The Astrophysical Journal, 637:91–95, 2006 January 20

EVIDENCE FOR EVOLUTION OR BIAS IN HOST EXTINCTIONS OF TYPE 1a SUPERNOVAE AT HIGH REDSHIFT

Pankaj Jain[1] and John P. Ralston[2]
Received 2005 June 13; accepted 2005 September 19

ABSTRACT

Type 1a supernova magnitudes conventionally include an additive parameter called the extinction coefficient. We find that the extinction coefficients of a popular "gold" set are well correlated with the deviation of magnitudes from Hubble diagrams. If the effect is due to bias, extinctions have been overestimated, which makes supernovas appear more dim. The statistical significance of the *extinction-acceleration correlation* has a random chance probability of less than one in a million. The hypothesis that extinction coefficients should be corrected empirically provides greatly improved fits to both accelerating and nonaccelerating models, with the independent feature of eliminating any significant correlation of residuals.

Subject headings: cosmological parameters — dust, extinction — galaxies: distances and redshifts — supernovae: general

Online material: color figures

1. INTRODUCTION

Type 1a supernovas (SNe) are candidates for standard astrophysical candles from which the relation of redshift z and distance can be estimated. In a universe of constant expansion the "Hubble plot" made from magnitudes and redshifts should be a straight line. Data are now available for a wide range of redshifts up to 1.755 (Schmidt et al. 1998; Garnavich et al. 1998; Perlmutter et al. 1998, 1999; Riess et al. 1998; Knop et al. 2003; Tonry et al. 2003; Barris et al. 2004). The Hubble diagrams derived from SNe have indicated an upward bending curve, interpreted as acceleration of the expansion rate, along with even more complicated features of "jerk." It is important to explore other interpretations, including possible evolution of SN or host galaxy characteristics with redshift. Many papers have explored noncosmological explanations (Coil et al. 2000; Leibundgut 2001; Sullivan et al. 2003; Riess 2000). Meanwhile, the high-redshift host galaxies have significantly different morphologies compared to those at low redshifts (Abraham & van den Bergh 2001; Brinchmann et al. 1998; van den Bergh 2001). Dust and related extinction characteristics may certainly depend on redshift (Totani & Kobayashi 1999). Furthermore, the abundance ratios of the progenitor stars may be different at different redshifts (Höflich et al. 2000). Several studies emphasize that evolution effects cannot be ruled out (Falco et al. 1999; Aguirre 1999; Farrah et al. 2004; Clements et al. 2004).

In this paper we find evidence for evolution or bias in the extinction parameters used to preprocess the data. If the effect is due to bias, extinctions have been overestimated, which makes SNe appear more dim. Yet just the same phenomenon could occur from a real physical effect in which the actual host extinctions are correlated with the deviation of magnitudes from model fits.

1.1. *Background*

Traditional Hubble diagrams represent the relation of observed flux F to the luminosity of the source L,

$$F = \frac{L}{4\pi d_L^2}, \tag{1}$$

where d_L is the so-called luminosity distance. The distance modulus $\mu_p = m - M$, where m and M are the apparent and absolute magnitudes respectively, is

$$\mu_p = 5 \log d_L + 25, \tag{2}$$

where the luminosity distance d_L is in megaparsecs.

The process of converting observed data into the SN magnitudes reported actually contains an additive parameter, called the extinction coefficient A. Extinction may depend on frequency, designated by A_B, A_R, etc. The units of A are magnitudes. In practice, A shifts the SN magnitude m_0 deduced from light curves to a reported magnitude ("extinction corrected magnitude") $m = m_0 - A$. Our galaxy contributes extinction, as do the additional extinction effects associated with SN host galaxies, which are more model dependent.

Riess et al. (2004) discovered 16 Type Ia SNe at high redshifts and compiled a 157 source "gold" data set held to be of the highest reliability. Extinctions are listed in Riess et al. (2004) for all except 24 sources among this "gold" set.

2. ANALYSIS

Riess et al. focus on the differences of magnitudes $\Delta\mu$ relative to the traditional Hubble plot. In Figure 1 we show the residuals $\Delta\mu$ versus the extinction coefficients A_V, for all the sources for which extinctions are known. There is a clear correlation. The sense of correlation is that points with $\Delta\mu > 0$, lying above the straight-line Hubble plot, tend to have small or even negative extinction, and points lying below the straight line tend to have large extinction. A precedent for examining correlations of residuals is given in Williams et al. (2003).

Residuals depend on the baseline model from which they are measured. Figure 1 uses the FRW model and "concordance" parameters $\Omega_M = 0.27$, with $\Omega_\Lambda = 0.73$ under the constraint $\Omega_k = 0$. This is one of the baselines cited by Riess et al. (2004). Here

[1] Physics Department, Indian Institute of Technology, Kanpur 208016, India.
[2] Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045.

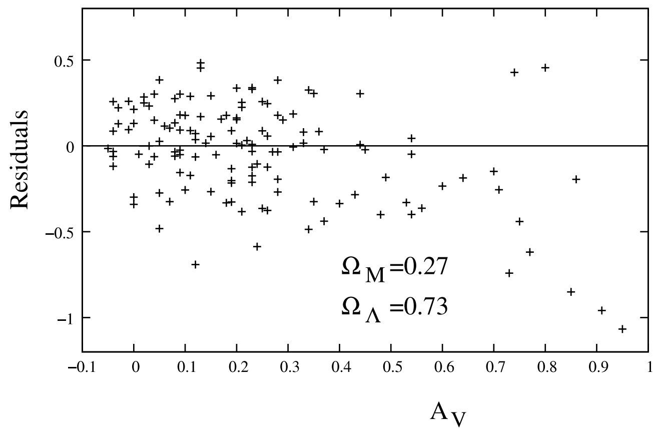

FIG. 1.—Residuals as a function of the host extinction for the concordance model $\Omega_M = 0.27$ and $\Omega_\Lambda = 0.73$.

Ω_M is the matter density, Ω_Λ the vacuum energy density, and $\Omega_k = 1 - \Omega_M - \Omega_\Lambda$. The class of FRW models predicts the luminosity distance as

$$d_L = \frac{c(1+z)}{H_0|\Omega_k|^{1/2}} \times \operatorname{sinn}\left\{|\Omega_k|^{1/2}\int_0^z dz\left[(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda\right]^{-1/2}\right\}. \quad (3)$$

Here sinn denotes sinh for $\Omega_k > 0$, sin for $\Omega_k < 0$, and is equal to unity for $\Omega_k = 0$. Parameters are fit by minimizing χ^2, defined by

$$\chi^2 = \sum_i \frac{(\mu_p^i - \mu_0^i - \mu_{p0})^2}{(\delta\mu_0^i)^2}, \quad (4)$$

where μ_p^i and μ_0^i are the theoretical and observed distance moduli, respectively, and $\delta\mu_0^i$ are the reported errors. Our notation includes the intercept parameter μ_{p0} (not always explicit in the literature). The Hubble constant H_0 and fit parameters such as the zero point are not reported in Riess et al. (2004), which states that they are irrelevant and arbitrarily set for the sample presented here. We verify (Riess et al. 2004) $\chi^2 = 178$ for the concordance parameters cited above, along with the other χ^2 values for several other studies, presented below.

2.1. *Quantification*

We quantify the correlation of extinctions with residuals with the correlation coefficient $R(\Delta\mu,\ A_V)$, also simply R, defined by

$$R(\Delta\mu,\ A_V) = \frac{\sum_i (\Delta\mu_i - \bar{\Delta}\mu)(A_{V,i} - \bar{A}_V)}{\sigma_{\Delta\mu}\sigma_{A_V}}, \quad (5)$$

where $\bar{\Delta}\mu$, $\sigma_{\Delta\mu}$ are the means and standard deviation of the $\Delta\mu$ set, with corresponding meaning for σ_{A_V}, $\bar{A}_V$. The correlation $R(\Delta\mu, A_V) = -0.439$ for the concordance parameters cited above, excluding the 24 sources for which extinctions are not known. The integrated probability (confidence level, P-value) to find correlations equal or larger in a random sample is 4.2×10^{-7}.

To investigate whether the correlation of extinctions with residuals might be a model artifact, we decided to fit several other models cited by Riess et al. (2004). The results of these fits are shown in Table 1. For example, under the best-fit model with $\Omega_M = 0.31$ and $\Omega_\Lambda = 0.69 = 1 - \Omega_M$, then $R(\Delta\mu, A_V) = -0.434$ with a probability $P = 5.6 \times 10^{-7}$.

From Figure 1 we see that the correlation is strongest for large values of A_V. For example, for the best-fit parameters ($\Omega_M = 0.31$ and $\Omega_\Lambda = 0.69$) we find that excluding the four sources with $A_V > 0.8$, the correlation coefficient goes down to $R(\Delta\mu,\ A_V) = -0.28$ with $P = 1.5 \times 10^{-3}$. Retaining the 139 points with $A_V \leq 0.5$ yields $R(\Delta\mu,\ A_V) = -0.18$. We do not have a particular reason to entertain these cuts except to make the correlation go away. At the risk of complicating the interpretation, one can try dividing the residuals by the data point's uncertainty. This is an uncertain trial because a fundamental issue is the *uncertainty in the extinction coefficients*, which is unavailable from the literature. Figure 2 shows the correlation with error bars assigned to the residuals.[3] The figure shows that most of the data with $A_V > 0.3$ lies below zero, indicating bias. Division by the uncertainty only reduces $R(\Delta\mu/\sigma,\ A_V) \to -0.37$ for the gold set, an effect of having introduced noise.

We next examine whether the correlation seen in the residuals depends on redshift. We divide the data as equally as possible in a high-redshift sample ($z \geq 0.41$; 78 sources) and a low-redshift sample ($z < 0.41$; 79 sources). (The cut $z \sim 0.46$ was identified by the Hubble team as a transition region.) For the low-redshift sample we find $R(\Delta\mu,\ A_V) = -0.509$ and $P = 1.2 \times 10^{-5}$, compared to the high-redshift sample yielding $R(\Delta\mu, A_V) = -0.378$ and $P = 3.7 \times 10^{-3}$. Although the statistics have been diluted, it is clear that the two samples show different behavior, with the correlation being much more significant in the low-redshift sample.

[3] We thank an anonymous referee for this suggestion.

TABLE 1
χ^2 VALUES

Model[a]	χ^2	R[b]	P[c]
$\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$	178.2	−0.439	4.2×10^{-7}
$\Omega_M = 0.31$, $\Omega_\Lambda = 0.69$ (best fit with $\Omega_M + \Omega_\Lambda = 1$)	177.1	−0.434	5.6×10^{-7}
$\Omega_M = 0.45$, $\Omega_\Lambda = 0.95$ (best fit)	175.1	−0.403	3.3×10^{-6}
$\Omega_M = 0.0$, $\Omega_\Lambda = 0.0$ (best fit with $\Omega_\Lambda = 0.0$)	191.7	−0.392	6.0×10^{-6}
$\Omega_M = 1$, $\Omega_\Lambda = 0$	324.8	−0.275	1.49×10^{-3}

[a] The different cosmological models come from Riess et al. (2004).
[b] Correlation statistic $R(\Delta\mu,\ A_V)$ between residuals and extinction.
[c] Confidence level (P) to find $R(\Delta\mu,\ A_V)$ in a random sample.

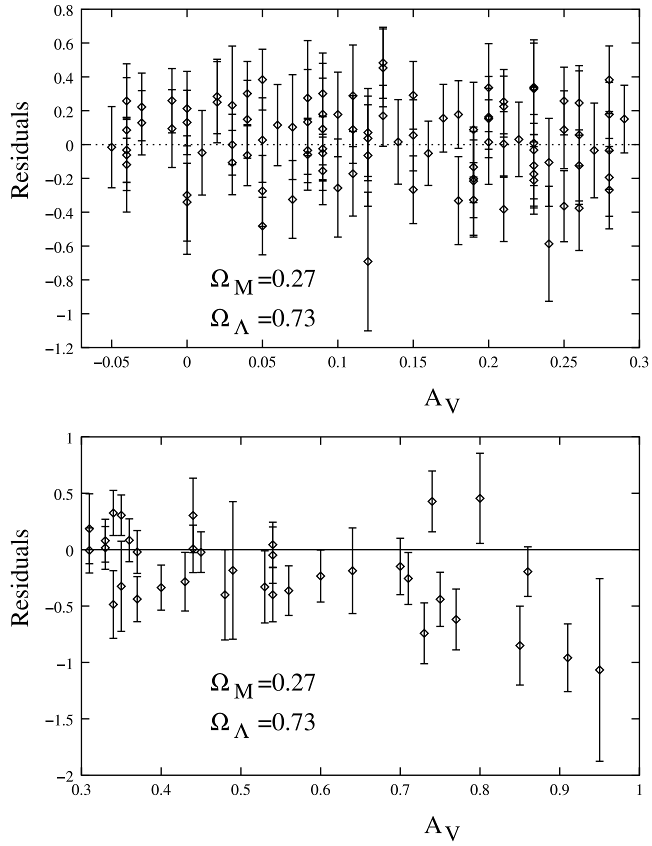

FIG. 2.—Residuals vs. A_V, including reported uncertainties assigned to the residuals. By inspection the region of $A_V \gtrsim 0.3$ shows systematic correlation.

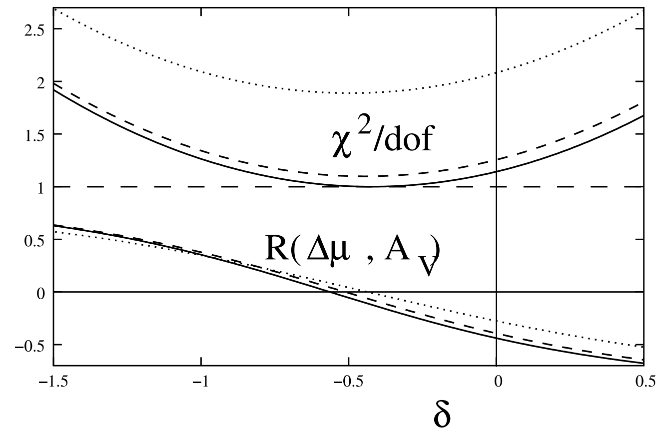

FIG. 3.—Best-fit values of χ^2/dof vs. parameter δ (*top curves*), $\Omega_M = 0.27$ (*solid line*), 0.5 (*dashed line*), 1 (*dotted line*) along with extinction correlation $R(\Delta\mu,\ A_V)$ (*bottom curves*). It is significant that $R(\Delta\mu,\ A_V)$ crosses zero just in the vicinity of the best-fit δ parameter. [*See the electronic edition of the Journal for a color version of this figure.*]

Questions then branch along three lines: (1) the assignment of extinctions by present schemes may contain hidden bias; (2) there may be a real physical effect at work; and (3) systematic errors might be reevaluated in order to ameliorate the significance of the correlation.

1. A seldom discussed but established bias exists in the assignment of A_V from the fits to light curves. We find it highlighted by the Berkeley group (Perlmutter et al. 1999, especially the Appendix). The scheme used starts with a conditional probability $P(A|A_{\rm dat})$, where $A_{\rm dat}$ is the extinction from the best fit to the light curve data. A prior probability $P_0(A_{\rm dat})$ is assumed, and from Bayes' theorem the probability of A after seeing the data is estimated. The value of A is chosen to "maximize the probability of A" given the combined information from the prior and the data.

The method introduces an extra dependence on the choice of priors. For prior distributions centered at small host extinction, the work of Hatano et al. (1998) is cited, based on Monte Carlo estimates from host galaxies of random orientation. Freedom is used to formulate a one-sided prior distribution with support limited to $A > 0$. This introduces a bias in the combination of assuming $A > 0$ for the priors (fluctuations could do otherwise) and the detailed way in which $A_{\rm dat}$ is assigned. This bias tends to cause the same signal as dimming or acceleration (Perlmutter et al. 1999). As of 1999, the outcomes of this bias were stated to be less than 0.13 mag.

Yet one would need an absolute standard to evaluate any bias reliably. Subsequently, the method itself has evolved, with Riess et al. (2004) citing an iterative "training procedure" we have not found described in detail. A few points now have $A_V < 0$.

There is evidently a further bias in taking data from the peak of the proposed distribution. It is not the same thing as sampling the proposed distribution randomly. Iteration of a procedure taking from the peak tends to drive a Bayesian update procedure toward a narrow distribution centered at the peak. In some renditions this may cause systematic errors of fluctuations to evolve in the direction of being underestimated.

2. It is possible that the extinction correlation signals physical processes of evolution with redshift. It is impossible to adequately summarize the literature discussing this possibility. Aguirre (1999) made a comparatively early study with a balanced conclusion that extinction models might cause some of the effects interpreted as acceleration. Drell et al. (2000) concentrate on this question, concluding that the methodology of using Type 1a SNe as standard candles cannot discriminate between evolution and acceleration. Farrah et al. (2004; see also Clements et al. 2004) cite a history of work scaling optical frequency extinction with the

TABLE 2

χ^2 VALUES INCLUDING CORRECTION TERM $A_V(\delta) = (1 + \delta)A_V$ (eq. [6])

Model	δ[a]	χ^2	R[b]	P[c]
$\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$	−0.42	156.0	−0.12	0.15
$\Omega_M = 0.32$, $\Omega_\Lambda = 0.68$ (best fit with $\Omega_M + \Omega_\Lambda = 1$)	−0.43	154.5	−0.10	0.23
$\Omega_M = 0.35$, $\Omega_\Lambda = 0.75$ (best fit)	−0.42	154.4	−0.11	0.22
$\Omega_M = 0.0$, $\Omega_\Lambda = 0.0$ (best fit with $\Omega_\Lambda = 0.0$)	−0.49	162.5	−0.11	0.20
$\Omega_M = 1$, $\Omega_\Lambda = 0$	−0.49	294.6	0.04	0.68

[a] Correction term in the distance modulus $A_V(\delta) = (1+\delta)A_V$ due to possible bias in the host extinction.
[b] The correlation statistic $R(\Delta\mu,\ A_V)$ between residuals and extinction.
[c] The confidence level (P) to find R in a random sample.

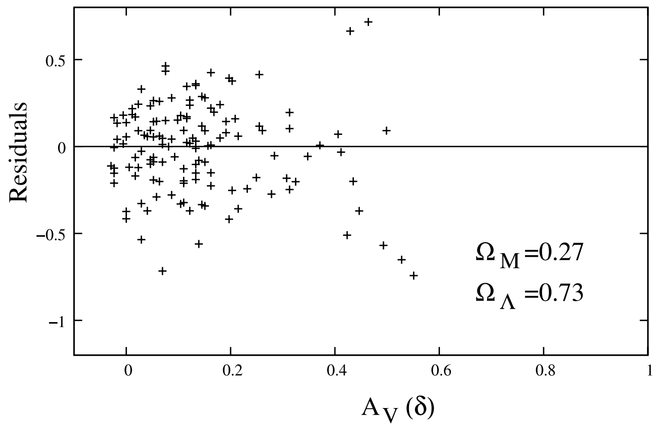

FIG. 4.—Magnitude residuals $\Delta\mu$ vs. host extinction after including a correction term $A_V(\delta) = (1+\delta)A_V$ (eq. [6]) in the distance modulus. Parameters are $\Omega_M = 0.27$, $\Omega_\Lambda = 0.73 = 1 - \Omega_M$, and $\delta = -0.42$, as in Table 2.

submillimeter wavelength observations (Hildebrand 1983; Casey 1991; Bianchi et al. 1999). They report extinction for 17 galaxies with $z = 0.5$ with submillimeter wavelengths. While claiming consistency with local extinctions at the 1.3 σ level, they add, "it does, however, highlight the need for caution in general in using supernovae as probes of the expanding Universe, as our derived mean extinction, $A_V = 0.5 \pm 0.17$, implies a rise that is *at face value* comparable to the dimming ascribed to dark energy. Therefore, our result emphasizes the need to accurately monitor the extinction toward distant supernovae if they are to be used in measuring the cosmological parameters." The trend of Farrah's observation is same as the correlation seen in the supernova data, and, remarkably, the corrections we obtain empirically in various fits (see below) almost all amount to 0.5 mag or less. The fact that low-redshift objects show higher correlation implies that there is a higher tendency to overestimate extinctions of these sources in comparison to the sources at higher redshifts. Since the estimated extinctions show no correlation with redshift, this suggests that the true low-redshift extinctions, on the average, may be smaller in comparison to the extinctions of high-redshift sources. Nevertheless, the question of evolution of the sources remains open and is not be resolved here.

3. Perhaps the means of assigning extinction coefficients are reasonable on average, but statistical fluctuations have given a false signal. Then the error bars on the extinction coefficients come to be reexamined. Inasmuch as this is coupled to the entire chain of data reduction, it is beyond the scope of this paper.

2.2. *Empirically Corrected Extinctions*

Without engaging in physical hypotheses of extinction, it is reasonable to test whether a different extinction model can give a satisfactory fit to the data. We studied a corrected value of $A_V(\delta)$ depending on the parameter δ by the simple rule

$$A_V(\delta) = (1+\delta)A_V. \tag{6}$$

We then determine δ by the best fit to the cosmological model. The best fit δ-values and the corresponding χ^2 values for different models are given in Table 2. Parameter δ produces a huge effect of more than 23 units of χ^2.

There are many ways to compare the new and old fits. As a rule, the model with χ^2 per degree of freedom (dof, the number of data points minus the number of parameters) closest to unity is favored. Since the new fits decrease χ^2 by about 20 units with one additional parameter, the significance of revising the extinction values is unlikely to be fortuitous. For example, the model with $\Omega_M = 0.27$ and $\Omega_\Lambda = 0.73$ gives $\chi^2/\mathrm{dof} = 1.14$ and 1.01 without correction ($\delta = 0$) and with correction ($\delta = -0.42$). As a broad rule in comparing data sets, the difference $\Delta\chi^2$ should be distributed by χ^2_ν, where $\nu = 1$ is the number of parameters added. The naive *p*-value or confidence level to find $\Delta\chi^2 = 23$ in χ^2_1 is 1.6×10^{-6}. Thus, introducing δ would be well-justified simply to improve the poor fit of $\chi^2 \sim 178$ without ever seeing the extinction correlation with residuals. Values of δ for all models are found to be negative, suggesting that the host extinction values given in Riess et al. (2004) are overestimates.

It is interesting and significant that the new residuals, computed relative to the revised fits, show negligible correlation with host extinction. This is seen in Figure 3, which shows the *R* values on the same plot as χ^2/dof. The fact that *R* vanishes when δ meets the best-fit value is significant. It is far from trivial, as *R* concerns an independent set of numbers, the A_V values, not directly used in calculating χ^2.

Figure 4 shows the residuals versus corrected host extinction after including the correction term. The reduction in correlation *R* comes with an increased scatter in $A_V(\delta)$ at large $A_V(\delta)$, which is not unexpected.

It is also interesting to ask whether host extinction might have some dependence on the luminosity distance d_L. It is hard to imagine no evolution at all, and we explored a linear ansatz. The linear model is

$$A_V(\delta,\ d_L) = (1+\delta)A_V + \delta_1 d_L. \tag{7}$$

We add that when a model of evolution is introduced, the cosmological interpretation might be disturbed, so that the outcomes must be taken in context. More cannot be anticipated because the fits themselves will choose δ_1. Fit parameters and χ^2 values are given in Table 3.

TABLE 3

χ^2 VALUES INCLUDING A CORRECTION TERM $A_V(\delta,\ d_L) = (1+\delta)A_V + \delta_1 d_L$ (eq. [7])

Model	δ[a]	δ_1[b]	χ^2	R[c]	P[d]
$\Omega_M = 0.27$, $\Omega_\Lambda = 0.73$	−0.42	−0.037	154.6	−0.11	0.18
$\Omega_M = 0.31$, $\Omega_\Lambda = 0.69$ (best fit with $\Omega_M + \Omega_\Lambda = 1$)	−0.42	−0.007	154.5	−0.11	0.20
$\Omega_M = 0.68$, $\Omega_\Lambda = 0.82$ (best fit)	−0.42	0.16	154.0	−0.079	0.36
$\Omega_M = 0.0$, $\Omega_\Lambda = 0.0$ (best fit with $\Omega_\Lambda = 0.0$)	−0.48	0.005	162.4	−0.11	0.19
$\Omega_M = 1$, $\Omega_\Lambda = 0$	−0.51	0.47	166.9	−0.03	0.73

[a] Correction term in the distance modulus $A_V(\delta) = (1+\delta)A_V$.
[b] Correction term in the distance modulus $A_V(\delta,\ d_L) = (1+\delta)A_V + \delta_1 d_L$ due to possible bias in the host extinction.
[c] The correlation statistic $R(\Delta\mu,\ A_V)$ between residuals and extinction.
[d] The confidence level (P) to find R in a random sample.

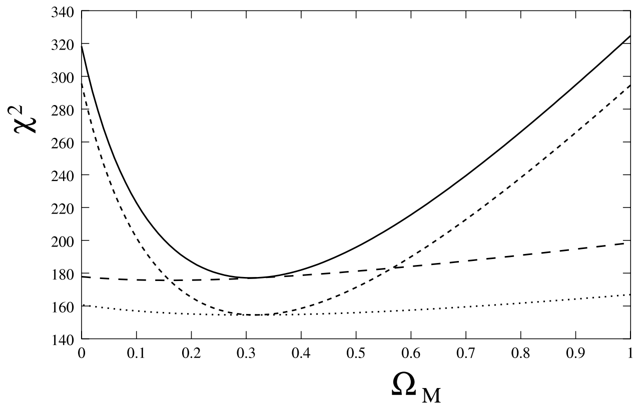

FIG. 5.—The χ^2 values vs. Ω_M parameter in various models: with uncorrected extinctions ($\delta = 0$, *solid line*), best-fit $A_V(\delta) = (1+\delta)A_V$ (*short dashed line*), best-fit $A_V(d_L) = A_V + \delta_1 d_L$ (*long dashed line*), and best-fit $A_V(\delta,\, d_L) = (1+\delta)A_V + \delta_1 d_L$ (*dotted line*). The models with $\delta_1 \neq 0$ are less sensitive to the value of Ω_M. [*See the electronic edition of the Journal for a color version of this figure.*]

2.2.1. *Is Acceleration Supported?*

Accelerating models show no need for the δ_1 term. Assuming acceleration, the fits (Table 3) show that reducing extinction values by about 40% explains the data better, and removes an alarming correlation. On the other hand, the matter-dominated model ($\Omega_M = 1$ and $\Omega_\Lambda = 0$) shows interesting sensitivity to δ_1. In Figure 5 we compare the sensitivity of different fits to the parameter Ω_M. With $\delta_1 = 0$ constrained, the effects of δ are rather orthogonal to those of Ω_M, so that the region $\Omega_M \sim 0.3$ is favored whether or not there is a significant correlation R. Yet varying δ_1 greatly broadens acceptable values of Ω_M, while maintaining the $R \to 0$ effect of δ. The significance depends on one's hypothesis: if one chooses $\Omega_M = 1$ a priori, the parameter δ_1 is traded for the parameter Ω_M. The overall probability of either hypothesis is only in part determined by the p-value of the data given the distribution: the rest depends on one's prior beliefs in evolution, which we do not pursue. It is fair to say that the revised fits give more leeway to matter-dominated models on statistical grounds.

In all cases fits are driven to $A_V \to A_V(\delta \sim -0.4) \sim 0.6 A_V$, either simply to improve χ^2/dof or to remove the correlation with residuals.

To conclude, analysis using reported extinction coefficients is well known to produce good fits to acceleration of the expansion rate. However, the extinctions show correlation with residuals with random chance probability using two independent tests, the extinction correlation and χ^2 values, both below the level of 10^{-6}. Our hypothesis is borne out, that extinction coefficients that are corrected empirically provide substantially improved fits to the data, while also eliminating significant correlation of residuals. A model of linear evolution yields interesting effects of high statistical significance correlated with redshift. The studies indicate either bias in host extinction assignments or evolution of the source galaxies. The significance of acceleration itself cannot be resolved on the basis of these studies but might be revised, depending on one's priors. We suggest that observers report uncertainties in their assignment of extinction parameters, both in the future and for existing data sets.

This research was supported in part under DOE grant DE-FG02-04ER14308. This work was completed when P. J. was visiting the National Center for Radio Astrophysics, Pune. He thanks V. Kulkarni for his kind hospitality. J. P. thanks Hume Feldman and Ruth Daly for discussions.

REFERENCES

Abraham, R. G., & van den Bergh, S. 2001, Science, 293, 1273
Aguirre, A. 1999, ApJ, 525, 583
Barris, B., et al. 2004, ApJ, 602, 571
Bianchi, S., Davies, J. L., & Alston, P. B. 1999, A&A, 344, L1
Brinchmann, J., et al. 1998, ApJ, 499, 112
Casey, S. C. 1991, ApJ, 371, 183
Clements, D. L., Farrah, D., Fox, M., Rowan-Robinson, M., & Afonso, J. 2004, NewA Rev., 48, 629
Coil, A. L., et al. 2000, ApJ, 544, L111
Drell, P. S., Loredo, T. J., & Wasserman, I. 2000, ApJ, 530, 593
Falco, E., et al. 1999, ApJ, 523, 617
Farrah, D., Fox, M., Rowan-Robinson, M., Clements, D., & Afonso, J. 2004, ApJ, 603, 489
Garnavich, P. M., et al. 1998, ApJ, 493, L53
Hatano, K., Branch, D., & Deaton, J. 1998, ApJ, 502, 177
Hildebrand, R. H. 1983, QJRAS, 24, 267
Höflich, P., Nomoto, K., Umeda, H., & Wheeler, J. C. 2000, ApJ, 528, 590
Knop, R., et al. 2003, ApJ, 598, 102
Leibundgut, B. 2001, ARA&A, 39, 67
Perlmutter, S., et al. (Supernova Cosmology Project Collaboration). 1998, Nature, 391, 51
———. 1999, ApJ, 517, 565
Riess, A. G. 2000, PASP, 112, 1284
Riess, A. G., et al. 1998, AJ, 116, 1009
———. 2004, ApJ, 607, 665
Schmidt, B. P., et al. 1998, ApJ, 507, 46
Sullivan, M., et al. 2003, MNRAS, 340, 1057
Tonry, J. T., et al. 2003, ApJ, 594, 1
Totani, T., & Kobayashi, C. 1999, ApJ, 526, L65
van den Bergh, S. 2001, AJ, 122, 621
Williams, B. F., et al. 2003, AJ, 126, 2608