Industrial Heterogeneity in Response to Factor Price Shocks: A Dynamic Framework of Production with Money Input

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Abstract

This thesis studied heterogeneity in firms' use of monetary assets and how it led to differential reactions to monetary shocks. I reviewed studies on the dynamics in industry-level and firm-level output, sales, employment, and investment in response to monetary policy shocks and during business cycle phases, summarized the implication to monetary transmission mechanism, to patterns in industrial and firm themselves' behaviors, and to welfare redistribution, and suggested new classification system of firms for better aggregation. I proposed a general framework of incorporating dynamics into modelling non-financial firms' multi-period production using flexible functional forms, a model family which was originally devised static and often undermined by dynamic misspecification issue in application. Then I applied the framework to the U.S. production data to model industry-level cost functions, and analyzed implication of dynamics in output, investment, and labor demand upon shocks from monetary asset prices, capital price, wage and so on. I identified a monetary transmission channel by examining the asset side of producer's balance sheet, different from any known channels which mainly affect real economic activities through financing and the liability side of producers' balance sheet. I call this mechanism the currency channel. In addition, I proposed the invariance of intermediate input price elasticity of output in production planning period horizon. Last, the application itself adds more empirical tests to assessing the competence of some flexible functional forms in modeling cost functions.

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Chapter 1

Introduction

1.1 Importance of Heterogeneity in Today's Macroeconomic Theory

Assuming all agents are identical in an economy, is a natural starting point to understand the economic laws regulating the dynamics in macroeconomic aggregates. The approach greatly simplifies analysis and makes many patterns stand out and relatively easy to apprehend. Identical agents leads to an equivalent condition: representative agent. That is all identical agents act collectively, as if they are only one agent, acting in representation of their aggregate behaviors. Thus the approach can be slightly generalized, to assume that there exist some types of agents, each having a representative. Agents of the same type are identical. But agents of different types differ in one way or several. That grants us to study what more complex interactions and dynamics emerge when all agents in the economy can be grouped by type, while the analysis is still kept manageable. This generalized approach, is called the representative agent approach, a dominating paradigm in macroeconomic studies.

However, macroeconomic models based on representative agent yield predictions and explanations far from satisfactory. There are something can be explained, and can be reliably predicted. But more can not. There are competing theories that seem to capture one facet of the economy's behavior each, but they cannot be consolidated due to contradictions in the presumption made on agents. It is reasonable for us to ask how to add more structure to our models, to reconcile the contradictions. Can a more generalized model develop a more self-consistent theory, to depict not only one facet of the macroeconomic world, but the whole scene, comprehensively and inclusively? One approach is to relax the homogeneous agent presumption to the heterogeneous case.

At least two reasons motivate us to explore macroeconomic theories with heterogeneous agents. First, this is the natural extension to studies using representative agent approaches. Second, introducing heterogeneity is backed by a solid theoretical ground. Researchers have established many sufficient and necessary conditions for the existence of a representative agent. The representative agent problem is equivalent to the admissibility of aggregating over agents. As for profit maximizing producers in competitive markets, the aggregation is simple. Aggregating over producers is adding all the production possibility frontiers of them. The representative firm is a mega firm that employs all the resources and technology that firms in the population have. But aggregating over utility maximizing consumers is not similar. Theories in this literature mainly around conditions for legitimately adding consumers together. The necessary conditions for the existence of representative agent are strict, and are not satisfied in many cases.

1.1.1 Micro Conditions

Theories on the existence of representative consumer, community preferences, and aggregate demand, along with implied macroeconomic conditions, developed extensively in the early studies of trade, efficiency, and distribution. These three topics (representative consumer, community preference, aggregate demand) are shown to be closely related. Samuelson [Sam56] summarized the relation between the representative consumer problem and the community preference problem well. We present some of the most general results here. A representative agent is said to exist if the collective behavior of a group of agents is the same as the behavior of an artificial agent attempting to achieve a similar goal under the aggregate constraint.

Gorman [Gor53] established the first conditions for the existence of community preferences, and thus the existence of representative consumer. Gorman's sufficient condition consists of two parts. First, no consumer is constrained by extremely low individual income. Second, the marginal propensity to consume for any good is the same across consumers. That is to say, the Engel curve expansion paths of all consumers are parallel.[1] The requirements can be stated as that, for every individual i in the cohort, his expenditure function is

$$e(u_i, \boldsymbol{p}) = a_i(\boldsymbol{p}) + b(\boldsymbol{p})u_i, \tag{1.1.1}$$

where u_i is the individual utility level, $\boldsymbol{p}$ is the prices of consumer goods, a_i, b are some functions that are concave, linearly homogeneous in $\boldsymbol{p}$. The derived community expenditure function will take the exactly same form, as the total of individual expenditures. Given those conditions, the community preference exist and income distribution does not affect the aggregate market demand any more.

Muellbauer [Mue76] further relaxed Gorman's condition and made it a special case. Muellbauer's conditions were developed from the opposite direction, in answer to the question that how a market demand can be equivalently decomposed in to a number of individual demands under a uniform income/wealth distribution, where the individual demands are derived from independent hypothetical utility maximizing consumers. The author used some auxiliary conditions to achieve so-called price independent generalized linearity (PIGL) generalization of the Gorman's condition. Budget (which is income or wealth in application cases) distribution "does not matter" in demand aggregation, if the representative consumer's total budget is equal to the average budget of individual consumers, and if for every individual i, his indirect utility function $v_i(y_i, \boldsymbol{p})$ is

[1] Originally by Gorman, the Engel curve expansion paths of all consumers are parallel straight lines. Unrealistic as it is, the straight line condition can be relatively easily relaxed to many smooth curves. The trick is to treat the curves piecewise linear, and then to take the limit of each piece to infinitely short.

either

$$v_i(y_i, \boldsymbol{p}) = \left(\frac{y_i}{b_i(\boldsymbol{p})}\right)^{\alpha(\boldsymbol{p})} \tag{1.1.2a}$$

or

$$v_i(y_i, \boldsymbol{p}) = \ln\left(\frac{y_i}{b_i(p)}\right), \tag{1.1.2b}$$

where y_i is individual total budget (income or wealth), α is some function homogeneous of degree zero, and b_i is some nonzero linearly homogeneous function. The representative consumer thus have indirect utility $v_{y_0,\boldsymbol{p}}$ in the macro form

either

$$v_{y_0,\boldsymbol{p}} = y_0^{\alpha(\boldsymbol{p})} \tag{1.1.3a}$$

or

$$v_{y_0,\boldsymbol{p}} = \ln y_0, \tag{1.1.3b}$$

where $y_0 = (\sum_{i=1}^{N} y_i)/N$ is the average budget.

Under slightly weaker conditions, Muellbauer derived a condition instantly comparable to Gorman's results. In order for a representative consumer to exist, for every consumer i, individual expenditure function should be either

$$e_i(u_i, \boldsymbol{p}) = \beta_i((a(\boldsymbol{p}))^\alpha + u_i(b(\boldsymbol{p}))^\alpha)^{1/\alpha} \tag{1.1.4a}$$

or

$$e_i(u_i, \boldsymbol{p}) = \beta_i(A(\boldsymbol{p}))^{u_i} B(\boldsymbol{p}), \tag{1.1.4b}$$

where $\alpha > 0, \beta_i > 0$ are scalar values, a, b, B are linearly homogeneous functions in price $\boldsymbol{p}$ and A is homogeneous of degree zero. The aggregate expenditure function derived from corresponding utility maximizing (and expenditure minimizing with community preference) representative consumer is

either

$$e(u_0, \boldsymbol{p}) = ((a(\boldsymbol{p}))^\alpha + u_0(b(\boldsymbol{p}))^\alpha)^{1/\alpha} \tag{1.1.5a}$$

or

$$e(u_0, \boldsymbol{p}) = (A(\boldsymbol{p}))^{u_0} B(\boldsymbol{p}), \tag{1.1.5b}$$

where scalar α and functions a, b, A, B satisfy the same set of conditions as (1.1.4). We can see from expressions (1.1.2)(1.1.4) that even the relaxed Muellbauer conditions are very restrictive. Translog and constant elasticity of substitution (CES) class expenditure functions appear to be the only model choices compatible with a representative consumer presumption. And the representative should be modelled with the corresponding form. Depending on the topic, these restrictions do not lead to far fetching macro level models. At the macro level, expenditures can be approximately

CES. Yet we need to be cautious that connection of models like such with microeconomic models, is excessively limited. It is quite improbable that the cohort of individual consumers have, for example, mean[2] preserving income distribution regime as well as uniform family of CES expenditure functions.

The existence of an aggregate demand[3] consistent with a utility maximizing consumer needs less restrictive conditions than the existence of community preference. Lewbel [Lew89] presented the most general form of representative agent compatible demand system

$$x_i = a_i(\boldsymbol{p}) + b_i(\boldsymbol{p})y + c_i(\boldsymbol{p})g(y, \boldsymbol{p}, \eta), \tag{1.1.6}$$

where i is the goods index[4], x the quantity demanded, $\boldsymbol{p}$ the price tuple, y the total budget, g an function to be specified, and η an agent-specific preference parameter. It is the most general form of a utility-derived demand system having Engel curves that are linear in income and one function of income. It encompasses at less the following preceding models as special cases:

- Almost Ideal Demand System (AIDS) model [DM80]
- Translog model [JLS+82]
- Generalized Cobb-Douglas functional form [BDD77]
- Homothetic demand system
- Quasi-Homothetic demand system
- PIGL, PIGLOG model
- Quadratic Expenditure System (QES) model
- Others

[2] Second and higher moments of the distribution.

[3] Potentially macroeconomic.

[4] Note the change in the meaning of index i. Without special mentioning, this subscript begins to represent goods (no longer agents) from now on.

Theorem 1 of the paper addressed how functions a, b, c, g are specified and parameterized to attain the demand function and indirect utility function in each special case.

Generally if we allow the aggregate demand function depend on the distribution of individual preferences, the existence of a utility-derived[5] demand function of a representative consumer. Let μ be the average of η over agents, X_i be the average demand for goods i (mean of x_i), y the mean income. Then these individual demands permit a representative consumer that maximizes $u(X, \mu)$ under the economy-wide budget constraint $\sum_i p_i X_i = y$. And the aggregate demand is

$$X_i = a_i(\boldsymbol{p}) + b_i(\boldsymbol{p})y + c_i(\boldsymbol{p})g(y, \boldsymbol{p}, \mu) \tag{1.1.7}$$

Here X_i is an aggregation function of x_i in a sense. Equation (1.1.7) holds as the actual demand derived from a representative consumer's utility maximization problem under a much looser condition called mean scaling, compared to the community preference conditions. When the aggregate holds, it also implicitly permits the representative agent modelling approach.

Except for the QES case, that the distribution of individual consumers' budgets y_i is "mean scaling", is a sufficient and necessary condition of representative consumer's existence [Lew89]. In general in equation (1.1.7), the preference like parameter μ has to be a function depending both on goods' prices $\boldsymbol{p}$ and total budget y. It was shown that under the mean scaling condition, μ is independent from $\boldsymbol{p}$ and y and can be viewed as a true preference parameter. Taking individuals as realizations of a random agent, even if the preference parameter η distribution is different from income y distribution, there is a scheme to construct the hypothetical representative consumer whose utility depends solely on a function of η.

Thus conditions for mean scaling property become central to the discussion. Here are a few easy to interpret economically. A variable y is mean scaling in its aggregate Y if and only if the ratio of Y to any quantile of the distribution of y, is independent of Y. Those ratios are constant over the "scaling" of the distribution, if and only if there ia a proportional distribution movement (each y value is scaled by a common scalar factor). Last we present a sufficient condition that adds

[5] Always be a solution to the utility maximization problem

a time dimension. If for each individual, $\ln y(t)$ is a random walk over time, and the distribution of the random innovations is independent of aggregate $Y(t)$, then the distribution of $y(t)$ for any fixed t will be mean scaled. If $y(t)$ is the instantaneous income of individual consumers[6], then that condition means personal income path innovations must have nothing to do with the systematic change in income distribution. In fact, mean scaling condition prohibits any systematic change in income distribution. And that we know, has never been observed in the macroeconomic dynamics.

If representative consumer in a macroeconomic model is a shortcut approximation at best, by greatly sacrificing the plausibility of the unstated underlying microeconomic ground, would measuring aggregate values using index numbers be able to reconcile the representative consumer approach and plausibility of the accompanying microeconomic assumptions? Reinsdorf [Rei98] answered the question by establishing conditions for an index number[7] based aggregate to hold in consistence with a hypothetical representative consumer.

If the income distribution among individual consumers is mean scaling, and if all individual preferences are homothetic, then a Divisia form of social cost of living index permits a representative consumer. The social cost of living index would behave as if it is the individual cost of living index of a hypothetical utility-maximizing consumer. Further more, if the individual utility functions are all linearly homogeneous, then the social utility function, which is the utility of this representative consumer, corresponding to the Divisia social cost of living function, is the geometric mean of individual utilities weighted by the income shares. The linear homogeneity condition can be relaxed to homothetic utility function in the risk free world, or when all individuals are risk neutral.

When individual utility functions are not homothetic, the Divisia index can exist, but its implied social cost of living index does not permit a representative agent, or a representative agent exists with violation of the Pareto principle. In either case, although we can calculate the index, it implies inconsistency of a preference in different areas of the commodity space. Reinsdorf's index number approach did give us more general space for using representative agents, but this improvement is

[6]Subscript representing agents is omitted.
[7]Or a compatible set of indices for price, quantity, etc.

minor. Especially when the model involves welfare evaluation, representative consumer approach could easily create much theoretical inconsistency.

1.1.2 Macro Applications

Caselli and Ventura [CV00] developed a scheme to model heterogeneity in growth models. In particular, after they introduced a bit heterogeneity in consumers' taste, initial income and wealth to two simple growth models, innumerable dynamics in consumption, income, and wealth emerged. They also established cases where some heterogeneity can be modelled under a RC framework, greatly simplifying the analysis. As stated before, the approach requires perfect certainty of the future, homothetic preference, and mean scaling income distribution. The synthetic model of growth and RC is somewhat pliable in the ability to generate various kinds of dynamics. Some of the paths can be consistent with the real world observation.

In the realm of asset pricing theory, Constantinides and Duffie [CD96] introduced heterogeneity in income flow to RC based asset pricing models, so as to explain several empirical puzzles (for example, equity premium puzzle, risk-free rate puzzle). Labor income shocks to consumers were uninsurable (incomplete consumption insurance), persistent, heteroscedastic. Individual income processes were specified such that aggregate income process matched the real world data, and the joint process of goods prices, bond prices, dividends matched, too. The joint hypothesis resolved those puzzles without restrictions of any kind that partially fixed the RC models (for example, borrowing constraints, short-sale restrictions, borrowing costs, transaction costs, restrictions on net supply of bonds).

As for studies on demand for consumer goods, Muellbauer [Mue75] established the most general form of aggregate demand (price independent generalized linearity) compatible with RC setting. Similar to [Mue76], those are cases where expenditure weight on each good is linear in the indirect utility or expenditure. And the linear coefficients are functions of prices. It is shown that generalized linearity is the most general case where any income redistribution is equivalent, for incomes above a certain lower limit, to some appropriately defined hypothetical change in average income. And PIGL is the only case where, if changes in incomes are equiproportional, no aggregation error is made by using RC (in the author's words, by fitting aggregate market equations). A special case of PIGL is shown to imply that the effects of redistribution on demand work entirely through mean and variance of the distribution. In the PIGLOG case these effects work entirely through mean and entropy measure of income dispersion.

Last, evidences show that heterogeneity is not an idiosyncratic feature that can be averaged out. We will elaborate in the next section.

1.2 Feasibility of Finer Modelling

1.2.1 Finer Observation

Growing collection of data with more details sets the ground for exploring the heterogeneity. Statistics authorities from major countries all have been collecting and providing data with many features about households and companies, for years. Just in the English speaking world, highly detailed national accounts and census data are regularly published by multiple agencies in every country. Similar data are assembled by international organizations like the UN (International Development Organization Industrial Statistics Database, INDSTAT), OECD (International Sectoral Database), IMF[8] that cover countries and regions whose data are less accessible in English. OECD maintains a database covering the widest range of aspects of economies and societies. UN's data focus on development and IMF's emphasize finance and trade.

Commercial databases now have collections to even more details for the recent several years. To name a few largest ones, Bureau Van Dijk (Moody's Analytics), Compustat, Dealscan have almost everything you'd like to know about listed companies. Plunkett Business Insight, SageWorks Analytics cover great details of 5-digit and 6-digit level NAICS sub-industries. And many more are specialized in one or two areas. Commercial data sets trade range for detail, and hence usually has shorter observation period coverage than the governmental ones. Leading investment banks,

[8]As we write, the World Bank does not publish any data more detailed than country level aggregation.

consulting, and rating agencies also built up in-house databases with considerable scales over the past few years. All those data enable us to find richer patterns and finer classification of economic agents.

1.2.2 Feature Difference Leads to Behavior Difference

countless general equilibrium models with heterogeneous settings, in households, in consumers, in producers' market powers, show that agents do diverge and their action rules in the economy can be very different. To name some recent ones, there are studies on household heterogeneity in reaction to monetary policy shocks.

Results from other topics also confirm that heterogeneity adds richness and depth in our understanding of the problems.

We are going to see plenty (more) examples from empirical studies in the next chapter.

The subsequent contents are arranged as follows. The rest part of the chapter 1.3 review all published studies up to the end of 2017 on industry-level or firm-level responses to monetary policy shocks. Some of those are dedicated to demonstrate the heterogeneity and to explore the reason. Some are studies on monetary transmission mechanism, using firms or industries as natural comparison groups. Chapter 2 generalizes the basic flexible function form into a multi-period model under the cost function context. The generalization enables one evaluate marginal dynamics production in a sequence of periods. Chapter 3 quantify industry-level cost functions of 26 industries in the U.S. over the past 30 years. It assesses how those industries respond to monetary shocks in adjusting their output level, investment, and employment.

1.3 Monetary Policy Effect Differentials across Industries and Firms (Related Literature)

Early macroeconomic literature showed some regularity of the economy in a neat and insightful way. In the light of those theories, researchers began to test for or against them empirically with highly aggregated variables corresponding to those in theories. However, when we tried to fit data to theoretical models, unexpected results rose. For example, the real output appeared to change too much after a rather small disturbance in interest rates. The aggregate price level appeared to rise under an unexpected contractionary monetary shock, before it depreciates for a few months, which is the well-known price puzzle. These irregularities led to new theories like the balance sheet transmission channel. And the new theories called for more detailed understanding of the economy structure, and thus required disaggregated data to validate.

The new need for validation has accumulated abundant disaggregated empirical evidences for the past thirty years. Although these studies were devised to verify certain theories, they themselves revealed some persistent patterns about different agent groups. Pooling the patterns together can help us develop a systematic understanding of how and why economic agents are different from their natures to their behaviors. That is what inspired this review. It is going to focus on the connections between monetary factors like policy shocks and credit supply, and the economic activities of firms, like production and employment. It is going to summarize the empirical evidences in the theoretical background. It will develop a general picture about how firms are behaviorally different or similar, whether the differences are significant aggregated, and what the significance means for one-size-for-all policies.

One may ask that with the knowledge of undesirable consequences, why we still use one-size-for-all monetary policies. One reason is the implementation simplicity. The less complex a policy is, the less likely that it is going to be wrongly implemented. Another more compiling reason is feasibility under some circumstances. Monetary authorities in large economies and in monetary unions will influence heterogeneous regions whatsoever. The different policy results and consequently different welfare effects are inevitable, for those affected local economies are highly integrated. Small economies that adopt hard pegging exchange rate regime and open financial market need to consider the impact of monetary policies from the anchoring currency country. Responses in the policy home economy are usually not what will happen to the small passively affected economy. The knowledge of disaggregate level regularity will assist large economy and monetary union policy makers, and passively affected policy makers better evaluate the regional effects.

When economic variables are disaggregated not by geological boundaries, but by economic attributes, they provide a different angle for us to understand the role of geological differences plays in determining the economy's characteristics. Some demographic features, political features may make a difference even in the short-run patterns while some may not. The economy's dependence on financial intermediation and credit market may or may not affect how a given industry reacts to a type of monetary shocks. We'd be able to prioritize institutional factors in country-level policy effectiveness comparison. Such comparative studies have always been important for micro and macro level modelling.

The studies in this review are involved with a wide range of literature topics. The monetary transmission mechanism studies are the primary sources of these evidences. Bernanke [B+93] introduced how the credit channel theory stemmed from information asymmetry and principal-agent problems, the early version of the theory, and early evidences to test for it. De Bondt [**?**] fully summarized all major transmission channels, evidences for and against them, and empirical studies on the asymmetry of monetary policy shocks[9]. Complementing them, a few surveys were dedicated to one of the specific mechanisms. They summarized evidences about mechanisms related to asset prices [Mis01], bank credit [S+02], trade credit [Mat05], exchange rate for emerging economies [MK+08], European transition after monetary unionization, securitization, and deregulation [Ban10], and transition economies [Ban06]. As for more fundamental patterns and theories, the review limitedly covers the corporate financial structure literature [PT08], inter-industry wage differentials and job- or skill-based wage structure [CLM+05, DCRT11, DCLP+10, GKLP05, Mon91, NOW17], goods market price structure [BN03, BC99, Las06, Rát01], inventory management and dynamics [BM91a, BM91b] and financial and regulation history [RR04, GK05, WY20].

[9]There has been a change in the meanings of these terms in this literature. Up to middle 1990s *credit channel* stood for the transmission channel through financial intermediation, which became the *narrow credit channel* or the *bank lending channel* afterwards. The term *asymmetry* was used for the economic agents' heterogeneity under monetary policy shocks up to late 1990s. But sometimes it also meant that agents respond to shocks during expansions and contractions differently. The first meaning was still occasionally in some literature in the early 2000s. Later and till now *asymmetry* only means the response differential to shocks during different phases in the business cycle.

1.4 Typical Empirical Methods

Monetary transmission mechanism studies began to use disaggregate data in tests for the existence and significance of the credit channel. The credit channel was first proposed by recognizing the macroeconomic influence and uniqueness of credit created by banks. It was later on called the bank lending channel or the narrow credit channel to distinguish from the broad credit channel. The broad credit channel extends the imperfect and asymmetric information essence that creates the channel through banks, to general credit creating process in loan making institutions and credit markets. Since the balance sheet stance of a borrower is the key observable that separates the affected and unaffected in this channel, it is also called the balance sheet channel.

The existence of credit channels, narrow or broad, implies that the real economic activities of some borrowers are affected significantly more than others by the same monetary shock. The real economic activities will be production, sales, and investment for firms in our context, and consumption for households. More precisely, firms vulnerable to shocks via credit channels are those that have the most severe asymmetric information problems in external financing. As a result, they usually receive the marginal credit in the economy. Their external financing premia are higher. They have less external financing sources or instruments and thus depend more on the few sources they have. Both the price and quantity of the credit they receive are the first to vary at the impact of monetary shocks.

Succinctly, the credit channels suggest that the information-opacity amplifies monetary policy shocks to firms in that (with a little notation abuse)

$$\frac{\partial^2 y}{\partial o \partial m} > 0, \tag{1.4.8}$$

if, everything else holds equal. In the inequality, y stands for a measure of real economic activities, o for a measure of information asymmetry (information opacity to creditors), and m for a measure of external monetary supply or the available internally generated cash flow at disposal. The measure of real economic activities can be output, sales, investment, employment, and so forth. The measure of information asymmetry can be the balance sheet strength, and so on, which is the most hunting practice targets for. Dependence on short-term debts, leverage ratio, coverage ratio, working capital intensity, credit rating, those commonly used measures in daily financial practice are all extensively studies. The measure of external monetary supply can be changes in benchmark interest rates, the credit yield spread, the Federal Reserve policy meeting and announcement dates, or otherwise identified policy changes.

1.5 Output Pattern Differentials

Naturally one would speculate, should production side of the economy be disaggregated, or classified in a way, sector and industry are the best way to separate homogeneity and heterogeneity. Determined by economic activities, which is how we define industries, firms in similar business will use similar technology, employ similar factors and production plans, have common market demand and competition environment, be under common regularity and common restrictions interacting with other businesses. Under monetary shocks, common patterns of firms in an industry is then likely to be fundamental. The differences shown within an industry is more likely to be idiosyncratic.

Ganley and Salmon [GS97] directly addressed this question with British data at the sector level. Sectors' responses to unexpected contractionary monetary shocks are different in at least three dimensions. The impulse responses of sector- and industry-level output have different initial response direction, maximum deviation (magnitude of trough), time to trough and time to recovery. For example, the U.K. agriculture output almost did not respond to monetary shock at all, while others did. The mining and quarrying industries uniformly expanded output a bit before falling below the trend growth. The manufacturing output curves plunged slowly, deep and for long.

Methods used by Ganley and Salmon were rudimentary and the data were coarse, but the significance of industry-level heterogeneity was confirmed by later more rigorous studies using data from other countries. Hayo and Uhlenbrock [HU00] inspected output and price dynamics after monetary shocks for German manufacturing and mining industries. The industry-level heterogeneity was evident. For real output and product price, impulse responses differ in reaction direction, maximum deviation from normal (magnitude), volatility (some oscillate around the projected normal level), time to maximum deviation (speed), and time back to normal (duration). Along these dimensions, industries clustered into groups. And the group patterns were systematically related to industry characteristics. Capital intensity measures had explanatory power over output patterns. The more capital intensive the industry was, the more likely that its output would be negatively affected, consistent with the interest rate channel prediction. Subsidy and export measures were explanatory over output and price patterns. Notably, subsidies and export were two critical factors in West Germany during the sampling period. German government heavily subsidized some manufacturing and mining industries considerably to support their development and export. The effect of subsidization was possibly strong enough to overwhelm regularities caused by labor market and investment agenda factors. In particular, investment measures appeared significantly pairwise correlated with some output and price pattern features, but became insignificant when they were modelled together with subsidy and export measures. This encompasses a subtle issue of modelling and comparing parallel empirical evidences from different countries and different historical periods. Some differences can be neglected and some need special treatments and tests to decide.

Findings on the importance of industry-level patterns are robust with expanded samples, and finer characterization of the response pattern. Arnold, Vrugt [AV04] examined factors causing regional response heterogeneity in the East and West Germany states. The study led to evaluation of sector- and industry-level output response heterogeneity in the complete German sovereignty and longer historical period. The pattern diversity was as apparent as Hayo and Uhlenbrock 's result. Expanding the geological area[10] Peersman and Smets [PS05] studied what factors characterized the degree of cyclical phase asymmetry[11] in output responses of manufacturing industries. Evidences

[10]That is Germany, France, Italy, Spain, Austria, Belgium, Netherlands.

[11]Cyclical phase asymmetry means the impulse response differential of economic variables under unexpected monetary shocks in expansion and recession phases of real business cycles. The phenomenon was predicted by all the credit channel transmission theories and documented empirically. Without specification cyclical phase asymmetry, or

indicated that the money channel[12] had the first order effect on the overall impact magnitude and the balance sheet channel had the first order effect on the cyclical phase differential. How the degree of cyclical asymmetry would be affected if monetary policies operated countercyclically was yet unknown.

The above three studies unanimously gave positive answers to the question that industry composition is the leading factor for regional different reactions to monetary policies, at least at state or province level within a country. Dedola and Lippi [DL05] addressed the question whether one industry in different countries reacted to monetary shocks differently. The answer they had was no for manufacturing industries in the U.S, U.K, Germany, France, Italy. In those developed economies, country did not make a difference to an industry. But conversely industry composition still made differences to an economy. This study marked the end[13] in the series of industry-based output heterogeneity in the literature.

1.5.1 Outstanding Credit and Credit Supply

Besides industry, there can be other equally powerful classifications of firms. Non-industry-based comparative evidences are even more abundant. Those studies mainly stem from and seek to test significance of credit channels and asset price (like exchange rate) channels. Thus their subjects are centered at credit patterns and the classifier searches unfold around factors like debt service ability measures and credit risk measures. How evidences support or counter the existence of, for example credit channels, is not this paper's interest. The review will only compare and syndicate empirical patterns they found.

From the nonfinancial firm (borrower) side, monetary shocks do negatively affect the marginal

cyclical asymmetry will be used as so throughout this paper.

[12] And possibly interest rate channel, too. Measures used in the study cannot distinguish effects from the two channels.

[13] Barth and Ramey [BIR01] attempted to reconcile three empirical puzzles and proposed the "cost channel" theory. But the study was based on a theoretical concept of monetary shocks that was outdated even by then. Ippolito, Ozdagli, and Perez-Orive [IOPO18] built a theory about the amplification effect of outstanding floating rate bank loans and supplemented empirical evidences. Recent as it is, the key mechanism was not new. It basically proposed that if interest rates were flexible, the associated debt service burden would become heavier in tight money time. That was readily part of the interest rate channel.

credit use. The credit evidences include but not limited to bank loans and their terms, corporate bonds, and trade credit and are mostly from manufacturing firms. Ashcraft and Campello [AC07] took a special angle into subsample comparison and showed evidences for the impact caused by using market credit, net of effects from the bank loans. Kashyap, Stein, Wilcox [KSW92] documented the aggregate substitution from bank loan to commercial paper shortly after monetary tightening. Oliner and Rudebusch [OR+95] supplemented the substitution behavior with a wider range of credit instruments. They also found absolute increase in bank loans used by large firms under the same circumstance.

Resonating results are found from studies on other developed economies. Atanasova, Wilson [AW04] found significant evidences using bank loan and inter-firm (nonfinancial firm) credit by small and medium enterprises in the Britain. Bougheas, Mizen, Yalcin [BMY06] looked at even more general credit creation in the U.K. and came about with more confirmation. They also confirmed many other credit creation hypotheses like credit rationing and the effects of relationship banking, though such practices, if existed, would reduce the credit sensitivity to monetary shocks. What was special with the British studies was that they used multiple factors to determine the degree of financially constrained-ness, as a correction to the prior commonly used ownership-based classification of monetary feature. The method is an improvement to the single factor classification system which plagued the transmission mechanism studies. But still, it does not satisfactorily solve it. Although all those factors are measures of credit risk or financial soundness, they, even used together, barely reveal enough information about the firms' financial soundness. They are not even monotonic measures. But those are the only statistics one can get from national accounting data or private databases.

From the commercial bank (lender) side, researchers worked on the hypothesis that loans on banks' balance sheets are sensitive to monetary shocks[14], which constitutes one of the two key frictions in the bank lending channel theory. Kashyap and Stein [KS00] showed that tight money

[14]To be specific, when the policy tightens total monetary supply, banks cannot unrestrictedly borrow from interbank market nor unrestrictedly issue new equities to issue new loans. Or on the credit side of the balance sheet, banks cannot substitute marketable securities for new loans unlimitedly.

did cause banks reduce loan supply and it affected banks differently. The more liquid a bank's balance sheet was, the less prone it was to contractionary shocks. And the liquidity was positively related to bank capitalization size. Only the top 1 percent of banks' loan supply appeared completely free from monetary tightening. The top 5 percent were likely to be invulnerable and the rest 95 to 99 percent banks' loan supply would decline. The cutoff percentage is very relevant in our later discussion of the differential between firms of different sizes.

But the case is never settled with however many confirmations if opposite evidences emerge from time to time with sample or method change. De Bondt [DB99] also used banking industry data to estimate the significance of credit channel effects for selective countries[15]. He also supported that regardless of country, smaller or less liquid banks' loan supply decreased more under contractionary shocks. But the conclusion on the significance of the bank lending channel and the balance sheet channel contradicted conclusions from [AW04, BMY06] .

As for country-level difference, which is a much more practically complicated but much less theoretically important issue, Ehrmann and Worms [EW01] pointed to one organizational reason. They evaluated the difference between the strength of the bank lending channel in the U.S. and in Germany. Commercial banks created a large proportion of all the credit in Germany whereas less than half in the U.S. One may speculate that the bank lending channel effect was stronger in Germany. But the reality turned out to be the opposite. Unlike in the U.S, German banks developed strong ties with other banks and clients. Small banks were usually affiliated to bank holding companies, which also owned substantial shares of the largest banks. Although most small banks did not have access to the credit market like their American counterparts, they could obtain credit via the parent holding company channeling liquidity from large banks. They essentially enjoy the low financing premium (especially when borrow internationally) like the large ones. The arrangement eased the financial friction for the entire banking industry. When banks lent to other industries, the prevailing relationship banking practice reduced total bank loan supply sensitivity further more. The system greatly offset the effects of heavy bank dependence for nonfinancial

[15] They are the U.K, Germany, France, Italy, Belgium, Netherlands.

producers in Germany.

1.5.2 Output and Sales by Firm Type

Financial strength, liquidity of asset holdings, and credit risks are very likely to dictate the credit received and banks' credit supply differentials, based on evidences from non-financial firms and financial intermediaries. To close and fully support the induction along the credit channel theory, one still need to know whether the differential in credit received under monetary shocks, will end up in substantial differences in producers' real economic activities like sales, output, investment, so on and so forth. The answer from studies so far is yes. But they disagree in which kind of financial strength to be influential in causing the differential.

Financial Strength

Dedola, Lippi [DL05] and Peersman, Smets [PS05] depicted a comprehensive picture of what characteristics explained industry-level differentials in reaction to monetary policy shocks. In fact the two studies are very comparable in that they both used industry-level data from manufacturing sector in multiple countries[16] spanning from late 1970s to late 1990s[17]. According to Dedola and Lippi, monetary policy shock effects were more pronounced for output in industries that had higher leverage (total debt to equity ratio). But dependence on short term debt (short term debt to total debt ratio) and interest burden (interest payment to gross operating profit ratio) or the reciprocal of the coverage ratio did not have explanatory power. Peersman and Smets looked into what characteristics of industries explained the degree of asymmetry of the industry-level output response to monetary shocks in different phases of real business cycles.[18] According to them, none of the financial strength or liquidity measures had explanatory power in the overall phase-

[16]The United States, United Kingdom, Germany, France, Italy in Dedola, Lippi. Germany, France, Italy, Spain, Austria, Belgium in Peersman, Smets.

[17]1975 to 1997 in Dedola, Lippi. 1980 to 1998 in Peersman, Smets

[18]In theory, and as the motivation to the Peersman, Smets' study, the average output responses of an industry to monetary policy shocks in expansion and contraction epitomizes the effects of the money channel and the interest rate channel. The asymmetry of output responses in the two cyclical phases symbolizes the effect of the bank lending channel and the balance sheet channel.

independent response differentials. As for the degree of phase asymmetry, more dependence on short term debt (short term debt to total liability ratio), lower coverage ratio (gross operating profit to interest payment ratio), and higher leverage (debt to total asset ratio) were associated with wider phase asymmetry.

While due work remains to investigate what the disagreement implies and what future research it entails, the two studies share a common weakness. Those financial indicators, even if used together, are barely enough to depict the whole picture of a business. For example, a firm holding low liquidity assets before identified contractionary monetary shocks may be a firm inclined to boldly and riskily investment during easy money periods, like ones in emerging industries. Or it can be a firm that is not able to grow into a profitable mode during a typical expansionary period. Or it can be a firm that does not do well in making enough operating revenue even during the expansionary time, like ones in declining industries. The profit is thin and paid out to cover the nonexpanding production costs and dividends. Even if one sets out to solely establish the causal relation between financial aspects or patterns with pattern differentials in the firms' real activity responses to monetary shocks, more information on firms' financial management is indispensable and case studies on typical business models may help develop some insight.

Firm Size and Age

Bearing the fact that direct measures of financial strength and balance sheet liquidity can ambiguously represent multiple types of firm strategies, some studies used other factors to differentiate firms in their responses to monetary policy shocks. At their time, they were commonly under the theme of verifying the existence of one or two of the credit channels. And what factor (best) proxied the state of being financially constrained was one of the top concerns in this research agenda. In essence all credit channels are amplification mechanisms stemmed from information asymmetry and financial market frictions. So the direct testimony for existence of credit channels should be causes of or observable behaviors due to information asymmetry. The notion of "financially constrained" is a less powerful compromise. A firm is said to be financially constrained if it increases investment with a windfall of additional funds. The concept serves well in studies about investment sensitivities to internally generated funds like operating profits and free cash flows. But it is not a well-behaving standard for monetary studies, nor a good proxy to any monetary policy sensitivity classifier. Separating firms to financially constrained and unconstrained ones does not imply the former group is more informationally opaque than the latter. Many factors in goods market condition, business model, development strategy, and management style strongly mix the two classification subsets. The confusion of relating financially constrained-ness (instead of the degree of information asymmetry) to production sensitivity under monetary shocks plagues many theoretical and empirical studies.

Regardless, firm size is the earliest and one of the most studied proxy in that search. The strong correlation between being large in capitalization and having access to credit market in companies makes firm size a terrific proxy to separate firms affected by the bank lending channel and those not. Gertler, Gilchrist [**?**] identified bank dependence among a majority of medium sized firms in the United States. Oliner, Rudebusch in a follow-up study [OR+95] noted that commercial papers were almost exclusively used by the very large firms. Atanasova, Wilson [AW04] constructed a measure of "borrowing constrained-ness" based on several financial factors from the bank and borrower sides. They found that firm size was strongly positively correlated with firms' ability to obtain loans under any monetary circumstance. Ehrmann, Worms [EW01] also documented that inter-bank credit market was almost exclusively used by very large banks in the United States and Germany.

Evidences showed that this correspondence did render small firms more vulnerable to adverse monetary shocks. Gertler and Gilchrist [GG94] first utilized this proxy property of firm size and established the contrast of sales under identified monetary policy shocks. The study also marked the beginning of a long strand of studies on differences of small and large firms under the monetary topic. The sample used by Gertler and Gilchrist only covered large manufacturing companies in then and today's standard. The small firms referred to by the paper were not exactly small. In their findings, sales of medium firms dropped times faster than that of large firms. At the peak value of sales response, medium firms' sales loss to a unit increase in the Fed funds rate was four times of large firms' loss. The recovery to normal also took several quarters longer for medium firms. Kudlyad, Price, and Sanchez [KPS+10] extended the study to cover the two recessions in the twenty-first century. In their reproduction part, the results still held but the contrast between two size groups was not as strong. Ehrmann [Ehr05] concluded that small firms felt more pressure and had dimmer business outlook than large ones under contractionary monetary shocks, using a monthly business opinion survey covering all sizes of German manufacturing firms.

In addition to that, the opposite evidences were just as strong and plenty. Arnold and Vrugt [AV04] in their study of German state-level cyclicality found that neither firm size distribution nor bank size distribution[19] alone had explanatory power over regional differentials in response to common monetary policy shocks. But industry composition could explain this evident state-level heterogeneity.

Results emphasizing the first order effect of industry emerged from country-level data. Dedola and Lippi [DL05] concluded that industry composition accounted for most of the country-level differences under a comparable monetary policy shock. Conditional on industry, firms of smaller size had output more sensitive to monetary policy shocks. Peersman and Smets [PS05] found that conditional on industry, firm size did not have explanatory power in the average output response sensitivity during different business cycle phases, but smaller size partly explained the higher sensitivity in recessions relative to the same industry's output sensitivity in expansions.

It is possible that methodological issues can undermine some studies and lead them to wrong conclusions given the complexity in the firm size concept itself. Firm size can be measured by at least three senses. It can be measured in their presence in the capital market or the amount of resources on command, by for example, capitalization (equity). Or it can be measured by its presence in the product markets, by for example sales. Or it can be measured by its presence in the labor market and the community, by the number of employees. Those standards each has subtle implications and complications, and is suitable for certain problems in particular. But all in all, the

[19] During this sample period, the two distributions can be seen as two measures of one variable, the firm size overall, in Germany.

choice of measure was not disturbing in aforementioned studies.

False positive error caused by regression bias and reclassification bias can somehow be devastating. Regression bias is a fallacy caused by that firms tend to become similar in the long-run. (Regression stands for the mean-reverting phenomenon.) That is small firms are more likely to grow faster than large ones, and large firms are likely to grow slower. In the end firms (of the same industry) can end up in similar sizes. Reclassification bias is caused by firms changing size class over time. During expansion, more firms grow into the larger size group, so that output or employment or investment of the large group appear to be relatively greater. In recessions the opposite happens. More firms drop into the smaller size group, and then output (or employment or investment) of the small group appear to be relatively weaker owing to the concentration of weaker firms. Both issues are best assessed and if in need, fixed by using longitudinal data. And some investigation of whether the subject of question has cyclical or countercyclical dispersion effects will help evaluate the regression bias.

Taking the possible error inevitably caused by data restriction, the mixed yet strong evidences make one wonder firm size's poor representation of the true financial friction and the degree of marginal financing cost increment may be in the way. After all, the nature of small and large firms is so different that they are analyzed as distinct species even within one industry or one region. And various aspects of the nature have implications of how the firm would adjust in reaction to monetary shocks in various directions. And many of them, working along the way with information asymmetry and financial friction, are not financial features. Small firms may employ more flexible technology and production agenda, which will result in lower cost of adjustment and swifter adaption. They may be more sensitive to notice market demand changes, too, taking the advantage of shallower management hierarchy and shorter decision chain. They are more prone to order losses and thus more conscious to such changes. They may operate in a more precautionary way for the same reason. Small firms can concentrate in more cyclical industries. In less cyclical industries they can be the marginal suppliers who buffer production capacities for large peers. During demand expansion periods, large manufacturers contract some orders out to small ones. When the market demand weakens, large firms will service all orders on their own and the order loss of small producers has nowhere to recover.

Hence a single classifier of size in a monetary problem may be telling too many stories all at once. A few employment studies have addressed the possibilities of expanding the classifier dimension by using size and age combinedly, which we are going to cover in the employment section. Unconditionally, a bivariate classifier of size and age can more unambiguously separate firms into ones susceptible to monetary shocks and ones that are not as much.

But still they fall secondary to the effects of being in a specific industry. However, industry is a rather dynamic standard, compared to firm size and firm age. New industries emerge out of existing ones when the market for the sub-kind of production activities grows. Existing industries can decline and get obsolete when the product is replaced by more advanced substitutes. The process takes place in the matter of years, which is not tremendously longer than a typical life span of a company.

If one attempts to abstract from specific industries, he cannot miss the industry maturity. Regardless of which industry a firm is in, its industry's maturity can substantially regulate the firm's financial management priority and policy, and how the financial sector interprets its balance sheet condition and production plans. Therefore, a tri-variate classifier of industry maturity, firm size, firm age is a strong candidate to a minimal classifier of firm's sensitivity to monetary policy shocks, which deserves investigation in the future studies.

Monetary transmission mechanisms aside, Monetary impacts on firms of different sizes have its own practical merits. It, along with firm age and possibly industry maturity, is easily and inexpensively observable to monetary authorities amongst a long array of firms' characteristics. Should monetary policies affect them considerably differently, which is most likely true, the authorities can more accurately predict policy outcomes using the firm cohort composition, and potentially design policy bundles to offset undesirable side effects. After all, small producers are often tragically and heroically portrayed in the common narrative. They are underrated, barely have external support and are second to everyone, yet despite hardship they persevere and prosper, sustain the economy's metabolism of job creation and innovation. They are individually weak and vulnerable to all sorts of adversity, often held back by top players and ignored by capital, but collectively contains and contribute vastly to the community. Is it true and what kind of role indeed do SME's play under the monetary context? Let us revisit the question in the welfare section.

Investment Intensity, Capital Intensity (Including Working Capital)

Since medium and large firm's investment projects are often externally financed, including very short-term investment like buying inventories, attention was paid to firms that ran at low and high investment intensity and capital intensity (including working capital whose majority variation attributes to variation in inventories). All three previously mentioned dedicated industry heterogeneity studies addressed the relations between such industrial characteristics and their output and price sensitivities to unexpected monetary policy shocks. In their study of West German (the Federal Republic of Germany) manufacturing and mining industries, Hayo and Uhlenbrock [HU00] found pairwise positive correlation of output sensitivity with two alternative measures of capital intensity (capital stock ratio, capital to labor ratio) and with two alternative measures of investment intensity (investment to labor ratio, investment to value added ratio). Without oversimplifying[20] neither investment intensity nor capital intensity significantly explained output sensitivity differences. According to Dedola and Lippi [DL05] based on pooled multiple-country sample, industries with more working capital (working capital to total asset ratio) appeared to adjust output by more in reaction to monetary policy shocks. But in the study of phase asymmetry also based on pooled multiple-country sample, Peersman and Smets [PS05] reached the contradictory conclusion that investment intensity (investment to value added ratio) and working capital (working capital to valued added ratio) had no systematic explanatory power in either average output response sensitivity or response sensitivity cyclical phase differential between in recession and in expansion.

[20]The study used eight dimensions to discretely quantify an industry's reaction to monetary policy shocks. They are reaction direction, magnitude (max deviation from normal), volatility (some oscillate between positive and negative), and duration (time to max deviation and time to normal) of real output and of product price. For simplicity in description one may use high sensitivity to represent large magnitude and long duration.

Product Durability and Price Rigidity

Product durability and price rigidity are the likely proxies for classifiers that separate firms vulnerable to the interest rate channel from those not. They are covered by studies verifying the interest rate channel (and sometimes the money channel) effects. In fact for various reasons price rigidity is implied by product durability, which Gwin and VanHoose [GV12] explicated theoretically and empirically. As for their explanatory power in the industry-level response differences, Dedola and Lippi [DL05] found durable goods producers experienced larger and longer adjustments in their output after monetary policy shocks, in accordance to conventional wisdom and what the interest rate channel would predict. Peersman and Smets [PS05] further confirmed with the result that product durability is the only relevant factor in this issue among many other factors like financial strength, firm size, capital intensity, investment intensity, and dependence on trade.

Location (Including Trade Dependence)

The last factor that received notable amount of research is location. It involves geological boundaries and associated development policies, social policies, and international policies. As previously mentioned with strong nonmonetary discriminating economic policies absent Within a country, industry composition explained most of regional differences in reaction to a common monetary policy shock ([AV04] [HU00]). Internationally, industry composition accounted for a majority of country differences in reaction to a comparable monetary policy shock and among the large developed economies, country has no significant influence on patterns of each industry's reactions ([DL05]).

Hayo and Uhlenbrock's study on West German manufacturing and mining sector [HU00] added an interesting case of the impact of sustained industry policies. Subsidies and export were two influential factors in West Germany during the sample period. German government subsidized some manufacturing and mining industries considerably to support their development and export. Subsidy and export measures unsurprisingly had strong explanatory power over both industrial output and price pattern differences. Those relations were in the direction consistent with conventional wisdom. The more heavily subsidized industries, and ones more involved in export, were the least affected industries by contractionary monetary shocks. The rest measures examined seemed to be overwhelmed by subsidy and export participation. They had no explanatory power but some appeared significantly pair correlated with the pattern differentials. In particular, investment intensity measures were insignificant, which falsifies some theories, at least in a heavily subsidized economy. When the sample was extended to other countries by Peersman and Smets [PS05], the significance of trade involvement disappeared.

Concluding Remarks on Patterns by Firm Type

On the margin, whether it's industry, or firm size, or leverage ratio, that determines the more sensitive group of firms, does not permit a single-factor answer. When we dig into the diversity of firms, all evidences lead to the conclusion that only a multi-dimensional classification can yield accurate and robust predictions of monetary shock effects. This is true for output. This is true for investment, employment, and any factor utilization as well. There are channels of the first order magnitude, and there are channels of the secondary. And which mechanism is on the first will vary with the class in which a firm is, in the multi-dimensional classification system. And of any economic shocks, all monetary transmission channels are secondary to demand and price shocks originating from noneconomic changes. That sets how we should apply the empirical results here and henceforward to any more general, more inclusive macroeconomic model.

1.6 Investment Pattern Differentials

Regularities of investment is not a primary concern in the monetary transmission mechanism evidences. Nevertheless they have been well documented using economy-wide aggregate data. Moreover, the relations between investment and firm's financial situation, financing policy and financial management, profitability and business outlook, stock prices, dividend policy and related factors, external credit supply, tax and bankruptcy costs, have even richer and more extensive evidences under the corporate finance theme. We shall only refer to the few studies under the monetary transmission mechanism theme using industry- or firm-level data here, to show that monetary policy shock influences investment in a way shadowing how it influences future output.

1.7 Employment Pattern Differentials

The numerous amount of empirical studies did not conclude the question of job change dynamics. The tide of such studies emerged from theories that mobility across occupations and industries can absorb economic shocks.

The connection between employment and monetary policy is not as clear as production. Monetary shocks may be an indirect factor to firms' employment consideration, like any other non-labor-market shocks that would affect the production process in general.

Skill specialty, or transferability determines which positions firms are most willing to create and destroy under financial distress. Project cancellation is another way that monetary shocks may affect employment. In this case, a firm cut off a whole project team or a whole division. Some staff can stay and transfer to other positions in the firm. The rest majority will have to find new jobs or leave the labor force. How that will affect the overall employment, for example, of an industry is different.

1.8 Distributional Effects

Large firms have higher productivity on average, if it is defined to be output per unit of labor use. But do they produce more efficiently than small competitors, or do they appear so because they enjoy more market power, is a long-standing doubt. The same debate applies to small firms, too. Small firms, and startups in particular, are the engine of job creation and employment growth. But small firms are on the risky margin of businesses, too. They fail and exit multiple times more than large ones. It is questionable whether resources helping building new firms promote overall employment.

These questions bring us back to the idea of welfare in the traditional complete market economics. In the frictionless market economy, production efficiency is welfare. Production equates consumption and whatever being produced will enter the end-consumer utility. Distribution is costless and information transmission is costless. But the simplicity does not extend to friction markets.

With incomplete and asymmetric information, distribution and information transmission are all costly. Sales can be a major issue blocking every chain from external financing to goods market signaling. Ownership structure affects production efficiency. Financial structure affects efficiency. Firm size affects efficiency not only by determining the technology options (economy to scale, economy to scope, etc.) but also by determining factor and good markets pricing and signaling. The variation of firm size alone, represents an array of production-sale aspects that factor into the efficiency consideration.

The complexity behind each observable attribute of firms implies the complex influence of even a single-target policy. For example, Upon an unexpected marginal change in the Federal funds rate, durable goods producers' demand is likely to reduce after the demand decline ripples off from fast pace consumer goods. But the majority of durable goods producers operates on high fixed capital intensity and often high working capital (which usually is large amount of inventories financed by short-term credit). Their high capital intensity and reliance on external credit makes their production the first vulnerable kind in the economy to even the most minimal contractionary shock. The factors operate in different directions and their compound effects become much less predictable.

The distribution effect of monetary policies does not purely redistribute income and wealth. If the resource transfer systematically benefit one kind of firms that are marginally more (or less) responsive in production, job creation, investment, or innovation, the distribution mechanism will amplify (or quieten) the first order effects of the policy shocks at the aggregate level. According to Auclert [Auc19], monetary policies redistributed income via wage, land rent, and capital profit, the earnings heterogeneity channel, via holding of nominal currency, the Fisher channel, and via nominal credit and liability position, the interest rate channel. And through all three channels expansionary policies benefit agents with higher marginal propensity to consume, they boost the real economy output. Thus the redistribution channels themselves are how monetary policies affect the aggregate real activities.

Arnold [Arn00] made a point on the welfare aspect of monetary shock response heterogeneity. (The empirical part then attempts to test for the hypothesis.) Positive correlation between wage and profit at industry level is well documented in the labor literature. The relation is very robust across time, region, and with or without considering fringe benefits, factors like working condition, social status, job stability, etc. Although the reason why this industry-level wage differential is persistent is yet to study, labor does share a part of the overall return with capital. As for stock prices, it is also well known that industry is an important explanatory factor of beta, and hence the stock returns. Based on these two facts, the author argue that the private sector may have factored heterogeneous risks caused by monetary shocks into capital and labor gains. If an industry's profit is more sensitive to monetary policy shocks, the industry will also pay higher average return to labor and capital owners. If so, and if regional heterogeneous sensitivity to monetary shocks is mainly a consequence of industry composition and industry heterogeneity, then the monetary authority may not need to worry about policies causing inter-region distribution effects.

Chapter 2

Multi-Period Cost Function in Modelling Dynamic Production with Flexible Functional Forms

There is not a broadly applicable generalization of flexible functional form to model intertemporal dynamics. Flexible functional forms are essentially truncated local or global functional expansions to approximate economic functions of interest. In particular, they are designed to model single-period demand systems, profit functions, and/or cost functions without considering forward looking behavior or multi-period planning. Nevertheless there are a few models incorporating various degrees of dynamics to address particular questions. The common practice is to estimate a suitable functional form approximation, and then evaluate its local values sequentially to represent values at different time points. However, when the functional form was evaluated, intertemporal connection among economic variables is not accounted for. The resulted estimation thus carry no information about such connections. And then the seemingly sequential inference can easily lose the intertemporal relations and the two-step model build is subjected to dynamic misspecification.

To resolve that potentially devastating danger, we are going to develop a framework to overcome the internal modelling inconsistency.

2.1 Capital Formation Process

We incorporate a capital formation process in the static form, and derive a multi-period cost function that models the present and planned future production. Let k_t be the amount of productive capital goods at the producers' disposal in period t. Let I_t be the investment made during the same period. Investment I and its installation add up to a total expenditure $G(I)$ to the firm, which is a down payment subtracted from the current period total revenue. Let p_I be the price index of investment. Then at period t, investment increases the same period total cost by

$$G_t(I_t) = p_{I,t} I_t. \tag{2.1.1}$$

And I_t more productive capital will be available to the producer at the beginning of period $t + 1$. Suppose capital goods depreciates at the rate δ over each period, due to the wear and outdating of equipments and facilities. Then the capital formation process is

$$k_{t+1} = I_t + (1 - \delta)k_t. \tag{2.1.2}$$

When a forward looking producer plans production over time horizon $1, \ldots, T$, it could derive the expected total discounted one-period cost from the cost minimization problem. The capital formation process will entire the problem as a set of constraints.

2.2 Cost Function with Capital Dynamics

Now we fully specify a producer's cost minimization problem when it plans forward over multiple periods. Then we derive properties of the multi-period cost function, which is the value function of the cost minimization problem.

2.2.1 Cost Minimization Over Multiple Periods

Suppose the cost minimizing firm considers production plan over current $t = 1$ and multiple future periods $t = 2, \ldots, T$. We first consider the case where the producer has complete knowledge over the entire planning horizon.

Let $y = f(\boldsymbol{x})$ be the production function, and $F(\boldsymbol{x}, y) = y - f(\boldsymbol{x}) \leq 0$ be production frontier. The producer makes investment I_t that will become productive capital immediately in period $t + 1$, at real installation cost $G_t(I_t)$. Without loss of generality, we let the first input x_1 be capital service, $\boldsymbol{x}_{-1}$ be the input vector with x_1 removed from $\boldsymbol{x}$. Subscripts of $\boldsymbol{p}_x, \boldsymbol{p}_{x_{-1}}, p_{x_1}$ have the same meanings. That notation is henceforth used throughout the subsection. Then the producer's cost minimization problem[1] over planning period is

$$\begin{aligned} \min_{\{\boldsymbol{x}_t\}_{t=1}^T, \{I_t\}_{t=1}^{T-1}} \sum_{t=1}^{T} & \beta^{t-1}(\boldsymbol{p}'_{x,t}\boldsymbol{x}_t + G_t(I_t) - I_t) + B_T \\ \text{s.t.} \quad & F(\boldsymbol{x}_t, y_t) \leq 0, \quad \forall t = 1, \ldots, T, \\ & x_{1,t+1} = I_t + (1 - \delta)x_{1,t}, \quad \forall t = 1, \ldots, T - 1, \end{aligned} \tag{2.2.3}$$

where $\boldsymbol{p}_x$ is the price of inputs, B_T is a terminal cost[2]. Now we look at the two types of constraints on the minimization problem. The equality constraints can be substituted into the objective function. And after substitution, the control variables $\{x_{1,t}\}_{t=1}^T$ will be replaced by $x_{1,1}, \{I_t\}_{t=1}^{T-1}$. Only the first period capital service remains and all planned future period capital service values are replaced by the planned investment.

[1]Here I_T is not one of the control variables. A production plan up to period T need not decide the investment plan in period T. However, we still write the summation in the objective function from 1 to T for simplicity. We can assume that $I_T \equiv 0$ regardless values of other variables. The same simplicity treatment will be used later in the exhibition of multi-period cost functions.

[2]Possibly the opposite number of the profit maximization problem terminal value, and independent from all control variables.

2.2.2 Multi-Period Cost Function With Perfect Information

Like the cost function of a one-period production plan, our multi-period cost function is the value function of problem (2.2.3). That is

$$c(\cdot) = \min_{\{\boldsymbol{x}_t\}_{t=1}^T, \{I_t\}_{t=1}^{T-1}} \left\{ \sum_{t=1}^{T} \beta^{t-1} (\boldsymbol{p}'_{x,t} \boldsymbol{x}_t + G_t(I_t) - I_t) + B_T \right. \\ \left. \middle| \; F(\boldsymbol{x}_t, y_t) \leq 0, x_{1,t+1} = I_t + (1-\delta) x_{1,t}, \forall t \right\}, \tag{2.2.4}$$

which is a function of $\{\boldsymbol{p}_{x,t}, y_t\}_{t=1}^T$, with predetermined parameters such as β and δ. Note that we can make two observations. First, the original objective function is linear in $\{x_{1,t}\}_{t=1}^T$. And $\{x_{1,t}\}_{t=1}^T$ are linear in $x_{1,1}, \{I_t\}_{t=1}^{T-1}$. Second, $G_t(I_t)$ is linear in I_t if we use a pair of exact price and quantity indices of investment. That is using exact price measure p_I and quantity I, $G_t(I_t) = p_{I,t} I_t$ by definition. Combining the two observations, we know that the objective function after substitution, is linear in $x_{1,1}, \{I_t\}_{t=1}^{T-1}$.

To establish the linearity, we substitute all the equality constraints into the objective function. First note that

$$\begin{aligned} x_{1,t} &= I_{t-1} + (1-\delta) x_{1,t-1} \\ &= I_{t-1} + (1-\delta) I_{t-2} + (1-\delta)^2 x_{1,t-1} \\ &= \cdots \\ &= \sum_{k=1}^{t-1} (1-\delta)^{t-1-k} + (1-\delta)^{t-1} x_{1,1}. \end{aligned} \tag{2.2.5}$$

Then using Equation (2.1.1) and (2.2.5), we have a linear[3] objective function

$$
\begin{aligned}
&\sum_{t=1}^{T}\beta^{t-1}(\boldsymbol{p}'_{x,t}\boldsymbol{x}_t + G_t(I_t) - I_t) + B_T \\
&= \sum_{t=1}^{T}\beta^{t-1}(p_{1,t}x_{1,t} + p_{I,t}I_t - I_t) + \sum_{t=1}^{T}\beta^{t-1}\boldsymbol{p}'_{x_{-1},t}\boldsymbol{x}_{-1,t} + B_T \\
&= p_{1,1}x_{1,1} + \sum_{t=2}^{T}\beta^{t-1}p_{1,t}x_{1,t} + \sum_{t=1}^{T}\beta^{t-1}(p_{I,t}-1)I_t \\
&\quad + \sum_{t=1}^{T}\beta^{t-1}\boldsymbol{p}'_{x_{-1},t}\boldsymbol{x}_{-1,t} + B_T \\
&= p_{1,1}x_{1,1} + \sum_{t=2}^{T}\beta^{t-1}p_{1,t}\left(\sum_{k=1}^{t-1}(1-\delta)^{t-1-k}I_k + (1-\delta)^{t-1}x_{1,1}\right) \\
&\quad + \sum_{t=1}^{T}\beta^{t-1}(p_{I,t}-1)I_t + \sum_{t=1}^{T}\beta^{t-1}\boldsymbol{p}'_{x_{-1},t}\boldsymbol{x}_{-1,t} + B_T
\end{aligned}
$$

Rearrange terms and swap the double summation, and then we have

$$
\begin{aligned}
&p_{1,1}x_{1,1} + \sum_{t=2}^{T}\beta^{t-1}p_{1,t}(1-\delta)^{t-1}x_{1,1} + \sum_{t=2}^{T}\sum_{k=1}^{t-1}\beta^{t-1}p_{1,t}(1-\delta)^{t-1-k}I_k \\
&\quad + \sum_{t=1}^{T}\beta^{t-1}(p_{I,t}-1)I_t + \sum_{t=1}^{T}\beta^{t-1}\boldsymbol{p}'_{x_{-1},t}\boldsymbol{x}_{-1,t} + B_T \\
&= \left(p_{1,1} + \sum_{t=2}^{T}\beta^{t-1}(1-\delta)^{t-1}p_{1,t}\right)x_{1,1} \\
&\quad + \sum_{t=1}^{T-1}\left(\beta^{t-1}(p_{I,t}-1) + \sum_{k=t+1}^{T}\beta^{k-1}(1-\delta)^{k-t-1}p_{1,k}\right)I_t \\
&\quad + \sum_{t=1}^{T}\beta^{t-1}\boldsymbol{p}'_{x_{-1},t}\boldsymbol{x}_{-1,t} + B_T.
\end{aligned}
\tag{2.2.6}
$$

Suffice it to show that the T-period cost minimization problem (2.2.3) has the same structure in $x_{1,1}, \{I_t\}_{t=1}^{T-1}, \{\boldsymbol{x}_{-1,t}\}_{t=1}^{T}$ as the 1-period cost minimization problem in $\boldsymbol{x}$.

[3] In $x_{1,1}, \{I_t\}_{t=1}^{T-1}, \{\boldsymbol{x}_{-1,t}\}_{t=1}^{T}$.

The linearity in the objective function implies that the multi-period cost function, as the value function of the minimization problem in $\boldsymbol{x}$ and I preserves all properties of a one-period cost function. It is dual to a multi-period profit maximization problem. The Hotelling-Shephard's Lemma holds. It is linear homogeneous in input prices. It is concave in input prices as a result of the second order conditions for minimization. Only that the "prices" associated with $x_{1,1}$ and $\{I_t\}_{t=1}^{T-1}$ are not their own price indices, but a value depending on price indices, discount rate, and capital goods depreciation rate. A rigorous proof will follow the same line of induction as [She70, M$^+$78, Die82].

To be exact, let $\boldsymbol{\eta}$ be the counterpart in a T-period cost function as of input prices $\boldsymbol{p}$ in a 1-period cost function. Elements in $\boldsymbol{\eta}$ are

$$
\begin{aligned}
&(\text{For } x_{1,1}) && \eta_1 = p_{1,1} + \sum_{t=2}^{T} \beta^{t-1}(1-\delta)^{t-1} p_{1,t}, \\
&(\text{For } I_1) && \eta_2 = p_{I,1} - 1 + \sum_{k=2}^{T} \beta^{k-1}(1-\delta)^{k-2} p_{1,k}, \\
&\dots && \dots \\
&(\text{For } I_{T-2}) && \eta_{T-1} = \beta^{T-3}(p_{I,T-2} - 1) + \sum_{k=T-1}^{T} \beta^{k-1}(1-\delta)^{k-T} p_{1,k}, \\
&(\text{For } I_{T-1}) && \eta_T = \beta^{T-2}(p_{I,T-1} - 1) + \beta^{T-1} p_{1,T}, \\
&(\text{For } x_{2,1}) && \eta_{T+1} = p_{2,1}, \\
&(\text{For } x_{2,2}) && \eta_{T+2} = p_{2,2}, \\
&\dots && \dots \\
&(\text{For } x_{2,T}) && \eta_{2T} = p_{2,T}, \\
&(\text{For } x_{3,1}) && \eta_{2T+1} = p_{3,1}, \\
&\dots && \dots \\
&(\text{For } x_{2,T}) && \eta_{3T} = p_{3,T}, \\
&\dots && \dots \\
&(\text{For } x_{n,T}) && \eta_{nT} = p_{n,T}.
\end{aligned}
\tag{2.2.7}
$$

Here we defer a full presentation of the multi-period cost function and simply conclude that it takes arguments as such

$$c(\boldsymbol{\eta}, y_1, \dots, y_T), \tag{2.2.8}$$

which has all properties in $\boldsymbol{\eta}$ as of properties of a one-period cost function $c^*(\boldsymbol{p}, y)$ in $\boldsymbol{p}$.

2.2.3 Multi-Period Cost Function With Risk

Therefore the flexible functional form approximation of a one-period cost function is generalized to a multi-period cost function without structural change, only in a larger scale. If a one-period cost function has n inputs, then its T-period flexible functional form generalization has nT inputs. And all the y in the one-period cost is substituted by $\sum_{t=1}^{T} y_t$ in the T-period cost, where $\sum_{t=2}^{T} y_t$ are planned future output levels.

The above structure and hence conclusions, apply as well in the case with uncertainty, where the future information and objectives are under conditional expectations. This is also a consequence of linearity, the linearity of expectation integral. Of course, additional regularity of input prices and the production function is required for generalization to the uncertainty case. We will discuss those additional regularity conditions and their implications subsequently.

2.3 Scope and Restriction

2.3.1 Weakly Separable Production Function

Our entire construction of the multi-period cost function requires the existence of aggregate functions of every kind of production inputs.[4] Equivalently, the production function is required to be weakly separable in all input categories, and weakly separable intertemporally. Let $F_{\text{total},T}(\{\boldsymbol{x}_t\}_{t=1}^{T}, \{y_t\}_{t=1}^{T}) \leq 0$ be the production function[5] spanning from planning period 1 to T. Then by definition,

$$F_{\text{total},T}(\{\boldsymbol{x}_t\}_{t=1}^{T}, \{y_t\}_{t=1}^{T}) = \sum_{t=1}^{T} F(\boldsymbol{x}_t, y_t), \tag{2.3.9}$$

which means intertemporal separability automatically holds. From definition (2.2.8) with price like arguments η taking form (2.3.9), we know that $F_{\text{total},T}(\cdot)$ will be weakly separable in all input

[4] The same set of conditions are required when we express the 1-period cost function as a function of all categorical input aggregates.

[5] Strict inequality represents cases where inputs are inefficiently used.

categories if and only if $F(\boldsymbol{x}, y)$ is weakly separable in all input categories. Details of weak separability condition are beyond our focus, we refer to [Bar87] for its definition and grounds in reality. It is worth noting that weakly separable production technology brings us a convenient equivalency. Both the profit maximization and cost minimization problems of the producer can be solved via two-stage budgeting. Take the cost minimization problem for example. In two stage budgeting, the producer can decide the optimal expenditure allocation to each kind of inputs at the first stage and then solve independent sub-cost minimization problems for optimal inputs in each category. The result would be the same as the solution to the original minimization problem. For rigorous definition, see [Bar00]. This behavior of the cost minimization problem will be extensively used in simplifying some of our empirical models.[6]

2.3.2 Time-Variant Discount Rate

Many time invariant variables in our model and derivation can be generalized to more realistic time variant cases. For the time variant discount rate case, all β^{t-1} in $\boldsymbol{\eta}$ expressions (2.2.7) are replaced by

$$\prod_{s=1}^{t-1} \beta_s. \tag{2.3.10}$$

Subjective discount rate is closely related to the rate of return paid on benchmark asset. A financial asset is the benchmark asset if it provides preservation and accumulation of wealth, forgoing all monetary services. The only benefit of holding benchmark assets is earning yield it pays over time. Let ρ_t be the return on equity (ROE) in period t, which is by definition the quotient of net income divided by equity. Then for all t. Then it can be an approximate to the yield of industry-wide or firm-wide benchmark assets

$$\beta_t = \frac{1}{1 + \rho_t}. \tag{2.3.11}$$

[6]We are going to revisit this property in the estimation method section, with more details present.

2.3.3 Time-Variant Depreciation Rate

For the time variant capital depreciation rate case, all $(1-\delta)^{t-1}$ in $\boldsymbol{\eta}$ expressions (2.2.7) are replaced by

$$\prod_{s=1}^{t-1}(1-\delta_s). \tag{2.3.12}$$

2.3.4 Competitive Rental Market for Capital Goods

The model requires existence of a competitive capital rental market for all industries examined. Capital input in the producer's cost function are capital services employed to produce outputs in each period. Thus capital input in each period can be chosen independently from one another, and coexist in the multi-period cost. By this "accounting" standard, production firms not really own and hold capital goods. They rent capital from a competitive market, pay rental prices, and utilize a part of the rental equipments and facilities in a perishable fashion. After production, all market value of the rental capital services consolidates to part of the output. And then a new set of capital will be rented in the next round of production.

This is reasonable in that businesses in continual operation are able to fully use the productive value of their installed capital goods. They spread out the expenditure of installation, maintenance, and upgrade over the equipments' life in book keeping. It is even true for industries like construction, whose most equipments are rental from the machinery manufacturers. In cases when businesses fail, depending on industry, more or less of the capital goods owned by those business can be recycled and sold to competitors. Sales and redeployment of depreciated specialized capital goods can take place in years and at greatly discounted prices, according to Ramey and Shapiro's study [RS01] based on aerospace manufacturers with equipment level data. But even some extremely specialized equipments can be redeployed in other industries, not surprisingly, sold at greater discount. These instances will be reflected as large depreciation rate in our data. As long as the industry is not in fast decline, we could assume the hypothetical rental capital system.

2.3.5 Case with Risk

To generalize formula (2.2.8) and (2.2.7) with all future prices p, future discount rates β and future depreciation rates δ replaced by expected values respectively, we need interchangeability of differentiation (of the total cost, the objective function of cost minimization problem) and integral (of the mathematic expectation). Since the objective function is fully affine in control variables, this condition is automatically satisfied.[7]

[7] Were the objective function is not affine, for instance in the expected utility maximization problem of risk averse investors, the interchangeability would require a complete market of all financial assets (the control variables in the maximization problem), or equivalently, existence of a stochastic discount factor.

Chapter 3

Industry-level Differentials in Demand for Money: Reassess Supply Side Heterogeneity in Monetary Policy Effects

We've seen that vector autoregression is a predominant way researchers use to evaluate policy effects. The method has its strengths and limits. VAR models approximate non-ergodic systems locally in a simple and effective way. The economy as a whole in the short-run can be seen as such kind of a system. Thus we'd be able to learn about its short-run dynamics using estimated VAR models, which is what VAR's are good at depicting.

Yet there are undesirable features of VAR in such applications. VAR models are sensitive to the ordering of variables. When estimated with the same sample, different variable orderings can yield different estimations. And that can make the estimated models behave fundamentally differently, which makes the model somewhat arbitrary. In the studies we reviewed, this drawback was partially resolved by validating the variable ordering by fitting the model to aggregate values. If the model fitted to the entire economy seemed to be right, it was likely to be right for each separate industries alone, more or less.

The big issue with those models is in the step applying industry-level data. In the industry-wise

VAR's, the aggregate output (or investment or any variable of primary interest) was replaced by the industrial output series and all other specifications of the models are left unchanged. It essentially took the industry output as the all the output in the economic system. The practice created two problems. The models were misspecified and the system defined by the model was incomplete (some dynamics were going on outside the model). Both problems would undermine the validity of the estimation and conclusions drawn thence.

People did this perhaps because the industry-level price index, employment, interest rate and monetary aggregates were not available at their time. So VAR with unmatched series was the best approximate they could get. And those models did reveal some patterns in accordance with conventional wisdom. But in today's standard, that can be substantially improved. And any inaccuracy due to that kind of model misspecification can be eliminated. But even with the disaggregated data, the second issue still creates error and bias. A VAR modelling one industry is only a slice of the economy. It takes the industry to be self-sufficient and ignores its interaction with other parts of the economy.

Since most, if not all, of such studies use the same family of models. We may well try to look at the question from a different angle. That will possibly help to resolve some conflicts among findings from previous studies. Instead of trying to model the entire economy, or model a slice of an economy, we model the characteristics of each industry, their production functions. And then we can evaluate how monetary policy shocks impact them differently. To connect policy shocks with production functions and firms' profit maximizing behavior, we introduce the concept of money in production.

3.1 Structural Model: Money in Production

Here we define the exact use of money in firm's production activities. We assume that firms hold monetary assets as one kind of production factor, to utilize the monetary services those assets provide. Monetary assets include money (or legal tender, equivalently) and goods or financial contracts that are considered to be close substitutes to money. Monetary services are the basic functions provided by monetary assets. Especially, we refer to the medium of exchange, or means of payments, the very high level of liquidity. Our notion of monetary asset is close to the same term in accounting. It is reported as balances in cash, in deposits, in certificate of deposits, etc. Assets from these accounts provide monetary services to firms' economic activities.

Like labor, capital, raw materials and energy, monetary service plays a role that can be partially substituted, but not be fully dispensed, in the production activities. It is one of the production factors, one of the inputs, in the production function we are going to model. We emphasize here, it is the monetary service provided by monetary assets, that enters production. To measure the service, we use functional monetary aggregates, like Divisia Index, quantifying the amount of this input. Sinai and Stokes [SS89] addressed theoretical motivation to model monetary asset holdings in the production in their survey of the real balance in the production function literature. From the empirical modelling perspective, they also summarized evidences supporting that aggregate production functions only with non-monetary inputs were misspecified.

We define the production function of an industry in analogy to that of a firm. The industry as a whole, buys factors from competitive markets, and produces output goods and services within limits of the available technology. The input factors comprise capital goods, labor, material (or intermediate goods), energy, service, and monetary service. The same strategy was used by Barnett, Kirova, Pasupathy [BKP95] in their innovative study of credit creation by modelling monetary service production function of financial firms and demand for monetary assets by nonfinancial manufacturing firms, and by Serletis, Isakin [SI+18] in their contemporary study of nonconventional monetary policy instruments' and new financial regulation's effects on the U.S. financial intermediaries.

After we estimate the industrial production function, we would be able to quantify the industry's response to monetary shocks in a way comparable to results from VAR models. Under a monetary shock, interest rate will change and hence will the price of monetary service. Change in price of one factor will change the quantities of all factors employed, with adjustments in the output level. Therefore we can quantify the industry-level responses and compare them, too.

Our approach is good in that it separate the effects like the VAR models do. It is invulnerable to errors out of modelling an incomplete system and pretending it's complete. It bypasses the complexity in capital structure adjustments, which some other static-point-of-view regression analysis are prone to. But it only captures the long-run relations, with annual data. It will be very inaccurate predicting short-turn reactions under monetary policy shocks.

3.1.1 Duality Theory

In practice, we don't directly estimate the production function. Estimating production function from observed data requires us to estimate the supply function first, and then retrieve the production function using the profit maximizing conditions. This method is subject to noises from the demand, and impractical. Instead, we make use of the duality theory to attain the information carried by the production function indirectly.

According to the duality theory established by [She53], the cost function derived from profit maximizing completely includes the technology information in the production function. Knowing the cost function is equivalent to knowing the production function. Modelling the industry's cost function is sufficient to capture the properties we want, about how the price of monetary services determines the production plans.

3.1.2 Flexible Functional Form and Factor Demand Equations

The empirical tool we use to model the industrial cost functions is the flexible functional form. Flexible functional forms are widely used to model consumer demand and producer production demand. It can model any kind of price elasticity combination in a demand system. Compared to other fixed form model families like CES, the flexibility comes at the cost of using far more parameters.

Flexible functional form is a function that approximates a target function to the second order derivatives. When it is estimated using a finite sample generated from the target function, the flexible functional form approximates at least in the region spanned by the sample points. In our study, the unknown sectoral cost function in production $\tilde{c}(\boldsymbol{p}, y, t)$ is the target function. Let $c(\boldsymbol{p}, y, t)$ be the flexible functional form. By definition,

$$c(\boldsymbol{p}, y, t) = \tilde{c}(\boldsymbol{p}, y, t), \tag{3.1.1a}$$

$$\boldsymbol{\nabla} c(\boldsymbol{p}, y, t) = \boldsymbol{\nabla} \tilde{c}(\boldsymbol{p}, y, t), \tag{3.1.1b}$$

$$\nabla^2 c(\boldsymbol{p}, y, t) = \nabla^2 \tilde{c}(\boldsymbol{p}, y, t), \tag{3.1.1c}$$

where y is the output level, $\boldsymbol{p}$ is the input price vector, and t is the parameter for technology. [1]

Flexible functional forms can be specified using many function families. The candidates differ mainly in their flexibility region and microeconomic regularity region. Studies have provided some insights on picking the suitable form for modelling application by making extensive comparison among forms. Griffin, Montgomery, Rister [GMR87] provided a guideline for integrating form selection into model building process. Their study covered most off-the-shelf nonflexible forms and many locally flexible forms. But globally flexible forms are not juxtaposed since none of them were posed yet. Kim [Kim05] compared form consistency with aggregation, that when firm-level production functions are aggregated, how each form of the representative agent's (e.g. industry's) production is consistent with the aggregation theory. Feng, Serletis [FS08] compared economic regularity and econometric regularity of forms in the economy production function context. They showed that locally imposing curvature conditions does not assure theoretical regularity. Serletis, Feng [SF15] compared regularity imposition methods (single point, in a neighborhood of a single point, point-wise on all sample points, and globally) and their impact on model flexibility. From single point to globally, the maintained regularity conditions get stronger and destroy forms' flexibility more and more. Imposing global curvature condition to a locally flexible form

[1] We use t to represent technology and time. Hopefully this variable overloading won't cause confusion, for the technology variable won't appear in our future estimations. Indeed time is one candidate of a technology index. And technological improvement would result in time variant production transformation and hence cost functions.

completely removes its flexibility and the estimates can be very biased. Kenkel, Signorino [KS13] compared variable selection methods (LASSO, adaptive LASSO, SCAD) in their merits of the oracle property and selecting the specification closeness to the true DGP. But they abstracted out all economic and econometric regularity aspects of the model family. Diewert, Wales [DW92] proposed a semi-parametric way to further reduce number of parameters in the NQ form without loss of flexibility by building linear or quadratic spline functions into the flexible functional form. The form concerning the splined variables would have local flexibility to the second order, while its global regularity could be maintained. A few subsequent studies applied and improved the technique. Hussian, Bernard [HB16] evaluated Canadian and the U.S. manufacturing industries' production functions, to compare performances of the Translog, the Generalized Leontief, and the Normalized Quadratic functional forms. Although the first two of their comparison subjects are locally flexible, NQ standed out, too, like what we find from our model comparison.

The plenty and heterogeneous subjects is central in shaping our modelling strategy and form pick. We choose three most flexible ones with regularities dictated by production theories being maintained. Those are Generalized Barnett (GB)[2], Generalized McFadden (GM)[3], and Normalized Quadratic (NQ). Generally speaking, they are all flexible enough to model arbitrary production functions, while have good properties suiting the underlying economic theory.

We further assume that there is no technological progress during the sample period. That is both $\tilde{c}$ and c are only functions of $\boldsymbol{p}$ and y. Unrealistic as the assumption may sound, it is not speculative. As production technology improves, scale of outputs and use of inputs also grow. The two changes almost always concur such that we cannot tell one from another from empirical data. When this is reflected in the fitted production function, we cannot tell if the production exhibits technological improvement, or economies of scale. Therefore we may well owe technology effects to economies of scale, or some learning by doing effects, and reduce the number of parameters to estimate.

We don't observe the total cost corresponding to a production plan a firm carries out. It is also

[2] Also called Minflex Laurent on few occasions.
[3] Also called Generalized Quadratic in some literature.

not possible to calculate this theoretical value from observable variables. Although the expenditure paid on all the input items are booked by companies, most of the data are not publicly available and difficult to collect. They are sensitive private information and the number of entities to survey is enormous. So regress the total input cost over a specified function of input prices, output level, and technology index is not feasible.

However, the available observations permit us to estimate factor demand equations. They carry just enough information for us to learn about the cost function. According to Shephard's Lemma, if the cost function is concave, the input demand of a cost minimizing firm will equal to the partial derivative of total cost with respect to the input price. That is

$$\boldsymbol{x}(\boldsymbol{p}, y) = \frac{\partial c(\boldsymbol{p}, y)}{\partial \boldsymbol{p}}, \tag{3.1.2}$$

where $\boldsymbol{x}$ is the input vector.

We are going to estimate the cost function parameters in a flexible functional form by estimating the system of equations

$$\boldsymbol{x}_t = \frac{\partial c(\boldsymbol{p}_t, y_t)}{\partial \boldsymbol{p}_t} + \boldsymbol{u}_t, \quad t = 1, \ldots, T, \tag{3.1.3}$$

where $\boldsymbol{u}_t$ are multivariate random variables representing unaccounted effects and errors. The sample has observation period up to T. The system of equations is also an SUR (seemingly unrelated regression) model. How exactly $\boldsymbol{x}_t$ depends on $\boldsymbol{p}_t, y_t$ is decided by the specific flexible functional form we set for $c(\boldsymbol{p}, y)$. They will be presented later.

3.1.3 Regularity Condition

For equation (3.1.2) to hold, the estimated cost function $c(\boldsymbol{p}, y)$ must be positive everywhere, increasing in input prices $\boldsymbol{p}$, increasing in output level y, and concave in input prices $\boldsymbol{p}$ in at least a reasonable region. These four properties together are called the regularity conditions in this literature. They must hold all together in order for the estimated factor demand equations to be truly derived from a profit maximizing behavior of the producer [Bar02].

Among the regularity conditions, the curvature condition (being concave) is most likely to be violated after estimation, and also the hardest to impose in any estimation methods. In early works, researchers used compromised versions such that the estimated cost functions were concave only at the sample points or even one point. And sometimes the positivity and monotonicity properties were violated in trade for local concavity. We choose the specific forms with the regularity property consideration. The GB form has global regularity by design. The GM form can satisfy the conditions globally with proper constraints imposed. The NQ form does not have globally uniform curvature property. We are going to impose regularity conditions in a region that at least covers the sample points. We are going to present the exact conditions we maintain when we introduce the forms later.

3.1.4 Specific Forms

Generalized Barnett

Diewert, Wales [DW87] extended the Minflex Laurent form to Generalized Barnett, by making the form homogeneous in prices yet still keeping its flexibility (the Laurent-expansion-like structure). They proposed a flexible functional form cost function

$$\begin{aligned} c(\boldsymbol{p}, y, t) = g(\boldsymbol{p})y + \sum_{i=1}^{n} b_{ii} p_i y + \sum_{i=1}^{n} b_i p_i + \\ \sum_{i=1}^{n} b_{it} p_i t y + b_t \left(\sum_{i=1}^{n} p_i \right) t + b_{yy} \left(\sum_{i=1}^{n} p_i \right) y^2 + b_{tt} \left(\sum_{i=1}^{n} p_i \right) t^2 y. \end{aligned} \tag{3.1.4}$$

It is Generalized Barnett form if function g is

$$g(\boldsymbol{p}) = \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} b_{ij} p_i^{\frac{1}{2}} p_j^{\frac{1}{2}} - \sum_{i=2}^{n} \sum_{j=2, j\neq i}^{n} d_{ij} p_1^2 p_i^{-\frac{1}{2}} p_j^{-\frac{1}{2}} - \sum_{i=2}^{n} \sum_{j=2, j\neq i}^{n} e_{ij} p_1^{-\frac{1}{2}} p_i^{-\frac{1}{2}} p_j^2, \tag{3.1.5}$$

where $b_{ij} = b_{ji} \geq 0$, $d_{ij} = d_{ji} \geq 0$, $e_{ij} \geq 0, \forall i, j = 1, \dots, n$. And this form is reasonably flexible. If we add one more term $\sum_{i=1}^{n} b_{ii} p_i$ to $g(\boldsymbol{p})$, then the cost function $c(\boldsymbol{p}, y, t)$ specified as such will be flexible except for that we don't know the flexibility with respect to the "numeraire input", which is input 1 in this specification.

As for regularity, linear homogeneity in $\boldsymbol{p}$ is automatically satisfied. Positivity, monotonicity (increasing in $\boldsymbol{p}$ and y), curvature (concave in a reasonable region) need to be imposed on the parameters. Here we have a sufficient condition for the form to be globally concave on the region where $\boldsymbol{p} \geq \mathbf{0}, y \geq 0, t \geq 0$. If the d and e parameters are non-negative, $d_{ij} \geq 0, e_{ij} \geq 0$, then the form is globally concave. This conclusion follows the fact that the summation of concave functions is concave.

If we drop the technology factor by setting all parameters subscripted t zero, a GB form cost function is

$$\begin{aligned} c(\boldsymbol{p}, y) = & \left(\sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} b_{ij} p_i^{\frac{1}{2}} p_j^{\frac{1}{2}} - \sum_{i=2}^{n} \sum_{j=2, j \neq i}^{n} d_{ij} p_1^2 p_i^{-\frac{1}{2}} p_j^{-\frac{1}{2}} - \sum_{i=2}^{n} \sum_{j=2, j \neq i}^{n} e_{ij} p_1^{-\frac{1}{2}} p_i^{-\frac{1}{2}} p_j^2 \right) y \\ & + \sum_{i=1}^{n} b_{ii} p_i y + \sum_{i=1}^{n} b_i p_i + b_{yy} \left(\sum_{i=1}^{n} p_i \right) y^2, \end{aligned} \tag{3.1.6}$$

where $b_{ij} = b_{ji} \geq 0, d_{ij} = d_{ji} \geq 0, e_{ij} \geq 0$, with other restrictions from regularity conditions applied.

Generalized McFadden

A GM form cost function is

$$\begin{aligned} c(\boldsymbol{p}, y, t) = & \frac{1}{2p_1} \sum_{i=2}^{n} \sum_{j=2}^{n} a_{ij} p_i p_j y + \sum_{i=1}^{n} b_{ii} p_i y + \sum_{i=1}^{n} b_i p_i \\ & + \sum_{i=1}^{n} b_{it} p_i t y + b_t \left(\sum_{i=1}^{n} p_i \right) t + b_{yy} \left(\sum_{i=1}^{n} p_i \right) y^2 + b_{tt} \left(\sum_{i=1}^{n} p_i \right) t^2 y, \end{aligned} \tag{3.1.7}$$

where $a_{ij} = a_{ji}, \forall i, j = 2, \dots, n$. There is no positivity or negativity restrictions on parameters in consequence of the curvature condition. Positivity and monotonicity need to be maintained in estimation. The form is linear homogeneous in input prices $\boldsymbol{p}$ by design. Note that all terms with parameter b are linear in $\boldsymbol{p}$. Thus those parameters will not enter the Hessian with respect to $\boldsymbol{p}$ and therefore will not be restricted by the concavity condition. The form is globally concave if and only if its Hessian with respect to $\boldsymbol{p}$

$$\nabla^2_{\boldsymbol{pp}} c = \frac{y}{p_1} \begin{pmatrix} \frac{1}{p_1^2}\boldsymbol{p}'_{-1} A \boldsymbol{p}_{-1} & -\frac{1}{p_1}\boldsymbol{p}'_{-1} A \\ -\frac{1}{p_1} A \boldsymbol{p}_{-1} & A \end{pmatrix}, \tag{3.1.8}$$

is negative semi-definite. Here $\boldsymbol{p}_{-1} = (p_2, p_3, \ldots, p_n)'$ and matrix $A = \{a_{ij}\}_{(n-1)\times(n-1)}$ (Both row and column indices of a_{ij} range from 2 to n).

Using results from Lau [Lau74], A being negative semi-definite is sufficient and necessary for $\nabla^2_{\boldsymbol{pp}} c$ to be negative semi-definite on the positive $\boldsymbol{p}$ half plain. We assert without showing, that the form is globally concave for all $\boldsymbol{p} > \boldsymbol{0}, y > 0, t > 0$ if and only if parameter matrix A is negative semi-definite.

We restrict $b_{it} = 0, b_t = 0, b_{tt} = 0$ and obtain the form we are going to estimate

$$c(\boldsymbol{p}, y) = \frac{1}{2p_1} \sum_{i=2}^{n} \sum_{j=2}^{n} a_{ij} p_i p_j y + \sum_{i=1}^{n} b_{ii} p_i y + \sum_{i=1}^{n} b_i p_i + b_{yy} \left(\sum_{i=1}^{n} p_i \right) y^2, \tag{3.1.9}$$

where $a_{ij} = a_{ji}, \forall i, j = 2, \ldots, n$, and the same regularity conditions apply.

Normalized Quadratic

The Normalized Quadratic form was originated by Diewert, Wales [DW88]. An NQ cost function is a generalization of the GM form to symmetry

$$\begin{aligned} c(\boldsymbol{p}, y) = & \left(\sum_{i=1}^{n} b_{ii} p_i + \frac{1}{2\sum_{i=1}^{n} \alpha_i p_i} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} p_i p_j \right) y \\ & + \sum_{i=1}^{n} b_i p_i + \sum_{i=1}^{n} b_{it} p_i t y + b_t \left(\sum_{i=1}^{n} p_i \right) t + b_{yy} \left(\sum_{i=1}^{n} p_i \right) y^2 + b_{tt} \left(\sum_{i=1}^{n} p_i \right) t^2 y, \end{aligned} \tag{3.1.10}$$

where $\boldsymbol{\alpha} > 0$ is usually predetermined and matrix $A = (a_{ij})_{n\times n}$ satisfies

$$A = A^T, \tag{3.1.11}$$

$$A\boldsymbol{p}^* = \mathbf{0}, \quad \exists \boldsymbol{p}^* > \mathbf{0}. \tag{3.1.12}$$

The choice of $\boldsymbol{\alpha}$ and $\boldsymbol{p}^*$ is much of a statistic issue. The purpose of $\boldsymbol{\alpha}'\boldsymbol{p}$ is to reduce the order of the quadratic term by 1, so that the entire form is linear homogeneous in $\boldsymbol{p}$. The existence of $\boldsymbol{p}^*$ reduces the rank of parameter matrix A and hence the quadratic form. This partially resolves the problem that when the concavity condition is maintained, the estimated form tends to be overly concave.[4]

Common in the literature, $\boldsymbol{\alpha}$ is often set to be an all-1 vector $\mathbb{1}$ or the sample mean of the production input vectors $(1/T)\sum_{t=1}^{T} \boldsymbol{x}_t$. As for the "tipping point" $\boldsymbol{p}^*$, it is often set to be $\mathbb{1}$ or some observation $\boldsymbol{p}_t$ in the sample. The choice of those parameters appear more to be a statistic issue. Making $\boldsymbol{p}'A\boldsymbol{p}/(\boldsymbol{\alpha}'\boldsymbol{p})$ be in the same magnitude of other linear terms (linear in $\boldsymbol{p}$) could help increase the speed of convergence in numeric estimation methods. And the choice of $\boldsymbol{p}^*$ will determine the direction in which the cost function is strictly affine. So our choice will adjust along with the estimation process.

As for regularity conditions, positivity and monotonicity need to be imposed as constraints in estimation. Linear homogeneity in prices is automatically satisfied. As for concavity, we first derive the $(k,m)^{th}$ element of its Hessian matrix

$$\frac{\partial^2 c}{\partial p_k \partial p_m} = \left(\frac{a_{km}}{\sum_{i=1}^n \alpha_i p_i} - \frac{\sum_{i=1}^n (\alpha_k a_{mj} + \alpha_m a_{kj}) p_j}{\left(\sum_{i=1}^n \alpha_i p_i\right)^2} + \frac{\alpha_k \alpha_m \sum_{i=1}^n \sum_{j=1}^n a_{ij} p_i p_j}{\left(\sum_{i=1}^n \alpha_i p_i\right)^3} \right) y. \tag{3.1.13}$$

The expression resembles the Hessian of the GM form. Using $A\boldsymbol{p}^* = \mathbf{0}$, we have the Hessian evaluated at $\boldsymbol{p}^*$

$$\nabla^2_{\boldsymbol{pp}} c(\boldsymbol{p}^*, y, t) = \frac{A}{\boldsymbol{\alpha}'\boldsymbol{p}^*} y. \tag{3.1.14}$$

[4] Why do we coerce the quadratic degenerate is unclear. But it is shown by [DW88] that with the constraint on A, the form has just enough parameters to be flexible of degree $K < n$, which means the form can't model the $n - K$ second order derivatives of the true cost function freely.

Therefore the form is concave at $\boldsymbol{p}^*$ if and only if $A \le 0$. Simulations [BU06] showed that making an NQ form globally concave is frequently at the cost of very biased estimation and failure in monotonicity. So we are going to impose the concavity condistions pointwisely and regionally.

Like before, removing all terms containing technological advancement, we have

$$c(\boldsymbol{p}, y) = \left(\sum_{i=1}^{n} b_{ii}p_i + \frac{1}{2\sum_{i=1}^{n}\alpha_i p_i}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}p_ip_j\right)y + \sum_{i=1}^{n} b_ip_i + b_{yy}\left(\sum_{i=1}^{n} p_i\right)y^2, \tag{3.1.15}$$

where $\boldsymbol{\alpha} > 0$ is usually predetermined and matrix $B = (b_{ij})_{n\times n}$ satisfies

$$A = A^T, \tag{3.1.16}$$

$$A\boldsymbol{p}^* = 0, \exists \boldsymbol{p}^* > 0, \tag{3.1.17}$$

where similar regularity conditions apply.

3.2 Data

3.2.1 Sources and Variables

The production input and output variable constructions come mostly directly from national account data sets collected and released by the Bureau of Economic Analysis (BEA) and the Bureau of Labor Statistics (BLS). Historical price and quantity (utilization in production) indices of labor, material, energy, and purchased business service. come from the combined release KLEMS[5] table by BEA and BLS. Intermediate input is an aggregate of material, energy, and purchased business service. It is directly available in this data set. For many industries, the observation period of material, energy, and purchased business service is about five to ten years shorter than all other variables. So intermediate input is an important substitute for those industries. In the empirical

[5]KLEMS stands for capital, labor, energy, material, and service. Along with output, the data set forms a system of national accounts. Data from many countries and regions are collected and published under a common standard for comparative study on productivity, development, trade, etc.

methods section, we will describe how models are adjusted to avoid missing data as well as to make use of the largest possible observation sample. All series from this data set are annual.

Monetary asset balances come from Quarterly Financial Report (QFR) released by the Census Bureau (CB), which are quarterly, in nominal terms, and not adjusted for seasonality. We use cash account, demand deposit account, etc. to construct the monetary serivce utilization variables at M1, MZM, and M2 scopes[6]. Interest rates, user cost indices of monetary services come from MSI[7] database. They are monthly, in nominal and real terms, available both seasonally adjusted and unadjusted. We use those variables to calculate the price of monetary services (or monetary asset user cost, in the corresponding literature) and to construct the quantity index of monetary service utilization. Producer price index (PPI) data come from BLS, which are annual and have a few alternative estimates. This price index is used as the deflation factor in the construction of monetary service price index. See Appendix A for construction of the monetary service price and quantity indices.

Capital service utilization quantity index and price index, investment quantity and price index, capital stock, accumulated total wealth, depreciation rate are from combined release capital tables by BEA-BLS. They all come at annual frequency. Note that these variables can be estimated or constructed via other data set releases from BLS. Values obtained in such ways are different from values in the combined release we use. They can be potentially used for validating our model specification and estimates.

Output quantity index in nominal and real terms, price index, and GDP deflator are from the national accounts set released by BEA. They are quarterly series.

Our estimate of subjective discount rate[8] of an industry uses net income and equity from the QFR data set[9], applying formulas (2.3.10)(2.3.11). It would be more desirable if we use the hurdle

[6]According to the Fed's standard, M1 consists of currency and travelers' checks, demand deposits, other checkable deposits, super NOW accounts held at commercial banks. M2 consists of all M1 components, overnight Repo, overnight Eurodollars, money market mutual fund shares, money market deposit accounts, savings accounts, small time deposits, retail Repo at commercial banks. MZM consists of all M2 components except for small time deposits.

[7]MSI is Monetary Services Index

[8]A plausible alternative approximate is to use the closure of ten year and longer term treasury bonds. This method assumes economy-wide homogeneity over all industries. We used this specification in the robustness check models.

[9]All items from QFR are originally quarterly, in nominal terms, and not adjusted for seasonality. See Appendix A

rate, which is the expected rate of return on equity set often by the managerial team or the board. National account surveys conducted by the Bank of England cover the hurdle rates of a sample of manufacturers in the U.K. But we doubt there is counterpart data collected by the U.S. economic statistics agencies.

It is worth noting that many alternative industry-level measures of technology or productivity are available along with aforementioned variables, from various sources. They are all estimates based on statistics agencies' production function estimation. The production function they model are in very parameter-stringent forms, and thus are far from flexible. And inputs in the model are highly aggregated, usually including only capital and labor. So the technology variables of their estimation are not consistent with the technology variable in our specification. Therefore we still choose to remove the technology terms from our models (3.1.4)(3.1.7)(3.1.10)[10] and resort to the economy to scale interpretation.

The longest common observation period for all variables and for the largest amount of industries is first quarter 1987 to last quarter 2017 (1987Q1–2017Q4). That gives us a sample of size 31. The small sample size leads us favor model specifications with less parameters. We are going to go over model settings for the sample size in the empirical method sector. Although the sample is small, pooling thirty years of observation is appropriate for our task. An industry's data from a too long period of observation cannot be treated as homogeneous any more. There are substantial changes in the economy structure over decades. Some industries emerge, some industries become obsolete and disappear, or decline from large to a small scale. Even for the very old mature industry, the same name would have very different contents, use new inputs and produce new outputs, and operate in unprecedented markets. We could possibly better off if all variables are available at a higher observation frequency. But longer period of observation is not of much help.

for the method of seasonality adjustment we use.

[10]They hence become (3.1.6)(3.1.9)(3.1.15).

3.2.2 Matching Industries across Data Sets

Without special mention, all the variables described above are at the industry level. We use NAICS[11] 3-digit level subsectors as our definition of industry. Also, we loosely follow the NAICS naming system. We call the 2-digit NAICS sectors *sector*, 3-digit subsectors *industry*, 4-digit industry groups *subindustry*; 5-digit and 6-digit level aggregates will not be used. The several data sets we use have different industry classification systems and some data sets change their own systems over time. How exactly industries are matched across data sets are presented in Appendix A. Here we only mention the important treatments.

The data sets we use, are designed and collected for different purposes. The diverse purposes define the wide variety in the industry classification systems they employ. Although seemingly, they cover many industries in common. But by scrutinizing their documentations, one will find that industries of common names are never exact matches.

Even industries themselves are not set still. Industry in essence, is a classification of economic activities based on their similarities. Over time, industries emerge, develop, mature, and decline. Businesses start small and subordinate to other businesses, and can grow into gigantic independent ones. Industry classification systems also adapt with those changes. A good classification can be quite obsolete every ten years for the past two centuries. QFR, the data set that has the longest time coverage among all we use, is an example. Although it tries hard to keep its industry definitions internally consistent over time, its classification system for the 1987–2018 sample turned from a loose alignment with SIC 1987, to with NAICS 2002.

To obtain the best match of industries over data sets, we consolidated the sets under the following procedure. We first map all classification systems to the NAICS 2007 system 4-digit or 5-digit level classes. NAICS 2007 is the system closest to the classification system used by QFR nowadays (least discrepancy between the two), and it is finer than all classification systems of the sets

[11] NAICS is the North America Industry Classification System. It has gone through a few revisions since first introduction, to accommodate structural changes in the economy. The current version NAICS is very different from the early versions, and even more different from its predecessor the SIC system. The version we use throughout the study is the 2007 NAICS.

we use[12]. Then we use the map to match industries over all observation series across data sets. We aggregate industries of some series whose classification systems are finer than those in other sets. We refrain ourselves from splitting series with coarser classifications, for it involves another layer of estimation and inaccuracy. We remove industries with missing values in any series[13] during 1987Q1–2017Q4.

Thus we end up with a set of 17 industries or sectors that includes more than half of the 3-digit level manufacturing industries, aggregated entire wholesale trade, aggregated entire retail trade; details see Appendix A. Agriculture, mining, service industries are dropped mainly because QFR does not cover those industries.

3.3 Estimation Method and Empirical Model Specification

3.3.1 One-Period Cost Function

Following the convention from previous sections, we have y_t the industry real output index in period t, x_{it} the quantity index of type-i input used in the period industry production, p_{it} the price index the industry face of type-i input, c the industry cost function, n the number of input types, and T the total amount of observations. Superscripts GB, GM, NQ mean models in the Generalized Barnett form, the Generalized McFadden form, and the Normalized Quadratic form. Let $\boldsymbol{u}_t$ be the estimation error whose elements are $u_{it}, i = 1, \ldots, n$. Our empirical system of equations are the factor demand equations from (3.1.2). In the Generalized Barnett form, the system of equations to

[12]There are more modern, and thus finer systems, but they are redundant for what we need.
[13]where no very good proxies are available.

estimate is

$$
\begin{aligned}
x_{1t}^{GB} &= \left(\sum_{j=2}^{n} b_{1j} p_{1t}^{-\frac{1}{2}} p_{jt}^{\frac{1}{2}} - 2 \sum_{i=2}^{n} \sum_{j=2, j\neq i}^{n} d_{ij} p_{1t} p_{it}^{-\frac{1}{2}} p_{jt}^{-\frac{1}{2}} \right. \\
&\quad \left. + \frac{1}{2} \sum_{i=2}^{n} \sum_{j=2, j\neq i}^{n} e_{ij} p_{1t}^{-\frac{3}{2}} p_{it}^{-\frac{1}{2}} p_{jt}^{2} \right) y_t + b_{11} y_t + b_1 + b_{yy} y_t^2 + u_{it}, \qquad (3.3.18) \\
x_{kt}^{GB} &= \left(\sum_{j=1, j\neq k}^{n} b_{kj} p_{kt}^{-\frac{1}{2}} p_{jt}^{\frac{1}{2}} + \sum_{j=1, j\neq k}^{n} d_{kj} p_{1t}^{2} p_{kt}^{-\frac{3}{2}} p_{jt}^{-\frac{1}{2}} \right. \\
&\quad \left. + \frac{1}{2} \sum_{j=2, j\neq k}^{n} e_{kj} p_{1t}^{-\frac{1}{2}} p_{kt}^{-\frac{3}{2}} p_{jt}^{2} - 2 \sum_{i=2, i\neq k}^{n} e_{ik} p_{1t}^{-\frac{1}{2}} p_{it}^{-\frac{1}{2}} p_{kt} \right) y_t \\
&\quad + b_{kk} y_t + b_k + b_{yy} y_t^2 + u_{kt}, \quad \text{for} \quad k = 2, \ldots, n. \qquad (3.3.19)
\end{aligned}
$$

In the Generalized McFadden form, the system of equations to estimate is

$$
x_{1t}^{GM} = -\frac{1}{2p_{1t}^2} \sum_{i=2}^{n} \sum_{j=2}^{n} a_{ij} p_i p_j y_t + b_{11} y_t + b_1 + b_{yy} y_t^2 + u_{1t} \qquad (3.3.20)
$$

$$
x_{kt}^{GM} = \frac{1}{p_{1t}} \sum_{j=2}^{n} a_{kj} p_{jt} y_t + b_{kk} y_t + b_k + b_{yy} y_t^2 + u_{kt}, \quad \text{for} \quad k = 2, \ldots, n. \qquad (3.3.21)
$$

In the Normalized Quadratic form, the system of equations to estimate is

$$
\begin{aligned}
x_{kt}^{NQ} &= \frac{\sum_{j=1}^{n} a_{kj} p_{jt}}{\sum_{i=1}^{n} \alpha_i p_{it}} y_t - \frac{1}{2} \alpha_k \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \frac{p_{it}}{\sum_{i=1}^{n} \alpha_i p_{it}} \frac{p_{jt}}{\sum_{i=1}^{n} \alpha_i p_{it}} y_t \\
&\quad + b_{kk} y_t + b_k + b_{yy} y_t^2 + u_{kt}, \quad \text{for} \quad k = 1, \ldots, n. \qquad (3.3.22)
\end{aligned}
$$

Superscripts of parameters and error term u are omitted and they are form-wisely, industry-wisely different and independent.

3.3.2 Multi-Period Cost Function

As was concluded in the multi-period cost function section, the multi-period flexible functional form approximation is the same form at a larger scale. Here we use index and subscript s to represent the observation points. A sample used for estimation has size S. Following the previous notation, index and subscript $t = 1, \dots, T$ represent the planning horizon. The cost function to be approximated $c(\boldsymbol{\eta}, y_1, \dots, y_T)$ is defined as (2.2.4) where $\{\eta_i\}_{i=1}^{nT}$ follow expressions (2.2.7).

Let $\boldsymbol{u}_t$ be the estimation error whose elements are $u_{it}, i = 1, \dots, n$. Our empirical system of equations are the factor demand equations from (3.1.2). Without causing confusion, we overload the factor demand notation x to stand for both the factors planned to use in all periods 1 to T, but also the investment plans $I_1, \dots, I_T$. The ordering of those variables can be arbitrary, as long as the corresponding η_i are correctly placed. In the Generalized Barnett form, the system of equations to estimate is

$$\begin{aligned}
x_{1,s}^{GB} = &\left(\sum_{j=2}^{n} b_{1j} \eta_{1,s}^{-\frac{1}{2}} \eta_{j,s}^{\frac{1}{2}} - 2 \sum_{i=2}^{n} \sum_{j=2, j \neq i}^{n} d_{ij} \eta_{1,s} \eta_{i,s}^{-\frac{1}{2}} \eta_{j,s}^{-\frac{1}{2}} \right. \\
&\left. + \frac{1}{2} \sum_{i=2}^{n} \sum_{j=2, j \neq i}^{n} e_{ij} \eta_{1,s}^{-\frac{3}{2}} \eta_{i,s}^{-\frac{1}{2}} \eta_{j,s}^{2} \right) \sum_{t=1}^{T} y_{s+t-1} \\
&+ b_{11} \sum_{t=1}^{T} y_{s+t-1} + b_1 + \sum_{t=1}^{T} b_{yyt} y_{s+t-1}^2 + u_{i,s},
\end{aligned} \tag{3.3.23}$$

$$\begin{aligned}
x_{k,s}^{GB} = &\left(\sum_{j=1, j \neq k}^{n} b_{kj} \eta_{k,s}^{-\frac{1}{2}} \eta_{j,s}^{\frac{1}{2}} + \sum_{j=1, j \neq k}^{n} d_{kj} \eta_{1,s}^{2} \eta_{k,s}^{-\frac{3}{2}} \eta_{j,s}^{-\frac{1}{2}} \right. \\
&\left. + \frac{1}{2} \sum_{j=2, j \neq k}^{n} e_{kj} \eta_{1,s}^{-\frac{1}{2}} \eta_{k,s}^{-\frac{3}{2}} \eta_{j,s}^{2} - 2 \sum_{i=2, i \neq k}^{n} e_{ik} \eta_{1,s}^{-\frac{1}{2}} \eta_{i,s}^{-\frac{1}{2}} \eta_{k,s} \right) \sum_{t=1}^{T} y_{s+t-1} \\
&+ b_{kk} \sum_{t=1}^{T} y_{s+t-1} + b_k + \sum_{t=1}^{T} b_{yyt} y_{s+t-1}^2 + u_{k,s}, \quad \text{for} \quad k = 2, \dots, n.
\end{aligned} \tag{3.3.24}$$

In the Generalized McFadden form, the system of equations to estimate is

$$x_{1,s}^{GM} = -\frac{1}{2\eta_{1,s}^2} \sum_{i=2}^{n} \sum_{j=2}^{n} a_{ij} \eta_i \eta_j \sum_{t=1}^{T} y_{s+t-1} + b_{11} \sum_{t=1}^{T} y_{s+t-1} + b_1 + \sum_{t=1}^{T} b_{yyt} y_{s+t-1}^2 + u_{1,s} \tag{3.3.25}$$

$$\begin{aligned}
x_{k,s}^{GM} = &\frac{1}{\eta_{1,s}} \sum_{j=2}^{n} a_{kj} \eta_{j,s} \sum_{t=1}^{T} y_{s+t-1} \\
&+ b_{kk} \sum_{t=1}^{T} y_{s+t-1} + b_k + \sum_{t=1}^{T} b_{yyt} y_{s+t-1}^2 + u_{k,s}, \quad \text{for} \quad k = 2, \dots, n.
\end{aligned} \tag{3.3.26}$$

Superscripts of parameters and error term u are omitted and they are form-wisely, industry-wisely different and independent. In this multi-period case, Normalized Quadratic form is no longer used and estimated. There is no compelling reason for us to believe that a quadratic-form-like function resembles how multi-period input prices enter the cost function.

The number of unknown parameters grows fast in a multi-period flexible form approximate. Recall that if the production has n kinds of inputs, there are (at least) $(n^2 + 3n - 2)/2$ unknown coefficients in the flexible functional form. When we estimate the input demand system of equations, there are n^2 elements in the unknown variance-covariance matrix of the error terms. Since a T-period generalization of and n input cost function in flexible function form is just like an nT input one in a 1-period form, adding the $T-1$ more coefficients of y^2 terms, the system of equations we are going to estimate have

$$\frac{(nT)^2 + 3nT - 2}{2} + (T - 1) + (nT)^2 = \frac{3(nT)^2 + (3n + 2)T - 4}{2}$$

unknown parameters. The number grows over one hundred if there are 4 kinds of production inputs and the planning horizon is 2 periods. That means we need over a century of observations to estimate such a model with annual data, and 27 years of quarterly data. To the best of our knowledge, there is no such records. Even if there are records or reliable ways to recover such a data set, using a observation period of this long is not plausible. As discussed earlier, an industry and the economy it is in must have gone substantial changes over a century. Its production and cost functions should be very different across the time.

We are not able to estimate even the simplest 4-input 2-period model with our data set. But we can still make a point if we twist the cost minimization problem a bit. That will change the definition of the cost function and considerably reduce the model scale. Let us revisit the multi-period cost minimization problem. In the 2-period case, the producer has multi-period *total* cost function

$$c(\cdot) = \min_{\boldsymbol{x}_1, \boldsymbol{x}_2, I_1} \Big\{ \boldsymbol{p}'_{x,1}\boldsymbol{x}_1 + \beta \boldsymbol{p}'_{x,2}\boldsymbol{x}_2 + G_1(I_1) - I_1 + B_2 \\ \Big| \, F(\boldsymbol{x}_1, y_1) \leq 0, F(\boldsymbol{x}_2, y_2) \leq 0, x_{1,2} = I_1 + (1-\delta)x_{1,1} \Big\}, \tag{3.3.27}$$

Using the condition that all inputs are separable in the objective function, the minimization problem can be reformulated to two-stage budgeting. At the first stage, the producer chooses optimal expenditure allocation over all periods of production. At the second stage, the producer solves two separate problems. One is the optimal capital goods expenditure, and input utilization plan $\boldsymbol{x}_2$ in period 2. The other is the optimal period 1 input utilization plan with preset $\boldsymbol{x}_2$. To be exact, the first stage solves

$$\min_{\boldsymbol{x}_2} \beta \boldsymbol{p}'_{x,2}\boldsymbol{x}_2 \\ \text{s.t. } F(\boldsymbol{x}_2, y_2) \leq 0. \tag{3.3.28}$$

Let its value function be $B_1(I_1, \boldsymbol{p}_{x,2}, y_2) = B_1(I_1)$. The second stage solves

$$\min_{\boldsymbol{x}_1, I_1} \boldsymbol{p}'_{x,1}\boldsymbol{x}_1 + G_1(I_1) - I_1 + B_1(I_1) \\ \text{s.t. } F(\boldsymbol{x}_2, y_2) \leq 0, \\ (1-\delta)x_{1,1} + I_1 = x_{1,2}. \tag{3.3.29}$$

Then we can define a partial multi-period cost function as the *value function* of Problem (3.3.29), net of the *value* of Problem (3.3.28),

$$c_{\text{partial}}(\boldsymbol{p}_{x,1}, p_{I,1}, y_1, y_2) = c_{\text{total}}(\cdot) - B_1(I_1^*).$$

This cost function has much less arguments than the total multi-period cost function. Yet it is still able to reflect the intertemporal relations among period 1 production plan and planned investment and output level in the subsequent periods. Moreover a functional form approximating the partial multi-period cost, will have much less parameters than the total cost approximate.

Generally, for a cost function with output-contingent investment plan over T periods, the producer solves the cost minimization problem backwards. She plans the period T input utilization at the first stage, and $T-1$ plan based on that, and then $T-2$, until the last stage period 1 input utilization plan. And the last stage value function gives us the partial cost function

$$c_{\text{partial}}\left(\boldsymbol{p}_{x,1}, \{p_{I,t}\}_{t=1}^{T-1}, \{y_t\}_{t=1}^{T}\right) = c_{\text{total}}\left(\{\boldsymbol{p}_{x,t}\}_{t=1}^{T}, \{p_{I,t}\}_{t=1}^{T-1}, \{y_t\}_{t=1}^{T}\right) - \sum_{t=1}^{T-1} B_t(I_t)|_{I_t=I_t^*}, \tag{3.3.30}$$

where $\{I_t^*\}_{t=1}^{T-1}$ are optimal investment levels solved from the sequence of backward cost minimization problems. Therefore estimating a flexible functional form of this partial cost function only involves

$$\frac{(n+T-1)^2 + 3(n+T-1) - 2}{2} + (T-1) + n^2$$

unknown parameters, where $n+T-1$ are coefficients of $\boldsymbol{x}_1$ and $I_1, \ldots, T-1$, second term $T-1$ are coefficients of $y_2^2, \ldots, y_T^2$.

3.3.3 Uniform Frequency Model Specifications

We specify six kinds of cost functions at the industry level by specifying the monetary asset aggregation scope, production input aggregation scope, and production plan time horizon. We estimate each of the following cost functions with monetary aggregation scope at MZM and M2 level:

- 4-input 1-year total cost,
- 4-input 2-year partial cost,
- 6-input 1-year total cost.

As for the 6-input ones, the input aggregates are capital goods[14], labor, materials and components,

[14]In the sense of the amount of service they provide in the modelling period.

energy, purchased business service[15], and monetary asset. These are the standard categorical inputs in any KLEMS data sets. Utilizing them gives us the most flexibility in estimated functional forms. But the flexibility comes at the price of burgeoning amount of unknown parameters, less accuracy and less power in statistical tests. The number of parameters becomes so large that a 6-input 2-year partial cost function is impossible to model in FFF with the data we have. Hence we trade the flexibility for reliability. We use the intermediate input, which is a higher level of aggregation over materials, components, energy, and purchased business service to specify the 4-input cost functions. The 4-input ones have capital goods, labor, intermediate input, and monetary asset as categorial inputs. The 2-year version of the 4-input cost functions will provide us insights on the dynamics with investment decisions involved. Unfortunately, a 3-year version will also introduce too many more parameters to sensibly estimate. We will have to find finer data sets to further expand the empirical model on input number (the n dimension) and time horizon (the T dimension).

For each kind of the six, we try three functional forms (GB, GM, NQ), with all possible asymmetric variations (which kind of input being the first input in the form). For each of these functional forms, we feed two variations of Divisia indices, one whose asset user cost derived assuming perfect certainty and one with uncertainty[16].

And for every variation above, we specify statistical variations of regressions with and without the constant term (the intercept term). This is done by using either the input demand level variables (x in previous sections) or the input demand share variable (x/y in previous sections) as regressands in the system of equations. Using the share variable is quite common (if not ubiquitous) in the FFF application literature, but they did not document a persuasive reason to do so.[17] Using the share variable changes essentially changes both the functional form as well as model assumptions about the error term. In the statistics sense, it is more appealing to include the constant term for the bias concern. Nevertheless, following the literature tradition we estimate all models both with and

[15] Including services from contractors.

[16] For their formulas and derivation, see [Bar80] the certainty case and [BW05] the uncertainty case.

[17] [FS08] makes a recent example. Estimating the production function is not the study's primary goal, but it tried to use the standard (i.e. most popular) model specification in this literature.

without intercepts.

3.3.4 MIDAS Model: Utilizing Mixed Frequency Data

Monetary asset holding and monetary asset prices are available quarterly throughout the observation period. This higher observation frequency enables us more accurately timing the shocks in monetary asset prices, and potentially more accurately quantify their impacts.

A shock early in the year may have a different impact on the same year output from a shock coming in late in the year.

3.3.5 MLE And Nested MCMC

The complicated nature of our empirical models averts us from almost all frequentist methods. The regression is a system of nonlinear equations with joint nonlinear non-closed-form parameter constraints. For the 4-input 2-year and 6-input 1-year cost functions, the number of parameters exceeds the number of observation points. So we mainly rely on Bayesian methods for estimation.

We first obtain the maximum likelihood estimate (MLE) without imposing regularity conditions. We call those estimates and economic variables based on them the irregular estimates. Very likely they will not satisfy one or multiple regularity conditions, hence do not have much economic meaning.

Then we obtain Bayesian estimators using nested Markov Chain Monte Carlo (MCMC) sampler. We divide parameters into groups that we call blocks. We use Gibbs sampler on blocks that enter the system of equations and parameter constraints linearly. We use random walk Metropolis Hastings sampler on blocks that enter either the system of equations or parameter constraints nonlinearly. A pure RWMH sampler is able to draw a equivalently good parameter sample, too. But it will take a much longer chain to reach a stable posterior. Given the amount of models to estimate, and the complexity of the models, we choose to use a carefully balanced hybrid of Gibbs and RWMH. We harness the efficiency of Gibbs and the generality of RWMH to accommodate the nonlinearity.

Since there is no theoretical reason to favor one value of any parameter, we use uninformative prior on all parameters. The Markov Chains start with the unconstrained MLE's as initial points. The sampling scheme and distributions of one parameter conditional on all others are presented in Appendix B.

We make several arrangements implementing the estimation process, which also affect the inference we can make. It is optimal for the acceptance rate of a RWMH sampler to be close to 0.234 ([RGG+97, SR+09]). An acceptance rate too far away from 0.234 means inefficient sampling. It then requires a much larger and finer sample to approximate the posterior distribution of the parameters. Were the sample is not large and fine enough, estimation based on it will be unreliable. The acceptance rate of an RWMH sampler can be tuned up and down by setting the scale parameter. The scale parameter is a hyperparameter of the algorithm that can be loosely understood as the step size of the proposing Markov chain. When the scale parameter is large, the chain will propose new sample points in a relatively large area. The proposed new sample point can jump wildly in the parameter space. When the scale parameter is small, the chain will stay in a small area and propose points nearby. Each parameter can have its own scale parameter to set the its own chain step size. The Markov chain will sample most efficiently when the joint acceptance rate of all parameters is close to the optimal value.

We try to take advantage of the efficient sampling using dynamic scale parameters. We monitor the acceptance rate of the sampler regularly and adjust the scale parameters such that the acceptance rate becomes stable in a narrow neighborhood of 0.23. After acceptance rate stabilizes, we monitor the convergence status of the parameter sample dimension-wise. That is, to evaluate the convergence of the marginal sample distribution of every parameter. After the marginal sample distribution stabilizes, we keep sampling until the parameter sample size reaches 0.2 million. All accepted points prior to them will be the burn-in part of the sampler.

In addition, we impose soft and hard constraints to the parameters. Hard constraints are conditions that all sample points satisfy. Soft constraints are conditions that sample points can violate with a likelihood penalty, at the acceptance step in the Metropolis Hastings algorithm. The motivation is again to save computation resource and the reason for using soft constraints lies in the admissible parameter space. The curvature condition among all regularity conditions, renders our admissible parameter spaces segmented, irregular in shape, and even possibly have isolated points. If we hard-impose it, chances are that at some points, the sampler keeps proposing new sample points outside the admissible space, rejecting them, and using the last accepted point as new sample points.[18] That will make the algorithm acceptance rate very low, and potentially the parameter sample very biased. We allow the sampling chain to temporarily step into and pass through the inadmissible area by soft-imposing those conditions. Eventually when the scale parameter is very small and sampling area very narrow, all sample points will be staying inside the admissible space.

As a result, the parameter sample covers an irregular, restrictive area with the posterior distribution mode in it. It takes a meandering but relatively direct path from the initial point, to the mode. But it misses a majority part of the entire distribution support set. Using the last points of the chain, we can make reliable estimate of the posterior mode, but nothing else. Since most part of the distribution is unknown to us, the sample mean can be far away from the distribution mean, so as other quantiles. That prevents us from evaluating standard errors, credible intervals, statistical significance, and so on.

3.4 Results

In this section, we evaluate and compare how reliable and credible our estimates are, from statistic and economic perspectives. Then we analyze the economic implications of functions and variables derived from estimated cost functions.

[18]The Metropolis-Hastings algorithm proposes a mixing sequence to the sampler. Our dynamic scale parameter scheme tends to make it even more mixing. With strict constraints sometimes the sampler can get stuck at a single point forever.

3.4.1 Statistical Regularity

Pooling all estimated models together, we find that the residuals are large in comparison to the magnitude of the number of parameters. This is probably the result of our scaling scheme for preprocessing the data variables. When the number of parameters is large and/or the parameter space is large, it is common to scale regressors and regressands to the $[-1, 1]$ intervals. That will usually reduce the search area in the parameter space in the sense that no parameters are exceptionally larger or smaller than the rest. For the ease of imposing monotonicity condition, one of the regularity conditions, we scale all regressors and regressands such that they are centered at 100. That can make the left and righ hand side deviate far off from each other with a tiny perturbation in parameters.

As a result, the mean square errors (MSE) of all models dominate their own number of parameters by at least 4 orders of magnitude. The differences between any information criteria thus become negligible. And if we order the models by information criteria, the model rankings are the same regardless which criterion is being used. Therefore we use AIC as our primary standard to compare models (that are comparable). The lower the AIC of a model, the more preferred it is to is alternatives. Likelihood is an important piece of information but it is not a useful standard in our comparison. We chose information criteria over likelihood, for that likelihood ratio tests only apply to nested models. But information criteria are not restricted so and are model-paradigm-free. Coefficients of determination are our secondary standard for model evaluation. We cast doubt to models with coefficients of determination, for example r^2, close to zero or one. Those models may have underfitting or overfitting issues.

Some patterns stand out when we compare those standards of all models. The 4-input 1-year models almost always have lower information criteria than 4-input 2-year models and 6-input 1-year models. The simpler form and less amount of parameters bring an edge in the sense of statistical behavior. Models of monetary aggregation at M2 scope almost always perform better than their MZM counterparts. Their MCMC samples converge faster, have higher likelihood, and lower information criteria evaluated at the Bayesian estimators. There are no particular patterns emerging from coefficients of determination, unless we compare a group of relevant models.

Model Selection

Our model selection task is to find the optimal model form among alternatives whose functional form and form first input are different. We find models without the intercept term (i.e. whose regressands are share variables) are universally worse than their alternatives. They have much higher[19] AIC and r^2 and adjusted r^2 very close to zero. We abort estimating every one of them and eliminate such specifications from our model pool. This result, and our model formulations henceforth, is the opposite to many existing studies in this literature. The reason for such a contrast is yet to investigate, but we observe low coefficients of determination in those studies, too.[20] Nevertheless our models are not comparable to those models in smaller observation sample size, lower economic aggregation level, larger amount of categorical inputs, inclusion of monetary assets, inclusion of intertemporal dynamics, imposition of a full, sufficient and necessary set of regularity conditions.

We also observe that the two methods of Divisia aggregation over the same scope of monetary assets produce very close price and quantity indices. Consequently the pair of model specifications which differ only in the monetary aggregation method have extremely close parameter estimates. In many cases, estimated economic variables using such a pair are the same up to the first two effective digits. In light of this, we regard those pairs models (and estimates) identical.

Holding everything else equal[21], NQ forms are most likely to have the lowest AIC and GM forms have the lowest AIC forms in some cases. As for cases where GM forms ranking first with the lowest AIC, monetary assets and capital goods are the most preferred first inputs, with few cases preferring labor. It appears that our small sample size is an important factor determining the

[19]Higher by 2 orders of magnitude.

[20]Their models have information criteria much lower than ours, lower by some orders of magnitude.

[21]That is, compare variations for every specification combination of a particular industry, a particular input aggregation scope (4-input, 6-input), a particular cost function type (1-year static, 2-year dynamic), and a particular monetary aggregation scope

form selection. Given the same particular aggregation scope and the same cost function type (static or dynamic), GB forms always have many more (even more than double sometimes) parameters than NQ and GM for better flexibility. However, our observation sample does not present enough information to make full use of the flexibility gain. Therefore GM and NQ, having the same amount of parameters for a given specification, are favored. The main difference between GM and NQ is the form symmetry in input prices. While NQ forms do not change with input ordering, GM forms distinguish its first input from the rest. The first input price is called the numeraire price and behaves just like a numeraire in the form. Cases preferring GM form are more likely to adjust the entire production plan under exogenous changes in the numeraire prices. In contrast, cases preferring NQ form are able to adjust only a pair of inputs (along with output) under exogenous change in any input prices. Why monetary assets and capital goods are most frequently selected numeraire becomes apparent. Those industries (in combination of the cost function type) are either industries that runs with more dependence on monetary asset holdings, or heavier dependence on capital goods.

Values of coefficients of determination for models with the lowest AIC do not suggest underfitting or overfitting. Their r^2 are safely in the range of 0.4 to 0.9. Adjusted r^2 are from -1.2 to 0.7. Those deeply negative values come from models whose number of parameters far exceeds the number of observations.

3.4.2 Economic Regularity

All economic variables derived from the flexible functional form costs are functions of input prices and output level. But comparing them as functions is better presented in graphs than verbally. It is clearer to compress some information, and to evaluate and compare them at local points. We localize the variables at the point where all input prices and output levels are set to 100. This point is convenient for two reasons. On one hand, it is an actual point in the observation sample since the 2012 values of all the series are normalized to 100. We don't have any other data point taking a common set of numbers across all industries in the observation sample. On the other hand, it equalizes some elasticities with corresponding partial derivatives, since the level values in those elasticities are the same and cancel off. Thus unless otherwise specified, all economic variables, implicit functions, derivatives in the subsequent sections are functions evaluated at the all-hundred point. Without causing confusion, we are going to refer to this point as the unity point, which is analogous to the actual unity point were the observation variables to be pre-scaled around one.

Self Price Elasticity

The diagonal elements of the Hessian matrix of the estimated cost function are the demand of inputs differentiated with respect to their own prices. For all industries, all input aggregation scopes, all cost function types and all monetary aggregation scopes, the selected functional forms have Hessian whose diagonals are all negative. In other words, the estimated self price elasticities from all models are negative. That is consistent with what microeconomic theory would predict. All inputs we consider are normal goods in production. The more expensive they are, the less producers will try to use them, by either substituting to other inputs in the range permitted by technology, or decreasing output.

In particular, the monetary assets held by firms are normal, too. The higher the opportunity cost of holding monetary assets is, the less firms are willing to keep them on book, if everything else hold constant.

Regularity Conditions

As for selected specifications, all the models with Bayesian estimators satisfy all the positivity, monotonicity, and concavity conditions, in the $\boldsymbol{p} > \delta, \boldsymbol{y} > \delta$ region. Here δ is a small[22], positive number that varies by model. The regular region is desirable, for all prices and outputs indices in the observation sample are far greater than the largest δ. It is worth noting that, although the functional forms we use are supposed to be concave globally by design, estimates become less accurate as $(\boldsymbol{p}, \boldsymbol{y})$ points move farther and farther away from the observation sample range. Ultimately,

[22] Less than 10, with many less than 1.

the specified models are effective locally but in a decently wide region. And we will never infer anything beyond their effective ranges. In effect, our estimates are economically regular globally.

As expected, the MLE's don't behave as good. Needless to check positivity and monotonicity, we find them having the wrong curvature mostly. Loosing the regularity condition constraints indeed sets loose the estimates. They are most likely to violate the curvature condition, as is also seen and documented by other flexible functional form and form application studies. Our great computational toll pays off. To maintain all three sets of regularity conditions, and to maintain them globally, one has to resort to Bayesian estimation[23], which is computationally costly both at the estimation stage and the inference stage. And it achieves what it intends to.

3.4.3 (Factor-)Price Elasticity of Output

How output responds to exogenous factor price change is best depicted by its price partial derivatives, and in more generally comparable senses, its price elasticities from our empirical models. They show the marginal adjustment of output plan when typical producers of every industry observe the factor price changes. We mainly examine and conclude from the 4-input 1-year cost with M2 scope model for its good statistical regularity. We also compare it with the 4-input 1-year cost with MZM scope model under some contexts. Contrasts between the pair usually show how economic variables change differently with user costs of different components of monetary assets. The components are coarsely grouped into money of zero maturity, M2 aggregately, and M2 components that are not money of zero maturity.

We are going to use price of monetary assets short for the price index of the monetary asset aggregate, so as those of other inputs, like we did in specifying the costs' functional forms. [24] Those prices are price indices dual to quantity measure indices of flow or service variables. For example, the capital goods quantity index is a measure of the capital service of exiting capital

[23] For the time being. Perhaps more sophisticated or dedicated methods will be developed in the future.

[24] Note that the KLEMS literature often interchangeably use the user cost of capital flow (or services) and the price of capital flow (or services). The practice does not apply to the monetary service index (or aggregation) literature. The user cost means the opportunity cost of holding *a particular kind of* monetary asset for its monetary services. It is similar to the price of a particular good instead of an aggregate of multiple goods. The price index of monetary assets, has the usual meaning.

goods and the labor quantity index is a measure of the labor service of hired labor. Notably, the price of monetary assets is not what people and the macroeconomic literature usually refer to, the expenditure of *obtaining* those assets. The price here can be understood as the foregone opportunity cost of holding those assets in liquid forms, rather than investing them and earn the economic return from the investment. It is thus loosely increasing in the yield from economic investing, or the rate of return of the benchmark asset in terms of the monetary quantity index literature. It is, generally and loosely speaking, decreasing in the interest rate paid on the corresponding monetary asset. This is also parent from the definition[25] of the Divisia monetary asset user cost. The definition and its implication are critical to correctly understanding our empirical results.

The dynamics properties of output and investment sensitivity to input prices, are summarized from the 4-input 2-year partial cost models. The construction is restricted to forward looking at a two-year horizon at annual frequency, due to characteristics of our model and observation data set. That means we are able to evaluate how output and investment would be affected by input price changes in the contemporaneous and subsequent year. We don't have enough information for assessing movements in a shorter time interval (quarter, for example) or a further future. Comparison of these main results with other models[26] is in the robustness study section.

Partial derivatives with respect to input prices are central to our assessment of price elasticities. We take capital price elasticity of output as a demonstration. Everything else are constructed in the same fashion. Our partial derivatives are obtained by assuming implicit functions in an equation where the total cost is equal to an arbitrary[27] constant. Suppose output y_t is function of all input prices $\boldsymbol{p}_t$, then differentiating

$$c(\boldsymbol{p}_t, y(\boldsymbol{p}_t)) = \text{Const.} \tag{3.4.31}$$

with respect to capital price $p_{1,t}$ yields

$$\frac{\partial c}{\partial p_{1,t}} + \frac{\partial c}{\partial y_t} \cdot \frac{\partial y}{\partial p_{1,t}} = 0, \tag{3.4.32}$$

[25]Any version under from the simplest to the most realistic conditions.
[26]All 4-input 2-year and 6-input 1-year models.
[27]Not exactly arbitrary. We are going to explain later.

from which $\partial y(\boldsymbol{p}_t)/\partial p_{1,t}$ can be solved. Since $\nabla c(\boldsymbol{p}; y)$ is a vector of factor demands, the price Hessian of the cost function is a matrix of demand price elasticity of all inputs. In the multi-period partial cost, investment enters the functional form like a production factor. The Hessian then is where its partial derivatives with respect to input prices come. When evaluated at a particular point of $(\boldsymbol{p}, y)$, the cost level constant is determined as a result. Therefore the constant is not purely arbitrary (in the usual sense), but truly predetermined. Elasticities based on those partial derivatives are point elasticities.

The Currency Channel

It turns out that monetary assets within the M2 scope *are* empirical production inputs. This is implied by the uniformly negative values of the monetary asset price elasticities in Table 3.6. On the asset side of the producers' balance sheets, the higher the opportunity cost of holding liquid monetary asset and utilizing their monetary services, the less they are able to produce, holding the total production expenditure equal. Sparing funds and keeping them for monetary services induces a true economic cost to firms. At the optimal production efficiency level, the higher that cost is, the less firms are able to produce.

The observation of monetary assets' production factor attribute can be contrary to what conventional monetary and macroeconomic theory would predict. It is also opposite to the prevalent, if not universal, empirical finding that an exogenous increase in the interest rates paid on monetary assets reduces the output aggregately or disaggregately. To see the possible contradiction, we need to know how exactly the price of monetary assets is related to the interest rates. The monetary asset price is a weighted average of the monetary service user costs associated to the aggregated asset components. The monetary service user cost is defined in equation 1.4.1 and under more general conditions, the right hand side of 1.4.1 plus a term about the second moments of interest rates. By definition, the monetary asset price is a function increasing in the benchmark rate of return, increasing in the true expenditure price index[28], and decreasing in interest rates paid on

[28]It can be taken as the producer price index in our context.

the component monetary assets. That means, everything else held equal, an increase in interest rates will decrease the monetary asset price, and thereafter increase the output, of every industries we model. And the use of one unit increase in the Federal Funds Rate as an unexpected exogenous contractionary monetary shock is standard in all prior VAR studies on monetary transmission mechanisms. Their empirical result is that will decrease output during the entire first year after the shock. Of course, an unexpected exogenous unit increase in the Federal Funds Rate does not cause the monetary asset price to decrease for sure. Both the producer price index and the rate of return on the benchmark asset are endogenously affected by the shock. The VAR models often predict an initial upturn of the price level at the shock and the benchmark rate probably will rise with the yield curve. Both make the direction of the monetary asset price development less predictable. From the comparative static point of view, the contradiction is apparent.

But our finding is not wrong. Interest rate changes affect firms and production much more strongly and dominatingly on the financing side and demand side. A contractionary monetary shock is adverse to production, investment, and sales via all those channels. On the asset side, where increasing interest rate promotes production, the effect of monetary shocks is minimal and obviously not substantial in comparison to the demand and financing channel effects. This is why when examined together, the overall effect of contractionary monetary shocks, are contractionary to the economy. Nevertheless, observing the asset side effect helps to distinguish cases and analyze with higher quantitative precise. We call such an effect of monetary shocks the currency channel.

Now we turn our sight from the sign of those elasticities to their magnitude.

Estimates of the first-order, direct reaction of output upon monetary asset price shocks turns out to be very mild. For 13 of the 15 manufacturing industries, the monetary asset price elasticities of output are less than 0.25, which means 1 percentage change in the monetary asset price will only cause a quarter of a percentage change in output within the same year, in turns of the direct effect. The apparel and leather product is the only industry deviating far from the cohort, with exceptionally high elasticity of 0.43. But it is still a small number in the absolute sense and in the relative sense in contrast to other input price elasticities of output. A few industries have very inelastic output towards monetary asset prices. The paper industry, printing and printing related product industry, petroleum and coal product, other (basically nonmetallic inorganic) materials, and nonmetallic mineral and their products, have elasticity less than 0.10 in absolute value. Although some of those industries are highly leveraged, like the nonmetallic minerals industry, and they run tight loan renewal cycles, monetary assets are not their primary resource of liquidity on the asset side, neither do they comprise a major proportion of the total expenditure. The implications are consistent with the conventional impression of those industries; they run heavily on fixed facilities and equipments, and trade credits for method of payments. Banks and credit markets usually roll over their credits with more regards to the downstream market demand than their spectacularly high leverage ratio. The trade sector exhibits a somewhat different pattern. Their elasticities are 10 percentage points larger in absolute value than the average manufacturing, but still small in the absolute sense.

For any given industry, the currency channel output sensitivity is systematically higher as the output level increases, if everything else remains unchanged. The same property exists with the increase in the price of monetary assets. In other words, the partial derivative of output with respect to monetary asset price is a decreasing function (get more negative) in the output level and in the monetary asset price. The result means monetary assets will not be fully substituted by other factors as the production scale becomes large. It also means that the larger scale production requires firms to keep more liquid assets on the book. For most industries, output will be reduced a bit more in the subsequent year by the increase in monetary asset price, as is shown in Table 3.11. The pattern is in line with the VAR findings, too, where output level usually reaches its tough in the sixth to eighth quarter after the initial monetary policy shock. Unfortunately, our data do not support us to model and plot into the further future and using our model in a rolling fashion is not meaningful.

Labor Price And Capital Price Elasticity

Labor price elasticity and capital price elasticity are universally negative and larger than the monetary asset price elasticity in absolute value, which are shown in Table 3.8 and Table 3.7. But for most of those values, they are still small in the absolute sense, for the negative values are almost all greater than -0.5. They also exhibit larger intertemporal differences when we look at the 2-year horizon in Table 3.13, 3.12. Pooling the results together, we find greater industry level diversity than that of the monetary asset price elasticities. For example, Petroleum and coal, all other electronic products have output most sensitive to labor prices and even more (while almost being the most sensitive) to capital prices. Foundry, electrical equipments and electrical components are on the small side. The retail trade and wholesale trade appear to be mixed right in the middle of the cohort of the manufacturing industries. They seem to be just like an average manufacturing sector in the sense of how labor and capital goods expenditures play roles in their production.

3.4.4 Factor-Price Elasticity of Investment

We use the previously introduced constant cost method to evaluate how demand for one factor changes when one input price exogenously changes. The presumption that the cost holds constant implies all price elasticities based on such derivatives are compensated. Signs of the compensated price elasticities show whether a pair of inputs are complements or substitutes in production (or equivalently, in factor demand). A pair of inputs are complements if their compensated cross price elasticity is positive. They are substitutes if their compensated cross price elasticity is negative. Zero elasticity means irrelevance but that is never a case found in our models.

But the equivalence between elasticity sign and substitute and complement relations can not establish on the ordinarily defined cross price elasticity. To establish meaningful, unambiguous pairwise partial elasticity of substitution of two inputs amongst many, we need to use the Morishima elasticity. The Morishima substitution elasticity of demand for input i against price of input

j is

$$\sigma_{ij}^{M} = s_i \left(\sigma_{ji}^{A} - \sigma_{ii}^{A} \right), \tag{3.4.33}$$

where s_i is the expenditure share of input i, (letting c be the total cost)

$$s_i = \frac{p_i x_i}{c}, \tag{3.4.34}$$

and σ_{ij}^{A} is the (input j price) Allen substitution elasticity of demand for input i, which is

$$\sigma_{ij}^{A} = \sigma_{ic} + \frac{\sigma_{ij}}{s_i}. \tag{3.4.35}$$

In the Allen substitution elasticity, the elasticities σ_{ic} and σ_{ij} are those in the standard sense. The demand price elasticity σ_{ij} of demand i against price j is

$$\sigma_{ij} = \frac{p_j}{x_i} \cdot \frac{\partial x_i}{\partial p_j}. \tag{3.4.36}$$

The "demand-cost elasticity" of input i, which does not have an economic meaning, is defined like the income elasticity in a demand system

$$\sigma_{ic} = \frac{c}{x_i} \cdot \frac{\partial x_i}{\partial c}. \tag{3.4.37}$$

Now when we refer to substitutes and complements we mean them in the Morishima sense. A pair of production factors are substitutes if and only if their corresponding σ_{ij}^{M} is positive, and they are complements if and only if the corresponding σ_{ij}^{M} is negative.

Monetary Asset Price Elasticity

The relationship between investment and monetary asset price is not clear in theory. We can find some connections from the construction of the monetary asset price. In the monetary asset price index, the rate of return on benchmark assets is directly related to investment gains. By definition the benchmark asset is a hypothetical kind of financial asset which only pays interest over time, without providing monetary services of any kind to its owner. Its rate of return represents the rate of return that a firm could possibly earn were the monetary asset it holds to be invested to generate future output. Since the monetary asset price is increasing in the benchmark asset rate of return, its increase might be the consequence of increasing benchmark rate of return. If it were the case, then investment and monetary assets should be substitutes. When the benchmark asset rate or return increase, monetary asset price gets higher, causing more incentive to invest, and less incentive to hold monetary assets.

However, that prediction is contradicted by the empirical evidence. For all industries, our result show that the complement aspect of investment and monetary asset holdings, is clear and strong. The monetary asset price elasticity of investment is negative, meaning the higher the price is, the less firms are willing to invest. The more liquid the balance sheet is, on the asset side especially, the more firms are inclined to investment. A very similar pattern is documented by the corporate finance literature, for example Fazzari, Hubbard, Petersen [FHP87, FHP00], Fazzari, Petersen [FP93], Gaiotti, Generale [GG02], Cooley, Quadrini [CQ06] and so on. They focused on the relation between investment and cash flow, or between investment and free cash flow[29]. Samples during different historical periods, in different countries, and of different firms or industries, all show that investment and cash flow are positively related.

Moreover, our finding mutually supports one another with the empirical results and the theory proposed by Gennaioli, Ma, Shleifer [GMS16]. They analyzed survey data from questionnaires answered by Chief Financial Officers' of listed companies all around the world, and found that investment decisions depend heavily on the expectation of company's future cash flow and profitability, which is formed on extrapolation of the company's current cash flow and profitability. Therefore the reinforcing relation of current cash flow and profitability (resulting in abundant liquidity balance) and investment is established. The motive behind the practice can be multiple. For example, forming expectation by extrapolation is simple and straightforward to the management

[29]Cash flow and free cash flow in the usual accounting sense.

and shareholders, and can be effective in most scenarios. Companies may need the liquidity to signal for more and cheaper external funds to finance investment projects. Investing in expansion into new technology and new market can be a learning process. Only companies with sufficient liquidity can take the risk to test the uncharted water. Only the working, profitable plans will receive more resources, which turn out to be new and larger investment.

The magnitude of the industries' elasticities indicates that investment of firms running with less cash flow (internal financing) or less liquidity (internal and external financing) is more affected by monetary policy. This is also predicted by the balance sheet channel. As for dynamics shown in Table 3.15, we find no pattern common to most industries.

Labor Price and Capital Price Elasticity

It is important to ask, does adding investment into the cost function change relations of other inputs? The answer is no. We are going to go into details in the robustness section. Moreover, our side products, the labor price elasticity of investment and capital price elasticity of investment are consistent with similar findings in the corporate finance literature[30]. In Table 3.17 the labor price elasticity is close to zero for all industries. It is thus not clear whether labor and investment are more like substitutes or complements. But investment is not sensitive to labor price overall, confirming both intuition and empirical evidences from regression models.

Table 3.16 shows that investment is marginally increasing in capital price. This is another confirmation of results from other empirical studies. In terms of elasticity, the current year investment is inelastic to capital price, but is overall much more elastic than to monetary asset price, and more to labor price. Over time, investment sensitivity will quickly drop down to close to zero in the subsequent year after a marginal change in capital price. This pattern is different from monetary price elasticity which shows that the second year investment can be more sensitive to monetary asset price than the first year investment for some industries.

[30]The investment we used here is the increment in physical capital, while sometimes investment in studies in the corporate finance has different meanings. Our results are not comparable to studies using the investment account on financial reports or the economic investment including inventory investments.

3.4.5 Substitution Elasticity of Labor Demand

Table 3.18 shows that the monetary asset price elasticity of labor demand is positive for most industries. Following the same decomposition of monetary asset price index as we did in the output sensitivity section, labor demand is approximately a decreasing function to interest rates paid on component monetary assets. In other words, labor demand tend to decrease as interest rate level increases. The observation confirms our intuition and is consistent with what all empirical and structural models on employment would predict[31]. Only machinery and wholesale trade have elasticities small and negative. They are not only different in sign but also in magnitude from other industries. While most industries show near unit elasticity of labor, machinery and wholesale trade show very inelastic labor demand to the monetary asset price.

Table 3.19 shows that capital price elasticity of labor demand is negative for most industries. It implies that labor input and capital input are empirical complements. Remarkably, we find the most industry-level heterogeneity in the labor-capital pair in all the economic variables[32]. As for substitution elasticity between labor and capital, there are the elastic industries like apparel and leather product, unit elastic industries like foundry, inelastic industries like plastics and rubber product and machinery, and extremely inelastic (close-to-zero elasticity) industries like all other electronic product and wholesale trade. The reason for the phenomenon deserves future investigation.

3.4.6 Currency Channel Revisit: Sensitivity to Each Kind of Monetary Asset

Comparison between model pairs that differ only in monetary asset aggregation scope gives us hints on the reasonable level of aggregation in monetary assets as one categorical production input. We observe that the partial derivatives of output, investment, and labor demand, with respect to monetary asset price, are substantially smaller in cost functions estimated with the MZM scope.

[31] Although our measure of labor is quite different from the concept and measures of employment, the two variables move closely and parallel.

[32] In fact, variables as functions, not just comparing at a particular point.

Evaluated at the unity point, the monetary asset price elasticity of investment and of labor demand are close to zero for many industries, although they have the same sign with their counterparts estimated with the M2 scope. It implies that investment and labor demand are extremely inelastic to changes in the MZM component monetary asset prices. The M2 but non-MZM components are the factors affect producer's decisions on investment and labor demand, and hence the output level as well. The comparison is convincing in that other elasticities, for instance the capital price elasticity of output, established by such model pairs are not as different in value.

The contrast is less conspicuous in the monetary asset price elasticity of output. We observe that the MZM scope price change induced output change is generally smaller in magnitude than that induced by the M2 scope changes. We device a variable[33] to measure the difference between the monetary asset price elasticity of output under MZM scope and M2 scope. Let $\sigma_{y,M2}$ and $\sigma_{y,MZM}$ be the monetary asset price elasticities of output under the two scopes respectively. Then

$$\psi_y = \frac{\sigma_{y,MZM}}{\sigma_{y,M2}} \tag{3.4.38}$$

measures the relative strength of the currency channel in the MZM components in contrast to the M2 components. The currency channel is less pronounced in the MZM components than the M2 but non-MZM components if and only if $\psi_y < 1$. It is more pronounced in the MZM components if and only if $\psi_y > 1$. And equality to one means equally strong.

Similarly, the relative strength or sensitivity measures ψ_I and ψ_L can be established for investment and labor demand, and $\psi_{y,t+1}$ for subsequent periods. We can further investigate how firms differ and resemble in the classification of those variables.

[33] Again, it is a function since all variables it depends on are functions.

3.5 Robustness

3.5.1 Results from Alternative Variable Construction

Given the same model specification, estimators of models using the two differently constructed monetary services, one under perfect certainty assumption and one under risk condition, turn out to be close. In our test estimation, those model pairs yield the same estimates up to the first two effective digits. This is probably the result of that monetary asset price enters industry cost functions with low impact. Thus we did not estimate every model with both measures of Divisia monetary service indices. The change of measure will not make a difference in the results.

3.5.2 Results from Alternative Functional Form

Estimates of models differing in form are very different. None of the three forms is a good one size for all to model industry level cost functions. Under the same system of model selection procedure, we find that no forms fits better for more industry-cost-type combination than another form. Generalized McFadden can be the best for some models but Normalized Quadratic is better in similarly many other cases. Generalized Barnett is less suitable for its formidable amount of unknown parameters while we are short of observation points. But regarding its greatest flexibility, it stands a chance to fit better were more data are available. We have the N-P problem, that the number of unknown parameters is greater than the sample size, for the 2-year 4-input cost functions even in GM and NQ form. It is reasonable to favor forms with fewer parameters as prioritizing statistical power is more awarding.

As for the asymmetric forms GB and GM, different specifications of the first input also make a difference. In many cases, putting capital service or monetary service the first input is preferred. But the arrangement is not suitable for all cases. For a few industries, labor input, intermediate input, purchased business services as the first input are better options under statistical model selection. No solution is universally suitable. And there is no systematic pattern about which input is more likely to fit in the first position, in either the selected GM forms, or disregarded GM and GB forms.

3.5.3 Results from Alternative Estimator

Our main results from the 1-year 4-input cost function models are robust using two estimators, the parameter sample mode and parameter sample mean. Their discrepancy is less than 10 percent (of the sample mode estimates' own values) on average, which is well in the acceptable region. Results from the 1-year 6-input cost function models are less robust when we switch from sample mode to sample mean estimators, despite the fact that estimation for those cost functions is universally not accurate. The mode and mean estimators from the 2-year 4-input cost functions have large discrepancy in some but not all economic variables. We shall be aware that estimation of the 2-year cost functions is even less accurate due to the enormous parameters to estimate with a rather small sample.

As we have explain, our computation approach does not admit us to calculate the true sample mean estimator for any of the models. Our Bayesian samples of parameters does only have a minimal coverage of a neighborhood around the posterior distribution mode, and some other areas of much less likelihood. The entire shape of the posterior support set and the entire distribution are obscure to us. Thus the "sample mean" here is only a mean based on an estimate of the truncated posterior distribution whose peripheral parts are coarsely assumed zero. Nor are we able to assess how wrong the assumption is. The neglected parts overall may have very small total likelihood, but can be substantial as well. Therefore information from the sample mean estimates are only suggestive. We don't have credible interval estimates for the same reason.

The unconstrained MLE's are what we will get if the economic regularity is not maintained. We call them the irregular estimators in the result tables. The irregular results are apparently different both in magnitude and sign. That means the economic regularity conditions are strongly binding and render a true constraint to estimation. Discarding them will produce cost functions in the opposite curvature, as many studies have documented, and even negative factor demand in some

$(\boldsymbol{p}, \boldsymbol{y})$ regions in the first quadrant. This phenomenon is in fact worth further investigation. Why the data usually suggest the opposite structure to what the producer theory predicts? Are agents not optimizing in action? Do they have different constraints or information set from the theory assumes? What other information should researchers to collect to study the problem?

3.5.4 Results from Alternative Cost Function

Cost functions with different time horizons are different functions by definition. The single period function considers only current period production plan and is complete in accounting for the economic cost of producing such amounts of output. The multi-period cost function we estimate considers the cost induced by both current and subsequent periods production, and is incomplete in not accounting for future prices of many inputs. So the 1-year 4-input and 1-year 6-input model pairs are alternative specifications but neither of them is comparable with any 2-year 4-input cost models. However, there are systematic similarities between the two 4-input cost functions. They may not be completely coincidental.

Estimates from the single period cost functions are not robust when we change the level of input aggregation. In the 4-input specifications, the intermediate input is a non-flexible index of materials, energy, and purchased business service aggregate. The three categories are modelled flexible to each other and to other inputs in the 6-input specifications. While the 6-input versions have more flexibility, the 4-input versions are desirably parsimonious. As reported in the statistical regularity section, it turns out that parsimoniousness has a significant edge over flexibility in fitting our data set. Although estimation based on 6-input cost functions are noticeably different for all cost function types, all monetary aggregation scopes, and all industries, we cast reasonable doubts on their accuracy. We believe they are not strong evidences opposing results from the 4-input cost models.

Estimates from the single period and multiple periods 4-input cost functions are not supposed to be close, and most derived economic variables are truly not. But a few appear to be similar in value for all industries. Regardless of time horizon, estimated intermediate input price elasticities of output are somewhat close. In models with M2 aggregation scope, they are almost identical for industries like foundry, electrical equipment and electrical components, motor vehicle and parts, retail trade. In models with MZM aggregation scope, they are almost the same for industries like food and beverage and tobacco products, petroleum and coal products, plastics and rubber products, foundry, machinery, motor vehicle and motor vehicle parts, wholesale trade. Other input price elasticities of output derived from two cost functions are similar with wider and industry-dependent gaps, unlike the patterns shown in the intermediate input price elasticity. The output sensitivity in intermediate input price is invariant to planning horizon, while output sensitivities to other input prices are not. The result is robust to altering industry, to altering modeled monetary asset aggregation scope and to altering estimators. This suggests that in production plans of any industry, intermediate input price does not interact with future period variables. Expenditures of those inputs are instant and transient but expenditure of labor is not. They will never affect forward looking firms' decisions beyond the current period.

3.6 Limitation and Extension

3.6.1 Limitation

Although the model is designed to be parsimonious, the number of parameters still ramps up quickly with the number of inputs, and the number of periods the total cost covers, quickly beyond the sample size. Its *n*-*p* problem makes the model really hard to estimate or impossible to estimate without variable selection. As another facet of its *n*-*p* problem, we cannot use a long time span of observations to estimate the model. Environmental factors, like technology changes, population changes, international competition and cooperation changes, industry changes, market changes, all render only a limited sample period grants us coherent results.

Then what can remedy it? We've explained that removing one or several of the inputs from production and thus from the cost function, undermines the economic meaning of our estimates.

More plausibly, we could implement variable selection methods to keep only some regressors from the factor demand equations. We force the insignificant terms to zero, reduce the smoothness (and possibly flexibility) of the cost function, and attain more robust estimators. Alternatively, we could shrink all coefficients. An appropriate shrinkage method can also improve estimation accuracy. It may be even more suitable than variable selection if we believe that all components in the cost function are contributing to the total cost, but at a very small magnitude.

One difficulty in either selection or shrinkage, is to maintain the consistency across equations. The system of equations, formed by factor demand equations, are bonded by the symmetry in the Hessian matrix of the cost function. Thus regressors in one equation must be kept or removed along with corresponding regressors in all other equations. That means the variable selection method we use must have taken account of these cross equation equality constraints. Nevertheless the selection or shrinkage direction is worth exploring.

Last, we make a criticism which can be a problem, but does not apply to our study. The estiamted model would not be widely applicable if there were little variation in the input prices (especially those of our concern) during the period of observation data we use in estimation. In that case, the estimated model only depicts cost function on a narrow region of input price(s). Its prediction outside the region can be very rough. And it will be even harder to quantify the variance of the out-of-sample-range prediction.

3.6.2 Extension

Other than the monetary aspects, many other implications can be drawn from our estimated model. Shocks originating from other factor markets alters the corresponding prices of those inputs. Our constant cost comparison applies to all factors included in the cost function. We can thus use the estimates to answer questions like, how a one-time shock in energy price, in upstream product price, an exogenous producer price index shift, an exogenous wage shift, affect the industries' output levels in the short future. In combination with Input-Output tables, we might be able to connect the aftermath in a sequence of chain reactions. We may sort out which industries are influential in transmitting the shocks, and which are peripheral.

In the economic domain, it is natural to classify industries according to the content economic activities. That one standard singles out most noneconomic factors from the activities we study. But one question is always present: are there alternative classification systems, that are equally useful as the activity-based system, in understanding the economic behaviors. Now we have developed one more option to answer the question. Using microly determined variable, we can look for patterns determining why industries are similar or different in reaction to monetary shocks. For instance, market structure factors like industrial concentration, technology factors like capital intensity, intellectual property intensity, and many more intensity variables, history factors like industry age, stage of development, financial technology and financial market factors like turnover rate, are all features worth examining. The alternative classification can and should apply across activity-based classification and country borders.

Even with variables from this data set, we could answer many questions. Because we have all the *local* estimates of cross elasticities, we can match them with other variables to form a new panel data set. For example, we can explore how the monetary asset price elasticity of output and investment, depend on the industries' output share in the entire economy or labor use share, or energy consumption share, or relative technological index. That will demonstrate systematic relations between the monetary cost sensitivity with the size and age of the industry.

This leads us to the luxury of going down to finer grains. It is tempting to apply the same method to larger scale comparisons. For example, apply the model family to sector and country level data, to make sector-wise and international comparisons. But this line of generalization is more a robustness study than a true generalization. However, going down the opposite direction, into the details, is different. With patterns fetched at finer level of industry classes, we would be able to cast light on alternative ways to grow economic activities, beyond the resemblance in products and services they provide, beyond their geometric proximity. Eventually, the alternative industry classification system may assist us in law making, and in designing more systematic, more efficient and effective industry policies.

Last, we point out the need for model selection insight, on the research agenda of MIDAS models. Theorists proposed MIDAS base specifications in the context of applying to general mixed frequency data sets. But in application, characteristics of observation series may render one MIDAS specification more suitable than another. Doing some analytic and experimental comparisons of specification applicability to series type could provide a guideline to non-theorist users of such models. It will have users to streamline their application of MIDAS models. Moreover finding optimal specific purpose MIDAS models can further promote our understanding of the general cases.

Figures

Figure 3.1: Partial derivative of output with respect to monetary price, fixing each non-monetary input price, evaluated at the all-100 local point, every industry

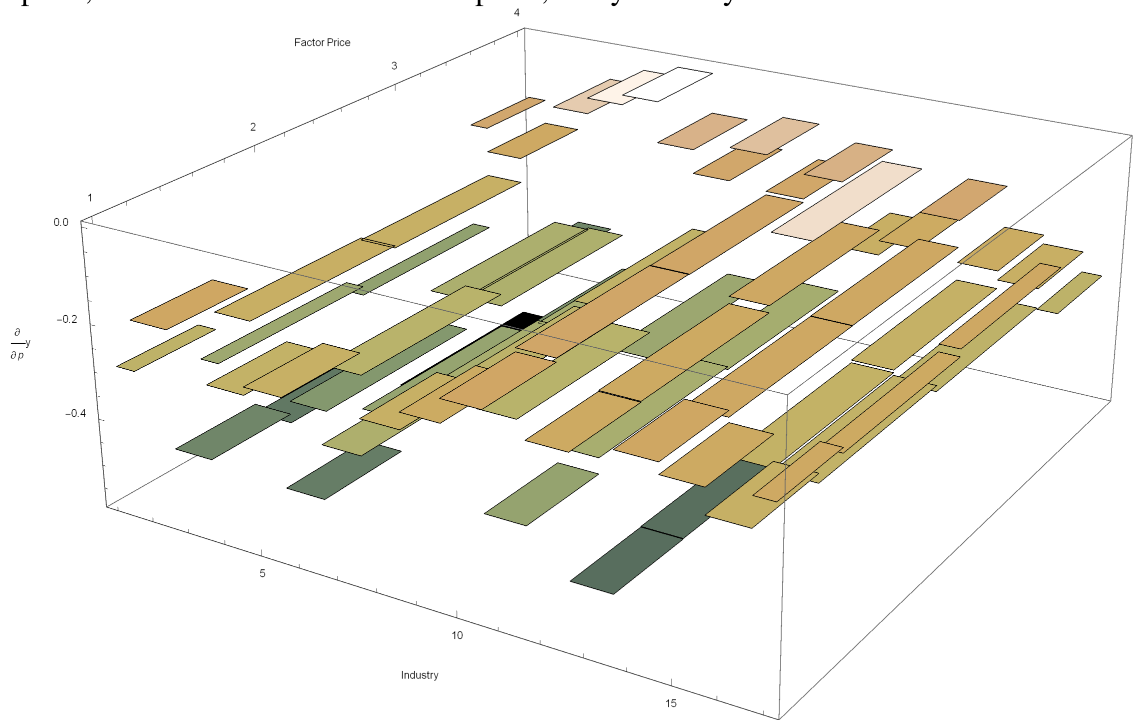


Figure 3.2: Partial derivative of output with respect to monetary price, fixing each non-monetary input price, evaluated at the all-110 local point, every industry

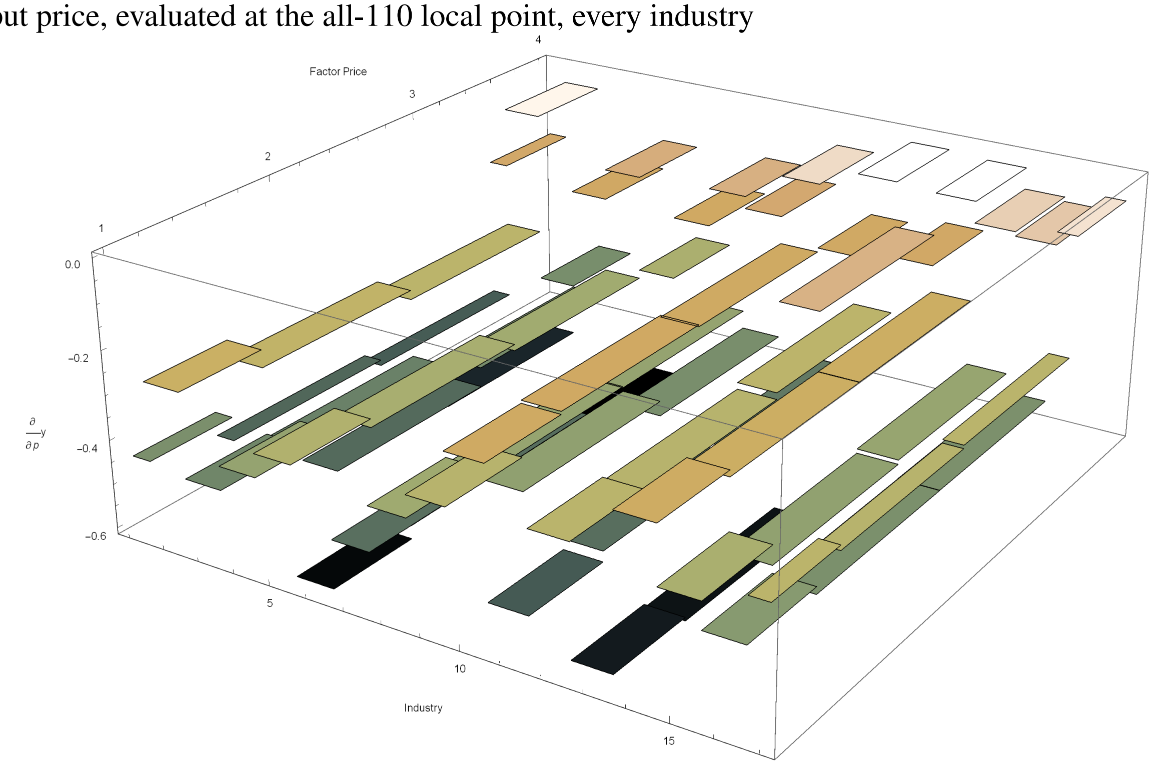


Figure 3.3: Partial derivative of output with respect to monetary price, fixing all non-monetary input prices, evaluated at each monetary price-output level coordinate local point, 4-input standard estimate

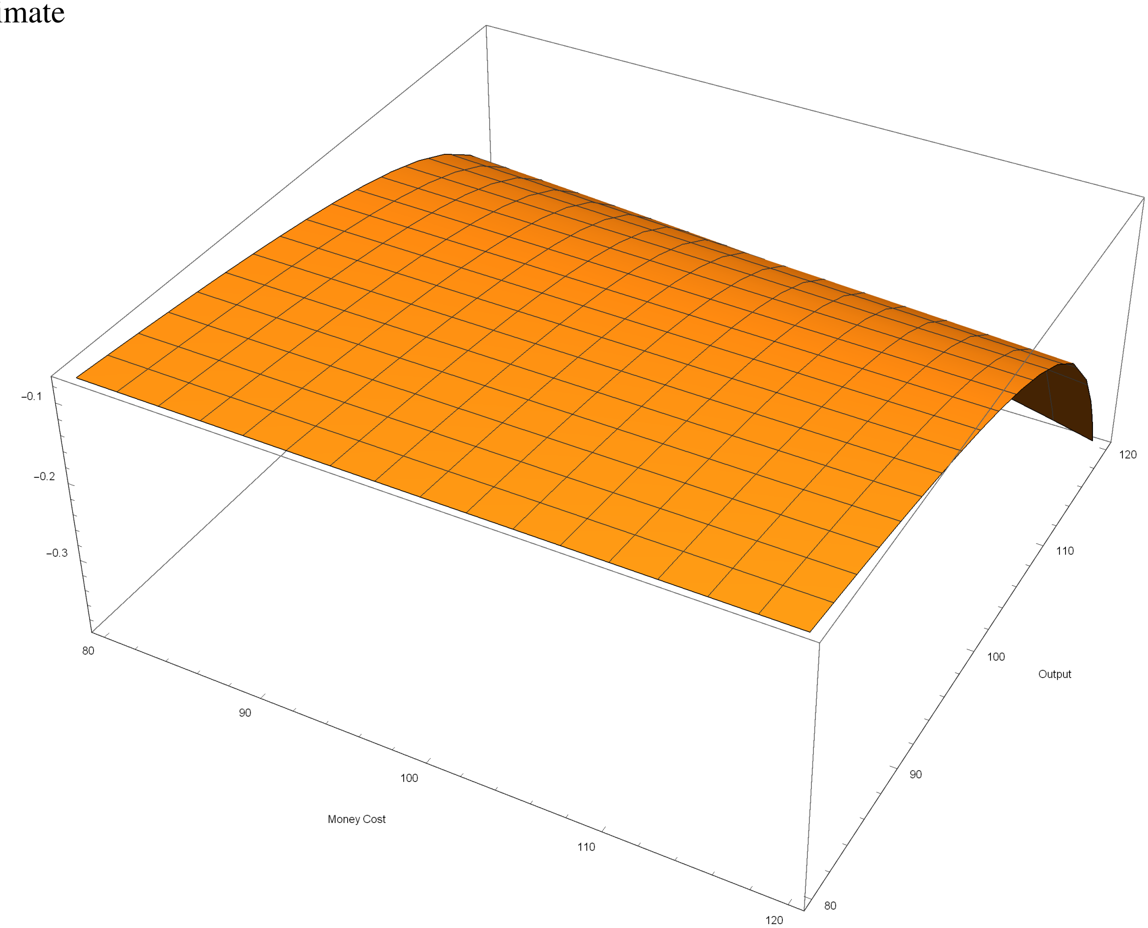


Figure 3.4: Partial derivative of output with respect to monetary price, fixing all non-monetary input prices, evaluated at each monetary price-output level coordinate local point, 6-input standard estimate

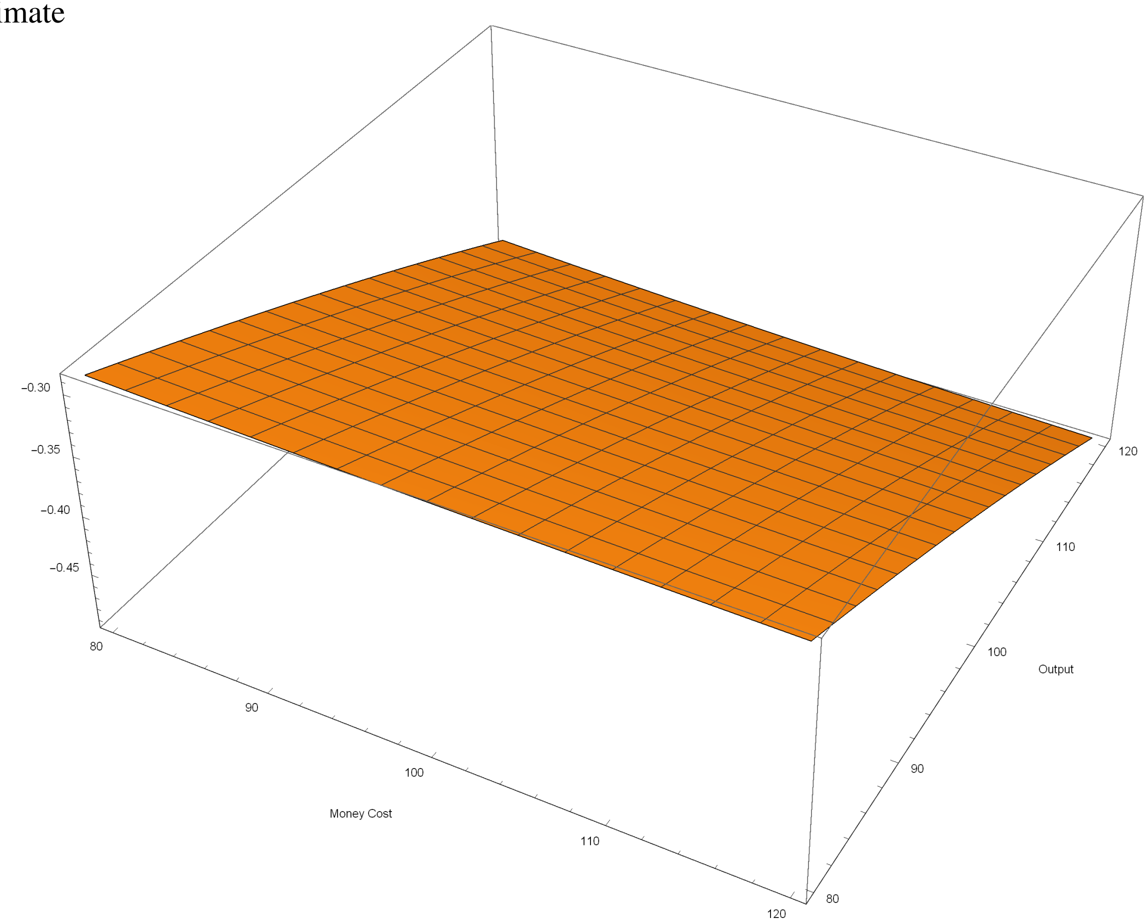
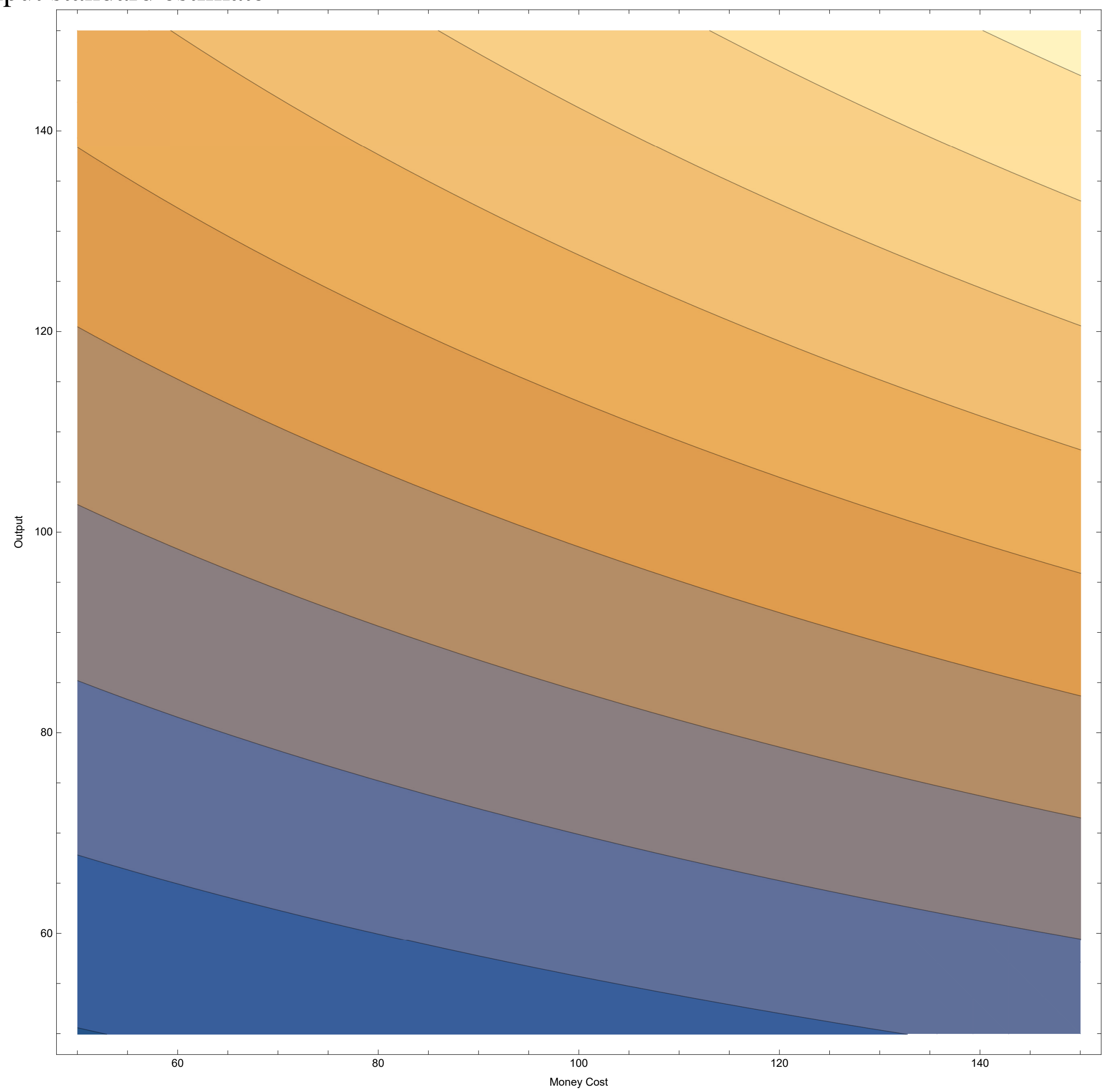


Figure 3.5: Partial derivative of output with respect to monetary price, contour graph, fixing all non-monetary input prices, evaluated at each monetary price-output level coordinate local point, 4-input standard estimate



Tables

Data

Table 3.1: Industry Code Correspondence across Data Sets

Industry	NAICS2007 Code	NAICS2007 QFR Modification	SIC1987 Code	KLEMS Code
Food and Kindred Products	3110 - 3119		(part of) 20	(part of) 311, 312
Beverage and Tobacco Products	3120 - 3129	Often combined to 311 in other sets	(part of) 20	(part of) 311, 312
Textile Mills and Textile Product Mills	3130 - 3149		22	(part of) 313, 314
Apparel and Leather Products	3150 - 3169		98.1	(part of) 315, 316
Wood Products	3210 - 3219		24	321
Paper	3220 - 3229		26	322
Printing and Related Support Activities	3230 - 3239		27	323
Petroleum and Coal Products	3240 - 3249		29	324
All Other Chemicals	3250, 3253, 3255 - 3259		28 (overlapping)	325
Plastics and Rubber Products	3260 - 3269		30	326
Nonmetallic Mineral Products	3270 - 3279		32 (and some other)	327
Foundries	3310, 3315 - 3319		(part of) 33	331
Fabricated Metal Products	3320 - 3329		34	332
Machinery	3330 - 3339		35	333
All Other Electronic Products	3340, 3343 - 3349		(part of) 36	334
Electrical Equipment, Appliances, and Components	3350 - 3359		(part of) 36	335
Furniture and Related Products	3370 - 3379		25	337
Miscellaneous Manufacturing	3390 - 3399		39 (overlapping)	339
Iron, Steel, and Ferroalloys	3311, 3312		33.1 - 33.2	
Computer and Peripheral Equipment	3341	SIC 3571, 3572, 3575, 3577	(small part of) 35	
Basic Chemicals, Resins, and Synthetics	3251, 3252		28.1 - 28.2	
Motor Vehicles and Parts	3361 - 3363		37.1	
Nonferrous Metals	3313, 3314		33.5 - 33.6	
Communications Equipment	3342	SIC 3661, 3663, 3669, 3679	(small part of) 36	
Pharmaceuticals and Medicines	3254		28.3	

Industry	NAICS2007 Code	NAICS2007 QFR Modification	SIC1987 Code	KLEMS Code
Aerospace Products and Parts	3364		37.7 (and some other)	
Wholesale Trade, Durable Goods	4230 - 4239		50	
Wholesale Trade, Nondurable Goods	4240 - 4259		51	
Food and Beverage Stores	4450 - 4459		(part of) 54	
Clothing and General Merchandise Stores	4480 - 4489, 4520 - 4529		(part of) 53	
All Other Retail Trade	4410 - 4449, 4460 - 4479, 4510 - 4519, 4530 - 4549	Part of SIC 54 - 57, 59	–	
Chemicals	3250 - 3259		28	
Computer and Electronic Products	3340 - 3349		(part of) 35 -36	
Primary Metals	3310 - 3319		33	
All Durable Manufacturing	3210 - 3219, 3270 - 3399		99	DM
All Manufacturing	3110 - 3399		20 - 39	MN
All Mining	2110 - 2139		10 - 14	
All Nondurable Manufacturing	3110 - 3169, 3220 - 3269		98	ND
Transportation Equipment	3360 - 3369		37	336
(residual included in Transportation Equipment)	3360, 3365 - 3369		–	
All Retail Trade	4410 - 4549		53 - 59	44,45
All Wholesale Trade	4230 - 4259		50 - 51	42

Parameter and Model

Table 3.2: Bayesian Parameter Estimation of 1-Period 4-Input Total Cost Function (Part 1)

Parameters, Statistics	***Food, Beverage, Tobacco***	***Textile, Textile Product Mill***	***Apparel, Leather Product***	***Paper***	***Printing and Related***	***Petroleum, Coal Product***	***All Other Chemical***	***Plastics, Rubber Product***
FFF	GM	GM	GM	GM	GM	GM	GM	GM
Input 1	mny	mny	mny	mny	mny	mny	mny	mny
a_{22}	0.324	0.45	0.945	0.47	0.53	0.614	0.576	0.449
	(0.137)	(0.115)	(0.0178)	(0.14)	(0.0975)	(0.0662)	(0.0743)	(0.108)
a_{23}	-0.0598	-0.0669	0.782	-0.051	0.14	-0.0287	0.025	0.0134
	(0.223)	(0.158)	(0.0402)	(0.221)	(0.178)	(0.209)	(0.221)	(0.219)
a_{33}	0.476	0.547	0.827	0.523	0.602	0.419	0.435	0.486
	(0.135)	(0.0772)	(0.0119)	(0.102)	(0.078)	(0.12)	(0.105)	(0.121)
a_{24}	-0.0853	-0.0467	0.941	-0.111	0.215	0.111	-0.0463	-0.147
	(0.22)	(0.202)	(0.0122)	(0.235)	(0.166)	(0.194)	(0.204)	(0.225)
a_{34}	-0.0147	0.102	1.03	0.2	0.242	0.0225	-0.0205	-0.0183
	(0.218)	(0.161)	(0.0313)	(0.166)	(0.177)	(0.215)	(0.222)	(0.204)
a_{44}	0.581	0.592	1.05	0.544	0.654	0.518	0.635	0.594
	(0.0916)	(0.0641)	(0.0063)	(0.11)	(0.0695)	(0.108)	(0.0724)	(0.0942)
b_{11}	1.13	1.83	0.871	1.14	1.54	1.08	1.26	1.31
	(0.031)	(0.125)	(0.029)	(0.0262)	(0.0955)	(0.0442)	(0.0554)	(0.0417)
b_{22}	0.899	0.61	0.738	0.919	0.689	0.928	0.811	0.878
	(0.0227)	(0.0742)	(0.0635)	(0.0102)	(0.0693)	(0.0127)	(0.04)	(0.0306)
b_{33}	0.918	0.787	0.916	1.09	0.898	0.992	0.972	0.877
	(0.0287)	(0.0538)	(0.0184)	(0.0248)	(0.0142)	(0.0172)	(0.0169)	(0.0158)
b_{44}	0.903	0.747	1.5	0.844	0.941	0.866	0.865	0.872

	Food, Beverage, Tobacco	Textile, Textile Product Mill	Apparel, Leather Product	Paper	Printing and Related	Petroleum, Coal Product	All Other Chemical	Plastics, Rubber Product
	(0.0268)	(0.0602)	(0.101)	(0.0428)	(0.00927)	(0.0301)	(0.0315)	(0.0254)
b_1	1.02	0.867	0.786	1.02	1.04	1.03	1.02	0.997
	(0.0164)	(0.0333)	(0.0193)	(0.0108)	(0.0082)	(0.0117)	(0.018)	(0.0116)
b_2	1.	1.12	0.99	0.98	1.04	1.01	1.06	0.871
	(0.00898)	(0.026)	(0.0114)	(0.0142)	(0.0162)	(0.0197)	(0.034)	(0.0149)
b_3	1.04	0.92	0.92	1.08	1.01	0.882	0.957	0.977
	(0.0183)	(0.0498)	(0.0128)	(0.0179)	(0.0175)	(0.0156)	(0.0168)	(0.0144)
b_4	1.01	1.22	1.02	1.1	1.06	1.03	1.05	1.01
	(0.00858)	(0.0297)	(0.0121)	(0.0114)	(0.0246)	(0.00894)	(0.0139)	(0.0208)
b_{yy}	-0.00145	0.0016	-0.000596	-0.000807	0.00123	-0.00191	-0.00116	0.000179
	(0.000909)	(0.000283)	(0.0000511)	(0.000515)	(0.00031)	(0.000836)	(0.000796)	(0.000697)
R^2	0.971	0.994	0.992	0.97	0.989	0.947	0.977	0.988
Adjusted R^2	0.946	0.989	0.984	0.943	0.979	0.9	0.956	0.977
AIC	2.32e4	2.71e5	1.58e6	7.23e4	2.03e5	3.52e4	4.13e4	1.32e5

Note: Standard deviations of each parameter estimation are in the parentheses. The number of observations is 31 and Bayesian sample size is 40,000 for all models in the table. Burn-in sizes vary from 0.5 million to 1.5 million depending on the convergence speed. Acceptance rates range from 0.19 to 0.31.

Table 3.3: Bayesian Parameter Estimation of 1-Period 4-Input Total Cost Function (Part 2)

Parameters, Statistics	**Nonmetallic Mineral Product**	**Foundry**	**Fabricated Metal Product**	**Machinery**	**All Other Electronic Product**	**Electrical Equipment, Component**	**Motor Vehicle, Part**	**All Retail Trade**	**All Wholesale Trade**
FFF	GM	GM	GM	GM	GM	GM	GM	GM	GM
Input 1	mny	mny	mny	mny	iip	mny	mny	mny	mny
a_{22}	0.377	0.615	0.666	0.405	-0.0112	0.524	0.486	0.452	0.317
	(0.122)	(0.0975)	(0.084)	(0.138)	(0.0362)	(0.0833)	(0.0895)	(0.116)	(0.145)
a_{23}	-0.106	0.218	-0.033	-0.107	-0.066	0.087	-0.284	-0.207	-0.254
	(0.229)	(0.185)	(0.226)	(0.24)	(0.19)	(0.188)	(0.263)	(0.252)	(0.266)
a_{33}	0.499	0.527	0.498	0.386	0.717	0.701	0.477	0.495	0.361
	(0.106)	(0.0993)	(0.102)	(0.134)	(0.079)	(0.0624)	(0.121)	(0.108)	(0.124)
a_{24}	-0.0118	0.214	-0.00639	-0.0884	-0.000285	0.182	-0.15	-0.0816	-0.0607
	(0.213)	(0.167)	(0.228)	(0.244)	(0.171)	(0.167)	(0.228)	(0.217)	(0.237)
a_{34}	0.234	0.184	-0.0233	-0.174	0.385	0.189	-0.0129	-0.145	-0.166
	(0.18)	(0.183)	(0.199)	(0.269)	(0.179)	(0.17)	(0.227)	(0.243)	(0.253)
a_{44}	0.622	0.527	0.508	0.379	0.593	0.676	0.469	0.462	0.404
	(0.0647)	(0.095)	(0.0962)	(0.126)	(0.104)	(0.0665)	(0.0905)	(0.0847)	(0.116)
b_{11}	1.2	2.74	1.31	1.47	1.28	2.23	1.29	1.28	1.48
	(0.0616)	(0.384)	(0.0696)	(0.094)	(0.0827)	(0.275)	(0.0731)	(0.0788)	(0.0834)
b_{22}	0.763	0.49	0.922	0.819	1.04	0.6	0.869	0.884	0.908
	(0.0453)	(0.132)	(0.0239)	(0.0367)	(0.0239)	(0.0968)	(0.0445)	(0.0139)	(0.0202)
b_{33}	0.813	0.473	0.986	0.914	0.82	0.581	1.05	0.976	1.03
	(0.0421)	(0.101)	(0.00997)	(0.0205)	(0.041)	(0.0995)	(0.0235)	(0.0155)	(0.0192)

	Nonmetallic Mineral Product	Foundry	Fabricated Metal Product	Machinery	All Other Electronic Product	Electrical Equipment, Component	Motor Vehicle, Part	All Retail Trade	All Wholesale Trade
b_{44}	0.864	0.491	0.972	0.814	0.875	0.501	0.893	0.932	0.707
	(0.0266)	(0.142)	(0.0231)	(0.0387)	(0.0396)	(0.116)	(0.0253)	(0.0194)	(0.0571)
b_1	1.05	1.06	0.953	0.923	1.04	0.986	0.91	1.01	1.1
	(0.0115)	(0.0293)	(0.0104)	(0.00586)	(0.0122)	(0.0207)	(0.0138)	(0.0153)	(0.0243)
b_2	1.01	1.06	1.11	1.08	1.06	1.05	0.98	0.998	0.91
	(0.0242)	(0.0167)	(0.0114)	(0.00926)	(0.00906)	(0.00965)	(0.0121)	(0.017)	(0.0288)
b_3	0.923	1.06	1.09	1.02	1.08	0.888	0.999	1.03	0.986
	(0.0093)	(0.0263)	(0.0271)	(0.0176)	(0.00928)	(0.0391)	(0.0116)	(0.0125)	(0.0165)
b_4	0.906	1.09	1.01	0.984	0.973	1.	1.07	1.02	0.986
	(0.0104)	(0.0254)	(0.00964)	(0.0158)	(0.0174)	(0.0278)	(0.0135)	(0.0167)	(0.0139)
b_{yy}	0.000247	0.0136	-0.00162	0.00194	-0.00166	0.00863	0.000243	-0.000738	0.00134
	(0.000578)	(0.000589)	(0.000636)	(0.00102)	(0.00571)	(0.000531)	(0.000934)	(0.000971)	(0.00151)
R^2	0.987	0.99	0.974	0.99	0.981	0.992	0.988	0.994	0.995
Adjusted R^2	0.975	0.981	0.952	0.981	0.964	0.984	0.978	0.988	0.99
AIC	6.51e4	3.65e5	2.96e6	2.42e4	9.36e4	2.26e6	2.26e4	1.13e4	1.02e4

Note: Standard deviations of each parameter estimation are in the parentheses. The number of observations is 31 and Bayesian sample size is 40,000 for all models in the table. Burn-in sizes vary from 0.5 million to 1.5 million depending on the convergence speed. Acceptance rates range from 0.19 to 0.31.

Table 3.4: Bayesian Parameter Estimation of 2-Period 4-Input Partial Cost Function (Part 1)

Parameters, Statistics	Food, Beverage, Tobacco	Textile, Textile Product Mill	Apparel, Leather Product	Paper	Printing and Related	Petroleum, Coal Product	All Other Chemical	Plastics, Rubber Product
FFF	GM	GM	GM	GM	GM	GM	GM	GM
Input 1	cpt	lbr	cpt	cpt	cpt	cpt	cpt	cpt
a_{22}	0.428	1.01	1.03	0.874	1.03	0.32	0.895	1.06
	(0.0288)	(0.00485)	(0.00836)	(0.0182)	(0.00919)	(0.0559)	(0.0167)	(0.00641)
a_{23}	0.403	0.582	0.919	1.04	0.634	0.792	0.795	0.871
	(0.077)	(0.0268)	(0.0238)	(0.0343)	(0.0128)	(0.0213)	(0.0318)	(0.0323)
a_{33}	1.21	0.772	0.96	0.719	0.954	1.03	0.813	1.06
	(0.0405)	(0.004)	(0.00947)	(0.00792)	(0.00951)	(0.0183)	(0.0303)	(0.0263)
a_{24}	0.782	0.773	0.98	0.914	0.446	0.666	0.382	0.674
	(0.0445)	(0.00798)	(0.0121)	(0.0228)	(0.00734)	(0.0647)	(0.0484)	(0.0189)
a_{34}	0.203	1.12	1.	0.669	0.829	0.927	0.588	0.384
	(0.0494)	(0.00627)	(0.0183)	(0.0442)	(0.0267)	(0.0344)	(0.0149)	(0.0107)
a_{44}	1.31	0.644	0.82	1.07	0.641	0.907	1.03	0.999
	(0.0207)	(0.0275)	(0.0475)	(0.016)	(0.0205)	(0.0786)	(0.011)	(0.0133)
a_{25}	0.93	-0.077	1.01	0.425	0.523	0.508	0.645	0.468
	(0.0538)	(0.00536)	(0.00893)	(0.0152)	(0.015)	(0.0181)	(0.0587)	(0.0285)
a_{35}	0.524	-0.0638	0.877	0.73	1.	0.561	0.69	0.518
	(0.0192)	(0.00465)	(0.0387)	(0.0285)	(0.0144)	(0.0519)	(0.0496)	(0.0135)
a_{45}	0.955	-0.0673	1.03	0.738	1.01	0.534	0.509	0.858
	(0.0647)	(0.00805)	(0.0246)	(0.0215)	(0.0233)	(0.0602)	(0.0375)	(0.0172)
a_{55}	1.03	0.00486	0.991	0.776	0.937	0.78	0.777	1.09

	Food, Beverage, Tobacco	Textile, Textile Product Mill	Apparel, Leather Product	Paper	Printing and Related	Petroleum, Coal Product	All Other Chemical	Plastics, Rubber Product
	(0.0211)	(0.000904)	(0.0173)	(0.0159)	(0.0137)	(0.0274)	(0.0276)	(0.0151)
a_{26}	-0.0697	0.93	-0.0935	-0.0468	-0.0598	-0.0314	-0.0691	-0.0643
	(0.0143)	(0.0231)	(0.259)	(0.00658)	(0.00815)	(0.00594)	(0.00873)	(0.00404)
a_{36}	-0.0504	0.888	-0.0368	-0.0812	-0.0612	-0.0494	-0.0646	-0.0701
	(0.0129)	(0.00865)	(0.251)	(0.00545)	(0.00735)	(0.00996)	(0.00845)	(0.0119)
a_{46}	-0.0687	0.945	0.0368	-0.0614	-0.051	-0.0537	-0.048	-0.0848
	(0.00829)	(0.0213)	(0.222)	(0.0087)	(0.0045)	(0.00797)	(0.00987)	(0.00744)
a_{56}	-0.0686	-0.0749	-0.0741	-0.0622	-0.106	-0.0604	-0.0486	-0.0764
	(0.00656)	(0.0075)	(0.249)	(0.00471)	(0.00744)	(0.0174)	(0.0111)	(0.00666)
a_{66}	0.00516	0.894	0.00243	0.00357	0.00585	0.00254	0.00362	0.00495
	(0.000992)	(0.0216)	(0.0362)	(0.000542)	(0.00082)	(0.000638)	(0.000552)	(0.000657)
b_{11}	1.17	1.41	1.04	1.16	1.26	1.02	0.759	1.14
	(0.0223)	(0.0298)	(0.011)	(0.0228)	(0.0149)	(0.072)	(0.0245)	(0.0121)
b_{22}	1.05	1.06	0.968	0.846	0.93	1.37	1.06	0.65
	(0.0145)	(0.00424)	(0.013)	(0.00673)	(0.00716)	(0.0222)	(0.0248)	(0.0145)
b_{33}	0.741	0.728	1.04	0.96	0.788	1.25	0.88	0.838
	(0.0405)	(0.025)	(0.01)	(0.016)	(0.0236)	(0.02)	(0.0127)	(0.0171)
b_{44}	0.92	0.44	0.965	0.907	0.766	0.848	0.908	1.16
	(0.0147)	(0.0309)	(0.0133)	(0.00736)	(0.00927)	(0.015)	(0.0373)	(0.011)
b_{55}	0.861	1.12	0.973	1.36	1.27	1.35	0.892	0.884
	(0.034)	(0.00904)	(0.0144)	(0.0183)	(0.00758)	(0.0406)	(0.0308)	(0.0169)
b_{66}	0.87	0.95	0.943	1.15	0.954	1.11	0.884	0.73
	(0.0125)	(0.00817)	(0.0104)	(0.0161)	(0.0101)	(0.0142)	(0.0137)	(0.0118)

	Food, Beverage, Tobacco	***Textile, Textile Product Mill***	***Apparel, Leather Product***	***Paper***	***Printing and Related***	***Petroleum, Coal Product***	***All Other Chemical***	***Plastics, Rubber Product***
b_1	0.92	1.25	1.04	1.13	1.06	0.759	1.4	1.13
	(0.018)	(0.0159)	(0.0151)	(0.0137)	(0.0216)	(0.0181)	(0.0226)	(0.0111)
b_2	0.811	1.26	0.921	0.812	1.37	0.867	1.08	0.827
	(0.0357)	(0.0105)	(0.0198)	(0.0264)	(0.0287)	(0.0395)	(0.0405)	(0.0113)
b_3	0.974	1.01	1.	1.38	1.23	1.35	1.22	1.43
	(0.044)	(0.00821)	(0.00686)	(0.0036)	(0.014)	(0.0271)	(0.00727)	(0.0119)
b_4	1.12	0.814	1.08	1.2	1.3	0.889	0.659	1.18
	(0.05)	(0.0128)	(0.00843)	(0.0186)	(0.0139)	(0.0447)	(0.0288)	(0.0133)
b_5	0.325	0.652	1.07	1.25	0.596	0.438	1.13	1.15
	(0.0237)	(0.0265)	(0.00814)	(0.00524)	(0.0168)	(0.0148)	(0.0266)	(0.0127)
b_6	0.661	0.769	1.	1.	0.94	1.25	1.44	1.19
	(0.0362)	(0.0103)	(0.0083)	(0.017)	(0.0243)	(0.022)	(0.0179)	(0.0147)
b_{yy1}	-0.0069	-0.00328	-0.000516	-0.00571	-0.00279	-0.00986	-0.00497	-0.00378
	(0.00141)	(0.000721)	(0.0105)	(0.000687)	(0.000852)	(0.000983)	(0.00116)	(0.000982)
b_{yy2}	-0.0767	-0.0612	-0.0243	-0.0703	-0.0504	-0.083	-0.0618	-0.0548
	(0.00661)	(0.0033)	(0.0163)	(0.00372)	(0.00374)	(0.00561)	(0.00593)	(0.00464)
R^2	0.657	0.87	0.996	0.572	0.831	0.737	0.619	0.634
Adjusted R^2	-2.43	-0.295	0.958	-3.28	-0.69	-1.63	-2.81	-2.66
AIC	1.26e6	2.33e6	9.98e6	1.51e6	1.69e6	1.09e6	1.13e6	1.46e6

Note: Standard deviations of each parameter estimation are in the parentheses. The number of observations is 31 and Bayesian sample size is 40,000 for all models in the table. Burn-in sizes vary from 0.5 million to 1.5 million depending on the convergence speed. Acceptance rates range from 0.19 to 0.31.

Table 3.5: Bayesian Parameter Estimation of 2-Period 4-Input Partial Cost Function (Part 2)

Parameters, Statistics	Nonmetallic Mineral Product	Foundry	Fabricated Metal Product	Machinery	All Other Electronic Product	Electrical Equipment, Component	Motor Vehicle, Part	All Retail Trade	All Wholesale Trade
FFF	GM	GM	GM	GM	GM	GM	GM	GM	GM
Input 1	cpt	cpt	cpt	cpt	cpt	cpt	cpt	mny	mny
a_{22}	0.592	0.927	1.11	1.23	0.534	1.07	0.367	0.691	0.866
	(0.0155)	(0.026)	(0.0389)	(0.0102)	(0.078)	(0.0183)	(0.0624)	(0.024)	(0.01480
a_{23}	0.641	0.546	0.843	0.233	0.172	0.599	0.655	0.571	0.611
	(0.0309)	(0.0188)	(0.0586)	(0.0528)	(0.0986)	(0.0295)	(0.0726)	(0.042)	(0.0293)
a_{33}	0.936	0.889	0.91	0.756	0.853	1.16	1.35	0.985	0.898
	(0.0191)	(0.0151)	(0.0201)	(0.0225)	(0.06)	(0.0214)	(0.0283)	(0.0228)	(0.0141)
a_{24}	1.02	0.756	0.656	0.656	0.119	0.678	0.427	0.508	0.726
	(0.0112)	(0.0255)	(0.0899)	(0.0375)	(0.148)	(0.0188)	(0.0373)	(0.0319)	(0.0235)
a_{34}	0.685	0.358	0.458	0.932	0.466	0.96	1.	0.589	0.704
	(0.0214)	(0.118)	(0.092)	(0.0528)	(0.141)	(0.0165)	(0.0486)	(0.0385)	(0.027)
a_{44}	1.01	0.897	0.515	0.604	0.288	0.716	0.747	0.728	0.816
	(0.0317)	(0.0262)	(0.0182)	(0.0181)	(0.0814)	(0.0094)	(0.0349)	(0.0279)	(0.0214)
a_{25}	0.833	0.727	0.327	0.719	0.415	0.824	0.744	0.636	0.399
	(0.0343)	(0.0303)	(0.0762)	(0.076)	(0.0911)	(0.0359)	(0.0252)	(0.0355)	(0.0231)
a_{35}	0.961	0.696	0.616	0.787	0.0262	0.696	0.335	0.449	0.425
	(0.0142)	(0.0616)	(0.0561)	(0.0158)	(0.123)	(0.0176)	(0.0836)	(0.038)	(0.0206)
a_{45}	0.689	0.596	0.955	0.445	0.48	0.311	0.444	0.719	0.367
	(0.0531)	(0.0632)	(0.0197)	(0.0239)	(0.0985)	(0.0459)	(0.0408)	(0.0394)	(0.0277)

	Nonmetallic Mineral Product	Foundry	Fabricated Metal Product	Machinery	All Other Electronic Product	Electrical Equipment, Component	Motor Vehicle, Part	All Retail Trade	All Wholesale Trade
a_{55}	0.666	1.26	0.541	0.479	0.405	1.22	1.01	0.759	0.421
	(0.0581)	(0.0156)	(0.0246)	(0.0564)	(0.0744)	(0.0179)	(0.0167)	(0.0265)	(0.0138)
a_{26}	-0.077	-0.0555	-0.0834	-0.0703	-0.0553	-0.0725	-0.0365	0.0657	0.082
	(0.0084)	(0.00932)	(0.0112)	(0.00885)	(0.0237)	(0.00659)	(0.00847)	(0.0163)	(0.00794)
a_{36}	-0.0624	-0.0653	-0.0893	-0.0728	0.00787	-0.0769	-0.0874	0.0443	0.0987
	(0.00474)	(0.0104)	(0.0115)	(0.0113)	(0.017)	(0.00864)	(0.0102)	(0.0144)	(0.00923)
a_{46}	-0.0631	-0.0539	-0.0678	-0.0449	0.141	-0.0436	-0.07	0.0634	0.0833
	(0.0118)	(0.0148)	(0.0135)	(0.0097)	(0.0307)	(0.0136)	(0.014)	(0.0166)	(0.0112)
a_{56}	-0.0711	-0.0743	-0.0358	-0.0666	-0.0462	-0.0691	-0.0767	0.0626	0.0620
	(0.0178)	(0.00905)	(0.0117)	(0.00801)	(0.0163)	(0.00912)	(0.0152)	(0.017)	(0.00829)
a_{66}	0.00493	0.00459	0.00633	0.0053	-0.00854	0.00491	0.00592	0.00466	0.009
	(0.000897)	(0.000954)	(0.00103)	(0.00103)	(0.00282)	(0.00104)	(0.00102)	(0.0016)	(0.00126)
b_{11}	1.48	0.956	0.959	1.41	1.71	1.23	1.18	1.16	1.18
	(0.0272)	(0.0284)	(0.00842)	(0.0115)	(0.0589)	(0.0163)	(0.0486)	(0.0222)	(0.0227)
b_{22}	1.08	1.32	0.649	0.867	1.06	1.27	0.763	0.931	1.02
	(0.0407)	(0.0237)	(0.0218)	(0.0228)	(0.022)	(0.0223)	(0.0233)	(0.0187)	(0.0141)
b_{33}	0.971	0.766	0.786	0.857	1.07	0.889	0.543	0.836	0.886
	(0.0205)	(0.0272)	(0.0565)	(0.025)	(0.0339)	(0.0206)	(0.014)	(0.0226)	(0.0212)
b_{44}	1.11	1.33	0.73	0.871	1.23	0.698	0.909	0.93	0.849
	(0.0121)	(0.0145)	(0.0926)	(0.0249)	(0.0249)	(0.0265)	(0.0554)	(0.0238)	(0.0167)
b_{55}	0.98	1.08	0.638	1.24	0.774	1.02	1.09	0.917	1.14
	(0.0125)	(0.0508)	(0.0691)	(0.0224)	(0.0517)	(0.0154)	(0.0515)	(0.0268)	(0.0213)

	Nonmetallic Mineral Product	Foundry	Fabricated Metal Product	Machinery	All Other Electronic Product	Electrical Equipment, Component	Motor Vehicle, Part	All Retail Trade	All Wholesale Trade
b_{66}	0.91	1.17	0.496	0.706	1.29	1.19	0.734	0.859	0.981
	(0.0245)	(0.0116)	(0.0394)	(0.0472)	(0.0256)	(0.0197)	(0.0136)	(0.0167)	(0.0157)
b_1	0.978	0.357	1.13	0.924	0.824	1.41	1.44	1.08	0.922
	(0.0134)	(0.02)	(0.00945)	(0.0162)	(0.0332)	(0.0217)	(0.0376)	(0.0195)	(0.0163)
b_2	0.598	0.158	0.704	1.05	0.969	1.1	0.725	0.87	0.75
	(0.0328)	(0.0231)	(0.0432)	(0.0456)	(0.0437)	(0.016)	(0.0434)	(0.0349)	(0.0215)
b_3	0.938	0.568	0.787	1.15	0.915	1.13	0.972	0.995	1.1
	(0.00634)	(0.0158)	(0.0179)	(0.0234)	(0.0419)	(0.0198)	(0.0282)	(0.0159)	(0.0132)
b_4	0.789	0.385	0.956	0.803	0.964	1.15	0.606	0.906	0.866
	(0.0142)	(0.0263)	(0.0237)	(0.0256)	(0.0631)	(0.0118)	(0.0241)	(0.0233)	(0.0196)
b_5	0.727	0.811	0.937	1.48	1.07	1.26	1.28	0.827	0.934
	(0.0174)	(0.0289)	(0.0132)	(0.0305)	(0.0428)	(0.0127)	(0.03)	(0.02)	(0.0162)
b_6	0.59	0.908	0.907	1.	0.789	1.24	0.882	0.872	1.04
	(0.0187)	(0.0123)	(0.0342)	(0.0543)	(0.0315)	(0.0153)	(0.0688)	(0.0255)	(0.0179)
b_{yy1}	-0.00862	-0.00366	-0.00227	-0.00579	-0.0036	-0.00603	-0.00383	0.00328	0.00459
	(0.000989)	(0.00109)	(0.00122)	(0.00123)	(0.00117)	(0.000732)	(0.00144)	(0.00153)	(0.000897)
b_{yy2}	-0.0892	-0.0594	-0.04	-0.0721	-0.0443	-0.0796	-0.0611	0.0523	0.0612
	(0.00522)	(0.0054)	(0.00657)	(0.00706)	(0.00869)	(0.00453)	(0.00654)	(0.00682)	(0.00476)
R^2	0.519	0.906	0.723	0.789	0.991	0.845	0.789	0.749	0.756
Adjusted R^2	-3.81	0.0646	-1.77	-1.11	0.905	-0.555	-1.11	1.53	0.774
AIC	1.83e6	1.41e6	1.07e6	8.21e5	6.7e5	1.43e6	1.01e6	1.54e6	1.38e6

Note: Standard deviations of each parameter estimation are in the parentheses. The number of observations is 31 and Bayesian sample size is 40,000 for all models in the table. Burn-in sizes vary from 0.5 million to 1.5 million depending on the convergence speed. Acceptance rates range from 0.19 to 0.31.

Functions and Values

M2 Money, $t + 0$ Output Sensitivity

Table 3.6: Estimated $\partial \text{output}_t / \partial \text{money price}_t$ with M2 Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model			6-Input 1-Period Model		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	-0.1453	-0.1756	-0.1925	-0.1258	-0.24	-0.1925	-0.03142	0.02539	-0.1293
Textile, Textile Product Mill	-0.2018	-0.03547	-0.2924	-0.1414	-0.02499	-0.2924	-0.04421	-0.04454	-0.1845
Apparel, Leather Product	0.4338	0.4429	0.4248	-0.1449	-0.2287	-0.2287	-0.06861	-0.2676	-0.059
Paper	-0.06376	-0.2036	-0.2233	-0.01399	-0.2389	-0.2233	-0.1005	-0.09619	-0.2162
Printing and Related	-0.02764	-0.1314	-0.235	0.004829	0.1729	-0.235	-0.08848	-0.08671	-0.1869
Petroleum, Coal Product	-0.005516	-0.361	-0.1712	0.0398	0.3938	-0.1712	0.007462	0.03691	-0.4077
All Other Chemical	-0.09917	-0.2117	-0.2631	-0.08811	-0.2163	-0.2631	-0.151	-0.1422	-0.1487
Plastics, Rubber Product	-0.1538	-0.122	-0.3526	-0.1453	-0.1451	-0.3526	-0.02222	-0.0139	-0.1283
Nonmetallic Mineral Product	-0.07812	-0.1508	-0.2339	-0.06319	-0.1982	-0.2339	-0.08141	-0.06564	-0.1222
Foundry	-0.1609	-0.05572	0.3288	-0.1447	-0.04751	0.3288	-0.06799	-0.06179	-0.2582
Fabricated Metal Product	-0.1054	-0.2059	-0.2314	-0.0704	-0.2027	-0.2314	-0.09135	-0.07831	-0.1429
Machinery	-0.2519	-0.01342	-0.3147	-0.2295	-0.0186	-0.3147	-0.1224	-0.09171	-0.1445
All Other Electronic Product	-0.2197	-0.1872	0.157	-0.2324	-0.2002	0.157	-0.09703	-0.06541	0.05041
Electrical Equipment, Component	-0.1385	-0.02026	-0.0775	-0.1266	-0.008161	-0.0775	-0.01873	-0.02083	-0.2081
Motor Vehicle, Part	-0.2317	-0.06943	-0.2477	-0.2302	-0.07207	-0.2477	-0.1382	-0.1241	-0.1058
All Retail Trade	-0.2516	-0.0796	-0.2646	-0.2564	-0.1165	-0.2646			
All Wholesale Trade	-0.2971	-0.05029	-0.5573	-0.3015	-0.03035	-0.5573			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.7: Estimated $\partial \text{output}_t / \partial \text{capital price}_t$ with M2 Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model			6-Input 1-Period Model		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	-0.2925	-0.4399	-0.2956	-0.3284	-0.4902	-0.2956	0.7807	1.413	-0.19
Textile, Textile Product Mill	-0.1852	-0.2614	-0.1432	-0.2363	-0.326	-0.1432	-0.4939	-0.4501	-0.168
Apparel, Leather Product	-0.4354	-0.4544	-0.3663	-0.5014	-0.765	-0.2246	0.5056	2.882	0.03248
Paper	-0.2782	-0.4005	-0.2464	-0.3386	-0.4331	-0.2464	1.445	1.487	-0.1596
Printing and Related	-0.2612	-0.3472	-0.189	-0.2946	-0.3635	-0.189	0.5998	0.5857	-0.1484
Petroleum, Coal Product	-0.4486	-0.5943	-0.3977	-0.5312	-0.6617	-0.3977	10.91	4.191	-0.3259
All Other Chemical	-0.3353	-0.4848	-0.2572	-0.357	-0.4892	-0.2572	0.9503	0.9508	-0.2019
Plastics, Rubber Product	-0.2601	-0.3838	-0.1747	-0.2876	-0.404	-0.1747	0.7771	0.997	-0.2252
Nonmetallic Mineral Product	-0.2248	-0.3339	-0.1915	-0.2726	-0.4033	-0.1915	0.711	0.9327	-0.2104
Foundry	-0.1761	-0.2175	0.1452	-0.1788	-0.2194	0.1452	0.4527	0.4638	-0.1215
Fabricated Metal Product	-0.3807	-0.5098	-0.2661	-0.3758	-0.4858	-0.2661	0.5741	1.416	-0.1999
Machinery	-0.2132	-0.3259	-0.1931	-0.2335	-0.3433	-0.1931	1.384	1.267	-0.1805
All Other Electronic Product	-0.4658	-0.5748	0.1769	-0.5216	-0.6671	0.1769	0.1695	0.308	0.07464
Electrical Equipment, Component	-0.1854	-0.2344	-0.02588	-0.1907	-0.2452	-0.02588	0.9496	0.8044	-0.1547
Motor Vehicle, Part	-0.2092	-0.3629	-0.2168	-0.2186	-0.358	-0.2168	0.7947	1.372	-0.2012
All Retail Trade	-0.2621	-0.4226	-0.2525	-0.2756	-0.4442	-0.2525			
All Wholesale Trade	-0.2008	-0.3226	-0.3739	-0.2054	-0.3334	-0.3739			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.8: Estimated $\partial \text{output}_t / \partial \text{labor price}_t$ with M2 Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model			6-Input 1-Period Model		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	-0.3676	-0.4945	-0.3352	-0.374	-0.5399	-0.3352	-0.5051	-0.8019	-0.2028
Textile, Textile Product Mill	-0.2546	-0.3021	-0.2236	-0.254	-0.3202	-0.2236	0.564	0.499	-0.1465
Apparel, Leather Product	-0.4543	-0.4622	-0.4147	0.1253	0.3971	-0.1628	-0.3521	-1.345	-0.08666
Paper	-0.4072	-0.4909	-0.3009	-0.3899	-0.4757	-0.3009	-0.906	-0.9139	-0.2267
Printing and Related	-0.3082	-0.3666	-0.2101	-0.3054	-0.3725	-0.2101	-0.4503	-0.4421	-0.1655
Petroleum, Coal Product	-0.3798	-0.564	-0.3304	-0.4241	-0.5675	-0.3304	-5.893	-2.279	-0.3314
All Other Chemical	-0.3471	-0.4869	-0.2819	-0.329	-0.4595	-0.2819	-0.6062	-0.5965	-0.202
Plastics, Rubber Product	-0.2958	-0.398	-0.2155	-0.3001	-0.4055	-0.2155	-0.4735	-0.5662	-0.2091
Nonmetallic Mineral Product	-0.3137	-0.4089	-0.2824	-0.3346	-0.4291	-0.2824	-0.4216	-0.5334	-0.2063
Foundry	-0.1679	-0.2111	0.1472	-0.1746	-0.2129	0.1472	-0.4358	-0.4328	-0.1375
Fabricated Metal Product	-0.3476	-0.4859	-0.3036	-0.3302	-0.4377	-0.3036	-0.4754	-0.9271	-0.2309
Machinery	-0.2115	-0.3339	-0.1812	-0.2239	-0.339	-0.1812	-0.9846	-0.8749	-0.1843
All Other Electronic Product	-0.4647	-0.5826	0.1492	-0.5336	-0.6771	0.1492	-0.2598	-0.3068	0.05669
Electrical Equipment, Component	-0.1988	-0.249	-0.03073	-0.197	-0.2445	-0.03073	-0.7041	-0.6134	-0.206
Motor Vehicle, Part	-0.276	-0.4063	-0.2456	-0.2593	-0.3989	-0.2456	-0.5659	-0.855	-0.1875
All Retail Trade	-0.2815	-0.4391	-0.2671	-0.2783	-0.4524	-0.2671			
All Wholesale Trade	-0.2118	-0.3342	-0.3846	-0.2231	-0.3521	-0.3846			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.9: Estimated $\partial \text{output}_t / \partial \text{intermediate input price}_t$ with M2 Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.3874	-0.5075	-0.3425	-0.4067	-0.5515	-0.3425
Textile, Textile Product Mill	-0.2593	-0.3239	-0.2211	-0.2713	-0.3152	-0.2211
Apparel, Leather Product	-0.5799	-0.5636	-0.678	-0.5843	-0.9567	-0.491
Paper	-0.3386	-0.4452	-0.2615	-0.3842	-0.4766	-0.2615
Printing and Related	-0.3342	-0.3814	-0.2124	-0.3265	-0.3886	-0.2124
Petroleum, Coal Product	-0.4152	-0.6021	-0.377	-0.5323	-0.6806	-0.377
All Other Chemical	-0.3532	-0.4723	-0.3036	-0.3985	-0.5078	-0.3036
Plastics, Rubber Product	-0.2832	-0.4006	-0.211	-0.3084	-0.4321	-0.211
Nonmetallic Mineral Product	-0.3706	-0.4428	-0.2888	-0.3953	-0.471	-0.2888
Foundry	-0.1688	-0.2152	0.1503	-0.1702	-0.213	0.1503
Fabricated Metal Product	-0.3536	-0.4735	-0.2735	-0.3487	-0.4513	-0.2735
Machinery	-0.1961	-0.3189	-0.159	-0.2168	-0.3279	-0.159
All Other Electronic Product	-0.04321	0.1168	0.2101	-0.01288	0.1697	0.2101
Electrical Equipment, Component	-0.1981	-0.2484	-0.02938	-0.1983	-0.2454	-0.02938
Motor Vehicle, Part	-0.2705	-0.3968	-0.255	-0.2713	-0.4236	-0.255
All Retail Trade	-0.2942	-0.4386	-0.2786	-0.2932	-0.4663	-0.2786
All Wholesale Trade	-0.1958	-0.3266	-0.3338	-0.1859	-0.3233	-0.3338

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.10: Estimated $\partial \text{output}_t / \partial \text{material, energy, business service price}_t$ with M2 Money Input

Industry	$\partial y_t/\partial mtr_t$			$\partial y_t/\partial ngy_t$			$\partial y_t/\partial svc_t$		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	-0.5651	-0.9181	-0.1889	-0.5872	-0.932	-0.2671	-0.7386	-1.151	-0.2371
Textile, Textile Product Mill	-0.3526	-0.3266	-0.2186	-0.5717	-0.5171	-0.2179	-0.3632	-0.3368	-0.2017
Apparel, Leather Product	-0.3663	-1.407	-0.212	-0.3769	-1.471	-0.4894	-0.3761	-1.428	-0.1615
Paper	-0.7259	-0.7333	-0.226	-0.9357	-0.9175	-0.25	-1.226	-1.197	-0.2118
Printing and Related	-0.3906	-0.3838	-0.2035	-0.4371	-0.4313	-0.1762	-0.4935	-0.4821	-0.1676
Petroleum, Coal Product	-6.518	-2.414	-0.6104	-4.69	-1.934	-0.3518	-6.307	-2.432	-0.3479
All Other Chemical	-0.6011	-0.6126	-0.1869	-0.5698	-0.5491	-0.2212	-0.654	-0.6469	-0.2225
Plastics, Rubber Product	-0.6162	-0.7418	-0.1989	-0.6225	-0.73	-0.1897	-0.6077	-0.7308	-0.2334
Nonmetallic Mineral Product	-0.5328	-0.6502	-0.2435	-0.5676	-0.6896	-0.2247	-0.4638	-0.5661	-0.1912
Foundry	-0.3772	-0.3648	-0.1328	-0.4211	-0.4276	-0.1552	-0.2578	-0.2694	-0.1423
Fabricated Metal Product	-0.4479	-0.8891	-0.2174	-0.334	-0.6651	-0.2532	-0.5301	-1.088	-0.2602
Machinery	-0.9093	-0.8122	-0.1631	-1.018	-0.9318	-0.1972	-0.9397	-0.8417	-0.1937
All Other Electronic Product	-0.359	-0.3865	0.06931	-0.2816	-0.307	0.06602	-0.4004	-0.4479	0.05517
Electrical Equipment, Component	-0.6248	-0.5383	-0.2422	-0.6779	-0.5937	-0.1982	-0.595	-0.5195	-0.2021
Motor Vehicle, Part	-0.6165	-0.9321	-0.2021	-0.6201	-0.9825	-0.241	-0.5699	-0.827	-0.1967

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

M2 Money, $t + 1$ Output Sensitivity

Table 3.11: Estimated $\partial \text{output}_{t+1} / \partial \text{money price}_t$ with M2 Aggregate As Money Input

Industry	$\partial y_t / \partial p_{mny,t}$			$\partial y_{t+1} / \partial p_{mny,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.1258	-0.24	-0.1925	-0.1308	-0.2803	-0.2061
Textile, Textile Product Mill	-0.1414	-0.02499	-0.2924	-0.1482	-0.02798	-0.3074
Apparel, Leather Product	-0.1449	-0.2287	-0.2287	-0.1536	-0.2505	-0.2454
Paper	-0.01399	-0.2389	-0.2233	-0.0101	-0.2666	-0.2383
Printing and Related	0.004829	0.1729	-0.235	0.006167	0.1855	-0.2479
Petroleum, Coal Product	0.0398	0.3938	-0.1712	0.05877	0.468	-0.1837
All Other Chemical	-0.08811	-0.2163	-0.2631	-0.08964	-0.2476	-0.282
Plastics, Rubber Product	-0.1453	-0.1451	-0.3526	-0.1528	-0.1626	-0.3749
Nonmetallic Mineral Product	-0.06319	-0.1982	-0.2339	-0.0634	-0.2204	-0.2481
Foundry	-0.1447	-0.04751	0.3288	-0.1523	-0.05248	0.2527
Fabricated Metal Product	-0.0704	-0.2027	-0.2314	-0.07131	-0.2288	-0.2472
Machinery	-0.2295	-0.0186	-0.3147	-0.2424	-0.02224	-0.3317
All Other Electronic Product	-0.2324	-0.2002	0.157	-0.2515	-0.2163	0.1099
Electrical Equipment, Component	-0.1266	-0.008161	-0.0775	-0.1335	-0.01104	-0.08745
Motor Vehicle, Part	-0.2302	-0.07207	-0.2477	-0.2437	-0.08365	-0.2627
All Retail Trade	-0.2564	-0.1165	-0.2646	-0.2737	-0.1375	-0.2825
All Wholesale Trade	-0.3015	-0.03035	-0.5573	-0.3187	-0.02896	-0.6373

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.12: Estimated $\partial \text{output}_{t+1} / \partial \text{capital price}_t$ with M2 Aggregate As Money Input

Industry	$\partial y_t / \partial p_{cpt,t}$			$\partial y_{t+1} / \partial p_{cpt,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.3284	-0.4902	-0.2956	-0.3562	-0.5406	-0.3176
Textile, Textile Product Mill	-0.2363	-0.326	-0.1432	-0.2502	-0.3455	-0.152
Apparel, Leather Product	-0.5014	-0.765	-0.2246	-0.5418	-0.8741	-0.2416
Paper	-0.3386	-0.4331	-0.2464	-0.3641	-0.4678	-0.2618
Printing and Related	-0.2946	-0.3635	-0.189	-0.312	-0.3858	-0.1997
Petroleum, Coal Product	-0.5312	-0.6617	-0.3977	-0.5935	-0.7473	-0.4336
All Other Chemical	-0.357	-0.4892	-0.2572	-0.3861	-0.5342	-0.2751
Plastics, Rubber Product	-0.2876	-0.404	-0.1747	-0.3071	-0.4344	-0.1847
Nonmetallic Mineral Product	-0.2726	-0.4033	-0.1915	-0.2916	-0.4339	-0.2031
Foundry	-0.1788	-0.2194	0.1452	-0.1883	-0.23	0.09816
Fabricated Metal Product	-0.3758	-0.4858	-0.2661	-0.4046	-0.5266	-0.2841
Machinery	-0.2335	-0.3433	-0.1931	-0.2469	-0.3655	-0.2038
All Other Electronic Product	-0.5216	-0.6671	0.1769	-0.5741	-0.7417	0.1304
Electrical Equipment, Component	-0.1907	-0.2452	-0.02588	-0.2006	-0.2572	-0.03875
Motor Vehicle, Part	-0.2186	-0.358	-0.2168	-0.2321	-0.3851	-0.2297
All Retail Trade	-0.2756	-0.4442	-0.2525	-0.2953	-0.4849	-0.2695
All Wholesale Trade	-0.2054	-0.3334	-0.3739	-0.2176	-0.3566	-0.4246

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.13: Estimated $\partial \text{output}_{t+1}/\partial \text{labor price}_t$ with M2 Aggregate As Money Input

Industry	$\partial y_t/\partial p_{lbr,t}$			$\partial y_{t+1}/\partial p_{lbr,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.374	-0.5399	-0.3352	-0.4066	-0.5965	-0.3608
Textile, Textile Product Mill	-0.254	-0.3202	-0.2236	-0.269	-0.3395	-0.2363
Apparel, Leather Product	0.1253	0.3971	-0.1628	0.1423	0.476	-0.1736
Paper	-0.3899	-0.4757	-0.3009	-0.4196	-0.514	-0.3201
Printing and Related	-0.3054	-0.3725	-0.2101	-0.3232	-0.3952	-0.2211
Petroleum, Coal Product	-0.4241	-0.5675	-0.3304	-0.4711	-0.6391	-0.3584
All Other Chemical	-0.329	-0.4595	-0.2819	-0.3548	-0.501	-0.301
Plastics, Rubber Product	-0.3001	-0.4055	-0.2155	-0.3204	-0.4359	-0.2277
Nonmetallic Mineral Product	-0.3346	-0.4291	-0.2824	-0.3581	-0.4615	-0.3
Foundry	-0.1746	-0.2129	0.1472	-0.1838	-0.2233	0.09951
Fabricated Metal Product	-0.3302	-0.4377	-0.3036	-0.355	-0.4741	-0.3243
Machinery	-0.2239	-0.339	-0.1812	-0.2365	-0.3606	-0.1908
All Other Electronic Product	-0.5336	-0.6771	0.1492	-0.5892	-0.7559	0.1035
Electrical Equipment, Component	-0.197	-0.2445	-0.03073	-0.2072	-0.2564	-0.04342
Motor Vehicle, Part	-0.2593	-0.3989	-0.2456	-0.2752	-0.4291	-0.26
All Retail Trade	-0.2783	-0.4524	-0.2671	-0.2982	-0.4939	-0.2846
All Wholesale Trade	-0.2231	-0.3521	-0.3846	-0.2359	-0.3763	-0.436

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.14: Estimated $\partial \text{output}_{t+1} / \partial \text{intermediate input price}_t$ with M2 Aggregate As Money Input

Industry	$\partial y_t / \partial p_{iip,t}$			$\partial y_{t+1} / \partial p_{iip,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.4067	-0.5515	-0.3425	-0.4447	-0.612	-0.3714
Textile, Textile Product Mill	-0.2713	-0.3152	-0.2211	-0.2879	-0.3351	-0.2347
Apparel, Leather Product	-0.5843	-0.9567	-0.491	-0.6301	-1.091	-0.5298
Paper	-0.3842	-0.4766	-0.2615	-0.4155	-0.5172	-0.28
Printing and Related	-0.3265	-0.3886	-0.2124	-0.3468	-0.4136	-0.2251
Petroleum, Coal Product	-0.5323	-0.6806	-0.377	-0.5974	-0.773	-0.4134
All Other Chemical	-0.3985	-0.5078	-0.3036	-0.4329	-0.5568	-0.3272
Plastics, Rubber Product	-0.3084	-0.4321	-0.211	-0.3307	-0.4663	-0.2249
Nonmetallic Mineral Product	-0.3953	-0.471	-0.2888	-0.4251	-0.509	-0.3085
Foundry	-0.1702	-0.213	0.1503	-0.1799	-0.224	0.1023
Fabricated Metal Product	-0.3487	-0.4513	-0.2735	-0.3769	-0.4909	-0.2939
Machinery	-0.2168	-0.3279	-0.159	-0.2304	-0.3503	-0.1692
All Other Electronic Product	-0.01288	0.1697	0.2101	-0.003003	0.2046	0.1523
Electrical Equipment, Component	-0.1983	-0.2454	-0.02938	-0.209	-0.258	-0.04216
Motor Vehicle, Part	-0.2713	-0.4236	-0.255	-0.2895	-0.4576	-0.2718
All Retail Trade	-0.2932	-0.4663	-0.2786	-0.3159	-0.5113	-0.2992
All Wholesale Trade	-0.1859	-0.3233	-0.3338	-0.198	-0.3467	-0.3787

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

M2 Money, $t + 0, t + 1$ Investment Sensitivity

Table 3.15: Estimated ∂investment/∂money price with M2 Aggregate As Money Input

Industry	$\partial ivm_t / \partial p_{mny,t}$			$\partial ivm_{t+1} / \partial p_{mny,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.08778	-0.419	0.02324	-0.2492	-0.5699	-0.02065
Textile, Textile Product Mill	-0.2844	-0.5683	-0.09624	-0.3617	-0.5428	-0.1824
Apparel, Leather Product	-0.1041	-0.07237	-0.002251	-0.04368	0.1377	0.02034
Paper	-0.2351	-0.5612	0.07216	-0.2943	-0.6334	0.01258
Printing and Related	-0.4524	-0.7484	-0.205	-0.4615	-0.7757	0.01137
Petroleum, Coal Product	-0.364	-0.6322	-0.05402	-0.349	-0.6536	-0.01775
All Other Chemical	-0.2671	-0.5559	-0.06045	-0.2968	-0.5885	-0.002528
Plastics, Rubber Product	-0.1992	-0.5423	0.1169	-0.242	-0.6187	0.1149
Nonmetallic Mineral Product	-0.2153	-0.5935	-0.02516	-0.3932	-0.6883	-0.07519
Foundry	-0.523	-0.7778	-0.7651	-0.4473	-0.7506	-0.8174
Fabricated Metal Product	-0.3104	-0.6138	-0.04043	-0.2632	-0.5814	-0.02138
Machinery	-0.1218	-0.4494	0.007541	-0.07876	-0.4164	0.08121
All Other Electronic Product	-0.06574	-0.07363	-0.01009	0.08273	0.1036	-0.02238
Electrical Equipment, Component	-0.4087	-0.6801	-0.5395	-0.4456	-0.7217	-0.5521
Motor Vehicle, Part	-0.05165	-0.4005	0.037	-0.13	-0.4873	-0.03716
All Retail Trade	-0.08633	-0.4213	-0.04765	-0.123	-0.4687	-0.04278
All Wholesale Trade	-0.03464	-0.3521	-0.0216	-0.05455	-0.3819	-0.1508

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.16: Estimated ∂investment/∂capital price with M2 Aggregate As Money Input

Industry	$\partial ivm_t / \partial p_{cpt,t}$			$\partial ivm_{t+1} / \partial p_{cpt,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	0.2132	0.3237	0.2328	-0.005286	0.1735	-0.124
Textile, Textile Product Mill	0.3461	0.4639	0.257	0.07472	0.1783	-0.0443
Apparel, Leather Product	0.7352	0.7363	-0.06178	0.7302	0.7516	0.1642
Paper	0.3167	0.3835	0.2238	0.02076	0.2237	-0.1361
Printing and Related	0.3668	0.45	0.3724	0.1626	0.3276	-0.05494
Petroleum, Coal Product	0.3868	0.4395	0.2885	0.1139	0.269	-0.0717
All Other Chemical	0.3017	0.3795	0.2957	0.1016	0.2451	-0.07241
Plastics, Rubber Product	0.2864	0.38	0.2119	0.03279	0.233	-0.1794
Nonmetallic Mineral Product	0.2799	0.3999	0.2484	0.07488	0.2705	-0.1028
Foundry	0.3914	0.4878	0.5687	0.1875	0.3244	0.4397
Fabricated Metal Product	0.3894	0.4673	0.2675	0.035	0.2178	-0.1014
Machinery	0.2577	0.352	0.2716	-0.04456	0.1462	-0.1104
All Other Electronic Product	0.6427	0.7829	0.1362	-0.9848	-1.353	-1.494
Electrical Equipment, Component	0.3631	0.4255	0.4115	0.09381	0.263	0.1351
Motor Vehicle, Part	0.2567	0.3478	0.2348	-0.04991	0.1569	-0.09524
All Retail Trade	0.2592	0.3657	0.278	-0.03714	0.1569	-0.06824
All Wholesale Trade	0.2457	0.3108	0.2438	-0.05296	0.1329	-0.01562

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.17: Estimated ∂investment/∂labor price with M2 Aggregate As Money Input

Industry	$\partial ivm_t/\partial p_{lbr,t}$			$\partial ivm_{t+1}/\partial p_{lbr,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.08302	0.09871	-0.1419	0.03199	0.2357	-0.11
Textile, Textile Product Mill	-0.01638	0.1661	-0.07582	0.12	0.1996	-0.01591
Apparel, Leather Product	-1.317	-1.385	-0.09927	-1.532	-1.814	-0.9428
Paper	-0.003156	0.1911	-0.1904	0.02261	0.2168	-0.1464
Printing and Related	0.1139	0.2868	-0.026	0.1342	0.3088	-0.1757
Petroleum, Coal Product	0.01707	0.1906	-0.14	0.002109	0.1889	-0.1628
All Other Chemical	-0.02342	0.166	-0.1373	-0.02347	0.1499	-0.1786
Plastics, Rubber Product	-0.03588	0.1583	-0.1987	-0.0000281	0.2094	-0.2022
Nonmetallic Mineral Product	-0.04865	0.1736	-0.1098	0.0788	0.2216	-0.06998
Foundry	0.165	0.2941	0.07974	0.0598	0.261	0.1899
Fabricated Metal Product	0.01711	0.1878	-0.1086	-0.008067	0.1772	-0.1232
Machinery	-0.0399	0.1411	-0.1719	-0.1341	0.06952	-0.2347
All Other Electronic Product	0.4356	0.6743	1.372	-0.9937	-1.334	-1.362
Electrical Equipment, Component	0.1243	0.2788	0.2207	0.1458	0.2853	0.2004
Motor Vehicle, Part	-0.1333	0.06492	-0.1922	-0.07199	0.1358	-0.137
All Retail Trade	-0.09929	0.07666	-0.142	-0.06488	0.1402	-0.1472
All Wholesale Trade	-0.1435	0.05706	-0.1975	-0.1127	0.08786	-0.1197

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

M2 Money, $t + 0$ **Labor Demand Sensitivity**

Table 3.18: Estimated $\partial \text{labor}_t / \partial \text{money price}_t$ with M2 Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model			6-Input 1-Period Model		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	0.4503	0.6248	0.4617	0.4296	0.5777	0.4617	0.784	0.7921	0.1959
Textile, Textile Product Mill	0.9451	0.8213	0.8579	3.122	3.638	1.184	0.2727	0.2766	0.05062
Apparel, Leather Product	0.4695	0.6782	0.3425	0.6544	0.7811	0.3425	0.302	0.3118	0.1501
Paper	0.53	0.6939	0.3763	0.6382	0.7777	0.3763	0.2783	0.2772	0.141
Printing and Related	0.614	0.5919	0.2552	0.3951	0.5318	0.2552	0.1876	0.1913	0.1309
Petroleum, Coal Product	0.5762	0.5911	0.2922	0.5066	0.6623	0.2922	0.2985	0.3112	0.144
All Other Chemical	0.4487	0.6272	0.3253	0.4918	0.6377	0.3253	0.2395	0.2359	0.1455
Plastics, Rubber Product	0.3772	0.6405	0.4391	0.566	0.7417	0.4391	0.1665	0.186	0.1406
Nonmetallic Mineral Product	0.6147	0.6512	1.129	0.5927	0.7097	1.129	0.3656	0.3678	0.1428
Foundry	0.6662	0.6521	0.3945	0.4877	0.6216	0.3945	0.3624	0.3418	0.1467
Fabricated Metal Product	0.4054	0.5836	0.2522	0.3838	0.5975	0.2522	0.2081	0.2155	0.1472
Machinery	-0.01119	0.7622	-0.0831	0.6339	0.7696	-0.0831	0.0667	0.08296	0.8795
All Other Electronic Product	0.5239	0.7836	0.3964	0.5186	0.6793	0.3964	0.3858	0.3851	0.1475
Electrical Equipment, Component	0.4862	0.6717	0.3173	0.5199	0.6655	0.3173	0.2248	0.2462	0.137
Motor Vehicle, Part	0.4521	0.6575	0.316	0.4693	0.6702	0.316	-0.05794	-0.0583	-0.02538
All Retail Trade	0.3173	0.4984	0.2356	0.4504	0.5986	0.2356			
All Wholesale Trade	-0.1789	0.1928	-0.227	-0.1328	0.1579	-0.227			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.19: Estimated $\partial \text{labor}_t / \partial \text{capital price}_t$ with M2 Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model			6-Input 1-Period Model		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	-0.1789	-0.1928	-0.227	-0.1328	-0.1579	-0.227	-0.05794	-0.0583	-0.02538
Textile, Textile Product Mill	-0.3367	-0.1444	-0.1213	-0.02621	-0.2657	-0.1213	-0.5349	-0.5396	-0.1585
Apparel, Leather Product	-2.668	-0.8177	0.6901	-1.531	-1.592	-0.1127	-0.07759	-0.1486	-0.003801
Paper	-0.3075	-0.2567	-0.3046	-0.00505	-0.3057	-0.3046	-0.07806	-0.07998	-0.03303
Printing and Related	-0.8845	-0.4188	-0.0416	0.1823	0.4589	-0.0416	-0.07076	-0.07088	-0.02892
Petroleum, Coal Product	-0.6965	-0.2579	-0.224	0.02731	0.3049	-0.224	-0.07514	-0.07369	-0.03068
All Other Chemical	-0.5549	-0.3376	-0.2198	-0.03748	-0.2657	-0.2198	-0.07605	-0.08267	-0.02857
Plastics, Rubber Product	-0.3146	-0.3226	-0.3179	-0.05741	-0.2532	-0.3179	-0.06936	-0.07022	-0.02877
Nonmetallic Mineral Product	-0.2592	-0.2018	-0.1757	-0.07783	-0.2778	-0.1757	-0.05368	-0.05574	-0.02633
Foundry	-1.047	-0.4522	0.1276	0.264	0.4705	0.1276	-0.09163	-0.09197	-0.02888
Fabricated Metal Product	-0.6268	-0.2922	-0.1738	0.02738	0.3005	-0.1738	-0.09146	-0.08974	-0.02891
Machinery	-0.2096	-0.2332	-0.2751	-0.06383	-0.2257	-0.2751	-0.05592	-0.0529	-0.03016
All Other Electronic Product	0.07748	0.6732	1.544	0.4901	0.7586	1.544	-0.008837	-0.02292	-0.2033
Electrical Equipment, Component	-0.7932	-0.3734	0.3531	0.1988	0.4461	0.3531	-0.0931	-0.09176	-0.03324
Motor Vehicle, Part	-0.05214	-0.0904	-0.3075	-0.2133	-0.1039	-0.3075	-0.05783	-0.06748	-0.02874
All Retail Trade	-0.1639	-0.1039	-0.2272	-0.1589	-0.1227	-0.2272			
All Wholesale Trade	-0.003076	-0.04718	-0.3159	-0.2296	-0.0913	-0.3159			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.20: Estimated $\partial \text{labor}_t / \partial \text{labor price}_t$ with M2 Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model			6-Input 1-Period Model		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	-0.324	-0.6271	0.3996	-0.5205	-0.6901	0.3996	-0.1688	-0.1745	0.1386
Textile, Textile Product Mill	-0.4503	-0.6248	0.4617	-0.4296	-0.5777	0.4617	-0.784	-0.7921	0.1959
Apparel, Leather Product	-0.9451	-0.8213	0.8579	-3.122	-3.638	1.184	-0.2727	-0.2766	0.05062
Paper	-0.4695	-0.6782	0.3425	-0.6544	-0.7811	0.3425	-0.302	-0.3118	0.1501
Printing and Related	-0.53	-0.6939	0.3763	-0.6382	-0.7777	0.3763	-0.2783	-0.2772	0.141
Petroleum, Coal Product	-0.614	-0.5919	0.2552	-0.3951	-0.5318	0.2552	-0.1876	-0.1913	0.1309
All Other Chemical	-0.5762	-0.5911	0.2922	-0.5066	-0.6623	0.2922	-0.2985	-0.3112	0.144
Plastics, Rubber Product	-0.4487	-0.6272	0.3253	-0.4918	-0.6377	0.3253	-0.2395	-0.2359	0.1455
Nonmetallic Mineral Product	-0.3772	-0.6405	0.4391	-0.566	-0.7417	0.4391	-0.1665	-0.186	0.1406
Foundry	-0.6147	-0.6512	1.129	-0.5927	-0.7097	1.129	-0.3656	-0.3678	0.1428
Fabricated Metal Product	-0.6662	-0.6521	0.3945	-0.4877	-0.6216	0.3945	-0.3624	-0.3418	0.1467
Machinery	-0.4054	-0.5836	0.2522	-0.3838	-0.5975	0.2522	-0.2081	-0.2155	0.1472
All Other Electronic Product	-0.01119	-0.7622	-0.0831	-0.6339	-0.7696	-0.0831	-0.0667	-0.08296	0.8795
Electrical Equipment, Component	-0.5239	-0.7836	0.3964	-0.5186	-0.6793	0.3964	-0.3858	-0.3851	0.1475
Motor Vehicle, Part	-0.4862	-0.6717	0.3173	-0.5199	-0.6655	0.3173	-0.2248	-0.2462	0.137
All Retail Trade	-0.4521	-0.6575	0.316	-0.4693	-0.6702	0.316			
All Wholesale Trade	-0.3173	-0.4984	0.2356	-0.4504	-0.5986	0.2356			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

MZM Money, $t + 0$ Output Sensitivity

Table 3.21: Estimated $\partial \text{output}_t / \partial \text{money price}_t$ with MZM Aggregate As Money Input

Industry	**4-Input 1-Period Model**			**4-Input 2-Period Model**			**6-Input 1-Period Model**		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	-0.2002	-0.2029	-0.2777	-0.1425	-0.09267	-0.2505	-0.3695	-0.3739	-0.0306
Textile, Textile Product Mill	-0.2312	-0.267	-0.1696	-0.02502	-0.04449	-0.2918	-0.358	-0.3608	0.1562
Apparel, Leather Product	-0.2098	-0.2325	-0.5047	-0.158	-0.2016	-0.174	-0.325	-0.3248	-0.3483
Paper	-0.2723	-0.2598	-0.2288	-0.09227	-0.05881	-0.2928	-0.3725	-0.3799	0.1364
Printing and Related	-0.2345	-0.2342	-0.2725	-0.1011	-0.07651	-0.2371	-0.3549	-0.3573	-0.2649
Petroleum, Coal Product	-0.1872	-0.178	-0.366	-0.04738	-0.0638	-0.2156	-0.3425	-0.2646	-0.3674
All Other Chemical	-0.2306	-0.222	-0.3504	-0.1288	-0.09442	-0.2189	-0.3605	-0.3661	0.07015
Plastics, Rubber Product	-0.2148	-0.2144	-0.2451	-0.1276	-0.07923	-0.2516	-0.3664	-0.3687	0.1267
Nonmetallic Mineral Product	-0.2379	-0.2362	-0.2145	-0.07445	-0.05545	-0.2209	-0.3564	-0.3569	0.0451
Foundry	-0.2757	-0.2559	-0.147	-0.2212	-0.1637	-0.3097	-0.3628	-0.3605	-0.2504
Fabricated Metal Product	-0.2398	-0.2535	-0.272	-0.1301	-0.08223	-0.2797	-0.364	-0.3684	0.02575
Machinery	-0.2331	-0.2444	-0.2705	-0.08102	-0.08706	-0.25	-0.376	-0.3797	0.1638
All Other Electronic Product	-0.3291	-0.2508	0.1374	-0.228	-0.2001	0.1508	-0.2847	-0.2977	-0.304
Electrical Equipment, Component	-0.2302	-0.2437	-0.2037	-0.0888	-0.05786	-0.2566	-0.3594	-0.3646	-0.01356
Motor Vehicle, Part	-0.1883	-0.1842	-0.2871	-0.0617	-0.07174	-0.1992	-0.3644	-0.3705	0.0259
All Retail Trade	-0.2064	-0.2181	-0.2827	-0.1917	-0.119	-0.2318			
All Wholesale Trade	-0.2482	-0.2351	-0.2689	-0.227	-0.1606	-0.2737			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.22: Estimated $\partial \text{output}_t / \partial \text{capital price}_t$ with MZM Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model			6-Input 1-Period Model		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	-0.277	-0.2808	-0.4396	-0.2872	-0.2738	-0.277	-0.3294	-0.3187	-0.4644
Textile, Textile Product Mill	-0.3049	-0.3214	-0.3807	-0.254	-0.2412	-0.1677	-0.3507	-0.3359	-0.1389
Apparel, Leather Product	-0.1472	-0.1278	0.1176	0.167	0.3195	0.1979	-0.3245	-0.3241	-0.3526
Paper	-0.3337	-0.325	-0.3941	-0.2372	-0.2343	-0.2154	-0.3376	-0.3232	-0.3848
Printing and Related	-0.2556	-0.2483	-0.2353	-0.2157	-0.2208	-0.1513	-0.3295	-0.3267	-0.3548
Petroleum, Coal Product	-0.3145	-0.3161	-0.2925	-0.2476	-0.2331	-0.3431	-0.2998	-0.234	-0.3137
All Other Chemical	-0.3908	-0.3557	-0.2258	-0.2649	-0.2505	-0.3005	-0.3412	-0.3257	-0.4342
Plastics, Rubber Product	-0.2782	-0.274	-0.4261	-0.2774	-0.2521	-0.2645	-0.345	-0.3248	-0.3118
Nonmetallic Mineral Product	-0.3048	-0.3186	-0.4376	-0.2564	-0.2474	-0.2425	-0.3375	-0.3319	-0.3822
Foundry	-0.1549	-0.1566	-0.1981	-0.1645	-0.1752	-0.127	-0.3371	-0.3347	-0.2953
Fabricated Metal Product	-0.3395	-0.3348	-0.4267	-0.2845	-0.2792	-0.2407	-0.3405	-0.3245	-0.4584
Machinery	-0.2645	-0.2571	-0.2903	-0.2338	-0.2184	-0.2489	-0.3316	-0.3248	-0.319
All Other Electronic Product	-0.2917	-0.2533	0.1853	0.1032	0.2031	0.1998	-0.3916	-0.3841	-0.3939
Electrical Equipment, Component	-0.2434	-0.2432	-0.3477	-0.2172	-0.2303	-0.1986	-0.3671	-0.3455	-0.3059
Motor Vehicle, Part	-0.2817	-0.2941	-0.2749	-0.254	-0.2421	-0.2856	-0.3449	-0.3349	-0.4078
All Retail Trade	-0.2449	-0.251	-0.2532	-0.2929	-0.2902	-0.2881			
All Wholesale Trade	-0.2365	-0.238	-0.2493	-0.2712	-0.2699	-0.2682			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.23: Estimated $\partial \text{output}_t / \partial \text{labor price}_t$ with MZM Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model			6-Input 1-Period Model		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	-0.3017	-0.3181	-0.3873	-0.4219	-0.3695	-0.45	-0.3509	-0.3515	-0.4005
Textile, Textile Product Mill	-0.3683	-0.3557	-0.3702	-0.3266	-0.2901	-0.367	-0.3529	-0.3539	-0.4717
Apparel, Leather Product	-0.299	-0.3139	-0.2583	-0.5298	-0.6993	-0.5589	-0.3536	-0.3522	-0.3318
Paper	-0.3697	-0.3566	-0.307	-0.3905	-0.3399	-0.4212	-0.3474	-0.3514	-0.4607
Printing and Related	-0.2679	-0.2705	-0.1412	-0.2862	-0.2611	-0.268	-0.3508	-0.3511	-0.3947
Petroleum, Coal Product	-0.2976	-0.3296	-0.3553	-0.342	-0.2896	-0.3122	-0.3384	-0.2626	-0.3562
All Other Chemical	-0.3778	-0.36	-0.2939	-0.3834	-0.3375	-0.3621	-0.3511	-0.3514	-0.4303
Plastics, Rubber Product	-0.3347	-0.34	-0.3675	-0.4158	-0.3472	-0.3996	-0.3486	-0.3536	-0.4842
Nonmetallic Mineral Product	-0.3584	-0.3599	-0.4098	-0.3943	-0.3444	-0.3911	-0.3688	-0.3714	-0.4449
Foundry	-0.166	-0.1695	-0.1713	-0.2264	-0.2245	-0.2096	-0.3535	-0.3507	-0.4311
Fabricated Metal Product	-0.3401	-0.3492	-0.3664	-0.459	-0.3706	-0.4759	-0.3558	-0.3564	-0.4161
Machinery	-0.2907	-0.3032	-0.2581	-0.3046	-0.2647	-0.3037	-0.3479	-0.3481	-0.4603
All Other Electronic Product	-0.3267	-0.2819	0.1721	-0.6303	-0.6953	0.175	-0.3382	-0.3368	-0.3287
Electrical Equipment, Component	-0.3008	-0.3049	-0.2962	-0.3343	-0.3042	-0.3533	-0.3576	-0.3606	-0.4422
Motor Vehicle, Part	-0.2905	-0.2952	-0.3348	-0.3353	-0.2861	-0.2674	-0.3441	-0.343	-0.4253
All Retail Trade	-0.2772	-0.3075	-0.3044	-0.3543	-0.324	-0.2526			
All Wholesale Trade	-0.2525	-0.2855	-0.2611	-0.3345	-0.3192	-0.2512			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.24: Estimated $\partial \text{output}_t / \partial \text{intermediate input price}_t$ with MZM Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.3492	-0.3469	-0.2421	-0.3695	-0.3298	-0.3966
Textile, Textile Product Mill	-0.3999	-0.2901	-0.2593	-0.3055	-0.2691	-0.3467
Apparel, Leather Product	-0.4169	-0.3886	-0.3542	-0.5822	-0.7681	-0.5687
Paper	-0.3651	-0.3307	-0.2562	-0.3075	-0.2832	-0.3262
Printing and Related	-0.2412	-0.2514	-0.2801	-0.2599	-0.2438	-0.2554
Petroleum, Coal Product	-0.3595	-0.3549	-0.2137	-0.3187	-0.2785	-0.3818
All Other Chemical	-0.386	-0.3743	-0.3794	-0.3503	-0.306	-0.3359
Plastics, Rubber Product	-0.3467	-0.3444	-0.2622	-0.3656	-0.324	-0.3412
Nonmetallic Mineral Product	-0.4733	-0.4394	-0.2401	-0.3323	-0.2998	-0.3609
Foundry	-0.1934	-0.197	-0.3029	-0.1928	-0.2006	-0.1842
Fabricated Metal Product	-0.3854	-0.3762	-0.2516	-0.3875	-0.312	-0.4124
Machinery	-0.2712	-0.2784	-0.236	-0.2584	-0.237	-0.2683
All Other Electronic Product	-0.3482	-0.3328	0.132	-0.6193	-0.6841	0.1649
Electrical Equipment, Component	-0.3099	-0.3031	-0.2444	-0.2953	-0.2809	-0.3024
Motor Vehicle, Part	-0.335	-0.3322	-0.2006	-0.3047	-0.2664	-0.3281
All Retail Trade	-0.3127	-0.3324	-0.2225	-0.3336	-0.3041	-0.3057
All Wholesale Trade	-0.2358	-0.2321	-0.2683	-0.2637	-0.2664	-0.2615

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.25: Estimated $\partial \text{output}_t / \partial \text{material, energy, business service price}_t$ with MZM Money Input

Industry	$\partial y_t/\partial mtr_t$			$\partial y_t/\partial ngy_t$			$\partial y_t/\partial svc_t$		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	-0.3489	-0.351	-0.3687	-0.3671	-0.3672	-0.3851	-0.3413	-0.3423	-0.4056
Textile, Textile Product Mill	-0.3493	-0.3575	-0.5298	-0.342	-0.3448	-0.5589	-0.3612	-0.3643	-0.4938
Apparel, Leather Product	-0.3579	-0.3593	-0.3323	-0.32	-0.3208	-0.3391	-0.3537	-0.3552	-0.3358
Paper	-0.3545	-0.358	-0.447	-0.358	-0.3622	-0.4202	-0.3604	-0.3636	-0.4695
Printing and Related	-0.3406	-0.3404	-0.3574	-0.371	-0.37	-0.3204	-0.3534	-0.354	-0.3917
Petroleum, Coal Product	-0.3253	-0.253	-0.3562	-0.3371	-0.2626	-0.3562	-0.3394	-0.2606	-0.3562
All Other Chemical	-0.3514	-0.3568	-0.3999	-0.3609	-0.3662	-0.4246	-0.354	-0.3564	-0.4389
Plastics, Rubber Product	-0.355	-0.3635	-0.4534	-0.361	-0.3655	-0.4429	-0.3347	-0.3388	-0.4955
Nonmetallic Mineral Product	-0.3573	-0.358	-0.4211	-0.3447	-0.3455	-0.4086	-0.347	-0.348	-0.4353
Foundry	-0.3392	-0.3374	-0.3447	-0.3393	-0.3373	-0.3804	-0.3493	-0.3461	-0.4054
Fabricated Metal Product	-0.3382	-0.342	-0.3979	-0.3511	-0.3549	-0.3972	-0.3661	-0.3679	-0.4116
Machinery	-0.3492	-0.3516	-0.512	-0.3505	-0.3506	-0.4684	-0.3486	-0.3511	-0.4572
All Other Electronic Product	-0.3404	-0.3515	-0.4515	-0.4212	-0.4074	-0.2111	-0.2853	-0.2801	-0.4652
Electrical Equipment, Component	-0.3532	-0.355	-0.4028	-0.3458	-0.3521	-0.4498	-0.3387	-0.3485	-0.4513
Motor Vehicle, Part	-0.3375	-0.3403	-0.4132	-0.3433	-0.3484	-0.3942	-0.3442	-0.344	-0.4408

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

MZM Money, $t + 1$ Output Sensitivity

Table 3.26: Estimated $\partial \text{output}_{t+1} / \partial \text{money price}_t$ with MZM Aggregate As Money Input

Industry	$\partial y_t / \partial p_{mny,t}$			$\partial y_{t+1} / \partial p_{mny,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.1425	-0.09267	-0.2505	-0.1529	-0.09871	-0.2748
Textile, Textile Product Mill	-0.02502	-0.04449	-0.2918	-0.02806	-0.04939	-0.3161
Apparel, Leather Product	-0.158	-0.2016	-0.174	-0.1694	-0.2195	-0.1868
Paper	-0.09227	-0.05881	-0.2928	-0.0985	-0.0637	-0.3193
Printing and Related	-0.1011	-0.07651	-0.2371	-0.1085	-0.0832	-0.2519
Petroleum, Coal Product	-0.04738	-0.0638	-0.2156	-0.05115	-0.06942	-0.2337
All Other Chemical	-0.1288	-0.09442	-0.2189	-0.1377	-0.101	-0.2369
Plastics, Rubber Product	-0.1276	-0.07923	-0.2516	-0.1363	-0.08466	-0.2737
Nonmetallic Mineral Product	-0.07445	-0.05545	-0.2209	-0.07917	-0.05984	-0.2391
Foundry	-0.2212	-0.1637	-0.3097	-0.2338	-0.1737	-0.3265
Fabricated Metal Product	-0.1301	-0.08223	-0.2797	-0.1392	-0.08759	-0.3086
Machinery	-0.08102	-0.08706	-0.25	-0.08741	-0.09415	-0.2684
All Other Electronic Product	-0.228	-0.2001	0.1508	-0.2496	-0.2182	0.105
Electrical Equipment, Component	-0.0888	-0.05786	-0.2566	-0.09526	-0.06311	-0.2764
Motor Vehicle, Part	-0.0617	-0.07174	-0.1992	-0.06638	-0.07775	-0.2137
All Retail Trade	-0.1917	-0.119	-0.2318	-0.2066	-0.1272	-0.2488
All Wholesale Trade	-0.227	-0.1606	-0.2737	-0.244	-0.1717	-0.2936

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.27: Estimated $\partial \text{output}_{t+1} / \partial \text{capital price}_t$ with MZM Aggregate As Money Input

Industry	$\partial y_t / \partial p_{cpt,t}$			$\partial y_{t+1} / \partial p_{cpt,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.2872	-0.2738	-0.277	-0.3121	-0.2937	-0.3045
Textile, Textile Product Mill	-0.254	-0.2412	-0.1677	-0.2695	-0.2547	-0.1802
Apparel, Leather Product	0.167	0.3195	0.1979	0.1825	0.3655	0.2159
Paper	-0.2372	-0.2343	-0.2154	-0.2537	-0.2488	-0.2334
Printing and Related	-0.2157	-0.2208	-0.1513	-0.2282	-0.2327	-0.161
Petroleum, Coal Product	-0.2476	-0.2331	-0.3431	-0.2635	-0.2464	-0.3735
All Other Chemical	-0.2649	-0.2505	-0.3005	-0.2853	-0.267	-0.3259
Plastics, Rubber Product	-0.2774	-0.2521	-0.2645	-0.3007	-0.2692	-0.2878
Nonmetallic Mineral Product	-0.2564	-0.2474	-0.2425	-0.275	-0.2631	-0.2626
Foundry	-0.1645	-0.1752	-0.127	-0.174	-0.1848	-0.135
Fabricated Metal Product	-0.2845	-0.2792	-0.2407	-0.3102	-0.299	-0.2642
Machinery	-0.2338	-0.2184	-0.2489	-0.2475	-0.2302	-0.2664
All Other Electronic Product	0.1032	0.2031	0.1998	0.1234	0.236	0.1466
Electrical Equipment, Component	-0.2172	-0.2303	-0.1986	-0.2309	-0.2438	-0.213
Motor Vehicle, Part	-0.254	-0.2421	-0.2856	-0.2701	-0.2559	-0.3063
All Retail Trade	-0.2929	-0.2902	-0.2881	-0.317	-0.3106	-0.3088
All Wholesale Trade	-0.2712	-0.2699	-0.2682	-0.2913	-0.2882	-0.2867

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.28: Estimated $\partial \text{output}_{t+1}/\partial \text{labor price}_t$ with MZM Aggregate As Money Input

Industry	$\partial y_t/\partial p_{lbr,t}$			$\partial y_{t+1}/\partial p_{lbr,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.4219	-0.3695	-0.45	-0.4598	-0.3963	-0.498
Textile, Textile Product Mill	-0.3266	-0.2901	-0.367	-0.3458	-0.3055	-0.3968
Apparel, Leather Product	-0.5298	-0.6993	-0.5589	-0.5713	-0.7773	-0.6028
Paper	-0.3905	-0.3399	-0.4212	-0.4177	-0.3599	-0.46
Printing and Related	-0.2862	-0.2611	-0.268	-0.3018	-0.2744	-0.2834
Petroleum, Coal Product	-0.342	-0.2896	-0.3122	-0.3633	-0.3053	-0.3406
All Other Chemical	-0.3834	-0.3375	-0.3621	-0.4138	-0.3594	-0.3939
Plastics, Rubber Product	-0.4158	-0.3472	-0.3996	-0.4517	-0.3703	-0.4362
Nonmetallic Mineral Product	-0.3943	-0.3444	-0.3911	-0.4227	-0.3655	-0.4252
Foundry	-0.2264	-0.2245	-0.2096	-0.238	-0.2356	-0.2208
Fabricated Metal Product	-0.459	-0.3706	-0.4759	-0.503	-0.3967	-0.529
Machinery	-0.3046	-0.2647	-0.3037	-0.3214	-0.2781	-0.3256
All Other Electronic Product	-0.6303	-0.6953	0.175	-0.7016	-0.7752	0.1254
Electrical Equipment, Component	-0.3343	-0.3042	-0.3533	-0.3545	-0.3211	-0.3799
Motor Vehicle, Part	-0.3353	-0.2861	-0.2674	-0.3561	-0.3017	-0.2872
All Retail Trade	-0.3543	-0.324	-0.2526	-0.3836	-0.3464	-0.271
All Wholesale Trade	-0.3345	-0.3192	-0.2512	-0.3594	-0.3406	-0.2691

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.29: Estimated $\partial \text{output}_{t+1}/\partial \text{intermediate input price}_t$ with MZM Aggregate As Money Input

Industry	$\partial y_t / \partial p_{iip,t}$			$\partial y_{t+1} / \partial p_{iip,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.3695	-0.3298	-0.3966	-0.4024	-0.3539	-0.4383
Textile, Textile Product Mill	-0.3055	-0.2691	-0.3467	-0.3238	-0.2838	-0.3749
Apparel, Leather Product	-0.5822	-0.7681	-0.5687	-0.6281	-0.8546	-0.6134
Paper	-0.3075	-0.2832	-0.3262	-0.3287	-0.3001	-0.3551
Printing and Related	-0.2599	-0.2438	-0.2554	-0.2744	-0.2566	-0.2704
Petroleum, Coal Product	-0.3187	-0.2785	-0.3818	-0.3387	-0.2938	-0.4167
All Other Chemical	-0.3503	-0.306	-0.3359	-0.378	-0.326	-0.3648
Plastics, Rubber Product	-0.3656	-0.324	-0.3412	-0.3966	-0.3455	-0.3721
Nonmetallic Mineral Product	-0.3323	-0.2998	-0.3609	-0.3565	-0.3185	-0.3924
Foundry	-0.1928	-0.2006	-0.1842	-0.2032	-0.211	-0.1943
Fabricated Metal Product	-0.3875	-0.312	-0.4124	-0.424	-0.334	-0.4576
Machinery	-0.2584	-0.237	-0.2683	-0.2731	-0.2495	-0.2871
All Other Electronic Product	-0.6193	-0.6841	0.1649	-0.6894	-0.7628	0.1169
Electrical Equipment, Component	-0.2953	-0.2809	-0.3024	-0.3132	-0.2967	-0.3249
Motor Vehicle, Part	-0.3047	-0.2664	-0.3281	-0.3238	-0.2812	-0.3518
All Retail Trade	-0.3336	-0.3041	-0.3057	-0.3609	-0.3251	-0.3274
All Wholesale Trade	-0.2637	-0.2664	-0.2615	-0.2826	-0.284	-0.279

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

MZM Money, $t+0, t+1$ Investment Sensitivity

Table 3.30: Estimated ∂investment/∂money price with MZM Aggregate As Money Input

Industry	$\partial ivm_t / \partial p_{mny,t}$			$\partial ivm_{t+1} / \partial p_{mny,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	0.02511	-0.0008756	0.05881	-0.004892	-0.03832	0.004054
Textile, Textile Product Mill	-0.02874	-0.08064	0.09227	-0.05574	-0.08666	0.02659
Apparel, Leather Product	0.03675	-0.05596	0.03799	-0.003799	0.05067	-0.003184
Paper	0.02411	-0.006009	0.07957	-0.0477	-0.07006	0.001608
Printing and Related	-0.0228	-0.06375	0.04961	-0.02272	-0.07628	0.02948
Petroleum, Coal Product	-0.03311	-0.08492	-0.05083	-0.08448	-0.125	0.0003691
All Other Chemical	-0.00802	-0.0308	-0.018	-0.02756	-0.04815	-0.005855
Plastics, Rubber Product	0.0576	0.02042	0.0669	-0.0302	-0.07276	0.01591
Nonmetallic Mineral Product	0.02338	-0.02277	0.05711	-0.01692	-0.05888	0.01553
Foundry	-0.04558	-0.07229	0.0069	-0.05299	-0.08529	-0.02393
Fabricated Metal Product	0.01145	-0.01955	0.05116	-0.02463	-0.05293	0.003384
Machinery	-0.0298	-0.08579	0.01883	-0.07992	-0.1234	-0.02234
All Other Electronic Product	-0.003043	0.01216	-0.06633	0.02501	0.0151	0.05407
Electrical Equipment, Component	-0.003303	-0.04028	0.04821	-0.06143	-0.09591	-0.003843
Motor Vehicle, Part	-0.03492	-0.09302	0.001087	-0.084	-0.1218	-0.03055
All Retail Trade	0.04834	0.001822	0.02619	-0.03314	-0.06461	-0.04139
All Wholesale Trade	0.03774	-0.00517	0.02368	-0.04631	-0.0736	-0.07986

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.31: Estimated ∂investment/∂capital price with MZM Aggregate As Money Input

Industry	$\partial ivm_t / \partial p_{cpt,t}$			$\partial ivm_{t+1} / \partial p_{cpt,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	0.07865	0.07058	0.05149	-0.08342	-0.06852	-0.07084
Textile, Textile Product Mill	0.0587	0.06856	0.05362	-0.04422	-0.01788	-0.08481
Apparel, Leather Product	3.29	3.49	3.282	-1.725	-1.771	-1.629
Paper	0.06455	0.04448	0.05103	-0.02556	-0.01962	-0.0646
Printing and Related	0.0852	0.08083	0.0734	-0.06396	-0.02999	-0.08248
Petroleum, Coal Product	0.04535	0.04785	0.1666	-0.01524	0.01062	-0.01813
All Other Chemical	0.08844	0.07741	0.1184	-0.05834	-0.0522	-0.03473
Plastics, Rubber Product	0.03831	0.02602	0.0539	-0.06098	-0.03752	-0.07533
Nonmetallic Mineral Product	0.04338	0.04346	0.05933	-0.07103	-0.04497	-0.07623
Foundry	0.1062	0.08183	0.08794	-0.02047	0.01038	-0.03734
Fabricated Metal Product	0.1029	0.08899	0.06341	-0.06381	-0.05	-0.06676
Machinery	0.04488	0.04933	0.09485	-0.01938	0.009454	-0.02547
All Other Electronic Product	2.347	2.743	1.847	-1.149	-1.358	-1.221
Electrical Equipment, Component	0.0642	0.06079	0.06555	-0.00876	0.002315	-0.05842
Motor Vehicle, Part	0.06539	0.07675	0.1237	-0.03557	-0.01055	-0.006958
All Retail Trade	0.03112	0.02327	0.08936	-0.0531	-0.03956	-0.01033
All Wholesale Trade	0.04985	0.05529	0.09133	-0.01991	-0.0127	0.02306

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.32: Estimated ∂investment/∂labor price with MZM Aggregate As Money Input

Industry	$\partial ivm_t/\partial p_{lbr,t}$			$\partial ivm_{t+1}/\partial p_{lbr,t}$		
	Sample Mode	Sample Mean	Irregular	Sample Mode	Sample Mean	Irregular
Food, Beverage, Tobacco	-0.01977	0.0007228	-0.04168	0.01496	0.03538	-0.004834
Textile, Textile Product Mill	0.01759	0.03694	-0.06528	0.01778	0.03383	-0.01688
Apparel, Leather Product	-1.498	-1.547	-1.586	0.7401	0.7383	0.7783
Paper	-0.06228	-0.01703	-0.06932	0.04299	0.05568	-0.01577
Printing and Related	0.005336	0.01925	-0.04153	0.02289	0.03826	-0.01507
Petroleum, Coal Product	0.006436	0.03323	-0.08849	0.03333	0.05077	-0.06236
All Other Chemical	-0.01893	0.009669	-0.06045	-0.002153	0.008185	-0.0349
Plastics, Rubber Product	-0.03747	-0.009552	-0.04811	0.01078	0.0419	-0.01258
Nonmetallic Mineral Product	0.003723	0.02667	-0.0421	-0.00617	0.02235	-0.009687
Foundry	-0.0337	-0.01253	-0.05495	0.01465	0.02636	-0.0145
Fabricated Metal Product	-0.04803	-0.01552	-0.04909	0.01056	0.03205	-0.009239
Machinery	0.00747	0.03388	-0.08591	0.03581	0.05387	-0.03733
All Other Electronic Product	-1.118	-1.309	-0.4993	0.4426	0.567	0.177
Electrical Equipment, Component	-0.04957	-0.01817	-0.05626	0.008664	0.02174	-0.01235
Motor Vehicle, Part	0.009152	0.03491	-0.1132	0.04534	0.06287	-0.0557
All Retail Trade	-0.02856	0.01476	-0.1033	-0.006806	0.02084	-0.04347
All Wholesale Trade	-0.06812	-0.03514	-0.1355	-0.01357	0.008877	-0.05094

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

MZM Money, $t + 0$ Labor Demand Sensitivity

Table 3.33: Estimated $\partial \text{labor}_t / \partial \text{money price}_t$ with MZM Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model			6-Input 1-Period Model		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	0.06731	0.07073	0.08978	0.09824	0.09709	0.1009	-0.01724	-0.01772	-0.00178
Textile, Textile Product Mill	0.1443	0.1275	0.0667	0.9114	0.9205	0.9712	-0.01533	-0.01539	-0.01316
Apparel, Leather Product	0.1017	0.1016	0.09948	0.08379	0.07875	0.1014	-0.01696	-0.01754	-0.007593
Paper	0.125	0.1246	0.1294	0.06884	0.07067	0.1023	-0.01845	-0.01847	-0.01289
Printing and Related	0.08272	0.09138	0.1087	0.1011	0.09576	0.0776	-0.01858	-0.01855	-0.01864
Petroleum, Coal Product	0.1371	0.1379	0.1555	0.07586	0.08432	0.09364	-0.01678	-0.01758	-0.009917
All Other Chemical	0.101	0.1015	0.08898	0.1015	0.08897	0.1024	-0.01775	-0.01826	-0.005492
Plastics, Rubber Product	0.1026	0.1082	0.08702	0.1231	0.1065	0.1017	-0.01747	-0.01818	-0.009476
Nonmetallic Mineral Product	0.101	0.1073	0.09975	0.08805	0.09707	0.09827	-0.01848	-0.01848	-0.006568
Foundry	0.0904	0.08836	0.09226	0.1196	0.1109	0.1019	-0.01778	-0.01828	-0.01063
Fabricated Metal Product	0.1094	0.119	0.1084	0.09902	0.09624	0.09149	-0.01777	-0.01805	-0.004373
Machinery	0.1363	0.1329	0.4698	0.8051	0.8862	0.355	-0.01439	-0.01533	0.004054
All Other Electronic Product	0.09773	0.1046	0.09439	0.1187	0.1184	0.09924	-0.01612	-0.01713	-0.003368
Electrical Equipment, Component	0.06469	0.06478	0.1191	0.09042	0.08717	0.08122	-0.01786	-0.01827	-0.008771
Motor Vehicle, Part	0.102	0.109	0.1186	0.0739	0.05993	0.08115	0.1762	0.1729	0.1838
All Retail Trade	0.1081	0.1119	0.138	0.09546	0.09864	0.07657			
All Wholesale Trade	0.08421	0.08057	-0.002093	-0.02633	-0.001479	-0.05263			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.34: Estimated $\partial \text{labor}_t / \partial \text{capital price}_t$ with MZM Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model			6-Input 1-Period Model		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	0.08421	0.08057	-0.002093	-0.02633	-0.001479	-0.05263	0.1762	0.1729	0.1838
Textile, Textile Product Mill	0.09937	0.09482	-0.02488	0.01924	0.04194	-0.08121	0.193	0.1905	0.03336
Apparel, Leather Product	-0.2687	-0.2786	-0.01821	-1.78	-1.835	-1.884	0.1167	0.1137	0.1684
Paper	0.07722	0.0801	-0.01743	-0.08004	-0.02406	-0.08655	0.1632	0.1655	0.1294
Printing and Related	-0.2311	-0.2337	0.02059	0.004067	0.02031	-0.05262	0.171	0.1703	0.1916
Petroleum, Coal Product	0.06591	0.06686	-0.06942	0.004919	0.03704	-0.113	0.1692	0.1686	0.1784
All Other Chemical	-0.2882	-0.2865	-0.03365	-0.02559	0.009499	-0.07761	0.1969	0.1905	0.1702
Plastics, Rubber Product	0.09481	0.09571	-0.01645	-0.04759	-0.0138	-0.06037	0.1717	0.169	0.1158
Nonmetallic Mineral Product	0.05465	0.0581	-0.009915	0.003286	0.03065	-0.05315	0.1898	0.1945	0.1338
Foundry	0.05386	0.05024	-0.01496	-0.04557	-0.01979	-0.07041	0.1904	0.1903	0.1563
Fabricated Metal Product	0.1188	0.1094	-0.01115	-0.0626	-0.02277	-0.06208	0.1833	0.1787	0.1811
Machinery	0.07947	0.0748	-0.05484	0.006263	0.03776	-0.1084	0.1812	0.1798	0.09832
All Other Electronic Product	-0.2081	-0.209	-0.2222	-1.326	-1.553	-0.5903	0.2063	0.1987	0.1771
Electrical Equipment, Component	0.09898	0.09498	-0.01523	-0.06445	-0.02616	-0.07104	0.1855	0.1838	0.1147
Motor Vehicle, Part	0.09069	0.09075	-0.06686	0.008001	0.03859	-0.1426	0.1974	0.1938	0.1458
All Retail Trade	0.09362	0.08444	-0.07639	-0.03581	0.0171	-0.1298			
All Wholesale Trade	0.06042	0.06975	-0.06323	-0.0859	-0.04608	-0.1699			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Table 3.35: Estimated $\partial \text{labor}_t / \partial \text{labor price}_t$ with MZM Aggregate As Money Input

Industry	4-Input 1-Period Model			4-Input 2-Period Model			6-Input 1-Period Model		
	Mode	Mean	Irregular	Mode	Mean	Irregular	Mode	Mean	Irregular
Food, Beverage, Tobacco	-0.09471	-0.09796	0.0909	-0.08518	-0.0768	0.1018	-0.01797	-0.01836	-0.0122
Textile, Textile Product Mill	-0.06731	-0.07073	0.08978	-0.09824	-0.09709	0.1009	-0.01724	-0.01772	-0.00178
Apparel, Leather Product	-0.1443	-0.1275	0.0667	-0.9114	-0.9205	0.9712	-0.01533	-0.01539	-0.01316
Paper	-0.1017	-0.1016	0.09948	-0.08379	-0.07875	0.1014	-0.01696	-0.01754	-0.007593
Printing and Related	-0.125	-0.1246	0.1294	-0.06884	-0.07067	0.1023	-0.01845	-0.01847	-0.01289
Petroleum, Coal Product	-0.08272	-0.09138	0.1087	-0.1011	-0.09576	0.0776	-0.01858	-0.01855	-0.01864
All Other Chemical	-0.1371	-0.1379	0.1555	-0.07586	-0.08432	0.09364	-0.01678	-0.01758	-0.009917
Plastics, Rubber Product	-0.101	-0.1015	0.08898	-0.1015	-0.08897	0.1024	-0.01775	-0.01826	-0.005492
Nonmetallic Mineral Product	-0.1026	-0.1082	0.08702	-0.1231	-0.1065	0.1017	-0.01747	-0.01818	-0.009476
Foundry	-0.101	-0.1073	0.09975	-0.08805	-0.09707	0.09827	-0.01848	-0.01848	-0.006568
Fabricated Metal Product	-0.0904	-0.08836	0.09226	-0.1196	-0.1109	0.1019	-0.01778	-0.01828	-0.01063
Machinery	-0.1094	-0.119	0.1084	-0.09902	-0.09624	0.09149	-0.01777	-0.01805	-0.004373
All Other Electronic Product	-0.1363	-0.1329	0.4698	-0.8051	-0.8862	0.355	-0.01439	-0.01533	0.004054
Electrical Equipment, Component	-0.09773	-0.1046	0.09439	-0.1187	-0.1184	0.09924	-0.01612	-0.01713	-0.003368
Motor Vehicle, Part	-0.06469	-0.06478	0.1191	-0.09042	-0.08717	0.08122	-0.01786	-0.01827	-0.008771
All Retail Trade	-0.102	-0.109	0.1186	-0.0739	-0.05993	0.08115			
All Wholesale Trade	-0.1081	-0.1119	0.138	-0.09546	-0.09864	0.07657			

Note: Values in the table are the function evaluated at the point where all related variables are set to 100. For each model, the economic variable is calculated and presented using three different estimators, Bayesian sample mean, Bayesian sample mode, and Maximum Likelihood Estimator (MLE, the Irregular column) that is attained without imposing regularity conditions. The MLE is also the initial value used in the corresponding Metropolis Hastings sampler, from which Bayesian estimators are derived.

Appendix A

Data Pre-Processing And Descriptive Statistics

A.1 Industry Code Correspondence

Industries in our sample are the first 15 entries in Table 3.1 (see details there), plus the trade sector. They are food and kindred products; beverage, tobacco products; textile mills and textile product mills; apparel and leather products; paper; printing and related support activities; petroleum and coal products; all other chemicals; plastics and rubber products; nonmetallic mineral products; foundries; fabricated metal products; machinery; all other electronic products; electrical equipment, appliances, and components; transportation equipment; all retail trade; all wholesale trade.

The following industries are omitted from Table 3.1 because they are missing from more than one data sets. They[1] are farms, crop and animal production; forestry, fishery, and related activities; oil and gas extraction; mining, except oil and gas; support activities for mining; air transportation; rail transportation; water transportation; truck transportation; transit and ground passenger transportation; pipeline transportation; other transportation and supportive activities; warehousing and storage; publishing industries, except internet (includes software); motion picture and sound recording; broadcasting and telecommunication; data processing, internet publishing, and other information services; legal services; computer system design and related services; miscellaneous professional, scientific, and technical services; management of companies and enterprises; administrative and support services; waste management and remediation services; educational services; ambulatory health care services; hospitals and nursing and residential care facilities; social assistance; performing arts, spectator sports, museums, and related activities; amusements, gambling, and recreation industries; accommodation; food services and drinking places; other services, except government.

[1]The entire finance sector is out of our scope, even though we have the data: Federal Reserve banks, credit intermediation, and related activities; securities, commodity contracts, and investments; insurance carriers and related activities; funds, trusts, and other financial vehicles; real estate; rental and leasing services and lessors of intangible assets.

A.2 Uniform Frequency Model: High Frequency Series Aggregation

All variables from Quarterly Financial Report (QFR) are quarterly series. All interest rate, monetary asset user cost variables are monthly data. In order to match them with other annual series to form a balanced sample, we aggregated the higher frequency series before using them. Generally,

- we take annual sums of quarterly or monthly observations, if the item is a “flow” variable;
- We take annual average of quarterly or monthly observations, if the item is a “stock” or “level” variable.

For example, depreciation, depletion and amortization of property, plant, and equipment is a flow variable. Its annual value is the total of its four quarterly in every year. Current asset and current liability, interest rates, total cash and deposit balances, are level variables. Their annual values are the average over the year. There might be reasons for use to take weighted average, and/or a moving average with a wider window, of level variables. But we did not as there is no clear benefit doing so and setting suitable weights and window width require additional estimation steps.

A.3 Mixed Frequency Model: QFR Series Seasonality Adjustment

Quarterly Financial Reports come as raw nominal data reported by surveyed companies. We remove seasonality of those variables so that the seasonal variation will not be captured by cost function models. Seasonality in corporate financial variables are naturally multiplicative. We first estimate the following regression to quantify the multiplicative factor of each quarter

$$\ln x_t = \theta_0 + \theta_1 t + \theta_2 D_{Q2,t} + \theta_3 D_{Q3,t} + \theta_4 D_{Q4,t} + u_t, \quad t = 1, \ldots, T,$$

where ln is the natural logarithm, D_{Q2}, D_{Q3}, D_{Q4} are dummy variables for the second, the third, and the forth quarter respectively, u is a residual term, and T is sample size.

Let x be the original series and x^a be the series after adjustment. Then for every t the seasonal adjustment is

$$x_t^a = \exp\{\ln x_t - \hat{\theta}_2 D_{Q2,t} - \hat{\theta}_3 D_{Q3,t} - \hat{\theta}_4 D_{Q4,t}\}.$$

The method is not our only choice. There are many ways commonly used to adjust for seasonality. ARIMA[2] with seasonal terms is the most widely used empirical-filter-based method and TRAMO-SEATS[3] is a typical model-based method. Many researches compared those alternative methods on simulated and real world data; [BH84, Mar03, HF03] are a few examples.

[2]ARIMA is the acronym for Autoregressive Integrated Moving Average model.

[3]TRAMO stands for Time series Regression with Autoregressive integrated moving average errors and Missing Observations. SEATS is short for Signal Extraction for Autoregressive integrated moving average Time Series.

A.4 Monetary Asset Balance

Quarterly Financial Reports provide sector and industry level balances of monetary assets of several kinds. The available items are

- Cash holding
- Demand deposits in the U.S.
- Time deposits (including negotiable certificates of deposit) in the U.S.
- Cash and deposits outside the U.S.
- U.S. Treasury and Federal agency securities: subject to agreements to sell
- U.S. Treasury and Federal agency securities: other, due in one year or less
- Commercial and finance company paper of U.S. issuers
- State and local government securities, due in one year or less
- Foreign securities, due in one year or less
- Other short-term asset (including marketable and government securities, commercial papers, etc.)

They are nominal and not seasonally adjusted. First, we adjust them by producer's price index to real values. As we use their annual values, we aggregate them using the aforementioned methods without adjusting for seasonality. Then we construct the amount of monetary services they provide using formulas developed in [Bar87] for one-period cost function estimation and multi-period cost function estimation with risk neural producers. To be exact, the user cost $p_{i,t}$ of holding type i monetary asset $m_{i,t}$ during period t is given by

$$p_{i,t} = p_t^* \frac{r_{B,t} - r_{i,t}}{1 + r_{B,t}}, \tag{1.4.1}$$

where p^* is an appropriate price index to deflate the nominal values, r_B is the rate of return on the benchmark asset, r_i is the rate of return paid on holding the asset m_i. Its corresponding real value is $p_{i,t}/p_t^*$.

The amount of monetary services, which is one of the x_{it} entering the cost function, is measured by the aggregate of all kinds of monetary asset holding according to a certain scope. The scope can be the base currency M0, the high liquidity set M1, or most balanced set M2, or wider scopes. Let $m_1, \ldots, m_S$ be the balance of the range of assets in the selected scope. The aggregate index $x_{m,t}$ then is defined as

$$\mathrm{d} \ln x_{m,t} = \sum_{i=1}^{S} \frac{p_{i,t}}{\sum_{j=1}^{S} p_{j,t} m_{j,t}} \mathrm{d} \ln m_{i,t}, \tag{1.4.2}$$

where $p_{i,t}$ are user costs defined in (1.4.1). We refrain from considering the case of risk averse producers with several reasons. It is acceptably close to reality to assume profit maximizing firms are risk neutral. And terms involving production function (or cost function) enter the monetary service user cost expression when the user is risk averse[4]. That means we need to have an estimate of production or cost function in the first place, denying the whole purpose of our study.

For definitions (1.4.2) and (1.4.1) to hold, the producers should be risk neutral, have weakly separable production technology[5], and homothetic categorical sub-production function of monetary services.

A.5 All Models: Normalization

All variables used in all models are normalized. We normalize them to eliminate units and more importantly to improve the convergence speed of MCMC samplers. Original series are centralized by sample mean and standardized by sample standard deviation. For each variable x and every observation point t

$$x_t^a = 100 + 10 \cdot \frac{x_t - \bar{x}}{s},$$

where the location factor 100 can bring all observations to positive values. The maintained regularity conditions are correct only in the first quadrant[6]. Later on, to further accelerate the convergence of MCMC sampling process in estimating some models, we normalize the data set by min-max feature scaling

$$x_t^a = 50 + \frac{100(x_t - x_{\min})}{x_{\max} - x_{\min}}.$$

The location factor 50 and scale factor 100 are used so that some program modules can be reused.

[4]See for example, [BLJ97, BZ+94]

[5]That is equivalent to the applicability of two-stage budgeting in profit maximization problem and cost minimization problem, which is a maintained presumption in the derivation of multi-period cost function.

[6]Quadrant in the generalized high dimensional Eulidean space sense

Appendix B

Nested Gibbs Sampler

This appendix shows the details of our Bayesian estimation method, its design and inference based on the parameter sample drawn by the MCMC samplers.

B.1 Multi-Block Implementation

B.1.1 Structure and Blocking Strategy

We put parameters into blocks by how they enter the regression, so as to be able to draw them from analytically known distributions. In this estimation, we generally follow the guideline

- Use Gibbs sampler for linear blocks and error term Var-Cov matrix
- Use sub-Gibbs for blocks who enter multiple equations at the same time
- Use Metropolis-Hastings (MH) sampler for nonlinear blocks (parameters who enter the model nonlinearly, including nonlinearity in constraints.)

We use the following simplest possible example, to show the exact blocking scheme and sample method. It preserves all the characteristics of the estimation problems we work on. Following the project variable name convention, a GM 3-input case writes as (time subscript omitted)

$$x_1 = a_{22} \cdot \left(-\frac{p_2^2}{2p_1^2}y\right) + a_{23} \cdot \left(-\frac{p_2 p_3}{p_1^2}y\right) + a_{33} \cdot \left(-\frac{p_3^2}{2p_1^2}y\right) + b_{11}y + b_1 + b_{yy}y^2 + u_1, \tag{2.1.1}$$

$$x_2 = a_{22} \cdot \frac{p_2}{p_1}y + a_{23} \cdot \frac{p_3}{p_1}y + b_{22}y + b_2 + b_{yy}y^2 + u_2, \tag{2.1.2}$$

$$x_3 = a_{32} \cdot \frac{p_2}{p_1}y + a_{33} \cdot \frac{p_3}{p_1}y + b_{33}y + b_3 + b_{yy}y^2 + u_3. \tag{2.1.3}$$

Symmetry conditions $a_{23} = a_{32}$ are readily used to merge terms. Error term $\boldsymbol{u} = (u_1, u_2, u_3)'$ follows multivariate Normal distribution $\mathcal{N}_3(\mathbf{0}, \Sigma)$, where Σ is unknown and to be estimated. Other than that, we have positivity conditions

$$\hat{x}_1 \geq 0, \hat{x}_2 \geq 0, \hat{x}_3 \geq 0, \quad \forall t; \tag{2.1.4}$$

$$\hat{c} \geq 0, \quad \forall t. \tag{2.1.5}$$

Then the cost function being modelled is

$$\begin{aligned}c = \frac{1}{2p_1}(a_{22}p_2^2 + 2a_{23}p_2p_3 + a_{33}p_3^2)y \\ + b_{11}p_1y + b_{22}p_2y + b_{33}p_3y + b_1p_1 + b_2p_2 + b_3p_3 \\ + b_{yy}(p_1 + p_2 + p_3)y.\end{aligned} \tag{2.1.6}$$

Let the parameter blocks be (a, b, Σ). Arrangements within groups will remain undecided for now. At the top level (Level 1), we use Gibbs sampler whose one typical loop is

1. Draw $\Sigma \mid a, b$. Conditional on a, b, Σ follows an inverse Wishart (IW) with known parameters. Assuming the whole Σ is unknown gives us simpler posterior than assuming it's diagonal.

2. Draw $a \mid b, \Sigma$. bȩvery a parameter enters multiple equations and their associated regressors are different, we use a second level (Level 2) Gibbs/Accept-Reject Metropolis sampler to draw them one by one, from normal distributions. Unlike at the top level, we will only get one set of draws (sample size of 1) in each loop of the **L1** sampler. To impose inequality constraint in this **L2** sampler (performing ARMH), we will draw from truncated distributions (linear constraints) and/or discard unsatisfying draws until a satisfying draw is made (nonlinear constraints).

3. Draw $b \mid \Sigma, a$. The step breaks down into a **L2** Gibbs sampler with block b_{yy} and all other $\boldsymbol{b}$. First draw b_{yy} conditional on all other parameters, from a normal distribution. Second draw $\boldsymbol{b}$ conditional on all other parameters, from a multivariate normal distribution.

B.1.2 Prior

We use uninformative prior for all parameters. Using informative prior will simply give us estimates that are mixtures of moments or modes of the true distribution and the prior. Since we don't have particular theoretical reasons to favor one value against all others, uninformative prior is more appropriate. Next, we present the drawing scheme in details.

B.1.3 Posterior of Σ (Level 1 Block 1)

We are going to repeatedly use the cyclic permutation property of the trace of a matrix in the derivation of the posterior of Σ. That is for all matrices A, B, C with proper dimensions, the trace of their product satisfies

$$\operatorname{tr}(ABC) = \operatorname{tr}(CAB) = \operatorname{tr}(BCA), \tag{2.1.7}$$

which in its application here is often

$$\operatorname{tr}(\boldsymbol{x}'A\boldsymbol{x}) = \operatorname{tr}(\boldsymbol{x}\boldsymbol{x}'A), \tag{2.1.8}$$

where $\boldsymbol{x}$ is a vector and A a matrix.

In **L1B1**, the problem is essentially observing $D = \{z_t\}_{t=1}^T$, $z_t \in E^d$, and

$$z_t \overset{iid}{\sim} \mathcal{N}_d(0, \Sigma) \tag{2.1.9}$$

to draw samples of unknown Var-Cov Σ. Here $d = 3$ in the illustrative example but we keep using d to establish relations more clearly. Conditionally, the sample has probability

$$\begin{aligned}\Pr(D \mid \Sigma) &\propto |\Sigma|^{-\frac{T}{2}} \exp\{-\frac{1}{2}\sum_{t=1}^{T} z_t'\Sigma^{-1}z_t\} \\ &= |\Sigma|^{-\frac{T}{2}} \exp\{-\frac{1}{2}\operatorname{tr}(\Lambda S)\},\end{aligned} \tag{2.1.10}$$

where $\Lambda = \Sigma^{-1}$ is the precision matrix and $S = \sum_{t=1}^{T}(z_t - \bar{z})(z_t - \bar{z})'$ is the scatter matrix. Combining uninformative prior (also called *Jeffrey's prior* for the Var-Cov matrix)

$$\Pr(\mu, \Sigma) \propto |\,\Sigma\,|^{-\frac{d+1}{2}}, \quad \text{(in our case } \mu \equiv 0) \tag{2.1.11}$$

we have the posterior

$$\Pr(\Sigma \mid D) \propto |\,\Sigma\,|^{-(\frac{T+d}{2}+1)} \exp\{-\frac{1}{2}tr(S\Sigma^{-1})\}. \tag{2.1.12}$$

That is, conditional on D, Σ follows an inverse Wishart distribution $\mathcal{IW}(S, T-1)$ with scale parameter S and degree of freedom $T - 1$, which is the drawing scheme. Derivation of this result involves finding the posterior under conjugate Normal-Inverse-Wishart prior, taking the limit of conjugate prior hyper-parameters, and setting distribution $\Pr(D \mid \boldsymbol{\mu}, \Sigma)$ mean $\boldsymbol{\mu}$ to zero.

The scatter matrix of a data set D is always positive definite. To see this, for arbitrary $\boldsymbol{x} \in E^d \backslash \boldsymbol{0}$,

$$\begin{aligned}\boldsymbol{x}'S\boldsymbol{x} &= \boldsymbol{x}' \sum_t (z_t - \bar{z})(z_t - \bar{z})'\boldsymbol{x} \\ &= \sum_t \boldsymbol{x}'(z_t - \bar{z})(z_t - \bar{z})'\boldsymbol{x} \qquad \text{(dimensions match)} \\ &= \sum_t (\boldsymbol{x}'(z_t - \bar{z}))^2 \qquad \text{(the product is a 1-by-d times d-by-1 scalar value)} \\ &> 0.\end{aligned}$$

Thus S is a proper scale parameter for the posterior.

B.1.4 Posterior of a (Level 1 Block 2)

The model at **L1B2** is essentially regression

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = a \cdot \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \overset{iid}{\sim} \mathcal{N}_2(\boldsymbol{0}, \Sigma_{sub}), \tag{2.1.13}$$

where y, x, a, ε are all scalar variables and Σ_{sub} is a submatrix of Σ with only the variance and covariance of $\varepsilon_1, \varepsilon_2$.

This is a typical regression that GLS deals with, whose residuals have correlation. We use GLS approach to find the distribution of a conditional on data $\{y_{1t}, y_{2t}, x_{1t}, x_{2t}\}_{t=1}^{T}$. Let

$$Y = \begin{bmatrix} y_{11} \\ y_{21} \\ y_{12} \\ y_{22} \\ \vdots \\ y_{1T} \\ y_{2T} \end{bmatrix}_{2T\times 1}, X = \begin{bmatrix} x_{11} \\ x_{21} \\ x_{12} \\ x_{22} \\ \vdots \\ x_{1T} \\ x_{2T} \end{bmatrix}_{2T\times 1}, \Omega = \begin{bmatrix} \Sigma_{sub} & & & \\ & \Sigma_{sub} & & \\ & & \ddots & \\ & & & \Sigma_{sub} \end{bmatrix}_{2T\times 2T}. \tag{2.1.14}$$

We assert without showing, for each parameter in Block a, the posterior with uninformative prior is a normal distribution

$$a \mid \Sigma, Y, X \sim \mathcal{N}((X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y, (X'\Omega^{-1}X)^{-1}), \tag{2.1.15}$$

which is the drawing scheme of **L1B2L2** Gibbs/ARMH sampler.

B.1.5 Posterior of b (Level 1 Block 3)

The b parameters can be alternatingly drawn in two blocks. **L1B3L2B1** contains b_{yy} and **L1B3L2B2** contains $b_1, b_2, b_3, b_{11}, b_{22}, b_{33}$.

Model at **L1B3L2B1** is essentially regression

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix} = b \cdot \begin{bmatrix} x_t \\ x_t \\ x_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} \overset{iid}{\sim} \mathcal{N}_3(\mathbf{0}, \Sigma), \tag{2.1.16}$$

where y, x, b, ε are all scalar variables and Σ is the matrix drawn in **L1B1** (and note the stacking same x). Again this is a typical situation to apply GLS arrangement. We stack data observations $\{y_{1t}, y_{2t}, y_{3t}, x_t\}_{t=1}^{T}$ vertically into new variables

$$Y_{3T\times 1} = b \cdot X_{3T\times 1} + \mathcal{E}_{3T\times 1}, \quad \mathcal{E} \sim \mathcal{N}_{3T}(\mathbf{0}, \Omega), \tag{2.1.17}$$

where $\Omega = diag(\Sigma)_{3T\times 3T}$ is blockwise diagonal. Then the posterior with uninformative prior is a normal distribution

$$b \mid \Sigma, Y, X \sim \mathcal{N}((X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y, (X'\Omega^{-1}X)^{-1}), \tag{2.1.18}$$

which is the drawing scheme of **L1B3L2B1** Gibbs sampler.

The model at **L1B3L2B2** is essentially regression

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} b_1 & b_{11} \\ b_2 & b_{22} \\ b_3 & b_{33} \end{bmatrix} \begin{bmatrix} 1 \\ x_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} \overset{iid}{\sim} \mathcal{N}_3(\mathbf{0}, \Sigma). \tag{2.1.19}$$

Rearranging it into a standard form, we have

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} 1 & x_t & & & & \\ & & 1 & x_t & & \\ & & & & 1 & x_t \end{bmatrix} \begin{bmatrix} b_1 \\ b_{11} \\ b_2 \\ b_{22} \\ b_3 \\ b_{33} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} \overset{iid}{\sim} \mathcal{N}_3(\mathbf{0}, \Sigma). \tag{2.1.20}$$

Using the same GLS approach, we stack all observations vertically and get

$$Y_{3T\times 1} = X_{3T\times 6}\beta_{6\times 1} + \mathcal{E}_{3T\times 1}, \quad \mathcal{E} \sim \mathcal{N}_{3T}(\mathbf{0}, \Omega), \tag{2.1.21}$$

where $\Omega = diag(\Sigma)_{3T\times 3T}$ is blockwise diagonal. Then the posterior with uninformative prior is a normal distribution

$$\beta \mid \Sigma, Y, X \sim \mathcal{N}_6((X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y, (X'\Omega^{-1}X)^{-1}), \tag{2.1.22}$$

which is the drawing scheme of **L1B3L2B2** Gibbs sampler.

B.1.6 Constraints

We maintain the following kinds of constraints in the sampler.

- Equality constraints

  Stem from the symmetry conditions and maintained by substituting them into the model.

- Inequality constraints

  Stem from the positivity conditions and curvature conditions. For linear single parameter inequality constraint, use truncated Normal to propose draws. For nonlinear and/or multiple parameter inequality constraint, use Accept-Reject MH (ARMH) to propose draws.

B.2 Inference Based on MCMC Estimates

From any MCMC implementation, we obtain an empirical *joint* distribution of all parameters. Let $\hat{\pi}(\boldsymbol{\theta})$ be this unnormalized estimate of posterior function. What we are interested in (those economic theoretical parameters) are some integral

$$\hat{f} = \mathrm{E}_{\hat{\pi}}[f(\boldsymbol{\theta})] = \frac{1}{Z(\boldsymbol{y})} \int_{\mathcal{D}} f(\boldsymbol{\theta})\hat{\pi}(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}\,, \tag{2.2.23}$$

where $\boldsymbol{y}$ is the observation sample, $Z(\boldsymbol{y})$ is the marginal likelihood of the observation sample, and $\mathcal{D}$ is a theoretically reasonable support set of parameters. The point we make here, is how we interpret the empirical pdf from MCMC sampling. It is not for point estimates of the model parameters, but for the joint estimation of theoretical parameters of interest.

Appendix C

Snapshot of Studies on Industry/Firm-Level Monetary Heterogeneity

We hereby list all related studies by theme, method, and scope.

C.1 Unexpected Change in Benchmark Interest Rate as Monetary Policy Shock

C.1.1 How output and/or product price of each industry are affected?

- Gertler M, Gilchrist S 1994 (empirical; only manufacturing)
- Ganley J, Salmon C 1997 (empirical; UK)
- Hayo B, Uhlenbrock B 1999 (empirical; Germany, only manufacturing and mining)
- Clark TE 1999 (empirical; input and output price focused)
- Ehrmann M 2000 (empirical; Germany)
- Arnold IJM 2000 (empirical; earning, wage, capital return focused)
- Kashyap AK, Stein JC 2000 (empirical; only banking)
- Ghosal V 2000 (empirical; markup focused; also compares with energy price shocks; industry concentration matters)
- Barth MJ III, Ramey VA 2001 (empirical; only sec 4 is relevant, sec 5 checking robustness)
- Owyang MT, Wall HJ 2003; Owyang MT, Wall HJ 2006 (empirical)
- Arnold IJM, Vrugt EB 2004 (empirical; Germany)
- Peersman G, Smets F 2005 (empirical; multiple Euro Area countries)

- Tena JD, Tremayne A 2006 (empirical; UK; linking asymmetric shock reponses with industry characters)
- Lastrapes WD 2006 (empirical; commodity price focused)
- Balke NS, Wynne MA 2007 (empirical; producer price focused)
- Bils M, Klenow PJ, Kryvtsov O 2003 (empirical; consumer price focused)
- Dedola L, Lippi F 2005 (empirical; US, UK, Germany, France, Italy; industry matters while country does not)
- Gilchrist S, Zakrajsek E 2007 (empirical; investment focused)
- Favero C, Giavazzi F, Flabbi L 1999 (empirical; only banking in 1992, France, Germany, Italy, Spain; bank loan channel focused)
- Ottonello P, Winberry T 2017 (theoretical part; investment focused; depnding on firm debt default risk distribution)
- Fares J, Srour G 2001 (empirical; Canada)

C.1.2 Which types of firms are more sensitive to monetary policy shocks?

Firm Size

- Gertler M, Gilchrist S 1994 (empirical; only manufacturing)
- Oliner SD, Rudebusch GD 1995 (empirical; denying dependence on bank loan transmits policy shocks to firms)
- Bernanke BS, Gertler M, Gilchrist S 1996 (empirical part; investment focused)
- Peersman G, Smets F 2005 (empirical; multiple Euro Area countries)
- Ehrmann M 2000 (empirical; Germany)
- Ehrmann M, Worms A 2001 (empirical; Germany; only banking, interbank loan focused)
- Arnold IJM, Vrugt EB 2004 (empirical; Germany)
- Dedola L, Lippi F 2005 (empirical; US, UK, Germany, France, Italy; industry matters while country does not)
- Zervou AS 2014 (theoretical, DSGE; mainly assume the larger firms are, the less binded they are by cash holding)
- Kudlyak M, Sanchez JM 2016 (empirical)
- Yu SE 2017 (empirical; balance sheet strength focused)
- Bahaj S, Foulis A, Pinter G, Surico P 2018 (empirical; UK; employment)

Firm Age

- Bahaj S, Foulis A, Pinter G, Surico P 2018 (empirical; UK; employment)

Firm Location

- Ramaswamy R, Slok T 1998 (empirical; multiple EU countries)
- Favero C, Giavazzi F, Flabbi L 1999 (empirical; only banking in 1992, France, Germany, Italy, Spain; bank loan channel focused)
- Owyang MT, Wall HJ 2003; Owyang MT, Wall HJ 2006 (empirical)
- Dedola L, Lippi F 2005 (empirical; US, UK, Germany, France, Italy; industry matters while country does not)

Involvement in Trade

- Hayo B, Uhlenbrock B 1999 (empirical; Germany, only manufacturing and mining)

Product Durability

- Fares J, Srour G 2001 (empirical; Canada)
- Dedola L, Lippi F 2005 (empirical; US, UK, Germany, France, Italy; industry matters while country does not)
- Peersman G, Smets F 2005 (empirical; multiple Euro Area countries)
- Erceg C, Levin A 2006 (theoretical w. data, DSGE)
- Barsky RB, House CL, Kimball MS 2007 (theoretical; 2-dim dichotomy)
- Bouakez H, Cardia E, Ruge-Murcia FJ 2011 (theoretical w. data, DSGE)
- Gwin C, VanHoose DD 2012 (empirical)
- Kim KH, Katayama M 2013 (theoretical, DSGE; closely related to [Barsky R, House CL, Kimball M 2003] and [Barsky RB, House CL, Kimball MS 2007])

Product Price Rigidity

- Bils M, Klenow PJ, Kryvtsov O 2003 (empirical; consumer price)
- Barsky RB, House CL, Kimball MS 2007 (theoretical; 2-dim dichotomy)
- Bouakez H, Cardia E, Ruge-Murcia FJ 2011 (theoretical w. data, DSGE)
- Bouakez H, Cardia E, Ruge-Murcia FJ 2009 (theoretical w. data, DSGE; w. real world production matrix and capital flow)

- Bouakez H, Cardia E, Ruge-Murcia F 2013 (theoretical w. data, DSGE)
- Gwin C, VanHoose DD 2012 (empirical)

Product Market Competition

- Ghosal V 2000 (empirical; markup focused; also compares with energy price shocks; industry concentration matters)

Firm P/E

Capital Intensity

- Hayo B, Uhlenbrock B 1999 (empirical; Germany, only manufacturing and mining)
- Dedola L, Lippi F 2005 (empirical; US, UK, Germany, France, Italy; industry matters while country does not)

External Finance Dependence

- Bernanke BS, Gertler M, Gilchrist S 1996 (theoretical part; investment focused)
- Favero C, Giavazzi F, Flabbi L 1999 (empirical; only banking in 1992, France, Germany, Italy, Spain; bank loan channel focused)
- Peersman G, Smets F 2005 (empirical; multiple Euro Area countries)
- Buca A, Vermeulen P 2015 (empirical; Germany, France, Belgium, Italy, Span, Portugal; investment focused)
- Ippolito F, Ozdagli AK, Perez-Orive 2016 (theoretical, DSGE; looking at stock price, cash holdings, sales, inventory, fixed capital investment)

Firm Leverage

- Dedola L, Lippi F 2005 (empirical; US, UK, Germany, France, Italy; industry matters while country does not)
- Peersman G, Smets F 2005 (empirical; multiple Euro Area countries)
- Cooley TF, Quadrini V 2006 (theoretical, DSGE; also modelled stock prices)
- Jeenas P 2017 (empirical; fixed capital, inventory, sales focused)
- Ottonello P, Winberry T 2017 (empirical part; investment focused)

Coverage Ratio

- Gertler M, Gilchrist S 1994 (empirical; only manufacturing)
- Bernanke BS, Gertler M, Gilchrist S 1996 (empirical part; investment focused)
- Peersman G, Smets F 2005 (empirical; multiple Euro Area countries)

Asset Liquidity

- Peersman G, Smets F 2005 (empirical; multiple Euro Area countries)
- Jeenas P 2017 (empirical; fixed capital, inventory, sales focused)

Risk Rating

- Ottonello P, Winberry T 2017 (empirical part; investment focused)

Bank Loan Availability or Availability of General External Financing

- Kashyap AK, Stein JC, Wilcox DW 1993 (empirical)
- Oliner SD, Rudebusch GD 1995 (empirical; denying dependence on bank loan transmits policy shocks to firms)
- Bernanke BS, Gertler M, Gilchrist S 1996 (theoretical, empirical; investment focused)
- Ehrmann M, Worms A 2001 (empirical; Germany; only banking, interbank loan focused)
- Chatelain JB, et al 2003 (summary, empirical; Euro Area; investment focused)
- Peydro JL, Jimenez G, et al 2009 (empirical; Spain; depending on firms' and banks' balance sheet strength)
- Ippolito F, Ozdagli AK, Perez-Orive 2016 (theoretical, DSGE; looking at stock price, cash holdings, sales, inventory, fixed capital investment)

Banking Industry Concentration

- Kashyap AK, Stein JC 2000 (empirical)
- Owyang MT, Wall HJ 2003; Owyang MT, Wall HJ 2006 (empirical)
- Arnold IJM, Vrugt EB 2004 (empirical; Germany)

Asset Tangibility

R&D Intensity

Cash Conversion Cycle

Technology or Productivity

- Bouakez H, Cardia E, Ruge-Murcia FJ 2009 (theoretical w. data, DSGE; w. real world production matrix and capital flow)

Historical Periods

- Owyang MT, Wall HJ 2003; Owyang MT, Wall HJ 2006 (empirical)
- Ehrmann M 2000 (empirical; Germany)
- Yu SE 2017 (empirical; balance sheet strength focused)

C.2 Business Cycle

- Rebelo S 2005 (literature survey)

C.2.1 Do aggregate fluctuations emerge from firm/industry level shocks?

- Horvath M 2000 (theoretical w. data, DSGE; w. real world production matrix)
- McCarthy J, Zakrajsek E 2007 (empirical; better inventory management reduces aggregate volatility)
- Anthonisen N 2016 (theoretical, DSGE; output)
- Aoki K 2001 (theoretical; product price)
- Carvalho VM, Grassi B 2017 (theoretical; emphasizing firm size distribution; matching output persistence, volatility, time variation of volatility)
- Barsky R, House CL, Kimball M 2003 (theoretical, DSGE; w. only durable goods but flexible price, the economy will behave the same as a model without durable goods distinction)
- Bergholt D, Sveen T 2014 (theoretical, DSGE; heterogeneous industry to explain international economy co-movement)
- Maeno Y 2013 (theoretical, graph theory; sparse network is likely to fail from sporadic firm failures)
- Moro A, Stucchi R 2015 (theoretical, empirical; Spain; productivity focused)

C.2.2 How all industries behave differently?

- Kashyap AK, Lamont OA, Stein JC 1992 (empirical; inventory focused; only 1981-1982 recession)
- Norrbin SC, Schlagenhauf DE 1996 (empirical; multiple countries; output focused; denying sectoral difference)
- Hornstein A 2000 (theoretical w. data, DSGE; output and productivity focased)
- McLaughlin KJ, Bils M 2001 (empirical; intersectoral employment movement and wage patterns)
- Fort T, Haltiwanger J, et al 2012 (empirical; employment focused; also checked housing price shocks)
- Casssou SP, Vazquez Perez J 2009 (empirical; employment focused)
- Chang Y, Hwang S 2015 (empirical; output focused)
- Crouzet N, Mehrotra NR 2017 (empirical w. structural model; only manufacturing; sales and investment focused; deny effects of heterogeneity in survivability, leverage, bank loan dependence, dependence on short term debt; size matters even if controlled for financing constraints like leverage and etc.)
- Coricelli F, Karadimitropoulou A, Leon-Ledesma MA 2012 (empirical; on value added, employment, productivity, concentration, structural change, distinguishing developed and emerging economies, financial and normal contractions)
- Coricelli F, Karadimitropoulou A, Leon-Ledesma MA 2016 (empirical)
- Kim KH, Kim YS 2006 (theoretical, DSGE; w. real world production matrix; employment focused; some form of cross sector re-employment frictions is always necessary)
- Bergholt D, Sveen T 2014 (theoretical, DSGE; heterogeneous industry to explain international economy co-movement)
- Lagoa S, Suleman F 2016 (empirical; wage focused) This is not a monetary or business cycle study. Nevertheless it is very informative on sectoral heterogeneity in cyclicality and in long term growth. Industries requiring special skills with different levels of transferability, would face differently labor supply elasticity, and pay different variable costs in short and long term downfalls. The labor cost channel may not be as strong as the capital costs and demand-supply movements, but can be substantial in sectors demanding very specific skills.
- Melolinna M, Miller H, Tatomir S 2018 (empirical; investment and hurdle rate focused)
- Peydro JL, Jimenez G, et al 2009 (empirical; Spain; banking and non-financial dichotomy)
- Matutinovic L 2005 (theoretical, small network; capital utilization focused)

C.2.3 Which types of firms are more sensitive?

Firm Size or Age

- Vermeulen P 2002 (empirical; Germany, France, Italy, Spain; investment focused)
- Fort T, Haltiwanger J, et al 2012 (empirical; only sec IV.D; employment focused; also checked housing price shocks)
- Moscarini G, Postel-Vinay F 2012 (empirical; US, France, Denmark; employment focused)
- Buera FJ, Fattal-Jaef R, Shin Y 2013 (theoretical, DSGE; TFP, employment focused)
- Midrigan V, Xu DY 2014 (theoretical, DSGE; TFP, output focused)
- Chari VV, Christiano LJ, Kehoe P 2013 (empirical)
- Becker B, Ivashina V 2014 (empirical)
- Kudlyak, Sanchez 2017 (empirical; robust version of Kudlyak, Price, Sanchez 2010)
- Crouzet N, Mehrotra NR 2017 (empirical w. structural model; only manufacturing; sales and investment focused; deny effects of heterogeneity in survivability, leverage, bank loan dependence, dependence on short term debt; size matters even if controlled for financing constraints like leverage and etc.)
- Yu SE 2017 (empirical; balance sheet strength focused)
- Jo IH, Senga T 2014 (theoretical, DSGE)
- Jo IH 2015 (theoretical, DSGE; resolve to major macro puzzles wølosing explanation power of phenomena under real shocks)
- Abo-Zaid S, Zervou AS 2016 (theoretical, DSGE; employment focused; size and age are proxy of financial constrainedness)

Firm Location

- Norrbin SC, Schlagenhauf DE 1996 (empirical; multiple countries; output focused; denying sectoral difference)
- Clark TE 1998 (empirical; employment focused) Clark TE 1998 and Arnold IJM, Vrugt EB 2004 comes to the opposite conclusions. The former says the heterogeneous behaviors have nothing to do with the regional industry composition, whereas the location itself appears to be important. However, the later says the industry composition is what really determines the heterogeneity, instead of the location.

Product Durability

- Barsky R, House CL, Kimball M 2003 (theoretical, DSGE; w. only durable goods but flexible price, the economy will behave the same as a model wødurable goods distinction)

Product Price Rigidity

- Barsky R, House CL, Kimball M 2003 (theoretical, DSGE; w. only durable goods but flexible price, the economy will behave the same as a model wødurable goods distinction)
- Bergholt D, Sveen T 2014 (theoretical, DSGE; heterogeneous industry to explain international economy co-movement)

Firm P/E

Leverage

- Buera FJ, Fattal-Jaef R, Shin Y 2013 (theoretical, DSGE; TFP, employment focused)
- Midrigan V, Xu DY 2014 (theoretical, DSGE; TFP, output focused)

External Finance Dependence

- Braun M, Larrain B 2005 (empirical)
- Chava S, Purnanandam A 2011 (empirical)
- Carvalho D, Ferreira MA, Matos P 2015 (empirical)

Bank Loan Dependence

- Kashyap AK, Lamont OA, Stein JC 1992 (empirical; inventory focused; only 1981-1982 recession) [Milne, A 1991 - financial effects on inventory investment] obtains similar results, based on British firm-level inventory data for several recessionary episodes. The paper is not digitally available.

Firm Leverage

- Carvalho D, Ferreira MA, Matos P 2015 (empirical)
- Giroud, Mueller 2015 (empirical; employment focused)

Bank Loan Availability or Availability of General External Financing

- Fisher JDM 1998 (theoretical, DSGE)
- Bernanke BS, Gertler M, Gilchrist S 1999 (theoretical, DSGE)
- Kohler M, Britton E, Yates T 2000 (empirical; UK)
- Chava S, Purnanandam A 2011 (empirical)
- Becker B, Ivashina V 2014 (empirical)
- Carvalho D, Ferreira MA, Matos P 2015 (empirical; focus on relationship loans)
- Braun M, Larrain B 2005 (empirical)

Banking Industry Concentration

Asset Tangibility

- Braun M, Larrain B 2005 (empirical)

R&D Intensity

Cash Conversion Cycle

Uncertainty Level

- Melolinna M, Miller H, Tatomir S 2018 (empirical; UK; investment and hurdle rate focused)

Industry Chain Structure or Business Network Structure

- Maeno Y 2013 (theoretical, graph theory; sparse network is likely to fail from sporadic firm failures)

- Bergholt D, Sveen T 2014 (theoretical, DSGE; heterogeneous industry to explain international economy co-movement)

Regulatorily Constrained

- Sen I, Humphry D 2018 (empirical; UK; only insurance)

Trade Exposure

- Bergholt D, Sveen T 2014 (theoretical, DSGE; heterogeneous industry to explain international economy co-movement)

C.2.4 Which types of firms make the output or price level more volatile?

Firm Size

- Arnold IJM, Vrugt EB 2004 (contrary to other works: more small firms, more stable growth)

Financial Sector

- Christiano L, Motto R, Rostagno M 2009 (theoretical, DSGE cal. w. Euro Area and US data; explicitly incorporate financial markets, types of producers, several financial frictions, interest rate spread)

Product Durability And Product Price Rigidity

- Gwin C, VanHoose DD 2012 (empirical)

Manufacturing Versus Other Industries

- Moro A 2012 (theoretical, DSGE)
- Ritschl A, Sarferaz S, Uebele M 2016 (empirical)

Inter-Sectoral Covariance

- Irvine O, Schuh SD 2005 (empirical; only general manufacturing and trade sectors)

C.2.5 Do business cycle phases affect industry structure or firm types?

Firm Size Distribution

- Arnold IJM, Vrugt EB 2004 (depend on sector but not on cyclicality)
- Crouzet N, Mehrotra NR 2017 (empirical w. structural model; only manufacturing; sales and investment focused; deny effects of heterogeneity in survivability, leverage, bank loan dependence, dependence on short term debt; size matters even if controlled for financing constraints like leverage and etc.)

C.3 Financial Distress and Financial Crisis

C.3.1 Which types of firms are more vulnerable?

- Milne AKL 1991 chp 4 (theoretical, empirical; UK; inventory focused; key factor net asset)
- Ivashina V, Scharfstein D 2008 (empirical; only 2007; credit ration focused)
- Carvalho D, Ferreira MA, Matos P 2015 (empirical)
- Duchin R, Ozbas O, Sensoy BA 2010 (empirical; only 2007; investment focused)
- Duygan-Bump B, Levkov A, Montoriol-Garriga J 2015 (empirical, only 1991, 2007; employment focused)
- Campello, Graham, Harvey 2010
- Kahle KM, Stulz RM 2013 (empirical, DID w. clustering by firm characteristics (called matching in the paper))
- Zamanian M 2014 (empirical; only 2007)
- Yu SE 2017 (empirical; balance sheet strength focused)
- Herrera A, Kolar M, Minetti R 2011 (empirical; credit reallocation)
- Giroud X, Mueller HM 2015 (empirical)

C.3.2 How credit shocks propagate differently?

- Chava S, Purnanandam A 2011 (empirical)
- Khawaja, Mian 2008
- Peek, Rosengren 2000

C.4 Patterns That Persist over Cycles

C.4.1 Inter-Industry Wage Structure (or Differential)

- Input-Output Analysis and the Structure of Income Distribution (1976).-.Kenichi Miyazawa
- Krueger AB, Summers LH 1986
- Krueger AB, Summers LH 1988
- Dickens WT, Katz LF 1987
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- Montgomery JD 1991
- Christensen BJ, et al 2000
- Gibbones R, et al 2005
- Dearden L, Reed H, Van Reenen J 2005
- Guadalupe M 2005
- Lallemand T, Plasman R, Rycx F 2005
- Dustmann C, Ludsteck J, Schoenberg U 2007
- Gannon B, et al 2007
- Du Caju P, et al 2009
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- Artuc E, Chaudhuri S, McLaren J 2010
- Konings J, Vanormelingen S 2010
- Sampson T 2015
- Neffke F, Otto A, Weyh A 2016
- Goldschmidt D, Schmieder J 2017
- Shim MK, Yang HS 2018

C.5 Firms' Financing (Industry or Firm Level Studies)

C.5.1 Do some types of firms' financial situation change more when (corporate-owned) real estate price changes?

- Bahaj S, Foulis A, Pinter G 2017 [sec 5.5] (empirical w. structural model; UK)

C.5.2 Do firms substitute one financing method for another?

- Kashyap AK, Stein JC, Wilcox DW 1993 (empirical)
- Kohler M, Britton E, Yates T 2000 (empirical; UK)
- Chava S, Purnanandam A 2011 (empirical)
- Becker B, Ivashina V 2014 (empirical)
- Carvalho D, Ferreira MA, Matos P 2015 (empirical)

C.5.3 Do firms restructure their balance sheet for operational purposes?

- Sen I, Humphry D 2018 (empirical; UK; only insurance)

C.5.4 Firm's uncertainty level (from the firm's perspective) and risk aversion reduces level of investment.

- Melolinna M, Miller H, Tatomir S 2018 (empirical; UK; investment focused)
- Saleheen J, et al 2017 (survey, empirical; UK; investment focused)

C.5.5 How hurdle rates connects to financial management?

- Poterba, J. and L. Summers (1995), "A CEO survey of U.S. companies' time horizon and hurdle rates", Sloan Management Review (Fall), pp. 43–53.
- Ben-David, I., J. Graham and C. Harvey (2013), "Managerial miscalibration", Quarterly Journal of Economics, Vol. 128, pp. 1547–1584 .
- Jagannathan, R., D.A. Matsa, I. Meier and V. Tarhan (2016), "Why do firms use high discount rates?", Journal of Financial Economics, 120, pp. 445-463.
- Jagannathan, R., I. Meier and V. Tarhan (2011), "The cross-section of hurdle rates for capital budgeting: an empirical analysis of survey data", NBER Working Paper 16770.

Appendix D

Acronyms

Acronym	Definition
	Variables, Functions, Algorithms, Models
AIDS	Almost Ideal Demand System model
ARIMA	Autoregressive Integrated Moving Average model
ARMH	Accept-Reject Metropolis-Hastings sampler
CD	Certificate of Deposit
CES	Constant Elasticity of Substitution function family
Cov	Covariance of two random variables
DSGE	Dynamic Stochastic General Equilibrium model
FFF	Flexible functional form
GB	Generalized Barnett flexible functional form
GM	Generalized McFadden flexible functional form
HANK	Heterogeneous Agent New Keynesian model
HJB	Hamilton-Jacobi-Bellman equation
KLEMS	Integrated production table of capital, labor, energy, material, and service
MCMC	Monte Carlo Markov Chain sampler
MH	Metropolis-Hastings algorithm or sampler
MIDAS	Mixed frequency Data model
MZM	Money of Zero Maturity
NQ	Normalized Quadratic flexible functional form
PIGL	Price Independent Generalized Linearity model
PIGLOG	Price Independent Generalized Logarithm model
PPI	Producer Price Index
QES	Quadratic Expenditure System model
QFR	Quarterly Financial Report
Repo	Repurchase agreement
ROE	Return on equity
RWMH	Random Walk Metropolis-Hastings sampler
SDE	Stochastic differential equation
SDF	Stochastic Discount Factor

Acronym	Definition
SEATS	Signal Extraction for Autoregressive integrated moving average Time Series
SME	Small and Medium Enterprises
TRAMO	Time series Regression with Autoregressive integrated moving average errors and Missing Observations
Translog	Transcendental Logarithm model
Var	Variance of a random variable, Variance-Covariance matrix of a random vector
VAR	Vector Autoregression model
VECM	Vector Error Correction Model
	Entities, Collections
BEA	Bureau of Economic Analysis
BLS	Bureau of Labor Statistics
CB	Census Bureau
FDIC	Federal Deposit Insurance Corporation
Fed	Federal Reserve
IMF	International Monetary Fund
MSI	Monetary Services Index
NAICS	North American Industry Classification System
OECD	Organization for Economic Cooperation and Development
SIC	Standard Industrial Classification
UN	United Nations
	Table, Figure Annotations
cbi	Combined input
cpt	Capital goods, capital flow or services
iip	Intermediate input
ivm	Investment
lbr	Labor
mmy	Money, monetary assets
mtr	Materials
ngy	Energy
svc	Purchased business services

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