THE EFFECTS OF THE FOUR-STEP PROBLEM-SOLVING MODEL ON ALGEBRA I STUDENTS' MATHEMATICAL ACHIEVEMENT AND OPINIONS

BY

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ABSTRACT

This study investigated the effects of experiences with a specific problem-solving model on students' ability to solve multi-step word problems and students' beliefs about problem solving. Algebra I students completed a test of the same 16 free response word problems and a beliefs survey before and two months after learning and using the problem-solving strategy.

The overall and subscore means for the word problems post-test were higher than the pre-test means; the difference was statistically significant. Results showed that students' abilities to approach word problems improved; the data showed that the number of problems that scored two or less points on the four-point, grading rubric decreased.

A Likert scale was used to gather quantitative data about four beliefs. The mean scores on the post-survey were equal to or slightly higher but not statistically significant. Responses to open-ended items on the post-survey indicated that students found the problem-solving strategy useful.

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CHAPTER 1 – THE RESEARCH PROBLEM

Introduction

"Ugh… not more word problems. I hate word problems!" Comments such as this one can be heard in any math class when students are asked to engage in problem solving using word problems. Many students struggle to interpret word problems or are apprehensive about applying the skills they have been taught to real world problems. These students typically have been taught in traditional math classrooms learning basic skills and algorithms to solve problems, and have not been provided with opportunities to think on their own.

In today's society, people are constantly faced with problems or dilemmas they must sort through to arrive at a solution. Many times the problems are complex with many different aspects, so there are not step by step algorithms for people to apply. Employers want to hire those people who are quantitatively literate and possess logical thinking skills (Schoenfeld, 2002). According to Schoenfeld (2002), "Children who are not quantitatively literate may be doomed to second-class economic status in our increasingly technological society" (p. 13).

Students are not going to become quantitatively literate and logical thinkers, and therefore be successful in today's society, without some changes occurring in the mathematics classroom (National Council of Teachers of Mathematics, 2000). As more research is published about problem solving, the issue is becoming harder to ignore. Teachers of all levels of mathematics need to start making problem solving the center of their classrooms.

In 1980, the National Council of Teachers of Mathematics (NCTM) published *An Agenda for Action* which called for "problem solving to be the focus of school mathematics in the 1980s" (as cited in Schoenfeld, 1987). In order to promote the problem solving initiative, NCTM dedicated their 1980 Yearbook publication to the topic and highlighted the four-step problem solving model devised by George Pólya in his book *How to Solve It* (Schoenfeld, 1987). According to Schoenfeld (1987), “For mathematics education and for the world of problem solving [the book *How to Solve It*] marked a line of demarcation between two eras, problem solving before and after Pólya” (p. 283).

Pólya (1945) broke down problem solving into four steps to consider and follow when approaching a problem: understanding the problem, devising a plan, carrying out the plan, and looking back. He suggested the first step to solving any problem is to understand the problem. Pólya explained, “It is foolish to answer a question that [one does] not understand” (p. 6). Therefore, before answering, the students must become familiar with what is being asked. This step is meant to focus on identifying what is the unknown, what information is provided, and what conditions need to be addressed. It is often helpful for students to illustrate the situation, if possible, and label the known data.

In Pólya’s (1945) model, the second step is devising a plan. The focus of this step is for the students to brainstorm and outline all of the calculations and/or constructions necessary for them to arrive at the solution. However, Pólya warned, “The way from understanding the problem to conceiving a plan may be long and tortuous” (p. 8). This step encompasses the majority of the work done in solving a problem and teachers may need to use some guiding questions to assist the students at the beginning.

Step three, carrying out the plan (Pólya, 1945), is pretty self-explanatory. In this step, the students need to implement the step by step outline they created. They need to complete each calculation and/or construction in the specific order they devised.

The final step of Pólya’s (1945) model is looking back. During this step, the students should reread the problem to check to see if their answer is logical and answers the question posed. They should also focus on checking their calculations by using methods such as estimation and working backwards. This step is often skipped by students, but Pólya believes that this step is the most important. Pólya argued, “By looking back at the completed solution, by reconsidering and reexamining the result and the path that led to it, [the students] could consolidate their knowledge and develop their ability to solve problems” (p. 15). The teacher must encourage students to consider this step and not allow students to bypass it in order to help them strengthen their logical thinking skills.

In the preface to his book, Pólya (1945) described the frustrations felt by many students about how to start and finish a problem. These frustrations motivated him to develop the suggestions that he presented as his four-step problem-solving model. He expressed his hope that teachers would use his model to help their students solve problems.

Statement of the Problem

The purpose of this study was to investigate the effects of teaching students a specific problem-solving strategy on students’ mathematical achievement and epistemological beliefs about mathematics. The Algebra I students from Santa Monica, California, involved in this research were taught how to correctly implement Pólya’s (1945) four-step problem-solving model. Throughout the study, they were provided with numerous opportunities to practice the model and engage in problem-solving situations.

The research questions posed in this study were:

1. Was there a change of students’ scores on a test of word problems after experiences with a four-step problem-solving model?
2. How did students’ beliefs about problem solving change after experiences with a

four-step problem-solving model?

At Santa Monica High School in Santa Monica, California, forty-two Algebra I students were taught Pólya’s (1945) four-step problem-solving model and provided with daily opportunities to practice applying the model to word problems. Prior to studying the model, students were asked to complete an academic achievement test consisting of sixteen word problems and a survey about mathematical beliefs. After two months of exposure to the problem-solving model, students were asked to take the same academic achievement test and complete a similar survey about mathematical beliefs.

Rationale for the Study

The National Center for Educational Statistics completed a study comparing the performance of students from the United States to other countries in various areas including mathematical problem solving. They found that the students from the United States performed lower in the area of problem solving than their peers from 25 of the other 38 countries involved (Lemke et al., 2004). They also noted there was no significant change in the scores of students from the United States from 2000 to 2003.

In today’s society, in order for students to have promising futures, they must be quantitatively literate. Due to developments in technology, the meaning of quantitatively literate has changed from being able to perform basic mathematical computations to being able to reason and communicate using mathematical ideas and logical thinking (Schoenfeld, 2002). For students to be able to reach this level, teachers need to provide the students with strategies to develop problem-solving skills, so students know how to approach or start a word problem (Manswell Butty, 2001; Montague & Applegate, 2000; NCTM, 2000; Shaw, Chambless, & Chessin, 1997). A trend found throughout the research is if a student is organized and determined when approaching a problem-solving situation, then he/she is more likely to experience higher mathematical achievement (Montague & Applegate, 2000; Shaw et al., 1997).

Researchers have investigated how students' mathematical achievement and problem-solving ability is impacted by their mathematical beliefs. The two most common beliefs found to impact students' achievement and ability are that mathematics requires time and effort, and mathematics is useful (Kloosterman & Cougan, 1994; Schoenfeld, 1989; Schommer-Aikins, Duell, & Hutter, 2005). In order to foster beliefs that lead to success in mathematics, teachers need to encourage persistence and students need to be exposed to challenging problems early with assistance from the teacher when the students need it (Carlson, 1999). Also, students need to be exposed to real world application problems on a regular basis in mathematics to demonstrate the usefulness of a concept (Kloosterman & Cougan, 1994; Schoenfeld, 1989). These beliefs, along with other affective issues, influence mathematical achievement. The most influential affective variables are confidence in learning mathematics and personal expectations regarding success (Hart Reyes, 1984; Schoenfeld, 1983).

Unfortunately, at Santa Monica High School, application problems are often times skipped in order to provide more time for students to practice basic computational skills. This being the case, during a testing situation, students do not have confidence in their ability and therefore struggle with application problems or do not even attempt them. Many students in Algebra I do not see any relevance in the concepts they are studying so they have low levels of motivation and bad attitudes towards mathematics class. The purpose of this study is to determine whether focusing on application problems and presenting Pólya's four-step problem-solving model will help students improve their ability to solve word problems and improve their attitude towards mathematics.

Assumptions

Some basic assumptions were made in order to be able to compare and interpret the results of this study. The first major assumption was that the students involved did not have a problem-solving strategy they were able to successfully apply on multi-step word problems. If the students did have a strategy, then they would not need assistance in the area of problem solving and the results would be skewed. An assumption was also made with regards to student effort. It was assumed that all students would try their best on the academic achievement test as both the pre-test and post-test. If students did not show consistent effort or did not attempt the problems, then the results would not show their true abilities. This assumption also applied to the surveys. It was assumed that the students read and understood each statement and answered truthfully. If this was not the case, then the results would not accurately reflect their mathematical beliefs. The same questions were used on both the pre-test and post-test. Since the tests were administered two-months apart, it was assumed that the students did not remember the problems. If students were familiar with the problems, then the results would not show true improvement in their problem-solving ability. The final assumption was that students would try to use and learn the new problem-solving model. If they did not put forth effort, then the results would not demonstrate if use of the model improved their ability to problem solve.

Limitations

This study was conducted with a specific population, so it is very important not to over generalize the results. All students involved in this study were from Santa Monica High School in Santa Monica, California, and had the same teacher. They were all taking Algebra I at the high school level, so they all have had some remediation in their mathematics classes at some point. Since this study was only conducted at the Algebra I level, the results may not be representative of what would occur for other levels of mathematics classes. Students at a higher level or on an advanced course of study may not need assistance with problem solving; therefore, these results would be invalid. It is important to understand these results and recommendations are only applicable to Algebra I students in similar settings.

CHAPTER 2 – REVIEW OF LITERATURE

Introduction

A review of the research on problem solving in the mathematics classroom will be discussed in this chapter. In research, problem solving involves the use of problems or "tasks where the solution or goal is not immediately attainable and there is no obvious algorithm for the students to use" (McLeod, 1988, p. 135). This review focuses on research in four main areas. The first area will focus on the current standards initiative and the requirement for problem solving to be incorporated into the mathematics curriculum. The second area concentrates on research about students' mathematical beliefs. The third area addresses affective issues and their influences in the mathematics classroom. Finally, the fourth area concentrates on student achievement and how students approach word problems.

Standards Calling for More Problem Solving

In today's society there is a strong concern about the quantitative literacy of the children. A civil rights leader, Robert Moses, stated, "Children who are not quantitatively literate may be doomed to second-class economic status in our increasingly technological society" (as cited in Schoenfeld, 2002, p. 13). This concern has driven educational leaders to begin to define what quantitative literacy is in the current society. With the presence of calculators and computers, the emphasis is no longer on being literate in basic mathematical facts and operations. Instead, in today's society, in order for students to be quantitatively literate, they need to learn to reason and communicate using mathematical ideas (Schoenfeld, 2002). This idea requires students to develop problem-solving strategies and logical reasoning. In 1980, the National Council of Teachers of Mathematics (NCTM) recognized this change in quantitative literacy and published

An Agenda for Action which called for “problem solving to be the focus of school mathematics in the 1980s” (as cited in Schoenfeld, 1987, p. 287).

Following their 1980 publication, NCTM has published several other documents calling for reform of the mathematics classroom. In 1989, NCTM issued *Curriculum and Evaluation Standards for School Mathematics,* which called for significant changes in the current curriculum to shift the emphasis to process instead of content and skills (Schoenfeld, 2002). It emphasized a “focus at all grade levels on problem solving, reasoning, [and] connections (between mathematical topics and to real world applications)” (as cited in Schoenfeld, 2002, p. 15).

The reform movement that followed the publication of the 1989 Standards was confronted with a great deal of skepticism and criticism. It was not until early in the twenty-first century that research could be done on some of the first large-scale implementations of the reform curriculum (Schoenfeld, 2002). Schoenfeld collected data on some reform students, who were being exposed to a reform curriculum emphasizing problem solving, and some traditional students, who were taught using the traditional curriculum that focused on the development of basic skills. The preliminary data gave hope to reform advocates because it indicated that the reform students outperformed their peers in the area of understanding concepts and problem solving, while maintaining the same level of performance on skills. The other major discovery among the reform students that arose from the data was that the traditional performance gap, between ethnically majority students and poor or underrepresented minority students, had diminished. Schoenfeld used the data to confirm the need to follow the suggested reform curriculum and place an emphasis on problem solving and applications.

After their publication of *Curriculum and Evaluation Standards for School Mathematics*, the National Council of Teachers of Mathematics (2000) realized "the ideas of the *Standards* have been interpreted in many different ways and have been implemented in varying degrees of fidelity" (p. 5). In order to clarify their ideas, the National Council of Teachers of Mathematics (NCTM, 2000) published *Principles and Standards for School Mathematics*. In this publication, NCTM focused one standard on problem solving. They stated that the importance of problem solving was to "equip students with knowledge and tools that enable them to formulate, approach, and solve problems beyond those that they have studied" (p. 335). NCTM described the teacher's role as a coach guiding the students as they worked on non-routine problems creating a stimulating environment showing mathematics as a sense-making discipline. NCTM noted that "research indicates that students' problem-solving failures are often due not to lack of mathematical knowledge but to the ineffective use of what they do know" (p. 54). NCTM suggested the way to overcome this deficiency is exposure. Students need to be engaged in non-routine problems that require problem-solving skills on a regular basis.

Students' Mathematical Beliefs

Many researchers have looked into mathematical beliefs to gain a better understanding of the students in the classroom. It is a widely accepted idea that mathematical beliefs play a role in students' attitudes in class and their overall achievement. Schommer-Aikins, Duell, and Hutter (2005) studied 1,296 students from two middle schools in the Midwest to test their hypothesis about students' epistemological beliefs about mathematics and their influence on the students' mathematical ability. They found that the "two strongest epistemological belief factors were quick/fixed learning and studying aimlessly" (p. 299). These two beliefs can have a heavy impact on the way students deal with problem-solving situations. The implication of students having the belief in quick/fixed learning is that students will assume that they should be able to complete all assignments in only a short amount of time. If the students do not complete the assignment in their allotted time, they will have the tendency to give up. Likewise, if the students hold the belief that mathematics is simply studying aimlessly and therefore not useful, the students may resist putting forth the time and effort needed to be successful. Schommer-Aikins et al. (2005) suggested that teachers can combat these implications by forewarning the students that "the task will be challenging and time-consuming" (p. 302).

Kloosterman and Cougan (1994) focused their attention on elementary school students' mathematical beliefs by interviewing 62 students in first through sixth grade. These students all attended the same elementary school which was participating in the second year of a project targeting the improvement of mathematical instruction by focusing on problem solving. The researchers focused on five different categories of beliefs including the extent to which students like mathematics, the perceived parental support of mathematics, the perceived usefulness of mathematics, the self-confidence in learning mathematics, and the existence of an inherent mathematical ability. The goal for the researchers was to gain a better understanding of elementary students' attitudes and beliefs about mathematics. After analyzing the data they collected through the interviews, Kloosterman and Cougan (1994) were encouraged to discover that most of the students voiced a belief that everyone has the ability to learn mathematics if they put forth effort and that mathematics was useful. The researchers did indicate that the setting of a school may have influenced the results since the school had a fairly strong mathematics program. Therefore, the researchers interpreted the results "in terms of the effects that good instruction can have on beliefs" (p. 386).

Schoenfeld (1989) conducted similar research on beliefs when he studied the mathematical beliefs of students enrolled in three highly regarded high schools in New York. He administered a questionnaire to 230 students following the academic, college-bound track who were enrolled in mathematics classes that included tenth grade geometry, eleventh grade pre-calculus, and twelfth grade calculus or problem-solving classes. The students reported beliefs that included "the subject matter can be mastered if they work at it" (p. 348) and "mathematics helps one to think logically" (p. 348). Schoenfeld's alarming findings were that the students believed that mathematics was best learned by memorization and in a majority of the classrooms real application problems were not evident. These conflicts in beliefs led Schoenfeld to conclude that "what counts in problem-solving situations is students' behavior, and that behavior seems to be driven much more by students' experiences than by their professed beliefs" (p. 349).

In order to get a different perspective on students' mathematical beliefs, Carlson (1999) investigated the beliefs of successful mathematics graduate students. In her research, Carlson collected qualitative data by interviewing six successful graduate students that had completed at least one graduate level mathematics class with a grade of an A. She learned from these students that they all had confidence in their ability to work through problems, they were willing to spend long periods of time attempting mathematical problems, and they enjoyed the challenge posed by complex mathematical tasks. Carlson also collected quantitative data by administering a beliefs survey to 34 mathematics graduate students at a large university. From this data, she concluded that persistence is a necessary trait for success in mathematics classes and students need to be exposed to challenging problems early with assistance from the teacher when the students need it.

Affective Issues in the Mathematics Classroom

In the past, researchers have investigated the effect of affective issues, such as attitude and emotion, on student achievement in the mathematics classroom. McLeod (1988) examined the theory behind affective issues and emotion that can influence a student's ability in the mathematics classroom. He suggested that there are four main factors of emotion that need to be considered in research: "the magnitude and direction of the emotion, duration of the emotion, level of awareness of the emotion, [and] level of control of the emotion" (p. 134). Students are affected by all four of these factors when they feel an emotion. The most common emotion when dealing with problem solving that students describe is a feeling of "getting stuck" (p. 136). The interruption in the student's plans to approach the problem, according to McLeod, launches the initial emotion of frustration. The magnitude of this emotion is often intense and the direction is negative. The duration of the emotion depends on the level of commitment the student has to the problem. This commitment also is influenced by the student's awareness of his/her emotion and his/her level of confidence in the ability to continue. This emotion could be brief if the student gives up immediately or it could be drawn out if the student tries to persevere through the problem. McLeod believed that teachers should be aware of these emotional reactions to problem solving in order to plan instruction that deliberately addresses these affective issues.

Schoenfeld (1983) argued that affective factors, such as personal expectations regarding success, confidence in one's mathematical ability, and drive to persevere through obstacles, influence the decisions made during problem solving. Boekaerts, Seegers, and Vermeet (1995) further investigated Schoenfeld's statement by studying three particular students and how they approached problem solving paying particular attention to gender. They studied one student with the affective factor that he had too much confidence in his ability to problem solve. This student applied an algorithm to find an answer without thoroughly reading the problem and circled his answer without ever checking to make sure it made sense. Another student they studied was influenced by the affective factor that he was not confident in his ability, so he did not have the drive to persevere. This student tried algorithm after algorithm in a fairly random order; when he did not feel like he was making progress, the student quit trying. The third student they analyzed was extremely unconfident in her abilities. She actually got the correct answer, but did not have confidence to mark it. She reported that she thought she missed the problem. This particular student demonstrated metacognitive skills and was able to choose an effective strategy, but failed to answer the question due to her lack in confidence. Boekaerts, Seegers, and Vermeet (1995) reported that there was a difference in affective factors based on gender. They concluded that "before starting with the math task, boys displayed more confidence, more pleasure, more positive emotions, and a higher learning intention than girls" (p.258). In their study, they concluded that boys are inclined to perform better on problem solving because they have more positive affective factors influencing them than girls.

Hart Reyes (1984) investigated which affective variables were most influential on mathematical achievement. She reported that confidence in learning mathematics and self-concept with respect to mathematics were the two most influential affective variables. She stated that a student with high confidence in his/her ability, or self-concept, will have higher mathematical achievement. Hart Reyes observed that "people who are sure of their ability in mathematics will probably choose tasks involving mathematics more often and persist longer than those who are not sure they will succeed" (p. 560). She also observed that "students high in self-confidence interact more with their teachers and spend more time on task than students who have lower self-concepts" (p. 562). Combined, these observations helped Hart Reyes conclude that students with high confidence and self-concept have the behaviors and habits to make them successful in studying mathematics. She noted that students who meet these criteria are typically better at problem solving and take more advanced mathematics courses.

Ma and Xu (2004) studied the causal order of attitude and achievement. Through their study, they determined that a reciprocal relationship exists between attitude towards mathematics and achievement in mathematics among students in a non-honors program. They concluded that this relationship, although it is circular in nature, starts with achievement which affects attitude. Ma and Xu found that "poor achievement in mathematics is one of the reasons leading to the decline in attitude" (p. 277). They believed the key to improving students' attitudes towards mathematics is improving achievement in mathematics. Their findings emphasized "the critical role that academic success plays in engaging students both affectively and cognitively in the learning of mathematics" (p. 276).

The Link between Student Achievement and Problem Solving

Researchers have studied the relationship between how students approach problem solving and student achievement. A trend found throughout the research is if a student is organized and determined when approaching a problem-solving situation, then he/she is more likely to experience higher mathematical achievement (Montague & Applegate, 2000; Shaw et al., 1997).

Montague and Applegate (2000) conducted a study comparing students' perception of problem-solving difficulty, knowledge of problem-solving strategies, and persistence when solving word problems among seventh and eighth graders of three different levels of problem-solving ability (learning disabled, average-achieving, and gifted). Before starting their study, Montague and Applegate examined the three groups for differences in reading achievement, cognitive ability, and computational accuracy in order to control these extraneous variables. Their findings revealed that when the average achievers rated a problem as more difficult, they persisted significantly longer than their gifted peers resulting in little to no significant difference in problem-solving accuracy. They also noted that students with a learning disability (LD) performed worse than their average and gifted peers on the mathematical problems. However, since the researchers controlled reading, cognitive ability and computational accuracy, they were able to conclude that the low performance was caused by other factors including the lack of persistence and problem-solving strategy deficits. They noticed that although the students with LD rated a problem as more difficult than their peers, they did not spend any more time on the problem than their peers. This led the researchers to the conclusion that the students with LD simply "shut down" on the more complex problems because they simply did not know how to start or approach the problem.

Another approach to study the link between student achievement and problem solving is to look at the classroom instruction and the problem solving incorporated into the everyday experience. Manswell Butty (2001) investigated this topic by looking at the effects of reform-based and traditional instructional practices on the mathematical performance of tenth and twelfth grade Black and Hispanic students. The students were in mathematics classrooms where the instructional practices were identified as either traditional or reform practices. In the traditional mathematics classrooms, instruction was teacher-centered with a heavy emphasis on lecture, guided practice, and individual practice. The reform-based mathematics classrooms centered more on conceptual understanding through inquiry-based activities as advocated by the National Council of Teachers of Mathematics (NCTM, 2000). In comparing these two particular teaching styles, the main difference to recognize was the focus of each type of instructional practice. In reform-based instruction, there was a heavier focus on problem-solving strategies and application. In contrast, traditional instruction was more focused on skills and rarely included applications with problem solving. Manswell Butty discovered from her data analysis that students receiving reform instruction had significantly higher achievement scores and better attitudes towards mathematics than the students receiving traditional instruction. The practical implication of this research is that mathematics teachers should focus more on inquiry-based, problem-solving instruction in order to get minority students motivated in the mathematics classroom and to increase their mathematical achievement.

One research team focused their efforts on analyzing the effects of using collaborative-group work and a specific problem-solving strategy in the mathematics classroom. Their research was conducted by teachers in a Professional Development School (PDS) site for the University of Mississippi (Shaw, Chambless, & Chessin, 1997). The teachers used the K-W-D-L technique to guide the problem solving of the fourth grade students in their rural school. In their technique, the "K" step required the students to identify what they knew about the problem, the "W" step prompted students to focus on what they were wanting to find, the "D" step called for students to narrate what they were doing to solve the problem, and the "L" step encouraged students to look back and defend their answer and approach. After incorporating this problem-solving technique in the classroom, the teachers noted it helped students to get started organizing and documenting their work while providing them with a starting point. They also credited the K-W-D-L technique for "helping their children develop study skills and for increasing their academic autonomy" (p. 484).

Summary

In today's society, in order for students to have promising futures, they must be quantitatively literate, meaning they need to learn to reason and communicate using mathematical ideas (Schoenfeld, 2002). The National Council of Teachers of Mathematics

(NCTM) realized this and called for a change in the curriculum for mathematics. In 1989, NCTM published *Curriculum and Evaluation Standards for School Mathematics.* This document emphasized a "focus at all grade levels on problem solving, reasoning, [and] connections (between mathematical topics and to real world applications)" (as cited in Schoenfeld, 2002, p. 15). The increased emphasis on problem solving has led to researchers looking into what influences problem solving and how to improve students' abilities to problem solve. In 2000, NCTM published *Principles and Standards for School Mathematics*, their clarification of the 1989 *Standards.* In this document, NCTM (2000) clarified problem solving as the use of non-routine problems to "enable [students] to formulate, approach, and solve problems beyond those that they have studied" (p. 335).

Research has shown that students' mathematical achievement and problem-solving ability is significantly impacted by their mathematical beliefs. The two most common beliefs found are that mathematics requires time and effort, and mathematics is useful (Kloosterman & Cougan, 1994; Schoenfeld, 1989; Schommer-Aikins et al., 2005). In order to foster beliefs that lead to success in mathematics, teachers need to encourage persistence and students need to be exposed to challenging problems early with assistance from the teacher when the students need it (Carlson, 1999). Also, students need to be exposed to real world application problems on a regular basis in mathematics to demonstrate the usefulness of a concept (Kloosterman & Cougan, 1994; Schoenfeld, 1989).

Beliefs, along with other affective issues, influence mathematical achievement. The most influential affective variables are confidence in learning mathematics and personal expectations regarding success (Hart Reyes, 1984; Schoenfeld, 1983). A student's commitment to persevere through difficulties and solve the problem is influenced by his/her attitude and level of confidence in his/her ability (Boekaerts et al., 1995; Hart Reyes, 1984; Ma & Xu, 2004; McLeod, 1988). Also, these affective factors influence decisions made during problem solving (Boekaerts et al., 1995; Schoenfeld, 1983). There is a reciprocal relationship between attitude and mathematical achievement; although it is circular in nature, it starts with achievement which affects attitude (Ma & Xu, 2004).

Many researchers have looked into problem solving and how to improve mathematical achievement. The major trend found throughout the research is if a student is organized and determined when approaching a problem-solving situation, then he/she is more likely to experience higher mathematical achievement (Montague & Applegate, 2000; Shaw et al., 1997). Students need to have increased exposure to real world application problems in order to practice approaching them (Manswell Butty, 2001; Montague & Applegate, 2000; NCTM, 2000; Shaw et al., 1997). Most importantly, teachers need to provide the students with strategies to develop problem-solving skills, so students know how to approach or start a word problem (Manswell Butty, 2001; Montague & Applegate, 2000; NCTM, 2000; Shaw et al., 1997).

CHAPTER 3 – METHODOLOGY

Introduction

Many students struggle in the area of mathematical problem solving. Often times, they complain of not knowing how to start or approach the problem. Research has proven that there is a link that exists between problem-solving ability and student achievement and beliefs. Knowing that, professionals in the field of mathematics education have begun to turn their attention to problem solving and teaching students the process of logical thinking. Many mathematicians have suggested models to use in approaching problem-solving situations; however, none of these attempts have been as systematic, concise, and practical as George Pólya's (1945) four-step problem-solving model.

This study was specifically designed to investigate the effects of a four-step problem-solving model on students' mathematical achievement and beliefs. The research questions addressed were:

1. Was there a change of students' scores on a test of word problems after experiences with a four-step problem-solving model?
2. How did students' beliefs about problem solving change after experiences with a four-step problem-solving model?

This chapter provides a detailed description of the subjects, instruments, and procedures that were used in this study.

Subjects

The subjects of this study were students from two classes of Algebra I taught by the researcher. The students attended Santa Monica High School during the 2007-2008 school year.

There were a total of forty-two students that participated in this study. Two students in one class did participate in all the activities, but their data were not used due to lack of parent consent. Of the forty-two participants, twenty were female and twenty-two were male. They ranged in age from fourteen to seventeen years old. These students included thirty-two freshmen, eight sophomores, and two juniors. All of these students had repeated either Pre-Algebra or Algebra in their education. Sixty percent of the students repeated Pre-Algebra for two consecutive years, fourteen percent repeated Pre-Algebra for three consecutive years, and twenty-six percent took Algebra the previous year. Thirty-one percent of the students involved in this study were classified as socioeconomically disadvantaged. Fifty-seven percent of the participants were Hispanic or Latino, twenty-four percent of the participants were Caucasian, ten percent were African-American, two percent were Asian, and seven percent were other ethnicities. Twenty-four percent of the students spoke Spanish as their primary language at home. Six of the students that participated in this study were enrolled in the English Language Learners Program with a ranking of intermediate to early advanced. Also, three students qualified for services from the Special Education Department.

Santa Monica High School is a very large campus located four blocks from the Pacific Ocean, in the heart of Santa Monica, California. During the 2007-2008 school year, there were 3,147 students enrolled in the school (Santa Monica High School SARC Committee, 2008). Approximately six percent of that entire student body was on interdistrict permits (students live within the boundary of another district in California, but meet one of the criteria to be allowed to attend Santa Monica High School). Twenty-four percent of the entire student body was considered socioeconomically disadvantaged.

In 2008, the city of Santa Monica had a population of 87,664 people and a median household income of $63,224 (U.S. Census Bureau). About six percent of the families living within Santa Monica in 2008 were below the poverty level. Twenty-six percent of the families reported speaking a language other than English at home. Additionally, seventy-one percent of the families reported living in a renter-occupied unit.

Instruments

In this research study, two types of instruments were used to gather data. Students were asked to complete both an academic achievement test and a survey. The academic achievement test was administered as both a pre-test and a post-test. This test consisted of sixteen free-response, multi-step word problems broken into three short parts; a copy can be found in Appendix A. These questions were selected from the Texas Assessment of Knowledge and Skills (TAKS) Exit Level test given to eleventh grade students in the state of Texas. Each question on the TAKS test was aligned to a Texas Mathematics Standard. The researcher compared the identified Texas Standard to the California Mathematics Standards (California Department of Education, 1997) in order to decide whether the question relied on prior knowledge from the middle school level curriculum or an algebraic concept from the Algebra I curriculum. Using this comparison process, the questions selected were categorized as either prior knowledge or algebraic questions. The questions from each category were coded for the test administrator, but were randomly sorted for construction of the student test.

The prior knowledge questions included multi-step, problem-solving situations that required the application of a formula (e.g. area, perimeter, volume, surface area, or Pythagorean Theorem). While applying the formula, students were required to use fractions, decimals, integers, and/or percents. The prior knowledge questions included problems one and five from test #1; problems one, three, and four from test #2; and problems three, four and six from test #3.

The algebraic questions included multi-step word problems that could be solved using several different approaches such as working backwards, guess-and-check, or setting up and solving an equation. The algebraic questions were problems two, three, and four from test #1; problems two and five from test #2; and problems one, two, and five from test #3.

As an example of the categorization of the problems, on test #1 (a copy can be found on page 51) question numbers four and five both deal with perimeter, however they were classified in different categories. Question four was classified as an algebraic question for this study, because, according to the Texas alignment, the focus of the question was on students being able to set up and solve a linear equation for a given word problem. The researcher located that same concept in the California Mathematics Standards in the Algebra I standards which stated, "Students solve multi-step problems, including word problems, involving linear equations in one variable" (California Department of Education, 1997, p. 38). On the other hand, the focus of question five was on computing perimeter according to the Texas alignment. The researcher located that same concept in the California Mathematics Standard in the Grade Seven standards under Geometry and Measurement which stated that students should "use formulas routinely for finding the perimeter and area of basic two-dimensional figures" (p. 32). Thus, question five was classified as a prior knowledge item in this study.

There were two forms of the student survey, a pre-survey and a post-survey. Both surveys were constructed by selecting and adapting statements from the Indiana Mathematics Belief Scales and the Fennema-Sherman Usefulness Scale (Kloosterman & Stage, 1992). These instruments were developed to test secondary students' opinions on beliefs about mathematics. Only four of the six beliefs pertained to this study. The original survey items were adapted by replacing "mathematics" with "problem solving".

The beliefs used addressed perseverance, complexity, importance and usefulness with regards to problem solving. The perseverance belief addressed students' perceptions of their ability to solve time-consuming mathematics problems. The complexity belief addressed students' opinions about the existence of word problems that cannot be solved with simple, step-by-step procedures. The importance belief addressed the students' opinions on the importance of word problems in mathematics. The usefulness belief addressed students' perception of the usefulness of problem solving in daily life.

The positive and negative statements assessing these four beliefs were mixed up in order to form the surveys used in this study. The statements were arranged in the same order on both the pre-survey and the post-survey. The perseverance belief was the focus of statements one, five, nine, thirteen, eighteen, and twenty-two. The complexity belief was the focus of statements four, eight, twelve, sixteen, twenty, and twenty-three. The importance belief was the focus of statements three, six, eleven, fourteen, seventeen, and twenty-four. The usefulness belief was the focus of statements two, seven, ten, fifteen, nineteen, and twenty-one.

The pre-survey was used to determine students' beliefs and feelings towards problem solving. The students marked their responses using a Likert scale, modified from the original five-point scale to a four-point scale, of strongly agree, agree, disagree, or strongly disagree. At the end of the pre-survey there was also an open-ended question to assess how students currently approach multi-step problem-solving situations. A copy of the pre-survey can be found in Appendix B.

The post-survey was similar to the pre-survey and can be found in Appendix C. The post-survey had identical statements from the pre-survey and the students marked their responses on the same Likert scale. In addition, the post-survey had five open-ended questions at the end asking the students about the usefulness of the model and how they might apply the model in other subject areas.

Procedures

On September 25th, one week before the research study began, the researcher provided the students with a consent form to take home to their parents for approval to participate in the study. The researcher also explained the study directly to any interested parents that night at an Open House event. If the parents gave consent, they signed the form and the student brought it back to class. Since the study was built in as part of the class activities, all students took part in the study. However, if the parent did not give consent, then data were not used from that student. A copy of the parent consent form can be found in Appendix D.

The study began on October 1, 2007. All students were given the pre-survey in class in order to gather information on their opinions and approaches to word problems. The academic achievement test was administered in three parts on October 2nd through 4th. The test was organized into three short parts to eliminate the students becoming too fatigued and quitting. Each part of five or six problems consisted of an assortment of questions from the two categories which were not arranged in any particular order. The researcher monitored the students' progress and concluded the test when all students had attempted the problems, about twenty minutes each day. The purpose of these tests was to assess their ability to approach and solve word problems. Calculators were made available to help students with basic computation skills so the focus would remain on the concept of word problems. Also, there was not an assigned time limit to allow the students to attempt all problems to their maximum ability. Students were encouraged to write what part of the problem confused them if they became stuck on a problem.

On October 5, 2007, the researcher formally presented George Pólya's (1945) four-step problem-solving model. This model was heavily emphasized for the following two months. On the first day, students were provided with a flow chart of the steps, and the researcher modeled how to apply each step to a word problem. A copy of this flow chart can be found in Appendix E.

For the first two weeks, the researcher focused on familiarizing the students with the four-step problem-solving model and providing numerous situations to allow the students time to practice correctly applying the process. The students were asked to practice on simple problems and real world situations. During the process orientation, students were given a problem a day and five minutes to work in their assigned groups applying the four-step model. After the allotted time, the researcher would lead a class discussion analyzing the problem. The researcher would prompt the class asking, "What is the first thing to do?" or "What should you look at next?" The questions were used as repetition to help the students become familiar with the steps and their order. Different groups were called on to explain how they used each step, and how the approach helped them solve their problem. During this orientation, the majority of the problem solving was done in small groups, followed by a class discussion, and then the researcher would model how to correctly apply the model.

After the initial process orientation was complete, the procedures were altered so students were required to try the problems individually. For the first five minutes of each day, students began working independently on solving a word problem by applying the four-step model. The daily word problems can be found in Appendix F. Following the students' individual attempts, the researcher led the discussion, calling on individual students to model the use of the four steps, and continually questioning the class on the steps they should be taking to approach the problem. Daily the researcher would ask the class, "What is the first thing you should do with this problem?" This question was followed by questions like, "What should you do next?" Students were required to keep a journal of these problems as a sample of how to effectively use the four-step problem-solving model. Also in the journal of these word problems, one or two times a week, the researcher would ask the students at the end of class to take a moment to reflect and write for the last three to five minutes of class. Each week, the researcher asked the students to write about a situation outside of class that they encountered which required some problem solving, and the thought process they used to come to a solution. Every other week, the researcher asked the students to journal about the problem-solving model they were focusing on and how it had helped them in or out of school.

Every other week students were involved in problem-solving presentations. The researcher provided the students with a set of problems about the current concepts being studied. The students worked in cooperative groups with the task of discussing the problems and effectively applying the four-step model in order to find the answer. The following day, each group was randomly assigned a problem to present to the class explaining how they used the four steps with their problem to find the answer.

On each formal assessment during the course of this study, students were required to answer an open-ended, problem-solving question. On this question, students were asked to identify the four steps and write an explanation of their approach to solving the problem. A heavier emphasis while grading this item was placed on the explanation and the approach as opposed to the correct answer alone. This was done to further emphasize the process and help motivate the students to continue trying the approach.

During the last week of the study, the post-test and post-survey were administered. The same three parts of the academic achievement test were used. This was done to eliminate the argument that the pre-test or post-test differed in difficulty level. The post-test was administered in the identical process as the pre-test. One part, consisting of five or six problems, was administered per day on December 11th through 13th. Students were allowed as much time as needed to attempt all the problems each day. The researcher monitored the students' progress and concluded the test when all students had attempted the problems, about twenty minutes each day. The students also had access to a calculator as needed to assist them in solving the problems. The post-survey was administered on Friday, December 14, 2007. The post-survey had identical statements from the pre-survey in order to gather data on any changes in attitude. The only difference from the pre-survey was the post-survey had five open-ended questions at the end asking the students about the usefulness of the model and how they might apply the model in other subject areas.

Data Analysis

The academic achievement test, pre-tests and post-tests, were mixed together when graded in order to conduct a blind study. The heading stating the student's name and whether the paper was a pre-test or post-test was covered up using fastened construction paper, then all of the tests were randomly shuffled together. This procedure helped keep the researcher from showing any bias while grading. Each question was graded using a four-point scale outlined in Appendix G. In order to receive the full credit of four points for a problem, the student needed to find the correct answer and show correct work to support his/her answer. Three points were awarded if a student correctly identified the key information and the procedures that needed to be carried out, but made a computational error while performing the steps resulting in an incorrect answer. A student received a score of two points if he/she correctly identified the key information and could verbally describe the steps, but did not know how to mathematically carry out the plan. A score of two was also awarded if the procedures were incomplete. One point was awarded if the student was only capable of identifying the key information. A solution assigned this score would completely lack an explanation of the steps or the steps outlined would be incorrect. No points were awarded if a problem was left blank, or an answer had no work to support it. This grading rubric was similar to the rubric used on other formal classroom assessments, so the students were familiar with the requirement of showing their work.

In order to address the research questions, the assessments were separated into pre-tests and post-tests after they were graded. The questions were also coded as either prior knowledge or algebraic questions. Since there were eight questions in each category, there was a total of 32 points possible for each type of question. Each student's total points were calculated for four categories: pre-test prior knowledge questions, pre-test algebraic questions, post-test prior knowledge questions, and post-test algebraic questions. This data can be found in Appendix H. The mean and standard deviation for each of these categories were also calculated as well as for the entire pre-test and post-test. A two-sample *t*-test was used to determine if there was a significant difference in the means of the pre-test and post-test. This calculation was considered to be statistically significant based on the accepted significance level of 0.05.

In order to see if there was improvement on how the students approached the problems, the researcher tallied the number of times a problem earned four points, three points, two points, one point, or no points. If a problem earned four points, then the student knew how to correctly approach and solve the word problem. If a problem earned three points, then the student knew how to approach the word problem, but made a minor mistake in solving it. A score of two points demonstrated that the student could decide what was needed to solve the problem, but did not know how to mathematically carry out the procedures. A score of no points or one point reflected that the student could at the most identify what they needed to solve for, but had no idea how to approach the problem. These tallies were compiled separately for both the prior knowledge and algebraic questions on the pre-test and the post-test.

In order to address the second research question, the surveys were separated into pre-survey and post-survey from the beginning. Points were assigned to each statement based on the student's response and whether the statement was positive or negative. If the statement was positive, strongly agree earned four points, agree earned three points, disagree earned two points, and strongly disagree earned one point. If the statement was negative, the points were awarded in reverse, so that strongly disagree received four points, disagree received three points, agree received two points, and strongly agree received one point. Once all the surveys were scored, the questions were coded based on the belief they belonged to. Four beliefs were assessed on the surveys focusing on perseverance, complexity, importance and usefulness with regards to problem solving. The totals per belief were tallied for each student for the pre-tests and post-tests. Those raw scores can be seen in Appendix I. The means and standard deviations for this data were calculated. A two-sample *t*-test was conducted for each of the four beliefs in order to determine if the means of students' opinions as measured by the post-test were significantly higher than on the pre-test. The accepted significance level of 0.05 was set.

The open-ended questions at the end of the surveys were analyzed as qualitative data. The open-ended question on the pre-survey was analyzed for trends in students' approaches to problem solving before the study. On the post-survey, there were two main themes in the questions posed to the students. The first theme dealt with the students' perceptions of the helpfulness of the problem-solving model, and the second theme addressed future uses of the problem-solving model. The answers were studied and similar responses were tallied.

CHAPTER 4 – RESULTS

Introduction

This study was designed to investigate the effects of a specific problem-solving model on students' ability to solve multi-step word problems and students' beliefs about problem solving. The research questions posed in this study were:

1. Was there a change of students' scores on a test of word problems after experiences with a four-step problem-solving model?
2. How did students' beliefs about problem solving change after experiences with a four-step problem-solving model?

The study began on Monday, October 1, 2007. Forty-two Algebra I students from Santa Monica High School located in Santa Monica, California participated in the study. Students were asked to respond to a pre-survey and attempt 16 word problems of two types (prior knowledge and algebraic questions), which made up the Academic Achievement Pre-test. Following the pre-test, the students were taught Pólya's (1945) four-step problem-solving model. This model is designed to help students approach word problems in a step-by-step process. The researcher provided the students with a word problem a day in order to help them become comfortable with the process and develop proficiency in applying it. Other classroom-based experiences with the model included class discussions, peer and instructor modeling, journal writing, and group presentations. The study spanned two months. At the end of the two months a post-test was administered that was identical to the Academic Achievement Pre-Test. The post-survey was administered on December 14th.

The results from this study are organized into four main sections. First, an analysis of the results of the Academic Achievement Pre-Test and Post-Test will be presented. Second, the tallied results indicating the points earned on the word problems will be shared. Next, a comparison of the pre-survey and post-survey results will be made. Finally, the qualitative data from the surveys will be discussed.

Academic Achievement

Pre-Test versus Post-Test

After the tests were graded, there was a total of sixty-four points possible, separated into thirty-two points for each category of questions, prior knowledge and algebraic. The data from the pre-test and post-test were analyzed in three different ways. First, the data from the prior knowledge questions were compared to identify any changes in the pre-test and post-test results. The mean score for the eight prior knowledge questions on the pre-test was approximately 10.929, while the mean score of the same questions on the post-test was approximately 16.786. These scores demonstrate that students did improve in this category after experiences with the problem-solving model. By the conventional criteria (p-value lower than 0.05), this difference is considered to be statistically significant. The results of the t-test are presented in Table 1.

The data from the algebraic questions were compared in a similar manner. The results for these specific questions in the pre-test and post-test were analyzed to see if any improvement was shown. The mean score for the eight algebraic questions on the pre-test was approximately 14.571, while the mean score for the same questions on the post-test was approximately 17.381. These mean scores indicate a slight improvement in the algebraic questions following experiences with the problem-solving model. This difference is considered statistically significant using the conventional significance level of a p-value less than 0.05. The detailed results for this t-test can be found in Table 1.

After comparing the results for the two problem categories, overall results on the pre-test and post-test were analyzed. The mean score for the entire pre-test was approximately 25.5, and the mean score for the entire post-test which was 34.167. These scores show a large improvement in overall performance when assessing students' abilities to approach and solve word problems. Using the conventional criteria of a *p*-value less than 0.05, this difference is considered to be statistically significant. Details for this *t*-test can be found below in Table 1.

Table 1

Comparison of Means on Pre-Test and Post-Test

	N	Mean	Std. Dev.	*t*-value	*p*-value
Pre-Test Prior Knowledge	42	10.929	4.027	4.74	0.000009
Post-Test Prior Knowledge	42	16.786	6.874		
Pre-Test Algebraic	42	14.571	5.023	2.173	0.0327
Post-Test Algebraic	42	17.381	6.706		
Pre-Test Total	42	25.5	6.833	3.975	0.0002
Post-Test Total	42	34.167	12.368		

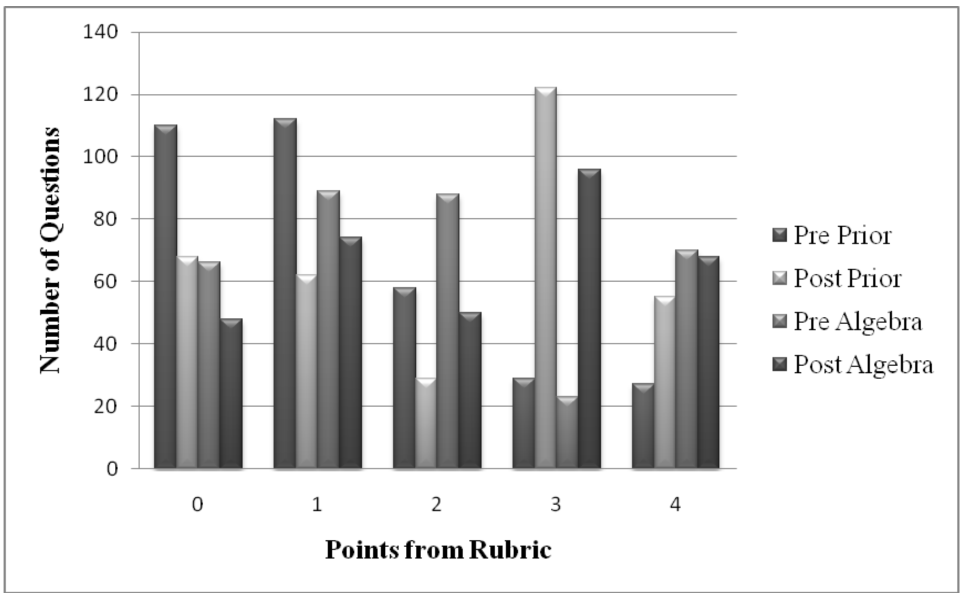
Comparing Points Earned

In order to see if there was improvement on how students approached the word problems, the researcher tallied the number of times a problem earned a specific score. Each problem was graded on a four point rubric earning zero points to four points based on how far the student was able to get in the solution process. The number of problems when students scored each point value was determined and comparisons were made among four categories: pre-test prior knowledge questions, post-test prior knowledge questions, pre-test algebraic questions, and post-test algebraic questions. The detailed data are represented below in Figure 1.

The bar graph shows that the lower scores, zero points up to two points, decreased when the pre-tests were compared to the post-tests. This demonstrates that the students knew how to approach the word problems better on the post-test since a student had to be able to identify the steps required by the problem and be able to correctly apply the majority of those steps in order to earn a score higher than two points. The score of three points had the largest increase in the post-test results. In order to score three points, students had to be able to identify the steps required to solve the problem and carry out the majority of those steps correctly. Students earned three points rather than four points if they missed one step previously stated or made a computational error in their calculations.

Figure 1

Number of Questions Earning Different Rubric Point Values



Pre-Survey versus Post-Survey

The surveys gathered data about students' opinions with regards to four beliefs from the Indiana Mathematics Belief Scales and the Fennema-Sherman Usefulness Scale (Kloosterman & Stage, 1992) including the perseverance belief, the complexity belief, the importance belief, and the usefulness belief. The perseverance belief was about the student's perception of his/her ability to solve time-consuming mathematics problems. After the results were totaled the mean scores were compared. The mean score on the pre-survey for the perseverance belief was approximately16.667, while the mean score on the post-survey was approximately 17.048. Although there was an increase in the mean score, it is not considered statistically significant based on the conventional significance level of a p-value less than 0.05. The results of this t-test can be found in Table 2.

Statements related to the complexity belief were used by the researcher to question students' opinions about the existence of word problems that cannot be solved with simple, step-by-step procedures. After the data were tallied based on the students' responses on the Likert scale, the mean scores for statements about the complexity belief were calculated. The mean score on the pre-survey for this belief was approximately 12.262, while the mean score on the post-survey was 12.5. Thus, there was only a slight overall increase in the mean score reflecting students' opinions about the complexity belief. Based on the conventional significance level of 0.05, this difference is not considered statistically significant. Details of this t-test are located in Table 2.

The next belief, the importance belief, dealt with the idea that word problems are important in mathematics. The importance belief statements were scored, and then the mean scores were calculated for comparison. The mean score on the pre-test for the importance belief was approximately14.476, while the mean score on the post-test was approximately 14.476 as well. The data showed no change in the students' opinion about word problems and their importance in mathematics. A t-test was completed on this data, and the results can be found in Table 2.

Statements related to the final belief, the usefulness belief, were used by the researcher to question students' perception of the usefulness of problem solving in daily life. The mean scores for this belief were calculated. On the pre-test, the mean score was approximately 16.690. On the post-test, the mean score rose to approximately 16.976. Although this was an increase, it was small enough that it is not considered to be statistically significant based on the conventional significance level of a *p*-value less than 0.05. The details of this *t*-test can be reviewed in Table 2.

Table 2

Comparison of Means on Pre-Survey and Post-Survey

	n	Mean	Std. Dev.	*t*-value	*p*-value
Pre-Survey Perseverance Belief	42	16.667	2.647	0.616	0.54
Post-Survey Perseverance Belief	42	17.048	3.012		
Pre-Survey Complexity Belief	42	12.262	1.926	0.568	0.572
Post-Survey Complexity Belief	42	12.5	1.916		
Pre-Survey Importance Belief	42	14.476	1.928	0	1
Post-Survey Importance Belief	42	14.476	1.864		
Pre-Survey Usefulness Belief	42	16.69	2.959	0.422	0.674
Post-Survey Usefulness Belief	42	16.976	3.25		

Qualitative Data

Pre-Survey

At the end of the pre-survey, there was one open-ended question that asked about students' current approaches to word problems. The students' responses to this question were read and analyzed for common terms or phrases. The responses were then read again and tallies were made for the common terms and phrases. If a response contained several of the common phrases, each phrase was tallied. This process generated more tallies than students taking the survey. The six common themes were looking for key words, reading and rereading until it makes sense, breaking down into steps, just trying operations, following examples, and skip them. The number of tallies for the themes can be seen in Table 3. There were six students who did not respond to this question.

As noted in Table 3, fifteen students commented about just trying operations making

Table 3

Comparison of Themes in Student Responses to Open-Ended Questions on Pre-Survey

Theme	Tallies
Looking for key words	10
Read and reread until it makes sense	9
Breaking down into steps	4
Try operations	15
Follow examples	2
Skip them	7
Left it blank	6

that the most common approach. One student wrote, "I do different things for different problems." Ten students mentioned looking for and using key words. Four students commented about breaking the problems into steps. One student commented, "I find [word problems] confusing. I try finding small steps to make them seem easier." Thirteen students wrote about not understanding the problems and not trying the problems. One particular student wrote, "I think [word problems] are dumb and I'll never use them."

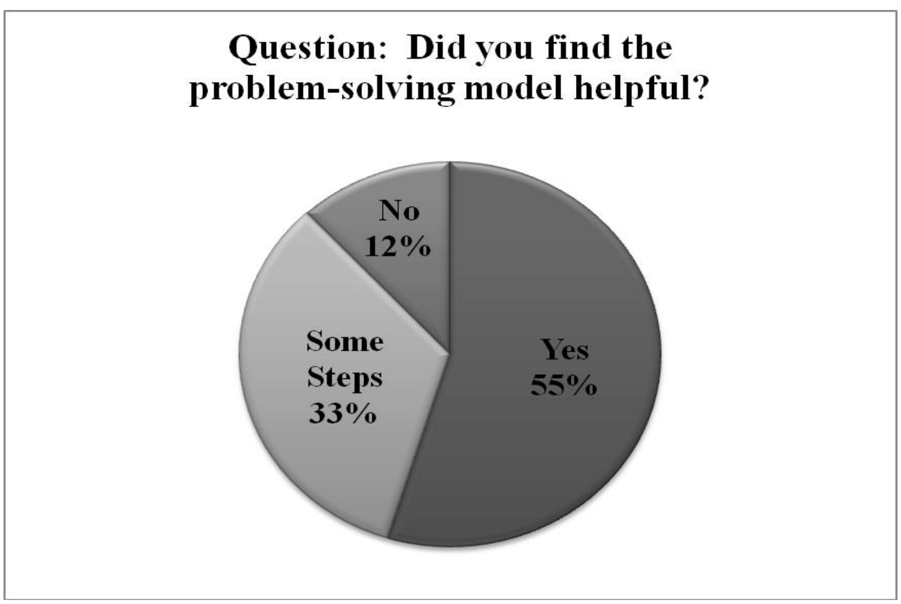
Post-Survey

At the end of the post-survey, there were five open-ended questions dealing with two main themes. The students' responses to these questions were read and analyzed for common responses which were tallied. The first theme in the open-ended questions dealt with students' perceptions of the helpfulness of the problem-solving model. There were three main answers with various reasons why or why not. Twenty-three students responded that the problem-solving model was very helpful to them. Fourteen students stated that some of the steps in the model were useful or that the model helped only on some problems. Five students reported that they did not feel the problem-solving model helped them at all. These data are represented in Figure 2.

The majority of the students that replied "yes," the problem-solving model was helpful, stated that the model helped them break down the problem. One student commented, "It helped me know how to approach the problem. It broke it into smaller, easier to manage steps, so the problem didn't seem as scary." Many of the students who stated that the model did not help them commented that they did not remember all the steps or simply did not want to write down all the information the steps required.

Figure 2

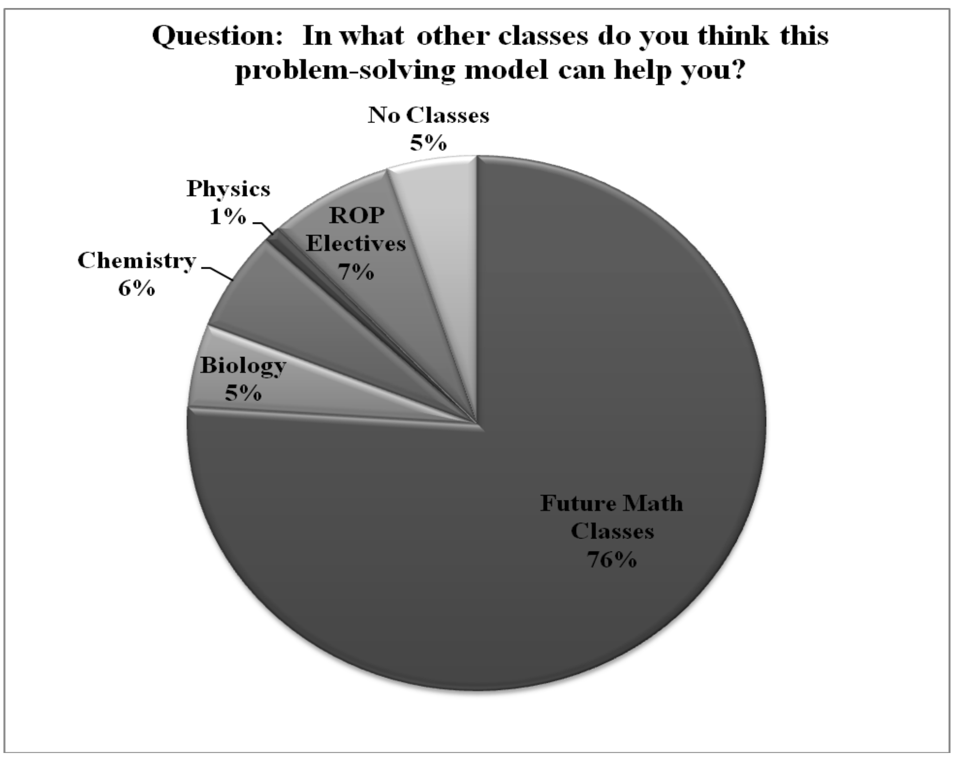
Student Responses to First Theme of Open-Ended Questions on Post-Survey



The second theme in the open-ended questions addressed future uses of the problem-solving model. These questions prompted the students to think about times, other than in their current math class, when they might use this problem-solving model. Sixty-seven percent of the students responded they would use it in their future math classes. Fourteen percent of the students replied they would use it in a science class. This was further broken down since two students specified Biology class, three students specified Chemistry class, and one student mentioned Physics. Seven percent of the students stated that the model could help them in their Regional Occupational Program (ROP) elective classes. These classes include courses such as Industrial Technology, Broadcasting, and Business. Twelve percent of the students mentioned they would not be applying this model in any other course. These data are represented in Figure 3.

Figure 3

Student Responses to Second Theme of Open-Ended Questions on Post-Survey



CHAPTER 5 – SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

In today's society, in order for students to have promising futures, they must be quantitatively literate, meaning they need to learn to reason and communicate using mathematical ideas (Schoenfeld, 2002). The National Council of Teachers of Mathematics (NCTM) realized this and called for a change in the curriculum for mathematics. In 1989, NCTM published *Curriculum and Evaluation Standards for School Mathematics.* This document emphasized a "focus at all grade levels on problem solving, reasoning, [and] connections (between mathematical topics and to real world applications)" (as cited in Schoenfeld, 2002, p. 15). The increased emphasis on problem solving has led researchers to investigate what influences problem solving and how to improve students' abilities to problem solve. In 2000, NCTM published *Principles and Standards for School Mathematics,* their clarification of the 1989 *Standards.* In this document, NCTM (2000) clarified problem solving as the use of non-routine problems to "enable [students] to formulate, approach, and solve problems beyond those that they have studied" (p. 335).

Research has shown that students' mathematical achievement and problem-solving ability is significantly impacted by their mathematical beliefs. The two most common beliefs found are (1) mathematics requires time and effort, and (2) mathematics is useful (Kloosterman & Cougan, 1994; Schoenfeld, 1989; Schommer-Aikins et al., 2005). In order to foster beliefs that lead to success in mathematics, teachers need to encourage persistence and students need to be exposed to challenging problems early with assistance from the teacher when the students need it (Carlson, 1999). Also, students need to be exposed to real world application problems on a regular basis in mathematics to demonstrate the usefulness of a concept (Kloosterman & Cougan, 1994; Schoenfeld, 1989).

Beliefs, along with other affective issues, influence mathematical achievement. The most influential affective variables are confidence in learning mathematics and personal expectations regarding success (Hart Reyes, 1984; Schoenfeld, 1983). A student's commitment to persevere through difficulties and solve the problem is influenced by his/her attitude and level of confidence in his/her ability (Boekaerts et al., 1995; Hart Reyes, 1984; Ma & Xu, 2004; McLeod, 1988). Also, these affective factors influence decisions made during problem solving (Boekaerts et al., 1995; Schoenfeld, 1983). There is a reciprocal relationship between attitude and mathematical achievement; although it is circular in nature, it starts with achievement which affects attitude (Ma & Xu, 2004).

Many researchers have looked into problem solving and how to improve mathematical achievement. The major trend found throughout the research is if a student is organized and determined when approaching a problem-solving situation, then he/she is more likely to experience higher mathematical achievement (Montague & Applegate, 2000; Shaw et al., 1997). Students need to have increased exposure to real world application problems in order to practice approaching them (Manswell Butty, 2001; Montague & Applegate, 2000; NCTM, 2000; Shaw et al., 1997). Most importantly, teachers need to provide the students with strategies to develop problem-solving skills, so students know how to approach or start a word problem (Manswell Butty, 2001; Montague & Applegate, 2000; NCTM, 2000; Shaw et al., 1997).

This study was designed to investigate the effects of experiences with a specific problem-solving model on students' ability to solve multi-step word problems and students' beliefs about problem solving. The research questions posed in this study were:

1. Was there a change of students' scores on a test of word problems after experiences with a four-step problem-solving model?
2. How did students' beliefs about problem solving change after experiences with a four-step problem-solving model?

In order to address these questions, the researcher created an Academic Achievement Test and a survey. The Academic Achievement Test was designed to help answer the first research question. The test consisted of sixteen free response questions selected from the Texas Assessment of Knowledge and Skills (TAKS) Exit Level test. This was the state test in Texas and had a heavy emphasis on multi-step word problems. Each question on the TAKS test was aligned to a Texas Mathematics Standard. The researcher compared the identified Texas Standard to the California Mathematics Standards (California Department of Education, 1997) in order to decide whether the question relied on prior knowledge from the middle school level curriculum or an algebraic concept from the Algebra I curriculum. Using this comparison process, the questions selected were categorized as either prior knowledge or algebraic questions. The prior knowledge questions included multi-step problem-solving situations that required the application of a formula (e.g. area, perimeter, volume, surface area, or Pythagorean Theorem) with fractions, decimals, integers, and/or percents. The algebraic questions were comprised of multi-step word problems that could be solved using several different approaches including working backwards, guess-and-check, or setting up and solving an equation. The test was administered in parts on three consecutive days in order to eliminate fatigue. This test was administered at the beginning of the study and again at the end of the study. A copy of this test can be found in Appendix A.

The survey was designed to help answer the second research question. It was constructed by adapting statements from the Indiana Mathematics Belief Scales and the Fennema-Sherman Usefulness Scale (Kloosterman & Stage, 1992). The survey was used to measure students' beliefs and feelings towards problem solving. The students marked their responses using a Likert scale of strongly agree, agree, disagree, or strongly disagree. Two forms of the survey were created. The pre-survey was administered at the start of the study. The other form was the post-survey that was administered at the end of the study. The only difference between the surveys was the open-ended questions. The pre-survey asked how students currently solved word problems. The post-survey asked for students' opinions about the specific model taught. A copy of each of these surveys can be found in Appendix B and Appendix C.

In between the pre-tests and post-tests, the researcher presented George Pólya's (1945) four-step problem-solving model to the forty-two Algebra I students participating in the study. Each day for two months, students began class trying to apply the model to a word problem. Following their individual attempts, the researcher would lead a class discussion on how to correctly apply the model. The researcher would prompt the class with the questions, "What do you look at first?" and "What should you do next?" The researcher would call on students to model how they applied the steps to solve the problem. At least once a week, students were asked to reflect and write about a situation outside of class which required them to do some problem solving. Additionally, they were asked to write about their thought process they used to come up with a solution to that problem. Outside of these routines, students were given other opportunities to participate in various activities familiarizing themselves with the model including group presentations and test problems.

Following the study, each problem on the Academic Achievement Tests was graded using a four point rubric outlined in Appendix G. In order to assess if students' scores had increased, the mean score of the pre-tests was compared to the mean score of the post-tests. The means reflected a very large increase in scores on the word problems. The difference was found to be statistically significant. A closer look was done to ensure scores rose in both the prior knowledge and algebraic categories. The means reflected that the post-test scores in both categories had increased.

Results were also tallied according to the points earned per problem. This was done to ensure that students were indeed consistently scoring higher on the rubric, therefore becoming more proficient at their approach on all problems. It was noted that more questions on the post-test received higher marks, three or four points on the rubric. On the post-test, the number of scores with two or fewer points, decreased. This can be interpreted to mean that students were getting better at setting up a plan and carrying it out.

The pre-survey and post-survey statements were categorized into the four beliefs that were represented. Statements related to the perseverance belief were used by the researcher to question students' perceptions of their ability to solve time-consuming mathematics problems. Statements related to the complexity belief were used by the researcher to question students' opinions about the existence of word problems that cannot be solved with simple, step-by-step procedures. Statements related to the importance belief were used by the researcher to question the students' opinions on the importance of word problems in mathematics. Statements related to the usefulness belief were used by the researcher to question students' perception of the usefulness of problem solving in daily life. Students' responses on a Likert scale were totaled, and mean scores were compared. It was found that the mean scores for all four beliefs increased or remained the same; none of the increases were considered statistically significant.

Qualitative data were gathered from students' responses to the open-ended questions on the pre-survey and post-survey. On the pre-survey, the responses were analyzed for trends in students' approaches to problem solving before the study. The top four common themes were just trying operations, looking for key words, reading and rereading until it makes sense, and skipping the problems all together.

On the post-survey, there were two main ideas in the questions posed to the students. The first idea dealt with the students' perceptions of the helpfulness of the problem-solving model. The second idea addressed future uses of the problem-solving model. The answers were studied and similar responses were tallied. Eighty-eight percent of the students found some, if not all, of the steps to be helpful. Sixty-seven percent of the students stated that they would use this problem-solving model in future math classes, while 21% suggested other classes including various science courses and business electives where they could use the model.

Conclusions

1. The students' experiences with the problem-solving model helped to increase students' scores on a test of word problems. The overall scores rose from a mean of 25.5 to 34.167. This increase was considered to be statistically significant, implying that experiences with a four-step problem-solving model did help the students to improve in solving word problems. This result is similar to the conclusion that Manswell Butty (2001) made in her study. Manswell Butty's data analysis demonstrated that students receiving reform instruction (instruction focused on problem-solving strategies and applications) performed significantly higher on an achievement test than the students receiving traditional instruction (skills oriented instruction). While there was much improvement by the students in the present study, the class average on the post-test was a 53%. This indicates that although the scores increased, there is still room for much more improvement in the area of multi-step word problems.

2. In this study, students' abilities to approach word problems did improve. By providing them with an organized, step-by-step procedure to apply on word problems, students raised their scores on individual problems. The number of problems on the post-test receiving a score two or less points decreased while the number of problems receiving a score of three points or higher increased. This result demonstrates that the students moved from not knowing how to approach the word problems, to a point where they could identify the steps that were required, because the score of three points indicates having knowledge of the steps to take to solve the problem, but not carrying out the steps correctly due to a computational error or missing a previously stated step. Schoenfeld (2002) collected similar data during a study which indicated that students following the reform curriculum (a curriculum emphasizing problem solving) outperformed their peers in understanding concepts and problem solving. Both studies indicate if students are given opportunities to practice problem-solving strategies, then they will improve in their ability to approach application problems.

3. Based on the results of this study, a conclusion can be drawn about students' mathematical beliefs. The four beliefs addressed in this study were the perseverance belief, the complexity belief, the importance belief, and the usefulness belief. The mean scores for each of these four beliefs remained the same or increased slightly, but not enough to be considered statistically significant. Ma and Xu's (2004) results are relevant to this conclusion. They found that a causal relationship existed between attitude towards mathematics and achievement in mathematics. Their data indicated that if student achievement improved then students' attitude towards mathematics would improve as well. This being stated, the students in this study only were given two months to see an improvement in mathematical ability. Although their ability to solve word problems improved, they had not yet mastered the process since their final mean

score was only 53% correct. Therefore, these students only saw a slight improvement in their mathematical achievement so it is not surprising that no significant change was found in the attitude and beliefs about problem solving.

4. A majority of the students found all or part of the problem-solving model helpful. On the post-survey, 37 of the 42 students replied that they found some or all steps in the model helpful. One student commented, “It helped me know how to approach the problem. It broke it into smaller, easier to manage steps, so the problem didn’t seem as scary.” These are similar to the results of Shaw, Chambless, and Chessin (1997) who researched a fourth grade class where the teacher exposed students to a specific problem-solving technique. They noted that the technique provided the students with a starting point and a way to organize and document their work.

Recommendations

The following recommendations will be shared with the Santa Monica High School Mathematics Department as well as the district mathematics coordinator. Others who may find these recommendations of interest include mathematics educators, researchers, and district coordinators.

1. One problem-solving model should be adopted and used at least school wide, preferably district wide. In this study, students had a hard time thinking of other classes or times they might use the problem-solving model. If students saw one problem-solving model, they would see the usefulness in all areas or subjects. This exposure would help them increase their motivation and academic achievement. This recommendation is in line with the results from a study by Shaw, Chambless, and Chessin (1997). They credited the specific problem-solving technique used in their study for “helping [the] children develop study skills and for increasing their academic autonomy” (p. 484). Also, according to the results of Manswell Butty’s (2001) research, inquiry-

based, problem-solving focused instruction helped to get minority students motivated in the mathematics classroom and to increase their mathematical achievement.

2. Teachers should provide opportunities for students to engage in problem solving on a regular basis. In this study, the students were exposed to a problem-solving situation daily for approximately two months. During that time, students made significant progress in their ability to solve word problems. Providing these numerous opportunities is in line with the curriculum emphasized by the National Council of Teachers of Mathematics (NCTM) in their 1989 publication *Curriculum and Evaluation Standards for School Mathematics* and their 2000 publication *Principles and Standards for School Mathematics.* The reason behind this recommendation is to help students see the usefulness of the mathematical concepts they are learning by exposing them to real world applications. In today's society, they can look up formulas or use calculators to help them with basic mathematical facts, but they must be able to think logically and solve non-routine problems. This recommendation is also supported by a study from Carlson (1999); she stated that students need to be exposed to challenging problems early and regularly with assistance, as needed, from the teacher in order to develop persistence.

3. Consideration for future studies similar to this one should be given. This particular study focused only on the effects of experiences with a problem-solving model used within an Algebra I course. A similar study should be conducted in a higher level mathematics class or in a different discipline such as science. Also, this study was done with a specific population in a high school Algebra I class. Different results might be found if a similar study was conducted with a different population, such as an Algebra I class taught to advanced students at the middle school level.

4. Finally, a longitudinal study should be conducted to see the influences of experiences

with a problem-solving model can have on students' mathematical achievement if that model is used consistently in several consecutive mathematics courses. This study was only carried out for two consecutive months. A statistically significant increase in students' mathematical beliefs might be seen if the study lasted for a longer timeframe. This potential change in beliefs could help the students pursue and excel in higher level mathematics studies.

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Appendix A

Academic Achievement Test: Part 1 Through 3
(Note: Only a copy of the pre-test is included in this Appendix. The only change made for the post-test was the title was changed on the student copy to state "Post-Assessment".)

Name ______________________________ Period _____

Pre-Assessment Part #1

Solve each problem in the space provided. Please clearly circle your answer.

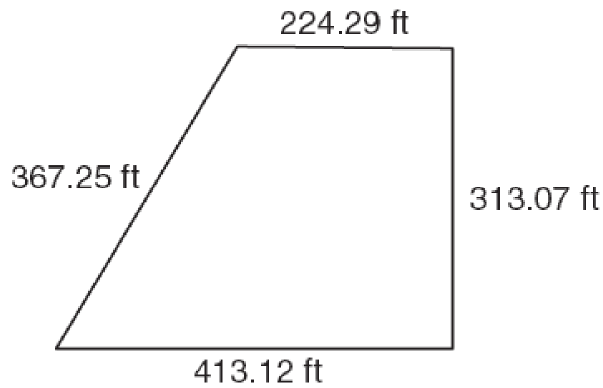
1. A cardboard box is 63 inches long, 18 inches wide and 2 feet high. What is the volume of the box in cubic feet?

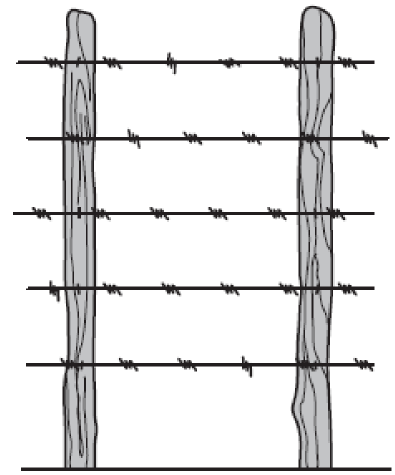
2. Harris has $20 to spend on video-game rentals at a local video store. The store charges $3.95 per video-game rental plus an 8% tax. What is the maximum number of video games that Harris can rent?

3. Two airplanes left the same airport traveling in opposite directions. If one airplane averages 400 miles per hour and the other airplane averages 250 miles per hour, in how many hours will the distance between the two planes be 1625 miles?

4. The perimeter of a rectangular wooden deck is 90 feet. The deck's length, l, is 5 feet less than 4 times its width, w. What is the width, in feet, of the wooden deck?

5. Mr. Rivera wants to build a barbed-wire fence containing 5 rows of barbed wire around the irregularly shaped area shown in the drawing below.





Barbed-wire fence

Mr. Rivera wants to purchase rolls of barbed wire that contains 1380 linear feet of wire per roll and purchase an extra 500 linear feet of wire for a gate for the fence. How many rolls of barbed wire does Mr. Rivera need to purchase?

Name ______________________________ Period _____

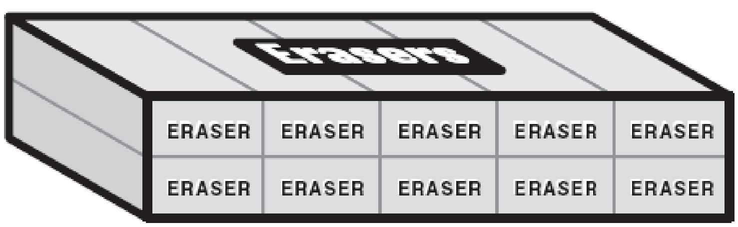
Pre-Assessment Part #2

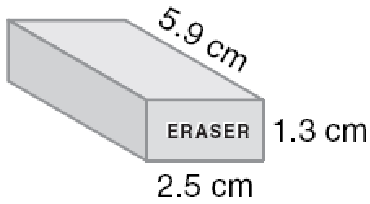
Solve each problem in the space provided. Please clearly circle your answer.

1. A recipe for 12 waffles calls for $1\frac{1}{2}$ cups of milk, $2\frac{1}{4}$ cups of flour, and $1\frac{1}{3}$ cups of other ingredients. How many cups of milk, flour, and other ingredients are needed to make 36 waffles?

2. Andy's average driving speed for a 4-hour trip was 45 miles per hour. During the first 3 hours he drove 40 miles per hour. What was his average speed for the last hour of the trip?

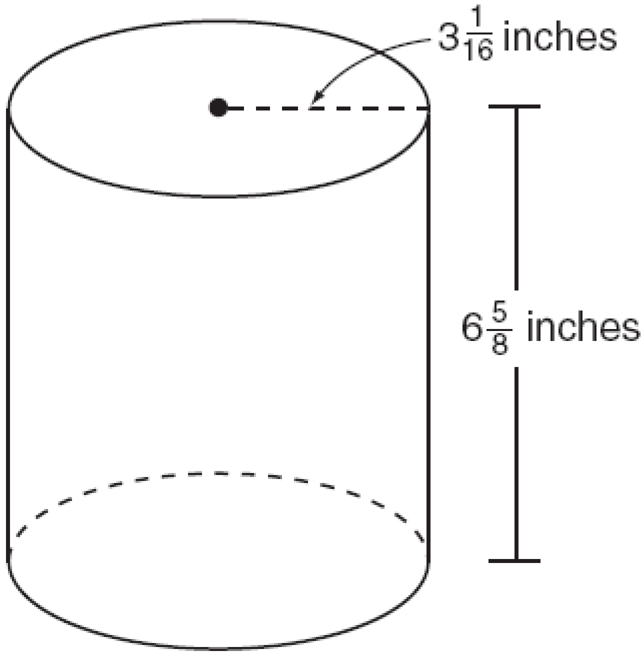
3. Mr. Lee bought a small rectangular box that contains 10 tightly packaged erasers shaped like rectangular prisms, as shown below.





What is the approximate volume in cubic centimeters of this rectangular box?

4. The student council members are making decorative labels to cover 32 identical empty coffee cans for a charity drive. Each label will cover the entire lateral surface area of a can.



What is the total lateral surface area the student council will be covering? (Use $\frac{22}{7}$ for π.)

5. A 120-foot-long rope is cut into 3 pieces. The first piece of rope is twice as long as the second piece of rope. The third piece of rope is three times as long as the second piece of rope. What is the length of the longest piece of rope?

Name ______________________________ Period _____

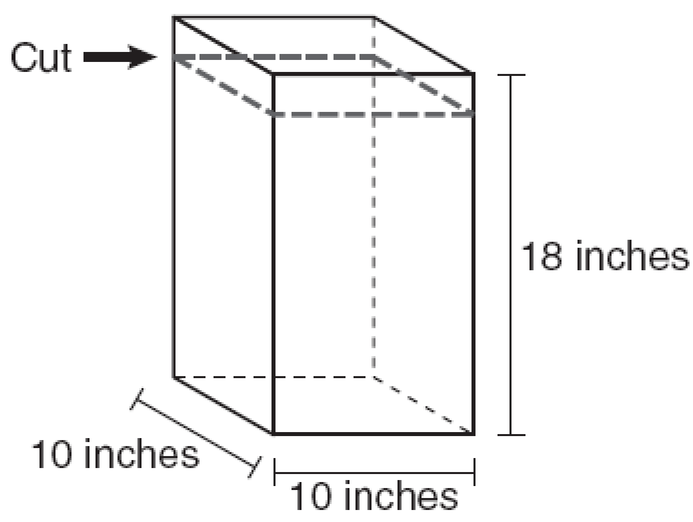
Pre-Assessment Part #3

Solve each problem in the space provided. Please clearly circle your answer.

1. The cost to rent a construction crane is $750 per day plus $250 per hour of use. What is the maximum number of hours the crane can be used each day if the rental cost is not to exceed $2500 per day?

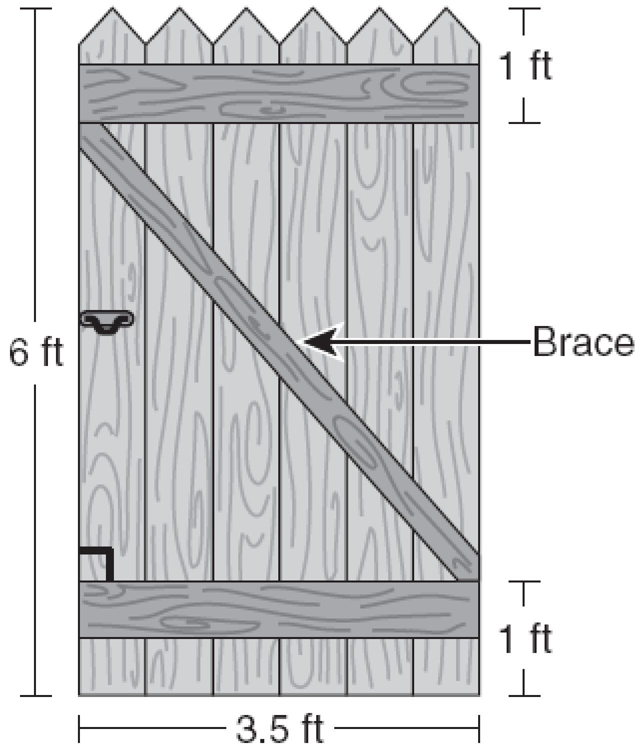
2. Ronald wants to buy a shirt that is on sale for 15% off the regular price. If he paid $51, what was the regular price of the shirt?

3. Deb has a rectangular storage box with a height of 18 inches, as shown below.



If Deb cuts off a 2-inch strip around the top of the box, what will be the new volume of the box in cubic inches?

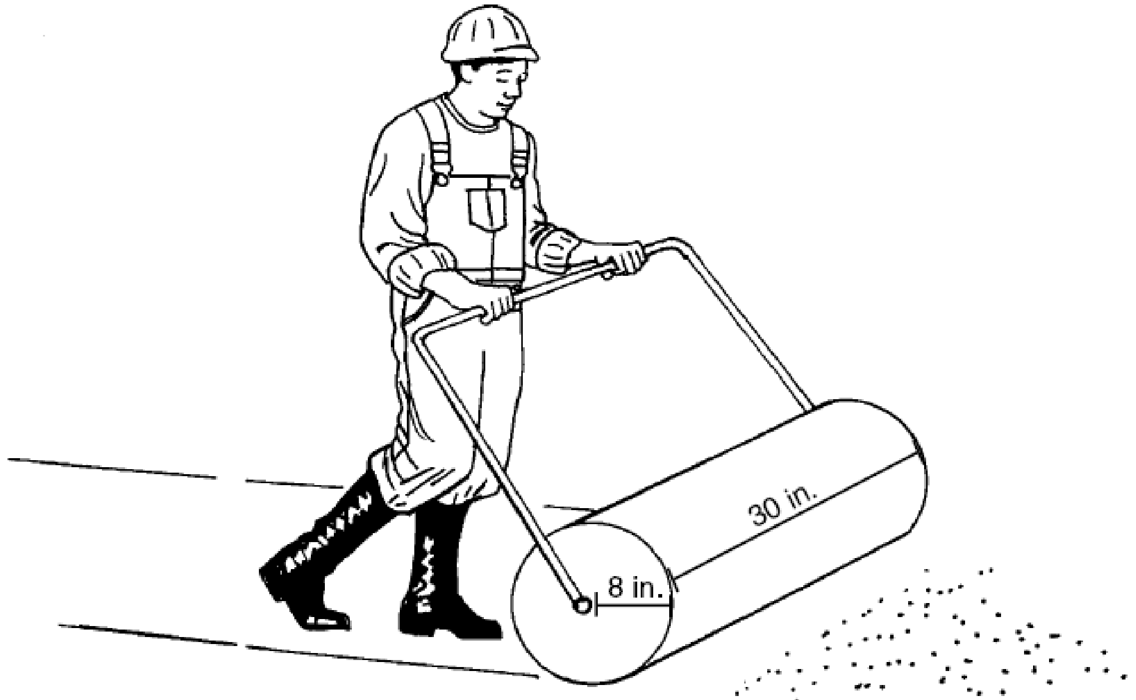
4. Mr. Carpenter built a wooden gate, as shown below.



What is the length rounded to the nearest whole foot of the diagonal board that Mr. Carpenter used to brace the wooden gate?

5. Marcy has a total of 20 dimes and quarters. If the total value of the coins is \$3.05, how many quarters does she have?

6. For small paving jobs, a contractor uses a roller pushed by a worker.



What is the area of pavement with which the surface of the roller will come into contact in two complete rotations? (Use 3.14 for π.)

Appendix B

Pre-Survey

Name ____ **Period ____**

Beliefs and Opinions About Mathematics Survey

Read the following statements carefully and respond Strongly Agree (SA), Agree (A), Disagree (D), or Strongly Disagree (SD) to each statement.

1. Math problems that take a long time don't bother me.	SA	A	D	SD
2. Studying problem-solving techniques is a waste of time.	SA	A	D	SD
3. A person who can't solve word problems really can't do math.	SA	A	D	SD
4. Memorizing steps is not that useful for learning to solve word problems.	SA	A	D	SD
5. If I can't do a math problem in a few minutes, I probably can't do it at all.	SA	A	D	SD
6. Math classes should not emphasize word problems.	SA	A	D	SD
7. Knowing problem-solving techniques will help me earn a living.	SA	A	D	SD
8. Most word problems can be solved by using the correct step-by-step procedures.	SA	A	D	SD
9. I feel I can do math problems that take a long time to complete.	SA	A	D	SD
10. Problem solving is of no relevance to my life.	SA	A	D	SD
11. Computational skills are useless if you can't apply them to real life situations.	SA	A	D	SD
12. There are word problems that just can't be solved by following a predetermined sequence of steps.	SA	A	D	SD
13. I'm not very good at solving math problems that take a while to figure out.	SA	A	D	SD
14. Learning computational skills is more important than learning to solve word problems.	SA	A	D	SD

15. I study problem-solving techniques because I know how useful it is.	SA	A	D	SD
16. Learning to do word problems is mostly a matter of memorizing the right steps to follow.	SA	A	D	SD
17. Word problems are not a very important part of mathematics.	SA	A	D	SD
18. I find I can do hard math problems if I just hang in there.	SA	A	D	SD
19. Problem solving is a worthwhile and necessary concept to study in mathematics.	SA	A	D	SD
20. Word problems can be solved without remembering formulas.	SA	A	D	SD
21. Problem solving will not be important to me in my life's work.	SA	A	D	SD
22. If I can't solve a math problem quickly, I quit trying.	SA	A	D	SD
23. Any word problem can be solved if you know the right steps to follow.	SA	A	D	SD
24. Computational skills are of little value if you can't use them to solve word problems.	SA	A	D	SD

Respond to the following open-ended question.

25. How do you currently approach word problems? What steps do you take?

Appendix C

Post-Survey

Name ____________________________________ **Period _____**

Beliefs and Opinions About Mathematics Survey

Read the following statements carefully and respond Strongly Agree (SA), Agree (A), Disagree (D), or Strongly Disagree (SD) to each statement.

1.	Math problems that take a long time don't bother me.	SA	A	D	SD
2.	Studying problem-solving techniques is a waste of time.	SA	A	D	SD
3.	A person who can't solve word problems really can't do math.	SA	A	D	SD
4.	Memorizing steps is not that useful for learning to solve word problems.	SA	A	D	SD
5.	If I can't do a math problem in a few minutes, I probably can't do it at all.	SA	A	D	SD
6.	Math classes should not emphasize word problems.	SA	A	D	SD
7.	Knowing problem-solving techniques will help me earn a living.	SA	A	D	SD
8.	Most word problems can be solved by using the correct step-by-step procedures.	SA	A	D	SD
9.	I feel I can do math problems that take a long time to complete.	SA	A	D	SD
10.	Problem solving is of no relevance to my life.	SA	A	D	SD
11.	Computational skills are useless if you can't apply them to real life situations.	SA	A	D	SD
12.	There are word problems that just can't be solved by following a predetermined sequence of steps.	SA	A	D	SD
13.	I'm not very good at solving math problems that take a while to figure out.	SA	A	D	SD
14.	Learning computational skills is more important than learning to solve word problems.	SA	A	D	SD

15. I study problem-solving techniques because I know how useful it is.	SA	A	D	SD
16. Learning to do word problems is mostly a matter of memorizing the right steps to follow.	SA	A	D	SD
17. Word problems are not a very important part of mathematics.	SA	A	D	SD
18. I find I can do hard math problems if I just hang in there.	SA	A	D	SD
19. Problem solving is a worthwhile and necessary concept to study in mathematics.	SA	A	D	SD
20. Word problems can be solved without remembering formulas.	SA	A	D	SD
21. Problem solving will not be important to me in my life's work.	SA	A	D	SD
22. If I can't solve a math problem quickly, I quit trying.	SA	A	D	SD
23. Any word problem can be solved if you know the right steps to follow.	SA	A	D	SD
24. Computational skills are of little value if you can't use them to solve word problems.	SA	A	D	SD

Respond to the following open-ended question.

25. Do you find the techniques to approaching word problems that we have studied to be helpful?

26. Why or why not?

27. Do you think you will continue to approach word problems using the steps we have used?

28. Why or why not?

29. In what other classes might you use this problem-solving model?

Appendix D

Parent Consent Letter

Approved by the Human Subjects Committee Lawrence Campus, University of Kansas. Approval expires one year from 9/18/2007.

The Influence of Problem-solving Strategies on Students' Mathematical Achievement and Beliefs

Dear Parent/Guardian:

The Department of Curriculum and Teaching at the University of Kansas supports the practice of protection for human subjects participating in research. The following is a description of a study that will be taking place in your student's Algebra I class. If you wish, you may elect not to allow your student's responses to be used in the analysis conducted for this study. Also, if you do sign the form and later change your mind, you may withdraw your consent at any time. Choosing not to participate in this study will in no way hurt your student's grade in the class or place your student at any kind of disadvantage.

PURPOSE OF THE STUDY

The purpose of this study is to determine if the four-step problem-solving model designed by George Pólya will help students be more successful on word problems. Previous research shows that teaching problem-solving strategies can enhance the performance of students when approaching word problems. This study will help show if this is true for this particular problem-solving model and in this specific school setting.

PROCEDURES

This study will take place during the first semester. Students will be asked to complete a pre-test at the beginning of the study that consists of 2 types of word problems at various difficulty levels. After the pre-test, the students will be exposed to the four-step problem-solving model designed by George Pólya. This model outlines steps for the students to follow when approaching a complex application word problem. Throughout the semester, opportunities for the students to practice applying the model will be incorporated into the curriculum through application word problems focused on the current concept being studied. Students will be asked to use these steps to explain how to solve the problems, as well as calculating the answer. At the conclusion of the study, students will be asked to take a post-test that is similar in length, types of word problems, and difficulty level to the pre-test. The scores will be compared to determine the impact of the problem-solving model on students' abilities to solve word problems.
This study involves no risk for the student participants.

BENEFITS

If the data collected in this study indicate that this problem-solving model helped students to be more successful when approaching word problems, this model may be adapted by all the Algebra I classes and other classes at a similar difficulty level. This study will provide information that is specific to our district about a strategy that we believe will improve students' abilities to solve application word problems similar to those seen on the California Standards Tests and the California High School Exit Exam.

PARTICIPANT CONFIDENTIALITY

Your student's name will not be associated in any way with the information collected about your student's ability or with the research findings from this study. If specific students are referred to when reporting results, they will be assigned a number or a pseudonym. By signing this form you give permission for the data collected in this study to be used at any time in the future.

If you have any questions about your student's rights as a research participant you may contact the Human Subjects Committee Lawrence Campus (HSCL) office at (785)864-7429 or (785)864-7385 or write the Human Subjects Committee Lawrence Campus (HSCL), University of Kansas, 2385 Irving Hill Road, Lawrence, Kansas 66045-7563, email dhann@ku.edu or mdenning@ku.edu.

If you have any questions about the procedures of this study, please feel free to contact me directly at (310) 395-3204 ext. 207 or email me at marae.cruce@smmusd.org.

Thank you,

Marae Cruce

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IF YOU AGREE TO ALLOW YOUR STUDENT'SRESPONSES TO BE USED IN THE ANALYSIS CONDUCTED FOR THIS STUDY, PLEASE SIGN WHERE INDICATED, TEAR OFF THE ATTACHED SHEET AND SEND IT TO CLASS WITH YOUR CHILD. KEEP THE CONSENT INFORMATION FOR YOUR RECORDS.

The Influence of Problem-solving Strategies on Students' Mathematical Achievement and Beliefs

HSCL #16861

PARTICIPANT CERTIFICATION:

I have read this Consent and Authorization form. I have had the opportunity to ask, and I have received answers to, any questions I had regarding the study. I understand that if I have any additional questions about my student's rights as a research participant, I may call (785) 864-7429 or write the Human Subjects Committee Lawrence Campus (HSCL), University of Kansas, 2385 Irving Hill Road, Lawrence, Kansas 66045-7563, email dhann@ku.edu.
I agree to allow my student's responses to be used in the analysis conducted for this study. By my signature I affirm that I have received a copy of this Consent and Authorization form.

______________________________ ____________________
Type/Print Student's Name Date

____
Parent/Guardian Signature

Appendix E

Flow Chart of Four-Step Problem-Solving Model

HOW TO SOLVE IT

UNDERSTAND THE PROBLEM

What is the unknown?

Read the quesiton through once.
Identify what you are trying to find.
Rewrite the question in your own words.

What is the condition? What are the data?

Highlight and list everything you know that is important to the situation.
Draw a picture or diagram of the situation including numbers and labels (inches, feet, meters, etc.).

DEVISE A PLAN

Do you know a related problem?

Reread the problem looking key words and concepts.

Do you know a concept that could be useful?

List all prior knowledge about the situation (formulas, rules, key things to remember, etc.).
Create a plan to solve the problem.
List out the steps you plan to take.

CARRY OUT THE PLAN

Did you complete each step of your plan?

Carry out your plan step-by-step.

Can you see clearly that the step is correct?

Verify each step and the order the steps were carried out.
If it is multiple choice, eliminate any answers you know are not correct.
Prove why eliminated answers do not work.
Identify your answer.

LOOKING BACK

Does your result answer the quesiton? Does it make sense?

Check your answer for reasonableness.

Can you check the result?

Rework the problem by substituting in your answer and working backwards.

Can you derive the result differently?

Check your work using a different method or strategy to find any errors in your calculations.

Four step problem-solving process developed by George Pólya.

Appendix F

Warm Up Word Problems

Friday, October 5th

A video game that regularly costs $29.95 is on sale for 15% off. What is the sale price of the video game?

Monday, October 8th

Your distance from lightning varies directly with the time it takes you to hear thunder. If you hear thunder 10 seconds after you see lightning, you are about 2 miles from the lightning. How far away is the lightning if you heard the thunder 27 seconds after seeing the lightning?

Tuesday, October 9th

A studio charges a $52 reservation fee and $26 per hour. Felipe paid a total of $130 to use the sound studio. How many hours did Felipe use the sound studio?

Wednesday, October 10th

To make a beaded necklace Jaime bought a bag containing 24 silver beads and 3 bags of colored beads. Each bag of colored beads contained the same number of beads. Jaime bought a total of 78 beads to make the necklace. How many beads were in each bag of colored beads?

Thursday, October 11th

Cathy ran for 30 minutes at a rate of 5.5 miles per hour. Then she ran for 15 minutes at a rate of 6 miles per hour. How many miles did she run in all?

Monday, October 15th

Ryan earns $16 for working 2 hours at his job. At this rate, how long will he have to work to earn $120?

Tuesday, October 16th

Suppose electricity costs 12 cents per kilowatts-hour. How much will it cost to use ten 75-watt light bulbs for 8 hours? (Hint: 1 kilowatt = 1000 watts)

Wednesday, October 17th

Beneath Earth's surface, the temperature increases 10˚C every kilometer. Suppose that the surface temperature is 22˚C, and temperature at the bottom of a coal mine is 45˚C. What is the depth of the coal mine?

Thursday, October 18th

Last season, Everett scored 48 points. This is 6 less than twice the number of points Max scored. How many points did Max score?

Friday, October 19th

A can is 4½ inches high and has a radius of 1¼ inches. What area does the label cover on the can? (Helpful formulas: $V = \pi r^2 h$, $SA = 2\pi r^2 + 2\pi rh$, Lateral $SA = 2\pi rh$)

Monday, October 22nd

The perimeter of a pool table is 30 ft. The table is twice as long as it is wide. What is the length of the pool table?

Tuesday, October 23rd

Lopez spent 1/3 of his vacation money on travel and 2/5 of his vacation money on lodging. He spent $1100 for travel and lodging. What is the total amount of money he spent in his vacation?

Wednesday, October 24th

Denise's cell phone plan is $29.95 per month plus $0.10 per minute for each minute over 300 minutes of call time. Denise's cell phone bill is $99.95. For how many minutes was she billed?

Thursday, October 25th

A cable television company charges $24.95 a month for basic cable service and $6.95 a month for each additional premium channel. If Sami's monthly bill is $45.80, how many premium channels is he receiving?

Monday, October 29th

The attendance at a baseball game was 400 people. Student tickets cost $2 and adult tickets cost $3. Total ticket sales were $1050. How many tickets of each type were sold?

Tuesday, October 30th

Two bicyclists ride in opposite directions. The speed of the first bicyclist is 5 miles per hour faster than the second. After 2 hours they are 70 miles apart. How fast is each one going?

Wednesday, October 31st

A bus traveling at an average rate of 30 miles per hour left the city at 11:45 A.M. A car following the bus at 45 miles per hour left the city at noon. At what time did the car catch up with the bus?

Thursday, November 1st

A triangle has a perimeter of 165 cm. The first side is 65 cm less than twice the second side. The third side is 10 cm less than the second side. Find the length of the three sides.

Friday, November 2nd

Two airplanes depart from an airport traveling in opposite directions. The second plane is 200 miles per hour faster than the first. After 2 hours they are 1100 miles apart. How fast is each plane traveling?

Monday, November 5th

In 2004, Lance Armstrong won the Tour de France, completing the 3391-km course in about 83.6 hours. Cycling at his average speed, about how long would it take Lance to cycle 185 km?

Tuesday, November 6th

A fire truck parks besides a building such that the base of the ladder is 16 ft from the building. The fire truck extends its 30-ft ladder and leans it against the building to reach a window above. The ladder starts 10 feet from the ground. How high is the window?

Wednesday, November 7th

A carpenter braces an 8-ft by 10-ft wall by nailing a board diagonally across the wall. How long is the bracing board?

Thursday, November 8th

At Movies 'R Us, it costs $3.00 for each new release you rent and $1.50 for each old favorite. You rented a total of 5 movies for $10.50. How many new releases did you rent? How many old favorites did you rent?

Tuesday, November 13th

A room has a perimeter of 52 feet. The length is 4 feet less than the width. What is the length and width of the room?

Wednesday, November 14th

At Fry Factory, two hamburgers and an order of fries cost $4.55. Four hamburgers and three orders of fries cost $10.15. Assuming the prices are constant, how much do they charge for each hamburger and each order of fries?

Thursday, November 15th

A 16-ft ladder is placed 4 ft from the base of a building. How high on the building will the ladder reach?

Friday, November 16th

A group of movie-goers paid $23.00 to see a matinee movie. Adult tickets cost $5.50 each and children's tickets are $3.25 each. The number of children's tickets are 2/3 the number of adult tickets. How many are adult tickets?

Monday, November 19th

At Party Warehouse, 3 rolls of crepe paper and 20 balloons cost $11.40. One roll of crepe paper and 10 balloons cost $4.20. Assuming the prices are constant, how much do they charge for each balloon?

Tuesday, November 20th

There are 8 bills in the bag consisting of twenty-dollar bills and five-dollar bills. The total value of the bills in the bag is $100. How many twenty-dollar bills are there in the bag?

Wednesday, November 21st

An artist is making a pair of earrings, a necklace, and a bracelet out of silver wire. The earrings take 2 inches of wire. The necklace is as long as the bracelet and twice the earrings. The bracelet is as long as 1/3 of the necklace and four times the length of the earrings. How much wire does the artist need to make all three pieces (the earrings, the necklace, and the bracelet)?

Monday, November 26th

There are 30 coins in a bag consisting of nickels, quarters and dimes. The total of the coins is $4.30. The number of dimes is 2 less than twice the number of nickels. How many quarters are in the bag?

Tuesday, November 27th

A cardboard box is 72 inches long, 18 inches wide and 3 feet high. What is the volume of the box in cubic feet?

Wednesday, November 28th

John has $30 to spend on video-game rentals at a local video store. The store charges $4.95 per video-game rental plus 9% tax. What is the maximum number of video games that John can rent?

Thursday, November 29th

Mr. Smith wants to build a fence containing 6 rows of barbed wire around a rectangular field that is 375 feet long and 340 feet wide. Mr. Smith wants to purchase rolls of barbed wire that contain 1400 linear feet of wire per roll. He needs an extra 500 linear feet of wire for the gate in the fence. How many rolls of barbed wire should Mr. Smith purchase?

Monday, December 3rd

A recipe for 12 cookies calls for $1\frac{1}{2}$ cups of milk, $2\frac{1}{4}$ cups of flour, and $1\frac{1}{3}$ cups of other ingredients. How many total cups of all the ingredients are needed to make 36 cookies?

Tuesday, December 4th

Levi's average driving speed for a 6-hour trip was 45 miles per hour. During the first 4 hours, he drove 40 miles per hour. What was his average speed for the last two hours of the trip?

Wednesday, December 5th

The MacNeills rented a moving truck for $49.95 plus $0.30 per mile. Before returning the truck, they filled the tank with gasoline, which cost $18.32. The total cost was $95.87. Find the number of miles the truck was driven.

Thursday, December 6th

A mountain bike tire has a diameter of 29 inches. A stop sign is 100 feet away. How many complete rotations will the tire make before you reach the stop sign?

Friday, December 7th

Suppose you earn $74.25 for working 9 hours. How much will you earn for working 15 hours?

Monday, December 10th

The sophomore class held a car wash to raise money. A local merchant donated all of the supplies. A wash cost $5 per car and $6.50 per van/truck. The number of cars was 3 less than twice as many vans/trucks. The students raised $614. How many cars did they wash?

Tuesday, December 11th

There are $3\frac{3}{4}$ cups of flour, $1\frac{1}{2}$ cups of sugar, $\frac{2}{3}$ cup of brown sugar, and $\frac{1}{4}$ cup of oil in a cake mix. How many total cups of ingredients are there in all?

Wednesday, December 12th

Inez buys a pair of boots on sale for $32.20. The sale price is 20% off the regular price. What is the regular price of the boots?

Thursday, December 13th

Lou is ordering books online for $4.99 each, plus $2.95 per order for shipping. He has a coupon for 25% off the cost of the books, but the coupon does not apply to shipping. If he orders 6 books, how much will he pay?

Friday, December 14th

Felipe is selling lemonade for $0.25 per cup. He bought the lemonade mix for $8.40 and the cups for $0.05 each. How much profit will he make if he sells 54 cups of lemonade?

Monday, December 17th

A Ferris wheel has a radius of 45 feet. If one ride consists of 3 full rotations, how far do you travel?

Appendix G

Scoring Rubric for Academic Achievement Test

Scoring Rubric for Academic Achievement Test

The Academic Achievement Test consists of sixteen free-response questions organized into three small tests. All the questions fit into two categories, prior knowledge or algebraic questions. The questions are to be scored on a four point rubric described below.

Score	***Criteria***
0 points	• No answer, no work • Incorrect answer, no work
1 point	• Correctly identified key information • No set up or plan • Incorrect set up
2 points	• Correctly identified key information • Correct set up, verbally • Incorrect or no mathematical set up • Incomplete procedures – missing several steps
3 points	• Correctly identified key information • Correct set up, verbally and mathematically • Incorrect calculations based on a computational error • Incorrect answer because missing a step (ex: not doubled)
4 points (full credit)	• Correctly identified key information • Correct set up, verbally and mathematically • Correct calculations • Correct answer with work

Appendix H

Raw Data for Academic Achievement Tests

Each row contains the data for one student.

	Pre-Test Prior Knowledge	Post-Test Prior Knowledge	Pre-Test Algebra	Post-Test Algebra	Total Pre-Test	Total Post-Test
	11	13	15	21	26	34
	13	22	6	11	19	33
	17	21	11	11	28	32
	15	22	13	18	28	40
	10	20	14	22	24	42
	9	18	15	23	24	41
	14	17	19	18	33	35
	17	15	5	5	22	20
	6	22	19	25	25	47
	7	18	11	19	18	37
	7	13	16	19	23	32
	17	30	24	26	41	56
	16	21	14	16	30	37
	12	12	15	12	27	24
	7	3	21	15	28	18
	7	25	16	25	23	50
	12	16	14	16	26	32
	8	16	14	16	22	32
	8	10	17	20	25	30
	7	29	24	24	31	53
	8	20	13	21	21	41
	14	14	12	11	26	25
	12	18	15	20	27	38
	22	21	15	25	37	46
	9	8	13	14	22	22
	10	6	15	16	25	22
	8	10	10	6	18	16
	10	13	13	9	23	22
	8	25	13	17	21	42
	13	28	18	27	31	55
	7	5	13	7	20	12
	8	12	15	15	23	27
	9	12	7	7	16	19
	11	23	26	30	37	53
	5	5	9	9	14	14
	10	17	18	24	28	41
	9	21	7	17	16	38
	10	26	17	24	27	50
	16	22	15	23	31	45
	8	7	11	7	19	14
	21	19	27	28	48	47
	11	10	7	11	18	21
Mean	10.9285714	16.7857143	14.5714286	17.3809524	25.5	34.16667
Standard Deviation	4.02669663	6.87354673	5.02259702	6.70621256	6.833383	12.36817

Appendix I

Raw Data for Surveys

Each row contains the data for one student.

	Pre Perseverance Belief	Post Perseverance Belief	Pre Complexity Belief	Post Complexity Belief	Pre Importance Belief	Post Importance Belief	Pre Usefulness Belief	Post Usefulness Belief
	16	20	13	10	14	16	19	20
	16	18	14	15	14	14	17	12
	16	17	13	12	13	14	18	15
	19	17	12	13	16	14	19	17
	14	18	13	13	20	17	16	19
	15	19	7	14	17	13	19	16
	14	16	10	13	12	16	18	19
	15	16	15	15	12	10	13	9
	17	16	13	14	13	11	12	13
	17	14	13	16	16	12	17	15
	19	19	10	9	15	16	17	21
	24	20	14	14	19	21	19	18
	20	19	9	11	15	12	20	20
	18	17	13	15	15	15	18	18
	16	17	14	11	13	14	17	18
	14	14	12	12	13	14	13	14
	15	20	10	12	13	15	16	18
	17	18	13	9	14	14	18	21
	22	18	11	7	18	16	23	23
	15	19	12	12	16	16	16	18
	14	19	13	12	12	13	14	17
	17	16	15	12	15	14	17	19
	15	16	15	15	16	15	16	16
	18	18	11	13	14	16	19	21
	14	15	12	13	14	15	9	10
	15	14	13	15	14	16	15	18
	18	15	14	14	16	16	19	18
	17	18	12	14	11	13	10	10
	19	17	13	13	16	14	17	18
	16	20	13	12	13	12	13	20
	11	7	14	13	13	13	15	12
	19	19	10	12	14	15	22	21
	15	11	15	13	12	12	16	19
	22	24	8	9	14	16	20	13
	17	15	15	15	15	14	21	19
	20	20	11	13	16	16	16	17
	13	14	11	12	15	15	17	17
	19	22	14	10	16	14	18	14
	15	12	12	12	15	15	12	19
	18	18	11	12	14	15	18	18
	16	19	10	12	13	15	17	17
	13	15	12	12	12	14	15	16
Mean	16.66666667	17.04761905	12.2619048	12.5	14.4761905	14.47619048	16.6904762	16.97619048
Standard Deviation	2.647287306	3.011784828	1.92619331	1.915915371	1.92845313	1.864142976	2.95897226	3.249676121