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A redshift-dependent colour–luminosity relation in Type 1a supernovae

Gopolang M. Mohlabeng★ and John P. Ralston★

Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045, USA

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ABSTRACT

Type 1a supernova magnitudes are used to fit cosmological parameters under the assumption that the model will fit the observed redshift dependence. We test this assumption with the Union 2.1 compilation of 580 sources. Several independent tests find that the existing model fails to account for a significant correlation of supernova colour and redshift. The correlation of magnitude residuals relative to the Λ CDM model and *colour* $\times$ *redshift* has a significance equivalent to 13 standard deviations, as evaluated by randomly shuffling the data. Extending the existing $B-V$ colour correction to a relation linear in redshift improves the goodness of fit χ^2 by more than 50 units, an equivalent 7σ significance, while adding only one parameter. The *colour*−*redshift* correlation is quite robust, cannot be attributed to outliers and passes several tests of consistency. We review previous hints of redshift dependence in colour parameters found in bin-by-bin fits interpreted as parameter bias. We show that neither the bias nor the change $\Delta\chi^2$ of our study can be explained by those effects. The previously known relation that bluer supernovae have larger absolute luminosity tends to empirically flatten out with increasing redshift. The best-fitting cosmological dark energy density parameter is revised from $\Omega_\Lambda = 0.71 \pm 0.02$ to $\Omega_\Lambda = 0.74 \pm 0.02$ assuming a flat universe. One possible physical interpretation is that supernovae or their environments evolve significantly with increasing redshift.

Key words: Supernovae: general – cosmological parameters – dark energy – dark matter.

1 INTRODUCTION

Observations of Type 1a supernovae (SN1a) provide evidence for an accelerating expansion of the universe and dark energy. Recent surveys have accumulated sufficient data that statistical uncertainties may be smaller than systematic ones. Supernovae are imperfect standard candles, and systematic corrections for their absolute luminosity have evolved over time. Besides a correction for time-stretch (Phillips 1993), there is now a colour (c) correction parameter (van den Bergh 1995; Tripp 1998) called β. Fits to the cosmological model replace the observed magnitude m_B by $m_B - \beta c$, representing the empirical fact that SN1a with bluer colours tend to be intrinsically brighter.

After such redshift-independent corrections, cosmological parameters are fitted using SN1a data under a hypothesis that the cosmological model describes the data. Despite the existence of publicly accessible SN1a data, we find few independent tests of the assumption. As noted by Vishwakarma & Narlikar (2010), for the past few years studies with high statistics have focused on parameter estimation using methods that are not designed to test the model.

Here, we pose simple and direct hypotheses tests comparing different models of SN1a magnitude corrections. In the existing null model, the correction parameters $\theta_i = (\alpha, \beta, \delta)$ for stretch, colour and galaxy type have no redshift dependence. We compare that to a model using a two term Taylor expansion $\theta_i \to \theta_i(z)$. For the colour parameter, the expansion is

$$\beta c \to \beta(z)c = \beta_0 c + \beta_1(cz - \overline{cz}). \quad (1)$$

We find the null model $\beta_1 \equiv 0$ fails with high statistical significance. We call the empirical correlation the '*colour* − *redshift* effect'. It is related to previous hints of SN1a evolution with redshift discussed below.

1.1 Assumptions and definitions

Before proceeding, we consider Malmquist (magnitude) selection bias and 'other' selection bias. The bias that observing higher redshift SN1a tends to select a brighter population is well known. A proper test is not concerned with the absolute magnitudes directly, but instead with the *differences* between the systematically corrected data and the cosmological model. When we extend parameters $\theta_i \to \theta_i(z)$, it stands on the same testable footing as testing redshift dependence of the dark matter and dark energy density parameters. Moreover, while a population bias may still contribute to the 'lever arm' of fitting parameters, it should not make a true model fitting the data appear to be false. Regarding other, unspecified 'bias' as

★E-mail: mohlabeng319@yahoo.com (GMM); ralston@ku.edu (JPR)

explanation for the effects we find: we regard that as fair game. When bias exists it needs to be explored, while the business of testing hypotheses is itself neutral on the interpretation.

We use the 580 points of the Union 2.1 compilation (Suzuki et al. 2012, henceforth S12) to test the current model. The data's distance moduli μ_B corrected for colour (c), stretch (x_1) and a certain probability $P_m = P(m_*^{\rm host} < m_*^{\rm Theshold})$ of the host galaxy are

$$\mu_B(\alpha, \beta, \delta, M_B) = m_B + \alpha\, x_1 - \beta\, c + \delta\, P_m - M_B. \quad (2)$$

Here, $x_1 = s - \bar{s}$, where s is the time stretch factor with redshift (z) corrections applied (Goldhaber et al. 2001; Guy et al. 2007). Symbol $c = colour - \overline{colour}$, where $colour = (B - V)_{\rm max} + 0.057$ and the Johnson–Cousins B stands for blue and V the visual magnitude. Overbars denote mean values, which have also been subtracted from P_m to remove a degeneracy with the value of M_B.

As mentioned earlier, we fit $\alpha(z)$, $\beta(z)$ and $\delta(z)$ to two-term polynomials. We believe that transparent and reproducible data analysis is of intrinsic value, and deliberately avoid statistically elaborate methods. Nevertheless, we have reproduced the analysis elements of our main references which are reproducible with the data that has been published. The comparison of our independent study with those produced with data compendia focused on cosmological parameter estimation should be a reason to extend those analyses to include redshift-dependent parameters.

Models assume a ΛCDM cosmology with zero radiation density, dark energy density Ω_Λ and dark matter density $\Omega_{\rm m}$. The predicted luminosity distance is (Weinberg 2008, p. 38):

$$d_{\rm L}(z) = \frac{c(1+z)}{H_0\sqrt{\Omega_k}} \sin h\left(\sqrt{\Omega_k}\int_{1/(1+z)}^{1} \frac{{\rm d}x}{x^2 H(x)}\right);$$

$$\text{where} \quad H(x) = \sqrt{\Omega_\Lambda x^{-3(1+w)} + \Omega_{\rm m} x^{-3} + \Omega_k x^{-2}}. \quad (3)$$

Here, $H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$ is the Hubble constant, which is degenerate in fits with M_B. The flat cosmology has $\Omega_k = 1 - \Omega_{\rm m} - \Omega_\Lambda = 0$. We use the standard value $w = -1$; little sensitivity of w to the new correlation appeared in preliminary work. The distance modulus $\mu_{\rm model}$ is defined by

$$\mu_{\rm model}(z, \Omega_\Lambda, \Omega_{\rm m}) = 5\log_{10}\left(\frac{d_{\rm L}(z)}{\rm Mpc}\right) + 25. \quad (4)$$

2 DATA AND ANALYSIS

We began with the 2008 Union 1 compilation (Kowalski et al. 2008, henceforth K08) before our main data set had been released. The data tables reported only magnitude uncertainties, omitting the systematic errors, which are much larger. We found a large correlation of *colour* $\times$ z and the residuals of the 307 point '3σ set' defined in the paper. One *colour* $\times$ *redshift* parameter β_1 also yielded a difference $\Delta_{\rm mag}\chi^2 > 30$, using the raw uncertainties. Next, we studied the Union 2.0 compilation of 557 SN1a (Amanullah et al. 2010, henceforth A10), which gave both raw and total uncertainties. The relative improvement of χ^2/df using β_1 was found to be quite comparable. Finally, the Union 2.1 compilation of S12 gave enough detail to reproduce its χ^2/df using its best-fitting parameters α, β, δ, $\Omega_{\rm m}$, M_B and $w = -1$. Our results here are confined to the '3σ set' of Union 2.1 data defined by S12.

In all work, data reversion and analysis were done twice, by independently written codes comparing outputs while sharing no common elements.

2.1 Correlation results

Residuals δ_μ are the differences between the distance moduli and the model:

$$\delta_\mu = \mu_B - \mu_{\rm model}. \quad (5)$$

Residuals are computed using the top line of table 6 in S12, except we impose $w = -1$. The correlation of residuals with *colour* $\times$ *redshift* is readily seen by eye in Fig. 1. Note that it extends over both positive (more red) and negative (more blue) colour.

The Pearson correlation coefficient r quantifies the correlation:

$$r(x, y) = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2}\sqrt{\sum_i (y_i - \bar{y})^2}}. \quad (6)$$

We find $r_{\rm SN} = r(\delta_\mu,\ c\ z) = -0.52$. We estimated the significance of $r_{\rm SN}$ with two Monte Carlo simulations. We randomly shuffled the (z_i, c_i) data elements and recalculated 10 000 random correlations $r_{\rm random}$. The distribution of $r_{\rm random}$ was consistent with a Gaussian with standard deviation $\sigma_{r-{\rm random}} = 0.042$ and mean $\sim\pm 0.001$. Numerically indistinguishable results were found when the data elements were combined into linear combinations (z_i', c_i') preserving the data's covariance matrix between redshift and colour. This was done to verify that the substantial correlation (equalling -0.22) between redshift and colour is not an issue with the correlation of residuals we observe. In both cases the data's correlation of $r_{\rm SN}$ is about $13\sigma_{r-{\rm random}}$ from the mean. The estimated P-value (of the order of 10^{-39}) is too small to simulate or interpret as a fluctuation. This correlation shows that the data has a redshift dependence that is not predicted by the model producing the residuals.

One might ask whether outliers produce the correlation. Actually outliers were already removed in the Union 2.1 data. Starting with 753 SN1a, data differing more than 3σ from the model were discarded. The detailed cuts listed in S12 are (1) requiring a cosmic microwave background (CMB)-centric redshift greater than 0.015; (2) requiring at least one point between -15 and 6 rest-frame days from B-band maximum light; (3) requiring at least five valid data points; (4) requiring the entire 68 per cent confidence interval for x_1 between -5 and $+5$; (5) requiring at least two bands with rest-frame central wavelength coverage between 2900 and 7000 Å; (6) requiring at least one band be redder than the rest-frame U band

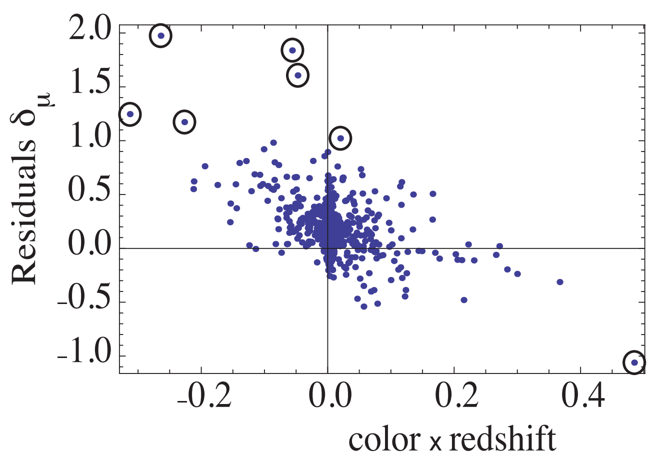

Figure 1. Distance modulus residuals $\delta_\mu = \mu_B - \mu_{\rm model}$ relative to the best-fitting ΛCDM model versus *colour* $\times$ *redshift*. Calculation assumes a flat universe and $w = -1$. The Pearson correlation coefficient of $r = -0.52$ has the significance of a 13σ effect, as evaluated by a simulation shuffling the data randomly. Seven points discussed in the text are indicated by circles. Data are the 3σ set of the Union 2.1 compilation.

Table 1. Comparison of fits without accounting for the *colour–redshift* effect ($\beta_1 = 0$) and including it ($\beta_1 \neq 0$). Parameters held fixed are indicated by an asterisk. The *colour–redshift* effect produces a highly significant improvement of the fit even with unphysical constraints such as $\Omega_m = \Omega_\Lambda = 0$. χ^2 is defined by equations (7) and (2), with σ_μ^2 the distance modulus uncertainties from the data tables, except for the last line (indicated by †) using parametrically fitted errors of the type found in current literature. χ^2_{min} and $\Delta\chi^2$ have been rounded to the nearest whole number.

Ω_m	Ω_Λ	α_0	β_0	δ_0	M_B	β_1	χ^2_{min}	$\Delta\chi^2$
0.291 ± 0.022	1-Ω_m	0.105 ± 0.007	2.31 ± 0.05	− 0.022 ± 0.03	− 19.133 ± 0.013	0*	550	0
0*	0*	0.10 ± 0.006	2.51 ± 0.07	− 0.11 ± 0.024	− 19.1 ± 0.012	− 1.35 ± 0.22	534	16
0*	0.31 ± 0.003	0.105 ± 0.000 04	2.61 ± 0.005	− 0.054 ± 0.0007	− 19.05 ± 0.0002	− 1.59 ± 0.051	511	39
0.260 ± 0.021	1-Ω_m	0.102 ± 0.007	2.62 ± 0.07	− 0.038 ± 0.03	− 19.14 ± 0.013	− 1.61 ± 0.23	500	50
0.259 ± 0.07	0.737 ± 0.13	0.102 ± 0.0065	2.62 ± 0.07	− 0.038 ± 0.027	− 19.16 ± 0.073	− 1.61 ± 0.227	500	50
0.236 ± 0.021	1-Ω_m	0.11 ± 0.0085	2.91 ± 0.074	− 0.05 ± 0.029	− 19.15 ± 0.014	− 2.40 ± 0.21	464	109†

(4000 Å). We note the extra attention to colour is more demanding than many previous studies. Using $w = -1$ there are seven points indicated with circles in Fig. 1 with $|\delta_\mu| > 3\sigma$ for the residuals shown. Removing those points reduced the correlation to $r_{SN,cut} = -0.45$, which equals 11σ.

2.2 A model with one new parameter

We define our first fit statistic χ^2 as

$$\chi^2 = \sum_{SNe} \frac{(\mu_B(\alpha, \beta, \delta, M_B) - \mu_{model}(z, \Omega_m, \Omega_\Lambda, w))^2}{\sigma_\mu^2}. \quad (7)$$

Here, σ_μ^2 are the complete (statistical and systematic) errors of the S12 data listed in full form on the *SCP* website.[1] The null model of a redshift-independent colour correction gives the best-fitting $\chi^2 = 550$ for $580 - 5$ degrees of freedom (*df*) (see Table 1).

We computed the change $\Delta\chi^2$ after extending α and δ to a Taylor series linear in z:

$(\text{fixing } \beta_1 = \delta_1 = 0): \quad \alpha \to \alpha_0 + \alpha_1 z; \quad \alpha_1 = -0.036;$

$\Delta\chi^2 = 3.8;$

$(\text{fixing } \alpha_1 = \beta_1 = 0): \quad \delta \to \delta_0 + \delta_1 z; \quad \delta_1 = -0.093;$

$\Delta\chi^2 = 2.$

Our reason for varying parameters one at a time was computational simplicity. These values do not indicate a highly significant redshift dependence of α or δ. However, $\Delta\chi^2 = 3.8$ slightly exceeds an equivalent 2σ confidence level disfavouring a constant stretch parameter.

On the other hand, varying β_1 (with $\alpha_1 = \delta_1 = 0$) produced a best fit with $\chi^2 = 500$ for $(580 - 6)df$. That is a 50 unit decrease in χ^2 from the addition of one parameter: see Table 1. When a null hypothesis is extended by one parameter, Wilk's theorem predicts that $\Delta\chi^2$ will be distributed by the χ_1^2 distribution, and be typically of order one. On that basis 50 units of χ^2 is equivalent to a Gaussian fluctuation of 7.1σ, which has a chance probability of the order of 10^{-12}. This rules out the null model on an independent basis.

The low value of $\chi^2/df \lesssim 1$ is unlikely on a statistical basis. A simple explanation exists. The data compilations assign systematic errors with a step $\sigma_\mu^2 \to \sigma_\mu^2 + \sigma_{int}^2$. Symbol σ_{int}^2 is an intrinsic scatter parameter adjusted to bring χ^2/df of each sample to unity. (See K08, A10, and especially S12 following equation 7). After β_1 has been fitted, the same procedure could readjust σ_{int}^2 to give $\chi^2/df \to 1$. Except for one test below, we make no adjustment or variation of errors, accepting the reported values. We also note the absolute magnitude M_B, as well as α, β, etc. have been called 'nuisance parameters' that are not reported in many papers. We believe fitted parameters have physical importance, and Table 1 reports everything needed to reproduce our results.

Figs 2 and 3 show χ^2 versus Ω_m and Ω_Λ with and without accounting for the *colour–redshift* effect. The plots use $\alpha_0\, \beta_0, \delta_0, M_B$ evaluated at their best-fitting values point by point

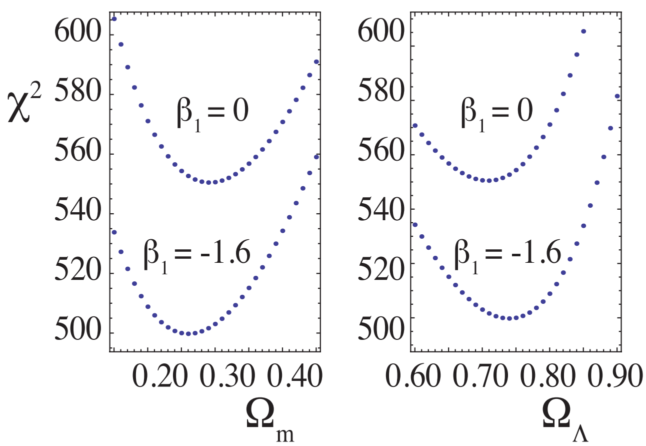

Figure 2. Left-hand panel: χ^2 versus Ω_m without *colour* × *redshift* parameter ($\beta_1 = 0$, upper points) and including it ($\beta_1 = -1.6$, lower points). Right panel: Same as left with Ω_Λ on the *x*-axis. Fits use $\Omega_m + \Omega_\Lambda = 1$ and best-fitting values of the remaining parameters point-by-point.

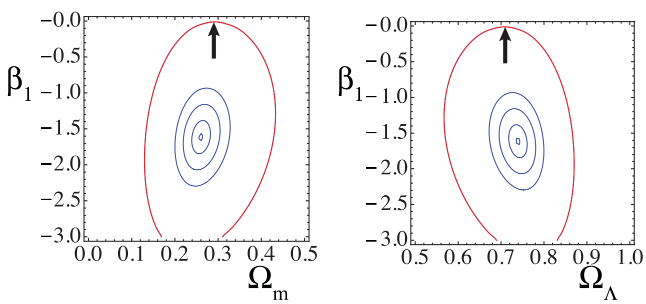

Figure 3. Left-hand panel: contours of constant χ^2 in the (Ω_m, β_1) plane. Inner contours are $\chi^2_{min} = 500 + j^2$ for $j = 0, 1, 2, 3$, corresponding to 1, 2, 3 units of Gaussian confidence levels σ. Outer contour (red line) is $\chi^2 = 550$, (7.1 σ equivalent), which intersects the null model ($\beta_1 = 0$, $\Omega_m = 0.29$), as indicated by the small arrow. Right-hand panel: same as left using the $(\Omega_\Lambda, \beta_1)$ plane with the outermost contour intersecting at ($\beta_1 = 0, \Omega_\Lambda = 0.71$). Fits use $\Omega_m + \Omega_\Lambda = 1$ and best-fitting values of the remaining parameters point-by-point.

[1] See http://supernova.lbl.gov.

and $\Omega_m + \Omega_\Lambda = 1$. As Table 1 shows, the important cosmological parameters Ω_m and Ω_Λ are sensitive to the value of β_1. The significance of the *colour−redshift* effect for Ω_m and Ω_Λ depends on how it is assessed. For example, fixing other parameters to the global best fit value (Mohlabeng & Ralston 2012) finds that Ω_m and Ω_Λ shift by more than their 99.95 per cent (3σ) confidence level uncertainties. That is appropriate when other parameters are known. The plots here showing χ^2 with other parameters floating to their best-fitting values are more conservative, and based on the supernova data alone. (They should not be compared with joint confidence intervals from CMB or galaxy distribution data.) Following the convention of S12 (and its table 7) the errors in the Table correspond to $\Delta\chi^2 = 1$.

3 DISCUSSION

The statistical significance of the *colour* × *redshift* correlation is sufficiently high that ordinary confidence levels fail to express it. The correlation is very robust and too large to be attributed to outliers.

Besides the test in Section 2, we computed a new difference $\Delta\chi^2(N) = \chi^2(\beta_1 = 0, N) - \chi^2(\beta_1, N)$, where N points were selected on the basis of their uncertainty-weighted residuals relative to the null-model fit. For this, the data were sorted in order of decreasing $\delta^2_{\mu\,1}/\sigma_1^2 > \delta^2_{\mu,\,2}/\sigma_2^2 > \cdots \delta^2_{\mu,\,N}/\sigma_N^2$ as computed from the null-model fit. Discarding the first N points and comparing $\beta_1 = 0$ with $\beta_1 \neq 0$ produces $\Delta\chi^2(N)$. This procedure is statistically unfair. Step by step it maximally selects data to confirm the null. Its purpose is to explore whether $\Delta\chi^2(N)$ might suddenly decrease for some value N, signalling a subset with anomalous statistical weight. Instead, $\Delta\chi^2(N)$ decreased smoothly with N from $\Delta\chi^2(N=0) = 51$ to $\Delta\chi^2(N=50) = 20$. Upon reaching $N = 50$, the value of $\chi^2(\beta_1 = 0, N = 50)/df$ had been artificially decreased from about 550/(550-6) to about 300/(500-6), a relatively large amount.

Two classes of questions naturally arise:

Data selection, parameter estimation bias, data processing: selection bias can strongly affect parameter estimation and has received much discussion. Many papers including K08, A10 and S12 extensively discuss the selection of intrinsically brighter supernovae at larger z. These papers dismiss (or limit) population bias effects in the statistics of likelihood-ratio tests (minimizing χ^2), supposing the model fits the data.

The numerical value of β has long been a topic of concern. van den Bergh (1995) observed that model calculations showed a bluer-brighter correlation, and suggested the colour correction of an effective magnitude parameter. Previously Branch & Tamman (1992) noticed that R_B found in data fits was smaller than expected from models concerned with dust. Yet it is common for best fit-parameters to disagree with 'true' parameters. Due to that, hints of SN1a evolution previously found were tempered by concerns over parameter bias. Astier et al. (2006) fit separate β parameters to selected redshift bins and observed an apparent decrease with z. This paper (and all we discuss here) used parametrically fitted errors $\sigma^2_\mu = \sigma^2_\mu(\alpha,\ \beta)$ as discussed below. Within such a procedure K08 noted a bias in fitting β, as suggested earlier by Wang et al. (2006). Kessler et al. (2009, henceforth K09) and Lago et al. (2012) also found β decreasing with z in bin-by-bin fits, and mentioned a possible signal of supernova evolution. After finding β again decreasing bin-by-bin with larger z (Guy et al. 2010, henceforth G10) explored uncertainties in *colour*. The result is sometimes described as explaining the findings of K09, as in e.g. Marriner et al. (2011), Sullivan et al. (2011), Conley et al. (2011). Actually, the issues were not cleanly resolved with G10 stating that 'we are not able to conclude on an evolution of β with redshift'.

Fits to binned data can be suggestive, but they dilute the statistical weight and should include statistical penalties depending on the number of parameters. Our one-parameter approach uses no binning, and is able to discriminate between parameter bias and correlation. To illustrate this, we repeated simulations of our data generated from a fixed cosmology with no corrections other than βc, plus random noise. Colour was generated with random Gaussian colour errors with adjustable variance σ_c^2. For $z > 0.7$, we found $\beta_{\rm fit}/\beta_{\rm true} = 1/(1 + \sigma_c^2/0.11^2)$ closely matched the mean simulation. The difference using the mean $\sigma_c = 0.091$ of our data subset is $\beta_{\rm true} - \beta_{\rm fit} \sim 1.2$, much like the previous study done by G10. Our entire data set of 580 points gave $\beta_{\rm fit}/\beta_{\rm true} = 1/(1 + \sigma_c^2/0.14^2)$. These results do not depend strongly on the variance of the noise added to m_B. With the smaller mean $\sigma_c = 0.052$ of this set we found $\beta_{\rm fit} - \beta_{\rm true} = 0.39$. That value is comparable to the $\beta_{\rm fit} - \beta_{\rm true} \sim 0.5$ shown for 307 Union points in fig. 5 of K08.

The actual value of β is a topic of great interest. Very recently, Scolnic et al. (2013) directly confront the question of β-bias, and find that the combination of residual scatter due to colour and a realistic colour distribution will bias β by roughly one unit lower than its true value. That work builds on Chotard et al. (2011) who suggest that the controversy over type 1a colours can be explained by the dispersion in colours, and by variable features observable in the spectra.

However, neither the bias nor the value of β is relevant to our particular task. The distribution of $\Delta\chi^2(\beta_1)$ from the simulations directly tests our methods independent of the bias. Our 1000-fit simulations adding random colour noise found $\Delta\chi^2(\beta_1)$ distributed by χ^2_1 just as statistical theory predicts. [Fitting χ^2_ν with ν a free parameter found $\nu = 1 \pm O(10^{-2})$.] This was found both for the 90 points of $z > 0.7$ and the full 580-point data set. The same was found when we made colour noise increase linearly with z. We are not aware of any previous studies that compared hypotheses using $\beta \to \beta_0 + \beta_1 z$ and global $\Delta\chi^2$ fits.

There is a question whether systematic error assignments might cause a false correlation. Our results for unweighted residuals δ_μ and $\Delta\chi^2/df$ using raw magnitude tends to contradict that possibility. We considered adjusting systematic errors to banish the *colour − redshift* effect, but soon realized it would be irresponsible. To explore systematic errors in an unbiased way, we padded magnitude errors σ_{m_B} by the rule $(\sigma_{m_B})^2 \to (\sigma_\xi m_B)^2 = (\sigma_{m_B})^2 + \xi^2$, and adjusted parameter ξ. The range of ξ spanned the differences of $\overline{\sigma_{m_B}} \sim 0.08$ to $\overline{\sigma_{\mu_B}} \sim 0.22$. The procedure tests whether points of small error might skew results without unfairly adjusting errors to fit the FLRW model. The recomputed best-fits comparing $\beta_1 = 0$ and $\beta_1 \neq 0$ yielded a smooth and nearly monotonic variation of $\Delta\chi^2(\beta_1) = 51$ ($\xi = 0.001$) to $\Delta\chi^2 = 35.5$ ($\xi = 0.2$). The result disfavours systematic error assignments causing the correlation.

We have focused on analysis based on errors as they are published, both for simplicity and so that our results can be reproduced. The recent compendia (K08, A10, S12, etc.) compute the errors by parametrically fitting error functions $\sigma^2_\mu(\alpha,\ \beta,\ \delta)$ to the cosmological model. The definition for each supernova is $\sigma^2_\mu(\alpha, \beta) = \theta_i C_{ij}\theta_j + \sigma^2_{int}$, where $\theta_i = (1, \alpha, -\beta)$. Here, C_{ij} are 580 3 × 3 covariance matrices from light-curve fitting. The diagonals $C_{ii} = \sigma^2_{ii}$ are reported as $(\sigma^2_{m_b}, \sigma^2_{x_1}, \sigma^2_c)$. We then followed K09 and Lago et al. (2012) in repeating the parametrically fitted error calculation using the diagonal covariance matrix elements. Fixing $\beta_1 = 0$ we found $\sigma^2_{\rm int} = 0.016$, as consistent with our references. We then compared the best fit with $\beta_1 = 0$ with the best-fitting β_1 to find $\Delta\chi^2(\beta_1) = 109$ (see Table 1). The value very strongly disfavours the null model. We doubt such a large change in χ^2 would be due to the (assumed small) unpublished off-diagonal elements of colour fitting, but if so it would cast a new light on the entire procedure of parametrically fitted errors. The observation of (Lago et al. 2012) that the statistic does not represent a likelihood may be relevant. For completeness, this fit gave $\Omega_m = 0.23 \pm 0.02$.

Physical interpretation: The possibility of SN1a evolving with redshift is well known. Tripp (1998) wrote that, 'by applying the same type of colour correction to cosmological supernovae *even without knowing whether reddening is intrinsic or due to dust*, one will be able to completely standardize the light output of each explosion...'. (Italics are ours.) The current practice of using one 'unique coefficient β ... for both dust extinction and any intrinsic colour–magnitude relation' (Menard, Kilbinger & Scranton 2011) allows for no evolution. The extraction and interpretation of the colour parameter is a very active topic. Besides the work already cited, a significant correlation between host galaxy extinction coefficients A_V and ΛCDM residuals suggested evolution or bias, quantified by finding the χ^2 value of the 'gold and silver' (Riess et al. 2004) data decreased by 23 units of χ^2 after varying one new parameter (Jain & Ralston 2006).

A positive value of β_1 would increase the redder-is-dimmer correction with increasing z, as expected from increasing dust. Instead, we find $\beta_1 < 0$, a decreasing colour correction and the opposite effect, which is unexpected and not attributable to conventional dust. The *colour − redshift* effect is, however, consistent with evolution of intrinsic luminosity of sources. Extrapolating $\beta(z) = 2.62 - 1.61z$ naively, the bluer-brighter, redder-dimmer relation would actually reverse for $z \gtrsim 1.6$. An unidentified bias in the observations or data reduction also cannot be ruled out. If a bias exists, it is an important issue and hardly a flaw of what we report, which can only be based on the data published.

In summary, straightforward tests using the Union 2.1 data finds that the current model using constant β parameter is ruled out compared to the model $\beta(z) = \beta_0 + \beta_1 z$. In as much as the fitting of cosmological parameters to SN1a data hinges on a model called into question, the values and errors of those parameters may be questioned. It seems premature to attempt the last word on the highly significant trend we have found.

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