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Final Report

EFFECT OF THE VARIATION OF STRUCTURAL PARAMETERS ON THE VIBRATIONAL CHARACTERISTICS OF STEEL JOIST-CONCRETE SLAB FLOOR SYSTEMS AND SUGGESTED DESIGNS

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EFFECT OF THE VARIATION OF STRUCTURAL PARAMETERS ON THE VIBRATIONAL CHARACTERISTICS OF STEEL JOIST-CONCRETE SLAB FLOOR SYSTEMS AND SUGGESTED DESIGNS

INTRODUCTION

Background

In 1962, Dr. Kenneth Lenzen of the University of Kansas issued a final report[1] summarizing therein all investigation completed at that time under sponsorship of the Research Committee of the Steel Joist Institute. That investigation set out to answer the questions: "How can annoying vibrations be eliminated in the initial design?" and "How can annoying vibrations be eliminated in a floor which is subject to them?" During the course of that investigation two test floors were constructed, various field investigations were conducted, human response was investigated, various damping means were designed and considered, and some analytical work on the general problem was completed. An appreciable store of information was gathered on floor vibrations.

However, for each question answered by that investigation new and important questions arose. As a result, several facets of the problem remained to be investigated more thoroughly. These included:

1. Verification or revision of the deflection formula;
2. Verification that joists and slab act as a single unit during vibration (that is, substantiation of the "T" beam analogy for sections);
3. Verification that bridging has little or no effect on the characteristics of floor vibration;

4. Determination of the effect of a slab supported on all edges (plate action rather than beam action);
5. Substantiation or revision of the impact effect;
6. Determination of the effect of the joist spacing and slab thickness on vibrational characteristics;
7. Determination of the number of joists (that is, area of a particular floor) which undergo deflection on initial impact, and
8. Determination of the distance between partitions that would cause the floor to act as an open floor rather than a partitioned one. (Previous tests had indicated that partitioned floors had no annoying vibrations).

Current Test Series

In order to further investigate these facets of the problem under controlled conditions a new test floor was constructed. This floor was much larger than either of the two previous floors and rested on a foundation which provided fixed support on three sides with a moveable wall supporting the fourth side. Bridging anchors were provided on the joists so that various bridging configurations could be installed. Also, the joists were installed in such a manner that they could be lowered or removed as desired. Thus, the new test floor lent itself both to the type of testing already completed and to the determination of essentially the complete set of fundamental properties of such floor systems.

Concurrent Analytical Developments

In the meantime, Dr. Marek Sokolowski of the Polish Academy of Sciences and Dr. Lenzen considered the analytical development of a more precise theory of steel joist-concrete slab floor systems. In their work they considered the problem of deflection and vibration of orthotropic plates, a generalization of the floor system. Rather than approaching the problem from the often used energy method (Ritz Method) they solved the deflection and vibrational problem from the more classical force-equilibrium viewpoint. As a result, a consistent theoretical background is available with which to predict the characteristics of any particular floor design.

Application of the Experimental and Analytical Developments

The overall objective of these investigations is, of course, to provide sufficient information to allow the design of better (less annoying) floor systems and to allow the elimination of annoying vibrations in present floor systems. This objective has been realized to a great extent by the application of the experimental and analytical developments by Mr. Ron Gibson[2] to both the calculation of the characteristics of the common floor systems which might be constructed with presently available joists and to the data already gathered on field installations by Mr. Joseph E. Keller.[3,4] A wealth of information has been compiled in tabular and figure form which should prove very useful in the setting of future specifications for the construction of floor systems, and also in better understanding the problems evident in floors for which complaints have been made.

CURRENT TEST SERIES

General

The test floor (Figure 1) constructed for the current test series consisted of twenty-five thirty-two feet, nine-inch, open-webbed, steel joists at twenty-four-inch spacing supported by a three-sided, fixed, concrete block foundation which was forty inches high. This supporting foundation was fabricated using eight by eight by sixteen-inch concrete blocks. Lateral step supports were build at three positions along each of the long sections of the foundation. Each end of each joist rested on 4 two by four by eight-inch concrete bricks. These bricks could be removed to allow removal or lowering of any joist.

Standard Corruform centering was spot-welded to the top of the joists and a two-and-one-half inch thick (maximum thickness) concrete slab was cast on the centering. This slab was made of three thousand psi ready-mix concrete. It was reinforced with six by six-inch # 6 wire mesh placed in the center of the slab. Casting the slab forced the wire mesh to the bottom and, although some effect was made to bring it back to the center, subsequent testing showed the wire to be just above the centering. The overall dimensions of the slab were thirty-two feet - nine inches by fifty-one feet.

To allow the active length of the floor to be varied, three angle-iron stands (Figure 2) were constructed on top of which a section of eight-inch, wide-flange I-beam was placed. The stands could be raised either by installed elevation screws or by placing hydraulic jacks under each end. The three sections spanned the entire width of the floor and could

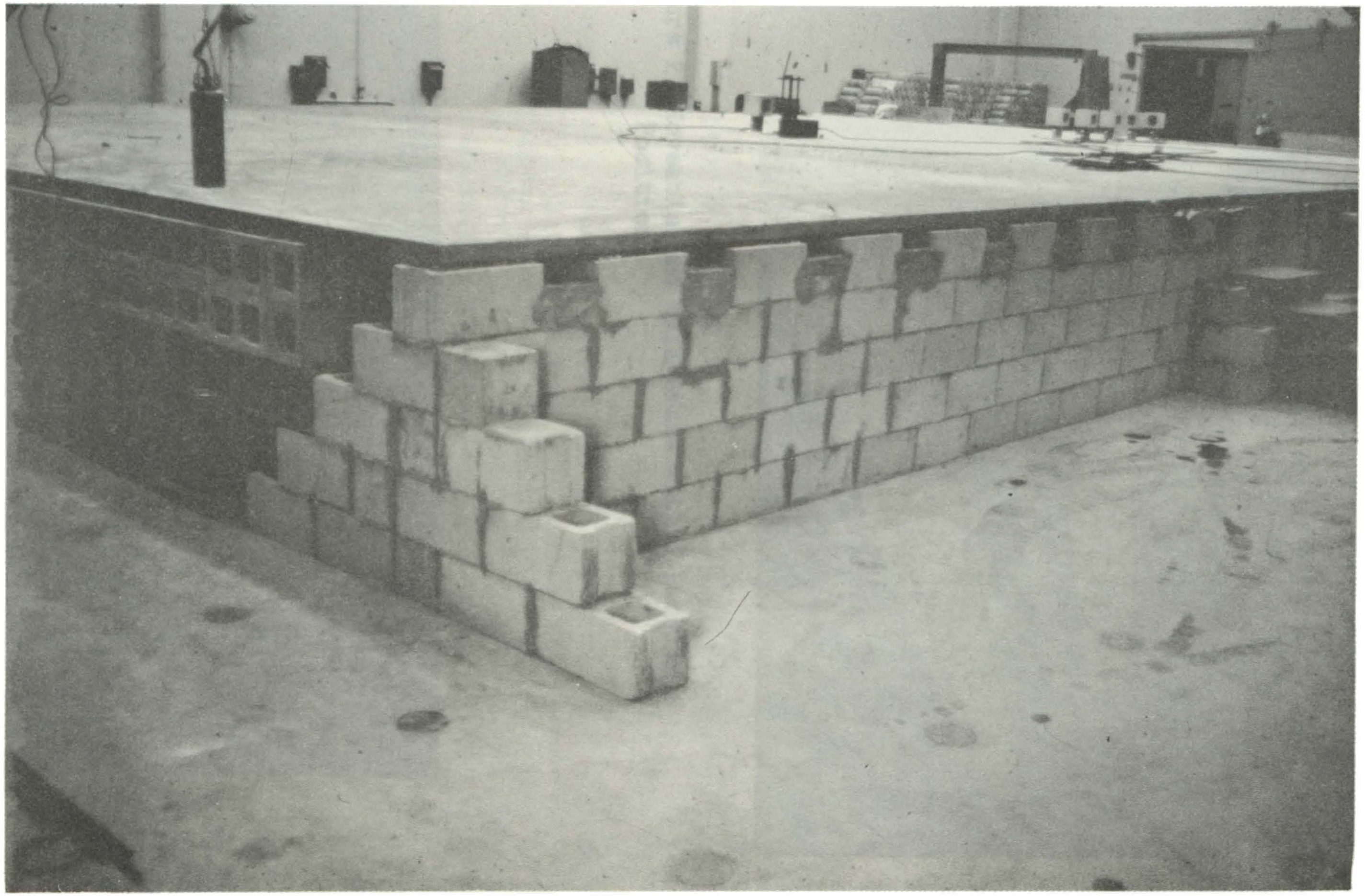


Figure 1. Test Floor Constructed For and Used in Current Test Series.

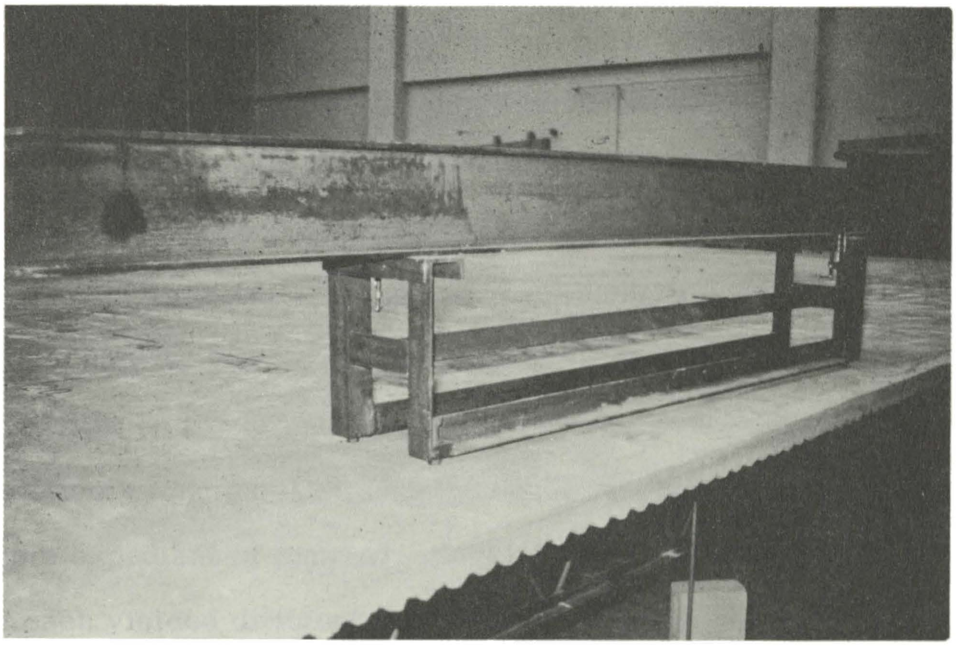


Figure 2. Moveable End Wall Stand with I-Beam in Position.

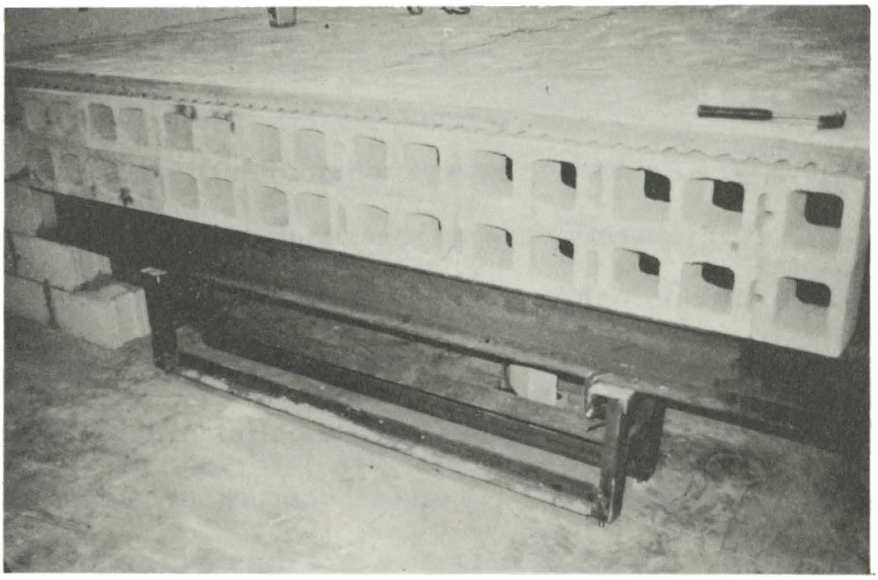


Figure 3. Moveable End Wall Section in Position at Extreme End of Floor for Test.

be moved beneath the floor, raised to come in contact with the bottom of the floor, and thereby, effectively shorten the length of the floor undergoing impact (Figure 3).

Three means of excitation were available. First, the floor could be continuously excited by a Calidyne Shaker. Secondly, a short duration impact could be applied by use of a mechanical impacter[5] (Figure 4). This device consists of a weight which can be raised to varying heights (depending on the magnitude of impulse desired), released and caught on the rebound so that only one impulse is propagated. Finally, a relatively long duration impulse could be applied to the floor by the simple expedient of jumping. Each of these excitation methods were used and each yielded different facets of the overall problem.

Five means were available to measure the various parameters of interest in the tests. First of all, static deflection was measured by dial strain gauges installed beneath the floor. These were used only for calibration purposes. Static deflection during the test series was calculated from strain measurements by electrical resistance strain gauges (Figure 5) installed on the lower chords of the joists. The signals from these strain gauges were amplified and recorded by a Sanborn pre-amplifier-amplifier-recorder system (Figure 6). Dynamic deflection was calculated from measurements by accelerometers (Figures 7 and 8), the output of which was fed through integrating circuits into the Sanborn equipment. Dynamic deflection could also be calculated from measurements by the installed strain gauges or directly by a seismic type Sprengnether instrument (Figure 9). Finally, in some tests, the acceleration of the slab was measured by accelerometers feeding directly into the Sanborn equipment.

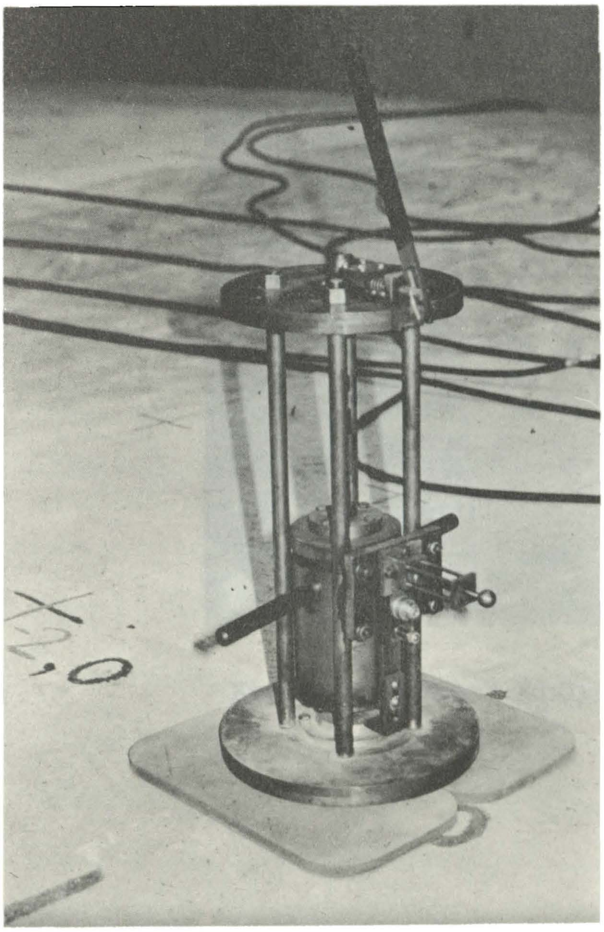


Figure 4. Mechanical Impacter.

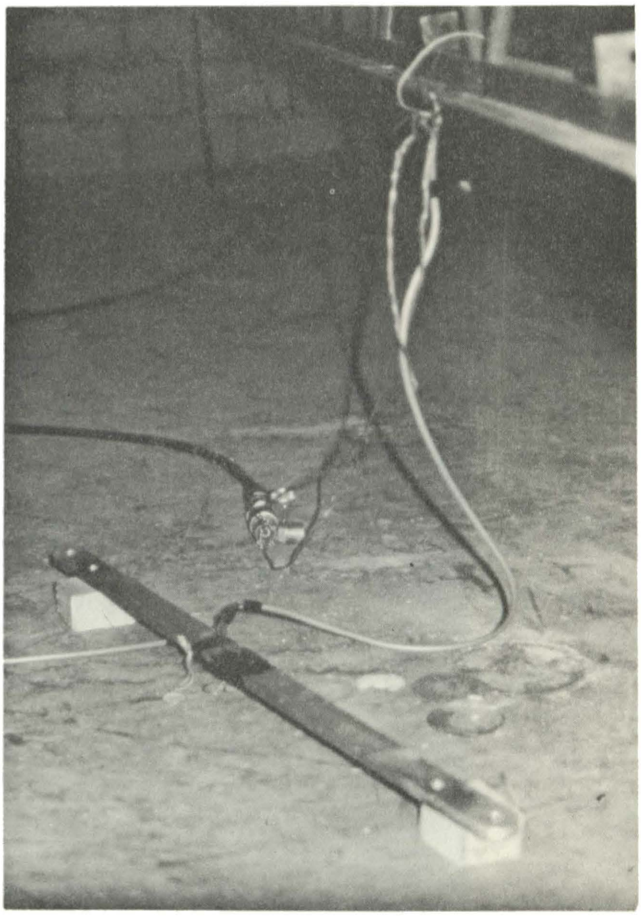


Figure 5. Strain Guage Installation.

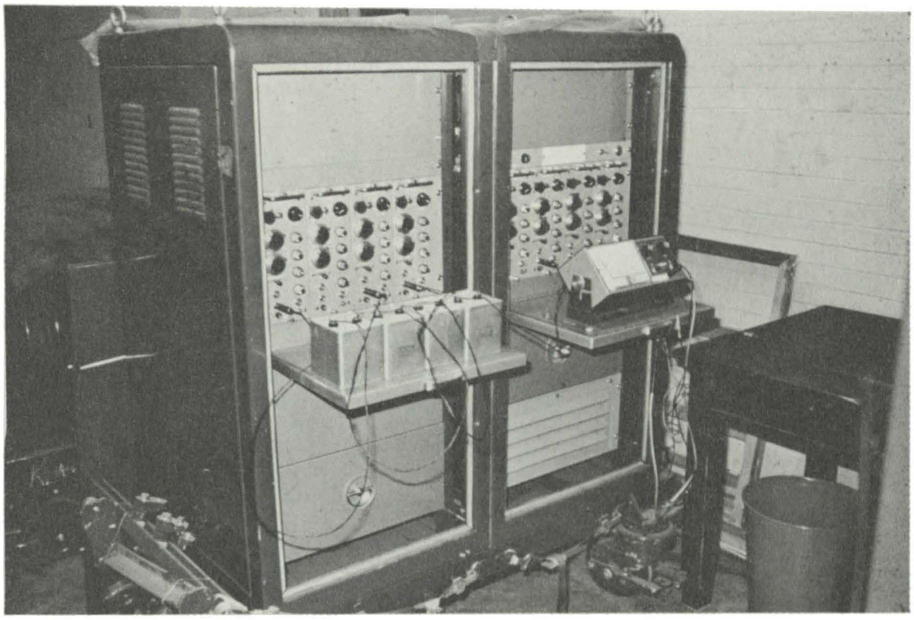


Figure 6. Sanborn Equipment.

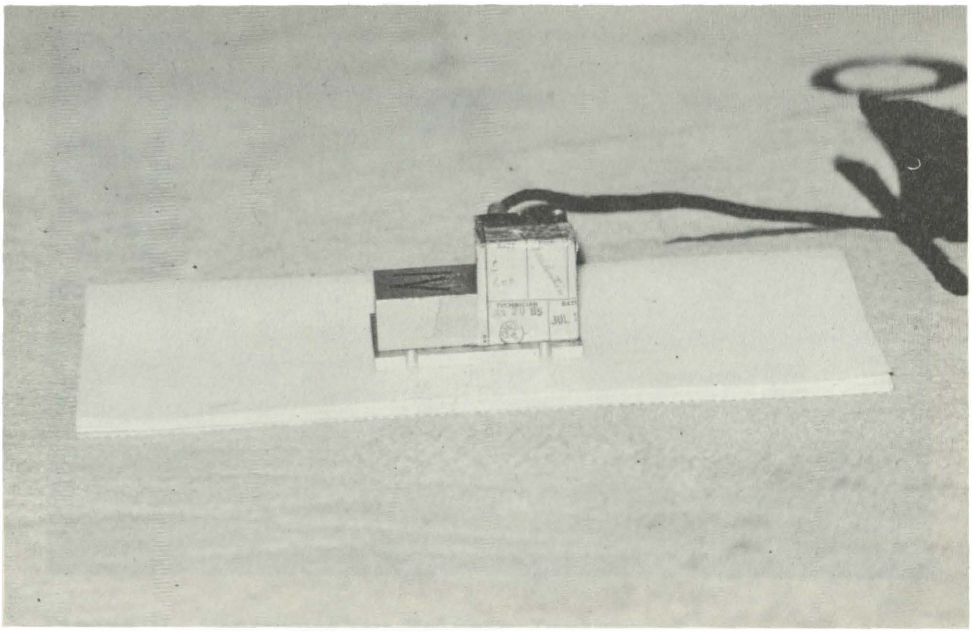


Figure 7. Accelerometer.

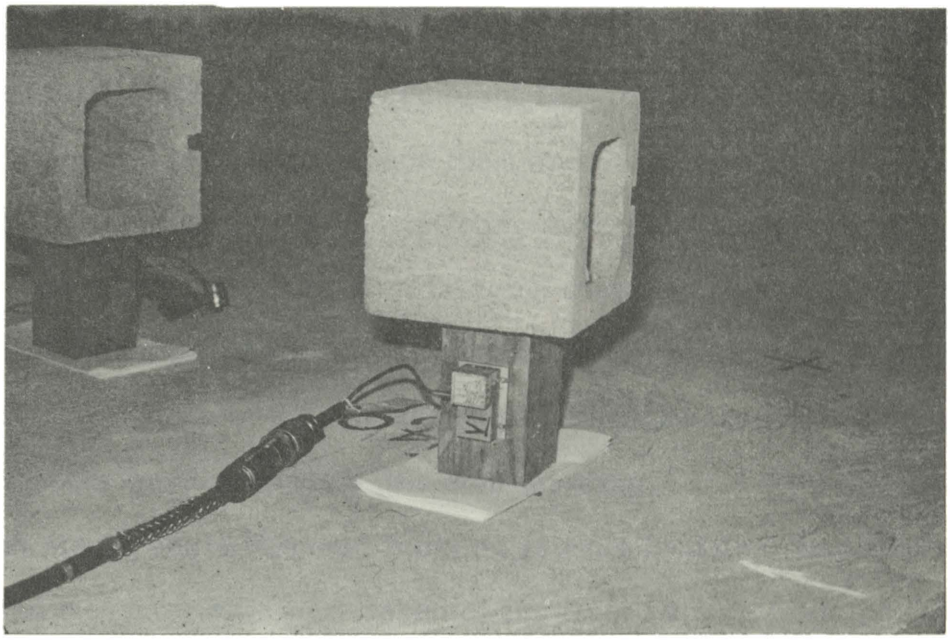


Figure 8. Accelerometer Positioned as for Testing.

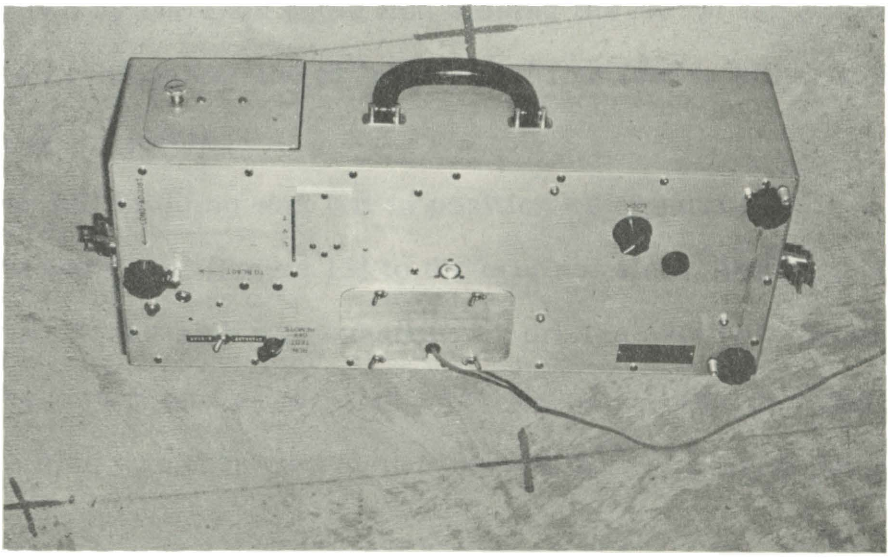


Figure 9. Sprengnether Seismic Instrument.

Floor Configurations Tested

When the test floor was built, single angle-iron bridging (Figure 10), one-eighth by one-and-one-quarter by one-and-one-quarter inch, was installed at the center and eight feet to either side of the center of the joists to properly space them and to provide some degree of stability during construction operations. After the slab was cast, tests were made with this particular bridging installed to develop the experimental techniques required, to check out the instrumentation, and to determine the nature of the characteristics to be measured. A full set of measurements were made with this configuration.

The single angle diagonal bridging (simple bridging) was then removed and measurements were taken of the floor with no bridging installed. There seemed to be no essential difference in the characteristics determined with and without simple bridging so the intermediate steps were bypassed and full bridging (Figure 11) was installed. The full bridging consisted of one-eighth by one-and-one-quarter by one-and-one-quarter inch angle iron forming and "X" pattern with horizontal one-quarter by two by two inch angle iron closing the "X" at top and bottom. It was placed in the center and at intervals of four feet to either side of the center of the joists.

The full bridging was left in position as an additional two inches of concrete were cast (Figure 13) to the original slab. Bonding between the two parts of the slab was enhanced by driving anchoring studs (Figure 12) into the bottom part at two-foot intervals with a stud gun (Figure 14). The bottom slab was then wetted and the additional concrete poured.

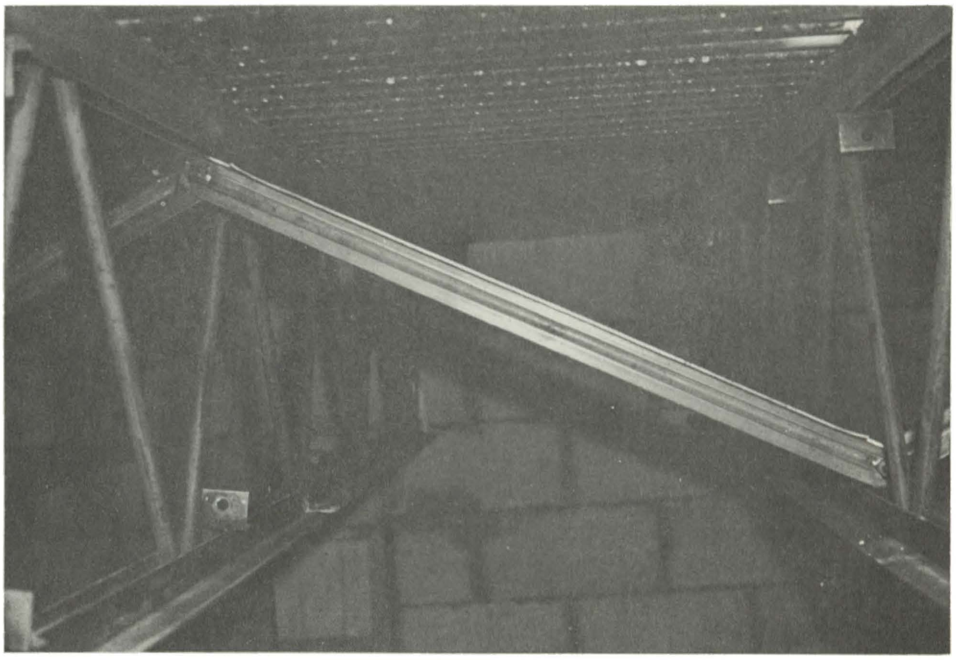


Figure 10. Simple Bridging.

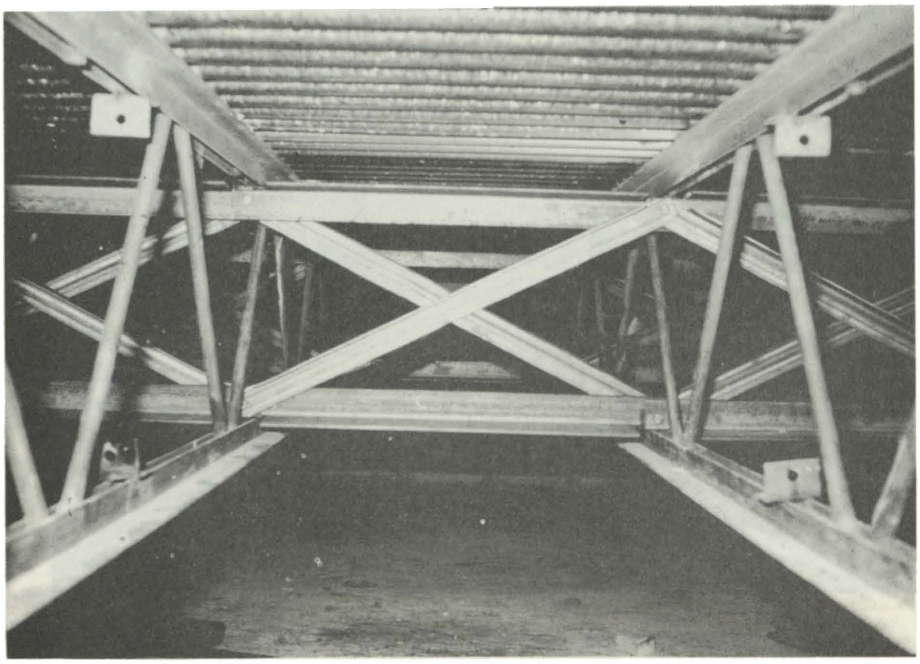


Figure 11. Full Bridging.

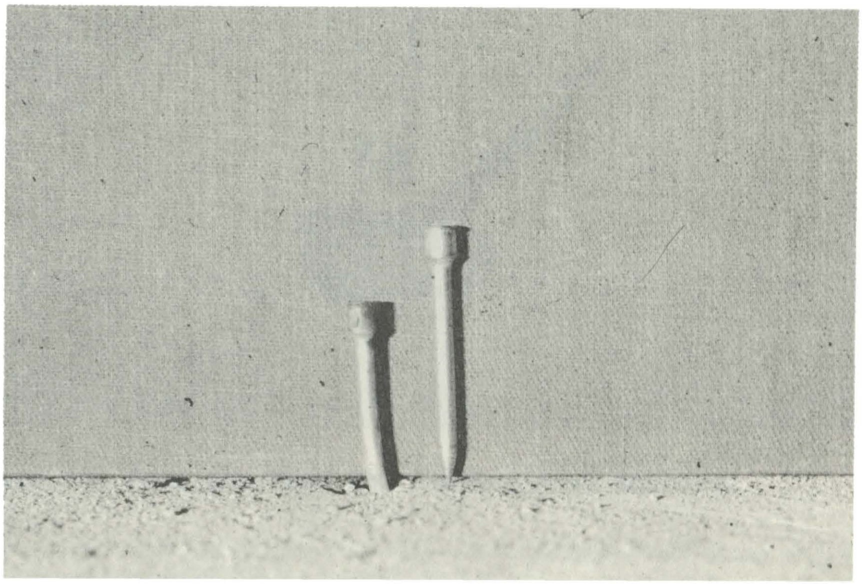


Figure 12. Anchoring Studs to Promote Bond Between Separate Slab Castings.

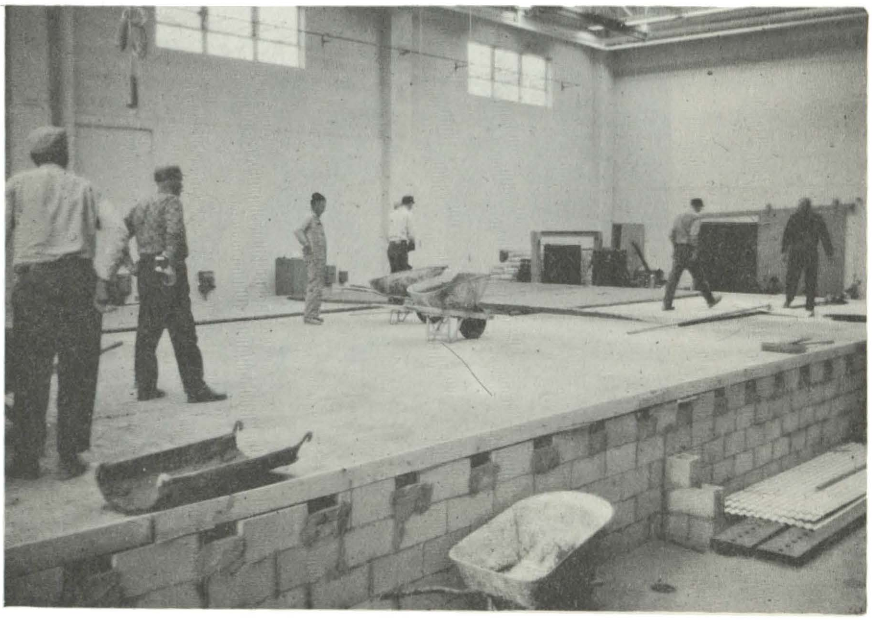


Figure 13. Addition of Two Inches of Concrete to Original Slab.

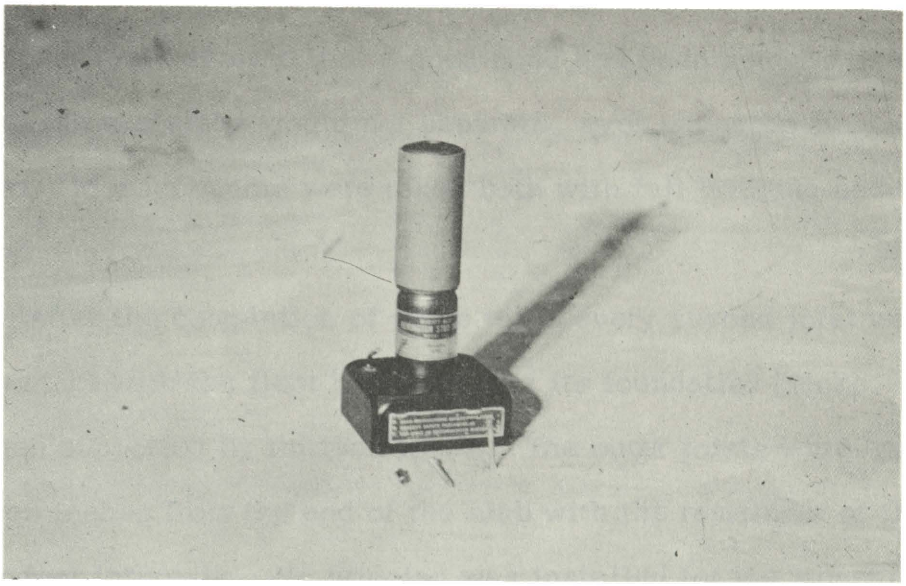


Figure 14. Stud Gun Used for Placing Slab Anchors.

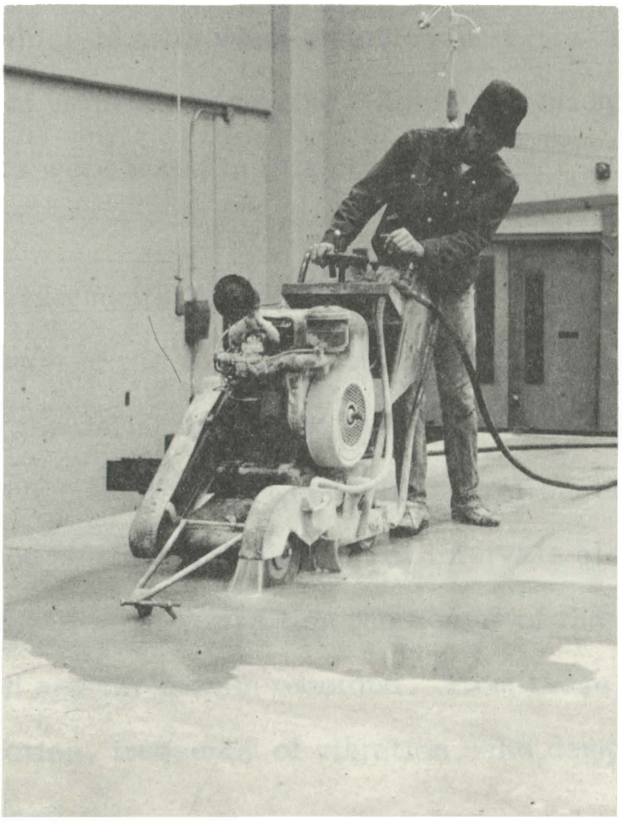


Figure 15. Removal of Four Feet from both Long Edges of Slab.

Subsequent tests showed that a good bond had been secured and that, in fact, the two slabs would not separate, even when the floor was dismantled. Measurements were taken both with full bridging and without any bridging.

After the completion of these tests every second joist was lowered from contact with the floor by removal of its foundation bricks. The floor was then supported by thirteen joists. The outer joists were located eighteen inches from the end of the slab with the remainder of the joists at four foot intervals. No bridging was installed for the measurements made during this series.

Finally, the slab was cut (Figure 15) four feet from the edge of either long side (perpendicularly to the line of the joists) and the concrete outside the cuts removed. This step was taken to simulate the floor (Figure 16) which results when a portion of it has been removed for the installation of ventilation ducting. Again, no bridging was used while measurements were taken in this configuration.

Specific Measurements Taken

For each of the various floor configurations tested a recording was made of the deflection of the center of the active floor under impact. That is, for each thickness of concrete and arrangement of bridging, the moveable, end wall was positioned at intervals along the length of the full floor and at each position the center of the resultant floor section was impacted and the motion recorded. From these recordings the initial impact deflection, frequency of vibration, and damping characteristics were determined.

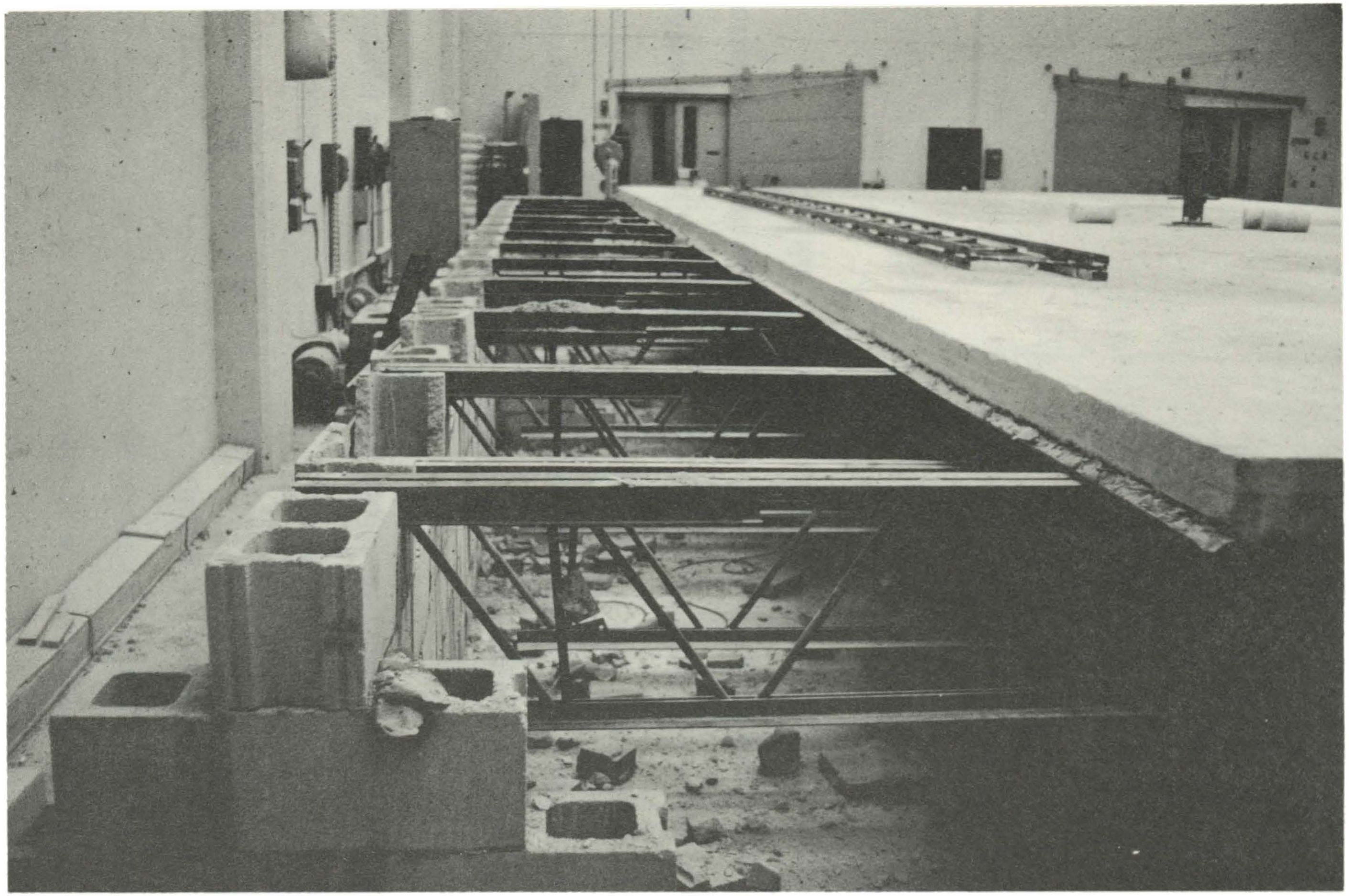


Figure 16. Test Floor with Four Feet of Slab Removed from Edge for Ventilation Duct Simulation.

For a few of the floor configurations the acceleration and the static deflection of the center of the active floor were measured. Acceleration was determined for the two modes of impact while the static deflection was determined for static point loading. In addition, for some floor configurations, the deflection profile under static point loading was determined. In this test the center of the full floor was loaded while the deflection of the floor was measured at two (or four--in configurations where half the joists were removed)-foot intervals along the center of the joists.

Finally, the original floor configuration was force vibrated by the Calidyne Shaker at the center of the full floor and at one-quarter of the length from the end. The shaker was operated at constant output over a range of frequencies from about one cps to ten cps in order to determine as accurately as possible the first few natural frequencies of vibration. Table 1 summarizes the tests made and measurements taken.

Table 1. Tests Made and Measurements Taken.

Floor Configuration	Impact Deflection	Frequency	Damping	Static Deflection Profile	Static Deflection of Center	Acceleration
Two-inch floor, simple bridging	X	X				X
Two-inch floor, no bridging	X	X	X	X	X	
Two-inch floor, full bridging	X	X				
Four-inch floor, full bridging	X	X				
Four-inch floor, no bridging	X	X	X	X	X	
Four-inch floor, thirteen joists	X	X	X	X	X	
Four-inch floor, thirteen joists, edges removed	X	X	X	X		

Note: Other tests were run and other measurements were, of course, taken. The above summary lists the set of tests considered most indicative of the characteristics to be investigated and, consequently, those tests for which sufficient readings were taken to allow a meaningful statistical average.

RESULTS OF THE TEST SERIES

Typical Recordings

Figure 17 shows a typical deflection-time recording after impact, whether by Sprengnether, accelerometer, or installed strain gauge, for the floor excited by the impacter. Indicated on the figure are the various measurements taken on each recording in order to gain the information necessary to make the plots that follow. As noted, each recording was analyzed for initial impact deflection, frequency of free vibration, logarithmic decrement, and the time required to establish the fundamental mode of vibration.

Figure 17 may be compared with Figure 18 which shows a typical deflection time recording after impact for the floor excited by a jump. Clearly apparent are beats which often occurred when the length of the active floor was more than thirty feet. The recordings of static deflection, whether for deflection profile or deflection of the center of the active floor, simply show a step discontinuity along the time axis. The acceleration recordings (recordings of accelerometer output with no integration) showed a rather high frequency signal superimposed on the basic floor vibration. These recordings were analyzed only for initial amplitude.

Composite Action of Slab and Joists

The instrumentation used for the current test series allowed the composite action of the slab and joists to be demonstrated. Perhaps the most clear demonstration is by Figure 19 where simultaneous recordings of the deflection of both the slab and joists after impact are depicted.

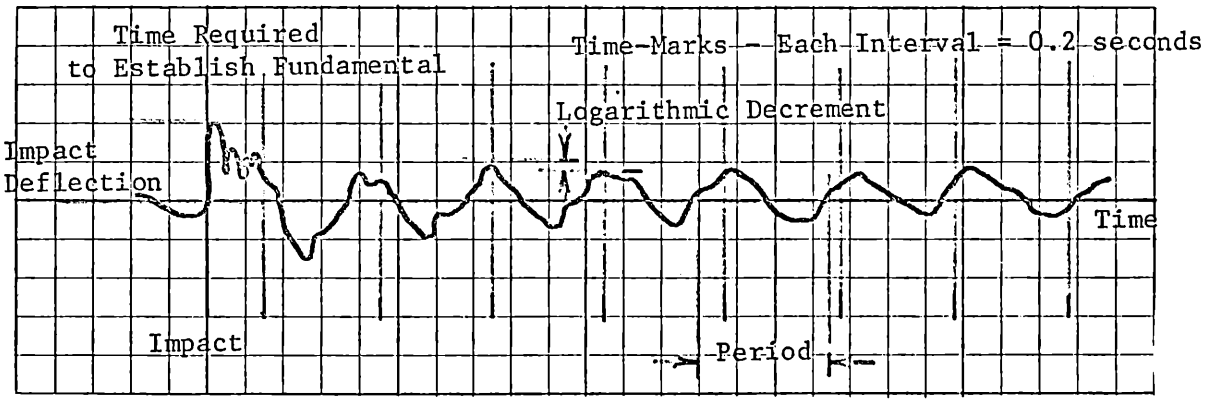


Figure 17. Typical Impact Deflection Recording Showing Measurements Taken on Recordings. Impacter Excited.

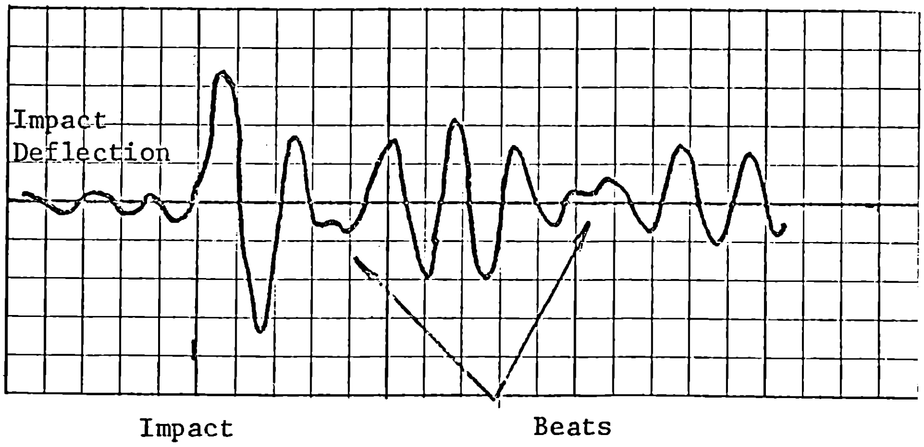
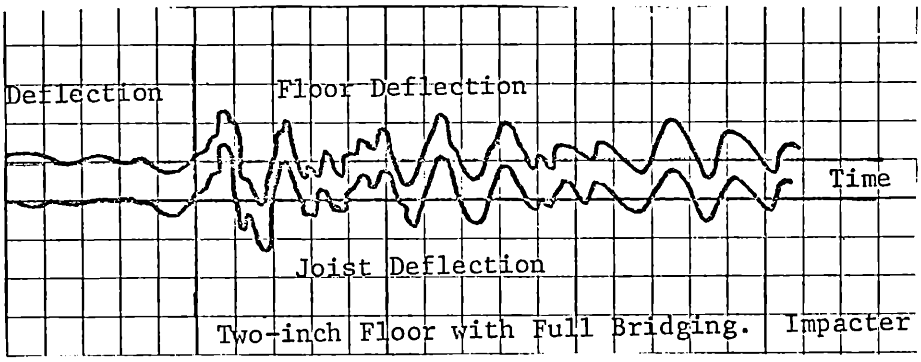
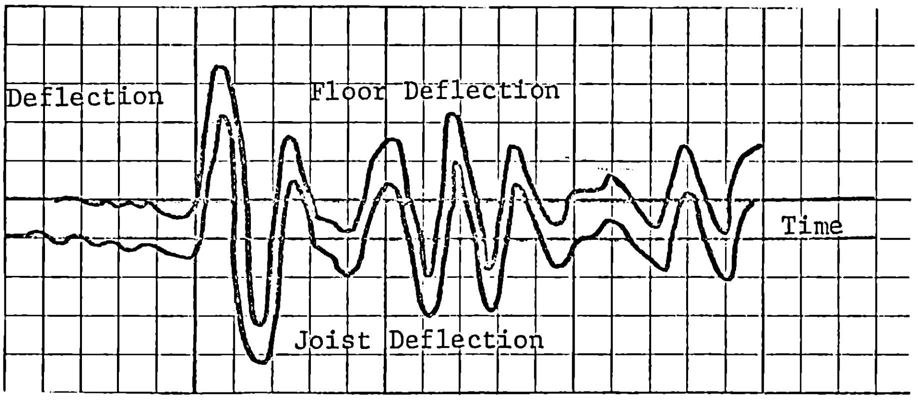


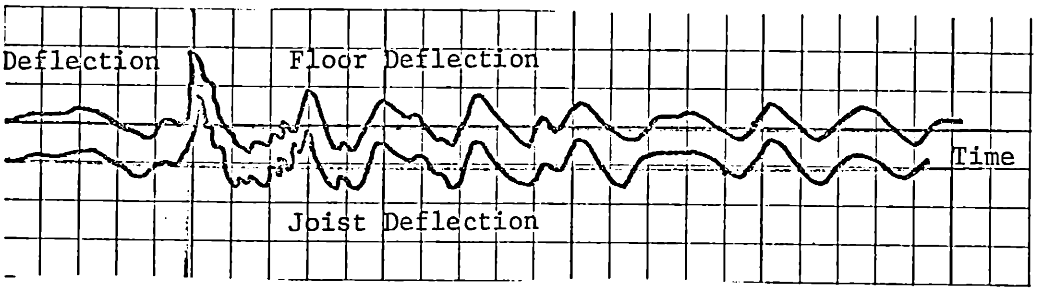
Figure 18. Typical Impact Deflection Recording Showing Beats. Jump Excited.



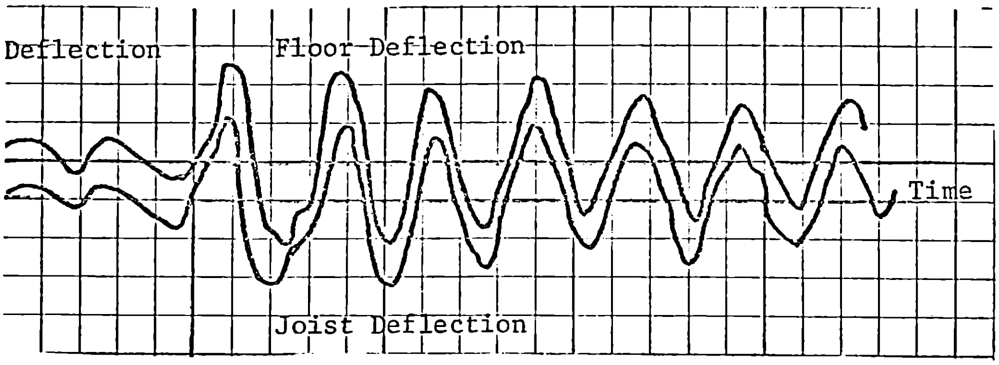
Two-inch Floor with Full Bridging. Impacter Excited.



Two-inch Floor with Full Bridging. Jump Excited.



Four-inch Floor with Full Bridging. Impacter Excited.



Four-inch Floor with Full Bridging. Jump Excited.

Figure 19. Comparitive Recordings of Floor (Slab) and Joist Deflections Taken Simultaneously.

In each instance the upper recording is an actual recording made by the Sprengnether instrument positioned on top of the slab; the bottom recording is a recording made by installed instrumentation (that is, by strain gauges fastened to the bottom chords of the joists) reproduced with time and amplitude scale corresponding to that of the associated Sprengnether recording. These strain gauges measured joist deflection while the Sprengnether instrument measured slab deflection.

It is apparent that both the slab and joists moved with almost identical motions. The differences that do appear on the comparative recordings may reasonable be explained by the different characteristics of the two sensing devices.

The deflections shown are those in the vertical direction, which is, of course, the direction of maximum amplitude. Some attempt was made to measure deflection in the horizontal direction. This deflection was more difficult to measure accurately, but the results indicate that the top chords of the joists and the bottom of the slab move together while the bottom chords of the joists have a horizontal amplitude about twenty percent greater than the top chords (with no bridging). Still, these horizontal amplitudes are small in comparison with the vertical amplitudes, being less than one-fifth the corresponding vertical amplitude.

General Discussion

The figures that follow (Figures 20 through 31) summarize the results of the current test series, depicting in plots the information taken from the deflection-time recordings. The first set of figures (Figures 20 through 23) shows the deflection of the center of the active floor under impact as a function of the length of the active floor. In each case an attempt is made to indicate the point at which the curve undergoes a distinct change. The floor length at that point is referred to as the "effective floor length."

The second set of figures (Figures 24 and 25) shows the fundamental frequency of free vibration of the active floor as a function of the length of the active floor. The third set of figures (Figures 26 through 28) shows the damping characteristics of the floor configurations tested. The first figure is a plot of the logarithmic decrement of the fundamental mode as a function of the length of the active floor. Since the logarithmic decrement is related to the damping coefficient, the same information may be presented in terms of the percentage of critical damping as is done in the second figure. The last figure of the set is a rather arbitrary attempt to indicate the relative damping of higher order vibrations. It is a plot of the time required to clearly establish the fundamental mode of vibration as a function of the active floor length.

The fourth set of figures (Figure 29) shows the static deflection profile of the floor loaded at the center with a point load. The oscillatory nature of the deflection is clearly visible. The fifth set of figures (Figure 30) shows the static deflection of the center of the active floor.as a function of the length of the active floor. Finally, the last set of figures (Figure 31) in this section shows the initial impact acceleration of the center of the active floor as a function of the length of the active floor.

Deflection of the Center of the Active Floor under Impact (Figures 20 through 23)

In all instances it may be noted that as the length of the active floor was decreased the impact deflection remained reasonably constant

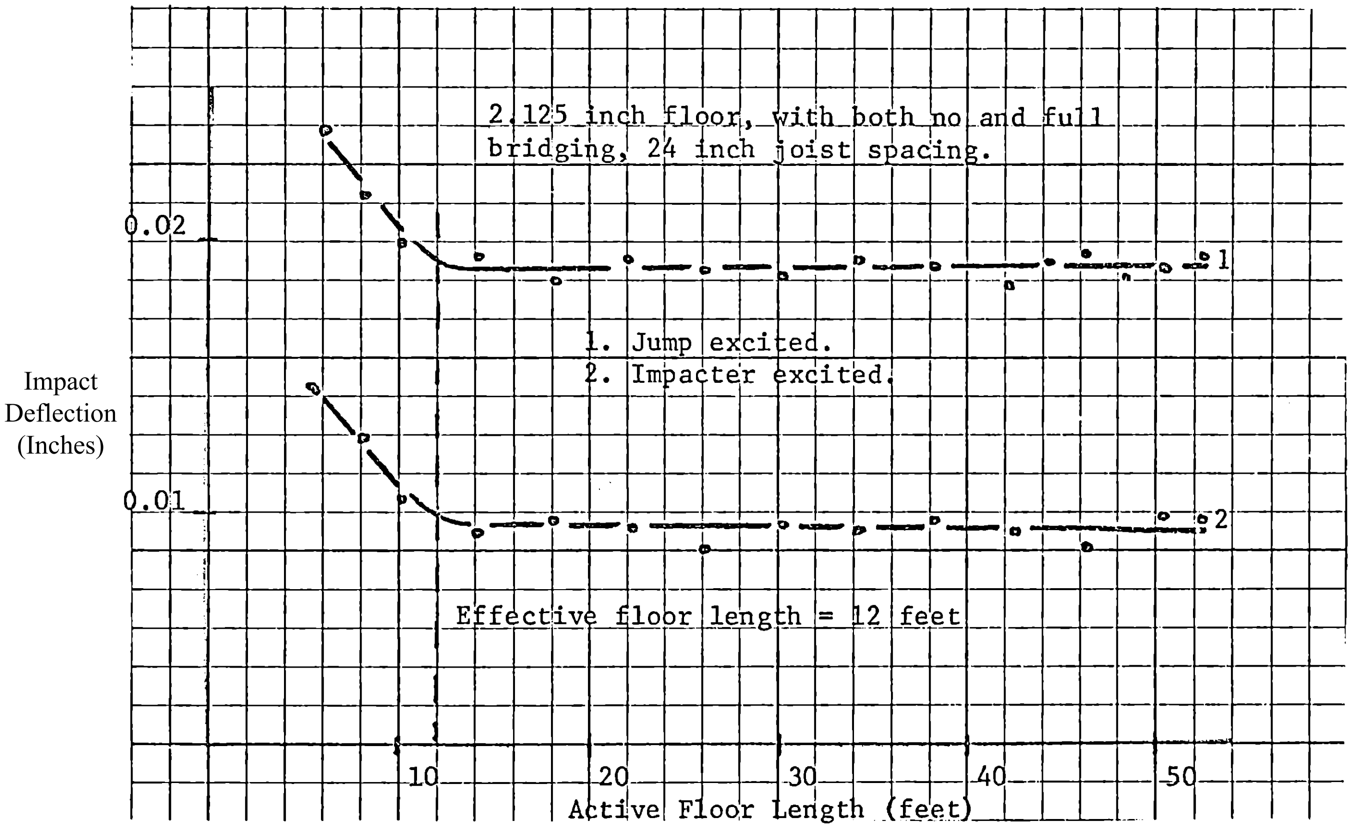


Figure 20. Impact Deflection of Center of Active Floor (2.125 inch Floor).

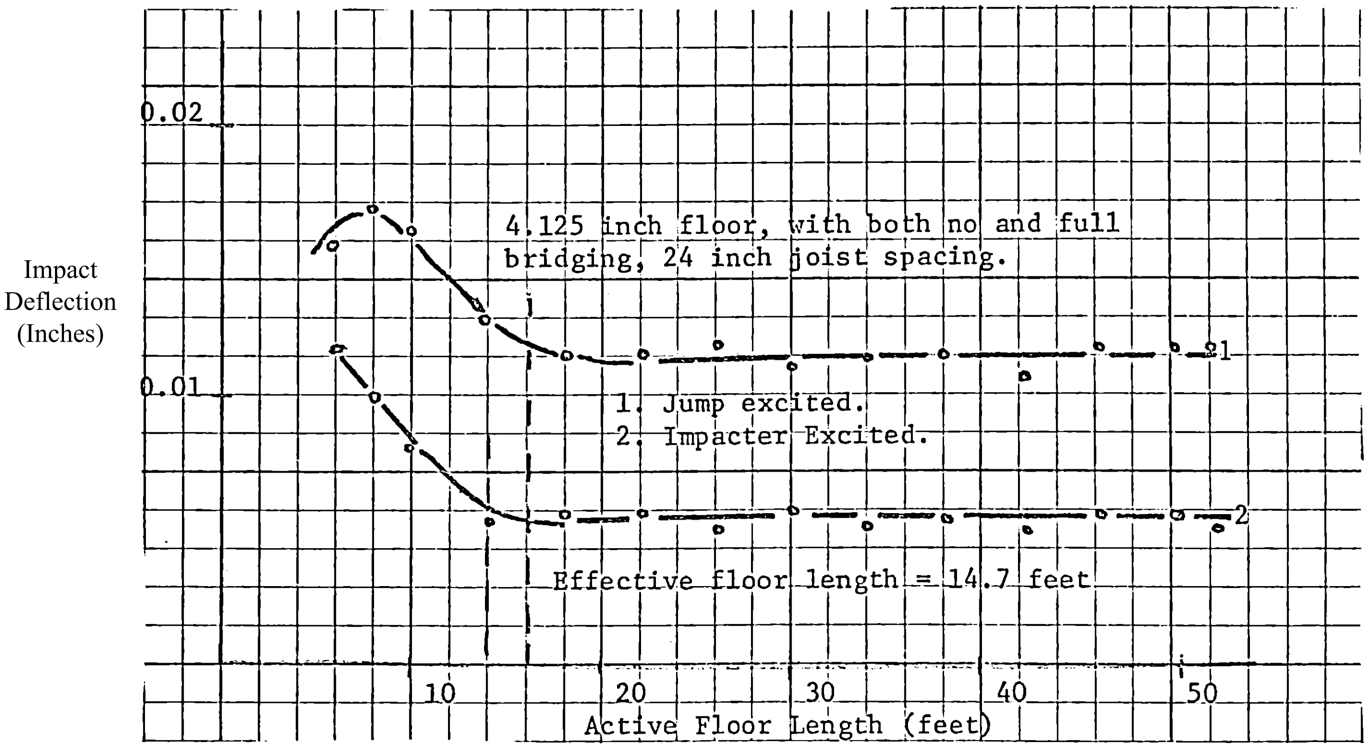


Figure 21. Impact Deflection of Center of Active Floor (4.125 inch Floor).

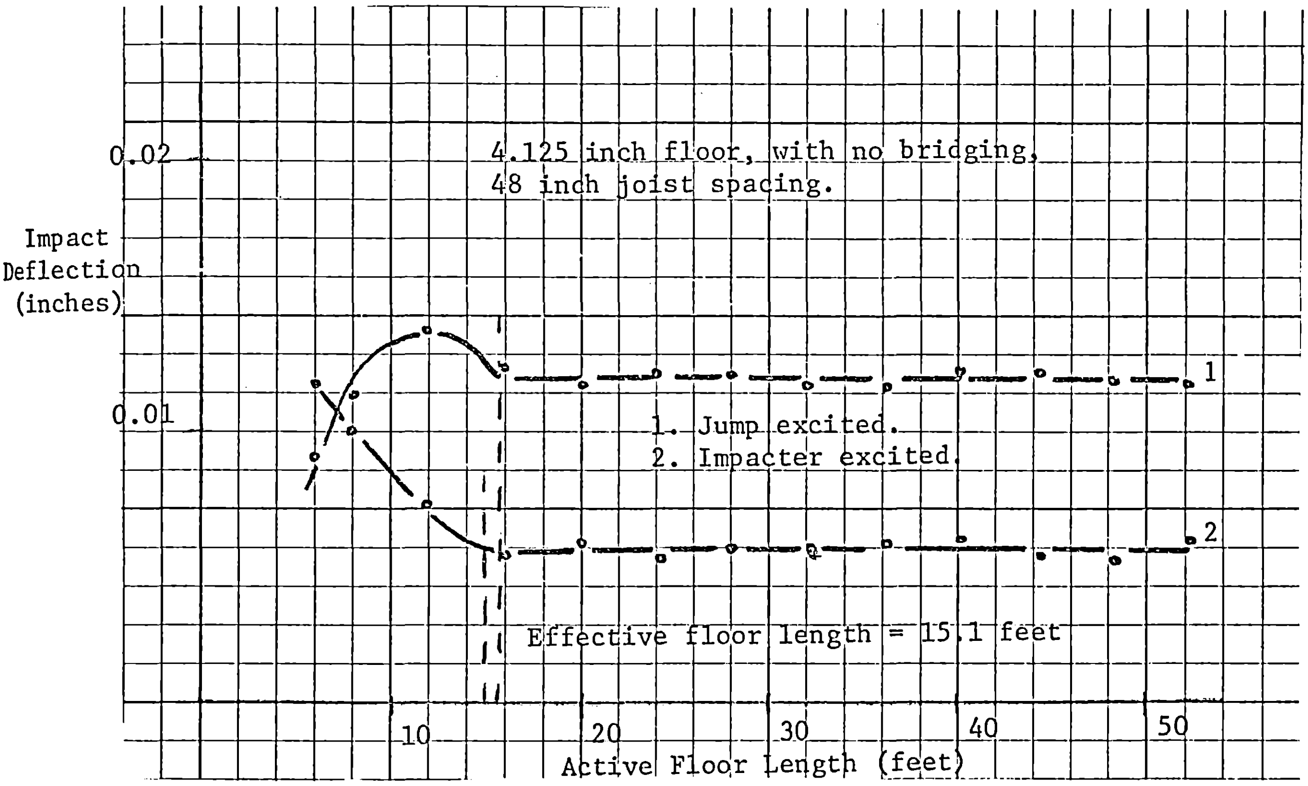


Figure 22. Impact Deflection of Center of Active Floor (4.125 inch Floor), 48 inch Joist Spacing.

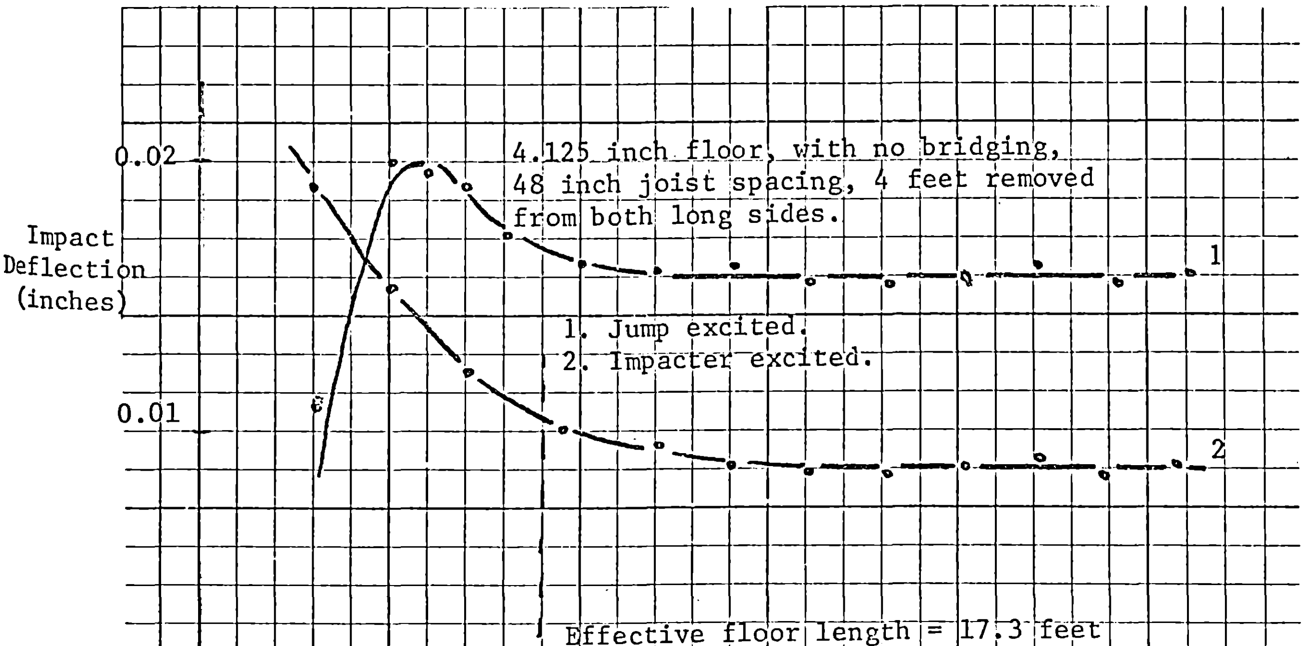


Figure 23. Impact Deflection of Center of Active Floor (4.125 inch Floor), 4 feet Removed from Both Long Sides.

until a readily apparent point was reached where the impact deflection sharply increased. This effect occurred both for the sharp impacts delivered by the impacter and for the longer duration impacts delivered by jumping. The point where these changes took place corresponds to a definite length of active floor referred to as the "effective floor length." For a given floor width the impact characteristics of a floor continuously change over floor lengths up to the effective floor length. For longer floors the characteristics are essentially constant.

Increasing the depth of the concrete added both flexural rigidity and weight to the system. As a result, the effective floor length increased somewhat while the impact amplitude decreased. It will be noted that the deflection excited by jumping decreases at an active floor length of about ten feet. This is due to the fact that the fundamental frequency of free vibration of the system becomes too high at that point to be effectively excited by the relatively long duration jump impact.

Removing half the joists naturally caused the effective floor length to increase with the change in flexural rigidity. Again, the impact amplitude increased, and the jump excited amplitude decreased at about twelve feet.

When four feet of the slab were removed from both long sides of the floor the general nature of the impact deflection curves remained the same but the point at which the characteristics changes became difficult to define. The effective floor length apparently increased again, although this observation is not certain. In any event, the impact deflection increased.

It is apparent, then, from these results that increasing the depth of concrete in the slab decreases impact deflection, while increasing the effective floor length. Reducing the number of joists (or, equivalently, increasing the joist spacing) increases the impact deflection and also the effective floor length. As may be noted, bridging has no discernable effect, either on effective floor length or impact deflection. The data obtained both with no and full bridging yield the same impact deflection curve.

Frequency of Free Vibration of the Floor System (Figures 24 and 25)

For the 2.125 inch floor system two different frequency curves are shown. The upper curve of Figure 24 shows the variation of frequency with active floor length soon after the slab was cast. At that time very little cracking had occurred. During the testing the slab sustained a number of large cracks. These had the effect of reducing the rigidity of the system and the lower frequency curve resulted. It is interesting to note that the slab cracked in a way to quickly lower the rigidity to the point where the lower curve resulted, but, even though further cracking occurred, no measurable lowering of the flexural rigidity occurred.

Again, it may be noted that bridging had no discernable effect on the frequency characteristics of the floor. Indeed, if there was any real difference between the frequencies obtained with no bridging installed and those obtained with full bridging installed, that difference could be explained and accounted for by the weight the bridging added to the system. Very heavy bridging was used to attain the maximum effect on

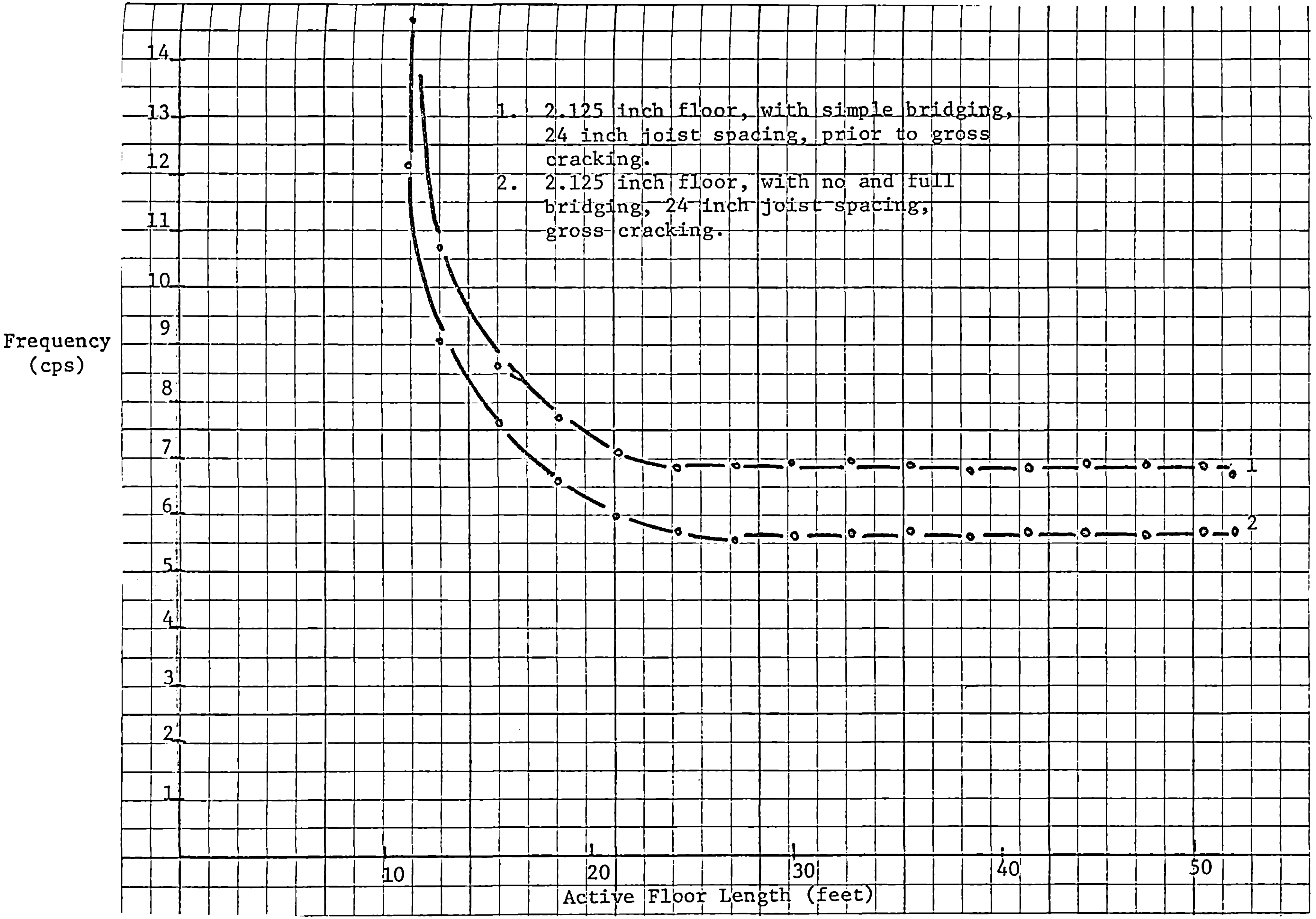


Figure 24. Experimentally Determined Fundamental Frequencies of Free Vibration (2.125 inch Floor).

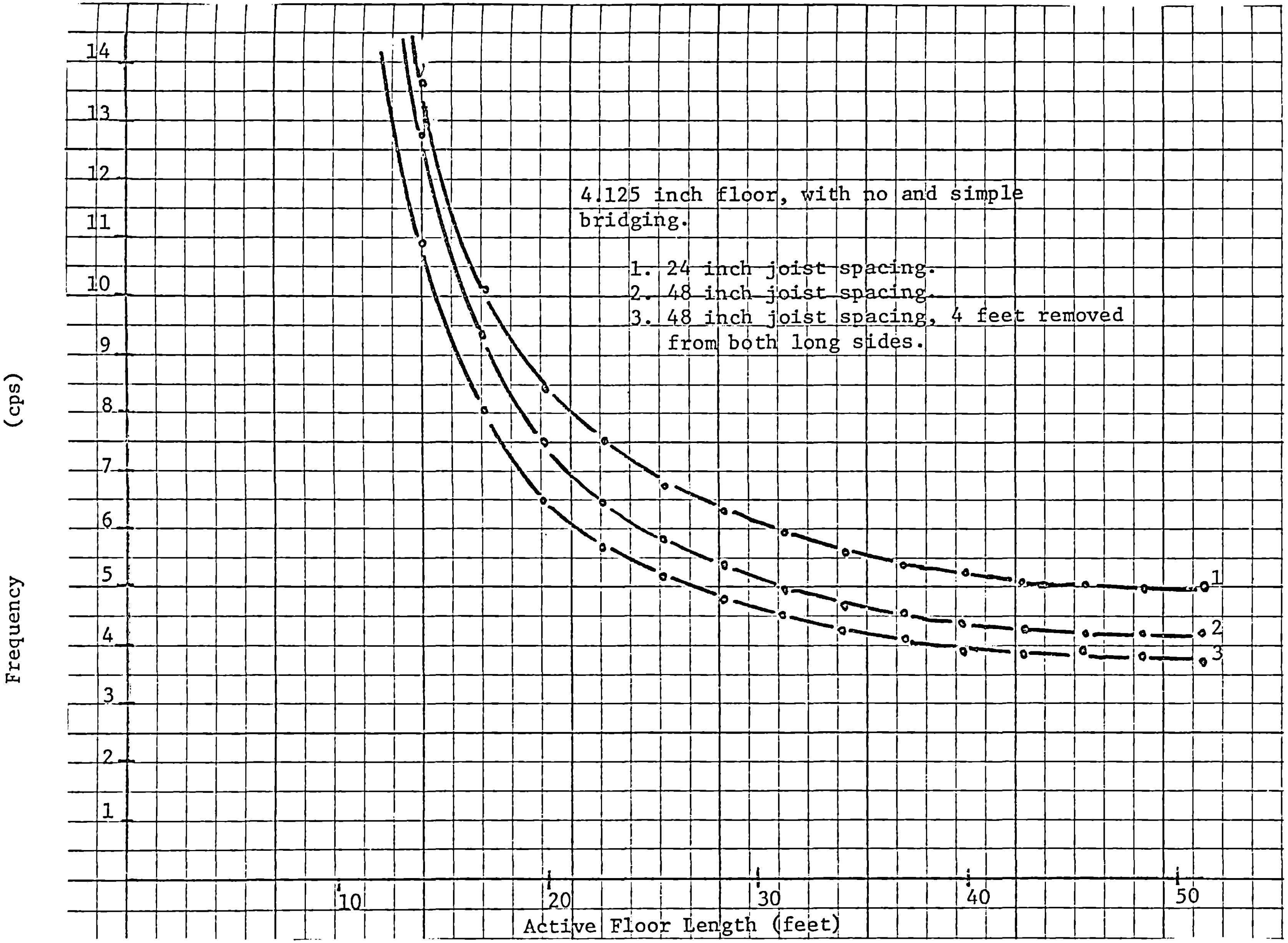


Figure 25. Experimentally Determined Frequencies of Free Vibration of Floor Configurations Tested (4.125 inch Floor).

rigidity that may be obtained by bridging. Calculations show that the weight of the bridging would decrease the fundamental frequency by about one percent. This difference was masked by the natural spread of the data.

Finally, the frequency of the fundamental mode changes very little for floors of length greater than the effective floor length, but increases for floors of smaller length because of increasing slab stiffness.

Damping Characteristics of the Floor System (Figures 26 through 28)

The damping of the fundamental mode of vibration, as evidenced in the change of the logarithmic decrement and the percentage of critical damping, decreased as the active floor length increased and as the rigidity of the system decreased (that is, thinner slab or more spacing between the joists). Especially noticeable is the low value of damping for the floor with four feet removed from both long sides of the slab.

Again, the damping of vibrational hormonics increases with an increase in the rigidity of the floor or a decrease in the active length of the floor. The time required to establish the fundamental mode of vibration, here used as a measure of the higher order damping, is not directly related to any particular parameter of the system. However, it seems to be the most convenient way to present the data.

Static Deflection Profile of Floor under Point Load at Center (Figure 29)

The static deflection profiles of the various floor configurations under point load at the center show the oscillatory nature of the floor deflection. The profile is quite similar to that of a beam on an elastic

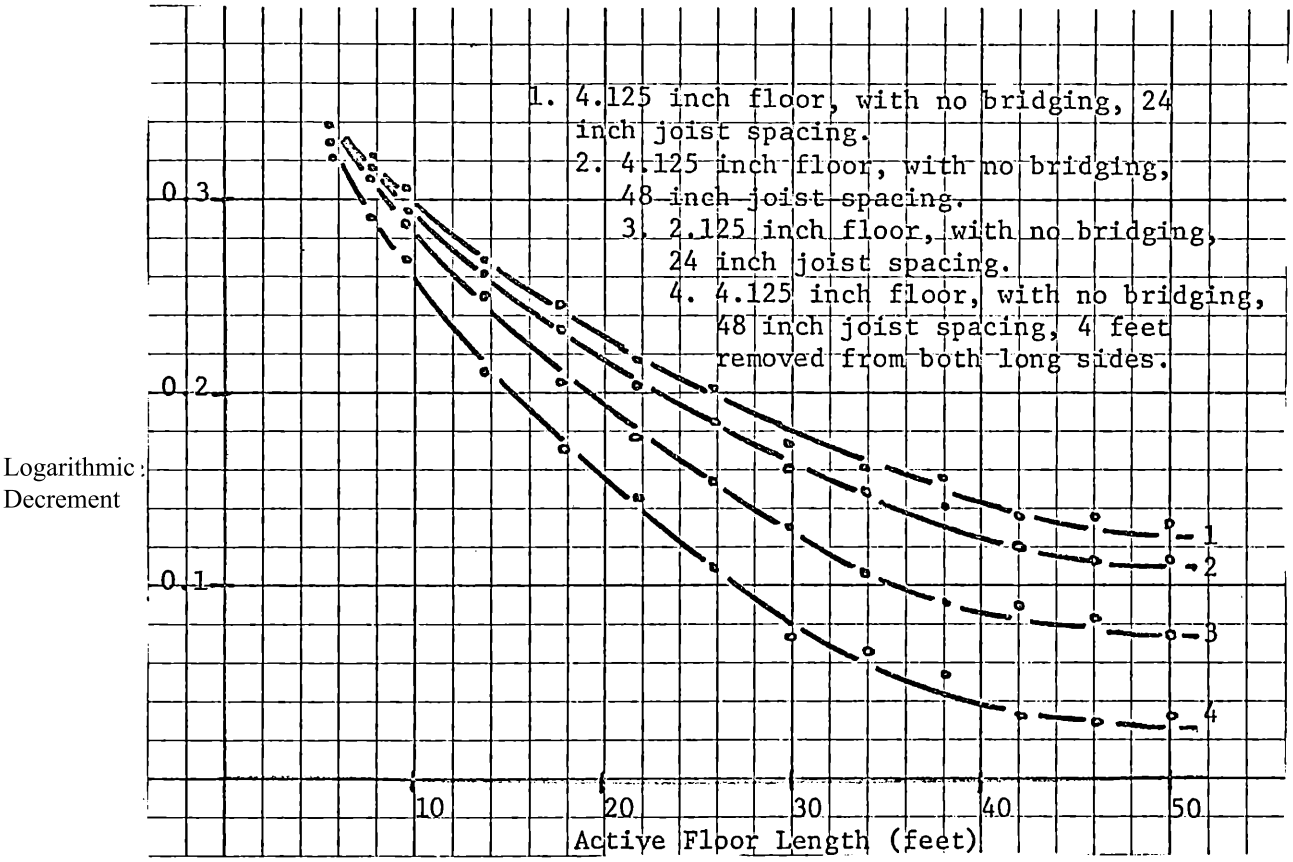


Figure 26. Damping of Fundamental Mode of Vibration: Logarithmic Decrement.

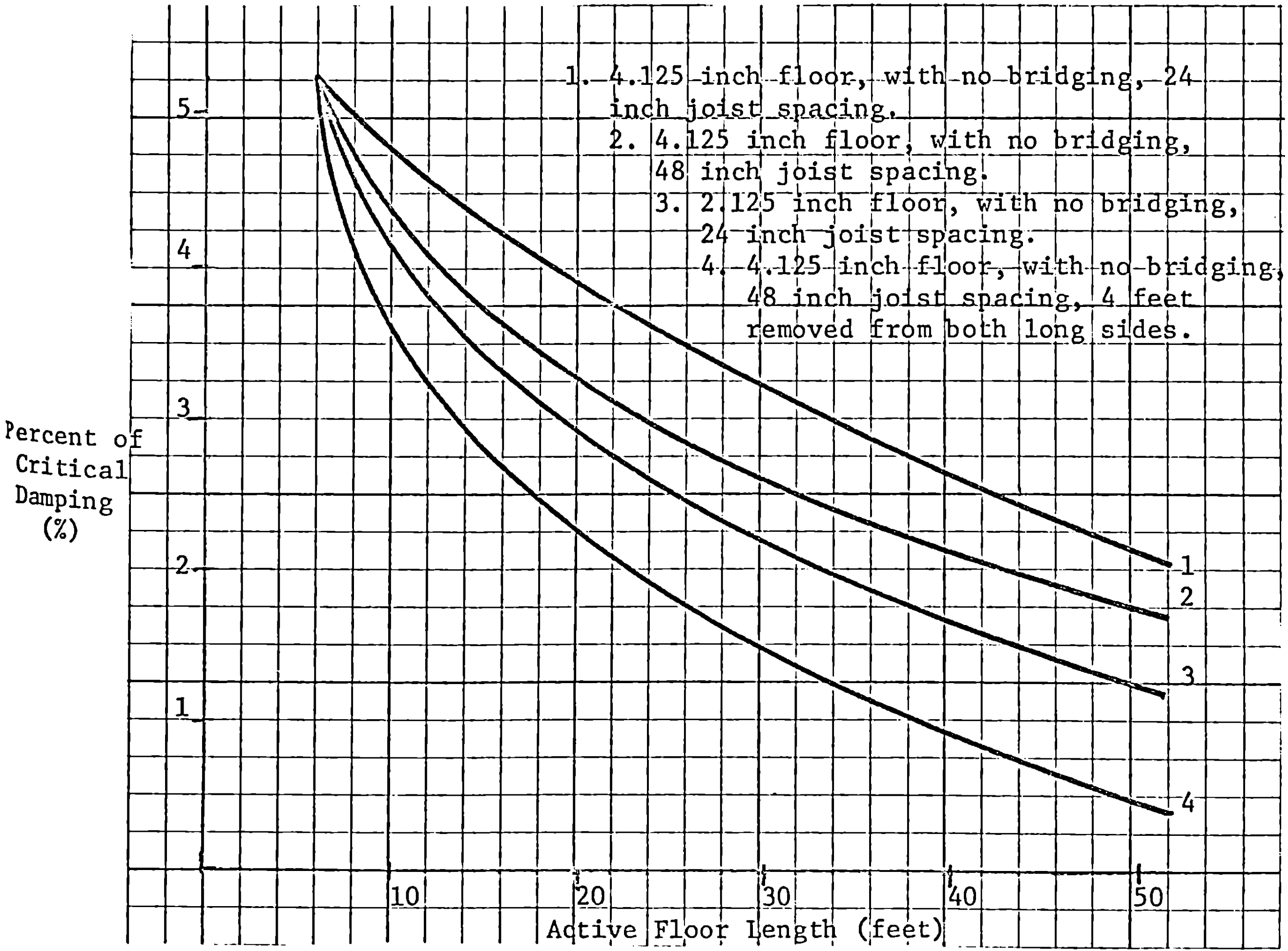


Figure 27. Damping of Fundamental Mode of Vibration: Percent of Critical Damping.

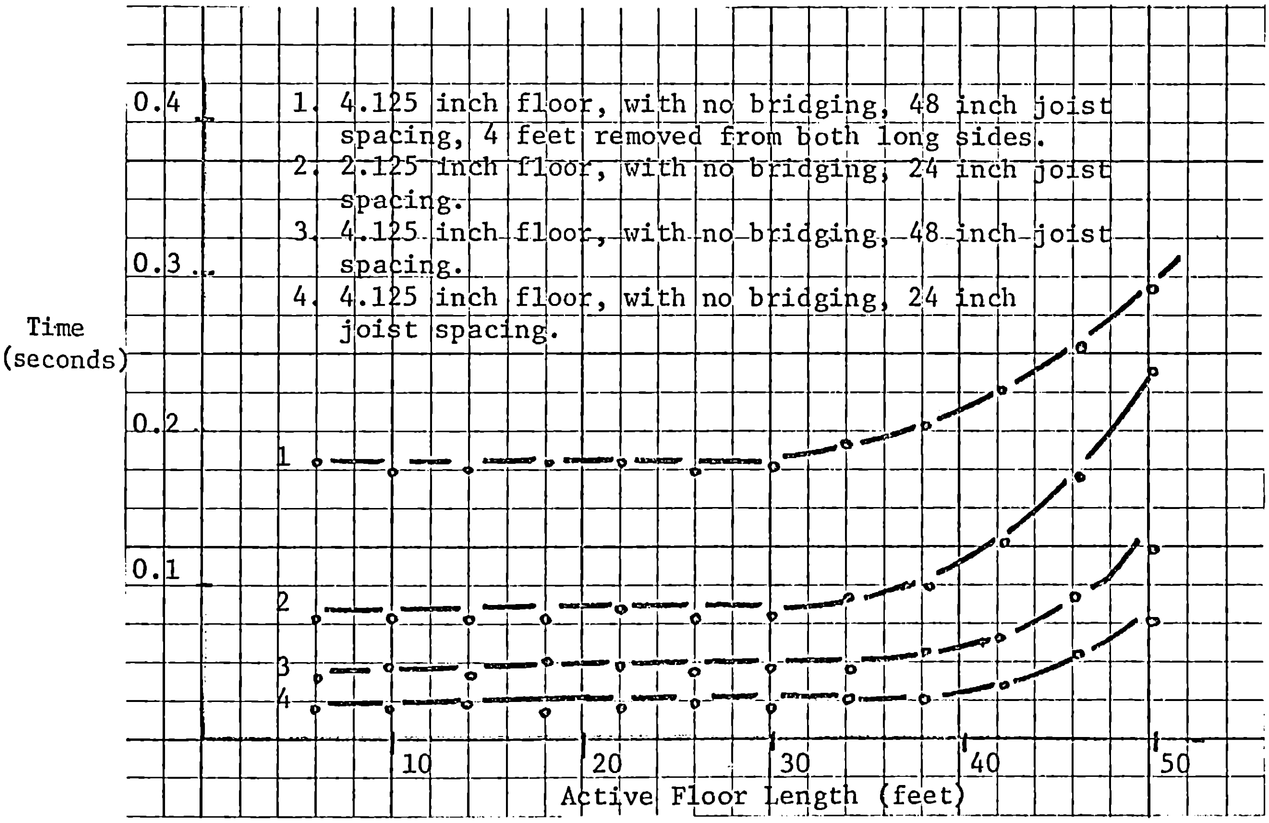


Figure 28. Damping of Vibration Harmonics: Time Required to Establish Fundamental Mode of Vibration.

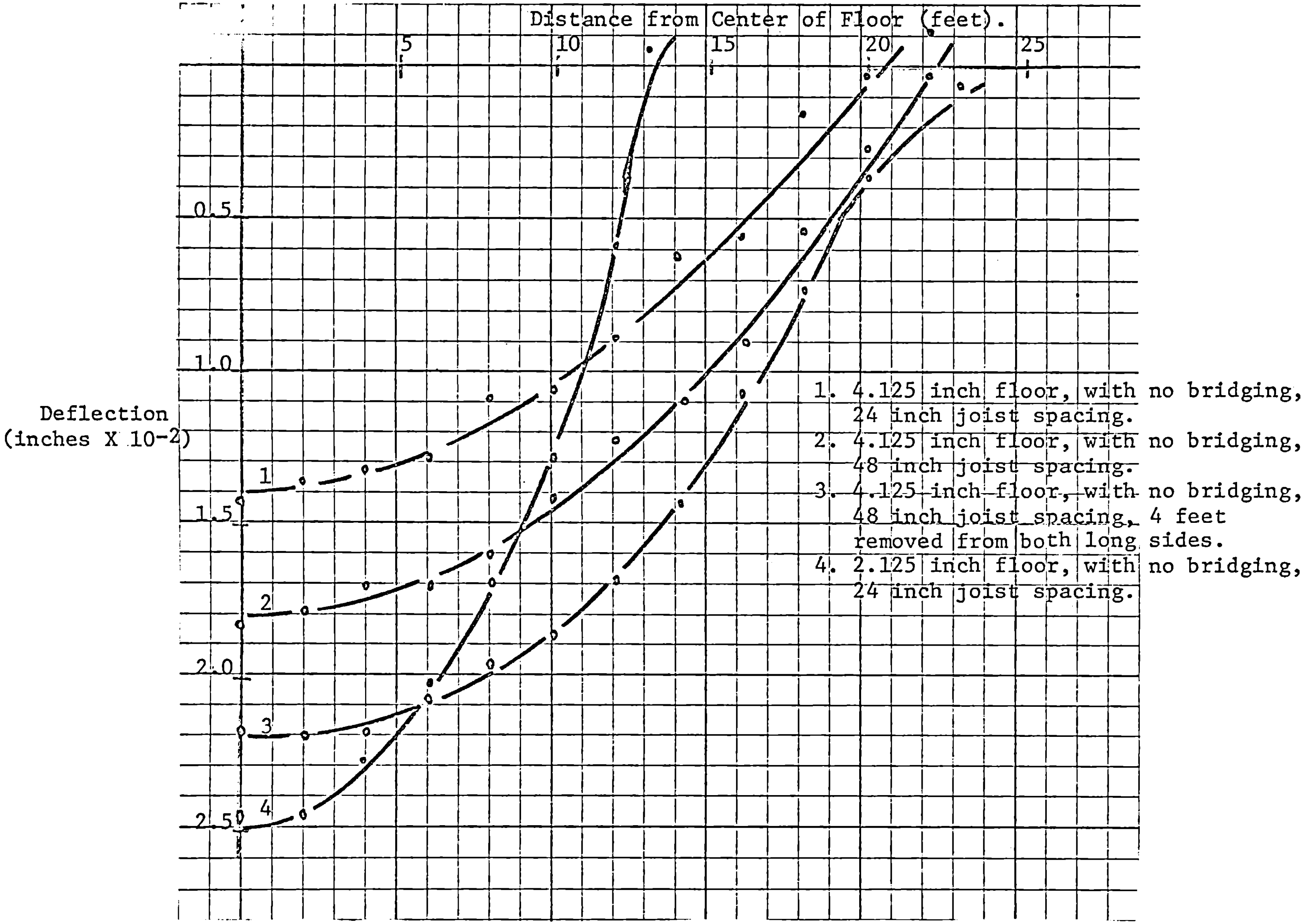


Figure 29. Experimentally Determined Static Deflection Profiles of Floor under Point Load at Center.

foundation. All floors showed some positive displacement (except that with four feet removed from both long sides), but, in general, it was very little, and the second reversal was never noted. The point at which the deflection changed signs occurred at greater distances from the load as the depth of concrete was increased and as the rigidity of the system in the direction of the joists was decreased.

The distance from the load to the point at which the deflection changes sign seems closely related to the effective floor size for a particular system. In fact, for the floors tested, the distance is approximately equal to the effective floor size. This suggests that such measurements may be used as an experimental measure of the effective floor size, at least on floors of greater length than the effective floor size for the particular system.

Finally, the total deflection of the center of the floor is inversely related to the rigidity as would be expected.

Static Deflection of the Center of the Active Floor under Point Load at the Center (Figure 30)

The static deflection of the center of the active floor under point load changed very little as the active floor length was varied. Only for short floor length was any significant difference noted. At these short lengths the deflection decreased somewhat. Again, the deflection was dependent on the rigidity of the system.

Acceleration of the Center of the Active Floor under Impact (Figure 31)

In the initial test runs recordings were made of the acceleration of the center of the active floor under impact. Since the acceleration is the

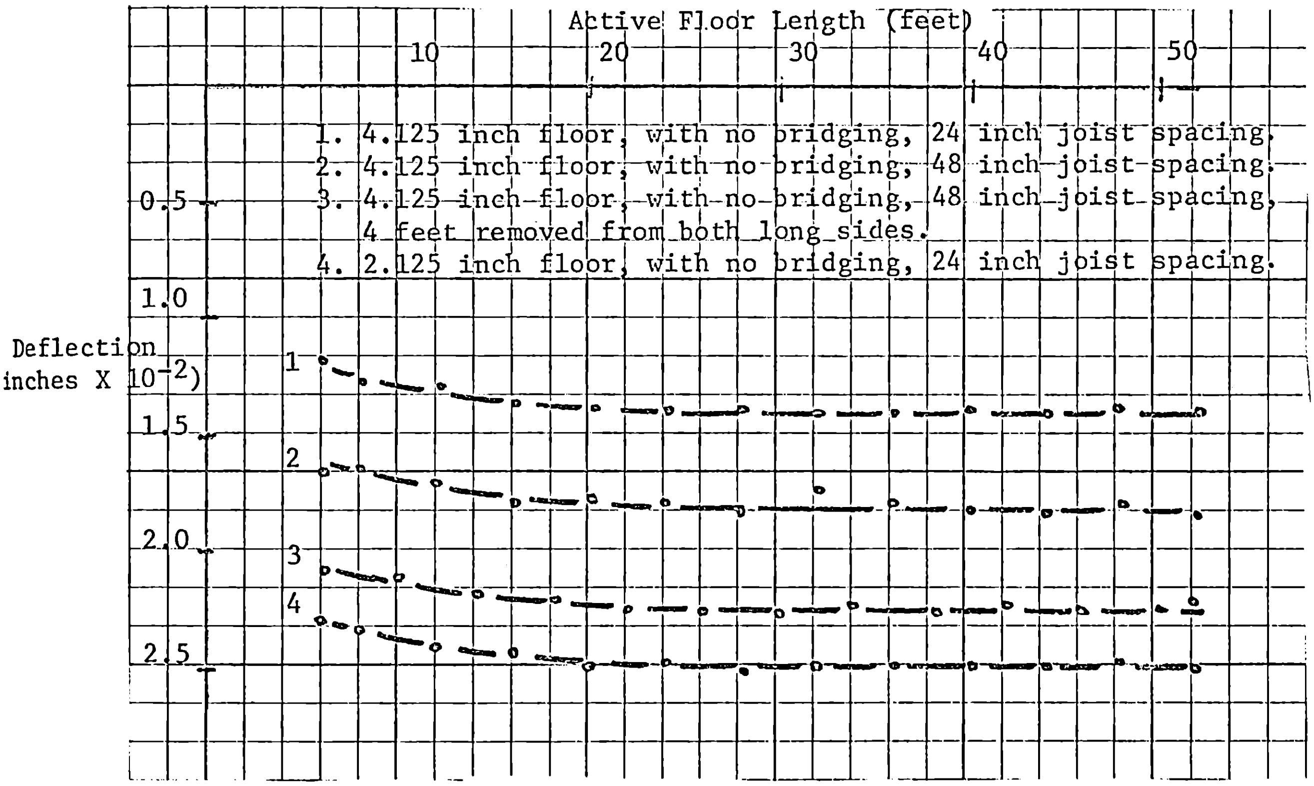


Figure 30. Static Deflection of Center of Active Floor under Point Load at Center.

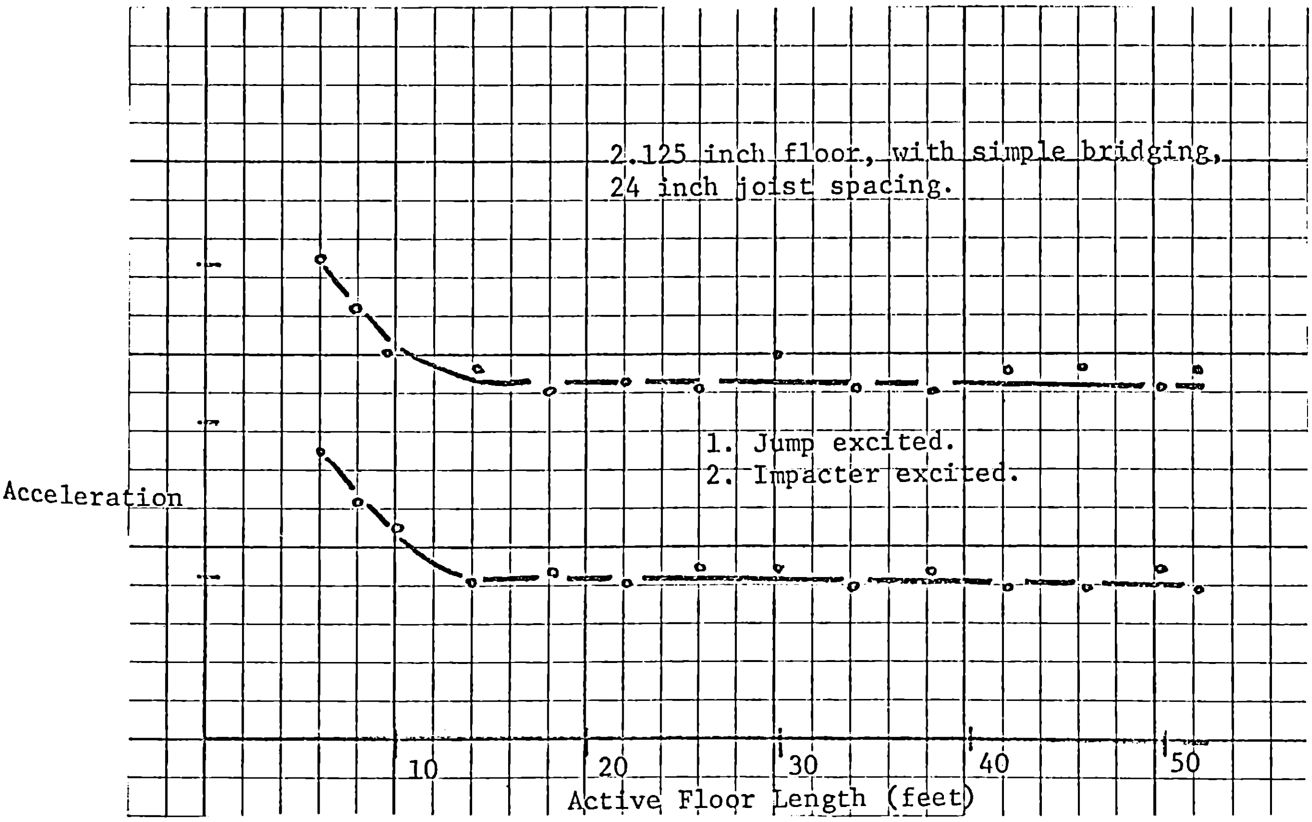


Figure 31. Impact Acceleration of Center of Active Floor.

second time derivative of displacement, the acceleration is proportional to the displacement. Therefore, measurement of the impact acceleration is a means to measure impact displacement. However, the other means available to more directly measure the impact deflection were sufficiently effective that measurement of the acceleration was abandoned after the initial runs. It may be seen from the figure that the acceleration curve is of the same shape as the impact deflection curve and yields approximately the same value of the effective floor size.

CONCLUSIONS FROM THE TEST SERIES

General

The following general conclusions seem apparent from the results of the current test series:

1. The slab and joists of a steel joist-concrete slab floor system act as a single unit during dynamic deflections. This conclusion is of particular importance as it verifies one of the fundamental assumptions of existing floor theory. As a result a floor system may be considered as an orthotropic plate (with less exact results as joist spacing increases), the bending rigidities of which are calculated by the standard "T" section analogy.
2. The basic characteristics of a steel joist-concrete slab floor system are almost completely unaffected by any of the commonly used forms of bridging. Bridging does, of course, give the joists some stability during construction operations, but it has no effect on the response of the floor other than that induced by its weight.
3. The basic characteristics of a steel joist-concrete slab floor system are almost invariant over a wide range of k ($k = b/a$, ratio of the dimensions of the slab). That is, for a given floor construction, the characteristics of any floor larger than one of the effective floor size for that construction are essentially identical. Floors smaller than the effective floor size exhibit a wide variation of parameters.

4. The addition of concrete to an existing floor reduces the vibrational amplitude and frequency. This result stems from the fact that the additional concrete has a greater effect on the weight of the system than it does on the flexural rigidity. The general effect is to improve the "feel" of the floor by reducing the amplitude of vibration, but it may not always be desirable to reduce the frequency of a particular system. Additional concrete also increases the damping of vibrational harmonics and the fundamental frequency.
5. Cracks in the slab of a floor system apparently have the effect of reducing the modulus of elasticity of the system. Considering the composite section as a whole, the modulus decreased about 20 percent.
6. It is highly undesirable to make the slab of a steel joist-concrete slab floor system discontinuous so as to accomodate ventilation ducting or wiring and plumbing lines. As may be seen from the information gained on the floor with four feet removed from both long sides, the frequencies of vibration and their associated amplitudes and damping characteristics are such as to make the floor very uncomfortable.
7. Use of the beam deflection formulas to calculate impact deflection formulas to calculate impact deflection of steel joist-concrete slab floor systems is reasonable although they seem to give about a ten percent higher deflection than is actually measured.

8. In large, light floors the phenomenon of "beating" occurs. That is, in addition to the natural frequency of the floor system, there can be felt a vibration of from one to three cycles per second. This very low frequency vibration is very noticeable and unpleasant.
9. Open floors are those with partitions further apart than the effective floor length.

Effective Floor Size

For a particular choice of joist, joist spacing, and concrete thickness for a steel joist-concrete slab floor system there can be defined an effective floor size, or, more precisely, an effective k ($k = b/a$, the ratio of the lengths of the sides of the floor). The basic characteristics of a floor with a k less than k_{eff} are essentially the same as those for a floor with $k = k_{eff}$. In addition, on impact, a floor system reacts over the area of the effective floor size. That is, to all intents and purposes, it may be assumed for a floor larger than the effective floor, the impact is being borne by a subfloor equal to the size of the effective floor for the given construction.

There are several ways to determine the effective floor size. First of all, the effective floor length seems to be about equal to the length from the center of the floor at which the floor deflection becomes zero for a concentrated load at the center. This point may be determined experimentally by suitable measurements. Secondly, the effective floor length is equal to the length of the active floor at which the basic characteristics begin to markedly change. This point may be experimentally determined as has been done in the current test series. Finally, an empirical formula may be developed connecting the data points found in the test series. Defining ϵ^4 as the square root of the ratio of the flexural rigidity perpendicular to the joists to the flexural rigidity parallel to the joists (that is, $\epsilon^4 = \sqrt{D_x/D_y}$) leads to the formula

$$k_{eff}^2 = -1 + \sqrt{1 + \frac{1.8}{\epsilon^4}} \tag{1}$$

Figure 32 is a plot of k_{eff} as a function of ϵ^4, derived on the basis of Eq. (1).

One of the most effective ways to eliminate undesirable vibrations is to subdivide a floor by office partitions or other means. This provides a mechanism for greater damping which often eliminates the undesirable effects. It might be thought possible to also subdivide a floor from below and achieve the same effect. This can be done so long as the subdivisions are all of smaller size than the effective floor size for the particular installation. Otherwise, no appreciable effect will be realized other than slightly greater damping.

Mechanical and Human Impact

Floors react very differently to human and mechanical impact. Mechanical impact is generally of short duration and excites not only the fundamental frequency of vibration but also some of the higher order harmonics. Human impact, on the other hand, excites only the fundamental frequency to any extent. Since mechanical vibration can be eliminated by other means, it is human impact which causes the most concern.

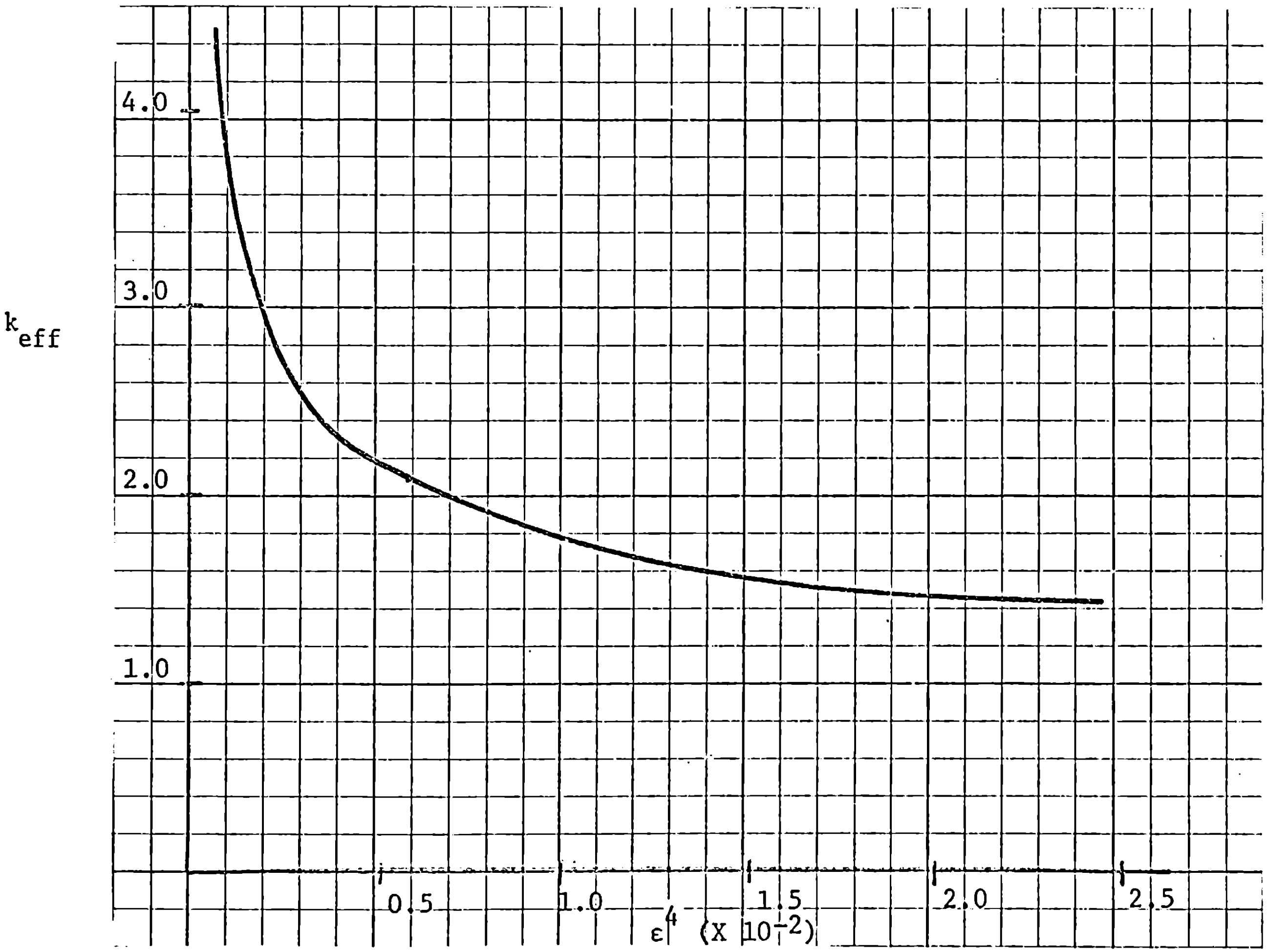


Figure 32. Variation of k_{eff} with ε^4.

Earlier experimental figures show that at about a natural frequency of fourteen or fifteen cycles per second human impact is of too long a duration to appreciably excite a floor. Thus floors with natural frequencies above about fifteen cycles should exhibit little problem due to human impact.

CONCURRENT ANALYTICAL DEVELOPMENTS

Introduction

The analytical investigation and determination of the vibrational characteristics of anisotropic plates in general and of steel joist-concrete slab floors in particular have primarily been developed by use of the various energy methods. In 1960, Mr. James A. Wiley[6], of the University of Kansas, considered such vibrations. This work, done as a part of the overall investigation for the Steel Joist Institute, produced a formula for the calculation of the frequencies of such systems. Frequencies calculated by this formula (and by energy methods in general) are somewhat higher than those measured in the laboratory because of the simplifying assumptions introduced into the general solution.

During the summer of 1966, Dr. Marek Sokolowski and Dr. Lenzen considered the problem on the basis of the more classical force-equilibrium method. Fewer assumptions need be made in deriving the solutions by this method so the results are, in general, more accurate.

Pertinent Formulas for the Calculation of Floor Characteristics

The detailed analysis of anisotropic plates will be contained in a forthcoming paper by Drs. Sokolowski and Lenzen. It is sufficient for the purposes of this report only to list the equations which apply to the characteristics of steel joist-concrete slab floor systems.

The natural frequencies of vibration of the floors is given by

$$\omega_{mn} = \sqrt{\frac{D_y}{\rho}}\ \frac{m^2\pi^2}{b^2}\ \sqrt{1 + \frac{2h\epsilon^2 n^2 k^2}{m^2} + \frac{n^4 k^4}{m^4}} \qquad (2)$$

in which

D_x = flexural rigidity of floor perpendicular to joists
D_y = flexural rigidity of floor parallel to joists
ρ = surface density
a = length of floor
b = width of floor
k = b/a
$h^n = \epsilon^2$
$\epsilon^4 = \sqrt{D_x/D_y}$

and m and n are integers indicating the mode of vibration of the plate. The fundamental frequency occurs when m = n = 1. Increasing m yields higher modes parallel to the joists whereas increasing n yields higher modes perpendicular to the joists. In practice it is difficult to excite modes other than m = 1, and n = 1, 2, 3, 4, 5.

Static deflection of the floor may be calculated by the formula

$$\omega(x, y) = \frac{4P}{abD_y} \sum \sum \frac{\sin \frac{m\pi}{2} \sin \frac{n\pi}{2} \sin\alpha_n \sin\beta_n y}{\frac{\pi^4}{b^4} (m^4 + 2h\epsilon^2 n^2 m^2 k^2 + \epsilon^4 m^4 k^4} \qquad (3)$$

in which

P = load

$\alpha_n = n\pi/a$, $\beta_n = n\pi/b$

This formula reduces, for deflections at the center, to

$$w(\frac{a}{2}, \frac{b}{2}) = \frac{Pb^2}{\sqrt{2}\, \pi D_y} \sqrt[4]{\frac{D_y}{D_x}} \qquad (4)$$

The deflection profile of the system is given by

$$w(x,y) = \frac{Pb^2}{\pi^3 \epsilon D_y} e^{\frac{-\beta_1 x}{\epsilon\sqrt{2}}} \sin\left(\frac{\pi}{4} + \frac{\beta_1 x}{\epsilon\sqrt{2}}\right) \tag{5}$$

into which is substituted y = b/2 to yield the deflection profile at the center. The first zero point of the deflection is obtained by substituting w(x,b/2) = 0 into Eq. (5) and solving for x. The resulting expression is

$$x_0 = \frac{3}{4}\sqrt{2} \;\epsilon b \tag{6}$$

Mr. Leslie D. Meyer derives a formula for the impact deflection generated by the impacter he designed. This formula has been revised somewhat by Dr. Robert D. Ohmart.[6] The formula is

$$A_o = \frac{4P_o b^3}{\pi^2 EI} \sin\left(\frac{\omega_{11}\gamma}{2}\right) \tag{7}$$

in which

A_o = initial impact deflection

P_o = value of experimentally determined force developed by impacter

γ = duration of impact force

When the constants are evaluated, the computational form of the formula is

$$A_o = 2.41 \times 10^{-6} \frac{b^3}{I} \sin\left[(1.57 \times 10^{-2})\, F_{11}\right] \tag{8}$$

COMPARISON OF THE RESULTS OF THE ANALYTICAL DEVELOPMENTS AND CURRENT TEST SERIES

Basic Characteristics

The results obtained by applying the formulas from the analytical developments are compared with the experimental results. Figure 33 compares the experimental and analytical frequencies, Figure 34 the static deflections at the center of the active floor, and Figure 35 the static deflection profiles.

Figure 33 is of prime importance. There is essentially no difference between the calculated curve and the experimentally determined curve. This figure, in itself, is strong substantiation for the analytical developments and thereby provides a second verification for the composite action of the slab and joists. That is, since the analytical developments are based on the supposition that slab and joists act as a single unit and the experimental results substantiate analytical predictions, then it may be assumed that the slab and joists do, in fact, act as a single unit.

The frequencies are predicted almost as closely for the 4.125 inch floor as well. The curve shown for the 2.125 inch floor represents data obtained prior to any substantial cracking of the slab. As the slab cracked the effect was to reduce the modulus of elasticity. In the course of the test series, the modulus dropped about 20%. When this correction is substituted into the analytical formulas the experimental frequencies are predicted almost exactly. Viest, Fountain, and Singleton in Composite Construction in Steel and Concrete note this change in the modulus with age and cracking although they appear to be considering less extensive cracking than encountered in this system.

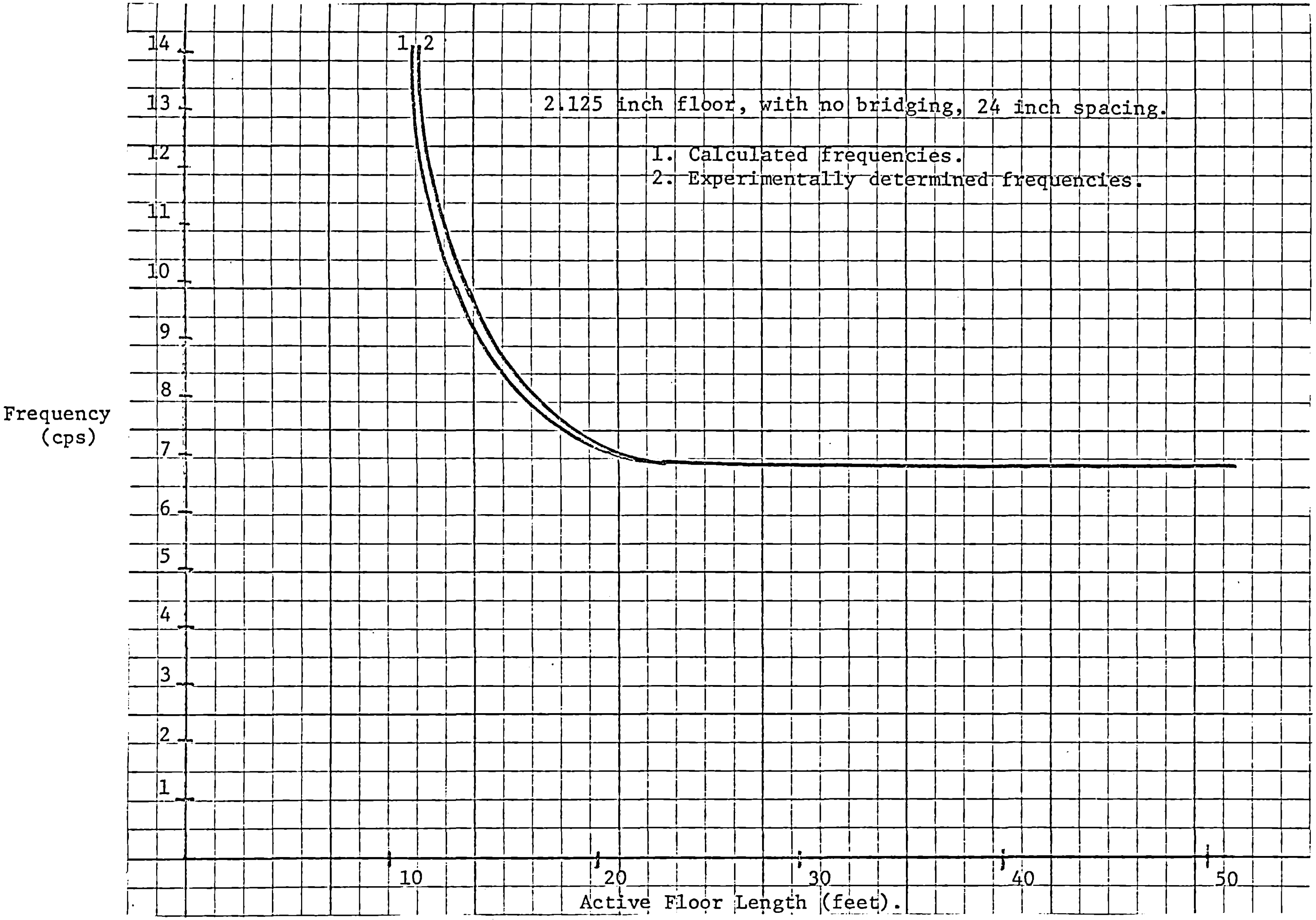


Figure 33. Comparison of Experimentally Determined and Analytically Calculated Frequencies of Vibration.

Figure 34 shows that the static deflections of the center of the active floor are also quite closely predicted. It may be seen that the experimental deflections decrease at short floor lengths. This is due primarily to the method of effecting a "point" load. For convenience, a cart was loaded with concrete test specimens and positioned at the center of the active floor. As a result, the loading was somewhat distributed rather than concentrated. This distributed effect was proportionately greater at shorter floor lengths and so the deflection decreased somewhat.

It may be noted that the curve for the 4.125-inch floor with forty-eight-inch joist spacing shows the most marked difference between theory and experiment. It is a possibility that this difference is an indication that the orthotropic assumption is not as good for the forty-eight inch joist spacing as for other floors.

The static deflection profiles shown in Figure 33 indicate again that the analytical developments closely predict the experimental results. There is some difference, of course, but in all instances the point of zero deflection is predicted within 10 percent. It should also be noted that it is difficult to precisely determine the experimental point of zero deflection. In no instance was the positive deflection very large (although it was readily apparent in all floors except the one with four feet removed from both long sides) and curve fitting introduced some uncertainty.

Impact Deflections

The actual impact deflections of the floor configurations tested were actually about 0.91 times the predicted values. This suggests a revision of the factor in the formula to 2.01×10^{-6} to allow prediction

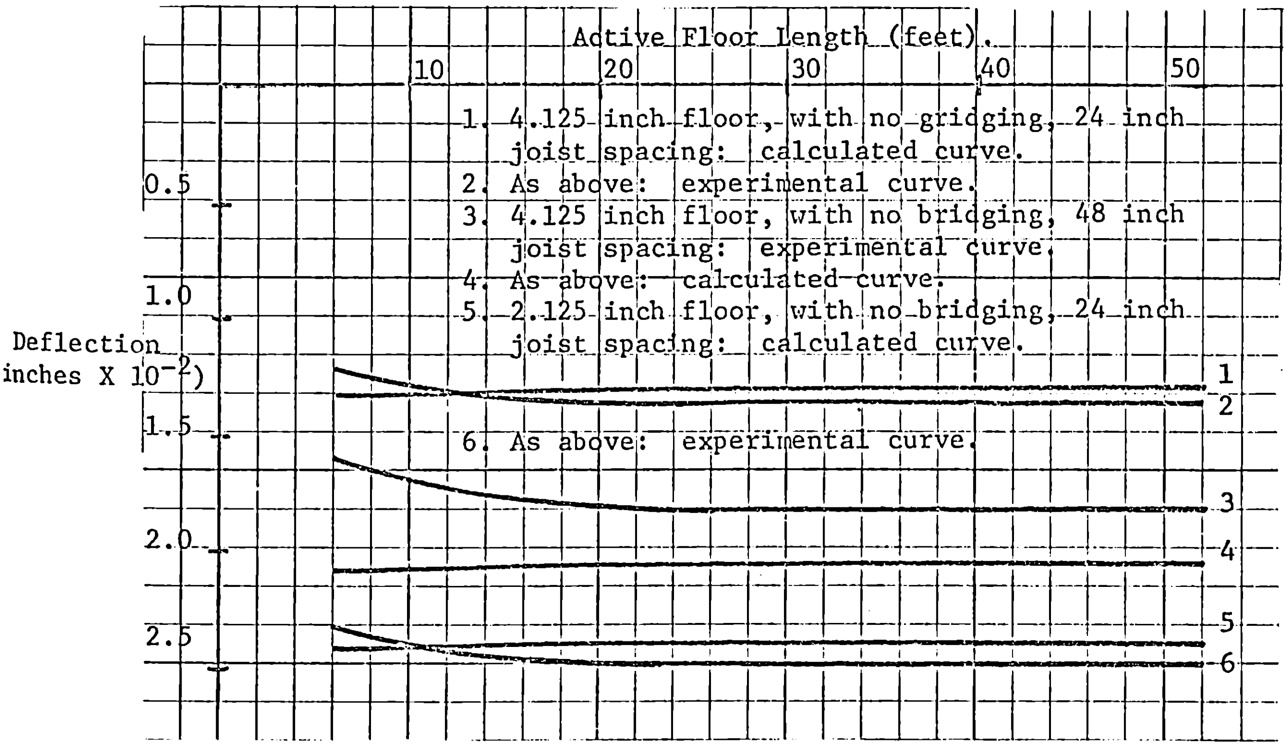


Figure 34. Comparison of Experimentally Determined and Analytically Calculated Deflections of Center of Active Floor under Point Load at Center.

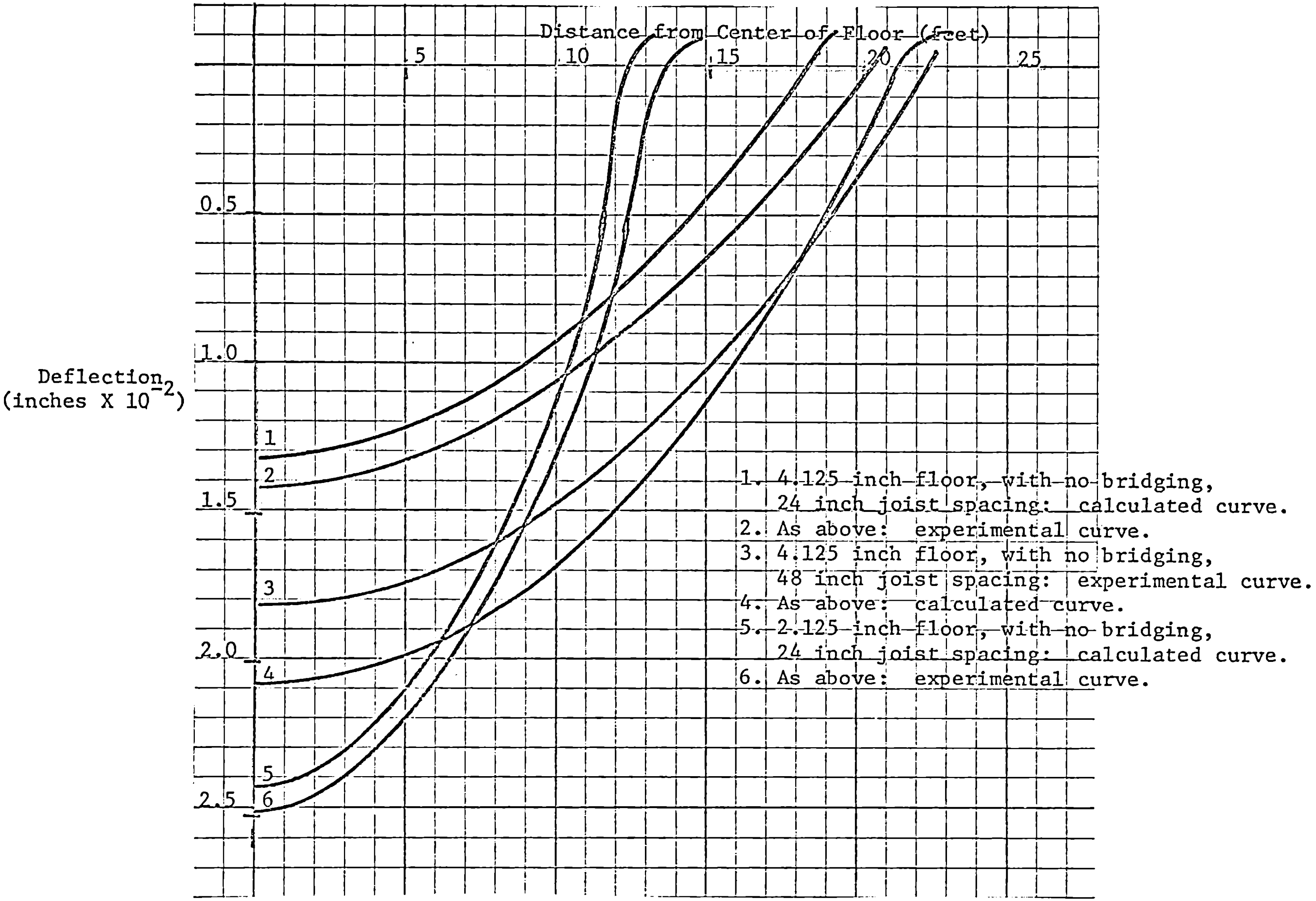


Figure 35. Comparison of Experimentally Determined and Analytically Calculated Static Deflection Profiles of Floors under Point Load at Center.

of the impact deflection of steel joist-concrete slab floor systems tested with the impacter. It should be noted that the constant given was not calculated specifically for this type of construction so that some revision might be expected. Further, the modulus of elasticity of the concrete was assumed to be 3,000,000 psi. A minor revision here would allow a calculation that would check the measured value. This is also true in the other calculations.

Conclusions

In view of the close agreement between the analytical and experimental results, it may be assumed that the analytical formulas presented may be used to predict the characteristics of steel joist-concrete slab floor systems. Applications indicate that use of these formulas yield results within about five percent of actual values.

Consideration of the frequency formula, Eq. (2) indicates the probable origin of "beats." In any application where the value of ϵ is very small the first two expected frequencies, ω_{11}, and ω_{13} will be very close to the same value (within one to three cycles per second). This proximity will yield the "beats" which will, in turn give the floor an uncomfortable feeling. Any design of a large floor should take this possibility into consideration.

APPLICATION OF EXPERIMENTAL AND ANALYTICAL DEVELOPMENTS

General

On the assumption of the validity of the analytical developments (as indicated by the experimental developments) a computerized study was made of the standard floor constructions that are now used. Rather than work with the concept of an effective floor size an effective number of joists was calculated for each particular construction. That is, the joist directly under the impact was assumed to be totally effective and each joist moving in a direction perpendicular to the joist and away from the impacted one is in some proportion less effective.

In actual application the deflection shape generated by Eq. (5) was approximated by a sine curve between the two points of zero deflection. The joist spacing for the particular construction is known and can be superimposed on the approximation. Then, the joist at the center is considered to be totally effective and those to either side are effective in proportion to the value of the sine function at their individual position. When all contributions from the joists within the approximation are added the result is known as the effective number of joists.

Figure 36 shows a typical deflection surface with a sine wave approximation. The effective number of joists is given by

$$N = 1 + 2 \ \Sigma \cos \pi x/2x_o \tag{9}$$

A sample calculation based on this figure is given in Appendix I. The other parameters of the system, fundamental frequency and impact deflection, are calculated directly from the formulas given earlier.

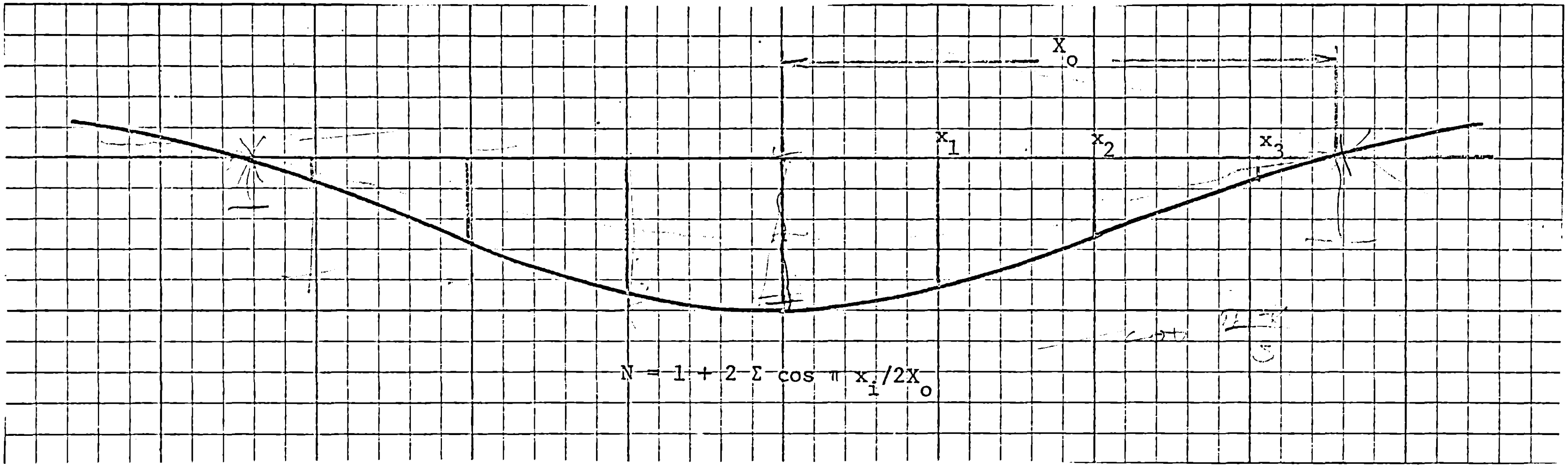


Figure 36. Sine Curve Approximation

In order to predict the occurrance of unpleasant vibration in design the natural frequency is plotted versus initial amplitude. The results thus obtained may be compared with existing response curves. These curves include those of Reiher and Meister, Figure 37, and Goldman, Figure 38. Goldman' s curve may be further simplified with a straight line approximation which is superimposed on Reiher's and Meister's response curves in Figure 39.

It must be noted that these response curves are based on steady state vibration. Investigations in the laboratory and field (with some forth-six floors tested in the Kansas City area) were based primarily on transient vibration induced by a single impact. Therefore, the amplitude scale has been multiplied by a factor of ten in order to account for the difference in the number of times the vibration occurs.

Characteristics of Generally Used Floor Constructions

Direct application of the formulas developed during the investigation to the standard joists now available yields the characteristics to be expected from the floors in which they are used. To complete the calculations it is necessary to assume a joist spacing and floor thickness. A complete catalog of the characteristics of existing joists in floors with twenty-four, thirty-six, and forty-eight inch joist spacing with concrete thicknesses of two-and-a-half, three-and-a-half, and four-and-a-half inches is given in Appendix III of this report.

Careful consideration of the information available in this catalog yields certain trends as various of the structural parameters are changed. For instance, when the length of the joists increases (that is, floor width

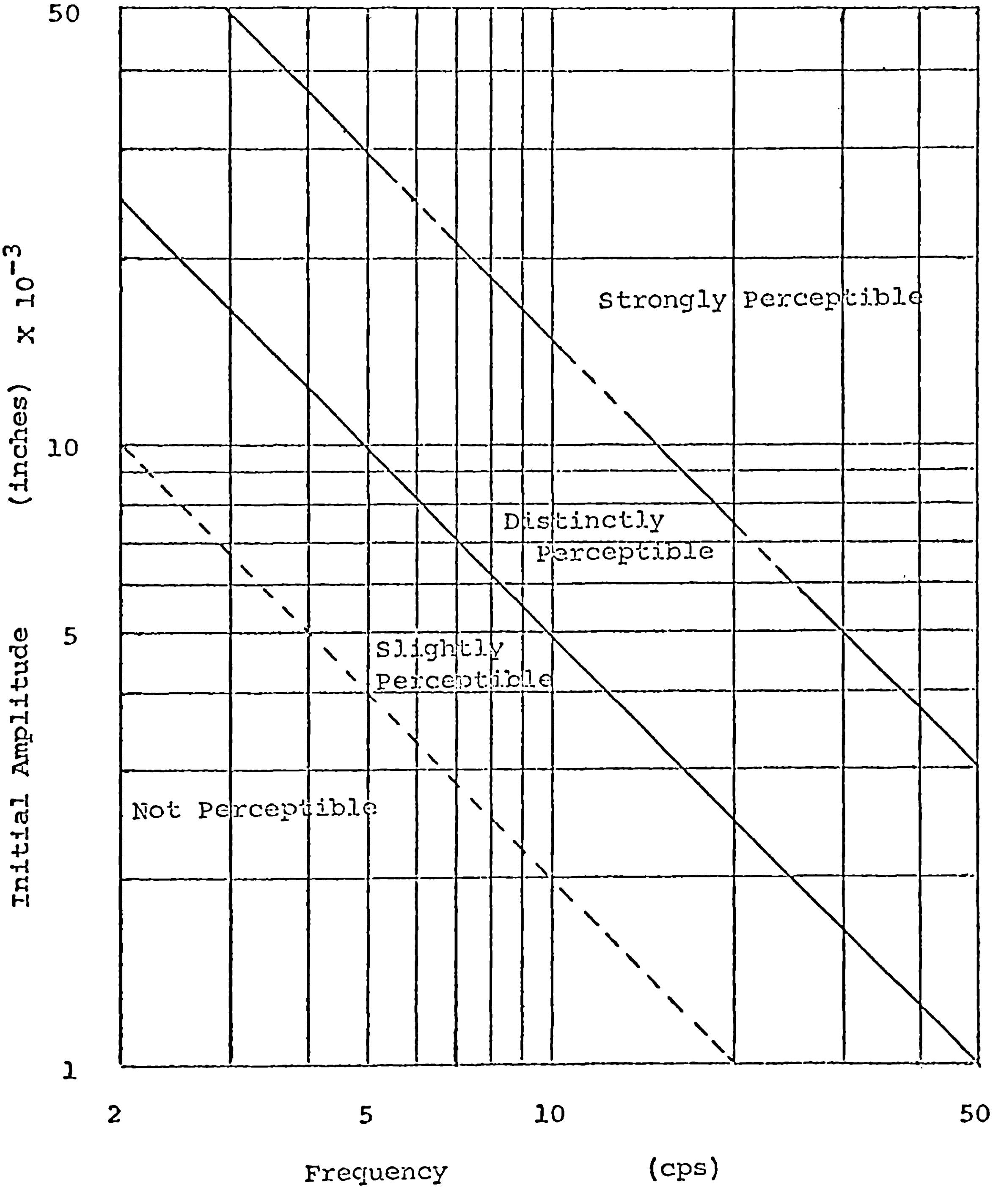


FIGURE 37. Reiher's and Meister's Human Response Curves

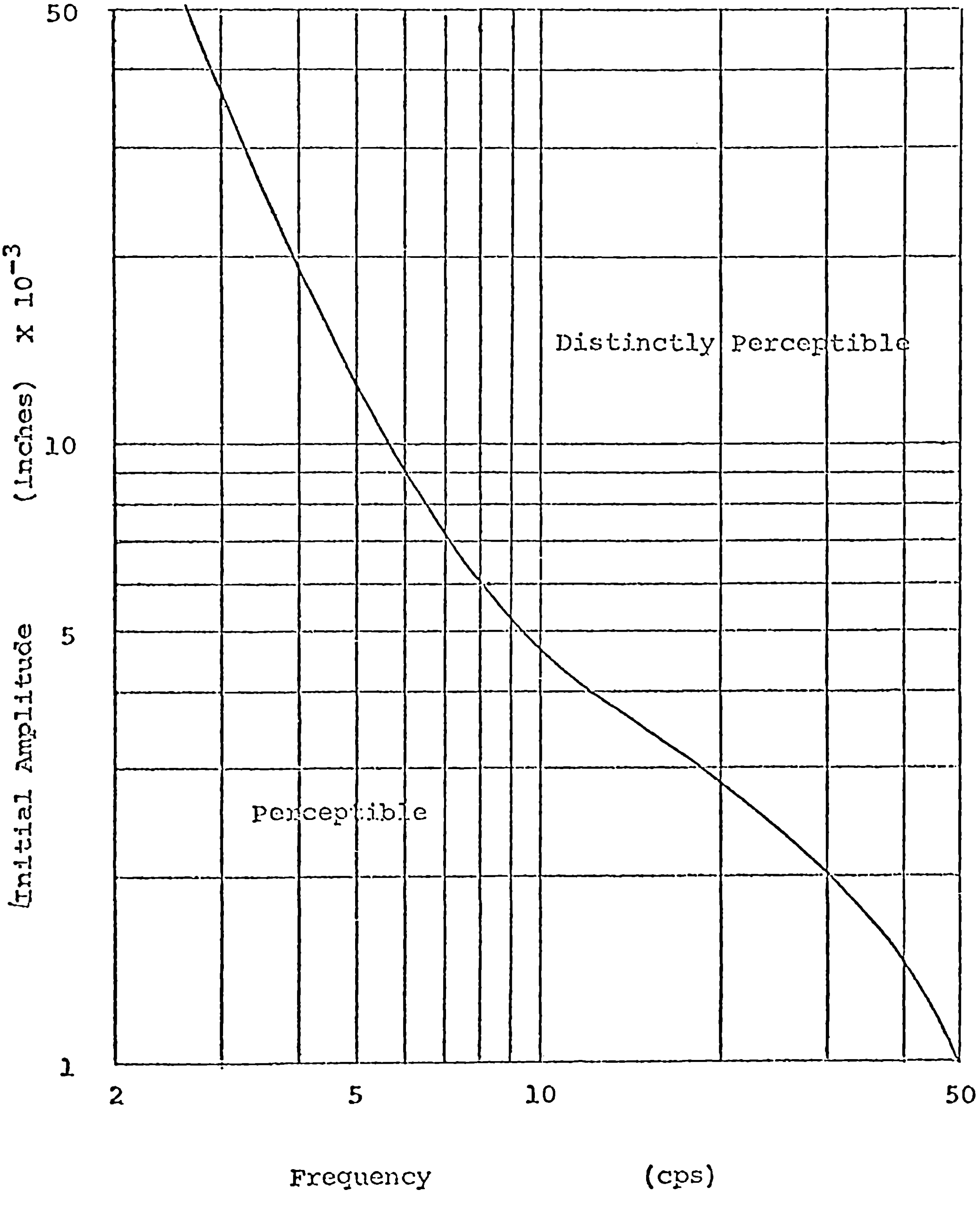


FIGURE 38 Goldman's Human Response Curve

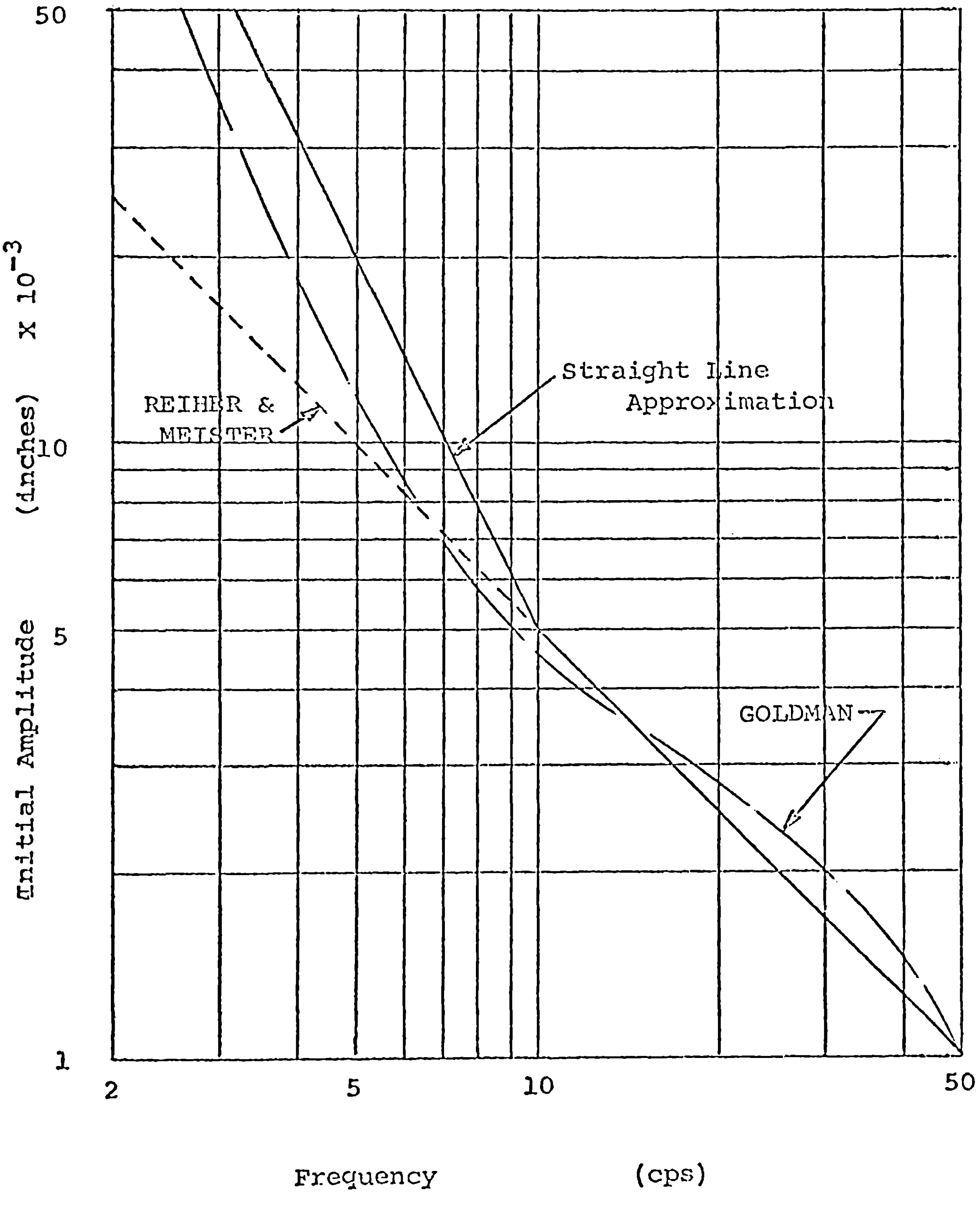


FIGURE 39. Superposition of the Human Response Curves

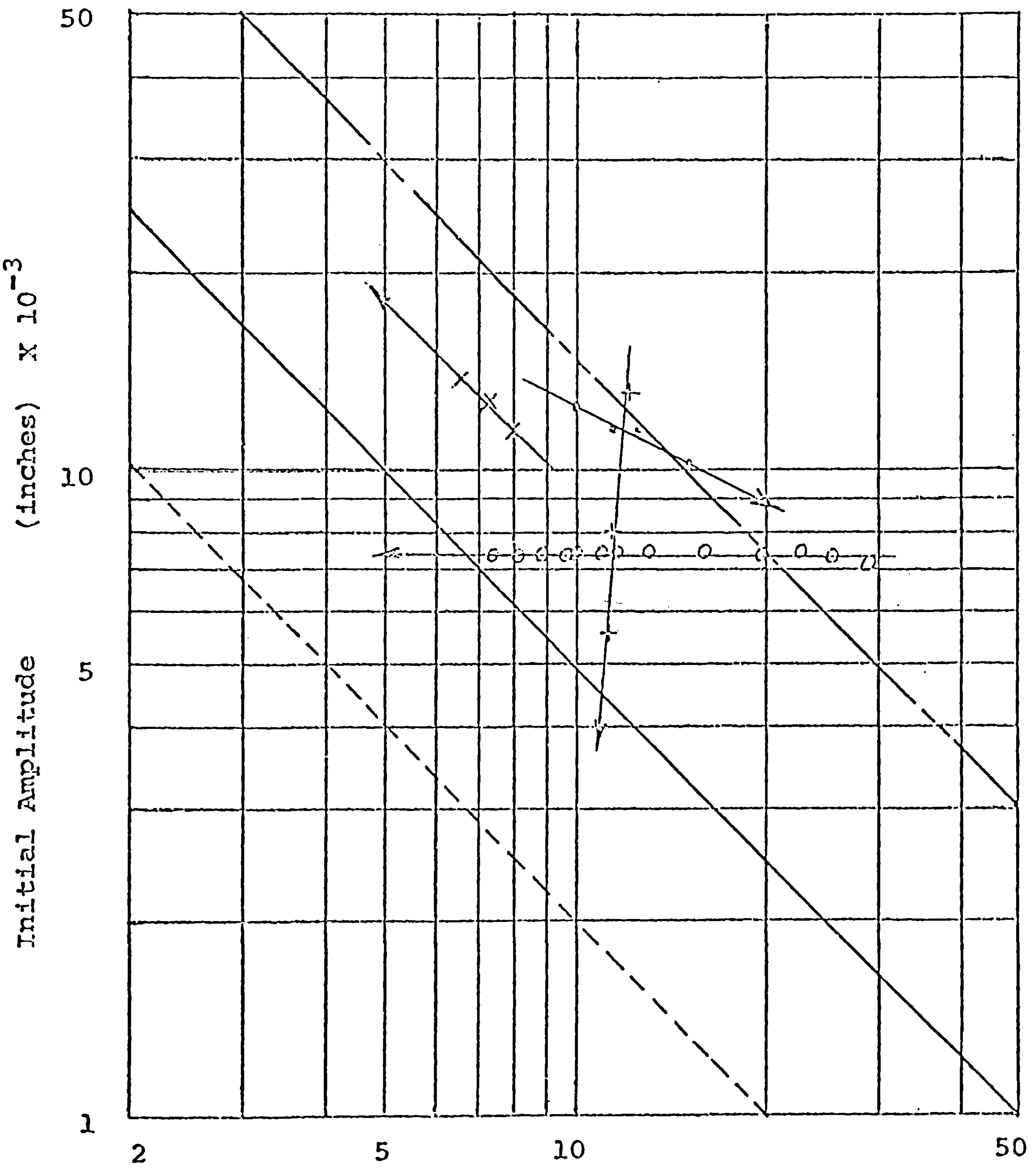
increased) for a given concrete thickness and joist spacing, the number of effective joists increases while the impact amplitude remains almost constant. Further, if the length of the joists and their spacing remain constant while the depth of concrete is increased, the number of effective joists increases while the amplitude decreases very rapidly. Finally, by increasing the spacing between the joists the number of effective joists decreases accompanied by a distinct increase in impact amplitude.

Figures 40 through 42 indicate these trends. Figure 40 shows the effects of parameters on the response curve. That is, it shows the effect on initial amplitude and frequency of changes in span, joist depth, center to center spacing, and concrete thickness. Figure 41 shows the effect of stiffness on amplitude and Figure 42 the effect of joist spacing on impact amplitude.

If the effective stiffness (NK) is multiplied by the square of the concrete thickness and divided by the spacing between joists and the resulting value plotted versus impact amplitude the data reduces to a limiting curve. This curve is shown in Figure 43.

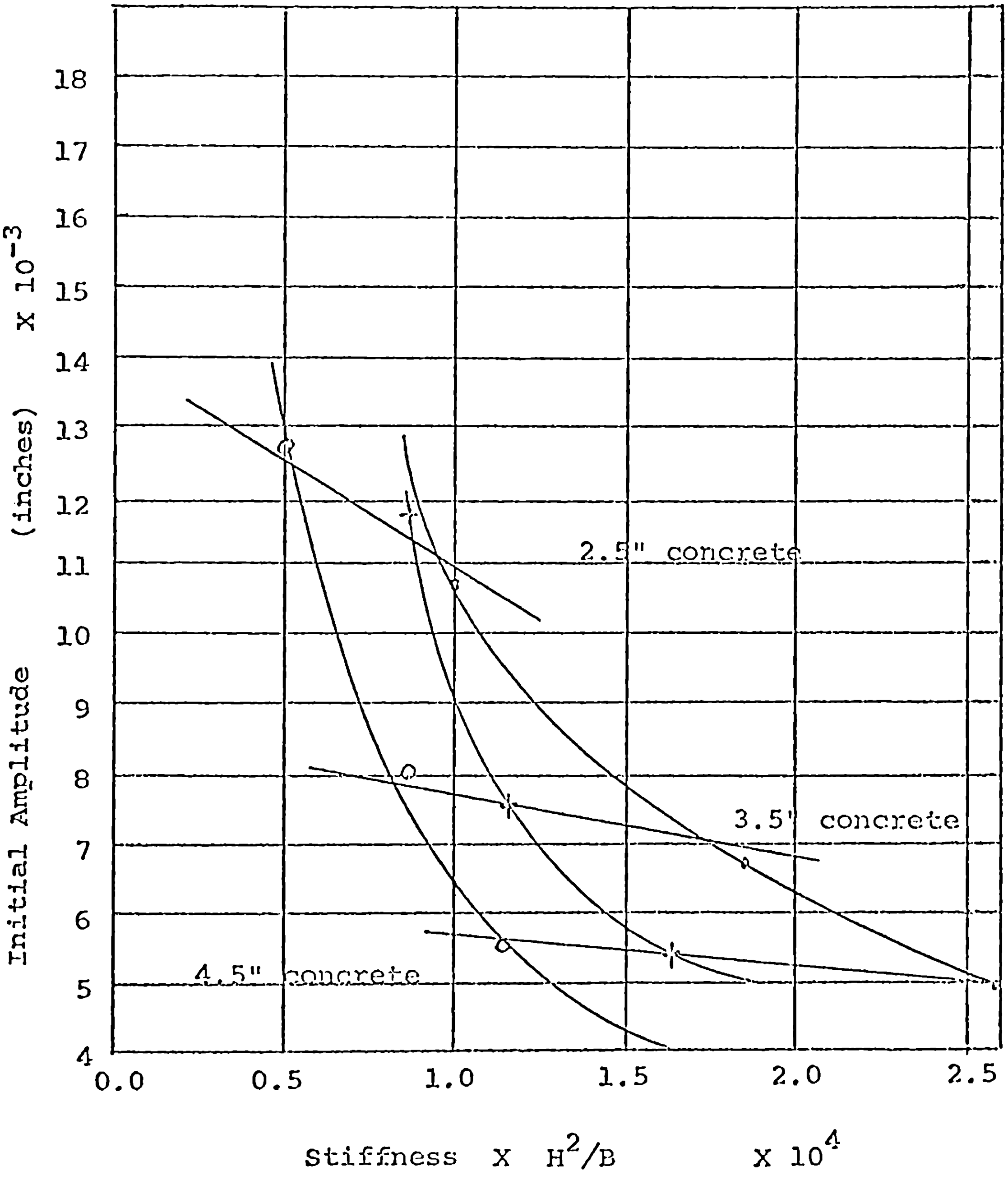
Application to Existing Floor Systems

The analysis thus far considered has been applied to the forty-six floors tested in the field.[4] Complete information is not available on the test data taken for these floors since at that time it was not known exactly what information was needed. However, as much information as possible has been taken from that data and compared with the predicted results.



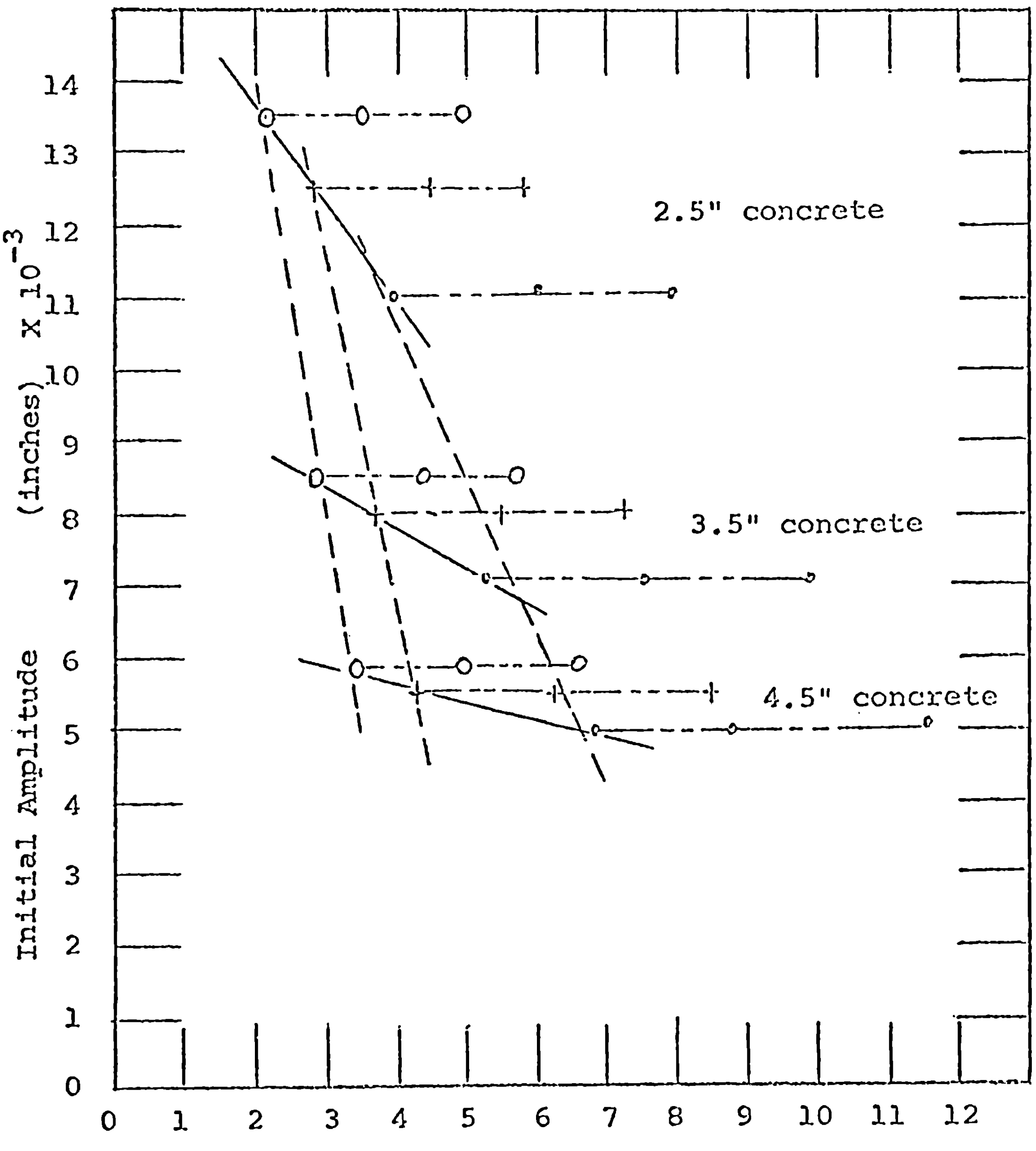
o = Increase in span from 12' to 24' for 12J4 with 3.5"x24" concrete.
• = Increase in joist depth with 2.5"x24"x20'concrete using 10J4, 12J4, 14J4, 16J4.
x = Increase in center to center spacing for 12J4 with 2.5" concrete and 23' span.
+ = Increase in concrete thickness for 12J4 with 36"c.c. concrete and 18' span.

FIGURE 40. Effects of Parameters on the Response Curves.



• = 24" center to center for a 14J4, span 14'.
+ = 36" center to center for a 14J4, span 14'.
o = 48" center to center for a 14J4, span 14'.

FIGURE 41. Stiffness - Amplitude Plot



• = 24" center to center for a 14J3.
+ = 36" center to center for a 14J3.
o = 48" center to center for a 14J3.

FIGURE 42. Effect of Parameters on Amplitude

Included in Appendix IV are the characteristics calculated for the buildings tested given both in terms of human and mechanical impact. This data, plotted against field test data yields the scatter shown in Figure 44. If the information were complete and the theory exact a forty-five degree line should result. However, it must be remembered that these tests were taken under field conditions and information such as partitioning and distributed loading were not noted so the data points show appreciable scatter. Despite this all the data points except two lie on or below the forty-five degree line. One of these two was a building subject to difficulties other than floor design and the other was tested in a narrow corridor where there were fewer joists than the number of effective joists for that particular construction. Thus, the data does show that the theory represents a limiting value. More complete data should only verify this conclusion.

Floors without Annoying Vibration

The basic purpose of this testing and research has been to aid in the design of floor systems not subject to annoying vibration. It is now possible to accurately predict the characteristics to expect from any open (non-partitioned) floor system. For instance, the set of figures 45 through 50 contained in Appendix II indicate the human response for J-series joist spans from twenty-four to forty-four feet, concrete thicknesses of two-and-a-half, three-and-a-half, and four-and-a-half inches for all depth joists from twelve to twenty-four inches. These curves are plotted from information contained in Appendix III and similar plots could be made at leisure from that information as desired.

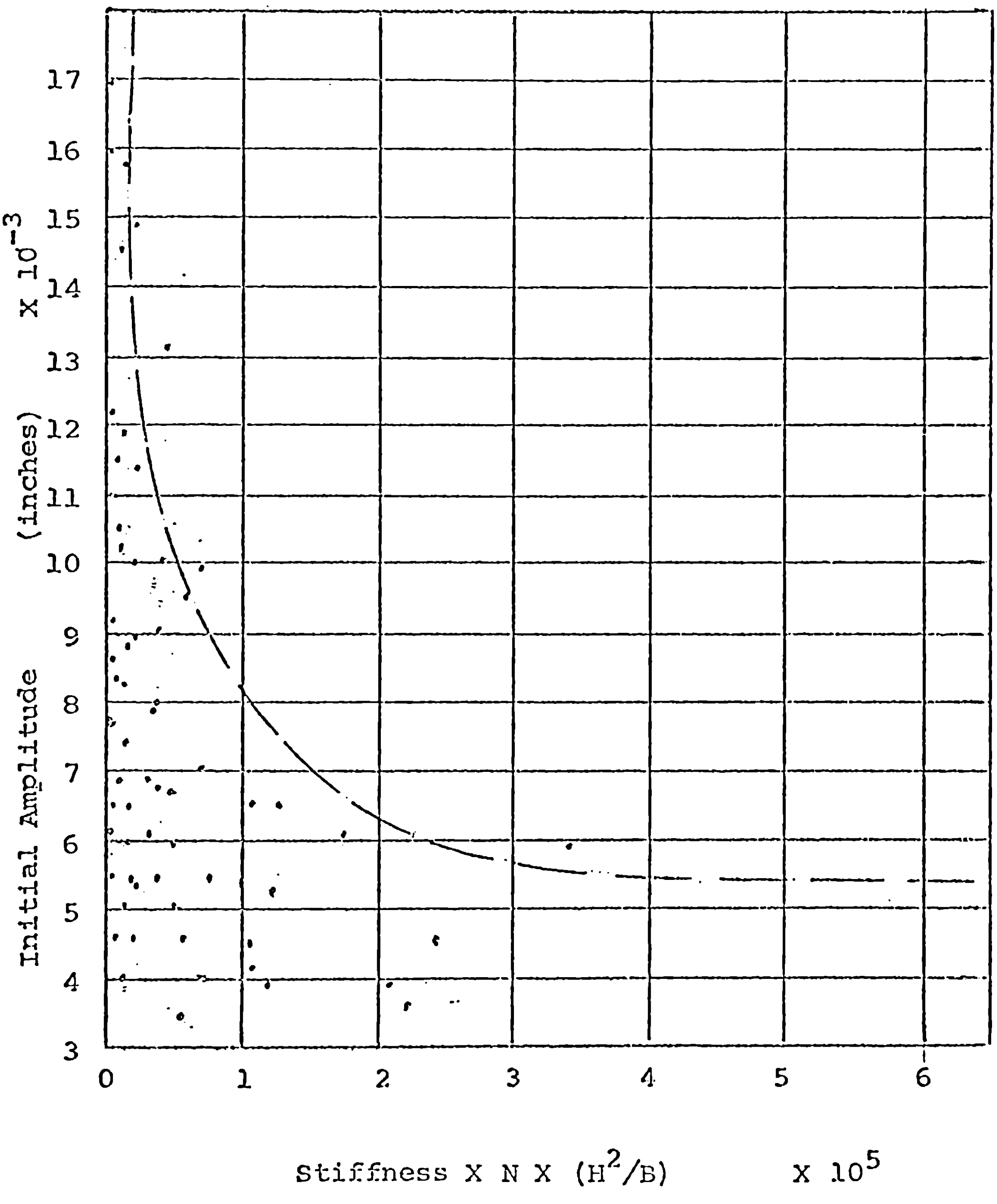


FIGURE 43. Limiting Value Curve

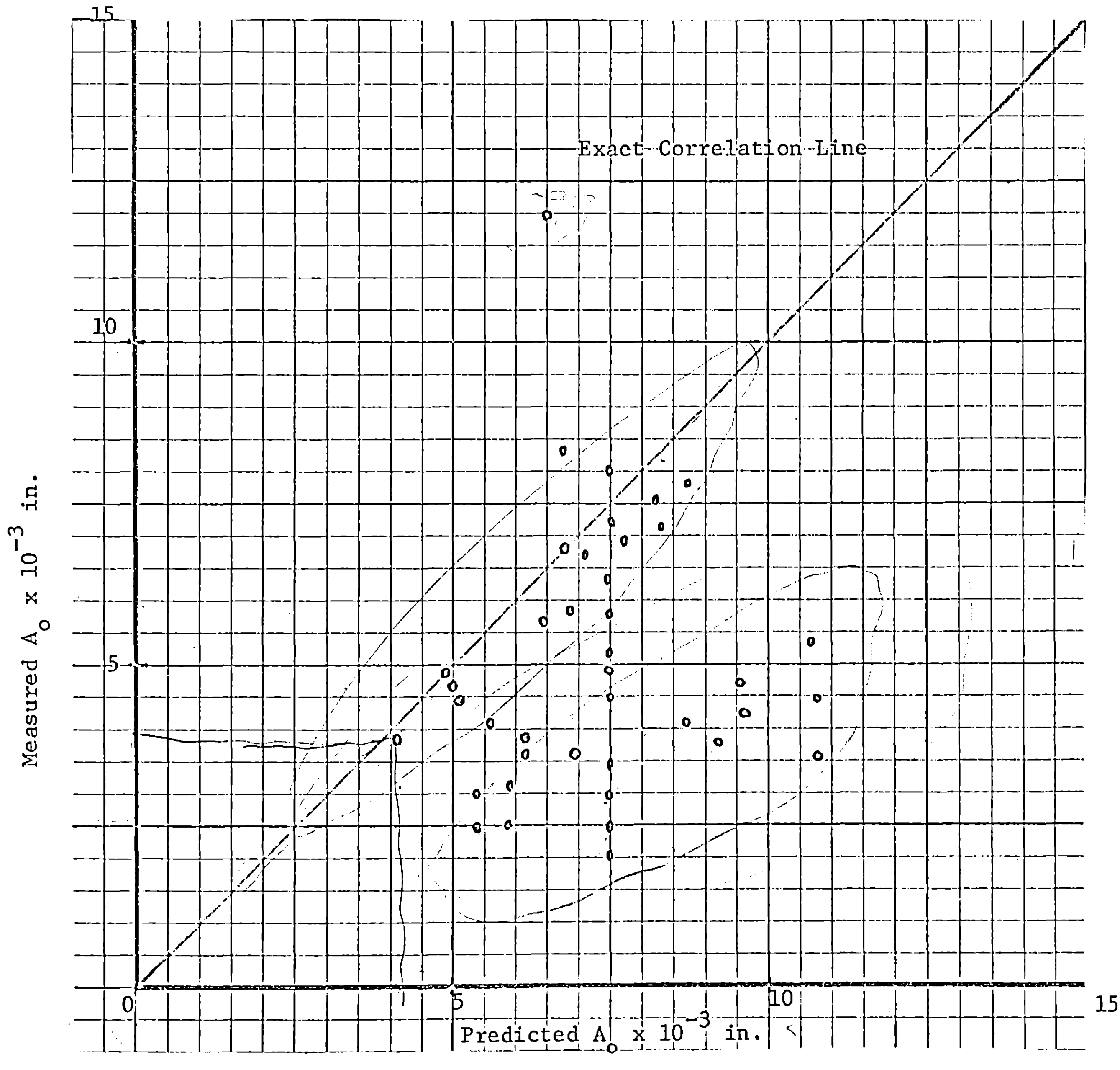


Figure. 44. Comparison of Measured and Predicted Impact Amplitude of Field Tests.

The fundamental difficulty, however, in designing floors without annoying vibration is the difficulty of defining just what constitutes annoying vibration. At present the response curves of Reiher and Meister and of Goldman are available. But these have been determined for steady-state vibrations. They still may be used if it is appreciated that they constitute a limiting case. That is, they represent the effect of the response of a system after a single impact with no damping. Damping is always present as evidenced by the experimental data. Therefore, the response curves represent very conservative limits for the design of floor systems.

It is still possible to define joist acceptability in terms of the curves. For instance, Tables 2, 3, and 4 list all available joists in various spans with two-and-a-half, three-and-a-half, and four-and-a-half inch concrete thicknesses and twenty-four, thirty-six, and forty-eight inch joist spacings. Blocked portions of the table indicate systems vibrationally acceptable (it is possible that a system vibrationally acceptable is not strong enough for the anticipated load, but there are other tables available for this determination) on the basis of Reiher and Meister's curves with the allowable amplitude increased by a factor of 10. Table 2 sets acceptance at the perceptable level, Table 3 at ten percent above the perceptable, and Table 4 at twenty percent above the perceptable level. This range is recommended since laboratory floor damping was used to set the limits. Field floors have higher damping and the added thickness would introduce additional damping. Thus it is felt that these are still conservative.

Just exactly what criteria will be chosen to determine acceptance of a system and under what conditions those criteria will be applied must be set by the industry. It is apparent, even though much remains to be learned of floor vibration, that such criteria may be established on the basis herein described. It may be appreciated that such criteria are likely to be conservative because of the limitations of knowledge now existent.

TABLE II

Tabulation of Vibrationally Acceptable Concrete Slab-Steel Joist Floor Systems Based on 0% Deviation from Reiher & Meister's Perceptible Curve.

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
12J2	24'-0"			▨			▨			▨
12J3				▨			▨			▨
12J4				▨			▨			▨
12J5				▨			▨			▨
12J6				▨			▨			▨
14J3				▨			▨			▨
14J4				▨			▨			▨
14J5				▨			▨			▨
14J6				▨			▨			▨
14J7				▨			▨			▨
16J4				▨			▨			▨
16J5				▨						▨
16J6				▨			▨			▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨						
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨						
20J6				▨			▨			
20J7				▨			▨			
20J8				▨			▨			
22J6				▨			▨			
22J7				▨			▨			
22J8				▨			▨			
24J6				▨						
24J7				▨			▨			
24J8				▨			▨			
14J3	25'-0"			▨			▨			▨
14J4				▨			▨			▨
14J5				▨			▨			▨
14J6				▨			▨			▨
14J7				▨			▨			▨

TABLE II

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
16J4	25'-0"			▨			▨			▨
16J5				▨			▨			▨
16J6				▨			▨			▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨			▨			▨
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨			▨			▨
20J6				▨			▨			▨
20J7				▨			▨			▨
20J8				▨			▨			▨
22J6				▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8				▨			▨			▨
14J3	26'-0"			▨			▨			▨
14J4				▨			▨			▨
14J5				▨			▨			▨
14J6				▨			▨			▨
14J7				▨			▨			▨
16J4				▨			▨			▨
16J5				▨			▨			▨
16J6				▨			▨			▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨			▨			▨
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨			▨			▨
20J6				▨			▨			▨
20J7				▨			▨			▨

TABLE II

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
20J8	26'-0"			▨			▨			▨
22J6				▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8				▨			▨			▨
14J3	27'-0"			▨			▨			▨
14J4				▨			▨			▨
14J5				▨			▨			▨
14J6				▨			▨			▨
14J7				▨			▨			▨
16J4				▨			▨			▨
16J5				▨			▨			▨
16J6				▨			▨			▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨			▨			▨
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨			▨			▨
20J6				▨			▨			▨
20J7				▨			▨			▨
20J8				▨			▨			▨
22J6				▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8				▨			▨			▨
14J3	28'-0"			▨			▨			▨
14J4				▨			▨			▨
14J5				▨			▨			▨

TABLE II

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
14J6	28'-0"			▨			▨			▨
14J7				▨			▨			▨
16J4				▨			▨			▨
16J5				▨			▨			▨
16J6				▨			▨			▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨			▨			▨
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨			▨			▨
20J6				▨			▨			▨
20J7				▨			▨			▨
20J8				▨			▨			▨
22J6				▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8				▨			▨			▨
16J4	29'-0"			▨			▨			▨
16J5				▨			▨			▨
16J6				▨			▨			▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨			▨			▨
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨			▨			▨
20J6				▨			▨			▨
20J7				▨			▨			▨
20J8				▨			▨			▨

TABLE II

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
22J6	29'-0"			▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8				▨			▨			▨
16J4	30'-0"			▨			▨		▨	▨
16J5				▨			▨			▨
16J6				▨			▨			▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨			▨			▨
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨			▨			▨
20J6				▨			▨			▨
20J7				▨			▨			▨
20J8				▨			▨			▨
22J6				▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8				▨			▨			▨
16J4	31'-0"			▨		▨	▨		▨	▨
16J5				▨		▨	▨		▨	▨
16J6				▨			▨		▨	▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨			▨		▨	▨
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨			▨			▨

TABLE II

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
20J6	31'-0"			▨			▨			▨
20J7				▨			▨			▨
20J8				▨			▨			▨
22J6				▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8				▨			▨			▨
16J4	32'-0"		▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6				▨			▨		▨	▨
18J7				▨			▨		▨	▨
18J8				▨			▨			▨
20J5				▨			▨		▨	▨
20J6				▨			▨			▨
20J7				▨			▨			▨
20J8				▨			▨			▨
22J6				▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8				▨			▨			▨
18J5	33'-0"		▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨

TABLE II

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
20J8	33'-0"		▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨			▨		▨	▨
22J8			▨	▨			▨		▨	▨
24J6				▨			▨		▨	▨
24J7				▨			▨		▨	▨
24J8				▨			▨			▨
18J5	34'-0"		▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
18J5	35'-0"		▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨

TABLE II

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
24J8	35'-0"		▨	▨		▨	▨		▨	▨
18J5	36'-0"		▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
20J5	37'-0"		▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
20J5	38'-0"		▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨

TABLE II

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
24J8	38'-0"		▨	▨		▨	▨		▨	▨
20J5	39'-0"		▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
20J5	40'-0"		▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
22J6	41'-0"		▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
22J6	42'-0"		▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨

TABLE II

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
22J6	43'-0"		▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
22J6	44'-0"		▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
24J6	45'-0"		▨	▨		▨	▨	▨	▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
24J6	46'-0"		▨	▨	▨	▨	▨	▨	▨	▨
24J7			▨	▨		▨	▨	▨	▨	▨
24J8			▨	▨		▨	▨	▨	▨	▨
24J6	47'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6	48'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J8		▨	▨	▨	▨	▨	▨	▨	▨	▨

TABLE III

Tabulation of Vibrationally Acceptable Concrete Slab-Steel Joist Floor Systems Based on 10% Deviation from Reiher & Meister's Perceptible Curve.

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
12J2	24'-0"			▨			▨			▨
12J3				▨			▨			▨
12J4				▨			▨			▨
12J5				▨			▨			▨
12J6				▨			▨			▨
14J3				▨			▨			▨
14J4				▨			▨			▨
14J5				▨			▨			▨
14J6				▨			▨			▨
14J7				▨			▨			▨
16J4				▨			▨			▨
16J5				▨			▨			▨
16J6				▨			▨			▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨			▨			▨
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨			▨			▨
20J6				▨			▨			▨
20J7				▨			▨			▨
20J8				▨			▨			▨
22J6				▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8				▨			▨			▨
14J3	25'-0"			▨			▨			▨
14J4				▨			▨			▨
14J5				▨			▨			▨
14J6				▨			▨			▨
14J7				▨			▨			▨

TABLE III

Joist Type No.	Clear Span	2.5" × 24"	3.5" × 24"	4.5" × 24"	2.5" × 36"	3.5" × 36"	4.5" × 36"	2.5" × 48"	3.5" × 48"	4.5" × 48"
16J4	25'-0"			▨			▨			▨
16J5				▨			▨			▨
16J6				▨			▨			▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨			▨			▨
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨			▨			▨
20J6				▨			▨			▨
20J7				▨			▨			▨
20J8				▨			▨			▨
22J6				▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8				▨			▨			▨
14J3	26'-0"			▨			▨		▨	▨
14J4				▨			▨			▨
14J5				▨			▨			▨
14J6				▨			▨			▨
14J7				▨			▨			▨
16J4				▨			▨			▨
16J5				▨			▨			▨
16J6				▨			▨			▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨			▨			▨
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨			▨			▨
20J6				▨			▨			▨
20J7				▨			▨			▨

TABLE III

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
20J8	26'-0"			▨			▨			▨
22J6				▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8				▨			▨			▨
14J3	27'-0"		▨	▨		▨	▨		▨	▨
14J4				▨		▨	▨		▨	▨
14J5				▨			▨		▨	▨
14J6				▨			▨			▨
14J7				▨			▨			▨
16J4				▨			▨			▨
16J5				▨			▨			▨
16J6				▨			▨			▨
16J7				▨			▨			▨
16J8				▨			▨			▨
18J5				▨			▨			▨
18J6				▨			▨			▨
18J7				▨			▨			▨
18J8				▨			▨			▨
20J5				▨			▨			▨
20J6				▨			▨			▨
20J7				▨			▨			▨
20J8				▨			▨			▨
22J6				▨			▨			▨
22J7				▨			▨			▨
22J8				▨			▨			▨
24J6				▨			▨			▨
24J7				▨			▨			▨
24J8			▨	▨			▨			▨
14J3	28'-0"		▨	▨		▨	▨		▨	▨
14J4			▨	▨		▨	▨		▨	▨
14J5			▨	▨		▨	▨		▨	▨

TABLE III

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
14J6	28'-0"		▨	▨		▨	▨		▨	▨
14J7			▨	▨		▨	▨		▨	▨
16J4			▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5				▨		▨	▨		▨	▨
18J6				▨			▨		▨	▨
18J7				▨			▨		▨	▨
18J8			▨	▨			▨			▨
20J5				▨			▨		▨	▨
20J6				▨			▨			▨
20J7			▨	▨			▨			▨
20J8			▨	▨			▨			▨
22J6				▨			▨			▨
22J7			▨	▨			▨			▨
22J8			▨	▨			▨			▨
24J6			▨	▨			▨			▨
24J7			▨	▨			▨			▨
24J8			▨	▨			▨			▨
16J4	29'-0"		▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨

TABLE III

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
22J6	29'-0"		▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨			▨
16J4	30'-0"		▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
16J4	31'-0"		▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨

TABLE III

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
20J6	31'-0"		▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
16J4	32'-0"		▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
18J5	33'-0"		▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨

TABLE III

Joist Type No.	Clear Span	2.5" × 24"	3.5" × 24"	4.5" × 24"	2.5" × 36"	3.5" × 36"	4.5" × 36"	2.5" × 48"	3.5" × 48"	4.5" × 48"
20J8	33'-0"		▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7										
22J8										
24J6			▨	▨		▨	▨		▨	▨
24J7										
24J8										
18J5	34'-0"		▨	▨		▨	▨		▨	▨
18J6										
18J7										
18J8										
20J5			▨	▨		▨	▨		▨	▨
20J6										
20J7										
20J8										
22J6			▨	▨		▨	▨		▨	▨
22J7										
22J8										
24J6			▨	▨		▨	▨		▨	▨
24J7										
24J8										
18J5	35'-0"		▨	▨		▨	▨		▨	▨
18J6										
18J7										
18J8										
20J5			▨	▨		▨	▨		▨	▨
20J6										
20J7										
20J8										
22J6			▨	▨		▨	▨		▨	▨
22J7										
22J8										
24J6			▨	▨		▨	▨		▨	▨
24J7										

TABLE III

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
24J8	35'-0"		▨	▨		▨	▨		▨	▨
18J5	36'-0"		▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
20J5	37'-0"		▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
20J5	38'-0"		▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨

TABLE III

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
24J8	38'-0"		▨	▨		▨	▨		▨	▨
20J5	39'-0"		▨	▨		▨	▨	▨	▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
20J5	40'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
20J6			▨	▨	▨	▨	▨	▨	▨	▨
20J7			▨	▨	▨	▨	▨	▨	▨	▨
20J8			▨	▨		▨	▨	▨	▨	▨
22J6			▨	▨		▨	▨	▨	▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
22J6	41'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
22J7			▨	▨	▨	▨	▨	▨	▨	▨
22J8			▨	▨	▨	▨	▨	▨	▨	▨
24J6			▨	▨	▨	▨	▨	▨	▨	▨
24J7			▨	▨		▨	▨	▨	▨	▨
24J8			▨	▨		▨	▨	▨	▨	▨
22J6	42'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
22J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J8		▨	▨	▨	▨	▨	▨	▨	▨	▨

TABLE III

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
22J6 22J7 22J8	43'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J6 22J7 22J8	44'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8	45'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8	46'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8	47'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8	48'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨

TABLE IV

Tabulation of Vibrationally Acceptable Concrete Slab-Steel Joist Floor Systems Based on 20% Deviation from Reiher & Meister's Perceptible Curve.

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
12J2	24'-0"			▨		▨	▨		▨	▨
12J3			▨	▨		▨	▨		▨	▨
12J4				▨		▨	▨		▨	▨
12J5				▨		▨	▨		▨	▨
12J6				▨			▨		▨	▨
14J3				▨			▨		▨	▨
14J4				▨			▨		▨	▨
14J5				▨			▨			▨
14J6				▨			▨			▨
14J7			▨	▨			▨			▨
16J4				▨			▨			▨
16J5				▨			▨			▨
16J6				▨			▨			▨
16J7			▨	▨			▨			▨
16J8			▨	▨			▨			▨
18J5				▨			▨			▨
18J6			▨	▨			▨			▨
18J7			▨	▨			▨			▨
18J8			▨	▨			▨			▨
20J5				▨			▨			▨
20J6			▨	▨			▨			▨
20J7			▨	▨			▨			▨
20J8			▨	▨		▨	▨			▨
22J6			▨	▨			▨			▨
22J7			▨	▨			▨			▨
22J8			▨	▨		▨	▨			▨
24J6			▨	▨			▨			▨
24J7			▨	▨		▨	▨			▨
24J8			▨	▨		▨	▨		▨	▨
14J3	25'-0"		▨	▨		▨	▨		▨	▨
14J4			▨	▨		▨	▨		▨	▨
14J5			▨	▨		▨	▨		▨	▨
14J6			▨	▨		▨	▨		▨	▨
14J7			▨	▨		▨	▨		▨	▨

TABLE IV

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
16J4	25'-0"			▨			▨			▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨			▨		▨	▨
18J7			▨	▨		▨	▨			▨
18J8			▨	▨		▨	▨			▨
20J5			▨	▨			▨		▨	▨
20J6			▨	▨		▨	▨			▨
20J7			▨	▨		▨	▨			▨
20J8			▨	▨		▨	▨			▨
22J6			▨	▨		▨	▨			▨
22J7			▨	▨		▨	▨			▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨			▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
14J3	26'-0"		▨	▨		▨	▨		▨	▨
14J4			▨	▨		▨	▨		▨	▨
14J5			▨	▨		▨	▨		▨	▨
14J6			▨	▨		▨	▨		▨	▨
14J7			▨	▨		▨	▨		▨	▨
16J4			▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨

TABLE IV

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
20J8	26'-0"		▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
14J3	27'-0"		▨	▨		▨	▨		▨	▨
14J4			▨	▨		▨	▨		▨	▨
14J5			▨	▨		▨	▨		▨	▨
14J6			▨	▨		▨	▨		▨	▨
14J7			▨	▨		▨	▨		▨	▨
16J4			▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
14J3	28'-0"		▨	▨		▨	▨		▨	▨
14J4			▨	▨		▨	▨		▨	▨
14J5			▨	▨		▨	▨		▨	▨

TABLE IV

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
14J6	28'-0"		▨	▨		▨	▨		▨	▨
14J7			▨	▨		▨	▨		▨	▨
16J4			▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
16J4	29'-0"		▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨

TABLE IV

Joist Type No.	Clear Span	2.5" × 24"	3.5" × 24"	4.5" × 24"	2.5" × 36"	3.5" × 36"	4.5" × 36"	2.5" × 48"	3.5" × 48"	4.5" × 48"
22J6	29'-0"		▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
16J4	30'-0"		▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
16J4	31'-0"		▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨

TABLE IV

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
20J6	31'-0"		▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
16J4	32'-0'		▨	▨		▨	▨		▨	▨
16J5			▨	▨		▨	▨		▨	▨
16J6			▨	▨		▨	▨		▨	▨
16J7			▨	▨		▨	▨		▨	▨
16J8			▨	▨		▨	▨		▨	▨
18J5			▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
18J5	33'-0'		▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨

TABLE IV

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
20J8	33'-0"		▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
18J5	34'-0"		▨	▨		▨	▨		▨	▨
18J6			▨	▨		▨	▨		▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
18J5	35'-0"		▨	▨		▨	▨	▨	▨	▨
18J6			▨	▨		▨	▨	▨	▨	▨
18J7			▨	▨		▨	▨		▨	▨
18J8			▨	▨		▨	▨		▨	▨
20J5			▨	▨		▨	▨		▨	▨
20J6			▨	▨		▨	▨		▨	▨
20J7			▨	▨		▨	▨		▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨

TABLE IV

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
24J8	35'-0"		▨	▨		▨	▨		▨	▨
18J5	36'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
18J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
18J7			▨	▨	▨	▨	▨	▨	▨	▨
18J8			▨	▨	▨	▨	▨	▨	▨	▨
20J5			▨	▨	▨	▨	▨	▨	▨	▨
20J6			▨	▨		▨	▨	▨	▨	▨
20J7			▨	▨		▨	▨	▨	▨	▨
20J8			▨	▨		▨	▨		▨	▨
22J6			▨	▨		▨	▨		▨	▨
22J7			▨	▨		▨	▨		▨	▨
22J8			▨	▨		▨	▨		▨	▨
24J6			▨	▨		▨	▨		▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8			▨	▨		▨	▨		▨	▨
20J5	37'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
20J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
20J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
20J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J6			▨	▨	▨	▨	▨	▨	▨	▨
22J7			▨	▨		▨	▨	▨	▨	▨
22J8		▨	▨	▨		▨	▨	▨	▨	▨
24J6			▨	▨		▨	▨	▨	▨	▨
24J7			▨	▨		▨	▨		▨	▨
24J8		▨	▨	▨		▨	▨		▨	▨
20J5	38'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
20J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
20J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
20J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J7		▨	▨	▨	▨	▨	▨	▨	▨	▨

TABLE IV

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
24J8	38'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
20J5	39'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
20J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
20J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
20J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
20J5	40'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
20J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
20J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
20J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J6	41'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
22J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J6	42'-0"	▨	▨	▨	▨	▨	▨	▨	▨	▨
22J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J7		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J8		▨	▨	▨	▨	▨	▨	▨	▨	▨

TABLE IV

Joist Type No.	Clear Span	2.5" x 24"	3.5" x 24"	4.5" x 24"	2.5" x 36"	3.5" x 36"	4.5" x 36"	2.5" x 48"	3.5" x 48"	4.5" x 48"
22J6 22J7 22J8	43'-0'	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
22J6 22J7 22J8	44'-0'	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8		▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8	45'-0'	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8	46'-0'	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8	47'-0'	▨	▨	▨	▨	▨	▨	▨	▨	▨
24J6 24J7 24J8	48'-0'	▨	▨	▨	▨	▨	▨	▨	▨	▨

SUGGESTIONS FOR FURTHER INVESTIGATION

In is readily apparent that a more difinite definition of annoying transient vibration must be made. Existing information deals almost entirely with steady-state vibrations. Furthermore, information (probably of a field installation nature) must be gathered to determine the damping effect of partitions, piping, and other structural additions. No clear picture of the relative effect of damping means now exists. Thus floors are designed to the open space layout which tends to be quite conservative in practice. Finally, there is little experimental and no theoretical information available to indicate the effect of partial floors (for instance, a floor partially removed to allow ventilation installation). It is clear that such floors are generally undesirable, but in some instances it may be practical to make such a design. However, without further information no reasonable prediction may be made of the characteristics of such a floor.

This type of an analysis can be extended to the long-span and the new intermediate span series. Same tests should be made to confirm values and to obtain same feeling of minimum damping.

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Ronnie W. Gibson made many of the calculations for the authors and contributed much to the project.

The contributions of the graduate students working on earlier portions of this problem are recognized. Most of their contributions are summarized in reference one. They include Joseph Keller, Gerald Barr, James Wiley, and William Lyons.

BIBLIOGRAPHY

1. Lenzen, Kenneth H. "Vibration of Steel Joist-Concrete Slab Floor Systems: Final Report," Studies in Engineering Mechanics, No. 16, August, 1962.

2. Gibson, Ronnie W. "Predicted Vibration Study in Steel Joist-Concrete Slab Construction," B.S. Thesis, University of Kansas, June, 1968.

3. Keller, Joseph E. "Damping Considerations in Vibration Response of Humans," Studies in Engineering Mechanics, No. 3, May, 1960.

4. Lenzen, Kenneth H., and Joseph E. Keller. "Vibration of Steel Joist-Concrete Slab Floor Systems," Studies in Engineering Mechanics, No. 3, May, 1960.

5. Meyer, Leslie. "Vibrations in Floor Systems of Steel Framed Buildings," Studies in Engineering Mechanics, No. 26, May, 1967.

6. Wiley, James A. "A Study of the Vibration of Rectangular Anisotropic Plates by the Ritz Method," Studies in Engineering Mechanics, No. 5, May, 1960.

7. Ohmart, Robert D. "An Approximate Method for the Response of Stiffened Plates to Aperiodic Excitation," Studies in Engineering Mechanics, No. 30, April, 1968.

8. Dorsett, Lawrence P. "Effect of the Variation of Structural Parameters on the Vibrational Characteristics of Steel Joist-Concrete Slab Floor Systems," M. S. Thesis, University of Kansas, May, 1968.

APPENDIX I

Sample Calculation of the Number of Effective Joists

The simplest demonstration of the calculation of the number of effective joists is by the graphical method. Suppose Figure 36 in the text represents the static deflection surface obtained for Raytown Jr. High School, data set no. 2. The joist spacing for this set is 23.5 inches. Substitution of the construction parameters into Equation (6) yields an x_o of about 84 inches (see Appendix IV). This means that a total of seven joists are located within the zeros of static deflection.

First assume that the center joist is totally effective. Then its contribution is 1.0. At a distance to either side or the center joist equal to the joist spacing the value of the sine approximation is about 0.9. Thus the contribution of the joists at these positions is taken to be 0.90. Similarly, the joists two spacings removed from the center each contribute about 0.6 and those at a distance of three spacings from the center each contribute about 0.2. Thus the total contributions are

center joist		=	1.0	
joists at 23.5 in. from center	2 x 0.9	=	1.8	
joists at 47.0 in. from center	2 x 0.6	=	1.2	
joists at 70.5 in. from center	2 x 0.2	=	0.4	
	Total		4.4	effective joists

This number compares favorably with the more exact computer calculation of 4.587 as listed in Appendix IV.

APPENDIX II

The following six figures (figures 45 through 50) illustrate the human response for J-series joist spans from twenty-four to forty-four feet, concrete thicknesses of two-and-a-half, three-and-a-half-, and four-and-a-half inches for all depth joists from twelve to twenty-four inches. These curves are plotted directly from information contained in Appendix III.

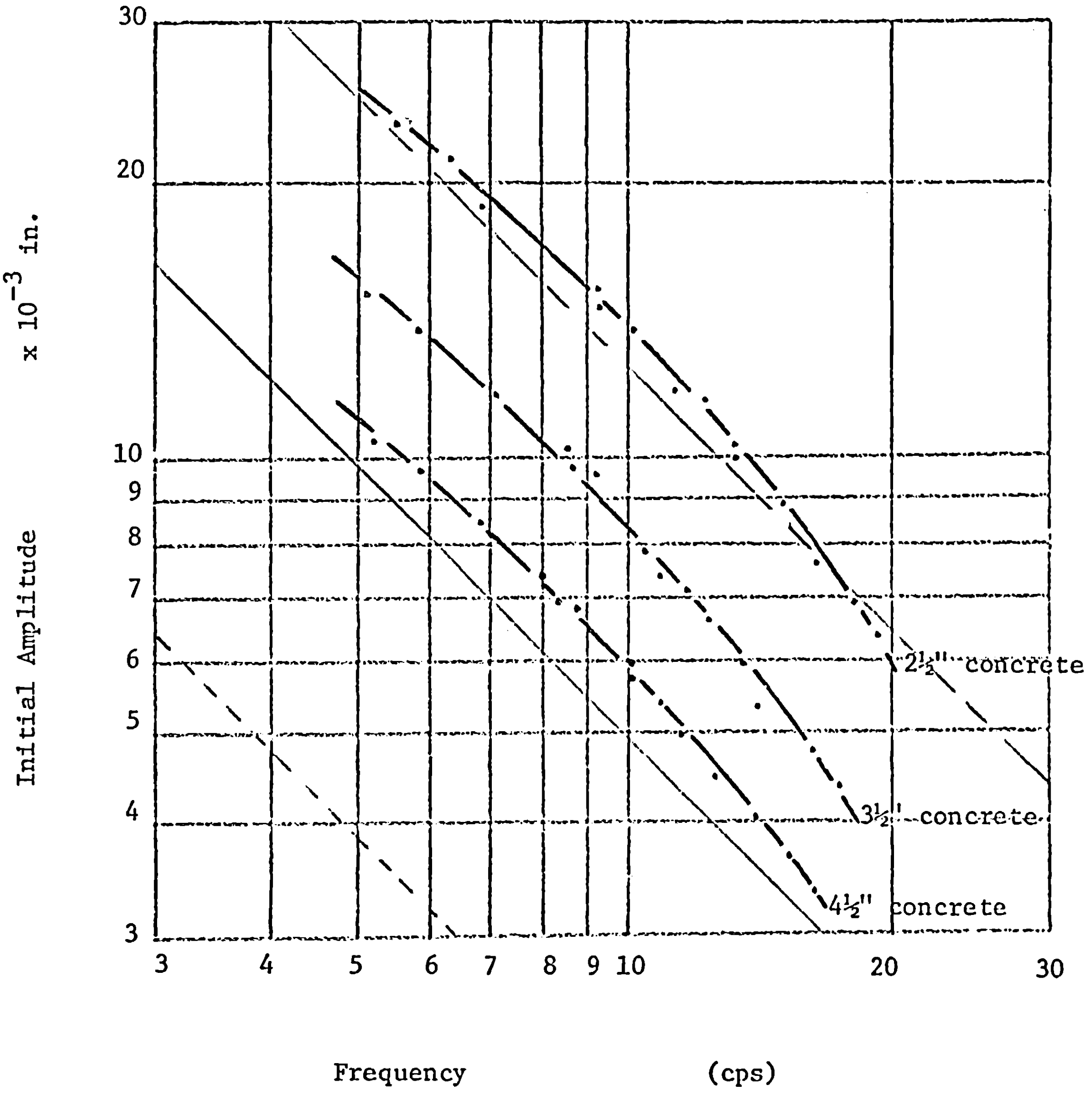

Figure 45. Human Response Curves, J-Series Joists, 24 Foot Span.

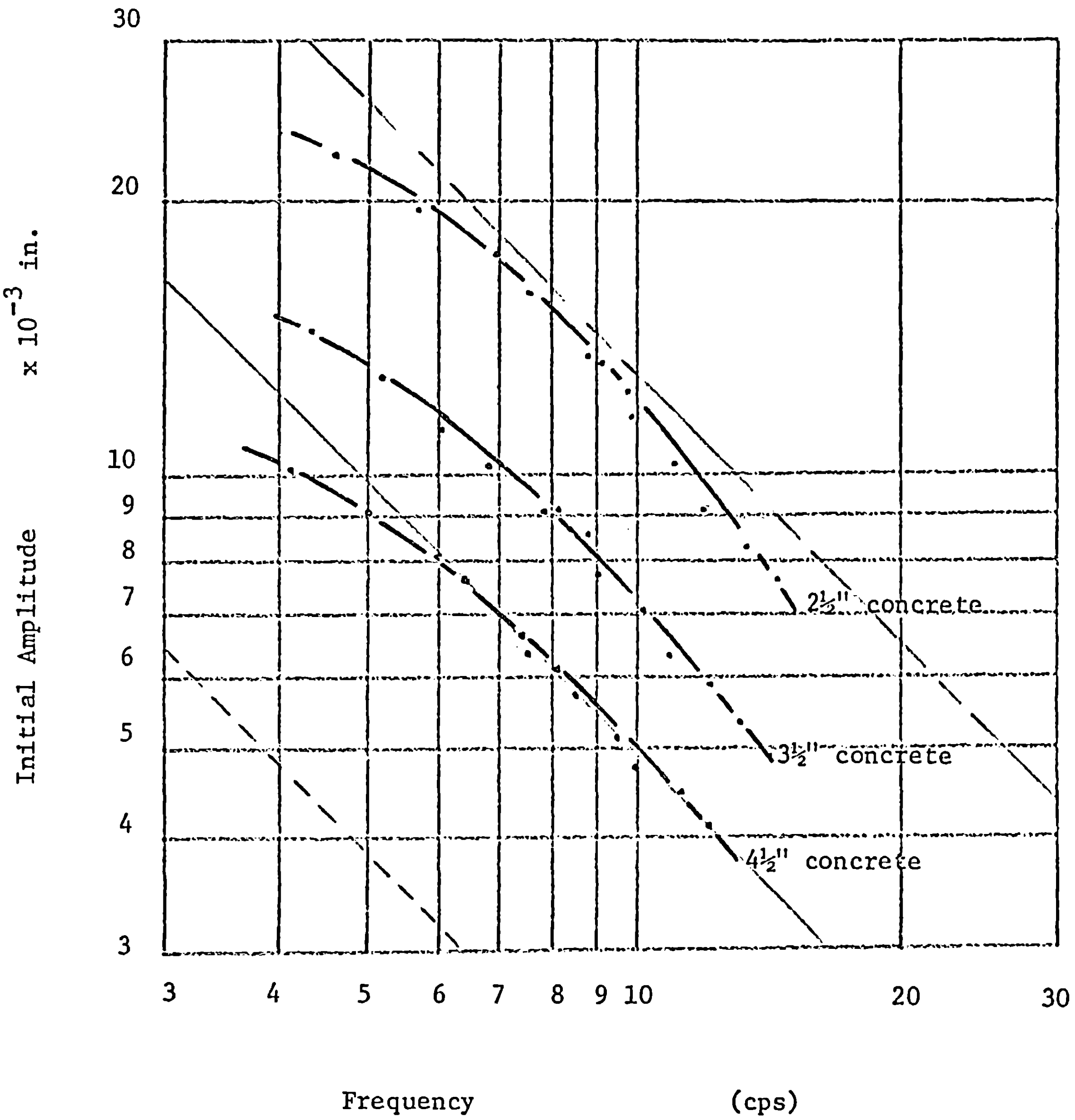

Figure 46. Human Response Curves, J-Series Joists, 28 Foot Span.

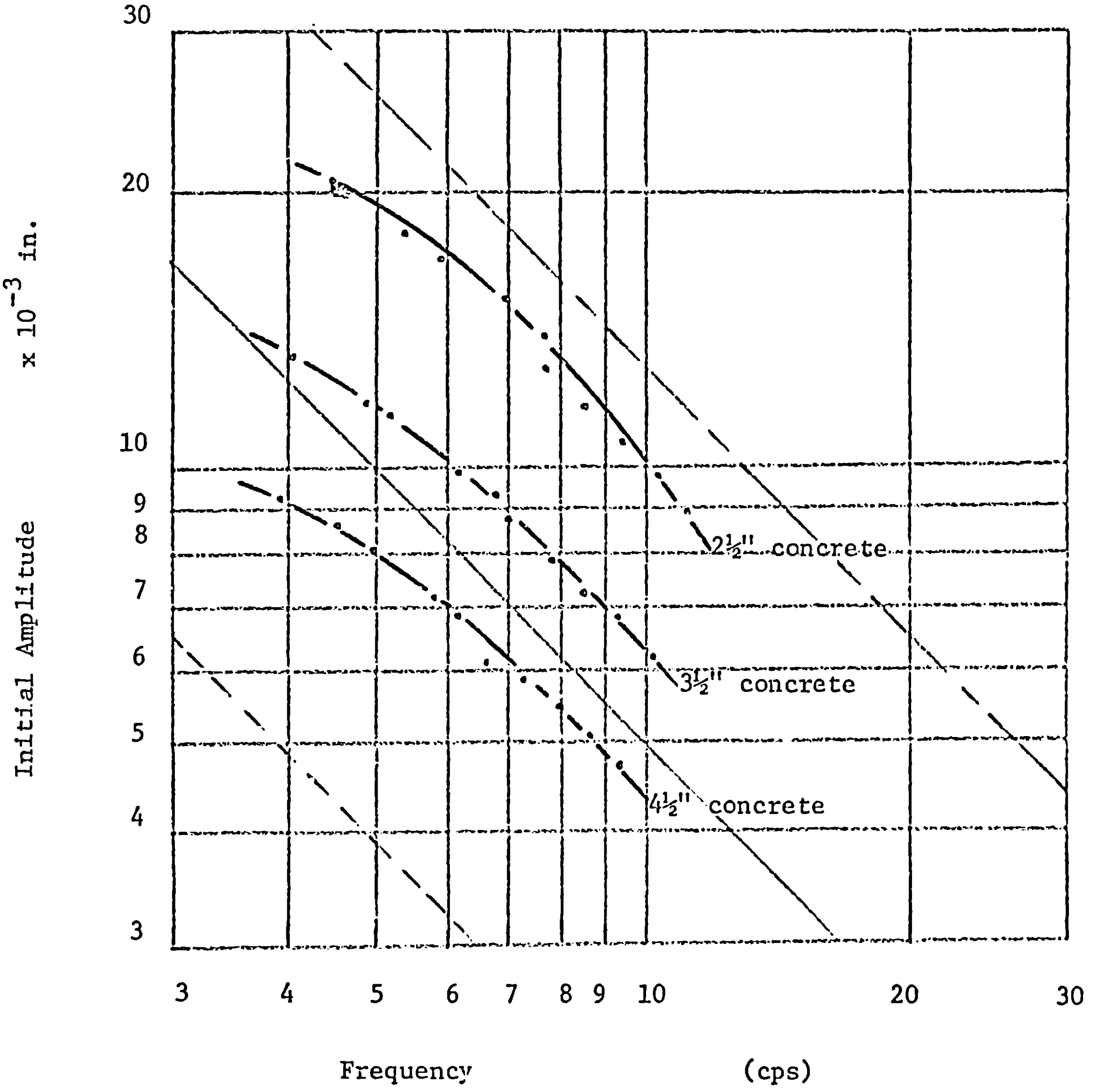

Figure 47. Human Response Curves, J-Series Joists, 32 Foot Span.

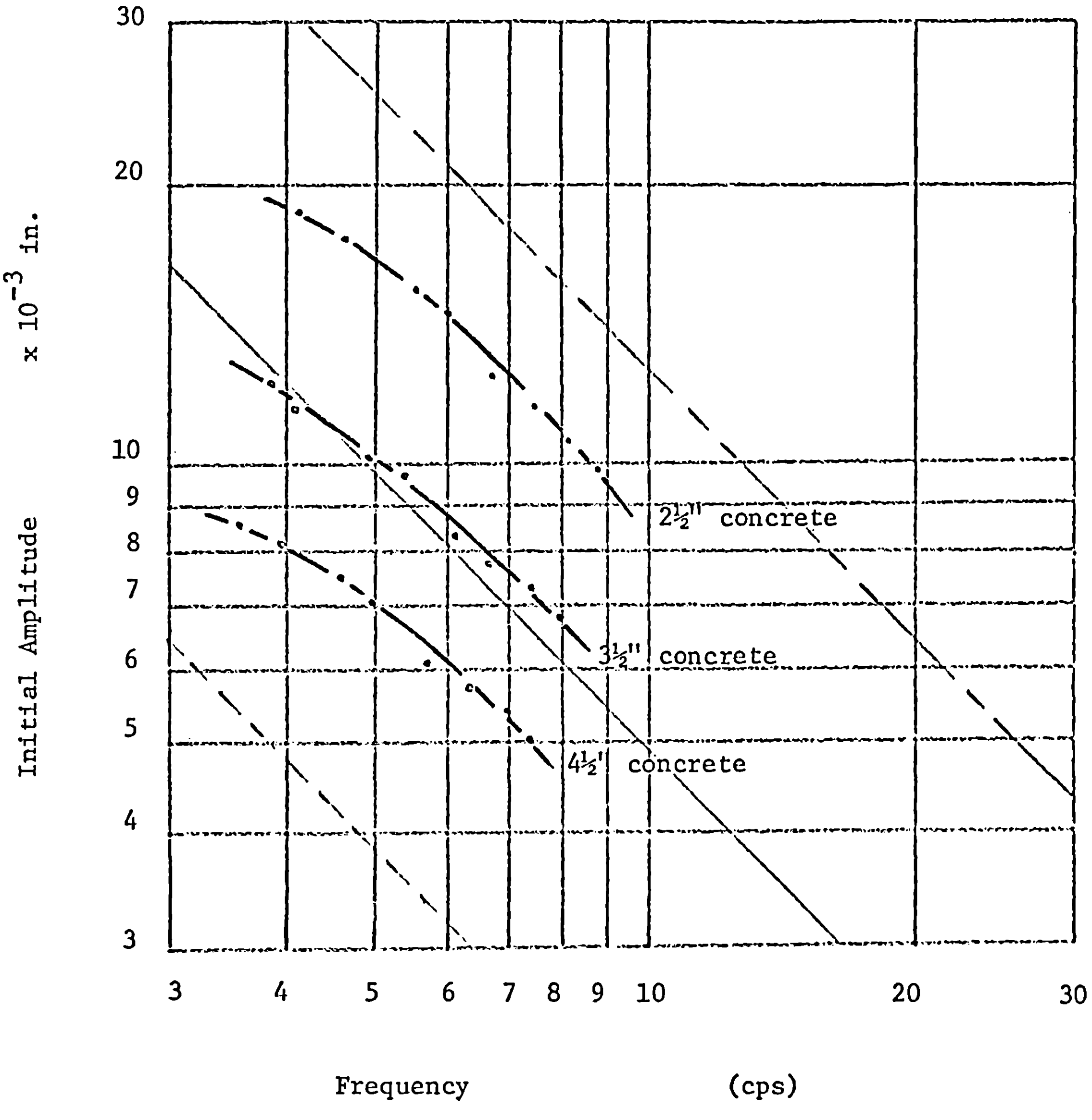

Figure 48. Human Response Curves, J-Series Joists, 36 Foot Span.

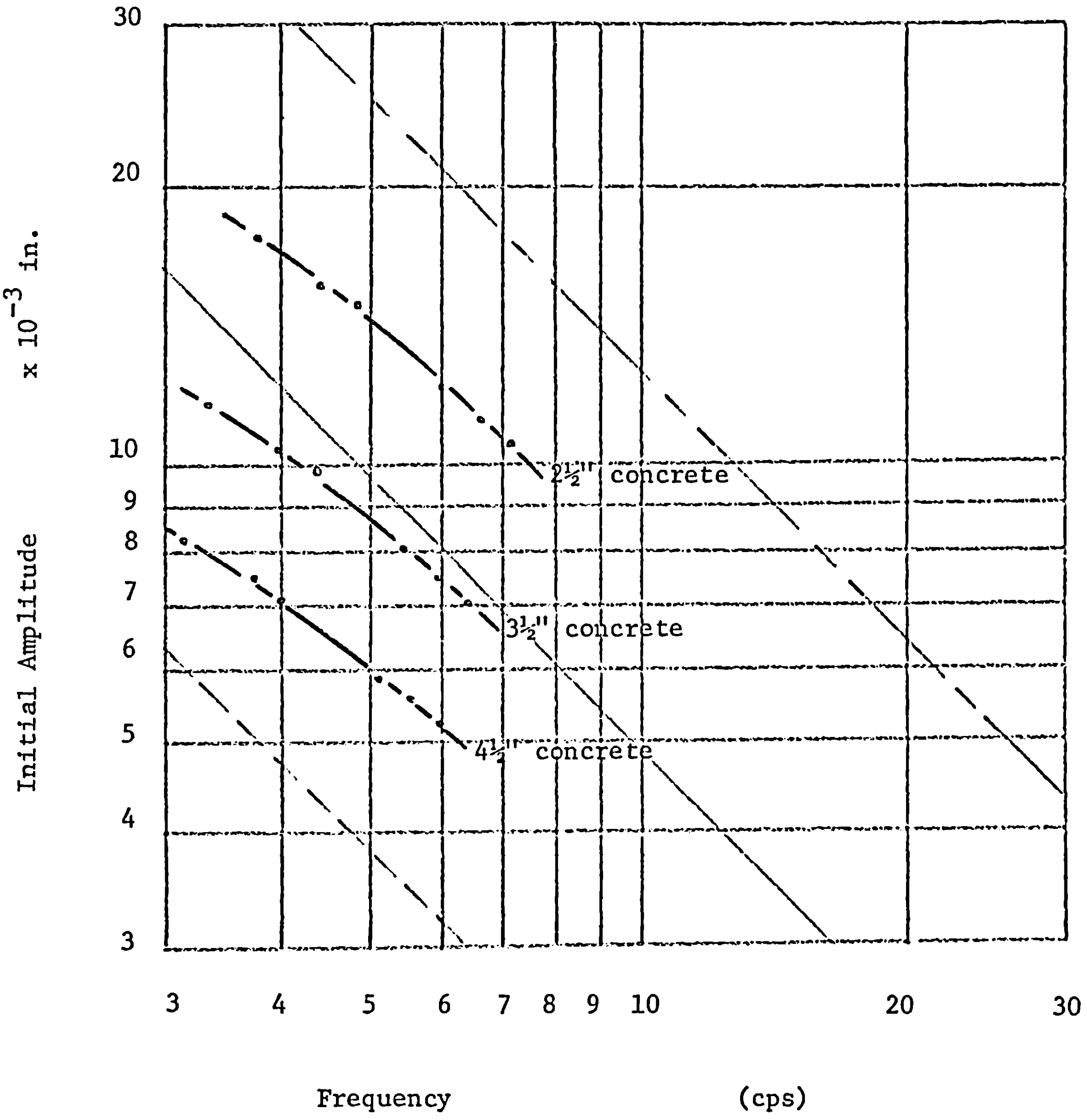

Figure 49. Human Response Curves, J-Series Joists, 40 Foot Span.

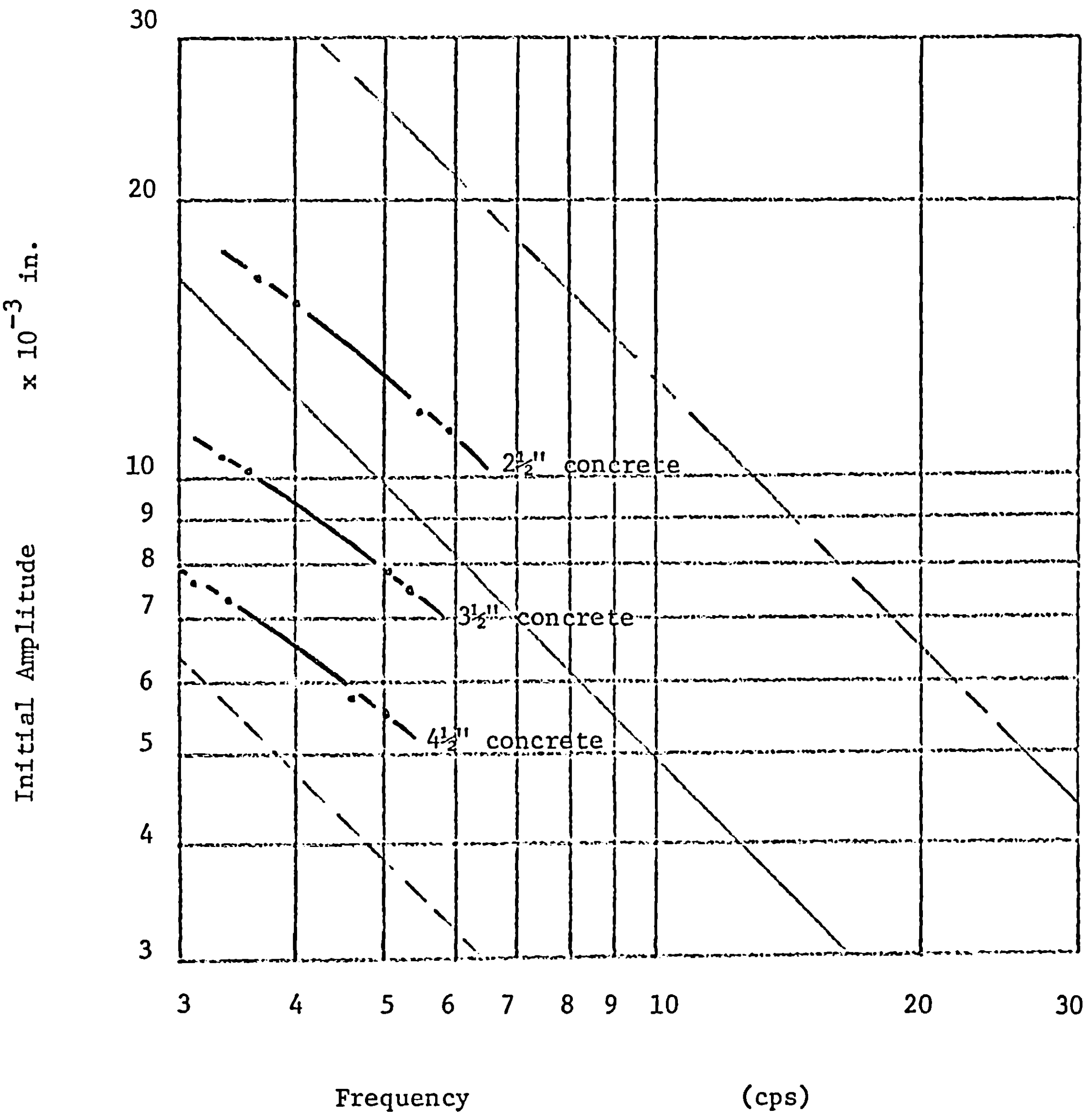

Figure 50. Human Response Curves, J-Series Joists, 44 Foot Span.

STUDIES IN ENGINEERING MECHANICS

1. "Vibration of Steel Joist-Concrete Slab Floor System--Part I," by Kenneth H. Lenzen and Joseph E. Keller, September, 1959.

2. "A Study of Flow Through Abrupt Expansions: Progress Report I, Mean Characteristics of Flow," by Neel Kanth D. Sharma, February, 1959.

3. "Vibration of Steel Joist-Concrete Slab Floor System--Part II," by Kenneth H. Lenzen and Joseph E. Keller, May, 1960.

4. "Damping Considerations in Vibration Response of Humans," by Joseph E. Keller, May, 1960.

5. "A Study of the Vibration of Rectangular Anisotropic Plates by the Ritz Method," by James A. Wiley, May, 1960.

6. "A Study of Flow Through Abrupt Expansions: Progress Report II, Pressure Fluctuations in Flow," by Charles L. Sanford and David W. Appel, June, 1961.

7. "A Study of Flow Through Abrupt Two-Dimensional Expansions: Progress Report III, Formation of Vortices," by Charles L. Sanford, June, 1961.

8. "A Study of the Dynamics of Flow Through Suction-Box Covers: Progress Report I, Flows Without Resistance," by Allen T. Hjelmfelt, June, 1961.

9. "Vibration Damping of Anisotropic Plates," by Gerald W. Barr, June, 1961.

10. "Stress in Unstiffened Cylindrical Shells Containing a Granular Material," by John T. Easley, June, 1961.

11. "A Study of the Dynamics of Flow Through Suction-Box Covers: Progress Report II, Flows with Resistance," by William H. Y. Lee, February, 1962.

12. "A Study of Various Damping Devices for Controlling Vibrating Floor Systems," by William C. Lyons, June, 1962.

13. "Characteristics of Flow Through Symmetrical Laterals: Progress Report I," by Carl T. Herakovich and John V. Otts, June 1, 1962.

14. "Separation of Flow at Interior Corners: Progress Report I, Geometry of Separation Zone," by Karl G. Maurer, June 1, 1962.

15. "A Study of Flow Through Abrupt Expansions: Progress Report IV, Effect of Gate Oscillation," by Svein Vigander, June, 1962.

16. "Vibration of Steel Joist-Concrete Slab Floor Systems: Final Report," by Kenneth H. Lenzen, August, 1962.

17. "Free-Streamline Analyses of Flow Nozzles, Flow through Side Inlets and Flow Past Corners," David W. Appel, March, 1963.

18. "Effect of Bearing Stresses on the Fatigue Strength of a Structural Joint," Tehyu Chu, May, 1966.

19. "Pilot Study--The Applicability of the AISC Formula to the Top Chords of Steel Joists," by Kenneth H. Lenzen, 1965.

20. "Separation of Laminar and Transitional Flows at an Interior Corner," by Mack H. Gray III, March, 1965.

21. "An Experimental Study of Wall-Pressure Fluctuations in a Cavitating Turbulent Shear Flow," by Svein Vigander, May, 1965.

22. "The Jayhawk Bender--A Structural Analysis Program," by J. B. Tiedemann, February, 1966.

22A. "Performance of Human Operators under Various System Parameters," by Hajime Akashi and Saad Mahmood, June, 1965.

23. "Computer Analysis of Two Dimensional Rigid Joint Frames," by V.S. Varadachary, June, 1966.

24. "Inelastic Behavior of the Compression Chord of Open-Web Steel Joists," by W. Scott McDonald, Jr., 1966.

25. "Uniform Load Testing of Open-Web Steel Joists," by Robert D. Ohmart, 1966.

26. "Vibrations in Floor Systems of Steel-Framed Buildings," by Leslie D. Meyer, May, 1967.

27. "Design Formulas for the Top Chords of Open-Web Steel Joists: Final Report," by Kenneth H. Lenzen, April, 1968.

28. "An Investigation of Parametric Stability of Stiffened Rectangular Plates," by Nicholas Willems and Roger C. Duffield, March, 1968.

29. "Vibration of Steel-Beam Concrete Slab Floor Systems," by Kenneth H. Lenzen and Thomas M. Murray, April, 1968.

30. "An Approximate Method for the Response of Stiffened Plates to Aperiodic Excitation," by Robert D. Ohmart, April, 1968.

31. "An Investigation of the Parametric Resonance of Rectangular Plates Reinforced with Closely Spaced Stiffeners," by Nicholas Willems and Roger C. Duffield, August, 1968.

32. "Effect of the Variation of Structural Parameters on the Vibrational Characteristics of Steel Joist-Concrete Slab Floor Systems and Suggested Designs," by Kenneth H. Lenzen and Lawrence P. Dorsett, August, 1968.

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