Divisia Monetary Aggregates and Monetary Policy in a Small Open Economy

by

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Abstract

The first chapter in my doctoral dissertation is co-authored with my advisor and it has been published in the Journal of Risk and Financial Management, August 2021. This paper applied the formula for user cost price of money and the recently developed credit-card-augmented Divisia monetary aggregates formula to construct monetary service indexes for Singapore. We produced state-of-the-art monetary service indexes from January 1991 to March 2021. We found that Divisia measures behave differently than simple sum measures in the period before the year 2000, when interest rates were high. Credit-card-augmented Divisia monetary services move closely with the conventional Divisia monetary aggregates, since the volume of credit card transactions in Singapore is relatively small compared with other monetary assets.

The second chapter uses the constructed Divisia indexes and other data to examine different instruments of monetary policy and their relevance in predicting real economic activity in Singapore. I apply the Hamilton based filter to extract the cyclical component of each time series and compute the cyclical correlation of different targets of monetary policy including interest rate, money supply and exchange rate with output and inflation. I find that while exchange rate and all money measures show a weakly contemporaneous correlation with output and inflation, interest rate is not contemporaneously correlated to either output or inflation. Among different money measures, Divisia always shows a stronger correlation with both output and inflation than simple sum. Credit-card-augmented Divisia is the most informative indicator to predict output and price.

To confirm the above statistical findings in a more rigorous framework, the last chapter in my dissertation revisits the issue of money measurement in the context of a small open economy using a recently developed micro founded DSGE model. I extend the New Keynesian model for a small open economy from the work of Faia and Monacelli in a similar manner to Belongia and Ireland by introducing private financial institutions, which create deposits as an imperfect substitute for government-issued currency. The banking sector allows the accommodation of multiple monetary assets like currency and interest-bearing-deposits. The central bank conducts its monetary policy via a simple interest rate rule. I explore the responses of different money measures including simple sum, monetary base, and Divisia quantity aggregate with respect to domestic and foreign shocks and compare these responses with those from a theoretical monetary aggregator. I find that Divisia tracks the movement of money most closely to the theoretical measure, followed by monetary base, while simple sum often does not match the correct trend. I analyze the impact of openness, which has an inverse relation with home-bias in consumption, on the volatility of macroeconomic variables. I find that as the small economy becomes more open, domestic inflation and nominal interest rate become more volatile while terms of trade and exchange rate become more stable. Among the different money measures, monetary base and Divisia follow the theoretical monetary aggregate to become less volatile as consumption becomes less home-biased, while simple sum, becomes more volatile.

Keywords: Divisia index; Divisia monetary aggregates; credit-card-augmented Divisia; cyclical correlation; open-economy macroeconomics; monetary policy analysis; New Keynesian model; Singapore; small open economy

JEL Classification: E31; E32; E40; E41; E47; E50; E51; E52; E58.

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William A. Barnett[1] and Van H. Nguyen[2]

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Abstract: Since Barnett derived the user cost price of money, the economic theory of monetary services aggregation has been developed and extended into a field of its own with solid foundations in microeconomic theory. Divisia monetary aggregates have repeatedly been shown to be strictly preferable to their simple sum counterparts, which have no competent foundations in microeconomic aggregation or index number theory. However, most central banks in the world, including that of Singapore, the Monetary Authority of Singapore (MAS), still report their monetary aggregates as simple summations. Recent macroeconomic research about Singapore tends to focus on exchange rates as a monetary policy target but ignores the aggregate quantity of money. Is that because quantities of money are irrelevant to economic activity? To examine the role of monetary quantities as potential monetary instruments, indicators, or targets and their relevance to predicting real economic activity in Singapore, this paper applies the user cost of money formula and the recently developed credit-card-augmented Divisia monetary aggregates formula to construct monetary services indexes for Singapore. We produce those state-of-the-art monetary services indexes from Jan 1991 to Mar 2021. We see that Divisia measures behave differently from simple sum measures in the period before the year 2000, while interest rates were

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high. Credit-card-augmented Divisia monetary services move closely with the conventional Divisia monetary aggregates, since the volume of credit card transactions in Singapore is relatively small compared with other monetary service assets. In future work, we plan to use our data to explore central bank policy in Singapore and to propose improvements in that policy. By making our data available to the public, we encourage others to do the same.

Keywords: Divisia index; Divisia monetary aggregates; credit-card-augmented Divisia; open-economy macroeconomics; monetary policy analysis; Singapore

JEL Classification: E32; E40; E41; E47; E50; E51; E52; E58.

1.1. Introduction

Since Irving Fisher (1922) published his classic book, *The Making of Index Numbers*, statistical indexes have been extensively applied in economic measurement. For instance, to measure real GDP, no one would today add apples and automobiles, since one apple is not a perfect substitute for one automobile. For the same reason, we cannot impute the same weight to percentage changes in the price of automobiles as to the percentage changes in the price of apples when measuring inflation. Although widely used in economic measurement since the appearance of Fisher's book, statistical index theory has not been applied in financial and monetary aggregation until recent decades.

Up until the 1980s, economists throughout the world measured different levels of monetary aggregation, such as M0/MB (monetary base), M1 (narrow money), M2 (broad money), and M3 and M4 (financial liquidity), by simply adding up the quantities of component assets. Simple summation assigns the same weights to different monetary assets and thereby implicitly assumes that all monetary assets are perfect substitutes. In modern economies, in which monetary assets possess different levels of liquidity and yield different interest rates, simple sum measures are misleading and can damage inferences about economic behavior and the economy. Chrystal and MacDonald (1994) coined the now well-known term "Barnett critique" to designate the resulting distortions of economic inferences.

To properly aggregate components in monetary service aggregation, we need both their quantities and prices. However, how to measure monetary service prices was not known to economists until the 1980s. Monetary asset services are not analogous to perishable consumer good services, such as apples, but to capital goods or durable goods, such as houses or automobiles. Hence, we need to measure their service prices in terms of their user cost prices.

The concept of user cost pricing of durable services was first introduced by Jorgenson (1963). He introduced user cost theory applicable to durable and capital goods for which perfect rental markets do not exist. When perfect rental markets exist for a good, the user cost price equals the market rental price. When a perfect rental market does not exist for a durable, the theoretically computed user cost price is sometimes called the "equivalent rental price" or shadow rental price. The theory of monetary aggregation was originated by Barnett in the 1980s, following his derivation of the user cost price of monetary services in Barnett (1978, 1980). Using the resulting user cost pricing, Barnett (1980, 1987) applied existing index number and aggregation theory to construct the Divisia index for monetary service aggregation. The famous Divisia index, originated by Francois Divisia (1925), measures the growth rate of a quantity (or price) aggregate as the weighted average of the growth rates of the quantities (or prices) of the component goods over which the index aggregates. The weights are the component expenditure shares.

Since the economic theory of monetary aggregation became available, the theory has been developed and extended substantially. Barnett et al. (1997) extended the theory to risk, based upon the consumption capital assets pricing model (CCAPM). That result extended Barnett's perfect certainty theory to the case of risk when consumers of monetary services are risk-averse and interest rates are not known at the beginning of the period. Barnett (2007) extended the theory to multilateral monetary aggregation over different countries. More recently, Barnett et al. (2016) and Barnett and Su (2016, 2017, 2018) have taken credit card transactions into account and produced the theoretical framework for the new credit-card-augmented Divisia monetary aggregates. Other extensions have included measurement of the economic capital stock of money, based on the expected discounted flow of monetary services, and extension of the risk adjustment to the case of intertemporal non-separability.

Hundreds of empirical papers from throughout the world have compared Divisia monetary aggregates with their simple sum counterparts. Key articles, books, and works on the topic can be found at the online library of Center for Financial Stability (CFS). Since simple sum monetary aggregation is theoretically inadmissible, having no competent theoretical foundations, it should be no surprise that in almost all cases, the Divisia monetary index has proven to be strictly preferable to its simple sum counterpart, relative to all available empirical tests (see, for example, Barnett et al. (1984), Barnett (2011), Belongia and Ireland (2014), and Ellington (2018)). Belongia and Ireland (2015) are critical of the omission of monetary quantities in recent mainstream macroeconomic models. These omissions are largely a result of empirical findings in such papers as Bernanke and Blinder (1988), who argue that the demand for money function has become unstable; but all such findings use simple sum monetary aggregates. Recent DSGE models often include an interest rate feedback rule as a basis for monetary policy, while totally ignoring monetary services in the economy. The most common interest rate rule in the literature is the Taylor rule, based on Taylor (1993). Replacing the traditional simple sum measure of money supply by Divisia measures, Belongia (1996), Barnett and Chauvet (2010), Barnett et al. (2013), and Liu et al. (2020), among many other researchers, have shown that money still shares a strong relationship with aggregate economic activity, and the demand for money function still exhibits stability. This simple solution has been found to be true in hundreds of publications throughout the world, since Divisia monetary aggregates became available.

Faced with all this theoretical and empirical evidence, central banks such as the Federal Reserve (FED) in the US, the Bank of England (BOE) in the UK, the European Central Bank (ECB), the Bank of Japan (BoJ), the National Bank of Poland, and the Bank of Israel, among others, have, at various times and in diverse ways, produced and maintained Divisia indexes for monetary aggregation. Some central banks choose to make it available to the public on an official basis, such as the BOE. Others choose to make those aggregates available only for internal use. However, the availability of the simple sum aggregates has continued. For good reasons, those incompetent simple sum aggregates are declining in usage by central banks and by the economics profession.

Many other central banks in the world, including the Monetary Authority of Singapore (MAS), continue to report their money supplies solely as simple sum measures. Singapore is a very small economy, the size of a medium city, with a population of about 5.8 million people, yet it has produced a remarkable success story as a major financial center in Southeast Asia. This success may be related to its unique and interesting monetary policy system, which has been centered on the management of the exchange rate since 1981. This approach is different from the conventional monetary policy targeting of interest rates or monetary aggregates. Nevertheless, the economy has not received much attention from academic scholars. Recent DSGE macroeconomic models often ignore aggregate quantities of money as possible instruments or targets of monetary policy. In the case of a small open economy such as Singapore's, exchange rates are often targeted to achieve goals for inflation and output gap, see McCallum (2007). Chow et al (2013) discuss the monetary regime choice in Singapore and compare its exchange rate rule with the Taylor rule but ignore money quantities.

Empirical work using Divisia monetary aggregates in Singapore is limited, with the only related work in the literature being Habibullah (1999). The focus of that paper was not primarily the case of Singapore, but rather the monetary policies in many Asian countries, including Indonesia, Malaysia, Singapore, Philippines, South Korea, Taiwan, and Thailand among others.

The data used in that research were mostly before the Asian financial crisis and did not use credit-card-augmented Divisia monetary aggregates, which were not yet known at that time.

Our paper constructs Divisia monetary aggregates for Singapore based on monthly data from Jan 1991 to Mar 2021. We find that the major contributions to the growth rates of Divisia monetary service flows come from demand deposits, fixed deposits, and savings (and other) deposits in commercial banks. Fixed deposits and savings deposits in finance companies provide moderate contributions, while the weights of other components such as negotiable CDs, repurchase agreements, and Treasury bills are negligible.

Although credit card transactions are augmented into our monetary aggregates, their weights are small. Therefore, we find their contributions at this time to the growth rate of Divisia monetary services in Singapore to be minor, although this could change in the future as money market institutional innovations continue. Another finding is that during the period before 2000, when interest rates were high and more volatile, Divisia monetary aggregates behaved significantly differently from the simple sum measures, while during the period after 2000, when interest rates on monetary assets have become close to each other at very low levels, Divisia monetary aggregates have behaved almost identically to the simple sum measures.

Providing the constructed data to the public, we would encourage others to do the same. We plan to use Divisia data to examine monetary policy in Singapore. Our planned first direction will be to examine the cyclical correlations and Granger causality relations between different measures of money and real economic variables. We also plan to build a New Keynesian model for a small open economy to be used to examine the potential role of money aggregates as a policy target in Singapore, in comparison with the central bank's current policy rule, targeting a trade-weighted exchange rate index.

1.2. Methodology

1.2.1. Conventional Divisia Monetary Aggregates

Barnett (1978, 1980) derived the user cost of monetary asset services from an intertemporal consumer utility maximization problem. Let U be the representative consumer's current intertemporal T-period utility function

$$U = U(u(\mathbf{m}_t), \mathbf{m}_{t+1}, ..., \mathbf{m}_{t+T}; \mathbf{x_t}, ..., \mathbf{x_{t+T}}; A_{t+T}) \tag{1.1}$$

for each period's consumption of goods $\mathbf{x}_s$ having prices $\mathbf{p}_s$, monetary assets $\mathbf{m}_s$, and bond holdings A_{t+T} for $s = t$, $t + 1$, …, T. In theory, the "bond" is called the benchmark asset, which formally is a pure capital investment held solely for its investment rate of return, and thereby providing no other services. The intertemporal utility function is assumed to be weakly separable in the current period consumption of monetary services, $\mathbf{m}_t$. The representative consumer maximizes utility subject to the constraint

$$\mathbf{p_s}' \ \mathbf{x_s} = w_s L_s + \sum_{i=1}^{n} [(1 + r_{i,s-1}) p^*_{s-1} m_{i,s-1} - p^*_s m_{i,s}] + [(1 + R_{s-1}) p^*_{s-1} A_{s-1} - p^*_s A_s] \tag{1.2}$$

for $s = t$, $t + 1$, …, T, where p_s* is the true cost of living index, w_s is the wage rate, L_s is the per capital labor supply, $r_{i,s}$ is the rate of return on monetary asset $m_{i,s}$, and R_s is the yield on the benchmark asset A_s.

Let $\mathbf{m}^*_t = (m^*_{1,t}, m^*_{2,t}, ..., m^*_{n,t})'$ be the solution for period t's monetary assets in the intertemporal decision. Barnett (1978, 1980) showed that $\mathbf{m}^*_t$ is also the solution to the current period conditional decision of maximizing $u(\mathbf{m}_t)$ subject to

$$\boldsymbol{\pi}_\mathbf{t}' \ \mathbf{m_t} = y_t \,, \tag{1.3}$$

where y_t is expenditure allocated to the portfolio of n monetary assets

$$\mathbf{m}_t = (m_{1,t}, m_{2,t}, ...m_{n,t})' \tag{1.4}$$

during the intertemporal decision and

$$\boldsymbol{\pi}_t = (\pi_{1,t}, \pi_{2,t}, ..., \pi_{n,t})' \tag{1.5}$$

is the vector of user costs of monetary asset services. To assure the existence of a current period monetary services aggregate, the category utility function, u, is assumed to be monotonically increasing, strictly concave, and blockwise weakly separable within intertemporal tastes.

Barnett (1978, 1980) proved that the resulting nominal user cost price of each monetary asset is

$$\pi_{i,t} = \frac{R_t - r_{i,t}}{1 + R_t}, \tag{1.6}$$

where the true cost of living index is used to deflate nominal quantities to real quantities, R_t is the expected one-period holding yield on the benchmark asset, and $r_{i,t}$ is the current-period rate of return on the i-th monetary asset. As emphasized in Barnett (1978, 2011), the user cost price of a monetary asset is not its interest rate but its opportunity cost, consisting of the interest rate forgone by consuming the services of the asset. For example, if the asset is currency, having an interest rate of zero, the forgone interest rate is the benchmark rate itself.

The corresponding real user cost price is

$$\frac{\pi_{i,t}}{p_t^*} = \frac{R_t - r_{i,t}}{1 + R_t}. \tag{1.7}$$

With availability of the user cost prices of monetary assets, both their quantities and their prices are well-defined. The economic theory of aggregation over monetary assets becomes available. Barnett (1987) proved that the exact monetary quantity aggregate, $M_t = M(\mathbf{m}_t)$, can be tracked without error in continuous time by the Divisia index, defining the growth rate of aggregate monetary services to be

$$\frac{d\log M_t}{dt} = \sum_{i=1}^{n} s_{i,t} \frac{d\log m_{i,t}^*}{dt}, \tag{1.8}$$

where

$$s_{i,t} = \frac{\pi_{i,t} m_{i,t}^*}{y_t} = \frac{\pi_{i,t} m_{i,t}^*}{\sum_{j=1}^{n} \pi_{j,t} m_{j,t}^*}. \tag{1.9}$$

The weight $s_{i,t}$ of monetary asset i is its share in the total expenditure on the portfolio. Since economic data are in discrete time, an approximation is needed. The Tornqvist–Theil approximation (often called the Tornqvist index or just the Divisia index in discrete time) is a second order approximation to the continuous Divisia index,

$$\log M_t - \log M_{t-1} = \sum_{i=1}^{n} \overline{s}_{i,t} (\log m_{i,t}^* - \log m_{i,t-1}^*), \tag{1.10}$$

where the discrete time share weights are approximated by

$$\overline{s}_{i,t} = \frac{1}{2}(s_{i,t} + s_{i,t-1}). \tag{1.11}$$

In short, the growth rate of a Divisia monetary quantity index is the share weighted average of the growth rates of its components. Barnett (1987) showed that the discrete time Divisia index is accurate to within three decimal places for monthly or weekly data. As a result, the remainder term in the Tornqvist approximation is less than the roundoff error in the available component data. Equation (1.9) can equivalently be written as

$$\frac{M_t}{M_{t-1}} = \prod_{i=1}^{n} \left(\frac{m_{i,t}^*}{m_{i,t-1}^*} \right)^{\bar{s}_{i,t}} . \tag{1.12}$$

The growth rate of the dual Divisia user cost price aggregate, $\Pi_t = \Pi\left(\boldsymbol{\pi}_t\right)$, in continuous time, is derived in a similar manner to be

$$\frac{d \log \Pi_t}{dt} = \sum_{i=1}^{n} s_{i,t} \frac{d \log \pi_{i,t}}{dt} , \tag{1.13}$$

with the corresponding Tornqvist discrete time approximation being

$$\log \Pi_t - \log \Pi_{t-1} = \sum_{i=1}^{n} \bar{s}_{i,t} (\log \pi_{i,t} - \log \pi_{i,t-1}) . \tag{1.14}$$

An alternative way to derive the dual user cost price aggregate is from Fisher's factor reversal test, in accordance with which

$$\Pi_t = \frac{\boldsymbol{\pi}_t' \ \mathbf{m}_t^*}{M_t} = \frac{\sum_{i=1}^{n} \pi_{i,t} m_{i,t}^*}{M_t} , \tag{1.15}$$

so that

$$\Pi_t M_t = \boldsymbol{\pi}_t' \ \mathbf{m}_t^* = \sum_{i=1}^{n} \pi_{i,t} m_{i,t}^* . \tag{1.16}$$

For a rigorous discussion, Barnett (1980) showed that in continuous time, the two methods of computing the dual user cost aggregate produce identical results. In discrete time, the two methods produce slightly different results, with the difference being less than the roundoff error in the component data and thereby negligible.

1.2.2. Credit-Card-Augmented Divisia Monetary Aggregates

In recent years, credit card payments have become increasingly common in modern economies worldwide. By accounting conventions, liabilities cannot be added to assets. Since credit card balances are liabilities, they cannot be added to monetary assets. Hence credit cards cannot be included in simple sum monetary aggregates. However, in economic theory, aggregation over services is possible, regardless of whether the services are produced from assets or liabilities. The deferred payment services of credit card transactions can be augmented into the Divisia monetary service aggregates.

The theoretical framework is provided in Barnett et al. (2016) and Barnett and Su (2016, 2017, 2018). Accordingly, the user cost price of a credit card's services is

$$\pi_{j,t}^{c} = \frac{p_t^*(e_{j,t} - R_t)}{1 + R_t}, \tag{1.17}$$

where $e_{j,t}$ is the interest rate charged by credit card type *j*, with *j* = 1, ... *k*, during time *t*. The consumer's optimal choice of the volume of purchases of goods and services during period *t* with credit card type *j* is $c_{j,t}^*$. The consumer's utility maximizing solution for the transaction services of the *k* credit card types is

$$\mathbf{c}_{i,t}^* = (c_{1,t}^*, c_{2,t}^*, ..., c_{k,t}^*)' \quad .$$

The growth-rate weight of monetary asset *i*'s services is

$$w_{i,t} = \frac{\pi_{i,t} m_{i,t}^*}{\boldsymbol{\pi}_t' \ \mathbf{m}_t^* + (\boldsymbol{\pi}_t^c)' \ \mathbf{c}_t^*}, \tag{1.18}$$

while the growth-rate weight of credit card *j*'s services is

$$w_{j,t}^{c} = \frac{\pi_{j,t} c_{j,t}^{*}}{\boldsymbol{\pi}_t{}' \ \mathbf{m}_t^{*} + (\boldsymbol{\pi}_t^{c})' \ \mathbf{c}_t^{*}}. \tag{1.19}$$

The credit-card-services-augmented Divisia monetary aggregate becomes

$$\frac{d \log M_t}{dt} = \sum_{i=1}^{n} w_{i,t} \frac{d \log m_{i,t}^{*}}{dt} + \sum_{j=1}^{k} w_{j,t}^{c} \frac{d \log c_{j,t}^{*}}{dt}. \tag{1.20}$$

The Tornqvist discrete time approximation is analogous to that for the conventional Divisia index.

1.2.3. User Cost and Interest Rate Aggregation

Divisia user cost price aggregates can be computed in a manner similar to the Divisia monetary quantity aggregates, using Equation (1.14) with the weights computed by Equations (1.9) and (1.11). However, in this paper, given that we already constructed the Divisia quantity index, M_t, the corresponding user cost price aggregate is derived from Fisher's factor reversed test as in Equation (1.15). Credit-card-augmented user cost price aggregates are also computed accordingly.

For interest rate aggregation, this paper follows Barnett et al. (2013) using accounting principles. Accordingly, the aggregated interest rate on a portfolio is the rate of return on the portfolio,

$$r_t = \frac{\sum_{i=1}^{n} r_{i,t} m_{i,t}}{\sum_{i=1}^{n} m_{i,t}}. \tag{1.21}$$

1.3. Data and Construction

To construct Divisia monetary aggregates for monetary services, we need data on both quantities and interest rates of each monetary asset. This section describes the data we used and the construction results.

1.3.1. Data Description

Table 1.1 provides a basic description for the data set. Data on levels and rates of return on monetary assets are monthly from Jan 1991 to Mar 2021, provided by the Monetary Authority of Singapore (MAS). The true cost of living index was measured by the consumer price index (CPI), which was from the Singapore department of statistics. For the United States, the Federal Reserve reports interest rates charged on credit card deposits averaged over all credit card users, including those who do not pay interest on their credit card balances, since they do not carry forward unpaid balances. That average interest rate is the one to use in modeling the decisions of the representative consumer, aggregated over all consumers, see Barnett and Su (2007).

Unfortunately, the central bank in Singapore (MAS) does not report those interest rates. However, ValueChampion in Singapore does report that interest rate averaged over time. Experiments with the United States data show negligible differences in the credit-card-augmented monetary aggregates, if that interest rate is averaged over the sample period and then treated as a constant, rather than being used as the actual interest rate each month. As a result, with our Singapore data, we used the interest rate averaged over time, as reported at the surprisingly high level of 25% per year by ValueChampion. Perhaps that high interest rate may partially explain why the share of credit card deferred payment services in Singapore is relatively low.

The central bank of Singapore (MAS) categorizes the primary components of M1 as currency in circulation and demand deposits in banks. The MAS simple sum M2 includes M1 and the banking sector components, fixed deposits (CDs), savings (and other) deposits, and negotiable certificates of deposits (NCDs). Simple sum M3 incorporates the non-banking sector by including net deposits in finance companies. Post Office Saving Bank (POSB) deposits existed in Singapore before Nov 1998. Up to Oct 1998, POSB was included by MAS in its non-banking sector data and included in M3, but not in M2. However, from Nov 1998, with the acquisition of POSB by the Development Bank of Singapore (DBS), POSB's data have been incorporated as part of the banking system in M1 and thereby also in M2 and M3.

Nesting components: In this paper, we follow Barnett et al. (2013) in clustering and nesting components of monetary assets. Our clustering of components into different levels of aggregation prior to computing the Divisia aggregates is slightly different from that of the simple sum measures reported by the MAS. Table 1.2 summarizes the components in the MAS simple sum aggregates and our Divisia aggregates. Accordingly, our Divisia M1 (DM1) has the same components as its counterpart simple sum aggregate, M1. Our Divisia M2 (DM2) aggregate includes components in DM1 along with fixed deposits (CDs) and savings (and other) deposits, both in the banking sector and the non-banking sector. Our Divisia M3 (DM3) includes the components of DM2 along with NCDs and repurchase agreements (repos). Finally, our Divisia M4 (DM4) incorporates Treasury bills into Divisia M3. Our credit-card-augmented Divisia indexes are computed by incorporating credit card transactions into each level of aggregation hence, we also report DM1a, DM2a, DM3a, and DM4a.

Data on levels: The data on levels of components are in current values, and the unit is million of Singapore dollars. Net deposits in finance companies are included in M2, but the break-down components, fixed deposits, and savings (and other) deposits, are not separately reported. We directly investigated the assets and liabilities of finance companies to obtain the break-down components. We recovered the fixed deposits and savings and other deposits in finance companies by using the proportion of net deposits in total deposits.

Since there were no available interest rates for deposits in POSB, we assumed that these rates are the same as those in the banking sector. The data on the POSB level (up to Oct 1998) are incorporated into the banking sector. We calculated the proportion of fixed deposits and savings (and other) deposits out of total deposits in both the banking sector and the non-banking sector. We then used those proportions to split the net deposits in POSB into fixed deposits and savings (and other) deposits and then added them into the banking sector.

In the series for the level of NCDs during the period from Aug 2009 to Jun 2010, the quantities reported are zeros, which does not seem credible. We smoothed the data on NCDs for the above period by approximating the zero-quantities by the average of the quantities 12 months before and 12 months after that period.

The benchmark rate: In theory, the benchmark rate is the rate of return on pure capital. Its rate of return cannot be less than the rates of return on any monetary assets that provide services to depositors along with investment yield. In this paper, we follow Barnett et al. (2013) in choosing the short-term lending rate as the benchmark rate. Banks cannot be expected to pay higher rates of interest to their depositors than they earn on their investments. Indeed, in the case of Singapore, the prime lending rate is always higher than the interest rates on the component monetary assets, as shown in Figure 1.1. It is noted that the credit card interest rates are much higher than the benchmark rate and all other rates of return. This is true for the cases of the US, Singapore, and other economies in the world. The reason is clear. Credit card interest rates are the interest rates charged to credit card users, who are borrowing money from credit card companies as unsecured loans with high default and fraud risk. Those interest rates are always higher than the rates of return on monetary assets, which are paid to the owners of monetary assets.

Data on rates of return: Since currency and demand deposits do not yield interest rate, we set their interest rates at zero. Short-term fixed deposits typically include 3-month CDs, 6-month CDs, and 12-month CDs. We do have data on their interest rates; however, we do not have the corresponding interest rates on fixed deposits in banks and finance companies. Hence, we used 3-month CD interest rates for banks and finance companies to represent the rates of return on fixed deposits in banks and non-banking institutions. For T-bills, we added together all T-bills and imposed the 12-month T-bill yield as their rate of return.

Since the rate of return on NCDs is not reported, we used the rate on 3-month commercial paper as a proxy. The rates of return series for 3-month commercial paper and repurchase agreements were discontinued from Jan 2014. The missing observations were estimated by a regression of each series on the 12-month T-bill yield.

1.3.2. Data Construction and Results

Based on our clustering of components into different levels of aggregation as presented in Table 1.2, we computed the growth rates of the Divisia monetary aggregates and their credit-card-augmented variants at the levels of aggregation we had chosen. Corresponding interest rate aggregates and user cost aggregates were also computed.

Divisia M1 (DM1) contains the same two components as its simple sum counterpart, and their corresponding interest rates are equal to zero. Hence, the user cost prices for these components are the same. Under those conditions, the Divisia quantity index becomes the simple sum. The growth rate of DM1 and its level (normalized to equal 100 in Jan 1991) are the same as those of M1, as shown in Figure 1.2. However, when we take into account the credit card transactions, the augmented Divisia indexes behave a little differently from the conventional Divisia indexes, which can be seen in Figure 1.3.

DM2, DM3, and DM4 have almost identical growth rates, but they are substantially different from the growth rate of DM1. As shown in Figure 1.2, DM2, DM3 and DM4 almost lie on top of each other. To explain this fact, refer to Tables 1.3 and 1.4 for the growth-rate weights of the components in the Divisia monetary aggregates. While DM1 contains only two components, currency and demand deposits, their weights for the latest months in our sample (Mar 2021) are 20.66% and 79.34%, respectively, as presented in Table 1.3. These weights are quite different from those of DM2, DM3, and DM4, which are about 7% and 28%, respectively. The major components that contribute to the growth rates of DM2, DM3, and DM4 are three components of the banking sector: demand deposits, fixed deposits in commercial banks, and savings (and other) deposits in commercial banks. These three components account for 60% of the fluctuation in the growth rates of DM2, DM3, and DM4. Finance companies provide a moderate contribution to the growth rates of those Divisia monetary aggregates. Although DM3 and DM4 incorporate additional components into DM2, namely NCDs, repos, and T-bills, their weights are almost negligible.

The credit-card-augmented Divisia aggregates behave similarly to the conventional Divisia monetary aggregates, since the volume of credit card transactions in Singapore is currently relatively small compared to other sources of monetary services. Hence, the growth-rate weight of credit card transaction volumes is currently small, about 9.5% at the M1 level of aggregation and 3.2% at broader levels of aggregation, as shown in Table 1.5. However, the role of credit card deferred payment services may grow in the future as financial services innovations continue to evolve.

For a comparison of the Divisia and simple sum aggregates, see Figure 1.2. It is noted that simple sum M2 experiences a sudden peak in Nov 1998, while simple sum M3 and Divisia indexes do not. It happened due to the acquisition of POSB into the banking sector. This is not an expansion of the money supply, but rather a structural change in nesting of components into different levels of aggregation. The Divisia monetary aggregates do not experience this sudden misleading spike.

The overall picture becomes clearer when we look at the year over year growth rate of money in Figure 1.4. The period before 2000 is particularly interesting, because interest rates were higher and more volatile compared to the later period (see Figure 1.1). There is a huge contraction of money supply during the Asian financial crisis, 1997-1998, as displayed very clearly in the DM3 growth rates, but simple sum M3 does not show it.

After 2000, the growth rates of the Divisia monetary aggregates and the simple sum versions are close to each other. Again, the reason is the behavior of interest rates in Singapore. After 2000, interest rates for fixed deposits and savings deposits in both commercial banks and finance companies are at very low levels, almost zero, and thereby very similar to each other (see Figure 1.1). As a result, the user cost prices for those assets are almost identical to each other, so that the Divisia indexes are close to their simple sum counterparts during that period. In the future, simple sum and Divisia measures will again diverge, if interest rates return to higher levels.

Interest rate aggregates and real user cost price aggregates are plotted in Figures 1.5 and Figure 1.6. Different levels of interest rate aggregation produce almost identical results and follow the common trend of interest rates in the world, with interest rates becoming very low after 2000. Real user cost aggregates are, in accordance with theory, always positive, and a higher level of aggregation shows higher real user cost.

1.4. Conclusion

Although aggregation theory and index number theory have been extensively applied in economic measurement for more than a century, monetary aggregation theory has appeared and been applied more recently. Large numbers of theoretical studies as well as empirical studies have repeatedly shown that Divisia monetary aggregates are superior to their simple sum counterparts, which have no competent foundations in economic theory. Nevertheless, many central banks in the world, including the Monetary Authority of Singapore, continue reporting money supply as a simple sum. This may be part of the reason that the quantity of money has been ignored in recent empirical macroeconomic research in Singapore. This paper provides the construction of Divisia monetary aggregates for Singapore and thereby will serve as the first step for research on the role of monetary services in the important Singapore economy.

We encourage others to use our Divisia Singapore data in their studies. We ourselves plan to use the data to examine cyclical correlations and Granger causality relation between different measures of money and real economic variables. Furthermore, we hope to build a New Keynesian model for a small open economy with a banking sector to examine the role of monetary aggregates as a possible policy target in Singapore, as opposed to the current trade-weighted exchange rate target.

A.1. Appendix 1

A.1.1. Figures

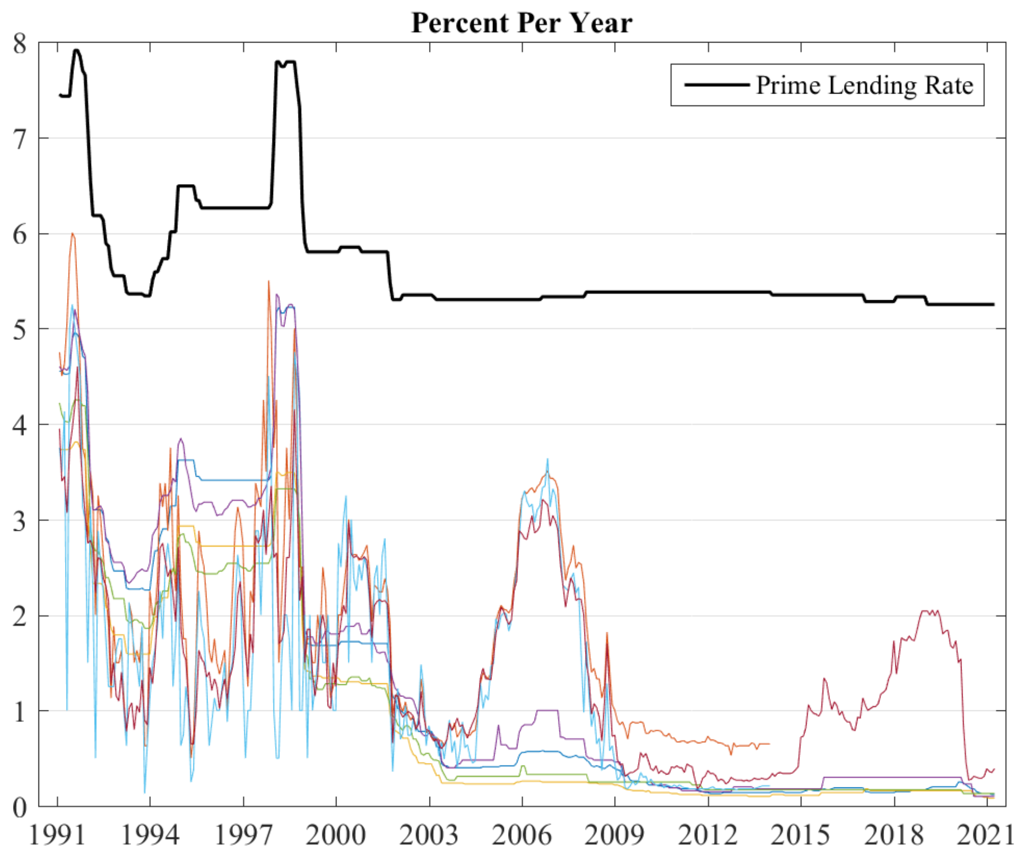

Figure 1.1. Benchmark Rate versus other Interest Rates

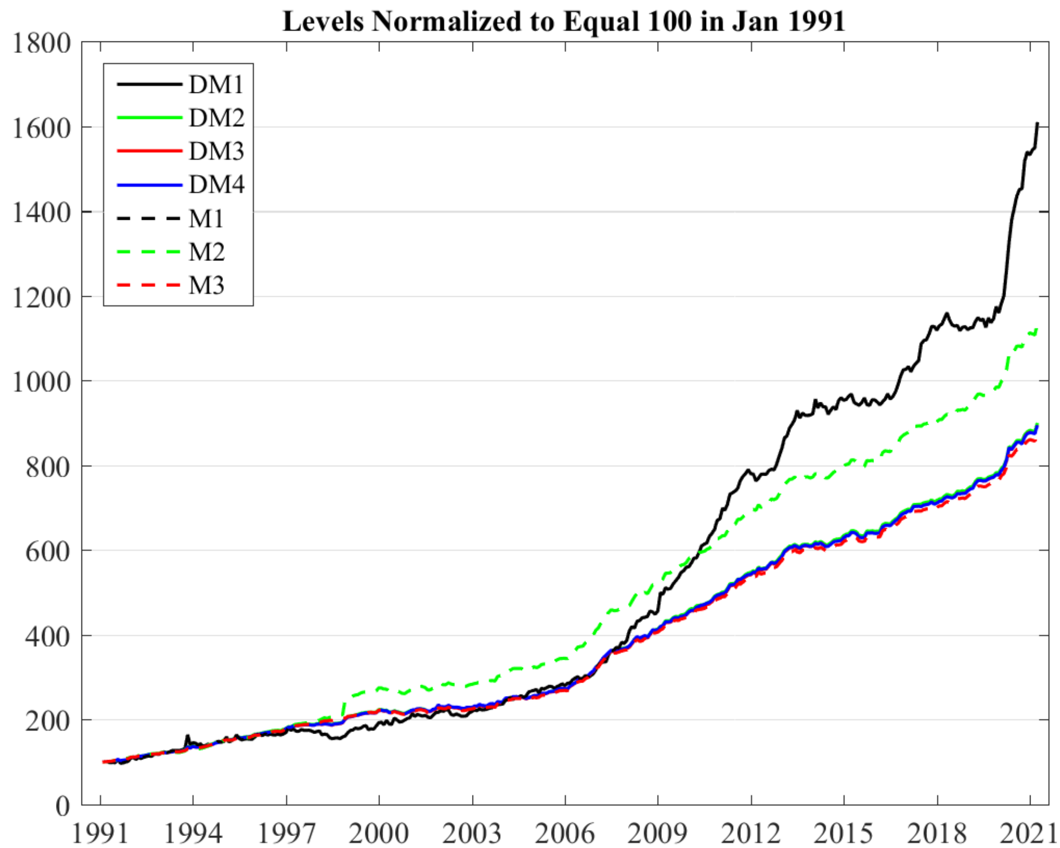

Figure 1.2. Divisia versus Simple-sum Aggregates, 1991-2021

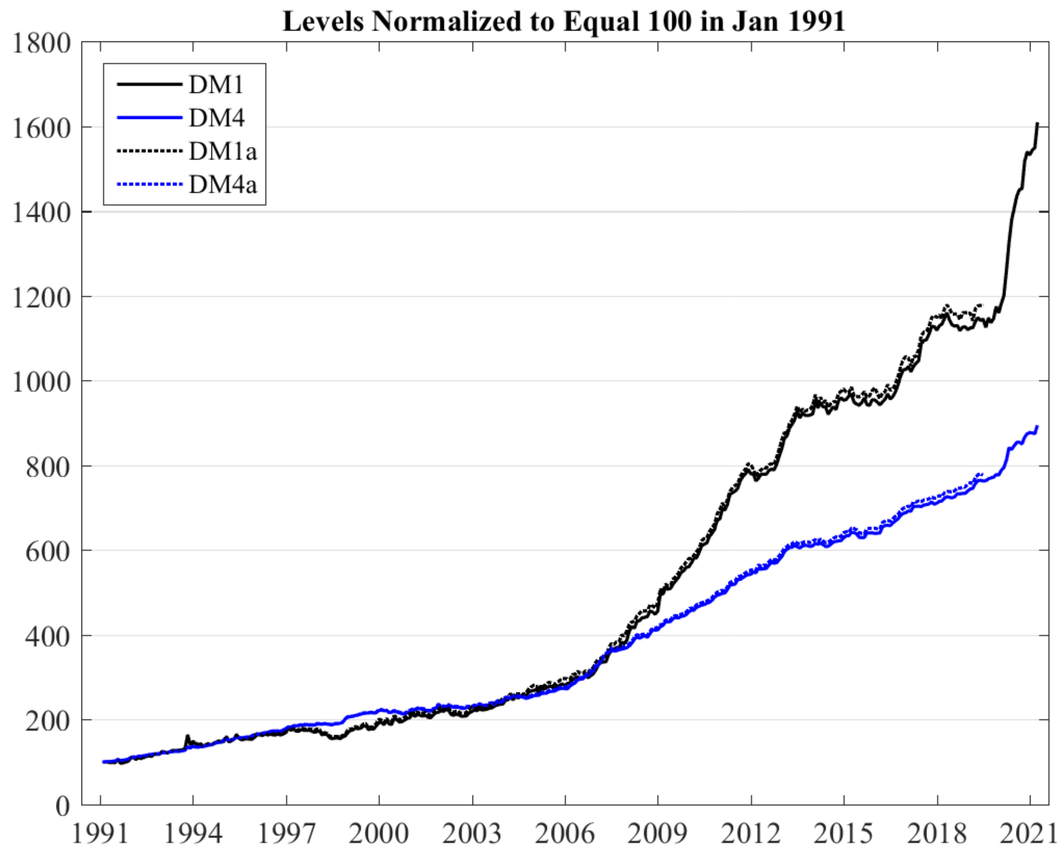

Figure 1.3. Divisia and Credit-card-augmented Divisia Monetary Aggregates, 1991-2021

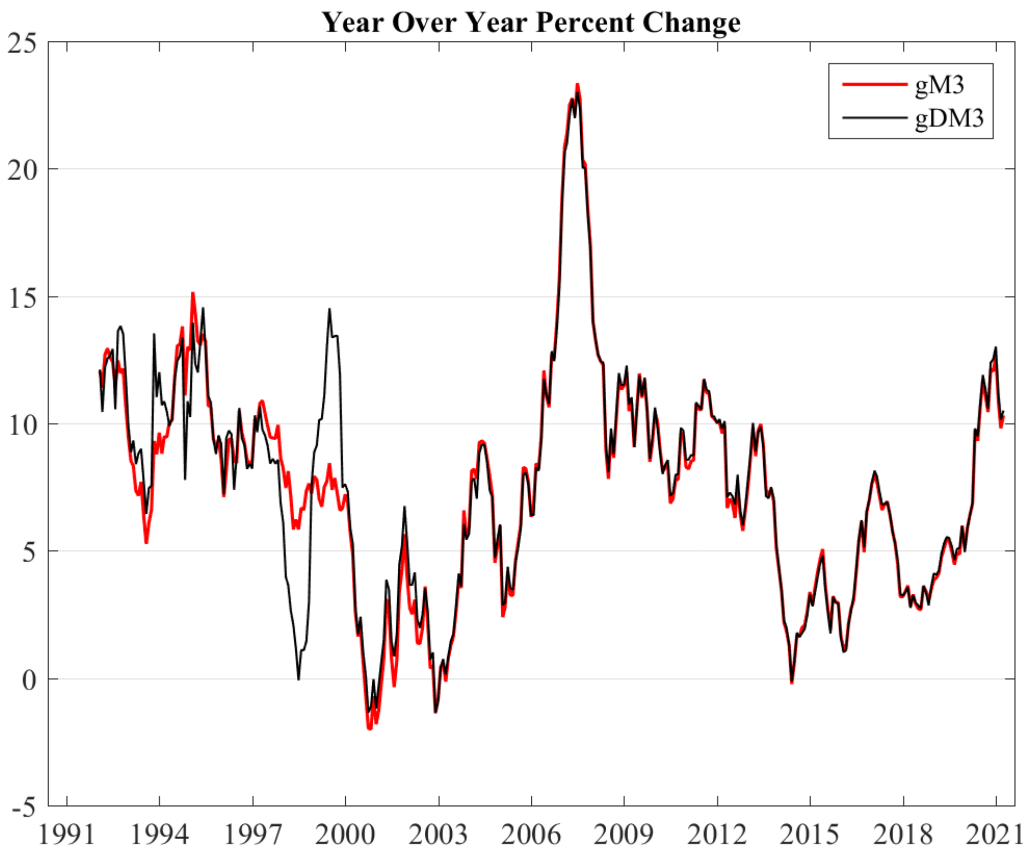

Figure 1.4. Divisia M3 versus Simple-sum M3 Growth Rates, 1992-2021

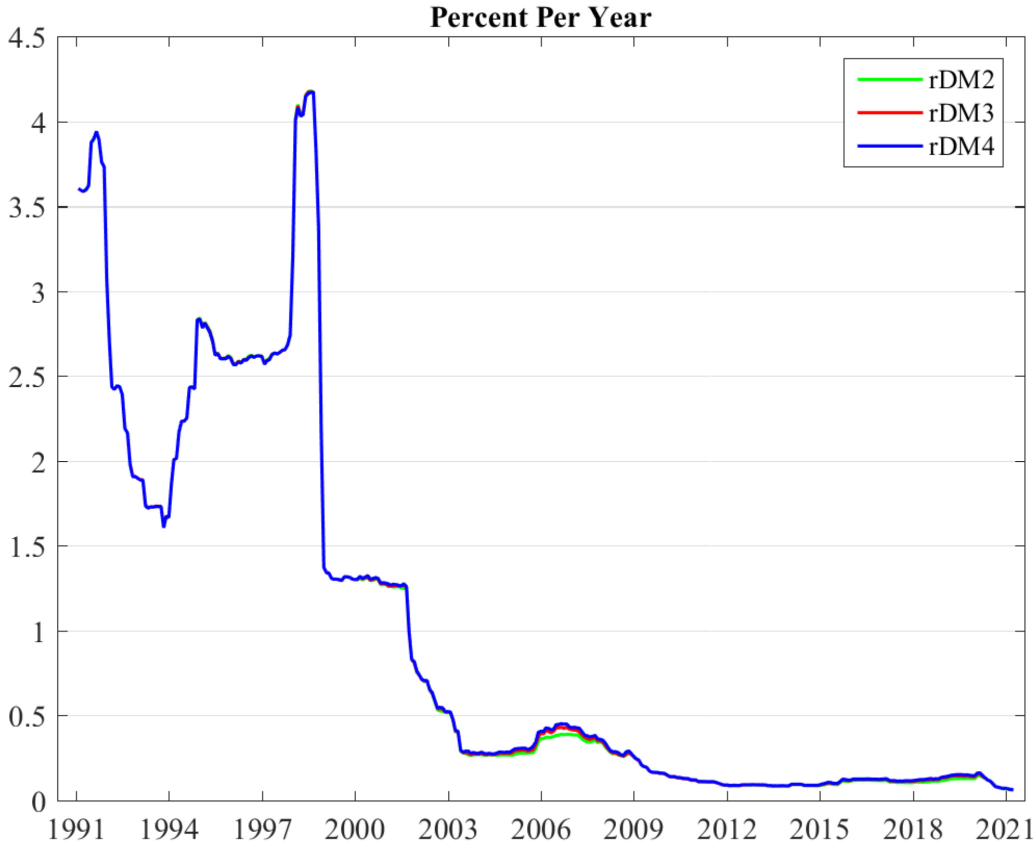

Figure 1.5. Interest Rate Aggregates, 1991-2021

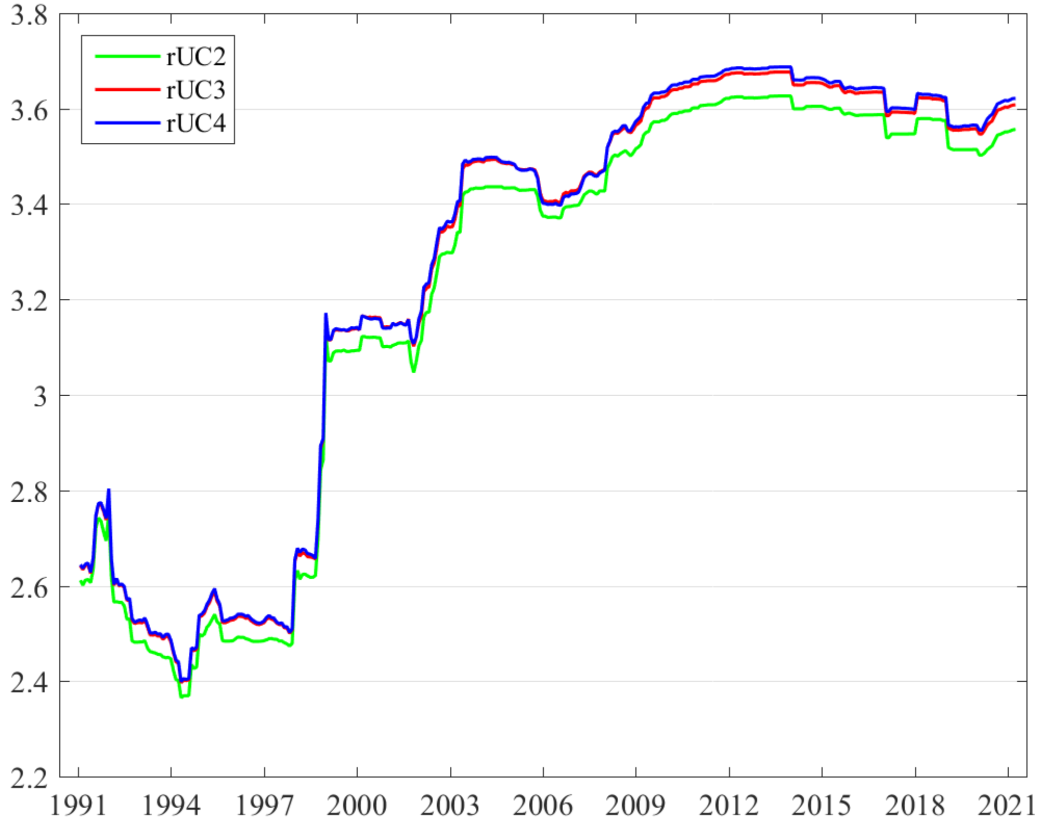

Figure 1.6. Real User Cost Aggregates, 1991-2021

A.1.2. Tables

Table 1.1. Monetary Asset Components

m	Asset	Rates of Return Used
1	Currency	0%
2	Demand Deposits	0%
3	Fixed Deposits in Commercial Banks	3-month CDs in banks
4	Negotiable CDs in Commercial Banks	3-month Commercial Bills
5	Saving and Other Deposits in Commercial Banks	saving rate in banks
6	Fixed Deposits in Finance Companies	3-month CDs in finance companies
7	Saving and Other Deposits in Finance Companies	saving rate in finance companies
8	Deposits in Post Office Saving Bank*	NA
9	Overnight and Term Repurchases	repos rate
10	Treasury Bills (all T-Bills and SGSs)	12-month T-bill yield
11	Credit Card Transaction Volumes**	average credit card interest rate

Data source: Monetary Authority of Singapore (MAS). The monthly dataset covers the period from Jan 1991 to Mar 2021.Data on levels are in millions of Singapore dollars (SGD). *Post Office Saving Bank was acquired by Development Bank of Singapore from Nov 1998. **SGSs stands for Singapore Government Securities. ***In Singapore, those volumes are called "Total Credit Card Billings".

Table 1.2. Nesting Components in Monetary Aggregates

m	Asset	M1	M2	M3	DM1	DM2	DM3	DM4
1	Currency	1	1	1	1	1	1	1
2	Demand Deposits	1	1	1	1	1	1	1
3	Fixed Deposits in Commercial Banks	0	1	1	0	1	1	1
4	Negotiable CDs in Commercial Banks	0	1	1	0	0	1	1
5	Saving and Other Deposits in Commercial Banks	0	1	1	0	1	1	1
6	Fixed Deposits in Finance Companies	0	0	1	0	1	1	1
7	Saving and Other Deposits in Finance Companies	0	0	1	0	1	1	1
8	Deposits in Post Office Saving Bank*	0	0	1	0	1	1	1
9	Overnight and Term Repurchases	0	0	0	0	0	1	1
10	Treasury Bills (all T-Bills and SGSs)	0	0	0	0	0	0	1

*Since the details on interest rates for POSB are not available, we split the quantities into fixed deposits and saving deposits and incorporate them into banking sector.

Table 1.3. Growth-rate Weights in the Last Month (Mar 2021), Percentages

m	**Asset**	**DM1**	**DM2**	**DM3**	**DM4**
1	Currency	20.658	7.622	7.555	7.526
2	Demand Deposits	79.342	29.281	29.024	28.913
3	Fixed Deposits in Commercial Banks	0	26.290	26.059	25.960
4	Negotiable CDs in Commercial Banks	0	0	0.018	0.018
5	Saving and Other Deposits in Commercial Banks	0	35.038	34.730	34.598
6	Fixed Deposits in Finance Companies	0	1.708	1.693	1.687
7	Saving and Other Deposits in Finance Companies	0	0.061	0.061	0.060
9	Overnight and Term Repurchases	0	0	0.860	0.857
10	Treasury Bills (all T-Bills)	0	0	0	0.381
		100	100	100	100

Table 1.4. Growth-rate Weights in the Last 12 Months (Apr 2020 - Mar 2021), Percentages

m	**Asset**	**DM1**	**DM2**	**DM3**	**DM4**
1	Currency	21.340	7.533	7.465	7.434
2	Demand Deposits	78.660	27.811	27.558	27.447
3	Fixed Deposits in Commercial Banks	0	29.046	28.781	28.663
4	Negotiable CDs in Commercial Banks	0	0	0.015	0.015
5	Saving and Other Deposits in Commercial Banks	0	33.737	33.430	33.295
6	Fixed Deposits in Finance Companies	0	1.823	1.807	1.799
7	Saving and Other Deposits in Finance Companies	0	0.050	0.049	0.049
9	Overnight and Term Repurchases	0	0	0.896	0.892
10	Treasury Bills (all T-Bills and SGSs)	0	0	0	0.405
		100	100	100	100

Table 1.5. Credit-card-augmented Growth-rate Weights (Jun 2019), Percentages

m	**Asset**	**DM1a**	**DM2a**	**DM3a**	**DM4a**
1	Currency	21.979	7.321	7.278	7.261
2	Demand Deposits	68.541	22.829	22.695	22.642
3	Fixed Deposits in Commercial Banks	0	33.354	33.159	33.081
4	Negotiable CDs in Commercial Banks	0	0	0.005	0.005
5	Saving and Other Deposits in Commercial Banks	0	31.355	31.171	31.098
6	Fixed Deposits in Finance Companies	0	1.937	1.926	1.921
7	Saving and Other Deposits in Finance Companies	0	0.046	0.046	0.046
9	Overnight and Term Repurchases	0	0	0.582	0.581
10	Treasury Bills (all T-Bills and SGSs)	0	0	0	0.235
11	Credit Card Transaction Volumes*	9.480	3.157	3.139	3.132
		100	100	100	100

*In Singapore, those volumes are called "Total Credit Card Billings."

2. Divisia Monetary Aggregates and Monetary Policy in Singapore

Van H. Nguyen[3]

Abstract: Recent New Keynesian DSGE models focus on interest rate, and often ignore aggregate quantity of money as an instrument or target of monetary policy. In the case of a small open economy like Singapore's, the effective exchange rate is managed to achieve goals for price stability and output growth. To examine the relevance of different tools of monetary policy in explaining real economy activity in Singapore, we examined the cyclical correlation between interest rates, exchange rates, and money supply with macro-economic variables like output and inflation. We also tested the marginal information content of these monetary policy instruments in predicting output and inflation. For money supply, we used different measures, including the official simple-sum measures reported by the Monetary Authority of Singapore, Divisia measures and recent credit-card-augmented Divisia monetary aggregates. We found that money shows a stronger relation with macroeconomic variables than both interest rates and exchange rates. Among different money measures, Divisia measures always show stronger correlation with both output and inflation compared with simple-sum measures. Credit-card-augmented Divisa measures are the most informative indicators for predicting output and price.

Keywords: Divisia monetary aggregates; credit-card-augmented Divisia; cyclical correlation; open-economy macroeconomics; monetary policy analysis; Singapore.

JEL Classification: E32; E40; E41; E47; E50; E51; E52; E58.

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2.1. Introduction

The role of nominal variables, like interest rate and money supply, are emphasized in recent New Keynesian literature to have impact on real economic variables like output and growth in short-run due to the stickiness of price and wage. However, recent DSGE macroeconomic models often ignore aggregate quantity of money as an instrument of monetary policy. They tend to solely rely on (nominal) interest rate, such as the federal fed fund rate in modelling the thrust of monetary policy. The entire intervention of the central bank is expressed via some kind of interest rate rule, like the Taylor rule (1993). The support for this view lies in the empirical evidence in 1980s by Bernanke and Blinder (1988) that demand for money is more unstable than the demand for credit, therefore, monetary policy would have better success in stabilizing output if it stabilized interest rate rather than money supply. Moreover, Sims (1980) argues that money loses its predictive power on output when interest rate is included in the regression.

As the world is getting more and more integrated, all economies are open and most of them are small. Macroeconomic research on small open economy has caught more and more attention lately. The so-called New Open Economy Macroeconomics literature has grown rapidly in recent years. The discussion of New Keynesian literature on optimal monetary policy in an open economy focuses on whether exchange rate stability should be part of a central bank's strategy, for example, see McCallum (2007). Again, the quantities of money are being ignored. In this context, Singapore is an interesting example to look at. It is a very small and highly open economy with a sophisticated financial system being a major financial center in Southeast Asia. Its successful story of economic development highlights its monetary policy, which has been centered on the management of the exchange rate since 1981. The monetary authority of Singapore (MAS) explicitly operates a managed float regime for the Singapore dollar. The trade-weighted exchange rate is allowed to fluctuate within a policy band to maintain price stability and economic growth. This choice of monetary regime was compared with a Taylor rule type of managing the interest rate in Chow et al (2013). The former is shown to be comparatively more advantage over the later for stabilizing both inflation and output growth.

In response to the lacking attention on money quantities in the New Keynesian literature due to some empirical evidence that the demand for money is unstable and money has low power in explaining output and other macroeconomic variables, other economists have brought up the issue of money measurement. Up until the 1980s, economists throughout the world measured different levels of monetary aggregation, such as M0/MB (monetary base), M1 (narrow money), M2 (broad money), and M3 and M4 (financial liquidity), by simply adding up the quantities of component assets. This simple sum assigns the same weight to every monetary asset and implicitly assumes all monetary assets are perfect substitutes. As the financial system becomes more and more sophisticated with different type of monetary assets, which produce different interest rates, possess different liquidity and are at different levels of risk, this simple sum measure is clearly not a proper way to aggregate money. Crystal and MacDonald (1994) used the term `Barnett critique' to refer to the misleading and potential distortion of economic inferences if the simple-sum measure of money keeps being used.

Barnett (1978, 1980) derived the formula to measure the user cost price of monetary services and proposed a method to aggregate money based on solid microeconomic theory foundation and index number theory. Accordingly, to properly aggregate components in monetary service aggregation, we need both their quantities and prices. Monetary asset services are not analogous to perishable consumer good services, such as apples, but to capital goods or durable goods, such as houses or automobiles. Hence, their prices are measured in term of user cost prices.

The Divisia measure of money that Barnett proposed is a weighted index whose weights are based on the expenditure shares of component assets. Since the theory of monetary aggregation became available, central banks such as the Federal Reserve (FED) in the US, the Bank of England (BOE) in the UK, the European Central Bank (ECB), the Bank of Japan (BoJ), the National Bank of Poland, and the Bank of Israel, among others, have, at various times and in diverse ways, produced and maintained Divisia indexes for monetary aggregation. Simply replacing the traditional simple-sum measure of money by Divisia measures, Belongia (1996), Barnett and Chauvet (2010), Belongia and Ireland (2015), among many other researchers have shown that money still shares a strong relationship with aggregate economic activity, and the demand for money function still exhibits stability.

Despite the availability of Divisia indexes and empirical evidence that they are superior to their simple-sum counterparts, the use of the latter has not been entirely eliminated. For example, get back to the case of Singapore, the monetary authority keeps reporting money supplies solely as simple sum measures. The notable point is the MAS conducts its monetary policy not by adjusting money supply but adjusting the Singapore dollar's effective exchange rate, which is managed against a basket of currencies of major trading partners and competitors. The various currencies are assigned weights in accordance with the importance of the country to Singapore’s trading relations with the rest of the world. The composition of the basket is revised periodically to take into account changes in trade patterns. What if money supply in Singapore were measured in a proper way similar to the trade-weighted effective exchange rate, would money be better than interest rates and exchange rates as an indicator of economic activity and an instrument of monetary policy?

2.2. Divisia Monetary Aggregates

The crucial point of monetary aggregation theory circles around the user cost price of monetary assets, which is more similar to a capital asset rather than a perishable consumption good. Accordingly, the user cost price of a monetary asset is not its interest rate but the forgone interest rate that could have been achieved by not consuming the service of that asset.

2.2.1. Conventional Divisia Index

Barnett (1978) derived the formula for the real user cost of a monetary service

$$\pi_{i,t} = \frac{R_t - r_{i,t}}{1 + R_t}, \tag{2.1}$$

where R_t is the interest rate on some benchmark asset, like one-period matured bond and r_t is the interest rate of the monetary asset itself. With the availability of the user cost prices of monetary assets, both their quantities and prices are well-defined. The economic theory of aggregation over monetary assets becomes available. Barnett (1980) proved that the exact monetary quantity aggregate, M_t can be tracked without error in continuous time by the Divisia index, defining the growth rate of aggregate monetary services to be

$$\frac{d \log M_t}{dt} = \sum_{i=1}^{n} s_{i,t} \frac{d \log m_{i,t}}{dt}, \tag{2.2}$$

where

$$s_{i,t} = \frac{\pi_{i,t} m_{i,t}}{\sum_{j=1}^{n} \pi_{j,t} m_{j,t}}. \tag{2.3}$$

The weight $s_{i,t}$ of monetary asset i is its share in the total expenditure on the portfolio. In short, the growth rate of a Divisia monetary quantity index is the share weighted average of the growth rates of its components. The discrete Divisia index is approximated by

$$\log M_t - \log M_{t-1} = \sum_{i=1}^{n} \overline{s}_{i,t} (\log m_{i,t} - \log m_{i,t-1}) \,, \tag{2.4}$$

where the discrete weights are approximated by

$$\overline{s}_{i,t} = \frac{1}{2}(s_{i,t} + s_{i,t-1}) \,. \tag{2.5}$$

2.2.2. Credit-card-augmented Divisia Index

In recent years, credit card payments have become increasingly common in modern economies worldwide. In the US, more than 80% of American households with credit cards are currently borrowing and paying interest on credit cards. In Singapore and other Asian economies, though the use of credit cards is not as that common/popular and the total volume of credit card transactions is relatively small, we see a clearly increasing trend. Credit card balances are liabilities, not assets; by accounting conventions, they cannot be added to monetary assets. Hence credit cards cannot be included in simple sum monetary aggregates. However, in economic theory, aggregation over services is possible, regardless of whether the services are produced from assets or liabilities. The deferred payment services of credit card transactions can be augmented into the Divisia monetary service aggregates.

Barnett et al. (2016) provided the theoretical framework for credit-card-augmented Divisia monetary aggregates. Accordingly, the user cost price of a credit card's service is

$$\pi^{c}_{j,t} = \frac{e_{j,t} - R_t}{1 + R_t} \,, \tag{2.6}$$

where $e_{j,t}$ is the interest rate charged by credit card type j, with j = 1, ... k. The growth-rate weight of monetary asset i's service is

$$w_{i,t} = \frac{\pi_{i,t} m_{i,t}}{\boldsymbol{\pi}_t{}' \, \mathbf{m}_t + (\boldsymbol{\pi}_t^c)' \, \mathbf{c}_t}, \tag{2.7}$$

where the volume of transactions using credit card type j is $c_{j,t}$. The growth-rate weight of credit card j's service is

$$w_{j,t}^c = \frac{\pi_{j,t}^c c_{j,t}}{\boldsymbol{\pi}_t{}' \, \mathbf{m}_t + (\boldsymbol{\pi}_t^c)' \, \mathbf{c}_t}. \tag{2.8}$$

The credit-card-augmented Divisia monetary aggregate becomes

$$\frac{d \log Ma_t}{dt} = \sum_{i=1}^{n} w_{i,t} \frac{d \log m_{i,t}}{dt} + \sum_{j=1}^{k} w_{j,t}^c \frac{d \log c_{j,t}}{dt}. \tag{2.9}$$

The analogue discrete credit-card-augmented Divisia index is

$$\log Ma_t - \log Ma_{t-1} = \sum_{i=1}^{n} \bar{w}_{i,t} (\log m_{i,t} - \log m_{i,t-1}) + \sum_{j=1}^{k} \bar{w}_{j,t}^c (\log c_{j,t} - \log c_{j,t-1}), \tag{2.10}$$

where the discrete weights are approximated by

$$\bar{w}_{i,t} = \frac{1}{2}(w_{i,t} + w_{i,t-1}) \text{ and } \bar{w}_{j,t}^c = \frac{1}{2}(w_{j,t}^c + w_{j,t-1}^c). \tag{2.11}$$

2.2.3. Interest Rate Aggregation

According to Barnett et al. (2013), aggregated interest rate can be computed using accounting principles. Simply speaking, aggregated interest rate on a portfolio is the rate of return on that portfolio,

$$r_t = \frac{\sum_{i=1}^{n} r_{i,t} m_{i,t}}{\sum_{i=1}^{n} m_{i,t}}. \tag{2.12}$$

2.3. Monetary Policy in Singapore

2.3.1. Data Description

To analyze monetary policy in Singapore, we collected data on output, price level, exchange rates, interest rates, and money supply. We also use the Divisia indexes for different levels of monetary aggregation that we computed and published in Barnett and Nguyen (2021). See Table 2.1 for a basic description of our data set.

To measure output, the department of statistics of Singapore (DOS) provides the quarterly series on real GDP from 1975. We would want monthly data on GDP since the rest of our dataset are monthly. Fortunately, the DOS also reports monthly Industrial Production Index (IPI) from Jan 1983, which is closely related to GDP. We used Chow and Lin (1971) method to interpolate quarterly GDP to monthly GDP using IPI as the related series. The summary of the procedure is as follow. Let y_q denote the quarterly series we want to interpolate and X_m be the monthly related series. In our case, y_q is quarterly GDP and X_m is monthly IPI.

(i) Convert $3n$ monthly observations of X_m into n quarterly observations X_q using $n \times 3n$ matrix C.

$$X_q = C \times X_m, \tag{2.13}$$

where

$$C = \frac{1}{3} \times \begin{pmatrix} 1 & 1 & 1 & 0 & . & . & . & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & . & . & . & 0 & 1 & 1 & 1 \end{pmatrix}. \tag{2.14}$$

(ii) Estimate the coefficient β from an OLS regression on the quarterly data,

$$y_q = X_m \beta + u \,, \tag{2.15}$$

and obtain the estimated coefficient and the residuals from regression (15)

$$\hat{u}_q = y_q - X_m \hat{\beta} \,. \tag{2.16}$$

Compute the monthly residuals using matrix C^T

$$\hat{u}_m = C^T \times \hat{u}_q \,. \tag{2.17}$$

(iii) Interpolate monthly data y_m using the estimated coefficient and the monthly residuals from step (ii).

$$y_m = X_m \hat{\beta} + \hat{u}_m \,. \tag{2.18}$$

Interpolated monthly GDP is plotted in Figure 2.1.

To measure price level, in addition to the DOS monthly series on Consumer Price Index from Jan 1983, the MAS reports the core CPI, which excludes accommodation and private transport costs from Jan 1990. See the comparison of these two series of inflation and core inflation in Singapore in Figure 2.2.

The MAS reports bilateral nominal exchange rates between the Singapore dollars (SGD) and the foreign currencies of major trading partners such as USD, GBP, FRA, JPY, RGI, AUD from Jan 1988. RMB was added in the list from Jun 1995, and EUR from Jan 1999. See Figure 2.3 for a visual image of these bilateral nominal exchange rates. As mentioned in the introduction, the MAS manages their currency against a basket of strong foreign currencies. Each foreign currency is assigned a weight based on its share in trade with Singapore. The MAS conducts its monetary policy by adjusting this trade-weighted effective exchange rate. Our data on Nominal

Effective Exchange Rate and Real Effective Exchange Rate Indexes are collected from the Bank of International Settlements (BIS). See Figure 2.4 for a graph of these series.

For interest rates and money supply, we looked into the monthly statistical bulletin of the MAS. Accordingly, MAS reports monthly data for money supply *M1, M2, M3* (which we refer to as the simple-sum measures) from Jan 1991. To promote our argument on the use of Divisia measures of money, we looked careful into the balance sheets of commercial banks and financial institutions to extract the information on quantities and interest rates of different monetary assets. We also collected data on the volume of credit-card transactions and the average credit-card interest rate is taken from ValueChampion (https://www.valuechampion.sg/about). We nested components of monetary assets into different levels of aggregation based on the asset's liquidity, according to Barnett et al (2013) and then computed the real user cost of each monetary asset by formula (2.1). The growth-rate weights are calculated based on Equation (2.3) and (2.5), and finally, the growth-rates of Divisia index are computed using Equation (2.4). For more details about the construction of Divisia monetary aggregates and credit-card-augmented Divisia monetary aggregates for Singapore, see Barnett and Nguyen (2021). We plotted different measures of money supply in Figure 2.4 and Figure 2.5. The aggregated interest rates are computed by accounting principle, as in Equation (2.12), see Figure 2.6 for a visual image.

2.3.2. Cyclical Correlation Analysis

Up to this point, we have gathered the data and get ready for the discussion of monetary policy in Singapore based on our data analysis. The monthly data covers the period from Jan 1991 to Mar 2021. Data in levels, i.e., output and money supply are in log scale.

To investigate the cyclical correlation between different monetary policy instruments and output and inflation, we first extract the cyclical components of each series. We decompose each series into trend and cyclical components using the Hamilton (2018) filter. Accordingly, the cyclical components of a non-stationary series can be captured by an OLS regression of the series on its *p* lags shifted back *h* horizons. As suggested by Hamilton (2018), for quarterly data, four lags and two-year horizon are recommended. Since our data are monthly over a period of 30 years, we decided to choose the number of lags of 4, and a horizon of 36 months in

$$X_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i X_{t-h-i+1} + u_t \, . \tag{2.19}$$

The residuals from regression (19) provides the cyclical component of the series

$$\hat{u}_t = X_t - \hat{\alpha}_0 - \sum_{i=1}^{p} \hat{\alpha}_i X_{t-h-i+1} \, . \tag{2.20}$$

We measure the cyclical correlation of two series X_t and Y_t by computing the contemporaneous correlation coefficient,

$$\rho(X_t, Y_{t+j}) = \rho(\hat{u}_{X,t}, \hat{u}_{Y,t+j}) \, , \tag{2.21}$$

for $j = \pm 36, \pm 30, \pm 24, \pm 18, \pm 12, \pm 9, \pm 6, \pm 3, \pm 2, \pm 1, 0$. If $\rho > 0$, X_t is pro-cyclical; if $\rho < 0$, X_t is counter-cyclical; if $\rho = 0$, X_t is acyclical. The degree of correlation is measured by the absolute value of ρ. If $|\rho| < 0.2$, X_t and Y_t are uncorrelated, if $0.2 < |\rho| < 0.5$, X_t and Y_t are weakly correlated, and if $|\rho| > 0.5$, X_t and Y_t are strongly correlated. Let j^* be the value of j at which the absolute value of ρ attains its maximum (ρ^*). This value provides the information about the phase shift of X_t. If j^* is positive or negative, we say that X_t is lagging or leading the cycle of Y_t by j^* period, respectively.

We computed the cyclical correlation between aggregated interest rates, different measures of money supply, effective exchange rates, and output and inflation. The results are plotted in

Figure 2.8, Figure 2.9, Figure 2.10, and Figure 2.11. We see that interest rates are uncorrelated to either output or inflation. Exchange rates and all money measures are pro-cyclical and leading the cycle of output of inflation. Looking at the magnitude of the correlation, we find that exchange rates show a weak correlation with both output and inflation, whereas all money measures demonstrate a weak correlation with output and a strong correlation with inflation. Among different measures of money, Divisia measures always show a stronger relation with both output and price than the simple-sum measure. Looking at the phase shifts of the series, we notice that for output, nominal effective exchange rate is leading the cycle for 24 months, while Divisia measures lead the cycle for less than 18 months. Similarly, while nominal effective exchange rate is leading the cycle of inflation for 6 months; most Divisia measures lead this cycle for 3 months, see Table 2.2 for details. On both correlation magnitudes and phase shifts, credit-card-augmented Divisia indexes are the most informative indicators for predicting output and inflation.

2.4. Conclusion

This paper examines different monetary policy instruments including interest rates, exchange rates and different measures of money supply and their relevance to predicting output and inflation in Singapore. Besides the monthly data on output, inflation, trade-weighted effective exchange rates, and simple-sum measures of money supply from the DOS, MAS and BIS, we also used the Divisa measures of money, and the aggregated interest rates that we computed by ourselves. We applied the Hamilton (2018) based filter to extract the cyclical component of each time series and compute the cyclical correlation of interest rates, money supply and exchange rates with output and inflation. We found that interest rates are not contemporaneously correlated to either output or inflation. Nominal effective exchange rate and all money measures show a weakly contemporaneous correlation with output and notably money measures show a strong correlation with inflation. Among different money measures, credit-card-augmented Divisa measures are the most informative indicators for predicting output and price. Our results support the argument that money still shares a strong relationship with aggregate economic activity. We advocate the use of Divisia monetary aggregates, which measures money in a more proper manner than the traditional simple-sum.

To confirm the above statistical findings in a more rigorous framework, in the future, we plan to examine different instruments of monetary policy for a small and open economy with sophisticated financial sector. We plan to develop a New Keynesian model for a small open economy with multiple financial assets. We borrow from Belongia and Ireland (2014) the way to introduce private financial institutions (like commercial banks), who create deposits as imperfect substitutes for government-issued currency. This framework allows us to construct and compare the behavior of different measures of monetary aggregates. It also allows us to check if money is more informative than interest rate and exchange rate in explaining/predicting output and inflation. Furthermore, to find out the optimal monetary policy instrument, we plan to evaluate the representative household welfare under different targets of monetary policy, i.e, interest rate rule, fixed growth rates for monetary base, fixed growth rates for simple sum, fixed growth rates for Divisia monetary aggregate, and fixed exchange rate.

A.2. Appendix 2

A.2.1. Figures

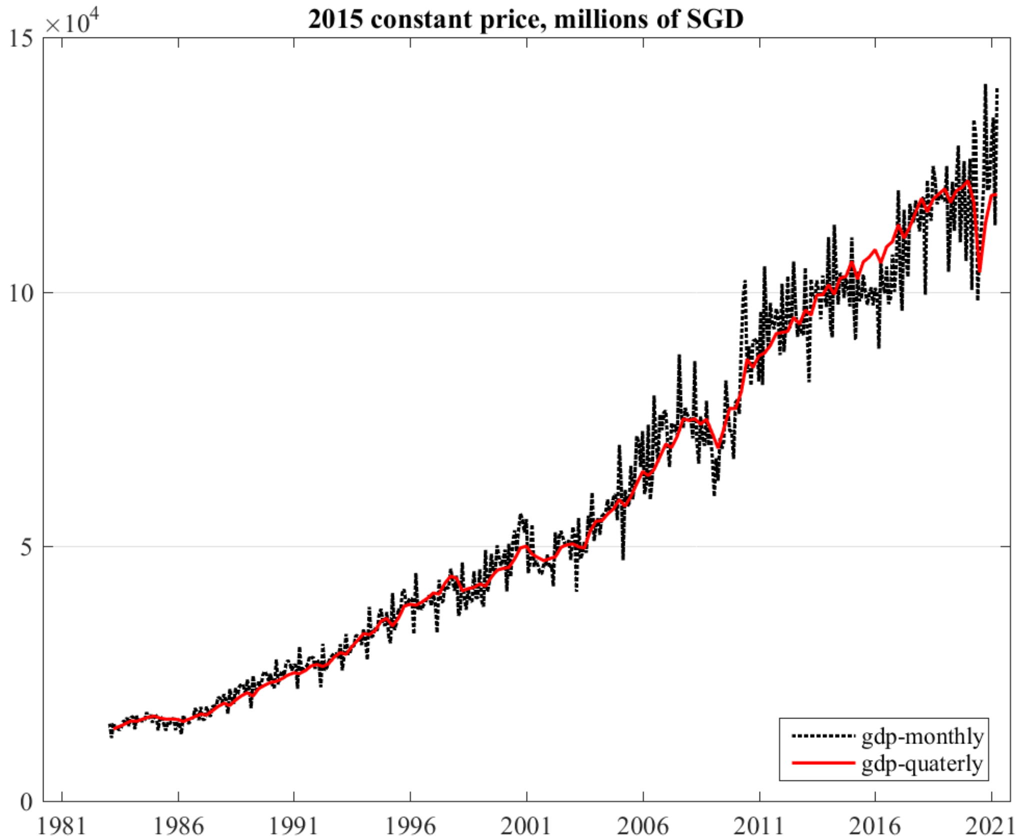

Figure 2.1. Chow-Lin (1971) Interpolation for Monthly GDP

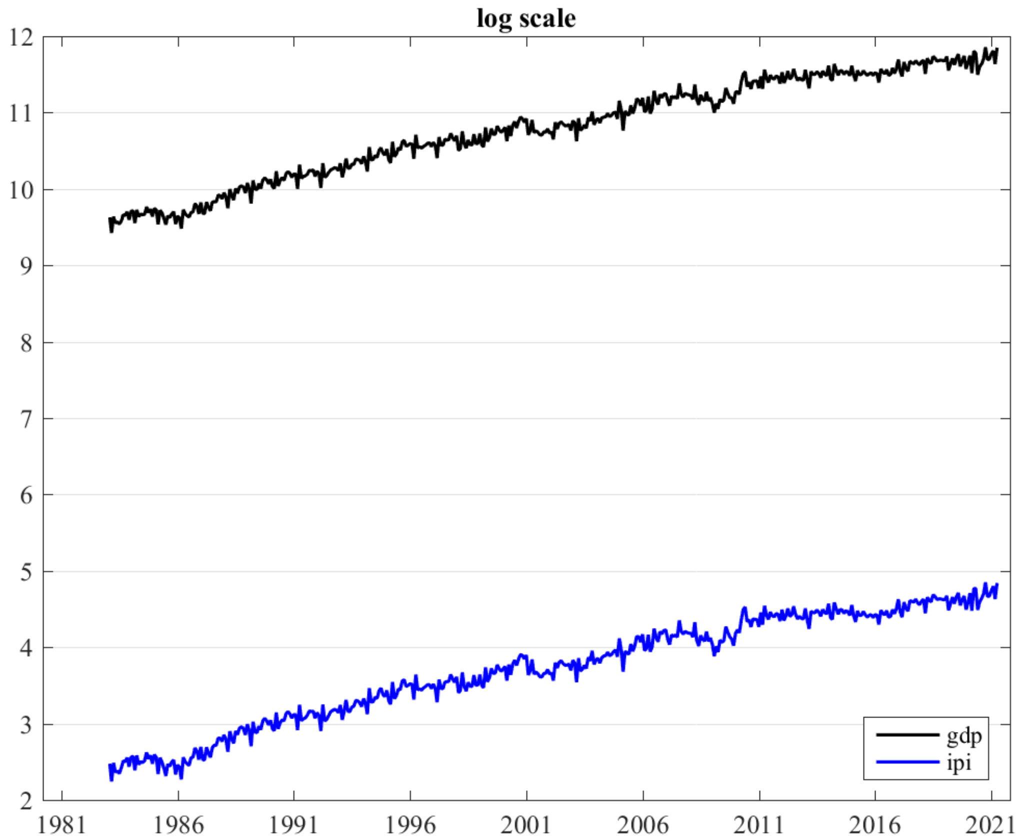

Figure 2.2. GDP (2015 Constant Price) and IPI (2014=100)

Figure 2.3. Inflation versus Core Inflation

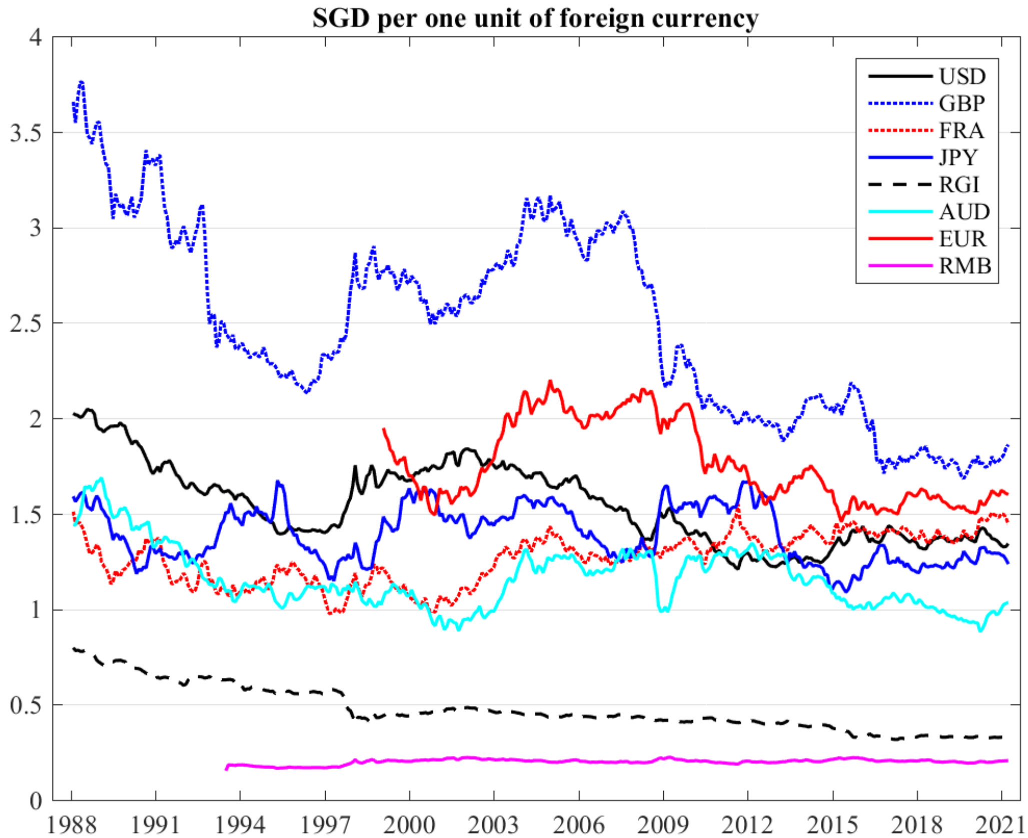

Figure 2.4. Bilateral Nominal Exchange Rates

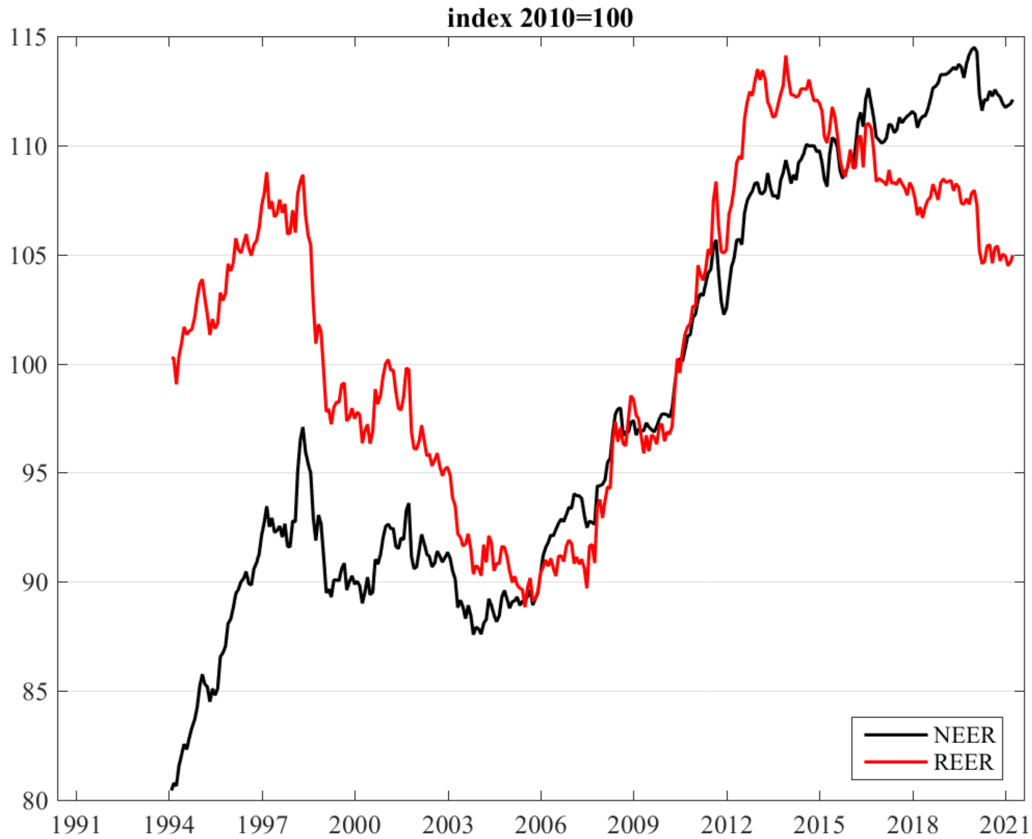

Figure 2.5. Effective (Trade Weighted) Exchange Rates, Nominal versus Real

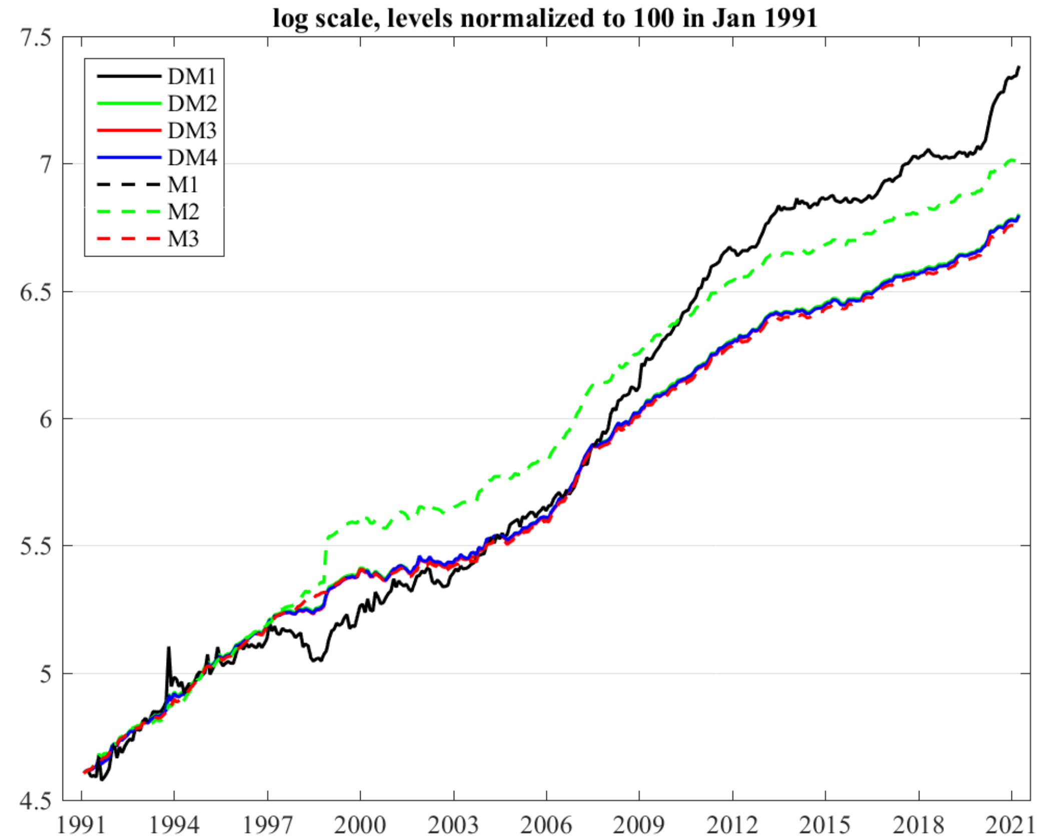

Figure 2.6. Divisia versus simple sum aggregates, 1991–2021

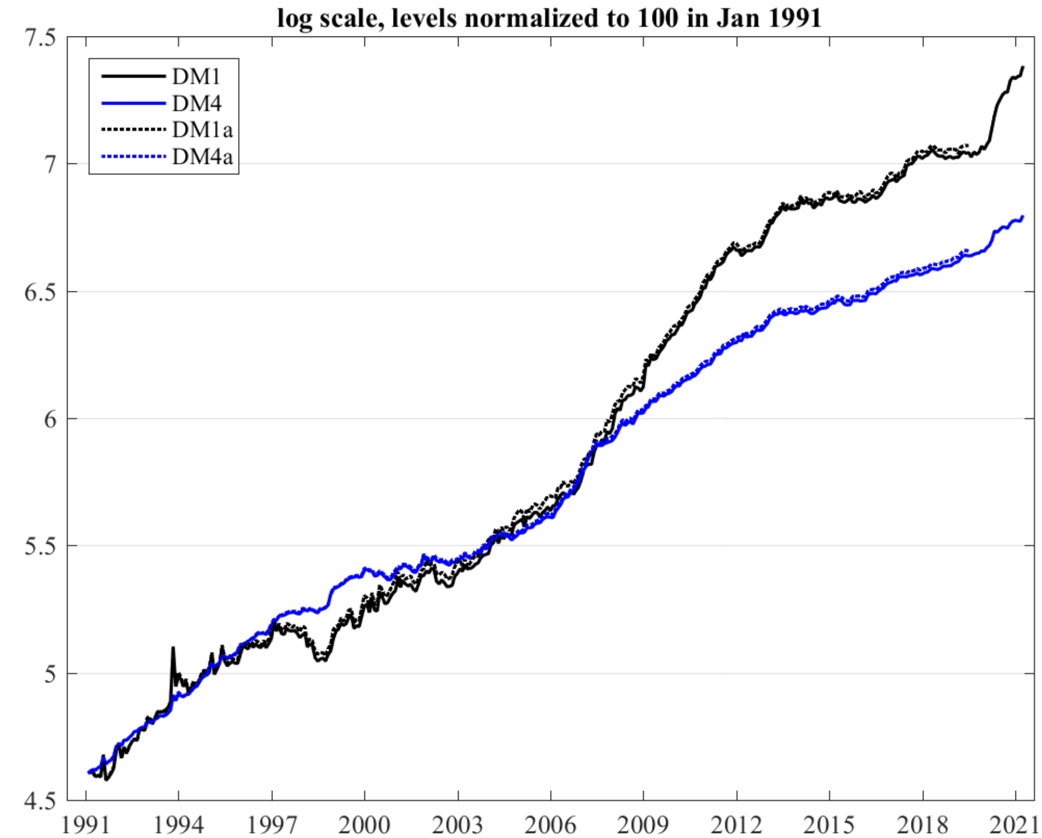

Figure 2.7. Divisia and Credit-card-augmented Divisia Monetary Aggregates, 1991-2021

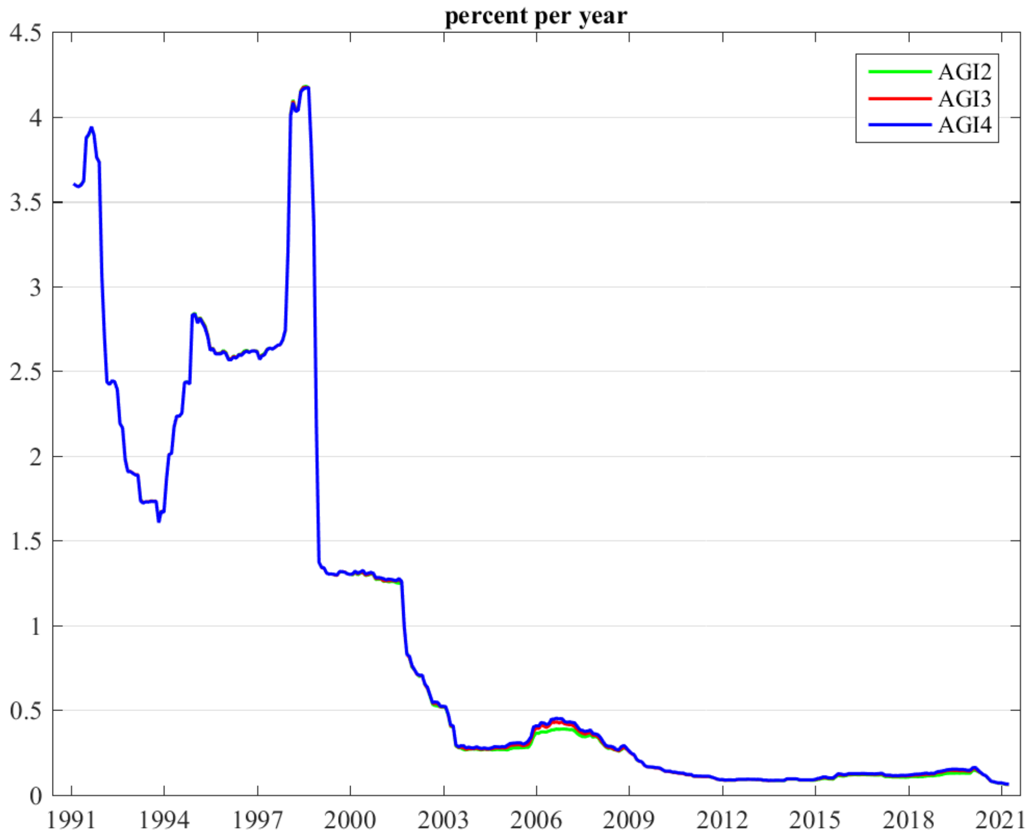

Figure 2.8. Aggregated Interest Rates 1991-2021

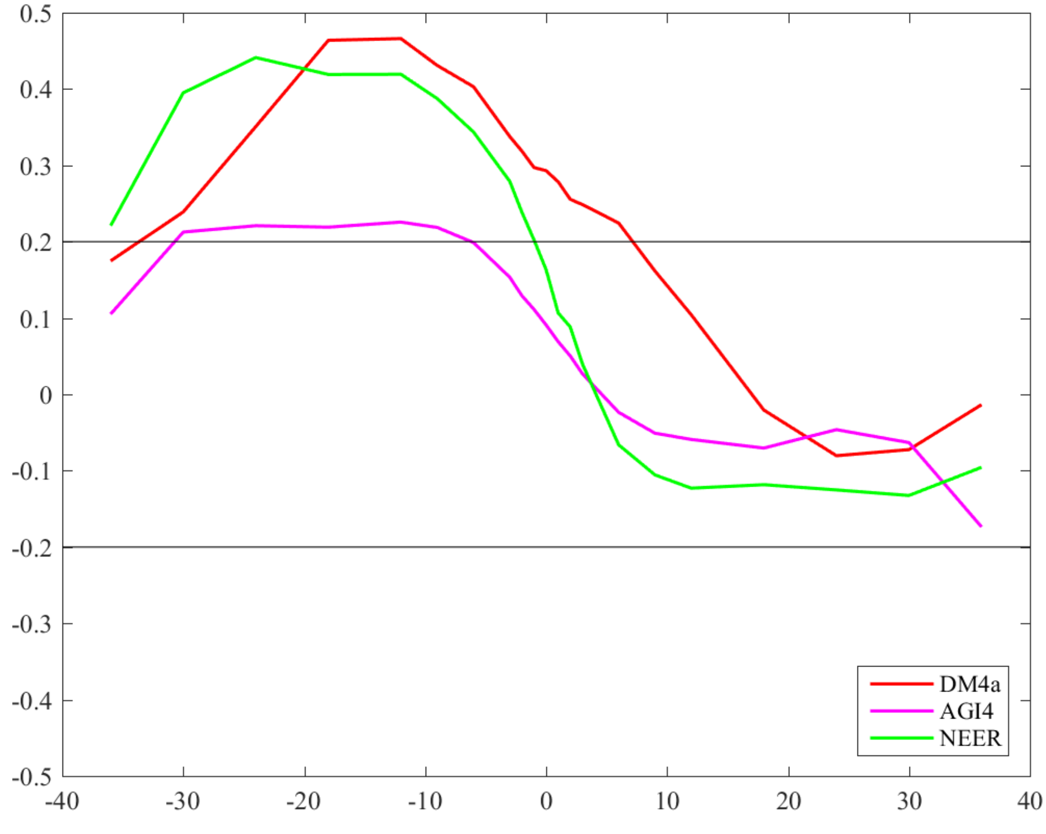

Figure 2.9. Cyclical Correlation between Money, Interest Rates, Exchange Rates and Output

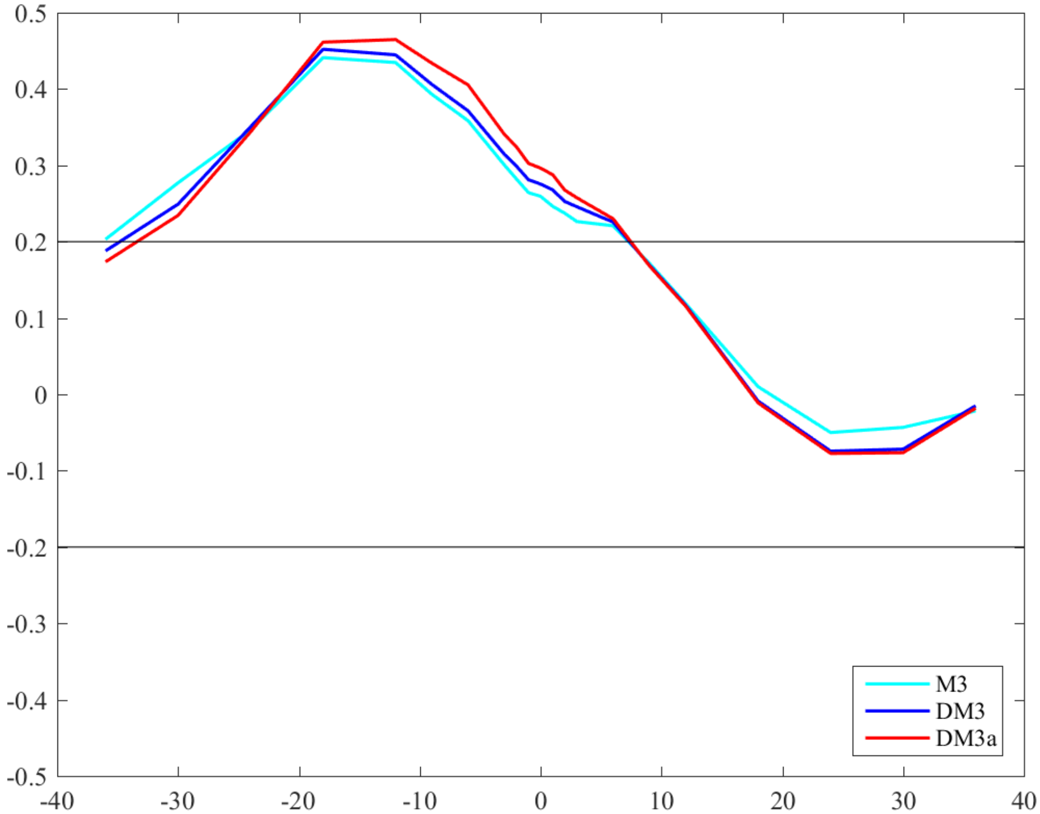

Figure 2.10. Cyclical Correlation between Different Money Measures and Output

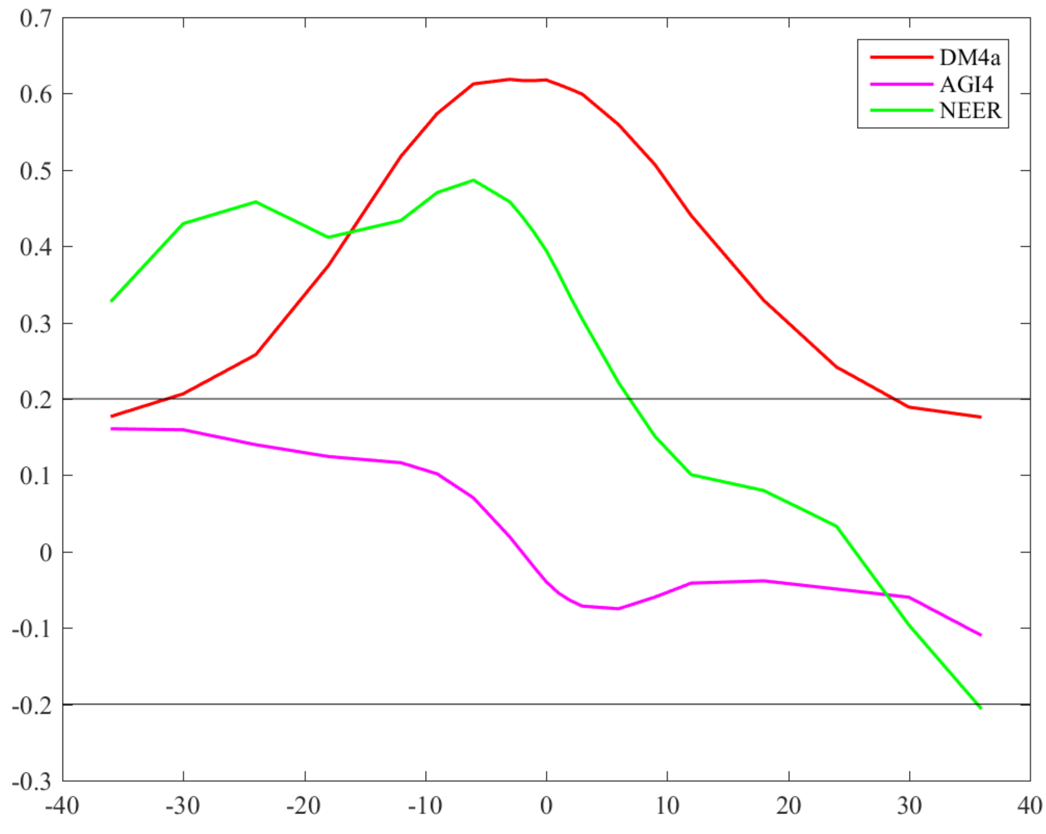

Figure 2.11. Cyclical Correlation between Money, Interest Rates, Exchange Rates and Inflation

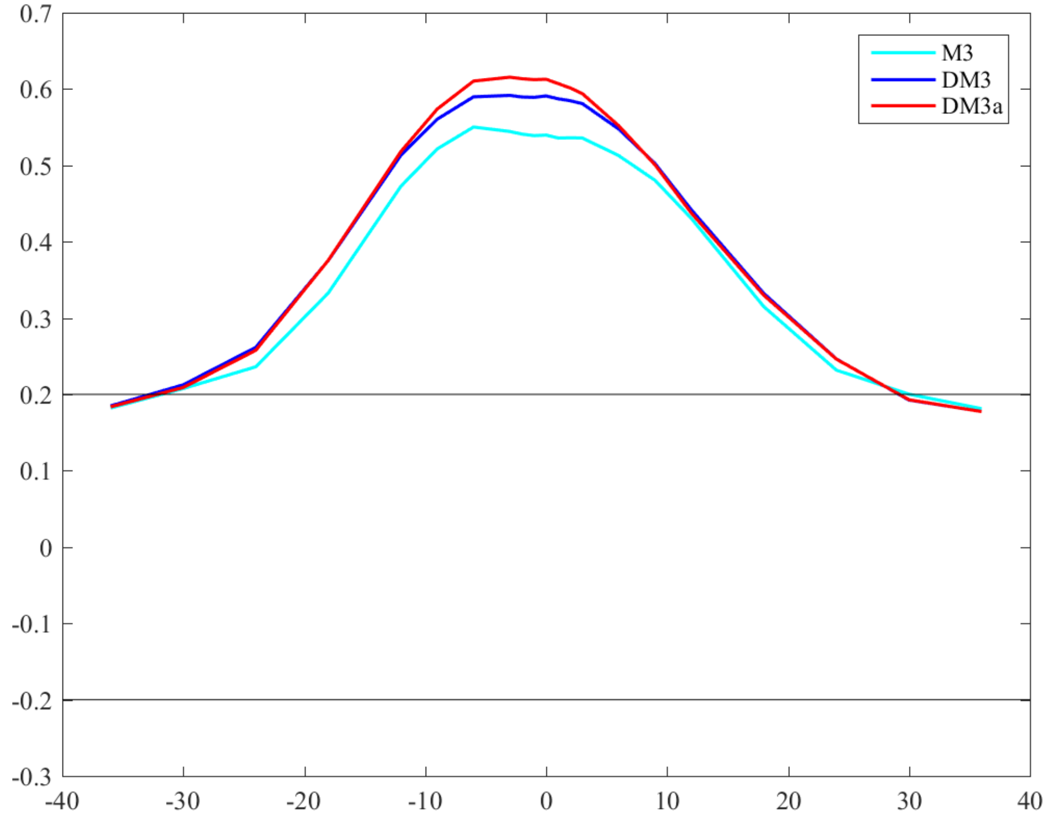

Figure 2.12. Cyclical Correlation between Different Money Measures and Inflation

A.2.2. Tables

Table 2.1. Basic Description of the Dataset

No	Series	Unit	Source
1	quarterly GDP (2015 constant price)	millions of SGD	DOS
2	monthly GDP (2015 constant price)	millions of SGD	self-computed
3	Industrial Production Index (IPI)	index, 2014=100	DOS
4	Consumer Price Index (CPI)	index, 2019=100	DOS
5	Core CPI	index, 2019=100	MAS
6	Simple sum M1 (current price)	millions of SGD	MAS
7	Simple sum M2 (current price)	millions of SGD	MAS
8	Simple sum M3 (current price)	millions of SGD	MAS
9	Divisia DM1	index, 1991=100	self-computed
10	Divisia DM2	index, 1991=100	self-computed
11	Divisia DM3	index, 1991=100	self-computed
12	Divisia DM4	index, 1991=100	self-computed
13	Credit-card-augmented Divisia DM1a	index, 1991=100	self-computed
14	Credit-card-augmented Divisia DM2a	index, 1991=100	self-computed
15	Credit-card-augmented Divisia DM3a	index, 1991=100	self-computed
16	Credit-card-augmented Divisia DM4a	index, 1991=100	self-computed
17	Aggregate interest rates	percent per year	self-computed
18	Effective trade-weighted exchange rates	index, 2010=100	BIS

Quarterly GDP is available from Q1 1975; IPI and CPI from Jan 1983; core CPI from Jan 1990. Nominal effective (trade-weighted) exchange rate from 1994; other series from Jan 1991 to Mar 2021.

Table 2.2. Maximum Absolute Values of Correlation Coefficients

No	Series	Output		Inflation	
		rho*	j*	rho*	j*
1	Simple sum M1	0.2861	6	0.6385	-6
2	Simple sum M2	0.3208	-18	0.4500	3
3	Simple sum M3	0.4411	-18	0.5502	-6
4	Divisia DM1	0.2861	6	0.6385	-6
5	Divisia DM2	0.4372	-18	0.5849	0
6	Divisia DM3	0.4521	-18	0.5917	-3
7	Divisia DM4	0.4552	-18	0.5951	0
8	Augmented Divisia DM1a	0.3149	-6	0.6607	-6
9	Augmented Divisia DM2a	0.4524	-12	0.6066	-3
10	Augmented Divisia DM3a	0.4648	-12	0.6155	-3
11	Augmented Divisia DM4a	0.4661	-12	0.6186	-3
12	Aggregated interest rate AGI2	0.2269	-24	0.1666	-36
13	Aggregated interest rate AGI3	0.2242	-12	0.1631	-36
14	Aggregated interest rate AGI4	0.2256	-12	0.1608	-36
15	Nominal effective exchange rate	0.4412	-24	0.4865	-6
16	Real effective exchange rate	0.3751	-24	0.4781	-24

Output and inflation from 1983; effective (trade-weighted) exchange rates from 1994; other series from Jan 1991 to Mar 2021.

3. Monetary Policy in a Small Open Economy with Multiple Monetary Assets

Van H. Nguyen[4]

Abstract: To examine the impact of openness on the volatility of macroeconomic variables in a small and open economy, we revisit the issues of money measurement. I compare the behavior of different money measures in the context of the New Keynesian framework with sticky price. I introduce the banking sector into the model, which allows the accommodation of multiple monetary assets like currency and interest-bearing-deposits. The central bank conducts its monetary policy via a simple interest rate rule. I explore the responses of different money measures, namely simple-sum, monetary base, and Divisia quantity aggregate with respect to domestic and foreign shocks and compare these responses with those from a theoretical benchmark. I find that Divisia tracks the movement of money most closely to the benchmark, followed by monetary base, while simple sum often does not match the correct trend. I analyze the impact of openness, which has an inverse relation with home-bias in consumption, on the volatility of macroeconomic variables. I find that as a small economy becomes more open, domestic inflation and nominal interest rate are more volatile while term of trade and exchange rate become more stable.

Keywords: Divisia monetary aggregates; open-economy macroeconomics; monetary policy; New Keynesian model; small open economy.

JEL Classification: E31; E32; E41; E47; E51; E52.

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3.1. Introduction

As the world is getting more and more integrated, all economies are open and most of them are small. Macroeconomic research on small open economy has caught more and more attention lately. The so-called New Open Economy Macroeconomics literature has grown rapidly in recent years. The discussion of New Keynesian literature on optimal monetary policy in an open economy focuses on whether exchange rate stability should be part of a central bank's strategy, for example, see McCallum (2007). Though the role of nominal variables like interest rate, money supply and exchange rate are emphasized in recent New Keynesian literature to have impact on real economic variables like output and growth in short-run due to the stickiness of price and wage. However, recent DSGE macroeconomic models often ignore aggregate quantity of money as an instrument of monetary policy. They tend to rely on (nominal) interest rate, such as the federal fed fund rate in modeling the thrust of monetary policy. In case of a small open economy, exchange rate is considered. The entire intervention of the central bank is expressed via some interest rate rule, like the Taylor rule (1993). The support for this view lies in the empirical evidence in 1980s by Bernanke and Blinder (1988) that demand for money is more unstable than the demand for credit, therefore, monetary policy would have better success in stabilizing output if it stabilized interest rate rather than money supply. Moreover, Sims (1980) argues that money loses its predictive power on output when interest rate is included in the regression.

In response to the lacking attention on money quantities in the New Keynesian literature due to some empirical evidence that the demand for money is unstable and money has low power in explaining output and other macroeconomic variables, other economists have brought up the issue of money measurement. Up until the 1980s, economists throughout the world measured different levels of monetary aggregation, such as M0/MB (monetary base), M1 (narrow money), M2 (broad money), and M3 and M4 (financial liquidity), by simply adding up the quantities of component assets. This simple sum assigns the same weight to every monetary asset and implicitly assumes all monetary assets are perfect substitutes. As the financial system becomes more and more sophisticated with different type of monetary assets, which produce different interest rates, possess different liquidity and are at different levels of risk, this simple sum measure is clearly not a proper way to aggregate money. Chrystal and MacDonald (1994) used the term "Barnett critique" to refer to the misleading and potential distortion of economic inferences if the simple-sum measure of money keeps being used.

Barnett (1978, 1980) derived the formula to measure the user cost price of monetary services and proposed a method to aggregate money based on solid microeconomic theory foundation and index number theory. Accordingly, to properly aggregate components in monetary service aggregation, we need both their quantities and prices. Monetary asset services are not analogous to perishable consumer good services, such as apples, but to capital goods or durable goods, such as houses or automobiles. Hence, their prices are measured in term of user cost prices. The Divisia measure of money that Barnett proposed is a weighted index whose weights are based on the expenditure shares of component assets. Since the theory of monetary aggregation became available, central banks such as the Federal Reserve (FED) in the US, the Bank of England (BOE) in the UK, the European Central Bank (ECB), the Bank of Japan (BoJ), the National Bank of Poland, and the Bank of Israel, among others, have, at various times and in diverse ways, produced and maintained Divisia indexes for monetary aggregation. Simply replacing the traditional simple-sum measure of money by Divisia measures, Belongia (1996), Barnett and Chauvet (2010), Belongia and Ireland (2015), among many other researchers have shown that money still shares a strong relationship with aggregate economic activity, and the demand for money function still exhibits stability.

Though the New Open Macroeconomics has been getting more attention during the past two decades, the discussion on monetary policy in an open economy centers around the matter of stabilizing the exchange rate and money supply is completely out of sight. In this paper, we would like to bring attention to the measurement of money in a context of small open economy with home-bias in consumption. We used the recent developed microfounded, dynamic and stochastic New Keynesian model to examine the responses of different measures of money supply, including the official simple-sum measure, monetary base and Divisia measure to various macroeconomic shocks. To do so, we extended the New Keynesian model for a small open economy from Faia and Monacelli (2008) in a similar manner to Belongia and Ireland (2014) by introducing private financial institutions, who create deposit as imperfect substitute for government-issued currency. This framework allows us to construct and compare the behavior of different measures of monetary aggregates. It also allows us to look at the impact of openness to the stability of macroeconomic variables. In such an environment, we showed that Divisia measure is strictly better than simple-sum measure and monetary base in tracking the movement of money. Our findings, consistent with many others in the literature, reemphasize the Barnett critique that simple-sum is misleading and using it can distort inference about the economy. We advocate the use of Divisia index in measuring money supply. In the future work, we plan to further examine the role of monetary aggregate as a potential intermediate monetary policy target and compare it with interest rate target and exchange rate target in the context of a small open economy.

The rest of the paper is organized as the following. Section 2 provides details of the theoretical model. Section 3 presents the parameters calibration for numerical solution of the model and discusses the results. Section 4 concludes. The full set of the equilibrium system can be found in the appendix.

3.2. The Model

The world consists of two countries: home country (Home) and the rest of the world (Foreign). Home country is of sized n, and relatively small compared to Foreign whose size is 1-n. Final goods are traded among two countries. The international financial market is accessible to residents in both economies. In addition to the interaction among traditional sectors in a small open economy, households, firms, and foreign sector, we introduce banking sector who receive deposits from and make loans to households. There also exists a central bank who conducts monetary policy. We character behaviors of each sector as below.

3.2.1. Household Sector

Households consume a composite final goods, which is a combination of domestic goods and imported goods. These goods can be substituted with the elasticity of substitution η. The first optimization problem of households is to allocate their expenditure among domestic and foreign goods, taking the prices as given.

3.2.1.1. First (Intra-temporal) Optimization Problem

The composite bundle for consumption (CES) in home country

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \qquad (3.1)$$

where 0≤ γ≤1 is the weight of domestic goods in the consumption bundle, γ =(1-n)α with 1-n is the relative size of the Foreign and α is the openness of the Home economy. The elasticity of substitution between domestic good and imported goods is η>0. In a similar manner, the composite consumption bundle in Foreign is

$$C_t^* = \left[(1-\gamma^*)^{\frac{1}{\eta}} C^{*\frac{\eta-1}{\eta}}_{H,t} + \gamma^{*\frac{1}{\eta}} C^{*\frac{\eta-1}{\eta}}_{F,t} \right]^{\frac{\eta}{\eta-1}}. \tag{3.2}$$

Home bias in consumption requires α <1. See Faia and Monacelli (2008) for more details. Also observe that

$$1-\gamma = 1-(1-n)\alpha > \gamma^* = n\alpha^*. \tag{3.3}$$

In final consumption stage, each consumption bundle C_H, C_F itself is composed of imperfect substitutable varieties with elasticity of substitution ε, where

$$C_{H,t} = \left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \left(\int_0^n C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3.4}$$

$$C_{F,t} = \left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \left(\int_n^1 C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{3.5}$$

The optimal allocation of expenditure between domestic and foreign goods depends on the relative price of domestic and foreign goods as follows

$$C_{H,t} = (1-\gamma)\left(\frac{P_{H,t}}{P_{H,t}}\right)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \gamma\left(\frac{P_{F,t}}{P_{F,t}}\right)^{-\eta} C_t. \tag{3.6}$$

The optimal allocation within each variety of goods depends on the relative price of goods for each variety,

$$C_{H,t}(j) = \frac{1}{n}\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \quad \text{and} \quad C_{F,t}(j) = \frac{1}{1-n}\left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}. \tag{3.7}$$

The general price level in the home country can be expressed as an index of domestic produced and imported prices (Consumer Price Index (CPI)),

$$P_t = \left[(1-\gamma) P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} . \tag{3.8}$$

3.2.1.2. Second (Inter-temporal) Maximization Problem

In the second optimization problem, households choose to allocate their resources to maximize the lifetime expected discounted utility,

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t h_t U\left(C_t, N_t, \frac{M_t}{P_t} \right), \tag{3.9}$$

where $0<\beta<1$ is the discount factor and h_t is a preference shock, which is assumed to follow an AR(1) process

$$\ln h_t = \rho_h \ln h_{t-1} + e_{h,t}, \tag{3.10}$$

with $0 \le \rho_h \le 1$ and $e_{h,t}$ ~ i.i.d $N(0,\sigma_h^2)$. Utility function is assumed to be increasing in (log of) consumption and decreasing in labor,

$$U_t = \ln C_t - \psi_N \frac{N_t^{1+\xi}}{1+\xi} + \psi_M \ln\left(\frac{M_t}{P_t} \right) . \tag{3.11}$$

with the inverse elasticity of labor $\xi>0$, and coefficients $\psi_M,\psi_N>0$. We also have money (real balance) entered in the utility function in log form. In this model, with the present of private financial institutions like commercial banks, households allocate their monetary assets between cash Ca_t and deposit D_t in commercial banks. Money in the utility function depends on a theoretical monetary aggregate relation,

$$M_t = \left[\nu^{\frac{1}{\omega}} Ca_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} D_t^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \quad (3.12)$$

where $0 \le \nu \le 1$ is the weight of cash among monetary assets, and ω is the elasticity of substitution between cash and deposit. The above true monetary aggregate can not be observed in real data. For the purpose of comparing the behavior of different money measures in the model, we assume that we know the true monetary aggregation's functional form.

Households enter each period with a portfolio of maturing bonds B_{t-1} and monetary assets A_{t-1}. In the first sub-period, they receive lump-sum transfer from the central bank and allocate their monetary assets between cash and deposit. They can also take loan from commercial banks to finance these transactions,

$$\frac{B_t}{1+r_t} + Ca_t + D_t = A_{t-1} + B_{t-1} + T_t + L_t \,. \quad (3.13)$$

In the second sub-period, households get wage from working. They also collect interest from their deposit r_t^D and pay interest on their loan r_t^L. At the end of each period, households receive nominal dividends from holding shares of intermediate firms. After all the payments are made, households carry A_t into the next period,

$$P_t C_t + A_t + (1+r_t^L) L_t = Ca_t + W_t N_t + F_t + (1+r_t^D) D_t \,. \quad (3.14)$$

Households choose the optimal sequences {C_t, N_t, B_t, A_t, M_t, Ca_t, D_t, L_t} for all *t* greater than or equal to zero, to maximize the expected discounted utility subject to constraints (3.12), (3.13), and (3.14). Let Λ_t^1, Λ^2_t, and Λ_t^3 be the Lagrangian multiplier on each constraint, we solved for the FOCs and obtained the Euler equation

$$\Lambda_t^3 = \beta \mathrm{E}_t \Lambda_{t+1}^3 (1+r_{t+1}) \frac{P_t}{P_{t+1}}, \tag{3.15}$$

where

$$\Lambda_t^3 = \frac{h_t}{C_t}. \tag{3.16}$$

3.2.2. Foreign Sector

Consumption goods is being traded among Home and Foreign. Export depends on the demand for domestic goods from the rest of the world. Assume that the structure of the Foreign economy is similar to the Home economy (except the size), hence, we can derive the Foreign demand for Home products as

$$C_{H,t}^*(j) = \frac{1}{n}\left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\varepsilon} C_{H,t}^*$$

$$\Rightarrow C_{H,t}^*(j) = \frac{1}{n}\left(\frac{P_{H,t}^*(j)}{P_{H,t}^*}\right)^{-\varepsilon} \gamma^* \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^*. \tag{3.17}$$

The general price level in Foreign (CPI) becomes

$$P_t^* = \left[(1-\gamma^*)(P_{F,t}^*)^{1-\eta} + \gamma^*(P_{H,t}^*)^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{3.18}$$

Define term of trade as the relative price of imported goods

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}}. \tag{3.19}$$

Compute the CPI-PPI ratios for Home and Foreign, we see that they depend on the term of trade

$$\frac{P_t}{P_{H,t}} = \left[(1-\gamma) + \gamma S_t^{1-\eta} \right]^{\frac{1}{1-\eta}} \equiv q(S_t) \,, \qquad (3.20)$$

$$\frac{P_t^*}{P_{H,t}^*} = \left[(1-\gamma^*) + \gamma^* S_t^{1-\eta} \right]^{\frac{1}{1-\eta}} \equiv q^*(S_t) \,. \qquad (3.21)$$

The *law of one price* tells us that goods must be sold at the same price everywhere, after being converted into the same unit of currency either in Home or Foreign,

$$P_{H,t}(j) = NE_t P_{H,t}^*(j) \quad \text{and} \quad P_{F,t}(j) = NE_t P_{F,t}^*(j) \quad \text{for} \quad \text{all} \quad j \in [0,1], \qquad (3.22)$$

where NE_t is the nominal exchange rate measured by the number of units of home currency per one unit of foreign currency. Real exchange rate is the relative price of one unit of domestic goods in term of imported goods,

$$Q_t \equiv NE_t \frac{P_t^*}{P_t} \,, \qquad (3.23)$$

where foreign price level P_t^* is exogenous. We see that term of trade and real exchange rate are linked through

$$Q_t = S_t \frac{P_t^*}{P_{F,t}^*} \left(\frac{P_t}{P_{H,t}} \right)^{-1} . \qquad (3.24)$$

We see that real exchange rate depends on the term of trade and other parameters that characterize the open economy,

$$Q_t = S_t \frac{q^*(S_t)}{q(S_t)} = S_t \frac{\left[(1-\gamma^*) + \gamma^* S_t^{1-\eta} \right]^{\frac{1}{1-\eta}}}{\left[(1-\gamma) + \gamma S_t^{1-\eta} \right]^{\frac{1}{1-\eta}}} \,. \qquad (3.25)$$

Notice that

$$\frac{\partial q(S_t)}{\partial S_t} > 0, \quad \frac{\partial q^*(S_t)}{\partial S_t} < 0, \quad \text{and} \quad \frac{\partial Q_t}{\partial S_t} > 0.$$

We are looking at a small economy, so *n* approaches 0, which implies $P_{F,t}^* = P_t^*$ and $q^*(S_t)=1$. Therefore,

$$Q_t = \frac{S_t}{q(S_t)}, \tag{3.26}$$

and

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} = NE_t \frac{P_t^*}{P_{H,t}}. \tag{3.27}$$

Risk sharing/ Interest rate parity: From the inter-temporal maximization problem of households, recall the optimal condition for bonds' holdings in Home is the Euler equation in (3.15). The analogue optimal condition for bond holding in Foreign is

$$\Lambda_t^{3*} = \beta \mathrm{E}_t \Lambda_{t+1}^{3*} (1 + r_{t+1}^*) \frac{P_t^*}{P_{t+1}^*}. \tag{3.28}$$

Perfect capital mobility means both domestic residents and foreigners can invest in the bond market, hence their expected return from this bond must be the same after being converted into domestic currency,

$$\beta \mathrm{E}_t \left(\frac{\Lambda_{t+1}^3}{\Lambda_t^3} \frac{P_t}{P_{t+1}} \right) = \beta \mathrm{E}_t \left(\frac{\Lambda_{t+1}^{3*}}{\Lambda_t^{3*}} \frac{P_t^*}{P_{t+1}^*} \frac{NE_t}{NE_{t+1}} \right). \tag{3.29}$$

Interest rate parity implies no arbitrage opportunity on bond market,

$$1 + r_{t+1} = (1 + r_{t+1}^*) \left(\frac{NE_t}{NE_{t+1}} \right). \tag{3.30}$$

Rearrange Equation (3.29), we get

$$NE_t \frac{P_t^*}{P_t} = \frac{\Lambda_t^{3*}}{\Lambda_t^3} \mathrm{E}_t \left(\frac{\Lambda_{t+1}^3}{\Lambda_{t+1}^{3*}} \frac{P_{t+1}^*}{P_{t+1}} NE_{t+1} \right) . \tag{3.31}$$

From households' FOCs, we know that $\Lambda_t^3 = h_t/C_t$, we further assume $\Lambda_t^{3*} = 1/C_t^*$ and other initial conditions for the two economies so that we can iterate the expectation in (3.31) and rewrite the interest parity as

$$Q_t = \kappa \frac{h_t C_t}{C_t^*} , \tag{3.32}$$

where Foreign demand is exogenously given. In our model, it follows that there is an exogenous process,

$$\ln C_t^* = \rho_c^* \ln C_{t-1}^* + e_{c,t}^* , \tag{3.33}$$

with $0<\rho_c^* <1$ and $e_{c,t}^* \sim$ i.i.d $N(0,\sigma_c^{*2})$.

3.2.3. Production Sector and Price Setting

Monopolistic intermediate production firms use labor to produce homogeneous goods under a constant return to scale technology. These outputs are used to assemble final goods for domestic consumption and export to foreign consumers. Each monopolistic firm *j* uses labor to produce homogeneous output with linear technology to meet the total demand for their product from the whole world,

$$Y_t(j) = Z_t N_t(j) \tag{3.34}$$

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} Y_t . \tag{3.35}$$

The level of technology Z_t follows a random walk process

$$\ln Z_t = \ln z + \ln Z_{t-1} + e_{z,t} \,. \tag{3.36}$$

with $e_{z,t}$ ~ i.i.d $N(0,\sigma_z^2)$, and z be the growth rate of productivity in steady state. Intermediate firms choose labor input to minimize their production cost,

$$-W_t N_t(j) + \varphi_t(j)\left[Z_t N_t(j) - \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t \right]. \tag{3.37}$$

FOC for cost minimization problem are

$$\varphi_t(j) = \frac{W_t}{Z_t} \quad \text{for all} \quad j \in [0,1], \tag{3.38}$$

where the Lagrangian multiplier can be interpreted as nominal marginal cost. Hence, real marginal cost is expressed as

$$MC_t(j) = \frac{W_t}{Z_t P_{H,t}} \quad \text{for all} \quad j \in [0,1]. \tag{3.39}$$

Monopolistic intermediate firms have the power to set their price to maximize profit. In a sticky price environment, we set an adjustment cost for price setting according to Rotemberg (1982). Accordingly, all production firms face the same quadratic cost of adjusting their nominal prices. This cost is measured in term of unit of domestic final goods

$$\frac{\phi}{2}\left(\frac{P_{H,t}(j)}{(1+\pi_H)P_{H,t-1}(j)} - 1\right)^2 Y_t \,,$$

and depends on the steady state of gross Home producer's inflation $1+\pi_H$. If $\phi=0$, prices are flexible. Each firm chooses to set its price $P_{H,t}(j)$ to maximize its expected discounted profit

$$\max E_t \sum_{i=0}^{\infty} \beta^j \Lambda_{t+i}^3 F_{t+i} \ ,$$

where nominal profit is

$$F_t(j) = P_{H,t}(j)Y_t(j) - W_t N_t(j) - \frac{\phi}{2}\left(\frac{P_{H,t}(j)}{(1+\pi_H)P_{H,t-1}(j)} - 1 \right)^2 P_{H,t} Y_t \, , \tag{3.40}$$

subject to

$$Y_t(j) \leq \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} Y_t \quad \text{and} \quad Y_t(j) = Z_t N_t(j) \, . \tag{3.41}$$

3.2.4. Financial Sector and Central Bank

Private financial firms like commercial banks accept households' deposit and make loan. They follow the required reserve set by the central bank. During each period, bank receives deposit D_t, and pays interest rate r_t^D on that deposit. They also give out loan L_t to households and charge interest rate r_t^L on the loans. The relation of loan and deposit depends on the actual reserve ratio $0 \leq \tau \leq 1$,

$$L_t = (1-\tau) D_t \, . \tag{3.42}$$

Commercial banks' operating cost depends on their real revenue linearly $x_t = \upsilon D_t / P_t$. Commercial banks' nominal profits during period t are

$$F_t^b = r_t^L L_t - r_t^D D_t - P_t \upsilon \frac{D_t}{P_t} \, . \tag{3.43}$$

Competition among commercial banks drive their profits to zero. Profit maximization condition (w.r.t D_t) requires that

$$r_t^D = r_t^L(1-\tau) - \upsilon . \tag{3.44}$$

The central bank injects/withdraws money in the economy via lumpsum transfer to households. Central bank's budget constraint

$$T_t = A_t - A_{t-1} . \tag{3.45}$$

Suppose the ultimate goal of the central bank is to stabilize price level, and it conducts monetary policy via targeting nominal interest rate. In this model, we follow the literature to employ a simple version of Taylor rule for monetary policy

$$r_t = (1-\rho_r)r + \rho_r r_{t-1} + (1-\rho_r)\rho_\pi(\pi_{H,t} - \pi_H) + e_{r,t} , \tag{3.46}$$

with $e_{r,t}$ ~ i.i.d $N(0,\sigma_r^2)$. $\rho_r > 0$ and ρ_π large enough to avoid indeterminacy. The Home producer's inflation target π_H is chosen by the central bank (exogenous).

3.2.5. Market Clearing Condition

Beside the assumption for a small economy, n approaches 0, we further assume symmetric openness among Home and Foreign $\alpha=\alpha^*$, so that $\gamma=\alpha$. Notice that the level of home-bias in consumption 1- γ is of inverse degree with the level of openness α.

Symmetric equilibrium implies that all intermediate firms end up setting the same price, produce the same level of output using the same amount of labor in equilibrium, $P_{H,t}(j)= P_{H,t}$, $N_t(j)=N_t$, and $Y_t(j)=Y_t$ for all j and t. Define $1+ \pi_{H,t}= (P_{H,t} / P_{H,t-1})$, and use it to rewrite the nominal profit of intermediate firms in equilibrium

$$F_t = P_{H,t}Y_t - W_t N_t - \frac{\phi}{2}\left(\frac{1+\pi_{H,t}}{1+\pi_H} - 1\right)^2 P_{H,t}Y_t , \tag{3.47}$$

and derive the FOC for optimal price setting

$$(1-\varepsilon)\Lambda_t^3 Y_t + \varepsilon\Lambda_t^3 \frac{W_t Y_t}{Z_t P_{H,t}} - \phi\left(\frac{1+\pi_{H,t}}{1+\pi_H} - 1\right)\left(\frac{1+\pi_{H,t}}{1+\pi_H}\right) Y_t$$

$$+\beta\phi \mathrm{E}_t \Lambda_{t+1}^3 \left(\frac{1+\pi_{H,t+1}}{1+\pi_H} - 1\right)\left(\frac{1+\pi_{H,t+1}}{1+\pi_H}\right) Y_{t+1}. \tag{3.48}$$

Market clearing condition for bonds requires B_t=0 for all t. Since the economy is open with trade, aggregate resource constraint meets the domestic and foreign demand and covers the cost from production sector and private financial sector,

$$Y_t = nC_{H,t} + (1-n)C_{H,t}^* + \frac{\phi}{2}\left(\frac{1+\pi_{H,t}}{1+\pi_H} - 1\right)^2 Y_t + \upsilon \frac{D_t}{P_{H,t}}. \tag{3.49}$$

The above condition can be written in term of S_t and $q(S_t)$ as

$$Y_t = (1-\alpha) q(S_t)^\eta C_t + \alpha (S_t)^\eta C_t^* + \frac{\phi}{2}\left(\frac{1+\pi_{H,t}}{1+\pi_H} - 1\right)^2 Y_t + \upsilon \frac{D_t}{P_{H,t}}. \tag{3.50}$$

3.2.6. Monetary Aggregation

In this economy, we have multiple monetary assets, cash and deposit, which produce different interest rates, possess different levels of liquidity and are at different levels of risk. Therefore, we have multiple ways to measure money supply. The traditional way is the official simple-sum measure that is often reported by central banks all over the world.

$$SM_t = Ca_t + D_t \tag{3.51}$$

Growth rate of simple-sum measure

$$1 + g_t^S = \frac{SM_t}{SM_{t-1}}. \tag{3.52}$$

The second measure is monetary base, which is also found in the central banks' reports. In our model, it is easy to see that monetary base is equivalent to monetary asset A_t by market clearing condition and bank's balance sheet,

$$MB_t = A_t = Ca_t + \tau D_t \,. \tag{3.53}$$

Growth rate of monetary base is

$$1 + g_t^B = \frac{MB_t}{MB_{t-1}} \,. \tag{3.54}$$

Another way to aggregate money, the Divisia measure, requires data not only on quantities but also on interest rates of each monetary asset. In this model, we have enough information to construct the Divisia measure. First, we compute the user cost price of currency and deposit

$$u_t^{Ca} = \frac{r_t - 0}{1 + r_t} \tag{3.55}$$

$$u_t^D = \frac{r_t - r_t^D}{1 + r_t} \,. \tag{3.56}$$

Then we compute the expenditure shares of currency and deposit,

$$s_t^{Ca} = \frac{u_t^{Ca} Ca_t}{u_t^{Ca} Ca_t + u_t^D D_t} \,, \tag{3.57}$$

$$s_t^D = \frac{u_t^D D_t}{u_t^{Ca} Ca_t + u_t^D D_t} \,. \tag{3.58}$$

The growth rate of Divisia quantity index is the weighted average growth rate of all components

$$1 + g_t^Q = \left(\frac{Ca_t}{Ca_{t-1}} \right)^{\frac{s_t^{Ca} + s_{t-1}^{Ca}}{2}} \left(\frac{D_t}{D_{t-1}} \right)^{\frac{s_t^D + s_{t-1}^D}{2}} \,. \tag{3.59}$$

Growth rate of true monetary aggregate is

$$1+g_t^M = \frac{M_t}{M_{t-1}}. \tag{3.60}$$

3.3. Calibration and Results

3.3.1. Calibration

In order to solve the model numerically, we first choose reasonable values for parameters. The households' discount factor β= 0.98525 indicates one period in the model as a month in real time, and an annual interest rate of 2% in equilibrium. The inverse elasticity of labor, ξ=1 implies that labor enters utility function in a quadratic form, and the coefficient ψ_N=3.5 orients the steady state value of labor to be in the range from 0.3 to 0.5, which can be understood as 8 hours to 12 hours per day. The elasticity of substitution among different variety of domestic goods reflexes the power of firms. Notice that the steady state of real marginal cost is equal to the inverse markup, MC=(ε-1)/ε. We choose ε=10, which implies a steady state markup of 11%. The degree of price stickiness ϕ is calibrated to match the slope of the New Keynesian Phillips Curve in another sticky price manner using Calvo (1983) approach. We set ϕ =105 which is equivalent to a probability of not resetting prices in a given period θ=0.75. The literature is quite varied in the value of η, the elasticity of substitution between domestic and foreign goods. In a special case, η=1, domestic and foreign goods are perfect substitutes. Other than that, most of the papers in the literature adopts a value of η above unity. As a benchmark case, we choose η =2, but we also do parameter variation for sensitivity analysis. In our model, the size of the small economy is assumed to be very small compared to the rest of the world, n ~ 0, hence the share of imported goods on composite consumption bundle becomes γ=(1-n)α ~ α. If α=0, the economy is closed. The level of home bias in consumption 1- γ has an inverse relation with the level of openness α and requires that α<1. We choose the benchmark value of α =0.4 and we vary it from 0 to 1 to see the impact of openness in our model. For the elasticity of substitution between cash and deposit, ω, we choose the benchmark value of 2, and set the weight of cash on monetary assets ν=0.625 to match the ratio of currency in circulation (Ca/SM) about 10%. Other values of ω, 1.16 and 3 are considered to resemble other scenarios in the economy with 25% and 3% currency in circulation. The financial sector cost υ=0.005 is measured in unit of final goods. It implies that banking activity accounts for about of total output in the steady state. The reserve ratio is set at τ=0.02 based on the average reserve ratio in a small economy such as Singapore. Other parameters, κ=1, ψ_M=0.01 are set for simplicity.

Table 3.1. Calibrated Values of Selected Parameters

Case	**omega**	**eta**	**alpha**	**ca/sm**	**N**
1	2	2	0.4	0.11	0.4305
2	1.16	2	0.4	0.27	0.1430
3	3	5	0.4	0.03	0.3438
4	2	5	0.1	0.11	0.4022
5	3	2	0.8	0.03	0.4089

Column 5 and 6 show the steady state values of the ratio of currency in circulation and labor w.r.t to each set of parameters.

We assume the central bank's ultimate goal is to stabilize price level, so exogenous inflation target in Home country is set at π_H=0; and similarly, we assume the central bank in Foreign succeeds in control inflation, so that exogenous Foreign inflation π^*=0. The calibrated value z=1.005 implies a growth of productivity (technology) of 6% per year in the model. The relative productivity of Home versus Foreign $z_t^f=Z_{t-1}/Z_{t-1}^*$ is assumed to be z^f=1, implying the same level of productivity all over the world in steady state.

For the coefficients of monetary policy rule (a simple version of Taylor rule), we followed Belongia and Ireland (2014) to set the coefficient on reaction to interest rate ρ_r=0.75. We need to set the reaction to inflation ρ_π large enough to avoid indeterminacy, i.e, to meet Blanchard and Kahn (1980) condition for unique equilibrium solution of the rational expected model. Given

ρ_r=0.75, we set ρ_π =1.5 to meet this condition. Other coefficients, ρ_h==0.9, ρ_c^f=0.95, ρ_z^f =0.95, ρ_π^f=0.95. All standard deviations are set at 0.01, which means 1% shock.

3.3.2. Results and Discussion

As the benchmark case, we choose the elasticity of substitution between cash and deposit, ω=2, the elasticity of substitution between domestic and foreign goods, η=2, and the openness, α=0.4, which implies a weight of 0.6 on domestic goods in consumption. With this setting of parameters, we are looking at a small economy with 10% currency in circulation in steady state. We shock this economy with various shocks including a preference shock, a monetary policy shock, a home-productivity shock, a shock to foreign demand, a shock to foreign productivity, and a shock to foreign inflation. The impulse responses of growth rate of different money measures, including the true monetary aggregate, Divisia quantity aggregate, monetary base and simple-sum measure are reported in Figure 3.1 and Figure 3.2. Notice that shock in foreign inflation is entirely absorbed by the exchange rate and all other macroeconomic variables do not response to this shock. This is due to the assumption of perfect capital mobility. The small economy chooses to let capital flow in and out freely, so that the domestic interest rate will be determined by the interest rate in the world. To ensure such an environment, the economy must float its exchange rate. As a consequence, the shock to foreign inflation does not impact other macroeconomic variables, except the exchange rate. For the left 5 stochastic shocks, while Divisa quantity aggregate tracks almost perfectly the movement of the true monetary aggregate, monetary base fails in the home-productivity shock and over-reacts with respect to the monetary policy shock. More seriously, simple-sum behaves differently with that of the true money in 3 out of 5 shocks, and for the other 2 shocks, its responses are not quite close to the movement of the true money.

In order to check for the robustness of our results, we vary parameters ω, η and α. Figure 3.3 and Figure 3.4 show the responses under the variation of parameter ω. A higher value of the elasticity of substitution between cash and deposit, a smaller ω, implies an economy with less cash in circulation and vice versa. See Table 3.1 for a numerical image. The behavior of Divisia quantity aggregate and monetary base does not change when we vary ω, however, simple-sum does. With a smaller value of ω, simple-sum is less likely to misbehave, but with bigger ω, the misbehavior becomes more and more serious. The intuition behind this result is that as cash and deposit becomes far away from perfect substitutes, people are willing to hold less cash. The simple-sum measure which implicitly assumes perfect substitution among components and assigns the same weight to different monetary assets gets more and more distorted.

In a similar manner, we vary the elasticity of substitution between domestic and foreign goods in Home's consumption, see Figure 3.5 and Figure 3.6. We later vary the openness (or the inverse degree of home-bias in consumption), see Figure 3.7 and Figure 3.8, for sensitivity analysis. Our conclusion is the same, in every case, Divisia index can track the movement of the true money very closely, monetary base is good in some cases but fails in some others, and simple-sum often behaves very differently compared with the theoretical monetary aggregate.

We further look at the volatility of macroeconomic variables under different values of α from 0 to 1. As α goes from 0 to 1, the economy becomes more open (it is a closed economy with α=0) and consumption becomes less home-biased. We find that nominal interest rate and domestic inflation fluctuate more as the economy is more open while the growth rate of exchange rate and true money become more stable, see Figure 3.9. Once again, Divisa index follows the correct trend of the theoretical money aggregate, while simple-sum behaves totally different. This pattern does not change under different financial structure with different value of ω.

3.4. Conclusion

Since Divisia monetary aggregate and the monetary aggregation theory became available from the 1980s, hundreds of theoretical and empirical work have been repeatedly showing that Divisa measure is strictly preferable to its official simple-sum counterpart, however, the availability of the simple-sum aggregates has continued. This paper revisits the issue of money measurement in a context of small open economy using the recent highly microfounded DSGE model. Our results are consistent with others such as Barnett and Chauvet (2010), Keating et al (2019). This paper is among the first work of Divisia measure in a small open economy. It introduced banking sector and Divisia measure of money in a New Keynesian framework for a small open economy. It is also the first paper to analyze the effect of openness (home bias in consumption) to the volatility of macroeconomic variables in a such an economy.

In the future work, we plan to continue our analysis on Divisia monetary aggregate in a small open economy. We want to check if money is more informative than interest rate and exchange rate in explaining/predicting output and inflation to see whether the relation of quantity of money and macroeconomic variables is still stable. Furthermore, to find out the optimal monetary policy rule, we plan to evaluate the representative household welfare under different targets of monetary policy, i.e, an interest rate rule, a rule for fixed growth rate for monetary base, fixed growth rate for simple sum, fixed growth rate for Divisia monetary aggregate, and fixed exchange rate.

A.3. Appendix 3

A.3.1. Equilibrium System

We have 31 endogenous variables in our original system: A_t, C_t, Ca_t, D_t, L_t, N_t, NE_t, M_t, $P_{H,t}$, P_t, Q_t, r_t, r_t^D , r_t^L, S_t, $q(S_t)$, W_t, Y_t, Λ_t^1, Λ_t^2, Λ_t^3, $\pi_{H,t}$, SM_t, g_t^S, g_t^B, g_t^M, g_t^Q, s_t^{Ca}, s_t^D, u_t^{Ca}, u_t^D, and 4 exogenous variables: h_t, Z_t, C_t^*, P_t^*.

Transforming this system into a stationary system with real effective variables, we have 30 endogenous variables: $c_t=C_t/Z_{t-1}$, $y_t=Y_t/Z_{t-1}$, $a_t=(C_t/P_t)/Z_{t-1}$, $ca_t=(Ca_t/P_t)/Z_{t-1}$, $d_t=(D_t/P_t)/Z_{t-1}$,

$l_t=(L_t/P_t)/Z_{t-1}$, $m_t=(M_t/P_t)/Z_{t-1}$, $w_t=(W_t/P_t)/Z_{t-1}$, $\lambda_t^1= Z_{t-1}\Lambda_t^1$, $\lambda_t^2= Z_{t-1}\Lambda_t^2$, $\lambda_t^3= Z_{t-1}\Lambda_t^3$,

$sm_t=(SM_t/P_t)/Z_{t-1}$, Q_t, N_t, r_t, r_t^D, r_t^L, S_t, $q(S_t)$, π_t, $\pi_{H,t}$, g_t^S, g_t^B, g_t^M, g_t^{NE}, g_t^Q, s_t^{Ca}, s_t^D, u_t^{Ca}, u_t^D, and 5 exogenous variables: h_t, $Z_t= C_t/Z_{t-1}$, $c_t^f=C_t^*/Z_{t-1}^*$, π_t^f, and $z_t^f= Z_{t-1}/Z_{t-1}^*$.

The full set of stationary equilibrium system is as follows.

(1) Theoretical monetary aggregation

$$m_t = \left[\nu^{\frac{1}{\omega}} ca_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}} .$$

(2) Households first sub-period budget constraint

$$ca_t + d_t = a_t + l_t .$$

(3) FOC for households

$$\frac{h_t}{c_t} = \lambda_t^3 .$$

(4) Labor supply

$$\lambda_t^3 w_t = \psi_N h_t N_t^{\xi} .$$

(5) Euler equation

$$\lambda_t^3 = \beta \mathrm{E}_t \left(\frac{\lambda_{t+1}^3 (1 + r_{t+1})}{1 + \pi_{H,t+1}} \right) \frac{q(S_t)}{q(S_{t+1}) z_t}.$$

(6)

$$r_t^L = r_t.$$

(7) Money demand

$$\lambda_t^1 m_t = \psi_M h_t.$$

(8)

$$\lambda_t^2 - \lambda_t^3 = \lambda_t^1 \left[\nu^{\frac{1}{\omega}} ca_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} \right]^{\frac{1}{\omega-1}} \nu^{\frac{1}{\omega}} ca_t^{-\frac{1}{\omega}}.$$

(9)

$$\lambda_t^2 - (1 + r_t^D) \lambda_t^3 = \lambda_t^1 \left[\nu^{\frac{1}{\omega}} ca_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} \right]^{\frac{1}{\omega-1}} (1-\nu)^{\frac{1}{\omega}} d_t^{-\frac{1}{\omega}}.$$

(11)

$$\frac{S_t}{S_{t-1}} = \left(\frac{1 + \pi_t^*}{1 + \pi_{H,t}} \right) (1 + g_t^{NE}).$$

(12) Term of trade

$$\frac{q(S_t)}{q(S_{t-1})} = \frac{1 + \pi_t}{1 + \pi_{H,t}}.$$

(13) CPI-PPI ratio

$$q(S_t)=\left[\ (1-\gamma)+\gamma S_t^{1-\eta}\ \right]^{\frac{1}{1-\eta}}.$$

(14) Real exchange rate

$$Q_t=\frac{S_t}{q(S_t)}.$$

(15) Production function

$$y_t=z_t N_t.$$

(16) Interest rate parity

$$\frac{S_t}{q(S_t)}=\kappa h_t\frac{c_t}{c_t^f}z_t^f\ .$$

(17) FOC for optimal price setting

$$(1-\varepsilon)+\varepsilon\frac{w_t q(S_t)}{z_t}-\phi\left(\frac{1+\pi_{H,t}}{1+\pi_H}-1\right)\left(\frac{1+\pi_{H,t}}{1+\pi_H}\right)$$

$$+\beta\phi \mathrm{E}_t\frac{\lambda_{t+1}^3 y_{t+1}}{\lambda_t^3 y_t}\left(\frac{1+\pi_{H,t+1}}{1+\pi_H}-1\right)\left(\frac{1+\pi_{H,t+1}}{1+\pi_H}\right)=0.$$

(18) Loans

$$(1-\tau)d_t=l_t.$$

(19) FOC for profit maximization of commercial banks

$$r_t^D=r_t^L(1-\tau)-\upsilon.$$

(20) Aggregate resource constraint

$$y_t=(1-\alpha)q(S_t)^\eta c_t+\alpha(S_t)^\eta\frac{c_t^f}{z_t^f}+\frac{\phi}{2}\left(\frac{1+\pi_{H,t}}{1+\pi_H}-1\right)^2 y_t+\upsilon d_t q(S_t).$$

(21) Simple-sum measure

$$ca_t + d_t = sm_t .$$

(22) Growth rate of simple-sum measure

$$1 + g_t^S = sm_t / sm_{t-1} .$$

Monetary base (equivalent to a by market clearing condition)

$$mb_t = a_t = ca_t + \tau d_t .$$

(23) Growth rate of monetary base

$$1 + g_t^B = mb_t / mb_{t-1} .$$

(24) User cost price of currency

$$u_t^{Ca} = \frac{r_t - 0}{1 + r_t} .$$

(25) User cost price of deposit

$$u_t^D = \frac{r_t - r_t^D}{1 + r_t} .$$

(26) Expenditure share of currency

$$s_t^{Ca} = \frac{u_t^{Ca} ca_t}{u_t^{Ca} ca_t + u_t^D d_t} .$$

(27) Expenditure share of deposit

$$s_t^D = \frac{u_t^D d_t}{u_t^{Ca} ca_t + u_t^D d_t} .$$

(28) Growth rate of Divisia quantity index

$$1+g_t^Q=\left(\frac{ca_t}{ca_{t-1}}\right)^{\frac{s_t^{Ca}+s_{t-1}^{Ca}}{2}}\left(\frac{d_t}{d_{t-1}}\right)^{\frac{s_t^{D}+s_{t-1}^{D}}{2}}.$$

(29) Growth rate of theoretical monetary aggregate

$$1+g_t^M=m_t/m_{t-1}.$$

(30) Monetary policy rule (a simple version of Taylor rule)

$$r_t=(1-\rho_r)r+\rho_r r_{t-1}+(1-\rho_r)\rho_\pi(\pi_{H,t}-\pi_H)+e_{r,t}.$$

(31) Preference shock

$$\ln h_t=\rho_h\ln h_{t-1}+e_{h,t}.$$

(32) Home technology/productivity shock

$$\ln z_t=\ln z+e_{z,t}.$$

(33) Shock to Foreign productivity

$$\ln z_t^f=(1-\rho_z^f)\ln z_t^f+\rho_z^f\ln z_{t-1}^f+e_{z,t}^f.$$

(34) Shock to Foreign demand

$$\ln c_t^f=\rho_c^f\ln c_{t-1}^f+e_{c,t}^f.$$

(35) Foreign inflation is exogenous. Assume the goal for central bank in Foreign is to stabilize the price level,

$$\pi_t^f=(1-\rho_p^f)\pi_t^f+\rho_p^f\pi_{t-1}^f+e_{p,t}^f.$$

A.3.2. Figures

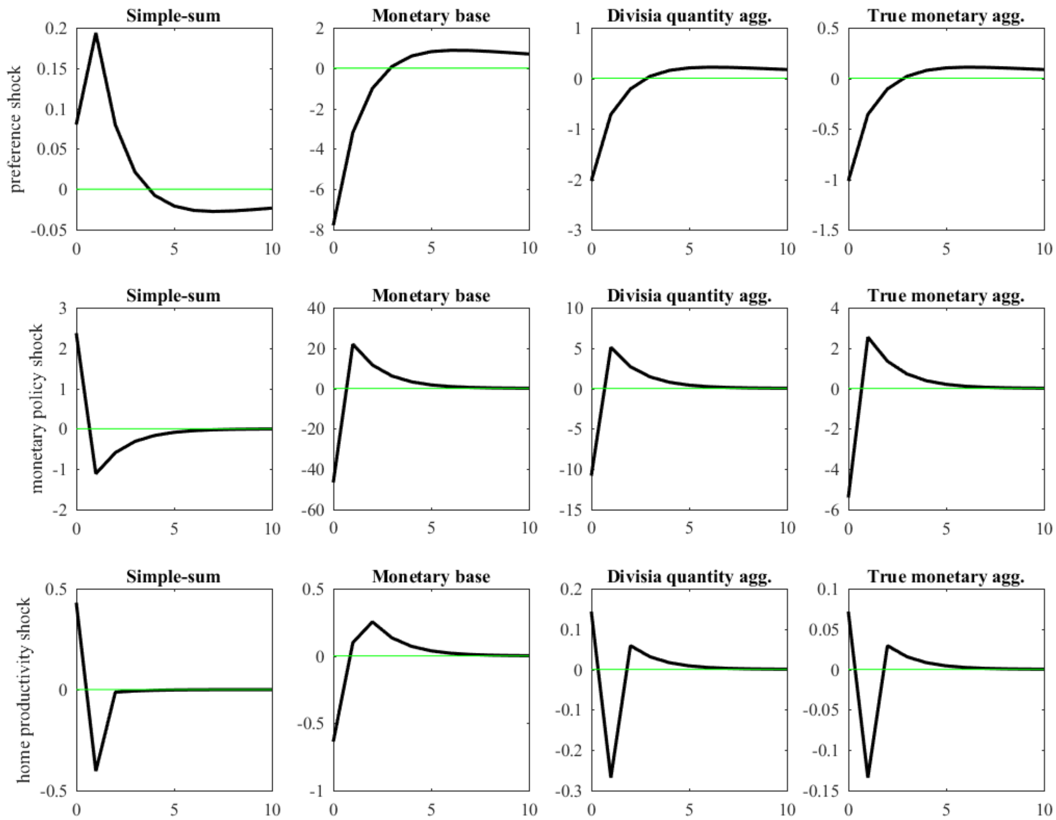

Figure 3.1. Impulse Responses of Growth Rate of Different Money Measures w.r.t Domestic Shocks, Benchmark ω=2, η=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

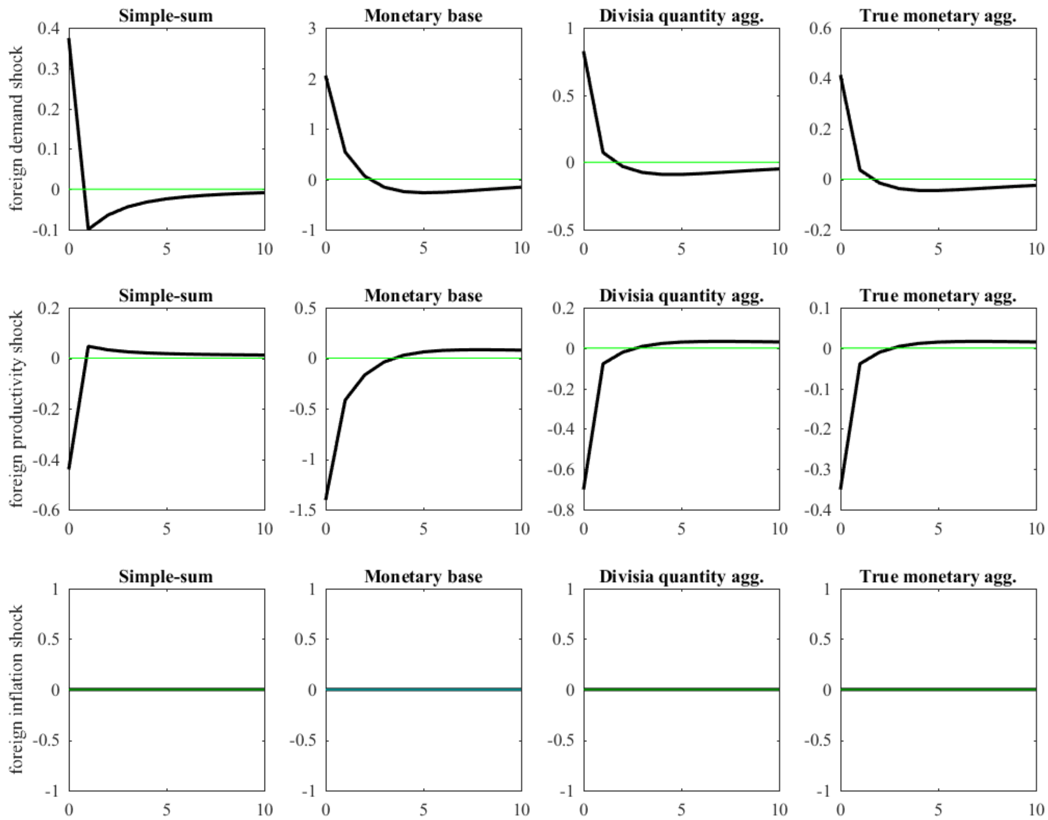

Figure 3.2. Impulse Responses of Growth Rate of Different Money Measures w.r.t Foreign Shocks, Benchmark ω=2, η=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

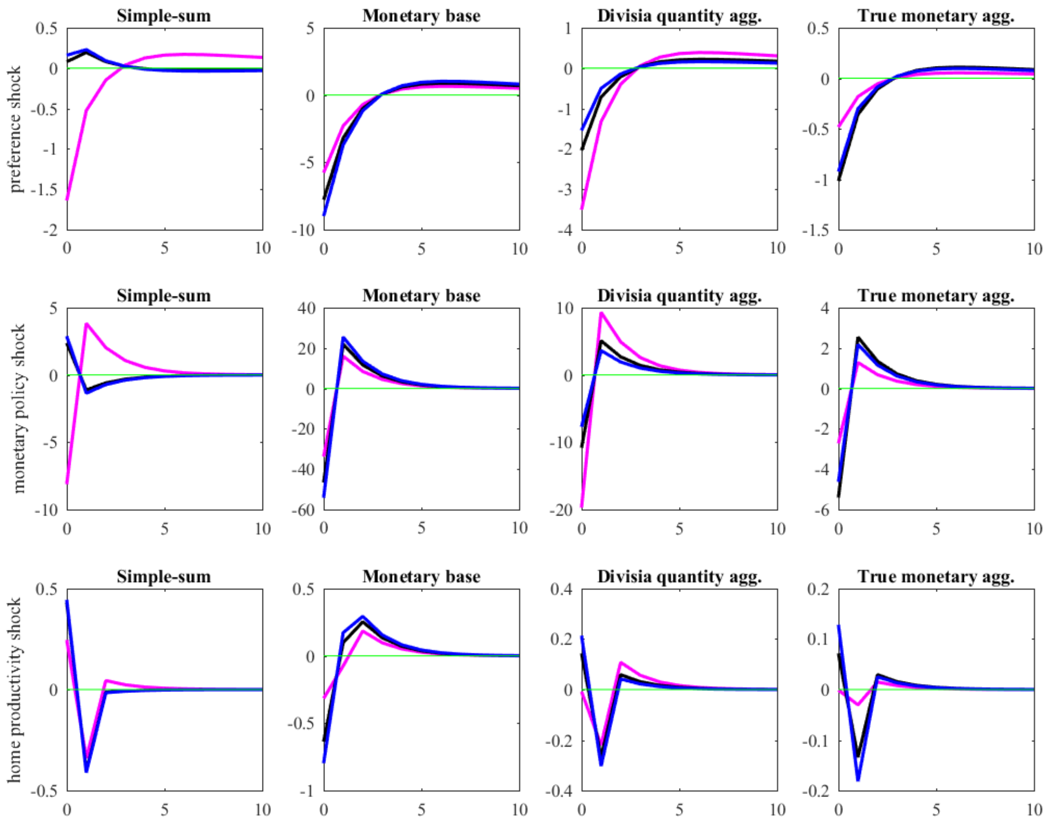

Figure 3.3. Impulse Responses of Growth Rate of Different Money Measures w.r.t Domestic Shocks under Variation of Parameter ω, pink for ω =1.16, black for ω =2, blue for ω =3, benchmark η=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

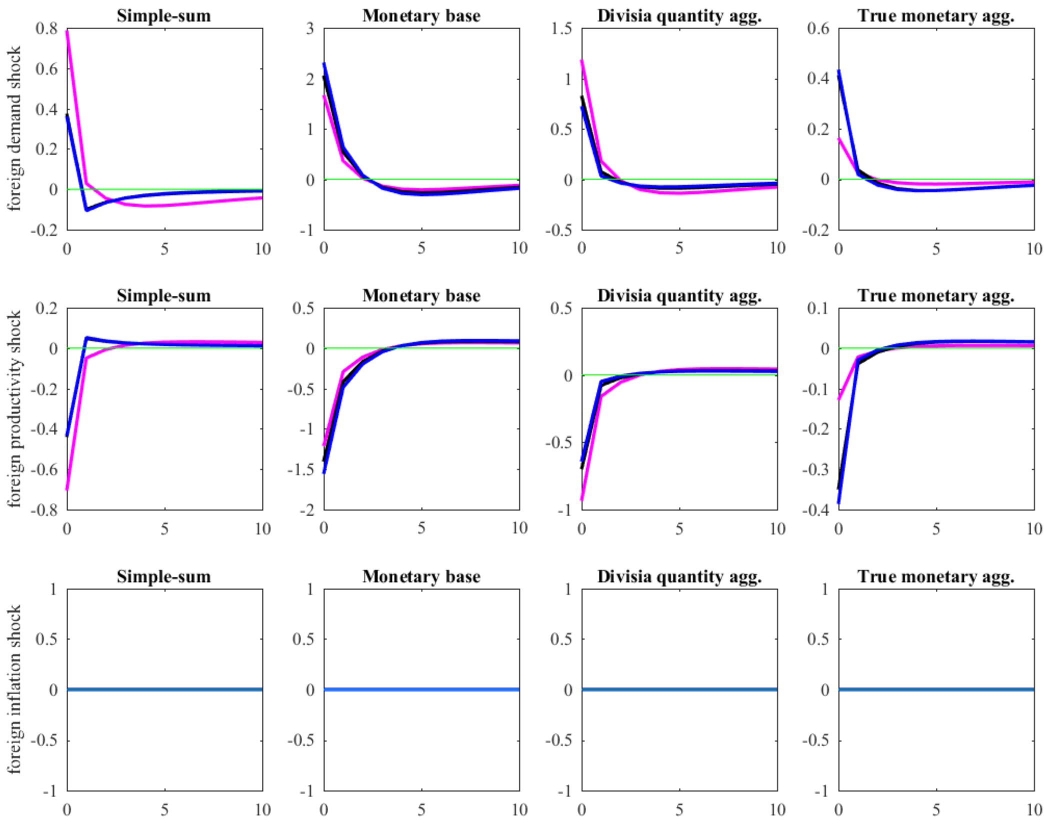

Figure 3.4. Impulse Responses of Growth Rate of Different Money Measures w.r.t Foreign Shocks under Variation of Parameter ω, pink for ω =1.16, black for ω =2, blue for ω =3, benchmark η=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

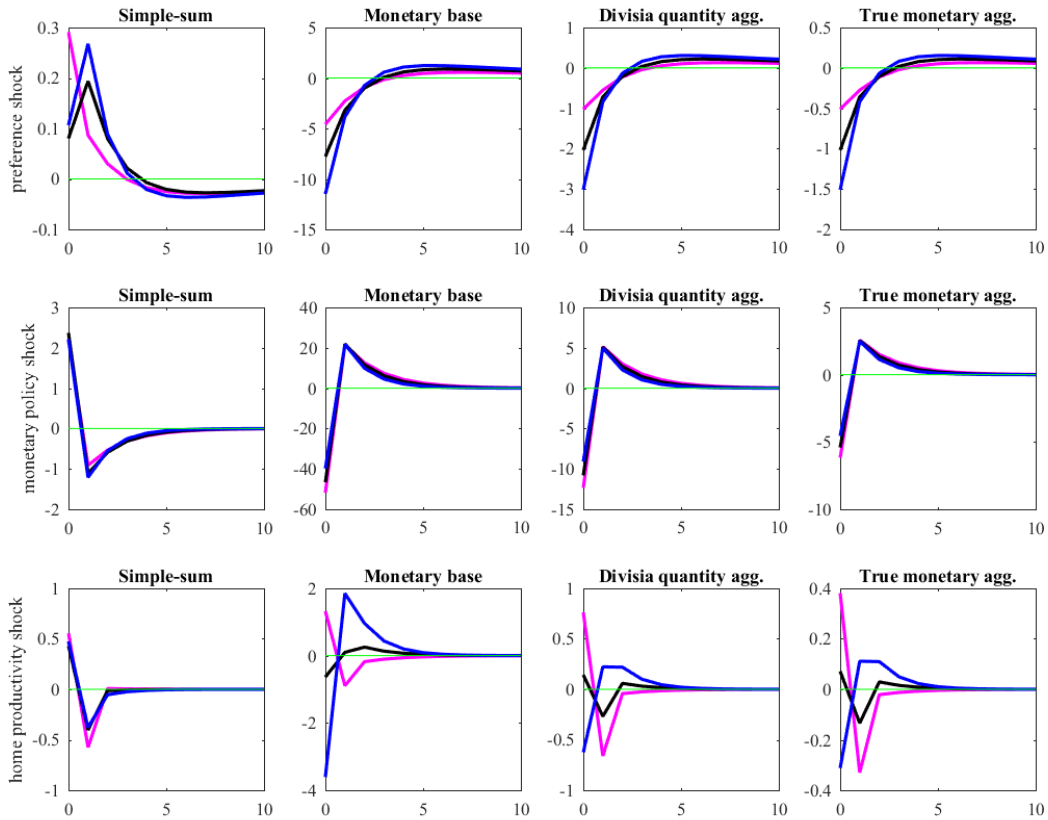

Figure 3.5. Impulse Responses of Growth Rate of Different Money Measures w.r.t Domestic Shocks under Variation of Parameter η, pink for η =1.1, black for η =2, blue for η =5, benchmark ω=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

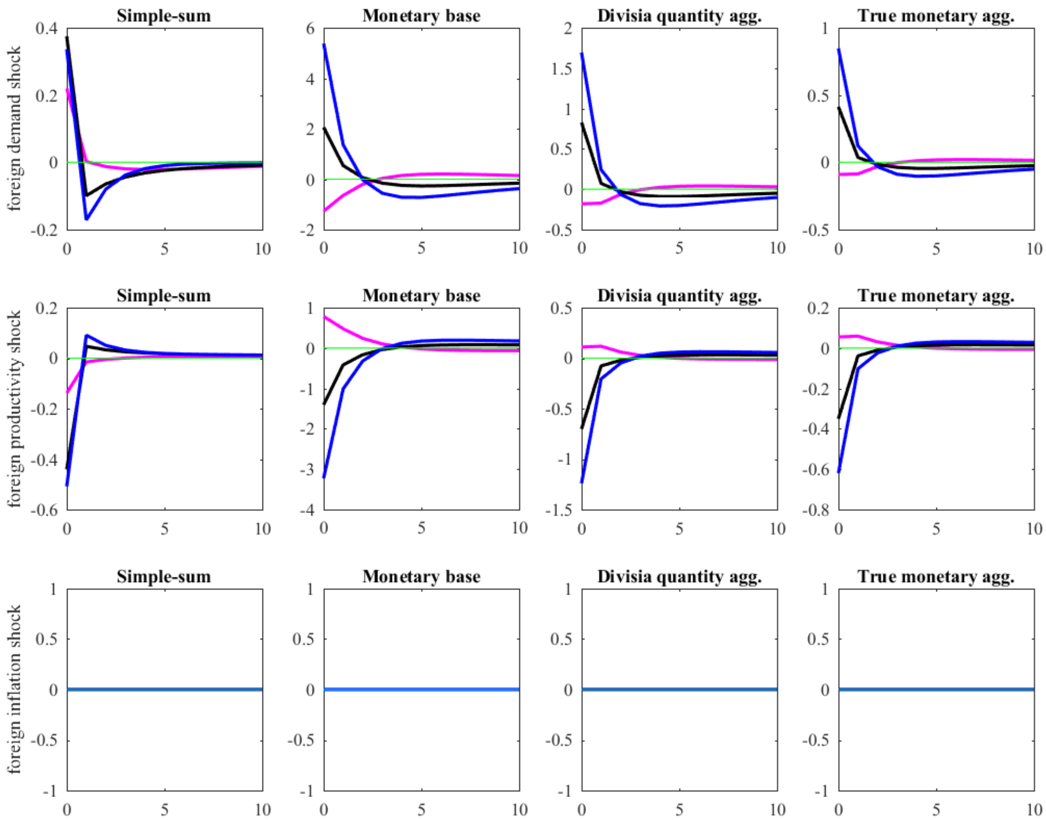

Figure 3.6. Impulse Responses of Growth Rate of Different Money Measures w.r.t Foreign Shocks under Variation of Parameter η, pink for η =1.1, black for η =2, blue for η =5, benchmark ω=2, α=0.4. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

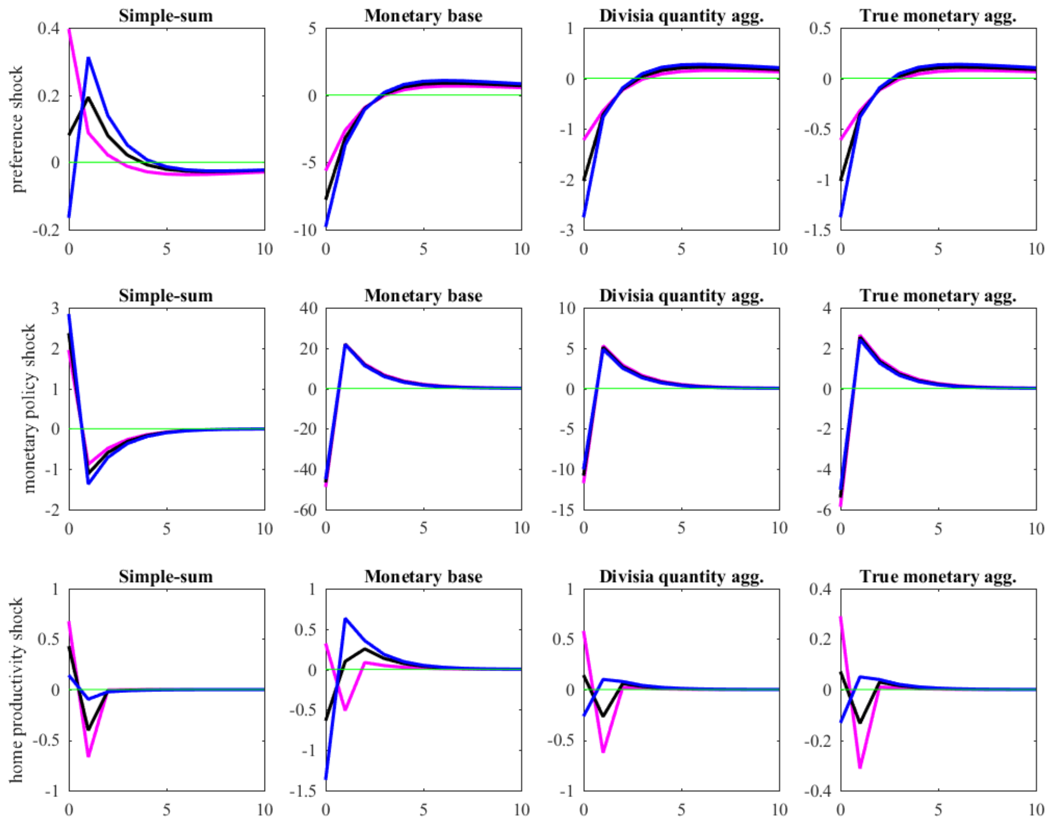

Figure 3.7. Impulse Responses of Growth Rate of Different Money Measures w.r.t Domestic Shocks under Variation of Parameter α, pink for α =0.1, black for α =0.4, blue for α =0.8, benchmark ω=2, η=2. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

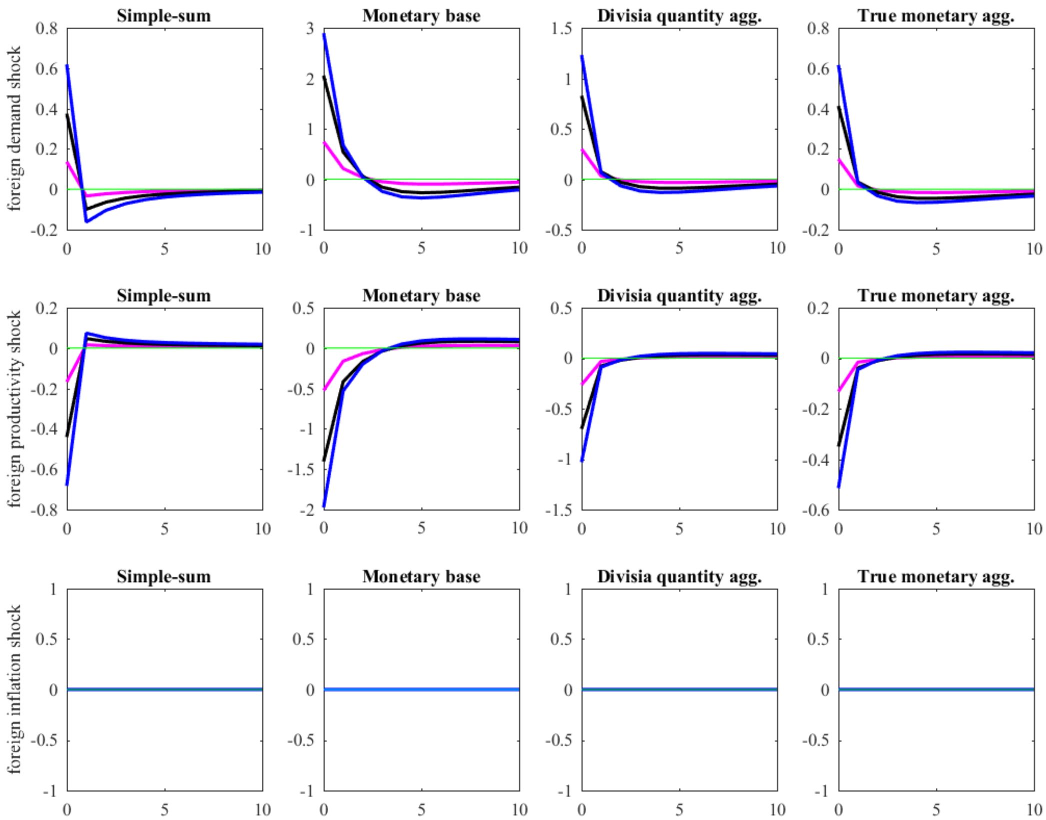

Figure 3.8. Impulse Responses of Growth Rate of Different Money Measures w.r.t Foreign Shocks under Variation of Parameter α, pink for α =0.1, black for α =0.4, blue for α =0.8, benchmark ω=2, η=2. Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

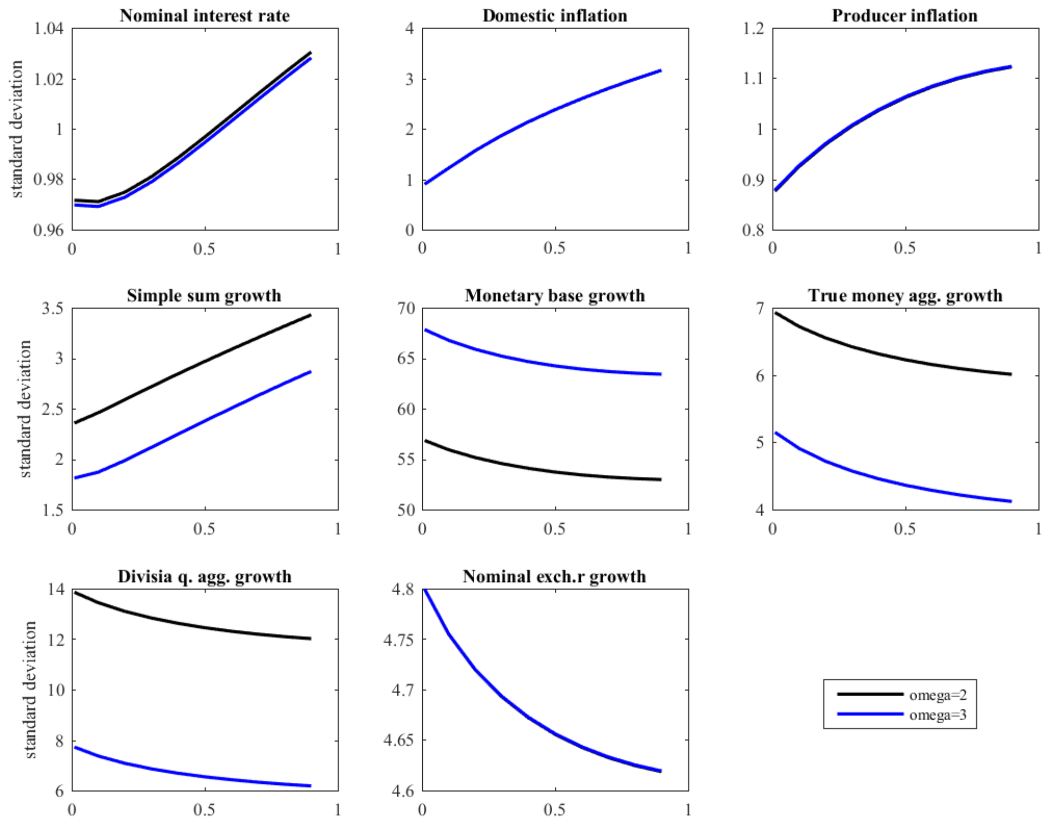

Figure 3.9. Volatility of Macroeconomic Variables under Variation of Parameter α. Each panel shows the standard deviation in percentage-point under different values of α from 0 to 1.

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