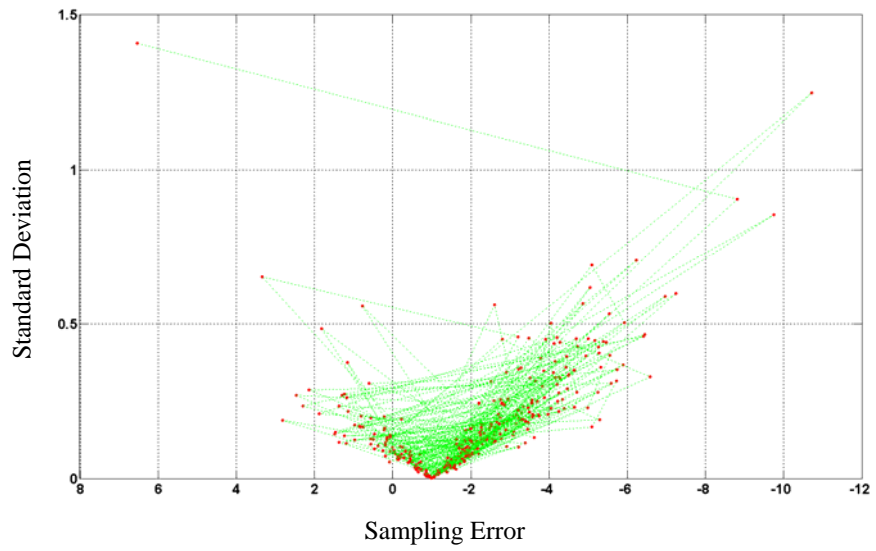
Table XII (Continue.1)

Equation,	h_0	h_1	h_2	h_3	h_4	h_5	h_6	h_7	Fval	J_T^{HJ}	J_{Dist}^{HJ}
$\hat{m}(x_t, \theta) = h^* \sum_{j=0}^L P_j(R_{pfs,t}^{FF}) + h^* \sum_{j=0}^L P_j(R_{pfs,t+1}^{FF}) + h^* \sum_{j=0}^L P_j(Solow_t) + h^* \sum_{j=0}^L P_j(Solow_{t+1})$ $+ h^* P_0(C_t) + h^* P_0(C_{t+1}) + h^* P_1(C_t) + h^* P_1(C_{t+1}) + h^* P_2(C_t) + h^* P_2(C_{t+1}) + h^* P_3(C_t) + h^* P_3(C_{t+1})$ $+ h^* Mkrtrf_t + h^* E(Smb_t) + h^* E(Hml_t) + h^* E(Umdl_t) \dots$											
$\hat{m}(x_t, \theta) = h_0 P_0(R_{pfs,t}^{FF}) + h_1 P_1(R_{pfs,t}^{FF}) + h_2 P_2(R_{pfs,t}^{FF}) + h_3 P_3(R_{pfs,t}^{FF}) + h_4 P_4(R_{pfs,t}^{FF}) + h_5 P_5(R_{pfs,t}^{FF}) + h_6 P_6(Solow_t) + h_7 P_7(Solow_t)$ $+ h_8 P_8(Solow_t) + h_9 P_9(Solow_{t+1}) + h_{10} P_{10}(Solow_{t+1}) + h_{11} P_{11}(Solow_{t+1}) + h_{12} P_{12}(C_t) + h_{13} P_{13}(C_t) + h_{14} P_{14}(C_t) + h_{15} P_{15}(C_{t+1})$ $+ h_{16} P_{16}(C_{t+1}) + h_{17} P_{17}(C_{t+1}) + h_{18} * Mkrtrf_t + h_{19} * E(Smb_t) + h_{20} * E(Hml_t) + h_{21} * E(Umdl_t)$											
Panel B: Order 1, Linear Kernel Augmented by Solow Residual Growth Rate and Fama French Three Factor Model											
Coefficient	1.820	-2.897**	0.609	-21.696	0.214	2.617	10.072	-7.601***	0.656	236.038***	0.810
P-value	(0.999)	(0.020)	(0.999)	(0.980)	(0.593)	(0.995)	(0.998)	(0.003)		(0.000)	
	h_8	h_9	h_{10}	h_{11}	h_{12}	h_{13}	h_{14}	h_{15}			
Coefficient	-1.444	2.254	0.062	2.396	11.714	5.803	-1.571	6.796			
P-value	(0.999)	(1.000)	(0.979)	(0.999)	(0.999)	(0.212)	(0.999)	(0.996)			
	h_{16}	h_{17}	h_{18}	h_{19}	h_{20}	h_{21}					
Coefficient	-7.964**	-0.483	-5.169	6.191***	8.053**	3.333***					
P-value	(0.044)	(0.999)	(0.134)	(0.000)	(0.016)	(0.000)					

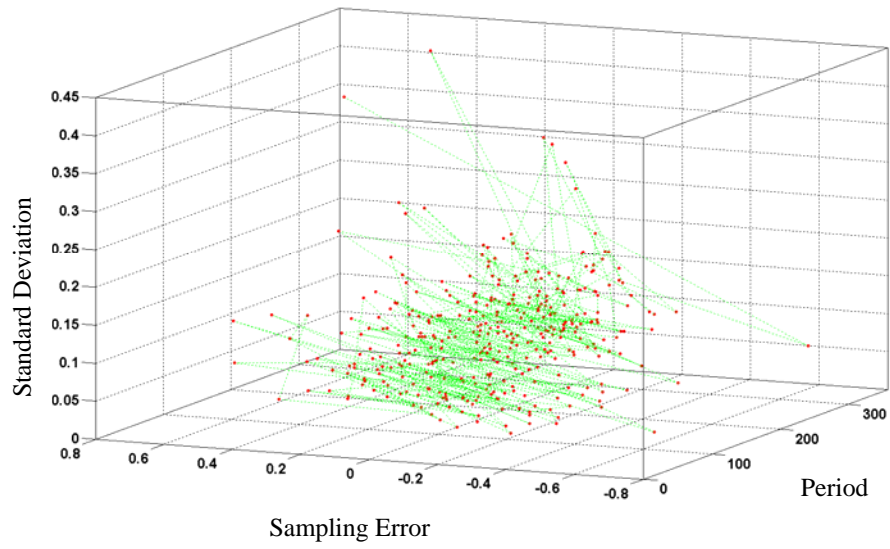
Plot.1a Pricing Error with Optimal Weighting Matrix



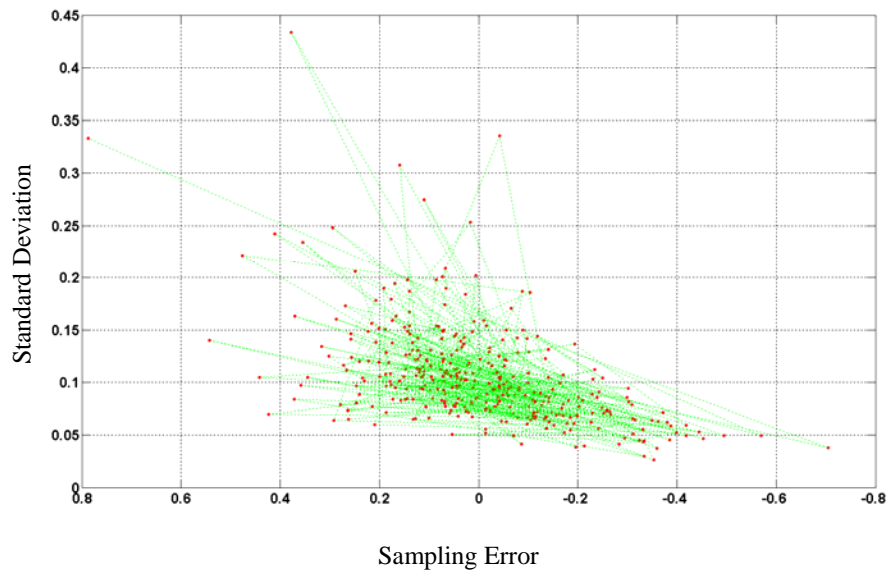
Plot.1b Pricing Error with Optimal Weighting Matrix



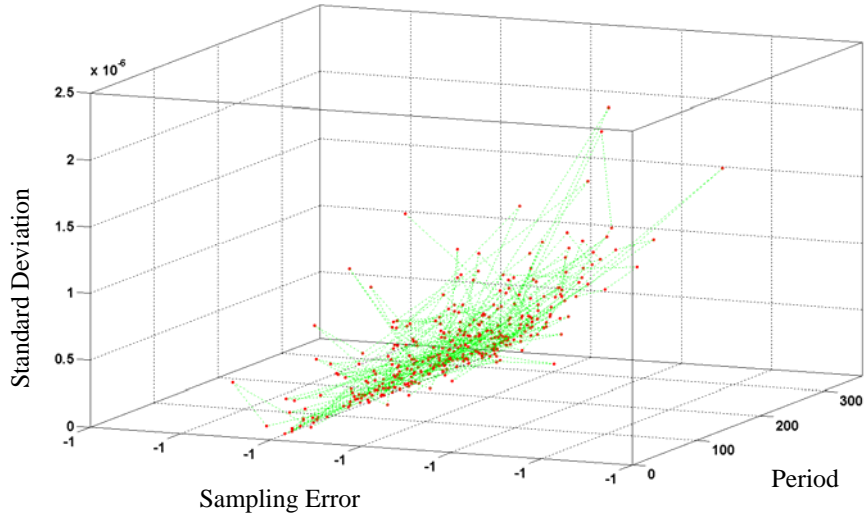
Plot.2a Pricing Error with Efficient H-J Weighting Matrix



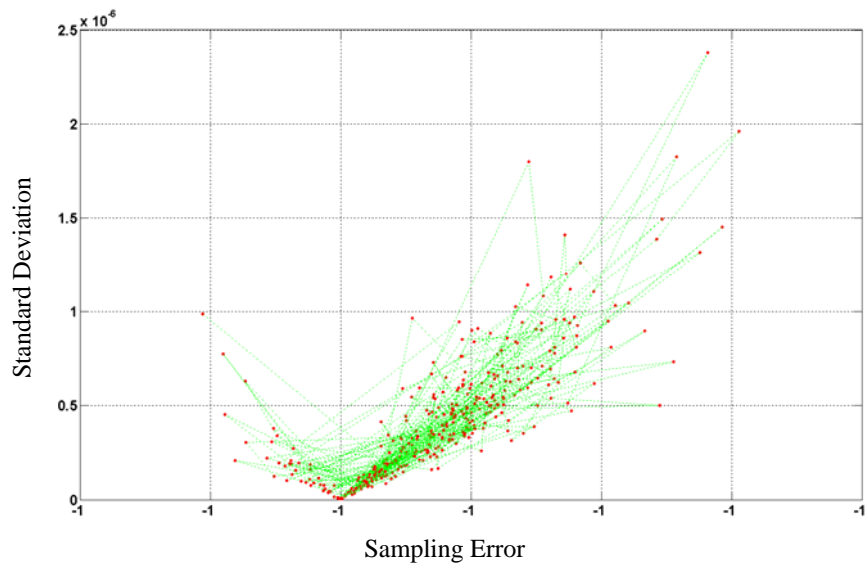
Plot.2b Pricing Error with Efficient H-J Weighting Matrix



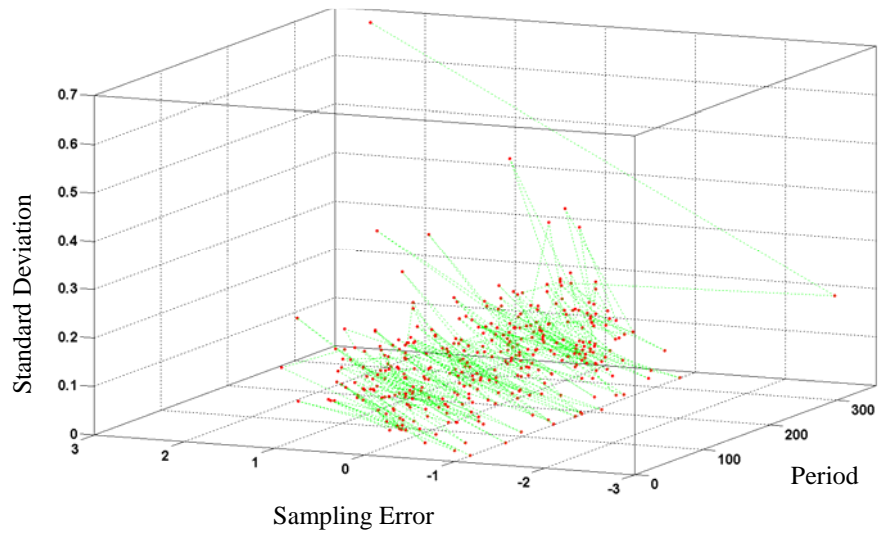
Plot.3a Pricing Error with Optimal Weighting Matrix



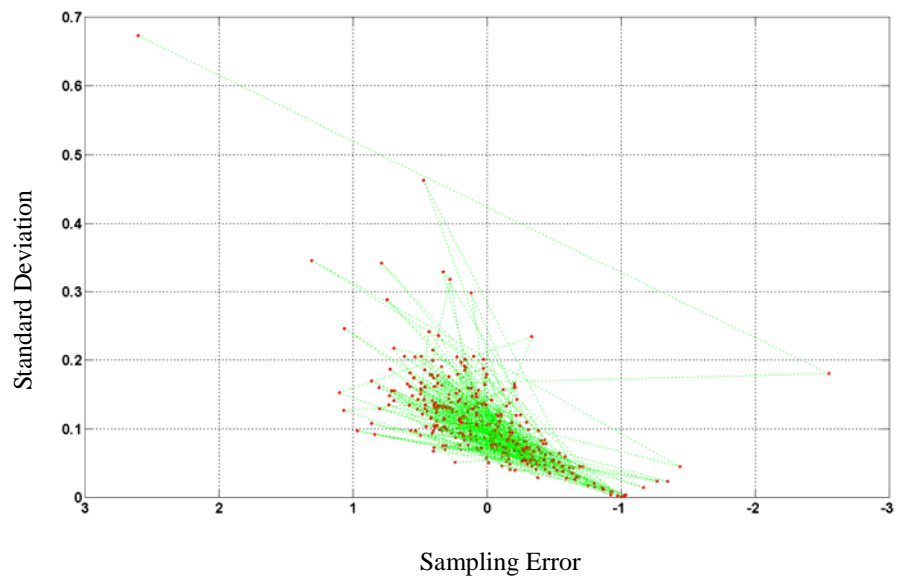
Plot.3b Pricing Error with Optimal Weighting Matrix



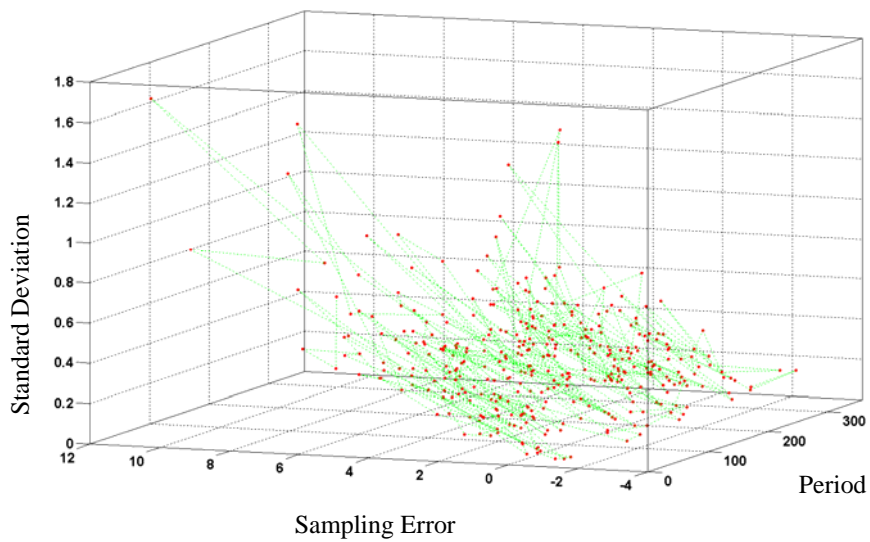
Plot.4a Pricing Error with Efficient H-J Weighting Matrix



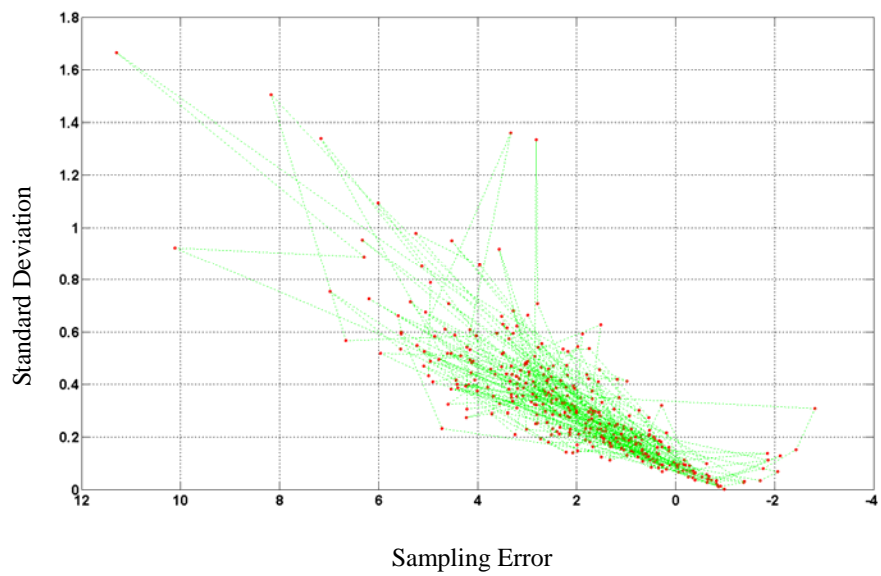
Plot.4b Pricing Error with Efficient H-J Weighting Matrix



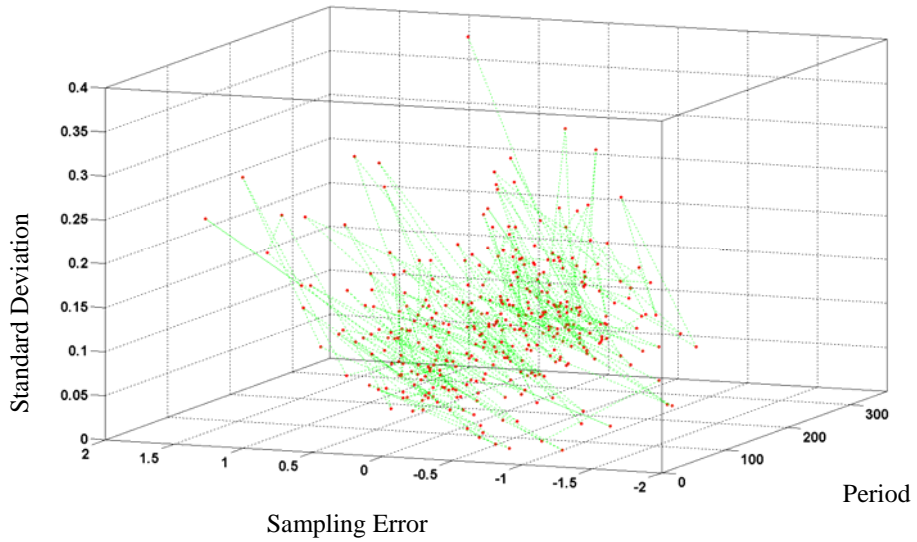
Plot.5a Pricing Error with Optimal Weighting Matrix



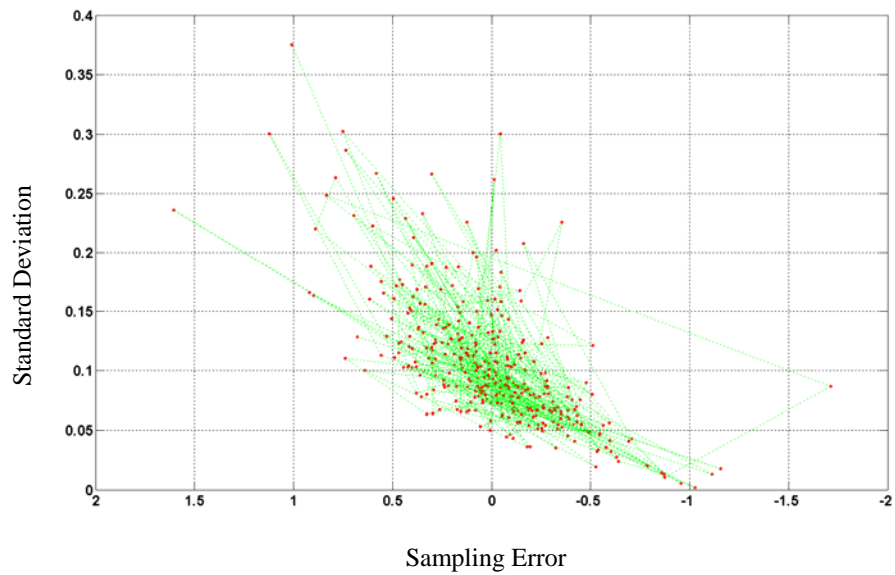
Plot.5b Pricing Error with Optimal Weighting Matrix



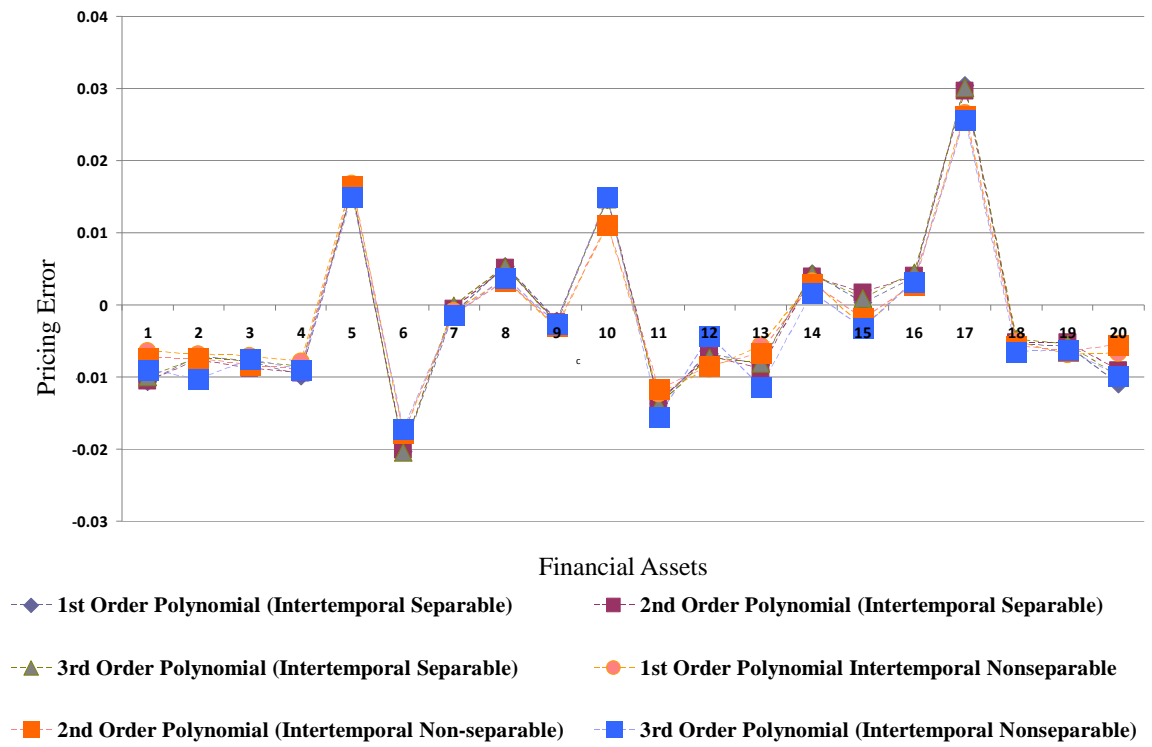
Plot.6a Pricing Error with Efficient H-J Weighting Matrix



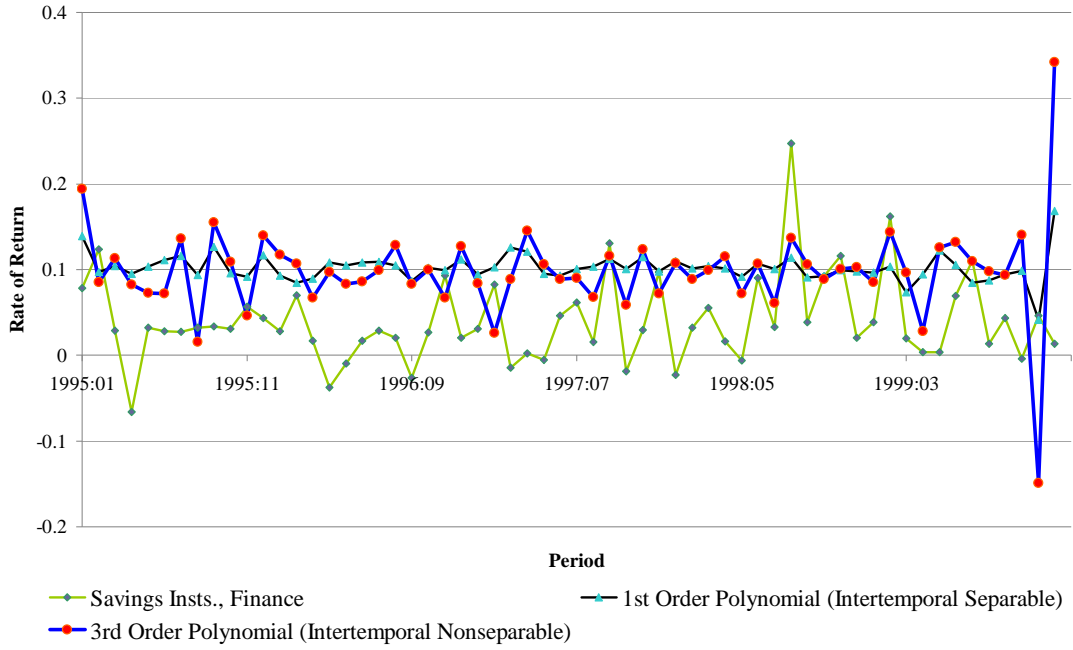
Plot.6b Pricing Error with Efficient H-J Weighting Matrix



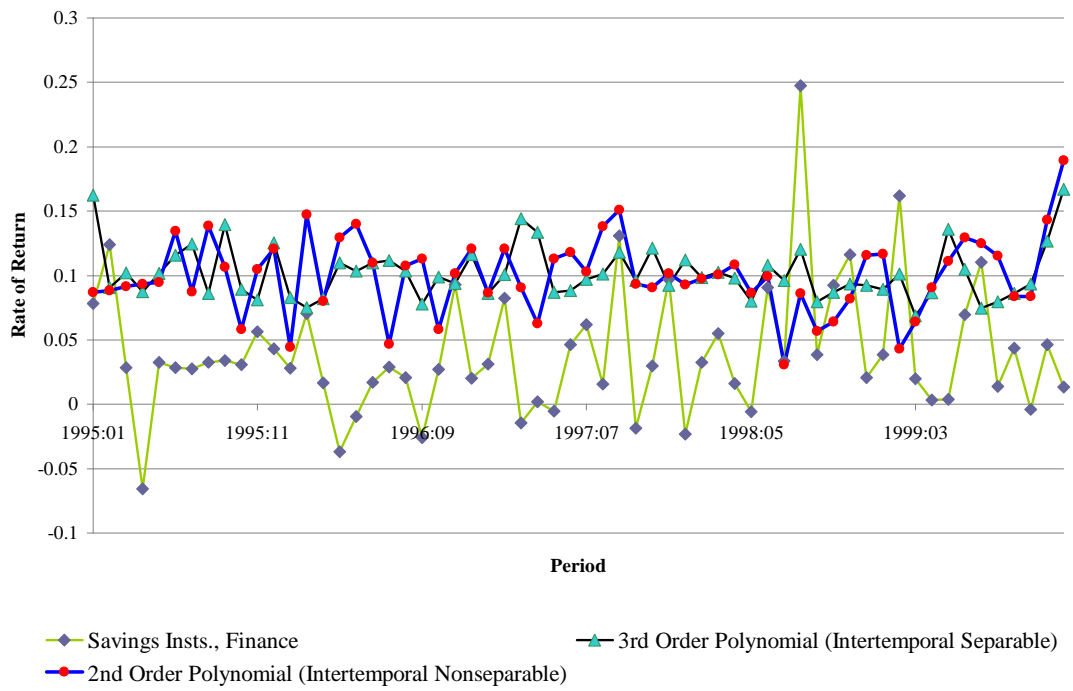
Plot.7 Average Pricing Error with Efficient H-J Weighting Matrix



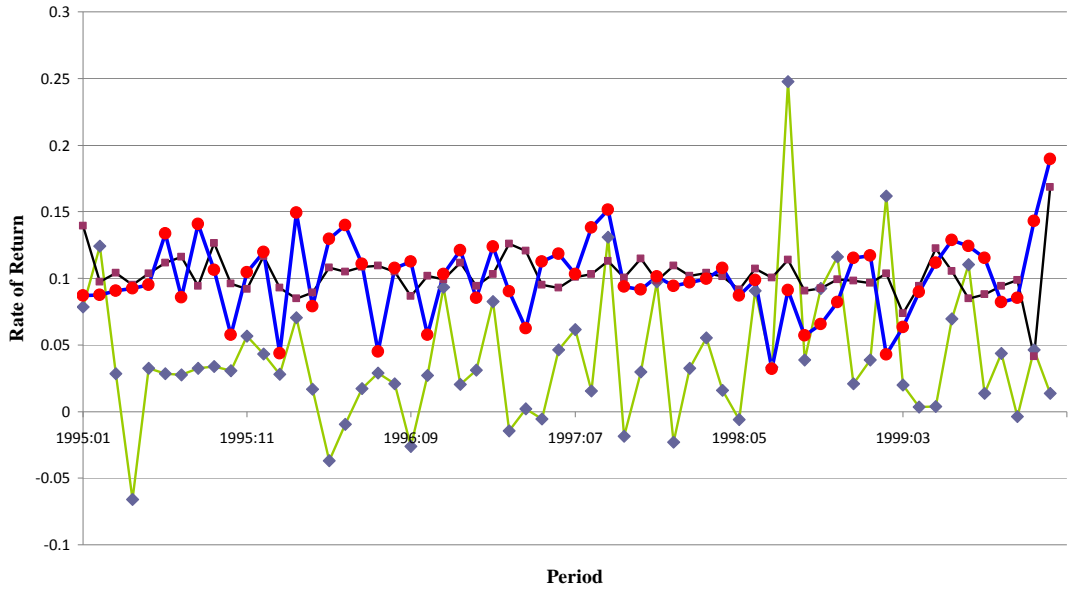
Plot.8 Fitted Asset Returns with Approximated Kernels



Plot.9 Fitted Returns with Approximated Kernels

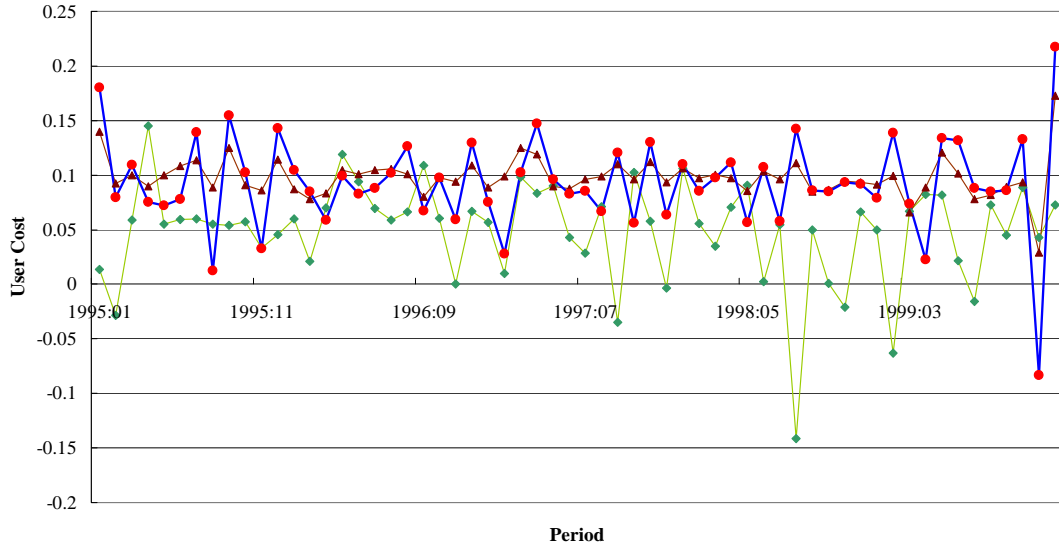


Plot.10 Fitted Returns with Approximated Kernels



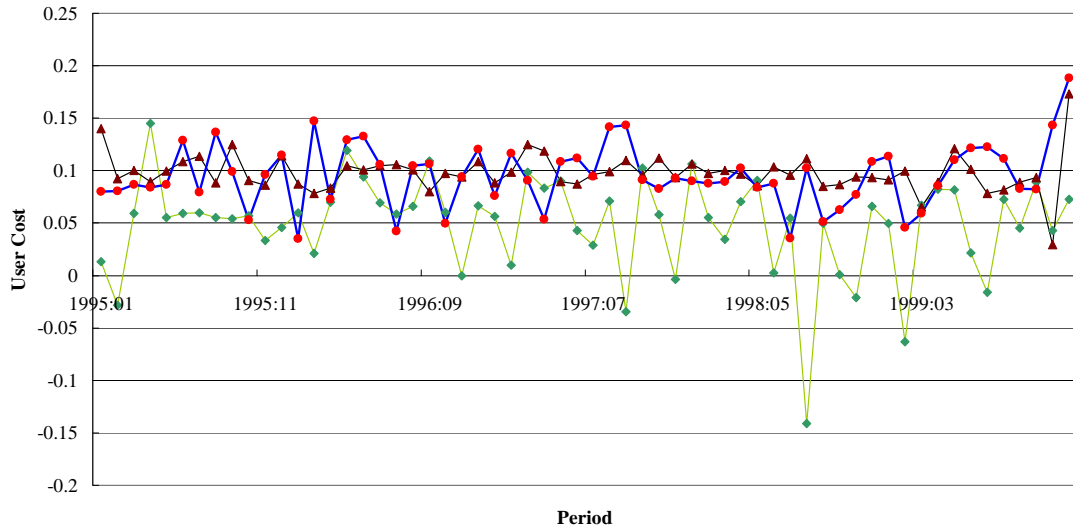
◆ Savings Insts., Finance ■ 1st Order Polynomial (Intertemporal Separable)
● 1st Order Polynomial (Intertemporal Nonseparable)

Plot.11 Fitted User Costs with Approximated Kernels



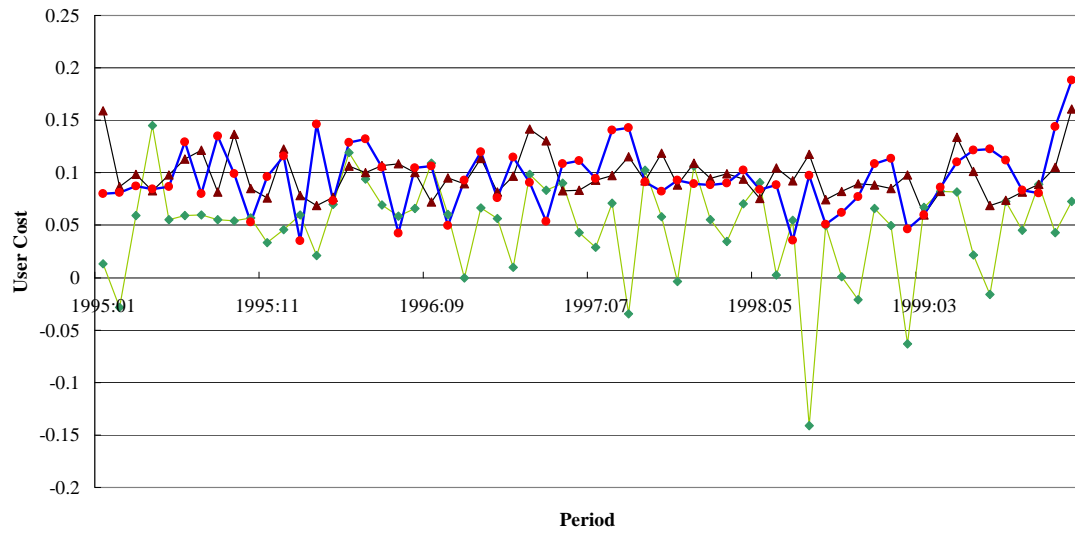
- ◆ Savings Insts., Finance
- ▲ 1st Order Polynomial (Intertemporal Separable)
- 3rd Order Polynomial (Intertemporal Nonseparable)

Plot.12 Fitted User Costs with Approximated Kernels



- ◆ Savings Insts., Finance
- ▲ 1st Order Polynomial (Intertemporal Separable)
- 1st Order Polynomial (Intertemporal Nonseparable)

Plot.13 Fitted User Costs with Approximated Kernels



- ◆ Savings Insts., Finance
- ▲ 3rd Order Polynomial (Intertemporal Separable)
- 2nd Order Polynomial (Intertemporal Nonseparable)