STUDIES IN ENGINEERING MECHANICS
REPORT NUMBER 16

FINAL REPORT
VIBRATION OF STEEL JOIST-CONCRETE SLAB FLOOR SYSTEMS

By

KENNETH H. LENZEN

THE UNIVERSITY OF KANSAS
CENTER FOR RESEARCH
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December 6, 1963

W. E. Bradbury, Chairman
Research Committee of S.J.I.
Sheffield Division
Armco Steel Corporation
Kansas City 25, Missouri

Dear Mr. Bradbury:

It has come to my attention that the Final Report on the Vibration of Steel Joist-Concrete Slab Floor Systems may be misleading as far as actually figuring the expected response of a joist floor with no damping. The example problem in Figure 6 on page 25 is the design of a joist floor. Everything is satisfactory in this figure until the last few lines in which the displacement is computed. Since in the previous computations the number of joists, n, would occur in the numerator and denominator and, therefore, cancel, it was also omitted in this calculation. Actually, the design is for a 2 foot wide floor, 25 feet long with only one joist. If this floor was 20 feet wide with 10 joists, the displacement, d, would be a tenth of the 0.031 inches or 0.0031 inches. This calculation is stated in Figure 5 which is the reference graph. The moment of inertia, I_t, is the moment of inertia of the composite T-section multiplied by the number of joists. With this alteration, the reference graph indicates a floor with vibrations which are slightly perceptible. The size of the joist should be increased until the deflection is 0.002 inches. The statements at the end of the first paragraph on page 18 are then incorrect since they refer to the single joist floor analyzed in Figure 6.

I would recommend that this be brought to the attention of your committee and anyone else who has received copies of the report. A single joist will always be in the strongly perceptible range.

Sincerely yours,

Redacted Signature

Kenneth H. Lenzen, Professor
Department of Mechanics and
Aerospace Engineering

KHL/eo

VIBRATION OF FLOORS

INTRODUCTION

Increased usage of lighter types of construction in floors in recent years for economy purposes has resulted in occasional floors with insufficient stiffness to prevent noticeable vibration induced by human impact. This problem has not been limited to any particular type of construction or construction material.

The Steel Joist Institute, in an attempt to develop basic knowledge and acquire test data in this floor vibration field, initiated and sponsored a research program at The University of Kansas. The program was started in 1958 and has developed into several successive projects. Six reports to the Steel Joist Institute have resulted from this research. This is a final report summarizing all of the previous work.

ACKNOWLEDGEMENTS

This investigation which was sponsored by the Research Committee of the Steel Joist Institute formed a part of the research program of The University of Kansas Center for Research in Engineering Science of which Dr. John S. McNown is Executive Director. Certain studies included in the investigation were thesis projects for advanced degrees in Engineering Mechanics in the Graduate School of The University of Kansas. The work of these students is gratefully acknowledged; Joseph E. Keller, James A. Wiley, Gerald A. Barr, and William C. Lyons all contributed much to the program.

OBJECTIVE

At the inception of the program, a full statement of the aims could not have been made other than "How can annoying vibrations be avoided in the initial design?", and "How can annoying vibrations be eliminated in a floor which is subject to them?" since too little was known to formulate the program. The project has evolved in several basic steps with the overall program aimed at answering these two questions. The determination of the following has been necessary:

1. The calculation of the natural frequency of the standard steel joist - concrete slab floor system.
2. The calculation of the natural frequency of a steel joist - concrete slab floor system in which the stiffness of the concrete slab laterally to the joists is important.
3. The determination of what constitutes annoying vibrations and what human response factors are involved.
4. The elimination of annoying vibrations.
5. An investigation of various means of introducing damping to the system.

NATURAL FREQUENCY OF FLOOR SYSTEMS

The method of computation of the natural frequency of the steel joist - concrete slab floor system was not apparent.

Although it was thought that the stiffness of the slab perpendicular to the joist would not be of major importance for typical construction, the question of interaction between the joists and the slab remained.

To investigate the amount of interaction which could be expected, a steel joist-concrete slab floor was constructed.

This floor consisted of six steel joists at twenty-inch centers. The joists supported a concrete floor cast on corrugated steel sheet centering. The joists had an overall length of 15 feet 4 inches with a clear span of 14 feet 8 inches. An effective length of 15 feet, center to center between supports, was used in all calculations. Three sets of joists were tested using different depths for each set. These were the standard 8S2 and 10S2, plus a specially fabricated 12S2 joist.

The concrete slab was cast with 3000 psi ready-mix concrete. The minimum thickness was $2\frac{1}{2}$ inches at the top of the centering corrugations and the maximum was 3 inches in the valley. Welded wire fabric, 6" x 6" - 6/6, was placed at the top and bottom of the 10'0" by 15'4" slab. This large amount of reinforcement in the slab was used in order to strengthen the slab for subsequent replacement of various sizes of joists. The forms were allowed to deflect with the joists in casting the slab.

The floor was supported on end walls 32 inches high built from 8" x 8" x 16" concrete blocks. The long sides of of the floor were not supported. Bolts were cast in the concrete to provide a method of clamping each joist to the slab at about 12-inch spacings. An insert was cast into the slab at the mid-point providing a connection to the Calidyne Shaker which was used to vibrate the floor.

The joists rested directly on solid concrete bricks which could be removed to change joists. Removable horizontal bridging was made of 1" x 1" x 1/8" angles. Figure 1 shows many of the details of construction.

The movement of the floor was sensed by a seismic type pickup and recorded on an oscillograph shown in Figure 2. Two methods of loading were used; (1) steady state vibration using a Calidyne Shaker and (2) impact vibration induced by the dropping of an iron ball and also by the jumping of a human.

Only in the completely unloaded condition with absolutely no attachment between the centering and the joists (a laboratory condition) did the natural frequency reduce to a value approaching that of independent action of the slab and the joist. Even here the measured natural frequency was higher than the value computed assuming no interaction. The procedure for computing the natural frequency is shown in Appendix A. In all other cases the measured natural frequency of the floor agreed with the theoretical value assuming full interaction between the steel joists and the concrete slab.

In fourteen buildings in the Kansas City area, dynamic measurements were made on 46 different floors. These readings were taken under various conditions. Some of the buildings were in the early stages of construction, some were recently completed, and others had been occupied for sometime. Loading conditions varied considerably for all floors. Loads other than dead loads could not be accurately included in the calculations since their magnitude could only be estimated. The frequency assuming full interaction was computed prior to the readings. It was found that in 39 floors the measured frequency agreed with the computed frequency within five cycles per second. In 27 floors the agreement was within 2 cycles per second, and in 14 floors the agreement was within 0.5 cycles per second.

The reasons for disagreement were many. Some floors were not regular in shape or support. Many of the floors were cracked, some so badly that it was questionable if the slab should be included for other than the weight. The load on the floors could only be a guess. At times, the type of support, and thickness of concrete were different than that shown in the drawings. Some of the floors damped so quickly that the natural frequency could only be estimated. Under the conditions, the agreement between the calculated and measured frequency was deemed to be excellent.

During the period in which the above readings were made, a method for computing the natural frequency of floors was determined in which the stiffness of the slab perpendicular to the joists could be taken into account. The method was based on Dana Young's energy analysis for computing the natural frequency of flat plates and was extended to the anisotropic condition. It appears that a steel joist - concrete slab floor in which the simple T-beam analysis is not applicable would not be subject to annoying frequencies. The stiffness of a floor is so great and the amplitude so small when the stiffness of the slab in the lateral direction is important that no vibration trouble should result. The analysis is therefore not included in this report.

HUMAN SENSITIVITY TO VIBRATIONS

Once it was known that the natural frequency of a floor could be computed with reliability as well as the amplitude of the oscillations, it had to be determined what constituted an annoying vibration. A paper by Wright and Green is included as Appendix B to show the effect of vibration on humans and their response to the vibration.

An excellent bibliography is included with this paper and many additional references are given in the bibliography of the first report by Lenzen and Keller.

The floor described earlier was used to substantiate the results obtained by earlier investigators. It was found that the prediction of the human response was reliable by placing one or more people on the floor and subjecting them to vibrations of varying frequency and amplitude. Reiher and Meister's work seemed to predict the human response reasonably well. These curves are shown in Figure 1 of Appendix B.

During the field work on the 46 floors in the Kansas City area, an attempt was made to correlate the work of Reiher and Meister to the vibrations found in these buildings. It was found that only five floors had a natural frequency exceeding 20 cycles per second and these were stair wells or halls. The natural frequency of the other floors varied from six to eighteen cycles per second with 12 cycles per second as a mean. The amplitude varied between .003 to .012 inches with an average of about .005 inches. When these values were checked on the human response graph it was found that they fell in the disturbing region. Yet, only three floors had vibrations which were perceptible to the occupants although the human response curves had been substantiated by many tests other than these. In addition, the amplitude and frequency of the annoying floors indicated that they should be slightly less disturbing than the average.

In originally deriving the graph and in all of the checks of it, a steady state vibration was used. In checking the floors, a transient vibration induced by impact was used. The difference was, therefore, the time or number of cycles to which the human was exposed to the vibrations. Additional tests on the test floor substantiated this conclusion.

The normal floor is not subjected to a steady-state vibration induced by machinery of any nature. If such is the case, the machinery can be easily damped from the floor system, and, thus, the vibrations eliminated. The main source of vibration is from the occupants themselves impacting the floor through normal usage. The problem then reduced to the elimination of vibrations induced by this source.

The search of literature gave no indication of how the human responded to transient vibrations. To substantiate the ideas formed from the various tests, a floor was designed on which the natural frequency, amplitude, and damping characteristics could be varied. Tests on students, and staff indicated that variation in frequency and amplitude had a minor effect. The main factor influencing the effect of vibrations on the human was the damping. If the floor was damped to a small amplitude prior to five cycles of vibration, the occupant felt only the initial impact, no vibration. If the vibration persisted above 12 complete cycles, the occupant responded to the vibration just as to a steady-state vibration. The response to the vibrations between these ranges was a function of the number of cycles before the amplitude became negligible. Reanalysis of the Kansas City floors substantiated these conclusions. The three floors on which the vibrations were definitely perceptible all had amplitudes after five cycles of vibration above 0.4 of the initial amplitude. All other floors had much lower amplitudes after five cycles. Those at 0.2 of the initial amplitude after five cycles and above had barely perceptible to perceptible vibrations. Amplitudes after five cycles below these values were barely or not at all perceptible.

The problem then is not one of frequency and amplitude as encountered in steady-state vibrations but of damping. If floors can be damped before 12 cycles of vibration the effect of the oscillatory motion is reduced, becoming zero at five cycles.

But what causes damping? Again, in reviewing the floors tested in Kansas City it was found that almost everything provided damping.

Human occupants provided excellent damping. Apparently, the human frame will absorb a large amount of energy. Four people increased the damping on the test floor 300% above that without them. In a school building annoying vibrations were reported by school teachers working after classes. Tests made in the evening indicated an initial amplitude of about .007 inches with a frequency of 9 cycles per second. After five cycles, the amplitude was 0.417 of the initial value. Later, the tests were repeated when the class was present. The initial amplitude remained 0.007 inches with a frequency of 8.75 cycles per second. The amplitude after five seconds was 0.087 of the initial value. The vibrations were not perceptible by the students or teacher even when looking for them when the room was full. The same vibrations but with much less damping were annoying when the room was empty. This also demonstrates the large effect of damping.

Simple loads other than humans do not increase the damping values. A floor loaded with concrete cylinders had much less damping than when unloaded. The activation of the dead load required more energy which had to be absorbed before the vibrations would decrease.

On the building floors investigated, it was found that most structural components contributed to the damping. Especially efficient were partitions of all types including the small wooden temporary partitions. The flooring, rugs, and ceilings assisted to a reduced extent. The three floors recorded earlier as having definite to annoying vibrations all had large open areas with no partitions and were in school buildings and a church. Normal construction provides the damping necessary to absorb the energy of the floor and, thus, remove any annoying sensation. Only when these normal values are totally absent as in a church is the vibration noticeable.

VIBRATION ABSORBING DEVICES

Since annoying vibrations would not usually be expected in a normal design, the focus of attention became that of providing damping if a floor had excessive vibration.

Also, since it seemed inadvisable to penalize all floors for the few that would be encountered with no partitions and consequently the possibility of insufficient damping, damping would have to be provided artificially if annoying vibrations resulted. Several methods have been tried but only one has been successful in this study.

A design of a dynamic vibration absorber was optimized to provide maximum damping with a minimum of weight. It was found that a device weighing .02 of the weight of the floor would dampen any floor in less than five cycles if it were mounted on springs which provided a natural frequency of the unit of about one cycle per second less than that of the floor to be damped. A dash pot must be provided to absorb the energy of the floor and must provide the damping unit with a damping equivalent to .075 of the critical value. Even this low weight ratio is a large value for the average floor. Rather than make one large unit, several small units which can be mounted at the top of joist and have a height less than the depth of the joists are more practical and easier to handle. The detailed design procedure for such units is given in Appendix C.

Another test floor was designed and built to evaluate the dynamic damping units and other methods of eliminating annoying vibrations from floor systems.

In constructing a floor with very little structural damping a maximum span to depth of joist ratio was used. In addition the concrete slab should not have a pronounced effect on the moment of inertia of the "T-beam" cross section.

In order to fulfill these conditions the selected joist was No. SJ-123 with a nominal depth of 12" and approximate weight per foot of 4.94 lbs. The joist had an overall length of 24' 8" and had a clear span of 24' 0" which is the maximum clear span allowable for this weight joist. The experimental test floor was constructed in the new "Center for Research in Engineering Science" laboratories. It consisted of nine (9) steel joists placed on 24" centers. The joists supported a concrete floor cast on corrugated sheet steel centering. The joists had an overall length of 24' 6", center to center between supports, was used in all calculations.

The floor was supported on end walls 42" high built from 8" x 8" x 16" concrete blocks. The end walls were prevented from tipping, due to the moment caused by the floor, by small pyramid shaped supporting walls placed perpendicular to the main end wall, one at each end and one in the middle. The long sides of the floor were not supported. The joists rested directly on concrete bricks placed on 24" centers throughout the top course of concrete blocks. The top chord of the joist was made level with the top of the end wall. The nine (9) joists were bolted together with removable horizontal bridging which was made of 1" x 1" x 1/8" steel angle. The corrugated sheet steel centering was bolted down to the top chord of the joist on approximately 2'-0" to 2'-6" centers. Wire mesh reinforcement, 6" x 6" - 6/6, was placed in the middle of the 18'0" x 24' -0" slab. The concrete slab was cast with 4,000 psi ready mix concrete. The minimum thickness was 2 1/4" at the top of the centering corrugations and the maximum was 2 3/4" in the valley. A mean thickness of 2 1/2" was used in all calculations. In casting the floor the forms were allowed to deflect with the joists. Due to the fact that the top chord of the joist was level with the top of the end wall and that the forms were allowed to deflect with the steel joist, the finished concrete slab rested only on the joist at the edge and not on the supporting end wall. This, in reality, gave a simply supported end condition with no internal structural damping due to the resting of the concrete slab on the end walls.

It was found that the damping units were as effective as predicted by the analog computer which was used to optimize the design. The test floor which was designed with annoying vibration characteristics became entirely satisfactory when the dynamic damping units were mounted on it. The damping units succeeded in damping the floor in less than four complete cycles.

Supports typical of normal construction, 8 I 13, were positioned on the long sides so that they could be engaged or disengaged from the floor. The dynamic damping units were equally efficient with these in place.

Since typical installations normally provided sufficient internal damping to avoid annoying conditions, the damping provided by ceilings was investigated. It was found that attached ceilings provided more damping than hung ceilings. The amount of energy which either would absorb was not significant as compared to that needed for a floor with so little damping as to be annoying. If the vibration is in the low perceptible range, a ceiling probably will damp the vibrations sufficiently.

An attempt was made to damp vibrations by various methods such as providing insulation of various types between the bridging members and the joists, providing cross bridging, prestressing the top and bottom chords of the joists, and providing a variety of cable installations which would cause interaction between members with a resulting absorption of energy. Since the motion of a floor is small, .003 to .010 inches, no effective damping method was found.

DISCUSSION OF THE TESTS

The results of the investigation clearly indicate the following:

1. Vibration due to steady-state excitation is not a bother in construction. If it is encountered the source can and should be insulated, thus removing the vibrations.

2. Transient vibrations are those which must be counteracted and these are only important if the vibrations induced persist more than five cycles.

3. Normal construction provides sufficient damping through such construction features as walls, partitions, flooring, floor covering, ceilings, etc., so that the transient vibrations are usually in the non-perceptible or barely perceptible range.

4. If unusual conditions exist, such as large school rooms and churches, in which damping is not inherent to the design, insufficient damping will usually not be supplied by bracing or ceilings. The damping can be provided by the dynamic damping units. (See Appendix C) If the structure is a borderline case sufficient damping may be obtained from flooring, floor covering or ceilings.

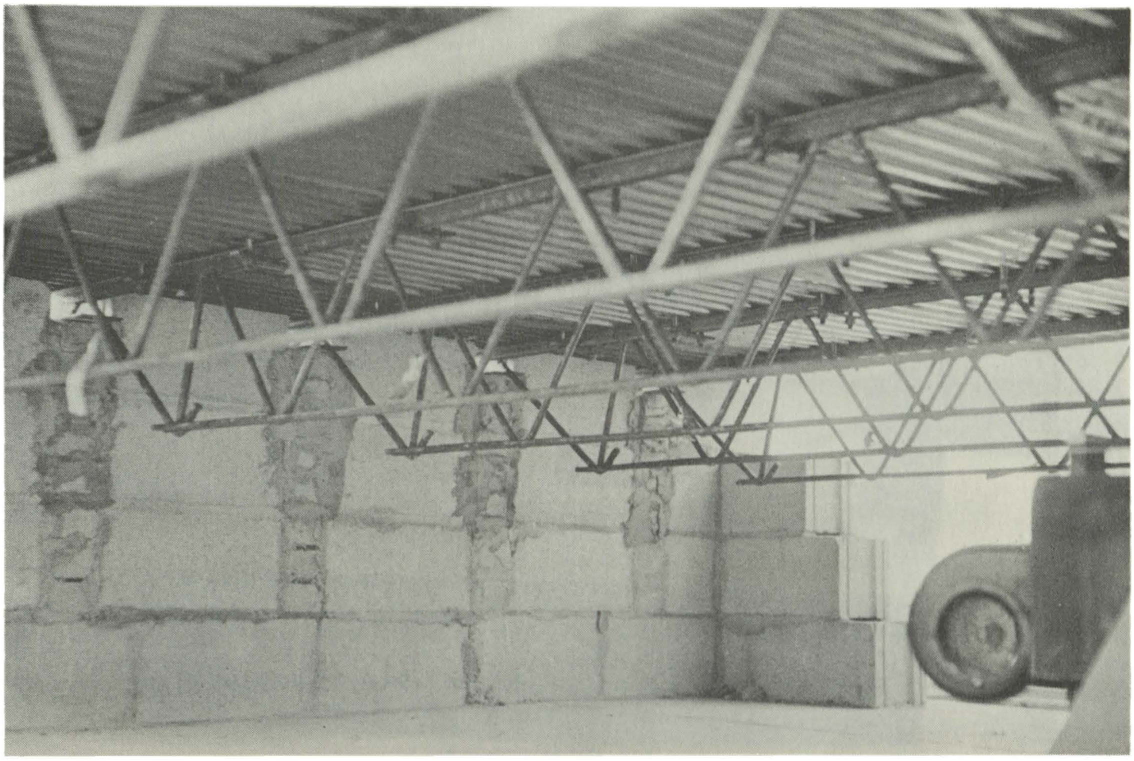
5. The human being was one of the best energy absorbers found.

Although considerable damping is inherent with even the worst design, it would be possible to design a floor satisfactory to the human occupants even if the structure had no damping. Such a design is possible because the natural frequency of vibration of the floor increases as the square root of the moment of inertia while the deflection decreases directly with the moment of inertia. (Figure 1, Appendix B). Actually, such a design will only remove the vibration from the perceptible range of the occupants and would require a natural frequency which would probably be at least 30 cycles per second. To comply with such a design requirement would necessitate a ridiculous size of joist and slab. It should be realized that a floor with no damping cannot be constructed.

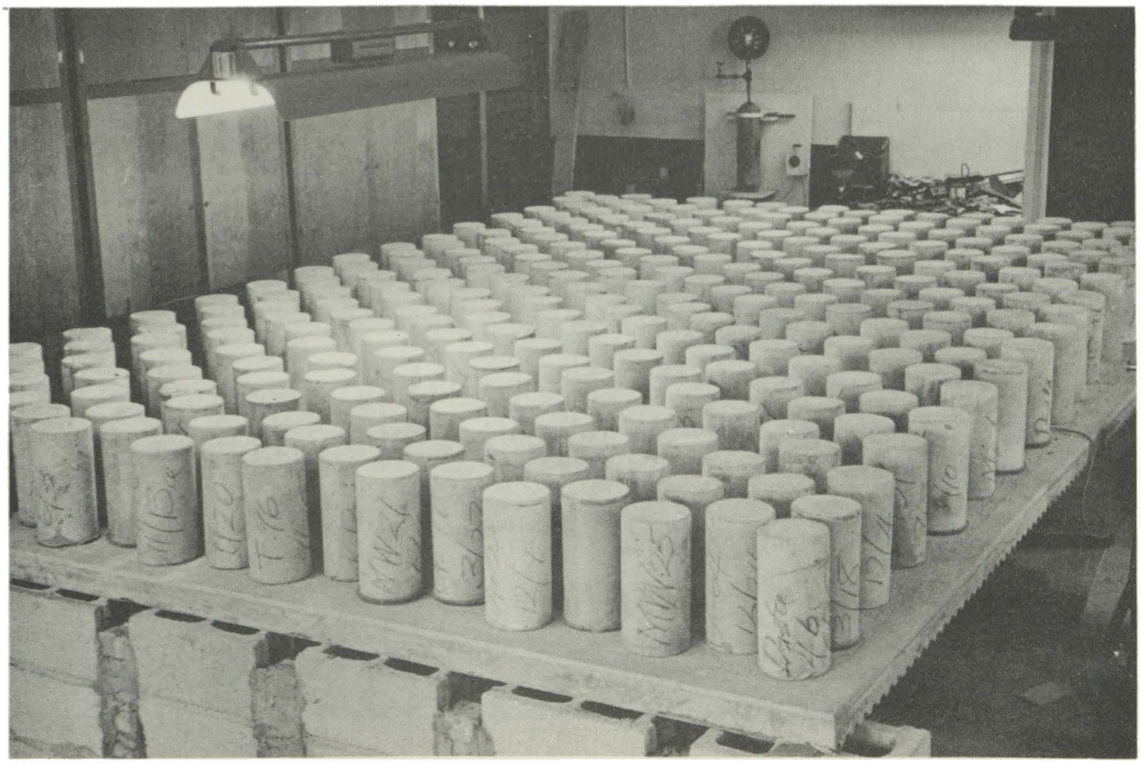
In the design of large open areas encountered in churches and school buildings, sufficient damping to avoid the vibration problem may usually not be realized. Increased stiffness may be used in combination with the inherent damping to insure that the vibrations are, at worst, in the low perceptible ranges. A graph is presented in Figure 5 which can be used for such designs. This graph originally presented in the first report to the Steel Joist Institute, has been confirmed in the field and laboratory tests under standard construction. The values are based on experience with the test floors and the open floors encountered in the field investigation. In general, the curve will require a stiffer floor structure than that usually used. The moment of inertia can be increased most efficiently by increasing the depth of the joist. Increasing the thickness of the concrete slab is not as efficient as increasing the joist size. The stiffness can be related to the L/d ratio but since several sizes of joists exist with each depth, the required stiffness or moment of inertia may be obtained by a small increase in weight by using a deeper joist, or, if headroom is important, a heavier joist of the same depth with added slab thickness can accomplish the same purpose. It will not be difficult to compute the natural frequency of floors by T-beam analogy (Appendix A) for every length, joist size, joist spacing, and slab thickness if this is encountered frequently. Such a design for a completely open floor with a clear span of 25 feet is shown in Figure 6. The design with the indicated natural frequency of 10.6 cps and displacement of .031 in. would be in the strongly perceptible range. This is an extreme case with a small dead load and no damping.

At the time that sufficient information is available, isoelastic pads between the joist and its end supports and between the joists and the centering may provide sufficient damping for this type of design so that increased size of joists are not necessary. Dr. Lazan of the University of Minnesota has been developing the constants necessary for the use of this material. Although his tests indicate a very usable material, insufficient data are available at present. This is a distinct possibility for future application.

It has been stated that vibrations can be eliminated by bracing. Although tests were only conducted using flexible side supports, the tests indicated no configuration of bracing that would provide damping or eliminate vibrations in any way. It can be argued that masonry supports would be different. This is, of course, possible. Since the deflection of most floors is a maximum amplitude of about .007 inches in vibration, the movement is insufficient to allow any important amount of damping. The author cannot visualize a condition in which bracing can be made important except when insufficient bracing is applied initially.

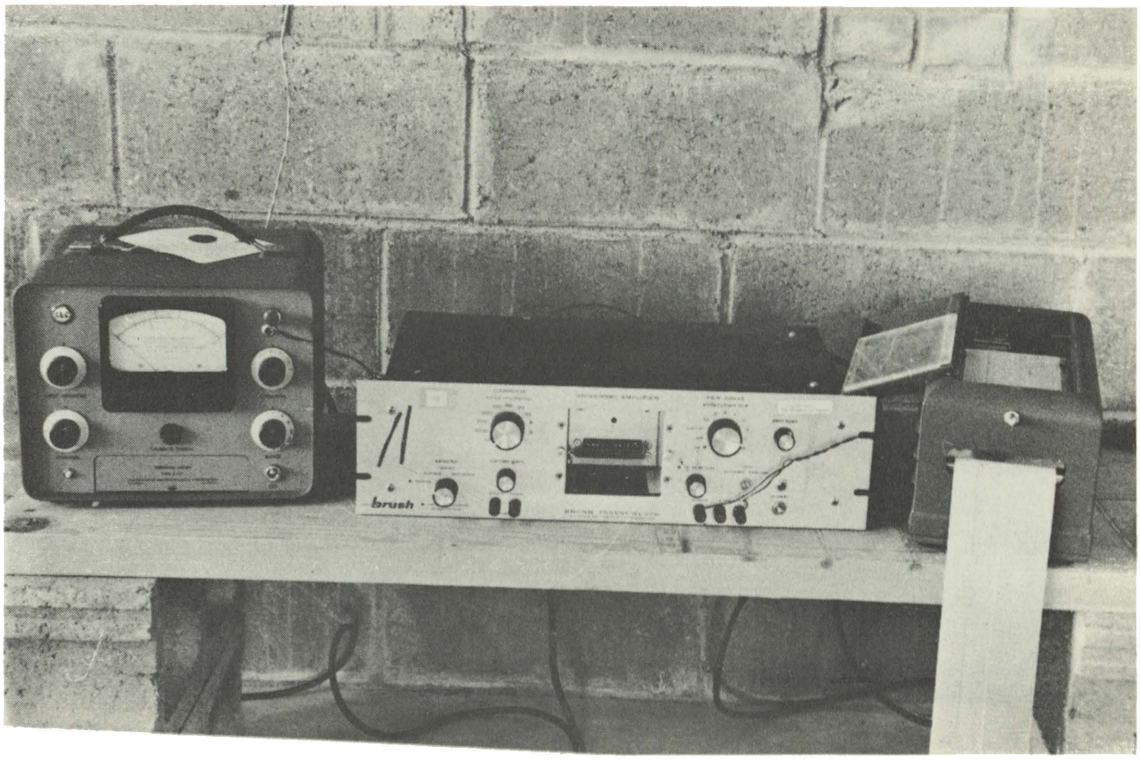


Underside of floor

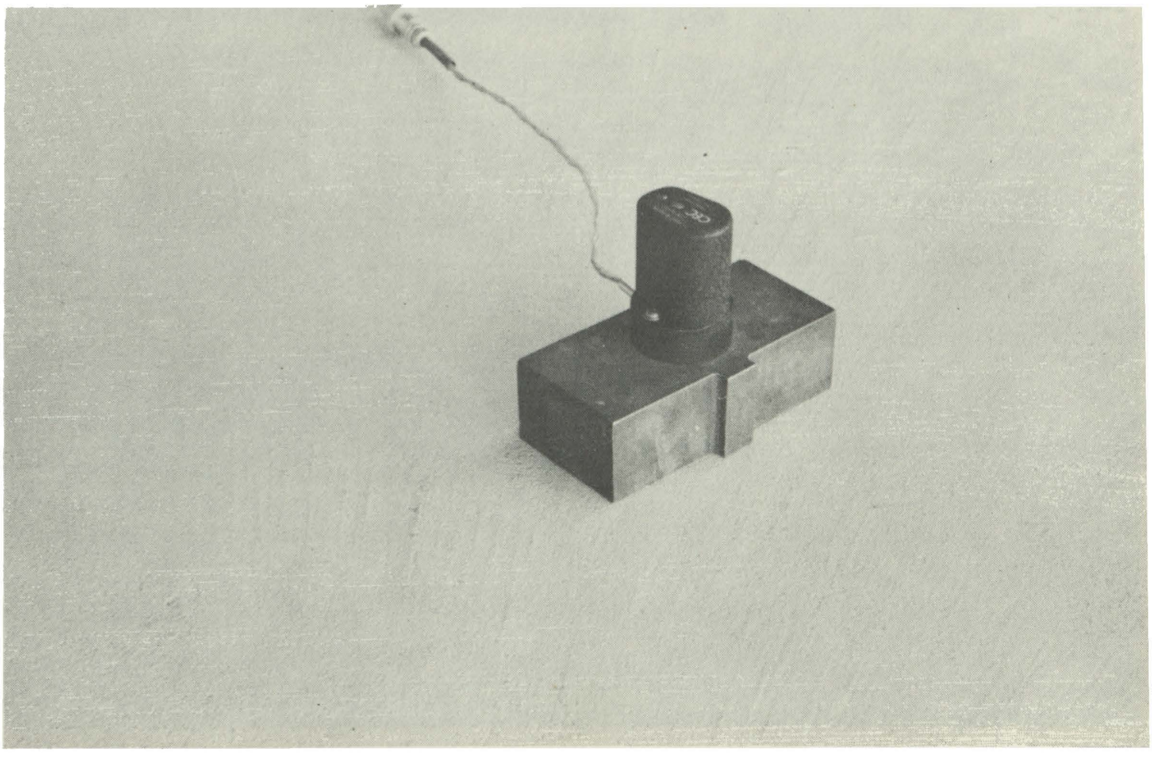


Loaded Floor

Figure 1
First Test Floor

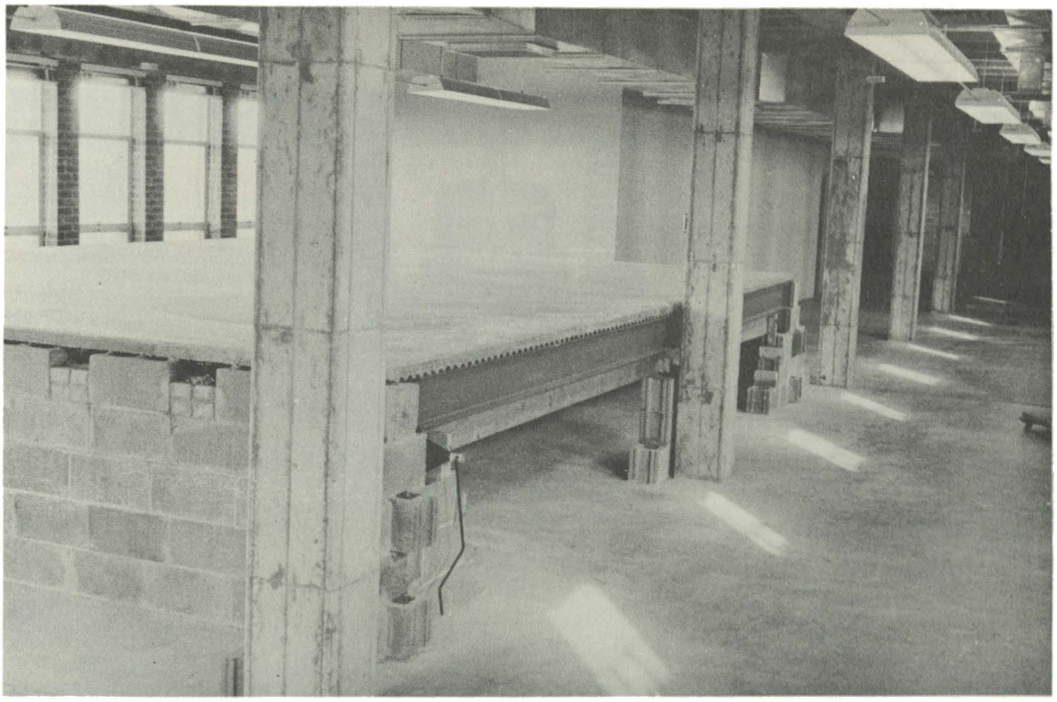


Amplifying and recording instruments used in experimental work. Consolidated Vibration Meter, Brush Universal Amplifier and Recorder.

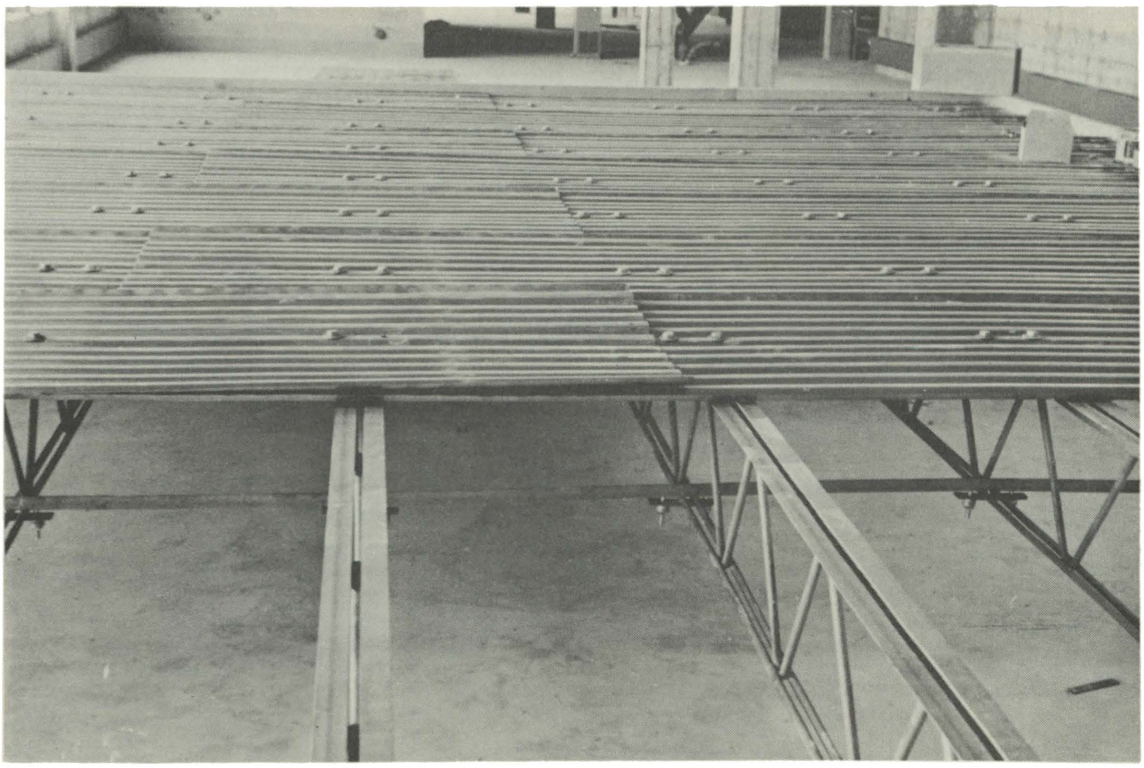


Seismic type instrument used to pick up amplitude and frequency of vibration.

Figure 2
Measuring Equipment

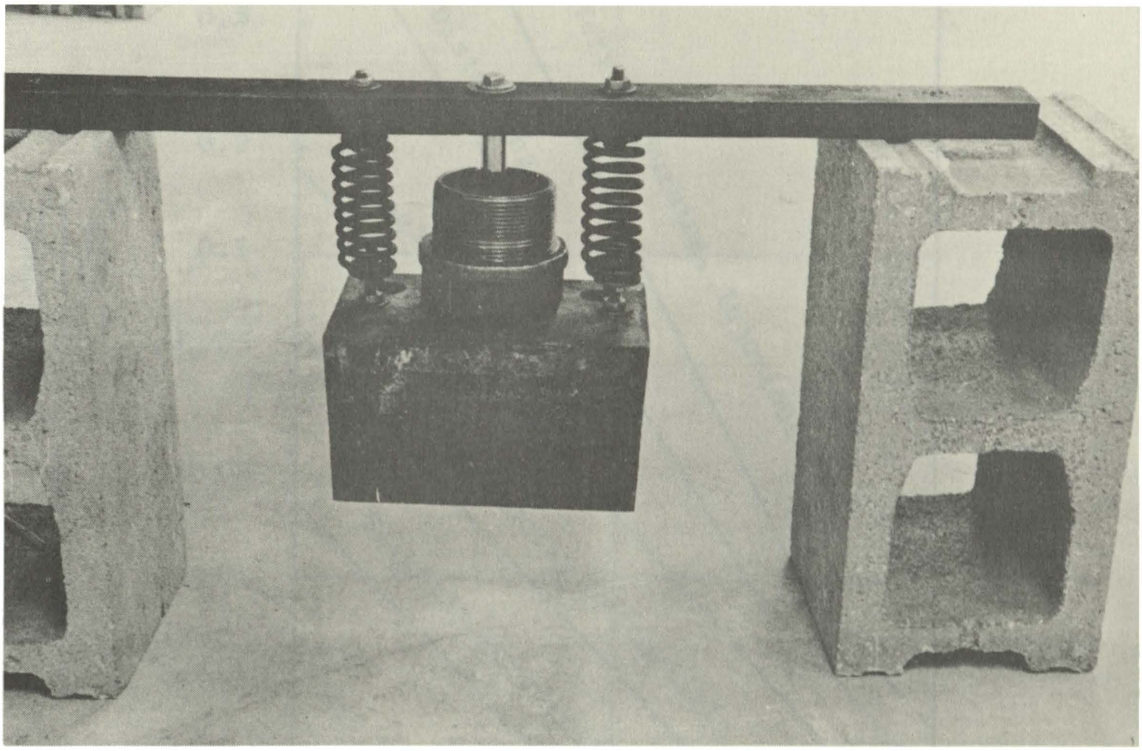


General view of test floor system with I-beam supporting free edge

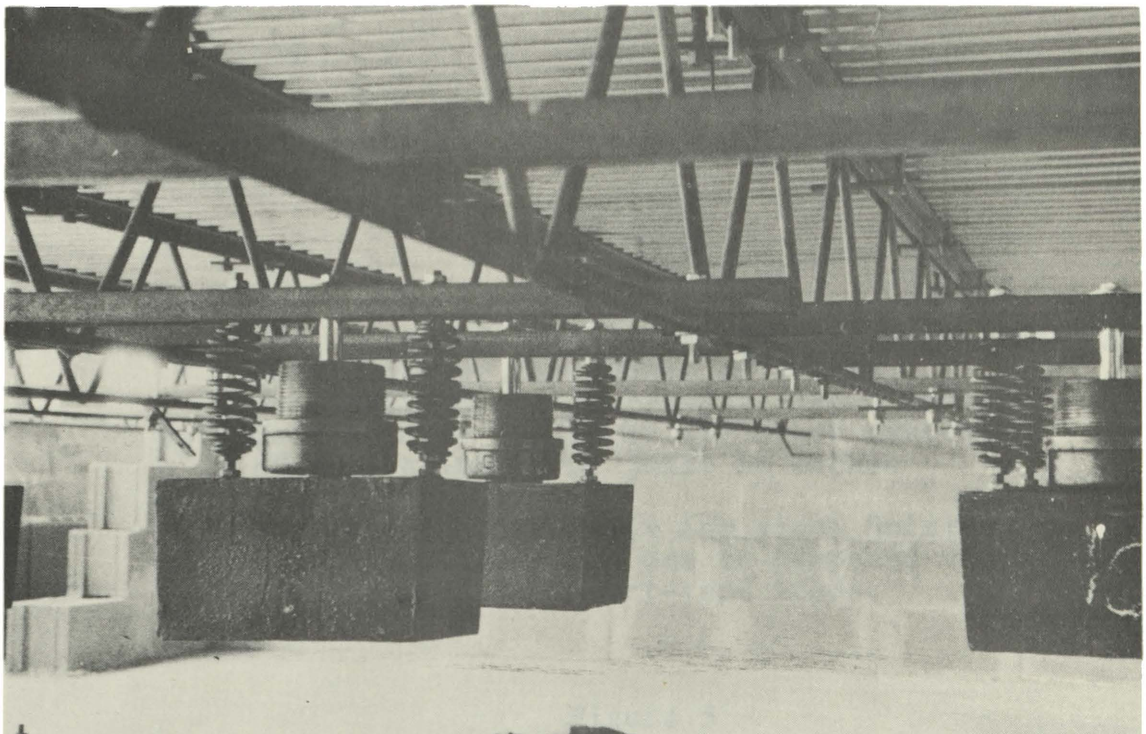


General view of centering installation showing horizontal bridging.

Figure 3
Second Test Floor

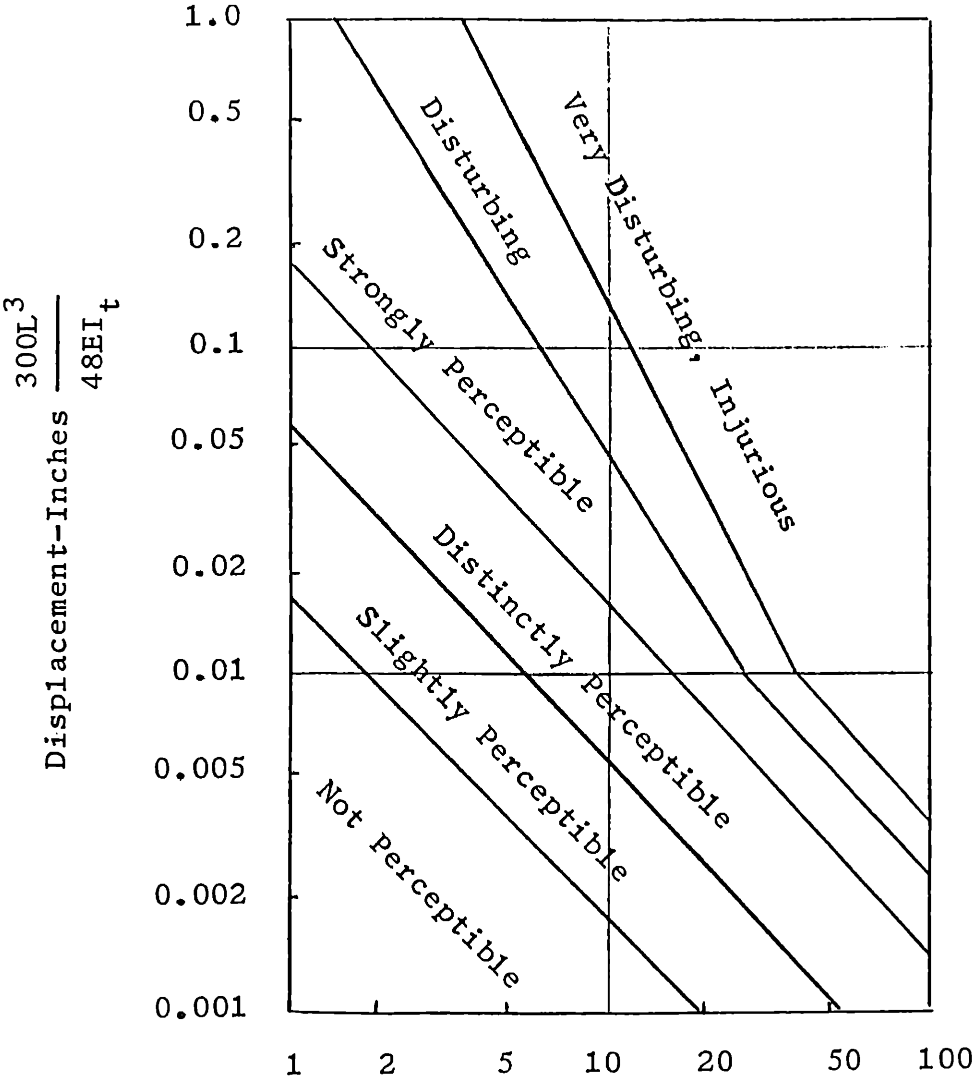


Detail view of Dynamic Vibration Absorber



General view of underside of floor with vibration absorbers installed.

Figure 4
Dynamic Vibration Dampers



Frequency - Cycles per Second =

$$1.57\sqrt{\frac{g\,EI_t}{w_d L^4}}$$

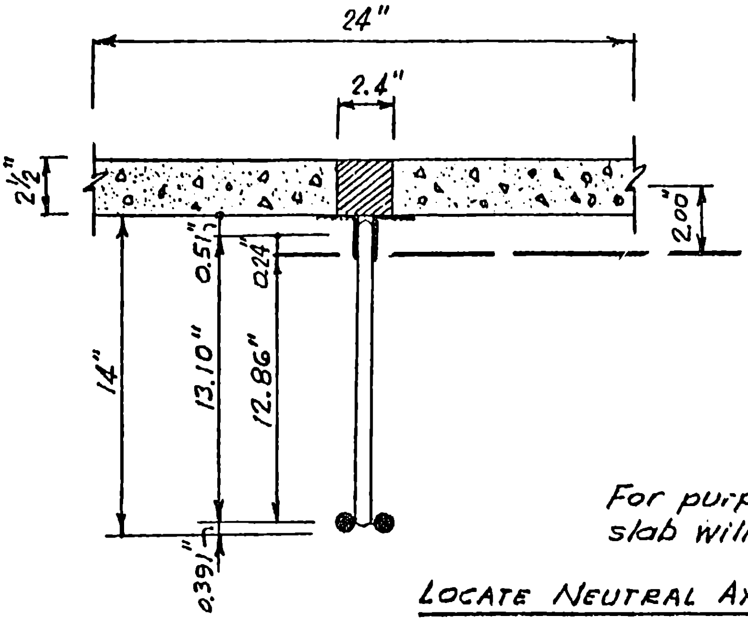
I_t = the moment of inertia of the composite T-section multiplied by the number of joists
g = acceleration due to gravity, 386.4 in/sec^2
w_d = dead load in lbs/in of the floor system
L = effective length of joist in inches

Suggested design curve for steel joist-concrete slab floors due to dampened vibrations caused by impact-type loads.

Figure 5

Graph of Reduced Human Response

Figure 6
Sample Calculations



JOIST TYPE : 14S7

Clear Span : 25'-0"

Top Chord: 2 Ls $1\frac{3}{4} \times 1\frac{3}{4} \times \frac{3}{16}$

Bottom Chord: 2- $\frac{25}{32}$" φ bars

Span/Depth Ratio: $\frac{(25)(12)}{14} = 21.4$

CONCRETE SLAB

2½" thick

Assume $f'_c = 3000$ psi, $n = 10$

Transformed Width: $\frac{24}{10} = 2.4$"

For purposes of vibration, joist and slab will act compositely.

DESIGN LOADS

Live Load 100 psf

Dead Load

Conc. 31 } 35 psf
Jsts. 4 }

TOTAL 135 psf

Capacity: 134 psf

LOCATE NEUTRAL AXIS OF COMPOSITE SECTION

Sum moments, centroid of bottom chord

SECTION	AREA	Y	A·Y
Bottom Chd.	0.959	0	0
Top Chd.	1.24	13.10	16.24
Slab	2.5 × 2.4	14.86	89.16
Sum	8.199 in.^{2}		105.40

$$\bar{Y} = \frac{105.40}{8.199} = 12.86 \text{ in.}$$

MOMENT OF INERTIA, COMPOSITE SECTION

SECTION	I_o	AREA	d	d^2	Ad^2	$I_o + Ad^2$
Bottom Chd.	0.04	0.959	12.86	165.38	158.60	158.64
Top Chd.	0.36	1.24	0.24	0.058	0.07	0.43
Slab	3.13	6.00	2.00	4.00	24.00	27.13

TOTAL : 186.20 in.^{4}

NATURAL FREQUENCY (DEAD LOAD ONLY)

$$f = 1.57\sqrt{\frac{gEI}{wL^4}} = \frac{4066}{L^2}\sqrt{\frac{I}{w}}, \text{ when } \begin{cases} \text{"}L\text{" is in ft.} \\ \text{"}I\text{" is in in.}^4 \\ \text{"}w\text{" is in lb. per lin. ft.} \end{cases}$$

$$= \left(\frac{4066}{(25)^2}\right)\sqrt{\frac{186.20}{2 \times 35}} = \underline{\underline{10.6 \text{ cycles per second}}}$$

DISPLACEMENT (½ AMPLITUDE)

Assume 300 lb. static load at center of span

$$d = \frac{PL^3}{48\,EI} = \frac{(300)(25)^3(1728)}{(48)(29{,}000{,}000)(186.20)} = \underline{\underline{0.031 \text{ in.}}}$$

From Fig. 5, vibration when no live load is on the floor may be expected to be "Strongly Perceptible"

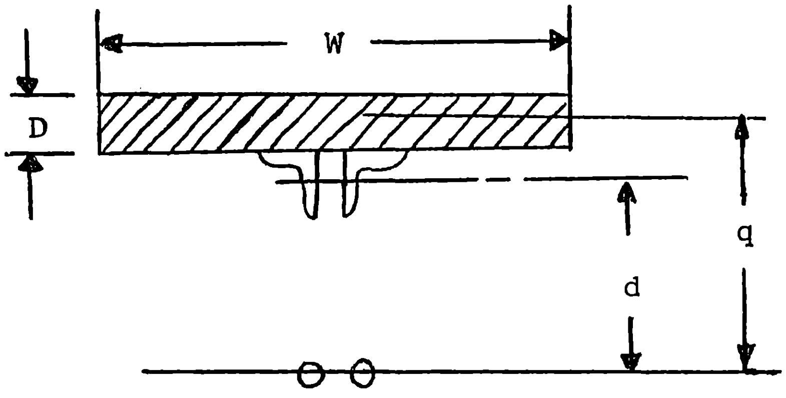
APPENDIX A

Computing Natural Frequency by T-beam Analogy

To find the moment of inertia for a steel joist-concrete slab floor with full interaction.

Notation:

Dimensions where listed are for the SJ-123 system.



	Cross section
d	effective depth joist, 11.358 inches
q	distance from centroid of lower chord to centroid of concrete, 12.97 inches
A_c	area concrete, (w) (D) = 60 square inches
A_u	area upper chord, .60 square inches
A_1	area lower chord, .497 square inches
E_j	modulus elasticity joist, 30×10^6 psi
E_c	modulus elasticity concrete, 3×10^6 psi
$\bar{r}$	distance from the centroid of the lower chord to the centroid of the composite section

	Transposed cross section
I	moment of inertia used in frequency calculation, full interaction
$\bar{s}$	distance from the centroid of the upper chord to the centroid of the composite section
$\bar{t}$	distance from the centroid of the composite section to the centroid of the transposed section
w	center distance of joists, 24 inches
D	maximum depth slab, 2 3/4 in.
I_u	moment of inertia of upper chord about its own centroidal axis, 0.8 inches^4
I_1	moment of inertia of lower chord about its own centroidal axis, negligible

n number of joists

N ratio $E_c/E_j = 0.1$

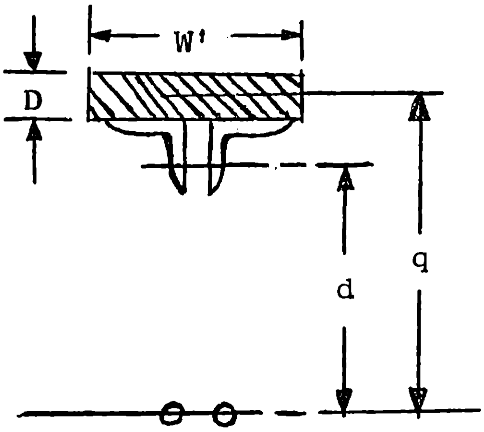
In calculations dealing with two different moduli of elasticity it is possible to transpose the cross section of one into an equivalent cross section of the other. In this case it is easier to transform the concrete into steel.

Since N is 0.1; the width w' of the transposed section will be $w' = 0.1w = wN$; the depth remains constant. In that the moment of inertia of a rectangular cross section is a function of the width of the first power, it is seen that

$$E_c\ I_c = E_s\ I'_c$$

where $$I'_c = I_c\ E_c/E_s = I_c N$$

As a result, for calculations, the cross section becomes



with A'_c area of transposed concrete section, $w'D = 6$ square inches

I'_c moment of inertia of transposed concrete section about its own centroidal axis,

$w'\ D^3.12 = 3.13$ inches^4

w' width of transposed concrete section, 2.4 inches

The case with full interaction is analogous to two beams one on top of another with shear connection between them.

$$I = n \left[I_1 + A_1\ (\bar{r})^2 + I_u + A_u\ (\bar{s})^2 + I'_c + A'_c\ (\bar{t})^2 \right]$$

where $\bar{r}$, the distance from the centroid of the lower chord to the centroid of the composite section is found from

$$(A_1 + A_u + A'_c)\,\bar{r} = A_u d + A'_c\,q$$

$$(.497 + .6 + 6)\,\bar{r} = .6\,(11.348) + (6)\,(12.97)$$

$$\underline{\bar{r} = 11.94 \text{ inches}}$$

and $\bar{s}$, the distance from the centroid of the upper chord to the centroid of the composite section is found from

$$\bar{s} = \bar{r} - d$$

$$\bar{s} = 11.94 - 11.358$$

$$\underline{\bar{s} = .582 \text{ inches}}$$

and $\bar{t}$, the distance from the centroid of the composite section to the centroid of the transposed concrete section is found from

$$\bar{t} = q - \bar{r}$$

$$\bar{t} = 12.97 - 11.94$$

$$\underline{\bar{t} = 1.03 \text{ inches}}$$

$$I = 9\,[0 + .497\,(11.94)^2 + (.08) + (.6)\,(.582)^2 + 3.3 + (6)\,(1.03)^2]$$

$$\underline{I = 726.00 \text{ inches}^4}$$

The moment of inertia resulting is the effective moment of inertia of one steel joist with its position of the concrete slab. The stiffness of the actual section is $E_j I$. The stiffness of the floor is the stiffness of one joist section times the number of joists, nIE_j.

The natural frequency of the floor if the stiffness of the floor lateral to the joist is small is that of a beam and is given in any basic vibration book.

$$f = 1.57\sqrt{\frac{gEI}{WL^4}}$$

in which

f = natural frequency of the floor in cycles per second

g = acceleration due to gravity = 386.4 inches/sec^2

EI = stiffness of the floor. For the steel joist-concrete slab floor it is the value defined above nIE_j

W = weight per inch of the floor and is equal to the weight per inch of one joist plus the weight per inch of the accompanying slab, 0.0868 WD, times the number joists,n

L = center to center distance between the joist support or clear length of the joist.

The frequency is effectively that of one joist with the accompanying slabs since the number of joists, n, occurs in both the stiffness factor, nIE_j, and the weight per length.

This method can be used for any joist configuration.

APPENDIX B

HUMAN SENSITIVITY TO VIBRATIONS

by

D. T. Wright
and
R. Green

A Report of an Investigation conducted in
THE DEPARTMENT OF CIVIL ENGINEERING
QUEEN'S UNIVERSITY
in co-operation with the
ONTARIO DEPARTMENT OF HIGHWAYS
as a part of
THE ONTARIO JOINT HIGHWAY RESEARCH PROGRAMME

Report No. 7.
Queen's University
Kingston, Ontario.
February, 1959

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I. INTRODUCTION

Human sensitivity to vibrations poses a serious technical problem concerning engineers in a wide variety of fields. The general nature of the problem, is, of course, easy to grasp; it is readily apparent that there are both physiological and psychological reactions when humans are subjected to vibration. When the exact nature of the reactions and the varying stimuli are studied, the very complex nature of the problem becomes apparent.

The present review of the susceptibility of a human being to mechanical vibration has been completed in connection with a study of highway bridge vibrations. In the course of attempts to discover suitable criteria for evaluating human sensitivity to vibrations it became apparent that some of the most readily obtainable information was incomplete and often misleading. The scope of the present study is limited to human reactions to mechanical vibrations - so-called simple harmonic motions. There is an inevitable overlapping between this and other types of motion, such as pure acceleration, and it is also difficult to discuss the effects of vibrations on humans without considering the effects of vibrations on structures. Such associated topics will however, only be mentioned inasmuch as they relate directly to human sensitivity.

This field of study may be considered part of the general subject of human engineering - a term which has been put to fairly wide use in the past few years. In human engineering there is an attempt to bridge the gap between the engineer and the physiologist and psychologist, in relating humans to their environment in a technical world. Human sensitivity to vibrations is important in a wide range of practical problems. In the field of transportation there is concern for comfort in automobiles,[8, 25, 48] the design of trucks for safety,[38, 39] considerations of comfort in civil aircraft[37], and in the design of military aircraft for maximum efficiency.[17,20] Blasting must be carefully controlled to limit both damage and undesirable human reaction.[4,12,44] There is concern for the residents of houses subjected to vibrations due to railway traffic[33] and industrial machinery.[15] It has even been suggested that human sensitivity to motion be used as a basis for the design of highway curves.[35] A fairly complete bibliography is appended to this report covering original researches, other reviews, applications,etc.

Many papers have been written on the subject, giving experimental information, scales of sensitivity, and various reviews of the problem. Though a large number of reviews have been published, even the most ambitious[15,27,32,45,50] were found to be inadequate in explaining the phenomenon completely. Most shortcomings in reviews have been due to limited interests related to specific fields of application. There has also been a tendency to over-simplify what is essentially a very complex matter. Finally in the usual way of things, some of the most valuable information has been published only in obscure esoteric publications.

II. EFFECTS ON HUMANS.

Available data cover a variety of motions including pure accelerations,[34] jerk, (the rate of change of acceleration),[42] and a wide range of simple vibrations from very low frequencies[49] (associated with motion sickness) to very high frequencies[10] of the order of thousands of cycles per second. Of most common interest is simple vibration with a range of frequencies from 1 to about 100 cycles per second.

The human perception of motion is the result of sensations received from the muscular and skeletal systems of the body. A distinction usually made between skin sensitivity, as touch and kinesthesis, the sense through which motion, weight, position etc. of the human body are perceived. A great controversy concerning the possible existence of a separate sense of vibration was settled by Geldard as recently as 1940. Proprioception, the mechanism of kinesthesis, refers to the means by which bodily positions and movements are discerned by the individual. The organs of proprioception are minute nerve endings in muscles, tendons and the inner ear which relay sensations to the central nervous system. Here the multiplicity of sensation is integrated to give an accurate picture of the circumstances of the body. Thus , one can be aware of the motion or position of a leg without being able to see it. The present review is most concerned however, with the "equilibrium" of the individual which is the integration of proprioception sensation from each portion of the entire body.

The human body is very sensitive to mechanical vibrations - so sensitive in fact, that vibrations considered to be almost intolerable to humans are required to do severe structural damage. As a result of this sensitivity and lack of experience, people are generally inclined to overestimate amplitudes or severity of vibration. Consequently human sensations are of little use in estimating the effects of vibrations on buildings or other structures.

Levels of human reaction may be established either on the basis of subjective sensations or from objective tests. Although subjective sensations cannot readily be treated quantitatively, most work has been done on this side through assessing degrees of annoyance, nervousness, or unpleasantness, as due to the subject's cognizance of resonance in the body organs, or when the subject experiences muscular tension due to tendencies to vibration of various parts of the body. Objective tests, indicating various malfunctions of the body are almost as hard to define quantitatively, and although many tests have been performed to study physiological reactions to vibrations, such reactions are rarely used for assessments of vibration strength.

The actual effects of vibrations on humans include changes in pulse rate, blood pressure, reaction time and acuity of vision, headache, fatigue nausea, and even death. The well-known patellar reflex will, in fact, disappear with moderate vibration after a very short time. A very special human reaction to vibration is a form of Raynaud's disease[13, 17] which is associated with the use of high speed grinding and chipping machines.

Many tests have been performed to determine the effects of vibrations on humans, both in terms of subjective sensations and physiological reactions. Most of the available data have been obtained from tests using shaking tables, where the subjects are seated or otherwise placed on a table which oscillates. It is obviously easy to do tests with varying frequencies and amplitudes, and varying lengths of time in this fashion. Environment is, of course, most important, and further data have been obtained from experiences in automobiles and aircraft, etc. under performance conditions.[17,20] Though such data are valuable, they are of course, much more difficult to obtain. One of the first complete experimental studies of the problem was completed by Reiher and Meister[46] who subjected some ten people, aged 20 to 37 years, to frequencies ranging from 3 to 70 cycles per second with amplitudes ranging from 4/ 10,000th of an inch to 4/ 10th of an inch. The subjects were placed in varying attitudes on the test table and Figure I shows the results for persons standing or sitting with vertical vibrations. It can be seen that the results are assessed in varying levels of sensitivity from perceptible or barely perceptible to discomforting and even intolerable severity. Sensitivity, of course, depends on the attitude of the body with respect to the source of vibration, and is most acute when the subject is lying down with vibrations in a horizontal direction perpendicular to the body axis. Minimum sensitivity occurs with the subject lying down and subjected to vertical vibrations. Figure I, as has been explained, shows the effects with vertical vibrations and the subject in a normal standing or sitting position. This case appears to be of most general interest and value in engineering applications and further discussions are limited to these conditions. Similar tests were made by Joplin[22,24] in connection with comfort of people in automobiles. In his tests, Joplin seated the subjects on car cushions. Suyehiro,[51] it might be noted, placed his subjects "kneeling, in the Japanese fashion", and provided useful data on lower frequencies. During the war, Coerman[9,10] made very exhaustive tests confirming Reiher and Meister's subjective scales and going on to study a variety of physiological reactions. Coerman showed that the patellar reflex can be paralyzed in a few seconds under moderate vibration which indicates that the main effect of vibrations lies in the province of neuro-physiology. He also made detailed studies of reaction times, visual effects, blood pressure and circulation, hand steadiness and pulse rates, etc. Coerman and later investigators, such as Franke,[18] have succeeded in explaining some of the confusing complexity of human reactions. They found that the elastic constants of various parts of the body - bone, cartilage, muscle tissues, etc. - vary considerably and that different parts of the body have resonant frequencies in the range of 2 to 20 or 30 cycles per second. For instance, the main body mass of the trunk resonates at about 2 1/2 cycles per second in most humans. The natural frequency of the suspended heart in the chest cavity is about 5 cycles per second. Similarly it has been found that the eyes tend to resonate in their sockets and that even the head tends to vibrate with respect to the body at certain exciting frequencies. Some additional complexities of the problem are shown in studies of motion sickness which is associated with frequencies somewhat lower than are of interest generally with respect to mechanical vibrations. It has been found that motion sickness is caused by angular accelerations that are barely on the threshold of perceptibility, and that motion sensed only visually will also produce sickness.

Experimental data produced by the principal investigators noted above, and others, was summarized by Goldman[21] in a most interesting and useful manner (see Figure 2). Taking all available data Goldman summarized results on the basis of the three fundamental levels of subjective sensation, i.e. first perception, discomfort and tolerance. By "discomfort" is meant pain or the sensation of muscular reaction, etc. The limit of "tolerance" is still below the level of vibration that would cause permanent injury to the human body. Though Goldman does not provide as many levels of sensitivity as Reiher and Meister, for instance, the completeness of his results makes them very valuable indeed. In Figure 2, the average values for each level of perception are indicated by solid lines with standard deviations plotted above and below in each case. It is noted that the standard deviation is about one half of the average value and on the logarithmic plot occupies half a log unit in width. This permits a very simple presentation and of course also shows the natural range of scatter in representative samples of humans. The complex shape of the curve - the fact that it does not fit any single straight line or set of geometric curves to which functions might be fitted --reflects the actual complex nature of the reaction. At low frequencies, for instance, the large looser portions of the body mass tend to vibrate, while at higher frequencies, resonances may occur here and there depending upon characteristics of various parts of the body. Figure 3 produces Goldman's results in a slightly different form, and shows even more clearly the difficulty of interpreting the results in terms of any single common parameter. The vector arrows indicate the directions the curves would have to follow to suit any of the ordinary functions: acceleration, velocity, etc. It is apparent that while for some parts of the frequency range, one or other parameter might be suitable, no single parameter is adequate to cover the general phenomenon.

It should perhaps be noted that the actual amplitudes of vibration required for unpleasant sensations are really quite small. It is also significant that, except in tests under controlled circumstances, it is not common to find vibrations of only a single frequency and amplitude existing. Coerman[9] showed that the effects of changing vibrations were considerably more severe than those due to simple continuing regular harmonic motion. It has been suggested that in the presence of complex harmonic motions the effective degree of intensity may be estimated by doubling the amplitude of the major component.[47]

III. SCALES AND PARAMETERS.

It is apparent that some quantitative scale or standard is required for the assessment of practical problems involving the human perception of vibrations, in order to secure objective evaluations for comparisons, for example, in cases of legal dispute. The difficulty in establishing scales is that no single criterion or function seems to provide an accurate approximation for the experimental values as they have been revealed (see Figures 2 and 3). In spite of these difficulties many parameters or scales have been suggested. In studies of the effects of vibrations due to subway trains, Mallock[33] suggested that for equal sensation "an^2" (where a is maximum acceleration and n is frequency) should be a constant. In studies of steamships, Melville[43] suggested that the parameter should be "an^3". Another early work by Digby and Sankey[16] suggested that "an" should be the parameter. Reiher and Meister carefully avoided suggesting parameters in describing the nature of their results, but another contemporary German investigator, Zeller[59], introduced the unit PAL (from the Greek "palleur" - to palpitate or vibrate). This was intended to be used in a similar way to the decibel or phon scale used for describing intensities of sound. The PAL is defined as equal to $10 \log_{10} 2x$ where x is equal to a^2/n (with 'a' in cm./sec.^{2}). The German standard for vibrations, DIN 4150[1] is based upon Zeller's PAL scale. Joplin[23] found that his curves of equal sensitivity or perception could be fitted by formula of the type $ae^{0.6n}$ = constant. Crandall[11] introduced the energy ratio which is $(a/n)^2$ (with 'a' in ft./sec.^{2}) and is used in connection with blasting. It is suggested that for an energy ratio less than 3 no damage will occur, that between 3 and 6 caution is required, and that with values over 6 damage is likely.

In comparing all these various scales and parameters which have been suggested, having in mind Goldman's results, it is apparent that none of the proposed scales can be recommended for general use.

Notwithstanding this conclusion, it is, of course, reasonable to use certain scales in various special applications where experience has shown them to be safe within a narrow range of frequencies or amplitudes experienced. The difficulty, of course, arises in extrapolations of a parameter derived for one kind of work, to other problems. In such applications the wrong parameter can give very misleading, if not dangerous, results.

In an effort to avoid some of these limitations, Janeway[3] has described levels of equal sensation in terms of a series of parameters. From one to six cycles per second he suggests that equal reaction is given by constant jerk; from 6 to 20 c.p.s.-constant acceleration; and from 20 to 60 c.p.s. - constant velocity. All of these are to be used only for simple harmonic motion. Janeway's results published by the Society of Automotive Engineers,[3] have come to be considered almost as a standard in the automotive industry (Figure 4). Though the shape of Janeway's curve, fitted as it is to various parameters, is reasonably good, Janeway's criterion is unfortunately not very useful for most practical problems - because only one limit or level of intensity is defined. For applications in the automotive industry in which Janeway is interested a single level of intensity is perhaps adequate for design purposes. However, in trying to assess the severity of vibrations in a given situation, there is no way of telling the relative significance of variations above or below the Janeway standard and with logarithmic plots a small deviation may, of course, have a large significance. It is apparent then, that a scale of values and intensities is needed for the sake of analysis and assessment.

Seismic scales appear to some investigators to be of use in the problem[14, 15] but upon examination it is apparent that most of the seismic scales are as subject to errors in evaluation as the scales already discussed. Quantitative levels are not often defined in earthquake scales: for instance, degrees of severity are defined as "when chimney pots fall off"[50], and the graduations in seismic scales are generally too coarse for application to problems of human perception of mechanical vibrations.

A recent contribution by Koch[30] introduced a scale of VIBRARS based on a logarithmic ratio, like Zeller's PAL. Koch's scale of vibrars is plotted in Figure 5 and enables a numerical assessment of the magnitude of a vibration to be made with ease. Since the lines of equal vibrar strength are roughly parallel to the Reiher-Meister scales, it is apparent that a scale of vibrars would be useful over a reasonable range of frequencies.

In the face of the existing diversity of scales and intent on the part of many investigators, one would be rash indeed to propose a new scale. It is apparent however, that none of the existing scales truly fits given experimental data as typified by Goldman's results. Yet it is equally apparent that for engineering and technical application a numerical value for the level of vibration would be eminently desirable. Since no functional relationship can be shown to fit the best curves available, and further, considering that there is no real need for a mathematical function to express this very complex human reaction, it would seem a very simple matter to fit a scale of numerical values to the curves produced by Goldman. Goldman defines three levels of perception and shows roughly a 2 to 1 range between the average and the standard deviation at each level. The upper two levels are more or less contiguous at their extremities and there is a gap between the first and second scales.

Taking these lines as contours of equal sensation ("isosensors" - ?) with arbitrary increments in tens, the results shown in Figure 6 are produced. The various levels have the following significance;

below 0	...	imperceptible
0 to 10	...	perceptible to some
10 to 20	...	perceptible to most
20 to 30	...	perceptible to all
30 to 40	...	unpleasant to some
40 to 50	...	unpleasant to most
50 to 60	...	intolerable to some
60 to 70	...	intolerable to most
above 70	...	intolerable to all

Though this approach is by no means ideal, it combines the usefulness of a quantitative assessment with the accuracy of Goldman's curves.

IV. CONCLUSIONS.

In summing up, it is clear that the human reaction to vibration is very complex indeed , as so well demonstrated by Goldman. It follows that, in spite of many proposals, no single parameter or function can give consistent descriptions of the severity of vibrations over a practical range of frequencies. Furthermore, it is also clear that human perceptivity can be thought of as a spectrum of subjective reactions through a variety of shades all the way from imperceptible to intolerable. Though a single level of vibration intensity can be readily used in design as a safe upper limit, analysis and evaluation of vibration require more explicit descriptions of severity. It is also evident that, for ready comparison, quantitative rather than qualitative descriptions are most useful.

From these considerations the chart shown in Figure 6 was evolved, based on Goldman's summaries, and giving the results as contours of equal sensitivity ("isosensors") with a set of arbitrary numerical values.

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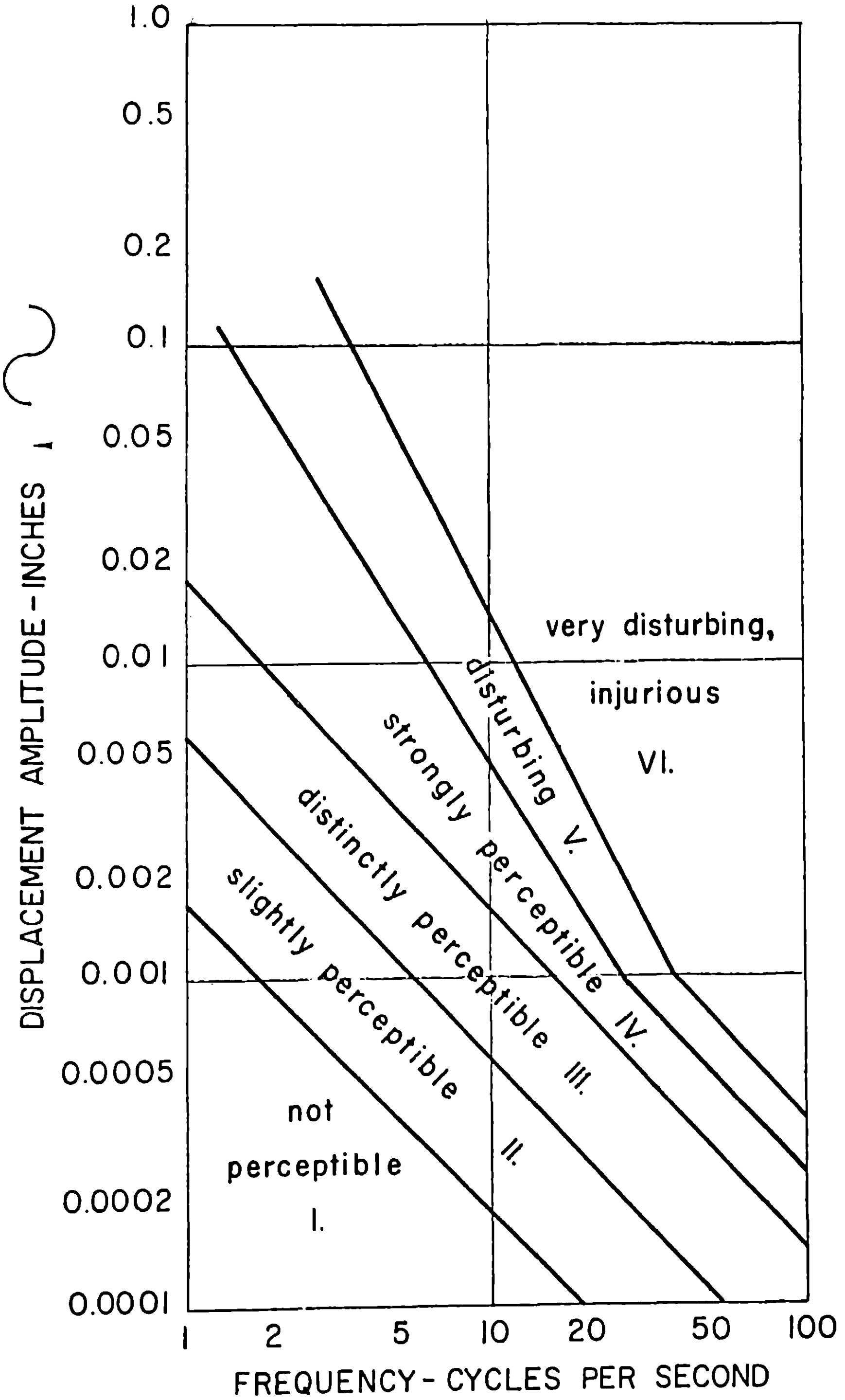

FIG. I. DOMAINS OF VARIOUS STRENGTHS OF SENSATIONS FOR STANDING PERSONS SUBJECT TO VERTICAL VIBRATION, AFTER REIHER AND MEISTER.

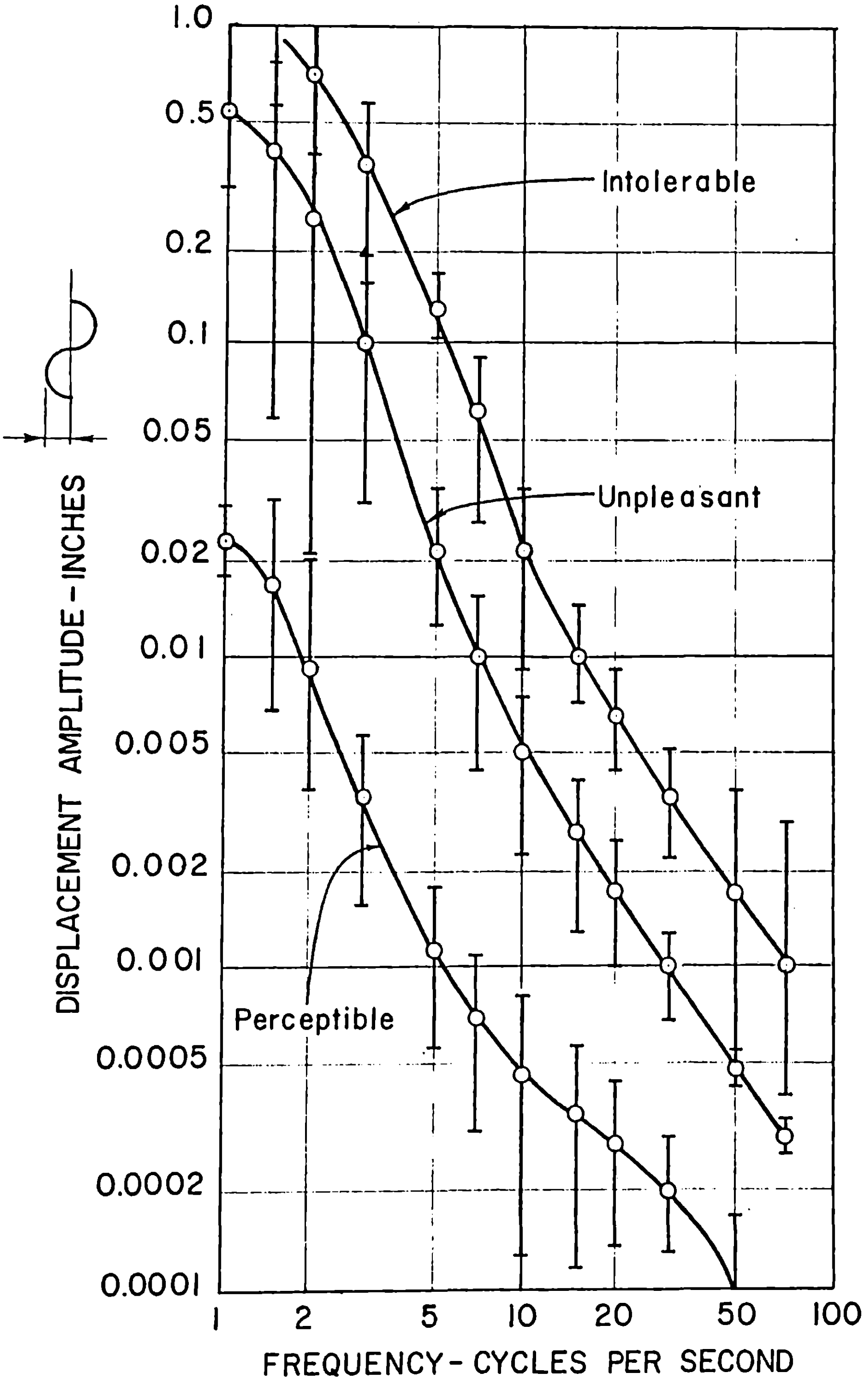

FIG. 2. SUBJECTIVE RESPONSES OF THE HUMAN BODY TO VIBRATORY MOTION, AFTER GOLDMAN.

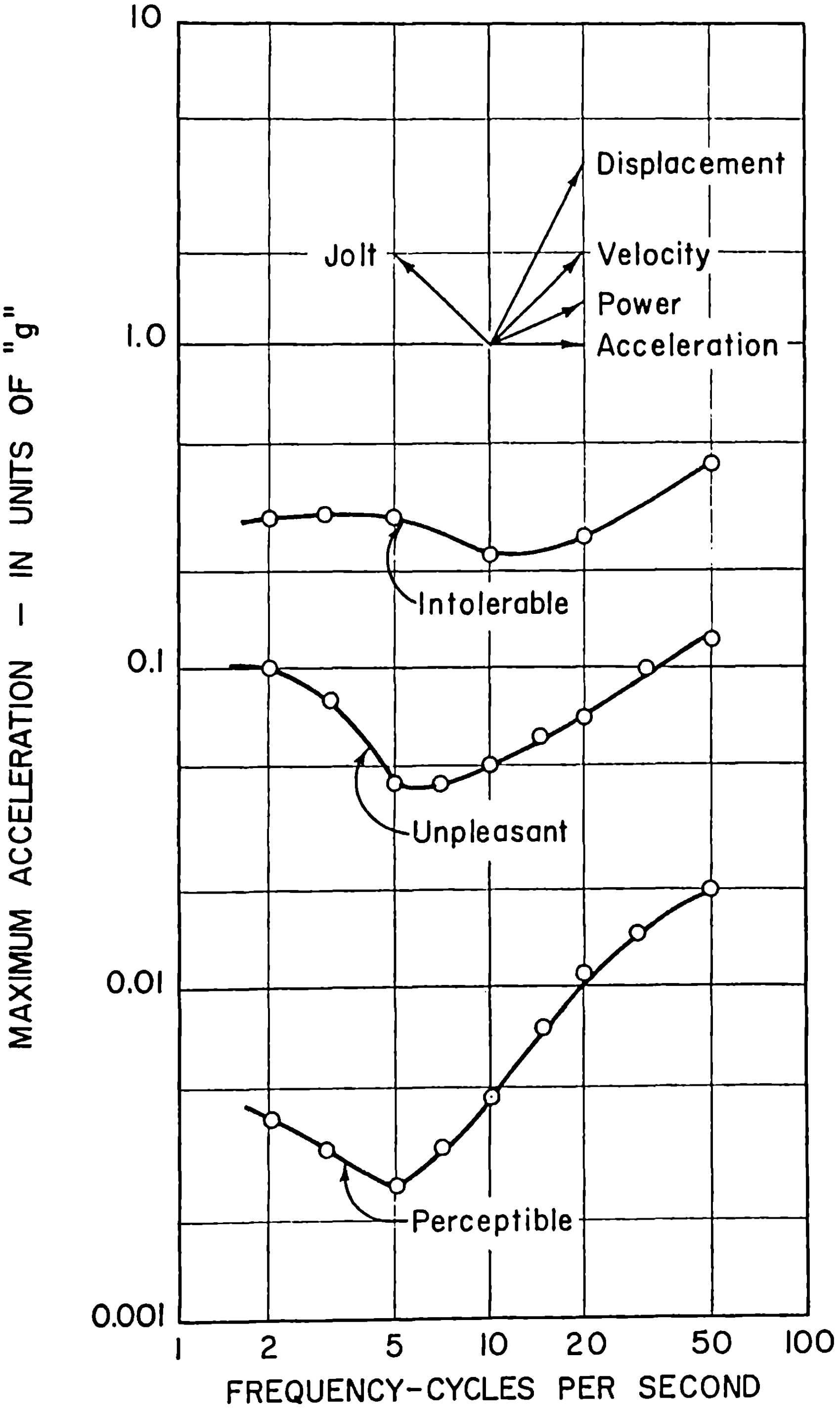

FIG. 3. SUBJECTIVE RESPONSES OF THE HUMAN BODY TO VIBRATORY MOTION, AFTER GOLDMAN.

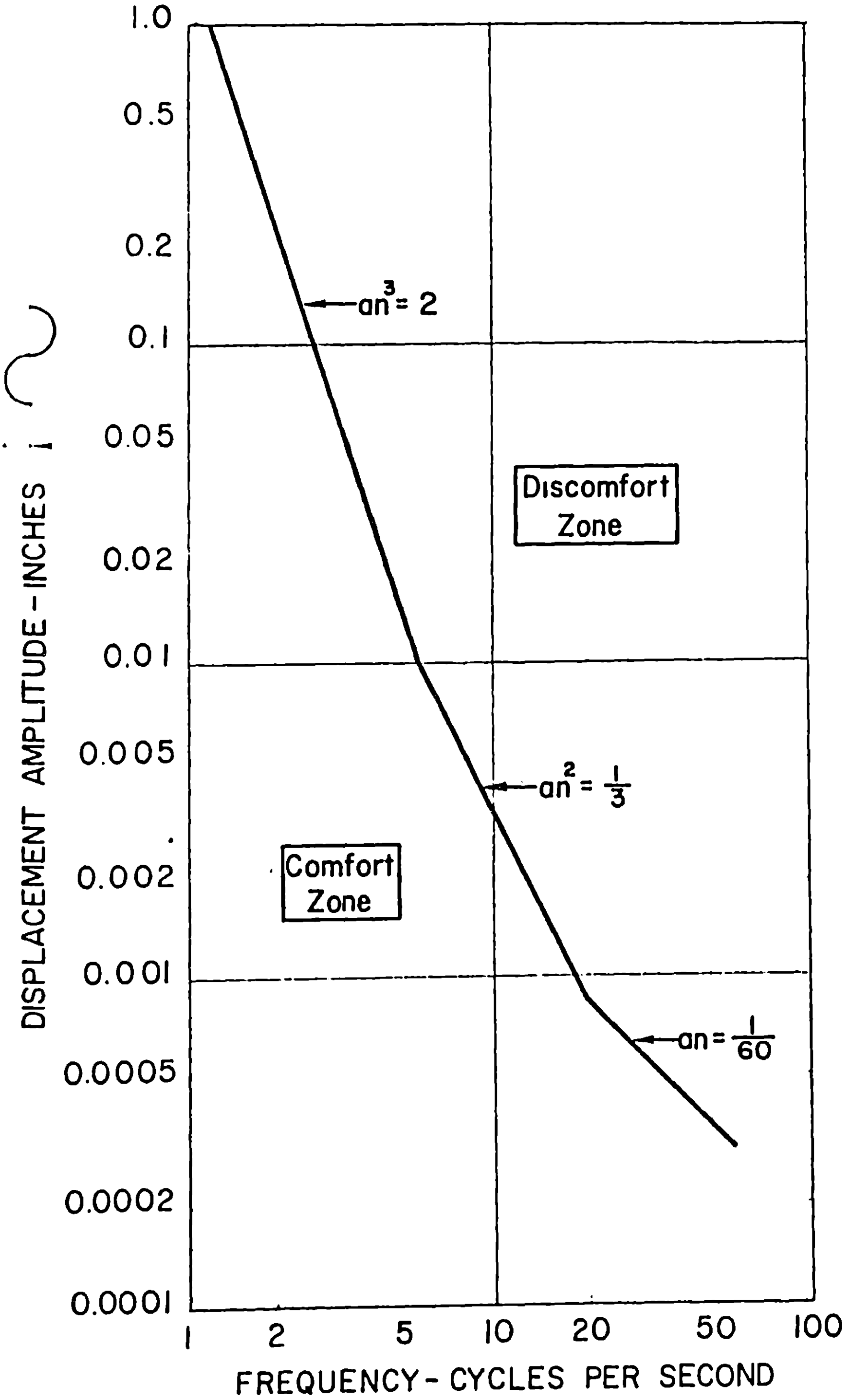

FIG. 4. VERTICAL VIBRATION LIMITS FOR AUTOMOBILE PASSENGER COMFORT, AFTER JANEWAY.

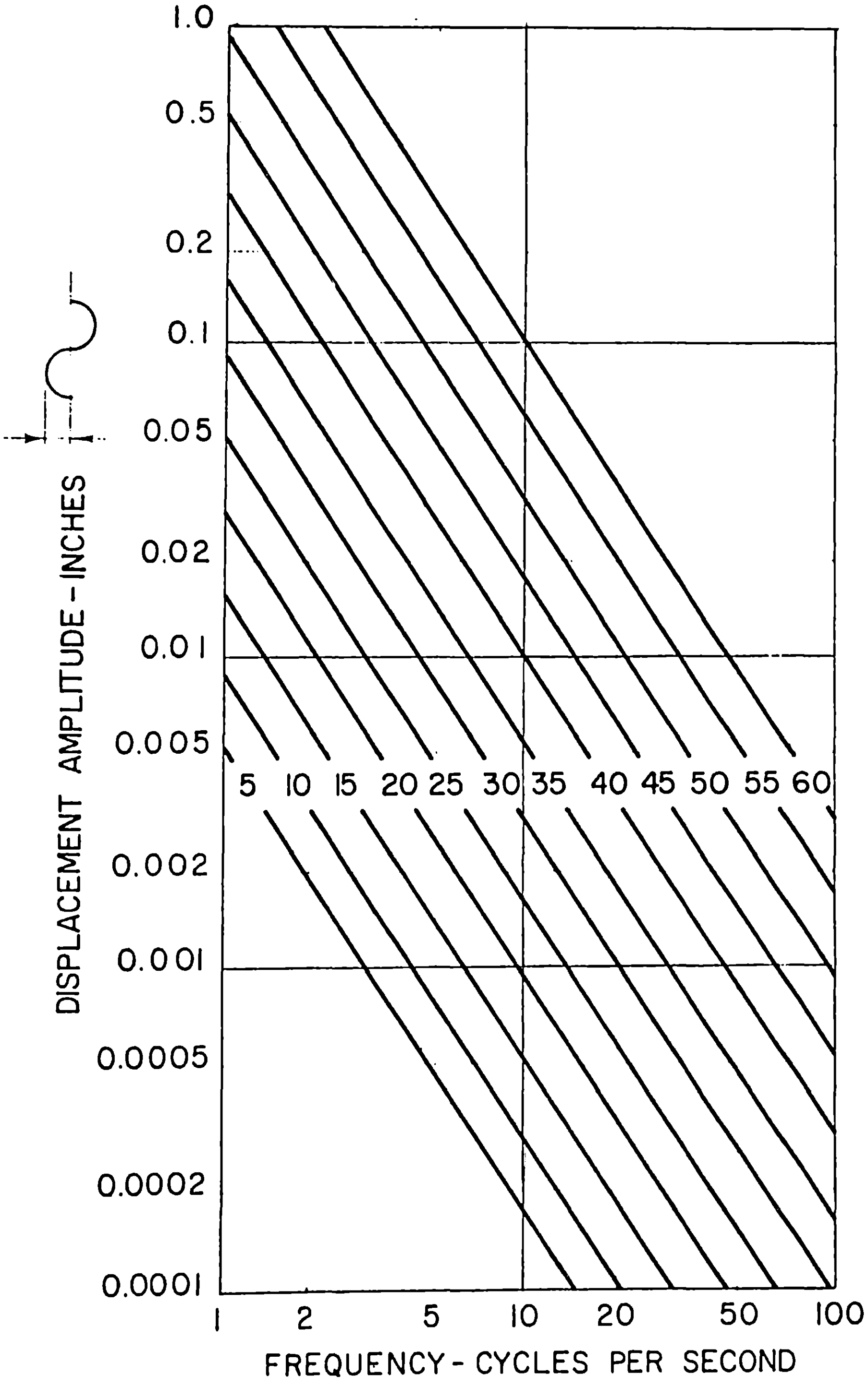

FIG. 5. VIBRATION STRENGTH IN VIBRARS, AFTER KOCH AND STEFFENS.

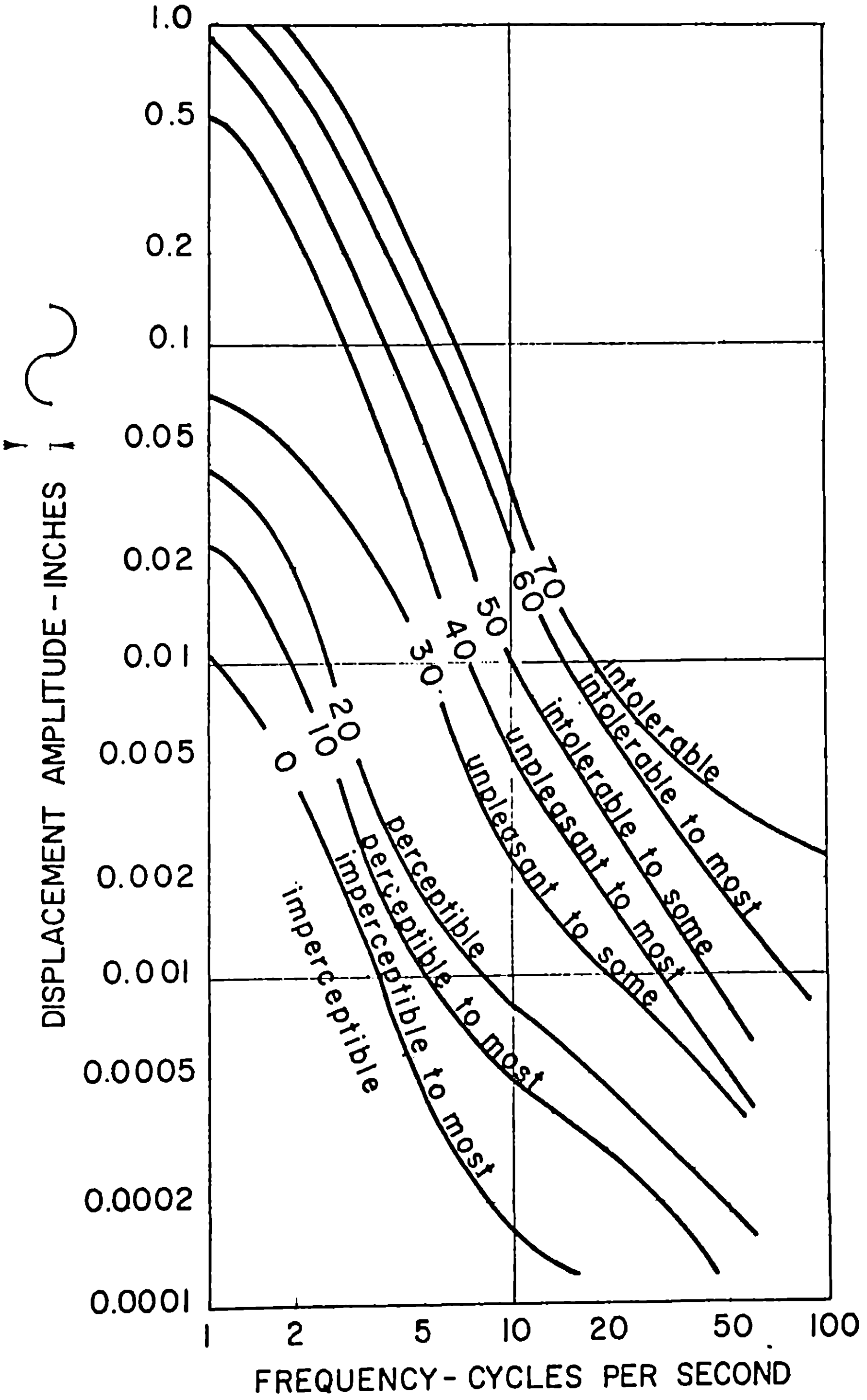

FIG. 6. CONTOURS OF EQUAL SENSITIVITY TO VIBRATION-"ISOSENSORS"

APPENDIX C

Design of Vibration Dampers

The dynamic vibration absorber used to damp the undesirable vibration response of the finished floor may be designed using the following procedure. (The subscripts 1 and 2 refer to the floor and vibration absorber respectively).

1. Determine the natural frequency, f_1, of the loaded floor in cycles per second by using recording equipment similar to the equipment described in this paper. Excite the floor systems with an impact. The amount of internal structural damping may be determined from the recorder trace. If this is not possible, use the method outlined in Appendix A.
2. Determine the weight of the floor system from the construction blue prints of the building and the weight of the office equipment supported by the floor. The mass, m_1, of the floor may be obtained by dividing the weight of the floor by the acceleration of gravity, 386 in/sec^2.

$$m_1 = W/386 = \text{lbs-sec}^2/\text{in}$$

3. Use a mass ratio of the vibration absorber to the floor of .02. The mass of the vibration absorber, m_2, will be

$$m_2 = .02\ m_1$$

The mass of the vibration absorber may be subdivided into smaller units, from 50 to 100 lbs. for example, for ease of manufacture and installation.

An even number of units should be used on a floor with an odd number of joists and vice versa for the sake of symmetry.

4. The natural frequency, f_2, of the vibrationaabsorber should be tuned from 1/4 to 1/2 cycles per second below the determined natural frequency, f_1, of the loaded floor. This will produce the beat frequency phenomenon as mentioned in Chapter I. Therefore, the natural frequency of the vibration absorber will be

$$f_2 = (f_1 - 1/2) = \text{cps}$$

5. Change the natural frequencies, f_1, and f_2, in cycles per second to circulate frequencies, ω_1 and ω_2, in radians per second.

$$\omega = 2\pi f = \text{Radians/second.}$$

6. To obtain the total spring constant of the springs to be used with each unit, multiply the mass of the unit, m_2, times the circular frequency of vibration absorber, squared, ω_2^2 .

$$k_2' = m_2\omega_2^2 = \text{lbs/in.}$$

The spring constant for each spring to be designed as in Chapter V will be the total spring constant, k_2, divided by the number of springs to be used. An even number of springs should be used to obtain a symmetrical unit. The design of the individual springs is based on the spring constant $K = d^4G/64R^3N$ in which the shearing modulus of elasticity, G, is 11.5×10^6 psi, N is the number of active coils in the spring, d is the diameter of the wire to be used in the spring, and R is the mean radius of the helix of the spring. At this point in the design of the vibration absorber the unit should be assembled and tested to determine if the calculated and actual frequencies, f_2, are the same.

If the natural frequency is greater or less than the expected natural frequency the mass of the unit may be adjusted to bring the calculated and actual frequencies to the same value by using the relationship

$$\omega_2 = \sqrt{k_2/m_2}$$

7. When designing the dash-pot, the damping fluid, the diameter of pipe for the cylinder, and the thickness of the material to be used as a piston must be arbitrarily selected.

a) Select a fluid having a fairly high dynamic viscosity, μ , so that the base clearance between the piston and cylinder will be sufficient to keep the piston from hitting the cylinder wall due to transverse motion of the unit. Heavy weight motor and fuel oils are good damping fluids.

b) The cylinder may be made of a steel pipe nipple with pipe caps to form the ends of the dash-pot. The inside iiameter of the cylinder should be machined to a smooth surface. cylinder diameters may vary from two to three inches.

c) Select the thickness of the piston to be used. The thickness may vary from .25" to 1.0". The thickness should not be less than .25 inches because this will introduce turbulence into the flow of fluid past the piston.

8. The base clearance, C_o, between the cylinder wall and the edge of the piston may be determined using the following theoretical formula,

$$C_o = R\sqrt[3]{12\pi\mu L/5c}$$

where the coefficient of viscous damping, c, is found by using a damping ratio of .075. L is the length of piston, R is mean radius of the cylinder, and μ is the dynamic viscosity of the damping fluid.

This damping ratio was found to be approximately the optimum value of damping to critical damping when using the dynamic vibration absorber with steel joist-concrete slab floor systems. If the damping ratio increases, the effect of the vibration absorber decreases until the relative motion between the floor and the unit has been eliminated. With the absence of relative motion between the two systems the damping effect of the vibration absorber will be lost.

$$c = 2\gamma m_2 \omega_2$$

Using the above procedure, a dynamic vibration absorber can be designed to eliminate the undesirable oscillatory motion in steel joist-concrete slab floor systems.