

Calculating Percentage Agreement Correctly but Writing Its Formula Incorrectly

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Hawkins (1983) recently called attention to inconsistencies in expressions for percentage agreement and in formulae for calculating this type of agreement. Unfortunately, Hawkins' note did not include an example of the correct formula for calculating percentage agreement, and disagreement still exists about the proper formula (Bell-Dolan, 1985).

After studying Hawkins' remarks and questioning a number of colleagues, we have come to the following (curious) conclusion: Nearly all of us can correctly calculate percentage agreement, but most of us cannot correctly write the formula used to produce the percentage. Thus, we have prepared this brief paper to report on the basis for this conclusion and to provide a proof for the correct formula for calculating percentage agreement scores.

THE MECHANISM FOR THE CONCLUSION

Five students and four faculty members from the Department of Human Development at the University of Kansas were asked to complete three simple exercises: (a) to calculate simple percentage agreement from two hypothetical observer records (Each record contained 10 cells, each cell contained either an "X" or an "O," and 5 of the same cells in both records contained an "X." The remaining 5 cells were not matched between records.), (b) to calculate percentage agreement when provided with the number of agreements and disagreements, and (c) to write the formula used to calculate the percentages in (a) and (b).

With one exception, all students and faculty successfully calculated the correct

percentages, but only one person wrote its formula correctly. That most people can correctly use the rules for calculating percentages but cannot write those rules correctly could be viewed as an interesting defect in the graduate program at Kansas, but this phenomenon is likely more pervasive than that. Four high school students (grades 10-12) also were asked to write a formula for calculating percentages and all four made the same error produced by our graduate students and faculty. After all, the error was not in determining agreement but, rather, in writing the steps for calculating percentages.

To estimate the ubiquity of the incorrectly written formula, we went to 12 books purporting to be statistics or behavior analysis texts and looked for formulae showing how to calculate percentages or percentage agreement scores. All of the formulae for percentage agreement were incorrect; one statistics book, though, did give the correct formula for calculating percentages. So, now we present the correct formula and its proof for calculating percentage agreement.

THE PROOF OF THE CORRECT FORMULA FOR CALCULATING PERCENTAGE AGREEMENT

According to *Webster's New World Dictionary of the American Language* (1962), "percent" (or %) means per 100 (or 1/100). Therefore, as an example, 60% means 60 per 100 or 60/100. Percentages are useful because expressions with a common denominator are easily compared. For example, because 43% and 23% both have a common denominator, 100, it is easy to see that 43 per 100 (or 43%) is larger than 23 per 100 (or 23%); for contrast, compare 3/7 and 7/31. Percentages are easily calculated by appending the percent symbol to a quotient after

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first *multiplying* it by 100. These are the steps most people correctly follow, but incorrectly describe, when writing the formula.

To take a specific example, suppose two observers scored 15 intervals alike, while 9 intervals were scored differently. To calculate the percentage agreement between these two observers, we would perform the following steps:

$$15/(15 + 9) = 0.625. \quad (1)$$

Now, to express 0.625 with respect to 100 (i.e., "with respect to 100" means per 100, which is the definition of percent) without changing its (i.e., 0.625) value, 0.625 is multiplied by 100/100:

$$0.625 \times 100/100 = 62.5/100. \quad (2)$$

Notice that the value has not been changed because the multiplier $100/100 = 1$. The product of (2) may be read as 62.5 per 100 or 62.5 "percent" (recall the definition of percent) and *written* as 62.5%. It is a subtle but important point to realize that 62.5/100 and 62.5% have the same *value* but different *forms*. The latter expression is a percentage while the former is a decimal fraction (or technically, a rational number). Hence, (2) can be rewritten in the following form without changing the value of the product

$$0.625 \times 100\% = 62.5\% \quad (3)$$

because $100\% = 100/100$ by the definition of percent. Thus, the correct formula for calculating percentage agreement is

$$(\text{agreements}/(\text{agreements} + \text{disagreements})) \times 100\% = P\%, \quad (4)$$

not the ubiquitous formula shown below in which the decimal fraction is multiplied by *only* 100:

$$(\text{agreements}/(\text{agreements} + \text{disagreements})) \times 100. \quad (5)$$

To recapitulate our proof, the most common error in writing the formula is to multiply expression (1) by 100 rather than 100%. The result of performing the former is simply to move the decimal point two places to the right, which is equivalent to increasing the value (of 0.625) 100 times while *not* setting the magnified value (i.e., 62.5) with respect to 100. Hence, the resulting value is not the same as (2) or (3). Thus, as we pointed out at the beginning of this proof, percentages are calculated correctly by multiplying a quotient by 100%.

We hope that these brief remarks will finally clear up writing the formula for calculating percentages and percentage agreement.

REFERENCES

- Bell-Dolan, D. (1985). In defense of the 100 in percentage agreement formulas. *the Behavior Therapist*, 8, 107.
- Hawkins, R. P. (1983). A frequent error in calculation or reporting of interobserver agreement. *the Behavior Therapist*, 6, 109.
- Webster's New World Dictionary of the American Language*. (1962). New York: The World Publishing Company.