FINAL REPORT
RESEARCH IMPROVEMENT AWARD FOR ECONOMIC MODELING

IPPBR/UNIVERSITY OF KANSAS
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EXECUTIVE SUMMARY

Purpose of the Report:

-- to summarize the results of the three year Research Improvement Award for Economic Modeling received in 1984 from the University of Kansas by the Institute for Public Policy and Business Research.

Purposes of the Award and the Project:

-- to support the initial conceptual development and the empirical implementation of a long term dynamic input-output model for the Kansas economy.

-- to provide seed money to assist the Institute in securing on-going funding for the model.

-- to invest in human and organizational capital directed to serving state government needs for economic policy analysis.

-- to improve the economic and business research capability of the University.

-- to expand the data-base of economic statistics available at the University and help make existing data more accessible.

Problem Statement:

-- There has been a substantial unmet demand for sophisticated policy analysis and modeling of the Kansas economy.

-- There are serious problems with conventional econometric time series modeling, which include practical restrictions on the size of the model and sensitivity of the estimated parameters to time, expectations, and policy changes.

-- Although conventional input-output analysis can overcome these problems, it has depended on unrealistic perfect foresight assumptions.

-- Dynamic instability is a recognized problem in all existing theory-based approaches to long term dynamic economic modeling.

-- Export base theory is generally accepted by regional economists. Any realistic regional model will reflect this.

New Modeling Insights Achieved:

-- Conventional approaches to economic modeling are very likely to fail when applied to long term models because of the problem of dynamic instability.

-- The introduction of nonlinearities is crucial for stabilizing the model.

-- Much greater realism can be achieved in dynamic input-output modeling if the assumption of perfect foresight is replaced with imperfect expectations. This change justifies the introduction of nonlinearities.

-- These ideas led to an innovative approach to dynamic regional modeling, embodied in a stable long term model.

-- The existence of a stable and realistic dynamic input-output model may lead to a powerful new confirmation method for regional economic theories. In particular, it is now possible to predict regional time-series data from regional cross-section data plus national time-series data.

-- The Cobb-Douglas approach to input-output modeling answers the traditional objection that Leontief technology is too inflexible. It also provides testable restrictions on the model.

-- Advances have been made in understanding several technical problems of input-output modeling, including methods of inferring the A-matrix and non-survey methods for updating parameters.

Major Modeling Results:

-- four different modeling approaches for Kansas were implemented on computers: a static Leontief model, a conventional dynamic Leontief model, a dynamic linear programming model, and a dynamic input-output model with imperfect foresight. The last model is referred to as the Kansas Long Term Model, or KLTM.

Organizational Development and Other Investments:

-- The project recruited and trained a core group of 7 personnel, most of whom have advanced degrees.

-- This core group became a basic resource for the Institute and has often been consulted on research projects.

-- An enhanced capability to perform economic policy analysis for the State of Kansas is in place; in other words, IPPBR is well equipped to analyze the impact of new taxes, government spending, transportation issues, etc.

-- A series of Economic Research Technical Notes was established so as to preserve and disseminate the knowledge gained.

-- Several computer procedures and programs were developed to execute mathematical procedures useful in regional modeling.

-- Three micro-computers were obtained, along with associated software.

-- An important large computer-readable data set was acquired (the MRIO model), together with other data.

-- Data base plans and proposals were formulated.

Papers and Presentations:

-- six articles and papers have been written which use the results of the model or deal with the theoretical aspects of the modeling effort.

-- In addition, 27 Technical Notes have been written.

-- three consulting contracts at the Institute have led to reports using the model and two others are under negotiation.

-- seven presentations and briefings have been made which drew on the model.

-- one dissertation is in progress using the model.

-- two funding proposals have been made in connection with the project.

Other Outputs:

-- Through the efforts of Dr. Anthony Redwood, the modeling project has obtained permanent funding as a line item from the state government.

-- The modeling project has generated good will for the University.

Plans:

-- The project will continue as an ongoing process of modeling the Kansas economy.

-- Confirmation, validation and improvement of the basic model will continue to be a major activity. The model will be expanded to 100 sectors from its present 11.

-- At least four additional papers are currently in progress. Many others are planned.

-- Future empirical investigations will study routine and non-routine investment, demand for sales to other states, the effect of government infrastructure on business, and the formation of business's expectations about the future.

-- Assumptions that were made to simplify and make the model more tractable will be investigated and improved structural accuracy will be sought.

-- Areas of future policy applications will necessarily depend on political interest, but are expected to include taxation and economic development.

-- Graduate students and professors will continue to use the model in their studies.

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1. INTRODUCTION

This is the Final Report for the three year Research Improvement Award for Economic Modeling (University of Kansas General Research allocation # 3461-0038), received in 1984 by the Institute for Public Policy and Business Research (IPPBR) from the University of Kansas division of Research, Graduate Studies, and Public Service. The grant was awarded to support the initial development of a dynamic input-output model for the Kansas economy. A major purpose of the Grant was to help the Institute secure on-going funding; since this effort was successful, some of the work reported below was accomplished under partial funding from another source (namely, a line item in the Kansas State Budget).

Although titled a "Final Report", this is actually a status report on an ongoing process -- the process of modeling the Kansas economy -- rather than a final report describing a completed project. The job of modeling the Kansas economy, by the very nature of the task, can never be completed until the Kansas economy stops changing. Thus, this report is a snap shot of the project as it stands today, a brief history of the successes and failures which led to this point, and an indication of some of the directions the project is currently headed. This report was written by David Burress, the current Project Coordinator, together with the current research staff: Mike Coen, Mike Eglinski, Bob Glass, Frank Hefner, and Pat Oslund; all were under the general direction of Mohamed El Hodiri, the Principal Investigator for the modeling project since its inception, and Dr. Anthony Redwood, IPPBR Executive Director. Amy Waddle provided indispensible secretarial assistance.

The report consists of nine sections. This introduction is the first section and a conclusion is the ninth. Section 2 describes the basic problem confronted by the modeling project and discusses previous attempts to confront similar problems. Sections 3 and 4 review the analytical progress of the modeling project from initial attempts at modeling the Kansas economy to the current status of the Kansas Long Term Model (referred to below as KLTM). Section 5 discusses some empirical results from the model. Section 6 describes the organizational and institutional forms which have evolved out of the modeling project. Section 7 enumerates the scholarly and social products of the modeling project. Section 8 outlines some new directions planned for the modeling project. Following the main body of the report are 17 appendices which provide additional details on many aspects of the modeling project. The appendices are described in the body of the report; the end plate gives a table of cross-references between the report and the appendices.

2. LITERATURE REVIEW AND PROBLEM STATEMENT

The Kansas Long Term Model has drawn substantially on the literature of applied and theoretical economics. The KLTM integrates results found in the literature with innovative modeling techniques developed over the course of the research improvement grant. This brief review of the literature will compare KLTM with other modeling efforts. (Sources for citations are given in the Bibiliography, Appendix Q.)

A. Goals of the Project

Over the last three years, IPPBR has worked to develop a dynamic model of the Kansas economy. The requirements of the modeling effort were twofold: the model must provide a useful tool for Kansas policy decisions while at the same time incorporate the best insights of economic theory. On the practical side, the goal of the Institute has been to provide a model which can be used to assist the public and private sectors with economic development, to simulate major policy issues under alternative scenarios, and to conduct economic impact studies. A model which accurately represents the important features of the Kansas economy is prerequisite to accomplishing any of these goals. The types of policy questions for which the model will be used dictate that the model be able to specify the relationships between a large number of industries. Furthermore, the model must be able to capture how an economy grows over time if it is to address economic development questions. It is essential to model an economy's investment behavior and the effect of investment on productive capacity. Despite the theoretical and practical difficulties of formulating large dynamic models, dynamic features are critical to the modeling task.

B. General Equilibrium Models

The KLTM belongs to the broad class of <u>applied general equilibrium</u> models. General equilibrium models share the following characteristics:

1. A requirement of simultaneous market clearing. This usually means that there exists a set of prices and quantities at which buyers and sellers are both satisfied.

2. A mechanism (such as price and quantity changes) which aloows the economy to achieve simultaneous market clearing.

3. A well specified description of how changes in any one market affect all other markets.

4. A budget equation which makes the expenditures of consumers consistent with the wage and profit payments payments made by firms.

The overriding theme of general equilibrium analysis is the interconnectedness of markets.

General equilibrium approaches have dominated theoretical work in economics but not, surprisingly, applied work. Empirical research has mainly relied on econometric methods. Several problems result from econometric modeling. These problems are detailed in a later section of this report, but to summarize, they often fail to provide reliable numerical estimates of the structural interrelationships in and economy.

C. Input-output Models

Input-output (I/O) modeling is the most common example of the use of general equilibrium concepts in applied work. The simplest type of I/O model consists of three components, a matrix of fixed coefficients representing the inputs used in production, a matrix vector of final demands, and a resulting vector of economy outputs. The components are related through market clearing equations, as characteristic of general equilibrium approaches. More realistic I/O models have added to the basic structure, as described in Section 3. The intellectual roots of I/O models extend back as far as the mid eighteenth century (Stone [1986] provides an excellent summary of the precursors of I/O analysis). I/O as a technique to model actual economies ignited with the work of Wassily Leontief in the late 1930s. Leontief's 1941 book [Leontief 1941] describing his work on the U.S. economy was seminal to the field.

Originally, I/O models were static, that is, they provided no mechanism for investment to augment the productive capacity of the economy. This was seen as a serious shortcoming if the models were to be used ot project the future path of the economy. In response to the perceived deficiencies of static modeling, Leontief developed a dynamic approach which attempted to incorporate investment and capacity changes in a consistent fashion [Leontief 1953]. Section 3 and Appendix C of this report analyze the problems encountered in attempts to uses the Leontief dynamic approach. Despite its deficiencies, the Leontief dynamic model focussed attention on the investment equation and its importance in developing reasonable input-output models.

Almon [1970] and Miernyk [1970] redirected attention to the investment equation in applied models. Commenting on Leontief dynamic models, Almon pointed out that

> ...such models produced some rather bizarre growth paths. It is these results that have, unfortunately. created the impression (under which I have long labored) that there is some 'problem' with investment peculiar to input-output models. ...Our present work, however, shows this view to be quite mistaken; any good investment function can be put into an input-output model that is patterned on the way the economy works. [Almon, 1970, p. 286].

Miernyk, working on a regional model of the West Virginia economy, adopted a modification of the Leontief model which made investment dependent on past output changes. Miernyk's approach is subject to some of the same criticism as the basic dynamic Leontief model, that is, substantial instability may appear over time. Furthermore, it allows investment to be negative, which means that capital goods can be converted costlessly to consumer goods. Almon, working on a model of the U.S. economy, made investment depend primarily on the previous year's output level. Almon's approach avoided some of the mathematical complications encountered by Leontief. Unfortunately, it is likely to give misleading results as the model moves over time. It is more realistic to assume that firms invest in <u>anticipation</u> of future output changes rather than to past output levels. Despite the emphasis that has been given to the role of expectations in the economic literature, there appear to be no examples of applied input-output which incorporate these variables explicitly. The KLTM, on the other hand, provides an important channel through which expectations affect investment behavior. Furthermore, the KLTM allows for a variety of different assumptions about expectations, including that decision makers are wrong in their predictions of the future.

D. Computational General Equilibrium Models

Computational general equilibrium (CGE) models can be defined as numerical specifications of the general equilibrium framework for the purpose of policy evaluation. Input-output models are the earliest examples of CGE models. In recent years, however, the original input-output framework been expanded to allow for more flexibility. Shoven and Whalley [1984] survey these important new developments. In contrast with most I/O work, the newer CGE models share the following characteristics:

1. They allow for substitution between labor, capital, and materials in response to price changes. Standard I/O models, on the other hand, generally assume that these inputs are used in fixed proportions.

2. They allow for consumers to alter their expenditures when relative prices change. Again, most I/O models assume that consumer expenditures are fixed.

3. They provide a framework sufficiently rich to solve for an equilibrium set of prices in the economy. The role of prices in I/O models appears to be secondary.

Unfortunately, dynamic CGE models have led to serious computational complexities which make them difficult to use.

The Kansas Long Term Model has already incorporated several of the advances of CGE analysis. The Cobb-Douglas framework discussed in Appendices G and H allows both firms and consumers to adjust their behavior when prices change. The Cobb-Douglas framework goes a long way toward capturing the flexibility of more general CGE models without introducing serious computational complexities.

Although the KLTM belongs in the category of CGE models, its orientation is somewhat different than that of much recent applied general equilibrium work. Major differences arise in the treatment of prices and of investment. Because KLTM describes a regional economy, it is reasonable to assume that most prices are determined exogenously (i.e. outside the model). KLTM asumes that the prices of most Kansas goods as highly dependent on national and world markets over which Kansas has little influence. Any temporary disequilibrium in a market is resolved through changes in production and imports rather than through the more standard mechanism of price changes. While KLTM's treatment of prices is simpler than that in many CGE models, the treatment of investment is much more developed. KLTM incorporates industry specific investment functions which respond to expected demand, the inflation rate, and the prices of capital goods. In contrast to many CGE models, there is no requirement that investment be financed by the savings of firms and consumers. The openness of the Kansas economy allows financial capital to be imported to the state.

E. Statistical Methods for I/O and Econometric Modeling

Statistical methods for I/O (and similarly, CGE) models are substantially different from conventional econometric time-series techniques. The underlying data source for an I/O model is largely cross-sectional (i.e. based on a single year), whereas time series methods require data across many years. I/O parameters are typically directly measured and easy to understand (e.g., the ratio of labor input to output for a sector), whereas time series parameters are typically based on "reduced form" equations with little intuitive meaning. The number of parameters describing inputs to a single I/O sector can be very large (up to 500 parameters is practical); whereas a time-series equation is severely restricted by the "degrees of freedom" problem (i.e. the number of time periods must be much larger than the number of parameters).

In consequence, dynamic I/O modeling potentially holds several important comparative advantages over econometric time series modeling. A much larger number of parameters can be estimated, leading to a much more detailed model. Parameters which depend on expectations about the future can be isolated from parameters which depend on technology or preferences, leading to a relative freedom from the "Lucas Critique" (i.e., the problem that parameter values may change when government policies change [Lucas 1976]). Matrix techniques and linear algebra provide an efficient means of control over large parameter sets, and lead to a parsimonious set of theoretical equations. Error checking can be simplified by the use of familiar or intuitive prior information about parameter values (e.g., the knowlege that labor costs should average around 70 per cent of value added).

However, these potential advantages have not been realized in the past because of offsetting disadvantages. The problem of dynamic instability had not been solved, so that I/O models were restricted to crude extensions of static analysis. Government data collection efforts are highly biased towards time series analysis: summary time series statistics are much more widely available than are detailed cross-section statistics. Econometric time series techniques are much more widely known, and also more highly developed, than are techniques related to I/O modeling. More importantly, time series techniques are fundamentally more flexible than I/O techniques. If a variable is missing in time series analysis, it can usually be proxied. If an entire structural equation is unknown, it can be replaced with an assumed reduced form. If an equation can not be solved using one functional form, then another functional form can be assumed. This flexibility is especially important for modeling the monetary side of the economy, and also for modeling the determination of prices. However, it is much less important for modeling regional sections of a national economy, because prices and monetary phenomena are assumed to be given exogenously by the national economy.

Moreover, this very flexibility of time-series modeling can be construed as a weakness. Time-series methods lead to so many possibilities that it is very hard to tell which one is best. The problem is compounded by the fact that time series data are often highly correlated with each other (the problem of "multi-collinearity"). In other words, using time series methods leads to a difficult problem of confirmation or verification of theories. A recent survey of this problem is given by Los [1986] who states "... econometrics can be used under only very rstrictive conditions. ...there exists an urgent need for a framework of quality control standards for good econometric analysis, to determine where it can be applied and to assess the quality of its results."

3. THE DEVELOPMENT OF THE KANSAS LONG TERM MODEL

During the three years of the Research Improvement Award, the development of KLTM proceeded in stages which at first recapitulated, and then went beyond, the historical development of input-output models in economics. A static Leontief model was developed first, followed in order by a conventional dynamic Leontief input-output model, a linear programming dynamic model, and finally an innovative dynamic input-output model with investment under imperfect foresight. (A fuller account of the earlier stages of KLTM is found in Appendix C.)

A. The Static I/O Model of Kansas

Static Leontief models, or input-output models, have been utilized in economic analysis for several decades. The data used by these models come from input-output tables, which give inter-industry flows, final demands, and total output. Final demands are further divided into consumption, investment, government purchases, and shipments out of the state ("exports") less shipments into the state ("imports"). When the economy is at equilibrium, supply equals demand, so that the total output equals the sum of the final demands plus the inter-industry flows.

The key idea of the model is to take final demands as given (i.e., "exogenous"), and use them to predict total output and employment. This can be accomplished using two main assumptions.

First, all sectors of production are assumed to obey constant returns to scale -- that is, inputs into a sector (inter-industry flows plus capital and labor) increase proportionately to its output. This assumption has a substantial amount of empirical and theoretical support. Second, and more problematically, each input is assumed to occur in a ratio to the sectoral output which is constant over time. The set of these fixed ratios or coefficients is referred to as the Leontief A matrix. (Section 4 and Appendix G discuss ways of relaxing the assumption of fixed proportions.) Under these assumptions, and assuming also that positive profits exist in every sector at the going prices, then there is a solution to the model (given by a formula referred to as the "Leontief inverse"). In other words, for any set of positive final demands, there is a unique set of positive outputs which will exactly satisfy the final demands plus the necessary inter-industry flows.

The solution to the static model for Kansas was worked out and used for policy analysis to determine the impact of changes in demand on the Kansas economy.

This basic model could be improved by noticing that state consumption, state investment, state imports and state and local government expenditures should not be given exogenously; instead, they all depend on state output and income. Using crude extensions of the fixed ratio assumption, the static model could be expanded so that only state exports are left exogenous; the other final demands are "endogenous". When this is done, the model agrees with conventional export base theory, which states that the income and output of a region are determined by a multiplier (e.g., the Leontief inverse) times its exports. Export base theory is widely accepted by economists.

However, rather than elaborating the static model, the modeling group began work on a fully dynamic model. At the same time, a new data source was acquired (the MRIO data set, described in Appendix L), which had the advantage of detailing trade flows in and out of Kansas.

B. The Dynamic Leontief Model of Kansas

The Research Improvement Award Proposal had proposed a dynamic model of Kansas, i.e., a model that accounts for developments over time, as well as one which endogenizes the various components of final demand.

In a static model, all inter-industry flows are viewed as serving as an input for current production. But in actuality some of these flows are not used up, as in the case of machines, buildings, etc. In addition to the static concept of Leontief coefficients, realism requires an inherently dynamic concept of capital coefficients. A capital coefficient measures the amount of the output of one sector that is held by another sector as capital stock. In addition to current capital, a growing economy needs different amounts of new capital of each type over time. A dynamic model takes into account the additional capital that will be needed in the next time period but must be purchased now. A "dynamic Leontief" model takes the investment demands and other demands of a given year as a given starting point. From this year's investment, the amount of potential output for next year is inferred. By assuming that all of next year's potential output will be realized, potential output for the following year can be inferred; and so on. The key assumption, and also the weak point of this model, is the idea that this year's investment contains perfect information about the entire future.

Whether a dynamic Leontief model can be solved or not depends on the nature of the capital coefficients matrix. The most serious problem is what is referred to as "causal indeterminacy" (for other problems, see Appendix C). Causal indeterminacy means that the capital coefficient matrix is unstable and can generate a model that fluctuates between positive and negative levels of output at an explosively growing rate. This is obviously not a realistic picture of the economy. Causal indeterminacy was observed in the dynamic Leontief model for Kansas.

C. The Dynamic Linear Programming Model of Kansas

After the (not unexpected) failure of the dynamic Leontief model, a dynamic linear programming model was developed.

Linear programming models require an objective function which is to be maximized subject to various constraints. The chosen objective function was the present value of future gross state product (however, the results proved insensitive to the choice of objective functions.) The initial model predicted declining output and negative investment in most sectors. Since it not very easy to trade unneeded machines and plant for money once they are in place, negative investment is not realistic. Therefore the model was given additional constraints against disinvestment. The revised model then exhibited infeasibilities (i.e., conflicts between constraints). In particular, the constraints required that all exogenous demands be supplied exactly each period, without ever causing either excess capacity or disinvestment; but this wasn't possible. In a final attempt, the equality constraint between supply and demand was relaxed to an inequality constraint; in other words, excess demands were allowed to be met through imports. This led to a feasible solution for the Kansas economy. However, the solution lacked empirical realism. In particular, Kansas was modeled as reaching a "corner solution", highly specialized in a very few sectors.

D. KLTM

Drawing on this body of experience, an innovative and more realistic model of Kansas was finally developed. First, in place of the "perfect-foresight" model of investment, it was assumed that investment is driven by short term expectations, and that temporary unplanned excess capacity can result. Second, to avoid instabilities and corner solutions, strategic (but theoretically justified) nonlinearities were introduced into the model. In particular, investment can not be negative, and income can not result from unused capacity. Third, the optimizing behavior of agents in the economy was decentralized by dispensing with an arbitrary objective function. These modifications led to the development of the current model, which is described more formally in Appendix D. The next two sections give an informal discussion of the important features of this model.

4. ACHIEVED MODELING INSIGHTS

The last section showed that creating a long term model of Kansas depended on finding solutions to some outstanding problems in regional economic modeling. This section summarizes some of what was learned.

A. Clearing the Underbrush

As in many research projects, much of the KLTM development effort involved showing that conventional approaches will not do the job. It was shown that neither a standard dynamic Leontief model, nor a dynamic linear programming model, will describe the Kansas economy realistically. These two types of model are both highly linear, and both are prone to suffering from unstable solutions. It was inferred from this that a nonlinear model was needed.

B. Nonlinear Stabilization of the I/O Model

KLTM is a regional dynamic I/O model which is stable over time. Its stabilization was achieved by the introduction of nonlinearities which actually increase economic realism. This is a significant accomplishment.

Nonlinearities not only can allow for stable solutions, but also can add realism since many real world relationships are not linear. However, models used in business and economics tend to be linear for ease of calculation. For the same reason, the use of nonlinearities in KLTM is carefully restrained. In the most practical version of the model (the version described in Appendix D), nonlinearities appear only in relationships between different times. Contemporaneous relationships are perfectly linear, leading to a simple solution method which is recursive across time. However, this version of the model depends on somewhat stylized assumptions (most importantly, the assumption that household consumption depends on last year's income rather than current or expected income). Solution techniques have also been worked out for less stylized (and consequently more nonlinear) versions of the model, but these solutions are more cumbersome.

C. An Alternative to Econometric Time Series Methodology

The existence of a stable dynamic I/O model raises the possibility of a powerful new method of validation or confirmation for theories of regional economics. That is, an I/O model which has been estimated almost entirely from non-dynamic (cross-sectional) data, can be tested dynamically. If this model succeeds in tracking real economies, then that will be convincing evidence that the theories underlying this model (e.g., export base theory, and the model of investment under imperfect foresight) are correct. In contrast, tests of time series econometric models tend to be inconclusive for reasons stated in Section 2.

D. Advanced Multipliers

Impact studies rely on a multiplier concept. That is, the impact of an increase in the export base on a given sector of the regional economy equals an appropriate multiplier times the amount of new exports. The KLTM has led to a series of new formulas for calculating sophisticated multipliers under a variety of scenarios. For example, the difference between the impacts caused by changes in export demand are different than those caused by spurts of investment demands and there are correspondingly different multiplier formulas. Further, since the model is dynamic, it leads to dynamic multipliers, i.e. ones which take into account the amount of time between cause and effect.

One of the original goals was to endogenize portions of what are generally treated as exogenous demands. State and local government has been made endogenous and multipliers have been formulated under that assumption.

E. A-matrix Methodology

The major data source for this kind of modeling is the A matrix (i.e. the ratios of inputs to outputs). Any economic model performs some aggregation. Errors are made in aggregation. Therefore, the kinds of biases that can develop in aggregating the A matrix have been formalized; see Appendix J. Further, there are many more methods for forming the A matrix from the raw data of the make and use matrices than are normally considered.

By categorizing the permutations of possible methods in forming the A matrix, it was possible to develop a clearer theoretical understanding of the conditions under which different methods are appropriate.

F. The Cobb-Douglas Approach to I/O Modeling

A theoretical innovation developed at the Institute has been the conceptualization of Cobb-Douglas technologies in production and investment in the Leontief type framework. In fact, the article by El-Hodiri and Nourzad (reproduced in Appendix G) points out the equivalence of the Leontief technology with Cobb-Douglas and under what conditions this equivalence holds.

G. Updating Methods

Since the model is sensitive to the level of exports from Kansas a method of updating the export data available is very important. However, acquiring new data directly is costly. A new method was developed to update the export data indirectly from published sources.

The A matrix which is the heart of the model also needs to be updated. Again, a complete survey to update the matrix would prove very costly. Therefore a non-survey method has been developed which will allow the Institute to generate an updated, more accurate A matrix. (Updating methods are described in Appendix K.)

5. DATA AND RESULTS

A. Potential Uses of KLTM

The Kansas Long Term Model has multiple goals. A primary purpose is to help Kansas policy makers understand the repercussions of state policy choices. A further purpose is to track the expected impacts of changes originating outside the state on the Kansas economy. For example, one would like to predict the effects of changes in federal tax policy or changes in foreign trade possibilities. The structure of the model allows it to be used to give long term (one to ten year) projections of the conditions of the Kansas economy given a set of assumptions about the course of the national economy and the structure of international trade.

B. Sketch of the Model

The basic structure of the model determines output by sector in the Kansas economy. Although data are available which describe about 100 detailed sectors, these sectors are currently aggregated into an 11 sector grouping to make the model of more manageable size. The aggregation scheme which has been chosen for the pilot stage of the model tries to group together industries which produce similar types of products. The resulting aggregation scheme is detailed in Appendix A.

As described in Appendix D, output in the Kansas Long Term Model is driven by final demand, primarily export demand. In simplified form, the model reduces to :

Output (by sector) = **(multiplier matrix)** x final demand

C. Data Requirements and Sources

The major data requirements of the model fall into two categories, those involving components of the multiplier matrix and those involving final demand. A large data set compiled by Jack Faucett and Associates supplies an important source of raw data in both categories (see Appendix M).

The Faucett data set is unique in that it provides the only available source of data on interstate flows for a complete set of goods and services. To construct this data set, Faucett and Associates compiled information from the Bureau of the Census, the Department of Transportation, and from private industry sources. The interstate sales data allowed calculation of Kansas import coefficients. State import coefficients, which reflect the amount of each good imported from out of state as a percentage of the total state use of that good, are an important component of the multiplier matrix. Accurate calculation of trade coefficients has been a stumbling block in regional modeling.

The Faucett data set also provided state specific data on the production and use of goods by each industry. This information, summarized in the Faucett make and use matrices, formed the basis for calculations of the technical requirements coefficients of the **A** matrix (see Appendix I). Finally, the

Faucett data set provided information on consumption, investment, and government spending for a specific year, 1977. Data on consumption and state government spending were used to calculate spending coefficients. The Faucett data on investment was used only as a check on independently calculated estimates of investment by commodity (in most cases, these estimates matched the Faucett estimates closely). Data on federal purchases was treated analogously with data on out of state sales; together these form the Kansas export base.

A complete description of all data used in the model is provided in Appendices L and M.

D. Projections from the Model

Parameters for the Kansas Long Term Model were calculated for 1977, the date of the Faucett data. 1977 became the initial base for the long term model (for updating plans see Section 8 and Appendix K). As a test of the structure of the model, projections of output from the model were compared with actual Kansas economic data for the years 1977 through 1986. As a basis for the projections, it was assumed that the export base grew 3% in real terms and 9% in nominal terms (including inflation). The projections contained no other information about the national or international economy after 1977. Projections were made using the computer software described in Appendix N. Based on expected growth in exports, the model predicted that Kansas firms would increase their capacity through investment. Investment demand sustained growth in investment sensitive sectors such as construction. The result of the projections of the Kansas model is a ten year series of output, investment, and wages for each of 11 sectors.

Ideally, one would want to compare projected output, investment, and wage bills for every sector with actual Kansas data. However, a complete set of Kansas specific data are available only for the wage bills. A comparison of the actual versus the projected Kansas wage bills by sector is presented in Appendix B. The graphs in the Appendix show that the model does a fairly good job of predicting overall Kansas wages. For some sectors, however, the performance of the model is quite poor. In particular, the model, with its assumption of steady 3% national growth in real demand in all sectors, fails to reflect important changes in the markets for aircraft, oil and gas, and automobiles which occurred during the ten year projection period.

Several sources of error contribute to inaccuracies in the projections of the model. First of all, the parameters of the model such as technical or import coefficients may be incorrect. Methods to update these coefficients over time are described in Appendix K. Secondly, some of the model equations may be mis-specified. Since the investment equation is a key to the model, it is planned to construct a time series of Kansas specific investment expenditures by industry so that the accuracy of this equation can be checked. Finally, the projected growth rates for the national economy may be incorrect. This source of error could be removed for runs of the model over a historical period such as 1977 - 1986. That is, actual annual growth rates for the U.S. economy by sector could be substituted for the 3% and 9% assumptions. However, for forecasting growth over a future time period (say 1987 - 1997), an independent estimate of the growth of the U.S. economy must be constructed. Alternatively, one could present conditional forecasts of the Kansas economy. For example, one could show how various sectors in Kansas would perform under the alternative assumptions of 10% growth in wheat exports or 2% growth in wheat exports. Eventually it is planned to obtain forecasts of important national and international variables which drive the Kansas economy, in cooperation with the Kansas Econometric Model.

E. Multipliers: A First Approach to Policy Analysis

Policy analysts frequently rely on multipliers as a short cut method of approximating the long range effects of changes in a region's economic environment. To this end, multipliers were developed which predict the impact of a sustained increase in demand for Kansas exports in each of the 11 aggregated sectors. In contrast to most regional multipliers, these numbers include the effect of the replacement investment necessary to sustain the economy's capacity at its new level. Kansas multipliers for output by sector and for Kansas value added (labor and property income) are reported in Appendix A. The multipliers for changes in output average a little over 2, meaning that each dollar of

Kansas exports generates an additional dollar of Kansas output. This occurs both because other Kansas industries supply the exporting industry and because the recipients of income due to exporting buy Kansas produced consumer goods. It should be noted that multipliers for subregions of the state, counties, or cities, will be considerably smaller than multipliers for all of Kansas. From the point of view of consumers and firms in a small region, purchases from Kansas locations outside the region are considered imports. Leakages from imports reduce the size of multipliers. Additionally, state tax payments are a leakage from the local region, although not from the state as a whole. This further reduces the size of local multipliers in comparison with state multipliers. The state multipliers put an upper bound on the benefits a region can hope to derive from attracting new export oriented firms.

6. ORGANIZATION AND DEVELOPMENT

One of the purposes of the 1984-87 Research Improvement Award was to create the capacity to do improved policy analysis at the University of Kansas. In other words, that grant was intended in part to be seed money for an investment in human, organizational, and physical capital.

A. Personnel

Human capital consists of personnel and their skills. In the process of developing the Kansas Long Term Model, IPPBR recruited a core group of personnel -- research associates, assistants and programmers. Prior training and on-the-job experience in developing KLTM has given these workers substantial amounts of knowledge and insight about regional economic modeling. They have also developed a familiarity with data sources (such as the Citibank data base, International Financial Statistics, the periodic publications of the Bureau of Economic Analysis, etc.) which are useful to KLTM and also to other investigations. A list of persons who have worked on the technical development of the KLTM is given in Appendix O.

B. Documentation

From the point of view of an organization, human capital can sometimes be an ephemeral asset: workers may come and go, or they may forget what they've learned. To preserve the gains and insights made, the modeling group has recorded its development history and methods in a series of miscellaneous documentation referred to as Economic Research Technical Notes (ERTNs). Using the ERTN series, new research assistants may learn relatively quickly what old assistants struggled to discover. Further, other members of the Institute interested in the methods and ideas of the KLTM can be referred to the appropriate ERTN. (A list of the ERTNs issued as of this report is given in Appendix P.)

C. Data Acquisition

With the help of the Research Improvement Award, the Institute has acquired valuable new data sets in support of KLTM, both in computer-readable media and on paper. The most important of these is the MRIO dataset described in Appendix L. Appendix M list other datasets which have been used in the model.

D. Hardware and Software

The ability to find and manipulate data is crucial to economic modeling. This ability depends not only on human skills and previously acquired data, but also on computer programs and equipment. Funded partly by the Research Improvement Award, the Institute has purchased three Zenith 150 PC-compatible micro-computers plus associated packages of commercial software for the KLTM group. These computers are used both independently and as remote terminals to an IBM 3031 mainframe running under VM/CMS.

Using these tools, the KLTM group has been developing the computer programs and data bases it needs. The existing software developed for the KLTM project is described in Appendix N.

E. Cross-fertilization

These skills and tools are not limited in their use just to the Kansas Long Term Model. For example, a program to aggregate matrices (based on the ideas in Appendices I and J) is expected to be useful to many projects at the Institute. Also, several projects have used the static version of the model to perform impact studies.

Members of the KLTM group have often been asked to consult with other Institute staff on policy studies and Kansas economic issues. On several occasions they have been involved in policy studies contracted with the Institute (as described in Section 7 below).

Another model at the Institute, called the Kansas Econometric Model (KEM), is a collateral or branch project to KLTM. Since data sources overlap for the two models, the modeling groups have developed a coding scheme that allows quick interaction on the computer of the various data bases used in both models. Further, the results from the KEM effort are used in Kansas Long Term Model and vice versa. On occasion the researchers on both projects have been consulted for input into research related to economic issues at the Institute.

The KLTM personnel have also been consulted with respect to improving the general library facilities and data resource management methods at the Institute. Conversely, the existing programming and data resources at the Institute were of great help in the KLTM development effort.

F. Capabilities

As a result of its development effort, the Institute now stands ready to perform policy analyses for the State of Kansas. For example, impact studies may be performed to analyze the results of new taxes, government spending, transportation issues, etc. (described further in Section 8). Key members of the legislature and the executive branches have been informed about this capability, as described in Section 7.

7. SCHOLARLY PRODUCTS AND SOCIAL IMPACTS

The Research Improvement Award has led to many direct and indirect products. Some products were completely funded by the grant. Other products were "spin-offs", funded partly or completely by other sources, but drawing on work funded by the grant (spinoffs are indicated below by an "*"). The most obvious kind of output consists of articles and working papers. However, results from the KLTM model have been communicated in several other ways, as detailed below.

A. Articles and Papers

Several articles have been written that either use the model in their analysis or deal with the theoretical under-pinnings of the model. These include:

1. "Kansas Exports and Economic Development," Gary Albrecht, Shirley Sicilian, and Kurt Krueger, Kansas Business Review, Fall 1985, pp. 3-9.

2. "The Kansas Economy," Bob Glass, Kansas Business Review, Spring 1986, pp. 27-30.

* 3. "Optimality in the Channel Model I," Mohamed El-Hodiri and F. S. Van Vleck, 1985, working paper.

* 4. "Optimality in the Channel Model II. " Mohamed El-Hodiri and F. S. Van Vleck, 1986, working paper.

5. "Leontief Technology and Input Substitution" Mohamed El-Hodiri and Farrokh Nourzad, forthcoming in the Journal of

Regional Science.

* 6. "Investment Demand Under Log-Linear Technology," Mohamed El-Hodiri, April, 1987, working paper.

B. Contract Reports

Several consulting contracts at the Institute have led to reports which used the results of the model in their analysis of various issues. The following is a list of project reports that have used the model:

* 1. "The Economic Impact of State Support for the Arts in Kansas. " Bob Glass and Shirley Sicilian. IPPBR, March 1986, report number 104, prepared for the Kansas Arts Commission.

* 2. "Costs and Benefits of Business Tax Incentives in Kansas. " Shirley Sicilian. IPPBR, Feb. 1987, report number 117, prepared for the Kansas Legislative Task Force for Tax and Capital Markets.

* 3. "Economic Impact of Cheyenne Bottoms. " Shirley Sicilian. Prepared for inclusion in the report "Cheyenne Bottoms: An Environmental Assessment. " Kansas Biological Survey and Kansas Geological Survey. Forthcoming.

C. Presentations

Several members of the modeling group have given presentations based on the work done for the modeling project. These include:

* 1. "The Kansas Input-output Model." David Burress, Bob

Glass, Frank Hefner, and Pat Oslund. Seminar given Oct. 22, 1986, to the Economics Department, University of Kansas.

* 2. "Optimality in the Channel Model II. " Mohamed El-Hodiri. Paper presented at the D. G. O. R. , Oct 3, 1986, Ulm, Germany.

* 3. "Optimality in the Channel Model I. " Mohamed El-Hodiri. Paper presented at the 11th Symposium on Operational Research, Darmstadt, West Germany, December 1-3, 1986.

* 4. "Status of the Kansas Input-output Model". David Burress, Bob Glass, Frank Hefner, Mike Eglinski and Pat Oslund. Workshop on Kansas modeling with economists from the six Kansas Regents Schools. Lawrence, Kansas, December 12, 1986.

* 5. "The Kansas Long Term Model." David Burress. presentation at the Seventeenth Annual Meeting of the Kansas Economic Association. Topeka, Kansas, April 24, 1987.

D. Dissertation

One of the proposed uses of the Kansas Long Term Model was to support graduate dissertations. Currently one dissertation in progress, supervised by Mohamed El-Hodiri, makes use of the data from the Kansas Long Term Model. The expected completion date is December 1987.

* 1. "Tax Analysis in a Dynamic General Equilibrium Model of Kansas," Frank Hefner.

E. Briefings of Government Officials

The KLTM has reached a stage of development such that it can begin to be applied to Kansas State policy issues. The Institute has begun an effort to inform government officials about the availability and capabilities of the model (and of KEM, its companion short term model).

* 1. Meeting with Tim Witsman, President of KANSAS, INC. Briefing by David Burress on KLTM modeling capabilities. Lawrence, Kansas, April 7, 1987.

* 2. Letter to Senator Dave Kerr from David Burress and Gary Albrecht. May 10, 1987. Attached report on capabilities and results from KLTM and KEM.

F. Funding Proposals

The Research Improvement Award has made it possible for IPPBR to seek additional funding from other sources, including:

1. "Kansas Economic Modeling Program", proposed to the Kansas Board of Regents, June, 1986. Joint with the Department of Economics and the School of Business. Due to the efforts of the Dr. Anthony Redwood, IPPBR Executive Director, the Modeling Program was accepted and subsequently made a line item of the IPPBR budget, currently funded at $180,000 per annum.

* 2. Informal proposal to KANSAS, INC on creating a prototype database of Kansas economic information at the county level. Discussions are in progress, with a formal proposal to follow.

8. CURRENT AND FUTURE ACTIVITIES

The Kansas Long Term Model is designed both for basic research and for policy applications. Much of the future development of KLTM will follow from the internal logic of the research effort. However, like any policy model, KLTM will follow a somewhat unpredictable path of development, as a result of shifts in attention to various issues on the part of state policy makers. Accordingly, the plans described below are necessarily tentative.

A. Confirmation and Validation of the Basic Model

Testing and verifying KLTM, and improving the ability of KLTM to track the Kansas economy, will continue to be major activities. While the planned improvements described below are being put into place, the predictions of the model will be compared to actual Kansas experience during the last decade (as in Appendix B). At the same time, the model will be expanded to show some 100 sectors (presently there are 11). The KLTM model will also be compared to KEM (a short term forecasting model of Kansas using very different statistical techniques).

B. Papers in Progress

The KLTM project has gone a long ways towards establishing a team of research workers who combine the ability to produce work which is publishable in first rate journals with the ability to address real life problems of immediate concern to the state of Kansas.

Several scholarly papers resulting from KLTM research are in various stages of progress. These include an overview of the basic model with results; a general framework for theories of inference of the A matrix from inter-sectoral flows; and an optimizing theory of Cobb-Douglas investment. A planned monograph on regional modeling under the Cobb-Douglas framework will draw on a number of the technical papers (ERTNs) listed in Appendix P.

C. Model Improvements in Progress

Current development of the model involves creating and expanding the database of U.S. variables, and altering the model to reflect the effect of sectoral price changes on investment. Activities in the near future will include replacing the present assumption of a constant capital-output ratio with the Cobb-Douglas putty-clay investment model described in Appendix H; developing data to measure the effect of imports of unearned income (such as social security payments) on consumption; and testing the methods described in Appendix K for updating coefficients (which are currently based on 1977 data).

D. Planned Empirical Studies

A main purpose of KLTM is supporting basic research on many aspects of the Kansas economy. A major area of research will be concerned with routine investment in existing firms, and would seek to understand such phenomena as changes in the capital-labor ratio over time; sectoral differences in the periods of "gestation" (time required to put capital into place) and "commitment" (the point in time at which a decision to install capital can not be reversed); the formation of expectations which influence investment; and sectoral differences in depreciation rates. A second major area would explore the effect of national and world economic conditions on the demand for existing Kansas exports. A third major area is concerned with non-routine investment and disinvestment, which is to say the birth, death, and relocation of firms. These processes are especially interesting in connection with creating new export markets, the loss of old export markets, and the process of import substitution. Other important areas include the relation of Kansas income by sectors to consumption and savings; the effects of government infrastructure on Kansas production; the relative accuracy of Cobb-Douglas vs. Leontief Technology modeling techniques (and the possible creation of hybrid techniques); and the exploration of methods for "bringing down" the statewide KLTM model to study local economies.

E. Major Improvements to the Model

After a certain point, improvements in both the structural accuracy, and the predictive accuracy, of the existing model will depend on improvements in the data being used. This can only be accomplished by means of direct surveys of Kansas firms. At the same time, the scope of the model should be gradually expanded so as to describe all of the major variables encompassed in the abstract flow charts contained in Appendix E. Some important variables not contained in the current version of KLTM include those related to labor markets and population; to minerals and water resources; to subregional effects on the region (especially in the Kansas City SMA); and to the effects which determine (i.e., endogenize) prices in Kansas.

F. Anticipated Applications

Although the specific policy applications chosen for KLTM will depend on future political interest, one can anticipate continued interest in state taxes and their effects on growth. The model is likely to be helpful in formulating state industrial policy, and in particular can be used to measure the key export-base and import-substitution industries. The model may be used to prepare 10 year forecasts of the Kansas economy. The model will almost certainly continue to be used actively for regional and sub-regional impact studies. It is hoped that graduate students and professors will continue to use the model in their studies.

9. CONCLUSION

The opinions of the authors of this report may appear to be colored by their personal involvement in the project; despite this risk, they want to express their view that the Research Improvement Award Grant has stimulated a highly successful modeling effort.

A. Tangible Results

In scholarly terms, Sections 7 and 8 list the publications which have grown and are growing out of this project. In financial terms, the most important development has been the Kansas state revenues allocated to the modeling project: $180,262 in FY87 and $182,280 in FY88. In terms of generating new projects, a preliminary design for a Kansas data base system has been completed. These developments, together with the creation of the Kansas Long Term Model, provide strong and tangible evidence for the success of the modeling project.

B. Institutional Benefits

Of equal importance are some of the less obvious, intangible results of the first three years of the modeling project. The modeling project has been fully integrated into the mission of IPPBR. As a result, the capacity of IPPBR to do sophisticated regional economic analysis has been enhanced in at least three ways. First, the existing staff now has the aid of the Kansas

Model in doing their research. For example, determining economic impacts of different events on the Kansas economy can now be done better, cheaper, and more quickly than before. Second, the size of the staff has been increased and the project has provided invaluable training for new staff -- the kind of training which only hands-on experience can provide. Finally, the increased staff and its specialized training has endowed IPPBR with greater diversity in staff abilities and, hence, greater flexibility in providing services to the university and to the state.

Not only has the modeling project aided IPPBR, but it has benefited the KU Department of Economics. This is true in obvious ways such as providing employment and experience for graduate students and increasing the number of positions on campus for economists. But possibly of more long-run significance, the Department of Economics is now working more closely with the Institute than it worked in the past (either with IPPBR or with IPPBR's predecessor, IEBR).

C. Intellectual Progress

Another area of progress has been in the analytical thrust of the modeling project. The Research Improvement Award proposal called for implementing a forward-moving empirical dynamic Leontief input-output model. The failure of the Dynamic Leontief Model of Kansas has actually led to two analytic developments which seem fertile areas for research. One development was to base the theoretical foundations of input-output modeling on

Cobb-Douglas technology rather than Leontief technology. The second development was to replace the strict linearity of the dynamic Leontief model with a model which incorporates optimizing behavior based on nonlinear relationships. In both of these cases, research has been pushed past an existing boundary.

D. Service to the State

One final result of this project may be the least measurable. The modeling project has helped to generate good will toward the University of Kansas both from the State Legislature and from the people of Kansas. The Research Improvement Award for the modeling project demonstrated concern for the state and its economy on the part of a university which had at times in the past been accused of aloofness to the needs of the state. As the modeling project has progressed it has increased the capacity of the University of Kansas to serve the state, which is especially appropriate under the adverse economic conditions faced by Kansas today. The modeling project should continue to generate good will because of the information and policy advice it can provide the State of Kansas.

CROSS REFERENCE TABLE

Appendix	is cited in Section
A	5,8
B	5
C	2,3
D	2,3,4
E	4,5,8
F	4
G	2,3,4
H	2,4,8
I	4,5
J	4
K	4,5,8
L	2,5,6
M	5,6
N	5,6
O	6,7
P	4,6,7,8
Q	2,7

APPENDIX A
KANSAS IMPACT MULTIPLIERS

I. PURPOSE:

This Appendix reports the aggregate 11 sector, steady-state multipliers predicted by the current version of the Kansas Long Term Model.

II. SUMMARY OF METHOD:

Two kinds of multipliers are reported below. One type shows the ratio of the amount of new Kansas outputs (by sector and total) to the increase in exports (or in exogenous federal government demands) which caused it. The other multipliers show the ratio of new Kansas income (in this case, only the totals) to the same increases in exogenous demands. The multipliers show the steady state effect of a sustained increase in exogenous demands; temporary effects are ignored. They take into account all of the following effects, followed through an infinite number of "rounds":

- first round demands generated by the exports
- intermediate product demands generated by other demands
- intermediate demands for wholesale and transport
- consumption demands generated by the associated increase in income
- the increased Kansas state and local government expenditures, assumed proportional to the increase in state income
- investment demands to cover the depreciation on the extra capacity needed for the extra output
- leakages due to imports of goods.

However, the multipliers could be improved somewhat by taking into account the following omitted effects, among others:

- leakages due to Kansas corporate income not consumed in Kansas
- over-stated consumption coefficients due to ignored social security benefits and other income imports
- differences in import coefficients between consumption goods, federal government demands, and intermediate products.

Also, the multipliers depend on many parameter estimates, some of which we expect to improve on in the future. In particular, they use 1977 data, which we plan to update; also, the depreciation estimates need further work.

III. SCHOLARLY APPARATUS:

The multiplier formulas used below are close to those given in ERTN 4v1.0, especially paragraph 6 Model C (steady state output multipliers) and paragraph 8 (income multipliers). However, they have been modified to account for the a-matrix assumption, so that (v-u) --> (I-A). They have also been modified for the structure of wholesale and transport margins described in ERTN 20v1.0. The parameter values will be given in a forthcoming ERTN; in particular, depreciation rates were chosen so as to replicate a measure of Kansas Investment for 1977 under steady state growth assumptions.

IV. OUTPUT-EXPORT MULTIPLIERS.

The column headers give the numbers which identify the commodity for which an increase in exports occurred. The rows refer to the sectors in which increases in output are observed. (The meanings of the commodity and sector numbers are given below.) Since exports are defined so as to exclude both inventory changes and transshipments of goods, a unit of exports in a sector must lead to at least one unit of production in that sector; i.e., the diagonals must exceed one. The column totals give the total value of additional outputs of all types resulting from an increase in exports by one unit of value. All entries have been rounded in accordance with an estimate of the number of significant figures; therefore the column totals do not sum exactly. (Note that, under our model, an increase in Federal government demands for a commodity would have the same effect as an increase in exports of that commodity.)

	Export Sector										
Output Sector	1	2	3	4	5	6	7	8	9	10	11
1	1.3	.03	.02	.02	.02	.02	.21	.03	.04	.03	.03
2	.02	1.0	.02	.01	.01	.01	.09	.05	.02	.01	.01
3	.14	.25	1.1	.05	.09	.09	.10	.21	.13	.26	.14
4	.00	.00	.00	1.0	.00	.00	.00	.00	.00	.00	.00
5	.00	.00	.00	.00	1.2	.00	.00	.01	.00	.00	.00
6	.06	.08	.13	.10	.11	1.1	.05	.07	.05	.06	.06
7	.17	.09	.09	.07	.08	.09	1.1	.11	.13	.11	.12
8	.22	.16	.10	.08	.12	.14	.17	1.2	.17	.14	.16
9	.29	.26	.26	.18	.25	.23	.23	.27	1.3	.29	.27
10	.16	.25	.14	.08	.14	.14	.13	.19	.23	1.3	.22
11	.18	.20	.17	.10	.18	.16	.14	.22	.27	.26	1.3
total	2.6	2.3	2.0	1.7	2.2	2.0	2.3	2.4	2.3	2.5	2.3

IV. TOTAL INCOME-EXPORT MULTIPLIERS.

The following multipliers give the total amount of value-added (both labor and property income) generated in Kansas for each unit of exports, under the same set of assumptions as above.

	Export Sector										
	1	2	3	4	5	6	7	8	9	10	11
Total	1.0	1.3	1.0	.6	1.0	1.0	.8	1.3	1.2	1.5	1.3

V. SECTOR AND COMMODITY DEFINITIONS.

As in ERTN 7v1.0, the sectors numbers refer to the following sectors:

1. Agriculture.
2. Mining (mainly gas and oil).
3. Construction.
4. Automobile manufacturing.
5. Aircraft manufacturing.
6. Manufacture of durables.
7. Manufacture of non-durables.
8. Transportation and utilities.
9. Wholesale and retail.
10. Finance, insurance, and Real Estate.
11. Other Services.

VI. DISCUSSION.

The total output multipliers were generally measured as a little over 2. The total income multipliers were generally around 1. In other words, a dollar of Kansas exports leads to about 2 dollars of Kansas output, and around 1 dollar of Kansas income. (However, future refinements to the model such as those described in Section II may lower these numbers slightly.) However, there is a certain amount of variation in the total income multipliers across sectors. (The amount of reported variation would be increased if the sectors were further disaggregated. Results of this type should be reported in a future ERTN.)

These numbers have important implications for state development policy. First, these numbers help place a limit on the amount of tax subsidies which the state of Kansas can profitably offer new businesses seeking to locate here (but the technical details are complex and deserve another ERTN). Second, these numbers refer to state-wide impacts. City and county multipliers are expected to be substantially smaller, as a result of the additional leakages resulting from income, tax revenue, and demand flows between the locality and the rest of Kansas. Accordingly, these numbers place upper bounds on the local multipliers which city governments should use when estimating the impacts of new business locations on the city.

Source: Economics Research Technical Note 25.

APPENDIX B
PROJECTED VS. ACTUAL GROWTH OF THE KANSAS ECONOMY (1977-1986)

1. PURPOSE.

This appendix illustrates a minimum level of projection accuracy which can be achieved using the Kansas Long Term Model in its current state. A higher level of accuracy can presumably be achieved if some additional structure is added to the model, as described below.

2. METHODOLOGY.

The following graphs show a comparison of a projection based on the model, to an estimate of what actually occurred, in Kansas during 1977-1986. The comparison shows wages and salaries for the 11 aggregate sectors. Wages and salaries are chosen for the comparison because they are the most reliable dollar flow data available for Kansas by sectors; they constitute two-thirds of personal income. All data are in current dollars. Each comparison is shown in two forms: first, the logarithms of the two wage bills are graphed together; second, the ratios of the two wage bills are shown. The graphs were prepared by Mike Coen.

The projection is based on the kind of information which might have been available in the late 1970's. In particular, the model assumed that nominal U.S. output would grow at a constant 9% per year in all sectors, and that real growth would be 3% per year in all sectors. Otherwise, the model contained no information about the actual U.S economy after 1977. (For detailed specifications of the model, see appendix D: the formal model. The current parameter values of the model are described in Appendix M: Data Sources and Parameter Estimates. The projection program was written by Pat Oslund and Mike Coen.)

The estimates of actual Kansas wage bills during the period were constructed by Mike Eglinski. The original BEA data are highly disaggregated by SIC codes, but some SIC-coded sectors cross the boundaries of our 11 sectors. In such cases, the wages were allocated into the two aggregate sectors using proration factors assumed constant over time. Also, a correction has been made so as to include fringe benefits.

The model coefficients are based on independent 1977 data on the wage bill, which differ from 1977 BEA data by some +/- 20 per cent. This essentially irrelevant difference has been removed from the comparison by means of an adjustment to the projected wages. In particular,

$$\text{projected wages (sector } i\text{, year } t) = \text{modeled wages}(i,t)\,\frac{\text{BEA wages}(i,1977)}{\text{modeled wages}(i,1977)}$$

3. COMMENTARY.

The model appears to perform relatively poorly in the key state export sectors of aircraft, mining (mainly gas and oil), and automobiles. In particular, the drop in automobile production during the recession of 1981, the oil price shock, and the boom in small aircraft during 1978-1980 were not predicted by the model. This is be expected: these sectors are very sensitive to national and world-wide market conditions, about which the model contained no specific information. In other words, accurate predictions in the export sectors will not be possible until accurate predictions of world-wide market conditions become available. This implies that any effort to form regional projections needs to pay careful attention to national and international modeling, as well as to regional modeling.

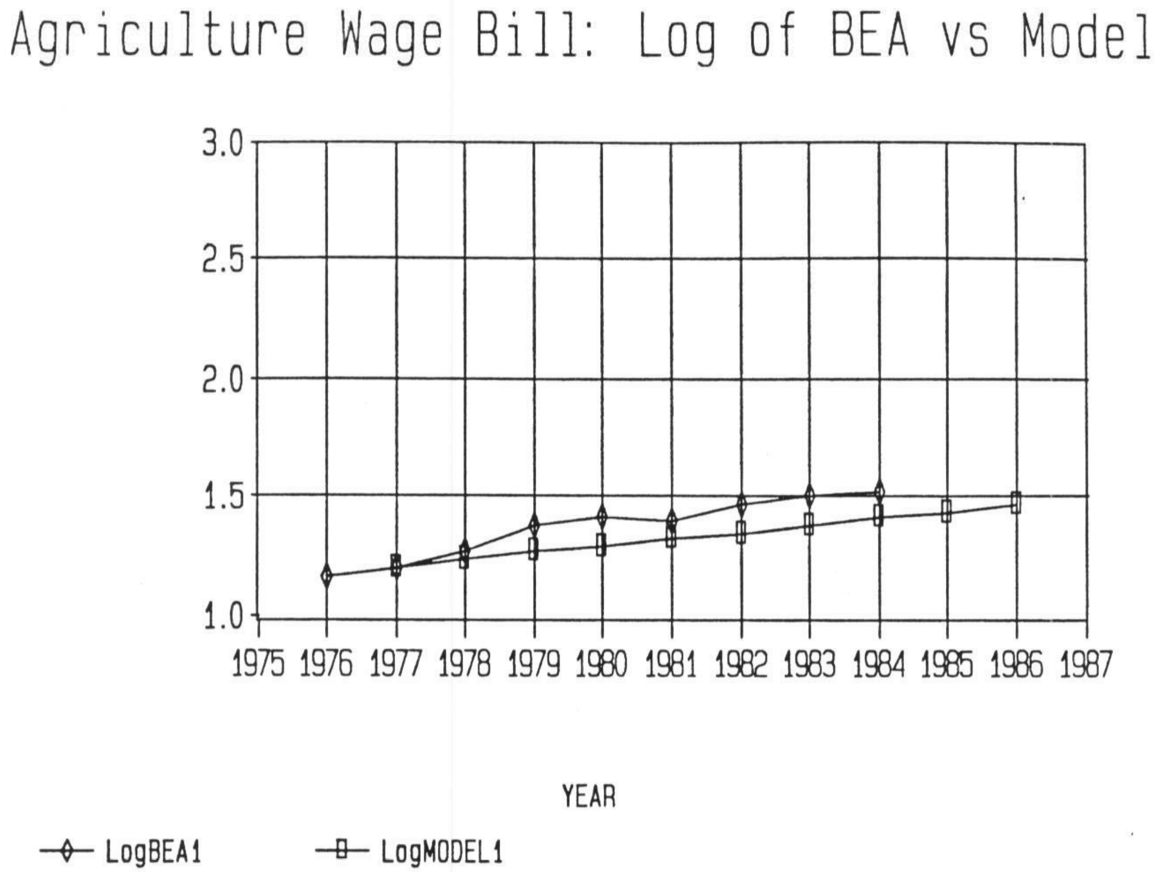
The services sector shows a steady growth in relative demands in Kansas which the model did not predict. This growth represents the Kansas version of a secular change which is occurring in the structure of the U.S. economy. The projection model used here contained no information about secular changes in the economic structure.

The other sectors experienced relatively steady nominal growth during the decade. Since the model predicted rather steady growth in all sectors, it performed well in those sectors.

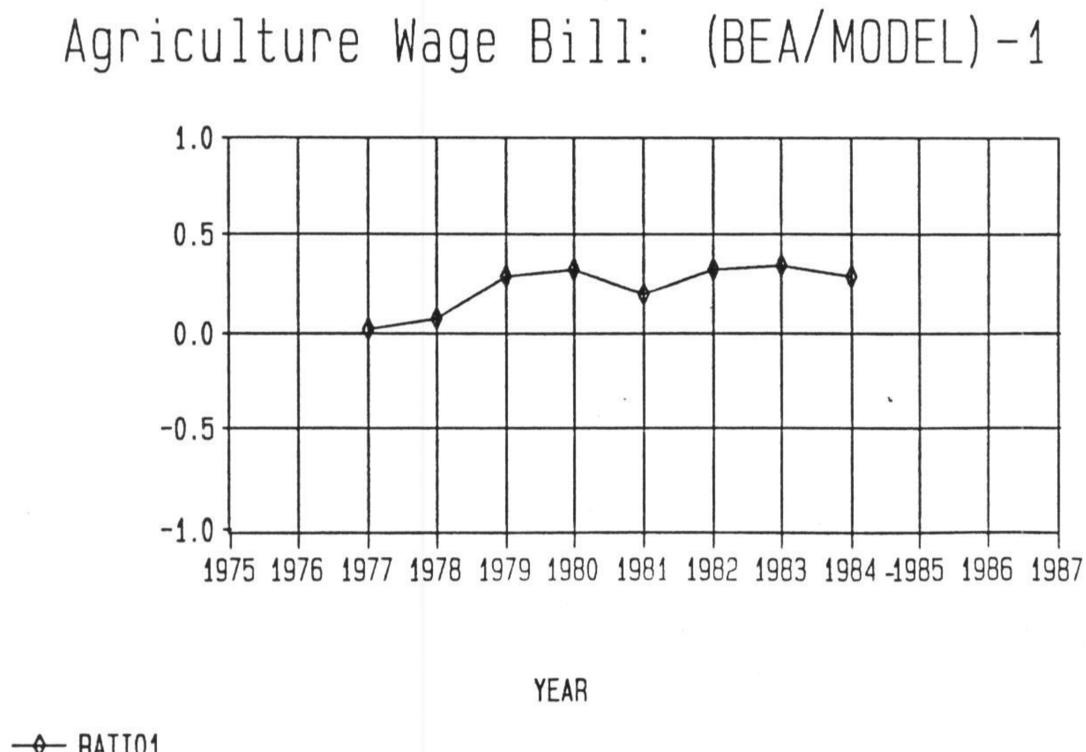
In general, these results seem quite encouraging. This model contains a comparable amount of theoretical economic structure to a typical econometric model, or a conventional dynamic input-output model, or an empirical linear programming model. However, it exhibits none of the long-run instability which these alternative types of models usually suffer from.

Moreover, it would be very satisfying to be able to track a real economy using a model with parameters based entirely on cross-section (and not time series) data. That is, if we can generate accurate regional dynamics using only statics plus a theory about the relation of the region to the nation (plus exogenous national variables), then we will have achieved a strong confirmation of the theory. The present results suggest that we may be able to approach that goal.

Source: Economic Research Technical Note 27, version 1.0.

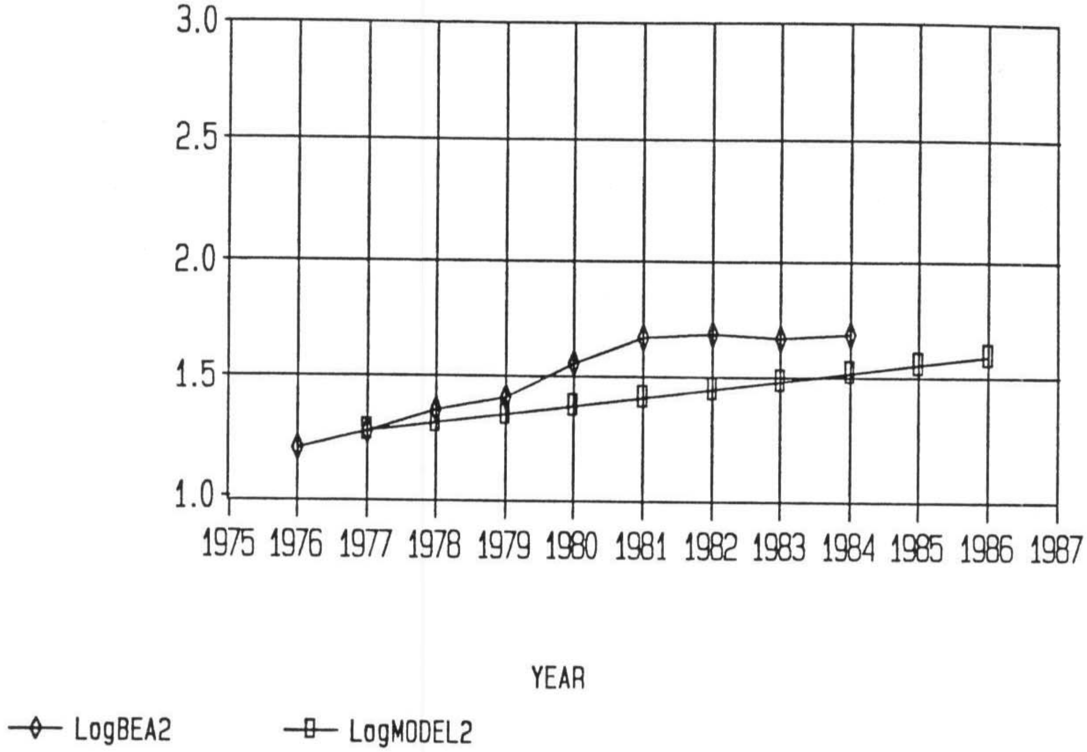


Agriculture Wage Bill: Log of BEA vs Model
3.0
2.5
2.0
1.5
1.0
1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987
YEAR
LogBEA1
LogMODEL1

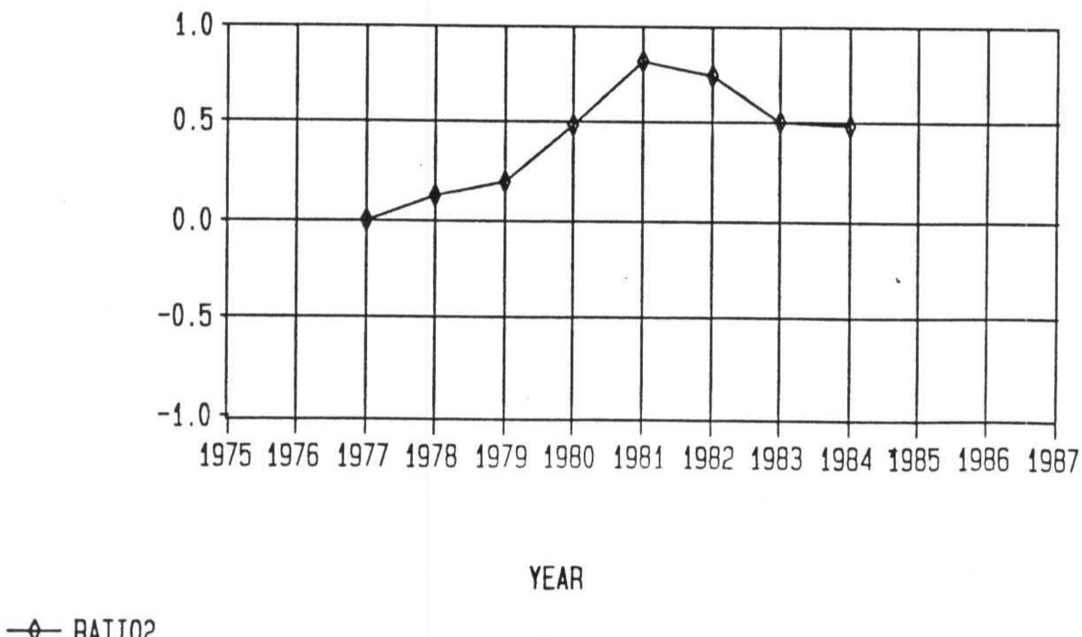


Agriculture Wage Bill: (BEA/MODEL)-1
1.0
0.5
0.0
-0.5
-1.0
1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987
YEAR
RATIO1

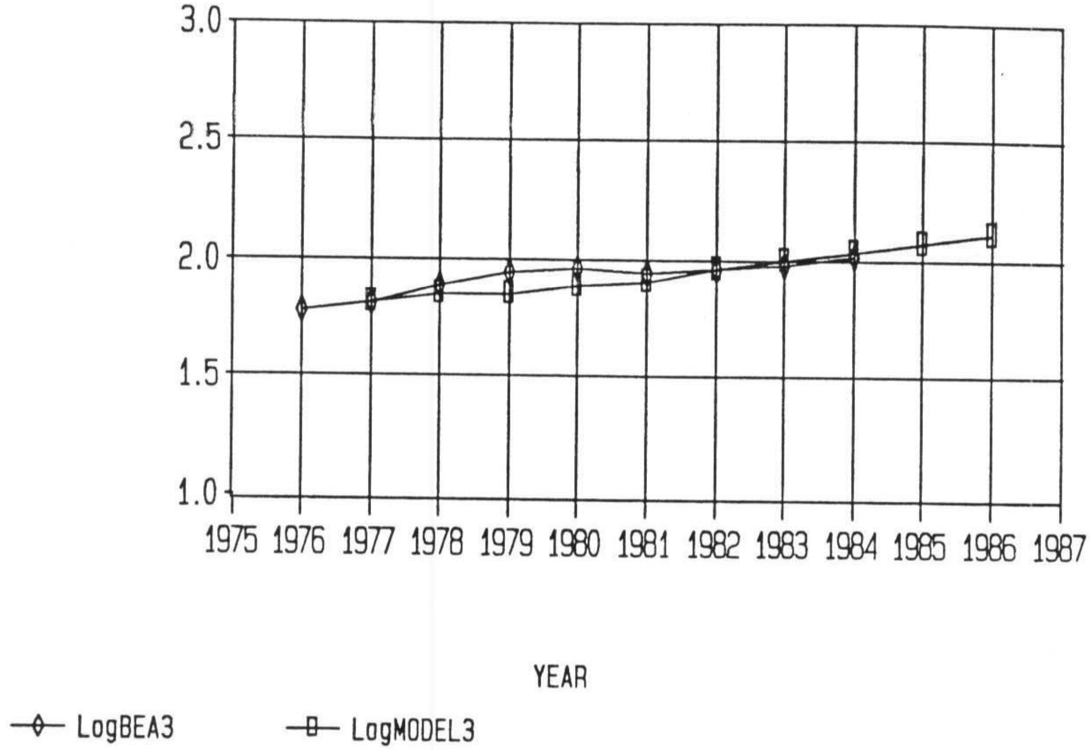
Mining Wage Bill: Log of BEA vs Model



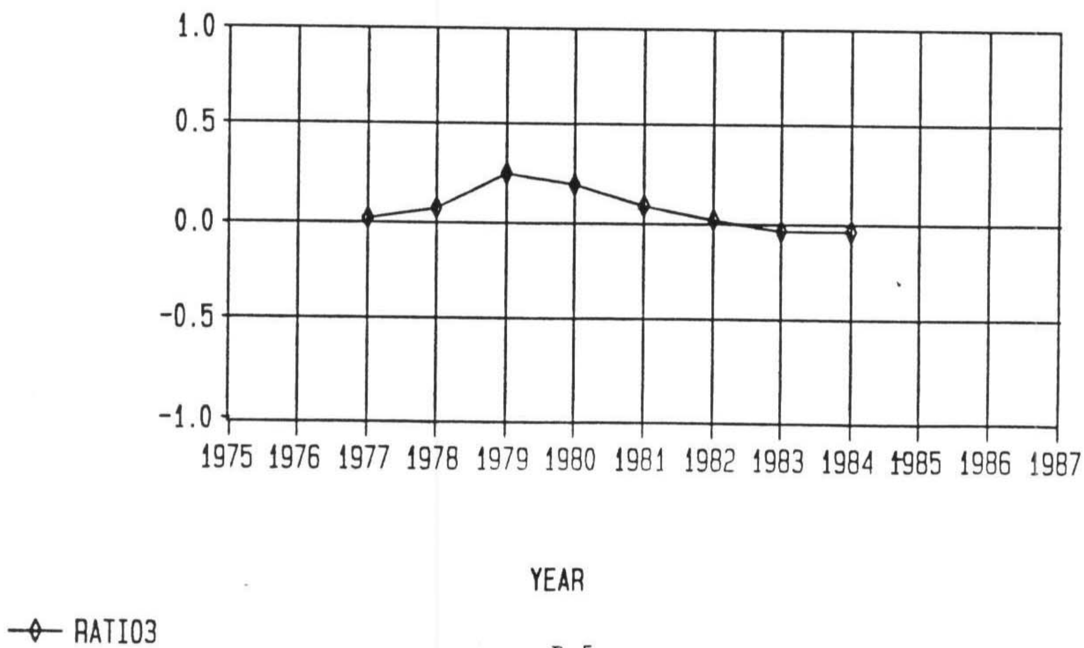
Mining Wage Bill: (BEA/MODEL)-1

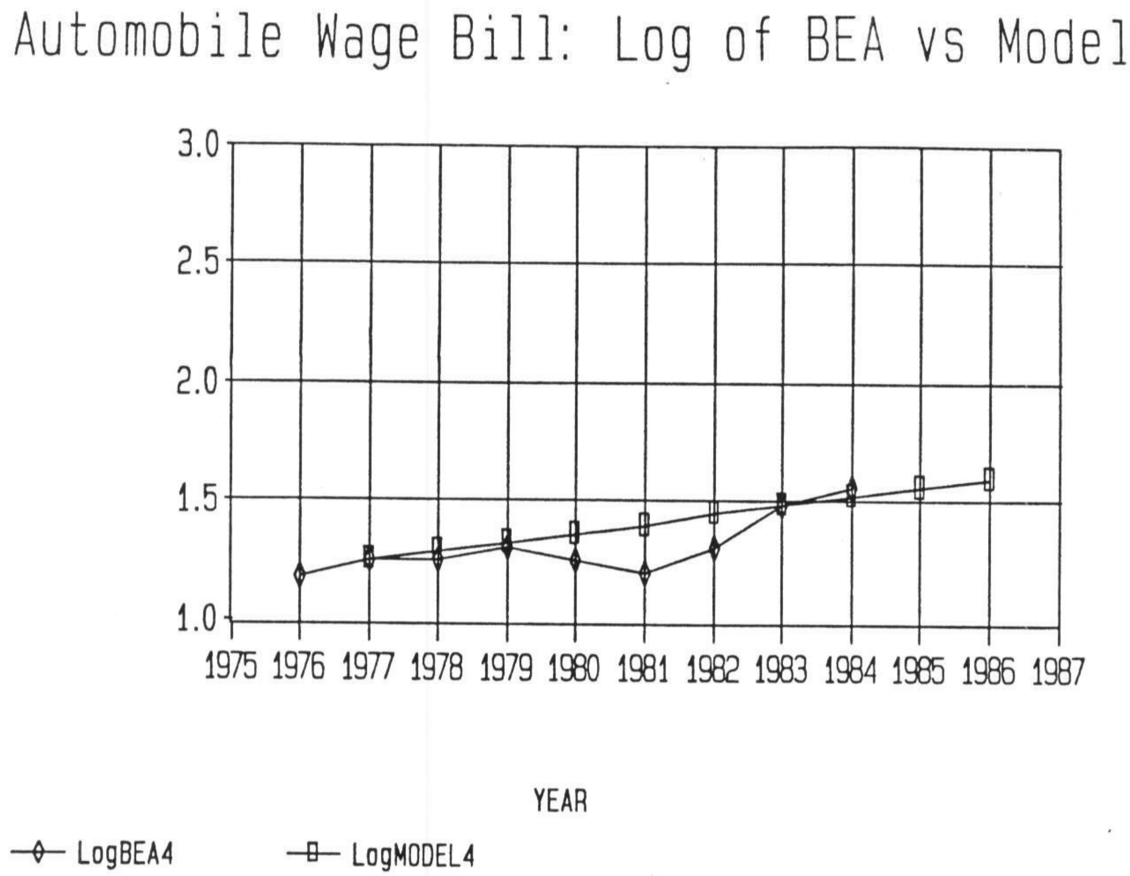


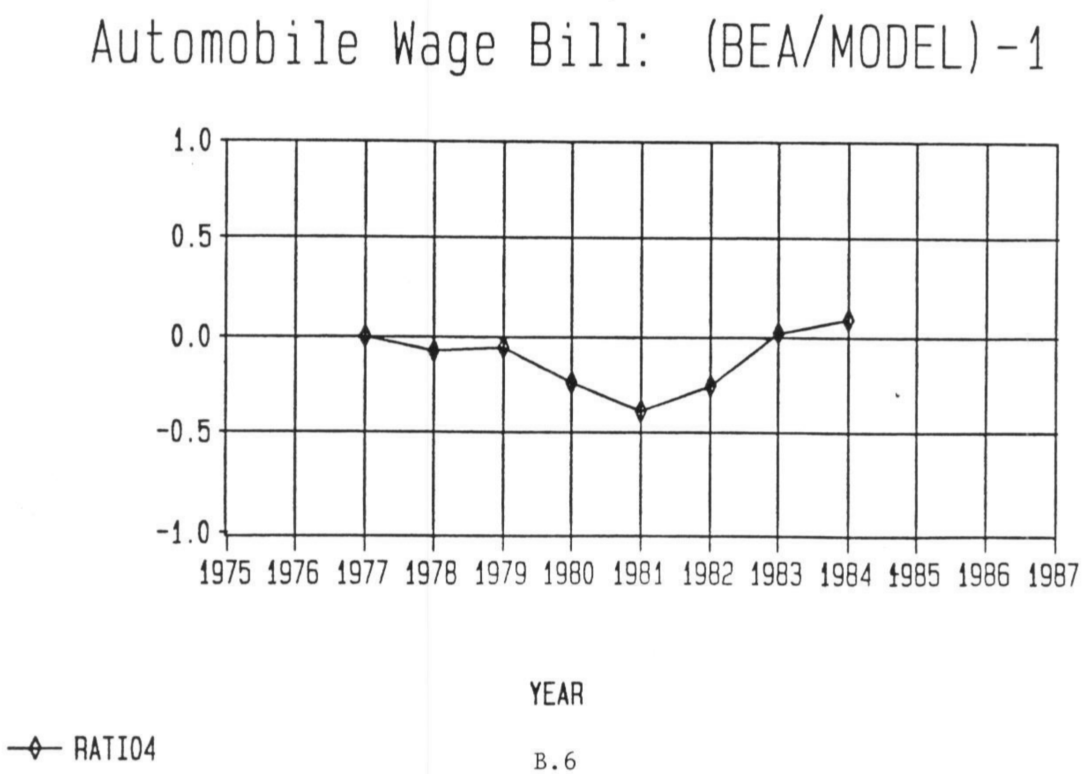
Construction Wage Bill: Log of BEA vs Model



Construction Wage Bill: (BEA/MODEL)-1




Automobile Wage Bill: Log of BEA vs Model
3.0
2.5
2.0
1.5
1.0
1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987
YEAR
LogBEA4
LogMODEL4


Automobile Wage Bill: (BEA/MODEL)-1
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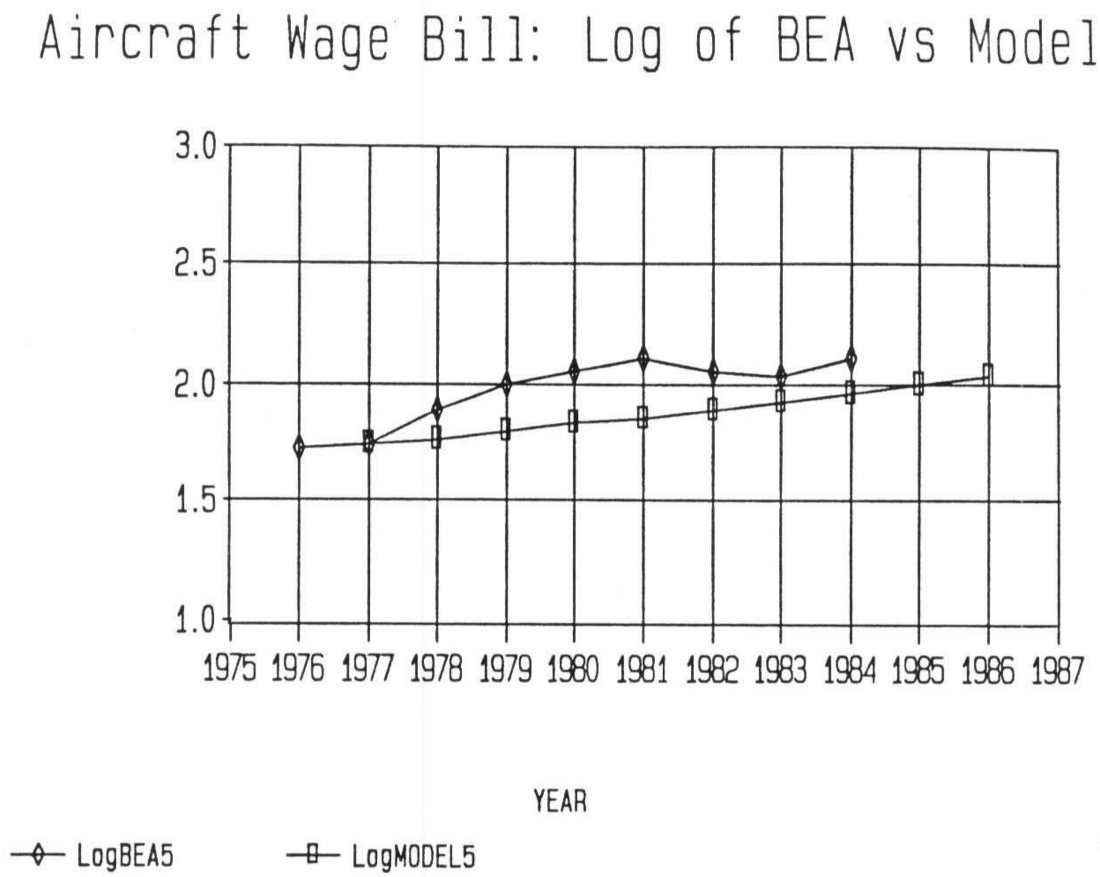
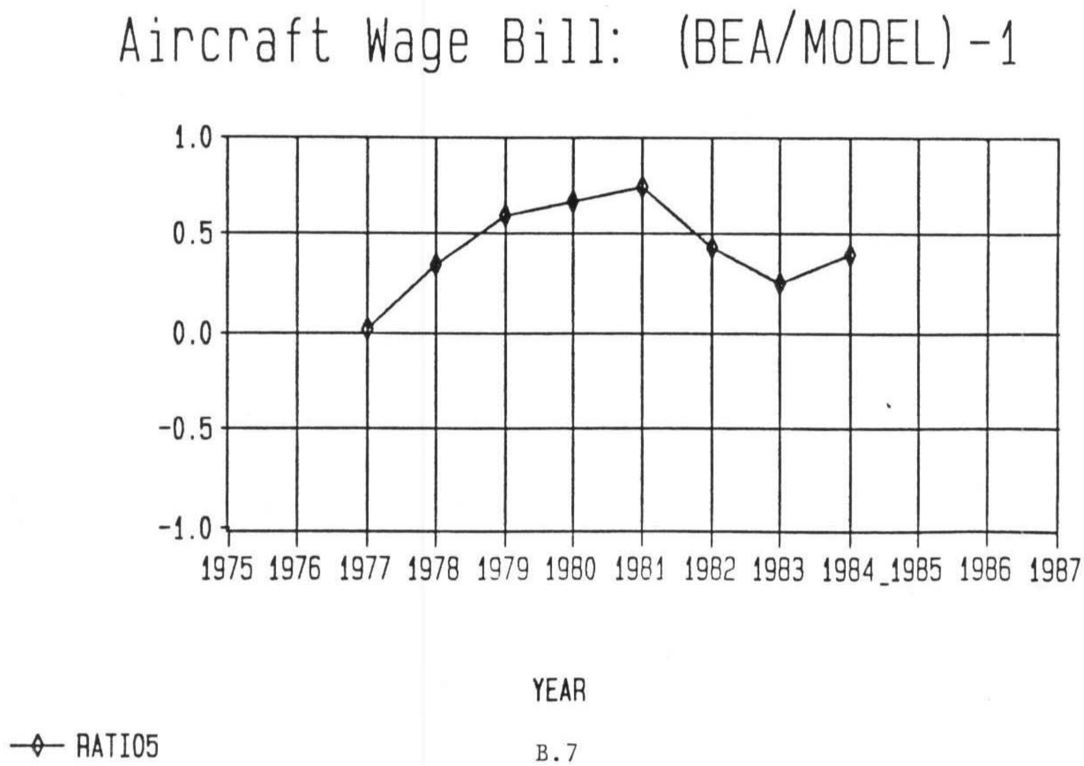
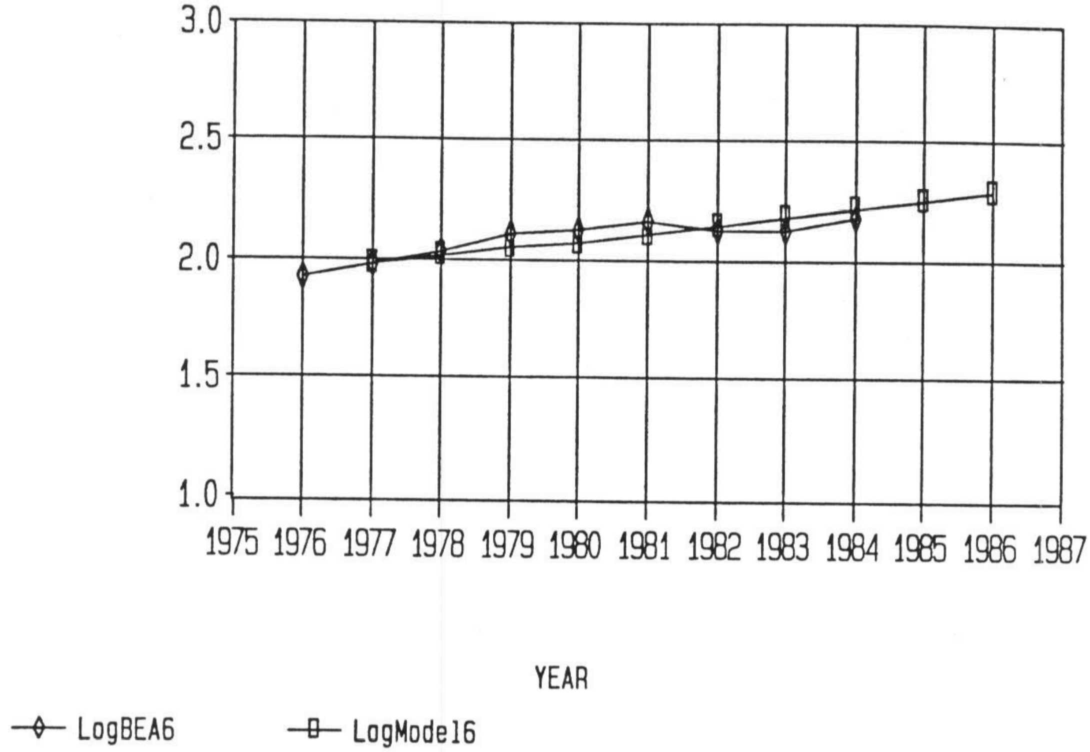
0.5
0.0
-0.5
-1.0
1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987
YEAR
RATIO4

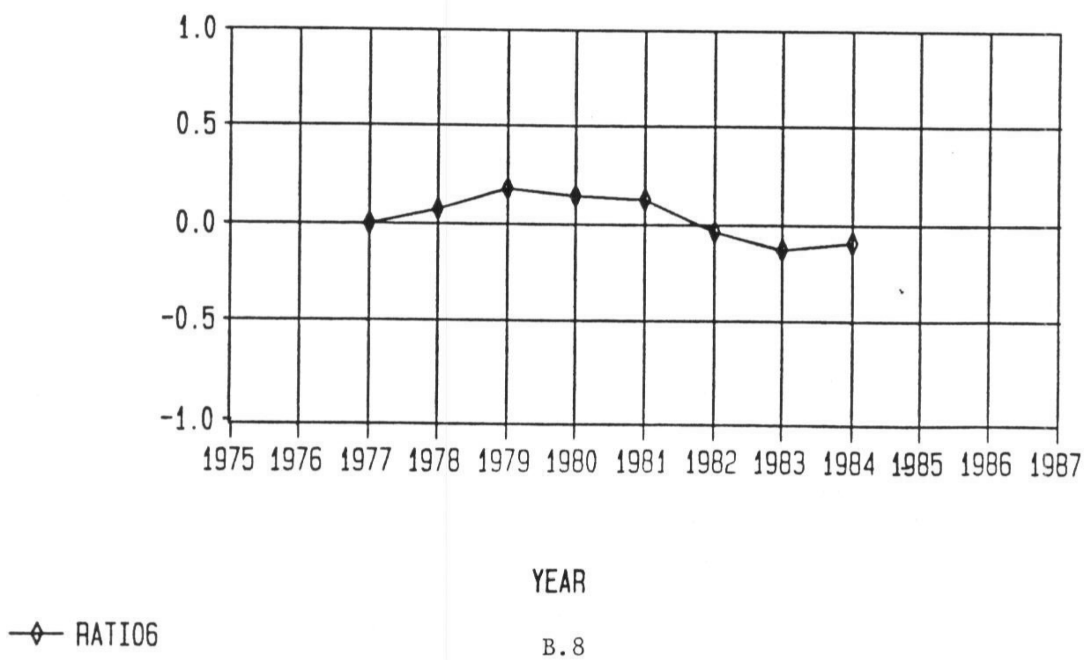

Aircraft Wage Bill: Log of BEA vs Model
3.0
2.5
2.0
1.5
1.0
1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987
YEAR
LogBEA5
LogMODEL5


Aircraft Wage Bill: (BEA/MODEL)-1
1.0
0.5
0.0
-0.5
-1.0
1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987
YEAR
RATIO5

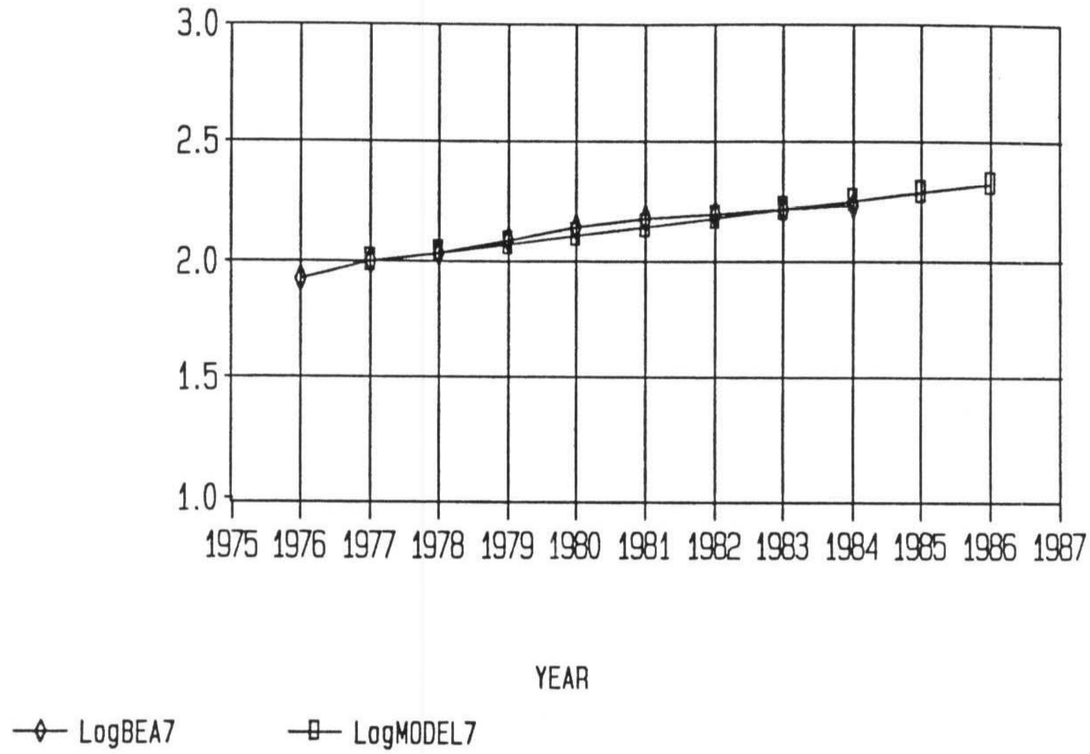
Durable Wage Bill: Log of BEA vs Model



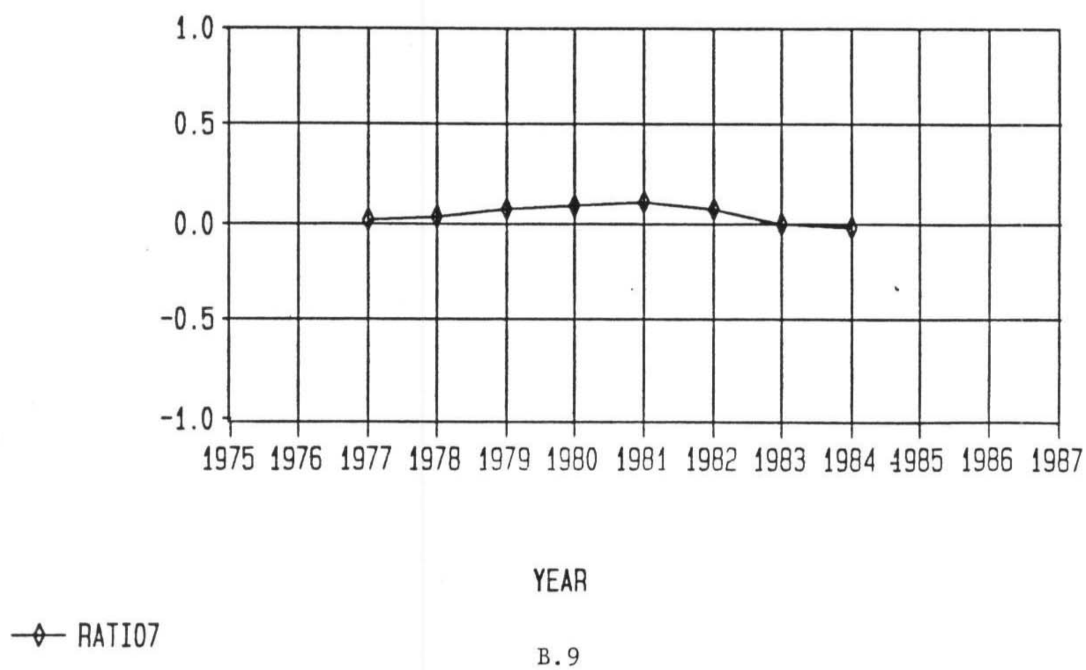
Durable Wage Bill: (BEA/MODEL)-1

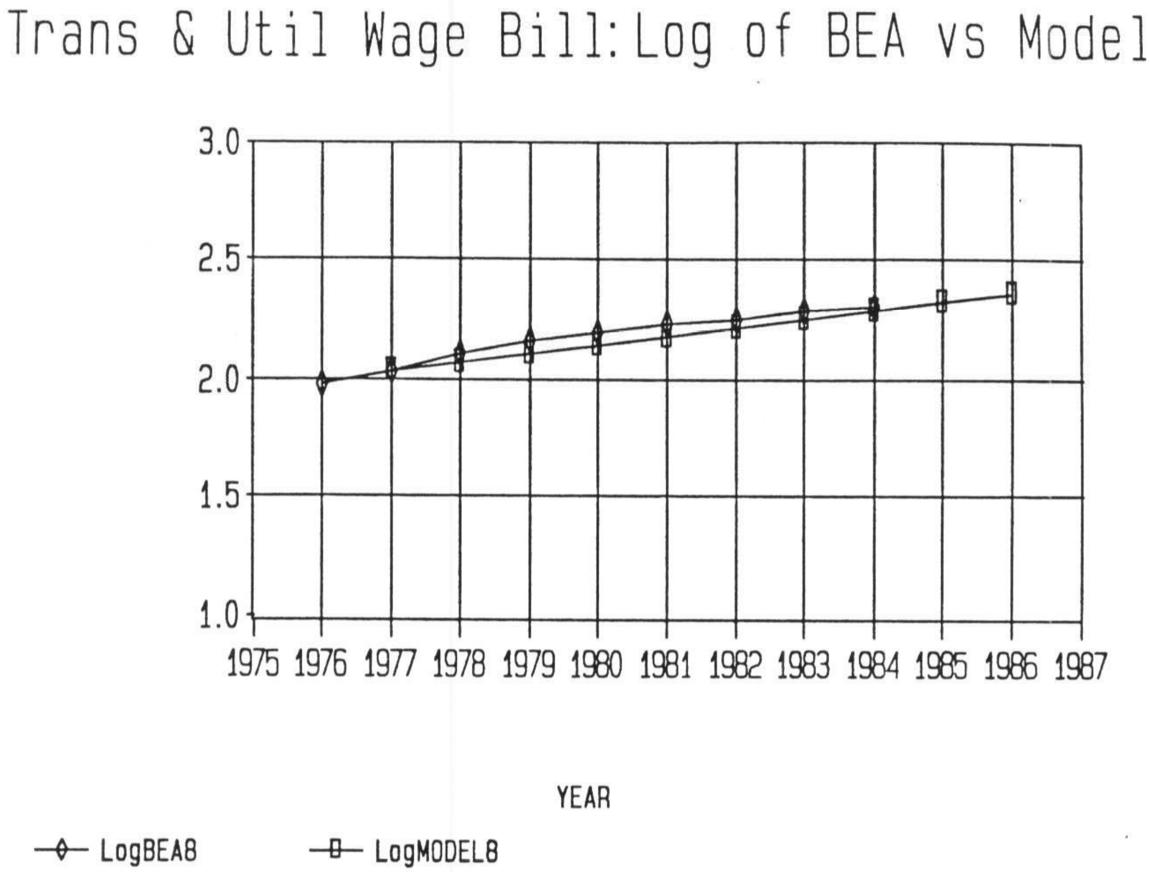


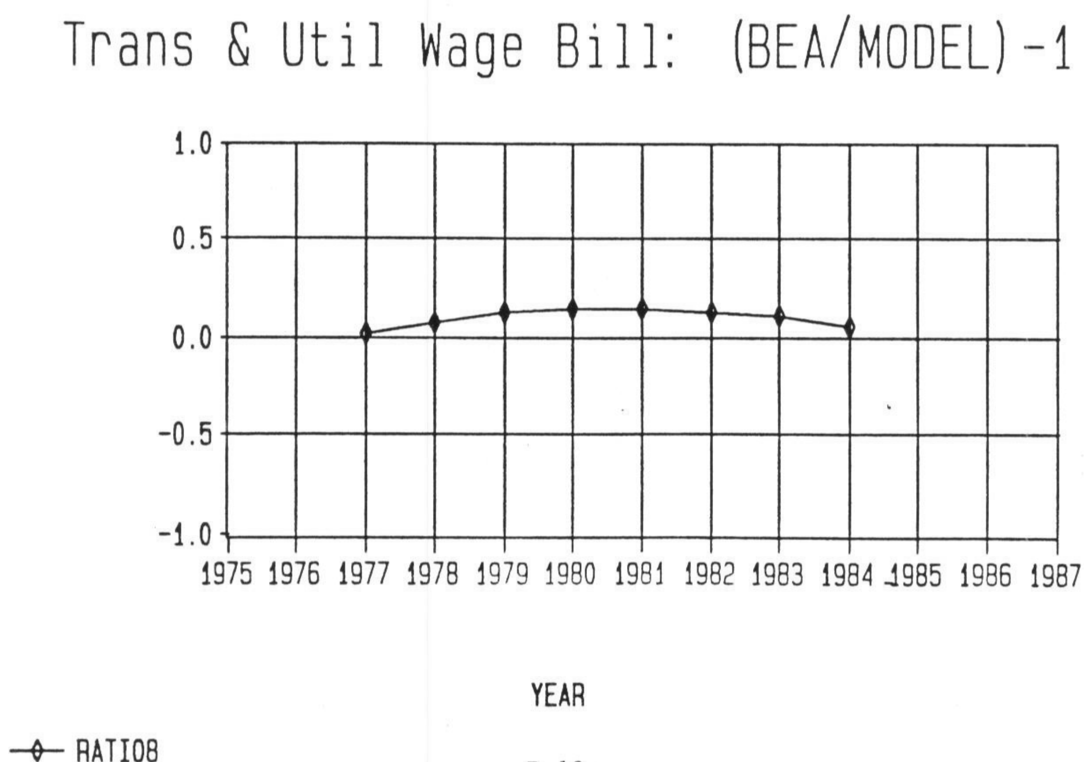
Nondurable Wage Bill: Log of BEA vs Model



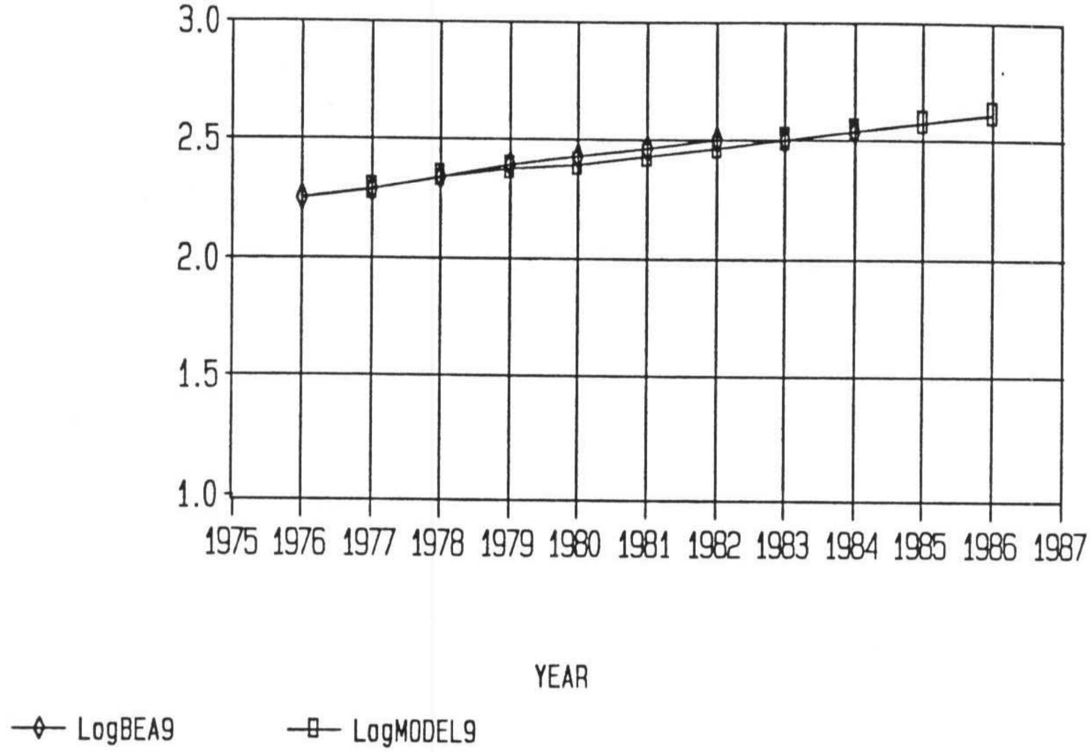
Nondurable Wage Bill: (BEA/MODEL)-1



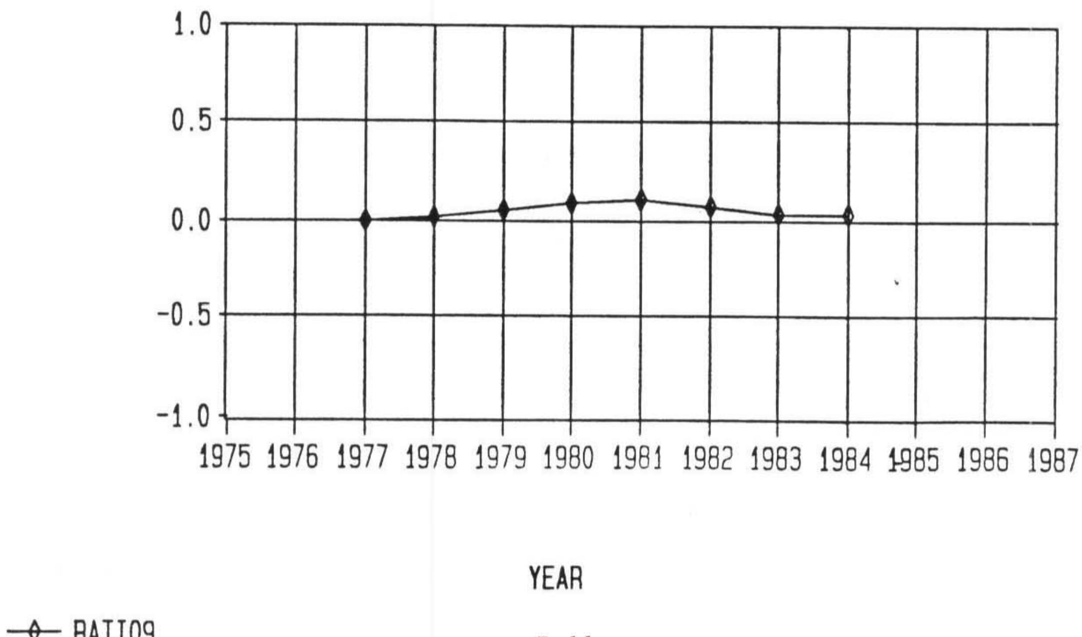

Trans & Util Wage Bill: Log of BEA vs Model
3.0
2.5
2.0
1.5
1.0
1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987
YEAR
LogBEA8
LogMODEL8


Trans & Util Wage Bill: (BEA/MODEL)-1
1.0
0.5
0.0
-0.5
-1.0
1975 1976 1977 1978 1979 1980 1981 1982 1983 1984 1985 1986 1987
YEAR
RATIO8

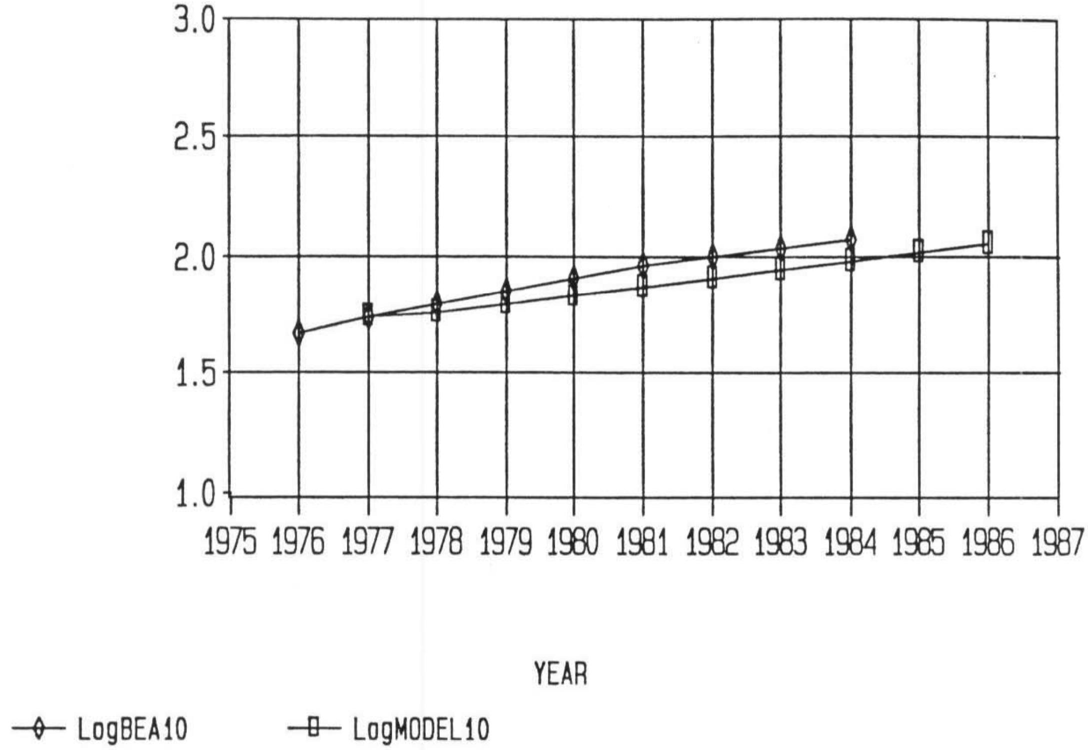
Whol&Retail Wage Bill: Log of BEA vs Model



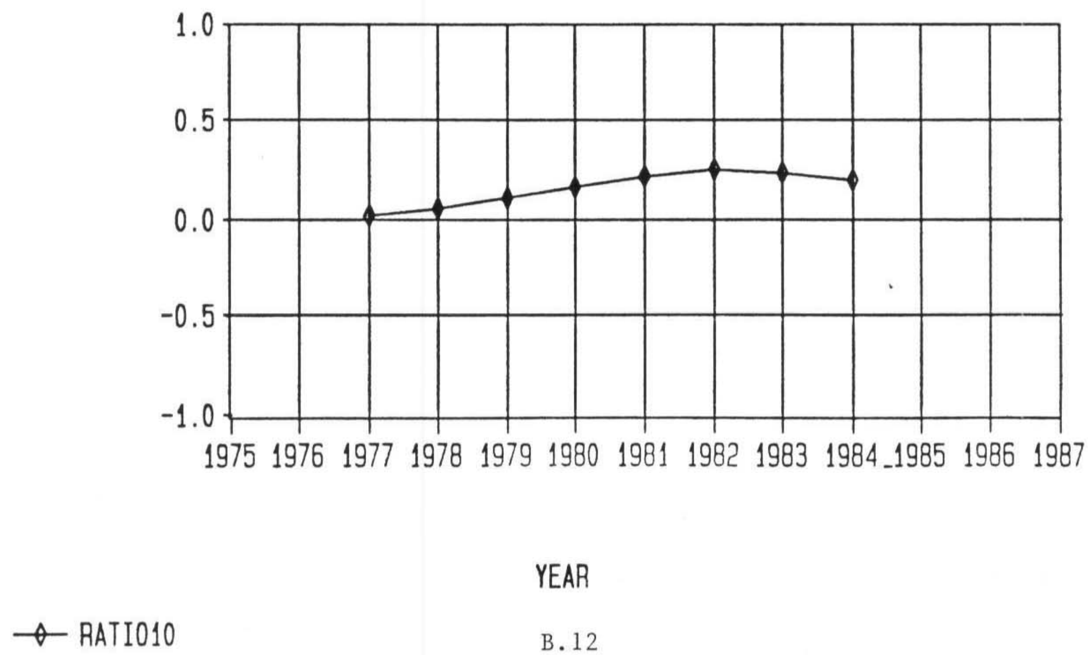
Whol & Retail Wage Bill: (BEA/MODEL)-1



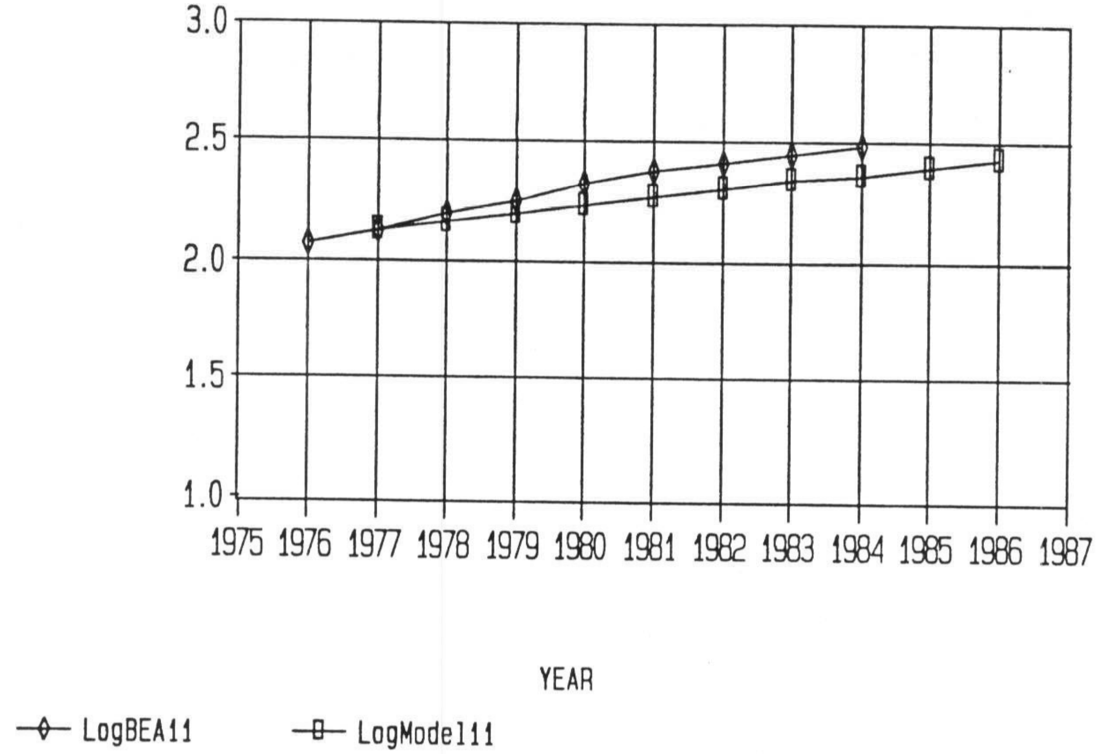
F.I.R.E. Wage Bill: Log of BEA vs Model



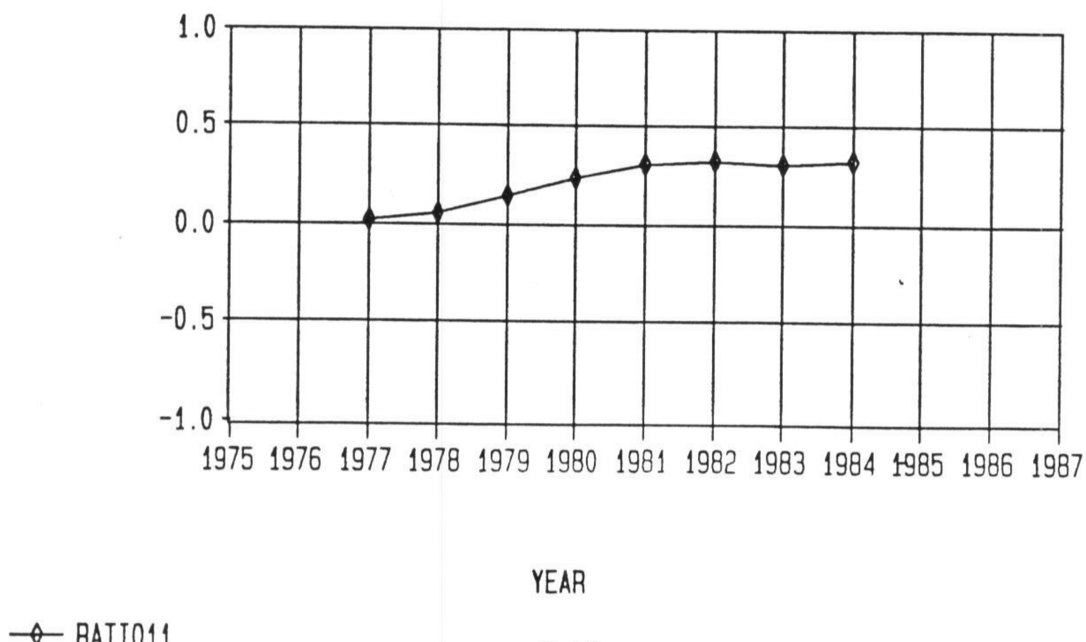
F.I.R.E. Wage Bill: (BEA/MODEL)-1



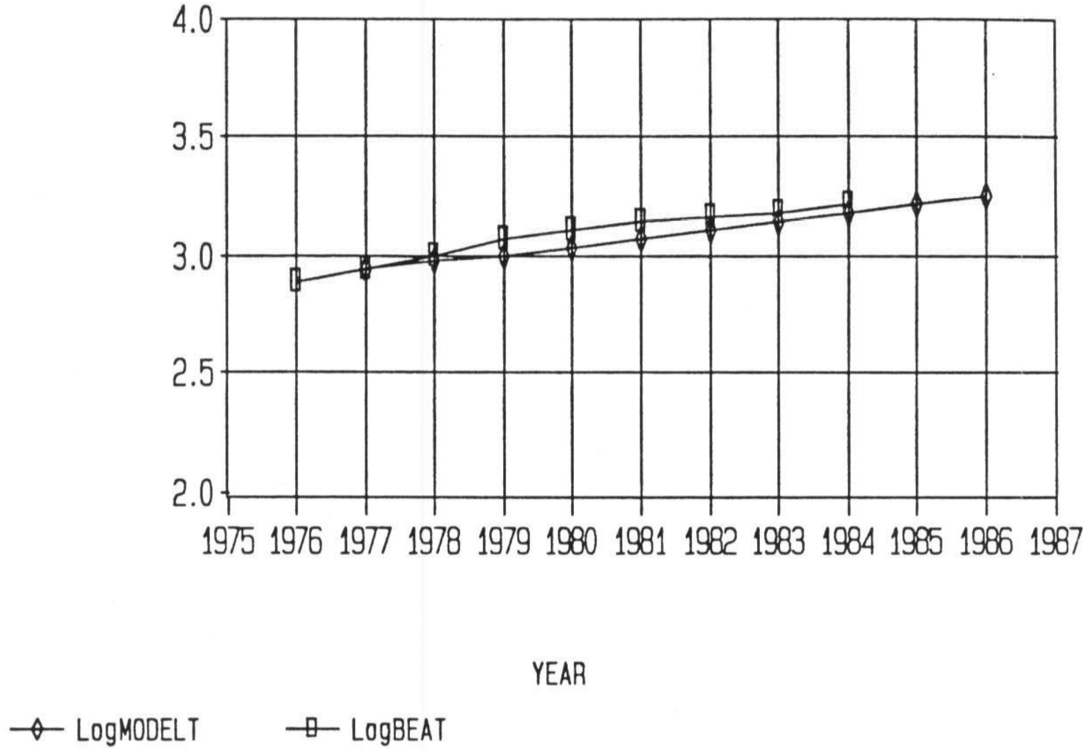
Service Wage Bill: Log of BEA vs Model



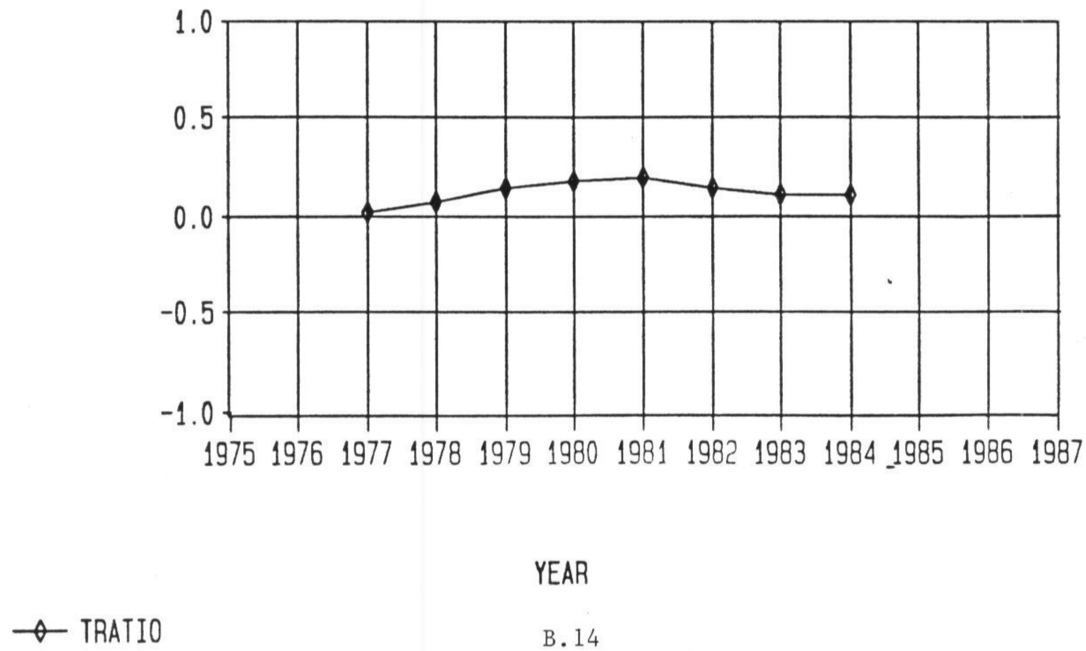
Service Wage Bill: (BEA/MODEL)-1



Total Wage Bill: Log of Model vs BEA



Total Wage Bill: (BEA/MODEL)-1



APPENDIX C

A HISTORY OF MODELS USED IN DEVELOPING THE KANSAS LONG TERM MODEL

Static Leontief, or input-output, models have been utililzed in economic analysis for quite some time. A static input-output model of Kansas was constructed by Emerson from his 1965 partial-survey input-output table. Since then, using an updating technique, Emerson has created new input-output tables which are the raw data for input-output models.[1] The innovations that were proposed for the IPPBR model were to develop a dynamic model, that is, a model that accounts for developments over time, and to endogenize various components of final demand. A new data source was acquired, which had the advantage of detailing trade flows in and out of Kansas, and a static model was developed[2]. Several policy studies utilized the static model. A brief description of the static type model and the dynamic model follows. Further, the dynamic model has inherent problems, which are incurable and which are also discussed. A standard solution to the instabilities exhibited by dynamic models is to develop a linear programming model. Linear programming models, as Leontief models, may be either static or dynamic. The results of the dynamic linear programming model are discussed here. Both of these approaches provide the background to understanding the current model which is described in Appendix D.

THE STATIC LEONTIEF MODEL:

Input-Output (I-O) tables provide the raw data on inter-industry flows, final demands, and total output. At equilibrium supply equals demand, so that the total output equals the sum of the final demands and the inter-industry flows.

Let X = a vector of output

Z = a matrix of inter-industry flows

F = a vector of final demands.

Then the statement that supply equals demand translates into the formula $X = Z + F$. Multiplying Z by the inverse of the matrix formed by diagonalizing the vector X, we obtain a matrix of I-O coefficients which we denote by A.

Then, AX = vector of intermediate demands.[3]

The model can be written as

$$AX + F = X$$

or intermediate demands + final demands = total output. Simple algebraic manipulation gives us the following:

$$X - AX = F$$

$$(I - A)X = F$$

$$X = (I - A)^{-1}F.$$

The expression $(I - A)^{-1}$ is called the Leontief inverse and gives us the total requirements needed to satisfy final demand. The column sums of $(I - A)^{-1}$ are called output multipliers, which indicate that total output needed to satisfy a unit change in final demand in the sector represented by the corresponding column.

The model is completely demand driven. Output is determined by F and F is exogenously given, consisting of consumer demand, investment demand, government demand, and exports to other states and abroad less imports.

It is well known that if (I - A) is nonsingular and if all of the elements of its inverse are positive, then there exists a positive X that solves the model for any positive F.[4]

In the static model used for Kansas, the inverse exists and the solution to this model has been worked out for policy analysis to determine the impact of changes in demand on the Kansas economy.

THE DYNAMIC MODEL:

In the static version, the inter-industry flows are viewed as serving as an input for current production, when in actuality some of these flows are not used up, as in the case of machines, buildings, etc. In fact, the production process requires a certain amount of this capital stock that is not completely used up in production. To the model presented above we need to add the concept of capital coefficients. A capital coefficient measures the value of the output of one sector that is held by another sector as capital stock. In addition to current capital, a growing economy needs different amounts of capital over time. A model that takes into account the amount of capital that will be needed in the next time period but must be purchased now is a dynamic model. It is this link between time periods which makes the model dynamic, thus differentiating it from static models, which do not take capital stock into account.

The standard dynamic Leontief model endogenizes the investment demand component of F. A matrix B, which is a capital coefficients matrix whose elements b_{ij} are coefficients which represent the amount of sector i's product held as capital stock for production of one dollar's worth of output by sector j, is used in conjunction with an accelerator assumption:

$$AX_t + B(X_{t+1} - X_t) + F_t = X_t \quad \text{or}$$

intermediate demand plus investment demand plus other final demand equals total output. The accelerator assumption presumes that the demand for capital depends on the changes in output levels. (Note that the gestation period for capital is assumed to be one year.)

Then, $X_{t+1} = B^{-1} [(I - A + B)X_t + F_t]$.

In addition to endogenizing investment, the IPPBR model sought to endogenize consumer demand by solving for disposable income. The Kansas Dynamic Model is characterized by the following equations:

$$X_t = AX_t + F_t^C + F_t^I + F_t^O$$

where F_t^C = consumer demand

c = vector of average propensities to consume

s = primary input coefficient vector (calculated from value added).

τ = average tax rate.

$F_t^C = c(s'X_t - \tau_t s'X_t)$, where the term in parentheses is disposable income.

B = capital coefficients matrix.

δ = depreciation vector diagonalized.

F_t^I = net investment demand (assuming the accelerator principle) plus replacement investment demand.

$F_t^I = B[X_{t+1} - X_t) + \delta X_t]$

F_t^O = other final demand (government and net trade)

Substituting and solving,

$$X_{t+1} = (B^{-1})\{[I - A -c(I - \tau_t)s' + B(I - \delta)]X_t - F_t^O\}$$

The above equation provides a forecast of next period's output for the economy taking into account the following:
1) technology via the technical coefficients matrix, A,
2) capital requirements of the economy through the use of the B matrix and the depreciation vector,
3) consumer demand which depends of the tax rate and the per centage of income that is spent,
4) and other demands.

A major feature of this model is endogenizing investment demand and consumer demand. The model itself is "self driving" in that output from one year becomes an input for the next year.

RESULTS:

Whether a dynamic Leontief model will work or not depends on the nature of B, the capital coefficients matrix. The first problem is that b may be singular (non invertible) which is generally the case and which means that solving for future X_t's is impossible. However, with some manipulations, the B matrix was forced to be invertible.[5] The model was then faced with a second problem, that of causal indeterminancy.

Takayama notes that "unless the initial output vector and stock vector are on a certain ray from the origin, it may so happen that the output and the stock of at least one good may become negative for sufficiently large t. Thus, waking up on a bright Monday morning, one may find that the dynamic Leontief economy which had started with a positive initial stock of commodities, has realized a negative stock of some commodity."[6]

Taylor states that whether or not output levels will converge to a balanced growth from arbitrary initial conditions depends on the characteristic value of B. If any correspond to growth rates exceeding that of the balanced growth characteristic value, the system will diverge and finally generate negative output levels in some sectors. In practice, B matrices tend to have predominantly unbalanced growth characteristic values."[7]

The modified B matrix used in the Kansas Dynamic I-O Model had complex eigen values with the real parts being both positive and negative. With these values a cyclical, exponentially explosive growth model was observed, which obviously does not describe the Kansas economy.

Given the instability of the B matrix, a dynamic linear programming model was developed. The advantage of such a model is that it does not rely on the stability of the B matrix since in its formulation the B matrix is not inverted.

THE BASIC DYNAMIC LINEAR PROGRAMMING MODEL:

Linear programming (LP) models are subdivided into static and dynamic models. The dynamic model was developed during the summer of 1986. A static model was not formulated since the static Leontief worked.

Linear programming models require an objective function which is to be maximized. The objective function that was chosen was the discounted sum of value added across industrial sectors.
Objective function:

$$\sum_i \sum_t s_i X_t^i / (1 + r)^t$$

s_i = the value added coefficient for industry i
r = the social discount rate.

It should be noted that several variations of the objective function were tried but that the results were not sensitive to these changes.
Supply and demand:

$$C_t + BI_t\ F_t + AX_t = X_t.$$

$$C_t = cY_t$$

$$I_t = B(Z_t^* - (1-\delta)Z_{t-1}^*)$$

where F_t = other final demand (government and net trade)
Y_t = personal income
c = average propensities to consume

Z^* = real capacity
δ = depreciation coefficients.

Note: This formulation was in industry by industry space. Capacity was assumed measured in dollar amounts. However, prices were assumed to be constant, so that real capacity equals Z_t. Solving the basic system:

$$F_t = (I - A)X_t - B(Z_t - (1 - \delta)Z_{t-1}) - cY_t.$$

The backward lag in investment gives the model its dynamic intertemporal nature.

The objective function was maximized with respect to the following constraints:

Capacity constraints: $X_t \leq Z_t^*$
This gives 2N variables and 2N equations and constraints.
Additional contraints:

Financial constraints: $1' (Z_t - (1 - \delta)Z_{t-1} \leq I_t$
Labor constraints: $\alpha' X_t \leq L_t$
where α = primary input coefficient vector.
Non-competitive import constraint: $m' X_t \leq m_{ot}$
where $1' = (1,1,\ldots,1)$ (i.e., a vector of ones)
m = import coefficients (assumed constant)
m_{ot} = total imports at time t.

Several modifications were attempted, including changing from an industry by industry space to a commodity by industry space. However, our primary focus was on investment. This was thought to be the key relationship in the model.

The model predicted declining output and negative investment in most sectors. Although dis-investment has actually happened in the Kansas economy in some sectors, we felt that it should not permeate the whole economy. Also, investment is not smoothly reversible in the real world, as the model assumes.

THE MODEL WITH IRREVERSIBLE INVESTMENT:

The solution to dealing with the disinvestment problem was to add an additional investment constraint:

either a) $Z_t \geq Z_{t-1}$,
or b) $I_t \geq 0$, which implies that $Z_t \geq (1 - \delta)Z_{t-1}$.

We chose (b) on the basis of realism.

The additional constraint gave us reasonable appearing results for an aggregated 4 x 4 model for ten years. However, when we moved to the 11 x 11 model, infeasibilities developed, preventing the model from running ten years. The infeasibilities were conflicts between constraints. In particular, the investment constraint and the supply and demand constraints conflicted.

THE MODEL WITH SUPPLY/DEMAND INDEPENDENCE:

In a final attempt, we changed the equality constraint of the supply and demand relationship to an inequality constraint:

either 1) supply greater than demand, making use of an assumpton of free disposal;
or 2) demand greater than supply, which amounts to endogenizing imports.

Relaxing the equality constraint produced the following results:

In case (1), infeasibilities still appeared in the 11 x 11 model, after eight years.

In case (2), the following were observed:

a) No infeasibilities were observed. Output is sectors diverged and went to zero in a couple of them.

b) The system binds on non-competitve imports and specializes in sectors which maximize the ratio of value added to import requirements.

c) There is a tendency to dis-invest in most sectors and move the proceeds into a few most productive sectors.

d) These problems appear very hard to remove from any linear programming framework.

CONCLUSION:

By endogenizing imports and constraining disinvestment, we succeeded in finding a feasible solution for the Kansas economy. However, the resulting model was lacking in empirical realism. In particular, Kansas was modeled as over specializing in a small number of sectors.

THE CURRENT MODEL:

To circumvent some of the problems discussed above, non-linearities were introduced into the model. Further, we have de-centralized the optimizing behavior of agents in the economy by dispensing with the arbitrary objective function. The formal model is described in appendix C. Appendix G describes the nature of assuming a Cobb-Douglas production technology in the model. Appendix A and B present the empirical results of the current model.

FOOTNOTES:

1. Emerson, Jarvin, The Input - Output Structure of the Kansas Economy 1973, KDED 1975.
2. Multiregional I/O Accounts 1977, Faucett Associates, 1981; see Appendix L for detailed description.
3. This formulation assumes that each industry produces a single unique commodity. Since the data was taken from Faucett's MRIO, which assumed otherwise, a certain amount of effort was required so as to infer the A matrix.
4. See Takayama, Mathematical Economics, pp. 359-366.
5. A method for accomplishing this is given in Kendrick, "On the Leontief Dynamic Inverse," QJE, 86(4), pp. 693-696. Another technique was to use the by-product assumption to eliminate the zeros in the B matrix.
6. Takayama, Mathematical Economics, p. 501.
7. Taylor, "Theoretical Foundations and Technical Implication," in Economy Wide Models and Development Planning, Blitizer, Clark, and Taylor, pp. 33-109.
8. Source: ERTN #13, #14.

APPENDIX D
SPECIFICATION OF A SIMPLE DYNAMIC INPUT OUTPUT MODEL

This project has developed a model of the Kansas economy which is intended to be used for both long run forecasts and for policy evaluation. The structure of the model can be summarized very simply. Written in algebraic form, the condensed version of the model becomes:

(1) $X_t = (\text{multiplier}) (FD_t)$

where

X_t = dollar value of Kansas output at time t.
FD_t = final demands for Kansas output time t.
multiplier = matrix of constants.

Equation (1) provides the foundation of the model, the relationship between supply and demand. Our model shares this basic structure with many regional models. However, our treatment of many terms in both the multiplier expression and in the final demand base are unique to the Kansas long term model and appear to be improvements on previously existing work. The discussion which follows looks behind the simplified model in an attempt to capture some of the complex interrelationships in a regional economy.

I. Preliminary View of Supply and Demand.

When a regional economy is in equilibrium, regional production of goods and services must exactly equal demand for those goods and services. Regional economies are almost by definition open economies. Many of the goods produced in a region will be sold to firms and consumers outside the region. Similarly, consumers and firms within the region are free to import the goods and services they desire rather than to buy them from local sources. Equation (2) shows that the supply and demand balance for the regional economy must account for imports and exports:

(2) X_t = intermediate demand + consumer demand + government demand + investment demand + inventory demand + exports - imports.

An examination of equation (2) can be grouped around four major topics, production, investment, trade, and resource use. These topics will be discussed individually, but this should not be taken to mean that each of the four parts is determined independently. In fact, a major focus of the following exposition of the model is the interaction between these four activities.

II. Production.

Production of goods and services requires labor, capital, and intermediate inputs. The production technology for any commodity is described by Cobb-Douglas assumptions with constant returns to scale. The dollar value of demand for the use of any type of input is a constant proportion of the dollar value of output. In particular, the Cobb-Douglas coefficients for intermediate inputs can be arranged into a matrix (the A matrix) such that:

(3) $A X_t$ = dollar value of intermediate demand.

Additionally, there exist constant coefficients w such that:

(4) $\hat{w} X_t$ = wage bill by commodity.
(^ indicates a diagonalized vector.)

Further discussion of the implications of Cobb-Douglas assumptions appears in appendices G and H.

III. Investment and Capacity.

Capacity plays an important role in determining the dynamic path of the long term model. Capacity can be defined formally as the smallest current output level for which an expected increase in real demand will stimulate net investment. More intuitively, firms which are already operating at capacity respond to demand increases by attempting to increase their stock of capital. Firms operating below capacity respond to improvements in demand by expanding utilization of their existing capital rather than by purchasing new capital. Firms operating above capacity are simultaneously attempting to expand their capital. Clearly, investment links capacity with future demand.

The relationship between investment, capacity, and expected demand is made explicit in equation (5):

(5) $I_t = \max \{ 0, \hat{k} [X_{t+1}^* - Z_t(1-\hat{d})(1+\hat{i}^*)] [1-\hat{i}^*]^{-1} \}$

where

$\hat{k}$ = diagonalized matrix of capital output ratios
X_{t+1}^* = expected next period demand for output
$\hat{d}$ = diagonalized matrix of depreciation coefficients (specific to each type of production)
$\hat{i}^*$ = expected inflation rates by commodity
I_t = gross investment by sector.
Z_t = capacity at time t in dollar value

Firms are assumed to make investment decisions at the beginning of time period (t). The decisions affect capacity at time (t+1). To make the proper decision, a firm must estimate next period's demand. The firm realizes that its current capital stock will depreciate at rate d. At the same time, it realizes that the value of output which its initial capital can produce will be inflated at rate i^*. In the absense of any investment, capacity in year (t+1) will be

(6) $Z_t(1-\hat{d})(1+\hat{i}^*)$ = estimated capacity in (t+1).

If estimated capacity is greater than estimated demand, no investment takes place. If estimated demand is greater than estimated capacity, then investment is undertaken to eliminate the gap. Investment as measured in output terms is simply estimated demand minus estimated capacity. To convert this to dollars of new capital, this difference is multiplied by the capital output ratio. Finally, it is realized that the value of new capacity, if contracted during time t, will be subject to inflation between t and t+1.

IV. Estimation of Future Demand.

At the time the investment decision is made, a firm knows neither its current or its future sales. The firm is assumed to use last periods' sales as a base on which to forecast future demand. We assume that the firm forms estimates of growth rates for two periods, perhaps based on estimates of industry growth done at the national level. Estimated demand is given by

(7) $X_{t-1}\ (1 + g_t^*)(1 + g_{t+1}^*)$ = estimated demand.

V. Nonlinearities in the Investment Equation.

Firms in any industry face a constraint on how fast they can disinvest; disinvestment can be no faster than depreciation. This is an important stabilizing force in the model. Models without this feature often exhibit unrealistic behavior. Suppose that in the face of an expected decline in demand, production in a particular sector is anticipated to be unprofitable. In the absence of constraints on disinvestment, firms would immediately divert their capital to other sectors or to consumption. Output would fall immediately to zero. Our assumption on disinvestment implies that it is difficult to convert a particular stock of capital to other uses.

VI. Investment Demand by Commodity.

Investment plays a dual role in this model. At the same time as investment alters the productive capacity of the economy , it also contributes to final demand. Let B be an NxN matrix of coefficients such that b_{ij} is the amount of commodity i purchased as investment goods for each dollar of investment which takes place in sector j. The B matrix translates investment by production sector into investment demand by commodity:

(8) $Jt = B\ I_t$.

The solution to many dynamic models requires that the **b** matrix be invertible. This has been problematic since data indicates that the **b** matrix is singular. In contrast, the current model never requires the **b** matrix to be inverted.

VII. Trade.

Exports and imports are critical features of the model. Exports contribute to final demand. Increases in exports and in federal government demand (which is treated analogously to exports and for our purposes can be lumped with private exports) are major sources of growth of the regional economy. Imports enter the model primarily as coefficients in the multiplier expression. Our preliminary work indicates that they are the second most important term in the multiplier after the **a** matrix coefficients.

Export demand is treated exogenously in the current model. Growth in exports is assumed to originate outside the regional economy. An important topic for further investigation is whether a regional economy can encourage the location of export oriented firms, and hence increase the growth rate of its export base.

Imports can be classified as competitive and noncompetitive. If the regional economy produces a good similar to an imported item, the imported item is a competitive import. On the other hand, if no similar good is produced in the region (for example, in Kansas), then an imported good is noncompetitive.

The desired level of a competitive import is assumed to be a constant proportion of the regional use of general category of good (the import plus its local counterpart). This assumption follows from the Cobb-Douglas framework for consumption and production which underlies the model. In matrix notation,

(9) $M = \hat{\mu}\ (X_t - E_t + M_t) = \hat{m}\ (X_t - E_t)$

where

M = imports
E = exports
$(X_t - E_t + M_t)$ = regional use
$m_i = \mu_i/(1 - \mu_i)$ = import coefficient.

Noncompetive imports enter the model in two places. First, costs for noncompetitive imports must be added to other input costs in order to calculate value added in regional firms. Second, spending on noncompetitive imports must be recognised in calculating marginal propensities to consume.

It should be noted that desired and actual imports may diverge in the model. Suppose that demand for Kansas output exceeds capacity in some sector. We assume that firms produce the capacity output level and that any gap between quantity supplied and demanded is filled by competitive imports. In this case the total amount of competitive imports consists of a desired and an unexpected component.

VIII. Resources and Income.

As a consequence of constant returns to scale, the revenue generated in an industry must equal the expenditures on inputs in the industry. Expenditures fall naturally into two categories, intermediate demand for products and value added. Intermediate demand appears when one firm consumes another firm's output as an input to current production. The value added in an industry is the expenditure to attract the resources (land, labor, capital) necessary for production. In our model, value added is either property income or wages and salaries. Wages and salaries pay for human resources while returns on property take the form of interest, accounting profits, or rent. Property income is a payment for the use of land and capital. Under our assumptions, value added in a particular industry remains a constant proportion of domestic production.

Personal income in the model is identical to value added; we have not accounted for income transfers into and out of Kansas. To find personal income it is only necessary to know the value added coefficients (ω) and value of output. Since firms cannot be forced to produce in excess of capacity, output is given by:

$$(10) \quad \min \{X_t , Z_t\}.$$

Consequently, personal income becomes

$$(11) \quad Y_t = \omega' [\min\{X_t , Z_t\}].$$

The income equation adds another nonlinearity to the model.

IX. Consumption.

Consumers maximize a Cobb-Douglas utility function each time period. It is assumed that consumers receive income earned in time (t-1) at the beginning of time t. State income taxes are subtracted from personal income to find the consumers' disposable income. Currently, we have not modelled federal taxes; we simply assume that federal taxes are equal to transfer payments, and hence have no effect on disposable income. Personal consumption can be stated as

$$(12) \quad C_t = c\ (Y_{t-1} - TAX)$$

where c is a column vector of marginal propensities to consume.

According to Kansas law, the state budget must be balanced. If state taxes are a constant proportion of income, we can conclude that

(13) $(C_t + St.Gov._t) = \gamma Y_{t-1}$

where γ is a vector of generalized consumption coefficients.

XI. Solving for Value of Output.

At this point it is useful to return to equation (2) and to incorporate the behavioral assumptions which have been made about the components of final demand. Equation (2) can be expanded to

(14) $X_t = A\ Xt + \gamma\ Y_{t-1} + B\ I_t + E_t - m(X_t - E_t) + F$

where F includes any additional exogenous final demands. Solving equation (13) for $(X_t - E_t)$ results in

(15) $X_t - E_t = (I - A + \hat{m})^{-1} + \gamma\ Y_{t-1} + B\ I_t + (I+A)E_t + F]$

where F contains any additional final demand. The multiplier matrix applying to changes in exports is given by:

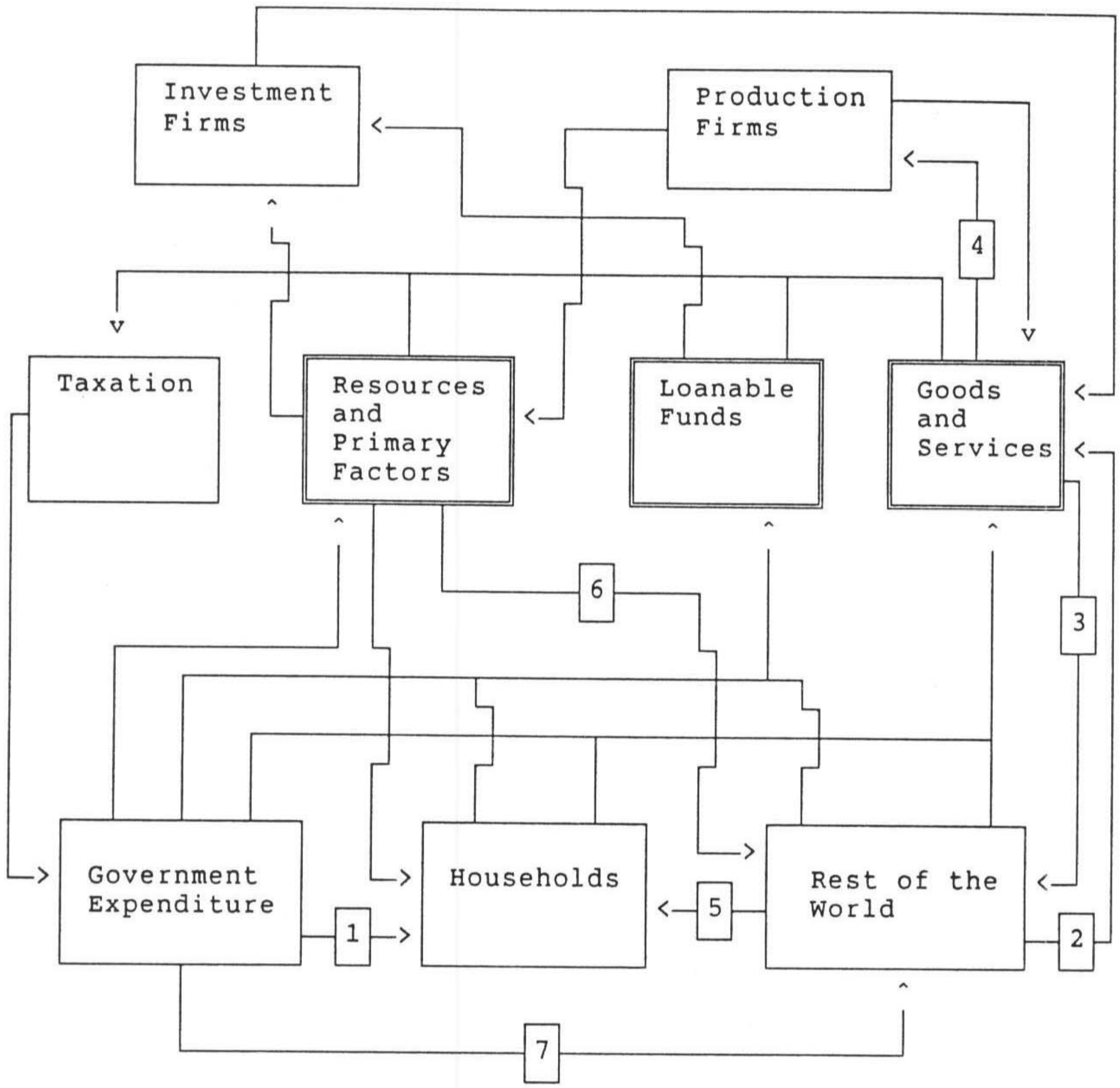
(16) $[I + (I - A + \hat{m})(I + A)^{-1}]$.

Source: Economic Research Technical Note 3,
Economic Research Technical Note 31.

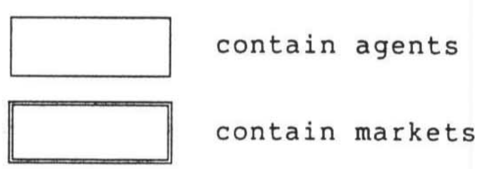
APPENDIX E
MODEL BLOCK DIAGRAM

This appendix shows a generalized block diagram of a dynamic regional economic model. Figure 1 describes a circular flow of funds in such a model. Figure 2 describes the material stocks and flows in such a model.

figure 1 - Circular Flow of Funds



Legend:



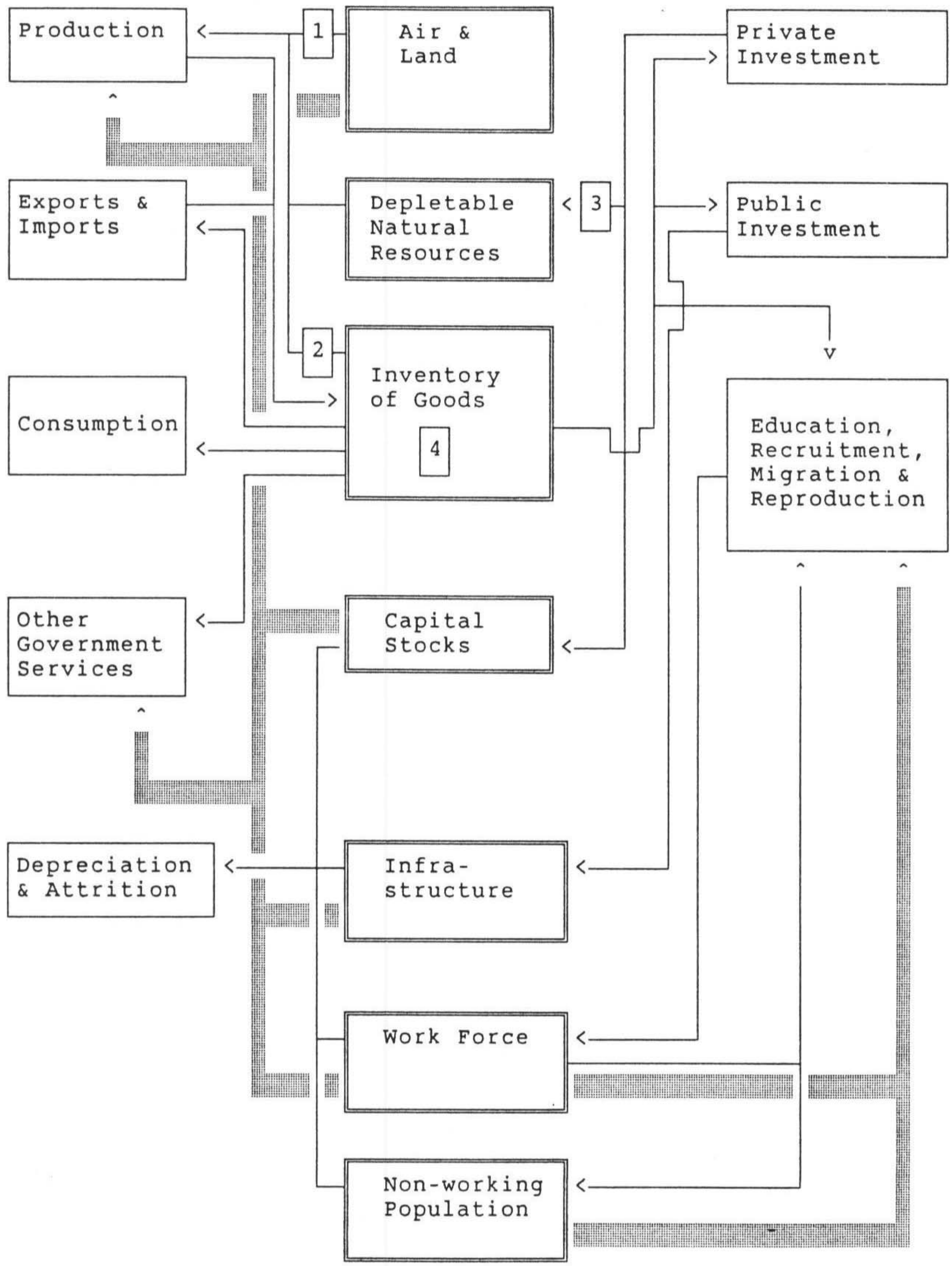
Arrows show the direction of dollar flows.

Loanable funds flows are net of interest and dividends.

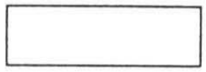
Notes:

1. Transfer payments.
2. Exports.
3. Imports.
4. Intermediate products.
5. Factor payments from Rest of the World.
6. Factor payments to Rest of the World.
7. Summarized transactions of Government and Rest of the World.

figure 2 - Material Stocks and Flows



Legend:

 contain processes

 contain stocks

————————> sources and sinks of stocks

 flows of services from primary products

Notes:

1. Pollution can deplete "non-depletable" resources, leading to the flow from air and land to production.

2. The flow from goods to production consists of intermediate products.

3. The flow from private investment to depletable natural resources represents exploration and development.

4. The flow of produced services is omitted from the diagram. It is similar to the flow of goods.

Source: Economic Research Technical Note 19, version 1.0.

APPENDIX F
TRADE MARGINS IN THE FORMAL MODEL

I. Purpose.

The purpose of this appendix is twofold. First, the appendix illustrates how trade and transportation margins can be introduced into an input-output model built on Cobb-Douglas assumptions. Second, it shows the ways in which trade and transport margins have been accounted for in the MRIO data set produced by Faucett and Associates.

II. Producer Prices and Purchaser Prices.

Commodity flows in an input-output model can be measured in several alternative sets of prices. Producer prices record the value received by firms for the sale of one unit of output. The producer price of a product excludes any margins (per unit costs) due to sales taxes, wholesaling, transportation, or retailing. Purchaser prices are the prices at which a wholesaler would sell and deliver goods and services. As such, purchaser prices include wholesale and transportation margins, and sales or excise taxes encountered at the wholesale level. Consumer prices can be thought of as the prices at which goods can be delivered to final demand in the consumption, investment, or government sectors. As such, they include retail margins and any remaining sales and excise taxes encountered at the retail level.[1]

III. Emphasis on Producer and Purchaser Prices

The primary emphasis of this appendix is on the distinction between producer and purchaser prices. Wholesale margins, transport margins, and taxes encountered at the wholesale level act as wedges between these two price vectors. The model which is presented can be solved for equilibrium commodity flows measured in either set of prices.

IV. Production of Delivered Products.

Delivered (purchasers') products are easily modelled as the result of a Cobb-Douglas production process which uses undelivered (producers') products, trade services, and transportation services as inputs. The delivery of product i can be represented as :

---

[1] The documentation for the data set from Faucett and Associates uses the inclusive term "purchaser prices" to refer to what we have distinguished as purchaser and consumer prices.

(1) $T\#_i^{pur} = (T\#_i^{pro})^{g_{io}} (H\#_1)^{g_{i1}} (H\#_2)^{g_{i2}} (H\#_j)^{g_{ij}} \ldots$

where:

$T\#_i^{pur}$ = units of good i after delivery

$T\#_i^{pro}$ = units of good i before delivery

$H\#_j$ = units of margin sector j services used as input

g_{ij} = Cobb-Douglas exponent, good i, margin input j.

g_{io} = Cobb-Douglas exponent, undelivered product i.

Note: Variables marked with # are physical quantities. The same letters marked with $ are prices. Finally, the letter with no marks are dollar flows or prices times quantities.

The process is assumed to be constant returns to scale, so that

(2) $$\sum_{j=0}^{k} g_{ij} = 1 \text{ (for all i).}$$

No value added is generated in the delivery process; instead, value added is associated with the production of good i and with production of the margin services. Therefore equation (1) does not include inputs for labor and capital.

Under the assumption of profit maximization, the Cobb-Douglas specification immediately leads to a linear relationship between delivered values, undelivered values, and marginal services:

(3a) $g_{io}\ T_i^{pur}\ (\equiv g_{io}\ T\$_i^{pur} T\#_i^{pur}) = T_i^{pro}\ (\equiv T\$_i^{pro} T\#_i^{pro})$

(3b) $g_{ij}\ T_i^{pur} = H^j\ (\equiv H\$_j H\#_j)$.

Equations 3a and 3b point up an important result. The value of shipments at producer prices is proportional to the value of shipments at purchaser prices.

As a consequence of the Cobb-Douglas specification of the delivery process, the value of each type of service used in delivery is also proportional to the value of the good delivered. The relationship between products and delivery services can be written as:

H_{ij} : Value of service flow from margin sector j used in delivery of good i.

T_i : Value of good i to be delivered, recorded in producer prices.

h_{ij} : H_{ij}/T_i.

The proportionality factor, hij, depends on the Cobb-Douglas exponents:

$$(4) \quad h_{ij} = g_{ij}/g_{io}.$$

IV. Transactions in Producer and Purchaser Prices.

The analysis of Part III can easily be extended to a case in which wholesale and transportation costs depend on the source and destination of a delivery. Suppose that h_{ijM}, h_{ijE}, and h_{ijK} represent the wholesale or transport margin factors for commodity i delivered as imports, as exports, and as shipments within Kansas, respectively. The values of shipments at producer prices and at purchaser prices are related by:

$$T_M^{pur} = (I + \hat{k}_M)T_M \quad \text{(imports)}$$

T_M : value of imports at producers' prices. (In the remainder this note, the "pro" superscripts are suppressed.)

$$T_E^{pur} = (I + \hat{k}_E)T_E \quad \text{(exports)}$$

$$T_K^{pur} = (I + \hat{k}_K)T_K \quad \text{(in-state)}$$

where the margin factors are:

$$k_M \equiv \sum_j h_{jM}$$ (vector equation with subscript i suppressed. Dimensions are N x 1.)

$$k_E \equiv \sum_j h_{jE}$$

$$k_K \equiv \sum_j h_{jK}$$

and where: j denotes margin sectors (except retail)
^ denotes a vector converted into a diagonal matrix.

V. Material Balance - Preliminaries.

Suppose that an economy produces N goods. The material balance equation requires that amount of each commodity used within the regional economy (i.e., domestic demand) be equal to the domestically available supply. Domestic demand consists of demand for intermediate products, consumption, investment, government spending, changes in purchaser's inventory, and demand for the services of the margin sectors. Available supply includes imports and domestic production minus exports. The next several sections of this note specify supply and demand carefully and describe the solution of the supply-demand equation for equilibrium output.

VI. Some Additional Notation.

X	:	value of output in producer prices (Nx1).
$A = \| a_{ij} \|$	:	matrix of input coefficients (NxN).
a_{ij}	:	value of input i in purchaser prices per dollar of output j in producer prices.
$n'X$	:	intermediate demand for non-competitive imports (1x1).
N_1	:	number of goods exclusive of transport, wholesale, and retail.
N_2	:	number of retail sectors.
N_3	:	number of wholesale and transport sectors.
$N=N_1+N_2+N_3$	:	total number of domestic sectors.
T_{NC}	:	value of noncompetitive imports in producers' prices (1x1).
h_{NC}	:	demand for margin sectors per unit of noncompetitive imports. Dimensions are N x 1. All entries are zero except in the margin sectors for wholesale and transport.
$k_{NC} \equiv \sum_j h_{jNC}$	:	margin factor for non-competitive imports (1x1).
FD	:	domestic final demand in purchaser prices (Nx1).
FD_{NC}	:	domestic final demand for non-competitive imports (1x1).
D	:	domestic demand in purchaser prices (Nx1).
H	:	vector of margin demands (Nx1, with zero entries in the non-margin industries).

XIII. Demand in the Margin Sectors-Wholesale and Transportation.

Demand for wholesale and transport services (sectors N_1+N_2+1 through N) consists primarily of margins on various types of trade. In matrix form,

$$(5)\quad \underset{(Nx1)}{H} = \begin{vmatrix} \underset{(N1+N2)x1}{0} \\ \hline \underset{(N3x1)}{\text{margins}} \end{vmatrix}$$

$$= \begin{vmatrix} \underset{(N1+N2)x\ N}{\mathbf{0}} \\ \hline \underset{(N3xN)}{\mathbf{h}_M'} \end{vmatrix} T_M + \begin{vmatrix} \mathbf{0} \\ \hline \underset{(N_3xN)}{\mathbf{h}_K'} \end{vmatrix} T_K + \begin{vmatrix} \mathbf{0} \\ \hline \mathbf{h}_E' \end{vmatrix} T_K + h_{NC}(T_{NC}).$$

Additionally, some transport and wholesale services may be demanded directly by firms, consumers, or government. This portion of demand for industries N1+N2+1 through N is already included in AX + FD.

IX. Demand in Retail Sectors.

Demand for retailing stems primarily from margins on consumption, investment, and government spending. Total retail demand from these sources could be calculated by multiplying retail margin factors times appropriate trade flows. However, this calculation has already been completed in the Faucett data. Total demand for retail services is entered in rows N1+1 through N1+N2 of FD + AX.

X. Sales and Excise Taxes.

Sales and excise taxes enter the model in two places. Taxes applied at the wholesale level affect the purchaser price of a good as well as its consumer price. Taxes applied at the retail level affect only consumer prices. We intend to treat taxes thoroughly in a future document. In this document, taxes will be suppressed to avoid further complication.

XI. Domestic Demand in Purchaser Prices.

Domestic demand is given by:

$$(6) \quad \underset{(N\times 1)}{D} = AX + FD + H$$

XII. Competitive Imports.

Imports, expressed in purchaser values, are assumed to be proportional to demand from domestic sources:

$$(7) \quad (I + \hat{k}_M)T_M = \hat{m}_O(FD + AX + H)$$

When the regional economy is in equilibrium, domestic demand must equal domestically available supply:

$$(8) \quad H + FD + AX = (I + \hat{k}_K)T_K + (I + \hat{k}_M)T_M$$

$$= (I + \hat{k}_K)\ (X - T_E) + (I + \hat{k}_M)T_M.$$

Combining (7) and (8) yields

$$(9) \quad T_M = \hat{m}\ (X-T_E), \text{ where}$$

(10) $\hat{m} \equiv (I + \hat{k}_M)^{-1} (I - \hat{m}_O)^{-1} \hat{m}_O(I + \hat{k}_K)$.
This shows that the marginal propensities to import $\hat{m}$ are diagonal. They can be calculated directly from the Faucett data, without using formula (10). In particular, we modify equation (9) to yield

(11) $\hat{m} = (\hat{X}-\hat{T}_E)^{-1}\hat{T}_M$.

XIII. Non-competitive Imports.

Since demand in purchasers' prices equals supply for non-competitive imports, we have:

(12) $n'X + FD_{NC} = (1 + k_{NC})T_{NC}$.

It follows that the demand generated in margin industries by the import of non-competitive goods is:

(13) $D_{MNC} = (1 + k_{NC})^{-1}(n'X + FD_{NC})h_{NC}$.

XIV. Solution of the Material Balance Equation.

Solution of equation (8) requires that domestic output be expressed in terms of estimated coefficients and exogenous demand. The coefficients a, m, h_M, h_E, and h_K are estimated from Faucett data. In the current model, domestic final demand FD and exports T_E are independent of current output (see ERTN no. 3). Exogenous demand includes these terms, plus some additional demand due to trade and transport margins. After some algebra, one can show that:

(14) X = [MULT] [EXOG], where

(15) $\mathbf{MULT} = \left[I+\hat{k}_K - \mathbf{a} + (I+\hat{k}_M)\hat{m} - \begin{vmatrix} 0 \\ \mathbf{h}_M' \end{vmatrix} \hat{m} - \begin{vmatrix} 0 \\ \mathbf{h}_K' \end{vmatrix} - (1 + k_{NC})^{-1} h_{NC} n' \right]^{-1}$

and

(16) $\mathrm{EXOG} = FD + (I+\hat{k}_K)T_E + (I+\hat{k}_M)\hat{m}\, T_E - \begin{vmatrix} 0 \\ \mathbf{h}_M' \end{vmatrix} \hat{m}\, T_E - \begin{vmatrix} 0 \\ \mathbf{h}_K' \end{vmatrix} T_E + \begin{vmatrix} 0 \\ \mathbf{h}_E' \end{vmatrix} T_E + (1+k_{NC})^{-1} FD_{NC} h_{NC}$.

Source: Economic Research Technical Note 20.

APPENDIX G

COBB-DOUGLAS AND I/O MODELLING

Leontief Technology and Input Substitution

1. Introduction

Input-output models have long been used in regional economic analysis. These models are specified around Leontief systems which describe technological processes. On a theoretical level, Leontief fixed-coefficient systems have been criticized for not allowing input substitution. In practice, however, the technical coefficients of Leontief systems are usually estimated using accounting data which are both behavioral and monetary.

In this communication, it is argued that if the Leontief system is considered as describing the result of optimizing behavior, then the constancy of monetary (value) input-output coefficients is equivalent to assuming technological processes that do allow input substitution. In fact, we show that under profit maximization, the Leontief fixed value coefficients are the same as the input shares associated with generalized Cobb-Douglas production functions.

2. The Equivalence of Leontief Value Coefficients and Cobb-Douglas Input Shares

Let x_i be the output of sector i, and let x_{ij} be the input of sector i used in sector j, all in physical units. The usual Leontief technology is given by:

$$x_{ij} = a_{ij}x_j \qquad i = 1,\ldots,n; \quad j = 1,\ldots,m. \tag{1}$$

Now assume a generalized Cobb-Douglas technology, i.e., let

$$x_j = \prod_{i=1}^{n} x_{ij}^{b_{ij}} \qquad j = 1,\ldots,m. \tag{2}$$

Let P_i be the price of good i and let $\sum_{i=1}^{n} b_{ij} \leq 1$. Then profit maximization is equivalent to:

$$P_i = P_j b_{ij} x_{ij}^{b_{ij}-1} \prod_{k \neq i} x_{kj}^{b_{kj}} \tag{3}$$

which can be rewritten as:

$$P_i x_{ij} = P_j x_j b_{ij} \tag{4}$$

or

$$a_{ij} = b_{ij}, \tag{5}$$

where a_{ij} is the Leontief coefficient of production in value terms, and b_{ij} is the input share associated with the Cobb-Douglas technology.

Conversely, assume (5) to hold and consider the isoquant:

$$f^j(x_{1j},\ldots,x_{nj}) = \text{constant.} \tag{6}$$

Then,

$$\sum_{i=1}^{n} f_i^j \, dx_{ij} = 0. \tag{7}$$

By profit maximization we must have:

$$P_j \, f_i^j = P_i. \tag{8}$$

By (7), (8), and (4) we have:

$$\sum_{i=1}^{n} \frac{b_{ij}}{x_{ij}} \, dx_{ij} = 0. \tag{9}$$

The solution to (9) is:

$$\prod_{i=1}^{n} x_{ij}^{b_{ij}} = \text{constant.} \tag{10}$$

Thus, profit maximizing behavior that is monetary Leontief implies a generalized Cobb-Douglas technology. It should be pointed out that input shares are often assumed to be constant when conducting comparative static analysis in general equilibrium frameworks other than input-output models. Further, the constancy of input shares does not imply lack of input substitution.

3. Conclusion

In this communication, it has been shown that estimating input-output coefficients from expenditure data, which are generally the only information available for this purpose, does not imply non-substitution among physical inputs. Furthermore, assuming generalized Cobb-Douglas technology, these value input-output coefficients are independent of prices.

APPENDIX H
COBB-DOUGLAS INVESTMENT.

I. PURPOSE:

This Appendix gives a model of regional investment under Cobb-Douglas production. Investment is conditional on expected output and prices next period; expectations about subsequent events are assumed myopic. Prices, expectations, and output are assumed exogenous, with some exceptions. Production and investment are assumed socially efficient, conditionally on expectations. (Therefore, constant returns to scale, competitive behavior, and/or profit maximization are NOT necessarily assumed, and there is no co-ordination problem.) Investment is driven by two main factors: increases in output demands, and depreciation on existing capital (assumed exponential). Investment also reponds to relative prices. Investment made this period comes on line next period. Putty-putty assumptions are used in the main part of the Appendix; the final section considers putty-clay assumptions.

II. VARIABLES:

We will concentrate our attention on investment occurring in period t. Expectations at time t will be denoted by E_t.

r_t is the nominal interest rate for loans made at the beginning of period t (payable at the beginning of period t+1).

Capital services $\mathbf{c}_t$ will be measured in the same units as capital stock $\mathbf{k}_t$, so that $\mathbf{0} \le \mathbf{c}_t \le \mathbf{k}_t$. In the putty-putty case, all capital is utilized if there is any output, so that $\mathbf{c}_t = \mathbf{k}_t$. The notation will allow intermediate product prices $\tilde{\mathbf{x}}_n$ to differ across industries (indexed by n), and also to differ from the corresponding output prices $\tilde{\mathbf{x}}$. This reflects the fact the prices of inputs depend on transport and wholesale margins and taxes, as well as on output prices.

For vector variables, lower case roman type refers to quantities; lower case roman with an overstrike ~ refers to the corresponding prices; and upper case roman type refers to the correponding nominal dollar values. Therefore, for most vectors $\mathbf{q}$ we have an identity

(1) $\mathbf{Q} = \hat{\mathbf{q}}\tilde{\mathbf{q}}$.

Vector variables are summarized in the following table.

variable	quantity	price	value
Vectors by industry:			
Capital stocks:			
Planned	$E_t k_{t+1}$	$E_t \tilde{k}_{t+1}$	$E_t K_{t+1} = E_t \tilde{k}_{t+1} \hat{k}_{t+1}$
Existing	k_t	$\tilde{k}_t$	$K_t = \tilde{k}_t \hat{k}_t$
Utilized	c_t		
Flows:			
Output	x_t	$\tilde{x}_t$	$X_t = \tilde{x}_t \hat{x}_t$
Gross Investment	i_t	$\tilde{i}_t$	$I_t = \tilde{i}_t \hat{i}_t$
Capacity (in output units)	z_t		
Labor services	l_t	$\tilde{l}_t$	$L_t = \tilde{l}_t \hat{l}_t$
Capital services	c_t	$\tilde{c}_t$	$C_t = \tilde{c}_t \hat{c}_t$
Vector flows by commodity:			
Intermediate product input demands in sector "n"	x_{nt}	$\tilde{x}_{nt}$	$X_{nt} = \tilde{x}_{nt} \hat{x}_{nt}$
Investment requirements	j_t	$\tilde{j}_t$	$J_t = \tilde{j}_t \hat{j}_t$

III. PARAMETERS, CONSTANTS, AND PRODUCTION.

Each output x_n is generated from labor services, capital services, and intermediate products according the Cobb-Douglas production function:

$$(2) \quad \log x_n = \alpha_n + \omega_n \log l_n + \rho_n \log c_n + \sum_m A_{mn} \log x_{mn} .$$

Equation (2) always describes production ex ante of the investment activity. Under putty-clay assumptions, the capital-labor ratio is fixed once the investment is put in place. Therefore, equation (2) must be replaced ex post with a more complex equation which keeps track of the capital-labor ratios of the various vintages of capital in place. However, under putty-putty assumptions, the capital-labor ratio can be freely modified ex post as well as ex ante. In that case, equation (2) holds ex post as well as ex ante.

Note that increasing returns to scale would imply

$$\boldsymbol{\omega} + \boldsymbol{\rho} + \mathbf{A}'\mathbf{1} = 1,$$

where $\mathbf{1}$ is a column vector of 1's. However, we do not assume this. Instead, we assume social efficiency, so that the value marginal product equals marginal cost, or

(3) $\tilde{x}_n \partial x_n / \partial q = \tilde{q}$, where q is any input.

In the case where q is c_n (capital services), (3) leads directly to a proportionality of value-flows:

(4) $C_t = \hat{\rho} X_t$.

We emphasize that, under putty-clay assumptions, this relation has been established only ex ante. In ex post terms, c_t would have to be replaced by some kind of aggregate over vintages. However, under putty-putty assumptions, (4) holds both ex ante and ex post.

IV. ASIDE ON SOCIAL EFFICIENCY:

We assumed social efficiency rather than constant returns and perfect competition, so as to achieve greater generality. This aside gives some sufficient conditions for social efficiency.

Suppose that (2) is interpreted to be the production function of any single firm, rather than the aggregate production function in the sector. If constant returns to scale does hold, then social efficiency follows from perfect competition.

But if decreasing returns to scale were to hold, then unconstrained perfect competition would lead to an infinite number of infinitely efficient firms, each producing an infinitesimal amount of output at an infinitesimal output price. Since we don't need to include explicit models for unpriced goods, this case is not relevent. Let us suppose instead that decreasing returns to scale holds, and that perfect competition takes place under the constraint that the number of firms is fixed, for example because there is a fixed and lumpy supply of an unmeasured input such as enterpreneurial talent. Then social efficiency would result at a positive output price.

And if increasing returns to scale were to hold, then unconstrained perfect competition would not be not credible, because it would lead to a single firm. However, social efficiency might still obtain, e.g. as result of government regulation.

In each case, equation (2) can be re-interpreted as the aggregate production function of the sector. If all firms are of equal size, then the only difference between the two interpretations is that the value of α_n differs by a constant times the logarithm of the number of firms.

A consequence of some of these interpretations is that observed property income can not be identified with the value of capital services, C_t; and the value of the firm can not be identified with the value of the capital stock, K_t. This implies that we should seek a model of investment in which the prices $\tilde{c}_t$ and $\tilde{k}_t$ are implicit rather than explicit.

V. AN INVESTMENT FRAMEWORK:

We assume that investment plans are expected to be fulfilled. Therefore, after accounting for exponential depreciation at a rate $\boldsymbol{\delta}$, we have

$$(5)\quad E_t\mathbf{k}_{t+1} = \mathbf{i}_t + \mathbf{k}_t(\hat{\mathbf{1}} - \hat{\boldsymbol{\delta}}),$$

where $\hat{\mathbf{1}}$ is the identity matrix. If any investment does occur in a sector, then full capacity utilization is expected in the next period. That is, in sectors where $\mathbf{i}_t > \mathbf{0}$ we have

$$(6)\quad E_t\mathbf{c}_{t+1} = E_t\mathbf{k}_{t+1}.$$

From (4) and (1) we derive

$$(7)\quad \mathbf{i}_t = E_t[\hat{\tilde{c}}_{t+1}{}^{-1}\hat{\rho}\mathbf{x}_{t+1}] - \mathbf{k}_t(\hat{\mathbf{1}} - \hat{\boldsymbol{\delta}}),$$

where $\hat{\tilde{c}}$ denotes the diagonalization of $\tilde{c}$. Note that this derivation appears to depend on putty-putty assumptions, since we appealed to equation (4); but see below.

We will now generalize this to include the case where $\mathbf{i}_t = \mathbf{0}$. We also translate the quantity of investment into the value of investment, leading to

$$(8)\quad \mathbf{I}_t = \mathbf{MAX}\{\ \mathbf{0}\ , \hat{\tilde{\imath}}_t E_t[\hat{\tilde{c}}_{t+1}{}^{-1}\hat{\rho}\mathbf{x}_{t+1}] - \hat{\tilde{\imath}}_t\mathbf{k}_t(\hat{\mathbf{1}} - \hat{\boldsymbol{\delta}})\},$$

where $\mathbf{0}$ is a column vector of zero's, $\hat{\tilde{\imath}}_t$ is a diagonalized vector of investment prices, and the MAX function is taken term by term.

DISCUSSION. This relation is rather general. It clearly applies to putty-putty investment. Depending on the exact concept of k_t, it could also apply to putty-clay investment. That is, we might be able to define k_t as a certain aggregate over previous vintages of capital investment such that (8) holds; this idea is pursued in a latter section.

Equation (8) may be taken as a complete model of investment, provided that the prices $\tilde{\imath}_t$ and $E_t\tilde{c}_{t+1}$ are known. However, in the real world, capital equipment is rarely leased; consequently, the rental price of capital $\tilde{c}$ is not directly observable in the market place. Therefore a model of inferred prices is needed.

VI. INFERRED PRICES OF CAPITAL ASSETS AND SERVICES:

We assume that no risk premia are implied by the structure of the expected market rates of return on capital. This would hold if all firms were risk neutral; it would also hold with risk aversion if fair insurance existed or if risks were otherwise diversifiable.

We assume that capital and investment purchases and bank deposits are made at the beginning of the period. Capital service and interest payments are made at the end of the period, or the beginning of the next period. In this section, we make no assumptions about technology.

If positive investment is taking place this period, and if there are no arbitrage opportunities, then the expected price of capital stock next period must be in equilibrium with the price of investment this period. This leads to the condition

$$(9)\quad \tilde{\imath}_t(1 + r_t) = E_t\tilde{k}_{t+1}.$$

In other words, it is just as good to purchase investment goods this period which will come on line next period, as to put the same money into the bank and use the proceeds to buy existing capital stock next period.

At the same time, the price of capital services must be in equilibrium with the price of capital stock. In particular, one can either purchase capital at the beginning of this period, rent it out, suffer some depreciation, and sell the remaining capital next period; or one can simply put the amount of the purchase price into the bank and draw a return next period. Hence

$$(10)\quad \tilde{c}_t + E_t\tilde{k}_{t+1}(1 - \delta) = \tilde{k}_t(1 + r_t).$$

Eliminating $E_t\tilde{k}_{t+1}$ between the two equations leads to the arbitrage condition between the current prices of capital stock and services and investment:

$$(11)\quad \tilde{c}_t + \tilde{\imath}_t(1 + r_t)(1 - \delta) = \tilde{k}_t(1 + r_t).$$

Taking expectations and using (9) again leads to

$$E_t\tilde{c}_{t+1} = [\tilde{\imath}_t(1 + r_t) - E_t\tilde{\imath}_{t+1}(1 - \delta)]E_t(1+r_{t+1}).$$

Let us define the expected rate of price inflation for investment goods

$$(12)\quad \epsilon_t \equiv \tilde{\imath}_t^{-1}E_t\tilde{\imath}_{t+1} - 1.$$

Then neglecting a small term $\epsilon\delta$, we have

$$(13)\quad E_t\tilde{c}_{t+1} = [\tilde{\imath}_t\{1r_t + \delta - \hat{\epsilon}_t\}]E_t(1+r_{t+1}).$$

Note that this derivation applies to both putty-putty and putty-clay investment. We will leave the model of expected investment prices and interest rates exogenous. But notice that the expected price of capital services would approach zero if the term in curly brackets approached zero; in that case the demand for investment would blow up. Therefore it is necessary to restrict any admissable model of price inflation so that

$$\{\hat{1}r_t + \hat{\delta} - \hat{\epsilon}_t\} > 0.$$

VII. PUTTY-PUTTY INVESTMENT:

Using (13), equation (8) becomes

$$(14)\quad I_t = \mathbf{MAX}\{\ 0\ ,\ (\hat{1}r_t + \hat{\delta} - \hat{\epsilon}_t)^{-1}E_t(1+r_{t+1})^{-1}\hat{\rho}x_{t+1} - \hat{i}_t k_t(\hat{1} - \hat{\delta})\},$$

a putty-putty model of investment with the price of capital services made implicit.

We do not give a model of capacity, because it is almost impossible to operate at less than full capacity under a putty-clay technology with optimizing behavior. Instead, capital stock is updated according to the equation

$$(15)\quad k_{t+1} = k_t(\hat{1} - \hat{\delta}) + i_t.$$

VIII. PUTTY-CLAY INVESTMENT:

We will assume in the following that changes in the capital-labor ratio over time are relatively small. In particular, we assume that no capital will be junked prematurely; capital of old vintages will continue to produce positive operating profits. Without this assumption, it would be necessary to keep track of the amount and type of capital of each vintage. (Another reason to keep track of previous investment history would be to measure the labor demand.)

Instead, we will summarize all previous investment history using a capacity variable z_t. As usual,

$$(16)\quad 0 \leq x_t \leq z_t.$$

Capacity consists of investments of many vintages, each with a fixed capital-labor ratio. However, each vintage is assumed to depreciate at the same exponential rate, and therefore capacity also depreciates at that rate. Investment is driven by the need to replace and expand capacity:

$$(17)\quad i_t = \mathbf{MAX}\{\ 0\ ,\ o_t[E_t x_{t+1} - z_t(\hat{1} - \hat{\delta})]\ \},$$

where o_t is the planned capital to output ratio for the new vintage of investment. But o_t can be derived from (4), since we are in the ex ante case. The result is

$$o_t = E_t\hat{\rho}\tilde{x}_{t+1}\tilde{c}_{t+1}{}^{-1}.$$

We can eliminate $\tilde{c}_{t+1}$ using (13), giving

$$(18)\quad o_t = E_t \hat{\rho} \tilde{x}_{t+1} [\tilde{i}_t(\hat{1} r_t + \hat{\partial} - \hat{\epsilon}_t)(1+r_{t+1})]^{-1}.$$

A comparison of this investment model to the putty-putty model (14) will reveal that the two models differ only in the z_t or k_t term. In fact, it is possible to define a capital aggregate for the putty-clay case such the two investment equations become formally identical. In particular, the required aggregate is given by

$$(19)\quad k_t = \tilde{i}_t^{-1} o_t z_t$$
$$= E_t \hat{\rho} \tilde{x}_{t+1} [\tilde{i} \hat{1} r_t + \hat{\partial} - \hat{\epsilon}_t)(1+r_{t+1})]^{-1} z_t.$$

However, this index number for physical capital is heavily sensitive to current prices, and may be of little use.

To complete the investment model, we point out the capacity updating equation:

$$(20)\quad z_{t+1} = z_t(\hat{1} - \hat{\delta}) + \hat{o}_t^{-1} i_t.$$

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APPENDIX I
INFERRING THE A MATRIX

I. Purpose.

This appendix shows a method to construct a Leontief type A matrix from available data, more specifically, the make and use matrices.

II. The A matrix.

Many of the basic concepts of input output analysis were developed by Wassily Leontief in the late 1930's. Since that time, models with an interindustry structure similar to that described by Leontief have been referred to as Leontief type models. A key feature of such a model is a matrix of coefficients which summarizes the use of inputs to produce various commodities. This matrix is commonly referred to as the Leontief A matrix. Each element a_{ij} is a technical coefficient which indicates the amount of commodity i used to produce one unit of output j.

Several interpretations of the A matrix technical coefficients are possible, as proved in detail in appendix G. A fixed coefficient interpretation requires that the physical quantities of inputs to produce a physical unit of output are fixed. A Cobb-Douglas interpretation specifies that the dollar amount of an input used to produce a dollar's worth of output is fixed. In either case, technology is assumed to be commodity specific. This means that there is a unique best combination of inputs required to produce a unit of a stated output. This best input combination is the same regardless of the firm or industry in which the commodity is produced. Furthermore, the best input choice for a given commodity is independent of the types and amounts of other goods being produced.

III. Available Data.

Surveys such as the Census of Manufactures collect data on the usage of inputs and the production of output by establishment. An establishment is roughly equivalent to a plant site. Establishments are classified into industries according to their major output. Establishments frequently produce several distinct outputs. Hence each industry will produce several commodities.

Information on inputs and outputs can be compiled into two matrices, the use matrix (**U**) and the make matrix (**V**). Each entry in the use matrix, U_{ij}, shows the dollar total of commodity i used by establishments in industry j. Each entry V_{ij} in the make matrix shows the output of good i by establishments classified in industry j. The following example indicates how establishment based data is used to construct **U** and **V**.

Example

Establishment #20

Inputs:		Outputs:	
good 1:	$100	good 2:	$250
good 3:	$40	good 4:	$50
labor:	$100		

The establishment in the example would be classified in industry 2 according to its major output. The establishment would contribute $100 to U_{12} , $40 to U_{32}, $250 to V_{22}, and $50 to V_{42}.

IV. Construction of the A matrix.

Suppose that the input output structure of an economy is truly represented by a commodity specific technology with an unknown A matrix of technical coefficients. Can the values of the a_{ij} be computed from the use and make matrices? The following discussion shows that the answer is yes.

The A matrix multiplied by the vector of outputs of a given industry provides the commodity inputs required by that industry. The A matrix multiplied by the successive column vectors of the make matrix **V** (outputs by industry) generates the matrix of commodity usages by industry, **U**:

(1) $\mathrm{A}\ \mathbf{V} = \mathbf{U}.$

If the make matrix is invertible, the A matrix will have the unique solution

(2) $\mathrm{A} = \mathbf{U}\ \mathbf{V}^{-1}.$

V. Applications.

Equation (2) has been used to compute the A matrix for the Kansas long term model. Kansas specific use and make matrices were provided in the data set from Jack Faucett, Assoc.

V1. Future Plans.

The Bureau of Economic Analysis (BEA) completely revises the U.S. use and make matrices at five year intervals. The revisions make use of new survey data. The future plans of this project call for the creation of a national A matrix from the most currrent make and use matrices. The national A matrix can then be aggregated using data of estimates for Kansas output, as discussed in appendix J. Ultimately, the project plan calls for the direct survey of selected Kansas industries. Information from a Kansas specific survey could supplement information brought down from the national data.

Source: Economic Research Technical Note 28.

APPENDIX J
AGGREGATING THE A-MATRIX

I. Purpose

This note compares two methods of aggregating the I/O coefficients (a) matrix. One method uses the transactions matrix (aQ) as a starting point. A second method starts with aggregated vesions of the use (U) and make (V) matrices. This note shows the superiority of the first method.

II. Aggregation in the absence of measurement error

Suppose that the data are perfectly disaggregated and perfectly measured, and that technology is described by fixed coefficients in commodity-commodity space. As was shown in forthcoming ERTN 28.

1) $a = UV^{-1}$

In theory, the entries of a are all nonnegative, the entries of a do not exceed 1, and the column totals of a do not exceed 1. Now let Q=VP be output by commodity and $T=a\hat{Q}$ be the true transactions matrix in commodity space. Let U_1, V_1, Q_1, T_1 be corresponding aggregations of U, V, Q, and T. Consider two possible formulas for an aggregated a matrix:

2) $a_2 = T_1 \hat{Q}_1^{-1}$ (starts with transactions matrix).

3) $a_3 = U_1 V_1^{-1}$ (starts with use and make matrices).

The matrix a_2 corresponds to aggregated transactions flows T_1. Since Q and a are nonnegative, it follows that T, T_1, Q_1, and a_2 are nonnegative. The individual entries and the column totals of a_2 do not exceed 1 due to the corresponding properties of the disaggregated a matrix. In summary, a_2 has the appropriate characteristics of a coefficients matrix.

In contrast, a_3 has no particular relation to T_1 or Q_1. Entries of a_3 may be negative or may exceed 1. Therefore, a_3 fails to provide a reasonable coefficients matrix. It is better to perform aggregation on the transactions matrix (as for a_2) than on U and V.

III. Measurement error

If U and V are measured with error, coefficients which are truly zero may appear as small positive or negative numbers in the calculated a matrix. Let U_0 and V_0 be measured values of U and V. The measured coefficients matrix can be defined as

4) $a_o = \max(0, U_o V_o^{-1})$.

In other words, n^2 nonnegativity constraints are imposed on the coeffients matrix.

Each of the n^2 nonnegativity restrictions can be thought of as a piece of information about the structure of the a matrix. A large amount of information will be lost if nonnegativity is imposed after a coefficient matrix is calculated through formula (3). In contrast, using formula (2) after imposing negativity, incorporates all nonnegativity restrictions at a disaggregated level; the nonnegativity restrictions apply to T and hence T_1.

IV. Losses due to errors in measurement

Let U and V be perfectly disaggregated but measured with error. Suppose that the true a matrix, $a = UV^{-1}$, has no negative entries. A loss function can be defined to reflect the seriousness of the difference between the true values of a and the estimates of a (such as a_o) based on U_o and V_o. The loss function should be nonnegative and should increase as any element of a gets further from its true value. An example of a loss function is

$$L = \sum_{i}^{n} \sum_{j}^{n} (a_{ij} - a_{oij})^2.$$

V. Nonnegativity constraints reduce losses

Consider two formulations of the a matrix based on the measured U and V matrices:

5) $a_o = \max[0, U_o V_o^{-1}]$

6) $a_o = U_o V_o^{-1}$

If the true a matrix is nonnegative, formula 5) dominates formula 6) for any loss function. That is, formula 5) losses are at least as small as formula 6) losses. If $U_o V_o^{-1}$ contains any negative terms, the loss resulting from formula 5) is strictly less than that from formula 6).

VI. Summary

The recommended procedure for aggregating a from given U and V is:

(1) infer a from U and V at the most disaggregated level.
(2) zero out any negative entries of a.
(3) form the disaggregated T from a.
(4) aggregate T.
(5) infer aggregated a from aggregated T.

VI. Numerical Example

This example compares the a matrix formed from the aggregated transactions matrix (formula 2) with the a matrix formed from the aggregate use and make matrices (forumula 3). Supose that an economy has three industries and three commodities, as shown in the initial data below:

$$a = \begin{pmatrix} 0 & .5 & .1 \\ .5 & 0 & .3 \\ .3 & .3 & 0 \end{pmatrix} \quad V = \begin{pmatrix} 12 & 0 & 6 \\ 0 & 10 & 0 \\ 8 & 0 & 12 \end{pmatrix} \quad U = \begin{pmatrix} .8 & 5 & 1.2 \\ 8.4 & 0 & 6.6 \\ 3.6 & 3 & 1.8 \end{pmatrix}$$

$$T = \begin{pmatrix} 0 & .5 & 2 \\ 9 & 0 & 6 \\ 5.4 & 3 & 0 \end{pmatrix} \quad Q = \begin{pmatrix} 18 \\ 10 \\ 20 \end{pmatrix} .$$

Next aggregate sectors 2 and 3, leading to

$$V_1 = \begin{pmatrix} 12 & 6 \\ 8 & 22 \end{pmatrix} \quad V_1^{-1} = \frac{1}{216} \begin{pmatrix} 22 & -6 \\ -8 & 12 \end{pmatrix}$$

$$U_1 = \begin{pmatrix} .8 & 6.2 \\ 12 & 8.4 \end{pmatrix} \quad Q_1 = \begin{pmatrix} 18 \\ 30 \end{pmatrix} \quad T_1 = \begin{pmatrix} 0 & 2.5 \\ 14.4 & 9 \end{pmatrix}$$

Using formula 2, the aggregated a matrix becomes

$$a_2 = \begin{pmatrix} 0 & .083 \\ .8 & .3 \end{pmatrix} .$$

Forumula 3 results in an aggregated a matrix of

$$a_3 = \begin{pmatrix} -.15 & .32 \\ .91 & .13 \end{pmatrix} .$$

Notice the negative entry and the large differences between a_2 and a_3.

Source: Economic Research Technical Note 28.

APPENDIX K
UPDATING METHODOLOGY

I. INTRODUCTION:

This appendix gives procedures for periodically updating the coefficients of the long-term model by making use of information from published time series data.

II. NOTATION:

$w_i(t) = W_i(t)/X_i(t)$ = wage bill coefficient for year t.
$a_{ji}(t)$ = input coefficient for commodity j into industry i
$p_i(t) = P_i(t)/X_i(t)$ = property income coefficient.
$s_i(t) = w_i(t) + p_i(t)$ = value added coefficient.
Some vectors of commodities in Kansas are:

- Q = state outputs.
- C = demands for household plus state and local government consumption.
- G = federal government demands.
- J = investment demands.
- E = exports.
- M = imports.

S_{US} = U.S. GNP (i.e., value added), vector by industry.
t and u denote time.

III. CITATION:

Garhart, Robert Jr. "The Role of Error Structure in Simulations of Input-Output Analysis", Journal of Regional Science, 25(3), 1985, pp.353-366.

In a static context, Grahart argues that it is best to allocate resources for updating coefficients in the following order of priority: first, wage bill coefficients; then technical requirements and trade coefficients in the largest sectors. He also provides a brief bibliography.

IV. THE A MATRIX:

a. Correcting for labor payment data:

The largest single coefficient in the A matrix is typically the wage bill coefficient. Since both wage bills and total output are available every five years for Kansas manufacturing industries and some other industries from the various business censuses done by the Bureau of the Census, an updated wage bill coefficient is available directly for these industries.

Since the A coefficients plus the value-added (wage and property) coefficients for an industry must sum to 1, the A coefficients must be updated when w is updated, according to the formula

$$a_{ji}(t) = a_{ji}(u)[1-w_i(t)]/[1-w_i(u)].$$

b. Correcting for property income data:

When profit data by industry are available, then we might update the property-income coefficient directly. However, this approach may not be optimal for all purposes. In particular, updating the property income coefficient for year t data, may not lead to the best predictor of year t+1 operating conditions. That is, any property income coefficient fluctuates a certain amount across the business cycle; it is less stable than the other coefficients, for the reason that business owners are the residual bearers of risk. Therefore, it might make sense to update the property income coefficient according to an autoregression, or a moving average.

In any case, if both property and labor income coefficients are updated from time u to time t, then the A matrix should be updated according to the formula

$$(2)\quad a_{ji}(t) = a_{ji}(u)[1-s_i(t)]/[1-s_i(u)].$$

V. IMPORTS AND EXPORTS:

Data on shipments to and from Kansas are not available on an annual basis. However, some information can be inferred from annual data on output, income, and government demands in Kansas. In particular, if investment and consumption coefficients are assumed stable, if the investment model is accurate, and if inventories are stable, then we have measures or inferences for Kansas production less all Kansas demands or

$$NI \equiv Q - aQ - C - J - G.$$

NI must also equal imports minus exports, where NI is net imports, i.e NI = M - E.

It seems reasonable to suppose that the propensities to import differ across the three goods aQ, J, and C. As an accounting identity, federal demands for Kansas goods are satisfied by goods produced locally in Kansas, and therefore the propensity to import corresponding to the the vector G is zero. Accordingly, a reasonable model of Kansas imports would be

$$(3)\quad M = \hat{m}_C C + \hat{m}_A aQ + \hat{m}_J J,$$

where $\hat{m}_k$ is the propensity to import by demand sector K (K = C, A, J) and any element $\hat{m}_{ki}$ of the three $\hat{m}$ vectors obeys

$$(4)\quad 0 < m_i < 1.$$

At the same time, exports will be modeled as

$$(5) \quad E = \hat{e}S_{US},$$

where any element e_i of $\hat{e}$ obeys

$$(6) \quad 0 < ei << 1.$$

Boiling all this down leads to a relation between unknown coefficients and known commodity vectors at any time t:

$$(7) \quad NI(t) = \hat{m}_C(t)C(t) + \hat{m}_A(t)a(t)Q(t) + \hat{m}_J(t)J(t) - \hat{e}(t)S_{US}(t).$$

A variety of estimators can make use of this relation.

(a) Minimimum quadratic changes in flows.

If the four coefficient vectors have been estimated to a reasonable accuracy at some previous time u, then one possible strategy would be to minimize the sum of squared changes of the four estimated flows on the RHS of (7), subject to the constraints (4) and (6). In the unconstrained case, this leads to the following formulas. First, compute a total discrepency Δ which needs to be distributed across the four coefficients:

$$(8a) \quad \Delta = NI(t) - [\hat{m}_C(u)C(t) + \hat{m}_A(u)a(t)Q(t) + \hat{m}_J(u)J(t) - \hat{e}(u)S_{US}(t)].$$

Then distribute the discrepency in the obvious way:

$$(8b) \quad \hat{m}_C(t) = \hat{m}_C(u) + \Delta/(4C(t)); \ldots$$

$$(8e) \quad \hat{e}(t) = \hat{e}(u) - \Delta/(4S_{US}(t)),$$

where the vector division is taken term by term. The extension to the case in which this formula would violate the constraints (4) or (6) is straight forward.

(b) Minimum quadratic changes in coefficients.

Rather than minimizing changes in flows, one might minimize the sum of squared changes in the coefficients, subject to the constraints (4) and (6). In the unconstrained case, this leads to the following formulas. Let Δ be defined as in (8a), and define

$$(9a) \quad \delta = \Delta/[C(t) + a(t)Q(t) + J(t) + S_{US}(t)].$$

Then the estimates of the updated coefficients are

$$(9b) \quad \hat{m}_C(t) = \hat{m}_C(u) + \hat{\delta}C(t); \ldots$$

$$(9e) \quad \hat{e}(t) = \hat{e}(u) - \hat{\delta}S_{US}(t).$$

Again, the extension to the constrained case is straight-forward.

How should one choose between the estimators (8) and (9)? Practically speaking, the estimator (9) will force most of the change onto the export coefficient, because the base S_{US} is so much larger than the others. For the same reason, the estimator (8) will lead to very small changes in the export coefficient, and larger changes in the other coefficients. Choosing between the two estimators in the absence of data amounts to choosing between two prior judgements: fluctuations of the national market share for Kansas exports are judged to be either relatively small, or relatively large, in comparison to fluctuations of the market share in Kansas consumption for imports to Kansas. I offer an aggregation and specialization story for preferring the latter judgement (and estimator (9)) over the former (and estimator (8)). That is, each industrial sector consists of a large number of subsectors. Of the subsectors which are heavily traded across state lines, Kansas probably specializes in only a few. Consequently, goods are imported in many subsectors, but exported in only a few. Imagine that fluctuations by percent of the flow are i.i.d. across sub-sectors; then by the law of large numbers, fluctuations in the aggregate sectoral flows will be larger in exports than in imports.

Naturally, one could construct a better estimator than either (8) or (9) if one had data on the variances of each of the four coefficients.

(c) A model of cross-shipments.

If no data exist for setting initial values of the m coefficients, then one must make a more extreme use of prior information. The following model seems reasonable:

Most of the manufacturing sectors are defined as aggregates over a large number of individual goods; the individual goods are typically poor substitutes for each other, resulting in a substantial amount of cross-shipment at the aggregate level. Cross-shipment occurs because individual goods in these sectors obey economies of scale such that a single factory will ship to a region of several or many states. In this case, it is reasonable to assume that:

(10a) $\hat{m}_C(t) = 1$ and

(10b) $\hat{m}J(t) = 1$,

which would be approximately true if most of the individual goods aggregated into these sectors were not produced in Kansas. However, we do NOT make any assumption about $\hat{m}A$. A supplier of an intermediate good may have an incentive to locate close to the plant it supplies, provided that the individual intermediate good produced by the supplier is a specialized demand of that single plant; and therefore $\hat{m}_A$ may be less than 1.

Under this model, $\hat{m}_A(u)$ and $\hat{e}(u)$ are exactly identified if imports and exports are known for the base year u; in particular, at time u (ignoring inventory changes), (10c) $\hat{m}_A = 1 - [Q - G - E]/[aQ]$, and

(10d) $\hat{e} = E/S_{US}$.

The prediction (4) then leads to a weak test of the model. If the m_A coefficient exceeds 1, then there is a data error leading to a negative measure of domestic shipments; it is appropriate to accept the model and set m_A to 1. If m_A is less than zero, then the model must be rejected. Otherwise, we are free to rely on a prior judgment that the model holds and accept the estimators given by (10).

If the model has been accepted, then updates can proceed by using an adaptation of estimator (9). In the unconstrained case, we would have

(11a) $\epsilon = \Delta/[a(t)Q(t) + S_{US}(t)]$

(11b) $\hat{m}_C(t) = 1$

(11c) $\hat{m}J(t) = 1$

(11d) $\hat{m}_A(t) = \hat{m}_A(u) + \hat{\epsilon}a(t)Q(t)$

(11e) $\hat{e}(t) = \hat{e}(u) - \hat{\epsilon}S_{US}(t)$.

If the previous model has been rejected in a sector because $m_A < 0$, then the data have shown conclusively that consumption and investment goods are partly local to Kansas. However, one might reasonably continue to believe that intermediate product goods are even more local than consumption and investment goods. We can generalize the previous import model in a continuous way by estimating at time u:

(12a) $\hat{m}_C = 1 - [Q - G - E - aQ]/[C + J]$

(12b) $\hat{m}J = \hat{m}C$

(12c) $\hat{m}A = 0$

(12d) $\hat{e} = E/S_{US}$.

The trade coefficients can then be updated to time t using (9).

VI. OTHER UPDATABLE COEFFICIENTS AND PARAMETERS:

Coefficients which were originally estimated from annual time series data, can obviously be re-estimated in the same fashion. In particular, depreciation rates and the capital-output ratio are parameters which can be estimated from a regression of national investment on output and lagged investment; they can be re-estimated.

To the extent that state and local government demands by commodity are available annually, the government consumption coefficients can be updated.

Household consumption coefficients might be updated using annual, national aggregate sales and disposable income. However, corrections would be needed for national intermediate demands, foreign trade, government demands, and investment.

Wholesale, retail, and transport margins can be updated from COUNTY BUSINESS PATTERNS data.

Capacity can be updated not only from the usual working of the model, but also from output data.

VII. NON-UPDATEABLE COEFFICIENTS:

There appears to be no obvious way to update the following coefficients on an annual basis:

The investment commodity requirements matrix.

The Kansas ownership share for property income.

Source: Economic Research Technical Note 30.

APPENDIX L
THE FAUCETT DATASET

This appendix provides a brief review of the contents of the Faucett dataset and information on the sources Faucett used in constructing state specific input-output accounts. The "Multi-Regional Input-Output Accounts, 1977," were compiled by Jack Faucett Associates for the U.S. Department of Health and Human Services. Faucett's MRIO was completed in 1983.

Contents of the Faucett Dataset

For each state Faucett has constructed the following for 1977;

1) Use Matrix - showing the use of commodities by industries.

2) Make Matrix - showing the commodity output by industry.

3) Trade Flows Matrix - showing the flows of commodities between states, within states, and to foreign nations, as well as sales taxes, wholesale costs, and transport costs associated with the flows.

4) Value Added - detailing value added by industry.

5) Final Demand - showing commodity expenditures by various final demanders (eg. personal consumption, investment, and government expenditures).

Commodity and industry detail for the above data are approximately 120 sectors.

Data Sources

The Use and Make matrices are based on the Bureau of Economic Analysis' (BEA) 1972 input-output tables. The BEA input-output tables were adjusted and brought down to the state level based mainly on data from the 1977 economic censuses.

Output and value added data at the state level were constructed from the 1977 economic censuses.

Final demand data for personal consumption expenditure is from the BEA National Income and Product Accounts (NIPA). Investment as a final demand is from the BEA. NIPA data is the basis for federal government demands. Personal consumption, investment, and federal government expenditures were brought down to the state level relying primarily on data from the 1977 economic censuses.

Trade flows of commodities are based primarily on the 1977 Census of Transportation, and federal government collected data on railroad transport, water transport, and agriculture shipments. Commodity flows data was adjusted to balance the supply and consumption of each commodity in a given state.

The data on commodity flows provide information on the method of transport (e.g., railroad, trucking, etc.) and on the origin and destination states of a given shipment. This data is the basis for the transportation service sectors trade flows. To calculated transportation service flows for all transport sectors except railroads, transportation revenues were assigned to origin and destination states according to national ratios. For railroad transportation, revenues were distributed to the origin and destination states and to states along the route of the shipment.

For other service sectors (including insurance, real estate, finance, trade, education, and other miscellaneous services) trade flows are difficult to handle. The is no data on inter-state sales of services, and inter-state service flows are difficult to quantify. Faucett chose to base service trade flows on national service sector import coefficients

According to Faucett's introductory volume, "the data on interregional flows are among the weakest in the input-output table." The data on service flows seems particularly suspect. Trade flows of other commodities and transportation services are based on survey data and are hence much more reliable.

This appendix lists only the major data sources used by Faucett, and very briefly describes some of the procedures used in the development of the MRIO accounts. Further details on the Faucett MRIO dataset are available in;

Multiregional Input-Output Accounts, 1977 (volumes 1 through 6),
Jack Faucett Associates,
U.S. Department of Health and Human Services, 1983.

These volumes include documentation of data sources, procedures used to develop the MRIO accounts, and descriptions of the MRIO account.

APPENDIX M
DATA SOURCES AND PARAMETER ESTIMATES

This appendix provides sources of data and descriptions of parameter estimations for the long term model. Within the first 7 sections of this appendix, data sources will be referred to by the letters A through F. In section VIII of this appendix, sources A through F are completely documented.

I. Technical Coefficients Matrix.

The data source for the technical coefficients matrix (or "A matrix") is Faucett's Multi-Regional Input Output model. The data is Kansas specific for the year 1977. See source A, section VIII of this note.

Appendix I:Inferring the A Matrix, explains how the A matrix was constructed from Faucett data.

II. Investment Coefficients Matrix.

The source of the investment coefficients matrix (or "B matrix") is the Bureau of Economic Analysis (BEA) "capital flows table." The capital flows table is for the U.S. in 1977. See source C.

Details of the procedure to convert the capital flows table to an investment coefficient matrix follow.

notation:

b = investment coefficients matrix.

b_{ij} = (dollar amount of commodity i used in investment in industry j)/ (dollar amount of investment in industry j)

C = capital flows table

C_{ij} = dollar amount of commodity i used in investment in industry j

1 = a row of N ones (N = number of sectors)

procedure:

$1C = I$ (I is a vector of the column sums of C)

let $\hat{I}$ be I diagonalized

then $b = C(\hat{I}^{-1})$

III. Trade Data.

Trade data consist of the values of 1977 commodity shipments to and from Kansas and the margins associated with transportation and wholesaling. Faucett was the source for the trade data. See source B.

Appendix F:Trade Margins in the Formal Model, describes the handling of trade margins in the long term model.

IV. Consumption and Final Demand.

Propensities to consume and final demands for Kansas commodities in 1977 are from Faucett. See source A.

V. Investment Data.

Investment by commodity for Kansas in 1977 is found in Faucett. See source A.

Investment by industry was estimated by a proration of US investment by industry for 1977. The procedure used is detailed below.

notation:

I_i^{US} = investment by industry i in the US in 1977.
X_i^{US} = output of industry i in the US in 1977.
X_i^{US} = output of industry i in the US in 1977.

procedure:

Investment by industry i in Kansas for 1977 is:

$$I_i^{KS} = I_i^{US} (X_i^{KS} / X_i^{US}).$$

U.S. investment by industry data is from the BEA. See source C. However, due to the BEA method of handling leased and rented capital equipment, it was decided to use a different source for data on the construction industry. U.S. investment in the construction industry is from the <u>1977 Census of Construction Industries</u>. See source F.

Output by industry data for the US in 1977 is from the BEA input-output model. See source D. Kansas output by industry is from Faucett. See source A.

VI. Capital-to-Output Ratios.

Capital-to-output ratios were calculated from US data on 1977 capital use and 1977 outputs published by BEA. Output is from the BEA input-output model. See source D. Capital use is from a BEA study on fixed private capital. See source E.

VII. Depreciation Rates.

Depreciation rates were inferred from investment data, capital-to-output ratios, and assumptions about expected growth and capacity. The procedure used is detailed below.

notation:

For any commodity i (subscript i suppressed),

Z_t = capacity in year t , measured in dollars of output.
K_t = capital in year t, measured as dollar value of plant and equipment.
g = expected real growth rate
d = depreciation rate
k = capital-to-output ratio
X_t = output in year t
I_t = investment in year t.

procedure:

Capital can be measured in dollars or in capacity (output units). The two measures are related by:

(1) $Z_t = K_t / k.$

Capital changes over time due to depreciation and investment. Equation (1) reflects the growth or decay of capital, measured in capacity units.

(2) $Z_{t+1} = Z_t \ (1-d) + I/k.$

Firms undertake investment to attempt to meet future demand for output. For the year 1977, it is assumed that $Z_t = X_t$. Furthermore, it is assumed that output is expected to grow at a rate g. Desired future capacity is given by:

(3) $Z_{t+1} = X_t(1+g) = Z_t(1+g).$

Combining equations (2) and (3) results in:

(4) $Z_t(g + d) = I/k.$

Depreciation is the only unknown value in equation (4). The solution to (4) provides the estimates of depreciation rates.

The procedure detailed above does not take inflation into account. The procedure used in developing the depreciation rates that are used in the long term model does account for inflation.

Output for Kansas in 1977 is from Faucett. See source A. See section VI of this note for the data sources for capital-to-output ratios.

VIII. Sources of Data.

A. "tape 1-MRIO Data Files, 1977, Use Matrix Data, Make Matrix Data, Output, Employment and Payroll Data." Jack Faucett Associates, U.S. Department of Health and Human Services, August, 1983.

B. "tape 2-MRIO Data Files, 1977, Trade Flow Matrices." Jack Faucett Associates, U.S. Department of Health and Human Services, August, 1983.

C. "New Structures and Equipment by Using Industries, 1977." "Survey of Current Business," U.S. Department of commerce-Bureau of Economic Analysis, vol. 65, no. 11, November, 1985.

D. "The Input-Output Structure of the U.S. Economy, 1977." "Survey of Current Business," U.S. Department of Commerce-Bureau of Economic Analysis, vol. 64, no. 5, May, 1984.

E. "Fixed Reproducible Tangible Wealth in the United States: Revised Estimates." "Survey of Current Business" U.S. Department of Commerce-Bureau of Economic Analysis, vol. 66, no. 1, January, 1986.

F. "Detailed Statistics for Establishments With Payroll, 1977-U.S. Summary-Establishments With Payroll." 1977 Census of Construction Industries-Industry and Area Statistics U.S. Department of Commerce-Bureau of the Census.

For further information on the Faucett data, see appendix L:The Faucett Dataset, and;

Multiregional Input-Iutput Accounts, 1977, (volumes 1-6), Jack Faucett Associates, U.S. Department of Health and Human Services, 1983.

It was often necessary to make adjustments to data to compensate for sectoral definition differences between the data sources. The procedure used involved proration based on industry employment figures for Kansas. The source of this data was,

"The State-Employees, Payroll, and Establishments, by Industry." County Business Patterns, Kansas U.S. Department of Commerce-Bureau of the Census.

Source: Economic Research Technical Note 29.

APPENDIX N
COMPUTER SYSTEM FLOW AND NARRATIVE

Computing activities for the Kansas Long Term Model rely primarily on the University of Kansas IBM mainframe computer and on the SAS software package. Prior to 1986, however, neither SAS nor the IBM were used. The dynamic Leontief input-output model was run in MINITAB, and the dynamic linear programing model was run in MPS. (See Appendix C:An Intellectual History of the Project, for details on early modelling activities.) Both MINITAB and MPS were on the University of Kansas' Honeywell mainframe.

SAS is used for both dataset management and the actual running of the long term model. All datasets used in the model are stored and maintained as SAS datasets. The SAS IML package, a software package for matrix algebra, is used for running the model itself. (See Appendix D:The Formal Model, for descriptions of the algebra involved in solving the model.)

A time series database is currently in the process of being designed and implemented . The variables contained in the database will be incorporated into the long term model as the theoretical structure of the model develops. Examples of variables in the database are, a time series of Kansas personal income, and a time series of price indexes. The database will be managed in SAS on the IBM.

There are four major types of SAS programs that have been written for the long term modelling project.

a) Programs to extract data from a given dataset. For example, a program to extract Kansas trade data from the Faucett trade flow matrices.(See Appendix L:The Faucett Dataset, for details of the contents of Faucett's model.)

b) Programs to aggregate data, in particular to translate Faucett's 121 sectors into the long term model's 11 sectors.

c) Programs to manipulate the data. For example, a program to convert Faucett's make and use matrices to an A matrix. (See Appendix I:Inferring the A Matrix.)

d) Programs to run the model itself.

The micro-computer software packages Lotus 1-2-3 and REFLEX have been used to some extent in the project. Lotus 1-2-3 has been used for dataset management, particularly in the early modelling efforts. REFLEX has been used to plot outputs of the long term model.

The following pages contain a greatly simplified flow chart showing the basic flow of datasets, the programs used to modify and preprocess the data to make it suitable for our modeling program and the major inputs/outputs from the modeling program.

Legend to data flow and program usage:

Intermediate input data and programs:

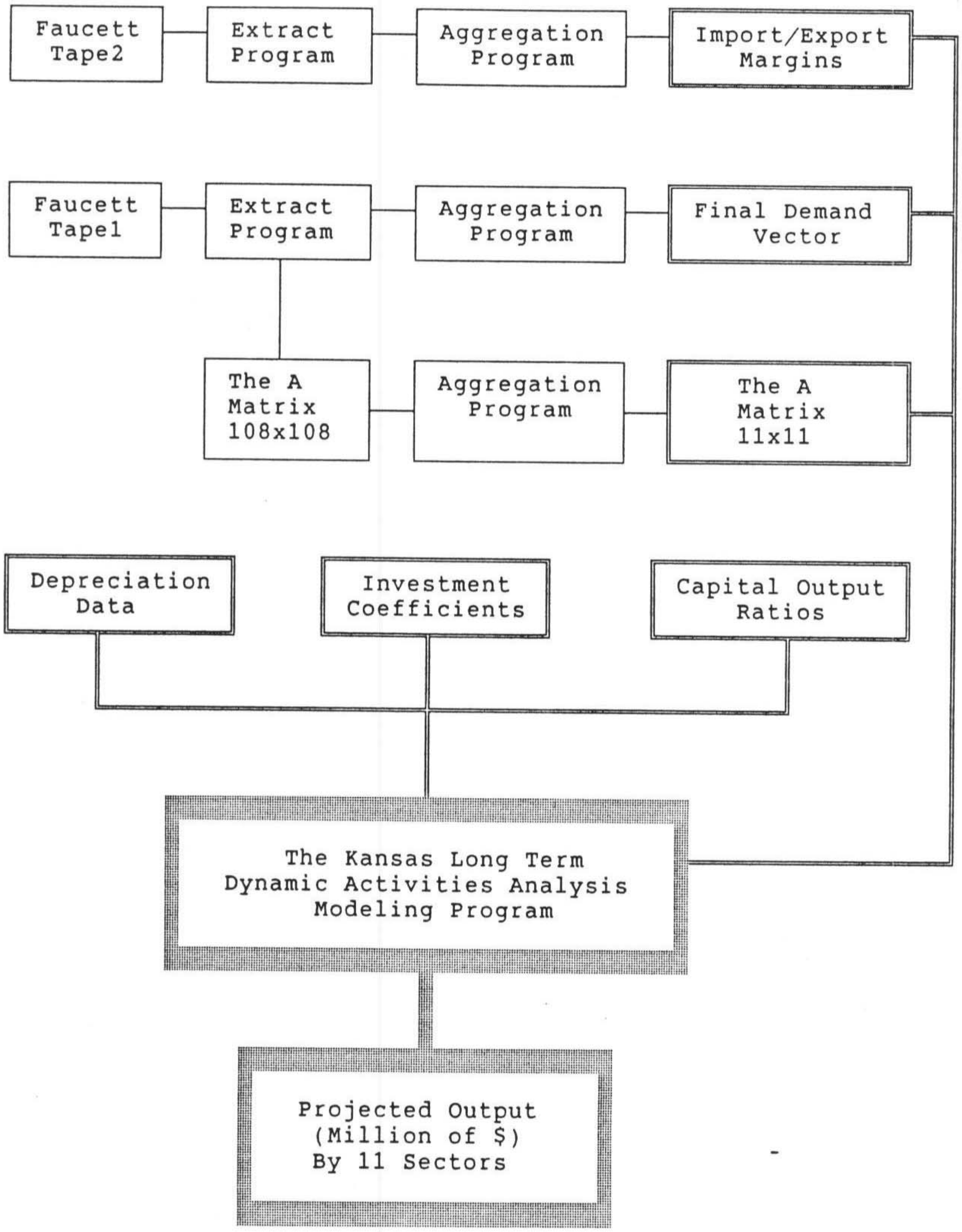
Flow between intermediate data and: preprocessing programs

Major input dataset to the modeling: program

Flow from major preprocessed datasets: to the modeling program

The modeling program and predicted output:

Kansas Long Term Dynamic Activities Analysis Model
Processing Flow Chart



APPENDIX O
PERSONNEL

Mohamed El-Hodiri: Professor El-Hodiri received his Ph.D. from the University of Minnesota and is currently Professor of Economics at the University of Kansas. He was the developer of the I/O Modelling Project and has been the director of the project from its inception.

Anthony Redwood: Professor Redwood received his Ph.D. from the University of Illinois and is currently Professor of Business at the University of Kansas and Director of IPPBR. He has been a co-invesigator of the I/O Modeling Project since its inception.

Farrokh Nourzad: Professor Nourzad received his Ph.D. from the University of Kansas and is currently Associate Professor of Economics at Marquette University. During the summer of 1986, Professor Nourzad was Project Coordinator.

David Burress: Dr. Burress received his Ph.D. from the University of Wisconsin and is currently a Research Associate at IPPBR. He has been the Project Coordinator since August 1987.

Pat Oslund: Ms. Oslund received an M.A. from the University of Kansas and is currently enrolled as a Ph.D. student at the University of Kansas. Ms. Oslund is also a Research Associate at IPPBR and has worked on the I/O Modelling Project since September 1986 as a Research Economist.

Bob Glass: Mr. Glass received an M.A. from the University of Kansas and is currently enrolled as a Ph.D. student at the University of Kansas. Mr. Glass is a Research Associate at IPPBR and has worked on the project since its inception.

Michael Coen: Mr. Coen received an M.A. from the University of Kansas. Mr. Coen is a Research Associate at IPPBR and has been the computing services coordinator since October 1986.

Donna Costello: Ms. Costello received a B.A. from the University of Oklahoma and is currently a Ph.D. student at Rochester University. She worked as a Research Assistant on the project from July 1984 to August 1985.

Michael Eglinski: Mr. Eglinski received a B.A. from the University of Kansas and is currently enrolled as a graduate student at the University of Kansas. Mr. Eglinski has worked as a Research Assistant on the project since August 1985.

Frank Hefner: Mr. Hefner received an M.A. from the University of Kansas and is currently enrolled as a Ph.D. student at the University of Kansas. Mr. Hefner is a Research Assistant at IPPBR and has worked on the project since August 1985.

APPENDIX P
CONTENTS OF ECONOMIC RESEARCH TECHNICAL NOTES (ERTN)

The Economic Research Technical Note series is an "in house" publication by the staff of the modelling project. Its purpose is to formalize thoughts about the model and to disseminate ideas among the researchers at IPPBR. Several of these ERTN's have been used at seminars and presentations outside of the Institute. This appendix consists of a listing of titles and authors of the ERTN's to date with a brief description of their contents.

1. Technical Note Series, David Burress; describes proposed procedures and formats for ERTN's.

2. Some Standard Notations, Frank Hefner and Mike Eglinski; describes notation to be used for ERTN's.

3. Specifications for a Simple Dynamic I/O Model, David Burress; discusses a simple dynamic input-output model to be implemented as an 11 sector pilot study.

4. I/O Impact Multipliers, David Burress; summarizes formulas for computing impact multipliers from a dynamic I/O model.

5. Program Specification-Routine to Aggregate I/O Technical Coefficients, David Burress. **

6. Price Effects in Cobb-Douglas Investment, David Burress. **

7. Aggregating to 11 x 11, Mike Eglinski; describes the aggregation schemes used to aggregate MRIO data and BEA investment data to an 11 sector economy.

8. Accounting Identities in the 1977 MRIO Data, David Burress; documents all known theoretical accounting identities for the Kansas level data of the 1977 MRIO data.

9. Summary of Emerson's Methodology for Constructing I/O Table, Frank Hefner; reviews the methodology Emerson used in constructing his 1965 I/O table for Kansas.

10. Time Series of Sectoral Value Added, Michael Eglinski; describes a procedure for developing a time series of sectoral value added for 11 sectors, including their data sources.

11. Capital Flows Matrix, Michael Eglinski; explains the construction of the capital flows matrix from the capital flows table of the BEA.

12. Aggregation Bias and Creation of the I/O Matrix, David Burress; compares two methods of aggregating the I/O coefficients matrix, the 'a' matrix.

13. Dynamic Leontief Model, Frank Hefner; explains the construction of the dynamic Leontief model and the results therefrom.

14. Dynamic Linear Programming Model, Frank Hefner and Bob Glass, documents the construction and results of the dynamic LP model at IPPBR.

15. void

16. void

17. Computer Program for Aggregation, Michael Eglinski; describes a program written by Mike Coen for aggregating a matrix.

18. Inferring I/O Coefficients from Industry Flows, David Burress; discusses techniques for inferring the A matrix from industry flows in the use and make matrices.

19. Generalized Block Diagram for Dynamic Regional Economic Models, David Burress; a flow diagram describing regional models.

20. unissued.

21. I/O Bibliography, Frank Hefner.

22. I/O Model Time Series Data Naming Conventions, David Burress; describes rules for naming annual time series data in the I/O model.

23. Internal Data Documentation, Pat Oslund; provides a procedure for documenting source records for the data used in the model.

24. 1977 Kansas Exports from Faucett, Mike Eglinski; provides the relative sizes of exports from the different commodity sectors from Kansas in 1977 as recorded in the Faucett data.

25. 11 Sector Multipliers, Pat Oslund and David Burress; reports the aggregate 11 sector, steady-state multipliers predicted by the current version of the Kansas Long Term Model.

26. Methodology for Time Series: Kansas Employee Compensation by 11 Sectors, Mike Eglinski.

27. Long Run Projection Accuracy of the I/O Model, David Burress; illustrates a minimum level of projection accuracy which can be achieved using the Kansas Long Term Model.

28. Inferring the a Matrix from Available Data, David Burress; demonstrates a method to construct a Leontief type a matrix from the make and use matrices.

29. Data Sources and Parameter Estimates, Michael Eglinski.

30. Non-Survey Means for Updating Coefficients Annually, David Burress; gives a procedure for periodically updating the coefficients of the long-term model.

31. Specification of a Simple Dynamic Input-Output Model, David Burress and Bob Glass; describes the mathematical structure of the Kansas Long Term Model.

Note: ** signifies that the ERTN is in the process of being written.

APPENDIX Q

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