Lap Splicing of Large High-Strength Steel Reinforcing Bars

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ABSTRACT

Use of large bars is advantageous in large structures like nuclear power plants. Nevertheless, ACI 318-19 prohibits tension lap splices for bars larger than No. 11 [36 mm] because limited test data are available. Eleven large-scale reinforced concrete beams with tension lap splices were tested under monotonically increasing four-point bending until failure to support design provisions for large diameter bars (No. 14 and 18 [43 and 57 mm]). Specimens had different amounts of transverse reinforcement, nominal concrete compressive strengths of 5 or 10 ksi [35 or 69 MPa], and target bar stresses at splice failure of 60 and 100 ksi [420 to 690 MPa] for No. 14 [43 mm] bars and 60 ksi [420 MPa] for No. 18 [57 mm] bars.

Test results show that lap splices of No. 14 [43 mm] bars can develop bar stresses up to 100 ksi [690 MPa] and that lap splices of No. 18 [57 mm] bars can develop bar stresses up to at least 60 ksi [420 MPa]. These limits reflect the scope of the text matrix, as there was no indication that higher bar stresses cannot be attained. Analyses showed that both the ACI 408R-03 and ACI 318-19 equations for lap splice lengths become less conservative as bar size increases. This trend was apparent when comparisons were made using prior test results between groups of specimens with No. 6 [19 mm] and smaller bars, No. 7 to No. 10 [22 to 32 mm] bars, No. 11 [36 mm] bars, and bars larger than No. 11 [36 mm]. Two methods are proposed for modifying ACI 408R-03 and ACI 318-19 development length equations to obtain similar levels of conservatism across all bar sizes: a bar size factor (1.15 for No. 11 [36 mm] bars and 1.25 for larger bars) in the numerator of the development length equations or an increased exponent on the bar diameter term (replacing $d_b$ with $d_b^{1.35}$ in ACI 408R-03 and $d_b^{1.25}$ in ACI 318-19). A minimum clear cover of one bar diameter is recommended for all lap splices. Transverse reinforcement should also be required throughout lap splices of No. 14 and 18 [43 and 57 mm] bars, as even small quantities ($K_{tr}/d_b = 0.5$) tended to reduce size effect. Other analyses showed that increasing the limit on $\left(\frac{c_b+K_{tr}}{d_b}\right)$ in the ACI 318-19 development length provisions from 2.5 to 3.5 results in similar conservatism for all amounts of confinement. It is also recommended that development length $\ell_d$ be limited to $50d_b$ when designing unconfined lap splices.
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CHAPTER 1 – INTRODUCTION

1.1 Motivation

Reinforced concrete members contain steel reinforcement for strength, toughness, and crack control. The bond between the steel reinforcement and concrete must allow forces to transfer between the materials for the reinforcement to be effective. In design, adequate bond strength is ensured by satisfying member-specific reinforcement detailing requirements and providing adequate lap splice and development lengths. Lap splices, which are the focus of this study, refer to the overlap between parallel discontinuous bars designed to allow for force transfer between the bars through bond with the concrete. The development length of a bar, which is also discussed in this report, is the length needed to transfer a given force between a reinforcing bar and concrete.

Research on bond has examined the effects of a wide range of variables, including bar size, bar stress, bar deformation geometry, concrete compressive strength, concrete density, clear cover, bar spacing, depth of concrete below the bar, and confinement. Design provisions and recommendations (ACI 318-19 and ACI 408-03) therefore account for these variables. However, because of very limited data from tests of large-bar lap splices, current design provisions do not permit lap splices of bars larger than No. 11. Large diameter No. 14 and 18 reinforcing bars are advantageous in very large members like foundation mats or thick walls in nuclear power plants. There is need to study the strength of lap splices of No. 14 and 18 bars to establish safe design criteria.

1.2 Scope & Objectives

The purpose of this study is to understand the behavior of large-diameter bar lap splices so that design provisions can be adapted for use with No. 14 and 18 [43 and 57 mm] bars. The authors are aware of only five studies of lap splices that included specimens with bars larger than No. 11 [36 mm]. The two most relevant are included in ACI Committee 408 (2021). Taken together, these studies include only 17
specimens with bottom-cast lap splices tested within a constant moment region. Furthermore, several of these 17 specimens had low-strength concrete or threaded bars that are not representative of conventional reinforcing bars. There is a need for more tests using materials representative of modern practice.

For this study, 11 tests of full-scale beams were conducted. There were seven specimens with No. 14 [43 mm] longitudinal bars and four specimens with No. 18 [57 mm] longitudinal bars. Each specimen was monotonically loaded in four-point bending until lap splice failure. Tension lap splices were located at midspan in a region with constant moment. The specimens were designed so that lap splice failures limited the beam strength and bar yielding did not occur. Test variables included the lap splice length (14 to 72 times the bar diameter), bar diameter (No. 14 and 18 [43 and 57 mm]), amount of transverse reinforcement (confinement), and nominal concrete compressive strength (5 and 10 ksi [35 and 69 MPa]). Specimens were designed to obtain bar stresses between 60 and 100 ksi [420 and 690 MPa].
CHAPTER 2 – LITERATURE REVIEW

2.1 Bond Mechanism

Force transfer between concrete and steel reinforcement (referred to as bond) is essential. The mechanisms of transfer are illustrated in Figure 2.1 and include bearing of the bar deformations (ribs) against concrete and frictional forces from the surface roughness of bars. These are balanced by both normal (perpendicular to bar) and shear (parallel to bar) forces. The force components perpendicular to the bar are prone to cause splitting cracks to form parallel to the bar. If cover, bar spacing, or transverse reinforcement are not adequate, a splitting failure will limit the bond strength. If clear cover, bar spacing, and transverse reinforcement are sufficient to prevent a splitting failure, a pullout failure will limit bond strength.

![Figure 2.1: Bond force transfer (ACI 408R-03)](image)

Bond strength is a structural property that depends on details such as transverse reinforcement, cover, bar size, and concrete strength and geometry. When a reinforced concrete member is loaded, there is initially strong adhesion between the steel and concrete that persists while there is relatively little slip. As loads are increased, cracks develop at the bar deformations and extend outward. As the bar slips, adhesion diminishes and force transfer occurs entirely through the bearing of bar deformations on concrete and friction. A further increase in loading causes longitudinal cracks to open, which increases slip. With low confinement, cracks may extend to the surface and result in a splitting failure (Figure 2.2). High amounts of confinement will resist concrete splitting, which can instead result in a bar pullout failure involving...
crushing of concrete between bar deformations (Figure 2.3). In commonly proportioned reinforced concrete members, splitting failures tend to be more common than pullout failures (Orangun, Jirsa, and Breen, 1977).

![Cross-sectional view of a splitting failure (ACI 408R-03)](image)

**Figure 2.2:** Cross-sectional view of a splitting failure (ACI 408R-03)

![Side view of a pullout failure (ACI 408R-03)](image)

**Figure 2.3:** Side view of a pullout failure (ACI 408R-03)

A critical bond condition in design occurs where bars are terminated or lap spliced. In either case, designers must ensure that bond force demands are low enough to prevent splitting or pullout failures under expected loads. ACI 318 approaches this using the concept of development length: the minimum embedment or overlap length needed for a terminated bar to develop a force equal to the bar area times the specified yield stress. Lap splices, which are used to transfer the force in one bar to another by overlapping them, are designed using the concept of development. The following sections describe key parameters affecting bond strength, and thus required development and lap splice lengths, followed by descriptions of relevant prior work and development length provisions in selected consensus documents including building codes/standards and committee reports.
2.2 Relevant Parameters

2.2.1 Transverse Reinforcement and Concrete Cover

Transverse reinforcement increases bond strength by resisting the opening of cracks parallel to the bar axis that may cause splitting failures (Orangun, Jirsa, and Breen, 1977) and carrying tension across cracks (Ferguson and Krishnaswamy, 1971). There is, however, a limit to the benefits of transverse reinforcement. If sufficient reinforcement is present to change the failure mode from splitting to pullout, further increases in transverse reinforcement are not correlated with bond strength (Orangun, Jirsa, and Breen, 1977).

These effects are demonstrated in results reported by Frosch, Fleet, and Glucksman (2020). Figure 2.4 shows results from tests of two specimens that were nominally identical except for the amount of transverse reinforcement. C3/60/2-40-5-50 (red) had transverse reinforcement equivalent to 50 psi (0.35 MPa), calculated as \( \left( A_v f_y \right) / (b_w s) \), where \( A_v \) is the area of shear reinforcement within stirrup spacing \( s \), \( f_y \) is the reinforcement grade, and \( b_w \) is the beam width. C3/60-40-5-100 (blue) had transverse reinforcement equivalent to 100 psi (0.69 MPa). Figure 2.4 shows that doubling the amount of transverse reinforcement produced a 12\% increase in lap splice strength. The longitudinal bar stresses at failure were 83 and 92 ksi (570 and 630 MPa) in this case.

![Figure 2.4: Effect of confinement on bond strength (Frosch, Fleet, and Glucksman, 2020)](image-url)
Increased concrete cover also increases bond strength. This is because larger concrete cover requires greater energy for splitting cracks to propagate from the bar (or bars) being developed to the concrete surface. Splitting cracks can occur between a bar and the concrete surface or within the clear spacing between bars. When $c_{bc}$ (bottom cover) is less than $c_{so}$ (side cover) and $\frac{1}{2}$ the clear spacing, $c_{sl}$, splitting cracks tend to initiate at the reinforcing bars and progress towards the bottom surface. Cracks tend to form through the side cover or between bars when $c_{bc}$ is greater than $c_{so}$ or $c_{sl}$. However, it is not just the minimum of $c_{bc}$, $c_{so}$, and $c_{sl}$ that will control behavior; Orangun, Jirsa, and Breen (1977) showed that relative values also have a substantial impact. This led to the development of the cover definitions in ACI 408-03 described in Section 2.4.2.

2.2.2 Bar Geometry

Increasing lap splice and development length will increase the force in the bar associated with bond failure, but the relationship is not proportional to length. Lap splice specimens have cracks that initiate at the end of the lap splice region, then propagate towards the center until failure. This is often accompanied by small amounts of concrete crushing near the bar deformations. Many studies show that “failure occur[s] by slow wedging action followed by rapid final slip of the bar at failure” (ACI 408R-03). Common design practice, as specified by ACI 318 codes, assumes a proportional relationship between length and force. This is conservative for short lap splices and development lengths but becomes more unconservative as the length or stress values increase because additional length does not produce a proportional increase in bond strength. Limited data show that very long unconfined lap splices ($\ell_s/d_b > 80$) may reach an asymptote where additional lap splice lengths do not provide any increase in bond strength (Glucksman et al., 2018).

As bar diameter increases, a longer lap splice length is required to reach a given bar stress. This is because the perimeter of a bar over which bond is active is a function of diameter whereas bar force is a function of diameter squared. For the same reason, larger bars will develop larger bar forces (but lower
stresses) than smaller bars using the same lap splice length when cover and confinement are constant. Bar size is therefore a key variable when calculating development length (Orangun, Jirsa, and Breen, 1977).

Most of what is known about bond is based on tests with bar sizes between No. 5 and No. 11 [16 and 36 mm] (inclusive). Only five previous studies have included specimens with larger bars: Ferguson and Thompson (1965), Ferguson and Krishnaswamy (1971), Thompson et al. (1975), Hassan et al. (2011), and Hegger et al. (2015). Between these five studies, 17 tests had large bar bottom-cast lap splices within a zero-shear region of a beam span and only 9 are included in ACI Committee 408 (2021). The remaining specimens were side casted and tested, so the correlation to modern practice is not as direct. Thompson et al. (1975) performed tests on two specimens with No. 14 [43 mm] bars and a lap splice length of 35 times the bar diameter. This study did not reach a failure bar stress of 60 ksi [420 MPa] in either test. Hassan et al. (2011) tested eight beam-splice specimens that used No. 20 [64 mm] threaded bars and had a lap splice over bar diameter ratio (ℓs/db) range from 52 to 94. The relative rib area of the threaded bars was reported as 0.16, which is much larger than the average value of 0.0727 provided in ACI 408R-03 for conventional reinforcing bars. Hegger et al. (2015) reported tests on specimens with bars with diameters of 1.56 and 1.97 in. [40 and 50 mm]. Results from Thompson et al. (1975), Hassan et al. (2011), and Hegger et al. (2015) will be used in comparisons with results from the current study.

A size-effect has also been observed in tests performed with large diameter bars. Ichinose et al. (2004) investigated size-effect on bond strength using both pullout and lap splice tests with short lap splice lengths. There were 18 beam-splice tests in this study, with and without transverse reinforcement. Ichinose et al. (2004) observed that less confinement (concrete cover and/or transverse reinforcement) results in a larger bar size-effect. The study also explained that splitting cracks, due to their inherent brittle nature, were the main factor in the size-effect. Ichinose et al. (2004) concluded that “bond strength is proportional to about -0.2 to -0.3 power of bar diameter.” A possible limitation to the test matrix of the study was lap splice length, as the only investigated value was ℓs = 12.6db. The fib Model Code (2010) appears to have accepted these findings, as the basic bond strength equation has a -0.3 power of bar diameter included.
The aforementioned large diameter study by Hassan et al. (2011) also reported similar observations. This study used both No. 9 [29 mm] deformed bars and No. 20 [64 mm] threaded bars. Hassan et al. (2011) concluded that ACI 318 design equations provided less conservative estimates of bar stress for large diameter bars. They also described a strong correlation between transverse reinforcement quantity and the apparent effect of size on lap splice strength.

Another feature of bar geometry that impacts bond strength is the size and spacing of bar deformations. A measure of this is called relative rib area (\(R_r\)). The relative rib area is a ratio of bearing area of deformations to the shearing area between as shown in Figure 2.4, which can be simplified to approximately the deformation height, \(h_r\), divided by rib spacing, \(s_r\). Higher relative rib areas tend to increase bond strength, especially in well-confined lap splices.

![Relative Rib Area (ACI 408R-03)](image)

**Figure 2.4: Relative Rib Area (ACI 408R-03)**

### 2.2.3 Concrete Compressive Strength

Bond strength increases with concrete compressive strength because higher strength concrete is more resistant to splitting and crushing near the bar deformations. Development and lap-splice lengths have traditionally been calculated as a function of \(\sqrt{f'_{c}}\). These functions apply to reinforcing bars in tension and are accepted for design for \(f'_{c} \leq 10,000 \text{ psi} \ [69 \text{ MPa}]\). The amount of contribution above this value has been the subject of many recent studies. Zuo and Darwin (1998) showed that the average bond strength normalized to \(\sqrt{f'_{c}}\) decreases with an increased concrete strength, and that the rate of decrease is more
pronounced as the lap splice length increases. Zuo and Darwin (2000) used statistical analyses of a large
database of lap splice test results to show that bond strength is more closely correlated to $f_{c}^{1/4}$ than $\sqrt{f_{c}}$.
ACI Committee 408 recommends using $f_{c}^{1/4}$ to represent the concrete contribution to bond strength (see
section 2.4).

2.2.4 Bar Stress

There is a lack of development length and lap splice test data that includes large bar diameters with
bond failures above 80 ksi [550 MPa]. Most precedent tests with bars larger than No. 11 [36 mm] have
failed below this threshold value, so there is a need to include tests that reach bar stresses of 100 ksi [690
MPa]. Longer lap splice lengths are needed to obtain higher bar stresses. However, evidence shows (refer
to 2.2.2 Bar Geometry) that very long lap splices, $\ell/d_b > 80$, may not be effective in reaching high stresses,
particularly when there is no transverse reinforcement. Some specimens in this study are designed for high
stresses.

2.2.5 Other Factors

ACI 318 and Committee 408 address a few other factors that have an impact on bond strength. One is
bar casting position. Top-cast bars, defined as having at least 12 in. [300 mm] of concrete below the bars at
the time of casting, exhibit lower bond strengths than bars bottom-cast bars located near the bottom of a
beam (Thompson et al., 1975). The negative effect is a result of bleed water migrating upwards and
materials like aggregate and cement paste settling downwards while the concrete is placed and consolidated.
These effects result in weaker concrete further from the bottom of a concrete placement. For design, a 30%
increase of development or lap splice length is recommended in ACI 318 if the longitudinal reinforcement
is top cast. Specimens can also be side-cast and as a result show varying bond strength along the height
(Ferguson and Krishnaswamy, 1971).

A reinforcement coating factor is applied in design to epoxy-coated bars. Epoxy coatings are usually
applied to prevent corrosion, but they also reduce the surface roughness and available friction and can affect
the effective $R_r$. The use of coatings can decrease bond strength up to 35%, so ACI 318 requires a factor ranging from 1.2-1.7 be applied to development and lap splice lengths. Bond strength depends on bar geometry factors in combination with coating type and casting position.

The type of aggregate used also affects bond strength. Lightweight aggregate such as blast furnace slag, pumice rock, or recycled materials are sometimes used in mixture designs. These aggregates tend to lower the tensile and compressive strengths of concrete, necessitating longer development and lap splice lengths.

2.3 Test and Specimen Setup

Many prior bond studies have been performed, but there are only a few ways to design a test setup that provides realistic results. The pullout-test schematic shown in Figure 2.5 has been widely used due to its simplicity, but the results are not representative of conditions in practice because the concrete surrounding the bar is compressed at the support where the bar is being pulled out. This setup provides apparent bond strengths that are much larger than can be achieved using other more realistic test setups. For this reason, ACI 408R-03 recommends to not use pullout specimens.

In practice, the bar and surrounding concrete are commonly both in tension parallel to the bar axis when the bar is being developed or lap spliced in tension. The only compression on the deformed bars should be the mechanical bearing of the concrete on the bar deformations. These boundary conditions are commonly achieved experimentally using a beam lap-splice specimen, shown schematically in Figure 2.6. Beginning with ACI 318-95, data used as a basis for ACI 318 bond requirements have been obtained from tests of lap splice specimens (ACI 408R-03).
Dozens of previous studies have used variations of the lap splice test setup, each of which is fundamentally similar to the schematic shown in Figure 2.6. Briggs et al. (2007) reported tests of beam-lap splice specimens to determine relationships between bond characteristics and high steel stresses. Each of the 22 specimens were cast with longitudinal bars on the bottom to avoid top-bar effects. The specimens were then inverted for testing, which allows easy (and safe) access to the concrete faces nearest the lap splice for crack marking and measurement. A photograph of the test setup is shown in Figure 2.7. Steel spreader beams were used to apply force to the ends of the specimen. The spreader beams were loaded through the laboratory strong floor with threaded rods and hydraulic jacks.
Glucksman (2018) also used a four-point bending setup to test lap splice specimens. It was important in this study for the setup to allow for the entire system to rotate and each beam end to translate as needed. This was achieved by placing a hydraulic ram directly above the concrete beam with a single point of contact. It was also important in design of the setup to locate the pin and roller supports far from the ends of the lap splice. Glucksman states that “supports were placed at least 1.5 times the beam height away from the lap splice ends” because bending stresses will distribute over a distance equal to the beam height.

2.4 Design Recommendations

2.4.1 Equation Development

Expressions for development and lap splice length are empirical (developed from test results) and not based on theoretical derivations. Orangun, Jirsa, and Breen (1977) used a statistical analysis to create equations for bond strength using results from tests of specimens with and without confinement. These equations were refined by Darwin et al. (1992), Darwin et al. (1996), and Zuo and Darwin (2000) using a larger database of test results and including a wider range of variables. The database developed and used by Zuo and Darwin (2000) was published, with some modifications, by ACI Committee 408 in 2001. This
database includes results from several hundred tests of lap splice specimens. An updated version of this database (ACI Committee 408 2021) will be used herein for some analyses.

Most equations proposed by researchers can be referred to as “descriptive” or “analysis” equations, which are meant to be best fits of a dataset with mean test/calculated results near 1.0. These equations can be very accurate for conditions similar to those represented by the specimens in the database and are useful for research purposes. Design equations used in practice must be straightforward for industry to systematically apply and safe to use in a wide range of conditions, so simplification of the descriptive equations and application of safety factors is required.

Development and lap splice requirements in ACI 408R-03, ACI 318-19, fib 2010 Model Code, and KCI Code 2018 are described below.

2.4.2 ACI 408R-03

The ACI 408R-03 descriptive equation for total bond force is shown in Equation 2.6. The last term is included only if confinement is present.

\[
\frac{T_b}{f'_c^{1/4}} = \frac{T_c + T_s}{f'_c^{1/4}} = \frac{A_b f_s}{f'_c} =
\]

\[
[59.9 \ell_d (c_{min} + 0.5d_b) + 2400A_b] \left[0.1 \frac{c_{max}}{c_{min}} + 0.90\right] + \left[30.88t_r t_d \frac{N_A t_r}{n} + 3\right] f'_c^{1/2} \quad \text{Equation 2.6}
\]

Where:

\(A_b\) = area of bar being developed or spliced
\(A_{tr}\) = total cross sectional area of each stirrup crossing splitting plane
\(c_{bc}\) = bottom concrete clear cover to bar being developed
\(c_{max}\) = maximum of \(c_{bc}\) and \(c_s\)
\(c_{min}\) = minimum of \(c_{so}, c_{bc}\,\text{and}\, c_{sl} + \frac{d_b}{2}\)
\(c_s\) = minimum of \(c_{so}\) and \(c_{sl} + 0.25\,\text{in.}\)
\(c_{sl}\) = half of clear bar spacing
\(c_{so}\) = side concrete clear cover to bar being developed
\( d_b = \text{bar diameter} \)

\( f'_{c} = \text{concrete compressive strength} \)

\( f_s = \text{bar stress} \)

\( \ell_d = \text{development or lap splice length} \)

\( n = \text{number of bars being developed or lap spliced} \)

\( N = \text{number of stirrups within the development or lap splice length} \)

\( t_d = 0.78d_b + 0.22 = \text{effect of bar size on } T_s \)

\( t_r = 9.6R_r + 0.28 = \text{effect of relative rib area on } T_s \)

\( T_b = \text{total bond force of a developed or lap spliced bar} \)

\( T_c = \text{concrete contribution to total bond force} \)

\( T_s = \text{transverse steel contribution to total bond force} \)

\( R_r = \text{relative rib area (see Section 2.2.2)} \)

The following restriction applies to Equation 2.6 because bond strength is not sensitive to transverse reinforcement quantity in addition to that required to prevent splitting failures:

\[
\frac{1}{d_b} \left[ (c_{min} + 0.5d_b) \left( 0.1 \frac{c_{max}}{c_{min}} + 0.90 \right) + \left( \frac{0.52t_r t_d A_{tr}}{sn} \right) f'_{c} \frac{1}{2} \right] \leq 4.0 \quad \text{Equation 2.7}
\]

ACI 408R recommends Equation 2.8 for both development and lap splice length calculations in design. The design equation was produced by solving Equation 2.6 for \( \ell_d/d_b \), substituting \( f'_{c} \) for \( f_s \), incorporating a phi-factor for safety, and simplifying the equation. Equation 2.9 is a form of Equation 2.8 that is used to calculate bar stress based on a lap splice length, with \( \ell_s \) substituted for \( \ell_d, f_{cm} \) for \( f'_{c} \), and \( \varphi \) set equal to 1.

\[
\frac{\ell_d}{d_b} = \frac{\frac{f_y}{\varphi f'_{c}^{1/4}} - 2400\omega}{76.3 \left( \frac{c\omega + R_{tr, 408}}{d_b} \right)} \quad \text{Equation 2.8}
\]
\[ f_s = \left( \frac{76.3 \, \ell_s \left( \frac{c \omega + K_{tr,408}}{d_p} \right)}{\alpha \beta \lambda} + 2400 \omega \right)^{1/4} f_{cm} \]  

Equation 2.9

where:  
\( c = c_{min} + d_b/2 \)
\( d_b = \) bar diameter
\( f'_c = \) concrete compressive strength
\( f_y = \) yield stress of bar being developed or lap spliced
\( K_{tr,408} = \frac{0.52 t_r t_d A_{tr}}{s n} f_{c}^{1/2} \)
\( \ell_d = \) development or lap splice length
\( \alpha = \) reinforcement location factor [1.3 for more than 12 in of concrete below bars]
\( \beta = \) reinforcement coating factor [1.2 for epoxy coated bars]
\( \lambda = \) concrete weight factor [0.75 for lightweight concrete]
\( \varphi = \) capacity reduction factor, to be taken as 0.82 for design. A value of 1.0 is used herein for comparisons with test data.
\( \omega = 0.1 \frac{c_{max}}{c_{min}} + 0.9 \leq 1.25 \)

ACI 408R-03 suggests using an upper limit of 4 for the confinement term \( \left( \frac{c \omega + K_{tr,408}}{d_p} \right) \) instead of the 2.5 value in ACI 318-19, although it’s important to note that ACI 408R-03 and ACI 318-19 define cover and confinement terms differently. The ACI 408R-03 recommendation is based on analyses that showed that bond strength is a function of \( \left( \frac{c \omega + K_{tr,408}}{d_p} \right) \) for values as large as 5 (Zuo and Darwin, 2000).

2.4.3 ACI 318-19

The ACI 318-19 equation for calculating development length is shown in Equation 2.10. The expression is a modification of the equation proposed in Orangun, Jirsa, and Breen (1977) with \( f_s \) substituted for \( f_c \). An analysis published in ACI 408R showed that this equation is generally conservative except for small bars.
ACI 408R recommends use of $\psi_s = 1.0$ for all bar sizes. Equation 2.11 is a form of Equation 2.10 that is used to calculate bar stress based on a lap splice length, with $\ell_s$ substituted for $\ell_d$ and $f_{cm}$ for $f'_{c}$.

\[
\ell_d \frac{d_b}{d_b} = \left( \frac{3}{40} \frac{f_y}{\lambda \sqrt{f'_{c}}} \psi_t \psi_e \psi_s \psi_g \right) \frac{c_b + K_{tr}}{d_b}
\]

Equation 2.10

\[
f_s = \frac{40 \ell_s \psi_t \psi_e \psi_s \psi_g}{\sqrt{f_{cm}} \frac{c_b + K_{tr}}{d_b}}
\]

Equation 2.11

where:
- $c_b$ = lesser of: (a) the distance from center of a bar or wire to nearest concrete surface, and (b) one-half the center-to-center spacing of bars or wires being developed
- $d_b$ = bar diameter
- $f'_{c}$ = concrete compressive strength
- $f_y$ = yield stress of bar being developed
- $K_{tr} = \frac{40A_{tr}}{sn}$
- $\ell_d$ = development length
- $s$ = spacing of transverse reinforcement
- $\lambda$ = lightweight aggregate concrete factor [0.75 for lightweight concrete]
- $\psi_t$ = reinforcement location factor [1.3 for more than 12 in of concrete below bars]
- $\psi_e$ = reinforcement coating factor [1.5 or 1.2 for epoxy-coated conditions]
- $\psi_s$ = reinforcement size factor [0.8 for No. 6 and smaller bars]
- $\psi_g$ = reinforcement grade factor [1.15 for Grade 80, 1.3 for Grade 100]

\[
\frac{c_b + K_{tr}}{d_b} \leq 2.5
\]
There is a restriction on both the concrete compressive strength and the bar yield stress values for Equations 2.10 and 2.11. Values are capped at $\sqrt{f'_{c}} \leq 100$ psi [0.69 MPa] & $f_{y} \leq 100,000$ psi [690 MPa]. These limits exist due to either limited test data or evidence that Equation 2.10 is unconservative outside these limits. ACI 318-19 also requires that $\ell_s = 1.3\ell_d$ for most cases, yet the development length is based on tests of lap splices. The 1.3 factor is not based on a strength criterion, rather it is intended to provide some measure of ductility and serves as a motivation for designers to stagger lap splice locations (ACI 318-19). The 1.3 factor is believed to allow steel reinforcing bars to yield instead of producing a sudden lap splice failure in the case of an overload.

2.4.4 fib Model Code 2010

The fib Model Code equation for development (anchorage) length is provided as Equation 2.12. The fib Model Code is used as a basis for many international building codes and incorporates similar factors as ACI 318, but with different variable terminology.

$$\ell_b = \frac{\varnothing \sigma_{sd}}{4f_{bd}} \geq \ell_{b,min}$$  \hspace{1cm} \text{Equation 2.12}$$

The fib Model Code equation for tension lap splice length is shown in Equation 2.13. Minimum lap splice length requirements are in Equation 2.14. Minimum cover is equal to one bar diameter.

$$\ell_b = \alpha 4 \frac{\varnothing f_{yd}}{4f_{bd}} \geq \ell_{b,min}$$  \hspace{1cm} \text{Equation 2.13}$$

$$\ell_{b,min} > max \left\{ 0.7 \frac{\varnothing f_{yd}}{4f_{bd}} ; 15 \varnothing ; 200 \text{ mm} \right\}$$  \hspace{1cm} \text{Equation 2.14}$$

where: \hspace{0.5cm} f_{bd} = \text{design bond strength}$$
\( f_{yd} \) = design yield stress of reinforcing bars in tension
\( \ell_b \) = design anchorage or lap splice length
\( \ell_{b,\text{min}} \) = minimum anchorage or lap splice length
\( \alpha_4 \) = reduction factor, may be taken as 0.7 where the calculated stress in reinforcement does not exceed 50% of the characteristic reinforcement strength, or no more than 34% of bars lapped at the section, otherwise = 1.0
\( \sigma_{sd} \) = steel stress to be anchored by bond over distance \( \ell_b \)
\( \phi \) = nominal bar diameter

2.4.5 KCI Code 2018

The current Korea Concrete Institute building code adopted development length and lap splice requirements from ACI 318-14. Requirements for large diameter bars are identical. The only difference between ACI 318-14 and ACI 318-19 is the addition of a reinforcement grade factor (\( \psi_g \)) because Grade 100 bars are now permitted.

2.5 Prior Studies of Lap Splices

2.5.1 Large (No. 14 and 18 [43 and 57 mm]) Bars

ACI 318-19, the fib Model Code, and the KCI Code do not allow tension lap splices of bars larger than No. 11 [36 mm] because of the limited test data available to provide a basis for safe design provisions. Despite the large number of studies on bond and lap splice strength, as evidenced by the ACI 408 Database (2021), the authors are aware of only five prior studies that reported tests of specimens with lap spliced No. 14 or 18 [43 or 57 mm] bars. Two of these studies used side-cast bars, so the analysis portion of this report will focus on the three studies that tested bottom-cast lap splices, which are described below. The two most relevant studies that are included in the ACI 408 Database (2021) are Thompson et al. (1975) and Hegger et al. (2015).
Ferguson and Krishnaswamy (1971) reported results from tests of nineteen specimens containing large bars. However, because the beams were side-cast, it is difficult to draw comparisons with bottom-cast bars to isolate effects of bar size. Specimens with Grade 60 [420 MPa] No. 14 [43 mm] lap-spliced bars had concrete compressive strengths of 2.7 to 3.6 ksi [19 to 25 MPa] and were either unconfined or had u-stirrup or spiral confinement. The Grade 60 [420 MPa] No. 18 [57 mm] lap-spliced bar specimens had concrete compressive strengths of 2.6 to 4.7 ksi [18 to 32 MPa] and either u-stirrups, spirals, or hairpin confinement. During testing, many specimens failed in flexure, while the remainder failed at the lap splice by “face and side split.” The report recommended that the use of transverse reinforcement be required for large lap-spliced bars. Ferguson and Krishnaswamy concluded with design recommendations and stated that lap splices with No. 14 and 18 [43 and 57 mm] bars should “no longer be prohibited.”

Thompson et al. (1975) reported tests of two specimens with lap spliced No. 14 [43 mm] bars. They stated that lap splice behavior is important to understand because of a lack of redundancy in structural members that contain lap splices. Wide sections, or wall type specimens, were the research focus in this study. The concrete compressive strength was approximately 3 ksi [20 MPa]. The study found that the lap splice provisions at the time were greatly conservative. Transverse reinforcement was shown to have many different benefits, including minimized crack widths and less brittle failures. The failure pattern described in Thompson et al. (1975) is very similar to the description reported in Orangun, Jirsa, and Breen (1977): flexural cracks initiated at the lap splice ends and then progressed towards the center of the lap splice as splitting cracks developed parallel to the lap spliced bars.

Hegger et al. (2015) reported tests of specimens with two different bar sizes larger than No. 11 [36 mm], seven of which are included in the ACI 408 Database (2021). The bars had diameters of 40 and 50 mm (1.58 and 1.97 in.). The specimens reached bar stresses up to 87 ksi [600 MPa] and included lap splice lengths of 23 to 44 times the bar diameter.
Although prior tests of lap spliced No. 14 and 18 [43 and 57 mm] bars appear to show that such lap splices can be effective, there is a need for more tests of specimens with bottom-cast bars that include a wider range of concrete compressive strengths, steel stresses, and amounts of confinement. This study is aimed at better understanding the behavior of large-diameter bar lap splices over a range of variables representative of modern practice.

2.5.2 Recent Studies of Lap Splices

Since ACI 408 published its database of lap splice test results in 2001, there have been several other important studies of lap splices within beam-lap splice specimens that inform the present work.

The ACI 408 database includes a large number of tests with high strength concrete and normal strength steel, but Seliem et al. (2009) showed that lap splices can be effective for high strength steel bars in normal strength concrete. MMFX Grade 100 [690 MPa] ASTM A1035 steel reinforcing bars were used in 69 beam-lap splice specimens to evaluate bond behavior of high-strength steel. Lap splices developed bar stresses between 68 and 155 ksi [470 and 1070 MPa] at failure. The report showed that the development length equation in ACI 408R-03 is accurate for both confined and unconfined specimens at high bar stresses. Seliem et al. (2009) also observed that unconfined specimens failed in a highly brittle (explosive) manner. The use of transverse reinforcement resulted in more gradual failures that approached bar yielding of the lap spliced bars. Although ACI 408R-03 design equations relate development length with bar stress and specify the use of conventional steel (yield stresses below 80 ksi [550 MPa]), they still proved to accurately estimate lap splice strength for high strength bars. Based on these results, and the recent addition of high strength steel to ACI 318-19, it is important to examine lap splices of large bars with high strength steel in combination with varying levels of confinement.

Large and high-strength bars may require long lap splices for the lap splice to be effective. It is therefore relevant that Richter et al. (2012) reported results from tests of specimens with long lap splices. Their focus on long lap splices was motivated by the fact that (1) most design provisions are based on tests with
specimens containing lap splice lengths shorter than $40d_b$ and (2) failure of long and unconfined lap splices has been observed in practice. Richter et al. tested prismatic coupons, T-beams, and rectangular beams. Their results show that strength does not increase proportionally with lap splice length, as discussed in Section 2.2.2 and ACI 408R, which states that bar stress is not proportional to lap splice length to bar diameter ratio. Furthermore, their work implies that unconfined lap splices may reach a point where increasing the splice length does not produce any increase in bar stress (Figure 2.8).

![Figure 2.8: Lap splice strength vs. lap splice length (Richter et al., 2012)](image)

Glucksman et al. (2018) reported results from tests of lap splice specimens with high strength reinforcement and lap splices longer than $40d_b$. Twenty-two beam specimens with No. 8 [25 mm] Grade 100 [690 MPa] longitudinal bars and a nominal concrete compressive strength of 5 ksi [35 MPa] were tested. It was found that lap splice strength is not proportional or linearly related to lap splice length. They argue that lap splice strength is a function of $\ell_{s0}^{0.5}$. It was also found that confinement is necessary for the lap spliced bars to achieve yielding when loaded, which supports the findings of Seliem et al. (2009). Frosch, Fleet, and Glucksman (2020) analyzed results from Glucksman et al. (2018) that focused on very long lap splices of No. 8 [25 mm] bars. They concluded that a predictable increase in bar stress occurs for unconfined lap splices less than $80d_b$. However, there is not an evident increase near or past this value.
Specimen U-60-5 ($ℓ/d_b=60$) reached a bar stress of 68.4 ksi [472 MPa], U-80-5 ($ℓ/d_b=80$) failed at 102.2 ksi [704.6 MPa], and U-100-5 ($ℓ/d_b=100$) failed at 103.7 ksi [714.9 MPa]. Specimen strengths increased drastically until near the $80d_b$ lap splice length threshold. This trend was shown across various specimen designs and in each large range of lap splice length.
CHAPTER 3 – TEST PROGRAM

3.1 Specimen Design

Eleven large-scale beam lap splice specimens with lap splices at midspan were tested in four-point bending under monotonically increasing force until failure. Elevation and section views of example specimens are shown in Figure 3.1 and 3.2. Specimen details are listed in Table 3.1. All specimens had either No. 14 (43 mm) or No. 18 (57 mm) lap spliced bars.

The specimens were designed to obtain a lap splice failure of the longitudinal bars at midspan. Beam cross-sectional dimensions (32 by 18 in. [810 by 460 mm] for specimens with No. 14 [43 mm] bars and 36 by 24 in. [910 by 610 mm] for specimens with No. 18 [57 mm] bars) were selected to avoid flexural compression damage prior to lap splice failure.

The overall length of each beam was either 25 or 31 ft. (7.6 or 9.5 m) and consisted of a support span, two shear regions, and two overhangs (Figure 3.1). The support span length was not less than the lap splice length plus two times the overall beam depth, \( h \). This allowed lap splice behavior to be observed without localized effects from supports. Shear region lengths and reinforcement spacing (S2) were designed to preclude shear failures. The overhang length was 24 in. (610 mm) for all specimens and was adequate to prevent longitudinal reinforcement anchorage failures near the beam ends.

![Figure 3.1: General elevation view of a test specimen](image-url)
Figure 3.2: General cross sections of the lap splice region

Lap splice lengths ($\ell_s$) were selected to obtain a nominal bar stress of either 60 or 100 ksi (420 or 690 MPa) to represent the range of bar stresses used in practice. The lengths, $\ell_s$, in Table 3.1 are mostly based on Equation 2.8 (from ACI 408R-03), although Equation 2.9 (from ACI 318-19) and some judgement were also used as a reference. The five specimens with No. 14 (43 mm) bars and a target stress of 60 ksi (420 MPa) had Gr. 80 (550 MPa) bars. The two No. 14 (43 mm) bar specimens with a target stress of 100 ksi (690 MPa) had Gr. 120 (830 MPa) bars. All specimens with No. 18 (57 mm) bars used Gr. 120 (830 MPa) bars. None of the specimens exhibited bar yielding before lap splice failure.

Other variables included concrete compressive strength, cover, spacing, and amount of confinement. Nominal concrete compressive strengths of 5 and 10 ksi (35 and 69 MPa) were used. Concrete cover values were set to realistic values to simulate construction methods in the field, but in no case was a clear cover less than one bar diameter used. No. 14 (43 mm) bar specimens had clear bottom and side cover to the longitudinal bars of 2 in. (51 mm), resulting in nominal cover to the bar center of 2.85 in. (72.4 mm). No. 18 (57 mm) bar specimens had clear bottom and side cover to the longitudinal bar of 2.5 in. (64 mm), resulting in nominal cover to the bar center of 3.63 in. (92.2 mm). These cover values coincided with
standard “chair” dimensions, easing construction, but also resulted in \( \text{(cover to bar center)} / d_b \approx 1.65 \) and \( \text{(clear cover to stirrup)} / d_b \approx 0.9 \) for all specimen designs, no matter the bar size. Conventional U-shaped hooked No. 4 (13 mm) Grade 60 (420 MPa) stirrups were used for transverse reinforcement within the lap splice and shear regions. The spacing of the confinement \( (S_1) \) was set to obtain \( K_{tr}/d_b \) ratios near target values of 0, 0.5, 1.0, or 2.0. A summary of variables is shown in Table 3.2.

### Table 3.1: Specimen properties

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Specimen Parameters</th>
<th>Transv. Bar Size</th>
<th>S1 ( d ) (in.)</th>
<th>Shear Bar Size</th>
<th>S2 ( d ) (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-a</td>
<td>h = 32, d = 29.2, b = 18</td>
<td>61</td>
<td>36</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>14-b</td>
<td></td>
<td>41</td>
<td>24</td>
<td>60</td>
<td>10</td>
</tr>
<tr>
<td>14-c</td>
<td></td>
<td>30</td>
<td>18</td>
<td>47</td>
<td>28</td>
</tr>
<tr>
<td>14-d</td>
<td></td>
<td>24</td>
<td>14</td>
<td>122</td>
<td>72</td>
</tr>
<tr>
<td>14-e</td>
<td></td>
<td>61</td>
<td>36</td>
<td>100</td>
<td>5</td>
</tr>
<tr>
<td>14-f</td>
<td></td>
<td>61</td>
<td>36</td>
<td>100</td>
<td>5</td>
</tr>
<tr>
<td>14-g</td>
<td></td>
<td>61</td>
<td>36</td>
<td>100</td>
<td>5</td>
</tr>
<tr>
<td>18-a</td>
<td>h = 36, d = 32.4, b = 24</td>
<td>81</td>
<td>36</td>
<td>60</td>
<td>5</td>
</tr>
<tr>
<td>18-b</td>
<td></td>
<td>63</td>
<td>28</td>
<td>60</td>
<td>5</td>
</tr>
<tr>
<td>18-c</td>
<td></td>
<td>54</td>
<td>24</td>
<td>60</td>
<td>5</td>
</tr>
<tr>
<td>18-d</td>
<td></td>
<td>41</td>
<td>18</td>
<td>60</td>
<td>5</td>
</tr>
</tbody>
</table>

\( ^a \) 1 in. = 25.4 mm, 1 in\(^2 \) = 645 mm\(^2 \), 1 ksi = 6.9 MPa, 1 psi = 0.0069 MPa

\( ^b \) Target bar stress at which lap splice failure is nominally expected. Bar yield stresses were always greater than \( f_y \)

\( ^c \) Clear cover to longitudinal bar, equals 1.18\( d_b \) and 1.11\( d_b \) for specimens with No. 14 and 18 bars. The clear side cover and clear bottom cover equaled \( c \) and clear spacing between lapped bars within the lap splice was \( 4c \).

\( ^d \) Stirrup spacing within lap-spline (\( S_1 \)) and shear span (\( S_2 \)) (see Figure 3.1)

### Table 3.2: Summary of variables of interest

<table>
<thead>
<tr>
<th>Group #</th>
<th>Specimens</th>
<th>Concrete Strength</th>
<th>Target Bar Stress</th>
<th>Variable ( K_{tr}/d_b ) Confinement Range</th>
<th>( \ell_{ls}/d_b ) [Within Group]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14-a, 14-b, 14-c</td>
<td>Normal</td>
<td>Regular</td>
<td>0, 1, 2.0</td>
<td>1.0, 0.7, 0.5</td>
</tr>
<tr>
<td>2</td>
<td>14-d, 14-e</td>
<td>High</td>
<td>Regular</td>
<td>0, 1, 2.0</td>
<td>1.0, 0.5</td>
</tr>
<tr>
<td>3</td>
<td>14-f, 14-g</td>
<td>Normal</td>
<td>High</td>
<td>0, 1, 2.0</td>
<td>1.0, 0.5</td>
</tr>
<tr>
<td>4</td>
<td>18-a, 18-b</td>
<td>Normal</td>
<td>Regular</td>
<td>0.5, 2.0</td>
<td>1.0, 0.8</td>
</tr>
<tr>
<td>5</td>
<td>18-c, 18-d</td>
<td>High</td>
<td>Regular</td>
<td>0.5, 2.0</td>
<td>1.0, 0.8</td>
</tr>
</tbody>
</table>
3.2 Materials

Concrete was provided from a local ready-mix plant using the specified mixture proportions in Table 3.3. The specified nominal compressive strength was 5,000 or 10,000 psi (35 or 69 MPa). The concrete contained Type I/II Portland cement, 0.75 in. (19 mm) maximum size crushed limestone aggregate, Kansas river sand, and a high-range water reducing admixture for the 10,000 psi (69 MPa) mixture. Each beam specimen was placed individually, but the specified mixture proportions were not varied.

Table 3.3: Concrete mixture proportions

<table>
<thead>
<tr>
<th>Material</th>
<th>Quantity (based on SSD aggregate conditions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Compressive Strength, psi (MPa)</td>
<td>5000 (35)</td>
</tr>
<tr>
<td>Type I/II Cement a, lb/yd³ (kg/m³)</td>
<td>590 (350)</td>
</tr>
<tr>
<td>Water b, lb/yd³ (kg/m³)</td>
<td>295 (175)</td>
</tr>
<tr>
<td>Crushed EUCA-5 Limestone c, lb/yd³ (kg/m³)</td>
<td>1670 (991)</td>
</tr>
<tr>
<td>Kansas River Sand d, lb/yd³ (kg/m³)</td>
<td>1395 (828)</td>
</tr>
<tr>
<td>Estimated Air Content, %</td>
<td>1</td>
</tr>
<tr>
<td>High-Range Water Reducer e, oz/yd³ (L/m³)</td>
<td>-</td>
</tr>
<tr>
<td>W/C Ratio</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>10000 (69)</td>
</tr>
<tr>
<td></td>
<td>750 (445)</td>
</tr>
<tr>
<td></td>
<td>218 (129)</td>
</tr>
<tr>
<td></td>
<td>1957 (1161)</td>
</tr>
<tr>
<td></td>
<td>1255 (744)</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>75 (2.9)</td>
</tr>
<tr>
<td></td>
<td>0.29</td>
</tr>
</tbody>
</table>

a Compliant with ASTM C150 requirements  
b Compliant with ASTM C1602 requirements  
c Compliant with ASTM C33 requirements, 0.75 in. (19 mm) maximum size  
d Compliant with ASTM C33 requirements  
e ADVA 600, compliant with ASTM C494 Type A and F and ASTM C1017 Type I requirements

Test-day and 28-day strengths are reported in Table 3.4. Measured test-day concrete compressive strengths ranged from 5,010 to 6,020 psi (34.6 to 41.5 MPa) for specimens with a nominal strength of 5,000 psi (35 MPa) and from 9,610 to 10,920 psi (66.3 to 75.3 MPa) for specimens with a nominal strength of 10,000 psi (69 MPa). Concrete properties, measured in accordance with ASTM standards, are given in Appendix C for each batch.

Steel reinforcement properties based on the test average of two samples of each bar type are reported in Table 3.5. Grade 60 (420 MPa) bars were used for all transverse reinforcement in the lap splice regions, with bars from a single heat used throughout the No. 14 (43 mm) bar specimen set and bars from a second
heat used throughout the No. 18 (57 mm) bar specimen set. The lap spliced No. 14 (43 mm) and No. 18 (57 mm) bars were either Grade 80 (550 MPa) or 120 (830 MPa). The grade of these bars was selected to ensure that lap splice failure occurred at bar stresses associated with linear-elastic behavior. Representative plots of stress versus strain results from tensile tests are in Appendix B for each bar type.

Table 3.4: Concrete compressive strengths

| Specimen ID | Target Nominal, $f'_{c}$ (psi) | Test-Day Measured, $f'_{cm}$ b (psi) | 28-Day Measured, $f'_{cm,28}$ b (psi) |
|-------------|-------------------------------|-----------------------------------|--------------------------------|---|
| 14-a        | 5000                          | 6020                              | 6420                           |   |
| 14-b        | 5000                          | 5990                              | 6640                           |   |
| 14-c        | 5000                          | 5920                              | 6480                           |   |
| 14-d        | 10000                         | 9610                              | 9710                           |   |
| 14-e        | 10000                         | 10920                             | 11750                          |   |
| 14-f        | 5000                          | 5540                              | 5900                           |   |
| 14-g        | 5000                          | 5010                              | 5580                           |   |
| 18-a        | 5000                          | 5670                              | 6290                           |   |
| 18-b        | 5000                          | 5410                              | 6230                           |   |
| 18-c        | 10000                         | 9730                              | 10700                          |   |
| 18-d        | 10000                         | 10190                             | 11210                          |   |

*1 ksi = 6.9 MPa, 1 psi = 0.0069 MPa*

*Measured from tests following ASTM C39 of 6 by 12 in. (150 by 300 mm) cylinders*
Table 3.5: Steel reinforcement properties  

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Grade</th>
<th>Steel Bar Use b</th>
<th>Yield Stress c, ksi (MPa)</th>
<th>Peak Stress, ksi (MPa)</th>
<th>Uniform Elongation d</th>
<th>Fracture Stress, ksi (MPa)</th>
<th>Fracture Elongation e</th>
<th>Relative Rib Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 4 (No. 13)</td>
<td>60 (420)</td>
<td>T, 14-a through 14-g</td>
<td>62.2 (429)</td>
<td>98.2 (678)</td>
<td>0.120</td>
<td>86.8 (599)</td>
<td>0.164</td>
<td>0.0693</td>
</tr>
<tr>
<td>No. 4 (No. 13)</td>
<td>60 (420)</td>
<td>T, 18-a through 18-d</td>
<td>74.0 (511)</td>
<td>91.3 (630)</td>
<td>0.102</td>
<td>73.8 (509)</td>
<td>0.136</td>
<td>0.0695</td>
</tr>
<tr>
<td>No. 14 (No. 43)</td>
<td>80 (550)</td>
<td>L, 14-a through 14-e</td>
<td>92.6 (638)</td>
<td>130.3 (898.4)</td>
<td>0.092</td>
<td>124.1 (855.6)</td>
<td>0.131</td>
<td>0.0642</td>
</tr>
<tr>
<td>No. 14 (No. 43)</td>
<td>120 (830)</td>
<td>L, 14-f through 14-g</td>
<td>134.6 (928.0)</td>
<td>169.6 (1169)</td>
<td>0.055</td>
<td>128.9 (888.7)</td>
<td>0.137</td>
<td>0.0843</td>
</tr>
<tr>
<td>No. 18 (No. 57)</td>
<td>120 (830)</td>
<td>L, 18-a through 18-d</td>
<td>129.4 (892.2)</td>
<td>163.8 f (1130.1)</td>
<td>0.0 f</td>
<td>123.9 (861.6)</td>
<td>0.131</td>
<td>0.0646</td>
</tr>
</tbody>
</table>

a Tests were conducted in accordance with ASTM A370  
b T = transverse reinforcement, L = longitudinal (lap-spliced) reinforcement  
c Determined with 0.2% offset method  
d Strain at peak stress  
e Calculated as the strain at fracture minus (fracture stress / 29,000 ksi) to approximate the fracture elongation defined in ASTM E8  
f Test terminated after bar yielding due to limitations of testing apparatus  

3.3 Construction Details

3.3.1 Formwork  

All specimens with No. 14 (43 mm) lap-spliced bars used the same set of wooden formwork, while the No. 18 (57 mm) specimens used a separate larger set. The set used for Specimens 14-a through 14-g is shown in Figure 3.4.  

Beams were cast individually to conserve space and allow each specimen to be tested near its respective target concrete compressive strength value. Formwork pieces were constructed in the laboratory out of dimensional lumber and 3/4 in. (19 mm) thick plywood in a fashion that was strong and stiff enough to keep the correct shape during concrete placement. To ensure the cross section remained at the desired width, 3/8 in. (9.5 mm) all-thread steel rods were used as through-width-reinforcement. The rods passed through the specimen at an elevation that was approximately 1/3 of the beam height from the top surface during
casting. These rods were placed at regularly spaced intervals along the beam length and cut flush with the side face after formwork removal. Interior faces of the formwork were coated with clear polyurethane to protect surfaces in contact with concrete to allow for the reuse of formwork. Exterior faces of the formwork were wrapped in plastic. Caulking was added to seams between adjoining plywood sections and painters’ putty was added around openings where threaded rods and lifting bars protruded through the formwork to prevent moisture loss and seepage. A form oil release agent was added before concrete placement as a final measure to aid the process of removing formwork. Care was taken to ensure no oil landed on the lap spliced bars.

Figure 3.4: No. 14 (43 mm) bar specimen formwork

3.3.2 Reinforcement Assembly

Specimens were fabricated and tested at the West Campus Structural Testing Laboratory at the University of Kansas. The No. 14 or 18 (43 or 57 mm) lap spliced longitudinal bars extended until 2 in. (51 mm) from beam ends (Figure 3.1) to allow for construction tolerances and ease of placement into formwork. Each specimen also had a pair of No. 7 (22 mm) longitudinal compression bars to support the transverse reinforcement. Shear reinforcement, which was used to prevent failure outside the lap splice region, initiated at the pin and roller support locations and was continued to the ends of the longitudinal bars. The
shear stirrups varied in quantity and spacing by specimen based on the expected shear demand. Figure 3.5 shows the reinforcement for Specimen 18-a after an inverted assembly.

![Figure 3.5: Reinforcement after assembly (18-a)](image)

Standard wire ties were used to assemble the reinforcement. Bars were cut with a band saw or large table saw, dictated by grade of steel and bar diameter. The clean-cut ends of large bars were used within the lap splice region to maintain consistent bar geometry at points of interest. During construction, the reinforcement was laterally supported using additional inclined bars (Figure 3.5) tied to the transverse reinforcement. These bars were removed before concrete placement if located in or near the center lap splice region. Although adequate length was provided to prevent bond failures at the beam ends, additional anchor bars were also welded between the ends of the longitudinal bars to ensure proper bar development. Standard metal bar chairs were used to provide correct concrete cover values and to prevent any movement during concrete placement.
The side and bottom cover were measured at several locations along the lap splices after the reinforcement was placed in and secured to the formwork. These measurements were used to identify deviations from target values before concrete placement and are not reported here. Concrete cover values reported in Chapter 4 were obtained from measurements made after testing.

3.3.3 Concrete Placement and Curing

Beams were cast individually using concrete from a local ready-mix concrete company. All beams had bottom-cast lap splices, as specimens were inverted prior to testing. Photos of example specimens before and after concrete placement are shown in Figures 3.7 and 3.8. The concrete was placed in layers using a process that started at the beam ends, progressed through the middle where the lap splice was located, then ended back at the beam ends. Both layers in the lap splice region were from the middle portion of the overall batch.

Samples were taken from the middle portion of the batch at regularly spaced intervals in accordance with ASTM C172. This concrete was used for standard tests including slump, temperature, and unit weight. Both 4 by 8 in. (75 by 150 mm) and 6 by 12 in. (150 by 300 mm) cylinders were cast using steel molds in accordance with ASTM C192. Specimens were cured in the formwork with wet burlap and plastic tarps
covering the open faces until the compressive strength was approximately 3,000 psi (21 MPa). Formwork was then removed from the specimen and cylinders were removed from the molds. Cylinders were stored next to the beam specimens while the concrete was dry cured until testing. Two 4 by 8 in. (75 by 150 mm) cylinders were tested every seven days until the concrete reached its target compressive strength, at which point the beam specimen was tested (approximately 14 to 21 days after concrete placement). Three standard 6 by 12 in. (150 by 300 mm) cylinders were then tested in accordance with ASTM C39 and reported as the test-day compressive strength in Table 3.4. Table 3.4 also reports the 28-day compressive strength of each batch.

Figure 3.7: Formwork before concrete placement (18-b)
3.4 Test Setup

Figure 3.9 shows 14-b set up for testing. A reverse four-point bending test setup was used wherein downward force was applied to the beam ends. This produced a condition of zero shear force within the middle portion of the beam span where the lap splices were located. Testing with the tension face upwards also provides easier and safer access for marking cracks during testing. As previously discussed, the specimens were cast with the longitudinal lap-sliced bars at the bottom to avoid top-bar effects. The beam was then rotated about its longitudinal axis and placed upon “pin” and “roller” supports. The “roller” consisted of a 2 in. (50 mm) diameter steel rod placed between steel plates. This rod was free to rotate, thereby allowing free horizontal translation of the beam at this support. Gypsum plaster cement was used between all steel and concrete surfaces.
Force was applied using four hollow-core hydraulic jacks acting on two steel built-up spreader beams that beared on the top face of the beam. Forces from the hydraulic jacks were transmitted to the strong floor by threaded rods with nuts above the setup and below the floor (Figure 3.10). Hollow-core load cells were placed above the hydraulic jacks to measure the imposed force. The steel spreader beams were constructed with C15x33.9 channels welded to 0.5 in. (13 mm) thick top and bottom plates. A 3 in. (76 mm) gap between the channels provided room for threaded rods to pass through the centroid of the member cross-section. To compensate for beam-end rotation during testing, a 4 in. (102 mm) diameter steel half-cylinder was welded to the bottom face of each spreader beam to form a “pin”. When the concrete beam ends deflected downwards, the threaded rods and components stayed vertical.
Figure 3.10: End view of test setup

The specimens were loaded using four hydraulic jacks, high-pressure hoses, and a manual pump. Force was applied at a slow rate until specimen failure, with pauses to allow for marking, measuring, and photographing cracks. Load was paused at half the estimated cracking load and at the estimated cracking load, followed by pauses at forces associated with increments in estimated lap-spliced bar stresses of 10 or 20 ksi (70 or 140 MPa), depending on the final target strength. For example, loading might be paused at loads associated with estimated bar stresses of 20, 30, and 40 ksi (140, 210, and 280 MPa) if the target stress at failure was 60 ksi (420 MPa). The final pause was at 60 to 80% of the expected failure force, at
which point the specimen was covered with a tarp for safety (this tarp contained the debris that can become airborne when a lap splice failure occurs for worker safety but did not affect test results).

At each load step, the beam sides and the top (tension) face were inspected and cracks were marked and measured. A crack comparator card was used and the largest crack at each load step within or near the lap splice region was recorded. Photographs of the marked cracks were taken during each pause and after failure. Load increments took approximately 5 minutes, while each pause took approximately 10 minutes.

3.5 Instrumentation

Hydraulic pressure was recorded using an electronic pressure gauge. Force applied by each jack was measured using load cells (Figures 3.9 and 3.10).

Bar strains were measured using four 120-ohm strain gauges with a gauge length of 5 mm applied to the lap spliced bars in each specimen. Each of the four strain gauges (for all 11 specimens) were located just outside the splice region, or a nominal distance of 1 ¾ - 2 ¾ in. (44 - 70 mm) from the splice ends (Figure 3.11). Strain gauges were located underneath the longitudinal bars and protected by multiple coating layers during concrete placement. Although bar force at failure was inferred from the beam moment (see Table 4.2), results from these gauges provide a secondary basis for estimating bar force at failure.

Figure 3.11: Strain gauge placement
In addition, 18-c and 18-d also had strain gauges on the confining reinforcement within the lap splice region. Specimen 18-c was instrumented with four additional gauges and 18-d had six additional gauges (Figure 3.12). Results from these gauges provide a basis for inferring the stress in the transverse reinforcement at failure of the lap splice. All longitudinal reinforcement gauges were affixed to the outermost surface of the bars (the bottom of the bar when concrete was placed) for protection and consistency throughout testing.

Vertical beam displacements were recorded using three potentiometers. One potentiometer was located below the beam at midspan, while the other two were each under the loading points near the beam ends (Figure 3.13). Beam deflection was taken as the difference between the mid-span (upward) displacement and the average of the (downward) displacements measured under the loading points.

An optical tracking system was used during the tests of 18-c and 18-d that allowed for measurement in three (x-y-z) dimensions of the positions of multiple markers fixed to the surface of the specimens. Figures 3.13 and 3.14 show the locations of the markers used for 18-c and 18-d, respectively. Seven columns of
two markers were attached to one face of each specimen at various points of interest. Points of interest included outside the lap splice end, inside the lap splice end, beam centerline, support location, and loading point. Among other things, these data can be analyzed to determine the relative rotation between vertical sections along the beam length, as described in Section 4.5.

One data acquisition system was used for the pressure gauge, load cells, strain gauges, and potentiometers. A separate data acquisition system was used for the position-tracking system. These data were combined after testing using a common event, such as failure, to align the results.

![Figure 3.13: Displacement instrumentation: 18-c](image1)

![Figure 3.14: Displacement instrumentation: 18-d](image2)
CHAPTER 4 – ANALYSIS

4.1 Load-Deflection Results

The applied load versus midspan deflection results had similar characteristics in all tests. Sample plots are shown as an example in Figure 4.1. Plots for all specimens are included in Appendix D. The applied load (kips) is the sum of the recorded forces in the four individual load cells, and the deflection (in.) is the vertical displacement of midspan minus the average vertical displacement at the supports (Section 3.5). The specimen self-weight and test frame weight are not included in the applied load versus deflection plots.

The general response shows an initial linear-elastic region that continues until beam cracking, followed by a change in slope. After cracking, the applied load and deflection were approximately linearly related. In specimens with no confinement (Figure 1(a)), lap splice failure occurred suddenly and with no apparent reduction in stiffness prior to failure. Specimens with confinement exhibited some minor softening prior to lap splice failure (Figure 1(b)) that was not a result of reinforcement yielding (Section 4.3). This softening may be a result of bar slip within the lap splice prior to lap splice failure. Load step pauses are also visible as drops in both load and deflection.

The maximum applied load and deflection values at failure are shown in Table 4.1. Specimens 14-f and 14-g, which nominally targeted bar stresses of 100 ksi (690 MPa), showed the largest deflections out of all the tests.
Figure 4.1: Sample load-deflection behavior: (a) 14-a and (b) 18-a
Table 4.1: Maximum applied load and deflection at failure

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Maximum Applied Load kip (kN)</th>
<th>Maximum Deflection in. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-a</td>
<td>196 (872)</td>
<td>0.64 (16.3)</td>
</tr>
<tr>
<td>14-b</td>
<td>200 (890)</td>
<td>0.81 (20.6)</td>
</tr>
<tr>
<td>14-c</td>
<td>218 (970)</td>
<td>0.89 (22.6)</td>
</tr>
<tr>
<td>14-d</td>
<td>190 (845)</td>
<td>0.57 (14.5)</td>
</tr>
<tr>
<td>14-e</td>
<td>196 (872)</td>
<td>0.72 (18.3)</td>
</tr>
<tr>
<td>14-f</td>
<td>247 (1099)</td>
<td>1.38 (35.1)</td>
</tr>
<tr>
<td>14-g</td>
<td>395 (1757)</td>
<td>1.69 (42.9)</td>
</tr>
<tr>
<td>18-a</td>
<td>370 (1646)</td>
<td>1.33 (33.8)</td>
</tr>
<tr>
<td>18-b</td>
<td>361 (1606)</td>
<td>1.28 (32.5)</td>
</tr>
<tr>
<td>18-c</td>
<td>347 (1544)</td>
<td>1.19 (30.2)</td>
</tr>
<tr>
<td>18-d</td>
<td>355 (1579)</td>
<td>1.18 (30.0)</td>
</tr>
</tbody>
</table>

4.2 Cracking Behavior

4.2.1 Description of Cracking and Failure

All specimens failed by splitting/spalling along the lap splice before yielding of the bars (Section 4.3); specimens with unconfined lap splices failed explosively whereas specimens with confined lap splices failed more gradually. Photos of all specimens after failure are in Appendix E.

Each test had load pauses for marking crack locations and measuring crack widths. Figures 4.2 and 4.3 show specimens with confined lap splices, and Figure 4.4 shows a specimen with unconfined lap splices. Cracking was largely similar across all specimens, but there were specific differences between specimens with confined and unconfined lap splices. The first observed cracks initiated at or near the ends of the lap splices. Cracks then propagated vertically down the beam sides and across the tension (top) face as the load increased during testing. For confined lap splices, cracks were commonly observed at locations where transverse reinforcement was present, as shown in Figure 4.2. For unconfined specimens, there was relatively less cracking along the lap splice length until failure (Figure 4.4b). Flexural-tension cracks (transverse to the beam axis) were the predominant crack type throughout most of the tests. Splitting cracks were also observed on the tension face parallel with the longitudinal reinforcement (Figure 4.5). At failure, specimens with confined lap splices usually exhibited spalling of the cover (Figure 4.3c). Specimens with
unconfined lap splices exhibited extensive spalling along with wide cracks at the lap splice ends perpendicular to the beam axis at failure (Figures 4.4c and 4.6).

Figure 4.2: Cracking along transverse reinforcement on tension face (14-b)
(a) Initial cracking

(b) Cracking during increased loading

(c) After failure

Figure 4.3: Photos of 14-g (heavily confined) during testing
Figure 4.4: Photos of 14-d (unconfined) during testing
4.2.2 Crack Widths

Crack widths were manually measured during each test when the loading was paused (Section 3.4). Figures 4.7 and 4.8 show the largest crack width measured at each pause in loading for specimens with No. 14 and 18 (43 and 57 mm) lap spliced bars, respectively, plotted versus estimated longitudinal bar stress. Values for bar stress in Figures 4.7 and 4.8 were estimated based on nominal section properties and concrete compressive strength measured on the day of testing. This includes the applied loads, weight of the test frame, and specimen self-weight. More detail is provided in Section 4.4.
Figure 4.7: Crack widths of specimens with No. 14 (43 mm) bars
As the load increased, so did the bar stress and crack widths. Specimens 14-f and 14-g, shown in Figure 4.7, follow the trends of the specimens designed to reach 60 ksi (420 MPa), but eventually reached higher bar stresses and thus larger crack widths. Specimens with the same nominal dimensions and reinforcement areas therefore exhibited similar crack widths at a bar stress of 40 ksi regardless of the target stress, as expected. However, specimens targeting a bar stress near 100 ksi (690 MPa) did have much larger crack widths at bar stresses near 2/3 of the target stress than specimens targeting bar stresses near 60 ksi (420 MPa). Plots of crack width versus bar stress in terms of a percentage of measured stress can be found in Appendix E.
Specimens with No. 14 (43 mm) bars showed similar trends as those with No. 18 (57 mm) bars for the most part. However, at a given bar stress, specimens with No. 18 (57 mm) bars had larger crack widths. At a load step correlating to approximately 40 ksi (280 MPa) of bar stress, the No. 14 (43 mm) bars had an average maximum crack width of 0.021 in. (0.533 mm) and the No. 18 (57 mm) bar specimens had an average maximum crack width of 0.027 in. (0.686 mm). These differences are statistically significant based on a two-sample Student’s t-Test assuming equal variances, with $p = 0.03$, where $p$ values less than 0.05 are taken as evidence of statistical significance. No other trends between crack width and other variables of interest (concrete compressive strength, amount of confinement, etc.) were observed. All maximum crack width data collected is also provided in Appendix E.

### 4.3 Measured Strains

Each specimen had strain gauges on the longitudinal bars near the ends of the lap splices and two specimens had strain gauges on transverse reinforcement within the lap splice (Section 3.5). Plots of measured strain versus time for each specimen are in Appendix F, along with a tabulated summary of bar stress at failure inferred from the strain gauge measurements.

Figure 4.9 shows longitudinal reinforcement strain versus time for 14-b. In general, the strains from all the gauges followed the same trend: strain increased with load, which increased with time except for the pauses in loading. However, a closer look shows differences. It appears that bar strains increased until failure at two of the gauges, whereas data from two of the gauges exhibited slight declines in strain shortly before failure. Interestingly, the gauges that showed declines were A-2 and B-2, located at each end of the splice on beam face 2. A possible explanation is that the lap splice on beam face 2 started to slip and shed force while the lap splice on beam face 1 continued to attract force as beam load increased. This implies that the lap splices did not fail simultaneously; rather, one lap splice started to slip and shed force to other lap splices until the group of lap splices could not take additional load. The plots in Appendix F show that this behavior is evident in several specimens with transverse reinforcement and is not evident in any
specimens without transverse reinforcement. Transverse reinforcement may therefore be necessary for this sharing of force among lap splices.

Figure 4.9: Longitudinal bar strain for 14-b

Figure 4.9 also shows that the largest recorded strain (0.00216) was smaller than the strain associated with yielding, which was 0.0032 based on the measured yield stress in Table 3.5 and an assumed modulus of 29000 ksi (200 GPa) for the bars used in 14-a through 14-e. This was the case for all specimens; there is no evidence of longitudinal bar yielding in these tests.

Figure 4.10 shows a plot of the transverse reinforcement strain versus time for 18-d. The figure shows that the transverse reinforcement strains were much smaller than the longitudinal (lap spliced) bar strains, and certainly much smaller than strains associated with transverse bar yielding (the strain at yield is approximately 0.00255 based on Table 3.5, an assumed modulus of 29,000 ksi (200 GPa), and assuming zero bar strain at the start of the test). It has been previously observed that transverse reinforcement confining a tension lap splice in a constant moment region does not yield (ACI 408R-03). The data in Figure
4.10 imply that this finding applies even to specimens with high strength concrete and large (No. 18, 57 mm) bars, which have large bar deformations that might be expected to force wider splitting cracks.

![Graph showing transverse reinforcement strains](image)

**Figure 4.10: Transverse reinforcement strain for 18-d (SG C1 not recorded)**

The two gauges that recorded the largest transverse reinforcement strains in 18-d were C2 and C5, which were located on transverse reinforcement 8.5 in. [220 mm] from the ends of the lap splice. It was expected that transverse reinforcement near the splice ends would be more stressed than transverse reinforcement near midspan because the longitudinal bar stresses and slip are largest near the lap splice ends. It is not clear why C6, located on the stirrup located at the end of the lap splice, recorded much smaller strains than C2 or C5.
4.4 Bar Stress Calculations

The stress at failure in the lap spliced bars was estimated for each specimen. The results are shown in Table 4.2. These bar stresses are based on the beam moment at the center of the lap splice (midspan), which was calculated based on the applied loads, test frame weight, and specimen self-weight. The stress in the longitudinal bars was estimated from the midspan moment using a moment-curvature analysis, assuming a concrete stress-strain relationship consisting of a parabolic shape up to peak stress and a linear descending tail from Hognestad (1951) (Equation 4.1). The peak stress was taken equal to the measured concrete compressive strength on the day of testing and the strain at peak stress, \( \varepsilon_o \), was from Darwin and Dolan (2021) or Bashur and Darwin (1978) (see ACI 408R-03). The concrete stress is shown as \( f_c \) and the strain is \( \varepsilon_c \).

\[
f_c = f_{cm} \left[ \frac{2\varepsilon_c}{\varepsilon_o} - \left( \frac{\varepsilon_c}{\varepsilon_o} \right)^2 \right], \varepsilon_c \leq \varepsilon_o \quad \text{Equation 4.1a}
\]

\[
f_c = f_{cm} \left[ 1 + \frac{0.15}{0.0038 - \varepsilon_o} (\varepsilon_o - \varepsilon_c) \right], \varepsilon_c > \varepsilon_o \quad \text{Equation 4.1b}
\]

The steel stress-strain curves were determined through tensile tests and representative plots are included in Appendix B. The constitutive relationships used for the steel were proportional for Grade 80 [550 MPa] bars up to the yield strain and an exponential best-fit for the Grade 120 [830 MPa] bars. General stress versus strain relationships for ASTM A1035 steel No. 5, 8, and 11 [16, 25, and 36 mm] bars were provided in Seliem et al. (2009). Following a similar equation form, the following two exponential best-fit equations were used for the Grade 120 [830 MPa] bars used in this study: \( f_s = 162(1 - e^{-212\varepsilon_s}) \) for No. 14 [43 mm] bars and \( f_s = 200(1 - e^{-175\varepsilon_s}) \) for No. 18 [57 mm] bars. This process was the same as described in ACI Committee 408R-03. The stresses estimated in this manner are within 10% of the bar stress values estimated based on strain gauges for all specimens except for 14-e (Appendix F).

Table 4.2 shows the bar stresses obtained from test results and bar stresses calculated based on ACI 408R-03 (Equation 2.9) and ACI 318-19 (Equation 2.10). The ACI 408R-03 descriptive equation is
intended to be a best fit of the available data, whereas the ACI 318-19 design equation is necessarily more conservative for use in design. For Equation 2.9, \( \frac{c_b + K_{tr}}{d_b} \) was taken as not greater than 4 as recommended in ACI 408R-03. Table 4.2 shows calculated bar stresses from Equation 2.11 with \( \frac{c_b + K_{tr}}{d_b} \) taken as either not greater than 2.5 or not greater than 4. The 2.5 limit is from ACI 318-19, but increasing the limit to 4, like Equation 2.9, provides a better fit to the experimental data in this study (COV of 0.18 compared with 0.27 in Table 4.2). Specimens 14-c, 14-e, and 14-g had \( 2.5 \leq \frac{c_b + K_{tr}}{d_b} \leq 4 \) and Table 4.2 shows that Equation 2.11 with \( \frac{c_b + K_{tr}}{d_b} \leq 4 \) provided a better estimate of the bar stress at failure for these specimens than Equation 2.11 with \( \frac{c_b + K_{tr}}{d_b} \leq 2.5 \). This is examined further in Section 4.4.1.

<table>
<thead>
<tr>
<th>Spec. ID</th>
<th>Beam Moment at Lap-Splice Center, kip-in.</th>
<th>( f_s ) c Estimated Bar Stress at Failure, ksi</th>
<th>Calculated Bar Stress d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ACI 408R-03 e, ksi</td>
<td>Test/Calc.</td>
</tr>
<tr>
<td>14-a</td>
<td>6,440</td>
<td>53.8</td>
<td>63.0</td>
</tr>
<tr>
<td>14-b</td>
<td>6,570</td>
<td>54.9</td>
<td>65.2</td>
</tr>
<tr>
<td>14-c</td>
<td>7,160</td>
<td>60.2</td>
<td>66.1</td>
</tr>
<tr>
<td>14-d</td>
<td>6,270</td>
<td>52.0</td>
<td>58.6</td>
</tr>
<tr>
<td>14-e</td>
<td>6,440</td>
<td>53.4</td>
<td>64.6</td>
</tr>
<tr>
<td>14-f</td>
<td>8,660</td>
<td>73.2</td>
<td>102.4</td>
</tr>
<tr>
<td>14-g</td>
<td>12,760</td>
<td>109.3</td>
<td>111.8</td>
</tr>
<tr>
<td>18-a</td>
<td>15,540</td>
<td>67.0</td>
<td>76.7</td>
</tr>
<tr>
<td>18-b</td>
<td>15,340</td>
<td>66.6</td>
<td>77.7</td>
</tr>
<tr>
<td>18-c</td>
<td>14,590</td>
<td>61.4</td>
<td>71.8</td>
</tr>
<tr>
<td>18-d</td>
<td>14,930</td>
<td>62.9</td>
<td>74.4</td>
</tr>
</tbody>
</table>

| | Mean: 0.86 | Coefficient of Variation: 0.07 | Mean: 0.99 | Coefficient of Variation: 0.18 | Mean: 1.06 | Coefficient of Variation: 0.27 |

\( a \) 1 ksi = 6.9 MPa
\( b \) 1 kip-in. = 0.113 kN-m
\( c \) Obtained from beam moment at failure using moment-curvature analyses per Section 4.4
\( d \) Minimum lap splice length requirements are neglected
\( e \) Equation 2.9
\( f \) Equation 2.11 with \( \psi \geq \left[ 1 + 0.15 \left( \frac{f_s - 60 \text{ksi}}{20} \right) \right] \text{ and } \sqrt{f_c'} \leq 100 \text{ psi} \)
Table 4.2 shows ratios of test / calculated (T/C) bar stresses. Equation 2.9, based on ACI 408R-03, provided the best fit of the data in terms of scatter, with a coefficient of variation (COV) of 0.07 compared with COVs of 0.18 and 0.27 for Equation 2.11 (based on ACI 318-19). Table 4.2 shows that 14-f had a very low test / calculated bar stress ratio regardless of which equation was used to calculate bar stress. This outlier will be discussed further in Section 4.4.2.

The T/C values in Table 4.2 were also frequently less than 1.0. All T/C values were less than 1.0 for Equation 2.9, with a mean of 0.86, and 5 of the 11 specimens had T/C less than 1.0 based on Equation 2.11 and \( \left( \frac{c_b + K_{tr}}{d_b} \right) \leq 2.5 \). This indicates that current ACI 318 design provisions are not safe for use in design of large-bar lap splices without modification. Bar size effects are examined further in Sections 4.4.3 and 4.4.4.

4.4.1 ACI 318-19 Limits on \((c_b + K_{tr})/d_b\)

The ACI 408 database was used to further investigate the effect of increasing the limit on \( \left( \frac{c_b + K_{tr}}{d_b} \right) \) in Equations 2.10 and 2.11. For this comparison, all bottom-cast lap splice specimens in the ACI 408 database were included except for those with a lap splice length less than 12 in. [300 mm]. Here Equation 2.11 is used with \( \psi_g \geq \left[ \left( 1 + 0.15 \left( \frac{f_{s-60ksi}}{20} \right) \right), 1.0 \right] \), \( \sqrt{f_c'} \leq 100 \) psi, and \( \psi_s = 1.0 \) for all bar sizes as recommended in ACI 408R-03.

Figure 4.11a shows calculated bar stress at failure from Equation 2.11 versus bar stress at failure estimated from tests. There is no limit applied to the \( \left( \frac{c_b + K_{tr}}{d_b} \right) \) term in this plot. The orange trendline is based on specimens with \( 2.5 \leq \left( \frac{c_b + K_{tr}}{d_b} \right) \leq 4 \) and the blue trendline is based on those with \( \left( \frac{c_b + K_{tr}}{d_b} \right) < 2.5 \). These two lines are indistinguishable in Figure 4.11a. The slopes of the trendlines, which were forced to intersect the origin, were 1.19 for the orange line (\( 2.5 \leq \left( \frac{c_b + K_{tr}}{d_b} \right) \leq 4 \)) and 1.19 for the blue line (\( \left( \frac{c_b + K_{tr}}{d_b} \right) < 2.5 \)). The scatter was also similar, with COV of 0.18 and 0.25 for the populations with \( 2.5 \leq \left( \frac{c_b + K_{tr}}{d_b} \right) \leq 4 \)
and \(\left(\frac{c_b + K_{tr}}{d_b}\right) < 2.5\), respectively. As expected, the green trendline that is based on tests with \(\left(\frac{c_b + K_{tr}}{d_b}\right) > 4\) has a much lower slope, and most (72%) of these results have test / calculated stress values below 1. It therefore appears that the limit on \(\left(\frac{c_b + K_{tr}}{d_b}\right)\) in the ACI 318-19 equation could increase from 2.5 to 4 without resulting in lower expected strengths than are already accepted for specimens with \(\left(\frac{c_b + K_{tr}}{d_b}\right) < 2.5\). This is demonstrated in Figure 4.11b, as the slope of the green trendline increases to 1.10 when a limit of 4 is applied. Further analysis shows that with \(\left(\frac{c_b + K_{tr}}{d_b}\right) \leq 3.7\), the green trendline would have a slope equal to that of the blue line (1.19). It therefore appears ACI 318-19 could be modified to allow \(\left(\frac{c_b + K_{tr}}{d_b}\right)\) values up to 3.5 while maintaining a similar conservatism for all \(\left(\frac{c_b + K_{tr}}{d_b}\right)\) values. For simplicity and consistency with ACI 408R-03, a limit of 4 is applied to \(\left(\frac{c_b + K_{tr}}{d_b}\right)\) for the rest of Chapter 4.

![Figure 4.11: ACI 408 Database (2021) bottom-cast specimens with \(\ell_s \geq 12\) in. [300 mm] versus stress calculated based on ACI 318-19](image-url)
4.4.2 Long Unconfined Lap Splices

The low T/C bar stress ratio for 14-f in Table 4.2 is an outlier from the other data in this study. Specimen 14-f had $\ell_s/d_b = 72$ and had a bar stress of 73.2 ksi [505 MPa] at failure, corresponding to T/C of 0.71 and 0.67 when compared with Equations 2.9 and 2.11, respectively. For comparison, 14-g with $\ell_s/d_b = 36$ and transverse reinforcement had a bar stress at failure of 109.3 ksi [754 MPa] and T/C of 0.98 and 1.39 when compared with Equations 2.9 and 2.11 (with $(c_b + K_{tr}d_b) \leq 4$), respectively. To understand whether the 14-f result was an anomaly or evidence of a problem with long and unconfined lap splices, the literature was reviewed to identify other tests of long and unconfined lap splices.

Seliem et al. (2009) tested several beam-splice specimens. Although the study focused on bar sizes No. 5, 8, and 11 [No. 16, 25, and 36 mm], high-strength steel was used in conjunction with normal-strength concrete, similar to 14-f of this study. Seliem et al. (2009) concluded that the presence of stirrups in the lap-splice region led to higher strengths at failure, as expected, but also that increasing unconfined splice lengths as a method to achieve a high bar stresses may not be as efficient as adding transverse reinforcement. Furthermore, a closer look at their results shows that specimens 11-5-XC0-2 and 11-8-XC0-2, which had No. 11 [36 mm] bars and $\ell_s/d_b = 56$ and 65 and were the only two specimens in the study with bars larger than No. 8 [25 mm] and $\ell_s/d_b \geq 50$, had bar stresses at failure between 70 and 80 ksi [480 and 550 MPa] and T/C less than 1 (0.96 and 0.79, respectively) based on Equation 2.9. This implies there may be a value of $\ell_s/d_b$ beyond which increases in unconfined lap splice length do not provide increased bond strength.

Results from Richter et al., described in Section 2.5.2 and shown in Figure 2.8, support this interpretation. Their tests showed that increases in unconfined lap splice length beyond $\ell_s/d_b = 60$ produced little increase in bar stress at failure. Results from Frosch, Fleet, and Glucksman (2020) further demonstrate that increasing the length of long unconfined lap splices does not produce an increase in bar stress at failure. Specimens U-80-5, U-100-5, and U-120-5, which had No. 5 [16 mm] bars, the same concrete, and lap splice lengths of 80, 100, and 120 times the bar diameter, exhibited bar stresses at failure of 102.2, 103.7, and
103.5 ksi [705, 715, and 714 MPa]. These results imply that $\ell_s/d_b = 80$ may be beyond the limit at which increases in unconfined lap splice length can be expected to provide increases in strength.

Given the results from this study, Seliem et al. (2009), Richter et al. (2012), and Frosch, Fleet, and Glucksman (2020), it seems appropriate to limit the unconfined lap splice length to approximately $60d_b$. However, a limit of $50d_b$ is recommended regardless of bar size because (1) there are few data available with lap splices longer than $50d_b$, and (2) the 11 test results in the ACI 408 Database (2021) that had unconfined bars with a lap splice length between 50 and $60d_b$ had a mean T/C of 0.90 based on Equation 2.9. For comparison, the mean T/C for specimens in the ACI 408 Database (2021) with bottom-cast unconfined lap splices with lap splice lengths shorter than $50d_b$ is 1.01. The authors therefore recommend prohibiting unconfined lap splices with $\ell_s/d_b > 50$.

4.4.3 Effects of Bar Size

The results in Table 4.2 show that the mean test / calculated values were 0.86 and near 1.0 based on Equations 2.9 and 2.11 (from ACI 408R-03 and ACI 318-19), respectively. The ACI 408 Database (2021) was used to examine how these results compare with a broader set of test results with smaller bar sizes to identify whether there is an effect of bar size. It is noted that Equation 2.9 (ACI 408R-03) is a “descriptive” equation designed to produce T/C near 1.0, whereas Equation 2.11 (ACI 318-19) is a design equation that should produce T/C greater than 1.0.

These comparisons used bottom-cast specimens from the ACI 408 Database (2021) with $\ell_s \geq 12$ in. [300 mm] and, for unconfined lap splices, $\ell_s \leq 50d_b$ (see Section 4.4.2 for rationale). The dataset used for comparisons against Equation 2.9 was further limited to $\ell_s \geq 16d_b$. Thus, 14-e and 14-f from the current study are not included in comparisons against Equation 2.9 in this section.

Figure 4.12 shows estimated bar stress at failure from tests versus bar stress calculated with Equation 2.9 for specimens from the ACI 408 Database (2021) and this study (gray circles and black triangles, respectively). The red dashed line represents a T/C of 1.0. Equation 2.9 provides an accurate estimation of
bond strength, with a mean T/C of 1.02 and a COV of 0.12 for the specimens from the ACI 408 Database (2021). Conversely, the nine tests from this study in Figure 4.12 had a mean T/C of 0.88. The mean T/C for specimens from the current study (0.88) is more than one standard deviation (0.12) less than the mean T/C for the entire database (1.02). This is a substantial difference and downward shift in mean T/C. These differences are also statistically significant based on a two-sample Student’s t-Test assuming equal variances, with $p$ less than 0.001, where $p$ values less than 0.05 are taken as evidence of statistical significance.

![Figure 4.12: Estimated bar stress at failure from tests versus bar stress calculated with Equation 2.9](image)

Figure 4.12 shows T/C based on Equation 2.9 (ACI 408R-03) versus four key variables: measured bar stress, concrete compressive strength, lap splice length to bar diameter ratio, and confinement ($\frac{c_\omega + K_{tr,408}}{d_b}$).

The figure includes the 24 specimens that the authors are aware of meeting the following criteria: bottom-cast lap spliced bars larger than No. 11 [36 mm], $\ell_s \geq 16d_b$, $\ell_s \geq 12$ in. [300 mm], and if unconfined $\ell_s \leq$
 Specimens from Hassan et al. (2012) are not included in the ACI 408 Database (2021) but are included here.

Figure 4.13 shows that 21 of 24 specimens with large bars had \( T/C \) less than 1.0 (mean = 0.89 and COV = 0.12), indicating that the low \( T/C \) values in Table 4.2 are not anomalies among tests with large bars. This is evidence of a large-bar size effect that is not captured by Equation 2.9.

There is no clear trend in Figure 4.13 between \( T/C \) and the four variables shown except perhaps that the lowest \( T/C \) (0.67) was for a specimen with a lap splice longer than \( 70d_b \) and that the three \( T/C \) over 1.0 were for specimens with lap splices shorter than \( 35d_b \).

Figure 4.13: \( T/C \) for large bar specimens based on Equation 2.9 (ACI 408R-03)
Figures 4.14 and 4.15 are corollaries to Figures 4.12 and 4.13 except with bar stresses calculated using Equation 2.11 based on ACI 318-19. Again, Equation 2.11 is used with \( \psi_g \geq \left[ \left(1 + 0.15 \frac{(f_s - 60 \text{ksi})}{20} \right) \right], 1.0 \), \( \sqrt{f'_c} \leq 100 \text{ psi}, \left( \frac{c_b + K_{tcr}}{d_b} \right) \leq 4 \), and \( \psi_s = 1.0 \) for all bar sizes as recommended in ACI 408R-03. The mean T/C for specimens from the ACI 408 Database (2021) is 1.27, with a COV of 0.23. The large bar tests included in Figure 4.15 have a mean T/C of 1.08 and a COV of 0.16. As with the comparison against Equation 2.9, these differences are statistically significant based on a two-sample Student’s t-Test assuming equal variances \( (p = 0.004) \). It therefore appears there is a large-bar size effect that is not captured by Equation 2.11.

![Figure 4.14: Estimated bar stress at failure from tests versus bar stress calculated with Equation 2.11](image)

Figure 4.15 includes 25 specimens that meet the following criteria: bottom-cast lap spliced bars larger than No. 11 [36 mm], \( \ell_s \geq 12 \) in. [300 mm], and if unconfined \( \ell_s \leq 50d_b \). Each data cluster is shifted up when compared to Figure 4.13, as ACI 318-19 is more conservative than ACI 408R-03. This is expected.
because ACI 318-19 is a design equation. However, the scatter is also larger for ACI 318-19. Figure 4.15 shows ACI 318-19 tends to be more conservative for high bar stresses and long lap splice lengths.

![Graphs showing T/C for large bar specimens based on Equation 2.11 (ACI 318-19)](image)

\( \frac{T}{C} \) for large bar specimens based on Equation 2.11 (ACI 318-19)

To further demonstrate the trend between T/C and bar size, Figures 4.16 and 4.17 show estimated bar stress from tests versus bar stress calculated with Equations 2.9 and 2.11 (based on ACI 408R-03 and ACI 318-19), respectively. These plots contain the same specimens as 4.12 and 4.14, but trends are shown for groups of specimens with No. 10 [32 mm] or smaller bars and specimens with No. 11 [36 mm] or larger bars (blue circles show results for “smaller” bars while the black circles and triangles show results for “larger” bars).
It is evident in Figure 4.16 that T/C values are larger for the smaller bars than for larger bars, with the slope for the data with No. 10 [32 mm] or smaller bars being 1.04 and the slope for the data with No. 11 [36 mm] or larger bars being 0.95. If a similar trendline were applied to only tests with bars larger than No. 11 [36 mm] (total of 19 specimens), the slope would be 0.90. The mean T/C for Equation 2.9 (ACI 408R-03) for bars No. 10 [32 mm] or smaller is 1.04 (COV = 0.12), while the mean for bars No. 11 [36 mm] or larger is 0.95 (COV is 0.11). These differences are statistically significant, with a p-value approaching zero (8.1E-11) when comparing test/calculated ratios of specimens with No. 10 [32 mm] or smaller bars against specimens with No. 11 [36 mm] or larger bars. Therefore, a size-effect seems evident.

![Graph showing test versus calculated bar stress by bar size for Equation 2.9 (ACI 408R-03)](image)

Figure 4.16: Test versus calculated bar stress by bar size for Equation 2.9 (ACI 408R-03)

Figure 4.17 shows similar trends as Figure 4.16, but with the ACI 318-19 expression for development length (Equation 2.11). Again, Equation 2.11 is used with \( \psi_d \geq \left[ \left( 1 + 0.15 \frac{(f' - 60kpsi)}{20} \right) , 1.0 \right] \), \( \sqrt{f'_c} \leq 100 \) psi, \( \left( \frac{c_b + K_{dr}}{d_b} \right) \leq 4 \), and \( \psi_z = 1.0 \) for all bar sizes as recommended in ACI 408R-03. Although both trendlines
have slopes greater than 1.0, the slope of the blue trendline (1.22 for No. 10 [32 mm] or smaller bars) is larger than that of the black trendline (1.12 for No. 11 [36 mm] or larger bars). These differences are also statistically significant, with a \( p \)-value less than 0.001 when comparing “smaller” bars versus “larger” bars.

![Figure 4.17: Test versus calculated bar stress by bar size for Equation 2.11 (ACI 318-19)](image)

Table 4.3 shows results that complement Figures 4.16 and 4.17 based on data from the ACI 408 Database (2021) and this study. Results are split into four different bar size groups in Table 4.3: smaller than No. 7 [22 mm] bars, No. 7 to No. 10 [19 to 32 mm] bars, No. 11 [36 mm] bars, and bars larger than No. 11 [36 mm]. A mean, minimum, and maximum T/C is reported for each bar size group. Unconfined and confined specimens were considered separately. Note that Equation 2.11 is based on a design equation, so relatively few specimens should have T/C below 1.0 since the tests are representative of conditions in practice. Equation 2.9, which has been used here with a phi-factor of 1.0, provides mean values much nearer to 1.0 as expected. Additional safety factors would need to be applied for Equation 2.9 to be used for design.
For application of Equation 2.11 in Table 4.3, $\psi_s = 1.0$ based on ACI 408R-03 recommendations. If $\psi_s = 0.8$ were indeed applied to the Table 4.3 bar group “smaller than No. 7 [19 mm]”, the mean T/C would drop from 1.37 to 1.10 for unconfined specimens (with 39% below 1.0) and from 1.26 to 1.00 for confined specimens (with 47% below 1.0). This agrees with the ACI 408R-03 recommendation to set $\psi_s = 1.0$, as having nearly half of specimens with T/C below 1.0 is not satisfactory for a design equation.

Table 4.3: Test to calculated bar stress at failure results based on current equations

<table>
<thead>
<tr>
<th>Type</th>
<th>Bar Size Group a</th>
<th>Equation 2.9 (ACI 408R-03) b</th>
<th>Equation 2.11 (ACI 318-19) c,d,e</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Count</td>
<td>Mean</td>
<td>T/C $&lt;$1.0</td>
</tr>
<tr>
<td>Unconfined</td>
<td>Smaller than No. 7</td>
<td>25</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>No. 7 to No. 10</td>
<td>75</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>No. 11</td>
<td>60</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>Larger than No. 11</td>
<td>4</td>
<td>0.89</td>
</tr>
<tr>
<td>Confined</td>
<td>Smaller than No. 7</td>
<td>41</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>No. 7 to No. 10</td>
<td>174</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>No. 11</td>
<td>74</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>Larger than No. 11</td>
<td>14</td>
<td>0.92</td>
</tr>
</tbody>
</table>

a No. 7, 10, and 11 bars in U.S. have diameters of approximately 22, 32, and 36 mm

b 467 specimens from the ACI 408 Database (2021) and this study that were bottom-cast and had $\ell_s \geq 12$ in. [300 mm], $\ell_s \geq 16d_b$, and for unconfined specimens $\ell_s \leq 50d_b$.

c 514 specimens from ACI 408 Database (2021) and this study that were bottom-cast and had $\ell_s \geq 12$ in. [300 mm] and for unconfined specimens $\ell_s \leq 50d_b$.

d $\psi_g \geq \left[ \left( 1 + 0.15 \frac{(f_{s-60ksi})}{20} \right), 1.0 \right]$, $\sqrt{T_c} \leq 100$ psi, $\frac{(c_p+K_{rs})}{d_b} \leq 4$, and $\psi_s = 1.0$ for all bar sizes.

e If $\psi_s = 0.8$ for specimens with bars smaller than No. 7, mean T/C = 1.10 for unconfined specimens (with 39% below 1.0) and 1.00 for confined specimens (with 47% below 1.0).

Table 4.3 shows a clear trend associated with bar size for both Equations 2.9 and 2.11. The mean T/C decreases in almost all cases as the bar size increases for both unconfined and confined specimens. Likewise, the percentage of tests with T/C less than 1.0 increases with bar size in almost all cases, except for the comparison between confined specimens with No. 7 to No. 10 [22 to 32 mm] bars and specimens with No. 11 [36 mm] bars using Equation 2.11 (15 and 14% of specimens had test / calculated below 1.0). Even though Table 4.3 reports results from only four unconfined and 14 confined (15 for ACI 318-19).
specimens that contained bars larger than No. 11 [36 mm], the evidence of a trend between T/C and bar size within the broader database is evident in Table 4.3.

4.4.4 Accounting for Effects of Bar Size in Design

Section 4.4.3 shows that the ACI 318-19 and ACI 408R-03 development length equations become less conservative as bar size increases. This finding builds on observations of a possible size effect in Thompson et al. (1975), Ichinose et al. (2004), and Hegger et al. (2015).

There are many possible methods of adjusting development and lap splice equations to account for bar size; two are described in this section: (1) a bar size factor can be added to the numerator of Equations 2.8 and 2.10 (see Section 4.4.4.1) or (2) the exponent on bar diameter can be increased (see Section 4.4.4.2). Both methods (1) and (2) can allow for T/C ratios to be independent of bar size.

4.4.4.1 Large-Bar Size Factor

ACI 408R-03 and ACI 318-19 development and lap splice length equations can be made independent of bar size by adding a bar size factor in the numerator of Equations 2.8 and 2.10 as shown in Equations 4.2 and 4.3. This is a relatively simple change, as the ACI 318-19 equation already has \( \psi_s = 0.8 \) for bars that are No. 6 [19 mm] and smaller, so it would only require a modification of \( \psi_s \). It is proposed to define this factor as shown in Equation 4.4. There are zero tests in the database with No. 10 [32 mm] bars, so some judgement is necessary here. It is proposed to assign \( \psi_s = 1.0 \) to No. 10 [32 mm] bars, although it could be argued that \( \psi_s = 1.15 \) would be appropriate until data are available.

\[
\ell_d = \left( \frac{f_y}{\psi f_y^t 1/4} - 2400 \omega \right) \alpha \beta \lambda \psi_s \left( \frac{c + K_{tr,408}}{d_b} \right) d_b \quad \text{Equation 4.2}
\]

\[
\ell_d = \left( \frac{3 f_y \psi_t \psi_s \psi g}{42 \lambda \sqrt{f_y^t} \left( \frac{c + K_{tr}}{d_b} \right)} \right) d_b \quad \text{Equation 4.3}
\]
\[
\psi_s = \begin{cases} 
1.0 \text{ for No. 10 [32 mm] and smaller bars} \\
1.15 \text{ for No. 11 [36 mm] bars} \\
1.25 \text{ for No. 14 [43 mm] and larger bars}
\end{cases} \quad \text{Equation 4.4}
\]

Table 4.4 is a repeat of Table 4.3, but with \(\psi_s\) defined according to Equation 4.4 included in both Equations 4.2 and 4.3. Table 4.4 shows that this modification produces similar mean T/C across bar sizes for Equation 4.2, with mean T/C between 1.01 and 1.11 for both unconfined and confined bars of all sizes. Also, the percent of specimens with T/C below 1.0 are no longer correlated with bar size in Table 4.4. The results in Table 4.4 for Equation 4.3 also show somewhat improved consistency in mean T/C across bar sizes. It could be argued that this modification is too conservative for confined No. 11 [36 mm] and larger bars with Equation 4.3, but Table 4.3 shows that Equations 4.3 and 4.4 may not be conservative enough for unconfined lap splices of bars larger than No. 11 [36 mm]. It is believed that, based on the available data, defining \(\psi_s\) defined according to Equation 4.4 provides a reasonable correction for both Equations 4.2 and 4.3 without being overly conservative for any group of bar sizes.

Table 4.4: Test to calculated bar stress at failure with proposed size effect factor

<table>
<thead>
<tr>
<th>Type</th>
<th>Bar Size Group a</th>
<th>Equation 2.9 (ACI 408R-03) b</th>
<th>Equation 2.11 (ACI 318-19) c,d</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Count</td>
<td>Mean T/C &lt;1.0</td>
<td>Min.</td>
</tr>
<tr>
<td>Unconfined</td>
<td>Smaller than No. 7 25</td>
<td>1.06</td>
<td>32%</td>
</tr>
<tr>
<td></td>
<td>No. 7 to No. 10 75</td>
<td>1.01</td>
<td>49%</td>
</tr>
<tr>
<td></td>
<td>No. 11 60</td>
<td>1.05</td>
<td>30%</td>
</tr>
<tr>
<td></td>
<td>Larger than No. 11 4</td>
<td>1.08</td>
<td>25%</td>
</tr>
<tr>
<td>Confined</td>
<td>Smaller than No. 7 41</td>
<td>1.11</td>
<td>12%</td>
</tr>
<tr>
<td></td>
<td>No. 7 to No. 10 174</td>
<td>1.03</td>
<td>47%</td>
</tr>
<tr>
<td></td>
<td>No. 11 74</td>
<td>1.07</td>
<td>24%</td>
</tr>
<tr>
<td></td>
<td>Larger than No. 11 14</td>
<td>1.08</td>
<td>36%</td>
</tr>
</tbody>
</table>

a No. 7, 10, and 11 bars in U.S. have diameters of approximately 22, 32, and 36 mm
b 467 specimens from the ACI 408 Database (2021) and this study that were bottom-cast and had \(\ell_s \geq 12\) in. [300 mm], \(\ell_s \geq 16d_b\), and for unconfined specimens \(\ell_s \leq 50d_b\)
c 514 specimens from ACI 408 Database (2021) and this study that were bottom-cast and had \(\ell_s \geq 12\) in. [300 mm] and for unconfined specimens \(\ell_s \leq 50d_b\)
d \(\psi_g \geq \left(1 + 0.15 \frac{f'c}{f_y}\right) \left(\frac{\ell_s}{20}\right), \sqrt{f'c} \leq 100\) psi, \(\frac{f'c + K_{tr}d_b}{d_b} \leq 4\), and \(\psi_s = 1.0\) for all bar sizes
Figures 4.18 and 4.19 are the same as Figures 4.16 and 4.17 except they include the large-bar size factor (Equation 4.4). Figure 4.18 shows the overall improved T/C trends; the slope for the data with No. 10 [32 mm] or smaller bars is still 1.04 and the slope for the updated data with No. 11 [36 mm] or larger bars is now 1.05. Use of the Equation 4.4 bar size factor therefore causes the ACI 408R-03 equation to be similarly conservative for all bar sizes. Figure 4.19 shows a similar outcome for the ACI 318-19 equation; the slope for the data with No. 10 [32 mm] or smaller bars is still 1.22 and the slope for the updated data with No. 11 [36 mm] or larger bars is now 1.30.

![Diagram showing test versus calculated bar stress by bar size for Equations 4.2 and 4.4](image_url)

Figure 4.18: Test versus calculated bar stress by bar size for Equations 4.2 and 4.4
4.4.4.2 Modified Exponent for Bar Diameter

Use of a modified exponent on bar diameter is examined as a means of accounting for effects of bar size. The concept is based on recommendations in Ichinose et al. (2004), which reported tests from short lap splice specimens with large diameter bars. They found that changing the exponent on bar diameter by 0.2 or 0.3 produced a more accurate estimate of bond strength with a similar level of conservatism across bar sizes.

Using the same data shown in Figures 4.18 and 4.19, the authors find that changing the exponent on bar is a viable solution for adjusting both the ACI 408R-03 and ACI 318-19 equations to produce similar conservatism across all bar sizes. It was found that using exponents of 1.35 and 1.25 on bar diameter in Equations 4.5 and 4.6 provided acceptable results, with mean T/C and % of T/C below 1.0 showing no clear
correlation with bar size. For this comparison, $\psi_s$ was taken as 1.0 for all bar sizes and thus is not shown in Equations 4.5 and 4.6.

$$
\ell_d = \left( \frac{f_y}{\phi f'_{c}} - 2400\omega \right) \frac{\alpha \beta \lambda}{76.3 \left( \frac{c_\omega + K_{tr,408}}{d_b} \right)} d_b^{1.35} \quad \text{Equation 4.5}
$$

$$
\ell_d = \left( \frac{3}{40} \frac{f_y}{\sqrt{f'_{c}}} \frac{\psi_c \psi_e \psi_g}{(c_b + K_{tr})} \right) d_b^{1.25} \quad \text{Equation 4.6}
$$

Table 4.5: Test to calculated bar stress at failure with proposed size effect exponent

<table>
<thead>
<tr>
<th>Type</th>
<th>Bar Size Group</th>
<th>Equation 2.9 (ACI 408R-03) $^{b}$</th>
<th>Equation 2.11 (ACI 318-19) $^{c,d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Count</td>
<td>Mean</td>
<td>T/C &lt;1.0</td>
</tr>
<tr>
<td>Unconfined</td>
<td>Smaller than No. 7</td>
<td>25</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>No. 7 to No. 10</td>
<td>75</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>No. 11</td>
<td>60</td>
<td>1.03</td>
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<tr>
<td></td>
<td>Larger than No. 11</td>
<td>4</td>
<td>0.98</td>
</tr>
<tr>
<td>Confined</td>
<td>Smaller than No. 7</td>
<td>41</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>No. 7 to No. 10</td>
<td>174</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>No. 11</td>
<td>74</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>Larger than No. 11</td>
<td>14</td>
<td>1.04</td>
</tr>
</tbody>
</table>

$a$ No. 7, 10, and 11 bars in U.S. have diameters of approximately 22, 32, and 36 mm

$b$ 467 specimens from the ACI 408 Database (2021) and this study that were bottom-cast and had $\ell_s \geq 12$ in. [300 mm], $\ell_s \geq 16d_b$, and for unconfined specimens $\ell_s \leq 50d_b$

$c$ 514 specimens from ACI 408 Database (2021) and this study that were bottom-cast and had $\ell_s \geq 12$ in. [300 mm] and for unconfined specimens $\ell_s \leq 50d_b$

$d$ $\psi_g \geq \left[ 1 + 0.15 \left( \frac{f_y - 60k_s}{20} \right) \right], \sqrt{f'_{c}} \leq 100$ psi, $\left( \frac{c_b + K_{tr}}{d_b} \right) \leq 4$, and $\psi_s = 1.0$ for all bar sizes

The results in Table 4.5 are based on the same data as Tables 4.3 and 4.4, but with a $d_b^{1.35}$ and $d_b^{1.25}$ replacing bar diameter in equations from ACI 408R-03 (Equation 4.5) and ACI 318-19 (Equation 4.6), respectively. Table 4.5 shows mean ratios between 0.98 and 1.04 for both unconfined and confined bars of
all sizes when calculated by Equation 4.5. The bar diameter exponent causes a slightly unconservative mean T/C for the unconfined “smaller than No. 7” and “larger than No. 11” groups, but there is overall improvement for Equation 4.5 and no clear trend between T/C and bar size. The results in Table 4.5 for Equation 4.6 also show less correlation between T/C and bar size, producing similar conservatism for all bar sizes.

Figures 4.20 and 4.21 are the same as Figures 4.16 and 4.17 except they are based on Equations 4.5 and 4.6, which include bar diameter exponents greater than 1.0. Figure 4.20 shows an improved test / calculated behavior for both small and large bars (No. 10 [32 mm] or smaller and No. 11 [36 mm] or larger). The slope for the data with No. 10 [32 mm] or smaller bars is now 1.02 and the slope for the updated data with No. 11 [36 mm] or larger bars is now 1.03. A bar diameter exponent of 1.35 therefore causes the ACI 408R-03 equation to be similarly conservative for all bar sizes. Figure 4.21 shows a similar trend for the modified ACI 318-19 equation; the slope for the data with No. 10 [32 mm] or smaller bars is now 1.20 and the slope for the data with No. 11 [36 mm] or larger bars is now 1.23.

![Figure 4.20: Test versus calculated bar stress by bar size for Equation 4.5](image-url)
4.4.5 Potential Explanations for Effects of Bar Size

The linear fracture mechanics size effect might explain the trends between T/C and bar size (Ichinose et al. 2004).

It might also be expected that larger bars, which have larger deformations and longer lap splices than smaller bars, will induce wider splitting cracks and cause yielding of transverse (confining) reinforcement that might compromise bond strength. However, there was no evidence of either wider cracks or transverse reinforcement yielding in this study.

Another potential explanation for the observed effect of bar size is related to the bar prying action that causes unconfined lap spliced bars to push the cover off at failure (Figure 4.4c). In beam tests, lap splices are subjected to bending as well as tension. Although generally negligible, bending/prying of the lapped bars does tend to induce tension in the concrete cover perpendicular to the lapped bars that could, in theory,
reduce the effectiveness of the cover for resisting bond-splitting failures. Data obtained with the optical sensors described in Section 3.5 were used to explore whether bar prying/bending could produce tension in the concrete cover large enough to affect bond strength. The analysis, described in Appendix G, demonstrates that bar bending/prying can produce concrete stresses large enough to compromise the splitting resistance of the concrete cover for No. 14 [43 mm] and No. 18 [57 mm] bars. Because of the larger bar diameter, bending of a No. 18 [57 mm] bar induces concrete splitting stresses that are approximately 16 times larger than a No. 9 [29 mm] bar and nearly seven times larger than a No. 11 [36 mm] bar. Furthermore, the splitting stresses induced by a No. 18 [57 mm] bar may be larger than 1/4 of the splitting strength of the concrete, depending on the assumptions made in the analysis. So, while prying appears negligible for No. 11 [36 mm] and smaller bars, it may be non-negligible for the larger bars considered in this study.

The size effect described by fracture mechanics and bar bending may, together, explain the lower bond strengths observed for No. 14 and 18 [43 and 57 mm] bars in this study.
CHAPTER 5 – CONCLUSIONS

Eleven large-scale beam specimens were tested to measure the strength of lap spliced No. 14 and 18 [43 and 57 mm] bars. Specimens had a clear cover to the lap splice of approximately 1.2 times the longitudinal bar diameter, varied amounts of transverse reinforcement, nominal concrete compressive strengths of 5 or 10 ksi [35 or 69 MPa], and target bar stresses at splice failure of 60 to 100 ksi [420 to 690 MPa] for No. 14 [43 mm] bars and 60 ksi [420 MPa] for No. 18 [57 mm] bars. The results justify the following conclusions:

1. Lap splices of No. 14 [43 mm] bars can develop bar stresses up to 100 ksi [690 MPa] and lap splices of No. 18 [57 mm] bars can develop bar stresses of at least 60 ksi [420 MPa]. These limits reflect the scope of the text matrix, as there was no indication that higher bar stresses cannot be attained.

2. Both the ACI 408R-03 and ACI 318-19 equations (Equations 2.8 and 2.10) become less conservative as bar size increases, and these differences in conservatism are statistically significant. The mean test / calculated value of 0.88 with Equation 2.8 for the tests reported herein is greater than one standard deviation (0.12) less than the mean test / calculated value of 1.02 with Equation 2.8 for the ACI 408 Database (2021). Bar size effects were also observed within the ACI 408 Database (2021) when comparisons were made between groups of specimens with No. 6 [19 mm] and smaller bars, No. 7 to No. 10 [22 to 32 mm] bars, No. 11 [36 mm] bars, and bars larger than No. 11 [36 mm]. This trend is evident for both unconfined and confined specimens, although it is more pronounced for unconfined specimens, particularly for Equation 2.10 (ACI 318-19).

3. Two alternatives are proposed for modifying ACI 408R-03 and ACI 318-19 development length equations to obtain similar levels of conservatism across all bar sizes.

   a. *Bar size factor in numerator of development length equation:* Adding a 1.15 factor for No. 11 [36 mm] bars and a 1.25 factor for bars larger than No. 11 [36 mm] to the numerator of the development length Equations 2.8 and 2.10 produces similar mean test / calculated
values across bar sizes, with mean ratios between 1.01 and 1.11 for both unconfined and confined bars of all sizes (ACI 408R-03). This also causes the percent of specimens with test/calculated values below 1.0 to no longer be correlated with bar size (Section 4.4.4.1). See conclusion 9 for discussion of small bar factor $\psi_s$.

b. *Exponent on bar diameter*: Replacing $d_b$ with $d_b^{1.35}$ in Equation 2.8 (ACI 408R-03) and $d_b^{1.25}$ in Equation 2.10 (ACI 318-19) results in similar conservatism across all bar sizes (Section 4.4.4.2). These replacements do not apply to $d_b$ in $\left(\frac{c_b + K_{tr} d_b}{d_b}\right)$. This recommendation is similar to that in Ichinose et al. (2004).

4. Transverse reinforcement should be required throughout lap splices on No. 14 and 18 [43 and 57 mm] bars, regardless of bar stress (as recommended by Ferguson and Krishnaswamy (1971)). Even small quantities of confinement ($K_{tr}/d_b = 0.5$) appeared to reduce the size effect and allowed for internal redistribution of forces before failure, evidenced by softening in the load-deflection results.

5. A minimum clear cover of one bar diameter should be required for all lap splices. There is insufficient evidence that lap splices of No. 14 and 18 [43 and 57 mm] bars are functional with less cover. A minimum clear spacing of two bar diameters is also recommended within a lap splice.

6. An unconfined lap splice with a length of $72d_b$ (14-f) failed at a much lower force than expected. A review of prior work indicates there may be a limit beyond which increases in the length of unconfined lap splices do not produce increased bond strengths. Based on this prior work, a review of the ACI 408 Database (2021), and the result obtained for 14-f, it is recommended that development length $\ell_d$ be limited to $50d_b$ when designing unconfined lap splices.

7. Increasing the limit on $\left(\frac{c_b + K_{tr}}{d_b}\right)$ in the ACI 318-19 development length provisions from 2.5 to 3.5 results in similarly conservative designs for all amounts of confinement.

8. Test results reported herein support prior observations that lap splices fail before yielding of the transverse reinforcement confining a lap splice.
9. Analyses support the ACI 408-03 recommendation that the $\psi_s$ term in the numerator of the ACI 318-19 development length equation be taken as 1.0 for No. 6 [19 mm] and smaller bars. When $\psi_s = 1.0$, ACI 318-19 is similarly conservative for No. 3 to No. 9 [10 to 29 mm] bars, with a mean test/calculated of 1.27 based on the ACI 408 Database (2021). When $\psi_s = 0.8$, a mean test/calculated of 1.02 is obtained for No. 6 [19 mm] and smaller bars. This is insufficiently conservative for a design equation.

10. The size effect described by fracture mechanics might adequately explain the observed trends. It was also shown that the greater bending stiffness of large bars might contribute to the lower observed bond strengths. This is because large bars have substantially larger moments of inertia than smaller bars and thus bending/prying of large bars might induce non-negligible tension stresses in the concrete cover.
CHAPTER 6 – REFERENCES

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APPENDIX A: SPECIMEN DRAWINGS

Figures A.1 through A.11 show profile (elevation) and plan views for each of the 11 specimens in this study. Cross-section drawings are shown in Figures A.12 through A.22. The drawings show the lap-spliced bars at the bottom of the beam, as they were for concrete placement. Specimens were then inverted and tested with the lap splice at the top of the specimens for convenience (Section 3.4). Values of $K_{tr}/d_b$ in Figures A.1 through A.11 are the target value set during initial design.

The drawings show the nominal specimen dimensions. Relevant as-built dimensions are in Table A.1. As-built dimensions were measured at multiple places throughout the lap splice region after testing was complete, then averaged. Measurements of clear cover were taken from the concrete face to the non-deformed bar edge (in between bar deformations). As-built beam cross-section height ($h$) and width ($b$), lap splice length ($\ell_s$), and transverse reinforcement spacing ($S1$) were within 0.25 in. [6 mm] of the nominal dimensions, so nominal dimensions are used throughout.

Figure A.1: Profile and plan of 14-a
Figure A.2: Profile and plan of 14-b

Figure A.3: Profile and plan of 14-c
Figure A.4: Profile and plan of 14-d

Figure A.5: Profile and plan of 14-e
Figure A.6: Profile and plan of 14-f

Figure A.7: Profile and plan of 14-g
Figure A.8: Profile and plan of 18-a

Figure A.9: Profile and plan of 18-b
Figure A.10: Profile and plan of 18-c

Figure A.11: Profile and plan of 18-d
Figure A.12: Cross section of 14-a

Figure A.13: Cross section of 14-b
Figure A.14: Cross section of 14-c

Figure A.15: Cross section of 14-d
Figure A.16: Cross section of 14-e

Figure A.17: Cross section of 14-f
Figure A.18: Cross section of 14-g

Figure A.19: Cross section of 18-a
Figure A.20: Cross section of 18-b

Figure A.21: Cross section of 18-c
Figure A.22: Cross section of 18-d

Table A.1: As-built dimensions

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>h, in.</th>
<th>b, in.</th>
<th>$c_{\text{min}}$, in. [clear]</th>
<th>$c_{\text{max}}$, in. [clear]</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-a</td>
<td>32</td>
<td>18</td>
<td>1.94</td>
<td>2.26</td>
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<td>14-b</td>
<td>32</td>
<td>18</td>
<td>2.03</td>
<td>2.21</td>
</tr>
<tr>
<td>14-c</td>
<td>32</td>
<td>18</td>
<td>2.02</td>
<td>2.40</td>
</tr>
<tr>
<td>14-d</td>
<td>32</td>
<td>18</td>
<td>1.84</td>
<td>2.25</td>
</tr>
<tr>
<td>14-e</td>
<td>32</td>
<td>18</td>
<td>1.98</td>
<td>2.34</td>
</tr>
<tr>
<td>14-f</td>
<td>32</td>
<td>18</td>
<td>2.01</td>
<td>2.35</td>
</tr>
<tr>
<td>14-g</td>
<td>32</td>
<td>18</td>
<td>2.08</td>
<td>2.28</td>
</tr>
<tr>
<td>18-a</td>
<td>36</td>
<td>24</td>
<td>2.30</td>
<td>2.95</td>
</tr>
<tr>
<td>18-b</td>
<td>36</td>
<td>24</td>
<td>2.47</td>
<td>2.86</td>
</tr>
<tr>
<td>18-c</td>
<td>36</td>
<td>24</td>
<td>2.52</td>
<td>2.75</td>
</tr>
<tr>
<td>18-d</td>
<td>36</td>
<td>24</td>
<td>2.34</td>
<td>2.89</td>
</tr>
</tbody>
</table>

* 1 in. = 25.4 mm
APPENDIX B: STRESS VERSUS STRAIN FOR REINFORCEMENT

Figure B.1: Stress versus strain for No. 4 Grade 60 [13 mm, 420 MPa] transverse reinforcement, 14-a to 14-g (nominal area of specimen: 0.2 in² [130 mm²], average sample yield stress: 62.2 ksi [429 MPa])

Figure B.2: Stress versus strain for No. 4 Grade 60 [13 mm, 420 MPa] transverse reinforcement, 18-a to 18-d (nominal area of specimen: 0.2 in² [130 mm²], average sample yield stress: 74.0 ksi [511 MPa])
Figure B.3: Stress versus strain for No. 14 Grade 80 [43 mm, 550 MPa] longitudinal reinforcement, 14-a to 14-e (nominal area of specimen: 2.25 in² [1450 mm²], average sample yield stress: 92.6 ksi [639 MPa])

Figure B.4: Stress versus strain for No. 14 Grade 120 [43 mm, 830 MPa] longitudinal reinforcement, 14-f and 14-g (nominal area of specimen: 2.25 in² [1450 mm²], average sample yield stress: 134.6 ksi [929 MPa])
Figure B.5: Stress versus strain for No. 18 Grade 120 [57 mm, 830 MPa] longitudinal reinforcement (nominal area of specimen: 4.0 in$^2$ [2600 mm$^2$], average sample yield stress: 129.4 ksi [893 MPa])
APPENDIX C: PLASTIC CONCRETE PROPERTIES

Table C.1 provides the plastic properties of each concrete batch used in this study. The unit weight, slump, and concrete temperature were determined in accordance with ASTM C138, C143, and C1064, respectively.

Table C.1: Specimen plastic concrete properties

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Unit Weight, lb/ft³</th>
<th>Slump, in.</th>
<th>Concrete Temperature, °F</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-a</td>
<td>147.9</td>
<td>6.50</td>
<td>73</td>
</tr>
<tr>
<td>14-b</td>
<td>147.6</td>
<td>4.25</td>
<td>60</td>
</tr>
<tr>
<td>14-c</td>
<td>146.9</td>
<td>5.75</td>
<td>60</td>
</tr>
<tr>
<td>14-d</td>
<td>149.4</td>
<td>6.50</td>
<td>62</td>
</tr>
<tr>
<td>14-e</td>
<td>150.7</td>
<td>9.50</td>
<td>69</td>
</tr>
<tr>
<td>14-f</td>
<td>145.4</td>
<td>8.00</td>
<td>70</td>
</tr>
<tr>
<td>14-g</td>
<td>146.7</td>
<td>6.00</td>
<td>62</td>
</tr>
<tr>
<td>18-a</td>
<td>147.8</td>
<td>3.75</td>
<td>68</td>
</tr>
<tr>
<td>18-b</td>
<td>147.7</td>
<td>4.25</td>
<td>71</td>
</tr>
<tr>
<td>18-c</td>
<td>151.8</td>
<td>6.50</td>
<td>79</td>
</tr>
<tr>
<td>18-d</td>
<td>151.8</td>
<td>6.00</td>
<td>83</td>
</tr>
</tbody>
</table>

* 1 lb/ft³ = 16.02 kg/m³, 1 in. = 25.4 mm, (32 °F-32) x 5/9 = 0 °C
APPENDIX D: APPLIED LOAD VERSUS DEFLECTION

Figure D.1: Load-Deflection for 14-a

Figure D.2: Load-Deflection for 14-b
Figure D.3: Load-Deflection for 14-c

Figure D.4: Load-Deflection for 14-d
Figure D.5: Load-Deflection for 14-e

Figure D.6: Load-Deflection for 14-f
Figure D.7: Load-Deflection for 14-g

Figure D.8: Load-Deflection for 18-a
Figure D.9: Load-Deflection for 18-b

Figure D.10: Load-Deflection for 18-c
Figure D.11: Load-Deflection for 18-d
APPENDIX E: PHOTOS AND CRACK WIDTHS

Appendix F contains a photo of each specimen after failure. The dashed line near the center of each image represents the centerline of the lap splice, while the solid lines represent the ends.

Figure E.1: Photo of 14-a after failure

Figure E.2: Photo of 14-b after failure
Figure E.3: Photo of 14-c after failure

Figure E.4: Photo of 14-d after failure
Figure E.5: Photo of 14-e after failure

Figure E.6: Photo of 14-f after failure
Figure E.7: Photo of 14-g after failure

Figure E.8: Photo of 18-a after failure
Figure E.9: Photo of 18-b after failure

Figure E.10: Photo of 18-c after failure
Figure E.11: Photo of 18-d after failure
Figure E.12: Crack widths versus % of estimated bar stress at failure for specimens with No. 14 bars

Figure E.13: Crack widths versus % of estimated bar stress at failure for specimens with No. 18 bars
Table E.1: Maximum crack widths for specimens with No. 14 bars

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Estimated Bar Stress at Failure (ksi)</th>
<th>Approximate Bar Stress (ksi)</th>
<th>Maximum Crack Width (in.)</th>
<th>% of Estimated Bar Stress at Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-a</td>
<td>53.8</td>
<td>17 ($M_{cr}$)</td>
<td>0.006</td>
<td>32%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.012</td>
<td>37%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>0.016</td>
<td>56%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>0.022</td>
<td>74%</td>
</tr>
<tr>
<td>14-b</td>
<td>54.9</td>
<td>17 ($M_{cr}$)</td>
<td>0.004</td>
<td>31%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.012</td>
<td>36%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>0.018</td>
<td>55%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>0.026</td>
<td>73%</td>
</tr>
<tr>
<td>14-c</td>
<td>60.2</td>
<td>17 ($M_{cr}$)</td>
<td>0.004</td>
<td>28%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.008</td>
<td>33%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>0.016</td>
<td>50%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>0.024</td>
<td>66%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>0.026</td>
<td>83%</td>
</tr>
<tr>
<td>14-d</td>
<td>52.0</td>
<td>20 ($M_{cr}$)</td>
<td>0.008</td>
<td>38%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>0.014</td>
<td>58%</td>
</tr>
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<td></td>
<td></td>
<td>40</td>
<td>0.024</td>
<td>77%</td>
</tr>
<tr>
<td>14-e</td>
<td>53.4</td>
<td>20 ($M_{cr}$)</td>
<td>0.004</td>
<td>37%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>0.010</td>
<td>56%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>0.014</td>
<td>75%</td>
</tr>
<tr>
<td>14-f</td>
<td>73.2</td>
<td>16 ($M_{cr}$)</td>
<td>0.004</td>
<td>22%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.006</td>
<td>27%</td>
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<td></td>
<td>40</td>
<td>0.018</td>
<td>55%</td>
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<tr>
<td></td>
<td></td>
<td>60</td>
<td>0.039</td>
<td>82%</td>
</tr>
<tr>
<td>14-g</td>
<td>109.3</td>
<td>16 ($M_{cr}$)</td>
<td>0.004</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.008</td>
<td>18%</td>
</tr>
<tr>
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<td></td>
<td>40</td>
<td>0.020</td>
<td>37%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>60</td>
<td>0.031</td>
<td>55%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>80</td>
<td>0.037</td>
<td>73%</td>
</tr>
</tbody>
</table>

$^a$ 1 ksi = 6.9 MPa, 1 in. = 25.4 mm
Table E.2: Maximum crack widths for specimens with No. 18 bars \(^a\)

<table>
<thead>
<tr>
<th>Specimen ID</th>
<th>Estimated Bar Stress at Failure (ksi)</th>
<th>Approximate Bar Stress (ksi)</th>
<th>Maximum Crack Width (in.)</th>
<th>% of Estimated Bar Stress at Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-a</td>
<td>67.0</td>
<td>14 ($M_{cr}$)</td>
<td>0.008</td>
<td>21%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.018</td>
<td>30%</td>
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<tr>
<td></td>
<td></td>
<td>30</td>
<td>0.024</td>
<td>45%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>0.030</td>
<td>60%</td>
</tr>
<tr>
<td>18-b</td>
<td>66.6</td>
<td>14 ($M_{cr}$)</td>
<td>0.004</td>
<td>21%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.010</td>
<td>30%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>0.016</td>
<td>45%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>0.028</td>
<td>60%</td>
</tr>
<tr>
<td>18-c</td>
<td>61.4</td>
<td>18 ($M_{cr}$)</td>
<td>0.004</td>
<td>29%</td>
</tr>
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<td></td>
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<td>0.008</td>
<td>33%</td>
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<td>49%</td>
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<tr>
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<td></td>
<td>40</td>
<td>0.024</td>
<td>65%</td>
</tr>
<tr>
<td>18-d</td>
<td>62.9</td>
<td>19 ($M_{cr}$)</td>
<td>0.004</td>
<td>30%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20</td>
<td>0.010</td>
<td>32%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>30</td>
<td>0.022</td>
<td>48%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40</td>
<td>0.026</td>
<td>64%</td>
</tr>
</tbody>
</table>

\(^a\) 1 ksi = 6.9 MPa, 1 in. = 25.4 mm
APPENDIX F: REINFORCEMENT STRAINS

Figure F.1: Longitudinal bar strains for 14-a
Figure F.2: Longitudinal bar strains for 14-b

Figure F.3: Longitudinal bar strains for 14-c
Figure F.4: Longitudinal bar strains for 14-d

Figure F.5: Longitudinal bar strains for 14-e (Side B: 1, not recorded)
Figure F.6: Longitudinal bar strains for 14-f (Side A: 2, not recorded)
Figure F.7: Longitudinal bar strains for 14-g

![Graph showing longitudinal bar strains for Side A: 1, Side A: 2, Side B: 1, Side B: 2, and Average.](image)

Figure F.8: Longitudinal bar strains for 18-a

![Graph showing longitudinal bar strains for Side A: 1, Side A: 2, Side B: 1, and Average.](image)
Figure F.9: Longitudinal bar strains for 18-b (Side B: 2, not recorded)

Figure F.10: Longitudinal bar strains for 18-c (Side B: 2, not recorded)
Figure F.11: Transverse reinforcement strains for 18-c (SG C1 not recorded)

Figure F.12: Longitudinal bar strains for 18-d (Side A: 1, not recorded)
Figure F.13: Transverse reinforcement strains for 18-d (SG C1 not recorded)

Table F.1: Bar stress at failure based on measured strains

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$E_{\text{nom}}$ (ksi)</th>
<th>$\varepsilon_{f_{\text{avg}}}^b$</th>
<th>$\sigma_{f_{\text{nom}}}$ (ksi)</th>
<th>% Error $^c$ [(+) means &gt; reference]</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-a</td>
<td>29,000</td>
<td>0.00177</td>
<td>51.3</td>
<td>(-) 4.6%</td>
</tr>
<tr>
<td>14-b</td>
<td>29,000</td>
<td>0.00205</td>
<td>59.5</td>
<td>(+) 8.4%</td>
</tr>
<tr>
<td>14-c</td>
<td>29,000</td>
<td>0.00198</td>
<td>57.4</td>
<td>(-) 4.7%</td>
</tr>
<tr>
<td>14-d</td>
<td>29,000</td>
<td>0.00167</td>
<td>48.4</td>
<td>(-) 6.9%</td>
</tr>
<tr>
<td>14-e</td>
<td>29,000</td>
<td>0.00157</td>
<td>45.5</td>
<td>(-) 14.7%</td>
</tr>
<tr>
<td>14-f</td>
<td>29,000</td>
<td>0.00239</td>
<td>69.3</td>
<td>(-) 5.3%</td>
</tr>
<tr>
<td>14-g</td>
<td>29,000</td>
<td>0.00416</td>
<td>109.0$^d$</td>
<td>(-) 0.3%</td>
</tr>
<tr>
<td>18-a</td>
<td>29,000</td>
<td>0.00243</td>
<td>70.5</td>
<td>(+) 5.2%</td>
</tr>
<tr>
<td>18-b</td>
<td>29,000</td>
<td>0.00225</td>
<td>65.3</td>
<td>(-) 2.0%</td>
</tr>
<tr>
<td>18-c</td>
<td>29,000</td>
<td>0.00209</td>
<td>60.6</td>
<td>(-) 1.3%</td>
</tr>
<tr>
<td>18-d</td>
<td>29,000</td>
<td>0.00226</td>
<td>65.5</td>
<td>(+) 4.1%</td>
</tr>
</tbody>
</table>

$^a$ 1 ksi = 6.9 MPa  
$^b$ Average of functioning strain gauges at time of failure (same moment in time as splice failure)  
$^c$ Bar stress calculated from beam moment method is used as reference (Table 4.2)  
$^d$ Stress was determined from measured stress-strain results plotted in Appendix B (due to nonlinearity)
APPENDIX G: ROTATION ANALYSIS

Figures G.1 and G.2 show the layout of optical sensors on 18-c and 18-d. The location of each sensor in 3-dimensional space was recorded throughout the tests. The data were used to calculate the relative rotations between each successive pair of vertical columns of optical sensors. The rotations calculated at loads just before failure are listed in Tables G.1 and G.2 and plotted in Figures G.3(a) and G.4(a). The average curvatures within each portion of the beam span (e.g. A-B, B-C, etc.) were calculated as relative rotation divided by nominal horizontal distance and are given in Tables G.1 and G.2 and Figures G.3(b) and G.4(b).

Figure G.1: Optical sensor layout for 18-c

Figure G.2: Optical sensor layout for 18-d
### Table G.1: Relative rotations for 18-c

<table>
<thead>
<tr>
<th>Location</th>
<th>A - B</th>
<th>B - C</th>
<th>C - D</th>
<th>D - E</th>
<th>E - F</th>
<th>F - G</th>
<th>C - F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Straddling splice end</td>
<td>Inside splice end to center line</td>
<td>Center line to inside splice end</td>
<td>Straddling splice end</td>
<td>Outside splice end to support</td>
<td>Support to load point</td>
<td>Center line to load point total</td>
</tr>
<tr>
<td>Relative Rotation radians</td>
<td>0.0050</td>
<td>0.0045</td>
<td>0.0025</td>
<td>0.0085</td>
<td>0.0021</td>
<td>0.0164</td>
<td>0.0295</td>
</tr>
<tr>
<td>Curvature 1/in. [1/mm] x10⁴</td>
<td>6.25 [0.25]</td>
<td>1.96 [0.078]</td>
<td>1.09 [0.043]</td>
<td>10.6 [0.43]</td>
<td>0.43 [0.017]</td>
<td>2.0 [0.079]</td>
<td>1.8 [0.072]</td>
</tr>
</tbody>
</table>

### Table G.2: Relative rotations for 18-d

<table>
<thead>
<tr>
<th>Location</th>
<th>A - B</th>
<th>B - C</th>
<th>C - D</th>
<th>D - E</th>
<th>E - F</th>
<th>F - G</th>
<th>C - F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Straddling splice end</td>
<td>Inside splice end to center line</td>
<td>Center line to inside splice end</td>
<td>Straddling splice end</td>
<td>Outside splice end to support</td>
<td>Support to load point</td>
<td>Center line to load point total</td>
</tr>
<tr>
<td>Relative Rotation radians</td>
<td>0.0184</td>
<td>0.0352</td>
<td>0.0110</td>
<td>0.0156</td>
<td>0.0050</td>
<td>0.0735</td>
<td>0.1051</td>
</tr>
<tr>
<td>Curvature 1/in. [1/mm] x10⁴</td>
<td>11.5 [0.45]</td>
<td>28.2 [1.10]</td>
<td>8.8 [0.34]</td>
<td>9.8 [0.38]</td>
<td>0.97 [0.04]</td>
<td>9.0 [0.35]</td>
<td>6.5 [0.26]</td>
</tr>
</tbody>
</table>
Based on these results, the following analysis was done to examine whether bar bending/prying induced by beam curvature might have contributed to the observed bond-splitting failures at loads smaller than expected based on prior tests with smaller bars. In other words, does the greater flexural stiffness of large diameter bars cause them to tend to pry the cover off in a manner that reduces the bond strength?

For this analysis, it was assumed that plane sections remain plane and reinforcing bars remain perpendicular to the beam section. Reinforcing bars therefore bend to remain compatible with beam bending, so beam curvature and bar curvature are equal. Figures G.3(b) and G.4(b) therefore represent beam curvature.
curvature and bar curvature. For the following analyses, only results from 18-c are used because 18-d had a prominent inclined crack within the lap splice that clearly violated the plane-sections remain plane assumption (Figure G.5).

Figure G.5: Specimen 18-d with inclined crack spanning from the compression fibers in B-C to the D-E tension face

To simplify the analysis and reduce the localized effects of concentrated rotations at a single crack, curvatures were averaged as follows: curvatures in A-B and D-E, which straddled the splice ends, were averaged, as were curvatures in B-C and C-D, which represent the middle portion of the lap splice. The resulting average curvatures are given in Table G.3 and Figure G.6(a). Figure G.6(a) shows average curvatures versus position within the lap splice. The distribution in Figure G.6(a) cannot be the real distribution of curvature because a discontinuity in bar curvature implies a discontinuity in moment on the bar. This is not possible since no moments were applied to the bar.
To obtain a more realistic estimate of bar curvature distribution, the following assumptions were made:

1. Bar curvature is a maximum at the start of the splice and decreases linearly over a length $x$. The total rotation over $x$ is equal to that measured in the test.
2. Bar curvature is zero at the end of a lap-spliced bar. This must be true because there is no moment applied to the bar end. But, this also requires some incompatibility in bending deformations near the end of the lap splice between the terminating end of one bar (zero curvature) and the continuous parallel bar (which experiences maximum curvature). The total rotation over $x$ at the terminating bar end is therefore not equal to that measured in the test.
3. Bar curvature varies linearly over a length $x$ from zero at the bar end towards mid-splice.
4. Bar curvature is constant over a length $(\ell_s - 2x)$ centered on the mid-length of the lap splice. This constant curvature is equal to $1.52 \times 10^4$ 1/in. [0.060 x 10$^4$ 1/mm] for 18-c (Table G.3).

Based on these assumptions, and an assumed $x$ of 6 in., the curvature distribution in one lap spliced bar within the lap splice is plotted in Figure G.6(b). Since the bars remained linear elastic, the bar moments in Figure G.6(c) can be obtained by multiplying curvatures in Figure G.6(b) by $EI$, where $E$ is 29,000 ksi [200 GPa] and $I$ is 1.27 in.$^4$ [530000 mm$^4$] assuming a No. 18 [57 mm] bar can be represented as a circle with a diameter of 2.25 in. [57 mm]. The shear in the bar (Figure G.6(d)) thus equals the slope of the moment distribution in Figure G.6(c).
The shear in the bar in Figure G.6(d), which is 5.7 and 0.9 kips [25 and 4 kN] at the continuous and terminating bar ends, respectively, can be a basis for estimating the force that must exist perpendicular to the bar axis within the lap splice to produce a bent-bar shape compatible with the shape of the flexing beam.

The specimens had two lap splices, each consisting of two parallel bars. So, the cover at one end of the lap splice must provide a total clamping force equal to \(2(5.7 \text{ kips} + 0.9 \text{ kips}) = 13.2 \text{ kips} \) [59 kN]. If the area of concrete in a plane parallel to the lap spliced bars that is active in providing this clamping force is the beam width, \(b\), times \(x\), this analysis implies that bar prying/bending induced a tensile stress in the concrete.
perpendicular to the bar axis equal to $13.2 \text{kips}/(b \cdot x) = 90 \text{ psi} [0.63 \text{ MPa}]$. The cracking stress of the cover is approximately $4\sqrt{f'_c} = 400 \text{ psi} [2.8 \text{ MPa}]$, so resisting bar prying required approximately $1/4$ of the available resistance to splitting provided by the cover concrete.

This analysis requires a series of assumptions that may not be valid, so it should not be interpreted as a means of accurately assessing the tension stress that bar prying/bending induces in the cover concrete. Nevertheless, this model is useful for comparisons and can be used to demonstrate how increasing bar diameter can result in substantial increases in the concrete stress required to resist bar prying/bending, and that the force required to resist bar prying/bending might be non-negligible for large bars.

Figure G.7 shows the estimated concrete stress required to resist bar prying/bending for different bar sizes and lengths $x$. The calculated concrete stress is heavily dependent on both $x$ and bar diameter. Regardless of $x$, and assuming that everything else is equal, prying of a No. 14 [43 mm] bar induces concrete splitting stresses that are 5 times larger than a No. 9 [29 mm] bar and twice as large as a No. 11 [36 mm] bar. Likewise, prying of a No. 18 [57 mm] bar induces concrete splitting stresses that are 16 times larger than a No. 9 [29 mm] bar and nearly seven times larger than a No. 11 [36 mm] bar.

Figure G.7 shows that although small bar bending may be negligible, it may not be appropriate to neglect the concrete stresses induced by bending of large diameter bars. Assuming this model produces an estimate of concrete stresses that are at least of the correct order of magnitude, prying/bending of large bars induces stresses in the concrete cover that are large enough to compromise the ability of the cover to resist bond-induced splitting. Figure G.7 shows that for $x = 3$ or 6 in. [75 or 150 mm] the No. 18 [57 mm] bars produce concrete tensile stresses that are approximately 0.25 and 1.5 times the splitting stress for 10,000 psi [70 MPa] concrete ($4\sqrt{f'_c} = 400 \text{ psi} [2.8 \text{ MPa}]$).
Figure G.7: Estimated stress in concrete cover perpendicular to bar axis required to resist bar prying/bending for varied bar size and length $x$. 

- $x = 3$ in. [75 mm]
- $x = 6$ in. [150 mm]
- $x = 9$ in. [225 mm]