WHERE HAS THE TIME GONE? THE ROLE OF TIME LAGS IN MODELS FOR LONGITUDINAL DATA

BY

James P. Selig

Submitted to the graduate degree program in Psychology and the Graduate Faculty of the University of Kansas in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

__Todd D. Little____________
Co-Chairperson

__Kristopher J. Preacher______
Co-Chairperson

__Pascal Deboeck____________

__Wei Wu__________________

__Neal Kingston____________

Date Defended: ___June 17, 2009______

The Dissertation Committee for James P. Selig certifies
that this is the approved version of the following dissertation:

WHERE HAS THE TIME GONE? THE ROLE OF TIME LAGS IN MODELS FOR LONGITUDINAL DATA

Committee:

Todd D. Little
Co-Chairperson

Kristopher J. Preacher
Co-Chairperson

Pascal Deboeck

Wei Wu

Neal Kingston

Date approved: June 18, 2009

Abstract

The purpose of this dissertation is to draw attention to a long neglected, yet very important issue in the statistical modeling of longitudinal data. The issue can arise in any analysis in which one variable, measured at a particular time, is modeled as a predictor or cause of another variable, measured at some later time. The problem is that the magnitude of the variable's effect can vary with the amount of time that elapses between the measurements, or the lag. The dissertation is divided into the following sections: 1) a brief discussion of the issue of causality in models for longitudinal data; 2) an examination of the fundamental role of time lag in any model for longitudinal data in which variables are depicted as predictors or causes; 3) a review of the existing solutions regarding the choice of lags for longitudinal models; 4) the introduction of an alternative strategy to addressing the lag issue: the lag as moderator (LAM) approach; and finally, 5) a demonstration of the potential of the LAM approach by applying it to the analysis of simulated and empirical data.

List of Tables and Figures

Where has the time gone? The role of time lags in models for longitudinal data

The purpose of this dissertation is to draw attention to a long neglected, yet very important issue in the statistical modeling of longitudinal data. The issue can arise in any analysis in which one variable, measured at a particular time, is modeled as a predictor or cause of another variable, measured at some later time. The problem is that the magnitude of the variable's effect can vary with the amount of time that elapses between the measurements, or the lag. The dissertation is divided into the following sections: 1) a brief discussion of the issue of causality in models for longitudinal data; 2) an examination of the fundamental role of time lag in any model for longitudinal data in which variables are depicted as predictors or causes; 3) a review of the existing solutions regarding the choice of lags for longitudinal models; 4) the introduction of an alternative strategy to addressing the lag issue: the lag as moderator (LAM) approach; and finally, 5) a demonstration of the potential of the LAM approach by applying it to the analysis of simulated and empirical data.

Description and Explanation

Scientists seek to describe and explain the world around them and in particular, the social scientist aims to describe and explain the behavior of people. (cf. Wold, 1954, 1956). Description relies upon careful observation, measurement, and the summarizing of what has been observed, while explanation requires a statement of the putative causes of behavior and the systematic evaluation of those causes. Prediction, while not usually requiring a formal statement of causation, is more similar to the goal of explanation in that it requires the structuring of an association in terms of time (earlier *X* predicts later *Y*) and role (*X* is a predictor and *Y* is a criterion). Given that the explanation of behavior hinges on finding the possible causes for that behavior, it follows that one of the primary pursuits of the social scientist, and any other scientist, is to discover causal or predictive relationships.

The Possibility of Causal Inference

There is considerable debate regarding what evidence is necessary to establish that a causal relationship exists. Philosophers, statisticians, and methodologists provide at times divergent guidelines regarding what criteria must be met in order to establish the existence of a causal relationship. Due to this longstanding controversy, the task of proving the existence of causal relationships seems impossible.

The problem with establishing the existence of a causal relationship has been described in many ways, but each hinges on the impossibility of ruling out other potential causes. Bollen (1989) describes this as the issue of *isolation*. To definitively show that *X* causes *Y* would require that *Y* be isolated from the influence of all other possible causes.

Rubin and Holland (Rubin, 1974, Holland, 1986) describe the problem of causal inference in a different way—stating the fundamental problem of causal inference as the impossibility of observing an effect both when the cause occurs and does not occur. This problem can be traced back at least as far as the work of David Hume (1977 [1739]) and is often stated as the problem of the *unobservable counterfactual*.

The scientist faces a serious dilemma. The progress of science relies upon the finding and evaluation of causal relationships, but it seems that these relationships are impossible to establish. The social scientist may be especially troubled as the possible determinants of behavior are myriad and there is little hope for complete isolation from other plausible causes.

I propose a pragmatic solution--that support for a causal inference should be viewed as a continuum rather than a threshold. We can stipulate that there will always be uncertainty regarding the existence of a causal effect, but it is possible to establish relative degrees of support for such an effect. The question then becomes not what qualifies as evidence of a causal relationship in the sense that either a threshold is crossed or not, but instead the question becomes what factors lead to stronger evidence for such a relationship. The various criteria proposed for establishing the existence of causal relationships may serve to define locations on this continuum.

Three of the most common criteria for showing that some variable, X, has a causal effect on some other variable, Y, are: 1) that X and Y covary, are associated, or co-occur; 2) that there are no other plausible causes for Y other than X; and 3) that X occurs prior to Y. (see e.g., Kenny, 1979; Mill, 1843; Shadish, Cook, & Campbell, 2002). The third criterion, often referred to as the principle of *temporal precedence*, is central to the current emphasis on time lags. In spite of the controversy surrounding the issue of causal inference, this principle is widely accepted by philosophers, statisticians, and methodologists alike (Holland, 1986; Hume, 1962[1739]; Mill, 1843; Shadish, Cook, & Campbell, 2002). Even those skeptical of the search for causal relationships, who instead emphasize association and prediction, can still apply the principle of temporal precedence to build a stronger case for a predictive relationship.

The Necessity of Longitudinal Data

Based on the principle of temporal precedence, that a cause must precede its effect, it follows that the data we collect to test causal models should be longitudinal. Even if one chooses to avoid the issues of causality by focusing instead on predictive relationships, measuring the predictor prior to measuring the criterion is necessary to make the strongest case for prediction. While it is difficult to formulate a strong argument against the necessity for longitudinal data, there has been considerable debate in past decades regarding when the analysis of non-longitudinal data or cross-sectional data will yield the same result as the analysis of longitudinal data (see, for example, Heise, 1975 and James, Mulaik, & Brett, 1982). The consensus from the examination of such 'equilibrium' assumptions is that it is extremely unlikely in practice to find a situation in which the analysis of cross-sectional and longitudinal data would lead to the same conclusions (see e.g., Cole & Maxwell, 2003; Gollob & Reichardt, 1987; Maxwell & Cole, 2007). Further, based only on the assumption that causes do not exert instantaneous effects, it can be demonstrated that the use of cross-sectional data to model longitudinal relationships can result in substantially biased estimates of effects (Cole & Maxwell, 2003; Gollob & Reichardt, 1987; Maxwell & Cole, 2007).

The Importance of Lag

To this point I have made the case that it is important to most scientists to provide explanations for behavior. Explanations are inextricably tied to causal influences. The previously mentioned approach to building a case for a causal influence is used here to avoid the pitfalls of the issue of causality. It is also possible to re-frame the pursuits of the scientist in terms of finding relationships between variables or predicting future behavior based on relevant variables. Regardless of wording, if results from the analysis of data are to support the effect of one variable (X) on another (Y), longitudinal data are required. The fact that observations must be separated by some interval of time leads to the conclusion that time lags play a key role in much research.

The Trouble with Choosing Lags

It follows from the above arguments that in many if not most investigations, some time lag is required. It is unclear just how much time should elapse between the measurements of variables. The choice of lag can be viewed from two perspectives: 1) so long as the principle of temporal precedence is followed, the choice of lag is trivial; or 2) the choice of lags can impact the magnitude of the estimated causal effect and therefore must be made with care.

If one argues that any amount of time is sufficient, then it follows that any lag between measurements is as good as any other. This view is implicit in the choice of lags in applied research when no justification is provided for the choice of lag used in a study. It also seems to be implied by the many treatises on causal inference that state the necessity of having a lag but ignore the question of what the length of that lag should be. The view that any lag is as good as another, however, carries with it some heavy implications. For purposes of illustration, consider a simple regression model where *X* is assumed to be a cause of *Y*. *X* is measured at some time (t_1) and *Y* can be measured at any subsequent time (t_{1+k}). To say that any lag will serve equally well in testing this relationship, it is implied that the effect of *X* on *Y* will be of the same magnitude regardless of the interval of time that passes. This would imply that either 1) shortly after the *X* is measured it exerts its effect on *Y* and the value of *Y* is then unchanging, or 2) that *Y* continues to change after being measured, but there is no interindividual variability in *Y* so that the individual standing on *Y* is unchanged though the mean level of *Y* changes. Both of these scenarios seem very unlikely.

To my knowledge, there are no published claims that the choice of time lag is inconsequential. The early descriptions of path models and structural equation models show a keen awareness of the possibility that the choice of time lags could be of fundamental importance. The father of path analysis, Sewall Wright (1960, p. 423), acknowledged that often the effect of a variable on another is not static and the effect, "...in most cases rises gradually to a peak and then gradually falls off…" Similarly, another early paper on the use of path analysis with cross-lagged panel data notes that using a poorly chosen lag can obscure causal relationships (Shingles, 1976). Finally, in an early treatment of the use of path analysis to model sociological variables, Heise (1970) notes that any such model must assume that the lag chosen corresponds to the true lag necessary to observe the causal effect.

This leads to the second view of choosing lags for longitudinal data collection, the view that lags matter and that different lags may lead to the finding of different effects (Gollob & Reichardt, 1987; Pelz & Lew, 1970; Wright, 1960). From this perspective, the collection of longitudinal data without first considering how much time should elapse between measurements may lead to serious problems. The effect of X on Y when Y is measured one year after X is likely to be different from the same effect measured after only one month or one day. This leads to the problem that within the context of any single study, the failure to find an expected effect may mean that that effect does not exist, or it may mean that the effect does not exist at the chosen lag. Similarly, the successful finding of an effect should be interpreted as the existence of an effect at the studied lag.

There is not, then, a question of whether the use of different lags can yield different estimates for a causal effect. This is clear both from the earliest examinations of models for longitudinal data and from an examination of the conditions required to produce a situation where lag choice did not affect the magnitude of the causal effect. In the following section I will expand upon the issue that effects may vary with the lag chosen by examining the issue of the choice of lag from a selection perspective.

Lag Choice as a Selection Issue

The issue of choosing a lag for a study can be usefully examined as a selection issue. Selection issues are those arising from the fact that any study, regardless of how large or lengthy, selects only a limited set of data values from a much larger possible number of such values (Nesselroade & Jones, 1991). Figure 1 depicts a version of Cattell's data box (1966) with dimensions of persons, variables, and occasions. This data box can be thought of as consisting of small cubes. Each cube represents a single datum representing a point on each of the respective dimensions—a person measured at a single occasion on a single variable. A single study can sample only a limited range of values from all persons, all variables, and all occasions. It is always possible to question whether the results for a study are representative because only a limited set of possible values are sampled. So we may ask whether the results of a study would apply to all persons in a population, whether the results would have been the same if other variables had been used, and whether the same results would have been found if different occasions had been sampled. The occasions dimension is relevant in at least three ways. First there is the issue of using longitudinal data (i.e., whether sampling a single occasion is sufficient). Next, there is the issue of effects changing across the lifespan (e.g., whether a relationship that exists in early childhood would exist in the same way in adulthood). Finally, the issue of lag choice can also be construed as relevant to the occasions sampled.

Nesselroade (1991) briefly makes this connection in describing how the sampling of occasions is related to the selection of *intervals*, the amount of time that elapses between adjacent observations. In fact, for any study the choice of occasions is integrally tied to the choice of intervals (lags) so consideration of one entails the other. For a two occasion study, one must choose explicitly which two occasions will be sampled. By doing so, the lag between observations is also chosen. If choice of occasions sampled is important and inadequate selection of occasions is a problem that may lead to different results being found when one pair of occasions are sampled rather than another, it follows that selection of lag, or the time that elapses between those occasions, is equally important.

With the exception of this mention by Nesselroade (1991), lag choice has not been viewed as a selection issue that could lead to selection bias. In contrast, there seem to be stalwart advocates for a full consideration of each of the other selection dimensions represented by the data box. For example, single-subject research designs are commonly criticized because such designs do not adequately sample individuals. Advocates of multivariate measurement strategies critique studies utilizing only a single variable to represent a construct. Finally, champions of the importance of longitudinal data collection point out that cross-sectional data are in many ways inadequate. It is unusual then that although effects may vary across different lags or the occasions sampled, the overwhelming majority of longitudinal studies consist of data assessed at only one fixed lag. After all, this means that because only some (or only one!) lag units were sampled from the many possible, the results may be different than they would be if all or different units were sampled. At the very least, there should be more concern regarding the question of whether the inferences based on the traditional approach to collecting data across only one fixed interval are on any stronger footing than inferences based on collecting data from only one person, only one occasion, or using only one variable.

Exploring How Effects Vary with Lag

To this point, I have argued that effects can vary with the lag chosen, and from the selection perspective that the choice of lag may affect results in the guise of selection effects. In the following section I will review the handful of studies that examined this variation. These studies include: 1) principled arguments in favor of studying varying lags; 2) simulation studies showing the results of using different lags; and 3) statistical analysis of models for longitudinal data. To these three, I will add analyses of existing data demonstrating how effects change with lag.

Though, as was previously mentioned, the importance of choosing lags was well documented in the early path analysis literature, after such early descriptions the issue was largely ignored in the social sciences until it was revived by Gollob and Reichardt (1987) nearly three decades later. Gollob and Reichardt proposed a number of principles regarding the issue of time and research design. Key among these principles is the statement that it should be expected that causal effects will change as a function of lag. They present a simple example of the effect of taking aspirin on reduction of headache pain. Anyone who has taken aspirin to relieve headache pain understands that the effect of the aspirin is not instantaneous (i.e., there is some lag between the cause and effect) and the effect is not constant across time (i.e., for a long-lasting headache, the pain reduction may slowly reach a peak and then diminish until additional aspirin are needed). Building on this example, it seems that the idea that effects vary as a function of lag has much intuitive appeal and is consistent with our everyday experience of the world. For example, the old adage, “time heals all wounds" reflects the common belief that the effect of a traumatic event on one's well being will diminish as the lag between the event and assessment of well being increases.

Evidence from Simulation and Analysis

Pelz and Lew (1970) provide the earliest simulation study demonstrating the importance of lag choice. Pelz and Lew use a two-variable two-wave panel model. For the demonstration, Pelz and Lew assume a particular value for the *causal interval*, or the precise interval required for X to affect Y and for Y to affect X. Through analysis and simulation, Pelz and Lew show that the use of lags different from the causal interval can lead to incorrect conclusions regarding the direction of causation (i.e., whether X causes Y or Y causes X); the magnitude of the causal effect; and even the sign of the causal effect (i.e., whether X acts to increase or decrease Y). Pelz and Lew note that this issue can be especially problematic when there are bidirectional effects (i.e., X causes Y and Y causes X). Therefore, based only on the assumption that some causal lag exists, Pelz and Lew demonstrate that the use of varying lags can lead to very different conclusions.

A number of decades later, the importance of time lags was explored by Cohen (1991). Cohen designed a simulation study to assess the degree of bias that may arise when covariates that change over time are measured at times that vary from an ideal interval. Cohen calls the issue the "problem of the premature covariate". Figure 2 depicts the model used by Cohen to describe this problem. In Cohen's scenario there are two variables and two occasions of measurement, Y is measured at time 1 and time 2 (e.g., at the beginning and end of a two-wave study). X is measured only at time 2. The variable X is a retrospective measure in that, according to Cohen's hypothetical description, participants report whether an event has occurred or not occurred since t_1. The focal analysis is the prediction of Y_2 by X_2. Cohen proceeds with the assumption that Y is changing and Y measured just prior to Y_2 is the value necessary for the estimates from the model to be unbiased (i.e., Y is described by a first-order autoregressive process, AR(1)). Thus, Y_t ($1 < t < 2$) measured just prior to t_2 is the desired value of the covariate; however, that value is not observed, so Y_1 will be used in its place. To the degree that Y_1 is not a good proxy for Y_t there will be bias in the estimate of the relationship between X and Y, particularly if X_2 and Y_t are correlated (e.g., in a situation where Y does not depend upon X but X depends upon Y). Cohen points out, however, that it may not be sufficient to include Y_1 in the equation predicting Y_2 if the autoregressive relationship of Y changes as a function of the length of the lag between measurements, as it will for any AR(1) process. In this instance, what is really required to eliminate potential bias is Y measured at a more proximal occasion, say time t that occurs after time 1 and just prior to time 2. Through simulation, Cohen shows that using Y_1 as a proxy for the desired value of Y_t results in biased estimates of the effect of X on Y where the bias increases as Y_1 is measured at occasions more distal from Y_t. Thus, Cohen showed that the choice of lag is important in that one's ability to detect a spurious relationship can be hindered by the choice of lag.

Though Cohen's study focuses on the how the (mis-)timing of a covariate may serve to bias estimates of other causal effects in a model, her results generalize to any causal relationship estimated using longitudinal data. To see this we can use a simple two variable example where X is measured at time 1 and Y measured at some time afterwards. We assume that Y can change over time thus the causal effect of X on Y may also change over time. Based only on the assumption that the effect of X on Y can change with the lag used, it follows that there will be some maximum effect of X on Y and for the moment we will assume that this maximum effect is the focus of our research. The maximum effect occurs at some time t_m when the value of Y is Y_m. If our goal is to estimate the effect of X on Y at t_m, and we measure Y at any other time, the quality of our results may be diminished to the degree that the measure Y is not a good proxy for the desired value of Y_m.

Collins and Graham (2002) reinforce the above point though the analysis of the three variable model depicted in Figure 3. In this model, X is a predictor of interest and Y^* is the outcome measured at precisely the time that X is said to have its effect on Y. The dashed line depicting Y^* means that Y^* could not be observed due to data collection constraints and Y', or Y measured at some time after Y^*, is used as a proxy. Collins and Graham use an analysis of the regression model where Y^* is expressed as a function of Y'. In this way, they show that the focus of the lag issue is on how well the desired variable (i.e., the value of Y measured at the time required to see the effect) is approximated by the observed Y. Collins and Graham thus show that the difference between the relationship of interest (i.e., $\beta_{Y^* \bullet X}$) and the observed relationship (i.e., $\beta_{Y'\bullet X}$) will vary as a function of the lag between Y^* and Y' because more proximal values of Y' are more highly correlated than more distal values, and the degree to which X is related to the residual variance in Y' not accounted for by Y^*.

Cole and Maxwell (2003) address the issue of poorly choosing lags in the context of mediation analysis, but the results apply equally well when direct, rather than indirect, relationships are examined. In many ways Cole and Maxwell are extending the results of Pelz and Lew (1970) to the case of a three variable panel model used to assess mediation hypotheses. Cole and Maxwell examined panel models in which three variables (X, M, and Y) are measured repeatedly at some fixed interval. The interval (I), similar to the causal interval previously mentioned, is the time required for one variable to have its effect on a subsequent variable. Cole and Maxwell focus on cross-lagged paths, but their analysis applies as well to autoregressive paths, so I will address both here. Figure 4 shows three possible scenarios for measurements of only X. In this model, we assume that X is stationary and therefore that the stability coefficients relating each X to the subsequent X are equal. In the first scenario X is measured repeatedly at intervals or lags equal to I. In the second scenario, the lag between X_1 and X_2 is lengthened to $2I$, and in the third scenario the lag is lengthened to $3I$. As the time of measurement of the second observation of X becomes more distal relative to the time of measurement of X_1, the stability coefficient, or the effect of X on itself, diminishes in a predictable way. This is also the pattern that would be expected from a series of repeated measures that showed a simplex correlation structure (Campbell & Kenny, 1999; Guttman &

Guttman, 1965). Therefore, when stationarity is assumed, choosing lags that exceed an a priori lag required to observe the effect of interest can lead to autoregressive effects that appear too small.

Figure 5 shows a similar panel model in which both *X* and *Y* are repeatedly assessed. The focus here is on the cross-lagged prediction of *Y* by *X* measured at a previous time. The top panel of Figure 5 shows that this relationship is equal to some value ($\beta_{y \bullet x}$) when the interval (*I*) is used. However, when a longer interval is chosen, the effect of *X* on *Y* depends also on the stability of both *X* and *Y* so that in some instances where *X* and *Y* are very stable, the effect of *X* on *Y* may appear larger when assessed using an interval longer than *I*. In other instances, when *X* and *Y* are only moderately stable, the effect of *X* on *Y* will likely be smaller when the chosen interval is longer than *I*.

Empirical Evidence for the Effect of Lag

It is also possible to find empirical evidence for the fact that effects may vary with the use of different lags. Such evidence may come from a systematic review, or meta-analysis, of existing studies examining a particular relationship in which it can be shown that effects vary as a function of the time that elapses between observations. Such evidence is presented in a very early example by Thorndike (1933). Thorndike showed that the test-retest correlation for IQ tests, essentially an autoregressive effect, from a number of studies showed a systematic decline in magnitude as the interval of time between the test and the re-test increased.

The examination of longitudinal data can also show that both autoregressive and cross-lagged effects will vary when the lag between two occasions is systematically varied. To illustrate I will use a data set from Kanfer and Ackerman (1989). These data were collected from 140 United States Air Force Trainees. The trainees were asked to complete a set of six air traffic control tasks in which the goal was to successfully land as many aircraft as possible in a ten minute session. The results from these six trials were used to define six repeated measures variables representing the number of successful landings for each respective trial. A seventh variable representing each trainee's intellectual ability was assessed some months prior to the collection of the other variables.

In order to examine how an autoregressive effect may change as a function of lag, I used the first score on the task from each trainee as the predictor in a series of five models where the criterion variable was one of the five different repeated measures of task success. This is similar to the earlier illustration based on the work of Cole and Maxwell (2003). The key difference is that real data are being used and no stationarity assumptions are used for this example. Figure 6 shows a plot of the standardized autoregressive coefficients from the five models. The figure shows that the standardized autoregressive effect starts at slightly less than 0.8 and declines as the number of lags increase to reach a final value of just under 0.5. The rate of decline diminishes as more lags are added. While it was not the goal of Kanfer and Ackerman (1989) to estimate the autoregressive effect of the trial variable, it is clear that if one were interested in such an effect for this set of repeatedly measured variables the amount of time that elapsed between subsequent measurements would affect one's results. In addition to time lag, there is also an issue of practice effects here. The diminished relationship between more distal trials can be accounted for by lag or the fact that there were intervening trials during which the trainees practiced the task.

Turing now to an example using the same data but focusing on an effect of one variable predicting another, I used the ability measure as a predictor of task performance in a set of six simple regressions where each of the six repeated measures of task performance was used as the criterion. Kanfer and Ackerman state only that the ability measure was assessed several months prior to the collection of the task performance, so the specific lag between the ability measure and the first trial is unclear, but assuming approximately equal intervals between the six trials allows us to interpret the respective lags from each of the regressions. Figure 7 shows the standardized estimates of the causal effect of ability on each of the repeated task performance variables. The figure shows that the standardized effect of ability on task performance starts at about 0.5 and declines in a very similar manner as the autoregressive effect to a value of just under 0.3. Once again, we can see that researchers using two-wave measurement strategies who chose different lags could come to quite different conclusions.

Summary of Evidence of the Importance of Lag

To this point I have reviewed several investigations showing that different lags may yield different estimates of effects. This was based on simple examples of causes and effects (e.g., Gollob & Reichardt's aspirin example) and folk wisdom (e.g., time heals all wounds). There is also evidence from simulation and analytical investigations showing that varying lags can produce different estimates of causal effects. For example, Cole and Maxwell (2003), Collins and Graham (2002), Cohen (1991), and Pelz and Lew (1970) took four different approaches to showing that choice of lag can have a serious impact on one's findings regarding causal relationships. Pelz and Lew (1970) used analysis and simulation to show that widely discrepant findings were possible when the same variables were examined at varying lags. Cohen (1991) approached the problem using a simulation study showing that bias occurs when one uses a covariate measured at any time other than the one closest to the measurement of the two focal variables. Collins and Graham (2002) approached the problem differently by using an algebraic analysis of regression models to show what factors are problematic when outcomes are measured at times that deviate from some optimal time. Cole and Maxwell (2003) used path models for panel data and tracing rules to illustrate that effects can differ substantially when measurements of variables are taken at varying times. Starting with only minimal assumptions (e.g., that variables can change over time) these studies show that choice of lag is very important. Finally, the illustrations using the empirical data from Thorndike (1933) and the re-analysis of the Kanfer and Ackerman (1989) data showed that both autoregressive and cross-lagged effects can vary when different lags are used.

Methods for Choosing Lags

While the various sources of evidence presented to this point provide a clear indication that the choice of lag can have an important impact on the results of any longitudinal study, it is not clear what steps should be taken to address the potential problem. Historically, it seems that the approaches taken to choosing lags can be divided into two categories. Individuals taking either of the following two approaches are often equally aware of the importance that the choice of lag can have on the results of a study. The first approach--the more popular--states that there is some *single best lag* to be used in assessing each causal effect. The single best lag has been called by a few different names. Wright (1960, p. 424) calls this the "interval of maximum effect". Shingles (1976, p. 102) simply calls it the "causal lag" and contrasts it with the "measurement lag" that may or may not coincide with the causal lag. Cohen (1991, p. 19), calls this lag the "effect period", and defines it as, "…the minimum period over which the predominant effect of a change in one variable on change in another takes place…" James, Mulaik, and Brett (1982) denote this optimal lag as the "true causal interval". In any formulation of this single best lag, it is clear that any departure from that optimal lag can lead to incorrect results. Thus, from the single best lag perspective, the task of the researcher planning a longitudinal study is to use all available resources to determine what the best lag for the study is.

The second approach also begins with the assumption that effects can vary with the lag chosen, but instead of focusing on finding the single best lag this approach places emphasis on studying effects across a variety of lags (Gollob &

Reichardt, 1987). The focus of this approach changes from finding a particular lag to use, to the study of an effect across a variety of lags. This strategy, at least in part, addresses the potential pitfall of poorly choosing a single lag and perhaps more importantly suggests a way to conduct future research that will improve the quality of findings and deepen our understanding of effects by seeing how such effects evolve over time. In many instances, the variable lag approach is superior to the single lag approach in that a selected variety of lags used is more likely to reveal information about an effect than any single chosen lag, but it is not a panacea in that only a range of lag values are used and it is still possible to miss important information that could be gotten by using other lags. It is clear that advocates of the variable lag approach would proceed in conducting research in a different manner than advocates of the single best lag approach. As a means of further exploration of this difference, I will next examine the appeal of the single best lag view and some recommendations for finding a single lag for a prospective study.

Finding the Single Best Lag

The approach that focuses on finding the single best lag for a study is the most common view. This can be shown in part by the fact that each of the analytical and simulation studies that examined lag can be classified as subscribing to the first strategy of searching for this lag. Pelz and Lew (1970), Cole and Maxwell (2003), Collins and Graham (2002), explicitly and Cohen (1991) implicitly begin with the premise that there is some optimal lag that will reveal the effect of interest and proceed by showing what will happen when we measure at times that differ from the optimal lag. Pelz and Lew do this by specifying the causal lag and then systematically deviating from that lag. Cohen does this by simulating data in which there is a first order autoregressive effect of Y on subsequent values of Y (i.e., each Y is a function of only the previous Y and some random component). In this way the quality of Y as a covariate in the X to Y relationship is at its peak when Y is measured just one lag prior to the Y of interest. Collins and Graham (2002) state their assumption that there is some interval of time that is required for one variable to have its effect on the other. For their analysis, this interval is the amount of time that passes between the measurement of X and the measurement of Y^*. Cole and Maxwell (2003) are also very clear on this point when they establish the interval required for one variable to have its effect on another (I).

Appeal of the Single Best Lag

In one sense, starting from the assumption that there is a single best lag is a natural way to proceed. If we assume that many effects (any effects where one or both of the variables are changing and there is interindividual variability in change) will vary as a function of lag, there will be a lag at which the effect will reach its peak. We may assume that a common goal of researchers interested in causal effects is to use a lag that will show this maximum effect. Further, all models examined to this point are discrete time models, meaning that time is represented in the model only indirectly as some fixed interval between measurements. Such models require that one specify one and only one interval of time for each causal effect, thereby reinforcing the view that it is necessary to find only a single best lag. So from both the perspective of finding the peak effect and the perspective that most models for longitudinal data use only one lag, it makes sense to pursue a single best lag.

Finding the Single Best Lag

Assuming there is some optimal lag, it is important to review the recommendations for choosing this lag . Many authors (e.g., Collins & Graham, 2002) point out that often the choice of lags for a particular study is based on convenience (i.e., this is the easiest time to measure), necessity (i.e., this is the only time it is possible to measure), or tradition (this is when the last study measured). Rarely is the choice of lags justified with theoretical considerations such as, we believe the change we seek will best be observed on this interval. This may be the case either because traditionally there is no such expectation for such a justification, or because much theory in the social sciences has not reached that level of specificity. However, if we assume that the choice of lags is important for any longitudinal model, choosing lags based on convenience, necessity, or tradition is not sufficient. The two most common recommendations for choosing lags are: 1) to use theory and an understanding of the processes under study to choose lag in an informed manner (Cole & Maxwell, 2003; Collins & Graham, 2002; Gollob & Reichardt, 1987); and 2) to measure very frequently to insure that an effect of interest is not missed due to an interval that is too long (Collins & Graham, 1991, 2002). To the first two recommendations for identifying the best lag, we can add the possibility of using empirical results from many studies using different lags to help in the choice of lag for future studies. For example, a meta-analytical study that summarizes the results of many studies or a series of empirical studies using varying lags could examine whether varying lags across studies moderates the effect size reported for each study.

Limitations of Recommendations

Of the three possible approaches to choosing lag, each has serious limitations. For example, we rarely have theory regarding the amount of time that must elapse for a cause to have its effect. Even in the case where there is strong theory and the researcher has a deep understanding of the processes of interest, it seems unlikely that such information would lead to a specific prediction of the best lag to use in planning a study. Therefore, an appeal to strong theory will only rarely be a sufficient basis for choosing lags.

The second strategy of measuring very frequently is a useful approach in that, in principle, it could eliminate any problem of poorly choosing lags because it includes most of the possible lags between the beginning and the end of the proposed study. In terms of selection, nearly all lags in a particular range will have been sampled. However, there are several limitations of frequent measurement. First, frequent measurements can be very expensive in terms of time and money. In some instances frequent measurements may lead to issues such as reactivity or practice effects and in the extreme may even change the characteristics of the variables being repeatedly measured (Collins & Graham, 1991). So while it is clear that frequent measurement is very promising in some respects, it cannot be used in all situations.

Conducting multiple studies where lags vary can also be very costly. Conducting multiple studies will almost always require more resources than conducting a single study and the variety of lags that can be used in a handful of studies is limited. In reviewing previous work, it may be quite difficult to find a sufficiently large sample of studies examining the same variables across different lags to draw any sound conclusions. On a practical note, given the current feeling towards the merit of replication studies, it is unlikely that a replication of an earlier finding using the same variables but with a different lag would ever be published, and that may be reason enough to call such an endeavor impractical.

Issues with the Single Lag Approach

In addition to the fact that the possible strategies for finding the single best lag are limited, the idea itself that the goal of the researcher is to find a single best lag is also problematic. The single best lag view leads researchers to face an inevitable quandary regarding the results of any longitudinal study. The issue is that it is never clear whether the results are due to the nature of the true relationship between the cause and effect or due to the fact that the specific lag chosen in part determined the result. In this way, the single best lag perspective will inevitably lead to indeterminacy regarding the status of any causal relationships. One may argue that this indeterminacy is not specific in any way to the lag issue. For example, there are untestable assumptions associated with every statistical model such as whether the failure to find the expected result may be due to a low power, measurement error, a misspecified model or any of a variety of other factors. The point here is not that the lag issue holds some special status in terms of its potential to muddle results, but instead that it is on par with these other factors. Therefore, the choice of lag should be given the same level of consideration as the issues of power, measurement error, and the specification of one's model. A key limitation of the single best lag approach is that it may severely hinder one's ability to properly consider the role of lag.

Further, the single best lag perspective becomes unwieldy when we consider the fact that the best lag may be different for: 1) every pair of variables measured (Wright, 1960); and 2) for the same variables measured at different time periods within the same study. Figure 8 shows a panel model in which two variables, X and Y, each are measured on three occasions. If we acknowledge that the single best lag for a causal effect can differ for different pairs of variables (e.g., $X_1 \rightarrow Y_2$ vs. $Y_1 \rightarrow X_2$) and for the same variables measured at different periods during the study (e.g., $X_1 \rightarrow X_2$ vs. $X_2 \rightarrow X_3$), there are eights possible best lags for this panel model, one for each causal effect. To see what complexity in data collection is implied, imagine modeling the effects of only X_1. One lag may be required for X_1 to affect X_2 and a different lag may be required for X_1 to affect Y_2. Thus, the use of the best lag for each causal effect could render one's ability to properly collect data for even a simple two-wave panel model impossible in that the relationships between the time one variables (X_1 and Y_1) and the time two variables (X_2 and Y_2) that are traditionally addressed using only one lag may in fact require four different lags in order for each effect to be measured at its single best lag.

In addition to the practical issues that arise from choosing only the best lag for one's research, the focus on the perfect lag may serve to diminish the overall quality of our theorizing by steering investigators away from an empirical examination of how causal effects change over time. For example, Gollob and Reichardt (1987, p. 82) state that, “Because different time lags have different effects, one must study many different lags to understand causal effects fully.” Currently, most theories state only that one variable will have an effect on another. A consideration of lag shows that what is implied by such a statement is, at some unspecified time, one variable will change and after some unspecified amount of time passes the second variable will change as a result. Theories will improve dramatically when the lags required for such change are specified. There is, however, even more room for theories to expand and improve. Imagine the theory that not only states that an effect is expected and specifies the time interval that is required for the cause to have its effect, but also describes the way in which this effect of one variable on another will evolve over time. Granted, theory building takes time and initially evidence from a single lag study will contribute to the collective knowledge. The goal here is to point out a possible route to continuing to advance such theory.

In some cases in the social sciences, knowing how the effect of one variable on another variable changes over time could be as important as, or more important than, knowing whether that effect exists at a single point in time. Intervention studies offer the perfect example of a situation in which knowing how long the intervention takes to have an effect and how long lasting that effect is may be far more useful than showing the existence of the effect at one particular time.

The Varying Lags Perspective

The argument to this point is that the assessment of causal and predictive effects should utilize longitudinal data measured at various lags. This is the case because the use of many lags ameliorates the possibility that a poorly chosen lag may negatively impact the results of a study, and the study of an effect across a variety of lags makes it possible to deepen our theoretical understanding of effects. There are two alternatives available if one wishes to study an effect across multiple lags. The first is similar to one of the proposed strategies presented in the description of the single best lag view. It is possible either to conduct or review multiple studies using the same variables each with a different lag between the variables. As stated above there are several limitations that make this strategy impractical. The other possible approach is to allow lags to vary across some range of possible values within a single study. Then it is possible, within a single study, to see how the effect of interest varies as a function of lag. In this section I will consider the use of statistical models that explicitly incorporate varying lags where the length of the lag will serve to moderate the causal relationship between variables.

The concept I present here is straightforward. A problem exists such that effects (e.g., the effect of *X* on *Y*) will likely vary as a function of the lag between *X* and *Y*. This is a description of an interaction effect. By choosing touse multiple lags in assessing whether *X* has an effect on *Y* in a single study, it is possible to directly test for such an interaction. In a simple two-variable two-wave study, this would mean that lag would be a third variable that could be different for each individual in the study. This approach--using the lag as a moderator (LAM) of an effect--has two noteworthy advantages similar in a sense to the respective advantages of the frequent measurement and multiple fixed lag approaches. Similar to the frequent measurement approach, the LAM strategy uses many lags in a single study; thus effects can be studied at a variety of lags, and the chance of problems arising due to using only a single lag is greatly reduced. In contrast to the frequent measurement strategy, the use of the LAM approach is far less likely to result in problems with attrition, reactivity, or the expense of measuring all participants many times.

Similar to the multiple fixed-lag studies approach, the LAM approach provides useful information regarding how effects may change as a function of the interval between measurements, thereby making it less likely that the results are misleading due to poor selection of a single lag. However, the LAM approach requires only a single study using varying lags to be conducted, thereby significantly reducing the costs associated with collecting data; eliminating potential problems that may arise when comparing effects from studies that were conducted at different time periods using different people; and avoiding the practical problem of not being able to publish replication studies where lag is varied.

Previous Relevant Work

Both the idea of conducting studies where lags vary across individuals and the idea of using lag as a moderator have been mentioned before. McArdle and Woodcock (1997), for example, utilize a variable lag design in the context of a latent growth model to control for test-retest effects. In this way McArdle and Woodcock are able to model ‘true’ change in the test score controlling for the change that would be due to the fact that individuals have recently taken the same test. In contrast to the proposed LAM model, McArdle and Woodcock utilize the information arising from varying lags to control for a ‘main-effect’ of practice (operationalized as lag) while the present suggestion is to focus on an interaction and conditional effects (i.e., either the way lag impacts the causal effect of interest or the effect of X on Y conditional upon a lag value).

In a paper describing the use of Ecological Momentary assessment in organizational research, Beal and Weiss (2003) propose the idea of using lag length as a moderator of effects for data arising from an intensive longitudinal design where times of data collection are randomly spaced. However, Beal and Weiss do not apply such a model and I am unaware of any other application of such a model.

A Multiple Regression LAM Model

In order to explore the meaning and utility of construing lag as a moderator, we can begin with a multiple linear regression model. Of course such a model will not be useful for most longitudinal studies in which multiple causes and effects are measured, but it serves here as a means of introducing the LAM approach. In general, the LAM approach can be adapted for use in any statistical model that can accommodate a moderator variable. Again, a key difference between the proposed model and any traditional causal model for longitudinal data is that, in the traditional model, it is assumed that lags are fixed to some value and that value is the same for each individual in the sample. In contrast, treating lag as a moderator of the causal effect requires that values of lag will vary across individuals. When treating lag as a variable, the possible values for lag can range from very small values that indicate only a short amount of time passed after the initial assessment to a value that is equal to the total duration of the study. The following equation is a multiple regression example of a lag as moderator (LAM) model. The earlier time subscript is omitted here for simplicity of presentation; however, we will assume that all causes are measured prior to effects.

$$\hat{Y}_i = b_0 + b_1 X_i + b_2 Lag_i + b_3 X_i \times Lag_i \tag{1}$$

In this model, X_i is the focal predictor or predictor of interest, Y is the outcome of interest, and Lag_i is the amount of time that elapses between the measurements of X_i and Y_i for individual i. The regression coefficients can be interpreted as follows: b_0 is the expected value for Y when both X and Lag take a value of zero; b_1 is the expected change in Y for a one unit change in X when Lag takes a value of zero; b_2 is the expected change in Y for a one unit change in Lag when X is equal to zero; and b_3 is the expected change in the <u>relationship</u> between X and Y for a one unit change of *Lag*. If an analyst has theory supporting a particular lag as being important, the *Lag* variable can be centered so that the b_1 coefficient can be interpreted as the effect of X on Y at that particular lag. In a situation where *Lag* is intentionally allowed to vary, b_2 may not be expected to be statistically significant. Exceptions may exist either: 1) when lags are assigned by the researcher and there is similar change in Y across individuals (i.e., in the case of growth, longer lags would tend to be associated with higher values of Y), or 2) when lags vary but are not assigned by the researcher so lag may be associated with some unmeasured predictor (e.g., when participants choose the time of the second observation and lag may be a proxy for a variable such as procrastination). The coefficient b_3 describes how the coefficient b_1 (the effect of *X* on *Y*) is expected to change as a function of *Lag*. Use of a LAM model such as this one does not require the investigator to commit to a single value of lag, but instead a range of lag values. It is assumed that the effect of interest will be best observed during the interval between the first and last times of observation and that the causal effect will vary as a function of lag length.

Following Aiken and West (1991) we can rearrange the above equation into the following form to focus more explicitly on the changing effect of *X* as a function of lag.

$$\hat{Y}_i = (b_0 + b_2 Lag_i) + (b_1 + b_3 Lag_i) X_i \qquad (2)$$

Now the compound coefficient (b_0+b_2*Lag*) can be interpreted as the intercept for the simple regression of *Y* on *X*, and the compound coefficient (b_1+b_3*Lag*) represents the simple slope of the effect of *X* on *Y*.

This relatively simple model may be an important step beyond fixed lag models in that it explicitly states that the causal effect of interest is expected to vary across lags. However, there is no reason to believe that this linear interaction model will accurately depict the way in which an effect will change as a function of lag. To be fair, there is also no evidence against this linear model. There is very little evidence whatsoever regarding how effects will change with lag.

Functional Forms for the LAM

I begin the investigation of what functional forms are most appropriate to model the change in effects with lag by using an example based on the work of Cole and Maxwell (2003). As previously mentioned, Cole and Maxwell's analysis focused on a restricted panel model in which variables were repeatedly assessed at fixed intervals and the autoregressive and cross-lagged effects were constant across all lags of the same length. Since this example is founded on the single best lag perspective, we will view the results as a preliminary investigation of what functional forms may be useful for the varying lag perspective. Figure 8 shows a similar representation of such a model. In this figure *X* and *Y* are repeatedly measured n times with a constant lag between measurements. Following Cole and Maxwell we can describe this lag as *I*, the lag required for each cause to have its effect. Focusing only on the autoregressive effect for *X*, when the lag = *I*, the autoregressive effect is x, when the lag is doubled (2*I*) the autoregressive effect is x^2, and when it is tripled (3*I*) the effect is equal to x^3. Such a progression of effects could be described by the following quadratic LAM model that would allow the value of the causal effect to show curvature as lags increased.

$$\hat{Y}_i = b_0 + b_1 X_i + b_2 Lag_i + b_3 Lag_i^2 + b_4 X_i \times Lag_i + b_5 X_i \times Lag_i^2 \qquad (3)$$

In this model the coefficient b_4 would describe the linear component of the trajectory and the coefficient b_5 would describe the degree and direction of curvature for the line.

An exponential function could also provide an accurate description of the size of an autoregressive effect as lags increase. Such a function would appear as follows.

$$Ae^{-kt} \tag{4}$$

In this expression, *A* is equal to the value of the autoregressive coefficient across a single lag, *e* is a constant (approximately 2.718), *k* is a constant describing the degree of change in *A* with change in *t*, and *t* represents the number of lags beyond lag *I* that are used to estimate the effect. Such a functional form may be introduced into a nonlinear regression model by starting with a very simple model:

$$\hat{Y} = b_0 + b_1 X \tag{5}$$

And then defining b_1, the effect of *X* on *Y*, to be a function of lag, so that

$$b_1 = Ae^{-kt} \tag{6}$$

And by substitution the regression model for *Y* would be the following;

$$\hat{Y} = b_0 + (Ae^{-kt})X \tag{7}$$

Cross-lagged effects of the sort where one variable predicts a different variable measured at a subsequent time may follow a different pattern. Cole and Maxwell (2003) show that in a bivariate system such as the one represented previously in Figure 8, again assuming stationarity for the system so that both the two autoregressive effects and the cross-lagged effect are the same for all lags of the same length, the respective stability of the two variables will impact the change in the magnitude of the cross-lagged effect as more than one interval is chosen as the lag. The analysis of Cole and Maxwell shows that cross-lagged effects can either increase or decrease as lags vary depending upon the magnitude of the autoregressive effects.

Forms for Various Contexts

The cursory examinations of a restricted panel model indicate that non-linearity may be expected in examining the change in effects across varying lags, but there is no reason to believe that this will always be the case. For models where the range of lags is not great, the linear interaction model may still adequately capture the functional form of change. Given our current state of knowledge, the linear form cannot be ruled out. Variations of polynomial regression models (e.g., using Lag^2, Lag^3, or beyond) may be useful for capturing the curvilinearity of the changing effects as a function of lag. These models have the advantage that they can be estimated in any software package that uses ordinary least squares regression. Cudeck and DuToit (2002) presented a re-parameterized variation on the quadratic model that may be particularly useful. For example, one of the parameters estimated in their version of the quadratic model is the maximum value. In the context of a LAM model, this parameter would be very useful in that it would indicate the peak causal effect expected over the observed range of lag values. As noted previously, nonlinear functions such as the exponential function may be very useful for LAM models in that many of the patterns of changing effects arising from idealized models seem to be captured well by the exponential function. Other contexts may call for other functional forms.

For example, the effect of an intervention on a desirable outcome may follow an S-shaped functional form such that the effect of the intervention is minimal on very short lags and as lags increase the effect increases up to some maximum value at which it reaches an asymptote. Such a form could be depicted by either a logistic or Gompertz function. We can assume that the effect of some interventions may fade even within the window of observation of a particular study, therefore functional forms that increase to a maximum and then decrease could be useful. The ubiquitous normal distribution could be useful in these situations.

It is worth stating again that the question of the best functional form to describe how an effect changes over time is open. The goal here is not to determine exactly which functional forms will be most useful, but instead to suggest that the form chosen should be suited to the phenomena studied and that there exist a variety of candidate forms.

Further Examination of the LAM

Having introduced the theoretical impetus behind the use of varying lag models, the LAM approach to analyzing such data, and the possible functional forms that may be useful in different modeling contexts, I will further examine the LAM approach in two ways. The first is a demonstration of the use of LAM models using simulated data. One purpose of the demonstration is to highlight aspects of LAM modeling that differ from traditional fixed lag analyses such as choosing the functional form for the LAM interaction; the number of lags to sample; and the location of the lags sampled. Using simulated data with a known population model allowed for the emphasis of the importance of using an analysis model that matches or closely approximates the population model. The demonstration was designed to show both the advantages and limitations of the LAM approach.

The second means of examining the LAM approach was the fitting of various LAM models to existing empirical data where lags vary across individuals. To my knowledge, no large longitudinal study has implemented a variable-lag research design. Therefore, these analyses used data from a fixed-lag research design in which lags unintentionally varied across individuals. In some ways this presented a more rigorous challenge for the use of the LAM approach because the design was not optimal for detecting LAM effects. Due to the fact that the data collection was not designed to find lag effects, that there is little theory regarding the type of LAM effects to expect, and that no a priori predictions are being made regarding the types of effects to be found, the findings should be regarded as preliminary. However, significant LAM effects provide evidence that such interactions are readily found and should be further examined. These effects also present an opportunity to explore the way that relationships between variables change over time and the possible functional forms for these changes.

Demonstration using Simulated Data

Data were simulated for two variables: Y and X. Ten values for Y (Y_1-Y_{10}) were simulated to represent ten repeated measures of the same variable measured at ten consecutive equally-spaced waves of data collection. Only one value for X was simulated at the ninth wave (X_9). In all models, Y had a first-order autoregressive relationship in which each Y was affected only by the previous value of Y. The X_9 variable was a function only of the concurrent value of Y. Figure 10 shows a diagram of the Y and X variables used in all simulations. A *Lag* variable used in the simulations represented the number of lags between Y_{10} and the Y_k ($1 \leq k \leq 9$) variable used as a predictor. Here the subscript k denotes the location of the Y variable used to predict Y_{10}. For ease of interpretation, the *Lag* variable is centered at 1 (e.g., *Lag* of 0 means that Y_9 is used as a predictor). Values for the *Lag* variable were assigned based on a random variable from a uniform distribution. For example, in the first two-lag condition, each subject was randomly assigned, with equal probability, either a 0 or a 1 value for the centered *Lag* variable. The *Lag* variable then determined which Y_k (Y_9 or Y_8) value was used for that subject.

Population Models

Two models were used to simulate data. The models can be distinguished by the way the autoregressive relationship for Y changes as *Lag* changes. In the first model, hereafter referred to as the exponential data model, the autoregressive relationship for Y_k predicting Y_{k+1} was set to 0.8 and the effect of Y_9 on X_9 was set at 0.5. The following equations were used to simulate these data.

$$\begin{aligned} Y_k &= 0.8Y_{k-1} + e_{Yk} \\ X_9 &= 0.5Y_9 + e_{X9} \end{aligned} \tag{8}$$

The values for Y_1 were randomly sampled from a standard normal distribution. The variables e_{Yk} and e_{X9} are residual terms also drawn from normal distributions with variances set to keep the variances for both X and Y equal to 1. This model is titled the exponential data model because the relationship between Y_k and Y_{10} follows an exponential functional form as k changes. Figure 11 shows this exponential pattern of effects. The second model, titled the linear data model, constrained the relationship between Y_k and Y_{10} as lag changes to have a linear form. The relationship between Y_9 and Y_{10} was set at 0.81 and declined by 0.10 with each change in lag. Figure 12 shows this pattern of autoregressive effects with change in lag. The effect of Y_9 on X_9 in the linear data model remained 0.5. The following equations were used to simulate the linear population data.

$$\begin{aligned} Y_{10} &= 0.81Y_k - .10Y_k \times Lag_k + e_{Y10} \\ X_9 &= 0.5Y_9 + e_{X9} \end{aligned} \tag{9}$$

As with the previous model, values for Y_1 were randomly sampled from a standard normal distribution and values for the residual terms were sampled from normal distributions with variances set to keep *X* and *Y* variances equal to 1. For each population model, 1,000 data sets containing data for 1,000 subjects were generated. Each *X* and *Y* variable was simulated to have a mean of 0 and a standard deviation of 1 in order to facilitate later interpretation.

Analysis Models

All analyses used multiple regression models with Y_{10} as the dependent variable. Two sets of parallel analyses were conducted for all conditions. The first set of analyses, the *Y* models, used Y_k, *Lag*, and interaction terms for Y_k and *Lag* as predictors. The *Y* models were intended to demonstrate the characteristics of simple LAM models in which the autoregressive effect of *Y* changes as the number of lags between observations increases. For all *Y* models, because the *Lag* variable is centered at 1, the object was to correctly estimate the AR coefficient between Y_9 and Y_{10}.

The second set of analyses, the *YX* models, also included X_9 as a predictor of Y_{10}. This scenario was patterned after Cohen's premature covariate study, in that the X_9 predictor has no effect on Y_{10} in the population model, but using any Y_k value as a predictor other than Y_9 will make it appear that X_9 does in fact have an effect on Y_{10}. The aim of the *YX* models was to see whether the LAM approach can in any way remediate the problem identified by Cohen.

For the exponential population data, the following analytical models were used: 1) traditional fixed lag analyses using values of Y_k from only one lag at a time as a predictor; 2) linear LAM models using Y_k values sampled from variable lags; and 3) quadratic LAM models also using Y_k values sampled from variable lags. The linear and quadratic LAM models used a variable-lag sampling strategy which varied by the number of lags sampled and the location of the lags sampled. For the linear population data, only fixed lag analyses and the linear LAM models were used.

Number of Lags Sampled. The number of lags sampled for the variable-lag models ranged from two to nine for the linear LAM analyses. For example, in one condition Y_k values used as a predictor of Y_{10} could come from either time 9 or time 8. A variable from a random uniform distribution was used as the *Lag* variable which in turn determined which value (i.e., Y_9 or Y_8) would be used as the predictor for each subject. Given that at least three points are required to fit a quadratic model, the quadratic LAM analyses used only conditions sampling four or more lags.

Location of Sampled Lags. In addition to varying the number of lags sampled, the location of the sampled lags was also varied. For example, there were eight possible ways to sample from two contiguous lags (i.e., Y_9 and Y_8; Y_8 and Y_7; Y_7 and Y_6; Y_6 and Y_5; Y_5 and Y_4; Y_4 and Y_3; Y_3 and Y_2; and Y_2 and Y_1). The number of combinations varied with the number of lags sampled so that while there were eight possible locations for the two lag condition, there was only one possible location for the nine lag condition.

Results for Exponential Population Data

The implied correlations among the eleven simulated variables were found using tracing rules for path diagrams and the relationships specified by the data generating model. The implied correlations are shown in Table 1.

Fixed Lag Analyses. In the fixed lag *Y* models, only lagged Y_k values from a fixed occasion were used as a predictor of Y_{10}. For the *YX* models, the lagged Y_k values and X_9 were used as predictors. This resulted in nine *Y* models, and nine *YX* models. The regression coefficients for the *Y* models are equal to the implied correlations between Y_k and Y_{10} and the standard errors can be calculated with the following formula from Cohen and Cohen (1983).

$$SE_\beta = \sqrt{\frac{1-r^2}{n-2}} \qquad (10)$$

Given that the *YX* models are two variable regression models with predetermined correlations among the variables, the standardized regression coefficients and standard errors were computed analytically using the following formulas adapted from Cohen and Cohen (1983).

$$\beta_{Y10X9\bullet Yk} = \frac{r_{Y10X9} - r_{Y10Yk} r_{X9Yk}}{1 - r_{X9Yk}^2}$$
$$\beta_{Y10Yk\bullet X9} = \frac{r_{Y10Yk} - r_{Y10X9} r_{X9Yk}}{1 - r_{X9Yk}^2} \tag{11}$$

$$SE_{\beta i} = \sqrt{\frac{1 - R_{Y10}^2}{n - k - 1}} \sqrt{\frac{1}{1 - R_i^2}} \tag{12}$$

The standardized regression coefficients were computed based upon the implied correlations among the variables. The standard error for the standardized regression coefficient used two R^2 values. The first, $R^2{}_{Y10}$, is the model R^2 when Y_{10} is the criterion and the second, $R^2{}_i$, is the R^2 for a model with the *i*-th predictor as the criterion and the other predictors as the predictors for that model.

Table 2 shows the analytical estimates for the regression coefficients and standard errors for the fixed lag analyses as well as the corresponding empirical estimates from the analysis of the simulated data for both the *Y*, and the *YX* models. The results from the analyses of the simulated data represent the average regression coefficients and average standard errors across the 1,000 replications. For the *Y* models, the effect of Y_k on Y_{10} diminishes as more distal values of *Y* are used as predictors. The standard error for the regression coefficient also increases with lag because of the decrease in $R^2{}_{Y10}$.

For the *YX* models, increasing lag results in smaller effects for Y_k and larger effect for X_9. The Y_k effects decrease because the correlation between Y_{10} and Y_k diminishes with increasing lags. The effect of X_9 increases because the more distal values of Y_k are not effective in controlling for the effect of Y and therefore the variance that X_9 shares with Y_{10}, due to their mutual dependence on Y_9, results in an inflated estimate of the effect of X_9 on Y_{10}. In contrast with the Y models, the Y_k effect for the YX models is even lower than before because the model controls for the effect of X_9. In essence, this is like having a weak proxy for Y_9 in the model. Because Y is specified to have an AR(1) relationship, a model with any Y_k and Y_9 as predictors of Y_{10} would show that only Y_9 had an effect. Similarly, having X_9 in a model with Y_k will diminish the effect of Y_k. Though X_9 has no true regression relationship with Y_{10}, only the use of Y_9 as a predictor can reveal this.

In the next series of models, variable lag sampling (i.e., sampling values of Y_k for use as a predictor of Y_{10} from more than one lag) will be used in conjunction with two types of LAM models, linear and quadratic.

Linear LAM Models. The following equation describes the linear LAM model fitted for the Y models.

$$\hat{Y}_{10} = b_0 + b_1 Y_k + b_2 Lag + b_3 Y_k Lag \tag{13}$$

The linear LAM model for the YX model is the same except for the addition of X_9 as a predictor. However, the interpretation of the other coefficients is now conditional on X.

$$\hat{Y}_{10} = b_0 + b_1 X_9 + b_2 Y_k + b_3 Lag + b_4 Y_k Lag \tag{14}$$

Results for both the Y models and the YX models using the linear LAM are presented in Tables 3 through 10. Intercepts for these models, due to the use of standardized data, are all approximately 0 and therefore not reported. As before, coefficients and standard errors are the average values across the 1,000 simulated data sets.

Y Model Linear LAM Results. The focus of the *Y* models is the accurate estimation of the AR coefficient (labeled b_{Yk} in Tables 3 through 10), here set at 0.8. The use of the linear LAM model to describe the exponential pattern of change in the AR coefficients in the population means the estimates for the AR effect can depend heavily upon the location of the sampled lags. Locations closer to Y_{10} yield estimates for the AR effect that are closer to 0.8 than those from more distal locations. Also, due to this mismatch between the population and analysis models, sampling only two lags at the location closest to Y_{10} actually outperforms all other models. The coefficients for the interaction terms are also dependent upon location because the slope of the line needed to describe different locations on the exponential curve changes with location.

The standard errors for the b_{Yk} coefficient increase much faster than those for the other coefficients. The increase can be explained by examining the previous formula for standard errors. The greater increase is due to the fact that the standard errors depend upon both R^2_{Y10} and R^2_i. Because Y_k is correlated with $Y_k \times Lag$ (this collinearity could be decreased by mean-centering *Lag* rather than centering it at a meaningful value, however this would change the interpretation of the coefficient for Y_k to be the effect of Y_k at the mean *Lag*), the R^2_i is quite large making the third part of the standard error formula large. Thus the increasing standard error due to the diminishing R^2_{Y10} is magnified by this value.

YX Model Linear LAM Results. While the *Y* model analyses show that some of the linear LAM analyses yielded reasonable estimates for the AR coefficient for *Y*, the *YX* models are not as successful. Recall that success for the *YX* models would be estimating the b_{Yk} coefficient as 0.8 and the b_{X9} coefficient as 0. The results in Tables 3 through 10 however, show that the LAM models were not able to reduce the upwardly biased estimates for the effect of X_9, nor were they able to accurately estimate the AR effect for Y_9 on Y_{10}. The reason for this is based on the fact that with both *X* and *Y* used as predictors, the coefficients now must reflect the unique effect of these variables when controlling for the other predictors. The effect of X_9 for any particular model represents the unique relationship between X_9 and Y_{10}. The implied correlation for X_9 and Y_{10}, regardless of the lagged *Y* values, is 0.40. As more distal lag values are chosen for *Y*, not only does the effect of Y_{k} diminish, but the shared variance between Y_{k} and X_9 also diminishes. This makes it appear that X_9 is a stronger predictor. Because the correlation between Y_{k} and Y_{10} decreases predictably with lag while the implied correlation between X_9 and Y_{10} remains constant, it follows that the effect of X_9 will increase with lag.

Quadratic LAM Models. From Figure 11 it is clear that while a straight line may describe some parts of this function, it cannot accurately describe the form because of the curvature of the line. Therefore, a quadratic LAM model should provide a better description of this relationship. The following equation describes the quadratic LAM model used for the *Y* models.

$$\hat{Y}_{10} = b_0 + b_1 Y_k + b_2 Lag + b_3 Lag^2 + b_4 Y_k Lag + b_5 Y_k Lag^2 \qquad (15)$$

The equation describing the quadratic *YX* analyses is as follows.

$$\hat{Y}_{10} = b_0 + b_1 X_9 + b_2 Y_k + b_3 Lag + b_4 Lag^2 + b_5 Y_k Lag + b_6 Y_k Lag^2 \qquad (16)$$

As before, the addition of the *X* predictor changes the interpretation of the other coefficients, making them conditional on *X*. Given that at least three lags are required to fit a quadratic model, the quadratic models were fit only to conditions in which four or more lags were sampled. Tables 11 through 16 show the results from the quadratic LAM analyses.

Y Model Quadratic LAM Results. The key difference between the previous linear LAM result and the current quadratic results is that the estimates for the Y_k coefficient are much better due to the closer match of the quadratic form to the form in the population model. The estimates still depend upon the sampled location; however the dependence is notably less than that for the linear LAM models. The same pattern of rapidly increasing standard errors as locations become more distal from Y_{10} is also seen here.

YX Model Quadratic LAM Results. Even the better fit between analysis and population models does nothing to remedy the poor estimates for the effect of X_9. Again, this is due not to the functional form specified, but instead to the fact that distal values of Y_k will inevitably have lower correlations with Y_{10} thus inflating the b_{X9} coefficient.

Results for the Linear Population Data

For the linear population data, the functional form for the way the effect of Y_k on Y_{10} changes over time was specified and this in turn determined the first order autoregressive effects. Although the functional form for change in the autoregressive Y relationship, rather than the autoregressive relationships were specified by the model, the autoregressive effect for each pair of Y variables can be solved for algebraically and the implied correlations can be found as before. Table 17 shows these correlations among the simulated variables. Specifying the functional form has interesting implications for the adjacent lag Y correlations. These correlations peak for Y_8 with Y_9 and grow smaller with the smallest being that for Y_1 and Y_2. This population model in which the stability of Y changes across the course of the hypothetical study therefore differs markedly from the previous population model in which Y showed the same stability across all intervals. Since the functional form is predetermined to be linear for these data, only the fixed lag and linear LAM models will be examined.

Fixed Lag Analyses. Table 18 shows the analytical and empirical coefficients and standard errors for the fixed lag analyses of the linear population data. The trends seen are very similar to those seen for the fixed lag analyses of the exponential population data. For the Y models, with increasing lag, the effect of Y_k diminishes. For the YX models, the effect of Y_k diminishes and the effect of X_9 increases. Tables 19 through 26 show the results from the linear LAM analyses.

Y Model Linear LAM Results. As expected given the perfect match between the population and analysis models, the estimates for the AR effect of Y_9 on Y_{10} are quite good. This is the case regardless of the location and the number of lags sampled with the possible exception that the effect may be slightly underestimated by the most distal two-lag models. The general pattern for the increasing standard errors for b_{Yk} is the same as for the previous analyses. In addition, the coefficient for the interaction term (labeled $b_{\mathrm{Yk} \times \mathrm{Lag}}$ in the tables) is also very close to the value specified by the simulation model.

YX Model Linear LAM Results. Even in this ideal case in which the match between population and analysis models is perfect, the estimates for b_{X9} are still problematic. It is clear that the improved point estimate of the AR effect of Y_9 on Y_{10} can in no way compensate for the fact that the relationship between Y_k and Y_{10} is not the same as that between Y_9 and Y_{10}.

Summary of Results from the LAM Analyses

The results for the *Y* model analyses show that given a reasonable fit between the population and analysis models, the LAM was successful at estimating the focal AR coefficient for *Y*. Even in the instances in which the match was not close, some estimates were still quite good depending upon the number of lags sampled and the location of the lags sampled. In many instances, even using distal locations for the sampled lags the LAM analyses yielded better estimates for the AR effect than the fixed lag models using similarly distal values of Y_k. These results suggest the LAM model is performing just as it should in accurately describing the effects as lags vary. The results also highlight the importance of the issues of: 1) choosing a functional form for the LAM interaction, and 2) always considering the number of lags sampled and the location of the sampled lags.

The results from the *YX* models show that there are some situations in which the LAM analysis will not be as useful. In the particular case of using distal values of a covariate to control properly for the effect of a proximal variable, the LAM is of no use. The failure of the LAM model to solve the problem of the premature covariate is in part due simply to the problem of using data from one set of occasions to draw inference about relationships at other occasions.

The pattern of increasingly large standard errors is of potential concern when using the LAM approach. Adding interaction terms that are highly collinear with other predictors to the model can make some standard errors quite large. However, this may not be as fundamental a problem as it first seems for the following reasons. First, part of the inflation in standard errors is due to the fact that the use of the distal Y_k predictors resulted in a diminished overall model R^2. Such a pattern is specific to the constraints of the simulation model and may not always be the case. Second, in all models *Lag* was centered at a meaningful value rather than mean-centered. It is well documented (see e.g., Cohen & Cohen, 1983) that mean-centering often reduces collinearity among variables thus resulting in lower standard errors. Finally, and perhaps most importantly, while the standard errors for b_{Yk} did increase sharply, the standard errors for the interaction terms increased much less. Though not the primary focus of this demonstration, usually it is the interaction term that is most important in an interaction model.

The issues of the number of lags to sample and the location of the lags are in some ways closely tied to the match between the analysis and population models. For example, the best estimates using the exponential population data in conjunction with a linear LAM model came from the two-lag condition closest to Y_{10}. However, such results seem entirely dependent on specific crossings of population and analysis models and it may be better just to conclude that the best location to sample will be based on the interest of the analyst and for most models using more lags will produce a better result because the additional information about the relationship between the size of the effect and lag will yield a better estimate of that effect. This is shown by the fact that increasing the number of lags used for the quadratic models tended to improve the estimate of the AR effect for Y.

As a final note on the results of the simulations, it is informative to examine the issue of the statistical significance of the interaction terms in the LAM models. The ratio of the average regression coefficient to the average standard error is a straightforward means of judging whether the interaction terms would likely be statistically significant. For the linear LAM models used for the exponential population data, most of the interaction terms would appear significant. The exception to this is when using Y values from the most distal lags in conjunction with sampling smaller numbers of lags. The slope of the lines describing change in the simple slopes for these models is rather small and therefore less likely to be significant.

For the quadratic LAM models fitted to the exponential population data, none of the highest order interaction terms would be statistically significant. This suggests that even a relatively large sample size of 1,000 yields insufficient statistical power to detect the interaction. Given that the ratio of the regression coefficient to the standard error must equal approximately 1.96 in order to be statistically significant, a sample size of well over 3,000 would be necessary to find the quadratic interaction terms for these analyses statistically significant.

LAM Analysis of Empirical Data

The previous demonstration highlighted both the utility of the LAM approach and its potential limitations by using simulated data. In the next section I applied the LAM model to empirical data. For these analyses, I used data from a large multi-site longitudinal study in which lags were planned to be the same for each individual, but varied considerably due to the practical issues related to collecting data on a fixed-lag schedule.

The data for the examples are from the Early Head Start Research and Evaluation study (EHSRE; Department of Health and Human Services: Administration for Children and Families, 1996-2001). The data are available from the Inter-university Consortium for Political and Social Research website (http://www.icpsr.org/). The intent of this study was to examine the impact of the Early Head Start Program on young children and their families. Data from three waves of the study were used in the following examples. These three waves were timed to coincide with the age of the focal child in each family. The data were to be collected when the child was 14, 24, and 36 months of age. The actual age ranges of the children at the three waves of data collection were respectively: 11 to 22 months; 20 to 32 months; and 33 to 54 months. The sample size for the analyses examining relationships between variables at the 14 and 24 month occasions was 1,823 and the sample size for the analyses using the 24 and 36 month occasions was 1,740. The next section briefly describes each of the measures used in the subsequent LAM analyses.

Measures and Variables

Home Observation for the Measurement of the Environment (HOME). The HOME (Caldwell & Bradley, 1984) is a semi-structured observational instrument that assesses the quality of stimulation provided to a child in his/her home. The HOME can be used as a total score assessing the overall quality of stimulation in the home, or as subscales designed to assess specific aspects of the home environment. For the present analyses, only the total HOME score was used.

Bayley Scales of Infant Development – Mental Development Index (MDI). The MDI (Bayley, 1969) can be used to measure the cognitive, language, and social development of children under the age of 42 months. Standardized scores can be computed based on the child's age. The Bayley was administered to children in the EHSRE study when the children were approximately 24 months of age.

Three-Bag Assessment (Semi-Structured Play). When the focal child in the EHSRE study was approximately 14 months of age and again when the child was approximately 36 months of age, parents were asked to play with their child using three bags of toys. Each play session was video-taped for later coding. Trained observers scored the parent and child on a number of subscales with possible scores ranging from 1 to 7. One such scale, *Child Negativity toward the Parent*, rated the child's negative behavior toward the parent. Negativity included anger, hostility, or dislike expressed toward the parent. Higher scores on this scale meant higher levels of negativity. Another subscale was called, *Parent Negative Regard toward the Child*. Negative regard could include disapproval, anger, or rejection expressed by the parent toward the child. Higher scores on this scale indicate higher levels of negative regard. A third subscale, *Parent Intrusiveness*, assessed the degree to which the parent attempted to control the play of the child rather than letting the child guide the play activities. Higher scores on this subscale mean that the parent did not allow the child much latitude in directing the play during the interaction. Finally, the *Child Sustained Attention to Objects* subscale measures the involvement of the child with the toys presented in the three bags. High scores on this subscale meant the child was more focused on the toys and explored the toys thoroughly.

Lag. Two lag variables for these analyses were constructed using the interview dates. One lag variable was constructed for the interval between the 14 and 24 month interviews, and a second lag variable was constructed for the interval between the 14 and 36 month interviews. The lag variables represent the number of months that elapsed between the waves of data collection. To facilitate interpretation of the regression coefficients, each lag was mean-centered. The distributions for the lag variables were approximately normal with most values close to the average lag. Table 27 shows the descriptive statistics for all variables used in the subsequent analyses.

LAM Models Examined

As previously remarked, there is very little existing theory regarding the expected functional forms for lag interactions. Therefore, the following analyses can be viewed as exploratory in nature. Using the previously described measures, three relationships were examined: 1) the HOME at 14 months predicting the MDI at 24 months; 2) Parent Intrusiveness at 14 months predicting Child Sustained Attention at 24 months; and 3) Parent Negative Regard toward Child at 14 months predicting Child Negativity toward Parent at 36 months. Each relationship was examined using three previously described LAM models: a) the linear LAM model described in equation 1; b) the quadratic LAM model described in equation 3; and c) the exponential LAM model described in equation 7. Statistically significant models are reported and described and when more than one model fit the data, results are compared. Due to the fact that these analyses are exploratory and fail to include other potentially important predictors, interpretation should be limited to the examination of the way the particular relationship changes as a function of lag length.

Simple Regressions by Lag Group. A statistically significant lag interaction should not be the sole evidence that a particular model is accurately describing a changing relationship. A similar question often arises when fitting growth curve models to a set of repeated measures and the question of the accuracy of the model's description can in part be addressed by comparing the means for the repeated measures to the implied trajectory from the growth model. Such an option is not available for the LAM analyses, however, because the focus is not on change in variables but instead on change in relationships.

In order to provide some descriptive measure of whether the interaction models provided a good description of the changing relationships between variables, a grouping procedure was used to form groups with similar lags values. Simple regression models were fitted within each group and the simple slopes from these models are plotted against the mean lag value for each group. Thus a scatter plot was obtained that describes the simple slopes as a function of lag.

Due to the fact that such estimates can be greatly influenced by the arbitrary cut points used to assign individuals to groups, two measures were taken to alleviate this potential problem. First, two different sets of lag groups were created: one set used groups of approximately 300 and the second set used groups of approximately 150. Second, group membership was not exclusive. Group assignments were made by creating an index variable ranging in value from 1 to *N* (*N* being the total sample size) where 1 represented the smallest value of lag and *N* represented the largest value of lag. For the groups of approximately 300, the first group contained the lowest values of lag with index variable scores from 1-300. The second group contained values of the index variable ranging from 200-500. The number of groups varied for different lags (i.e., 14m to 24m or 14m to 36m) because the last group was created so that it would have as close to 300 members as possible. In this way, all adjacent groups shared 100 common members. A similar procedure was used to form the groups of 150. This method was used to make the results from the simple regressions more consistent and less sensitive to the fact that group results were determined by the specific members of the particular group. The adjacent groups of 300 share 100 members in common and the adjacent groups of 150 shared 50 members in common. For the 14 month to 24 month lag, there were either eight groups (the first seven with 300 and the last with 423) or 18 groups (the first 17 with 150 and the last with 123). For the 14 month to 36 month lag, there were either 8 groups (the first seven with 300 and the last with 340) or 17 groups (the first 16 with 150 and the last with 140). For all models reported below, scatter plots of the simple regression slopes for these two sets of groups are presented in which each point represents the simple regression slope for a group plotted at the mean lag for that group.

HOME at 14 months predicting MDI at 24 months. The first set of analyses used a measure of the global quality of stimulation in the home environment from the Home Observation for the Measurement of the Environment (HOME, Caldwell & Bradley, 1984) assessed when the child was approximately 14 months of age as a predictor of the child's developmental status as measured by the Mental Development Index from the Bayley Scales of Infant Development (MDI; 1969) assessed when the child was approximately 24 months of age. Both the linear LAM and the exponential LAM models showed a statistically significant interaction, but not the quadratic LAM model. The following equation shows the parameter estimates from the linear LAM model.

$$MDI_{24} = 88.96 + 1.26HOME_{14} - .065Lag - .15HOME_{14} \times Lag \qquad (17)$$

All parameters in this model are statistically significant. Initial results from this model showed that the *Lag* variable was a significant predictor of MDI scores. To account for the possibility that the Lag effect may be due to the fact that the agestandardization of the MDI scores did not completely eliminate age effects (i.e., longer Lags meant children were older and thus scored higher on the MDI), mean-centered child age at the 24 month assessment was added as a covariate and the effect of Lag on MDI scores was no longer significant. This highlights an important issue for LAM models, that of relative time effects versus absolute time effects, which will be addressed later in the dissertation.

The following equation shows the parameter estimates from the exponential LAM model for this relationship.

$$MDI_{24} = 89.37 + 1.30e^{-.10Lag} HOME_{14} \tag{18}$$

All coefficients were statistically significant for the exponential model. Figure 13 shows a plot of the lines depicting change in the simple slopes for HOME at 14 months predicting the MDI score at 24 months for both the linear and exponential models. The solid line in the figure depicts the implied simple slopes for the linear model while the dashed line represents the simple slopes for the exponential model. In addition, the figure shows a scatter plot of simple regression slopes for the previously described lag groups. The empty circles represent the slope estimates for the eight groups of 300 and the black diamonds represent the slope estimates for the 18 groups of 150. The mean-centered values along the *x*-axis represent lags ranging from approximately three months to approximately seventeen months. The figure shows that for shorter lags the effect of the HOME on the MDI is larger and this effect decreases as lags grow longer.

These results may mean that there is a positive effect of stimulation in the home on child development, but the effect diminishes over time. Alternatively, these two variables may be correlated due to their common reliance on some other factor such as family socioeconomic status and their relationship weakens with increased lag in the same way that a test-retest correlation diminishes with increased intervals between tests. In comparing the lines for the linear and exponential models, it is unclear whether the more complex nonlinear model adds substantially to the interpretation of the interaction.

Parent Intrusiveness at 14 months predicting Child Sustained Attention at 24 months. The next series of models examines whether the tendency for the parent to control the play activities of the child at 14 months predicts the degree to which the child attends to and explores the toys available at 36 months. Here again, the linear and exponential models yielded statistically significant interactions, but the quadratic model did not. The parameter estimates for the linear model are shown below.

$$CSA_{24} = 5.02 - .15 PINT_{14} + .01 Lag + .03 PINT_{14} \times Lag \qquad (19)$$

The coefficients for all predictors are statistically significant except that for *Lag*. The parameter estimates for the exponential model are as follows.

$$CSA_{24} = 5.02 - .15 e^{-.18 Lag} PINT_{14} \qquad (20)$$

All parameters from this model are statistically significant. Figure 14 shows the implied simple slopes for the linear (solid line) and exponential (dashed line) models. The scatter plots of the simple regression slopes by lag group are also included on the figure. These results indicate that there is an effect of how intrusive a parent is at 14 months and the ability of the child to attend to toys at 24 months and that this effect appears to increase with longer lags. While this phenomenon may seem counterintuitive, it is possible that there is a cumulative effect of intrusive parenting such that consistently intrusive parenting eventually leads to lower sustained attention or that there is a developmental pattern present such that the effect of intrusive parenting on sustained attention does not emerge until the child is older than 14 months. Regardless of the interpretation, it is clear that there is a pattern present such that longer lags tend to yield larger slopes; however, as with the previous example the necessity of the more complex exponential model is questionable.

Parent Negative Regard at 14 months predicting Child Negativity at 36 months. The next analyses examine the relationship between the parent's negative regard toward the child at 14 months and the child's negativity towards the parent at 36 months. For this relationship both the quadratic and exponential LAM models resulted in statistically significant interactions, but not the linear model. The following equation shows the parameter estimates from the quadratic model.

$$\begin{aligned} CNEG_{36} &= 1.27 + .069 PNEG_{14} - .001 Lag + .004 Lag^2 + \\ &.022 PNEG_{14} \times Lag + .013 PNEG_{14} \times Lag^2 \end{aligned} \tag{21}$$

The coefficients for all predictors were statistically significant except those for *Lag* and *Lag^2*. The parameter estimates for the exponential LAM model are as follows.

$$CNEG_{36} = 1.28 + 0.083 e^{0.358 Lag} PNEG_{14} \tag{22}$$

All coefficients from this model were statistically significant. Figure 15 shows the lines depicting the implied simple slopes for the quadratic (solid line) and exponential (dashed line) models. The scatter plot of the simple regression slopes is also included. These results show that either: 1) the effect of parental negative regard on child negativity is large, decreases, and then increases again, or 2) the effect is very small and becomes larger with increasing lag. Based upon the scatter plots it appears that the latter explanation may be better. It is interesting to note that this effect too seems to increase with longer lags suggesting either a cumulative effect of negative parenting or some age-related developmental effect.

In contrast to the previous analyses, the information depicted by the points in the scatter plot is more ambiguous as to the accuracy of the descriptions provided by the interactions. It appears that the majority of the points close to the average lag show little relationship between the slope and lag and a very small number of points at the extreme lag values are responsible for determining the nonlinear shape.

Empirical Analysis Conclusions. The results from the analysis of the empirical data are very encouraging. It was shown that even in a study not designed for use with a LAM approach it is possible to find relationships that are moderated by lag length and to find a variety of functional forms for the moderation. The ability to move from examining an effect as a single fixed effect to examining the same effect as changing according to the length of time that passes between observations is a great advance. Given that such results were found in this case in which the research design was not ideal for the use of a LAM model, it seems likely that a study designed specifically to test for moderation of effect by lag length would have great potential to better understand how such relationships change over time. The ambiguity of some of the previous results regarding the pattern of the lag interaction near the extremes of sampled lags also suggest that it may be important to sample more heavily at the extremes.

The results also show that one must be cautious in interpreting the LAM interactions to insure the patterns described by the interaction models are supported by descriptive analyses of the data. The method of forming lag groups may is not ideal in that the results are based on arbitrarily forming groups based on values of a continuous variable, however, the information provided by this methods does seem to be a valuable addition.

Relative versus Absolute Time. One very important issue, not addressed in the previous empirical analyses but hinted at by the need for the child age covariate in the first model is that of relative versus absolute time. The dates for the interviews for 14 month, 24 month, and 36 month interviews could vary. This variation produced variation in the lags between observations (relative time between occasions), but the variation also meant that children were different ages when the interviews occurred. It is possible that some effects that appear to be due to lag may be due to the differing ages of the children. Further investigation is needed in order to insure that LAM methods can tease apart these relative and absolute time effects.

Conclusions

The purpose of this dissertation was to introduce readers to the important role that time lags play in models for longitudinal data. Based on a wide variety of sources, demonstrations, a simulation study, and several analyses of empirical data, I have argued that no longitudinal data collection should be undertaken without a careful consideration of time lags. The LAM approach offers a new and potentially very useful strategy for addressing the perennial problem of choosing lags for a longitudinal study. However, the true appeal of the LAM approach is the fact that it provides a straightforward means of extending and enhancing our current understanding of phenomena in the social sciences.

Limitations of the LAM model

While I argue that treating the lag in a longitudinal study as a moderator of any causal effect, there are some clear limitations of this approach. First, while it can be argued that the LAM is better than a single lag design in terms of decreasing the chance of misrepresenting an effect due to lag choice and in terms of better understanding the way an effect changes over time, it would be difficult to use more than a small range of lags for any one study. In the context of the previous selection argument, it can be said that while we cannot sample the entire population of lags, using a few lags is better than using only one.

A second limitation of the LAM approach is the fact that even simple models are made complex by considering the lag as a potential moderator for each effect. For the linear LAM regression model, one predictor yields four parameters to estimate, and each additional predictor adds at least two more parameters. When any longitudinal effect is an interaction effect, the complexity of any model will be considerably greater than a non-LAM model. On the other hand, this sort of complexity is not unwarranted if one wishes to accurately describe a relationship and the way it changes over time.

Related to the complexity issue, another potential limitation of LAM models is that the sample sizes required for even simple LAM analysis may be quite large. The results of the simulation study, particularly those for the quadratic LAM analyses underline the fact that even LAM models using only multiple regression can require very large sample sizes in order to find statistically significant moderation by lag.

Finally, the simulation results also make it clear that the LAM approach cannot be used to draw inference outside of the range of observed data. Thus, while the LAM approach offers a unique way to understand changes in relationships as a function of lag length, it cannot be reliably used to understand such change outside of the range of observed values.

Future Directions

The present exposition of the LAM model, because it introduces a new way of thinking about effects in longitudinal models that entails major changes in the way the effects are assessed and in the way longitudinal data are collected, is by necessity preliminary and incomplete. Viewing lag as a potential moderator of any effect offers many opportunities for further study. LAM models could be used for any existing data set in which lags vary to some degree and the lag variable could be constructed.

Such secondary analyses may show that previous conclusions regarding relationships were either incorrect or at least incomplete. Further, these secondary analyses could potentially be the foundation for a growing body of knowledge regarding the characteristic way that effect change with lag. However, the range of lags values found in fixed lag designs is likely to be limited.

LAM models also have the potential to be employed in choosing lags for future studies. A LAM model showing the point at which an effect is expected to become statistically significant, the point at which an effect reaches its peak, or the point at which an effect becomes no longer statistically significant, may provide very useful information for an investigator planning a large longitudinal study. However, the very large sample sizes required for many LAM models may make this use prohibitively expensive in most instances.

In general, the LAM model has the potential to improve virtually any longitudinal investigation. Its key limitations, namely that it increases the complexity of any model, that it requires a very large sample size, and is most useful when the functional form of the interaction is properly specified do not seem to be severe flaws of the approach as much as they are coincident with the fact that often we are seeking to describe very complex phenomena.

References

Aiken, L. S., & West, S. G. (1991). *Multiple regression: Testing and interpreting interactions*. Newbury Park, CA: Sage.

Baltes, P. B., Reese, H. W., & Nesselroade, J. R. (1977). *Life-Span developmental psychology: Introduction to research methods*. Belmont, CA: Wadsworth.

Bayley, N. (1969). *The Bayley Scales on Infant Development*. New York: Psychological Corp.

Beal, D. J., & Weiss, H. M. (2003). Methods of ecological momentary assessment in organizational research. *Organizational Research Methods*, *6*, 440-464.

Bollen, K. A. (1989). *Structural equations with latent variables*. New York: Wiley.

Caldwell, B., & Bradley, R. (1984). *Home Observation for the Measurement of the Environment*. Unpublished manuscript, University of Arkansas.

Campbell, D. T., & Kenny, D. A. (1999). *A primer on regression artifacts*. New York: Guilford.

Cattell, R. B. (1966). The data box: Its ordering of total resources in terms of possible relational systems. In R. B. Cattell (Ed.), *Handbook of multivariate experimental psychology* (pp. 67-128). Chicago: Rand McNally.

Cohen, J., & Cohen, P. (1983). Applied multiple regression/correlation analysis for the behavioral sciences (2nd ed.). Hillsdale, N J: Erlbaum.

Cohen, P. (1991). A source of bias in longitudinal investigations of change. In L. M.

Collins & J. L. Horn (Eds.), *Best methods for the analysis of change: Recent advances, unanswered questions, future directions* (pp. 18–25). Washington, DC: American Psychological Association.

Cole, D. A., & Maxwell, S. E. (2003). Testing meditational models with longitudinal data: Questions and tips in the use of structural equation modeling. *Journal of Abnormal Psychology*, *112*, 558-577.

Collins, L. M., (2006). Analysis of longitudinal data: The integration of theoretical model, temporal design, and statistical model. *Annual Review of Psychology*, *57*, 505-528

Collins, L. M., & Graham, J. W. (2002). The effect of the timing and spacing of observations in longitudinal studies of tobacco and other drug use: Temporal and design considerations. *Drug and Alcohol Dependence*, *68*, S85-S96.

Cudeck, R., & du Toit, S. H. C. (2002). A version of quadratic regression with interpretable parameters. *Multivariate Behavioral Research*, *37*, 501-519.

Department of Health and Human Services, Administration for Children and Families. (1996-2001). Early Head Start Research and Evaluation Study. [Computer File]. Available from Inter-university Consortium for Political and Social Research Web site, http://www.icpsr.org

Gollob, H. F. & Reichardt, C. S. (1987).Taking account of time lags in causal models. *Child Development*, *58*, 80-92.

Gollob, H. F., & Reichardt, C. S. (1991). Interpreting and estimating indirect effects assuming time lags really matter. In L. M. Collins & J. L. Horn (Eds.) *Best* *methods for the analysis of change: Recent advances, unanswered questions, future directions* (pp. 243-259). Washington D. C.: American Psychological Association.

Guttman, R., & Guttman, L. (1965). A new approach to the analysis of growth patterns: the simplex structure of intercorrelations of measurements. *Growth*, *29*, 219.

Heise, D. R. (1970). Causal inference from panel data. *Sociological Methodology*, *2*, 3-27.

Heise, D. R. (1975). *Causal Analysis*. New York: Wiley-Interscience.

Holland, P. W. (1986). Statistics and causal inference. *Journal of the American Statistical Association*, *81*, 945-971.

Hume D. (1962[1739]). *A treatise of human nature. Book I. Of the understanding*. Repr. Glasgow:William Collins Sons & Co.

James, L.R., Mulaik, S.A. & Brett, J.M. (1983). *Causal analysis: assumptions, models, and data*. Sage Publications, Inc.

Kenny, D. A. (1979). *Correlation and causality*. New York: Wiley-Interscience.

Kanfer. R. & Ackerman. P. L. (1989). Motivation and cognitive abilities: An integrative aptitude-treatment interaction approach to skill acquisition. *Journal of Applied Psychology Monograph*. *74*, 657-690.

Maxwell, S. E., & Cole, D. A. (2007). Bias in cross-sectional analyses of longitudinal mediation. *Psychological Methods*, *12*, 23-44.

McArdle, J. J. & Woodcock, R. W. (1997). Expanding test-retest designs to include developmental time-lag components. *Psychological Methods*, *2*, 403-435.

Mill, J. S. (1843). *A system of logic, ratiocinative and inductive*. London: Longmans.

Nesselroade JR. 1991. Interindividual differences in intraindividual change. In *Best methods for the analysis of change*, ed. LM Collins, JL Horn, pp. 92–105. Washington, DC: Am. Psychol. Assoc.

Nesselroade, J. R. & Jone, C. J. (1991). Multi-modal selection effects in the study of adult development: A perspective on multivariate, replicated, single-subject, repeated measures designs. *Experimental Aging Research*, *17*, 21-27.

Pelz, D. C. & Lew, R. A. (1970). Heise's Causal Model Applied. in E. F. Borgatta & G. Bohrnstedt (eds) *Sociological Methodology*, pp. 28–37. San Francisco, CA: Jossey-Bass.

Radloff, L. (1977). The CES-D Scale: A self-report depression scale for research in the general population. *Applied Psychological Measurement*, *1*, 385-401.

Rubin, D. B. (1974). Estimating causal effects of treatments in randomized and nonrandomized studies. *Journal of Educational Psychology*, *66*, 688-701.

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference*. Boston, MA: Houghton Mifflin.

Shingles, R. D. (1985). Causal inference in cross-lagged panel analysis. In H. M. Blalock Jr. (Ed.), *Causal models in panel and experimental designs*. New York: Aldine.

Thorndike, R. L. (1933). The effect of the interval between test and retest on the constancy of the IQ. *Journal of Educational Psychology, 24,* 543-549.

Wold, H. (1954). Causality and econometrics. *Econometrica*, *22*, 162-177.

Wold, H. (1956). Causal inference from observational data: A review of ends and means. *Journal of the Royal Statistical Society*, *119*, 28-61.

Wright, S. (1960). The treatment of reciprocal interaction, with or without lag, in path analysis. *Biometrics*, *16*, 189-202.

Table 1.

Correlations for exponential data model

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	X_9
Y_1	1.00										
Y_2	0.80	1.00									
Y_3	0.64	0.80	1.00								
Y_4	0.51	0.64	0.80	1.00							
Y_5	0.41	0.51	0.64	0.80	1.00						
Y_6	0.33	0.41	0.51	0.64	0.80	1.00					
Y_7	0.26	0.33	0.41	0.51	0.64	0.80	1.00				
Y_8	0.21	0.26	0.33	0.41	0.51	0.64	0.80	1.00			
Y_9	0.17	0.21	0.26	0.33	0.41	0.51	0.64	0.80	1.00		
Y_{10}	0.13	0.17	0.21	0.26	0.33	0.41	0.51	0.64	0.80	1.00	
X_9	0.08	0.10	0.13	0.16	0.20	0.26	0.32	0.40	0.50	0.40	1.00

Table 2.

Results for fixed lag analyses exponential data

	Y Models		*YX* Models			
	Analytical	Simulation	Analytical		Simulation	
Lag (Y_k)	b_{Yk} (SE)	b_{Yk} (SE)	b_{X9} (SE)	b_{Yk} (SE)	b_{X9} (SE)	b_{Yk} (SE)
0 (Y_9)	0.800 (0.019)	0.801 (0.019)	0.000 (0.022)	0.800 (0.022)	0.001 (0.022)	0.801 (0.022)
1 (Y_8)	0.640 (0.024)	0.640 (0.024)	0.171 (0.026)	0.571 (0.026)	0.173 (0.026)	0.571 (0.026)
2 (Y_7)	0.512 (0.027)	0.513 (0.027)	0.263 (0.027)	0.428 (0.027)	0.265 (0.027)	0.428 (0.028)
3 (Y_6)	0.410 (0.029)	0.410 (0.029)	0.316 (0.028)	0.329 (0.028)	0.318 (0.028)	0.329 (0.028)
4 (Y_5)	0.328 (0.030)	0.329 (0.030)	0.347 (0.029)	0.257 (0.029)	0.349 (0.029)	0.257 (0.029)
5 (Y_4)	0.262 (0.031)	0.263 (0.031)	0.367 (0.029)	0.202 (0.029)	0.369 (0.029)	0.203 (0.029)
6 (Y_3)	0.210 (0.031)	0.211 (0.031)	0.379 (0.029)	0.160 (0.029)	0.381 (0.029)	0.162 (0.029)
7 (Y_2)	0.168 (0.031)	0.169 (0.031)	0.387 (0.029)	0.127 (0.029)	0.389 (0.029)	0.128 (0.029)
8 (Y_1)	0.134 (0.031)	0.135 (0.031)	0.391 (0.029)	0.101 (0.029)	0.394 (0.029)	0.102 (0.029)

Table 3.

Results for two-lag, linear LAM models using exponential data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)
0,1 (Y_9,Y_8)	0.801 (0.031)	-0.001 (0.044)	-0.161 (0.044)	0.091 (0.024)	0.756 (0.033)	-0.001 (0.043)	-0.152 (0.044)
1,2 (Y_8,Y_7)	0.767 (0.082)	0.001 (0.052)	-0.127 (0.052)	0.221 (0.027)	0.662 (0.080)	0.001 (0.050)	-0.110 (0.050)
2,3 (Y_7,Y_6)	0.718 (0.144)	-0.002 (0.056)	-0.103 (0.056)	0.292 (0.028)	0.588 (0.137)	-0.002 (0.053)	-0.084 (0.054)
3,4 (Y_6,Y_5)	0.659 (0.209)	0.003 (0.059)	-0.083 (0.059)	0.334 (0.028)	0.520 (0.196)	0.003 (0.055)	-0.065 (0.055)
4,5 (Y_5,Y_4)	0.598 (0.275)	0.002 (0.061)	-0.067 (0.061)	0.359 (0.029)	0.460 (0.256)	0.001 (0.056)	-0.051 (0.056)
5,6 (Y_4,Y_3)	0.529 (0.341)	0.000 (0.062)	-0.053 (0.062)	0.375 (0.029)	0.398 (0.315)	0.000 (0.057)	-0.039 (0.057)
6,7 (Y_3,Y_2)	0.479 (0.407)	0.001 (0.062)	-0.044 (0.062)	0.385 (0.029)	0.366 (0.375)	0.001 (0.057)	-0.034 (0.058)
7,8 (Y_2,Y_1)	0.392 (0.473)	-0.004 (0.063)	-0.032 (0.063)	0.391 (0.029)	0.298 (0.435)	-0.002 (0.058)	-0.024 (0.058)

Table 4.

Results for three-lag, linear LAM models using exponential data

Lags (Y_k)	*Y* Models: b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk\times Lag}$ (SE)	*YX* Models: b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk\times Lag}$ (SE)
0,1,2	0.796	0.001	-0.145	0.155	0.719	0.000	-0.131
(Y_9,Y_8,Y_7)	(0.038)	(0.030)	(0.030)	(0.026)	(0.040)	(0.029)	(0.029)
1,2,3	0.753	0.000	-0.116	0.255	0.634	0.000	-0.098
(Y_8,Y_7,Y_6)	(0.073)	(0.034)	(0.034)	(0.027)	(0.071)	(0.032)	(0.032)
2,3,4	0.690	0.000	-0.091	0.312	0.556	0.001	-0.073
(Y_7,Y_6,Y_5)	(0.112)	(0.036)	(0.036)	(0.028)	(0.106)	(0.034)	(0.034)
3,4,5	0.618	-0.002	-0.071	0.346	0.482	-0.002	-0.055
(Y_6,Y_5,Y_4)	(0.152)	(0.037)	(0.037)	(0.029)	(0.143)	(0.035)	(0.035)
4,5,6	0.557	0.000	-0.058	0.367	0.430	0.000	-0.045
(Y_5,Y_4,Y_3)	(0.193)	(0.038)	(0.038)	(0.029)	(0.180)	(0.035)	(0.035)
5,6,7	0.506	0.000	-0.049	0.380	0.391	-0.001	-0.038
(Y_4,Y_3,Y_2)	(0.234)	(0.039)	(0.039)	(0.029)	(0.217)	(0.036)	(0.036)
6,7,8	0.424	-0.001	-0.036	0.388	0.318	-0.001	-0.027
(Y_3,Y_2,Y_1)	(0.275)	(0.039)	(0.039)	(0.029)	(0.254)	(0.036)	(0.036)

Table 5.

Results for four-lag, linear LAM models using exponential data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk\times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk\times Lag}$ (SE)
0,1,2,3 (Y_9,Y_8,Y_7,Y_6)	0.785 (0.043)	0.000 (0.023)	-0.130 (0.023)	0.200 (0.027)	0.687 (0.044)	0.000 (0.022)	-0.114 (0.023)
1,2,3,4 (Y_8,Y_7,Y_6,Y_5)	0.733 (0.069)	0.001 (0.025)	-0.104 (0.025)	0.280 (0.028)	0.604 (0.067)	0.001 (0.024)	-0.086 (0.024)
2,3,4,5 (Y_7,Y_6,Y_5,Y_4)	0.671 (0.098)	0.001 (0.027)	-0.084 (0.027)	0.327 (0.028)	0.534 (0.093)	0.001 (0.025)	-0.066 (0.025)
3,4,5,6 (Y_6,Y_5,Y_4,Y_3)	0.600 (0.128)	0.001 (0.028)	-0.066 (0.028)	0.355 (0.029)	0.464 (0.120)	0.001 (0.026)	-0.051 (0.026)
4,5,6,7 (Y_5,Y_4,Y_3,Y_2)	0.530 (0.158)	0.001 (0.028)	-0.052 (0.028)	0.373 (0.029)	0.409 (0.147)	0.001 (0.026)	-0.041 (0.026)
5,6,7,8 (Y_4,Y_3,Y_2,Y_1)	0.479 (0.188)	-0.001 (0.028)	-0.044 (0.028)	0.383 (0.029)	0.367 (0.173)	-0.001 (0.026)	-0.034 (0.026)

Table 6.

Results for five-lag, linear LAM models using exponential data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)
0,1,2,3,4 (Y_9,Y_8,Y_7,Y_6,Y_5)	0.772 (0.046)	0.000 (0.019)	-0.117 (0.019)	0.234 (0.027)	0.659 (0.046)	0.000 (0.018)	-0.100 (0.018)
1,2,3,4,5 (Y_8,Y_7,Y_6,Y_5,Y_4)	0.714 (0.068)	0.001 (0.020)	-0.094 (0.021)	0.299 (0.028)	0.580 (0.065)	0.001 (0.019)	-0.076 (0.020)
2,3,4,5,6 (Y_7,Y_6,Y_5,Y_4,Y_3)	0.643 (0.091)	-0.001 (0.021)	-0.075 (0.021)	0.339 (0.028)	0.506 (0.086)	-0.001 (0.020)	-0.059 (0.020)
3,4,5,6,7 (Y_6,Y_5,Y_4,Y_3,Y_2)	0.579 (0.114)	0.000 (0.022)	-0.060 (0.022)	0.362 (0.029)	0.448 (0.107)	0.000 (0.020)	-0.047 (0.020)
4,5,6,7,8 (Y_5,Y_4,Y_3,Y_2,Y_1)	0.509 (0.138)	0.000 (0.022)	-0.048 (0.022)	0.377 (0.029)	0.390 (0.128)	-0.001 (0.021)	-0.037 (0.021)

Table 7.

Results for six-lag, linear LAM models using exponential data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk\times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk\times Lag}$ (SE)
0,1,2,3,4,5	0.756	0.000	-0.106	0.260	0.633	0.000	-0.088
(Y_9,Y_8,Y_7,Y_6,Y_5,Y_4)	(0.049)	(0.016)	(0.016)	(0.027)	(0.049)	(0.016)	(0.016)
1,2,3,4,5,6	0.691	0.000	-0.085	0.314	0.555	0.000	-0.068
(Y_8,Y_7,Y_6,Y_5,Y_4,Y_3)	(0.067)	(0.017)	(0.017)	(0.028)	(0.064)	(0.016)	(0.016)
2,3,4,5,6,7	0.621	0.000	-0.068	0.348	0.487	0.000	-0.053
(Y_7,Y_6,Y_5,Y_4,Y_3,Y_2)	(0.086)	(0.018)	(0.018)	(0.029)	(0.081)	(0.017)	(0.017)
3,4,5,6,7,8	0.553	-0.001	-0.055	0.368	0.425	0.000	-0.042
(Y_6,Y_5,Y_4,Y_3,Y_2,Y_1)	(0.105)	(0.018)	(0.018)	(0.029)	(0.098)	(0.017)	(0.017)

Table 8.

Results for seven-lag, linear LAM models using exponential data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)
0,1,2,3,4,5,6	0.742	0.000	-0.097	0.279	0.613	-0.001	-0.080
(Y_9,Y_8,Y_7,Y_6,Y_5,Y_4,Y_3)	(0.051)	(0.014)	(0.014)	(0.028)	(0.050)	(0.013)	(0.014)
1,2,3,4,5,6,7	0.667	0.000	-0.077	0.326	0.529	0.000	-0.061
(Y_8,Y_7,Y_6,Y_5,Y_4,Y_3,Y_2)	(0.066)	(0.015)	(0.015)	(0.028)	(0.064)	(0.014)	(0.014)
2,3,4,5,6,7,8	0.597	0.000	-0.062	0.355	0.464	0.000	-0.048
(Y_7,Y_6,Y_5,Y_4,Y_3,Y_2,Y_1)	(0.083)	(0.015)	(0.015)	(0.029)	(0.078)	(0.014)	(0.014)

Table 9.

Results for eight-lag, linear LAM models using exponential data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)
0,1,2,3,4,5,6,7 ($Y_9,Y_8,Y_7,Y_6,Y_5,Y_4,Y_3,Y_2$)	0.721 (0.052)	0.000 (0.013)	-0.087 (0.013)	0.295 (0.028)	0.587 (0.051)	0.000 (0.012)	-0.071 (0.012)
1,2,3,4,5,6,7,8 ($Y_8,Y_7,Y_6,Y_5,Y_4,Y_3,Y_2,Y_1$)	0.649 (0.066)	0.000 (0.013)	-0.071 (0.013)	0.336 (0.028)	0.513 (0.063)	0.000 (0.012)	-0.056 (0.012)

Table 10.

Results for nine-lag, linear LAM models using exponential data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)
0,1,2,3,4,5,6,7,8 (Y_9,Y_8,Y_7,Y_6,Y_5, Y_4,Y_3,Y_2,Y_1)	0.707 (0.053)	0.000 (0.011)	-0.081 (0.011)	0.307 (0.028)	0.572 (0.052)	0.000 (0.011)	-0.065 (0.011)

Table 11.

Results for four-lag, quadratic LAM models using exponential data

	Y Models					*YX* Models					
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag}^2 (SE)	$b_{Yk \times Lag}$ (SE)	$b_{Yk \times Lag}^2$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag}^2 (SE)	$b_{Yk \times Lag}$ (SE)	$b_{Yk \times Lag}^2$ (SE)
0,1,2,3 (Y_9,Y_8,Y_7,Y_6)	0.801 (0.050)	-0.002 (0.079)	0.001 (0.025)	-0.176 (0.079)	0.015 (0.025)	0.200 (0.027)	0.701 (0.051)	-0.002 (0.077)	0.001 (0.025)	-0.154 (0.077)	0.014 (0.025)
1,2,3,4 (Y_8,Y_7,Y_6,Y_5)	0.794 (0.157)	-0.001 (0.141)	0.000 (0.028)	-0.165 (0.142)	0.012 (0.028)	0.280 (0.028)	0.655 (0.150)	0.001 (0.135)	0.000 (0.026)	-0.136 (0.135)	0.010 (0.027)
2,3,4,5 (Y_7,Y_6,Y_5,Y_4)	0.782 (0.339)	0.009 (0.207)	-0.001 (0.029)	-0.154 (0.207)	0.010 (0.029)	0.327 (0.028)	0.620 (0.319)	0.009 (0.194)	-0.001 (0.028)	-0.121 (0.195)	0.008 (0.028)
3,4,5,6 (Y_6,Y_5,Y_4,Y_3)	0.730 (0.592)	0.005 (0.273)	0.000 (0.030)	-0.128 (0.274)	0.007 (0.030)	0.355 (0.029)	0.562 (0.551)	0.005 (0.255)	0.000 (0.028)	-0.097 (0.255)	0.005 (0.028)
4,5,6,7 (Y_5,Y_4,Y_3,Y_2)	0.742 (0.912)	0.000 (0.340)	0.000 (0.031)	-0.133 (0.341)	0.007 (0.031)	0.373 (0.029)	0.574 (0.845)	-0.001 (0.315)	0.000 (0.029)	-0.103 (0.316)	0.006 (0.029)
5,6,7,8 (Y_4,Y_3,Y_2,Y_1)	0.690 (1.296)	0.001 (0.406)	0.000 (0.031)	-0.111 (0.407)	0.005 (0.031)	0.383 (0.029)	0.527 (1.195)	0.000 (0.374)	0.000 (0.029)	-0.084 (0.375)	0.004 (0.029)

Table 12.

Results for five-lag, quadratic LAM models using exponential data

	Y Models					*YX* Models					
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag}^2 (SE)	$b_{Yk\times Lag}$ (SE)	$b_{Yk\times Lag}^2$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag}^2 (SE)	$b_{Yk\times Lag}$ (SE)	$b_{Yk\times Lag}^2$ (SE)
0,1,2,3,4	0.799	0.001	0.000	-0.168	0.013	0.234	0.682	0.001	0.000	-0.144	0.011
(Y_9,Y_8,Y_7,Y_6,Y_5)	(0.057)	(0.066)	(0.016)	(0.067)	(0.016)	(0.027)	(0.055)	(0.064)	(0.015)	(0.063)	(0.015)
1,2,3,4,5	0.786	0.002	0.000	-0.155	0.010	0.299	0.638	0.002	0.000	-0.126	0.008
(Y_8,Y_7,Y_6,Y_5,Y_4)	(0.139)	(0.105)	(0.017)	(0.105)	(0.017)	(0.027)	(0.130)	(0.099)	(0.016)	(0.098)	(0.016)
2,3,4,5,6	0.778	0.003	0.000	-0.151	0.010	0.338	0.611	0.001	0.000	-0.118	0.007
(Y_7,Y_6,Y_5,Y_4,Y_3)	(0.269)	(0.145)	(0.018)	(0.145)	(0.018)	(0.028)	(0.251)	(0.134)	(0.017)	(0.136)	(0.017)
3,4,5,6,7	0.722	-0.004	0.000	-0.122	0.006	0.362	0.564	-0.007	0.001	-0.097	0.005
(Y_6,Y_5,Y_4,Y_3,Y_2)	(0.441)	(0.185)	(0.018)	(0.186)	(0.018)	(0.029)	(0.421)	(0.171)	(0.017)	(0.176)	(0.017)
4,5,6,7,8	0.677	-0.010	0.001	-0.107	0.005	0.377	0.514	-0.012	0.001	-0.080	0.004
(Y_5,Y_4,Y_3,Y_2,Y_1)	(0.654)	(0.225)	(0.019)	(0.226)	(0.019)	(0.028)	(0.618)	(0.206)	(0.017)	(0.214)	(0.018)

Table 13.

Results for six-lag, quadratic LAM models using exponential data

	Y Models					*YX* Models					
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag}^2 (SE)	$b_{Yk\times Lag}$ (SE)	$b_{Yk\times Lag}^2$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag}^2 (SE)	$b_{Yk\times Lag}$ (SE)	$b_{Yk\times Lag}^2$ (SE)
0,1,2,3,4,5	0.798	-0.003	0.000	-0.166	0.012	0.259	0.669	-0.003	0.000	-0.140	0.010
(Y_9,Y_8,Y_7,Y_6,Y_5,Y_4)	(0.062)	(0.058)	(0.011)	(0.058)	(0.011)	(0.027)	(0.061)	(0.055)	(0.011)	(0.055)	(0.011)
1,2,3,4,5,6	0.777	0.000	0.000	-0.148	0.009	0.314	0.623	0.002	0.000	-0.118	0.007
(Y_8,Y_7,Y_6,Y_5,Y_4,Y_3)	(0.130)	(0.084)	(0.012)	(0.084)	(0.012)	(0.028)	(0.124)	(0.079)	(0.011)	(0.080)	(0.011)
2,3,4,5,6,7	0.762	0.001	0.000	-0.141	0.008	0.347	0.597	0.001	0.000	-0.110	0.006
(Y_7,Y_6,Y_5,Y_4,Y_3,Y_2)	(0.230)	(0.111)	(0.012)	(0.111)	(0.012)	(0.029)	(0.215)	(0.104)	(0.011)	(0.104)	(0.011)
3,4,5,6,7,8	0.701	-0.007	0.001	-0.114	0.005	0.368	0.542	-0.006	0.000	-0.089	0.004
(Y_6,Y_5,Y_4,Y_3,Y_2,Y_1)	(0.359)	(0.138)	(0.012)	(0.139)	(0.013)	(0.029)	(0.333)	(0.128)	(0.012)	(0.129)	(0.012)

Table 14.

Results for seven-lag, quadratic LAM models using exponential data

Lags (Y_k)	*Y* Models b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag^2} (SE)	$b_{Yk \times Lag}$ (SE)	$b_{Yk \times Lag^2}$ (SE)	*YX* Models b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag^2} (SE)	$b_{Yk \times Lag}$ (SE)	$b_{Yk \times Lag^2}$ (SE)
0,1,2,3,4,5,6	0.796	0.000	0.000	-0.160	0.011	0.278	0.659	0.000	0.000	-0.133	0.009
($Y_9,Y_8,Y_7,Y_6,Y_5,Y_4,Y_3$)	(0.066)	(0.051)	(0.008)	(0.051)	(0.008)	(0.028)	(0.064)	(0.048)	(0.008)	(0.049)	(0.008)
1,2,3,4,5,6,7	0.771	0.001	0.000	-0.145	0.009	0.326	0.612	0.000	0.000	-0.115	0.007
($Y_8,Y_7,Y_6,Y_5,Y_4,Y_3,Y_2$)	(0.124)	(0.070)	(0.009)	(0.071)	(0.009)	(0.028)	(0.118)	(0.066)	(0.008)	(0.066)	(0.008)
2,3,4,5,6,7,8	0.746	0.000	0.000	-0.132	0.007	0.354	0.583	-0.001	0.000	-0.104	0.006
($Y_7,Y_6,Y_5,Y_4,Y_3,Y_2,Y_1$)	(0.206)	(0.090)	(0.009)	(0.090)	(0.009)	(0.029)	(0.192)	(0.084)	(0.008)	(0.084)	(0.008)

Table 15.

Results for eight-lag, quadratic LAM models using exponential data

	Y Models					*YX* Models					
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag}^2 (SE)	$b_{Yk\times Lag}$ (SE)	$b_{Yk\times Lag}^2$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag}^2 (SE)	$b_{Yk\times Lag}$ (SE)	$b_{Yk\times Lag}^2$ (SE)
0,1,2,3,4,5,6,7	0.790	0.000	0.000	-0.155	0.010	0.294	0.644	0.000	0.000	-0.126	0.008
(Y_9,Y_8,Y_7,Y_6,Y_5,Y_4,Y_3,Y_2)	(0.069)	(0.046)	(0.006)	(0.046)	(0.006)	(0.028)	(0.067)	(0.043)	(0.006)	(0.044)	(0.006)
1,2,3,4,5,6,7,8	0.768	-0.001	0.000	-0.141	0.008	0.335	0.608	-0.001	0.000	-0.112	0.006
(Y_8,Y_7,Y_6,Y_5,Y_4,Y_3,Y_2,Y_1)	(0.120)	(0.061)	(0.007)	(0.061)	(0.007)	(0.028)	(0.113)	(0.057)	(0.006)	(0.057)	(0.006)

Table 16.

Results for nine-lag, quadratic LAM models using exponential data

	Y Models					*YX* Models					
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag}^2 (SE)	$b_{Yk\times Lag}$ (SE)	$b_{Yk\times Lag}^2$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	b_{Lag}^2 (SE)	$b_{Yk\times Lag}$ (SE)	$b_{Yk\times Lag}^2$ (SE)
0,1,2,3,4,5,6,7,8	0.788	0.000	0.000	-0.149	0.009	0.306	0.638	0.000	0.000	-0.121	0.007
(Y_9,Y_8,Y_7,Y_6,Y_5,Y_4,Y_3,Y_2,Y_1)	(0.072)	(0.041)	(0.0005)	(0.042)	(0.005)	(0.028)	(0.069)	(0.039)	(0.0005)	(0.039)	(0.005)

Table 17.

Correlations for linear data model

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	X_9
Y_1	1.00										
Y_2	0.09	1.00									
Y_3	0.05	0.52	1.00								
Y_4	0.03	0.35	0.68	1.00							
Y_5	0.02	0.27	0.51	0.76	1.00						
Y_6	0.02	0.22	0.41	0.61	0.80	1.00					
Y_7	0.02	0.18	0.34	0.51	0.67	0.84	1.00				
Y_8	0.01	0.15	0.30	0.44	0.58	0.72	0.86	1.00			
Y_9	0.01	0.14	0.26	0.38	0.51	0.63	0.75	0.88	1.00		
Y_{10}	0.01	0.11	0.21	0.31	0.41	0.51	0.61	0.71	0.81	1.00	
X_9	0.01	0.07	0.13	0.19	0.25	0.31	0.38	0.44	0.50	0.41	1.00

Table 18.

Results for fixed lag analyses linear data

	Y Models		*YX* Models			
	Analytical	Simulation	Analytical		Simulation	
Lag	b_{Yk}	b_{Yk}	b_{X9}	b_{Yk}	b_{X9}	b_{Yk}
(Y_k)	(SE)	(SE)	(SE)	(SE)	(SE)	(SE)
0	0.810	0.810	0.000	0.810	0.000	0.810
(Y_9)	(0.019)	(0.019)	(0.021)	(0.021)	(0.021)	(0.021)
1	0.710	0.710	0.116	0.659	0.115	0.659
(Y_8)	(0.022)	(0.022)	(0.025)	(0.025)	(0.025)	(0.025)
2	0.610	0.610	0.204	0.533	0.203	0.534
(Y_7)	(0.025)	(0.025)	(0.026)	(0.026)	(0.026)	(0.026)
3	0.510	0.510	0.271	0.425	0.271	0.425
(Y_6)	(0.027)	(0.027)	(0.027)	(0.027)	(0.027)	(0.027)
4	0.410	0.410	0.322	0.329	0.321	0.328
(Y_5)	(0.029)	(0.029)	(0.028)	(0.028)	(0.028)	(0.028)
5	0.310	0.309	0.359	0.241	0.358	0.241
(Y_4)	(0.030)	(0.030)	(0.028)	(0.028)	(0.028)	(0.028)
6	0.210	0.208	0.384	0.160	0.384	0.159
(Y_3)	(0.031)	(0.031)	(0.029)	(0.029)	(0.029)	(0.029)
7	0.110	0.109	0.399	0.083	0.399	0.081
(Y_2)	(0.031)	(0.031)	(0.029)	(0.029)	(0.029)	(0.029)
8	0.010	0.011	0.405	0.008	0.404	0.008
(Y_1)	(0.032)	(0.032)	(0.029)	(0.029)	(0.029)	(0.029)

Table 19.

Results for two-lag, linear LAM models using linear data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk\times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk\times Lag}$ (SE)
0,1	0.809	-0.001	-0.100	0.059	0.780	-0.001	-0.096
(Y_9,Y_8)	(0.029)	(0.041)	(0.041)	(0.023)	(0.031)	(0.041)	(0.041)
1,2	0.808	0.001	-0.099	0.160	0.728	0.002	-0.089
(Y_8,Y_7)	(0.075)	(0.047)	(0.047)	(0.026)	(0.075)	(0.047)	(0.047)
2,3	0.812	-0.001	-0.101	0.238	0.693	0.000	-0.086
(Y_7,Y_6)	(0.134)	(0.052)	(0.052)	(0.027)	(0.129)	(0.050)	(0.051)
3,4	0.814	-0.002	-0.101	0.296	0.669	-0.001	-0.084
(Y_6,Y_5)	(0.199)	(0.056)	(0.056)	(0.028)	(0.189)	(0.053)	(0.053)
4,5	0.807	0.001	-0.099	0.340	0.638	0.000	-0.079
(Y_5,Y_4)	(0.267)	(0.059)	(0.059)	(0.028)	(0.250)	(0.055)	(0.055)
5,6	0.821	0.000	-0.102	0.371	0.626	-0.001	-0.078
(Y_4,Y_3)	(0.338)	(0.061)	(0.061)	(0.029)	(0.313)	(0.056)	(0.057)
6,7	0.790	-0.002	-0.097	0.392	0.597	-0.002	-0.073
(Y_3,Y_2)	(0.408)	(0.062)	(0.063)	(0.029)	(0.375)	(0.057)	(0.057)
7,8	0.798	0.001	-0.098	0.402	0.599	0.001	-0.074
(Y_2,Y_1)	(0.475)	(0.063)	(0.063)	(0.029)	(0.435)	(0.058)	(0.058)

Table 20.

Results for three-lag, linear LAM models using linear data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk×Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk×Lag}$ (SE)
0,1,2 (Y_9,Y_8,Y_7)	0.811 (0.035)	0.001 (0.028)	-0.100 (0.028)	0.110 (0.024)	0.756 (0.037)	0.001 (0.027)	-0.094 (0.027)
1,2,3 (Y_8,Y_7,Y_6)	0.807 (0.067)	0.001 (0.031)	-0.099 (0.031)	0.199 (0.026)	0.708 (0.067)	0.001 (0.030)	-0.086 (0.030)
2,3,4 (Y_7,Y_6,Y_5)	0.818 (0.105)	0.001 (0.034)	-0.103 (0.034)	0.267 (0.027)	0.683 (0.102)	0.000 (0.032)	-0.086 (0.032)
3,4,5 (Y_6,Y_5,Y_4)	0.812 (0.147)	0.000 (0.036)	-0.101 (0.036)	0.318 (0.028)	0.652 (0.139)	0.000 (0.034)	-0.081 (0.034)
4,5,6 (Y_5,Y_4,Y_3)	0.801 (0.190)	0.000 (0.037)	-0.098 (0.037)	0.355 (0.028)	0.627 (0.177)	0.000 (0.035)	-0.077 (0.035)
5,6,7 (Y_4,Y_3,Y_2)	0.812 (0.234)	0.001 (0.038)	-0.101 (0.039)	0.381 (0.029)	0.622 (0.216)	0.001 (0.036)	-0.077 (0.036)
6,7,8 (Y_3,Y_2,Y_1)	0.809 (0.276)	0.002 (0.039)	-0.100 (0.039)	0.396 (0.029)	0.610 (0.254)	0.003 (0.036)	-0.075 (0.036)

Table 21.

Results for four-lag, linear LAM models using linear data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)
0,1,2,3 (Y_9,Y_8,Y_7,Y_6)	0.810 (0.040)	0.001 (0.021)	-0.100 (0.022)	0.155 (0.025)	0.733 (0.041)	0.001 (0.021)	-0.090 (0.021)
1,2,3,4 (Y_8,Y_7,Y_6,Y_5)	0.812 (0.065)	-0.001 (0.024)	-0.101 (0.024)	0.232 (0.027)	0.696 (0.064)	-0.001 (0.023)	-0.086 (0.023)
2,3,4,5 (Y_7,Y_6,Y_5,Y_4)	0.818 (0.094)	-0.001 (0.025)	-0.102 (0.026)	0.291 (0.028)	0.671 (0.090)	-0.001 (0.024)	-0.084 (0.024)
3,4,5,6 (Y_6,Y_5,Y_4,Y_3)	0.816 (0.124)	0.000 (0.027)	-0.102 (0.027)	0.335 (0.028)	0.649 (0.117)	0.000 (0.025)	-0.081 (0.025)
4,5,6,7 (Y_5,Y_4,Y_3,Y_2)	0.820 (0.156)	-0.001 (0.028)	-0.102 (0.028)	0.366 (0.029)	0.635 (0.145)	-0.001 (0.026)	-0.079 (0.026)
5,6,7,8 (Y_4,Y_3,Y_2,Y_1)	0.802 (0.188)	0.000 (0.028)	-0.099 (0.029)	0.387 (0.029)	0.605 (0.174)	0.000 (0.026)	-0.075 (0.026)

Table 22.

Results for five-lag, linear LAM models using linear data

Lags (Y_k)	*Y* Models b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)	*YX* Models b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)
0,1,2,3,4	0.809	0.001	-0.100	0.192	0.713	0.001	-0.088
(Y_9,Y_8,Y_7,Y_6,Y_5)	(0.044)	(0.018)	(0.018)	(0.026)	(0.044)	(0.017)	(0.018)
1,2,3,4,5	0.809	0.000	-0.100	0.260	0.679	0.000	-0.084
(Y_8,Y_7,Y_6,Y_5,Y_4)	(0.064)	(0.019)	(0.020)	(0.027)	(0.063)	(0.019)	(0.019)
2,3,4,5,6	0.814	0.001	-0.101	0.311	0.658	0.001	-0.082
(Y_7,Y_6,Y_5,Y_4,Y_3)	(0.087)	(0.021)	(0.021)	(0.028)	(0.084)	(0.019)	(0.020)
3,4,5,6,7	0.808	-0.001	-0.100	0.349	0.634	0.000	-0.078
(Y_6,Y_5,Y_4,Y_3,Y_2)	(0.112)	(0.022)	(0.022)	(0.028)	(0.105)	(0.020)	(0.020)
4,5,6,7,8	0.808	-0.001	-0.100	0.375	0.622	-0.002	-0.077
(Y_5,Y_4,Y_3,Y_2,Y_1)	(0.137)	(0.022)	(0.022)	(0.029)	(0.127)	(0.020)	(0.021)

Table 23.

Results for six-lag, linear LAM models using linear data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)
0,1,2,3,4,5	0.810	0.001	-0.100	0.222	0.698	0.000	-0.086
(Y_9,Y_8,Y_7,Y_6,Y_5,Y_4)	(0.046)	(0.015)	(0.015)	(0.027)	(0.047)	(0.015)	(0.015)
1,2,3,4,5,6	0.811	-0.001	-0.101	0.283	0.669	-0.001	-0.083
(Y_8,Y_7,Y_6,Y_5,Y_4,Y_3)	(0.064)	(0.017)	(0.017)	(0.028)	(0.063)	(0.016)	(0.016)
2,3,4,5,6,7	0.812	0.000	-0.100	0.327	0.649	0.000	-0.080
(Y_7,Y_6,Y_5,Y_4,Y_3,Y_2)	(0.084)	(0.017)	(0.017)	(0.028)	(0.080)	(0.016)	(0.016)
3,4,5,6,7,8	0.814	0.000	-0.101	0.358	0.635	0.000	-0.079
(Y_6,Y_5,Y_4,Y_3,Y_2,Y_1)	(0.104)	(0.018)	(0.018)	(0.029)	(0.097)	(0.017)	(0.017)

Table 24.

Results for seven-lag, linear LAM models using linear data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)
0,1,2,3,4,5,6	0.810	0.000	-0.100	0.248	0.686	0.000	-0.085
(Y_9,Y_8,Y_7,Y_6,Y_5,Y_4,Y_3)	(0.048)	(0.013)	(0.014)	(0.027)	(0.048)	(0.013)	(0.013)
1,2,3,4,5,6,7	0.812	0.000	-0.101	0.301	0.661	0.000	-0.082
(Y_8,Y_7,Y_6,Y_5,Y_4,Y_3,Y_2)	(0.064)	(0.014)	(0.014)	(0.028)	(0.062)	(0.014)	(0.014)
2,3,4,5,6,7,8	0.809	0.000	-0.099	0.339	0.639	0.000	-0.079
(Y_7,Y_6,Y_5,Y_4,Y_3,Y_2,Y_1)	(0.081)	(0.015)	(0.015)	(0.028)	(0.077)	(0.014)	(0.014)

Table 25.

Results for eight-lag, linear LAM models using linear data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)
0,1,2,3,4,5,6,7 (Y_9,Y_8,Y_7,Y_6,Y_5,Y_4,Y_3,Y_2)	0.811 (0.050)	0.000 (0.012)	-0.101 (0.012)	0.270 (0.027)	0.677 (0.050)	0.000 (0.012)	-0.084 (0.012)
1,2,3,4,5,6,7,8 (Y_8,Y_7,Y_6,Y_5,Y_4,Y_3,Y_2,Y_1)	0.809 (0.064)	0.000 (0.013)	-0.100 (0.013)	0.315 (0.028)	0.652 (0.062)	0.000 (0.012)	-0.081 (0.012)

Table 26.

Results for nine-lag, linear LAM models using linear data

	Y Models			*YX* Models			
Lags (Y_k)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)	b_X (SE)	b_{Yk} (SE)	b_{Lag} (SE)	$b_{Yk \times Lag}$ (SE)
0,1,2,3,4,5,6,7,8 (Y_9,Y_8,Y_7,Y_6,Y_5, Y_4,Y_3,Y_2,Y_1)	0.809 (0.052)	0.000 (0.011)	-0.100 (0.011)	0.287 (0.028)	0.666 (0.051)	0.000 (0.010)	-0.082 (0.011)

Table 27.

Descriptive statistics for variables used in LAM models

Variable	Mean	Min.	Max.	Std. Dev.
Lag 14m-24m	9.67	3.0	17.0	1.92
Lag 14m-36m	22.05	14.0	38.0	2.21
HOME Total 14m	25.98	6.46	31.00	3.61
Child Negativity 14m	2.11	1.00	7.00	1.11
Parent Negative Regard 14m	1.46	1.00	7.00	0.79
Parent Intrusiveness 14m	2.49	1.00	7.00	1.24
MDI 24m	89.08	49.00	134.00	13.68
Child Sustained Attention 24m	5.01	1.00	7.00	0.95
Child Negativity 36m	1.28	1.00	7.00	0.57

Figure 1.

A version of Cattell's data box

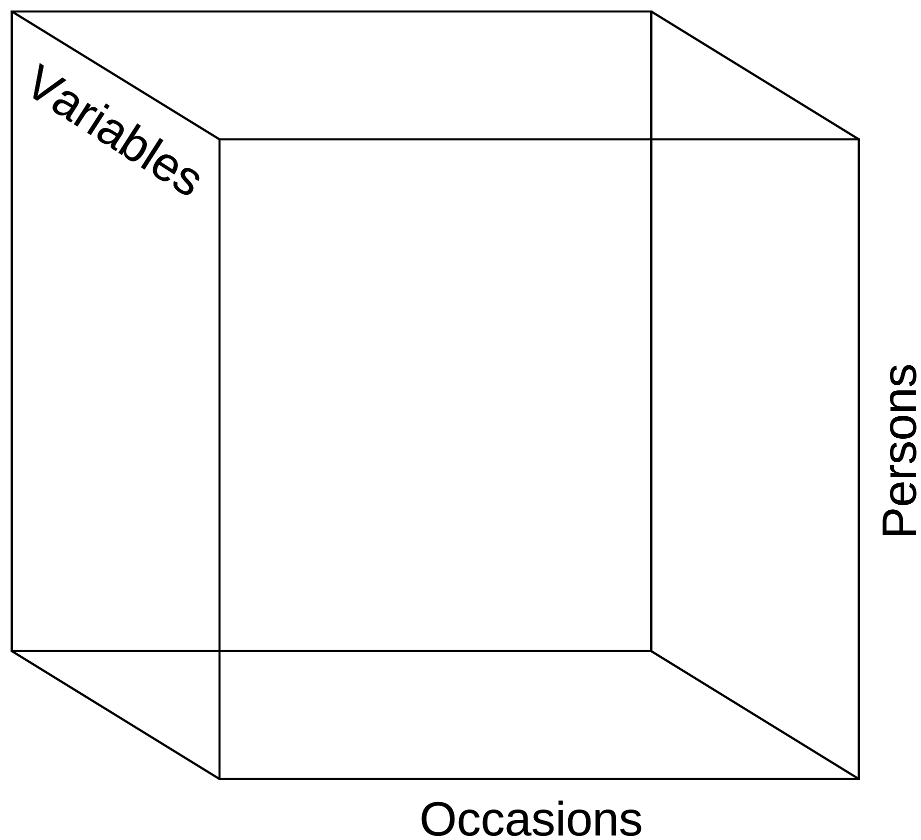

Figure 2.

Diagram of Cohen's premature covariate scenario

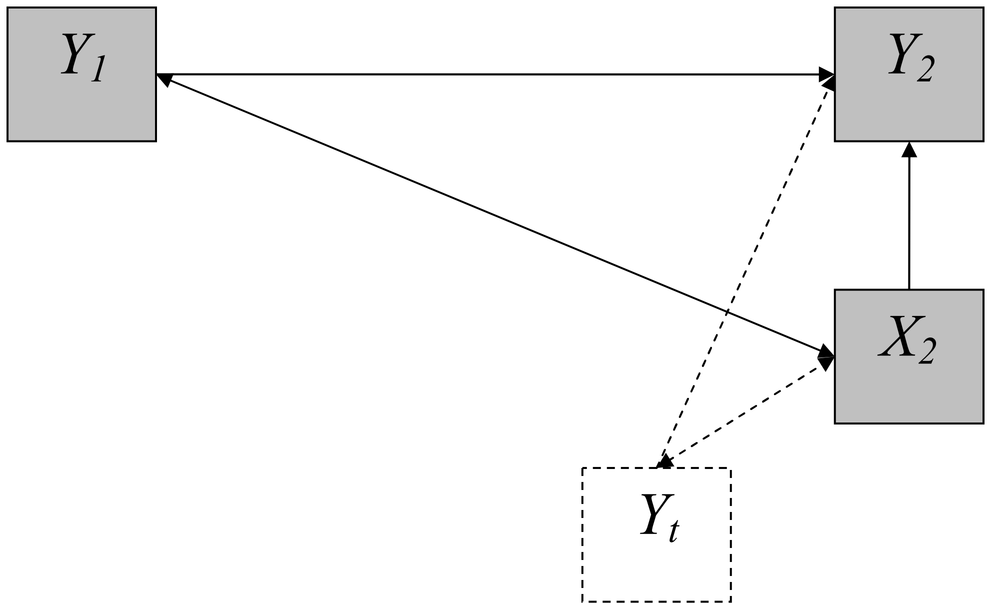

Figure 3.

Three-variable model from Collins & Graham (2001)

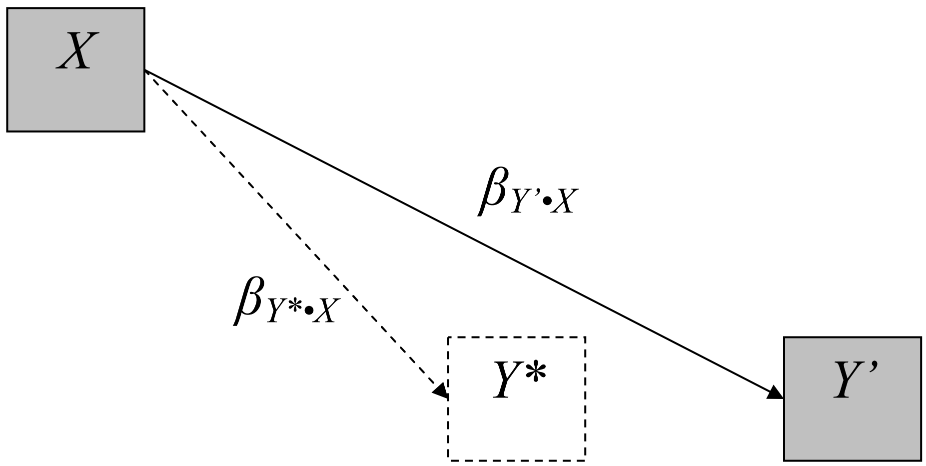

Figure 4.

Repeated measures of X

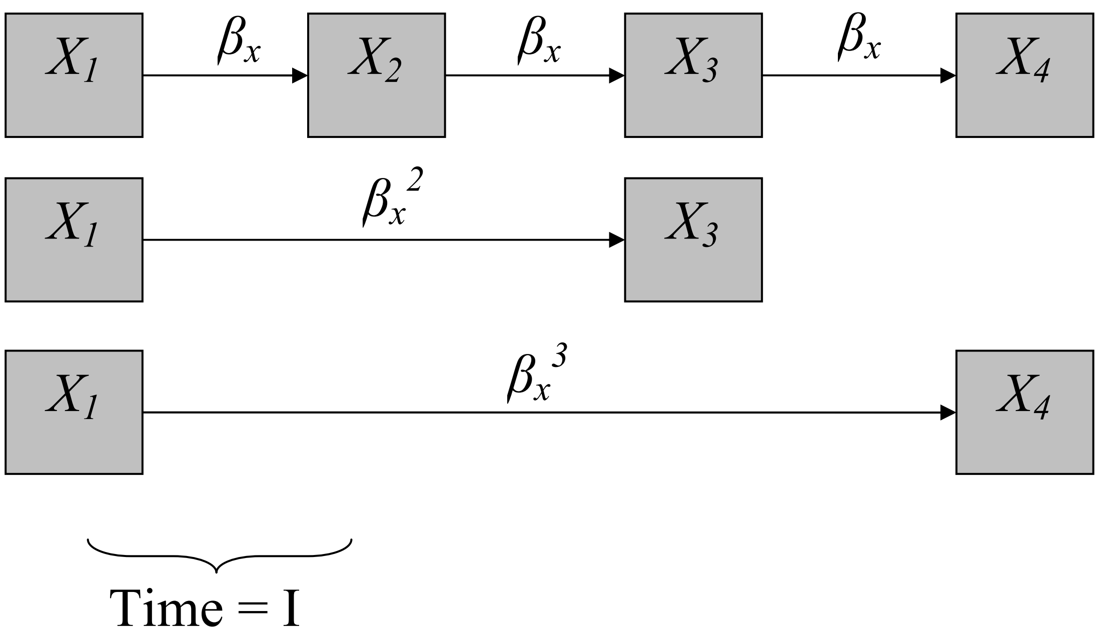

Figure 5.

Two cross-lagged panel models using different lags lengths

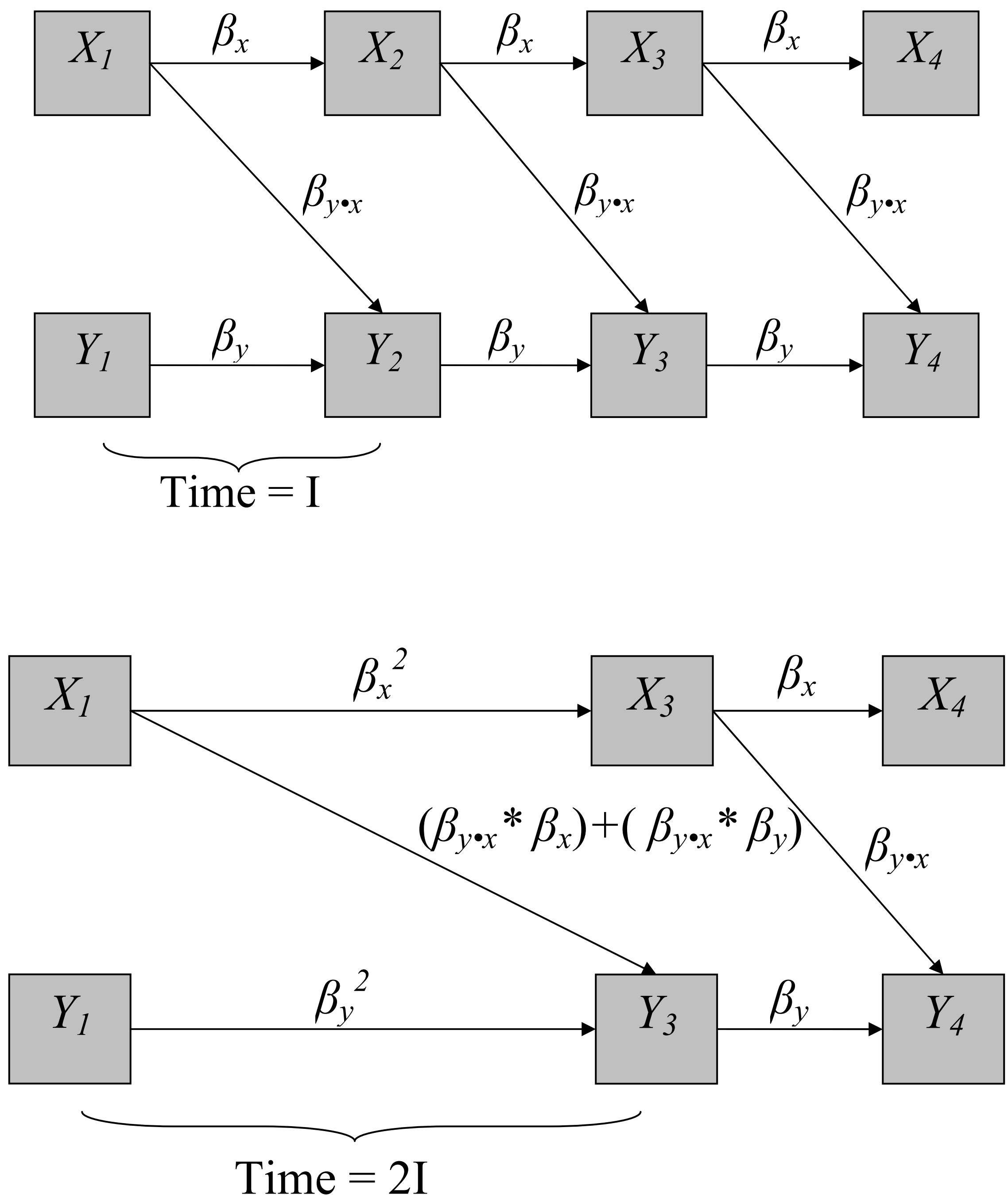

Figure 6.

Standardized autoregressive effects from Kanfer & Ackerman (1989)

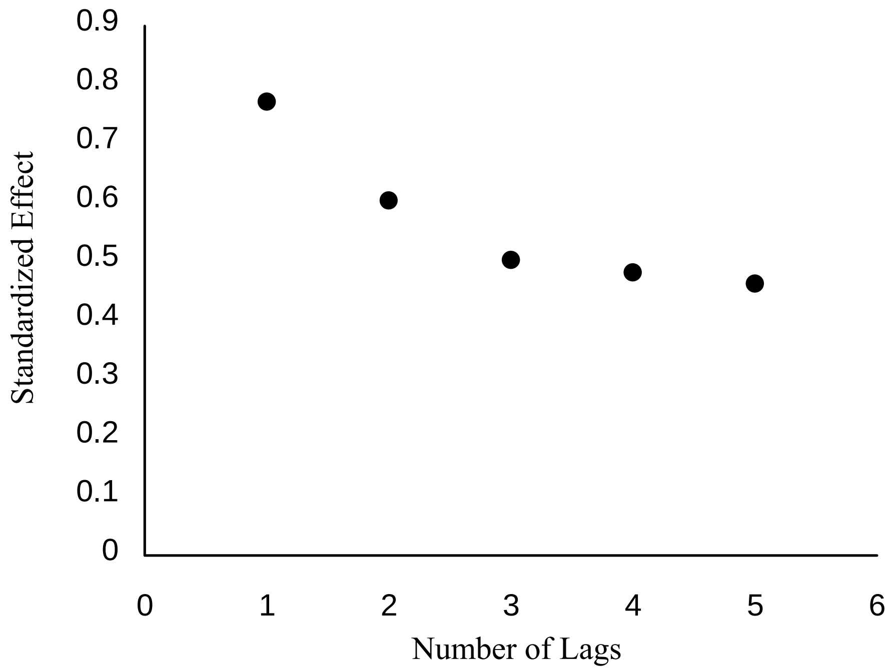

Figure 7.

Standardized cross-lagged effects from Kanfer & Ackerman (1989)

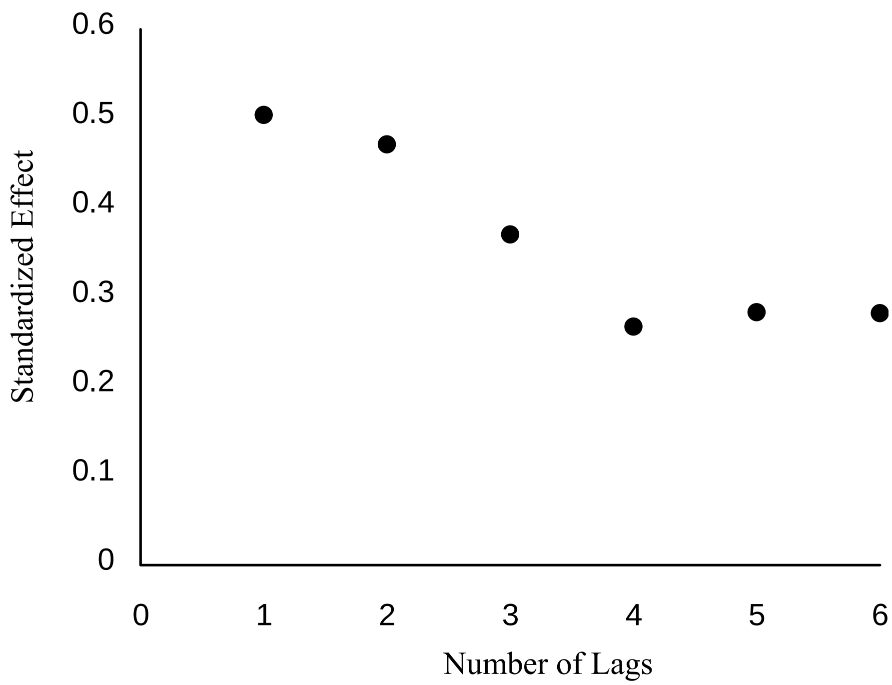

Figure 8.

Hypothetical multi-wave two-variable panel model

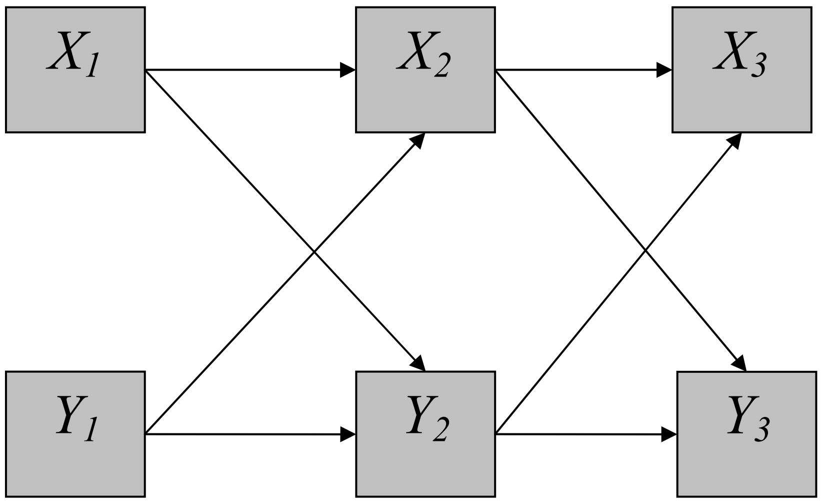

Figure 9.

A two-variable multi-wave panel model

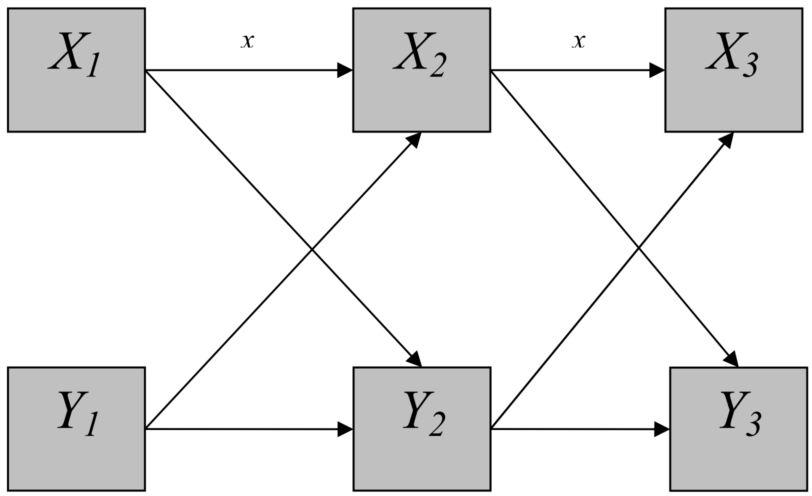

Figure 10.

Diagram of variables used in simulation study

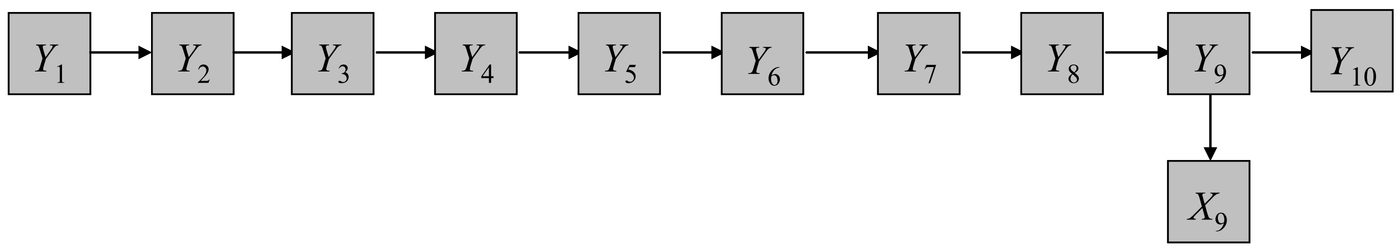

Figure 11.

Change in the autoregressive effect for Y as lag changes

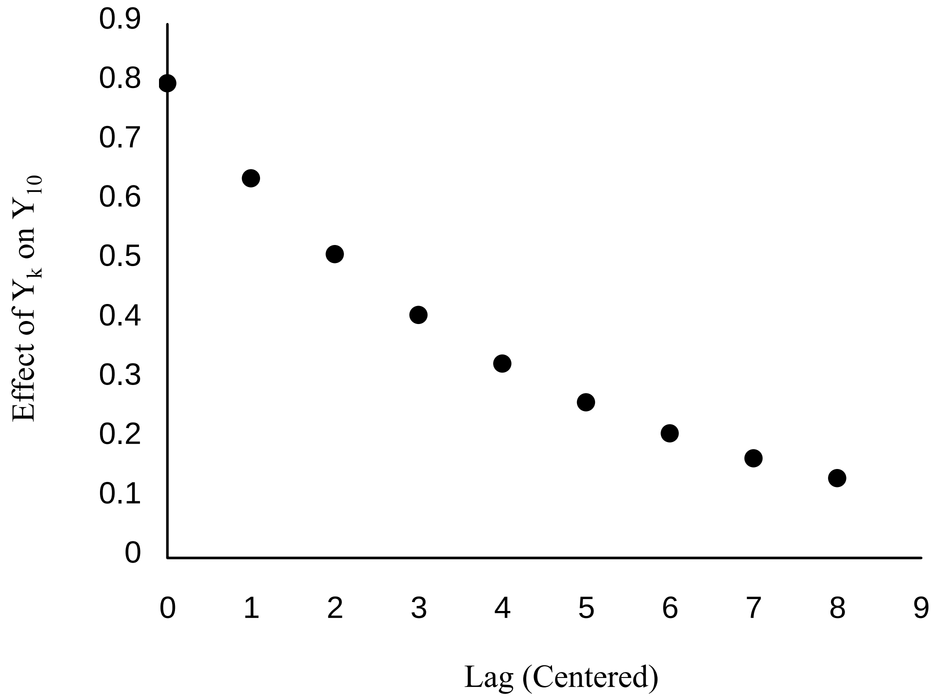

Figure 12.

Change in the autoregressive effect for Y as lag changes

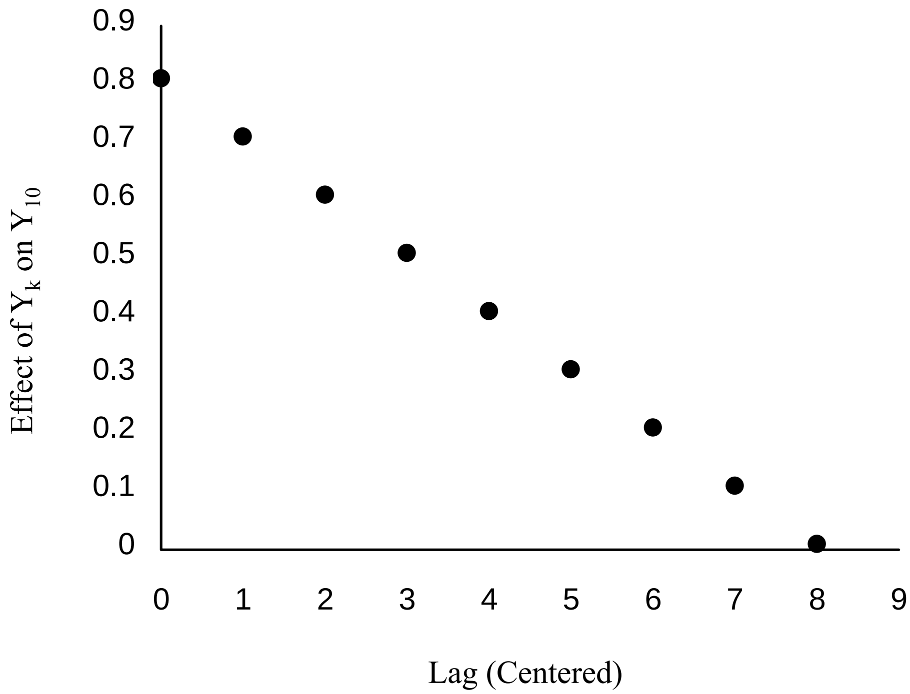

Figure 13.

Home environment and Mental Development Index

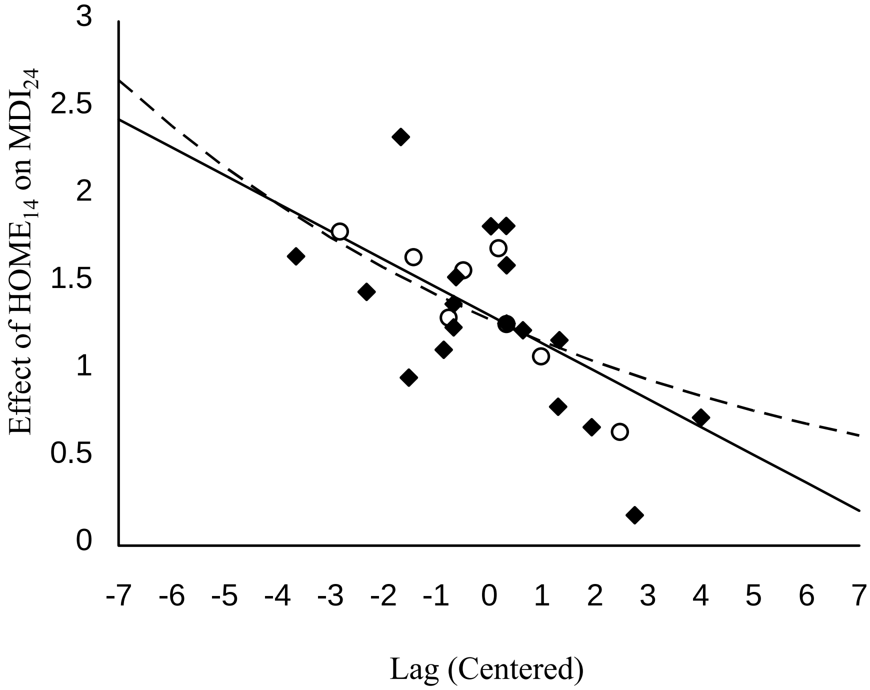

Figure 14.

Parent intrusiveness and child sustained attention

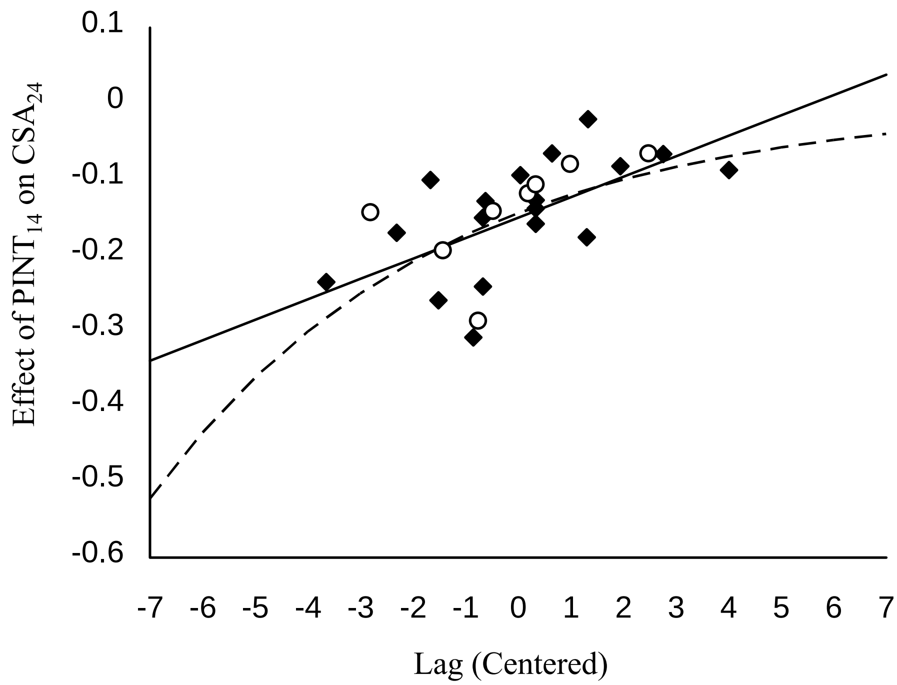

Figure 15.

Parent negative regard and child negativity