

Three Essays on Industrial Organization

By

Yang Seung Lee

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Chairperson

Committee members* _____*

_____*

_____*

_____*

Date defended: _____

The Dissertation Committee for Yang Seung Lee certifies
that this is the approved version of the following dissertation:

Three Essays on Industrial Organization

Committee:

Chairperson

Committee members* _____*

_____*

_____*

_____*

Date approved: _____

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Chapter 1. Public Projects and Innovation Game in Private Sector

1. Introduction

What influence does the government have on the outcome of innovation games? During the last ten years of rapid industrial progress and globalization, the governments in many emerging economies have attempted to actively help the private sector's innovative efforts. For example, Korean government established KOSEF (Korea Science and Engineering Foundation) to motivate more innovate activities in a private sector. KOSEF actually runs some programs such as Basic Science Programs, National R&D Programs, Nuclear R&D programs, Research Promotion Programs, and so on. Chinese government also runs Torch program (one National program for science and technology) to establish high-tech industrial development areas for more advanced economy. This program is known to involve a number of projects in various fields of new technology.

If the government provides better infrastructure or any other type of specialized inputs to the firms competing for innovation, does it increase or decrease the firms' expenditure on innovative activities? At first glance, it appears that since provision of better infrastructure will increase all firms' profits at present and in the future, it may not have any incremental effect on the firms' innovative activities. However, in many emerging economies, there is substantial uncertainty about completion of government projects because of budget problems. Sometimes, a project remains incomplete due to lack of funding. A firm investing on innovative activities may find that the government project is completed before the firm successfully innovates, in which case its profits before and after innovation both will go up. On the other hand, a firm may find that the government project is finished after it innovates, thus increasing post-innovation profits.

In this research, we consider innovation games, where two firms are spending money on innovative activities. The money spent on innovation affects the probability of success and either firm may be the first to innovate. In section 2, the innovation considered is non-

drastic, both firms are incumbent duopolists. The standard innovation race literature (Bhattacharya (1986), Reingaum (1981)) examines the nature of dynamic Nash equilibria in this game. We consider a modified version of this game where the profits of both firms, both pre-and post-innovation may be affected by the successful completion of a government project. Our main enquiry is about the effect of the timing of completion of the government project on the equilibria of an innovation game. In this model, the firms play an innovation game where the probability of success follows a standard stochastic process, but the pre- and post-innovation profits are affected by the timing of completion of a government project.

After characterizing a dynamic Nash equilibrium for this game, it is shown that both firms' equilibrium expenditure will depend on the probability of completion of the project. It is shown that if the probability that the government will complete the project before either firm successfully innovates increases, then, in a somewhat paradoxical way, both firms will spend smaller amount on innovation because the government sponsored projects mainly enhances their duopoly profits. However, it is shown that under certain circumstances, the reverse can be true, i.e., a higher probability of completion of the government project will inspire innovative activities of both firms. Therefore, government sponsored projects that provide infrastructure or specialized inputs to innovating firms may inspire the level of innovative activities even though the success of government-projects mainly improve their duopoly profits rather than monopoly profits. The intuitive reasoning behind this result is as follows: Under uncertainty of a completion of the government-sponsored projects, the average of monopoly profits might increase for both firms. Thus, it could happen that firms competitively increase their R&D expenditures to win the innovation game.

In section 3, the same innovation game is considered for a drastic innovation (Gilbert and Newberry (1982)), where one firm is and incumbent and has a pre-innovation monopoly. The entrant firm has no pre-innovation profits, but will replace the incumbent if it innovates first. The conclusion obtained here is that a higher probability of completion of the government's project will increase the incumbent firm's innovative expenditure more than the entrant's expenditure. This result implies that government support can actually

lead to a higher degree of persistence of monopoly in emerging economies. The intuition behind the result is as follows: The entrant spends more R&D expenditure than the incumbent. The uncertainty of a completion of government-sponsored projects makes both firms to increase their expenditure when the probability of completion of the government's project increases because the average of post-innovation monopoly profits would increase for both firms. In fact, the incumbent increases expenditure more than the entrant since the entrant's larger R&D expenditure deepens the incumbent's concern for its future.

2. Non-Drastic Innovation

In this section, both firms are incumbent duopolists, and any innovation from either side will eventually change the duopoly into a monopoly.

The government-sponsored projects

A public sector and a private sector seem to be vertically-structured in some aspects because many private companies are provided a supportive service from a government. For example, a professional research base is commonly found in most nations and many private firms actually do their R&D works in that base. In this context, one government has projects to provide better infrastructure or any other type of specialized inputs for private firms. There are two ways by which the government project is completed. First, the project is completed before a private firm innovates. Second, the project is completed after a private firm innovates. The first case mainly enhances both firms' duopoly profits and also monopoly profits. The second case improves only the monopoly profits for both firms. Thus, both firms play an innovation game in which they strategically invest on R&D projects for their futures. For example, many governments support the Nanotechnology project or have a grand plan to provide the better infrastructure for the project that would lead innovations from all science fields.

A. The government's project is completed before a private firm innovates

The government's innovation or other beneficial upstream projects will shift private firms' duopoly profits. The enhanced duopoly profits improve private firms' competence so an instant monopoly profit would also be increased when they actually innovate themselves. Both private firms invest on R&D to bring an innovation earlier by their own hands. As they spend some money on R&D, the timing of innovation would be faster because their innovation occurrences are exponentially distributed, and their instant success rates depend on the R&D expenditures.

Formally, if b is Firm B's instantaneous R&D spending, then the instant success rate for innovation u is $\beta(b)$. The time of occurrence of u is denoted as τ_u , which is exponentially distributed with a density function.

$$\psi_u(\tau_u) = \beta(b)e^{-\beta(b)\tau_u}, \quad 0 < \tau_u < \infty$$

Similarly, if Firm C spends c as an instantaneous investment on R&D, then the instant success rate for innovation n is $\gamma(c)$. The arrival timing of n is denoted as τ_n , which is exponentially distributed with a density function.

$$\psi_n(\tau_n) = \gamma(c)e^{-\gamma(c)\tau_n}, \quad 0 < \tau_n < \infty$$

1) Final Stages

An innovation race terminates in this final stage by either private firm's success. It means that both firms actually stop spending for R&D projects. If a firm succeeds in an innovation then a firm would gain a monopoly profit because of its overall dominance over the market. However, the rival would gain no more profit in this market. Depending on the result of the innovation race, a firm's instant profit should be either a monopoly profit

or zero by the new technology that can dominate over the market. There are two different final stages.

(1) Final Stage d_1

In this stage, Firm B successfully innovates so the innovation race is finished because both firms would cease further spending on R&D. In this case, Firm B acquires a patent by an innovation and becomes a monopolist. The rival (Firm C) gives up its R&D projects. Thus, no private firm needs to spend R&D expenditure any more. Both firms' Nash Equilibrium strategies are to choose zero R&D expenditure in this stage. The instant profit vector is $(B(d_1 + \Omega), 0)$. From the timing of an innovation, the final stage begins and the expected discounted total profits for both firms are as follows.

$$H_B^{d_1} = \int_0^{\infty} e^{-rt} [B(d_1 + \Omega)] dt = \frac{[B(d_1 + \Omega)]}{r}$$

$$H_C^{d_1} = \int_0^{\infty} e^{-rt} [0] dt = 0$$

(2) Final Stage d_2

When the other private firm, Firm C, accomplishes an innovation and monopolizes all market shares by using the technological advance, the rival firm (Firm B) has nothing to gain in this market. Accordingly, the profit vector becomes $(0, C(d_2 + \Omega))$. At this stage, the expected discounted total profits for both firms are as follows.

$$H_B^{d_2} = \int_0^{\infty} e^{-rt} [0] dt = 0$$

$$H_C^{d_2} = \int_0^{\infty} e^{-rt} [C(d_2 + \Omega)] dt = \frac{[C(d_2 + \Omega)]}{r}$$

2) Non-Final Stage

Different from final stages, both firms' Nash Equilibrium R&D strategies are not zero in a Non-Final Stage. They continue to spend money on R&D for an innovation as early as possible. This stage goes on unless either firm declares a success of an innovation.

(1) The Initial Stage

In this stage, no firm achieves an innovation and both participants make their efforts to develop the new technology for their own sakes. This Initial Stage will last until either Firm B or Firm C succeeds in an innovation, and the stage turns into a final stage in which the firms stop R&D investment. There are two final stages that would occur by the first-innovator. The timing that either final stage arrives is randomly determined by exponential distribution. The instant profit vector is $(B_0 + \Omega, C_0 + \Omega)$. At this Initial Stage, the expected discounted total profits for both firms are as follows.

The probability that no firms have succeeded in innovation until time τ , and Firm B has its own innovation u at time τ is

$$\begin{aligned}\psi_u(\tau) &= \Pr ob(\tau < \tau_u < \tau + d\tau, \tau_n > \tau) = \Pr ob(\tau < \tau_u < \tau + d\tau) \cdot \Pr ob(\tau_n > \tau) \\ &= \Pr ob(\tau < \tau_u < \tau + d\tau) \cdot [1 - \Pr ob(\tau_n \leq \tau)] = \beta(b)e^{-\beta(b)\tau} [1 - (1 - e^{-\gamma(c)\tau})] = \beta(b)e^{-[\beta(b) + \gamma(c)]\tau}\end{aligned}$$

Similarly, the probability that no firms have succeeded in innovation until time τ , and Firm C has its own innovation n at time τ is

$$\begin{aligned}\psi_n(\tau) &= \Pr ob(\tau < \tau_n < \tau + d\tau, \tau_u > \tau) = \Pr ob(\tau < \tau_n < \tau + d\tau) \cdot \Pr ob(\tau_u > \tau) \\ &= \Pr ob(\tau < \tau_n < \tau + d\tau) \cdot [1 - \Pr ob(\tau_u \leq \tau)] = \gamma(c)e^{-\gamma(c)\tau} [1 - (1 - e^{-\beta(b)\tau})] = \gamma(c)e^{-[\gamma(c) + \beta(b)]\tau}\end{aligned}$$

An innovation happens at time τ by either Firm B or Firm C. Intuitively speaking, Firm

B can calculate its expected discounted profit under the possibility of its own innovation, and under the possibility of its rival firm's innovation. The sum of those two expected discounted profits under both cases represents Firm B's real expected discounted profit in the Initial Stage. If Firm B's expectation is formalized then it can be shown as below.

$$H_B^0 = \int_0^\infty \left[\int_0^\tau e^{-rt} (B_0 + \Omega - b) dt + \int_\tau^\infty e^{-rt} (B(d_1 + \Omega)) dt \right] \psi_u(\tau) d\tau \\ + \int_0^\infty \left[\int_0^\tau e^{-rt} (B_0 + \Omega - b) dt + \int_\tau^\infty e^{-rt} (0) dt \right] \psi_n(\tau) d\tau$$

From the function above, it seems that Firm B's instant profits should be discounted first and those discounted profits are actually expected. In discounting, the randomly distributed timing of innovation plays an important role as below.

$$H_B^0 = \int_0^\infty \left[\frac{[B_0 + \Omega - b]}{r} [1 - e^{-r\tau}] \right] \beta(b) e^{-[\beta(b) + \gamma(c)]\tau} d\tau + \int_0^\infty \left[-\frac{[B(d_1 + \Omega)]}{r} [e^{-r\tau}] \right] \beta(b) e^{-[\beta(b) + \gamma(c)]\tau} d\tau \\ + \int_0^\infty \left[\frac{[B_0 + \Omega - b]}{r} [1 - e^{-r\tau}] \right] \gamma(c) e^{-[\gamma(c) + \beta(b)]\tau} d\tau + \int_0^\infty [0] \gamma(c) e^{-[\gamma(c) + \beta(b)]\tau} d\tau$$

First of all, all integrals can be found by the random variable τ , which actually represents the timing of innovation. Then, Firm B's expected discounted profit function entirely depends on the random variable τ . Because of the probability densities, the function above can be simplified as follows, and it should be a function of two different success rates. In this case, the success rates are not constant. They actually change with the R&D expenditures. The changing success rates provide the main reason why private firms join the innovation race.

$$H_B^0 = \left[\frac{\left[\frac{[B_0 + \Omega - b]}{r} \right] \beta(b)}{\beta(b) + \gamma(c)} - \frac{\left[\frac{[B_0 + \Omega - b]}{r} \right] \beta(b)}{r + \beta(b) + \gamma(c)} \right] + \frac{\left[\frac{[B(d_1 + \Omega)]}{r} \right] \beta(b)}{r + \beta(b) + \gamma(c)}$$

$$+ \left[\frac{\left[\frac{[B_0 + \Omega - b]}{r} \right] \gamma(c)}{\beta(b) + \gamma(c)} - \frac{\left[\frac{[B_0 + \Omega - b]}{r} \right] \gamma(c)}{r + \beta(b) + \gamma(c)} \right] + 0$$

By arranging all terms, then it can be simplified as below.

$$H_B^0 = \frac{[B_0 + \Omega - b] + \beta(b)[H_B^{d_1}]}{r + \beta(b) + \gamma(c)} \quad \text{where} \quad H_B^{d_1} = \frac{[B(d_1 + \Omega)]}{r}$$

Intuitively, Firm B's simplified profit informs that an overall expected profit should be discounted by the new discount factor $[1 + r + \beta(b) + \gamma(c)]$ rather than $[1 + r]$. That is, the randomly distributed timing of innovation induces an overall expectation over an instant profit $[B_0 + \Omega - b]$ and the obtained discounted profit from a final stage $[H_B^{d_1}]$, and eventually discounts an overall expected profit by the new discount factor.

Firm C can expect its discounted profit as similar as Firm B. That is, its expected discounted profit can be calculated under both cases, the possibility of its own innovation and the possibility of its rival's innovation.

$$H_C^0 = \int_0^\infty \left[\int_0^\tau e^{-rt} (C_0 + \Omega - c) dt + \int_\tau^\infty e^{-rt} (C(d_2 + \Omega)) dt \right] \psi_n(\tau) d\tau$$

$$+ \int_0^\infty \left[\int_0^\tau e^{-rt} (C_0 + \Omega - c) dt + \int_\tau^\infty e^{-rt} (0) dt \right] \psi_u(\tau) d\tau$$

By integrating, Firm C's expected discounted profit function depends on the random variable τ as below.

$$\begin{aligned}
H_C^0 &= \int_0^{\infty} \left[\frac{[C_0 + \Omega - c]}{r} [1 - e^{-r\tau}] \right] \gamma(c) e^{-[\gamma(c) + \beta(b)]\tau} d\tau + \int_0^{\infty} \left[-\frac{[C(d_2 + \Omega)]}{r} [e^{-r\tau}] \right] \gamma(c) e^{-[\gamma(c) + \beta(b)]\tau} d\tau \\
&+ \int_0^{\infty} \left[\frac{[C_0 + \Omega - c]}{r} [1 - e^{-r\tau}] \right] \beta(b) e^{-[\beta(b) + \gamma(c)]\tau} d\tau + \int_0^{\infty} [0] \beta(b) e^{-[\beta(b) + \gamma(c)]\tau} d\tau
\end{aligned}$$

With the density functions, the expected value can be found as follows.

$$\begin{aligned}
H_C^0 &= \left[\frac{\left[\frac{[C_0 + \Omega - c]}{r} \right] \gamma(c)}{\beta(b) + \gamma(c)} - \frac{\left[\frac{[C_0 + \Omega - c]}{r} \right] \gamma(c)}{r + \beta(b) + \gamma(c)} \right] + \frac{\left[\frac{[C(d_2 + \Omega)]}{r} \right] \gamma(c)}{r + \beta(b) + \gamma(c)} \\
&+ \left[\frac{\left[\frac{[C_0 + \Omega - c]}{r} \right] \beta(b)}{\beta(b) + \gamma(c)} - \frac{\left[\frac{[C_0 + \Omega - c]}{r} \right] \beta(b)}{r + \beta(b) + \gamma(c)} \right] + 0
\end{aligned}$$

The obtained expected value above is simplified as follows.

$$H_C^0 = \frac{[C_0 + \Omega - c] + \gamma(c)[H_C^{d_2}]}{r + \beta(b) + \gamma(c)} \quad \text{where} \quad H_C^{d_2} = \frac{[C(d_2 + \Omega)]}{r}$$

The two expected forms of profit functions show that, by the innovation game, each firm's instant profit in the Initial Stage eventually evolves into the profit of either final stage by the success rate. In fact, this innovation race is a huge sequential game in which both participants play by R&D expenditure under an infinite horizon. Furthermore, the entire game has two sub-stages by an innovation, which randomly occurs from the competition between two firms. For each sub-stage, a discounted profit can be obtained. Since it is

already known that both firms would not spend any R&D expenditure in either final stage, a huge sequential game can be reduced to the Initial Stage in which both firms still spend the R&D expenditure for their own innovations. By using discounted profits from both final stages, the profit functions for both firms can be derived as H_B^0 and H_C^0 , and Nash Equilibrium (b^*, c^*) can be found in the reduced game.

A Nash equilibrium can be obtained in the Initial Stage because both firms try to maximize their expected discounted profit functions.

From the first order condition $\frac{\partial H_C^0}{\partial c} = 0$, a best response function is found such as $c^R = c(b)$. Similarly, a best response function is found such as $b^R = b(c)$ from the first order condition $\frac{\partial H_B^0}{\partial b} = 0$. From two conditions above, a Nash equilibrium is derived as (b^N, c^N) .

By deriving the Nash equilibrium, the backward induction is complete. The derived Nash equilibrium implies that the private firms determine the levels of the R&D expenditure when the innovation race begins, and their choices remain the same levels during the Initial Stage. The firms' R&D expenditures of the Initial Stage affect an arrival of an innovation, and the timing of sub-stage is actually determined.

Firm B's equilibrium expected total discounted profit in the Initial Stage is

$$H_B^0 = \frac{\left[B_0 + \Omega - b^N \right] + \beta(b^N) \left[\frac{B(d_1 + \Omega)}{r} \right]}{r + \beta(b^N) + \gamma(c^N)}$$

Firm C's equilibrium expected total discounted profit in the Initial Stage is

$$H_C^0 = \frac{\left[C_0 + \Omega - c^N \right] + \gamma(c^N) \left[\frac{C(d_2 + \Omega)}{r} \right]}{r + \beta(b^N) + \gamma(c^N)}$$

B. The government's project is completed after a firm innovates

Unlike the previous case for mainly enhancing the private firms' duopoly profits, this case entirely focuses on improving the monopoly profit. That is, the first innovator will be compensated for its efforts by the specially improved monopoly profit.

1) Final Stages

(1) Final Stage d_1

In this stage, Firm B succeeds in an innovation so the innovation game actually has a sub-stage by Firm B's innovation. The entire sequential game possibly has one final stage but the timing of the final stage is unknown due to randomness of innovation. As explained before, there are two final stages. Here, Firm B innovates and turns to be a monopolist by its patent right. Accordingly, the rival firm (Firm C) abandons its investment on R&D project. Thus, both firms stop spending for R&D. Firm B already achieved its goal, and Firm C finds no meaning to spend more on the R&D project. That is, the Nash equilibrium strategy for both firms is to spend nothing for R&D in this stage. An instant profit vector is $(B(d_1) + \delta, 0)$. At this stage, the expected discounted total profits for both firms are as follows.

$$H_B^{d_1} = \int_0^{\infty} e^{-rt} [B(d_1) + \delta] dt = \frac{[B(d_1) + \delta]}{r}$$

$$H_C^{d_1} = \int_0^{\infty} e^{-rt} [0] dt = 0$$

(2) Final Stage d_2

This stage represents that other private firm (Firm C) accomplishes an innovation and eventually possess an exclusive patent right. So, the innovating Firm C becomes a monopolist to roll over all market shares by the technological advantage in the market. Meanwhile, the rival firm (Firm B) gains nothing by the technological disadvantage. Accordingly, the profit vector is much favorable to Firm C such as $(0, C(d_2) + \delta)$. At this stage, the expected discounted total profits for both firms are as follows.

$$H_B^{d_2} = \int_0^{\infty} e^{-rt} [0] dt = 0$$

$$H_C^{d_2} = \int_0^{\infty} e^{-rt} [C(d_2) + \delta] dt = \frac{[C(d_2) + \delta]}{r}$$

2) Non-Final Stage

(1) The Initial Stage

As explained before, both firms make their efforts to be the first-innovator for the monopoly profit when no firm has achieved an innovation yet. The Initial Stage will remain until either Firm B or Firm C succeeds in an innovation, and the stage turns into another stage, Stage d_1 or Stage d_2 . In this stage, an instant profit vector is (B_0, C_0) . The total expected discounted profit for both firms can be obtained in the same way as before.

Intuitively, Firm B separately calculates its expected discounted profits under either its own innovation or its rival's innovation, and adds up those two expected values.

$$H_B^0 = \int_0^\infty \left[\int_0^\tau e^{-rt} (B_0 - b) dt + \int_\tau^\infty e^{-rt} (B(d_1) + \delta) dt \right] \psi_u(\tau) d\tau$$

$$+ \int_0^\infty \left[\int_0^\tau e^{-rt} (B_0 - b) dt + \int_\tau^\infty e^{-rt} (0) dt \right] \psi_n(\tau) d\tau$$

where $\psi_u(\tau) = \beta(b)e^{-[\beta(b)+\gamma(c)]\tau}$ and $\psi_n(\tau) = \gamma(c)e^{-[\gamma(c)+\beta(b)]\tau}$

Firm B's entire expectation is eventually simplified as follows.

$$H_B^0 = \frac{[B_0 - b] + \beta(b)[H_B^{d_1}]}{r + \beta(b) + \gamma(c)} \quad \text{where} \quad H_B^{d_1} = \frac{[B(d_1) + \delta]}{r}$$

Firm C also finds its expected discounted profit in a similar way as Firm B.

$$H_C^0 = \int_0^\infty \left[\int_0^\tau e^{-rt} (C_0 - c) dt + \int_\tau^\infty e^{-rt} (C(d_2) + \delta) dt \right] \psi_n(\tau) d\tau$$

$$+ \int_0^\infty \left[\int_0^\tau e^{-rt} (C_0 - c) dt + \int_\tau^\infty e^{-rt} (0) dt \right] \psi_u(\tau) d\tau$$

where $\psi_u(\tau) = \beta(b)e^{-[\beta(b)+\gamma(c)]\tau}$ and $\psi_n(\tau) = \gamma(c)e^{-[\gamma(c)+\beta(b)]\tau}$

By expectation, Firm C's profit is simplified as a function, depending on two different success rates.

$$H_C^0 = \frac{[C_0 - c] + \gamma(c)[H_C^{d_2}]}{r + \beta(b) + \gamma(c)} \quad \text{where} \quad H_C^{d_2} = \frac{[C(d_2) + \delta]}{r}$$

As similar as the previous case, a Nash equilibrium will be found in this Initial stage because one firm maximizes its expected discounted profit function while the other firm maximizes in the same manner.

From the first order condition $\frac{\partial H_B^0}{\partial b} = 0$, a best response function is derived such as $b^R = b(c)$. Similarly, a best response function $c^R = c(b)$ is derived from the first order condition $\frac{\partial H_C^0}{\partial c} = 0$.

From two conditions above, a Nash equilibrium is derived as (b^N, c^N) . The derived Nash equilibrium supports the completion of backward induction since the entire innovation game was reduced to the Initial Stage. As explained before, the innovation race is a huge sequential game under an infinite horizon, and an innovation randomly occurs. That is, no one knows when an innovation may realize, and, as soon as either firm succeeds in innovating, the stage changes into a final stage in which firms stop spending for R&D. The fact that an arrival of a final stage depends on the randomly distributed variable τ requires the new form of discount factor $[1 + r + \beta(b) + \gamma(c)]$, constituting of two success rates. The equilibrium expected total discounted profits for both firms are as follows.

Firm B's expected total discounted profit in the Initial Stage is

$$H_B^0 = \frac{[B_0 - b^N] + \beta(b^N) \left[\frac{B(d_1) + \delta}{r} \right]}{r + \beta(b^N) + \gamma(c^N)}$$

Firm C's expected total discounted profit in the Initial Stage is

$$H_C^0 = \frac{[C_0 - c^N] + \gamma(c^N) \left[\frac{C(d_2) + \delta}{r} \right]}{r + \beta(b^N) + \gamma(c^N)}$$

Uncertainty of the government-sponsored projects

As discussed, the government's project can be completed either before or after a firm innovates. A completion of the government's project before a private innovation mainly

enhances duopoly profits. It also affects both firms' potential discounted monopoly profits in the final stage. Meanwhile, a completion after a private innovation provides the firms the specially enhanced monopoly profits. However, a private firm does not know when a completion might happen. Let π_1 denote the probability of a completion before a private innovation, and similarly, π_2 denotes the probability of a completion after a private innovation. Both firms can average the derived expected profits over two states and find the overall expected profits.

Sometimes, a completion of the government project is not guaranteed because of budget problems even though the government makes all efforts. So, private firms face two situations. First, the government makes sure that the project will be complete either before or after a private innovation. Second, the government does not make sure a completion of the project despite its all efforts. The implication is that the government may not be responsible for uncertain circumstances such as unexpected costs behind the project. Usually, financing a public project is limited due to the constrained budget. In the case that the government has the difficulty to raise additional funds, the government's project might make no progress and remain incomplete.

1) Certainty case ($\pi_1 + \pi_2 = 1$)

In this case, the government's project must be completed either before or after a private innovation. That is, the government makes firms sure that a supportive innovation will happen despite some restrictive situations such as budget problems. A completion of the project may require much expenditure however the government guarantees a private sector a completion of the project. Thus, private firms can calculate their overall expected profits. Both firms' overall expected profits are following.

Firm B's overall expected total discounted profit is

$$H_B^{t_0} = \pi_1 \left[\frac{[B_0 + \Omega - b] + \beta(b) \left[\frac{B(d_1 + \Omega)}{r} \right]}{r + \beta(b) + \gamma(c)} \right] + (1 - \pi_1) \left[\frac{[B_0 - b] + \beta(b) \left[\frac{B(d_1) + \delta}{r} \right]}{r + \beta(b) + \gamma(c)} \right]$$

Firm B's overall expected total discounted profit is

$$H_C^{t_0} = \pi_1 \left[\frac{[C_0 + \Omega - c] + \gamma(c) \left[\frac{C(d_2 + \Omega)}{r} \right]}{r + \beta(b) + \gamma(c)} \right] + (1 - \pi_1) \left[\frac{[C_0 - c] + \gamma(c) \left[\frac{C(d_2) + \delta}{r} \right]}{r + \beta(b) + \gamma(c)} \right]$$

In the situation that each firm averages the expected profit functions over two states, the Nash equilibrium can be found because each still tries to maximize its calculated overall expected profit. Intuitively, the Nash equilibrium would depend on the distributions of innovations.

For example) $\beta(b) = b^{\frac{1}{2}}$, $\gamma(c) = c^{\frac{1}{2}}$

$$H_B^{t_0} = \left[\frac{\pi_1 \Omega + [B_0 - b] + b^{\frac{1}{2}} \left[\pi_1 \left[\frac{B(d_1 + \Omega)}{r} \right] + (1 - \pi_1) \left[\frac{B(d_1) + \delta}{r} \right] \right]}{r + b^{\frac{1}{2}} + c^{\frac{1}{2}}} \right]$$

$$H_C^{t_0} = \left[\frac{\pi_1 \Omega + [C_0 - c] + c^{\frac{1}{2}} \left[\pi_1 \left[\frac{C(d_2 + \Omega)}{r} \right] + (1 - \pi_1) \left[\frac{C(d_2) + \delta}{r} \right] \right]}{r + b^{\frac{1}{2}} + c^{\frac{1}{2}}} \right]$$

The R&D expenditure, as a firm's strategy in the innovation game, should be at least as well as zero. The Nash equilibrium strategy promises the greatest profit at the opponent's given strategy. Thus, each firm can take a derivative the overall expected profit with respect to its own strategy.

$$\frac{\partial H_B^0}{\partial b} = \left[-r - \left(\frac{1}{2}\right)b^{\frac{1}{2}} - c^{\frac{1}{2}} + \left(\frac{1}{2}\right)b^{-\frac{1}{2}} \left[(H_B^*)r - (B_0 + \pi_1\Omega) \right] + \left(\frac{1}{2}\right)b^{-\frac{1}{2}}c^{\frac{1}{2}} \left[H_B^* \right] \right] \left[r + b^{\frac{1}{2}} + c^{\frac{1}{2}} \right]^{-2} = 0$$

$$H_B^* = \pi_1 \left[\frac{B(d_1 + \Omega)}{r} \right] + (1 - \pi_1) \left[\frac{B(d_1) + \delta}{r} \right]$$

In a similar manner, the opponent maximizes its overall expected profit.

$$\frac{\partial H_C^0}{\partial c} = \left[-r - \left(\frac{1}{2}\right)c^{\frac{1}{2}} - b^{\frac{1}{2}} + \left(\frac{1}{2}\right)c^{-\frac{1}{2}} \left[(H_C^*)r - (C_0 + \pi_1\Omega) \right] + \left(\frac{1}{2}\right)c^{-\frac{1}{2}}b^{\frac{1}{2}} \left[H_C^* \right] \right] \left[r + b^{\frac{1}{2}} + c^{\frac{1}{2}} \right]^{-2} = 0$$

$$H_C^* = \pi_1 \left[\frac{C(d_2 + \Omega)}{r} \right] + (1 - \pi_1) \left[\frac{C(d_2) + \delta}{r} \right]$$

Since $r + b^{\frac{1}{2}} + c^{\frac{1}{2}} \neq 0$

$$-r - \left(\frac{1}{2}\right)b^{\frac{1}{2}} - c^{\frac{1}{2}} + \left(\frac{1}{2}\right)b^{-\frac{1}{2}} \left[(H_B^*)r - (B_0 + \pi_1\Omega) \right] + \left(\frac{1}{2}\right)b^{-\frac{1}{2}}c^{\frac{1}{2}} \left[H_B^* \right] = 0 \quad (1)$$

$$-r - \left(\frac{1}{2}\right)c^{\frac{1}{2}} - b^{\frac{1}{2}} + \left(\frac{1}{2}\right)c^{-\frac{1}{2}} \left[(H_C^*)r - (C_0 + \pi_1\Omega) \right] + \left(\frac{1}{2}\right)c^{-\frac{1}{2}}b^{\frac{1}{2}} \left[H_C^* \right] = 0 \quad (2)$$

Let X denote $b^{\frac{1}{2}}$, and let Y denote $c^{\frac{1}{2}}$.

$$-rX - \left(\frac{1}{2}\right)X^2 - YX + \left(\frac{1}{2}\right) \left[(H_B^*)r - (B_0 + \pi_1\Omega) \right] + \left(\frac{1}{2}\right)Y \left[H_B^* \right] = 0 \quad (1)'$$

$$-rY - \left(\frac{1}{2}\right)Y^2 - XY + \left(\frac{1}{2}\right) \left[(H_C^*)r - (C_0 + \pi_1\Omega) \right] + \left(\frac{1}{2}\right)X \left[H_C^* \right] = 0 \quad (2)'$$

Since the firms, as the incumbent duopolists, have equal market shares under the same circumstances in the industry, the market profits should be equally distributed to both firms. Furthermore, the potential monopoly profit in a final stage would be the same for both firms because they target the same type of technological improvement through R&D project. Then, the symmetry condition can be derived from (1)' and (2)' because the duopoly profits and the potential monopoly profits are identical for both firms. The symmetric condition actually implies that one firm chooses its R&D expenditure based on the expectation that its opponent would make the same choice under all the same circumstances.

Under a symmetry condition ($X = Y$), Firm B chooses its Nash Equilibrium R&D expenditure from the following equation.

$$3X^2 + [2r - (H_B^*)]X - [(H_B^*)r - (B_0 + \pi_1\Omega)] = 0 \quad \text{where } B_0 = C_0, \quad H_B^* = H_C^*$$

The equation above implies that Firm B still needs to find the profit-maximizing R&D choice from the clue that Firm C would actually choose the same as Firm B. In the opposite way, Firm C obtains the profit-maximizing R&D choice by the same speculation. The symmetric condition simply tells that both firms provide a clue for each other and they equally reach the Nash Equilibrium.

$$X' = \frac{[(H_B^*) - 2r] \pm \sqrt{[2r - (H_B^*)]^2 + 12[(H_B^*)r - (B_0 + \pi_1\Omega)]}}{6} = Y'$$

$$X', Y' > 0$$

If $(H_B^*) - 2r \geq 0$

$$X' = \frac{[(H_B^*) - 2r] + \sqrt{[2r - (H_B^*)]^2 + 12[(H_B^*)r - (B_0 + \pi_1\Omega)]}}{6} = Y'$$

$$b' = \left[\frac{\left[(H_B^*) - 2r \right] + \sqrt{\left[2r - (H_B^*) \right]^2 + 12 \left[(H_B^*)r - (B_0 + \pi_1 \Omega) \right]}}{6} \right]^2 = c'$$

(b', c') is the Nash Equilibrium. As both firms expect, they actually obtain the same equilibrium R&D choices.

The government tries to figure out whether the public projects have an effect on the innovative activities of private firms. First of all, the government aims in inspiring innovative activities. The government still has a question if any increased probability of a completion of the project before a private innovation has an impact on innovative activities. To see what effects the increased probability (π_1) would bring in the industry, a comparative study is suggested for π_1 and the Nash Equilibrium R&D expenditures. Depending on the size of π_1 , the Nash Equilibrium R&D expenditures of both firms would change. If we actually take a derivative X' with respect to π_1 then the following result is obtained.

$$\frac{dX'}{d\pi_1} = \frac{1}{6} \left[\frac{dH_B^*}{d\pi_1} + \frac{1}{2} \left[\left[2r - (H_B^*) \right]^2 + 4 \left[(H_B^*)r - (B_0 + \pi_1 \Omega) \right] \right]^{\frac{1}{2}} \left[(-2) \left[2r - (H_B^*) \right] \frac{dH_B^*}{d\pi_1} + 4r \frac{dH_B^*}{d\pi_1} - 4\Omega \right] \right]$$

$$\text{where } \frac{dH_B^*}{d\pi_1} = \left[\frac{B(d_1 + \Omega)}{r} \right] - \left[\frac{B(d_1) + \delta}{r} \right] < 0$$

$$\therefore \frac{dX'}{d\pi_1} < 0 \quad \Rightarrow \quad \frac{db'}{d\pi_1} < 0$$

The inequality above tells that a raised probability for a completion of the project actually reduces the R&D expenditures for both firms. The implication is that as the government tries harder to complete the project before a private innovation both private firms are not so motivated for their innovative activities because a completed government-project

mainly enhances both firms' duopoly profits. More intuitively, the increased probability (π_1) reduces the averages (H_B^* and H_C^*) of discounted monopoly profits. The direct reason is that an increase in π_1 necessarily implies a decrease in π_2 under certainty. It means that an increased chance in one state reduces the chance of the other state. Unfortunately, both firms can expect higher monopoly profits at π_2 than at π_1 . Thus, private firms' average of (discounted) monopoly profit would be decreased as π_1 increases. That is,

$$\frac{dH_B^*}{d\pi_1} = \left[\frac{B(d_1 + \Omega)}{r} \right] - \left[\frac{B(d_1) + \delta}{r} \right] < 0$$

$$\text{where } H_B^* = \pi_1 \left[\frac{B(d_1 + \Omega)}{r} \right] + (1 - \pi_1) \left[\frac{B(d_1) + \delta}{r} \right]$$

$$\frac{dH_C^*}{d\pi_1} = \left[\frac{C(d_2 + \Omega)}{r} \right] - \left[\frac{C(d_2) + \delta}{r} \right] < 0$$

$$\text{where } H_C^* = \pi_1 \left[\frac{C(d_2 + \Omega)}{r} \right] + (1 - \pi_1) \left[\frac{C(d_2) + \delta}{r} \right]$$

Meanwhile, both firms' averages ($B_0 - b + \pi_1\Omega$ and $C_0 - c + \pi_1\Omega$) of duopoly profits definitely increase as π_1 increases. So the firms are less motivated for their own innovation.

2) Uncertainty case ($\pi_1 + \pi_2 < 1$, where $0 \leq \pi_1 < 1$ and $0 \leq \pi_2 < 1$)

This case represents that the government-project might not be completed for private firms due to some reasons, different from the certainty case. In a private sector, firms would calculate their overall expected profits under uncertainty by averaging both states as follows. Both firms' overall expected profits are following.

Firm B's overall expected total discounted profit is

$$H_B^{t_0} = \pi_1 \left[\frac{[B_0 + \Omega - b] + \beta(b) \left[\frac{B(d_1 + \Omega)}{r} \right]}{r + \beta(b) + \gamma(c)} \right] + \pi_2 \left[\frac{[B_0 - b] + \beta(b) \left[\frac{B(d_1) + \delta}{r} \right]}{r + \beta(b) + \gamma(c)} \right]$$

Firm C's overall expected total discounted profit is

$$H_C^{t_0} = \pi_1 \left[\frac{[C_0 + \Omega - c] + \gamma(c) \left[\frac{C(d_2 + \Omega)}{r} \right]}{r + \beta(b) + \gamma(c)} \right] + \pi_2 \left[\frac{[C_0 - c] + \gamma(c) \left[\frac{C(d_2) + \delta}{r} \right]}{r + \beta(b) + \gamma(c)} \right]$$

For example) $\beta(b) = b^{\frac{1}{2}}$, $\gamma(c) = c^{\frac{1}{2}}$

$$H_B^{t_0} = \left[\frac{\pi_1 \Omega + (\pi_1 + \pi_2) [B_0 - b] + b^{\frac{1}{2}} \left[\pi_1 \left[\frac{B(d_1 + \Omega)}{r} \right] + \pi_2 \left[\frac{B(d_1) + \delta}{r} \right] \right]}{r + b^{\frac{1}{2}} + c^{\frac{1}{2}}} \right]$$

$$H_C^{t_0} = \left[\frac{\pi_1 \Omega + (\pi_1 + \pi_2) [C_0 - c] + c^{\frac{1}{2}} \left[\pi_1 \left[\frac{C(d_2 + \Omega)}{r} \right] + \pi_2 \left[\frac{C(d_2) + \delta}{r} \right] \right]}{r + b^{\frac{1}{2}} + c^{\frac{1}{2}}} \right]$$

Firm B tries to find its best R&D choice. So, the firm takes a derivative with respect to its own R&D expenditure and has the following first order conditions, which represents that Firm B actually finds a Nash Equilibrium at a given rival firm's R&D choice.

$$\frac{\partial H_B'^0}{\partial b} = \frac{\left[-r + \left(\frac{1}{2} [\pi_1 + \pi_2] - 1 \right) b^{\frac{1}{2}} - c^{\frac{1}{2}} + \left(\frac{1}{2} \right) b^{-\frac{1}{2}} \left[(H_B^*)r - ([\pi_1 + \pi_2] B_0 + \pi_1 \Omega) \right] + \left(\frac{1}{2} \right) b^{-\frac{1}{2}} c^{\frac{1}{2}} \left[H_B^* \right] \right]}{\left[r + b^{\frac{1}{2}} + c^{\frac{1}{2}} \right]^2} = 0$$

$$H_B^* = \pi_1 \left[\frac{B(d_1 + \Omega)}{r} \right] + \pi_2 \left[\frac{B(d_1) + \delta}{r} \right]$$

In a similar manner, the opponent maximizes its overall expected profit by the first order condition.

$$\frac{\partial H_C'^0}{\partial c} = \frac{\left[-r + \left(\frac{1}{2} [\pi_1 + \pi_2] - 1 \right) c^{\frac{1}{2}} - b^{\frac{1}{2}} + \left(\frac{1}{2} \right) c^{-\frac{1}{2}} \left[(H_C^*)r - ([\pi_1 + \pi_2] C_0 + \pi_1 \Omega) \right] + \left(\frac{1}{2} \right) c^{-\frac{1}{2}} b^{\frac{1}{2}} \left[H_C^* \right] \right]}{\left[r + b^{\frac{1}{2}} + c^{\frac{1}{2}} \right]^2} = 0$$

$$H_C^* = \pi_1 \left[\frac{C(d_2 + \Omega)}{r} \right] + \pi_2 \left[\frac{C(d_2) + \delta}{r} \right]$$

From the two first order conditions, the new necessary conditions for both firms' overall expected profits are found as follows since $r + b^{\frac{1}{2}} + c^{\frac{1}{2}} \neq 0$.

$$-r + \left(\frac{1}{2} [\pi_1 + \pi_2] - 1 \right) b^{\frac{1}{2}} - c^{\frac{1}{2}} + \left(\frac{1}{2} \right) b^{-\frac{1}{2}} \left[(H_B^*)r - ([\pi_1 + \pi_2] B_0 + \pi_1 \Omega) \right] + \left(\frac{1}{2} \right) b^{-\frac{1}{2}} c^{\frac{1}{2}} \left[H_B^* \right] = 0$$

(1)

$$-r + \left(\frac{1}{2} [\pi_1 + \pi_2] - 1 \right) c^{\frac{1}{2}} - b^{\frac{1}{2}} + \left(\frac{1}{2} \right) c^{-\frac{1}{2}} \left[(H_C^*)r - ([\pi_1 + \pi_2] C_0 + \pi_1 \Omega) \right] + \left(\frac{1}{2} \right) c^{-\frac{1}{2}} b^{\frac{1}{2}} \left[H_C^* \right] = 0$$

(2)

Let X denote $b^{\frac{1}{2}}$, and let Y denote $c^{\frac{1}{2}}$ for simplicity.

$$-rX + \left(\frac{1}{2}[\pi_1 + \pi_2] - 1\right)X^2 - YX + \left(\frac{1}{2}\right)\left[(H_B^*)r - ([\pi_1 + \pi_2]B_0 + \pi_1\Omega)\right] + \left(\frac{1}{2}\right)Y[H_B^*] = 0$$

(1)'

$$-rY + \left(\frac{1}{2}[\pi_1 + \pi_2] - 1\right)Y^2 - XY + \left(\frac{1}{2}\right)\left[(H_C^*)r - ([\pi_1 + \pi_2]C_0 + \pi_1\Omega)\right] + \left(\frac{1}{2}\right)X[H_C^*] = 0$$

(2)'

As similar as the certainty case, both firms would choose their R&D expenditures by expectation that the rival would determine its choice based on the same expectation since both firms make their decisions under identical circumstances. They would reach a symmetric equilibrium R&D choice. Under a symmetry condition, Firm B has the new condition for maximizing its overall expected profit as follows. The simplified form implies that Firm B narrows its choices by a clue that its rival Firm C would choose the same as itself.

$$\left[4 - [\pi_1 + \pi_2]\right]X^2 + \left[2r - (H_B^*)\right]X - \left[(H_B^*)r - ([\pi_1 + \pi_2]B_0 + \pi_1\Omega)\right] = 0$$

$$X' = \frac{\left[(H_B^*) - 2r\right] \pm \sqrt{\left[2r - (H_B^*)\right]^2 + 4\left[4 - [\pi_1 + \pi_2]\right]\left[(H_B^*)r - ([\pi_1 + \pi_2]B_0 + \pi_1\Omega)\right]}}{2\left(4 - [\pi_1 + \pi_2]\right)} = Y'$$

$$X', Y' > 0$$

$$\text{If } (H_B^*) - 2r \geq 0$$

$$X' = \frac{\left[(H_B^*) - 2r\right] + \sqrt{\left[2r - (H_B^*)\right]^2 + 4\left[4 - [\pi_1 + \pi_2]\right]\left[(H_B^*)r - ([\pi_1 + \pi_2]B_0 + \pi_1\Omega)\right]}}{2\left(4 - [\pi_1 + \pi_2]\right)} = Y'$$

Both Firm B and Firm C actually choose the same R&D expenditure by the same expectation toward its rival's strategic behavior. The expectation actually leads to the symmetric result.

$$b' = \left[\frac{\left[(H_B^*) - 2r \right] + \sqrt{\left[2r - (H_B^*) \right]^2 + 4 \left[4 - [\pi_1 + \pi_2] \right] \left[(H_B^*)r - ([\pi_1 + \pi_2]B_0 + \pi_1\Omega) \right]}}{2 \left(4 - [\pi_1 + \pi_2] \right)} \right]^2 = c'$$

(b', c') is the Nash Equilibrium.

If the government enlarges the budget for the public project and increases the probability of a completion, how do private firms respond to this favorable government's support? Do they increase or decrease their innovative activities?

$$\frac{dX'}{d\pi_1} = \frac{1}{6} \left[\frac{dH_B^*}{d\pi_1} + \frac{1}{2} [M]^{-\frac{1}{2}} \left[\frac{dM}{d\pi_1} \right] \right] \left[2 \left(4 - (\pi_1 + \pi_2) \right) \right]^{(-1)} + \left[(H_B^* - 2r) + [M]^{\frac{1}{2}} \right]^{(-1)} \left[2 \left(4 - (\pi_1 + \pi_2) \right) \right]^{(-2)} \quad (-1)$$

$$\text{where } M = \left[2r - (H_B^*) \right]^2 + 4 \left[4 - [\pi_1 + \pi_2] \right] \left[(H_B^*)r - ([\pi_1 + \pi_2]B_0 + \pi_1\Omega) \right]$$

and

$$\frac{dM}{d\pi_1} = 2 \left[(H_B^*) - 2r \right] \frac{dH_B^*}{d\pi_1} + \left[16 - [\pi_1 + \pi_2] \right] r \frac{dH_B^*}{d\pi_1} - 4(H_B^*)r - 16\Omega + 8\pi_1\Omega + 4\pi_2\Omega - 16B_0 + 8\pi_1B_0 + 8\pi_2B_0$$

Different from the certainty case, $\frac{dX'}{d\pi_1}$ is not necessarily less than zero. It is actually

ambiguous. $\frac{dX'}{d\pi_1}$ could be greater than zero for some cases. For example, if $\frac{dM}{d\pi_1}$ is

greater than or equal to zero then the private firms' R&D choices would increase in π_1 .

Implicatively, the private firms may spend more expenditure for R&D, under uncertainty, although a completion of the government project mainly enhances the firms' duopoly profits.

$$\frac{dM}{d\pi_1} \geq 0$$

implies

$$2\left[(H_B^*) - 2r\right] \frac{dH_B^*}{d\pi_1} + [16 - [\pi_1 + \pi_2]] r \frac{dH_B^*}{d\pi_1} - 4(H_B^*)r - 16\Omega + 8\pi_1\Omega + 4\pi_2\Omega - 16B_0 + 8\pi_1B_0 + 8\pi_2B_0 \geq 0$$

$$\text{where } \frac{dH_B^*}{d\pi_1} = \left[\frac{B(d_1 + \Omega)}{r} \right] \text{ and } H_B^* = \pi_1 \left[\frac{B(d_1 + \Omega)}{r} \right] + \pi_2 \left[\frac{B(d_1) + \delta}{r} \right]$$

Rearrange all the terms by π_1 and π_2 .

$$\begin{aligned} \pi_1 \left[B(d_1 + \Omega) \left[\frac{2B(d_1 + \Omega)}{r^2} - 5 \right] \right] + \pi_2 \left[\frac{2(B(d_1) + \delta)(B(d_1 + \Omega))}{r^2} - 4[B(d_1) + \delta] - [B(d_1 + \Omega)] \right] \\ + 12[B(d_1 + \Omega)] - 16\Omega - 16B_0 \geq 0 \end{aligned}$$

The following conditions actually satisfy the inequality above since $\pi_1 \geq 0$ and $\pi_2 \geq 0$.

$$(1) \quad B(d_1 + \Omega) \geq [5r^2/2]$$

$$(2) \quad (B(d_1) + \delta)(B(d_1 + \Omega)) \geq [r^2/2][4[B(d_1) + \delta] + [B(d_1 + \Omega)]]$$

$$(3) \quad B(d_1 + \Omega) \geq (4/3)(B_0 + \Omega)$$

Actually, the conditions above can be simplified. First, if $[5r^2/2] \geq (4/3)(B_0 + \Omega)$ then the intersection between (1) and (3) should be $B(d_1 + \Omega) \geq [5r^2/2]$. In this case, we can conclude that $r \leq -\sqrt{(8/15)(B_0 + \Omega)}$ or $r \geq \sqrt{(8/15)(B_0 + \Omega)}$. In fact, $r \geq \sqrt{(8/15)(B_0 + \Omega)}$ since $0 < r < 1$. $(B_0 + \Omega)$ should be less than $(15/8)$.

Second, if $[5r^2/2] \leq (4/3)(B_0 + \Omega)$ then the intersection between (1) and (3) will be $B(d_1 + \Omega) \geq (4/3)(B_0 + \Omega)$. So, $-\sqrt{(8/15)(B_0 + \Omega)} \leq r \leq \sqrt{(8/15)(B_0 + \Omega)}$. r actually takes

some value between 0 and $\sqrt{(8/15)(B_0 + \Omega)}$ since $0 \leq r < 1$.

If $(B_0 + \Omega)$ is at least as great as $(15/8)$ then the imposed constraint

$\left[5r^2/2\right] \geq (4/3)(B_0 + \Omega)$ is meaningless in this argument and the three conditions above

should be reduced to the following two conditions,

$$(B(d_1) + \delta)(B(d_1 + \Omega)) \geq \left[r^2/2\right] \left[4[B(d_1) + \delta] + [B(d_1 + \Omega)]\right]$$

and $B(d_1 + \Omega) \geq (4/3)(B_0 + \Omega)$.

So far, we have discussed under what cases the private firms may increase their R&D expenditures though their enhanced duopoly profits by a completion of the government-project.

Under certainty case, an increased funding for completing the project reduces R&D expenditures of the private firms. In fact, an increased probability (π_1) of completion of the project improves the firms' averages of duopoly profits but reduces averages of (discounted) monopoly profits for both firms. It results from the fact that an increased probability of one state necessarily means a reduction of probability of the other state under certainty case. Due to the reduced averages of monopoly profits and the improved averages of duopoly profits, the firms would lower their levels of R&D expenditures. However, both the averages of duopoly profits and the averages of monopoly profits improve under uncertainty case when π_1 increases. Unlike the certainty case, the averages of monopoly profits increase in π_1 as below.

$$\frac{dH_B^*}{d\pi_1} = \left[\frac{C(d_1 + \Omega)}{r} \right] > 0 \quad \text{where} \quad H_B^* = \pi_1 \left[\frac{B(d_1 + \Omega)}{r} \right] + \pi_2 \left[\frac{B(d_1) + \delta}{r} \right]$$

$$\frac{dH_C^*}{d\pi_1} = \left[\frac{C(d_2 + \Omega)}{r} \right] > 0 \quad \text{where} \quad H_C^* = \pi_1 \left[\frac{C(d_2 + \Omega)}{r} \right] + \pi_2 \left[\frac{C(d_2) + \delta}{r} \right]$$

The averages of monopoly profits would make the firms confused about their R&D

decisions. That is, the firms would be willing to reduce their R&D spending by the improved averages of duopoly profits and be ready to increase the R&D spending for the improved averages of monopoly profits. In this case, the firms may increase or decrease their R&D expenditures, depending on the situation. If an increase in π_1 more strongly impacts the averages of duopoly profits then the private firms would be less motivated for an innovation so their R&D expenditures would be decreased. On the contrary, if the increased π_1 more strongly impacts the averages of monopoly profits then they would be more motivated to innovate earlier. Thus, their R&D expenditures would be increased.

Intuitively, the uncertainty makes both firms to remind that they are involved in an innovation race for their future. Thus, if the government increases the fund to complete the project and enhances the duopoly profits under uncertainty then both firms might increase their R&D expenditures, in some cases, for their monopoly profits in the future. Implicatively, they are forced to spend more for their innovative activities under uncertainty because the uncertainty reminds them their competition for an innovation.

3. Drastic Innovation

Unlike the previous case that both firms equally share the market, this innovation race starts by a monopoly and ends up with a monopoly. In detail, the incumbent firm has a monopoly profit before the entrant firm succeeds in an innovation. The entrant's innovation would actually replace a monopolist. However, the entrant would gain nothing until it successfully innovates through its continuous R&D expenditure. Up to the timing of any innovation, the incumbent firm still obtains its monopoly profit. With its own innovation, the incumbent firm improves its monopoly profit. With the entrant's innovation, the incumbent firm will be replaced for good by the entrant firm. For example, a medicine represents the drastic innovation well. Usually, a newly-invented medicine monopolizes all market shares and is replaced by the new medicine that has smaller side effect. In this way, a medicine evolves to the smallest side effect (Obesity, Anti-depression, and so on). The pharmacy industry is closely related with the government project because of a lot of R&D costs.

Here, Firm B is the incumbent firm and Firm C is the entrant firm. Both firms' overall expected profits are following.

Firm B's overall expected total discounted profit is

$$H_B^{t_0} = \pi_1 \left[\frac{[B_0 + \Omega - b] + \beta(b) \left[\frac{B(d_1 + \Omega)}{r} \right]}{r + \beta(b) + \gamma(c)} \right] + \pi_2 \left[\frac{[B_0 - b] + \beta(b) \left[\frac{B(d_1) + \delta}{r} \right]}{r + \beta(b) + \gamma(c)} \right]$$

Firm C's overall expected total discounted profit is

$$H_C^{t_0} = \pi_1 \left[\frac{[0 - c] + \gamma(c) \left[\frac{C(d_2 + \Omega)}{r} \right]}{r + \beta(b) + \gamma(c)} \right] + \pi_2 \left[\frac{[0 - c] + \gamma(c) \left[\frac{C(d_2) + \delta}{r} \right]}{r + \beta(b) + \gamma(c)} \right]$$

For example) $\beta(b) = b^{\frac{1}{2}}$, $\gamma(c) = c^{\frac{1}{2}}$

$$H_B^{t_0} = \left[\frac{\pi_1 \Omega + (\pi_1 + \pi_2)[B_0 - b] + b^{\frac{1}{2}} \left[\pi_1 \left[\frac{B(d_1 + \Omega)}{r} \right] + \pi_2 \left[\frac{B(d_1) + \delta}{r} \right] \right]}{r + b^{\frac{1}{2}} + c^{\frac{1}{2}}} \right]$$

$$H_C^{t_0} = \left[\frac{(\pi_1 + \pi_2)[0 - c] + c^{\frac{1}{2}} \left[\pi_1 \left[\frac{C(d_2 + \Omega)}{r} \right] + \pi_2 \left[\frac{C(d_2) + \delta}{r} \right] \right]}{r + b^{\frac{1}{2}} + c^{\frac{1}{2}}} \right]$$

From the first order conditions,

$$-r + \left(\frac{1}{2}[\pi_1 + \pi_2] - 1\right)b^{\frac{1}{2}} - c^{\frac{1}{2}} + \left(\frac{1}{2}\right)b^{-\frac{1}{2}}[(H_B^*)r - ([\pi_1 + \pi_2]B_0 + \pi_1\Omega)] + \left(\frac{1}{2}\right)b^{-\frac{1}{2}}c^{\frac{1}{2}}[H_B^*] = 0 \quad (1)$$

$$-r + \left(\frac{1}{2}[\pi_1 + \pi_2] - 1\right)c^{\frac{1}{2}} - b^{\frac{1}{2}} + \left(\frac{1}{2}\right)c^{-\frac{1}{2}}[(H_C^*)r] + \left(\frac{1}{2}\right)c^{-\frac{1}{2}}b^{\frac{1}{2}}[H_C^*] = 0 \quad (2)$$

Let X denote $b^{\frac{1}{2}}$, and let Y denote $c^{\frac{1}{2}}$.

$$-rX + \left(\frac{1}{2}[\pi_1 + \pi_2] - 1\right)X^2 - YX + \left(\frac{1}{2}\right)[(H_B^*)r - ([\pi_1 + \pi_2]B_0 + \pi_1\Omega)] + \left(\frac{1}{2}\right)Y[H_B^*] = 0 \quad (1)'$$

$$-rY + \left(\frac{1}{2}[\pi_1 + \pi_2] - 1\right)Y^2 - XY + \left(\frac{1}{2}\right)[(H_C^*)r] + \left(\frac{1}{2}\right)X[H_C^*] = 0 \quad (2)'$$

By comparative static analysis, we can recognize how private firms would respond to the government's favorable policy such as enlarging the budget for the public project and increasing the probability of a completion of the public project. By total differentiation,

$$\left[\frac{1}{2}X^2 - \frac{1}{2}B_0 - \frac{1}{2}\Omega\right]d\pi_1 + \left[\frac{1}{2}X^2 - \frac{1}{2}B_0\right]d\pi_2 + 2\left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{Y}{2} - \frac{r}{2}\right]dX + \left[\frac{1}{2}[H_B^*] - X\right]dY = 0 \quad (3)$$

$$\left[\frac{1}{2}Y^2\right]d\pi_1 + \left[\frac{1}{2}Y^2\right]d\pi_2 + 2\left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{X}{2} - \frac{r}{2}\right]dX + \left[\frac{1}{2}[H_C^*] - Y\right]dY = 0 \quad (4)$$

By transformation, the equations (3) and (4) can be rewritten as (3)' and (4)'.

$$Ad\pi_1 + Bd\pi_2 + CdX + EdY = 0 \quad (3)'$$

$$A'd\pi_1 + B'd\pi_2 + C'dX + E'dY = 0 \quad (4)'$$

where $A = \left[\frac{1}{2}X^2 - \frac{1}{2}B_0 - \frac{1}{2}\Omega \right]$, $B = \left[\frac{1}{2}X^2 - \frac{1}{2}B_0 \right]$, $C = 2 \left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{Y}{2} - \frac{r}{2} \right]$
 $E = \left[\frac{1}{2}[H_B^*] - X \right]$, $A' = \left[\frac{1}{2}Y^2 \right]$, $B' = \left[\frac{1}{2}Y^2 \right]$, $C' = 2 \left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{X}{2} - \frac{r}{2} \right]$,
and $E' = \left[\frac{1}{2}[H_C^*] - Y \right]$

From (4)', the following is obtained.

$$dY = -\frac{A'}{E'}d\pi_1 - \frac{B'}{E'}d\pi_2 - \frac{C'}{E'}dX \quad (5)$$

(5) is plugged into (3)' and it can be shown as below.

$$dX = \frac{\left[\frac{A - EA'}{E'} \right]}{\left[\frac{EC'}{E'} - C \right]} d\pi_1 + \frac{\left[\frac{B - EB'}{E'} \right]}{\left[\frac{EC'}{E'} - C \right]} d\pi_2 \text{ where } \frac{\partial X}{\partial \pi_1} = \frac{\left[\frac{A - EA'}{E'} \right]}{\left[\frac{EC'}{E'} - C \right]} \text{ and } \frac{\partial X}{\partial \pi_2} = \frac{\left[\frac{B - EB'}{E'} \right]}{\left[\frac{EC'}{E'} - C \right]}$$

$$dY = \frac{\left[\frac{A - CA'}{C'} \right]}{\left[\frac{CE'}{C} - E \right]} d\pi_1 + \frac{\left[\frac{B - CB'}{C'} \right]}{\left[\frac{CE'}{C} - E \right]} d\pi_2 \text{ where } \frac{\partial Y}{\partial \pi_1} = \frac{\left[\frac{A - CA'}{C'} \right]}{\left[\frac{CE'}{C} - E \right]} \text{ and } \frac{\partial Y}{\partial \pi_2} = \frac{\left[\frac{B - CB'}{C'} \right]}{\left[\frac{CE'}{C} - E \right]}$$

$$\frac{\partial Y}{\partial \pi_1} = \frac{\left[\frac{A - CA'}{C'} \right]}{\left[\frac{CE'}{C'} - E \right]} = \frac{\left[\frac{\frac{1}{2}X^2 - \frac{1}{2}B_0 - \frac{1}{2}\Omega}{\left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{Y}{2} - \frac{r}{2} \right]} - \frac{\left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{Y}{2} - \frac{r}{2} \right] \left[\frac{1}{2}Y^2 \right]}{\left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{X}{2} - \frac{r}{2} \right]} \right]}{\left[\frac{\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{Y}{2} - \frac{r}{2}}{\left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{X}{2} - \frac{r}{2} \right]} \left[\frac{1}{2}[H_C^*] - Y \right] - \left[\frac{1}{2}[H_B^*] - X \right] \right]}$$

If $Y > X$ under a condition $[H_B^*] = [H_C^*]$, $\frac{\partial Y}{\partial \pi_1} > 0$

As explained for the non-drastring case, an increased probability of a completion of the project would improve the averages of post-innovation monopoly profits under uncertainty. Intuitively, the entrant would increase its R&D expenditure when the government enlarges the budget for completing the public projects because the entrant has spent more on R&D to challenge the entire market. Both firms' discounted post-innovation monopoly profits are reasonably assumed to be the same in the final stage.

The incumbent's response to the increased probability can be found as similar as the entrant.

$$\frac{\partial X}{\partial \pi_1} = \frac{\left[A - \frac{EA'}{E'} \right]}{\left[\frac{EC'}{E'} - C \right]} = \frac{\left[\frac{\frac{1}{2}X^2 - \frac{1}{2}B_0 - \frac{1}{2}\Omega}{\left[\frac{1}{2}[H_C^*] - Y \right]} - \frac{\left[\frac{1}{2}[H_B^*] - X \right] \left[\frac{1}{2}Y^2 \right]}{\left[\frac{1}{2}[H_C^*] - Y \right]} \right]}{\left[\frac{\left[\frac{1}{2}[H_B^*] - X \right] 2 \left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{X}{2} - \frac{r}{2} \right]}{\left[\frac{1}{2}[H_C^*] - Y \right]} - 2 \left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{Y}{2} - \frac{r}{2} \right] \right]}$$

If $Y > X$, under conditions such as $\left[\frac{1}{2}[H_B^*] - X\right] > 0$, $\left[\frac{1}{2}[H_C^*] - Y\right] > 0$, and

$$[H_B^*] = [H_C^*]$$

$$\frac{\partial X}{\partial \pi_1} > 0$$

Implicatively, an increased probability of a completion of project would increase both the average of pre-innovation monopoly profit and the average of post-innovation monopoly profit for the incumbent. The incumbent would decrease its R&D expenditure by an increase in the average of pre-innovation monopoly profit however the firm would increase its R&D expenditure by an increase in the average of post-innovation monopoly profit. In this situation, if the increased probability impacts the average of post-innovation more strongly than the average of pre-innovation then the incumbent increases its R&D expenditure.

From above, both firms would respond to the government's enlarging budget for a completion by increasing their R&D expenditures. Then, which firm would increase more R&D expenditure than the other? That is, is it the incumbent or the entrant that would be more inspired to increase its innovative activities?

$$\frac{\partial X}{\partial \pi_1} = \frac{\left[A - \frac{EA'}{E'}\right]}{\left[\frac{EC'}{E'} - C\right]} \quad \text{and} \quad \frac{\partial Y}{\partial \pi_1} = \frac{\left[A - \frac{CA'}{C'}\right]}{\left[\frac{CE'}{C'} - E\right]}$$

$\frac{\partial X}{\partial \pi_1}$ and $\frac{\partial Y}{\partial \pi_1}$ can be compared as below.

$$\frac{\partial Y}{\partial \pi_1} - \frac{\partial X}{\partial \pi_1} = \frac{\left[A - \frac{CA'}{C'}\right]}{\left[\frac{CE'}{C'} - E\right]} - \frac{\left[A - \frac{EA'}{E'}\right]}{\left[\frac{EC'}{E'} - C\right]} = \left[\frac{AC' - CA'}{CE' - EC'}\right] - \left[\frac{E'A - EA'}{EC' - CE'}\right]$$

$$= \frac{A(C' + E') - A'(C + E)}{CE' - EC'} < 0$$

$$= \frac{\left[\frac{1}{2}X^2 - \frac{1}{2}B_0 - \frac{1}{2}\Omega \right] \left[2 \left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{X}{2} - \frac{r}{2} \right] + \left[\frac{1}{2}[H_C^*] - Y \right] \right] - \left[\frac{1}{2}Y^2 \right] \left[2 \left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{Y}{2} - \frac{r}{2} \right] + \left[\frac{1}{2}[H_B^*] - X \right] \right]}{2 \left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{Y}{2} - \frac{r}{2} \right] \left[\frac{1}{2}[H_C^*] - Y \right] - \left[\frac{1}{2}[H_B^*] - X \right] 2 \left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{X}{2} - \frac{r}{2} \right]} < 0$$

$$\text{where } A = \left[\frac{1}{2}X^2 - \frac{1}{2}B_0 - \frac{1}{2}\Omega \right], \quad C = 2 \left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{Y}{2} - \frac{r}{2} \right], \quad E = \left[\frac{1}{2}[H_B^*] - X \right]$$

$$A' = \left[\frac{1}{2}Y^2 \right], \quad C' = 2 \left[\frac{1}{2}[\pi_1 + \pi_2] - 1 - \frac{X}{2} - \frac{r}{2} \right], \quad E' = \left[\frac{1}{2}[H_C^*] - Y \right]$$

Under the condition that $Y > X$, $[H_B^*] = [H_C^*]$, and $Y + X > \frac{1}{2}[H_B^*] + (\pi_1 + \pi_2) - 2 - r$

$$\frac{\partial X}{\partial \pi_1} > \frac{\partial Y}{\partial \pi_1}$$

The incumbent firm should be more responsive to the increased probability of a completion of the project than the entrant. Implicatively, if the government enlarges the budget for completing the public projects under uncertainty, the averages of post-innovation monopoly profits improve for both firms. Here, as the entrant spends larger R&D expenditure and intimidates the incumbent, the incumbent's concern for the future grows. So the incumbent should sensitively respond to any change in circumstances such as an increased probability of a completion of the public projects, which could be favorable to its own R&D project. Thus, the incumbent increases its R&D spending more than the entrant.

4. Conclusion

We have discussed how the government project actually inspires innovative activities in a private sector for two different cases, a standard case and a drastic case. The main result is that if the probability of a completion of the government-project increases then both firms' R&D expenditures will decrease. Implicatively, in the case that the government raises the budget for a completion of the project, both firms find that their duopoly profits seem to be enhanced more than their monopoly profits so they would lose their motivation for innovative activities. However, we could find some different results under the uncertainty case. That is, if the government does not make sure a completion of the project then a raised probability of a completion does not necessarily reduce the R&D expenditures in a private sector. An implication is as follows. Because a raised probability would increase both the averages of duopoly profits and the averages of monopoly profits in the future, both firms are confused about their R&D choices. Different from the certainty case, the uncertainty reminds the firms that they are involved in the innovation race for acquiring the monopoly profit at their future so both firms may increase their R&D expenditure for earlier innovation. In this case, they are actually forced for more innovative activities although a completion of the government-project implies their enhanced duopoly profits rather than their potential monopoly profits.

For a drastic case in which the entrant would replace the incumbent when the entrant succeeds in innovation, we also found the similar result. Under the situation that the government does not make sure a completion of the government-sponsored project, both firms may increase their R&D spending for innovation when the probability of a completion of government project is raised. In fact, the raised probability of a completion increases the average of post-innovation monopoly profit for the entrant so the entrant would increase its R&D expenditure. The raised probability of a completion also improves the average of pre-innovation monopoly profit and the average of post-innovation monopoly profit for the incumbent. The incumbent would decrease its R&D expenditure by an increase in the average of pre-innovation monopoly profit and would increase its R&D expenditure by an increase in the average of post-innovation monopoly profit. If the increased probability impacts the average of post-innovation more strongly than the

average of pre-innovation then the incumbent increases its R&D expenditure.

The situation that the entrant spends larger R&D expenditure makes the incumbent concern about its future. The point is that the incumbent increases its R&D expenditure more than the entrant. The incumbent seems more responsive to any change in its research infrastructure because the entrant consistently challenges the entire market by larger R&D spending, and any favorable change, from research infrastructure, makes a positive impact on its post-innovation monopoly profit in the future. In this context, the incumbent is never less motivated for innovative activities than the entrant under uncertainty. In another word, if the entrant is no more motivated than the incumbent then it seems hard that the entrant can replace the incumbent. That is why a persistent monopoly could be found in emerging economies.

Chapter 2. Are Homeruns Overemphasized in Baseball?

1. Introduction

In the major league, the highest payroll team spends for employing players almost ten times larger than the lowest payroll team. However, the size of team payroll does not necessarily represent winning rate or well-performance in a tournament of fall classic. Surprisingly, some financially small-sized teams often advance to championship series. For example, only Boston was the only leading high-payroll team among the four teams (American league - Boston and Cleveland, National league – Colorado and Arizona) that joined the championship series in the last 2007. In detail, Boston and Cleveland ranked the 2nd and the 24th out of 31 teams, respectively. The National league rivals, Colorado and Arizona ranked the 26th and the 27th for each.

Interestingly, there has been an argument that directly relates a team performance to a team's overall salary. Recently, Hakes and Sauer (2006) found the main reason of low winning rates in high-payroll team from a 'mis-pricing'. With empirical findings, they insisted that many baseball players have been actually 'mis-priced'. The main point was that the on-base percentage contributes more to improve a team's winning rate than the slugging percentage but, in most cases, the owners do not pay much attention on the on-base percentage.

Why does not the team's payroll represent the team's performance well? This paper finds a reason of this 'unbalanced payroll and performance' from 'Overemphasis' of homeruns. Usually, a homerun contributes much to increase a total base in each game and also helps ballgame become more excited. However, the overemphasis of homerun may lead to larger uncertainty of expected total base. To discuss the overemphasis problem, we need to define two different 'playing spirits' such as 'Egoism (Self-discipline)' and 'Altruism' and theoretically explain how an owner implements different types of hitters, expensive hitters and inexpensive hitters. Expensive hitters could have larger expected total bases at 'Egoism' than at 'Altruism' so an owner implements 'Egoism (Self-discipline)' from

expensive hitters in section 2. In fact, implementing ‘Egoism’ increases an expected total bases and also increases the uncertainty of total base itself because hitters would focus on bigger hits and necessarily raise the probability of ‘Out’. Meanwhile, ‘Altruism’ is implemented from inexpensive hitters because they have larger expected total bases at ‘Altruism’ than at ‘Egoism’. Overall, expensive hitters have larger expected total base and larger uncertainty at ‘Egoism’ than inexpensive hitters at ‘Altruism’.

In ‘Egoism’, a homerun, among other hits, is mostly emphasized by an owner because of its contribution to increase total base and to attract more fans. The empirical result supports that the number of homerun actually plays a major role in determining sluggers’ salaries. In section 3, it will be shown that the over-emphasis of the biggest hit causes a concept of ‘opportunity cost’ in producing the relatively smaller hit such as a triple, a double, and a single. Due to ‘opportunity cost’, the order of preference could be changed. Any changed order between smaller hits ironically verifies that a homerun is over-emphasized. According to the empirical finding from 2SLS estimation, a double seems to be interestingly preferred to a triple.

A homerun acquires four bases, which is the largest among available hits. It would help increase the expected total base however substantially increases the uncertainty of total base since the probability of ‘Out’ also increases. In summary, owners do not count larger uncertainty but concentrates only on larger expected total bases. No concern about larger uncertainty seems to be directly related with ‘unbalanced team’s payroll and team’s performance’. Overall, this paper is constituted of two parts, a theoretic description and an empirical study. The theory part shows that an expensive hitter necessarily swings hard due to an incentive scheme imposed on his salary contract. In the empirical part, all types of hits will be discussed for salary decision. A homerun is actually over-weighted.

2. Theoretical Background

A. Salary Contract for a hitter

An owner has to determine his player’s salary without observing efforts in making a

contract. A salary-negotiation would happen before a season so an owner faces a risk of his player's hidden action during a pennant race. To avoid any hidden action, an incentive scheme is needed from an owner's perspective. First of all, a hitter's salary is distributed based on the number of total bases that provide a good approximation for how a hitter contributes to win. A team's revenue is assumed to increase by improved winning rate. A total base can be mainly produced by two types of hits, a single and a bigger hit including a homerun, a triple, and a double. A single and a bigger hit are random outputs, and they have some probability densities. Depending on a hitter's effort, the densities would be raised. An owner makes a contract by expecting a hitter's total base. There are two different playing spirits, '*Altruism*' and '*Egoism (Self-discipline)*' by which a player contributes to win a game in some ways. In '*Altruism*', a hitter plays for his organization so he tries to make a timely hit for a run. Due to a light and a precise swing, a probability of single is larger, and a probability of Out is smaller. In '*Egoism*', a hitter swings hard with personal ambition so a probability of a big hit is larger. However, a probability of Out is also larger.

Different from other sports such as soccer, basketball, and football, an owner can not observe under what spirit a hitter plays. An owner actually implements a state of '*Altruism*' for inexpensive hitters and a state of '*Egoism*' for expensive hitters. Usually, inexpensive hitters are not fitted in flying a ball away. If they play themselves in '*Egoism*' then the expected total base is even decreased. Meanwhile, expensive hitters are talented to make a big hit. It means that they are efficient to play themselves in '*Egoism*' more than '*Altruism*' since they are paid much. For inexpensive hitters, an owner obviously chooses to implement '*Altruism*' since the expected total base is larger at '*Altruism*' than at '*Egoism*' and the uncertainty is smaller at '*Altruism*' than at '*Egoism*'. Unlike inexpensive hitters, implementation of '*Egoism*' for expensive hitters is a bit arguable because the expected total base is larger at '*Egoism*' than '*Altruism*', and the uncertainty is also larger at '*Egoism*' than at '*Altruism*'. However, an owner accepts the increased uncertainty because of efficiency.

To understand the concepts of expected total base and the uncertainty, a 'sacrificing bunt' can be exemplified. In the late innings, a coach usually directs a burnt to score one or to

break a tie because the uncertainty is reduced despite smaller expected total base. Occasionally, a coach technically utilizes the larger variance by allowing the larger expected total base for a team's defense. In the situation that runners are on the third base and the second base, a pitcher intentionally walks a hitter for a double play. They focus on the increased uncertainty for the next hitter although his expected total base also increases. A 'sacrificing bunt' and a 'loading bases' start from the recognition that an increased expected total base does not necessarily help a team win under the larger uncertainty.

Due to the larger uncertainty of total base, a team's winning rate also faces larger uncertainty. High-payroll teams are constituted of many expensive hitters, and low-payroll teams of mainly inexpensive hitters. An owner's different implementation provides a reason why high-payroll teams possibly have a low winning rate, and low-payroll teams have a good winning rate.

1) A player's playing spirit is observable

Here, TB denotes a total base, X denotes a single, and Y denotes a big hit (including a homerun, a triple or a double). According to the model below, each hitter contributes to the team's total revenue and he receives his salary as a reward for his contribution. An owner maximizes the difference between a hitter's contribution ($R(TB)$) to the team's total revenue and the payment to a hitter ($s(TB)$).

$$\begin{aligned} & \underset{e \in \{e_{SC}, e_{SF}\}, s(TB)}{\text{Max}} \int (R(TB) - s(TB)) \left[f(X|e) + f(Y|e) \right] dTB & \text{s.t.} \\ & \int v(s(TB)) \left[f(X|e) + f(Y|e) \right] dTB - c(e) \geq \bar{u} \quad \text{where} \quad f(X|e) + f(Y|e) = f(TB|e) \end{aligned}$$

The constraint above represents that a salary must meet a hitter's reservation utility to make him to join the ball club and to play. The maximization problem above should be equivalent as the minimization problem below to find the optimal salary.

$$\underset{s(TB)}{\text{Min}} \int s(TB) \left[f(X|e) + f(Y|e) \right] dTB \quad \text{s.t.} \quad \int v(s(TB)) \left[f(X|e) + f(Y|e) \right] dTB - c(e) \geq \bar{u}$$

$$\text{Min}_{s(TB)} L = \int (s(TB)) [f(X|e) + f(Y|e)] dTB + \gamma [\bar{u} - \int v(s(TB)) [f(X|e) + f(Y|e)] dTB + c(e)]$$

The first order condition can be found for the optimum of salary.

$$[f(X|e) + f(Y|e)] - \gamma v'(s(TB)) [f(X|e) + f(Y|e)] = 0$$

$$\frac{1}{v'(s(TB))} = \gamma$$

Let $\hat{s}(TB)$ denotes an optimally determined salary (the observable-effort-salary) for a hitter according to the optimization above. Intuitively, $\hat{s}(TB)$ can be found by the given utility function v . Actually, the salary will be a constant.

2) A player's playing spirit is not observable

In all cases, the general manager signs the salary contract without any revealed spirits from a player's side. The salary distributor would consider a player's potential capability for the upcoming season, and eventually conclude that he deserves the contract. Unlike the case of the observable playing spirit, the optimization problem under the unobservable playing spirit should have two constraints, the participation constraint and the incentive constraint as below.

$$\text{Min}_{s(TB)} \int (s(TB)) [f(X|e) + f(Y|e)] dTB \quad (2-4)$$

$$\text{s.t. (1) } \int v(s(TB)) [f(X|e) + f(Y|e)] dTB - c(e) \geq \bar{u} \quad \text{Participation constraint} \quad (2-5)$$

$$(2) \quad e \text{ solves } \text{Max}_e \int v(s(TB)) [f(X|e) + f(Y|e)] dTB - c(\tilde{e}) \quad \text{Incentive constraint}$$

(2-6)

By the incentive constraint, a hitter would have a reason to play under one playing spirit rather than under the other playing spirit.

3) Implementing a state S_i for higher total base

An owner would implement either state of ‘Egoism’ or ‘Altruism’ from hitters.

$S_i \in \{AL, EG\}$ where AL denotes ‘Altruism’, and EG denotes ‘Egoism’. The incentive constraint is follows.

$$\int v(s(TB)) \left[f(X|S_i) + f(Y|S_i) \right] dTB - c(S_i) \geq \int v(s(TB)) \left[f(X|S_{-i}) + f(Y|S_{-i}) \right] dTB - c(S_{-i})$$

The given incentive constraint, an owner finds the optimal salary for a hitter as below.

$$\begin{aligned} \underset{s(TB)}{Min} L = & \int (s(TB)) \left[f(X|S_i) + f(Y|S_i) \right] dTB + \gamma \left[\bar{u} - \int v(s(TB)) \left[f(X|S_i) + f(Y|S_i) \right] dTB + c(S_i) \right] \\ & + \mu \left[\int v(s(TB)) \left[f(X|S_{-i}) + f(Y|S_{-i}) \right] dTB - c(S_{-i}) - \int v(s(TB)) \left[f(X|S_i) + f(Y|S_i) \right] dTB + c(S_i) \right] \end{aligned} \quad (2-8)$$

The first order condition can be found as below.

$$\begin{aligned} & \left[f(X|S_i) + f(Y|S_i) \right] - \gamma v'(s(TB)) \left[f(X|S_i) + f(Y|S_i) \right] \\ & + \mu \left[\left[f(X|S_{-i}) + f(Y|S_{-i}) \right] - \left[f(X|S_i) + f(Y|S_i) \right] \right] v'(s(TB)) = 0 \end{aligned} \quad (2-9)$$

Due to the incentive constraint, the derived condition seems to be different from the derived condition under an observable playing spirit.

$$\frac{1}{v'(s(TB))} = \gamma + \mu \left[1 - \frac{[f(X|S_{-i}) + f(Y|S_{-i})]}{[f(X|S_i) + f(Y|S_i)]} \right] \quad (2-10)$$

The simplified condition actually represents that the optimal salary under an unobservable playing spirit is not constant. As mentioned before, the optimal salary under an observable playing spirit is constant. Unlike the optimal salary under a revealed spirit, this case intuitively informs that the salary would depend on the probability densities of products.

$\hat{s}(TB)$ denotes an optimal salary under an observable playing spirit ($(1/v'(\hat{s}(TB))) = \gamma$).

$$\begin{aligned} \frac{[f(X|S_{-i}) + f(Y|S_{-i})]}{[f(X|S_i) + f(Y|S_i)]} < 1 &\Rightarrow \gamma + \mu > \gamma \Rightarrow \frac{1}{v'(s(TB))} > \frac{1}{v'(\hat{s}(TB))} \\ &\Rightarrow v'(s(TB)) < v'(\hat{s}(TB)) \Rightarrow s(TB) > \hat{s}(TB) \end{aligned} \quad (2-11)$$

The salary under an unobservable playing spirit is greater than the salary under an observable playing spirit if $f(X|S_i) + f(Y|S_i) > f(X|S_{-i}) + f(Y|S_{-i})$

There are four cases for the inequality above.

- 1) $f(X|S_i) > f(X|S_{-i}), f(Y|S_i) > f(Y|S_{-i}) \Rightarrow s(TB) > \hat{s}(TB)$: Impossible
- 2) $f(X|S_i) > f(X|S_{-i}), f(Y|S_i) < f(Y|S_{-i}) \Rightarrow$ Ambiguous
- 3) $f(X|S_i) < f(X|S_{-i}), f(Y|S_i) > f(Y|S_{-i}) \Rightarrow$ Ambiguous
- 4) $f(X|S_i) < f(X|S_{-i}), f(Y|S_i) < f(Y|S_{-i}) \Rightarrow s(TB) < \hat{s}(TB)$: Impossible

In fact, the first and the fourth cases are impossible. A hitter can not have larger probability densities for both a single and a big hit at one state than at the other state. So, we focus only on the second and the third cases.

For the second case,

$$s(TB) > \hat{s}(TB) \text{ if } \left| f(X|S_i) - f(X|S_{-i}) \right| \text{ is greater than } \left| f(Y|S_i) - f(Y|S_{-i}) \right|.$$

This is true only when an owner implements a state of ‘Altruism’ because a probability density of a single at ‘Altruism’ should be larger than at ‘Egoism’, and a probability density of a big hit at ‘Egoism’ should be larger than at ‘Altruism’. Inexpensive players are not very talented to make a big hit so an owner considers them to produce more singles. A hitter lightly swings to make on-base by a spirit for a team’s winning more than a desire for his own record. So, he can make a timely single for a leadoff or a run.

For the third case,

$$s(TB) > \hat{s}(TB) \text{ if } \left| f(Y|S_i) - f(Y|S_{-i}) \right| \text{ is greater than } \left| f(X|S_i) - f(X|S_{-i}) \right|.$$

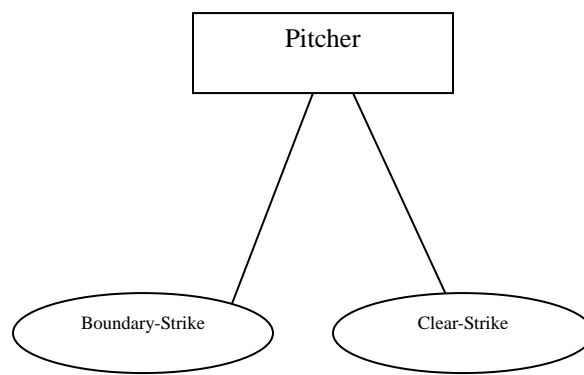
This is true only when an owner implements a state of ‘Egoism’. From an owner’s view, expensive players are efficient to make a big hit. In this case, they are exempted from the spirit of ‘Altruism’. To produce a big hit, a batter desirously swings to fly a ball away so it raises a probability of ‘Out’. In a state of ‘Egoism’, expensive hitters are expected to produce more total bases. This is the reason why an owner implements a state of ‘Egoism’ for expensive hitters.

B. Slugger’s Choice in a ‘Component Sequential game’

Unlike other sports, baseball is a huge sequential game, which is comprised of a lot of small component games between a hitter and a pitcher. Total base is recorded in two ways, for a team and for an individual. An increase in individual’s total base implies an improvement in team’s total base. This is why a ‘component game’ plays an important role. By the incentive scheme, expensive hitters would concentrate on self-discipline to make more total bases. When a slugger is at bat, a ‘component game’ starts by a pitcher’s throw. A slugger can choose a strategy, ‘Swing Light’, ‘Swing Hard’, or ‘Don’t Swing’. For simplicity, a strategy of ‘Swing Light’ can produce only a single with higher probability. By ‘Swing Hard’, a hitter can make various outputs. He may not swing for a pitched ball. Then, it possibly adds a strike-ball count.

From a pitcher's side, he would choose either 'Clear-strike' or 'Boundary-strike'. As soon as a pitcher throws a ball, a component sequential game begins and a hitter would immediately determine his action.

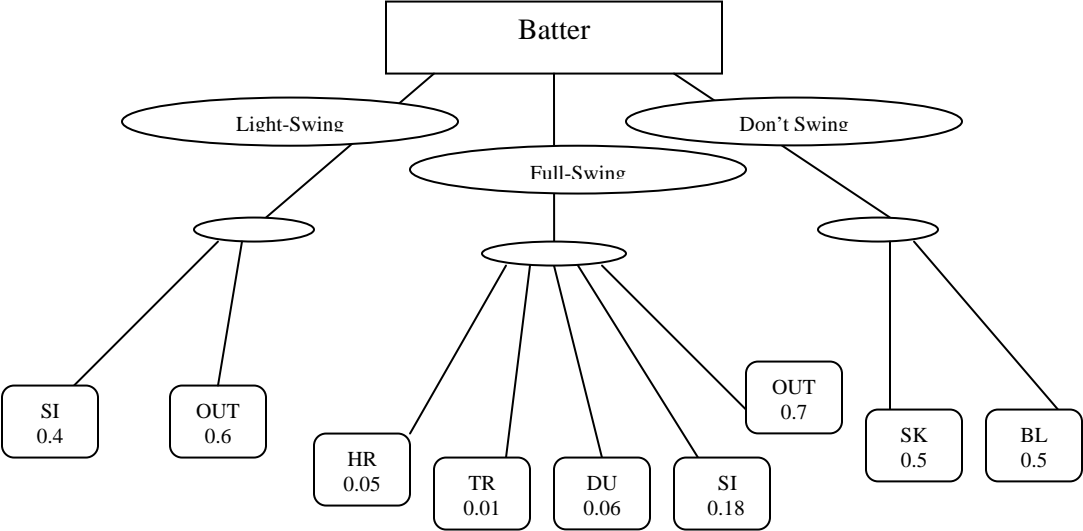
<Figure 1>



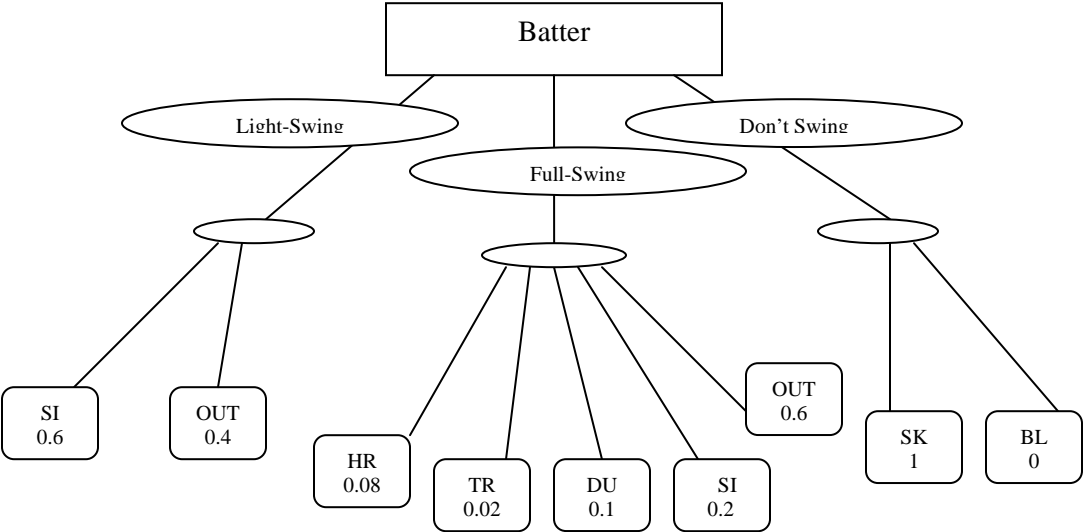
A clear-strike means that a pitcher throws a ball 'clearly' inside a strike zone. Thus, if a hitter does not swing then it should be definitely counted as a strike. However, this clear-strike is riskier in the sense that the centered location is easier for batters to hit. A boundary-strike represents a tricky strike, which is successful by 50% chance. This 'strike-alike' ball is less risky because of lower probabilities of allowing base hits. In the case that a batter does not swing, it can be counted as a strike by 50% or as a ball by 50%.

Among 'Swing Light', 'Swing Hard', and 'Don't Swing', 'Swing Light' enables a batter to precisely hit a pitched ball and have him higher probability of a base hit. That is, he could reduce the probability of 'Out' by increasing more singles. 'Full-swing' has diverse outputs such as a homerun, a triple, a double, and a single. However, the probability of 'Out' is higher than 'Swing Light'.

<Figure 2> In a subgame that a pitcher throws a 'Boundary-Strike'



<Figure 3> In a subgame that a pitcher throws a 'Clear-Strike'



<Figure 2> represents a batter is situated in a subgame when a pitcher throws a ‘Boundary-Strike’. The indicated values below the final outputs inform the probability densities. According to the densities, a batter has an expected base of 0.4 from ‘Swing Light’, an expected base of $0.53(=0.05*4+0.01*3+0.06*2+0.18*1)$ from ‘Swing Hard’, and an expected base of $0.125(=0.5/4)$ from ‘Don’t swing’. Thus, a Nash Equilibrium strategy should be ‘Swing Hard’ for a batter in the situation that a pitcher throws a ‘Boundary-Strike’.

Similarly, <Figure 3> shows a batter’s subgame when a pitcher tries a ‘Clear-Strike’. By the given probability densities with the final outputs, an expected base from ‘Swing Light’ is 0.6, and an expected base from ‘Swing Hard’ is $0.78(=0.08*4+0.02*3+0.1*2+0.2*1)$. ‘Don’t swing’ has 0. In this case, a strategy ‘Swing Hard’ is a Nash equilibrium strategy in the subgame. For a pitcher’s reduced game, ‘Boundary-Strike’ should be a Nash equilibrium strategy because he prefers less expected base. Thus, the SPNE is ((a pitcher throws ‘Boundary-Strike’) and (a batter chooses ‘Swing Hard’)). An expected base is 0.5.

Intuitively, a pitcher already knows that a batter is going to swing hard since ‘Swing Hard’ should be a Nash equilibrium strategy in either subgame displayed above. All pitchers try to reduce allowed bases by tricky strikes as possible. Even in unfavorable situation, a batter swings hard because he knows that ‘Swing hard’ is still better than ‘Swing Light’ in any situation. Basically, the SPNE tells how uncertainty of total base is deepened and how baseball entertains fans. Despite a pitcher’s effort to prevent any bases, a batter is still able to achieve bases. In fact, most slugger’s remarkable records are achieved under challenging situation of many tricky balls.

In summary, the uncertainty of total base is even more increased due to a hitter’s sequential game-theoretic situation.

C. Valuation of Outputs after a Season

As mentioned, each ‘component sequential game’ constitutes an entire baseball game. Due to a Nash equilibrium chosen for an expected base, final outputs will be still randomly distributed by their probability densities. Since an owner implements ‘Egoism (Self-

discipline)' from expensive hitters for larger expected total bases, he would assess sluggers' achievements by focusing on big hits. At least, he will compare sluggers' records and what they are already paid. An owner will consider those records for the next contracts. In this context, he has his own valuation method.

First of all, one can think that an owner develops a salary function as a weighted average over all possible outputs, homeruns, triples, doubles, and singles. The following equation represents what an owner would have in his mind.

$$Y = e^{\sum_{i=1}^4 \alpha_i X_i} \quad (3-1)$$

Y denotes a batter's salary, X_1 the number of homeruns, X_2 the number of triples, X_3 the number of doubles and X_4 the number of singles.

$$\ln Y = \sum_{i=1}^4 \alpha_i X_i \quad (3-2)$$

Simply, the preference of base hit can be found as 4 (homerun) > 3 (triple) > 2 (double) > 1 (single) according to the order. First, an argument about contribution makes a homerun have more weight in the salary equation because the advantage of automatic runs. Unlike other types of base hits, a homerun automatically scores by itself. For example, a triple or a double requires at least another statistical event to make a run. Second, a homerun mostly excites fans. In most cases, people remember the homerun records and the famous homerun hitter such as, Babe Ruth, Hank Aaron, and Barry Bonds. Even, media continuously reports or forecasts who will possibly replace the on-going Homerun record of Bonds based on the projected results. The mentioned hitters are Alex Rodriguez (New York Yankees), David Ortiz (Boston Redsox), Albert Pujols (Saint Luis Cardinals), Manny Ramirez (L.A. Dodgers), Ryan Howard (Philadelphia Phillies), and so on. If one hitter is in a homerun race for a season or for his entire career, baseball fans would have more interests on their team and would be more tempted to join ball games. A good homerun hitter actually brings a positive shock on a team to raise the total revenue.

Furthermore, an owner would strongly prefer a homerun in the situation that a batter has a limited capacity to hit bigger hits. Usually, a hitter's total number of at-bats is approximately restricted so the number of homeruns is supposed to decrease in the number of other base hits. So, the more 'emphasis' on homeruns is supposed to be true in salary equation above.

Given capacity 100, a hitter can allocate his capacity for three types of big hits by efforts. For example, a good homerun hitter allocates 50% of capacity for homeruns, 10% for triples, and 40% for doubles. One can find that there exists proximity among big hits. In this case, the proximity between homeruns and triples is higher than that between homeruns and doubles. Intuitively, it means that the triples could have been a homerun by a little stronger impact. However, a double would not have been a homerun by the same impact. For this hitter, a double needs an additional strength to become a homerun. Conclusively, the athlete records fewer homeruns as he hits more triples. An opportunity cost would follow a triple due to a stressed weight on a homerun. In the same manner, an opportunity cost can be found from a double.

Considering the proximities, the function (3-2) can be rewritten as (3-3).

$$\ln Y = \alpha_1 X_1(X_2, X_3) + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_4 \quad (3-3)$$

In (3-3), X_1 (Homerun) depends on both X_2 (Triple) and X_3 (Double). An implication is that the number of homerun should be affected by the number of triples and doubles under a limited capacity. In the same sense, the number of triples or doubles would depend on the number of homeruns. However, a homerun has no opportunity cost at this moment because of the order of preference. In fact, an owner does not concern any proximity except for the proximities affecting the number of homeruns.

The followings are obtained by taking a derivative with respect to X_1 , X_4 , X_2 , and X_3 .

$$\frac{d \ln Y}{d X_1} = \alpha_1 \quad (3-4)$$

$$\frac{d \ln Y}{dX_4} = \alpha_4 \quad (3-5)$$

$$\frac{d \ln Y}{dX_2} = \alpha_2 + \alpha_1 \frac{\partial X_1}{\partial X_2} \quad \text{where} \quad \frac{\partial X_1}{\partial X_2} < 0 \quad (3-6)$$

$\frac{\partial X_1}{\partial X_2} < 0$ implies that triples have a negative effect on homeruns under a limited capacity.

Intuitively, a negative effect informs that the proximity exists between a triple and a homerun. According to (3-6), an overall effect of triples on a salary is composed of two effects, a direct effect (α_2) and an indirect effect that is represented by the coefficient (α_1)

times the proximity ($\frac{\partial X_1}{\partial X_2}$). If an indirect effect dominates a direct effect, then an overall

effect could be theoretically negative although a triple acquires three bases. An interpretation is that triples' total return is not sufficient to cover its cost of losing homeruns from an owner's view since the proximity represents an opportunity cost.

$$\frac{d \ln Y}{dX_3} = \alpha_3 + \alpha_1 \frac{\partial X_1}{\partial X_3} \quad \text{where} \quad \frac{\partial X_1}{\partial X_3} < 0 \quad (3-7)$$

$\frac{\partial X_1}{\partial X_3} < 0$ shows that the number of homeruns are negatively related the number of doubles

under a limited capacity. As similar as triples, an actual effect of doubles on salaries is made up of two parts, which are a direct effect (α_3) and an indirect effect ($\alpha_1 \frac{\partial X_1}{\partial X_3}$). The

implication is the same as the case of triples. However, it is presumed that the proximity between doubles and homeruns is smaller than the proximity between triples and homeruns. Accordingly, a direct effect is supposed to outweigh an indirect effect so a

positive effect of doubles on a salary would be expected. An implication is that a double contributes the same as a triple with less opportunity cost so an owner is ready to positively accept a double since it does not substantially hurt the number of homeruns.

$$\left| \frac{\partial X_1}{\partial X_2} \right| > \left| \frac{\partial X_1}{\partial X_3} \right| \quad (3-8)$$

The inequality (3-8) shows that the proximity between triples and homeruns is greater than the proximity between doubles and homeruns. More details will be discussed by empirical findings in the next chapter.

Empirically, X_1 must be an endogenous variable since it depends on both X_2 and X_3 in (3-3). Thus, the variable of homerun should be treated as an endogenous variable in a regression analysis. Without considering an endogenous property, a simple regression must produce some errors. 2SLS estimation is suggested due to an endogenous variable.

3. Empirical Findings

A. American League

Data description 28 hitters and 289 observations

Each player has the same number of observations as his career years.

Gary Shaffield has the longest career by 17 years (1989-2006), and Justin Morneau has the shortest career by 3 years (2004-2006).

To investigate determining factors of salaries, the variable of Salary could be regressed onto all types of base hits including the variable of Homerun.

$$Salary = X\beta + u \quad \text{where } X = [Con \ Single \ Double \ Triple \ Homerun]_{N \times 5} \quad (4-1)$$

According to the theory explained in the last section, Homerun should be an endogenous variable since an owner considers opportunity costs behind triples and doubles. Instrumental variables are available for homeruns. Walk and Strikeout seem to have a high correlation with homeruns. Thus, Homerun should be regressed onto all base hits and the suggested instrumental variables, Walk and Strikeout, for the second stage least square estimation.

The following table actually shows the proximities and the correlations between homeruns and walks or strikeouts.

Table 1-1

Homerun	Coefficient	S.E.	t-Value	P-Value
Single	.2139245	.0152737	14.01	0.000
Double	-.4841474	.0621964	-7.78	0.000
Triple	-.7394117	.1957041	-3.78	0.000
Walk	.113987	.016615	6.86	0.000
Strikeout	.0898962	.0143799	6.25	0.000

Homerun seems to be correlated with various outputs. The obtained estimated coefficients provide some implications. Double has a negative estimate (-0.484), and Triple also has a negative estimate (-0.739) that is greater in an absolute term than -0.484. These two different estimates represent different levels of proximities. The estimates strongly support the following inequality, which was already drawn in the last chapter.

$$\left| \frac{\partial X_1}{\partial X_2} \right| > \left| \frac{\partial X_1}{\partial X_3} \right|$$

The next step is to test if Homerun is indeed endogenous or not. Here, two instrumental variables, Walk and Strikeout, can be implemented by the fact that a good homerun hitter records a lot of walks and strike-outs as well. For example, David Ortiz hit 54 homeruns in 2006. He recorded 119 walks and 117 strikeouts at the same year. As discussed, a pitcher would avoid a slugger's powering batting by an intentional walk, or by throwing a 'strike-

alike' ball, which is actually fishing a batter.

A pitcher implements the subgame perfect equilibrium strategy by pitching a 'strike-alike' ball according to the theory in the previous section. The derived SPNE supports that expensive hitters prefer 'Swing Hard' to 'Swing Light', and batters actually have strike-outs more times because of their attempts to put an exceedingly strong impact on a pitched ball. This physically overwhelming effort raises a possibility of a strike-out. Thus, walks and strike-outs can be good instrumental variables for homeruns. For implementing these instrumental variables, the residual should be obtained through the regression (4-2) as below.

$$\text{Homerun} = Z\alpha + \varepsilon \quad (4-2)$$

where $Z = [\text{Single Double Triple Walk Strikeout}]_{N \times 5}$

Eventually, the variable 'Salary' will be regressed onto all the explanatory variables including the residuals obtained from the regression (4-2) as above.

$$\text{Salary} = X\beta + \gamma\hat{\varepsilon} + u \quad (4-3)$$

The null hypothesis $H_0 : \gamma = 0$ is tested. By rejecting the null hypothesis, it can be concluded that Homerun is endogenous. The test result is as follows.

Table1-2

Ln_Salaries1	Coefficient	S.E	t-Value	P-Value
Single	.0016813	.0047315	0.36	0.723
Double	.0115548	.013084	0.88	0.378
Triple	-.093309	.037516	-2.49	0.013
Homerun	.0297622	.0105339	2.83	0.005
Residual	.0379959	.0167703	2.27	0.024
Constant	12.87462	.2012876	63.96	0.000

Apparently, the null hypothesis of $H_0 : \gamma = 0$ should be rejected by the obtained results. Homerun is concluded as an endogenous variable. Furthermore, the test results inform that a simple OLS would result in some errors. Therefore, the second stage estimation is recommended to avoid any errors. However, both test results will be provided cohesively for a comparison and a contrast in this research.

1) With a simple OLS

The test results are as follows.

Table1-3

Ln_Salaries1	Coefficient	S.E	t-Value	P-Value
Single	.0086116	.0036362	2.37	0.019
Double	.0004604	.012222	0.04	0.970
Triple	-.1145622	.0365905	-3.13	0.002
Homerun	.0447534	.0082566	5.42	0.000
Constant	12.86299	.2026966	63.46	0.000

The relatively smaller hits such as singles and doubles lead to very small estimates 0.009 and 0.0005 for Single and Double, respectively. However, the relatively bigger hits, triples and homeruns have bigger estimates. Remarkably, there is a huge difference between an estimate for Double and that for Triple in an absolute value. The question is if the abnormally big difference between two estimates rises from the errors, and the difference would be reduced by 2SLS.

2) With a 2SLS

The second stage estimation shows the results in the following table. Comparing to the OLS, the estimated coefficients seem to be aligned by 2SLS estimation. That is, the

estimates for Double and Homerun are actually increased, but the estimate for Triple is decreased in an absolute term.

Table1-4

Ln_Salaries1	Coefficient	S.E	t-Value	P-Value
Single	.0016813	.0048315	0.35	0.728
Double	.0115548	.0133607	0.86	0.388
Triple	-.093309	.0383094	-2.44	0.015
Homerun	.0677582	.013325	5.09	0.000
Constant	12.87462	.2055443	62.64	0.000

Different from the OLS, the p-value of Single substantially rises from 0.019 to 0.728 by 2SLS. The p-value of Double falls to 0.388, and the p-value of Triple rises to 0.015. However, the estimate of Triple is still significant. By either estimation, Homerun is most significant.

Table 1-4 exhibits an interesting result that the p-values are actually arranged in an order by Single, Double, Triple and Homerun. That is, Homerun is most significant in determining the salary with the estimated coefficient, 0.068. The second is Triple with the estimated coefficient, -.093. The third is Double with the estimate, 0.012, and the fourth is Single with 0.002.

Notably, the most significant variable, Homerun, has the largest positive estimated coefficient, and Double has the second. Single has the third. Triple has the negative estimate.

All of these estimates indicate that an owner concentrates on homeruns rather than any other types of hits at his decision for a hitter's salary. According to the test result, Double has a positive effect on a salary however it is insignificant. Triple has a negative impact on a salary with sufficient significance even though a triple acquires more bases than a double or a single. It seems that a double is strictly preferred to a triple. An insignificant positive estimate of Double implies that doubles do not play any negative roles in salary. Moreover, its insignificance might imply zero effect on a salary. In another word, doubles would

bring some effects at least as much as 0 despite its unclearness. Meanwhile, triples significantly have a negative effect on a salary. The implications support a double's priority to a triple in an owner's mind.

More intuitively, the obtained estimates tell that a boss actually puts the top priority on the number of homerun, and the second priority on a slugger's contribution to win. That is, he concerns about the number of homeruns mainly because a homerun acquires four bases, which actually scores one without any runners on bases and entertains fans most excitedly. As explained before, a double has less opportunity cost than a triple and contributes the same as a triple. With considering both the number of homeruns and contribution from a base hit, an owner has in his mind that a double might be better than a triple. Empirically, the negative estimate of Triple actually shows that an owner finds less efficiency from a triple due to its greater opportunity cost and its same level of contribution as a double.

B. National League

Data description 39 hitters and 369 observations

Each player has the same number of observations as his career years.

Bary Bonds has the longest career by 20 years (1987-2006), and Ryan Howard has the shortest career by 2 years (2005-2006).

Here, National League is discussed. Homerun is regressed onto fewer hits, and Walk and Strikeout as similar as before. As the result, the estimates are shown in Table 2-1. Comparing Table 2-1 to Table 1-1, Triple is observed to have a bigger proximity (-1.03) in National League than in American League (-0.74). Double has a smaller proximity (-0.27) than in (-0.48) American League. The some differences in proximities may arise from batting-pitchers in National League. In fact, the proximities play a crucial role in explaining how a player's salary is determined in both leagues.

Table 2-1

Homerun	Coefficient	S.E.	t-Value	P-Value
Single	.1910018	.0178108	10.72	0.000
Double	-.2701738	.0640572	-4.22	0.000
Triple	-1.031126	.1553716	-6.64	0.000
Walk	.1381923	.012932	10.69	0.000
Strikeout	.057763	.0109794	5.26	0.000

Also, the bigger estimated coefficient (0.14) for Walk and the smaller estimated coefficient (0.06) for Strikeout are found in Table 2-1, compared to Table 2-1(0.11 and 0.09, respectively for Walk and Strikeout). Baseball fans acknowledge a reason. Traditionally, both leagues are mainly different by the designated hitting system, which allows one hitter not to join in fielding. American League accepts the designated hitting rule but National League does not. Thus, all fielders including a pitcher have to join a batting line-up in National League. In this case, there exists the weakest spot in the line-up because of a pitching-hitter so an opponent team's pitcher can intentionally walk power hitters more times to face a pitching-hitter in National League. Sluggers tend to have more walks and fewer strike-outs in National League than in American League.

Under the presumption that Homerun is an endogenous variable, Salary is regressed onto all other explanatory variables including the residuals obtained from the simple OLS above. The test results are contained in the following Table 2-2.

Table 2-2

Ln_Salaries1	Coefficient	S.E	t-Value	P-Value
Single	-.0050084	.0051403	-0.97	0.331
Double	.027438	.012578	2.18	0.030
Triple	-.0413302	.0348975	-1.18	0.237
Homerun	.0231364	.009739	2.38	0.018
Residual	.0418539	.0177585	2.36	0.019
Constant	13.23069	.1802799	73.39	12.87617

There is no big difference for the estimated coefficient for Residual between Table 1-2 (0.038) and Table 2-2 (0.042). It is largely because the effect from the bigger estimate of Walk has been offset by the effect from the smaller estimate of Strikeout in National League. Accordingly, Residual has the low P-value (0.019), which sufficiently supports the significance of the estimate (0.042). Thus, Homerun is concluded as an endogenous variable.

1) With a simple OLS

By the endogenous variable of Homerun, 2SLS method is recommended to prevent any errors from occurring in estimation. However, a simple OLS estimation results are provided for comparison and contrast.

Table 2-3

Ln_Salaries1	Coefficient	S.E	t-Value	P-Value
Single	.0032037	.0038026	0.84	0.400
Double	.0193204	.0121727	1.59	0.113
Triple	-.0814523	.0306536	-2.66	0.008
Homerun	.0357242	.0081946	4.36	0.000
Constant	13.19992	.1809281	72.96	0.000

By a simple OLS, Triple seems to have the biggest effect on a salary by the negative estimate (-0.081) with the P-value of 0.008. Homerun has the second biggest effect by the positive estimate (0.036) with P-value of 0.000. Double has the third biggest effect however the estimate is not significant (p-value of 0.113). Interestingly, the estimated coefficient of Homerun is much smaller than the estimate of Triple in an absolute term. The problem may rise from the endogenous Homerun.

2) With a 2SLS

The estimators should be biased and inconsistent by a simple OLS largely because of one endogenous variable. Instead, the 2SLS estimation results follow in Table 2-4.

Table 2-4

Ln_Salaries1	Coefficient	S.E	t-Value	P-Value
Single	-.0050084	.0052622	-0.95	0.342
Double	.027438	.0128763	2.13	0.034
Triple	-.0413302	.0357251	-1.16	0.248
Homerun	.0649903	.015202	4.28	0.000
Constant	13.23069	.1845552	71.69	0.000

The 2SLS estimation produces some pattern of estimates as similar as the observed in American League. That is, Homerun has the largest positive estimate (0.065), and Double has the second largest (0.027438). Triple has the negative estimate (-0.0413). The important point is that Double becomes significant in National League while it is insignificant in American League. On the contrary, Triple is significant in American League however it turns to be insignificant in National League.

Some inferences are possibly found from the estimate of Triple. Theoretically, an overall effect of triples on a salary is comprised of two parts, which are a direct effect (α_2) and an indirect effect ($\alpha_1 \frac{\partial X_1}{\partial X_2}$). Intuitively, a direct effect can be represented by the estimated coefficient of Triple from a simple OLS. An indirect effect can be actually calculated by the OLS estimate of Homerun times the proximity between homeruns and doubles in the league. National League has bigger estimate (-0.041) with bigger P-value than American League (-0.093 with P-value, 0.015). In absolute terms, triples have a bigger effect on a salary in American League.

There are two reasons. First, American League has much more emphasis on homeruns since its designated hitting rule can support more professionalism in the league. In fact, one expensive slugger entirely specializes in batting without any fielding. By the allowed specialty, an owner would anticipate more homeruns from a designated hitter. Thus, it is reasonable to suppose that the coefficient α_1 is greater in American League than in

National League. Actually, α_1 is known as 0.045 in American League, and it is 0.036 in National League. With the given proximity, the bigger α_1 implies the bigger estimate of Triple. Second, the two different proximities between a homerun and a triple are found in both American League and National League. The proximities are -0.739 and -1.031 for American League and National League, respectively.

C. Two Digit Triples

A triple rarely happens in a ball game. Since it seems hard to obtain three bases by one base hit, most players normally record less than ten triples per season. However, some players can advance to the third base by a double with their good base-running. These talented runners tend to produce abnormally many triples by their inborn skillfulness. In fact, these players have the history of recording two digit triples for many years. Two digit triples are unusual for other sluggers.

In some senses, those triples of talented runners are not appropriate to measure the proximity. Some level of attention should be paid on these abnormal observations to see if they might have affected the entire regression results. Unlike hitters in American League, many hitters are found to have recorded two digit triples in National League. Players' outstanding-base running might exogenously influence the proximity between homeruns and triples. This could be a good reason why a big difference exists between two proximities in both leagues. So, new estimation is suggested after eliminating all the two digit triples from National League. The test results are as follows.

National League (Without two digit triples)

First of all, Homerun is regressed on all explanatory variables, and Walk and Strikeout.

Table 3-1

Homerun	Coefficient	S.E.	t-Value	P-Value
Single	.1970244	.0178051	11.07	0.000
Double	-.299278	.0643885	-4.65	0.000
Triple	-.9059067	.1781947	-5.08	0.000
Walk	.136978	.0128841	10.63	0.000
Strikeout	.0556785	.010892	5.11	0.000

From Table 3-1, Triple turns out to have the larger estimated coefficient (-0.906) than the estimate (-1.031) in Table 2-1 after eliminating the two digit triples. The proximity between homeruns and triples is reduced. It means that a marginal effect of a triple on the number of homeruns has been weakened. On the other hand, the proximity between homeruns and doubles is strengthened. Two different proximities seem more balanced by removing two digit triples.

Table 3-2

Ln_Salaries1	Coefficient	S.E	t-Value	P-Value
Single	-.0063998	.005304	-1.21	0.228
Double	.0345286	.0129834	2.66	0.008
Triple	-.0839387	.0375094	-2.24	0.026
Homerun	.0252853	.0099478	2.54	0.011
Residual	.0421195	.018147	2.32	0.021
Constant	13.24066	.1811125	73.11	0.000

The endogeneity of Homerun should be tested. By the result shown in Table 3-2, Homerun is concluded as endogenous. Since Homerun is an endogenous variable, a simple OLS would produce a biased and an inconsistent estimator. As similar as previous studies, the OLS estimates are still provided for a comparison with the 2SLS estimates.

1) With a simple OLS

Table 3-3

Ln_Salaries1	Coefficient	S.E	t-Value	P-Value
Single	.0020497	.0038811	0.53	0.958
Double	.0252138	.0124235	2.03	0.043
Triple	-.1181063	.0347119	-3.40	0.001
Homerun	.0379424	.0083711	4.53	0.000
Constant	13.20513	.1815728	72.73	0.000

As seen in Table 3-3 above, the estimates of all the base hits are found to diminish after eliminating two digit triples. The estimate of Triple is actually decreased (increased in an absolute term). Here, the argument is focused more on a marginal effect of a base hit on a salary so the changes in estimates have interpretations that marginal effects of base hits have been affected by removing two digit triples. In fact, it indicates that two digit triples have played some roles in the regression results. Homerun has quite a small estimated coefficient 0.038. The marginal effect of homeruns on a salary seems weak. The relatively small effect strongly supports that the estimates are actually biased and inconsistent.

2) With a 2SLS

Table 3-4

Ln_Salaries1	Coefficient	S.E	t-Value	P-Value
Single	-.0063998	.0054286	-1.18	0.239
Double	.0345286	.0132885	2.60	0.010
Triple	-.0839387	.0383908	-2.19	0.029
Homerun	.0674049	.015534	4.34	0.000
Constant	13.24066	.1853682	71.43	0.000

Usually, good base-runners show relatively many triples largely due to their techniques but fewer homeruns, compared to other sluggers. It means that most of their triples would have

been a double to a normal-base-runner. However, the skillful hitters can obtain one more base by their tactics and their timely run. This point was not taken into account in the previous regressions. As a result, a triple seems to have less opportunity cost for a homerun. In American League, the estimated coefficient of Triple is -0.093, and the estimate substantially rises up to -0.041 in National League. Without two digit triples, the estimate restore the lower level -0.084 in National League. As discussed, the changed estimates insinuate that two digit triples of National League have affected the regression results.

4. Conclusion

‘Over-emphasis’ of homerun was the arguing point in the beginning of this research. Different from other sports, higher payroll does not necessarily improve team performances. Theoretically, owners hire expensive hitters and implement ‘*Egoism (Self-discipline)*’ from those hitters to have larger expected total bases by increasing the number of ‘big hits’ and by decreasing the number of singles. Thus, the uncertainty of total base increases. At bats, the expensive hitters choose ‘Swing Hard’ in either case that a pitcher throws a ‘Clear-strike’ or a ‘Boundary-strike’. In a sequential game, a pitcher throws a ‘Boundary-strike’ as the first mover. So, the uncertainty of total bases increases even more by these strike-alike balls. When a season ends, an owner collects all hitter’s records, reviews them, and compares them to his payments before he makes new contracts for the next year. He must have his own salary function.

Empirically, an owner’s salary function is estimated by the second stage least square. As the theory points out in section 2, owners totally disregard the number of singles to determine expensive hitters in both American League and National League. The P-values of Single are very high in both leagues. Meanwhile, bigger hits such as Homerun, Triple, and Double have significant estimates. Specially, Homerun always has the largest positive estimate in every case with 0 P-value. An implication is that the number of homeruns makes a big impact on an owner’s decision for a salary. Double also has positive estimates in all cases. The positive estimates of Homerun and Double support that owners actually emphasize big hits and implement ‘*Egoism (Self-discipline)*’. Interestingly, Triple has negative estimates in all cases. The negative estimate of Triple strongly supports that an

owner 'over-emphasizes' a homerun. As explained in section 2, the negative estimate could happen because of the opportunity cost for a homerun. The opportunity costs from a triple and a double actually change an owner's preference toward both types of hits. A double is preferred to a triple with less opportunity cost. In fact, the negative estimates of Triple show that owners put 'over-emphasis' on the number of homeruns.

Due to overall strength on big hits and over-emphasis on homeruns, the expected total bases increase and the uncertainty of total bases also increases. By 'over-emphasis' on a homerun, an owner does not successfully reduce the uncertainty of total bases but actually increase the uncertainty even more despite a lot of payments for expensive hitters. Under the larger uncertainty, the results of ball games seem more random.

Chapter 3. Any Subsidy Converges to a Direct Quantity Control Over Infinite Horizon

1. Introduction

A subsidy and a direct quantity control are leading trade policies over many countries. Many researchers have tried to figure out how these two main policies would support the domestic exporters and, furthermore, develop national welfare since these policies are mostly useful for helping domestic firms improve their profits. Interestingly, Cooper and Riezman (1989) found a general conclusion under what condition one policy is more useful than the other based on a game-theoretic approach. According to them, a subsidy is a dominant policy when a foreign market is placed under a sufficiently large uncertainty. The paper also shows that a dominant policy becomes a direct quantity control if uncertainty reduces to a sufficiently lower level in a foreign market. The result intuitively makes sense. Under no market uncertainty, an oligopolist reasonably would earn more profit by a constrained quantity. However, it is unsure if a constrained quantity still helps earn more profit under large demand variability. This is what Cooper and Riezman have mainly found through their four-stage long sequential game frame.

An important question can be raised when we extend their results to an infinite horizon model. That is, if we let the governments choose their policies through an infinite horizon then the subsidy policy is not so efficient. For example, if a government individually subsidizes exporters for every period infinite times then a social cost would be substantially big although a future subsidy should be discounted. Instead, if the policy makers reallocate a part of subsidy for infrastructure, in purpose, to stabilize demand variability for their own exporters and successfully induce a direct quantity control then the overall social cost could be saved.

This paper shows how a government improves total discounted social benefit by reallocating a subsidy through an infinite horizon and any dominant subsidy policy is changed to a direct quantity control over time. This paper eventually shows how the C-R

(referring Cooper and Riezman)'s SPNE, under the sufficiently large demand variability, changes over time for two different cases, one-sided intervention and both-sided intervention. In section 4, one-sided intervention represents that only one government involves in the project of stabilizing demand. In this case, the SPNE changes only once because one government actually changes its dominant policy to a quantity control by the time that the demand variability is stabilized. The SPNE might change more than once if both governments join the project of stabilizing demand.

In section 5, we discuss the both-sided intervention case. Depending on the number of exporting firms, the SPNE would change only once or more than once. That is, if both countries have the same number of exporting firms then the demand stability is realized at the same time for both countries. Under this symmetric case, the SPNE would change only once over time because both governments coincidentally choose a quantity control as a dominant policy. Instead, if both countries have different number of exporting firms then their demand stabilities are realized by the different timings because of asymmetric condition. Under this asymmetric condition, the SPNE would change twice over time because both governments differently change their dominant policies to a quantity control.

2. Foreign market under market uncertainty

According to Cooper and Riezman (1989), both governments involving trade policy game face four stages to reach a final output. At the first stage, the two public authorities determine an intervention mode. Secondly, the rivals decide intervention levels. At the third, market uncertainty is revealed to the firms. Finally, the firms maximize their profits with the given government's intervention at the fourth stage. Most of all, the firms' optimal quantities under the given trade policies must be found in the fourth stage.

Cooper and Riezman introduced the concept of social benefit, the sum of all domestic firms' profits. By using the derived the optimal quantities in the fourth stage, an expected social benefit function or formula can be investigated from a domestic government's view. The problem is to find the ex post subsidy that maximizes this expected social benefit.

Simply speaking, the policy makers need to find a Nash equilibrium of the reduced game as an optimal subsidy. This Nash equilibrium actually represents the SPNE. The authors have different SPNE according to the size of variance, which is denoted by θ . In fact, when the variance of θ is sufficiently large, the government's policy mode must be chosen as a subsidy. When the variance of θ becomes sufficiently small, the government's policy mode should be chosen as a direct quantity control.

Intuitively, a government anticipates how domestic firms would obtain their profits from the trade policy or it can be foreseen how the firms will be benefited by the selected governmental service. To choose any policy, a government considers an expected social benefit, the sum of all domestic firms' expected profits.

The values below show the obtained expected social benefits in an oligopolistic situation from both governments' perspectives by Cooper and Riezman. According to the tables, each government would reach an expected social benefit by choosing a trade police such as a subsidy or a direct quantity control.

(Table 1) Expected social benefit for a Country 1's firm from Cooper and Riezman (1989)

		Country 2	
		Subsidies	Quantities
Country1	Subsidies	$\frac{(a-c)^2(1+N_2)}{bN_1(F+3)^2} + \frac{Var(\theta)}{b(F+1)^2}$	$\frac{(a-c)^2}{bN_1(N_1+3)^2} + \frac{Var(\theta)}{b(N_1+1)^2}$
	Quantities	$\frac{(a-c)^2(N_2+1)}{bN_1(N_2+3)^2}$	$\frac{(a-c)^2}{9bN_1}$

(Table 2) Expected social benefit for a Country 2's firm from Cooper and Riezman (1989)

		Country 2	
		Subsidies	Quantities
Country1	Subsidies	$\frac{(a-c)^2(1+N_1)}{bN_2(F+3)^2} + \frac{Var(\theta)}{b(F+1)^2}$	$\frac{(a-c)^2(N_1+1)}{bN_2(N_1+3)^2}$
	Quantities	$\frac{(a-c)^2}{bN_2(N_2+3)^2} + \frac{Var(\theta)}{b(N_2+1)^2}$	$\frac{(a-c)^2}{9bN_2}$

Both governments simultaneously select either subsidy or direct quantity control as a policy mode, and four different sequential game starts with four nodes if the entire police game is depicted. So, there are supposed to be four different expected social benefits in a reduced form at which both governments choose their police modes. In a game theoretic sense, the four-stage long extensive game has been reduced to a one-shot simultaneous game by backward induction.

According to the expected social benefits matrix above, the SPNE is $((Policy\ mode\ is\ Subsidy, Policy\ mode\ is\ Subsidy), (s_1^*, s_2^*), (\theta\ is\ revealed), (q_1^*, q_2^*))$ when $Var(\theta)$ is sufficiently large. This SPNE changes to $((Policy\ mode\ is\ Direct\ Quantity\ Control, Policy\ mode\ is\ Direct\ Quantity\ Control), (q_1^{**}, q_2^{**}), (\theta\ is\ revealed), (q_1^{**}, q_2^{**}))$ when $Var(\theta)$ is sufficiently small.

3. Expenditure for Infrastructure and Variance Assessment

Any exporting firms are supposed to encounter a substantial level of demand variability in a newly developed foreign market. Sometimes, foreign customers have biased preferences toward the origins of products or the nationalities. If they originate from the same country then they are assumed to meet the same demand variability. Under a volatile demand, a subsidy should be a dominant policy for both governments according to Cooper and Riezman because a policy helps firms individually adjust to unstable circumstances. However, if policy makers think over their choices under an infinite horizon, they would realize that stabilizing market demand eventually helps improve their social benefit more than subsidizing firms for every period. First, they assess demand variability and search for any possibility to lower the variability for domestic firms. Accordingly, a government spends some expenditure for infrastructure to stabilize demand.

For example, Korean government has a special institution such as KOTRA to bolster domestic exporters, and the public institution has local agencies over many countries. Its main objective is to help Korean firms have more information about a market or a local area. In fact, highly skilled manpower performs a high quality of research about a market

and regularly has an open seminar in which consumer reports and market details are covered. In some cases, they are requested a market survey or forecasting. Since KOTRA successfully achieved its goal of supporting Korean exporters over all countries, some local governments of Korea have their own institutions or facilities to benefit mid-sized firms originating from their local provinces these days.

Korean exporting firms used to be provided a financial support (a subsidy) for their exported products. However, they are more benefited from well-founded governmental agencies over the world. Intuitively, Korean firms would become oligopolists when they eventually have stable demand, and they are suggested to constrain the exported quantity for raising their profitability.

1) Variance Assessment

A government collects all information about a market and concludes that the demand variability could be overcome by a public-led effort. The variance of θ_i is a decreasing function of expenditure for infrastructure (g_i) such as $Var(\theta_i) = \sigma_i^2 - kg_i$. As mentioned, the market level of variance is a sum of all individual levels of variances such as $Var(\theta) = \sum_{i=1}^N Var(\theta_i)$. This idea is from the total demand $Q = \sum_{i=1}^N q_i$. The variance of total

demand is constituted of the variances of individual demands such as $Var(Q) = \sum_{i=1}^N Var(q_i)$.

Products are originated from different countries so the differently-originated products would face different demand variabilities due to prejudices for their origins. For example, Japanese cars, Korean mobile phones. Customers usually generalize a product's quality by its origin.

2) Expenditure for infrastructure

A government actually spends some expenditure (g_1) on establishing public infrastructure within a limited budget ($0 < g_1 < s_1^*$).

4. *One-sided Intervention*

Here, only one government has a motivation to lower demand variability for helping its domestic firms by a public infrastructure. In the following section, two different governments simultaneously establish a public infrastructure for their own domestic firms.

Formally, government 1 reallocates the pre-determined budget s_1^* for subsidy into \bar{s}_1 (a subsidy for exported good) and g_1 (expenditure for infrastructure). That is, $s_1^* = \bar{s}_1 + g_1$. Simply speaking, government 1 finds the optimal value, g_1^* , maximizing the total expected discounted social benefit by reducing the demand variability to the efficient level under the constraint, $s_1^* = \bar{s}_1 + g_1$.

However, government 2 spends no expenditure for infrastructure ($g_2 = 0$) because of no reallocated subsidy. Thus, $s_2^* = \bar{s}_2$.

As government 1 spends some money for infrastructure over time to bring demand stability, a stable demand becomes present earlier. Importantly, Cooper and Riezman have not provided the information about the value of variance by which an unstable demand could be turned into a stable demand. A government would recognize 'by chance' that the individual variability for its domestic firms has been sufficiently reduced. A stable demand is assumed to arrive by exponential probability density.

A government's expenditure for infrastructure affects the timing of a stable demand. If g_1 denotes an instantaneous spending for infrastructure by government 1 then the instant success rate for realization of a stable demand, u is $\alpha(g_1)$. The time of occurrence of

u is denoted as τ_u , which is exponentially distributed with a density function.

$$\psi_u(\tau_u) = \alpha(g_1)e^{-\alpha(g_1)\tau_u}, \quad 0 < \tau_u < \infty$$

The dominance of a subsidy policy still remains until θ_1 's variance diminishes to the sufficiently small level. As mentioned, the value of the sufficiently small variance is still unknown. Implicatively, a public-led effort can not stop due to unawareness of the boundary of variance although the variance continuously diminishes. Rather, a government would recognize an arrival of a stable demand for its domestic firms by chance.

Once individual demand variability is sufficiently reduced, a direct quantity control becomes a dominant policy for a government while the rival still chooses a subsidy policy. The **SPNE** changes by the timing of a stable demand for country 1.

As mentioned before, government 1 takes a part (g_1) from the pre-determined level (s_1^*) and reallocates for each period. With expenditure for infrastructure ($g_1 > 0$), the new expected social benefit can be found as follows.

$$E\pi_1'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*) = \frac{(a-c)^2(1+N_2)}{bN_1(F+3)^2} + \frac{Var'(\theta_1)}{b(F+1)^2}$$

$$+ \left[\frac{(a-c)(1+N_2)}{b(F+1)(F+3)} \right] g_1 - \left[\frac{bN_1}{b(F+1)} \right] \left[\frac{1+N_2}{b(F+1)} \right] g_1^2$$

where $s_1' < \bar{s}_1$ and $Var'(\theta_1) = \sigma^2 - kg_1$

With an increased expenditure (g_1) for infrastructure, an individual-financial support (subsidy) level for an exported product should be reduced, and $Var(\theta)$ continuously diminishes for every time period while the government's overall expenditure ($s_1^* = \bar{s} + g_1$) for domestic firms should remain the same.

$$\begin{aligned}
E\pi'_1(s_1^*(s_1 = s'_1, g_1 > 0), s_2^*) &= E\pi_1(s_1^*, s_2^*) - \frac{kg_1}{b(F+1)^2} \\
&+ \left[\frac{(a-c)(1+N_2)}{b(F+1)(F+3)} \right] g_1 - \left[\frac{bN_1}{b(F+1)} \right] \left[\frac{1+N_2}{b(F+1)} \right] g_1^2 \\
E\pi_1(s_1^*, s_2^*) - E\pi'_1(s_1^*(s_1 = s'_1, g_1 > 0), s_2^*) \\
&= \\
\left[\frac{bN_1}{b(F+1)} \right] \left[\frac{1+N_2}{b(F+1)} \right] g_1^2 - \left[\frac{(a-c)(1+N_2)}{b(F+1)(F+3)} \right] g_1 + \frac{kg_1}{b(F+1)^2}
\end{aligned}$$

As a subsidy is reduced, total social benefit decreases. Interestingly, the marginal change of total social benefit from reducing a subsidy and establishing infrastructure can be minimized for each period. With continuous spending for demand stability, a stable demand will be realized. However, the timing of a stable demand is randomly distributed as mentioned. By an arrival of a stable demand, two separate stages (the Initial Stage and Stage One) can be defined. In the Initial Stage, a government still spends some money for infrastructure to bring a stable demand and no firms have a stable demand. The Initial Stage continues until demand stability is achieved by government 1's support. By an occurrence of demand stability, the Initial Stage changes to Stage One. Both governments have different total expected social welfares in each stage. At first, the total expected discounted social benefits in Stage One for both governments can be found.

1) In Stage One

In Stage One, the demand variability remains sufficiently small for the firms originating from country 1. A stable demand is assumed to take place at $t = N$. However, the firms originating from the other country still face larger demand variability.

By the timing of a stable demand, government 1 would deviate from a subsidy to a direct quantity control. By government 1's deviation, both governments face the new expected social benefits as below. By referring to the expected social benefit matrix (Table 1) and (Table 2),

$$E\pi_1(q_1^{**}, s_2^*) = \frac{(a-c)^2(N_2+1)}{bN_1(N_2+3)^2}$$
$$E\pi_2(q_1^{**}, s_2^*) = \frac{(a-c)^2}{bN_2(N_2+3)^2} + \frac{Var(\theta_2)}{b(N_2+1)^2}$$

Government 2 still has a dominant policy of a subsidy because $Var(\theta_2)$ has never changed. Actually, the new bilateral game starts at $t = N$ and new social benefits are $E\pi_1(q_1^{**}, s_2^*)$ and $E\pi_2(q_1^{**}, s_2^*)$ for country 1 and country 2, respectively, in every time period. By comparing $E\pi_1(q_1^{**}, s_2^*)$ and $E\pi_2(q_1^{**}, s_2^*)$, an interesting point can be found. Unlike country 1, the variance plays a major role in the new social benefit for country 2. It ironically supports that a subsidy policy is proper under larger uncertainty and the firms from country 2 still has larger demand variability. An intuition is that total social welfare for country 2 has larger uncertainty while total social welfare for country 1 has less uncertainty.

With a government-supported infrastructure, a trade policy converges to a direct quantity control. A government always considers the best policy for its own domestic exporters. Under an unstable demand, a subsidy helps domestic firms most efficiently because funded firms can individually adjust to dynamic market circumstances. Meanwhile, a direct

quantity control policy must be more efficient under a stable demand. In fact, oligopoly firms tend to sell smaller amount at a higher price for larger total revenue in a market. This is the main reason why a government chooses a direct quantity control as a dominant policy under demand stability.

Let $H_{N,1}^*$ denote the government 1's total expected discounted social welfare at $t = N$.

$$H_{N,1}^* = \int_0^{\infty} e^{-rt} E\pi_1(q_1^{**}, s_2^*) dt = \frac{E\pi_1(q_1^{**}, s_2^*)}{r}$$

Let $H_{N,2}^*$ denote the government 2's expected discounted total social welfare at $t = N$.

$$H_{N,2}^* = \int_0^{\infty} e^{-rt} E\pi_2(q_1^{**}, s_2^*) dt = \frac{E\pi_2(q_1^{**}, s_2^*)}{r}$$

2) In the Initial Stage

As remarked earlier, the Initial Stage lasts unless market uncertainty is removed, and government 1 still spends expenditure for infrastructure to support domestic exporters. To avoid any confusion, the Initial Stage is assumed to start at $t = 0$.

Let $H_{0,1}^*$ denote government 1's total expected discounted social benefit at $t = 0$. No one knows when a stable demand is realized for the firms from country 1. A stable demand causes a substantial change on an exporter's profit and eventually on a government's expected discounted total social benefit. Government 1 is able to derive $H_{0,1}^*$ by the randomly distributed timing of a stable demand and its probability density function, depending on g_1 .

$$\begin{aligned}
H_{0,1}^* &= \int_0^\infty \left[\int_0^\tau e^{-rt} (E\pi_1(s_1^*(s_1 = s_1', g_1 > 0), s_2^*)) dt + \int_\tau^\infty [e^{-rt} (E\pi_1(q_1^{**}, s_2^*)) dt] \right] \psi(\tau) d\tau \\
&= \int_0^\infty \left[\frac{1}{r} E\pi_1'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*) [1 - e^{-r\tau}] + \frac{1}{r} e^{-r\tau} (E\pi_1(q_1^{**}, s_2^*)) \right] \psi(\tau) d\tau \\
&= \frac{E\pi_1'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*)}{r} \left[1 - \frac{\alpha(g_1)}{r + \alpha(g_1)} \right] + \frac{\alpha(g_1)}{r + \alpha(g_1)} \left[\frac{E\pi_1(q_1^{**}, s_2^*)}{r} \right]
\end{aligned}$$

where $\psi(\tau) = \alpha(g_1) e^{-\alpha(g_1)\tau}$, $0 < g_1 < s_1^*$, $\alpha(g_1) = g_1^{\left(\frac{1}{k}\right)}$

Thus, $H_{0,1}^* = \frac{E\pi_1'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*) + \alpha(g_1)[H_{N,1}^*]}{r + \alpha(g_1)}$, where $H_{N,1}^* = \frac{E\pi_1(q_1^{**}, s_2^*)}{r}$

$$E\pi_1'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*) = \frac{(a-c)^2(1+N_2)}{bN_1(F+3)^2} + \frac{\text{Var}(\theta_1)}{b(F+1)^2}$$

$$+ \left[\frac{(a-c)(1+N_2)}{b(F+1)(F+3)} \right] g_1 - \left[\frac{bN_1}{b(F+1)} \right] \left[\frac{1+N_2}{b(F+1)} \right] g_1^2$$

Interestingly, government 1 can determine the optimal level of g_1 in each period. The intuition is that when the government reduces an individual financial support (a subsidy) and allocates the reduction for a public infrastructure the expected social welfare is not linearly decreased. As mentioned before, a marginal decrease of the expected social benefit from reducing a subsidy could be minimized.

$$\text{Min}_{g_1} E\pi_1(s_1^*, s_2^*) - E\pi_1'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*)$$

$$\text{where } E\pi_1(s_1^*, s_2^*) - E\pi_1'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*)$$

$$= \left[\frac{bN_1}{b(F+1)} \right] \left[\frac{1+N_2}{b(F+1)} \right] g_1^2 - \left[\frac{(a-c)(1+N_2)}{b(F+1)(F+3)} \right] g_1 + \frac{kg_1}{b(F+1)^2}$$

$$g_1^* = \frac{\left[\frac{(a-c)(1+N_2)}{b(F+1)(F+3)} \right] - \frac{k}{b(F+1)^2}}{\left[\frac{2bN_1}{b(F+1)} \right] \left[\frac{1+N_2}{b(F+1)} \right]}$$

With government 1' optimal expenditure (g_1^*) for infrastructure, the instant success rate is determined. As government 1 spends a determined expenditure to reduce $\text{Var}(\theta_1)$ for more time periods, the probability of reaching a stable demand rises.

Country 1's total expected social welfare can be calculated as follows.

$$H_{0,1}^* = \frac{E\pi_1'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*) + (g_1^*)^{\frac{1}{k}} [H_{N,1}^*]}{r + (g_1^*)^{\frac{1}{k}}}$$

$$\text{where } E\pi_1'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*) = \frac{(a-c)^2(1+N_2)}{bN_1(F+3)^2} + \frac{\sigma^2 - kg_1^*}{b(F+1)^2}$$

$$+ \left[\frac{(a-c)(1+N_2)}{b(F+1)(F+3)} \right] g_1^* - \left[\frac{bN_1}{b(F+1)} \right] \left[\frac{1+N_2}{b(F+1)} \right] (g_1^*)^2$$

Let $H_{0,2}^*$ denote government 2's total expected discounted social benefit at $t = 0$.

$$H_{0,2}^* = \int_0^{\infty} \left[\int_0^{\tau} e^{-rt} \left[E\pi_2'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*) \right] dt + \int_{\tau}^{\infty} e^{-rt} \left[E\pi_2^*(q_1^{**}, s_2^*) \right] dt \right] \psi(\tau) d\tau$$

where $\psi(\tau) = \alpha(g_1) e^{-\alpha(g_1)\tau}$

$$H_{0,2}^* = \frac{E\pi_2'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*) + (g_1^*)^{\frac{1}{k}} [H_{N,2}^*]}{r + (g_1^*)^{\frac{1}{k}}} \quad \text{where} \quad H_{N,2}^* = \frac{E\pi_2^*(q_1^{**}, s_2^*)}{r}$$

$$E\pi_2'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*) = \frac{(a-c)^2(1+N_1)}{bN_2(F+3)^2} + \frac{Var(\theta)}{b(F+1)^2} + \left[\frac{N_1}{F+1} \right] \left[\frac{(a-c)(1+N_1)}{bN_2(F+3)} \right] g_1^* + \frac{1}{b} \left[\frac{N_1}{F+1} \right]^2 (g_1^*)^2$$

Since the optimal choice of expenditure for infrastructure is made by government 1, SPNE should be revised for each period.

The SPNE, ((*Policy mode is Subsidy, Policy mode is Subsidy*), (s_1^*, s_2^*) , (θ_1 and θ_2 are revealed), (q_1^*, q_2^*)), lasts until a stable demand is realized for the firms from country 1.

Then it turns to New SPNE, ((*Policy mode is Direct quantity control, Policy mode is Subsidy*), (q_1^{**}, s_2^*) , (θ_1 and θ_2 are revealed and $Var(\theta_1)$ turns out to be sufficiently small), (q_1^{**}, q_2^*)).

This SPNE remains forever.

5. Both-sided Intervention

Only one government has been discussed so far. Here, both governments simultaneously reduce their levels of subsidies for increasing total social benefits under infinite horizon. They do prefer less uncertainty of total social welfare so stabilizing demand for their domestic firms would help. There are two different cases, symmetry and asymmetry. The

symmetry supports that both countries have equal number of firms in a foreign market. The asymmetry represents the case that one country has larger number of firms than the other country in a foreign market. Thus, demand variability for the firms from the country should be larger than demand variability for the firms from the other country. A stable demand takes more time in this case.

1. Symmetric case ($N_1 = N_2 = N$)

This case represents that both countries would have equal number of firms in a foreign market. Unlike the asymmetric case, Nash equilibrium expenditure for infrastructure is identical for both governments. An intuition is that both governments' choices would be the same under the same circumstances. The identical choices lead all the firms to have a stable demand at the same time regardless their origins since the instant success rates become the same in this case.

As similar as the asymmetric case, there are two different stages, Initial Stage and Stage One. In the Initial Stage, both governments spend money for infrastructure and a stable demand is not realized in this stage. This stage maintains until both governments successively stabilize the demands for their own domestic firms. By the timing of stabilizing demands, the Initial Stage changes into Stage One. No governments spend money for infrastructure in Stage one since a stable demand coincidentally happens for all the firms regardless the origins.

1) Stage One

Both governments stop spending for infrastructure. By stable demands for all the firms from both origins, both governments would change a trade policy from a subsidy to a direct quantity control. In this stage, the total expected discounted social benefits for both governments are as follows. It is assumed that stable demands happen for the firms from both countries at $t = N$.

Let $H_{N,1}^*$ denote the government 1's total expected discounted social welfare at $t = N$.

$$H_{N,1}^* = \int_0^{\infty} e^{-rt} E\pi_1(q_1^{**}, q_2^{**}) dt = \frac{E\pi_1(q_1^{**}, q_2^{**})}{r}$$

Let $H_{N,2}^*$ denote the government 2's total expected discounted social welfare at $t = N$.

$$H_{N,2}^* = \int_0^{\infty} e^{-rt} E\pi_2(q_1^{**}, q_2^{**}) dt = \frac{E\pi_2(q_1^{**}, q_2^{**})}{r}$$

2) The Initial Stage

In this stage, no stable demand has been realized for the firms from either country. Both governments continuously spend money for infrastructure to reduce the market variances for their own domestic firms. By backward induction, the entire sequential game can be reduced to the Initial Stage. Both governments' total expected discounted social benefits are found as below.

Let $H_{0,1}^*$ denote the government 1's total expected discounted social welfare at $t = 0$.

$$\begin{aligned} H_{0,1}^* &= \int_0^{\infty} \left[\int_0^{\tau_1} e^{-rt} \left[E\pi_1'(s_1'(g_1), s_2'(g_2)) \right] dt + \int_{\tau_1}^{\infty} e^{-rt} \left(E\pi_1(q_1^{**}, q_2^{**}) \right) \right] \psi(\tau_1) d\tau_1 \\ &= \frac{E\pi_1'(s_1'(g_1), s_2'(g_2))}{r} + \frac{\alpha(g_1)}{[r + \alpha(g_1)]} \left[\frac{E\pi_1'(s_1'(g_1), s_2'(g_2))}{r} - \frac{E\pi_1(q_1^{**}, q_2^{**})}{r} \right] \end{aligned}$$

Let $H_{0,2}^*$ denote the government 2's total expected discounted social welfare at $t = 0$.

$$\begin{aligned}
H_{0,2}^* &= \int_0^{\infty} \left[\int_0^{\tau_2} e^{-rt} \left[E\pi_2'(s_1'(g_1), s_2'(g_2)) \right] dt + \int_{\tau_2}^{\infty} e^{-rt} \left(E\pi_2(q_1^{**}, q_2^{**}) \right) \right] \psi(\tau_2) d\tau_2 \\
&= \frac{E\pi_2'(s_1'(g_1), s_2'(g_2))}{r} + \frac{\beta(g_2)}{[r + \beta(g_2)]} \left[\frac{E\pi_2'(s_1'(g_1), s_2'(g_2))}{r} - \frac{E\pi_2(q_1^{**}, q_2^{**})}{r} \right]
\end{aligned}$$

where $\alpha(g_1) = \beta(g_2)$

In determining the optimal expenditure for infrastructure, both governments would face the simultaneous game situation. As reviewed in one-sided intervention case, when a government cuts a subsidy to support infrastructure then a marginal decrease of the expected social welfare can be minimized. In this context, Nash Equilibrium (g_1^*, g_2^*) can be found.

Due to the expenditures for infrastructure, both governments reduce subsidies as below.

$$s_i' = s_i^* - g_i \quad \text{where} \quad s_i^* = \frac{(a-c)(1-N_i-N_{-i})}{N_i(F+3)} \quad \text{for } i=1,2$$

The corresponding quantities are found as follows.

$$\begin{aligned}
q_1' &= q_1^* - \left[\frac{1+N_2}{b(F+1)} \right] g_1 + \frac{N_2}{b(F+1)} g_2 \\
q_2' &= q_2^* - \left[\frac{1+N_1}{b(F+1)} \right] g_2 + \frac{N_1}{b(F+1)} g_1 \quad \text{where} \quad q_i^* = \frac{(a-c)(1+N_{-i})}{bN_i(F+3)} + \frac{\theta_i}{b(F+1)} \quad \text{for } i=1,2
\end{aligned}$$

Both firms' new social welfare functions are derived as below.

$$E\pi_1'(s_1'(g_1), s_2'(g_2)) = E\pi_1(s_1^*, s_2^*) + \left[\frac{N_1 g_1 + N_2 g_2}{F+1} \right] \left[\frac{(a-c)(1+N_2)}{bN_1(F+3)} - \left(\frac{1+N_2}{b(F+1)} \right) g_1 + \frac{N_2}{b(F+1)} g_2 \right]$$

$$E\pi_2'(s_1'(g_1), s_2'(g_2)) = E\pi_2(s_1^*, s_2^*) + \left[\frac{N_1 g_1 + N_2 g_2}{F+1} \right] \left[\frac{(a-c)(1+N_1)}{bN_2(F+3)} - \left(\frac{1+N_1}{b(F+1)} \right) g_2 + \frac{N_1}{b(F+1)} g_1 \right]$$

where $E\pi_i(s_1^*, s_2^*) = \frac{(a-c)^2(1+N_{-i})}{bN_i(F+3)^2} + \frac{(\sigma_i^2 - kg_i)}{b(F+1)}$ for $i = 1, 2$

By minimizing the marginal decrease of total social benefits from the reduced subsidy, Nash Equilibrium (g_1^*, g_2^*) can be found.

From the first order condition, $\frac{d \left[E\pi_1(s_1^*, s_2^*) - E\pi_1'(s_1'(g_1), s_2'(g_2)) \right]}{dg_1} = 0$

$$-k + b(F+1)N_1 \frac{(a-c)(1+N_2)}{bN_1(F+3)} - 2N_1(1+N_2)g_1 + N_2(N_1 - 1 - N_2)g_2 = 0$$

From the first order condition $\frac{d \left[E\pi_2(s_1^*, s_2^*) - E\pi_2'(s_1'(g_1), s_2'(g_2)) \right]}{dg_2} = 0$

$$-k + b(F+1)N_2 \frac{(a-c)(1+N_1)}{bN_2(F+3)} - 2N_2(1+N_1)g_2 + N_1(N_2 - 1 - N_1)g_1 = 0$$

Under the symmetric condition $N_1 = N_2$, each government considers that its rival would make the same choice as itself.

$$g_1^* = g_2^* = \frac{\left[\frac{(a-c)(F+1)(1+N)}{(F+3)} - k \right]}{2N(1+N)}$$

The equilibrium total expected discounted social benefits for both governments are found as below, and they are actually equal.

$$H_{0,1}^{**} = \frac{E\pi_1'(s_1'(g_1^*), s_2'(g_2^*))}{r} + \frac{\alpha(g_1^*)}{[r + \alpha(g_1^*)]} \left[\frac{E\pi_1'(s_1'(g_1^*), s_2'(g_2^*))}{r} - \frac{E\pi_1(q_1^{**}, q_2^{**})}{r} \right]$$

$$H_{0,2}^{**} = \frac{E\pi_2'(s_1'(g_1^*), s_2'(g_2^*))}{r} + \frac{\beta(g_2^*)}{[r + \beta(g_2^*)]} \left[\frac{E\pi_2'(s_1'(g_1^*), s_2'(g_2^*))}{r} - \frac{E\pi_2(q_1^{**}, q_2^{**})}{r} \right]$$

Thus, $H_{0,1}^{**} = H_{0,2}^{**}$

The SPNE, ((*Policy mode is Subsidy, Policy mode is Subsidy*), (s_1^*, s_2^*) , (θ_1 and θ_2 are revealed), (q_1^*, q_2^*)), lasts until a stable demand is realized for the firms from country 1. Then it turns to New SPNE, ((*Policy mode is Direct quantity control, Policy mode Direct quantity control*), (q_1^{**}, q_2^{**}) , (θ_1 and θ_2 are revealed, and $\text{Var}(\theta_1)$ and $\text{Var}(\theta_2)$ coincidentally turn out to be sufficiently small), (q_1^{**}, q_2^{**})). This SPNE remains forever.

2. Asymmetric case (the number of firms from country 2 is greater than the number of firms from country 1, $N_2 > N_1$)

In this case, a stable demand for the firms from country 1 should be realized earlier than country 2. There are also two different stages, Initial stage and Stage One, by the timing of a stable demand for country 1.

1) Stage One

Government 1 spends nothing for infrastructure since a stable demand for firms from country 1 was already realized in the past. However, a stable demand for the other country

is still in a process so government 2 spends for infrastructure. In this stage, total social benefits for both countries can be found as below.

First, government 1 would consider two different instant social benefits $E\pi_1(q_1^{**}, s_2^*)$ and $E\pi_1(q_1^{**}, q_2^{**})$ by the timing τ_2 to calculate its overall social welfare as below.

$$H_{N,1}^* = \int_0^{\infty} \left[\int_0^{\tau_2} e^{-rt} (E\pi_1(q_1^{**}, s_2^*)) dt + \int_{\tau_2}^{\infty} e^{-rt} (E\pi_1(q_1^{**}, q_2^{**})) dt \right] \psi(\tau_2) d\tau_2$$

The integral above actually represents its own expected discounted social benefit after government 1 stabilizes demand for its own domestic firms and can be solved as follows

$$\begin{aligned} &= \int_0^{\infty} \left[\frac{1}{r} (E\pi_1(q_1^{**}, s_2^*)) [1 - e^{-r\tau_2}] + \frac{1}{r} (E\pi_1(q_1^{**}, q_2^{**})) e^{-r\tau_2} \right] \psi(\tau_2) d\tau_2 \\ &= \frac{1}{r} (E\pi_1(q_1^{**}, s_2^*)) - \frac{1}{r} (E\pi_1(q_1^{**}, s_2^*)) \left[\frac{1}{r + \beta(g_2)} \right] + \frac{1}{r} (E\pi_1(q_1^{**}, q_2^{**})) \left[\frac{1}{r + \beta(g_2)} \right] \end{aligned}$$

Similarly, government 2 can find total social benefit as below. From government 2's perspective, its own demand stability would be realized no earlier than government 1's. After government 1's demand stability at τ_1 , government 2's expected discounted social benefit can be found by the integral as below.

$$\begin{aligned} H_{N,2}^* &= \int_0^{\infty} \left[\int_0^{\tau_2} e^{-rt} (E\pi_2(q_1^{**}, s_2^*)) dt + \int_{\tau_2}^{\infty} e^{-rt} (E\pi_2(q_1^{**}, q_2^{**})) dt \right] \psi(\tau_2) d\tau_2 \\ &= \int_0^{\infty} \left[\frac{1}{r} (E\pi_2(q_1^{**}, s_2^*)) [1 - e^{-r\tau_2}] + \frac{1}{r} (E\pi_2(q_1^{**}, q_2^{**})) e^{-r\tau_2} \right] \psi(\tau_2) d\tau_2 \\ &= \frac{1}{r} (E\pi_2(q_1^{**}, s_2^*)) - \frac{1}{r} (E\pi_2(q_1^{**}, s_2^*)) \left[\frac{1}{r + \beta(g_2)} \right] + \frac{1}{r} (E\pi_2(q_1^{**}, q_2^{**})) \left[\frac{1}{r + \beta(g_2)} \right] \end{aligned}$$

2) Initial Stage

As explained before, there two demand variability by different countries. Each government can reduce its own demand variability by expenditure for infrastructure. Two different governments would stabilize their demand variability at a different time. If government 1 successfully reduces the variance arising for the country 1 earlier than government 2 then its total expected discounted social welfare can be calculated as follows.

$$H_{0,1}^* = \int_0^{\infty} \left[\int_0^{\tau_1} e^{-rt} [E\pi_1'(s_1'(g_1), s_2'(g_2))] dt + \int_{\tau_1}^{\infty} e^{-rt} [H_{N,1}^*] \right] \psi(\tau_1) d\tau_1$$

$$\text{where } E\pi_1'(s_1'(g_1), s_2'(g_2)) = E\pi_1'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*(s_2 = s_2', g_2 > 0))$$

That is, government 2 still tries to reduce the variance arising for the country 2 after government 1 already has demand stability. By the randomly distributed timing τ_1 , government 1 chooses q_1^{**} while the rival chooses s_2^* . Later on, government 2 would succeed in stabilizing its individual market demand by τ_2 . It means that government 1's total expected social welfare will be changed from $E\pi_1(q_1^{**}, s_2^*)$ to $E\pi_1(q_1^{**}, q_2^{**})$ by the timing τ_2 .

So, government 1's total expected discounted social welfare is derived as below.

$$H_{0,1}^* = \frac{1}{r} [E\pi_1'(s_1'(g_1), s_2'(g_2))] - \left[\frac{\alpha(g_1)}{r + \alpha(g_1)} \right] + \left[\frac{\alpha(g_1) \left[\frac{H_{N,1}^* [\beta(g_2)]}{r} \right]}{r + \alpha(g_1)} \right]$$

Similarly, government 2 can find its total expected discounted social welfare as below.

$$H_{0,2}^* = \int_0^{\infty} \left[\int_0^{\tau_1} e^{-rt} [E\pi_2'(s_1'(g_1), s_2'(g_2))] dt + \int_{\tau_1}^{\infty} e^{-rt} [H_{N,2}^*] \right] \psi(\tau_1) d\tau_1$$

$$\text{where } E\pi_2'(s_1'(g_1), s_2'(g_2)) = E\pi_2'(s_1^*(s_1 = s_1', g_1 > 0), s_2^*(s_2 = s_2', g_2 > 0))$$

Eventually, government 2's total expected discounted social welfare is found as follows.

$$H_{0,2}^* = \frac{1}{r} [E\pi_2'(s_1'(g_1), s_2'(g_2))] - \left[\frac{\alpha(g_1)}{r + \alpha(g_1)} \right] + \left[\frac{\alpha(g_1) \left[\frac{H_{N,2}^*[\beta(g_2)]}{r} \right]}{r + \alpha(g_1)} \right]$$

Both governments' total expected discounted social benefits are actually functions of instant success rates, which depends on expenditures for infrastructure. To find the Nash Equilibrium (g_1^*, g_2^*) , the first order conditions should be reconsidered.

$$\text{From the first order condition, } \frac{d \left[E\pi_1(s_1^*, s_2^*) - E\pi_1'(s_1'(g_1), s_2'(g_2)) \right]}{dg_1} = 0$$

$$-k + b(F+1)N_1 \frac{(a-c)(1+N_2)}{bN_1(F+3)} - 2N_1(1+N_2)g_1 + N_2(N_1-1-N_2)g_2 = 0$$

$$\text{From the first order condition } \frac{d \left[E\pi_2(s_1^*, s_2^*) - E\pi_2'(s_1'(g_1), s_2'(g_2)) \right]}{dg_2} = 0$$

$$-k + b(F+1)N_2 \frac{(a-c)(1+N_1)}{bN_2(F+3)} - 2N_2(1+N_1)g_2 + N_1(N_2-1-N_1)g_1 = 0$$

$$g_1^* = \frac{2(1+N_1) \left[\frac{(a-c)(F+1)(1+N_2)}{(F+3)} - k \right] + (N_1-1-N_2) \left[\frac{(a-c)(F+1)(1+N_1)}{(F+3)} - k \right]}{4N_1(1+N_2)(1+N_1) - [N_1(N_2-1-N_1)](N_1-1-N_2)}$$

$$g_2^* = \frac{\left[\frac{(a-c)(F+1)(1+N_1)}{(F+3)} - k \right]}{2N_2(1+N_1)} + \frac{N_1(N_2-1-N_1)}{2N_2(1+N_1)} g_1^*$$

The equilibrium total expected discounted social benefits for both governments are follows.

$$H_{0,1}^{**} = \frac{1}{r} \left[E\pi_1'(s_1^*(g_1^*), s_2^*(g_2^*)) \right] - \left[\frac{\alpha(g_1^*)}{r+\alpha(g_1^*)} \right] + \left[\frac{\alpha(g_1^*) \left[\frac{H_{N,1}^*[\beta(g_2^*)]}{r} \right]}{r+\alpha(g_1^*)} \right]$$

$$H_{0,2}^{**} = \frac{1}{r} \left[E\pi_2'(s_1^*(g_1^*), s_2^*(g_2^*)) \right] - \left[\frac{\alpha(g_1^*)}{r+\alpha(g_1^*)} \right] + \left[\frac{\alpha(g_1^*) \left[\frac{H_{N,2}^*[\beta(g_2^*)]}{r} \right]}{r+\alpha(g_1^*)} \right]$$

The Nash Equilibrium tells that both firms spend a determined expenditure for every period however the timing of demand stability is still random. By their expenditures, the total social welfares would be expected.

The SPNE, ((Policy mode is Subsidy, Policy mode is Subsidy), (s_1^*, s_2^*) , (θ_1 and θ_2 are revealed), (q_1^*, q_2^*)), lasts until a stable demand is realized for the firms from country 1. Then it turns to New SPNE, ((Policy mode is Direct quantity control, Policy mode is Subsidy), (q_1^{**}, s_2^*) , (θ_1 and θ_2 are revealed, and $\text{Var}(\theta_1)$ turns out to be sufficiently small), (q_1^{**}, q_2^*)). This SPNE lasts until a stable demand is realized for the firms from

country 2. Then it turns to New SPNE, ((Policy mode is Direct quantity control, Policy mode Direct quantity control), (q_1^{**}, q_2^{**}) , $(\theta_1$ and θ_2 are revealed, and $\text{Var}(\theta_2)$ turns out to be sufficiently small), (q_1^{**}, q_2^{**})). This SPNE remains forever.

6. Conclusion

The argument has been centered on if a dominant trade policy should converge to a direct quantity control under infinite horizon. The answer was ‘yes’ because a government has a vision to improve total social benefit for infinite horizon and might be ready to take a loss of total social benefit occurring from a reduction of a subsidy for some periods. As a result, the demand variability is successively reduced while the firms from the other country still struggle with large demand variability. Under a government-led effort to remove the market variance, government 1 has a dominant policy of direct quantity control. The rival government still chooses a subsidy as dominant policy since the demand variability for its domestic exporters are unchanged. Under the new dominant equilibrium policies, the total expected discounted social benefits for country 1 and country 2 can be compared. Unlike country 1, country 2 has large uncertainty for its total expected discounted social benefit. In the sense of less uncertainty, the total social benefit for country 1 is more efficient.

If both governments involve in the removal of market variance, there would be two different cases, the symmetry and the asymmetry. In the symmetric case, both firms have equal number of firms in a market. Intuitively, both governments have the same choices under the identical circumstances. Thus, stable demands for all the firms coincidentally happen regardless the origins. In this case, the dominant equilibrium trade policy changes from (subsidy, subsidy) to (direct quantity control, direct quantity control). The total expected discounted social benefits for both governments can be found in this sense. They are actually equal.

In the asymmetric case, each country has different number of firms operating in a market. For example, if country 2 has larger number of firms than country 1 then it would take more time for government 2 to stabilize demand for all the firms. Accordingly, government

2 successively stabilizes the demands for its own domestic exporter after government 1 succeeded. In this situation, the total expected discounted social benefits for both governments can be obtained. Unlike the symmetric case, the dominant equilibrium trade policy changes from (subsidy, subsidy) to (direct quantity control, subsidy). Depending on government 2's expenditure for infrastructure, the equilibrium changes to (direct quantity control, direct quantity control) again.

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