AN EXPLANATION AND EXPLORATION OF IANNIS XENAKIS'S WORKS FOR SOLO PERCUSSION: REBONDS AND PSAPPHA IN PERFORMANCE AND ANALYSIS.

By

Winston Gregory Haynes II

M.M., University of Georgia 2006

Submitted to the graduate program in the Department of Music and Dance
Graduate Faculty of the University of Kansas
In partial fulfillment of the requirements for the degree of
Doctor of Musical Arts

Date defended May 14, 2009
The Document Committee for Winston Gregory Haynes II certifies that this is the approved version of the following dissertation:

AN EXPLANATION AND EXPLORATION OF IANNIS XENAKIS’S WORKS FOR SOLO PERCUSSION: REBONDS AND PSAPPHA IN PERFORMANCE AND ANALYSIS.

Date approved: **May 21, 2009**
An Explanation and Exploration of Iannis Xenakis's Works for Solo Percussion: 

*Rebonds* and *Psappha* in Performance and Analysis.

Iannis Xenakis (1922-2001), 20th century architect turned composer, is an influential figure among musical thinkers, highly regarded for his developments in compositional processes, multi-media performances, and electro-acoustics. Although Xenakis’s music is often extolled for its uniquely rigorous craft and complexity, its governing processes are difficult to understand and often require knowledge of advanced mathematics and number theory. This situation can be especially troublesome in the musical performance context for all parties involved. As the performer seeks to express and the audience to make musical connections, Xenakis’s material often presents a conceptual roadblock. While program notes can be helpful, they are usually not sufficient to generate the level of understanding requisite to explain the composer’s detailed aesthetic ideas and compositional processes. An analytic pre-talk providing an appropriate background with focus on the works to be performed may function well to invoke an appreciation and accessible rendering of Xenakis’s music that may otherwise be impossible in the performance context. Henceforth, this stands as the purpose of the ensuing presentation.

Particular to this presentation, the young, yet burgeoning solo percussion idiom is considered, with focus on the composer’s total applicable output. Xenakis’s two solo percussion works, *Psappha* (1975) and *Rebonds* (1988)
influenced and inspired a new generation of musicians, including percussion virtuoso Steven Schick, who would seek to establish the solo percussion genre as a respected musical medium in the art music context.¹ These works are seminal with respect to contemporary musical developments and, appropriately, have attracted a high level of attention from music theorists and musicologists alike. \textit{Psappha}, in particular, will receive primary attention in this presentation due to its theme construction using known sieves, which are logical representations stemming from an ancient mathematical process Xenakis has pioneered for the purpose of music composition. In realizing the theory behind the composer’s methods, we aim to come closer to an appreciation of the material produced.

Before descriptions of musical techniques and compositional processes can begin in earnest, it is necessary to consider some biographical background and context. Thus, a brief description of the composer’s early life and career establishes a framework for the music in question. Born on May 29th, 1922 to a Greek couple in Romania, Xenakis moved to the island of Spetsai in Greece at age ten to attend boarding school.² Here, he studied both science and Greek literature, entering the Athens Polytechnic School to study civil engineering at age 16.³ In the course of his education, emphasizing architecture and


mathematics, Xenakis took an active role as a participant in the Greek Nazi-resistance movement during the early 1940's. Xenakis, characterizing his political passion, states in a conversation with Bálint András Varga:

We, the younger generation, hoped not only that the war would one day end; we wanted social changes - a more just society. We wanted the land to be cultivated, the mines to be exploited more effectively, to get rid of foreign influence, to be free.\(^4\)

It was during this period that Xenakis was famously hit in the face by a tank shell, disfiguring the left side of his head. After a miraculous recovery and being drafted into the Greek military, Xenakis escaped the political turmoil and vacated illegally to France where he found work in the studio of the famous architect, Le Corbusier.\(^5\)

While working in Paris, and contributing to major building projects\(^6\), Xenakis turned his attention to the serious study of music and composition at the late age of 25, having received only a modicum of prior training from his parents and elective courses taken in piano and music theory at the Polytechnic School.\(^7\) He sought out many great composers and teachers with whom to study music including Arthur Honegger, Nadia Boulanger (who turned him away), and

---

\(^3\)Ibid.


\(^5\)Hoffmann, "Xenakis, Iannis."

\(^6\)A career culminating in the design of the Philips Pavilion in Brussels for which Xenakis also composed the electro-acoustic work *Concret PH* for the 1958 World Fair installation

\(^7\)Hoffmann, "Xenakis, Iannis."
Olivier Messiaen. Xenakis, as he reports in a variety of interviews regarding this time in his life, found himself disappointed in the limiting nature of his training with these major figures. While owing a great debt to their influences, Xenakis sought to discover his own musical language, pulling from the resources associated with his background and cultivating an interest in electronic music production as well as in acoustic instrumental forms.

In the early 1950's, Xenakis set into serious composition with *Metastasis* for 60-piece orchestra, generally recognized as being his first major work. From the late 1950's through the 1970's, Xenakis worked in a variety of musical forms, including orchestral, chamber, solo, and electro-acoustic media. Several experimental pieces color this early musical yield, reflecting Xenakis's unusual technical training. These include such pieces as *Duel* (1959) and *Stratégie* (1962), utilizing two separate orchestras with basis on game theory, and the *Polytope de Montréal* (1967), the first of several works combining elements of architectural design, electronic sound production, and light manipulation. An examination of his compositions reveals Xenakis's love of theoretical mathematics, especially conceptually elegant integer sets such as the Fibonacci sequence and the set of prime numbers. In 1971, Xenakis completed his book

---


10these two instances to be discussed in detail later with regard to *Psappha*
*Formalized Music*, which articulates his ideas in detail regarding mathematics and its role in his composition. While approaching this text without a theoretical mathematical training proves an exercise in futility, a guided explanation of specific techniques can prove a fruitful endeavor with approach to the works that utilize them. One particular process Xenakis utilizes, that of sieving numbers, is credited to Eratosthenes, a Greek mathematician who lived from 276-194 B.C.¹¹ Thus, this technique proves a natural choice for Xenakis, encapsulating his interests in both elegant mathematical design and Greek nationalism. In approaching the particulars of Xenakis's early life and career, several significant compositional elements have already become apparent. As this discussion delves deeper into the construction of particular works, it may be helpful to refer to Appendix A, which provides an explanation and instance of sieve theory, detailing the famous Sieve of Eratosthenes as an example of a logically encapsulated process.

Highly relevant to the discussion is the composer's individual aesthetics of music. Xenakis articulates his musical philosophy in different ways at different times in his life, often in interviews, updating and tempering his ideas through the passing years. One common thread, however, is that Xenakis believes in the necessity of utilizing what he calls "outside-time" and "temporal" structures in conjunction with one another to render "inside-time" musical

entities that are rich and meaningful.\textsuperscript{12} By using these terms, Xenakis means to articulate that there must exist an abstract thematic element that is independent of its own musical realization on a two-dimensional time scale. Thus, a sieve expressed as a logical statement, such as the one detailed in the appendix, satisfies such requisite time independence conditions for Xenakis. To more fully understand why classifying entities in a time-relevant manner is important to the composer, one may consider his criticism of serialism. In his 1975 interview with Michael Zaplitny, Xenakis states:

\begin{quote}
Serial music is a typical in-time structure. The relationship of the notes in the twelve-tone scale is one of the simplest outside-of-time structures because you are repeating exactly the same chromatic interval creating the twelve tones. When you take these notes which are outside-of-time and, so to speak, put them in a bag or pick them up in a certain order, you are doing a time ordering, you are putting them in-time. So any serial string of notes is something which is in-time, not outside-of-time. That's very important.\textsuperscript{13}
\end{quote}

Xenakis goes on to express that serialism produces music without richness due to this compositional reliance on rows, necessitating ordering without expressing interesting relationships existent abstractly and independently from time.\textsuperscript{14} Regardless of considering whether or not this criticism holds value, it is

\begin{itemize}
  \item \textsuperscript{12}Ellen Rennie Flint, "An Investigation of Real Time as Evidenced by the Structural and Formal Multiplicities in Iannis Xenakis' \textit{Psappha}," (Ph.D. diss., University of Maryland College Park, 1989), 57.
  \item \textsuperscript{13}Michael Zaplitny and Iannis Xenakis, "Conversation with Iannis Xenakis," \textit{Perspectives of New Music} 14, no. 1 (Autumn-Winter 1975): 96.
  \item \textsuperscript{14}Ibid.
\end{itemize}
clear that Xenakis means to differentiate his music markedly from serialism methods and aesthetics, creating something that, to him, exudes some greater, higher dimensional compositional richness. One might even compare Xenakis's compositional ideal as an analog to Plato's cave analogy from *The Republic*, in which the representations of perceived entities are truly just the derivatives from abstract forms. The usage of sieves, then, seems quite attractive then, functioning as abstract time-independent forms for the composer, satisfying a complex musical philosophy in addition to aspects related to nationalistic and intellectual agendas.

As we move forward now to consider the particulars relating to Xenakis's piece *Psappha*, we realize an overwhelming congruence with the general ideas expressed thus far. Completed for percussionist Sylvio Gualda in 1975 and premiered in London in May of 1976, *Psappha* takes compositional inspiration in accordance with its namesake, an archaic spelling of the Greek lyric poetess Sappho.15 Sappho proves an especially suitable muse for Xenakis's work as he credits her with being the first of the poets to utilize changes and transformations among textural rhythmic patterns.16 Xenakis designates *Psappha* as an intuitively composed work, utilizing sieves only to create

---

15Flint, "Investigation," 5, 222.
materials with which to later manipulate directly. Thus, the piece as a whole is less rigorous in mathematical determinacy than some of the composer's previous works governed entirely by sieves. Xenakis employs a concept he refers to as creating arborescences, freely branching and developing structures, which corresponds well with his aforementioned claim. To invoke the sense of a changing and transforming poetic meter, Xenakis creates a process that results in the scattering of iambic feet, two-part cells containing a short syllable followed by a long syllable, throughout his primary theme.

Another important feature particular to Psappha regards its interesting and unusual system of notation. Xenakis uses a grid-like template, in which dots represent musical attacks, to represent the typical elements of musical concern: instrumentation, time, dynamics, and pitch (relative in this case). In standard fashion, time is represented horizontally, with each grid unit representing one unit of time as defined by the section's tempo; both pitch and instrumentation are represented vertically. For example, an excerpt of this notation follows:

\[\text{\ldots}\]

17Ibid., 24.

18Flint, "Investigation," 368.

19Ibid., 243.
An important aspect to notice from Figure 1 is the absence of regular metric dividers, or any structural equivalent; this maintains for the entire piece. As shown on the left side of this grid, the horizontal lines are stratified into different groupings, represented by capital letters, and are further identified using Arabic numerals. Each line represents a different single-toned percussion instrument. The numerals indicate relative pitch, ‘1’ being highest and ‘3’ being lowest, and the letters indicate the instrumental groupings. These groupings are disclosed on the final page of the score, breaking down thusly:

**Figure 2. Legend from Xenakis’s Psappha**

- **Group A:** high wooden or membrane instrument (1-3)
- **Group B:** medium wooden or membrane instruments (1-3)
- **Group C:** low membrane instruments (1-3)
- **Group D:** medium metallic instruments (1-3)
- **Group E:** neutral metallic instrument (single line)
- **Group F:** very high metallic instruments (1-3)
Hence, there are a total of sixteen percussion instruments required for the piece. The number of groups in use in any given section within the piece varies, contributing to the formal breakdown described later in this presentation.

We now move to consider Xenakis's usage of sieves in a definitive manner for Psappha. In order to capture an explanation of this method, I will largely be presenting and interpreting the work of Ellen Rennie Flint, who succeeded in accessing Xenakis's original compositional sketches in research for her Ph. D. dissertation, "An Investigation of Real Time as Evidenced by the Structural and Formal Multiplicities in Iannis Xenakis' Psappha." This document comprises, to date, the most in-depth and comprehensive analysis of Psappha from a theoretical and compositional perspective. While the work makes several significant strides in its 590 pages of text, the identification of Xenakis's sieves and their usage becomes the most significant aspect for our present purpose, especially considering that a distillation of sieve methods without disclosure from the composer himself would be close to impossible.²⁰ The complex sieve instances and derivation tables expressed in the next section, therefore, come directly from Flint's document.

In overview, Xenakis's process of generating the theme for Psappha entails the use of some existing mathematical entities in combination with a couple creative choices and sieve statements produced directly from the

---

²⁰barring the use of such statistical technique as described by theorist Evan Jones in his 2001 Perspectives article, "Residue-Class Sets in the Music of Iannis Xenakis: An Analytical Algorithm and a General Intervallic Expression"
composer's imagination. These elements in cooperation yield a sub-set of integers that, when put into a time-ordered context, create the building blocks from which *Psappha* flourishes. As the process unfolds, it is important to consider the origin of the materials and how Xenakis interfaces with them. In this way, we can consider two types of out-of-time structures Xenakis uses in combination to construct these primary thematic materials. Structures of the first type are elementary sieves called residue classes. A residue class is defined as "a set of integers containing exactly those integers which are congruent, modulo *m*, to a fixed integer."\(^{21}\) Another way of describing this, though, is to consider a residue class as a set of integers, in which each member yields a common remainder when divided by the modulus. Thus, the number of possible residue classes existing for a modulus *m* is, in fact, identical to *m*, since there are only that many numbers in the module with which to establish congruence.

Consider the following partial residue classes for the modulus 8, in the following table:

\(^{21}\)Shockley, *Introduction*, 43.
Although a complete residue class continues infinitely into both positive and negative integers, the table above indicates only those 40 set members limited to the integer domain between 0 and 39, inclusive. The notation here indicates modulus 8 simply with the congruence integer in subscript. Now consider the next table detailing the residue classes for the modulus 5 within the same domain range.

It is precisely these two moduli, 8 and 5, that Xenakis creatively chooses to establish a partitioned set on which his more complex sieve will operate.

---

22Flint, "Investigation," 229.

23Ibid.
To determine why Xenakis would have selected these numbers, we recall the composer's fascination with both prime numbers and the Fibonacci sequence, which is defined as starting with 0 and 1 with each consecutive member being composed from the sum of the two previous members. Consider the following sketch made by Xenakis as it is reproduced from Flint's dissertation.

Figure 5. Xenakis sketch from Flint dissertation.24

While 8 is not a prime number, 2 is one. Considering that Xenakis likely conceived of the number 8 as being equivalent to $2^3$, he falls back on both 2 and 5 as prime sources, while also representing four sequential numbers in the Fibonacci sequence: 2, 3, 5, 8.25

Returning our attention to the residue classes at hand, each class modulo 8 intersects with exactly one member from each class modulo 5 across the 0 - 39 integer domain. The following table displays these intersections:

---

24Ibid., 228.

25A more compelling example for Xenakis's use of the Fibonacci sequence comes at the end of Psappha, where the sequence is represented thusly in the accent pattern of the C3 line starting at 2252T: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55.
Figure 6. Residue class intersections for 8 and 5.\textsuperscript{26}

\begin{tabular}{|c|c|c|c|c|}
\hline
 & 5_0 & 5_1 & 5_2 & 5_3 & 5_4 \\
\hline
8_0 \rightarrow & 0 & 16 & 32 & 8 & 24 \\
8_1 \rightarrow & 25 & 1 & 17 & 33 & 9 \\
8_2 \rightarrow & 10 & 26 & 2 & 18 & 34 \\
8_3 \rightarrow & 35 & 11 & 27 & 3 & 19 \\
8_4 \rightarrow & 20 & 36 & 12 & 28 & 4 \\
8_5 \rightarrow & 5 & 21 & 37 & 13 & 29 \\
8_6 \rightarrow & 30 & 6 & 22 & 38 & 14 \\
8_7 \rightarrow & 15 & 31 & 7 & 23 & 39 \\
\hline
\end{tabular}

Indeed, the residue class sets of both 8 and 5 can be said to be congruent to each other modulo 40. This simply means that the same intersection table structure will reproduce itself in modules of 40-member sets across any integer domain. Using these residue classes and their intersections, Xenakis generates the following sieve representation, establishing his primary creative out-of-time structure:

Figure 7. Sieve $\alpha_1$ from Flint dissertation.\textsuperscript{27}

$\alpha_1 \rightarrow [(8_0 \cup 8_1 \cup 8_7) \cap (5_1 \cup 5_3)] \cup [(8_0 \cup 8_1 \cup 8_2) \cap 5_0] \cup (5_3 \cup (5_0 \cup 5_1 \cup 5_2 \cup 5_3 \cup 5_4)) \cup (8_4 \cap (5_0 \cup 5_1 \cup 5_2 \cup 5_3 \cup 5_4)) \cup (8_5 \cup 8_6) \cap (5_2 \cup 5_3 \cup 5_4)) \cup (8_1 \cap 5_2) \cup (8_6 \cap 5_1)$

It is important to note here that the Figure 7 statement is created entirely from the composer’s own stipulation; any other possible means of its generation are unknown. Notice that the statement composes itself entirely of the unions and

\textsuperscript{26}Flint, "Investigation," 230.

\textsuperscript{27}Ibid., 228.
intersections between the residual classes displayed in Figure 6. To find the group of integers this sieve yields, the distributive property can be employed within each set of square brackets. This produces the following table:

Figure 8. Sieve \( \alpha_t \), distributed.

<table>
<thead>
<tr>
<th>1st Bracket</th>
<th>2nd Bracket</th>
<th>3rd Bracket</th>
<th>4th Bracket</th>
<th>5th Bracket and rest</th>
</tr>
</thead>
<tbody>
<tr>
<td>8051 = 16</td>
<td>8050 = 0</td>
<td>8350 = 35</td>
<td>8450 = 20</td>
<td>8552 = 37</td>
</tr>
<tr>
<td>8053 = 8</td>
<td>8150 = 25</td>
<td>8351 = 11</td>
<td>8451 = 36</td>
<td>8553 = 13</td>
</tr>
<tr>
<td>8151 = 1</td>
<td>8250 = 10</td>
<td>8352 = 27</td>
<td>8452 = 12</td>
<td>8554 = 29</td>
</tr>
<tr>
<td>8153 = 33</td>
<td></td>
<td>8353 = 3</td>
<td>8453 = 28</td>
<td>8652 = 22</td>
</tr>
<tr>
<td>8154 = 31</td>
<td></td>
<td>8354 = 19</td>
<td>8454 = 4</td>
<td>8653 = 38</td>
</tr>
<tr>
<td>8253 = 23</td>
<td></td>
<td></td>
<td></td>
<td>8654 = 14</td>
</tr>
</tbody>
</table>

These integers, when listed in the following ascending order determine the exact placement of attacks in line B2 of the first 40 time units in Psappha: 0, 1, 3, 4, 6, 8, 10, 11, 12, 13, 14, 16, 17, 19, 20, 22, 23, 25, 27, 28, 29, 31, 33, 35, 36, 37, 38.

Thus, the first primary theme is realized in real time as evidenced from the notation shown below.

Figure 9. Excerpt from Psappha, 0-40T.
Notice the exact correspondence between line B2 and the set of integers produced from the sieve $\alpha_1$. Additionally, note that lines B1 and B3 simply highlight the iambic qualities present in B2, B1 filling in the gaps while B3 emphasizes the long syllable attacks. Regarding his theme, it becomes clear that Xenakis operates on his own aesthetic terms, using a combination of out-of-time and temporal structures.

Xenakis also composes a variation to this theme using the same mathematical process directly thereafter, but substitutes different moduli into the sieve. Specifically, he replaces modulus 8 with modulus 7 and modulus 5 with modulus 6. Hence, the residue classes for both 7 and 5 within the integer domain range 0-41 and their points of intersection are shown in the following tables, also represented in Flint’s dissertation:

Figure 10. Partial residue class table for modulus 7.28

<table>
<thead>
<tr>
<th>$7_0$</th>
<th>0</th>
<th>7</th>
<th>14</th>
<th>21</th>
<th>28</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7_1$</td>
<td>1</td>
<td>8</td>
<td>15</td>
<td>12</td>
<td>29</td>
<td>36</td>
</tr>
<tr>
<td>$7_2$</td>
<td>2</td>
<td>9</td>
<td>16</td>
<td>23</td>
<td>29</td>
<td>37</td>
</tr>
<tr>
<td>$7_3$</td>
<td>3</td>
<td>10</td>
<td>17</td>
<td>24</td>
<td>30</td>
<td>38</td>
</tr>
<tr>
<td>$7_4$</td>
<td>4</td>
<td>11</td>
<td>18</td>
<td>25</td>
<td>31</td>
<td>39</td>
</tr>
<tr>
<td>$7_5$</td>
<td>5</td>
<td>12</td>
<td>19</td>
<td>26</td>
<td>32</td>
<td>40</td>
</tr>
<tr>
<td>$7_6$</td>
<td>6</td>
<td>13</td>
<td>20</td>
<td>27</td>
<td>33</td>
<td>41</td>
</tr>
</tbody>
</table>

---

28Ibid., 233.
Figure 11. Partial residue class table for modulus 6.29

<table>
<thead>
<tr>
<th>60</th>
<th>61</th>
<th>62</th>
<th>63</th>
<th>64</th>
<th>65</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
</tr>
<tr>
<td>61</td>
<td>7</td>
<td>13</td>
<td>19</td>
<td>25</td>
<td>31</td>
</tr>
<tr>
<td>62</td>
<td>8</td>
<td>14</td>
<td>20</td>
<td>26</td>
<td>32</td>
</tr>
<tr>
<td>63</td>
<td>9</td>
<td>15</td>
<td>21</td>
<td>27</td>
<td>33</td>
</tr>
<tr>
<td>64</td>
<td>10</td>
<td>16</td>
<td>22</td>
<td>28</td>
<td>34</td>
</tr>
<tr>
<td>65</td>
<td>11</td>
<td>17</td>
<td>23</td>
<td>29</td>
<td>35</td>
</tr>
</tbody>
</table>

Figure 12. Residue class intersections for 7 and 6.30

<table>
<thead>
<tr>
<th>60</th>
<th>61</th>
<th>62</th>
<th>63</th>
<th>64</th>
<th>65</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td>28</td>
<td>35</td>
</tr>
<tr>
<td>71</td>
<td>1</td>
<td>8</td>
<td>15</td>
<td>22</td>
<td>29</td>
</tr>
<tr>
<td>72</td>
<td>30</td>
<td>37</td>
<td>2</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>73</td>
<td>24</td>
<td>31</td>
<td>38</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>74</td>
<td>18</td>
<td>25</td>
<td>32</td>
<td>39</td>
<td>4</td>
</tr>
<tr>
<td>75</td>
<td>12</td>
<td>19</td>
<td>26</td>
<td>33</td>
<td>40</td>
</tr>
<tr>
<td>76</td>
<td>6</td>
<td>13</td>
<td>20</td>
<td>27</td>
<td>34</td>
</tr>
</tbody>
</table>

Xenakis’s new sieve representation also reflects the change in moduli as indicated below:

Figure 13. Sieve $\alpha'_1$ from Flint dissertation.31

$$\alpha'_1 \rightarrow [(7_0 \cup 7_1 \cup 7_2) \cap (6_1 \cup 6_3)] \cup [(7_0 \cup 7_1 \cup 7_2) \cap 6_0] \cup [7_3 \cap (6_0 \cup 6_1 \cup 6_2 \cup 6_3 \cup 6_4 \cup 6_5)] \cup [7_4 \cap (6_0 \cup 6_1 \cup 6_2 \cup 6_3 \cup 6_4 \cup 6_5)] \cup [(7_5 \cup 7_6) \cap (6_2 \cup 6_3 \cup 6_4)] \cup (7_1 \cap 6_2) \cup (7_6 \cap 6_1)$$

\[29\]Ibid., 233.

\[30\]Ibid., 234.

\[31\]Ibid., 235.
It is important to realize when evaluating this statement that adjustments have been made to account for the altered quantifications in the moduli. Note that equivalent residue classes are present in the $\alpha_1'$ statement (i.e. $7_0 = 7_7$) and the existence of $(n + 1)$ residue classes of modulus 6, compared with $n$ residue classes of modulus 5 is accounted for in the 3rd and 4th bracketed intersection sets. To attempt a more generalized sieve to account for these two variants, I present the following statement independent from Flint’s work:

Figure 14. Xenakis’s sieve, generalized.

For two moduli, $n$ and $m$, with residue classes $0, \ldots, j-1$ and $k-1$, respectively, where $n=j$ and $m=k$:

$$\alpha_s \rightarrow [(n_0 \cup n_1 \cup n_7) \cap (m_1 \cup m_3)] \cup [(n_0 \cup n_1 \cup n_2) \cap m_0] \cup [n_3 \cap (m_0 \cup \ldots \cup m_{k-1})] \cup [n_4 \cap (m_0 \cup \ldots \cup m_{k-1})] \cup [(n_5 \cup n_6) \cap (m_2 \cup m_3 \cup m_4)] \cup (n_1 \cap m_2) \cup (n_6 \cap m_1)$$

This merely observes some of the adjustments Xenakis makes to his sieve instances. It is interesting to note here, however, that Xenakis insists on the usage of several specific residue classes without proportional adjustment to the modulus. One can only hypothesize that he perhaps foresaw set yield characteristics relevant to the iambic nature of the resulting integers.

The resulting integers from the $\alpha_1'$ sieve using the moduli 7 and 6 intersections are indicated in the following table:
Figure 15. Sieve $\alpha_1^*$, distributed.

<table>
<thead>
<tr>
<th>1st Bracket</th>
<th>2nd Bracket</th>
<th>3rd Bracket</th>
<th>4th Bracket</th>
<th>5th Bracket and rest</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7661 = 7$</td>
<td>$7660 = 0$</td>
<td>$7360 = 24$</td>
<td>$7460 = 18$</td>
<td>$7562 = 26$</td>
</tr>
<tr>
<td>$7663 = 21$</td>
<td>$7160 = 36$</td>
<td>$7361 = 31$</td>
<td>$7461 = 25$</td>
<td>$7563 = 33$</td>
</tr>
<tr>
<td>$7161 = 1$</td>
<td>$7260 = 30$</td>
<td>$7362 = 38$</td>
<td>$7462 = 32$</td>
<td>$7564 = 40$</td>
</tr>
<tr>
<td>$7163 = 15$</td>
<td></td>
<td>$7363 = 3$</td>
<td>$7463 = 39$</td>
<td>$7662 = 20$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$7364 = 10$</td>
<td>$7464 = 4$</td>
<td>$7663 = 27$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$7365 = 17$</td>
<td></td>
<td>$7664 = 34$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$7162 = 8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$7661 = 13$</td>
</tr>
</tbody>
</table>

From here we can order the integers as follows: 0, 1, 3, 4, 7, 8, 10, 11, 13, 15, 17, 18, 20, 21, 24, 25, 26, 27, 30, 31, 32, 33, 34, 36, 38, 39, 40. Considering their usage to determine attack points in line B2, we compare this new integer yield to the score notation.

Figure 16. Excerpt from *Psappha*, 40-80T.
Upon comparison, it becomes obvious that, in addition to the sieve adjustments described earlier, other surface adjustments have also been made. Flint suggests in her dissertation that most of the adjustments are likely present to enhance the iambic nature of the passage and account for the inequality between the 40 and 42 time units ranges realized by the two distinct residue class intersection tables in Figures 6 and 12.\textsuperscript{32} The most noticeable of these adjustments, especially in a performance context, is the sole quintuplet rhythm occurring at 70T. This quintuplet condenses the total temporal space of the second sieve realization, causing the number of time points to become equal with that of the initial 0-40T section.\textsuperscript{33}

Following these methods described above for determining the primary iambic passages, Xenakis engages in his arborescences, thus departing from the earlier compositional rigors of sieves and developing materials congruent to his intentions as previously stated. While internal rhythmic transformations are difficult to assert due to the lack of higher metric partitions, a formal description proves highly useful for the performance context, as compositional significances will become apparent on the macro scale. In examining different ways that scholars have sectionalized Psappha, the criteria determining formal breaks

\textsuperscript{32}Ibid., 238-241.

\textsuperscript{33}Ibid., 238.
generally consist of marked tempo changes, instrument group changes, and changes in modules of activity. At the highest level, tempo changes divide the work clearly into five macro sections. These sections, their starting points, and tempi are indicated in the table below:

Figure 17. Macro sections in *Psappha* as determined by tempo changes

<table>
<thead>
<tr>
<th>Section</th>
<th>Starting point (T)</th>
<th>Tempo</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>152 MM</td>
</tr>
<tr>
<td>B</td>
<td>740</td>
<td>272 MM</td>
</tr>
<tr>
<td>C</td>
<td>990</td>
<td>110 MM</td>
</tr>
<tr>
<td>D</td>
<td>1720</td>
<td>134 MM</td>
</tr>
<tr>
<td>E</td>
<td>2175</td>
<td>152 MM</td>
</tr>
</tbody>
</table>

Since Xenakis indicates *Psappha* as "a purely rhythmical composition" in his interview with Simon Emmerson, it makes sense that the work should be divided with accord to the passage of time.35

Beyond this large-scale division, both Flint in her dissertation and James Harley in his book *Xenakis: His Life in Music* make an effort to define sectional subdivisions according to shifts in timbral or modular activity. A brief

---


comparison of these subdivisions via the following charts provides an overview of how the piece can be interpreted.

Figure 18. Comparison of sectional subdivisions in *Psappha* by Harley and Flint

18.a. Harley subdivisions

<table>
<thead>
<tr>
<th>Subdivision</th>
<th>Starting point (T)</th>
<th>Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>No division</td>
</tr>
<tr>
<td>B</td>
<td>745</td>
<td>No division</td>
</tr>
<tr>
<td>C1</td>
<td>1000</td>
<td>New tempo</td>
</tr>
<tr>
<td>C2</td>
<td>1238</td>
<td>Introduction of groups D and E</td>
</tr>
<tr>
<td>C3</td>
<td>1614</td>
<td>Start of Accelerando</td>
</tr>
<tr>
<td>D1</td>
<td>1723</td>
<td>New tempo</td>
</tr>
<tr>
<td>D2</td>
<td>2023</td>
<td>Roll figures begin, replacing single attacks</td>
</tr>
<tr>
<td>E</td>
<td>2175</td>
<td>No division</td>
</tr>
</tbody>
</table>

18.b. Flint subdivisions

<table>
<thead>
<tr>
<th>Subdivision</th>
<th>Starting point (T)</th>
<th>Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0</td>
<td>Start</td>
</tr>
<tr>
<td>A2</td>
<td>525</td>
<td>Fragmentation and transformation of themes</td>
</tr>
<tr>
<td>B</td>
<td>745</td>
<td>No division</td>
</tr>
<tr>
<td>C1</td>
<td>1000</td>
<td>New tempo</td>
</tr>
<tr>
<td>C2</td>
<td>1205</td>
<td>&quot;Multiple successive attack formations begin&quot;36</td>
</tr>
<tr>
<td>C3</td>
<td>1411</td>
<td>Increased density and</td>
</tr>
</tbody>
</table>

36Flint, "Investigation," appendix 1.
simultaneous usage of all instrument classes

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>C4</td>
<td>1614</td>
<td>Start of Accelerando, new rhythmic modules</td>
</tr>
<tr>
<td>D1</td>
<td>1723</td>
<td>New tempo</td>
</tr>
<tr>
<td>D2</td>
<td>2023</td>
<td>Roll figures begin, replacing single attacks</td>
</tr>
<tr>
<td>E</td>
<td>2175</td>
<td>No division</td>
</tr>
</tbody>
</table>

From these charts, we can gather a sense of what elements are compositionally important while the rhythmic arborescences on the surface level are freely developing.

We now move to consider Xenakis’s latter solo percussion work *Rebonds*, first performed, again, by percussionist Sylvio Gualda in 1988 at the Festival Roma Europa in Villa Medici, almost 15 years after premiering *Psappha*. Upon first glance, *Rebonds* appears more conventional than *Psappha*, utilizing traditional notation and specifying existing percussion instruments to be used as different voices. The work is partitioned into two movements, $a$ and $b$, which are to be performed in either possible order. Both movements call for a similar instrumentation; $a$ requires 2 bongos, 3 concert toms, and 2 bass drums, while $b$ requires 2 bongos, tumba, concert tom, bass, and 5 woodblocks. Thus, the piece can be rendered without a significant reset time between movements.

---

37Varga, *Conversations*, 236.

One feature of the work attracting immediate attention is the inclusion of drums that carry a cultural connection, the bongos and tumba being associated with African and South American musical contexts. Despite the assertion by Jacques Lonchampt, music critic for *Le Monde*, that *Rebonds* is divorced from what he calls "folkloristic contamination," Xenakis did observe players while in Africa prior to composing *Rebonds*.\(^3\) Xenakis apparently took a keen interest during his trip as he states in an interview, "I was intrigued to observe players in Senegal. I recorded them on my portable machine and studied their technique on my return home."\(^4\) Corroborating the composer's likely influence from these sounds and instruments is the nature of the first movement's development, which focuses on polyrhythmic nuances as a means of building intensity and saturating a textural space.

Although there are no known sieves or mathematical paradigms controlling the compositional process in *Rebonds*, both the \(a\) and \(b\) movements develop in a similar way, but utilize different techniques. Movement \(a\) presents in the first measure a registral and temporal divide as the following figure illustrates:

---

\(^3\)Schick, *Percussionist's Art*, 204.

\(^4\)Varga, *Conversations*, 179.
Steadily throughout the course of the movement, this space is filled rhythmically and texturally using polyrhythms in different voices in a continuous, non-sectional fashion. The level of density and complexity continues to increase as multi-layered structures evolve like those illustrated in the next figure:

The movement concludes with the last seven measures reopening the register and timbral space; measure 55 particularly recalls the opening figure of the work, but with an inverse registration.

---

41 Schick, *Percussionist's Art*, 209.
Although the movement is simple with regard to overall concept, the rhythmic complexities that result are vastly challenging to the audience and the performer alike.

While the second movement of *Rebonds* is similar in its conceptual design to the first, sections that imitate a form can be more easily identified. The first measure of *b*, like *a*, establishes a framework against which later developments will occur:
The regular sixteenth notes in the high bongo persist throughout the opening section of the piece as the rhythmic melodic line in the lower drums transforms and develops, rubbing against the common time meter. The grace notes and accents in the high bongo also eventually shift in metric placement, contributing to the destabilization of a regular pattern. Amidst this primary activity, periodic linear passages, first in the woodblocks and later in the drums themselves, become present as a competing thematic idea. Measures 31 and 32 from the first linear interruption are shown in the following figure:

Figure 23. *Rebonds b*, measures 31-32.

As the movement continues to progress with these linear interruptions, a merger starts to occur between the two thematic ideas, mixing the different sonorities and rhythmic figures in such a way as to compose an entirely different sort of texture. Thus, movement b bears a similarity to movement a in that one element gradually permeates and saturates the other. 42 A short passage illustrating this mixture is shown in the following figure:

---

42 Ibid.
The movement concludes with a rapid, polyrhythmic linear passage in two simultaneous voices followed by a multiple-bounce passage around all of the available surfaces.

This concludes the presentation concerning Xenakis's solo percussion works and their concept driven compositional processes. As Xenakis achieved a productive musical output among a vast array of media, there remain many more theoretical facets which would benefit from a detailed, common sense explanation in context with the performance of the works for which they apply.
APPENDIX A

Turning our attention toward some principals of number theory, we will examine, in particular, sieve theory, which Xenakis makes frequent use of in composition. The sieving process itself can be best understood by looking at one of its early applications in constructing a table of prime numbers. The famous Sieve of Eratosthenes produces the following table for integers between 2 and 49:

![Sieve of Eratosthenes, partial integer table](image)

- *indicates multiples of 2
- ^indicates multiples of 3
- _ indicates multiples of 5
- ' indicates multiples of 7

The process of sieving prime numbers in the table above occurs by beginning with the lowest prime number, 2, and eliminating all of its non-identical multiples contained in the table. These eliminated multiples are indicated with a
symbol, *, as shown. Moving to the right, the next remaining number, 3, must also be a prime, since it was not eliminated in the previous step. Hence, it is used as the next point of departure, and all non-identical multiples of 3 still present are eliminated with a symbol, ′, as shown. This process continues through the numeric table, skipping the already eliminated 4, and moving to 5 as the next prime. Upon arriving at the end of the table, we are left solely with primes, which are unmarked: 2, 3, 5, ..., 47.

This sieving process can be realized in the production of an equation to determine the number of primes less than or equal to \( n \), provided that the primes less than or equal to the square root of \( n \) have already been determined.\(^{43}\) This can be accomplished due to the following observation:

If the integer \( n \) is composite, it must have a prime factor that is less than or equal to \( \sqrt{n} \). If this were not the case, we could write \( n = ab \) where \( a > \sqrt{n} \) and \( b > \sqrt{n} \). This would imply that \( n = ab > \sqrt{n}\sqrt{n} = n \), which is impossible.\(^{44}\)

With this information, we turn to the following logical statement representing the Sieve of Eratosthenes:


\(^{44}\)Ibid., 118.
Although the formula looks unwieldy and overly complex at first, we find that
the process of sieving, as described earlier, is completely present, yet
represented in a way that does not necessitate the temporal aspect of an ordered
series of events.

As we break the equation in Figure 26 into its components, we see that
(n-1) represents the number of integers in the set excluding 1, since 1 is not
considered a prime number. Skipping over the $\pi(\sqrt{n})$ expression\(^{47}\), which will be

\[
\pi(n) = n - 1 + \pi(\sqrt{n}) - \left\{ \frac{n}{P_1} + \frac{n}{P_2} + \ldots + \frac{n}{P_k} \right\} \\
+ \left\{ \frac{n}{P_1P_2} + \frac{n}{P_1P_3} + \ldots + \frac{n}{P_1P_k} + \frac{n}{P_2P_3} + \ldots + \frac{n}{P_{k-1}P_k} \right\} \\
- \left\{ \frac{n}{P_1P_2P_3} + \ldots + \frac{n}{P_{k-2}P_{k-1}P_k} + \ldots + (-1)^{k-1} \frac{n}{P_1P_2\ldots P_k} \right\}
\]

\(\pi\) indicates the function\(^{46}\)
\(n\) indicates all natural numbers of the set
\(p_1, p_2, \ldots, p_k\) indicate primes less than or equal to \(\sqrt{n}\)

\(^{45}\)Ibid.

\(^{46}\)It is important to recognize that \(\pi\) represents the function of the sieve and does not, in
this paper, represent the irrational constant used in Euclidian geometry

\(^{47}\)This will add all the primes of the \(\sqrt{n}\) set back in when we finish our sieve
explained later, the first expression contained in the curly braces is subtracted from the total number of integers in the original set.

Figure 27. First expression in braces.
\[
\left\{ \left\lfloor \frac{n}{p_1} \right\rfloor + \left\lfloor \frac{n}{p_2} \right\rfloor + \ldots + \left\lfloor \frac{n}{p_k} \right\rfloor \right\}
\]

This represents the removal of all prime number multiples less than or equal to $\sqrt{n}$ in the set. By dividing the total number of members in the set by each prime member less than or equal to $\sqrt{n}$, we are essentially extracting the number of members containing each of these primes as a factor. Note that this also removes the prime numbers themselves, as 2 is, in fact, divisible by itself. The next expression in curly braces, shown below, is then added back into the set.

Figure 28. Second expression in braces.
\[
\left\{ \left\lfloor \frac{n}{p_1p_2} \right\rfloor + \left\lfloor \frac{n}{p_1p_3} \right\rfloor + \ldots + \left\lfloor \frac{n}{p_2p_k} \right\rfloor + \left\lfloor \frac{n}{p_3p_k} \right\rfloor + \ldots + \left\lfloor \frac{n}{p_{k-1}p_k} \right\rfloor \right\}
\]

This expression represents all of those integers that were removed more than once on account of their divisibility by two prime numbers less than or equal to $\sqrt{n}$. An example of this would be the number 6, which was removed twice by the Figure 27 expression, due to its divisibility by both 2 and 3. Since each integer
can be removed from the set only once, the Figure 28 expression cleans up such
double removals. The rest of the expression serves this same process, cleaning
up all possible invalid removals and reinstatements of the original set of
members. After the process is complete, we must add back in the original prime
numbers less than or equal to $\sqrt{n}$ that we have been working with and, in the
process, have removed from the set. This, then, explains the addition of $\pi(\sqrt{n})$
expression we intentionally skipped over earlier. From this explanation, it is
most important to realize that, while a sieve can be understood in terms of a
process, it can also be represented with a freestanding logical expression, not
referencing temporal activity or an order of events. Despite the fact that the
expression’s explanation occurs as an order of events in writing, the sieve
expression itself does not imply temporal activity. This proves an essential
feature considering Xenakis’s aesthetic philosophy as it applies to music.

\*\*\*\*

\*\*\*\*\*

48 Shockley, Introduction, 120.
Bibliography


Xenakis, Iannis. *Arts/Sciences: Alloys*. Translated by Sharon Kanach. Aesthetics in


