AERODYNAMIC MODELING OF AN AIRCRAFT
IN ATMOSPHERIC TURBULENCE
AND CORRELATION TO HAZARD

By

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ABSTRACT

Improvingaviationsafetyhasbecomeafocalpointofpresentaeronautics research. Quantifying and predicting the hazards of flight into atmospheric turbulence is one area of interest. The present research investigates and extends the use of aerodynamic modeling techniques to better enhance the representation of nonlinear, unsteady effects in a turbulence encounter. The focus of the research is on flight dynamic, versus structural loads, aspects. Flight data from an intentional atmospheric turbulence penetration was used along with fuzzy logic techniques to develop and enhance longitudinal and lateral-directional aerodynamic coefficient models. These models indicated the presence of nonlinear and unsteady aerodynamic effects, including lateral-directional coupling into the longitudinal axis. Effective mass and damping were proposed as one means to correlate loads-induced hazards to the aerodynamic response of the aircraft, which were compared with results from an actual passenger flight. The results suggest that the cause of fast plunging motion may be shock-induced stall in largely static motion, i.e., low reduced frequency, whereas in oscillatory motion with higher reduced frequencies, dynamic stall may inhibit fast plunging motion. Therefore, some form of hazard index may relate to the magnitude of effective damping in plunging motion, or alternatively to the measure of unsteadiness in the aerodynamics of the encounter. A control strategy for countering a rapid plunge may benefit from means to artificially drive unsteady aerodynamic effects.
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LIST OF SYMBOLS AND ABBREVIATIONS

Abbreviations

ACARS: Aircraft Communications and Reporting System
ADC: Air Data Computer
CFD: Computational Fluid Dynamics
FAA: Federal Aviation Administration
(D)FDR: (Digital) Flight Data Recorder
FOQA: Flight Operations and Quality Assurance
IRU: Inertial Reference Unit
MAC: Mean Aerodynamic Chord
NACA: National Advisory Committee for Aeronautics
NASA: National Aeronautics and Space Administration
NEXRAD: Next-Generation Weather Radar
NTSB: National Transportation Safety Board
PID: Parameter Identification
PIREPS: Pilot Reports
rms: Root Mean Square
s/s: Samples per Second
TDWR: Terminal Doppler Weather Radar

Symbols

\( a_{x,y,z} \) acceleration (m/sec^2)
\( a \) lift curve slope
\( a, b \) angle-of-attack vane coefficients
\( a_N \) vertical acceleration (m/sec^2)
\( \bar{c} \) mean aerodynamic chord (m)
CG  center of gravity
C_L  lift coefficient
C_z  normal force coefficient
C_N  normal force coefficient
C_N1  normal force coefficient residual
C_N0  initial normal force coefficient
C_m  pitching moment coefficient
C_Y  side force coefficient
C_n  yawing moment coefficient
C_i, C_roll  rolling moment coefficient
D, J, ..., T  intermediate terms in motion equations
e  distance to aircraft CG from wing, tail
e_mass  effective mass (normalized if with bar)
e_damping  effective damping (normalized if with bar)
g  acceleration due to gravity
h, Δh  altitude, change in altitude (m)
H  filter
I  moments of inertia (xx,yy zz,xz; kg m²)
J  cost function
k_α, k_φ  reduced frequency in angle of attack, roll
L_w, L_t  lift of wing, tail
m  mass (kg)
M_m0  maximum operating Mach
n_z  normal acceleration (g)
N  normal force
N_a, E_a, D_a  earth-axis velocity components
N_w, E_w, D_w  earth-axis wind components
p  roll rate (rad/sec or deg/sec)
q  pitch rate (rad/sec or deg/sec)
r  yaw rate (rad/sec or deg/sec)

r  pitch radius of gyration

R  correlation coefficient

S  wing area (m²)

T  thrust component

T  time (sec)

t  time (sec)

u, v, w  local flow velocities

U  airspeed

V  airspeed (m/sec)

V_{m0}  maximum operating speed

v, w  turbulence component

w  vertical wind relative to airplane

\bar{w}  angle of attack approximation for vertical wind relative to airplane

W  weight

x, y, z  longitudinal, lateral, vertical dimensions (m)

\alpha  angle of attack (rad or deg)

\beta  angle of sideslip (rad or deg)

\varepsilon_{\alpha}  downwash angle

\varepsilon  turbulence eddy dissipation rate

\theta  pitch angle (rad or deg)

\phi  roll angle (rad or deg)

\Phi  power spectral density

\Psi  heading angle (rad or deg)

\gamma  glide slope angle (rad or deg)

\Delta  change

\delta e  elevator angle (deg)

\delta a  aileron angle (deg)
δr  rudder angle (deg)
δs  stabilizer angle (deg)
ρ  air density (kg/m³)
ω  circular frequency

**Superscript**

., ..  time derivatives
_  vector, or average value
~  turbulence component
^  mean value
^  output of fuzzy-logic model
^  intermediate term for motion equations
*  rollup term

**Subscript**

A  amplitude
ADC  air data computer
b  body
cor  corrected
g, gust  gust
i, j  time index
IRU  inertial reference unit
local  local flow velocities
m  motion (airplane)
o  at α=0
osc  oscillatory
q  pitch rate
t  tail
v  vane
\( w \)    wing
\( w \)    wind (gust)
\( \bar{w} \)  angle of attack approximation for vertical wind
\( x, y, z \)  orthogonal components

\( \alpha \)  angle of attack
\( \dot{\alpha} \)  angle of attack rate
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I. INTRODUCTION: BASIC PROBLEM AND REVIEW OF THE LITERATURE

A. General Overview -- Aviation Safety Initiatives

Within about the last decade, aviation safety has received considerable attention as flying as a mode of transportation continues to become more readily available, affordable, and flexible. With the planning and purchasing capabilities offered by the internet, the advent of more leisure time and flexible work schedules, and the opening of many worldwide destinations previously restricted by the political climate of the Cold War era, the flying public is enjoying more travel options than ever and is taking advantage of them in record numbers.

Although advances in aviation-related technology and aircraft design have made air travel relatively safe, aircraft accidents and incidents still continue to occur. The instant communications capabilities enjoyed today, which include live satellite broadcasts from around the world and over the internet, coupled with unprecedented access to regulatory and investigative arms of the Government (e.g., the U.S. Federal Aviation Administration [FAA], and the National Transportation Safety Board [NTSB]) continue to keep the flying public aware of the potential dangers of air travel. Also, with the increase in the amount of air travel, i.e., number of flights, if the accident rate were to remain at present levels the overall number of accidents
would unacceptably increase. Thus, greater demands are being put upon aircraft manufacturers, airlines, and the FAA to continue to improve the safety of air travel.

In 1997, the White House announced a major aviation safety initiative aimed at reducing the fatal aircraft accident rate by a factor of five by the year 2007, and by a factor of ten within twenty years. In response, the U.S. Government, including the FAA, NTSB, and The National Aeronautics and Space Administration (NASA) – as a major “supplier” of advanced technology to the aviation industry -- began organizing programs aimed at helping achieve this goal. These were combined with initiatives within other entities such as the Commercial Aircraft Safety Team (CAST) and the commercial aircraft manufacturers. Instrumental in this process was the identification of several targeted areas of research that were based on objective and comprehensive “data mining” and analysis of recent aviation accidents and underlying causal factors [1]. High on the list of contributors to the fatal accident rate were those due to loss-of-control and those where weather was a significant player.

Major research projects were subsequently organized to address these contributors. Where weather was a factor, encounters with atmospheric turbulence were noted as both a nuisance and a hazard, and special attention was given to deal with that problem. Specifically, severe turbulence was noted as a particular hazard that occasionally results in loss of control. Over the next several sections of the present report, the objectives of both of these initiatives are reviewed and described in some detail in order to lay the foundation for the present research.
B. Loss-of-Control Studies

1. Overview of the Problem

While loss-of-control is not a significant contributor in the fatal accident rate for small, professionally-flown aircraft (e.g., business jets), it is a major contributor for small, personally-flown aircraft (e.g., general aviation) and large, transport-category aircraft [2-4]. Loss-of-control can be caused by several factors, including natural or man-made atmospheric disturbances such as wake turbulence or atmospheric turbulence [5-11]. The present author has had some involvement in the investigation of one of these types of accidents [5, 6].

Given that accidents of large, transport-category aircraft are typically more spectacular and thus more visible in the public eye, research was established to provide technology which would extend present capabilities for addressing unusual attitudes and loss-of-control scenarios, particularly for those types of aircraft. Specifically, goals centered around:

(a) Analyzing the flight dynamic elements associated with loss-of-control.

(b) Providing vehicle health monitoring and automatic control schemes to prevent loss-of-control.

(c) Developing capabilities for assisted or automated recovery from unusual attitudes and upset scenarios.

The discussion to follow, including one element of the present research, will focus on the first item (a) above, particularly the aerodynamic modeling aspects.
At about the same time as the national aviation safety program was being organized, and recognizing that potential hazards existed that typical airline pilots may not be adequately prepared to handle [e.g., 6, 9], several initiatives began in the late 1990s to provide effective training for upsets and unusual attitudes. In some cases, small, private companies began offering in-flight training courses using aircraft ranging from a highly modified business jet [12, 13] to small, propeller-driven trainer aircraft [14]. The programs have received a significant amount of recognition and have been featured in several aviation and flying periodicals ranging from *Flying*, *NBAA Journal of Business Aviation Safety*, *Aviation Week & Space Technology*, *PrivatePilot*, *Business Aviation*, and *Aviation International News*.

The major U.S. airlines also became interested and began incorporating advanced maneuvering training or upset-recovery training into their regular, recurrent, simulator-based pilot qualification programs [11, 15]. At least one highly-publicized loss-of-control accident initiated interest in this training [6]. Although each airline has treated the training a little differently, in many cases the initiative was to provide the flight crew with exposure to the “unusual” or “upset” environment so that they may recognize it and take the proper initial actions (realizing that with present hiring trends airline flight crews are no longer being drawn primarily from the ranks of the military-trained fighter and tactical pilots who are used to aerobatic maneuvers). The focus was on stressing the “startle factor” and training the crew to follow established procedures, e.g., attitude declaration, then instrument cross check, then autopilot disengage; and implementation of basic problem solving techniques,
e.g., solve the roll problem first, the pitch problem next, and the thrust problem last. Examples of maneuvers that had been deemed critical for this training, at least in the early phases of implementation, were wind shear events, unusual attitudes (e.g., nose high/stalls, banked nose-low, near-inverted recoveries), wake turbulence encounters, and rudder hardovers [16, 17].

Naturally, since in-flight unusual-attitude and upset-recovery training can be both expensive and hazardous, all parties became interested in understanding the limitations of and, where practical, extending the capabilities of present simulators for conditions beyond the normal flight operating envelope. It is not uncommon for aircraft to reach flight conditions, during loss-of-control situations, that are beyond the experimentally-validated range of an otherwise regulatory-qualified training device [4]. Interestingly, and recognizing the limits of simulators (at that time) and available training resources, the objective of upset-recovery training was generally not to train a pilot to fly a perfect recovery from an unusual or upset condition. The fact that a training device is not qualified for these extreme conditions is due to the fact that present qualification guidelines do not call for that level of capability [18, 19], nor is it easy or presently practical to capture simulator databases for that purpose [20]. Of course, it is possible to provide “negative training”, i.e., depicting an aircraft response that isn’t real, if an inaccurate simulator aerodynamic database is employed. To that end, operators attempted to devise training maneuvers that could be flown and recovered within the acceptable simulator model envelope [17]. Mostly, these envelopes are defined by limits on angle of attack ($\alpha$) and angle of
sideslip ($\beta$), typically for static or quasi-steady assumptions. At worst, an accepted practice was and still is to simply freeze the simulator, or display a visual cue to the trainee, if an envelope parameter is exceeded. Thus, there is a significant amount of interest in at least understanding if not correcting the issue of database envelope limitations.

One difficulty is defining the threshold for an acceptable simulator model envelope. Typically, there are four subsets of a manufacturer’s simulation aerodynamic database, all normally established in the context of $\alpha$ and $\beta$:

(a) Flight derived (often flight validated as well, e.g., using parameter identification (PID) or coefficient matching);

(b) Wind-tunnel derived;

(c) Analytically derived (e.g., empirical methods) extrapolation from experimentally-derived or theoretically-based models;

(d) “No data” or “Hold last known value”.

In the case of (d), and sometimes (c), the typical practice is to “hold last value” beyond experimentally-validated, or analytically-derived, database envelope limits [4, 17, 21]. The key for properly addressing upsets or unusual attitudes is developing a database that can be validated. Database validation is difficult, risky, and costly for unusual attitudes, particularly in an aircraft not designed for that regime (i.e., general aviation or transport configuration) [4], with the difficulty coming in defining a suitable set of maneuvers to properly capture the extreme conditions [20].
An opportunity was identified to leverage major research initiatives to address improved capabilities for aerodynamic flight prediction, particularly as it relates to stability and control [21, 22]. In the past, these had been geared more toward fighter aircraft configurations, and considerable progress had been made in predicting and/or at least understanding aerodynamic phenomena at the more extreme conditions through a variety of wind-tunnel, flight, and simulation studies. Recently, however, an unprecedented wind-tunnel investigation of a typical commercial transport configuration has been used as a starting point for developing improved simulator and aerodynamic analysis capabilities [4]. The present author had some involvement in laying out the objectives for these tests, in the context of simulator math-model requirements. These initiatives began to explore the generation of aerodynamic databases to high angle-of-attack (α) and -sideslip (β), including dynamic testing, i.e., forced-oscillation and rotary balance testing, of large transport configurations, the latter which is rarely if ever accomplished.

Regarding the issue of validation, given the difficulty and expense of capturing flight maneuvers geared specifically toward high α and β flight conditions for transport aircraft, there has been discussion toward development of scale flying models for aerodynamic database extraction [23]. Scale model testing has been used successfully for years to conduct qualitative handling qualities assessments. Any kind of flight into extreme conditions implies some type of real-time model or PID capability [e.g., 24]. At worst, some capability to extract aerodynamic models from nonlinear, unsteady aerodynamic databases is needed [e.g., 25]. This also becomes
important in addressing applications which involve vehicle self “health monitoring”
and “self healing” automatic control schemes, and developing capabilities for assisted
or automated recovery from upset scenarios. These imply the capability to accurately
identify and describe the real “aerodynamic state”, i.e., model, of the aircraft. The
present research proposes to extend a technique that has been previously introduced
for capturing a nonlinear, unsteady transport configuration aerodynamic model
utilizing flight data in order to benefit the overall objectives of the national research
initiatives.

2. Technical review of aerodynamic math modeling approaches

Categories

Most aerodynamic practitioners recognize that for the majority of design
applications, aircraft typically stay within the regime of the flight envelope
characterized by steady and linear assumptions, shown as region A in Figure 1. For
purposes of the discussion to follow, steady refers to either slowly varying or wholly
invariant flow, and implies only knowledge of the instantaneous aerodynamic state,
i.e., there is no consideration for the past history of the motion. The term linear
generally refers to the region of small angles of attack and sideslip.

The regime of steady but nonlinear flow, region B of Figure 1, might include
analysis such as high-lift design or perhaps steady spins. Flutter analysis is typically
conducted in the unsteady but linear regime as depicted in region C of Figure 1.
These analyses typically include assumptions about small- amplitude motions,
Figure 1: Regimes of aerodynamic modeling

and linear unsteady doublet-lattice techniques work well in predicting the aerodynamic characteristics of the motion provided that the motion remain small amplitude.

In all the preceding applications, methodologies are well in place to conduct satisfactory and reliable analysis. The target area for the present research is that in region D of Figure 1, particularly unsteady and bordering on nonlinear behavior. This region could be characterized by rapid or relatively large amplitude maneuvers,
particularly at high angles of attack and sideslip, or even motion about multiple axes, such as a yaw-pitch coning motion. Proper analysis requires that the aerodynamic model include some characterization of the history of the motion.

**Quasi-Steady**

For regions A and B of Figure 1, a quasi-steady approach can be and typically is used to model the aircraft-level motion characteristics. This approach includes some consideration of the motion, but only for the instant in time under analysis. A typical quasi-steady formulation for the longitudinal axis might look like:

\[
\begin{align*}
C_L &= C_{Lo} + C_{La}\alpha + C_{Lq}q + C_{La}\dot{\alpha} + C_{La\dot{\alpha}}\dot{\alpha} + C_{La\ddot{\alpha}}
\end{align*}
\]

\[
C_m = C_{mo} + C_{ma}\alpha + C_{mq}q + C_{ma\dot{\alpha}}\dot{\alpha} + C_{ma\ddot{\alpha}}
\]

where \(C_L\) and \(C_m\) are the lift and pitching moment coefficients, respectively, and the other terms are defined in the list of symbols. Note that all derivatives here are evaluated about the trim conditions. The key terms here are those dynamic terms (coefficients) related to pitch rate \(q\) and rate of change of angle of attack \(\dot{\alpha}\). The dynamic terms are either estimated using empirical techniques or developed using small-amplitude, forced-oscillation wind-tunnel techniques, at constant mean angles of attack and at a fixed reduced frequency. If the latter approach is used, then it is necessary to lump the aerodynamic terms into equivalent oscillatory terms since it is not possible to break out individual contributions. It is possible to assess in-phase
and out-of-phase components. For example, in the longitudinal axis, these terms are

\[ (C_{zq})_{osc} = C_{zq} + C_{zq} = -\frac{2}{\pi k_1 \Delta \alpha} \int_0^{2\pi} C_z \sin \theta d\theta \]  
(2)

\[ (C_{z\alpha})_{osc} = C_{z\alpha} - k^2 C_{zq} = \frac{1}{\pi \Delta \alpha} \int_0^{2\pi} C_z \cos \theta d\theta \]  
(3)

\[ (C_{mq})_{osc} = C_{mq} + C_{m\alpha} = -\frac{2}{2\pi k_1 \Delta \alpha} \int_0^{2\pi} C_m \sin \theta d\theta \]  
(4)

\[ (C_{m\alpha})_{osc} = C_{m\alpha} - k^2 C_{mq} = \frac{1}{\pi \Delta \alpha} \int_0^{2\pi} C_m \cos \theta d\theta \]  
(5)

where

\[ k_1 = \frac{\omega \bar{c}}{V} \]  
(6)

is the reduced frequency; \( \omega \) is the frequency of oscillation, \( \bar{c} \) is the wing mean aerodynamic chord, \( V \) is the flight speed, “z” takes the place of “L” from the previous equations, and the term under the integral represents one cycle of the oscillation. The derivatives are defined as, for example,

\[ C_{mq} = \frac{\partial C_m}{\partial (q \bar{c} / 2V)} \]  
(7)

Note, in the lateral-directional axis, the reduced frequency would be

\[ k_2 = \frac{\omega b}{(2V)} \]  
(8)

where \( b \) is the wing span.
An equivalent type of analysis is often used for steady rotary balance data, except in this case there are no in-phase versus out-of-phase differences and the motion is a constant rotation instead of an oscillation. Regardless, the strength of this overall approach is that for many applications in the linear range it provides a reliable estimate of the force or moment coefficient with respect to roll, pitch, and yaw rates (but not $\dot{\alpha}$ or $\dot{\beta}$), and is relatively simple to capture with the proper experimental setup. However, it is not adequate at higher angles of attack (i.e., out of the linear range), nor when considering a range of equivalent reduced frequencies of motion. In its present form, it is incapable of providing any information about the history of the motion on the aerodynamic forces and moments.

Figure 2, from [21], shows one example of the effect of not carrying the estimates of roll damping to more fully replicate the full flight envelope. In this case, an estimate presently in a training simulator database, versus one from recent wind-tunnel tests, is shown. The damping actually changes sign, which of course impacts the overall flight dynamic characteristics.

Indicial and State-Space Response

The next level of complexity attempts to include information about the history of the aerodynamic response to motion. It uses a nonlinear indicial response function. This formulation was developed starting in about the 1950s for flight dynamics research, and effectively models the “response of the aerodynamic flowfield to a step change in a set of defined boundary conditions such as a step change in airfoil angle...
Figure 2: Examples of limited roll damping estimates for a transport (Ref. 21)

of attack, a step change in pitch rate about some axis, or a step change in control surface deflection” [27]. Effectively, a series of these step changes are integrated to form a general unsteady time history response. While it is only nonlinear in the sense that it can take an infinite number of “directions” (e.g., with bifurcations) depending upon the past history of motion, it can be used to model the nonlinear regime. The roots of this type of analysis trace back to some of the early aerodynamic research (e.g., Wagner, and the so-called “Wagner effect”, which represents the aerodynamic lag due to shed vortices due to aircraft motion [28]; note, this is not the same as the Kussner unsteady lift function, which models the effect of gust penetration [29] in,
for example, the well-known Pratt gust formula). The general form of this approach can be represented by, for example,

\[
C_L = C_L(0) + \int_0^t \left[ \frac{\partial C_L(t - \tau)}{\partial \alpha} \right] \frac{d\alpha}{d\tau} d\tau
\]

(9)

where

\[
\frac{\partial C_L(\alpha(\xi); t, \tau)}{\partial \alpha} = \lim_{\Delta \alpha \to 0} \left[ \frac{C_L(\alpha(\xi) + H(\xi - \tau)\Delta \alpha) - C_L(\alpha(\xi))}{\Delta \alpha} \right]
\]

(10)

The term under the integral provides the step change to, in this example, angle of attack in a functional form. Discussion of this method can be found in several references in the literature [cf., 27, 30-34]

An extension of this approach using a state-variable formulation has also been used to represent unsteady flow [cf., 35-38]. In essence, a formulation such as

\[
dx/dt = f(x, h)
\]

(11)

\[
C = g(x, h)
\]

(12)

where \( x \) is the flow state variable;

\( dx/dt \) is its rate of change;

\( h \) = system inputs;

\( C \) = aerodynamic force and moment coefficients.

uses a term to represent the flow state in this linear representation.

A summary of the indicial method [33, 34] also notes that while the indicial integral method is theoretically capable of modeling nonlinear and time-dependent aerodynamic responses, it suffers from several practical limitations which impact its
usefulness for flight dynamics applications. Primarily, it is a relatively complex method, owing to its technique of using series of functional representations, which provides at least theoretically infinite dimensions to the problem. This causes difficulties when the time history of one variable is dependent upon the time history of another. Applying this to a system of equations of motion, for each variable, is therefore practically impossible.

Improvements to this technique came about through the use of Fourier functional analysis to visualize the solution representation as a series of superimposed harmonics in the frequency domain [39]. At the same time, the analysis was extended from two to three dimensions.

**Computational and PID Methods**

From a different perspective, other efforts are targeting the use of computational methods for building stability and control parameters and simulation databases [21, 40-42]. These techniques employ automatic differentiation schemes coupled directly into computational fluid dynamics (CFD) solvers to extract stability derivatives and coefficients. With maturity, these offer at least the possibility of providing the equivalent of dynamic wind-tunnel results with the benefit of a more controlled environment.

In the context of extracting models from existing dynamic data, progress continues to be made in the development and application of classical PID techniques. One approach has proposed building nonlinear models using multivariate orthogonal
functions [25]. In the present research associated with this proposal, this approach perhaps most closely identifies the numerical limitations and challenges associated with deriving nonlinear, unsteady aerodynamic models from specific flight test maneuver sets.

Fuzzy Logic Models

Keying off of work with Fourier functional analysis, indicial functions and state-space representations, more recent work focused on building suitable simulation databases for studying maneuvering capabilities [43-50, 26] by analyzing nonlinear, unsteady results from a series of wind-tunnel experiments. Results have been formulated for representative fighter configurations such as the F-16XL and the F/A-18. The regime of nonlinear, unsteady analysis, has maintained international interest, and other collaborative efforts continued through the 1990s to experimentally and analytically study approaches for characterizing these effects [cf., 51]. Of particular interest has been means to drive and extract information from various motions in the wind-tunnel, such as multi-axis oscillatory motion, coning, and dynamic pitch and plunge.

Focusing on the present research, an approach that is beginning to show promise essentially ignores any attempt to model specific flow physics (i.e., vortex shedding), and instead uses relational datasets to most closely approximate a representation of the characteristics of a system. This technique, using fuzzy logic, has enjoyed some measure of success in being able to predict and model nonlinear,
unsteady aerodynamic characteristics [52-58, 28]. As a more global approach, instead of being a purely “parameter identification” technique (e.g., $C_{ma}$), it tends to be more of a “system identification” technique for each axis (e.g., $C_m$) but can include the functional effect of several independent variables simultaneously. It has the ability to capture nonlinear and unsteady behavior, including time dependencies via reduced frequencies, and its ability to capture “localized” effects such as aerodynamic bifurcations is presently being investigated. The obvious disadvantage to this approach is that the resulting model is contained in the equivalent of a “black box”, i.e., a relational database that lends no direct insight into specific aerodynamic phenomena as in classical analysis and it is a “curve fit” to the modeling data. However, the advantages to this approach are that the implementation of a model in a simulation or other analysis environment is relatively straightforward, effectively nothing more than a multi-dimensional table lookup; and with proper “data mining” tools (e.g., an $\alpha$-sweep “wind-tunnel” mode as used in present simulators) it is possible to extract the classical aerodynamic coefficients for analysis.

Application of the fuzzy logic approach is the subject of the present research, with a view toward contributing to the advanced aerodynamic modeling facets of the aviation safety initiatives. The next major section will describe the context within which this will be accomplished.
1. Overview of the Problem

While encounters with atmospheric turbulence do not contribute substantially to the fatal aviation accident rate, they do cause a significant number of injuries and cost major airlines millions of dollars per year in compensation to passengers and fuel costs for rerouting around the turbulence. Additionally, any encounter can often be frightening, and impacts at least the perception of safety needed to continue customer confidence in flying as a safe means of travel [59, 60]. Typically, the danger associated with a turbulence encounter results from the additional positive and negative load factors within the aircraft, particularly in the aft end, for passengers and crew not restrained by seatbelts. Occasionally it results from either the pilot or autopilot overcontrolling the aircraft.

As a result, most if not all airlines have some active incorporation of turbulence forecasts and predictions in their regular flight planning and dispatch operations. Some of these are relatively sophisticated and include special flight crew training and procedures to plan for, deal with, and report turbulence [59, 61, 62]. For example, one major airline has a “Turbulence Plot” system that dates back to an industry-leading weather hazards initiative begun in the late 1960s. It includes measures for dealing with turbulence, such as special navigational charts custom-tailored to include high-likelihood areas for encountering turbulence, and marked special deviation routes. The system has evolved over the years to cover several
facets of atmospheric hazards, and the flight crew training and resource material typically covers thunderstorm, clear-air (CAT), and mountain wave turbulence; low-altitude frontal and convective windshear; and other atmospheric hazards such as volcanic ash, icing, and ozone.

Flight training also includes some means for handling the airplane in a turbulence encounter, and was begun several years ago with the introduction of the swept-wing jet upset issue [cf., 63, 64]. Transport-aircraft flight manuals contain guidance on flying in turbulence, i.e., turbulence penetration speeds as a tradeoff between high- and low-speed buffet limits, use of autopilot, engine ignition procedures, and control strategies in turbulence. A general rule of thumb suggests attempting to maintain pitch attitude, primarily because that is the only reliable cockpit indication to the pilot. Airspeed, rate of climb, etc., are typically affected by the direction of the gust and do not provide reliable guidance [cf., 63].

Airline meteorology staffs are actively involved in any process dealing with turbulence [61, 65, 66]. As of the late 1990s, the airline industry as a whole was well aware of how to deal with turbulence [67], and the U.S. Government had initiated active programs in addressing weather hazards, in general, with interest in turbulence in particular.

One national research program has been focused on [1, 68-70]

(a) In-situ detection and reporting of turbulence, as a supplement to current pilot reports (PIREPS) for real-time reporting and “maps” of turbulence areas for
air-traffic routing and flight planning, and for updating production forecast/nowcast weather tools.

(b) Remote detection and sensing of turbulence, to provide better warning to flight crews of impending turbulence from enhanced airborne Doppler weather radar and laser radar (lidar) capabilities and from ground-based next-generation weather radar (NEXRAD) and terminal Doppler weather radar (TDWR) capabilities.

(c) Better forecasting/nowcasting capabilities, which key off of improvements to weather prediction models and tools in order to provide better flight planning capabilities.

(d) Mitigation of turbulence effects, including improved cockpit/cabin procedures and the possible development of automatic load-alleviation control systems.

For each of the areas noted above, it is recognized that it is desirable, if not critical, to establish an aircraft hazard level associated with any turbulence encounter. This need follows directly in the footsteps of what had been successfully accomplished in addressing the wind shear problem, and what had been investigated for wake turbulence hazards in the airport environment. Validation and verification of any new technologies requires the use of analysis, simulation, and ultimately flight trials.

The hazard associated with a turbulence encounter is obviously coupled to the aircraft response to that turbulence. The practical quantification of a hazard sometimes could be as simple as an integer numerical scale [59]. But key to any hazard assessment is that accurate forecasting and/or reporting of turbulence is
inherent on the ability to “scale” its inherent hazard from one aircraft type or size to another. In other words, what might be very hazardous to a light twin might be only a subtle bump for a heavy transport, or vice-versa. Thus, the hazard prediction must include accurate considerations for the expected response of a given airplane to that turbulence patch. Furthermore, effective mitigation strategies or systems must be closely coupled to the actual hazard, and the capabilities of the airplane and crew to deal with them. This, of course, raises issues when considering particularly strong turbulence in the context of modeling the aircraft response. Several major airline accidents have been the result of loss of control initiated, in part, by natural or man-made atmospheric disturbances. These accidents have taken their respective airplanes and flight crew well beyond the “normal” flight regime.

Within the scope of the present research initiatives, it has also become highly desirable, given the capabilities of the digital databus systems on modern commercial airliners, to leverage and exploit those capabilities as much as possible. This plays into the ability to conduct real-time reporting of turbulence using, for example, an Aircraft Communications Addressing and Reporting System (ACARS). Recorded datastreams can also aid in reconstruction of an incident or accident after the fact, and in fact are already used by airlines in working with airframe manufacturers to better address maintenance and operational issues.

2. Technical review of turbulence hazard problem and present initiatives

Dealing with turbulence is nothing new. In fact, early National Advisory Committee for Aeronautics (NACA; the predecessor of NASA) research dealt with
the subject. Approaches have subsequently been developed to deal with, from the
aircraft design perspective, both continuous turbulence and discrete gusts [cf., 71]. A
more recent summary of the evolution of gust loads design requirements [29] makes
several key points with some relevance to the proposed present research. Among
them are a comparison of the effects of incorporating various modeling assumptions
for estimating the loads due to discrete gusts. For the conditions noted in the
comparison, there is greater than a 0.5g “spread” in the induced, normal load factor
when using the classic Pratt formula (incorporating the Kussner lag function)
compared to a more sophisticated approach employing considerations for flexibility
and a tuned gust. The author also notes that the U.K. Civil Aviation Authority (CAA)
implemented discrete gust criteria because it was noted that “the majority of gust load
accidents appeared to have been more related to ‘sudden events’ than to continuous
turbulence.” This is indicative that perhaps the most hazardous turbulence is
eSSentially a discrete event. Regarding nonlinearities, “if the response is nonlinear,
then neither the equations of (Matched Filter Theory) nor random process theory are
valid, and an explicit solution for worst-case input is not possible. Equally, the linear
summation of 1-cosine ramps in the (Statistical Discrete Gust) method is no longer
feasible.” In addition, “dynamic elastic effects can be large and should be accounted
for by the inclusion of appropriate dynamic factors .” Finally, “all things
considered, we are, no doubt, fooling ourselves with analytical oversophistication,
therefore, periodic checks of actual flight test results against dynamic analysis results
will continue to be required to properly scope design analysis efforts .” [29].
Having noted these comments, several excellent texts are available and continue to aid the designer [e.g., 72], and provide the classic techniques for dealing with gust loads and aeroelastic effects. Additionally, a significant amount of research on gust loads was produced by the NACA and NASA after World War II [cf., 73]. The interest in understanding gust and turbulence effects on aircraft during this period of research laid the groundwork for present initiatives.

**Direct Measurement**

An initial attempt to establish a measured turbulence intensity coupled to something useful for flight applications began in the 1960s [74, 75]. The turbulence eddy dissipation rate (ε) was proposed as the key parameter, which describes the rate of energy transfer in going from large eddies to small eddies in the atmosphere. This keys off of the Kolmogorov hypothesis that the turbulence energy spectrum is tied to the eddy dissipation rate for frequencies within the inertial subrange. This subrange is important in the context of aircraft response. Calculation of this parameter from flight data supported a proposed coupling back to some expected severity in the aircraft response.

In the mid-1990s there was interest in continuing this work and addressing, with improved computer and communications capabilities, the opportunity to measure and report turbulence from commercial aircraft in real time. Means to achieve this again appealed back to keying off of the aircraft response in the presence of atmospheric turbulence [76]. One investigation proposed by researchers at the
National Center for Atmospheric Research (NCAR) began implementation of the first practical turbulence monitoring and reporting system based upon in-situ aircraft measurements. While some measure of success had been previously attained in the purely research environment, key to this approach was a simple and efficient means to calculate the eddy dissipation rate from the measurement of aircraft normal acceleration. This particular methodology employed a homogeneous, continuous, linear von Kármán turbulence model coupled with a linear transfer function of the aircraft dynamics. The equations of motion in pitch and plunge for the simplified aircraft model had the form

\[ m\ddot{z}(t) = L_w(t) + L_i(t) \]  
\[ mr^2\ddot{\theta}(t) = eL_w(t) - e_iL_i(t) \]

where the wing lift was represented by

\[ L_w(t) = \frac{1}{2} a \rho S V^2 \{ \alpha - [e - (c/2)]\dot{\theta} + w_g \} \]

and the tail lift as

\[ L_i(t) = \frac{1}{2} a \rho S_i V (VF_i\delta_e + (\dot{\alpha} - \dot{\theta} + w_g)(1 - \epsilon_\alpha) + \epsilon_i + (c/2) + \epsilon_d [e - (c/2)]\dot{\theta}) \]

and the embedded symbols are defined in the list of symbols. Of note is that these equations represent a linear, quasi-steady aircraft response. These equations are then manipulated into a form which ultimately allows for the estimation of \( \epsilon \) through the proper assessment of
\[ \sigma_z^2 = 0.7v^{2/3} \varepsilon^{2/3} \int_0^\infty |H_{bp}(\omega_1, \omega_2, \omega)|^2 \left| \frac{\ddot{y}(i\omega)}{w_g(i\omega)} \right|^2 \omega^{-5/3} d\omega \]  

(17)

where \( H_{bp} \) is a bandpass filter having cutoff frequencies \( \omega_1 \) and \( \omega_2 \), and \( w_g \) is the gust input.

The “NCAR algorithm” was embedded in special on-board processors. Since the going-in idea was that this would be installed on a fleet of commercial airliners, the overall desire was to have a real-time assessment of turbulence that was independent from the aircraft measuring it, with the application to report some level of turbulence intensity that could be properly interpreted for any size aircraft. Several years ago, major airlines began implementing this algorithm on a trial basis, for the purposes of recording and reporting turbulence in real time [77]. Obviously, by the nature of the model it is intended for continuous turbulence environments and cannot handle the single-event type of gust.

More recent efforts have partnered with the previous and ongoing initiatives for in-situ turbulence measurements from fleet aircraft. These key off of the premise that these measurements are useful for downlinking and reporting turbulence locations and levels, in real time, to flight dispatchers, meteorologists, and other aircraft operational support functions. One approach has been to tune and optimize the prior (linear) NCAR approach for applicability to a variety of aircraft [78] using flight simulation capabilities. This investigation has focused on using (simulated) flight measured normal accelerations to improve the robustness of the turbulence extraction algorithms to noise, discrete events, and the effects of aircraft maneuvering.
and control inputs; in other words, to “tune” the simplified flight dynamics and turbulence models to the “real world”. The flight simulation approach has been used to demonstrate that two different aircraft types, having vastly different operating, geometric, and aerodynamic characteristics, can at least in theory recover the same turbulence eddy dissipation rate parameter from an assumed spatial turbulence field. However, this approach still relies on a linear aircraft model transfer function and the assumption of the continuous turbulence model within the inertial subrange. Presently, research flight datasets, to be discussed later, are being utilized in the tuning process. One important note relative to the present research, as captured from these prior efforts is that all algorithm assumptions hold well until the angle of attack gets into the nonlinear range, “at which point the linear model was no longer representative of the aircraft’s response and the algorithm’s accuracy was degraded. Based on flight and simulation data, this would result from severe turbulence” [78].

Current, unpublished work is leaning toward the thought of skipping the measure of turbulence itself (i.e., eddy dissipation rate) and reporting (or deriving) the aircraft response (hazard) instead. The hazard is, essentially, what is most important to a flight crew. Figure 3 depicts the two approaches used by in-situ techniques. In the “normalization” and scaling steps shown in Figure 3, these assume some form of linear relationship.

An alternative to the NCAR formulation has been separately proposed [79]. This work suggests a more direct measure of gust intensity based on very fundamental linear aerodynamic theory, essentially employing a much simpler
implementation of the extraction of turbulence components, as derived from aircraft response, with no prior assumption about the turbulence field. This approach has the form

\[
\Delta C_N = C_{N\alpha} \Delta \alpha + C_{N\delta} \Delta \delta + C_{Nq} \Delta \dot{q}
\]  
(18)

\[
\Delta \alpha = \Delta \theta - \Delta \gamma + \Delta \alpha_g
\]  
(19)

where \(\Delta\) represents a change from nominal, the subscript “g” is the gust (turbulence) term, and these two equations are manipulated to end up with a measure of \(\Delta \alpha_g\). This approach, too, had captured some measure of success in utilizing NASA flight data, although results have been contaminated by the effects of maneuvering and control inputs. The rationality for this algorithm is that it is a much simplified, linear
approach as within the realm of most turbulence and at least as good as the NCAR approach. The simplification results in, at least in theory, a more straightforward on-board installation of the real-time algorithm. While the model itself is straightforward, other more complex effects, such as rate of change of angle of attack $\dot{\alpha}$, can be included in the model if desired.

Remote Detection

Research is also geared to develop turbulence detection technologies [119]. The idea is to provide an advance warning to a flight crew that turbulence is up ahead some distance to the aircraft, in order to properly prepare the passengers and crew for the encounter. The majority of the most recent efforts have been focused on weather radar applications, with some flight evaluations already complete. There has also been some continued interest in the ability of using airborne laser radar (lidar) as a means to measure clear-air turbulence (through particulate backscatter) and perhaps even activate some sort of active load alleviation system. With detection comes inherently some ability to predict the resulting aircraft response to the turbulence and define a hazard index. The difficulty is in identifying a suitable “observable” that can be formulated into parameters meaningful to and measured by the detection (e.g., radar) system itself. An approach that is presently being investigated keys off of the idea that

1. aircraft g-loads can be predicted as a function of turbulence intensity;
(2) sensor observables can be developed that are a function of turbulence intensity;
(3) these two can be combined, to remove turbulence intensity, and result in a prediction of aircraft g-load as a function of the sensor observable;
(4) the remaining task is then to define some suitable threshold for hazard severity and safety criteria.

This approach is summarized in [80, 81], and is depicted in Figure 4. Step (2) above is particularly mathematically intensive in terms of development of the equations, and well beyond the scope of the present research. Step (1) assumes simple, linear, quasi-steady aerodynamic models.

Characterization and Forecasting

Along with the abilities to capture and report turbulence, in-situ, and detect it ahead of the aircraft, a significant amount of effort has gone into studying the character, signature, and underlying causal nature of hazardous atmospheric turbulence.

Studies were initiated in the 1970s and continued through the mid-1990s to use available meteorological data and/or flight data recorder (FDR) information to study the causal factors of airline accidents pertaining to atmospheric events [8, 82-87]. Some measure of success was enjoyed in terms of being able to accurately reconstruct some of the underlying factors. Key to these investigations was working through the problems involved with limitations in the FDR, primarily low sampling
rates, minimal numbers of data channels recorded, and overall data quality, particularly with early FDRs. One of these studies concluded that “perhaps the most important and immediate application of DFDR information is the quantification of turbulence intensity” [85].

Since then, a meteorological team working within the scope of the national aviation safety initiative has been working to tie all of the present turbulence research efforts together [88-93], and is following the approach used to successfully characterize windshear and wake turbulence hazards. The idea is to characterize the meteorological aspects of turbulence encounters, both from the underlying
atmospheric (mesoscale) conditions characteristic of significant hazardous turbulence [92, 93] as well as the (microscale) phenomenological signatures of specific turbulence events [88-91]. This broad approach, in a sense, follows along from the work conducted in earlier attempts to identify hazardous turbulence from airliner accidents [e.g., 82-87], and in fact keys off of accident/incident databases from the NTSB.

Significant computational modeling of atmospheric physics is being employed to characterize these events and to support the development of turbulence detection algorithms and to assess flight dynamics hazards, as previously mentioned. One of the key benefits to having the results of these computational models is the ability to theoretically “shoot” the radar at the turbulence field and then, using simulation, “fly” an aircraft through it to study the response. In this approach, the turbulence fields and resulting aircraft response can be characterized as [91]

\[
\sigma_w(x, y) = \left[ \frac{1}{L_x L_y} \int_{x=\frac{L_x}{2}}^{L_x} \int_{y=\frac{L_y}{2}}^{L_y} \left\{ w(x', y') - \bar{w}(x, y) \right\}^2 dx' dy' \right]^{1/2}
\]

(20)

\[
\bar{w}(x, y) = \frac{1}{L_x L_y} \int_{x=\frac{L_x}{2}}^{L_x} \int_{y=\frac{L_y}{2}}^{L_y} w(x', y') dx' dy'
\]

(21)

\[
\sigma_{aw}(x, y) = \sigma_w(x, y) [a(z) - v(z) \log(l)] * \left( \frac{180}{W_a} \left( \frac{V}{\sqrt{\nu(n(z))}} \right) \right) * K
\]

(22)
where equations (20) and (21), in this case, are used to define gust fields, and equation (22) results from a linear representation of the flight dynamics of a turbulence encounter. The constants are derived from linear state-space models generated using a flight simulator, and their details are not shown here. In addition, a relationship, based upon empirical data, relates a maximum expected gust response back to $\sigma_{\alpha n}$ [91]. An early form of this is shown in Figure 5. This approach is practical from the standpoint of defining triggering thresholds for alerting passengers and flight crew of hazardous turbulence. The idea of having these models is to drive toward establishing standards that could be used for certifying hazard prediction systems. However, whether this approach is applicable to cases where the fundamental aerodynamics are changed due to turbulence resulting in high-speed, or dynamic stall, remains to be seen. Such cases are possibly abundant during encounters with severe turbulence.

Mitigation

At the tail end of these efforts, mitigating the hazards of turbulence is the desired outcome [94, 95], i.e., determining how to properly use the threat information. This problem is compounded by the variety of aircraft in the operational airspace, and becomes a scaling problem to map a turbulence measurement or induced load or hazard from one aircraft to another.

One of the more practical problems is making turbulence hazard information useful to flight crews. In the late 1990s, the present author was a technical monitor
for an effort directed at determining the best ways to present turbulence information to cockpit crews [96, 97]. Through a series of usability surveys to airline flight crews, including pilots and flight attendants, display content requirements were established that were then rolled into a simulator study using the facilities of a major airline. Subject pilots were asked to evaluate these display concepts, and the subsequent information provided to them, in an environment where turbulence was simulated in the motion-based cockpit and warning areas of various levels of
turbulence were presented in a simple display. The research was essentially one of human factors but important in the context of bringing a major airline aboard the turbulence research initiative and laying some of the ground work for adequate cockpit warning protocols [98-100].

Other considerations for mitigation have to do with cabin procedures, which tie back into some perceived hazard severity [e.g., 96]. Automatic load alleviation systems are also being proposed [94, 95, also 69, 70]. But some very practical and physical limitations will make successful implementation difficult on commercial transports. These key off of prior attempts to improve the ride quality of flight through turbulence [101-104].

Handling Significant Turbulence

Meanwhile, regulatory agencies continue to wrestle with the right approach for handling discrete gusts [e.g., 29, 105]. In addition, initiatives for addressing aging aircraft issues are focusing on better tracking of significant loading events by operators. A recent airline accident [106], in which the vertical tail of a large transport aircraft broke off of the airplane, has led to speculation that excessive tail loads may have been involved following a wake turbulence encounter. Almost immediately it was recognized that present design criteria, per certification regulations, do not account for dynamic loads effects (although the critical maneuvers are dynamic) [107].
One approach that was conducted long before this accident, and completely independent of the organized national initiatives, investigated the effects of nonlinear, unsteady gust loads on aircraft [108, 109]. This effort extended research conducted by aeroelasticity specialists that employed matched filter theory. This study concluded that nonlinear, unsteady effects could cause significantly larger loads.

More recently, efforts have been initiated to study the general problem of modeling aerodynamics during turbulence encounters using research flight data [110, 120], and during windshear events using FDR data [123]. These studies successfully employed an application of the fuzzy logic approach previously applied to fighter aircraft flight dynamics problems and described in the previous section.
II. INTRODUCTION TO THE PRESENT APPLICATION

In designing aircraft for turbulence encounters, the focus is predominantly centered on protecting the aircraft structure from excessive loads, i.e., predicting the loads conservatively in order to design an adequate structure. Handling qualities and stability and control considerations often receive secondary attention.

In the context of aviation safety, where a given structure is certified to handle extremes of loading conditions, there is a need for understanding the hazards of turbulence encounters from the standpoint of the aircraft dynamic response and the subsequent impact to passengers and crews. In other words, the gust-induced loads are only part of the problem. Therefore, one key is a better understanding of the gust-induced or pilot-induced response of the aircraft and whether present modeling assumptions can be improved. One potential outcome is an improved recommendation for how a pilot should, or an autopilot should be designed to, control the aircraft in turbulence, or alternatively the limitations in doing so.

In the present research, fuzzy logic is utilized in order to develop an aerodynamic model of an aircraft in a turbulence encounter. Improvements are made to earlier research results, with the primary efforts to (1) study the estimated aerodynamic characteristics of an airplane at key points in a turbulence encounter; (2) study and show the effect of a nonlinear, unsteady model compared to classic assumptions; and (3) draw conclusions regarding the behavior of these models and
their significance. The goal is to propose means to reduce the significance of the normal load response of the aircraft to turbulence – often the cause of passenger injuries in particular – by understanding the impacts to aerodynamic stiffness and damping.
III. TECHNICAL DEVELOPMENT

A. Mathematical and scientific development

Process for Building the Aerodynamic Model

The process for developing a suitable aerodynamic model from turbulence-encounter flight data can be broken into several steps:

a) Flight data capture
b) Flight data conditioning
c) Data compatibility analysis
d) Identification of coefficient dependencies
e) Parameter calculation using equivalent harmonic motion
f) Identification of optimum fuzzy logic structure
g) Refinement of model to optimum structure
h) Generation of aerodynamic coefficients from model
i) Analysis against test data, harmonic comparisons, breakout of stability derivatives

Step (a) is discussed in detail later. Steps (b) and (c) are straightforward and techniques are presently available. In particular, flight data conditioning for this
purpose entails the application of an M-point moving average filter of the form [110, 120]

\[ y[n] = \frac{1}{M} \sum_{k=1}^{M-1} x[n + k - M / 2 - 1] \]  \hspace{1cm} (23)

where \( y \) is the filtered, discrete data point at index \( n \), \( x \) is the original measurement, and \( M \) is the index of the averaging interval. This filter removes spurious signals and noise from the datastream. Data compatibility analysis ensures that each datastream parameter physically “fits” together with all the others and removes any biases due to sampling offsets or errors. The technique employed here follows exactly that of [110, 120] and uses an embedded optimization scheme to minimize the bias error in the measurement after stripping out a turbulence component. An alternative might be to use a Kalman filter for the compatibility analysis [111]. The kinematic equations that are satisfied include consideration for removal of (fast) turbulence variations to the parameter, and include

\[ \dot{V} = (\overline{a}_x - g \sin \theta) \cos \overline{a} \cos \overline{b} + (\overline{a}_y + g \sin \phi \cos \theta) \sin \overline{b} \]
\[ + (\overline{a}_z + g \cos \phi \cos \theta) \sin \overline{a} \cos \overline{b} \]  \hspace{1cm} (24)

\[ \dot{\overline{a}} = [(\overline{a}_z + g \cos \theta \cos \phi) \cos \overline{a} - (\overline{a}_x - g \sin \theta) \sin \overline{a}] / (V \cos \overline{b}) \]
\[ + \frac{\overline{a}}{V} - \tan \overline{b} (\overline{p} \cos \overline{a} + \overline{r} \sin \overline{a}) \]  \hspace{1cm} (25)

\[ \dot{\overline{b}} = \cos \overline{b} (\overline{a}_x + g \cos \theta \sin \phi) / V + \overline{p} \sin \overline{a} \cos \overline{a} - \overline{r} \sin \overline{a} - \sin \overline{b} [(\overline{a}_z + g \cos \theta \cos \phi) \sin \overline{a} - (\overline{a}_x - g \sin \theta) \cos \overline{a}] / V \]  \hspace{1cm} (26)

\[ \dot{\theta} = \frac{\overline{a}}{V} \cos \phi - \frac{\overline{r}}{V} \sin \phi \]  \hspace{1cm} (27)

\[ \phi = \frac{\overline{p}}{V} + (\overline{q} \sin \phi + \overline{r} \cos \phi) \tan \theta \]  \hspace{1cm} (28)
\[ \psi = (\bar{q} \sin \phi + \bar{r} \cos \phi) \sec \theta \]  

(29)

where

\[ a_x = \bar{a}_x + \tilde{a}_x, a_y = \bar{a}_y + \tilde{a}_y, a_z = \bar{a}_z + \tilde{a}_z \]

\[ \alpha = \bar{\alpha} + \tilde{\alpha}, \beta = \bar{\beta} + \tilde{\beta} \]

(30)

\[ p = \bar{p} + \tilde{p}, q = \bar{q} + \tilde{q}, r = \bar{r} + \tilde{r} \]

and the bars (-) represent average (mean) values, the (~) represent turbulent values, and the biases resulting from an optimization scheme operating on these equations is removed from each measured term.

Once the biases have been estimated, the turbulence components are added back into the aerodynamic coefficients in order to have a corrected parameter set and to estimate an equivalent harmonic motion for each time step. Each coefficient is calculated from [26, 58]

\[ ma_z = C_z \bar{q} S + T_z \]  

(31)

\[ ma_y = C_y \bar{q} S + T_y \]  

(32)

\[ ma_x = C_x \bar{q} S + T_x \]  

(33)

\[ C_i \bar{q} S \bar{b} + T_i = I_{xx} \bar{p} - I_{xz} (\bar{r} + p \bar{q}) - (I_{yy} - I_{zz}) qr \]  

(34)

\[ C_m \bar{q} S \bar{c} + T_m = I_{yy} \bar{q} - I_{xz} (r^2 - p^2) - (I_{xx} - I_{yy}) rp \]  

(35)

\[ C_n \bar{q} S b + T_n = I_{zz} \bar{r} - I_{xx} (\bar{p} - qr) - (I_{xx} - I_{yy}) pq \]  

(36)

where \( I_{xx}, I_{yy}, I_{zz}, \) and \( I_{xz} \) are the moments of inertia, \( \bar{q} \) is dynamic pressure here, and the “\( T \)” terms represent thrust components.
Identification of coefficient dependencies is next, step (d). In [110], for the longitudinal aerodynamics, pitch and normal force were assumed to be a function of
\[ [\alpha, \dot{\alpha}, q, k_a, \beta, \delta \dot{\varepsilon}] \].

In the present research, an improvement was pursued to this model by incorporating the roll rate parameter, \( p \), as a dependency as a means to further couple in lateral-directional motion, resulting in
\[ [\alpha, \dot{\alpha}, q, k_a, \beta, \delta \dot{\varepsilon}, p] \].

Variation in Mach number for the present flight data set is small and is therefore of little consequence. In the lateral-directional axis, the model in the present research contains nine variable dependencies including
\[ [\alpha, \dot{\alpha}, \beta, \phi, p, r, k_a, \delta \alpha, \delta \tau] \]

The turbulence component was similar to previous studies [110]; a lateral component was studied, having a functional dependency of
\[ V = f(p, q, r, \theta, \phi, a_s, a_x, a_y, a_z) \]  \hspace{1cm} (37)

but was subsequently disregarded after it was observed that the lateral component of turbulence was relatively insignificant for the purposes of the present research. Note that this component and the longitudinal component are independent of any direct induced turbulence component, such as \( \tilde{\alpha} \) and \( \tilde{\beta} \). Descriptions of the gust field and turbulence model are given in Appendix A and Appendix B, respectively.

The calculation of the reduced frequency in the airplane coefficient model is accomplished by assuming a local equivalent harmonic motion and minimizing a cost
function in order to estimate the necessary parameters (step (e)). For the rolling moment coefficient, for example, this process may be written as

\[
C_i = f(\alpha, \dot{\alpha}, \beta, \phi, p, r, k, \delta_a, \delta_r)
\]

\[
\beta(t) = \hat{\beta} + \beta_A (\cos \omega t + \phi)
\]

\[
J = \sum_{i=1}^{M_i} (\beta_i - \hat{\beta} - \beta_A \cos(\omega t + \phi))^2
\]

\[
k = \omega \beta / 2V
\]

where the “i”, “A”, and “^” indices on β represent the present time value, the oscillatory amplitude, and the mean value of the amplitude, respectively; and J indicates a cost function in the optimization of the harmonic motion fit.

The fuzzy logic modeling approach has been summarized in several of the references [cf., 52, 53, 58, 110, 120] and will not be discussed in detail here. A recent paper [123] has shown that results using this approach are suitable and in acceptable agreement with other modeling approaches such as the feed-forward neural network. In the present application, the models that are extracted are continuous given the use of internal functions. The coefficient models are derived by using a series of optimizations to minimize the sum of squared errors of the output variables. In these steps, the time history being analyzed is split into a training set (to establish the model) and a testing set (to evaluate the prediction accuracy of the model), normally by partitioning the time history by every other data point.

The first portion of the fuzzy logic analysis establishes an optimum structure for the model (step (f)). This structure is based directly upon the model coefficient.
dependencies. For example, from [110], one model for the pitching moment coefficient had a model structure of

\[ \{\alpha_5, \dot{\alpha}_2, q_4, k_2, \beta_2, \delta e_2\} \,.
\]

For each variable, the number of membership functions is represented by the subscript.

Once a suitable structure has been determined, the coefficients of the model are then optimized through a series of successive iterations and numerical optimizations (step (g)). The routine continues to maximize the multiple correlation coefficient \( R^2 \) in an attempt to converge to a value at or very close to 1.0. The closer to 1.0, the better the model fit for the training time history. From [126], the correlation coefficient is defined as

\[
R^2 = 1 - \frac{\left\{ \sum_{j=1}^{m} (\hat{y}_j - y_j)^2 \right\}}{\left\{ \sum_{j=1}^{m} (\bar{y} - y_j)^2 \right\}}
\]

where \( \hat{y}_j \) is the output of the fuzzy logic model, \( y_j \) the measured data, and \( \bar{y} \) the average value of all data. A recent discussion [128] provides a good description of processes for filtering data and for improving the value of the correlation coefficient. In the present case, every other data point in the time history is utilized to fit the model, and in some cases supplemental discrete points from other time segments of a data set are incorporated.
One complication of attempting to identify suitable model fits to nonlinear or largely unsteady datasets is proper formulation and correlation of the model components in order to avoid numerical instabilities [25]. The present approach lends some insight into the correlation during the model fit process via output of intermediate steps.

After the model coefficients have been optimized, it is a relatively straightforward step to then analyze a given motion to extract coefficient values at points of interest (steps (h) and (i)). Of use is extracting an equivalent harmonic analysis to study aerodynamic lags and dependencies based upon reduced frequencies, motion amplitudes, etc. An extension of this enables the study of specific stability derivatives. For example, the extraction of pitch damping, $C_{mq}$, might be accomplished by using a central difference scheme and two estimates of pitching moment to obtain

$$
C_{mq} = \frac{C_m(\ldots, q_2, \ldots) - C_m(\ldots, q_1, \ldots)}{(q_2 - q_1)\bar{c}} \frac{1}{2V}
$$

(40)

There is one important distinction between derivatives calculated in this manner and those estimated in the conventional linear sense. The present derivatives are not evaluated about a trim condition, but instead are instantaneous or local values, because the aircraft is not trimmed in dynamic and unsteady motions.
Fuzzy Logic Comments

Several comments are in order regarding the fuzzy logic approach. First, it does provide the capability to capture the general features of a model, and with a suitable input and a suitable governing set of parameters can capture a model that describes a broad frequency content. In theory, a turbulence event is an ideal input set for model building because of the large frequency content of the input and response.

However, the technique has limitations in that the model it produces is only as good as the data used to build it, in terms of both range of parameters and in terms of specific combinations of those independent variables. And it is just a model representative of the specific study of interest, not a general database. Therefore, in the present research the focus will be on analyzing about specific, critical points. In addition, a fuzzy logic model is generally regarded as a “black box” with no insight into specific contributions of coefficients or parameters. This drawback, however, can be offset by simply putting the model into a “wind tunnel” mode and sweeping certain parameters while holding others fixed, so long as these combinations are generally within the bounds of the modeling set. For example, this could be used to strip out $C_m\alpha$, as implied by equation (40). Also, relative to modern simulation databases, the complexity and the inability for physical insight into the model is not appreciably different.

While PID is satisfactory for the linear and steady regime, as is “engineer in the loop” coefficient extraction, fuzzy logic enables capture of the nonlinear and
unsteady “time history” effects in a model. In the context of identifying airplane-specific effects, such as flexibility and autopilot or control system effects, there can be challenges. Augmented by wind-tunnel results, it is possible to add Reynolds number, Mach number, and dynamic pressure to show the former two effects plus structural flexibility. This is beyond the scope of the present research. The contributions of the autopilot and control system, however, should be accounted for by inclusion of the control surface deflections in the modeling data.

In theory, fuzzy logic does lend itself to a progression of modeling approaches that might be planned for a given research program. Most notably, it is suitable to an extension from wind-tunnel, to scale model research, to flight data. Wind-tunnel results might show dynamic and static high $\alpha$ and $\beta$ effects, where the dynamic terms are from either well-developed forced oscillation or rotary balance techniques [112, 113]. Scale model research, which is not a new technique to flight dynamics research [cf., 114], could be used to extract coefficients and models in extreme flight regimes, and could even be used to capture full-scale Reynolds number similarity [115]. From flight data, it should be possible to capture linear, steady and nonlinear, unsteady effects.

B. Computational development

Several FORTRAN computer codes have been developed and utilized, starting in about 1997, to aid in the fuzzy logic model extraction process. They have supported several research studies, and credit for their creation is given to prior
investigators [cf. 53, 110]. In general, they contain either modules or stand-alone programs to mechanize the equations and processes described in the previous section, and are used to conduct:

a) Flight data conditioning;
b) Flight data compatibility analysis;
c) Definition and extraction of fuzzy logic model structure;
d) Fuzzy logic model optimization;
e) Output prediction using the fuzzy logic models, and equivalent harmonic analysis of the nonlinear, unsteady model;
f) Extraction of specific stability derivatives.

C. Experimental Development: Aircraft and Flight Tests

The benefits of establishing a flight test program to aid in addressing aviation safety initiatives, utilizing a representative transport-category research aircraft, were recognized early on [116]. An initial project was established to fly several experiments associated with weather hazards research, and included experiments to safely evaluate several of the turbulence technologies in development at that time [117, 118] using the most up-to-date results of meteorological and turbulence research as an aid [cf. 92, 93].

The research aircraft selected for these flight experiments was typical of a modern single-aisle, twin-engine commercial transport aircraft. A photograph of it is shown in Figure 6. Having a wing span of 125 ft, a mean aerodynamic chord (MAC)
of 16 ft, a wing area of 1994 ft², a maximum takeoff weight of 230,000 lb, an in-flight center of gravity range of 7% to 39% MAC, the aircraft has an operational altitude envelope of from sea level to 42,000 ft, with an envelope speed limit of 350 KIAS at \( V_{mo} \) or 0.86 \( M_{mo} \).

The instrumentation suite was relatively comprehensive in that it had to cater to the needs of several different experiments simultaneously. In many cases, parameters already on the aircraft databusses were utilized, and ones that were particularly useful for the present application included parameters originating from the inertial reference units (IRU) and the air-data computer (ADC). In other cases, special equipment was installed on the aircraft, including the ability to conduct real-time processing of in-situ turbulence algorithms and weather radar turbulence algorithms through a special computer workstation arrangement. The aircraft was

Figure 6: Photograph of the research airplane.
equipped with a research angle-of-sideslip ($\beta$) vane located on the lower forward fuselage, and the normal low-pass filtering of the standard angle of attack ($\alpha$) vanes was bypassed in order to better measure the actual turbulence inputs to the aircraft. No nose boom or other free-air measure of $\alpha$ was available, which made the extraction of turbulence information more difficult but, on the other hand, more realistic in the context of evaluating the capability for extracting turbulence parameters from fleet aircraft systems. Several special pieces of instrumentation were installed, including accelerometers and rate gyros mounted near the aircraft center of gravity (in lieu of the standard IRUs, which are near the front of the airplane), plus accelerometers installed in the cockpit and in the rear of the airplane. Autopilot parameters/discretes were available and recorded since the autopilot tends to factor into the aircraft response.

The data system was capable of sampling and recording some channels at relatively high rates. While the databusses on commercial aircraft can operate this fast, they typically are not programmed to be interrogated that quickly for applications such as the DFDR. The experimental systems enabled the simultaneous look at in-situ calculated parameters and parameters available from radar detection algorithms. It also tied together real-time weather uplinks and the weather radar from the cockpit.

The test program [117, 118] was initially established, for the turbulence research of interest, to
a) Run and evaluate updated, tuned real-time in-situ turbulence algorithms
   (following a similar effort in unpiloted, desktop simulators);

b) Correlate airborne radar turbulence products with in-situ measurements,
   i.e., determine if the turbulence indication detected by weather radar was
   accurate in the context of what the airplane subsequently flew through;

c) Display radar turbulence products within the aircraft;

d) Provide forecast/nowcast products for flight planning and execution;

e) Acquire phenomenological turbulence information.

The focus of the research was on weather penetration, and as a result several
flights were conducted that have been summarized in several references [79, 88-91,
121]. The weather targets of interest were typically found in and around convective
activity, using the guidance provided by the conclusions of phenomenological studies
[i.e., 92, 93].
IV. RESULTS AND DISCUSSION

A. Flight Data

Several significant turbulence events were captured over the course of flight research. One in particular generated high positive and negative g-loads on the airframe (event 191.3), and the time histories of several (unprocessed) parameters are shown in Figure 7. This case has been used in previous analyses [79, 121] and is utilized presently as representative of a severe turbulence encounter. This particular flight condition is representative of cases in NTSB accident and incident databases [88]. Sample data plots and descriptions of these turbulence penetrations are described in detail in the references.

Figure 7: Flight data of a turbulence encounter, research event 191.3
Of particular interest is roughly the last third of the condition. Figures 8 and 9 show several more parameters for this particular time segment, with the ones used for modeling later on having passed through the smoothing and kinematic compatibility checks as previously accomplished. As a means to expedite the initial phases of this investigation, and after a considerable effort was expended in understanding the steps leading to the results in the prior work [110, 120], the specific numerical results were obtained from the prior investigator and largely used to establish the starting point for the fuzzy-logic analysis presented here.

The vertical gust parameter, \( w_{\text{gust}} \) (or \( w_{\text{g}} \)), was processed and recorded aboard the aircraft utilizing several air-data and inertial data sources. The algorithm is provided in Appendix A.

Referring to Figure 7, the aircraft had been in a region of choppy air until about 100 seconds before the time period of the present analysis. At that time the air had become smoother, although it was still a little bumpy. Observing Figures 8 and 9, at about 189 seconds the air began getting turbulent again as indicated by the oscillations in load factor, \( n_z \).

The point at about 188 seconds, for the purposes of the discussion to follow, is considered relatively steady – although not necessarily in true “trim” – and is denoted by Point A. Straddling the time of 191 seconds, load factor increases, as does vertical gust and airplane angle of attack. This general time period is called Point B. At about 199 seconds, the aircraft experiences an almost 2.0g load factor as a large vertical gust is encountered, followed by a rapid decrease in load factor then another
Figure 8: Time histories of independent variables
Figure 9: Time histories of independent variables.
rapid increase. This is Point C. Point D follows this sequence immediately, and is the point where the aircraft hits a large downward shear on the gust and experiences a negative load factor. Past this point, the aircraft loses about 150 ft (about 50 m) of altitude over about 12 seconds as the gust slowly builds back up from negative, and the load factor again is slightly elevated above 1.0g, as the descent is arrested at Point E. At this latter point the air is relatively smooth although the aircraft is slowly settling back to steady values.

The points described above are denoted on Figures 8 and 9, along with dashed lines through each subplot to aid in visualization. Table 1 shows the details of those points.

For the fuzzy-logic model, the reduced frequency in pitch, $k_\alpha$, was calculated previously [110, 120] and is shown in Figure 10. For convenience, the lateral-directional reduced frequency, $k_\phi$, is also shown and its use is described in later discussion.

It is important to note that units of measure shown on several upcoming plots are in the metric system. While a bit awkward to combine with English units, that is the way the flight data was presented to the author. Because many of the results are coefficients that are independent of units, and to retain some consistency with the data as provided, the results are shown in those units.
Table 1: Values of discrete points in turbulence encounter time history.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time  (sec)</td>
<td>188.375</td>
<td>191.102</td>
<td>198.695</td>
<td>200.922</td>
<td>216.750</td>
</tr>
<tr>
<td>(\alpha) (deg)</td>
<td>2.01</td>
<td>2.79</td>
<td>5.40</td>
<td>-0.70</td>
<td>2.64</td>
</tr>
<tr>
<td>(\alpha)-dot (deg/sec)</td>
<td>0.28</td>
<td>-5.13</td>
<td>0.47</td>
<td>-5.67</td>
<td>-0.13</td>
</tr>
<tr>
<td>(\beta) (deg)</td>
<td>0.18</td>
<td>0.96</td>
<td>-1.30</td>
<td>4.49</td>
<td>0.16</td>
</tr>
<tr>
<td>(p) (deg/sec)</td>
<td>-1.93</td>
<td>-4.66</td>
<td>-1.21</td>
<td>-3.15</td>
<td>1.10</td>
</tr>
<tr>
<td>(q) (deg/sec)</td>
<td>0.27</td>
<td>0.058</td>
<td>-0.0067</td>
<td>-1.42</td>
<td>0.36</td>
</tr>
<tr>
<td>(r) (deg/sec)</td>
<td>-0.49</td>
<td>-0.06</td>
<td>-0.62</td>
<td>-0.86</td>
<td>-0.64</td>
</tr>
<tr>
<td>(\phi) (deg)</td>
<td>-14.1</td>
<td>-20.6</td>
<td>-24.2</td>
<td>-26.5</td>
<td>-16.3</td>
</tr>
<tr>
<td>(\delta\varepsilon) (deg)</td>
<td>-2.18</td>
<td>-1.49</td>
<td>-2.44</td>
<td>-2.68</td>
<td>-2.81</td>
</tr>
<tr>
<td>(\delta\alpha) (deg)</td>
<td>0.35</td>
<td>0.22</td>
<td>0.24</td>
<td>0.24</td>
<td>0.35</td>
</tr>
<tr>
<td>(\delta\beta) (deg)</td>
<td>0.35</td>
<td>0.26</td>
<td>0.35</td>
<td>0.26</td>
<td>0.40</td>
</tr>
<tr>
<td>(k_\alpha)</td>
<td>0.041</td>
<td>0.078</td>
<td>0.052</td>
<td>0.088</td>
<td>0.042</td>
</tr>
<tr>
<td>(k_\phi)</td>
<td>0.92</td>
<td>0.59</td>
<td>1.28</td>
<td>0.51</td>
<td>1.14</td>
</tr>
</tbody>
</table>

Figure 10: Time histories of reduced frequency values.
B. Aerodynamic Characteristics

Longitudinal Models

Roll rate, $p$, was added to the model dependency published previously [110, 120], resulting in a normal force and pitching moment setup having the following form:

$$ C = f [\alpha, \dot{\alpha}, q, k_\alpha, \beta, \dot{\beta}, p] $$

Recent research into unsteady, nonlinear modeling suggests that lateral-directional coupling may impact traditional longitudinal dynamic responses. The sideslip term had already been present in the model. Physically, this implies the possibility of a non-uniform loading, in the spanwise direction, for a condition that is already experiencing unsteady or nonlinear behavior in the lift or pitch axes. Another way to state it is that at an angle of attack that is high enough, or at high speed, or with a significant and non-uniform gust input, sideslip or the effects of rolling may promote early flow separation or shocking on one side of the airplane but not the other. These effects can be checked using sensitivity derivatives resulting from the model formulation above.

Normal Force, $C_z$

For normal force, $C_z$, the following structure was determined:

$$ C_z = f [\alpha_3, \dot{\alpha}_2, q_2, k_{\alpha_2}, \beta_4, \dot{\beta}_4, p_2] $$
This model structure resulted in a convergence of $R^2 = 0.9607$, an indication of the fit of the model to the training data, where a value of 1.000 indicates a perfect fit. Previous results [110, 120] indicated $R^2 = 0.934$. The present models were trained to a much longer time history than before, from 185-240 seconds, plus several additional points in an earlier portion of the total time segment.

The ranges of the variables for building these models encompassed:

- $\alpha$: [-5, 10 deg]
- $\dot{\alpha}$: [-40, 40 deg/sec]
- $q$: [-6, 6 deg/sec]
- $k_\alpha$: [0, 0.6]
- $\beta$: [-4, 7 deg]
- $\delta_e$: [-5, 5 deg]
- $p$: [-15, 15 deg/sec]

Figure 11 shows a plot of the time history of the normal force coefficient, $C_z$, as extracted from the flight data and the resulting model, for the flight data set used to establish the model. Figure 11 also shows a check against an independent flight data set, as a means to check the reasonableness of the model in predicting general trends. In both cases, the model showed a good ability to track the general characteristics of the flight data.

The computational tool used to extract a fuzzy-logic model fit to a set of flight conditions also has the ability to conduct central-difference derivatives at specific points, or all points, along a trajectory. In the present research, stiffness, rate, and control derivatives are of particular interest. Figure 12 presents $C_{z\alpha}$, $C_{z\dot{\alpha}}$, $C_{zq}$, and $C_{z\delta_e}$ for the entire time history, as well as sensitivity to roll rate, $C_{zp}$. Also depicted are Points A through E as described earlier. As a means to better interpret the trends in the first two derivatives for discussion later on, 3- and 5-second moving averages
were applied to these time histories to smooth them, and the results are depicted in Figure 13. Additionally, Table 2 presents the values of the derivatives at each of the discrete points A through E.

Figure 11: Flight and model of normal force coefficient, $C_z$. 
Figure 12: Normal force coefficient derivatives extracted from the model.
Figure 13: Smoothed values of stiffness and plunge normal force derivatives.
Table 2: Coefficients and derivatives at discrete points in turbulence encounter time history.

<table>
<thead>
<tr>
<th></th>
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<tr>
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<td>191.102</td>
<td>198.695</td>
<td>200.922</td>
<td>216.750</td>
</tr>
<tr>
<td>$C_{z}$ (flight)</td>
<td>0.409</td>
<td>0.605</td>
<td>0.797</td>
<td>0.063</td>
<td>0.480</td>
</tr>
<tr>
<td>$C_{z}$ (model)</td>
<td>0.407</td>
<td>0.535</td>
<td>0.792</td>
<td>-0.178</td>
<td>0.473</td>
</tr>
<tr>
<td>$C_{za}$ (rad$^{-1}$)</td>
<td>7.27</td>
<td>8.40</td>
<td>13.71</td>
<td>42.02</td>
<td>6.69</td>
</tr>
<tr>
<td>$C_{\alpha}$ (rad$^{-1}$)</td>
<td>48.76</td>
<td>1.94</td>
<td>11.08</td>
<td>511.37</td>
<td>58.10</td>
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<tr>
<td>$C_{z\alpha}$ (rad$^{-1}$)</td>
<td>559.76</td>
<td>731.03</td>
<td>1557.50</td>
<td>-392.93</td>
<td>645.53</td>
</tr>
<tr>
<td>$C_{z\alpha-osc}$ (rad$^{-1}$)</td>
<td>608.53</td>
<td>732.97</td>
<td>1568.57</td>
<td>118.45</td>
<td>703.63</td>
</tr>
<tr>
<td>$C_{z\delta e}$ (rad$^{-1}$)</td>
<td>0.62</td>
<td>-0.30</td>
<td>12.67</td>
<td>-12.25</td>
<td>1.00</td>
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<tr>
<td>$C_{z\delta p}$ (rad$^{-1}$)</td>
<td>0.350</td>
<td>-0.794</td>
<td>0.796</td>
<td>-19.91</td>
<td>-2.68</td>
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<tr>
<td>$C_{m}$ (flight)</td>
<td>-0.0028</td>
<td>-0.0146</td>
<td>-0.0970</td>
<td>0.1379</td>
<td>-0.0059</td>
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<tr>
<td>$C_{m}$ (model)</td>
<td>-0.0008</td>
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<td>-0.0938</td>
<td>-0.0097</td>
<td>-0.0015</td>
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<tr>
<td>$C_{ma}$ (rad$^{-1}$)</td>
<td>-1.32</td>
<td>-2.45</td>
<td>-1.80</td>
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<td>-1.04</td>
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<td>$C_{ma}$ (rad$^{-1}$)</td>
<td>9.77</td>
<td>-132.50</td>
<td>-108.60</td>
<td>-135.53</td>
<td>75.65</td>
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<tr>
<td>$C_{mq}$ (rad$^{-1}$)</td>
<td>-192.48</td>
<td>81.51</td>
<td>-934.51</td>
<td>135.74</td>
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<td>$C_{mq-osc}$ (rad$^{-1}$)</td>
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<td>-1043.11</td>
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<td>$C_{m\delta e}$ (rad$^{-1}$)</td>
<td>2.52</td>
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<tr>
<td>$C_{m\delta p}$ (rad$^{-1}$)</td>
<td>0.207</td>
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<tr>
<td>$C_{roll}$ (flight)</td>
<td>0.00367</td>
<td>0.0215</td>
<td>-0.0333</td>
<td>-0.04015</td>
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<tr>
<td>$C_{roll}$ (model)</td>
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<tr>
<td>$C_{lp}$ (rad$^{-1}$)</td>
<td>-0.65085</td>
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<td>1.5330</td>
<td>4.2555</td>
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<tr>
<td>$C_{lp}$ (rad$^{-1}$)</td>
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<td>-28.7062</td>
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<td>$C_{l\delta}$ (rad$^{-1}$)</td>
<td>14.4413</td>
<td>-26.685</td>
<td>-20.897</td>
<td>10.7281</td>
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<tr>
<td>$C_{l\delta e}$ (rad$^{-1}$)</td>
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<td>-6.910</td>
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<td>$C_{n}$ (flight)</td>
<td>0.000375</td>
<td>0.01039</td>
<td>0.0223</td>
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<td>0.00004</td>
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<td>$C_{n}$ (model)</td>
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<td>0.02137</td>
<td>0.02250</td>
<td>0.00094</td>
</tr>
<tr>
<td>$C_{ndl}$ (rad$^{-1}$)</td>
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<td>-0.6252</td>
<td>0.5775</td>
<td>-4.7282</td>
<td>-0.1492</td>
</tr>
<tr>
<td>$C_{np}$ (rad$^{-1}$)</td>
<td>-3.8625</td>
<td>-3.2364</td>
<td>6.2527</td>
<td>2.3078</td>
<td>-1.6494</td>
</tr>
<tr>
<td>$C_{nr}$ (rad$^{-1}$)</td>
<td>-1.4542</td>
<td>4.3036</td>
<td>10.8545</td>
<td>63.935</td>
<td>-6.3609</td>
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<tr>
<td>$C_{n\delta}$ (rad$^{-1}$)</td>
<td>0.9415</td>
<td>-1.6975</td>
<td>-1.1006</td>
<td>5.8131</td>
<td>1.0759</td>
</tr>
</tbody>
</table>
By observation of the time histories, particularly those shown in Figure 13, it appears that as the average magnitude of the upgust increases and angle of attack increases, the average lift-curve slope also gradually increases until the point of the two strong positive load factor spikes right around t = 199 seconds. It peaks as the sharp downgust progresses and, on average, eventually comes back to nearly its starting value. However, in this period there are two sharp decreases in gust, between C and D and at D, which could possibly be causing suction to reduce the lift-curve slope.

The damping, on the other hand, first decreases slightly in the gradual upgust, then increases, then decreases noticeably, going negative at the times of the sharp up then down wind shear. It then returns to positive at a level higher than at “trim” as the airplane is descending, then eventually smooths out. For most of the time history, the changes in the lift-curve slope and the damping appear to be out of phase with each other.

The effect of roll rate on normal load factor appears most evident at point D, as the large change in this derivative, $C_{zp}$, corresponds to a large change in roll rate (Figure 9). The implication is that portions of the wing may either symmetrically or asymmetrically load or unload due to localized stalling (e.g., dynamic stall at a high value of $k_\alpha$), shocking, or separation in rolling. Alternatively, asymmetric stalling or separation may also explain why aircraft tend to roll and yaw in significant turbulence, even when there are not appreciable horizontal gusts. In the present case, horizontal winds in the inertial sense were relatively steady, but while not shown here
the aircraft was generally wings-level and experiencing appreciable roll-rate
colorations prior to the time shown in Figure 9. Figure 14, which shows the pressure
colorants estimated using CFD around a representative supercritical airfoil typical of
the kind used on modern transonic jets, indicates the presence of a strong shock
formation on this airfoil. Dynamic lift effects offer the possibility that a similar shock
on the research aircraft may be changing characteristics during unsteady periods and
driving the asymmetries noted above.

Curiously, the sensitivity of $C_z$ to pitch rate is most pronounced between
points B and C at about $t = 195$ sec, and at point D, although there is no appreciable
pitch rate change (Figure 8). During the former, the aircraft is in a period where it is
briefly settling down around a 1.0g flight condition and the gust magnitude is
transitioning in sign. The effect of pitch rate on airplane lift is typically and
predominantly a tail effect, so the trends depicted in Figure 12 are possibly related to
changes in tail lift characteristics due to wing downwash changes.

The impact of these changes in normal force coefficient are analyzed and
discussed in more detail in a later section.

**Pitching Moment, $C_m$**

The following structure was determined for pitching moment, $C_m$:

$$
C_m = f [\alpha_2, \dot{\alpha}_3, q_2, k_{a_2}, \beta_3, \delta e_3, p_4]
$$

This model structure resulted in a convergence of $R^2 = 0.965$. As noted earlier, the
present models were trained to a much longer time history than before, from 185-240 seconds. The variable ranges used to tune this model were:

\[ \alpha: [-4.5, 9 \text{ deg}] \quad \dot{\alpha}: [-34, 16 \text{ deg/sec}] \quad q: [-6, 5 \text{ deg/sec}] \]

\[ k_\alpha: [0, 0.6] \quad \beta: [-7, 8 \text{ deg}] \quad \delta e: [-8, 3 \text{ deg}] \]

\[ p: [-19, 8 \text{ deg/sec}] \]

Figure 15 shows a plot of the time history of the pitching moment coefficient, \( C_m \), as extracted from the flight data and the resulting model, for the flight data set used to establish the model. Figure 15 also shows the check against an independent
flight data set. As before, the model showed a good ability to track the general characteristics of the flight data.

Figure 16 presents $C_{m\alpha}$, $C_{m\alpha}$, $C_{mq}$, and $C_{m\delta_e}$, as well as sensitivity to roll rate, $C_{mp}$, for the entire time history, and Points A through E are again depicted. The 3- and 5-second moving averages were again applied to the first two derivatives, and the results are depicted in Figure 17. As before, Table 3 presents discrete values of the derivatives at points of interest.

Similar to the trend in $C_z$, the sensitivity of pitching moment to pitch rate, $C_{mq}$, has its largest change between points B and C in the time history, although there is nothing remarkable in the value of pitch rate. Evidently, these two are coupled and could be the effect of an artificial (aerodynamic) pitch rate as the gust input changes sign, or a change in the downwash characteristics at the tail as a result of unsteady changes in wing lift. There is some effect of roll rate, possibly the result of localized stalling on the swept wing (e.g., dynamic stall at high $k_{\alpha}$ at the wingtips aft of the center of gravity) or the tail or both, as discussed in the section on lift.

Analyzing Figure 17, it appears that average static stability appears to remain negative throughout the time history, except for near the very end during the aircraft altitude recovery. $C_{m\alpha}$ appears to become most negative during the time of the sharp up-then down-shear of the wind. The average sign of the damping, $C_{m\alpha}$, appears to start out negative (positive damping) but then turns strongly positive except for a return during the strong up-then down-windshear. But then it tends to positive for the remainder of the maneuver, indicating a loss in pitch damping. It is important to
Figure 15: Flight and model of pitching moment coefficient, $C_m$. 

(a) Training set.

(b) Independent set.
Figure 16: Pitching moment coefficient derivatives extracted from the model.
Figure 17: Smoothed values of stiffness and plunge pitching moment derivatives.
note that the oscillatory variations in values of $C_{m\dot{\alpha}}$ in the present results are not unlike those seen in the rolling moment due to sideslip rate, $C_{i\dot{\beta}}$, in other studies to investigate wing-rock, or nonlinear lateral oscillation, unstable motion on a fighter aircraft [128].

**Longitudinal Harmonic Oscillation**

Figure 18 shows the small oscillation harmonic motion for normal force coefficient, at values of +/- 1 degree of sweep in alpha (Figure 18(a)). The points near the center show good characteristics in lift curve slope relative to what would be expected for this configuration. At the two extreme points, note the marked increase in lift curve slope. These points are at the “edges” of the modeling set, and therefore the model predictor is extrapolating to capture the full sweep. As a check, the sweep was reduced to +/- 0.5 degree, as shown in Figure 18(b), with very similar results. The points were also subsequently shifted “inward” so that their “center point”, and subsequently the edges of the sweep, were within the model limits. Although not shown, the results were very similar.

As a means to possibly explain and understand the extreme slopes, several sweeps through the minimum and maximum range of the model, for angle of attack, were conducted at several values of $k_\alpha$. These are shown in Figure 19. For these plots, $q = \beta = p = 0$, and $\delta e = -2$ deg.

For condition C of the main time history, in Figure 19 the value of $k_\alpha$ was approximately that of the $k = 0.06$ plot. For condition D, $k_\alpha$ was 0.088. It appears
Figure 18: Normal force coefficient harmonic oscillations about specific conditions, 
\( q = \beta = p = 0, \delta e = -2 \) deg.
Figure 19: Normal force coefficient harmonic oscillations for several values of $k_\alpha$

$q = \beta = p = 0$, $\delta e = -2$ deg, $\alpha_A = 4$ deg.

(continued)
Figure 19: Normal force coefficient harmonic oscillations for several values of $k_\alpha$, $q = \beta = p = 0$, $\delta e = -2$ deg, $\alpha_4 = 4$ deg (concluded).
that for Condition C, the condition is along the far right-hand side of that oscillation, where the local slope on the increase in alpha takes a sharp turn upward. Similarly, for Condition D it is on the far left-hand side of that sweep where there is the possibility for a large slope. These same effects can be seen for both Conditions C and D on the \( k = 0 \) plot of Figure 19, although caution should be exercised that this value of \( k \) is outside the range of the training data. Near the center of each of the four plots in Figure 19 the lift-curve slope appears to be reasonable. However, it is important to note that if the hysteresis loop of the normal force with angle of attack is clockwise, then the corresponding \( C_{z\alpha} \) should be positive, and vice-versa. Although not shown in Figure 18, the loops for points C and D are both counterclockwise. The instantaneous values of the damping derivative are positive in Table 3, yet from Figure 13 the average values during these time periods are negative.

Figures 20 and 21 show similar characteristics for pitching moment. In the case of Conditions C and D again, trends depicted in Figure 20 can be observed in the right-hand side of the \( k = 0.088 \) plot for Condition D, and in the left-hand side of the \( k = 0.06 \) graph for Condition C, on Figure 21. Again, for these plots, \( q = \beta = p = 0 \), and \( \delta \dot{\epsilon} = -2 \) deg, so the sweeps aren’t necessarily taken “on” any particular condition. On average, at each of these conditions the airplane appears to be stable as depicted by the negative slope. This is an important point because in Table 3, Condition D shows a positive value of \( C_{ma} \) at this specific discrete point. In the oscillation about that point, the average slope is calculated as negative from the results used to generate Figure 20.
(a) +/- 1 degree oscillation

(b) +/- 0.5 degree oscillation

Figure 20: Pitching moment coefficient harmonic oscillations about specific conditions, \( q = \beta = p = 0, \delta e = -2 \) deg.
Figure 21: Pitching moment coefficient harmonic oscillations for several values of of $k_\alpha$, $q = \beta = p = 0$, $\delta e = -2$ deg, $\alpha_t = 4$ deg

(a) $k_\alpha = 0$

(b) $k_\alpha = 0.04$

(continued)
Figure 21: Pitching moment coefficient harmonic oscillations for several values of $k_\alpha$, \( q = \beta = p = 0 \), \( \delta_e = -2 \text{ deg} \), \( \alpha_4 = 4 \text{ deg} \) (concluded).
Lateral-Directional Models

The lateral-directional models were assumed to have the form

\[ C = f [\alpha, \dot{\alpha}, \beta, \phi, p, r, k_{\phi}, \dot{\alpha}, \dot{\phi}] \]

Where \( k_{\phi} = \frac{\omega_b}{2V} \) represents a reduced frequency in bank angle. In all cases, the time segment used for modeling was extended from that used in the longitudinal model, to the segment from 175 to 240 seconds plus several discrete points from the time period prior to that. The reason for extending the start to the 175 second mark was that the original segment, starting at 185 seconds, did not include a period when the bank angle was either zero or positive, as shown in Figure 9. This led to the potential problem of analyzing a model (e.g., for a harmonic analysis) outside of the range of its model fit capability. The time period between 40 and 65 seconds in the flight data was again used to check the capability of the model to predict an independent set of data.

An example of \( k_{\phi} \) as derived from an equivalent harmonic analysis similar to that used for the reduced frequency in pitch, is shown in Figure 10.

Rolling Moment, \( C_{\text{roll}} \)

For rolling moment, \( C_{\text{roll}} \) (or equivalently \( C_i \)), the following structure was determined:

\[ C_{\text{roll}} = f [\alpha_5, \dot{\alpha}_4, \beta_2, \phi_2, p_2, r_2, k_{\phi_2}, \dot{\alpha}_2, \dot{\phi}_2] \]
The fact that a rolling-moment model has a dependency on at least two pitch terms, in particular $\dot{\alpha}$, may support the point discussed in the normal force ($C_z$) model section regarding significant rolling motion in severe turbulence. The physical driver could be localized, asymmetric dynamic stall, shocking, or separation on a wing surface. This point would also extend to yawing moments.

This model structure resulted in a convergence of $R^2 = 0.947$. The ranges of the variables in the models encompassed:

- $\alpha$: [-5, 10 deg]
- $\dot{\alpha}$: [-34, 16 deg/sec]
- $\beta$: [-7, 9 deg]
- $\phi$: [-75, 16 deg]
- $p$: [-19, 8 deg/sec]
- $r$: [-8, 5 deg/sec]
- $k_{\phi}$: [0, 3.0]
- $\delta\alpha$: [-2, 3 deg]
- $\delta r$: [-11, 5 deg]

Figure 22(a) shows a plot of the time history, showing the model fit to the flight data for the set used to establish the model, within the time period consistent with the longitudinal results. Figure 22(b) shows a check against an independent flight data set. As seen, the model fit the magnitudes of the independent set fairly well although overpredicted some peaks. This may be indicative of a need to incorporate additional important variables in the model in future or related work. In other words, there may be other effects that are not accounted for here, possibly either an aeroelastic term or perhaps a sideslip rate term. The critical points from the longitudinal discussion are depicted and, except for perhaps point C, appear to show some correlation to the normal force coefficient time history events (i.e., peaks and benign areas).
Figure 22: Flight and model of rolling moment coefficient, $C_{roll}$. 

(a) Training set.

(b) Independent set.
Figure 23 presents the local linearized rolling-moment derivatives $C_{I\beta}$, $C_{lp}$, $C_{lr}$, and $C_{l\delta}$. Table 3 also summarizes discrete values of the coefficient and derivatives at the critical points. It is important to note from Figure 9 that the pilot was instructed to “ride out” this turbulence encounter with minimal control inputs. Therefore, the results for the aileron contribution in Figure 23 must be viewed with some amount of caution because there were no substantive aileron inputs from which to extract a clean model.

The typical sign conventions on $C_{I\beta}$ and $C_{lp}$ are negative, and $C_{lr}$ positive. Curiously, during this portion of the time history, $C_{I\beta}$ correctly began and ended that way (Figure 23 and Table 3), but generally developed a positive bias once the significant turbulence began. On the other hand, $C_{lp}$ was just the opposite, with the model predicting unstable damping to start with then biasing toward stability in the time period of most interest. $C_{lr}$ showed the same trend as $C_{I\beta}$ albeit with opposite sign. It is important to note that this is not inconsistent with the results of other investigations of nonlinear lateral dynamics [cf., 128]. These derivatives do not necessarily describe the conventional stability conditions, but instead describe the nonlinear nature of the dynamic motion itself.

**Yawing Moment, $C_n$**

For yawing moment, $C_n$, the following structure was determined:

$$C_n = f [\alpha_2, \dot{\alpha}_2, \beta_2, \phi_2, p_2, r_2, k_{\beta_2}, \dot{\beta}_2, \dot{r}_2]$$

This model structure resulted in a convergence of $R^2 = 0.958$. The ranges of the
Figure 23: Rolling moment coefficient derivatives extracted from the model.
variables in the models encompassed:

\[\alpha: [-5, 10 \text{ deg}] \quad \dot{\alpha}: [-40, 40 \text{ deg/sec}] \quad \beta: [-6, 6 \text{ deg}]\]

\[\phi: [-45, 45 \text{ deg}] \quad p: [-15, 15 \text{ deg/sec}] \quad r: [-5, 5 \text{ deg/sec}]\]

\[k_\phi: [0, 2.5] \quad \delta\alpha: [-2, 2 \text{ deg}] \quad \delta r: [-8, 5 \text{ deg}]\]

Figure 24(a) shows a plot of the time history, showing the model fit to the flight data for the set used to establish the model, again within the time period consistent with the longitudinal results. Figure 24(b) shows a check against an independent flight data set. As seen, the model fit the magnitudes of the independent set fairly well although overpredicted some peaks while underpredicting others, again possibly indicative of a need to incorporate more independent parameters in the model. The critical points from the longitudinal discussion are depicted and appear to show some correlation to the normal force coefficient time history events (i.e., peaks and benign areas), except that peaks following points C and D appear to lag the event.

Figure 25 presents the local linearized yawing-moment derivatives \(C_{n\beta}, C_{np}, C_{nr},\) and \(C_{n\phi}.\) Table 3 also summarizes discrete values of the coefficient and derivatives at the critical points. As noted earlier, the pilot was instructed to “ride out” this turbulence encounter with minimal control inputs, with rudder activity shown in Figure 9 likely resulting from the yaw damper. Therefore, as before, the results for the rudder contribution in Figure 25 should be viewed with some amount of skepticism although there was more rudder activity than on the aileron channel.
Figure 24: Flight and model of yawing moment coefficient, $C_n$.
Figure 25: Yawing moment coefficient derivatives extracted from the model.
The typical sign convention on $C_{n\beta}$ is positive, on $C_{nr}$ negative, and on $C_{np}$ dependent upon configuration and speed. As seen in Figure 25, during this portion of the time history, $C_{n\beta}$ began and ended with the correct sign, but developed what appears to be a negative bias in the portion of significant turbulence. On the other hand, like $C_{lp}$ the value of $C_{nr}$ was just the opposite, with the model predicting unstable damping to start with then biasing toward stability in the time period of most interest. The value of $C_{np}$ appears to stay mostly negative.

**Side Force, $C_Y$**

Investigation of the relatively minor side force term, $C_Y$, was initially intended as part of this effort. A plot of the flight values for a time segment that was originally analyzed is shown in Figure 26. The model dependencies were the same as rolling moment and yawing moment.

For unknown reasons, fitting a model for this coefficient, having a suitably high value of $R^2$, proved particularly difficult. In the interest of time, it was concluded that the relative unimportance of this degree of freedom to the present study warranted disregarding any further consideration, perhaps to be studied in a future effort.

**Lateral-Directional Harmonic Oscillation**

Figures 27 and 28 show results for harmonic motion in roll, for rolling moment. General characteristics of these types of plots in the quasi-steady, linear
sense indicate stability by the ±φ slope of the curve, and damping by the vertical dimension of the resulting ellipse [123]. For these plots, \( q = 0, \alpha_0 = 2 \text{ deg}, \delta a = \delta r = 0 \), and \( \beta, \alpha, \rho, \) and \( \dot{\alpha} \) vary during the body-axis roll as if on a wind-tunnel sting.

Figure 27 is a check of the static case, \( k_\phi = 0 \), where of course no elliptical shape to the curve is expected since there would be no vortex lag. As shown, the configuration appears stable in roll although there is a curious positive bias to the rolling moment through zero bank angle. Recall, however, that this value of \( k_\phi \) is at the very limit of the modeling range, and a bias is not necessarily uncommon as seen in wind-tunnel results for this configuration [123].

Figure 28 shows the result of sweeping the aircraft model, for various values of \( k_\phi \), in a manner that captures a consistent and constant value of maximum roll rate,

Figure 26: Flight values of side force coefficient.
that is within the bounds of the modeling range. This is consistent with means for conducting wind-tunnel forced-oscillation testing [123]. As $k_\phi$ is increased, the bank angle range of the sweep must be reduced accordingly in order to hold $p_{max}$ constant.

There are at least two interesting observations from Figure 28. The first is that damping is apparent by the thickness of the “ellipse”, although it seems to change as a function of bank angle, particularly noted for positive values of bank angle. While this is consistent with observed wind-tunnel results at high angles of attack [e.g., 123], such a phenomena is typically symmetric with respect to positive and negative bank angle in the linear sense. Therefore, non-linear effects are prevalent, and it is quite possible that more flight data at positive bank angles could improve this aspect of the model in a future effort.

The other interesting facet of these results is that the sign convention of the rolling moment coefficient trend with the bank angle sweep is opposite of positive damping. In other words, the trace of the curves is clockwise, instead of counter-clockwise, except for the smallest value of $k_\phi$ at large positive angle of bank. While this doesn’t explain the damping trends observed in the flight time history, it does lend some consistency in interpreting those results.

Initial investigations into roll harmonic motion incorporated the yawing moment as a secondary check. For the purposes of the present study, investigation of yawing moment as a function of roll sweep did not appear to offer any particular
benefit or insight into the aircraft motion, so no further action with the final model was undertaken.

Figure 27: Rolling moment harmonic oscillation, $k_{\phi} = 0$. 
Figure 28: Rolling moment harmonic oscillation, various $k_\phi$, constant $p_{max}$.

(a) Small $k_\phi$.

(b) Large $k_\phi$. 

90
C. Correlation to Hazard

The purpose of this section is to tie the preceding discussion together in order to propose some relevance in predicting turbulence hazards.

As previously stated, in designing aircraft for turbulence encounters, the focus is predominantly centered on protecting the aircraft structure from excessive loads, with handling qualities and stability and control considerations often a secondary consideration. But the aircraft response is dictated by its damping and stiffness characteristics, and as observed from the results of previous sections those appear to be affected in a severe turbulence encounter. It appears possible that the aerodynamic characteristics of the aircraft can change, albeit momentarily, as a result of the unsteady and non-uniform flowfield and perhaps localized flow separation or shocking. The implication is that these effects may or may not drive an increase in overall load factors in turbulence by affecting the response of and the basic ability to fly the aircraft in a predictable manner.

Stated differently, it may not necessarily be the excessive loads that are the hazard, but instead it may be the reversal or changes in excessive loads resulting from the dynamic response of the aircraft. One way to visualize this is the unbelted passenger who may be lifted off of their seat or the aisle with a negative \( g \)-increment from the aircraft in a plunging motion, then thrown back down in a recovery. In order to reduce these types of events that cause injuries to passengers and flight crews, then, minimizing the response of the aircraft will be the objective. By far the most direct means to address this problem is with the design and implementation of an active load
alleviation system [cf. 101-104]. Practically speaking, however, unless these systems are designed into new airplanes they tend to be mechanically and structurally difficult to implement. Other means to control the airplane then need to be investigated.

Conceptually, the one direct approach in studying this problem is to write a simple set of equations of motion for the aircraft in plunging, assuming an estimated aerodynamic model, then proposing means to minimize or control that motion.

**Plunging Motion**

Consider the plunging motion of the aircraft. From first principles, normal acceleration results in the normal force coefficient, which enables the estimation of variations in pressure altitude:

\[ \frac{W}{g} a_N = N = C_N q S \]  \hspace{1cm} (41)

where \( W \) is the aircraft weight, and \( g \) is the acceleration due to gravity. From a flight dynamics formulation [cf. 122]

\[ -a_N = a_z = \frac{dw}{dt} - Uq \]  \hspace{1cm} (42)

where \( w \) is the vertical wind velocity component due to aircraft motion, \( q \) is the pitch rate, and \( U \) is the forward speed. In a plunging motion, \( w \) can also represent the vertical plunging velocity relative to some reference system. This leads to,

\[ \frac{dw}{dt} = Uq - C_N g \frac{q S}{W} + g \cos \theta \cos \phi \]  \hspace{1cm} (43)
where $\theta$ is the pitch attitude, $\phi$ is the bank angle, and the gravity term has been added as an inertial component to accommodate the investigation of changes in accelerations. Note that Eq. (43) can be integrated directly, with known terms on its right-hand side, to result in the velocity, which can in turn be integrated to result in an inertial altitude variation with a proper correction for bank angle.

The form of Eq. (43) can be presented slightly differently in order to study the controlling components of altitude variation. From discussion in previous sections, it is evident that large changes in vertical winds did not drive an appreciable change in altitude for the subject research aircraft. It is instructive to study whether that is a result of inertial effects or aerodynamic damping effects. Therefore, consider two components of the aircraft normal force,

$$\frac{W}{g} \left( \frac{\text{dw}}{\text{dt}} - Uq \right) = -N_m - N_g + W \cos \theta \cos \phi = -C_{N_m} q S - C_{N_g} q S + W \cos \theta \cos \phi \quad (44)$$

where the subscripts “m” and “g” stand for motion and gust, respectively.

The normal force can be modeled as a function of several variables, including pitch rate. In this case, however, consider that the model is limited to a function of $\alpha$ and $\dot{\alpha}$ to get

$$C_N (\alpha, \dot{\alpha}) = C_{N1} + C_{Na} (\alpha, \dot{\alpha}) \alpha + C_{Na} (\alpha, \dot{\alpha}) \dot{\alpha}$$

$$= C_{N1} + C_{Na}(t)\left(\frac{w}{V(t)}\right) + C_{Na}(t)(\dot{\alpha}) \frac{\bar{c}}{2V(t)} \quad (46)$$
where the component $C_{N_f}$ is a reminder that $C_N$ is not just a function of $\alpha$ and $\dot{\alpha}$ and to ensure that

$$C_{N_m} + C_{N_g} = C_N$$  \hspace{1cm} (47)

Note also that in this formulation the $C_N$ derivatives are equivalent to the $C_z$ derivatives from prior discussion.

The rate of change of angle of attack can be written as

$$\dot{\alpha} = \frac{d}{dt} \left( \frac{w}{V} \right) = \frac{\dot{w}}{V} - \frac{w V'}{V^2}$$  \hspace{1cm} (48)

which leads to Eq. (44) becoming

$$\frac{W}{g} \left( \frac{dw}{dt} + C_{N_{a_x}} \frac{\bar{q}S}{V(t) 2V(t)} \bar{c} + C_{N_{a_y}} \frac{\bar{q}S}{V(t)} - C_{N_{a_z}} \frac{\bar{q}SV'}{V^2 2V} \right)$$

$$= -(C_{N_1} + C_{N_g}) \bar{q}S + \frac{W}{g} Uq + W \cos \theta \cos \phi$$  \hspace{1cm} (49)

$$\approx -(C_{N_1} + C_{N_{a_x}} \alpha_w + C_{N_{a_y}} \dot{\alpha}_w \frac{\bar{c}}{2V}) \bar{q}S + \frac{W}{g} Uq + W \cos \theta \cos \phi$$

or,

$$\left[ \frac{W}{g} + C_{N_{a_x}} \frac{\bar{q}S \bar{c}}{V(t) 2V(t)} \right] \frac{dw}{dt} + \left[ C_{N_{a_y}} \frac{\bar{q}S}{V(t)} - C_{N_{a_z}} \frac{\bar{q}SV'}{V^2 2V(t)} \right] w$$

$$= -(C_{N_1} + C_{N_{a_x}} \alpha_w + C_{N_{a_y}} \dot{\alpha}_w \frac{\bar{c}}{2V(t)}) \bar{q}S + \frac{W}{g} Uq + W \cos \theta \cos \phi$$  \hspace{1cm} (50)

This is a first-order differential equation, with the terms on the left-hand side describing the physical motion of the aircraft, and the terms on the right-hand side describing the external forcing contribution of the gust and the inertial body-force contributions. Since the aircraft motion is of interest, focus is primarily on the left-hand-side components.
Equation (50) can be adjusted to result in

\[
\frac{dw}{dt} + \frac{C_{N\alpha} \bar{q}S}{V(t)} \frac{\bar{q}SV}{V^2} \frac{\bar{c}}{2V(t)} w
\]

\[
= \frac{W}{g} C_{N\alpha} \bar{q}S \frac{\bar{c}}{V(t) 2V(t)}
\]

\[
- [C_{N1} + C_{N\alpha, \alpha_w} \bar{c} + C_{N\alpha, \dot{\alpha_w}} \bar{c} \frac{\bar{q}S}{2V(t)} + \frac{W}{g} Uq + W \cos \theta \cos \phi]
\]

\[
= \frac{W}{g} C_{N\alpha} \bar{q}S \frac{\bar{c}}{V(t) 2V(t)}
\]

that of course can be simply represented as

\[
\frac{dw}{dt} + f(t)w = g(t) \quad (52)
\]

and can be solved by numerically integrating, time-step by time-step, using simple techniques, the exact solution. Utilizing the method of integrating factors, this solution is

\[
w = e^{-\int f(t)dt} \int e^{\int f(t)dt} g(t)dt \quad (53)
\]

or numerically this solution can also be represented by

\[
w = \exp[-\int_{t_0}^{t} f(t)dt]\{w_0 + \int_{t_0}^{t_2} g(t_2) \exp[\int_{t_0}^{t_2} f(t_2)dt_2]dt_2\} \quad (54)
\]

where \(w_0\) reflects the initial condition on the motion variable. Since

\[
\frac{d^2z}{dt^2} = \frac{dw}{dt} = gn_z \quad (55)
\]

this can be used to obtain the altitude variation as

\[
dz = \int \frac{dw}{dt} dt \rightarrow z = \int wdt = \sum \Delta h_i \quad (56)
\]
which can be numerically integrated using simple techniques, after accounting for the initial condition and constants.

Equations (43) and (51) are fundamentally the same, but the obvious differences arise in how the normal force coefficient is handled and subsequently integrated. The term $C_N$ in Eq. (43) results from a direct measure of normal acceleration, so it is without interaction in the sense of the aerodynamic terms that are affected by the vertical motion. The form of Eq. (51), however, utilizes specific components of the aerodynamic variables in order to develop the normal force, so it is with interaction effects.

**Application to the Present Research Aircraft**

Utilizing the normal force coefficient derivatives shown in Figure 12, Eq. (51) has been integrated and the results are shown in Figure 29 for the intermediate calculation of $w$ and the change in altitude $h$. While a direct comparison will be shown later, the altitude calculation follows that shown in Figure 9.

It is interesting to break the plunge motion into angle-of-attack components resulting from the external gust input and from the motion of the aircraft. Figure 30 shows these contributors, where the wind component is from the direct measure of wind, and the measured component is the result of flight measurements. The sum of the motion component, i.e., the contribution due to $w$, and the wind component results in the total angle of attack. Figure 30 indicates a very good agreement in total measured versus calculated total angle of attack, which is indicative that
representation of the motion by Eq. (51) is reliable. Curiously, utilizing constant coefficients (except for $C_{Nf}$), which is to say, the quasi-steady assumption, from initial average values in Table 3 yielded very similar results that are not shown here. This may be simply the result of the aircraft mass acting as an inertial filter to any unsteady inputs.

On the other hand, the calculated components of Eq. (43) utilizing flight measurements, i.e., without interaction, are presented in Figure 31. In smooth, stable air, the $C_N$ term and the gravity term should offset each other, and the pitch rate term should not be a factor. However, in the case of the turbulence penetration, near the end of the time history the load factors are elevated slightly (Figure 8), either because
of recovery in aircraft roll attitude (Figure 9) or because of a gradual increase in upward gust (Appendix B), or both. Regardless, Figure 32 shows that the integration of a slight negative bias in $dw/dt$ from Eq. (43), resulting from the elevated load-factor term, leads to divergence in $w$. This subsequently leads to a large altitude excursion as that term is integrated, as shown in Figure 33, which is clearly not accurate.

The altitude calculations resulting from both Eq. (43) and Eq. (51) are both shown in Figure 33, which also depicts the flight-measured altitude variation. The case with interaction effects is a relatively good match (regardless of whether the coefficients are variable or assumed quasi-steady). The obvious conclusion from the case without interaction effects is that inertial measurements alone evidently do not tell the whole story in the presence of moving air masses that are rapidly changing.

To look at this differently, consider the following formulation. Starting with the $dw/dt$-term, there are two components of it in Eq. (50) related to the inertial characteristics of the airplane. The first one is the physical mass, and the second one is the apparent mass. The effective mass can be defined as the sum of the two:

$$\frac{e_{mass}}{w} = \frac{W}{g} + C_{N\alpha} \frac{qS}{V(t)} \frac{\bar{c}}{2V(t)}$$  \hspace{1cm} (57)

This can be normalized to the physical mass of the aircraft, which does not appreciably change over very short time durations, to result in

$$\frac{\bar{e}_{mass}}{w} = \frac{W}{g} + C_{N\alpha} \frac{qS}{V(t)} \frac{\bar{c}}{2V(t)} = 1 + \frac{C_{N\alpha}}{W} \frac{qS}{W} \frac{\bar{c}}{2V(t)}.$$  \hspace{1cm} (58)
Figure 30: Components of angle of attack from case with interaction.

Figure 31: Components of calculation from without-interaction case.
Table 3: One-second average “steady” values for use in simplified coefficient prediction.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (sec)</td>
<td>187.375 - 188.375</td>
</tr>
<tr>
<td>$\alpha$ (deg)</td>
<td>1.92</td>
</tr>
<tr>
<td>$\dot{\alpha}$ (deg/sec)</td>
<td>0.16</td>
</tr>
<tr>
<td>$p$ (deg/sec)</td>
<td>-2.35</td>
</tr>
<tr>
<td>$q$ (deg/sec)</td>
<td>0.28</td>
</tr>
<tr>
<td>$\dot{\delta}e$ (deg)</td>
<td>-2.18</td>
</tr>
<tr>
<td>$C_z$ ($C_{No}$)</td>
<td>0.40</td>
</tr>
<tr>
<td>$C_{z\alpha}$ (rad$^{-1}$)</td>
<td>7.46</td>
</tr>
<tr>
<td>$C_{z\alpha-dot}$ (rad$^{-1}$)</td>
<td>34.87</td>
</tr>
<tr>
<td>$C_{zq}$ (rad$^{-1}$)</td>
<td>584.8</td>
</tr>
<tr>
<td>$C_{zq-osc}$ (rad$^{-1}$)</td>
<td>619.7</td>
</tr>
<tr>
<td>$C_{z\delta}$ (rad$^{-1}$)</td>
<td>-1.27</td>
</tr>
<tr>
<td>$C_{zp}$ (rad$^{-1}$)</td>
<td>0.844</td>
</tr>
<tr>
<td>$C_m$ (model)</td>
<td>0.00</td>
</tr>
<tr>
<td>$C_{m\alpha}$ (rad$^{-1}$)</td>
<td>-1.29</td>
</tr>
<tr>
<td>$C_{m\alpha-dot}$ (rad$^{-1}$)</td>
<td>-3.37</td>
</tr>
<tr>
<td>$C_{mq}$ (rad$^{-1}$)</td>
<td>-152.4</td>
</tr>
<tr>
<td>$C_{mq-osc}$ (rad$^{-1}$)</td>
<td>-155.8</td>
</tr>
<tr>
<td>$C_{m\delta}$ (rad$^{-1}$)</td>
<td>2.73</td>
</tr>
<tr>
<td>$C_{zp}$ (rad$^{-1}$)</td>
<td>0.138</td>
</tr>
</tbody>
</table>
Figure 32: Intermediate calculations from without-interaction case.

Figure 33: Altitude calculation comparison.
This implies that the effective mass, in this case, is really simply a measure of the aerodynamic normal force damping. The effective aerodynamic damping is represented by the coefficient of “\( w \)” in Eq. (50), because

\[
\frac{dw}{dt} = \frac{1}{2} C_{\text{V}} \frac{V}{C_{\text{V}}} \frac{\bar{V}^2}{2V(t)}
\]  

which is the velocity term in the context of airplane motion. Therefore,

\[
e_{\text{damp}} = C_{Na} \frac{\bar{q}S}{V(t)} - C_{Na} \frac{\bar{q}S\dot{V}}{V^2} \frac{\bar{e}}{2V(t)}
\] (60)

This can be normalized to give

\[
\bar{e}_{\text{damp}} = \frac{C_{Na} \frac{\bar{q}S}{V(t)} - C_{Na} \frac{\bar{q}S\dot{V}}{V^2}}{\frac{W}{g}} \frac{\bar{e}}{2V(t)}
\] (61)

Figures 34 and 35 show the effective mass and damping, respectively, for the present research aircraft. For comparison, the aircraft normal acceleration is shown alongside.

From Figure 34, it is evident that in periods of elevated aircraft normal acceleration, the effective mass is reduced, whereas the opposite is generally true during periods of reduced normal acceleration. Equivalently, this is saying that there is less loading when there is more inertial resistance, or that an increase in aerodynamic damping essentially contributes to the resistance of motion.

Curiously, Figure 35 indicates similar trends in effective damping, particularly at the time (approx. 201 seconds) previously labeled as condition D where there is a
large negative change in load factor. In this case, however, the general (smoothed) trend of the aerodynamic stiffness is increasing and that of the aerodynamic damping is decreasing (Figure 13). From Eq. (61), this is indicative of the aerodynamic stiffness term being dominant, particularly noting the minor peak in effective damping near condition B (approx. 191 seconds). Physically, this implies that as airplane total angle of attack is increasing with an upward gust, the aerodynamic stiffness (lift-curve slope) is decreasing, possibly due to localized stall break or shock formation (e.g., Figure 14). This is not uncommon at transonic speeds on the typical supercritical airfoils used on this type of aircraft, and this can be inferred from the harmonic oscillation plots of Figure 18. But with a strong downward gust, driving less angle of attack, there is a strong increase in lift-curve slope, possibly due to flow reattachment, and the aircraft responds accordingly.

One correlation to the hazard, that is, a marked change in aircraft load factor, may manifest itself by a corresponding change in effective mass and effective damping.

A different way to view the motion problem is to consider the coupled pitch and plunge characteristics. While an analysis in this manner was initiated, the results did not provide any more useful insight into the problem. However, the development of those equations is documented in Appendix C for reference.
Figure 34: Effective mass for the present research aircraft in flight test.
Figure 35: Effective damping for the present research aircraft in flight test.
Comparison to a Second Aircraft

In comparison, flight data and subsequent analysis from an incident involving a revenue passenger flight of another large transport aircraft has been made available, and used here with permission, for comparative purposes and in the interest of improving aviation safety research [125]. Figures 36 through 43, and Figure 45, that are shown here are from that source. The aircraft was flying at 33,000 ft over the ocean near Japan when it reportedly encountered turbulence, resulting in injuries to several passengers and crewmembers. The “passenger flight” aircraft weighed 321,940 lbs at the time, had a span of 147.1 ft, a reference chord of 21.68 ft, and two engines.

Figure 36 presents the basic flight parameters from the passenger flight. Several key points are denoted by the markers A through D. At point A, the load factor is the highest, which becomes nearly zero at point B, then peaks again at point C, followed by another local minimum at point D. Both angle of attack and pitch attitude $\theta$ follow the trends in load factor. At point A, the airplane is seen to lose approximately 100 feet of altitude in a relatively brief period of time. Figure 37 presents other parameters from this event, including pitch rate, derived $\dot{\alpha}$, roll rate, yaw rate, elevator deflection, and aileron deflection. Figure 38 presents the aircraft bank angle. The elevator angle appears to be correlated in time with the other pitch-axis parameters, while the aileron angle appears to be correlated with the roll rate and bank angle. There was no measure of winds and turbulence available, and it is noted that the sample rate on this FDR was eight samples per second for the normal
acceleration. For the angle of attack the sampling rate was only one per second, and that for
the pitch angle was four per second.

Figure 39 presents the resulting normal force coefficient model derived as a result of
calculating the same parameter from direct flight measurements using the process
described earlier.

Observing the trends in Figures 36, 37, and 38, as well as the parameters plotted in
Figures 40 and 41, it appears that most if not all of the aircraft motion after about
3930 seconds in the time history is driven by the control inputs. It is possible that at
point A, and again at point C, the aircraft may have momentarily stalled, perhaps as a
result of a strong gust, followed by the large drop in load factor and angle of attack as
reverse elevator was applied, with the altitude loss occurring at the first stall.

As a check, the estimated lift-force stiffness and damping were calculated and are shown
in Figure 40. It is also noted in that figure that there was a relatively significant change in
airspeed and Mach number. Corresponding to these aerodynamic characteristics are
the effective damping and effective mass, shown in Figures 42 and 43. As with the research
aircraft, it appears that effective damping is correlated most closely to the change in
lift-curve slope, while the minimum effective mass at time 3932 appears to be correlated
with a local minimum in the aerodynamic damping.

However, the trends in effective mass and effective damping for the passenger
flight, at the time interval where aircraft load factor is changing the most, appear to be
opposite of that seen in the research aircraft. In other words, for the passenger flight,
the change in load factor appears to be more in phase (same sign) with the change in effective mass and effective damping. One might then be inclined to conclude that perhaps the load factor changes are not turbulence-induced, particularly since there was a comparatively large amount of control input. The magnitudes of effective mass and effective damping are also lower than those of the research airplane.

Figure 36: Passenger flight turbulence encounter parameters (Ref. 125).
Figure 37: Dynamic characteristics and control input of passenger flight (Ref. 125).
Figure 38: Bank angle for passenger flight during turbulence encounter (Ref. 125).

Figure 39: Normal force coefficient for passenger flight during turbulence encounter (Ref. 125).
Figure 40: Normal force characteristics for passenger flight during turbulence encounter (Ref. 125).
Figure 41: Other parameters for passenger flight during turbulence encounter (Ref. 125).
Figure 42: Effective damping for the plunging motion of the passenger flight in turbulence encounter (Ref. 125).
Figure 43: Effective mass for the plunging motion of the passenger flight in turbulence encounter (Ref. 125).
In order to investigate, the estimated wind model derived from the research aircraft (Appendix B) is employed here. Figure 44 shows the estimated vertical winds for the passenger flight, utilizing the parameter time histories shown in the previous figures. Recall from those figures that prior to a time of 3929 seconds, there was minimal control input by the pilot. Therefore, the vertical wind prediction in that time period may be useful, since that was a basic premise of the research aircraft model. From Figure 44, no large single gust input is apparent to drive the change in angle of attack and altitude of the passenger flight, but the aircraft may have experienced a series of smaller gusts. The spikes in the figure are likely just noisy
calculations, perhaps even the result of time lags in the instrumentation as discussed
in Appendix B, but a negative bias can be observed with careful study. Combined,
these may have contributed to the large excursion in altitude from which recovery
was initiated.

For example, further analysis reveals that from the traces of load factor,
altitude, angle of attack, and pitch attitude, that at around 3928 the airplane
experienced a reduction in load factor and altitude, but a nearly constant angle of
attack while pitch attitude was gradually increasing. This is indicative of the airplane
pitching up, and maintaining angle-of-attack trim, possibly in response to a mild net
downward gust (probably in choppy air since the flight crew reported turbulence) as
described in an operational bulletin [63, 64]. If the downward wind did exist, it may
have been a gradual event that caught the pilot off guard. When the loss of altitude
was experienced it may have been aggravated as a stall by rapid pull on the column.
It appears that the large reduction in g-load, after the initial increase, was the result of
a rapid reversal in the pitch control in recovery. Since the turbulence model is not
tuned to predict the gust input under these conditions, the resulting prediction is not
reliable. It appears that the aircraft response after the initiating event may have
simply been due to efforts to recover from it.

Therefore, the correlation of the hazard from the research aircraft, as well as
the derived turbulence model, appears inconclusive in the analysis of a related but
much different turbulence encounter. The reason is that most of the aircraft motion
appears to be driven by control inputs rather than external gusts. This may explain
why, although there are changes in effective mass and effective damping correlated to aircraft load factor changes, they are opposite in sign to those of the research aircraft where control inputs were minimal.

It is quite possible that the nature of the turbulence encountered in the passenger flight was much different than that for the research flight. Recent development efforts supporting airborne weather radars [127] suggest marked differences in weather radar observables depending upon if the weather is over water or over land, and its latitude. One indicator of this might be contained in a comparison of the reduced frequency, \( k \), for each event. The relative contribution of unsteady aerodynamic effects can be estimated from the values of \( k \), with higher values generally indicative of less of a reduction, or even an increase, in wing lift near stall. Figure 45 presents a comparison of the values of \( k \) for both the research aircraft and the passenger flight. The figure shows much higher values of \( k \), and hence unsteadiness, for the case of the research aircraft.

Another Means to Compare the Two Events

If the load factors from the passenger event were indeed driven primarily by control inputs, resulting in aircraft motion, then it should be possible to analyze the motion component from the research aircraft and draw some comparison.

From the intermediate calculations in the conduct of either Eq. (53) or (54), the relationship of \( dw/dt \) to load factor noted in Eq. (55), and by moving the \( f(t)w \)
term in Eq. (52) to the right-hand side of that equation,

\[
\frac{dw}{dt} = g(t) - f(t)w
\]  

(62)

it is possible to directly calculate the load factor for the aircraft.

To accomplish this and remain consistent with normal acceleration as measured on the aircraft \(n_z\), first note that in the calculation of Eq. (54) there is inherently a 1-g component factored in, and in Eq. (55) there is a difference in sign from that measured on the aircraft. In other words, in order to compare the load
factor calculated from the estimated motion, the following adjustment to Eq. (62) must be incorporated:

\[ n_{z_{\text{motion}}} = -\left(\frac{1}{g} \frac{dw}{dt} - 1.0\right) + 1.0 = -\frac{1}{g} \frac{dw}{dt} + 2.0 \]  \hspace{1cm} (63)

where the 1.0 and 2.0 correct for 1-g and 2-g, respectively. To check the effects of the components of Eq. (62), since the \(g(t)\) term already has the 1-g term included in it, the following is appropriate:

\[ n_{z_{g(t)}} = -\frac{1}{g} g(t) + 2.0 \]  \hspace{1cm} (64)

\[ n_{z_{f(t)}} = \frac{1}{g} f(t) \]  \hspace{1cm} (65)

so that

\[ n_{z_{\text{motion}}} = n_{z_{g(t)}} + n_{z_{f(t)}} \]  \hspace{1cm} (66)

where the subscripts \(f(t)\) and \(g(t)\) simply refer to that contribution from Eq. (62), and \(g\) is the gravitational acceleration.

Figure 46 shows normal acceleration calculated from Eq. (63), compared to the aircraft-measured, for the research aircraft, utilizing the derivatives in Eq. (51). With time-varying values of the derivatives, the calculations are very “noisy”; in order to make the trends easier to visualize, Figure 46 incorporates the constant-derivative values from Table 3. While the estimates of load factor as derived from Eq. (62) and (63) are still very noisy, they do tend to match the trends from those measured aboard the aircraft.
Figure 46: Research aircraft load factors as measured and estimated from plunge motion with constant aerodynamic derivatives.

In order to understand the contributors to these load factor estimates, Figure 47 shows the effects of the two components from Eq. (64) and (65) along with the total load factor, Eq. (66), again using the constant coefficients. It is important to note that the $f(t)w$ contribution is a change in acceleration from some nominal (i.e., zero) value. The two components effectively decouple the motion of the airplane and the induced motion due to the gusts, aerodynamically-speaking, although the former is largely the result of the latter. In the lower plot of Figure 47, the $f(t)w$ component is shown along with flight-measured normal acceleration, on a slightly different scale, this time calculated with the time-varying coefficients in order to allow sharp peaks to stand out.
What is important to note in either case, for the time immediately before and after 200 seconds, is that the changes in load factor due to aircraft motion still appear to be more out of phase with the effective damping trends shown in Figure 35 and effective mass trends shown in Figure 34, and in phase, albeit slightly lagging, with the sharp measured aircraft load factor changes. This is again inconsistent with the observations from the passenger flight for the effective mass and damping trends. The implication is that the gust-induced effects on the research aircraft motion may have been far more significant than the turbulence encountered by the passenger flight. This supports the observation that much of the passenger aircraft motion in the latter portion of the time history may have been driven by control inputs.

In terms of mitigating the hazard of a turbulence encounter, then, future efforts should consider devising strategies for either controlling or nullifying the aerodynamic effect (in terms of changes to the aerodynamic characteristics) of the gust input.
Figure 47: Components of estimated research aircraft load factor calculated from plunge motion, constant coefficients (top) and non-constant coefficients (bottom).
D. Additional Studies Related to Atmospheric Turbulence

Two additional studies were conducted related to atmospheric turbulence, and are discussed in detail in Appendix B. In the first, the previous [110, 120] model of atmospheric turbulence was reconstructed and used as a check to show that it is possible to adequately fit a time-history sample that is constructed from equivalently reduced sample rates, much like would be available on a conventional flight data recorder. A psd depiction, however, shows the predictable result that input power is reduced at high equivalent frequencies, which may make it difficult to predict the effects of high-frequency, sharp-edged gusts on the aircraft. Similarly, it was next shown that a model of a pair of key parameters on the airplane, namely the “truth” angle-of-attack and the normal-force acceleration, could be created from higher-quality research measurements but accurately predicted using lower-quality, often filtered, measurements. The importance of this capability is in being able to reliably and accurately estimate these important parameters from sources that are not often of the best quality. The fuzzy-logic model structures were remarkably simple and, after accounting for a time offset between high-quality research measurements and lower-quality databus measurements, were shown to result in a reasonable and useful model.
V. CONCLUSIONS AND RECOMMENDATIONS

Conclusions and Contributions

Research has been conducted to study the nonlinear, unsteady aerodynamic characteristics of a typical transport aircraft configuration in atmospheric turbulence, and to study the resulting effects relative to the hazard of a turbulence encounter. In the context of improving aviation safety, the following conclusions and contributions are offered as a result of this effort:

Upon incorporating a lateral-directional (roll rate) term, improvements were made to a previous longitudinal (lift and pitch) model. Non-zero, time-varying values of the derivatives $C_{zp}$ and $C_{mp}$ showed possible asymmetric separation, shock movement, or stalling on lifting surfaces, which in turn strongly suggest nonlinear and unsteady lift effects, possibly driving the aircraft rolling motion itself. In both lift and pitch, estimates of both aerodynamic stiffness and damping varied rapidly throughout the time history, indicative of similar physical effects.

Lateral-directional models were developed for the first time, for rolling moment coefficient and yawing moment coefficient, to study the nonlinear, unsteady aerodynamic characteristics of an aircraft in turbulence. While the results showed trends in the stiffness and damping derivatives that do not necessarily describe conventional stability conditions, they do describe the nature of the dynamic motion itself that is not inconsistent with the results of other research on limit-cycle
oscillations and wing rock. Unstable and time-varying roll damping may be a reason for rolling motion in a turbulence encounter.

Equivalent harmonic analysis in lift, pitch, and rolling were used as means to check the models against more conventional linear observations. Hysteresis was observed as indicated by the direction of the harmonic loops, while the thickness of the loops confirmed the presence of unsteady aerodynamics in damping. In some cases, large unsteady aerodynamic effects were apparent.

The aerodynamic lift model was then analyzed in the context of stability and control to better study the hazard to the airplane, via a plunging-motion analysis. Fast plunging motion of an aircraft in turbulence is often the cause of passenger injuries. The measured altitude change of the research aircraft in turbulence, which occurred over a moderate period of time, was reasonably replicated utilizing a form of the equation of motion having discrete aerodynamic derivatives. Effective mass and effective damping, characterized by aerodynamic stiffness and damping terms and normalized by aircraft mass, constitute the coefficients of this equation. Comparison with a passenger flight event, having a rapid plunge motion, showed marked differences in the characteristics of these terms compared to aircraft normal load factor. This was possibly due to differences in the type of input driving the motion, i.e., turbulence versus control, respectively, characterized, in part, by differences in reduced frequency in angle of attack.

Specifically, the plunge-motion observations identify that the cause of fast plunging motion may be shock-induced stall in more static motion, i.e., low reduced
frequency, whereas in oscillatory motion with higher reduced frequencies, dynamic stall may inhibit fast plunging motion. In the former case, effective damping was observed as largely decreased relative to the latter case. Therefore, some form of hazard index may relate to the magnitude of effective damping in plunging motion, or alternatively to the measure of unsteadiness in the aerodynamics of the encounter. A control strategy for countering a rapid plunge may benefit from means to artificially drive unsteady aerodynamic effects.

**Recommendations**

Future, detailed study and research are suggested in order to verify the results here and expand on the capability to analyze and characterize aircraft aerodynamics in severe atmospheric turbulence. The following recommendations are offered:

Unsteady and nonlinear effects are apparent in longitudinal and lateral-directional coefficient models. It appears that in turbulence the stiffness and damping characteristics of the airplane are affected, and means to validate these effects should be investigated. One method might be by simply analyzing more of the flight data available from the research aircraft described here, and verifying repeatability. Supplemental research could entail establishing sensitivity factors to model parameters as a means to quantify relative effects.

More efficient methods to generate models could be a substantial aid. Means should be investigated to improve the fuzzy-logic modeling scheme, in the sense of
improving the efficiency of the codes and the ability to utilize much more flight data points in the process, therefore improving the robustness of the models.

Other experimental datasets are available. Consideration should be given to modeling the results of recent wind-tunnel results of forced-oscillation testing of a transport configuration and either combining them with, or comparing them to, flight-data-derived models. The former, of course, will not be in turbulence, but could provide a critical link in validation of the models derived from flight data that is.

The modeling technique has potential for future applications in accident or incident investigations. Further detailed studies should investigate the robustness of models built utilizing limited sample rate (i.e., FDR type) time histories, and comparing them to high-rate flight test results.

Finally, practical control schemes for dealing with the aerodynamic influence of turbulence on the subsequent motion of the aircraft should be investigated, as a means to reduce the hazards to passengers and crews from turbulence encounters. One approach might be to incorporate some means to artificially introduce unsteady aerodynamic effects, such as oscillating surfaces.
REFERENCES

1. NASA Aviation Safety Program, see http://larc.nasa.gov, search on “avsp”.


17. Stuever, R.A.: Personal notes from visit to Delta Airlines to observe simulator testing for upset recovery training, Atlanta, GA, April 20, 1999.


67. Turbulence Education and Training Aid, Produced by the U.S. Department of Transportation, Federal Aviation Administration; Air Transport Association of America; and McDonnell Douglas Corporation; July 1997.


END OF MAIN BODY OF REPORT
APPENDIX A: GUST COMPONENT ALGORITHM

In the case of the turbulence components, the specific $w$ (vertical), $u$ (longitudinal), and $v$ (lateral) components, in inertial space, were available from the flight research database. A system aboard the research aircraft was programmed to calculate these components based upon the following sequence of calculations and transformations (steps below courtesy E. C. Stewart, NASA Langley Research Center):

i. Begin with corrected $\alpha$, $\beta$, and $V$ using calibration and position-error tables.

ii. Calculate local flow velocities $u_{\text{local}}$, $v_{\text{local}}$, and $w_{\text{local}}$.

$$
\begin{bmatrix}
    u_{\text{local}} \\
    v_{\text{local}} \\
    w_{\text{local}}
\end{bmatrix} = \begin{bmatrix}
    V_{\text{cor}} / (\tan^2 \alpha_{\text{cor}} + \tan^2 \beta_{\text{cor}} + 1) \\
    u_{\text{local}} \tan \beta_{\text{cor}} \\
    u_{\text{local}} \tan \alpha_{\text{cor}}
\end{bmatrix}
$$

where $\beta$ here is a “flank” angle, not the customary sideslip angle.

iii. Correct local flow velocities for aircraft angular velocities

$$
\begin{bmatrix}
    u_b \\
    v_b \\
    w_b
\end{bmatrix} = \begin{bmatrix}
    u_{\text{local}} - (z_q y - y_r) \\
    v_{\text{local}} + (z_p x - x_p) \\
    w_{\text{local}} - (y_p x - x_q)
\end{bmatrix}
$$

where $p$, $q$, and $r$ are rates, and the $x$, $y$, and $z$ values are linear position constants for the angle of attack and sideslip instruments.

iv. Transform the body-axis velocity components to the earth axis system

$$
\begin{bmatrix}
    N_a \\
    E_a \\
    D_a
\end{bmatrix} = \begin{bmatrix}
    c \theta \psi & s \phi s \theta \psi & -c \phi s \psi & c \phi s \theta \psi + s \phi \psi \\
    c \theta \psi & s \phi s \theta \psi + c \phi c \psi & c \phi s \theta \psi - s \phi c \psi \\
    -s \theta & s \phi c \theta & c \phi c \theta
\end{bmatrix}
\begin{bmatrix}
    u_b \\
    v_b \\
    w_b
\end{bmatrix}
$$

v. Calculate winds, including turbulence components, in inertial coordinates
\[
\begin{bmatrix}
\dot{N}_w \\
\dot{E}_w \\
\dot{D}_w
\end{bmatrix} = \begin{bmatrix}
\dot{N}_i \\
\dot{E}_i \\
\dot{D}_i
\end{bmatrix} - \begin{bmatrix}
\dot{N}_a \\
\dot{E}_a \\
\dot{D}_a
\end{bmatrix}
\]

where the \(N_i\), etc., terms are inertial velocities.

In this case, the non-vertical components of turbulence were described as east-west and north-south components, not as aligned with and aligned across the aircraft body axis. Typically, the assumption used in developing turbulence loads is that the aircraft remains aligned with its starting flight path angle and heading. In the case of the present research aircraft, the pilots intentionally allowed the aircraft to respond naturally and they provided only limited inputs to adjust its attitude and heading. Thus, the three components of turbulence could vary in time with respect to the aircraft but not with respect to inertial space. Plots of the vertical (up-down) and horizontal components of turbulence are shown in Figure A-1.

For the particular condition of interest (event 191.3), the lateral and longitudinal components of turbulence were generally unremarkable, in that they were nearly smooth and of little variation compared with the vertical gust component. This is shown in Figure A-1, and because of this any further efforts to analyze these components was determined to be of little value.
Figure A-1: Turbulence time history.
Note, as shown for $w_g$, positive is a downward gust.

APPENDIX A: CONCLUDED
APPENDIX B: TURBULENCE, $\alpha$ VANE, AND $n_z$ SENSOR MODELS

The purpose of this appendix is to document for reference the development of a model of the turbulence gust field, the $\alpha$ vane, and the $n_z$ sensor. This is in support of research is to study means to better reconstruct and predict useful information from typical FDR output. For example, one use would be in supporting accident and incident investigations.

Vertical Turbulence Model

For the vertical turbulence model, the effort was essentially repeated from that in [110, 120] as an independent check of the model. The objective was to model the vertical turbulence component in the form

$$w_g = f[p, q, r, \phi, a_x, a_y, a_z]$$

In the case of the turbulence components, the specific $w$ (vertical), $u$ (longitudinal), and $v$ (lateral) components, in inertial space, were available from the flight research database. A system aboard the research aircraft was programmed to calculate these components based upon the sequence of calculations and transformations presented in Appendix A.

The non-vertical components of turbulence were presented as east-west and north-south components, not as aligned with and aligned across the aircraft body axis. Typically, the assumption used in developing turbulence loads is that the aircraft remains aligned with its starting flight path angle and heading. In the case of the
present research aircraft, the pilots intentionally allowed the aircraft to respond naturally and they provided only limited inputs to adjust its attitude and heading. Thus, the three components of turbulence could vary in time with respect to the aircraft but not with respect to inertial space. Plots of the vertical (up-down) and horizontal components of turbulence are shown in Appendix A. As shown, the non-vertical components were generally unremarkable in terms of variation, both holding nearly constant biases. As a result, it was concluded that modeling these components using fuzzy logic offered little benefit and insight into the present research.

The resulting structure of vertical turbulence was of the form

\[ w_g = f [p_z, q_z, r_z, \theta, \phi, a_{xz}, a_{yz}, a_{zz}] \]

providing an \( R^2 \) of 0.952 with the model construction, which was nearly identical to the previous model.

The following ranges of input parameters were used in setting up the model structure:

- \( p: [-10, 5 \text{ deg/sec}] \)
- \( q: [-3, 5 \text{ deg/sec}] \)
- \( r: [-3, 3 \text{ deg/sec}] \)
- \( \theta: [0, 5 \text{ deg}] \)
- \( \phi: [-38, 38 \text{ deg}] \)
- \( a_x: [-5, 5 \text{ m/sec}^2] \)
- \( a_y: [-5, 5 \text{ m/sec}^2] \)
- \( a_z: [-18, 18 \text{ m/sec}^2] \)

Dimensional accelerations, versus non-dimensional \( g \), were utilized to provide numerical stability and consistency in the fuzzy-logic modeling codes.

Figure B-1 shows the time history for the portion of the data that was used to build the model, along with the resulting model fit. It should be noted that the sign convention for \( w_g \) here is opposite to that in Appendix A, in order to make it
consistent with other published work [e.g., 79, 121]. For this purpose, a positive sign means an up-gust relative to the airplane.

The usefulness of this model compared to the previous studies is that it can be utilized to study the effect of a practical limitation with FDR data, namely, limited sampling rates.

To this end, Figure B-2 shows the effect of reducing the “effective” sample rate of all the components in the model above to a typical FDR sampling rate of 8 samples/second (s/s). To accomplish this, the original time history was first adjusted to a uniform sampling interval of 50 s/s using spline functions available in the Matlab® software package (it was discovered well into these studies that the sampling intervals of the flight data stream were of about this order, but not uniform). Points were then culled out of this set to represent 8 s/s. Since the model would likely successfully be able to fit to these points by themselves, offering no real advantage (since they were effectively culled out of the modeling set to start with), the time history was then rebuilt to 50 s/s with splines in order to check its capability to match the original flight time history. In other words, the rebuilt model contained intermediate combinations of the parameters above which could be used to test the robustness of the model.

Figure B-2 simply shows the resulting fit of the reconstructed time history to the original time history, while Figure B-3 shows the result of running the latter through the $w_x$ model and its performance against the flight data. As shown, it appears to perform about as well as against the original flight data (Figure B-1) and
captures the main features of the turbulence, although it is important to note that it missed many of the high-frequency peaks.

The significance of reducing the sampling interval can be seen in Figure B-4, which are power spectral density (psd) plots of the two data traces. The psd plots were generated using Matlab® and running a standard function that was available, using the splined time history. Previously [79], a comparison of the present flight data psd was shown relative to the “-5/3” slope of the von Karman turbulence spectrum model, with generally good agreement. Figure B-4(a) shows the original time history of $w_g$ splined to a uniform sample rate of 50 s/s, with the slight relative peak in power at the higher frequencies. The obvious and expected effect of reducing the sampling interval is to remove the power at the higher frequencies, as verified in Figure B-4(b). Beyond about 4 Hz (Nyquist frequency for 8 s/s), the power is reduced and the frequency overlap can be seen. One conclusion from this is that strong, sharp-edged, high-frequency gusts like those missed by the model (Figure B-3) may not be possible to reconstruct using typical FDR data.
Figure B-1: Flight and modeling of vertical turbulence.

Figure B-2: Reduced effective sample rate of vertical turbulence.
Figure B-3: Result of model fit to reduced effective sample rate of turbulence.
(a) Original flight data splined to 50 samples/sec.

(b) Flight data stripped to 8 samples/sec then splined to 50 samples/sec.

Figure B-4: Plot of psd of flight and reduced model turbulence component, \( w_g \).
Model of $\alpha$ and $n_z$ Sensors

A practical problem in utilizing flight data from airliners is that the parameter sources are not always recorded at a high rate nor properly corrected. This is particularly true when using results from FDRs, almost always employed in accident investigations and often in incident investigations. FDR data is often sampled at a minimal rate, normally just a few samples per second, which of course is an artificial means of filtering the data. Modern systems are also available for recording typical “health” information of the aircraft to be later supplied to aircraft maintenance crews, such as the aircraft condition and reporting system (ACARS), and these offer the capability for a higher-quality recording than most FDRs. For the discussion to follow, it was assumed that an improved system such as ACARS would be available for future applications. It is presently being utilized as a host system for the in-situ turbulence algorithms being investigated by government researchers. While the previous section discussed one aspect of reduced sample rates, a further investigation into the very low sample-rate realm of the FDR is recommended as a future research topic.

On the research aircraft flown in the present investigation, a high-resolution and high-rate measurement of angle of attack was utilized in place of the standard ship’s measurement across the air-data computer (ADC) databus. In effect, the filtering and corrections implemented in the path from the $\alpha$-vane to the ship’s databus (and subsequently the pilot’s cockpit display, or the autopilot system) were by-passed in lieu of a direct measurement of the $\alpha$-vane angle. Similarly, a special
research-quality accelerometer package at the aircraft center of gravity (CG) was used to supplement the ship’s inertial reference unit (IRU) acceleration measurements transmitted across the IRU databus. For both signals that get transmitted across the databusses, there are inherent lags and filtering in the signals relative to the “truth” measurements from the research system. The filtering in the context here also encompasses sampling/updating at a lower rate than the standalone research measurements.

An investigation was undertaken to determine if it would be possible to model the “truth” $\alpha$, as supplied by the research measurement, using the results of the ship’s databus measurements. Since $\alpha$ is a primary variable in the models, it is important to be able to use an alternate source if required; however, a model of $n_z$ was also investigated as well since it’s essentially the same type of problem. Common sense indicates that the resulting models will have some degradation and aliasing. However, recall that the parameters being fed into the fuzzy-logic algorithms to model the aerodynamic coefficients are smoothed by time averaging and further processed to match an equivalent harmonic motion.

The models were set up to have the form:

$$\alpha \text{ or } n_z = f[\alpha_{ADC}, n_z_{IRU}]$$

where the models of $\alpha$ and $n_z$ are built to the research-quality measurements, and ADC and IRU simply denote ship’s databus measurements.
The model was fit to the data sequence of 100 to 240 seconds in both cases, and was evaluated for the time history of 0 to 100 seconds. The models were fit within the ranges of:

\[ n_{z,\text{IRU}}: [-1.0, 1.0 \, \text{g}] ; \quad \alpha_{\text{ADC}}: [-20, -5 \, \text{deg}] \]

It is important to note that the angle of attack measurements used here both represent uncorrected data, in the sense of applying a bias and position error correction. Actual aircraft angle of attack is obtained by applying a scale factor and bias offset to these raw measurements, i.e.,

\[ \alpha = a \alpha_{\text{ADC}} + b \]

so that a typical trim \( \alpha \) generally represents a positive number. The terms \( a \) and \( b \) in the preceding expression are unknown to the present investigator (although they are applied in the ship’s ADC, and are normally functions of speed/Mach and sometimes altitude and flap/gear configuration); it is assumed that if this approach were to be applied at some later time, the values for these terms would be obtained and utilized. For now, the idea is to determine if the raw measurements can be adequately modeled.

The best fuzzy logic model structures were remarkably simple, and resulted in

\[ \alpha = f[\alpha_2, nz_4] \]

\[ nz = f[\alpha_2, nz_2] \]

The model for angle of attack resulted in a value for \( R^2 = 0.965 \), and for normal acceleration \( R^2 = 0.869 \). However, this was obtained only after removing a time offset between the research measurements and the databus measurements. While
obtaining a fit of $R^2 = 0.899$ for $\alpha$ for the unadjusted data, it was apparent in the
resulting time history that the fit was very poor, as shown in Figure B-5. There was
no point in examining an independent time history. Through trial and error, it was
determined that about a seven-frame time lag existed between the two data sources.
Once this was removed, the results improved dramatically. It was assumed that this
could also be done in any future applications.

Figure B-6 presents results for the training set of angle of attack, while Figure
B-7 presents the test case, all with adjustments made in the input set to account for
the time lags. Similarly, Figure B-8 and Figure B-9 show the time-adjusted results
for $n_z$. For both variables, the fit and prediction are very reasonable, remembering
that the databus measurements are updated at a lower rate than the research
measurements. The conclusion is that such a model might be useful for predicting the
actual input to sensors, and a detailed study is recommended for future work.
Figure B-5: Time history of $\alpha$ measurement without time-lag adjustments.

Figure B-6: Time history of $\alpha$ measurement with time-lag adjustments for modeling set.
Figure B-7: Time history of $\alpha$ measurement with time-lag adjustments for independent set.

Figure B-8: Time history of $n_z$ measurement with time-lag adjustments for modeling set.
Figure B-9: Time history of \( n_z \) measurement with time-lag adjustments for independent set.
The purpose of this appendix is to document for reference the formulation of coupled pitch and plunge equations of motion (courtesy [125]). The original intent was to utilize this approach to re-create the angle-of-attack, altitude, and load factor time histories using first-principles and the results of the non-linear, unsteady aerodynamic models from the main body of this report as a means to understand the contributing components to these variables.

Starting in a manner similar to the plunge motion, define

\[ w = \frac{dz}{dt} \]

but considering the effect of pitch motion, \( w/V = \bar{w} \approx \alpha \). Then

\[ \frac{d\bar{w}}{dt} = \frac{1}{V} \frac{d^2 z}{dt^2} \approx \frac{d\alpha}{dt} \tag{C-1} \]

so that similar to the expression for the plunge, but also including terms important for pitching motion while ignoring rolling and yawing motion, is

\[ \frac{VC_{N0}}{g} \left( \frac{d\bar{w}}{dt} - q \right) = -C_{Nw} \bar{w} - C_{Nq} \bar{w} - C_{Nq} q - C_{Na} \alpha_w - C_{Na} \dot{\alpha}_w - C_{Nqw} q_w \tag{C-2} \]

or

\[ \left( \frac{VC_{N0}}{g} + C_{Nw} \right) \frac{d\bar{w}}{dt} + C_{Nq} \bar{w} + C_{Nq} q + C_{Na} \dot{\alpha}_w + C_{Na} \alpha_w + C_{Nqw} q_w = 0 \tag{C-3} \]

where

\[ C_{Nq}^* = C_{Nq} - \frac{VC_{N0}}{g} \tag{C-4} \]
and \( \dot{q}_w \) is the apparent pitch rate due to gust. The derivative of Eq. (C-3) is

\[
\left( \frac{VC_{Nq0}}{g} + C_{N\dot{w}} \right) \frac{d^2 \dot{w}}{dt^2} + (\dot{C}_{N\dot{w}} + C_{N\dot{\pi}}) \frac{d\dot{w}}{dt} + \dot{C}_{N\dot{w}} \dot{w} + \dot{C}_{Nq} q + C_{Nq} \frac{dq}{dt} + C_{Na} \ddot{\dot{w}} \\
+ (\dot{C}_{Na} + C_{Na}) \ddot{\alpha}_w + \dot{C}_{Na} \alpha_w + \dot{C}_{Nq} q_w + C_{Nq} \dot{q}_w + \dot{C}_{Nk} \dot{\delta}e + C_{Nk} \dot{\delta}e = 0
\]

(C-5)

where the derivatives are time-varying.

The pitch equation, ignoring rolling and yawing motion, can be written as

\[
\frac{d^2 \theta}{dt^2} = \frac{dq}{dt} = \frac{\bar{q}S\bar{c}}{I} \left( C_{M\dot{\pi}} \frac{d\dot{w}}{dt} + C_{M\dot{w}} \dot{w} + C_{Mq} q + C_{Ma} \ddot{\alpha}_w + C_{Ma} \alpha_w + C_{Mq} \dot{q}_w + C_{Mk} \dot{\delta}e \right)
\]

(C-6)

Defining

\[
K_2 = \frac{\bar{q}S\bar{c}}{I}
\]

(C-7)

solving Eq. (C-5) for \( dq/dt \), setting the result equal to the right-hand side of Eq. (C-6), and then solving for \( -q \) results in

\[
- q = \frac{1}{D} \left( \frac{VC_{Nq0}}{g} + C_{N\dot{\pi}} \right) \frac{d^2 \dot{w}}{dt^2} + (\dot{C}_{N\dot{\pi}} + C_{N\dot{w}} + K_2 C_{Nq} C_{M\dot{w}}) \frac{d\dot{w}}{dt} + (\dot{\dot{C}}_{N\dot{w}} + K_2 C_{Nq} C_{M\dot{\pi}}) \dot{w} \\
+ C_{N\dot{\pi}} \ddot{\alpha}_w + (\dot{\dot{C}}_{Na} + C_{Na} + K_2 C_{Nq} C_{Ma}) \ddot{\alpha}_w + (\dot{\dot{C}}_{N\dot{\pi}} + K_2 C_{Nq} C_{M\dot{\pi}}) \alpha_w \\
+ K_2 C_{Nq} C_{Mq} \dot{q}_w + C_{Nq} \dot{q}_w + (\dot{\dot{C}}_{Nk} + K_2 C_{Nq} C_{Mk}) \dot{\delta}e + C_{Nk} \dot{\delta}e \right]
\]

(C-8)

where

\[
D = K_2 C_{Nq} C_{Mq} + \dot{\dot{C}}_{Nq}
\]

(C-9)

Similarly, solving Eq. (C-3) for \( -q \) results in

\[
- q = \frac{1}{C_{Nq}^*} \left( \frac{VC_{Nq0}}{g} + C_{N\dot{\pi}} \right) \frac{d\dot{w}}{dt} + \frac{C_{N\dot{\pi}}}{C_{Nq}^*} + \frac{C_{N\dot{w}}}{C_{Nq}^*} \dot{w} + \frac{C_{Na}}{C_{Nq}^*} \alpha_w + \frac{C_{Nq}}{C_{Nq}^*} q_w + \frac{C_{Nk}}{C_{Nq}^*} \delta_e
\]

(C-10)
Equating (C-8) and (C-10) results in an expression of the form

\[ J \frac{d^2 \bar{W}}{dt^2} + K \frac{d\bar{W}}{dt} + L\bar{W} = P\ddot{\alpha}_w + M\dot{\alpha}_w + N\alpha_w + Q\dot{q}_w + R\ddot{q}_w + S\ddot{e} + T\dot{e} \]  

(C-11a)

or

\[ \frac{d^2 \bar{W}}{dt^2} + \dot{\bar{K}} \frac{d\bar{W}}{dt} + \dot{L}\bar{W} = \dot{P}\ddot{\alpha}_w + \dot{M}\dot{\alpha}_w + \dot{N}\alpha_w + \dot{Q}\dot{q}_w + \dot{R}\ddot{q}_w + \dot{S}\ddot{e} + \dot{T}\dot{e} \]  

(C-11b)

where

\[ \dot{K}, \dot{L}, \ldots = \frac{K}{J}, \frac{L}{J}, \ldots \text{ in Eq. (C-11b)} \]

\[ J = \left( \frac{VC_{N0}}{g} + C_{\bar{W}} \right) \]  

(C-12)

\[ K = (\dot{C}_{Nw} + C_{N\dot{w}} + K_2 C_{Nq}^* C_{M\dot{w}}) - \frac{D}{C_{Nq}} \left( \frac{VC_{N0}}{g} + C_{\bar{W}} \right) \]  

(C-13)

\[ L = (\dot{C}_{Nw} + K_2 C_{Nq}^* C_{M\dot{w}}) - \frac{DC_{\bar{w}}}{C_{Nq}^*} \]  

(C-14)

\[ M = \frac{DC_{N\dot{w}}}{C_{Nq}^*} - (\dot{C}_{N\dot{w}} + C_{N\dot{w}} + K_2 C_{Nq}^* C_{M\dot{w}}) \]  

(C-15)

\[ N = \frac{DC_{N\dot{w}}}{C_{Nq}^*} - (\dot{C}_{N\dot{w}} + K_2 C_{Nq}^* C_{M\dot{w}}) \]  

(C-16)

\[ P = -C_{N\dot{w}} \]  

(C-17)

\[ Q = \frac{DC_{Nq}}{C_{Nq}^*} - (\dot{C}_{Nq} + K_2 C_{Nq}^* C_{Mq}) \]  

(C-18)

\[ R = -C_{Nq\dot{w}} \]  

(C-19)

\[ S = \frac{DC_{N\dot{w}}}{C_{Nq}^*} - (\dot{C}_{N\dot{w}} + K_2 C_{Nq}^* C_{M\dot{w}}) \]  

(C-20)

\[ T = -C_{N\dot{w}} \]  

(C-21)

For the practical purposes,
\[ C_{Na} = C_{N\bar{\alpha}} \]
\[ C_{Na} = C_{N\bar{\alpha}} \]
\[ C_{M\bar{\alpha}} = C_{M\bar{\alpha}} \]
\[ C_{M\bar{\alpha}} = C_{M\bar{\alpha}} \]
\[ C_{Nq} = C_{Nq} \]
\[ C_{Mq} = C_{Mq} \]

and under certain conditions [121, 122], \( q_w = -\dot{\alpha}_w \). Eq. (C-11b) can be expressed as

\[
\frac{d^2\bar{W}}{dt^2} + \hat{K}\frac{d\bar{W}}{dt} + L\bar{W} = h(t) \quad \text{(C-23)}
\]

and the following definitions can be established so that

\[
\bar{W}_1 = \bar{W} \\
\dot{\bar{W}}_1 = \bar{W}_2 = \frac{d\bar{W}}{dt} \quad \text{(C-24)}
\]

\[
\dot{\bar{W}}_2 = -\hat{K}\bar{W}_2 - L\bar{W}_1 + h(t) = \frac{d^2\bar{W}}{dt^2}
\]

The second and third elements of Eq. (C-24) can be solved simultaneously, using a numerical integration scheme, in order to result in the first element. That first element and its derivative can be used in Eq. (C-6) to solve for pitch rate, which in turn may be integrated a second time for pitch attitude. The plunge variable can also be integrated using

\[
z = \int w dt = V \int \bar{W} dt \quad \text{and} \quad h = -z \quad \text{(C-25)}
\]

to estimate the altitude change in the maneuver.

**APPENDIX C: CONCLUDED**