The impact of quality perception and consumer valuation change on manufacturer's optimal warranty, pricing and market coverage strategies

Kunpeng Li
Email:Kunpeng.li@csun.edu
Department of Systems and Operations Management, California State University, Northridge
18111 Nordhoff St, Northridge, CA 91330, USA
Phone: (818) 677-2992, Fax: (818) 677-6264

Lan Wang
Email: lan.wang@csueastbay.edu
Department of Management, College of Business and Economics
California State University, East Bay
Hayward, CA 95452, USA
Phone: (510) 885-2922

Dilip Chhajed
chhajed@illinois.edu
Department of Business Administration, University of Illinois at Urbana-Champaign
350 Wohlers Hall, 1206 South Sixth Street, Champaign, IL 61820, USA

Suman Mallik
suman@ku.edu
School of Business, University of Kansas
1654 Naismith Dr, Lawrence, KS 66045, USA

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ABSTRACT

We consider a manufacturer who produces and sells a new product in a monopoly market. The new product quality is unobservable to consumers before purchase. Consumers make purchase decisions based on their perception of product quality. In addition, consumer valuation towards the product will be reduced if a product failure is experienced. We formulate a two-period model to analyze the impact of consumer quality perception and consumer valuation change on manufacturer's optimal decisions over quality, warranty, price and market coverage strategies. We find that it is optimal for the manufacturer to offer warranty compensation that is higher than the purchase price when he has a low quality reputation but is offering a high quality product. We also identify the optimal strategy for the manufacturer in terms of market coverage. Specifically, when the consumer valuation discount factor due to product failure experience is high, it is optimal for the manufacturer to serve only the high segment in the second period. Otherwise, serving both the high and the low segments generates more profit. We further investigate consumer welfare and identify win-win conditions, where the optimal strategy of the manufacturer benefits the consumers as well.

Keywords: ***Product Design, Quality, Self-selection, Market Segmentation, Game Theory, Retail Marketing, Pricing, Multi-period Modeling***

1. Introduction

The quality of many consumer goods (e.g., a new model of a smartphone or an appliance) cannot easily be evaluated before purchase (Shapiro, 1982). Product quality, however, plays a very important role in purchase decisions, as consumers are concerned about the risk of product failure and the high expense of repair and replacement. Manufacturers typically offer a warranty to insure consumers against this risk and to signal unobservable product quality (Pindyck and Rubinfeld 2012). Extensive warranty coverage is often associated with high quality, due to the warranty costs in the event of product failure. On the other hand, a manufacturer with poor quality product and high failure rate will not be able to afford extensive warranty coverage. Sahin and Polatogu (1998), Boulding and Kirmani (1993), and Spence (1977) also have similar findings; they demonstrate that both warranty length and warranty contents could be used as signals of product quality.

Quality reputation is another important element of consumer's quality perception towards unobservable quality. Quality reputation is regarded as a perception of product quality associated with a brand name (Li et al. 2011). Cabral (2008) states that a firm's quality history is encapsulated in the value of its brand. Shapiro (1982) has shown that brand reputation can be modeled as an expectation of quality when consumers have imperfect information about product quality. A branded manufacturer is expected to claim the true value of the unobservable product quality. A false statement would lead to unattractive future profits, according to Erdem and Swait (1998). Hence, rational consumers will associate reputable brand with good quality, and ill-reputable brand with questionable quality. For example, the poor brand image of Saturn, an automobile brand of General Motor, has led consumers to form unfavorable quality perceptions, as evident in an experiment: when a sedan model of Saturn without the nameplate was shown, the car got a score of 3.4/4.0; with the Saturn badge, the same car scored only 2.0 (Welch, 2009).

Consumers often rely on information of warranty and quality reputation to infer unobservable quality, and to make purchase decisions, as demonstrated in empirical research (Blair and Innis, 1996; and Soo et al., 2001). Manufacturers with different quality reputation can also use warranties as their marketing strategies. In the US during the early-1990s, the newly founded matress manufacturer Tempur-Pedic (without prior quality reputation) assured consumers the quality of its revolutionary memory foam mattress with an impressive 25 years warranty, while industry standard warranty was 10 years. However, the highly reputable company Apple only offered industry standard one-year warranty on iPod when it was first launched in 2001.

The existence of different warranty strategies, such as the aforementioned examples, raises our first set of research questions:

- *Why do some manufacturers offer industry standard warranties, while the others offer above-industry-standard warranties? How would consumer quality perception affect a manufacturer's warranty strategies? Under what conditions would it be optimal for a manufacturer to offer industry standard warranty or above-industry-standard warranty?*

A product failure will negatively affect a consumer's purchasing decision in the future, even under the protection of warranty. Specifically, the consumer's valuation towards the product would decrease. Cabral (2008) has discussed that a consumer values a product with high failure probability much lower. Consumer's willingness to pay will be diminished when product failure occurs or an inferior quality perception is involved (Grewal et al., 1994, Teo and Yeong, 2003, Jiang et al., 2017, Wang et al., 2017). Jiang and Guo (2015) find that a company's optimal review system design and pricing strategy depend on the product quality (valuation) and consumer valuation update after experiencing the product. According to Customer Loyalty Statistics (2015), 90% of consumers experiencing a product failure spent less with that brand during the following year. In addition, not all consumers would experience product failures; their valuation towards the product would not be affected by such failure. This would segment the market and provide challenges and opportunities for the manufacturer in terms of market strategies.

The above discussion inspires our next research question:

- *How would the change of consumer valuation caused by product failures affect a manufacturer's marketing strategies and quality decisions in a repeat-purchase setting?*

In our paper, we formulate a two-period model to analyze the impact of consumer quality perception and consumer valuation change on manufacturer's optimal decisions over quality, warranty, price and marketing strategies. We consider a monopolist manufacturer who produces and sells a new product. The new product quality is unobservable to consumers before purchase. In the first period, consumers form quality perception based on the manufacturer's quality reputation and the product warranty offered. In the second period, consumers observe the first-period product quality and use it to update quality perception for the second period. In addition, the consumers who experienced a product failure in the first period discount their valuations of the product in the second period. This creates two segments of consumers in the second period market: the high segment, that has the original valuation, and the low segment, that has a reduced valuation. The manufacturer hence determines the optimal market coverage strategies, as well as optimal decisions of quality, warranty and price in each period.

We first reveal interesting implication about the relationship between the optimal quality decision and the consumer quality perception. We show that, generally, the optimal quality choice of the manufacturer can be either higher or lower than the consumer quality perception depending on the production cost. We also identify and explain when the optimal quality choice of the manufacturer equals to the consumer quality perception in the second period. We then analyze the relation between the optimal warranty and the optimal price. We find that it is optimal for the manufacturer to offer warranty compensation that is higher than the purchase price when he has a low quality reputation but is offering a high quality product. Through this action, the manufacturer is able to signal and convince consumers about the product's high quality despite his low quality reputation. Under a different set of conditions, it is optimal for the manufacturer to offer warranty compensation lower than the purchase price.

We further identify the optimal strategy for the manufacturer in the second period. We find that either full market coverage or high market coverage can be optimal. We confirm the intuition that the full market coverage strategy is more likely to be optimal when manufacturer's quality reputation is high, the production cost is high, or the consumer valuation is low. Otherwise, serving only the high segment generates more profit.

We investigate the strategy that would benefit the consumers in terms of higher quality and better warranty, as well as higher consumer welfare. We also illustrate a win-win region where both the manufacturer and the consumers would be better off with higher profit and higher consumer welfare respectively. Finally we describe an extension of our model where we let the second period valuation of the consumer to depend both on first period quality experience and the second period warranty.

The rest of the paper is organized as follows. Section 2 introduces the related literature. The basic model setup and the detailed discussion of the manufacturer's marketing strategies are presented in section 3. Model results are analyzed and managerial insights are presented in section 4. Section 5 explores an extension of our model while Section 6 concludes the paper and summarizes the limitations and future research directions.

2. Literature Review

This paper is closely related to two streams of literature. One stream focuses on signaling unobservable product quality. The other stream examines optimal strategies in a segmented market using self-selection method.

There are many *empirical* research papers on product quality signaling through reputation, warranty or price (or all of them). Wiener (1985) provides verification that consumers do consider warranty as a signal of product quality. Boulding and Kirmani (1993) test propositions about when high warranties may be perceived as signaling higher, lower, or the same quality as low warranties. Other studies examine the relation between warranty, quality and brand reputation. Innis and Unnava (1991) investigate how warranty impacts consumers' product evaluations for new and established brands. They conclude that warranty has a positive impact on new brands but little effect on established brands. Shimp and Bearden (1982) conduct five experiments to examine the effect of warranty, price, and warrantor reputation on consumer perceptions of the financial and performance risks associated with a new product. Price and Dawar (2002) empirically study the interaction of brand information and warranty information in determining quality perception. They find that warranties can enhance brand signal credibility, and the joint signaling effects of brands and warranties depend on both inherent information content and relative credibility. Blair and Innis (1996) study the effect of consumers' product knowledge on the evaluation of product quality through brands and warranties. They divide the consumers into experts and non-experts according to their knowledge about the product. They conclude that a warranty is more likely used by non-expert consumers in forming quality perceptions, especially if the product brand is not well known. Akdeniz et al (2013) use two experimental studies to examine interactions among market cues such as brand reputation, warranty and price. The results of experiments demonstrate that brand reputation has a greater impact on consumer quality perceptions than warranties and price.

The empirical research demonstrates the importance of the joint effect of quality reputation and warranty in quality signaling. However, theoretical work on interactions among warranties, quality reputation, and product quality is limited. Most of the theoretical work studies the quality relation either with warranties (Cooper and Ross, 1985; Lutz and Padmanabhan, 1998; Balachandran and Radhakrishnan, 2005) or with reputation (Shapiro, 1982; Chu and Chu, 1994), but hardly with both.

In the quality and warranty literature, Cooper and Ross (1985) propose a general warranty and quality model to study moral hazard problem when product quality is not directly observable. They find that the optimal warranty and quality increase in the buyer's maintenance cost of the product. However, for the cost of warranties that does not go to the buyer, (such as the cost of verifying that the product is broken and of processing the claim) the optimal quality increases in it and the optimal warranty decreases in it. We analyze the sensitivity of the warranty and quality from a different perspective. Specifically, we find that the optimal warranty and quality decrease in the cost of quality and in manufacturer's quality reputation.

Lutz and Padmanabhan (1998) model a situation where consumers infer unobservable quality from warranties. Their paper focuses on how the availability of extended warranties affects a monopoly manufacturer's warranty policy in a one-period setting. They show that an entry of extended warranties is blockaded with homogenous consumers. With heterogeneous consumers, an independent insurer can enter the market, and this competition may increase or decrease the manufacturer warranty policy designed for low valuation consumers.

The utility function in our paper is similar to that in Cooper and Ross (1985) and Lutz and Padmanabhan (1998). Differently, in our paper, the unobservable quality perception is derived not only from warranty information, but also from quality reputation. More importantly, our paper has a totally different focus from the above two papers. We do not consider consumer moral hazard or warranty competition from an extended warranty provider. Instead, our paper focuses on how consumer valuation change due to product failure experience in the first period affects manufacturer's market coverage strategies in the second period.

Balachandran and Radhakrishnan (2005) study quality implications of warranties in a supply chain setting. They examine warranty/penalty contracts between a buyer and a supplier based on internal component inspection and external product failure under two moral hazard settings (single and double). This research is not closely related to our paper, but is worth mentioning because of its unique supply chain perspective.

In the quality and reputation literature, Shapiro (1982) studies the optimal quality choices of a monopolist when reputation is viewed as an expectation of quality. The paper concludes that the best quality choice is less than consumer's initial expectation of quality. In contrast, our paper shows that the optimal quality can be either higher, lower or equal to the quality expectation of the consumers.

Chu and Chu (1994) discuss a scenario where a manufacturer's product quality is signaled through the reputation of a retailer, instead of his own. They find that manufacturers of high quality products distribute through reputable retailers, while manufacturer of low quality products distribute through retailers with no reputation. Differently, consumers in our paper form quality perception of the product based on the manufacturer's own quality reputation.

Another work that is related to ours is Li et al. (2011). They investigate a one-period duopoly scenario where consumers use both warranty and reputation to form quality perception in their purchase decisions. The assumption of quality perception is similar to our work. However, our work is fundamentally different from theirs because (1) we examine a two-period monopolist setting by incorporating market segmentation in the second period; (2) the quality in our work is a decision variable, while it is exogenously given in their work; (3) the warranty is a monetary compensation in our work and is modeled as a length of coverage in their work; (4) the focus of our work is to identify the optimal market strategy for the manufacturer, while the focus of their work is to find optimal warranty and price decisions.

The methodology of our models is related to the self-selection literature that is dealing with segmented market. Decision making in a segmented market is often solved using self-selection method, similarly as in Moorthy and Png (1992), Kim and Chhajed (2002), Mallik and Chhajed (2006), etc. In the literature, many research about segmented market focuses on product's time to market issues, and two consumer segments are given with differing valuation of product quality. For example, Wilson and Norton (1989) consider the problem of deciding the optimal timing of introducing new products. Cohen et al. (1996) consider the tradeoff between time to market and product performance. Unlike these papers, we focus on manufacturer's marketing strategies, when considering consumer segments are created due to product failure experience previously. In our paper, consumers who experienced a product failure will discount their valuations towards the product in future purchases, while there is no valuation change for consumers who did not experience a product failure. This creates a segmented market in the second period and affects the manufacturer's market strategy decisions.

3. Model

3. 1 Basic Model Setup

We consider a manufacturer (he) who produces a new product in a monopoly market, and sells the product to consumers (she) in a two-period horizon. Following the literature (Cooper and Ross, 1985; Bigelow et al., 1993; Lutz and Padmanabhan, 1995 and 1998), we assume that each unit of the product sold either "works" or "not works" upon purchase. A working product generates v dollars of value for the customer, which is also the customer's willingness to pay or monetary valuation for the product. A non-working product generates no value. However, the customer receives a monetary warranty compensation w_i from the manufacturer, where $i = 1 \; or \; 2$ denotes for Period I or Period II. For the ease of exposition, we will refer v as consumer valuation, and refer to a product that performs as per its specifications upon purchase as a working product, while the opposite scenario will be referred to as a product failure.

The product is produced at quality q_i during period i, $i = 1, 2$. The quality in our paper is viewed as an attribute that represents the probability of the product working as per its specifications. Thus, we normalize the quality to be between 0 and 1. Consequently, the probability of product failure in period i is $1 - q_i$. Similar assumptions can be found in Matthews and Moore (1987), and Lutz and Padmanabhan (1998). The product is priced at $p_i, i = 1, 2$. If the purchased product works, a consumer obtains utility $v - p_i$; if the purchased product fails, the consumer receives warranty compensation w_i and gets utility $w_i \; - p_i$. Note that our model does not require $w_i \; - p_i$ to be either positive or negative. This implies that the consumer utility for a failed product does not necessarily equal to zero and may even be positive. Our utility functions are consistent with the literature (Lutz and Padmanabhan, 1995 and 1998; Cooper and Ross, 1985; Bigelow et al., 1993).

Suppose the market size for the product in Period I is M, then $(1 - q_1)M$ number of consumers would experience product failure in Period I. Consistent with the literature (Moorthy and Png, 1992; Kim and Chhajed, 2000; Mallik and Chhajed, 2006), we assume the marginal production cost to be cq_i^2, where $c > 0$. The cost can be interpreted as the cost of achieving a certain level of probability for the product to work as per its specifications.

Consumers buy a new product in each of the two periods, as long as they get non-negative surplus. The consumer valuation towards a working product in Period II remains at v if she experiences no product failure in Period I. If the consumer experiences product failure in Period I, she will discount her valuation towards the working product to av in Period II, where $a \in [0,1]$ is the discounting factor. Note that, when $a = 0$, the consumer would have no willingness to purchase in Period II, and will exit the market at the end of Period I. Even when $a > 0$, the consumer may not purchase if the surplus is negative. This creates two segments of consumers in Period II. We denote the segment with valuation v as high (h) segment, and the other segment with valuation av as low (l) segment. Note that the market size is q_1M for the high segment, and $(1 - q_1)M$ for the low segment in Period II.

The customer cannot observe the product quality at the beginning of the first period because of the newness of the product. This is different from the assumption made in Bigelow et al. (1993) and Lutz and Padmanabhan (1998) where it is assumed that the customer knows the quality in Period I. In the absence of quality information, consumers measure perceived quality of the product from quality signaling information, which are product warranty and the manufacturer's quality reputation.

We next derive consumer quality perception for Period I, q_1^e, from warranty and the manufacturer's quality reputation. We assume the manufacturer has a quality reputation r in the minds of the consumers. This quality reputation reflects, for instance, historical quality levels of the manufacturer's products or brand image, and gives the consumer a perception of the quality of the new product the firm is introducing in Period I. For example, a highly reputable manufacturer (such as Apple) is believed to provide high quality new products and enjoys high value of r for its products. Previous studies (Blair and Innis, 1996; Akdeniz et al., 2013) have shown quality reputation can communicate unobservable quality. False quality will prevent firms from recouping the investments of brand equity (Erdem and Swait, 1998). Studies of consumer quality perception show that consumers are more likely to perceive a good warranty to be associated with a higher product quality if the manufacturer has a good quality reputation; on the other hand, if the manufacturer has a poor quality reputation, then warranty has less impact on the consumer quality perception (Boulding and Kirmani, 1993; Purohit and Srivastava, 2001). We therefore express the consumer quality perception in Period I, $q_1^e = rw_1$, which is increasing in warranty w_1 at a rate of quality reputation r. Thus the customer forecasts the product will not fail with probability rw_1. This expression captures the fact, as demonstrated in Akdeniz et al. (2013), that warranty has more (less) impact on consumer quality perception when the manufacturer's quality reputation is high (low). Therefore, the expected consumer utility in Period I is

$$EU_1 = q_1^e(v - p_1) + (1 - q_1^e)(w_1 - p_1). \tag{1}$$

The probability that the product will work in Period II, q_2, is still unobservable to consumers before the second period purchase. Consumers use quality perception for Period II, q_2^e, to make purchase decisions. The utility of the product is $v - p_2$ for customers who did not experience a failure in Period I. For customers who experienced a failure will derive a utility of $av - p_2$, where there is a reduction in consumer's willingness to pay because of the particular negative experience in Period I.

Therefore, the expected consumer utilities of the two segments of consumers in Period II are

$$EU_2^h = q_2^e(v - p_2) + (1 - q_2^e)(w_2 - p_2) \tag{2}$$

$$EU_2^l = \begin{cases} q_2^e(av - p_2) + (1 - q_2^e)(w_2 - p_2), & a \in (0,1] \\ 0, & a = 0. \end{cases} \tag{3}$$

Note that $EU_2^l = 0$ refers to the situation where consumers, who experienced product failure in Period I and discounted their valuations to zero ($a = 0$), will not purchase in Period II.

We now derive the expression for q_2^e. At the end of Period I, we assume the product quality q_1 is known to all consumers due to personal consumption experience, word of mouth, or public information such as consumer reports. Furthermore, we assume consumers use the Period I quality q_1 to update their quality perceptions in Period II, i.e., the consumer quality perception in Period II is $q_2^e = q_1$. Shapiro (1983) and Choi (1998) state that consumers update their quality expectations to the quality experienced after trying the product. This assumption of quality perception captures the influence of Period I quality in the updating of quality perception for Period II, and makes the model results tractable. In Section 5 of the paper, we will relax this assumption, and present a more general form of consumer quality perception for the second period.

We summarize the model notations in Table 1. Note that our model formulation in equations (1)-(3) assume that the consumers are homogenous and that the utility functions are linear. The linear utility function is commonly used in the literature (Moorthy and Png, 1992; Moorthy and Srinivasan, 1995) and it also keeps the model tractable to focus on deriving insights. Similarly, the homogeneous consumers assumption allows us to simplfy our model formulation and improve our exposition. It could, however, be shown that the results for heterogenous consumers case are qualitatively similar to those in our paper. This development is provided in Appendix B.

Table 1: Summary of Model Notation

Notation	Description
i	Index for period ($i = 1$ for Period I; $i = 2$ for Period II)
j	Index for market segment ($j = h$ for high; $j = l$ for low)
S	Strategy type ($S = F,\ H$)
M	Initial market size
c	Coefficient of marginal production cost
r	Manufacturer's quality reputation
v	Consumer's initial product valuation
a	Discount factor of consumer valuation in Period II, if product failure is encountered in Period I, and $a \in [0,1]$
w_i	Warranty
q_i	Quality of the product (also the probability of the product working)
q_i^e	Consumer quality perception of the product (also, the *perceived* probability of the product working)
p_i	Product price
EU_i^j	Expected utility
π_i^S	Manufacturer's profit under Period i and strategy S

3. 2 Manufacturer's Market Coverage Strategies

At the beginning of Period I, the manufacturer anticipates the market response, makes the optimal decisions for both periods, and simultaneously maximizes the total profit over Periods I and II. In each period, the manufacturer makes decisions in terms of product price, product quality and the warranty payment. The manufacturer incurs two costs: unit production cost cq_i^2, and the warranty payment in case of product failure. With full knowledge of the product failure probability, the manufacturer's expected warranty payment is $(1 - q_i)w_i$.

There are two decision-making strategies for the manufacturer in the two-period horizon, based on market coverage decisions in Period II. There are two consumer segments in Period II. The manufacturer can choose to either serve only the high segment or both the high and low segments. He cannot serve the low segment alone because when the price is low enough to attract the low segment, the high segment will buy as well due to a positive utility. Therefore, the manufacturer has two market coverage strategies in Period II: full market coverage (Strategy F) versus high market coverage (Strategy H).

We next present the model and its optimal solutions under each of the two market coverage strategies for the manufacturer.

Case 1: Full Market Coverage Strategy (Strategy F)

Under strategy F, the forward-looking manufacturer optimizes profit over the two periods and covers the full market in Period II. In Period I, the market size is M. The consumers purchase the product if their expected utilities are non-negative, i.e., $EU_1 \geq 0$. Thus, the maximum price customers are willing to pay in the first period is $p_1 = q_1^e v + \left(1 - q_1^e\right) w_1$, where $q_1^e = r w_1$. This is also the optimal price the manufacturer charges.

The market size is the same as in Period I, when the manufacturer sells to both segments in Period II. This assumption is also used in Bigelow et al. (1993). We, however, do allow for the possibility that the manufacturer may target only the high segment (Strategy H), thus selling a lower volume in Period II.

The manufacturer's profit function is as follows.

$$\underset{p_1, p_2, q_1, q_2, w_1, w_2}{\text{Max}} \pi^F = [p_1 - (1 - q_1) w_1 - c q_1^2] M + [p_2 - (1 - q_2) w_2 - c q_2^2] M, \quad (4)$$

Subject to:

$$EU_1 \geq 0, \quad (5)$$

$$EU_2^h \geq 0, \quad (6)$$

$$EU_2^l \geq 0, \tag{7}$$

$$q_i \leq 1, i = 1, 2, \tag{8}$$

$$q_i^e \leq 1, i = 1, 2. \tag{9}$$

Note that the constraints (8) and (9) esure that the quality and the quality perception could be expressed as a probability.

The price in Period II needs to guarantee both $EU_2^h \geq 0$ and $EU_2^l \geq 0$ in order to serve both segments. This ensures that each segment will buy the product rather than not buy anything at all. Note that, when low segment consumers discount their valuations to zero ($a = 0$) in Period II, the manufacturer has only the high segment to serve in Period II, since the low segment would exit the market at the end of Period I. Manufacturer's decision making in this scenario is mapped into the high market coverage strategy (Strategy H) presented in case 2. If the manufacturer makes the utility of high segment in equation (6) binding by setting price at $p_2 = vq_2^e + (1 - q_2^e)w_2$, and tries to extract all the consumer surplus from the high segment, then the utility of the low segment will be negative and the low segment will not buy. Therefore, in order to serve both segments, the manufacturer can only make the utility of the low segment in equation (7) binding and let the high segment enjoy a positive consumer surplus. The optimal price the manufacturer can charge in Period II when selling to both segments is thus $p_2 = avq_2^e + (1 - q_2^e)w_2$, where $q_2^e = q_1$.

By making the consumer utility in Period I (5), and the low segment consumer utility in Period II (7) binding, we obtain the respective product prices, i.e., $p_1 = rw_1v + (1 - rw_1)w_1$ and $p_2 = avq_1 + (1 - q_1)w_2$. We substitute these prices into the manufacturer's profit function, and maximize with respect to quality and warranty over two periods. In particular, we start with Period II problem by solving optimal decisions w_2 and q_2. We then maximize the total profit over two periods by solving for optimal decisions in Period I, i.e., w_1 and q_1, given the second period optimal decisions. Parameter conditions are applied, in order to make sure that the interior optimal solutions are feasible under constraints (8) and (9), i.e., $q_i \leq 1$ and $q_i^e \leq 1$, $(i = 1, 2)$. The optimal interior solutions are summarized in Table 2 in Appendix A. Note that the boundary solutions describe the situation where the product never fails, which is not of our interest. We focus our result analysis based on the optimal interior solutions.

Case 2: High Market Coverage Strategy (Strategy H)

In Strategy H, the manufacturer maximizes two-period profits and covers only the high market in Period II. When the manufacturer sells to only the high segment in Period II, the market size is $q_1 M$, since the high segment is the group of consumers who did not experience product failure during Period I. To ensure the low segment will not buy, the high segment's utility EU_2^h should be non-negative and the low segment's utility EU_2^l should be non-positive. Therefore, the manufacturer can set price to make the high segment utility binding and charge the maximum price that the high segment is willing to pay, $p_2 = vq_2^e + (1 - q_2^e)w_2$, where $q_2^e = q_1$.

The manufacturer's profit over the two selling periods is as follows.

$$\underset{p_1, p_2, q_1, q_2, w_1, w_2}{\text{Max}} \pi^H = [p_1 - (1 - q_1)w_1 - cq_1^2]M + [p_2 - (1 - q_2)w_2 - cq_2^2]q_1 M, \tag{10}$$

Subject to:

$$EU_1 \geq 0, \tag{11}$$

$$EU_2^h \geq 0, \tag{12}$$

$$EU_2^l \leq 0, \tag{13}$$

$$q_i \leq 1, i = 1, 2, \tag{14}$$

$$q_i^e \leq 1, i = 1, 2. \tag{15}$$

The binding constraints here are the first period consumer utility in (11) and the high segment utility in (12), from which the product prices, $p_1 = rw_1v + (1 - rw_1)w_1$ and $p_2 = vq_1 + (1 - q_1)w_2$, are derived. The rest of the solution procedures are similar as in the full market coverage strategy and the optimal interior solutions can be found in Appendix A.

4. Results

This section is divided into two sub-sections. Section 4.1 discusses characteristics of the two market coverage strategies. Section 4.2 identifies and analyzes the optimal strategy. We also elaborate on the interest alignment between the manufacturer and the consumers.

4.1 Characteristics of the Two Strategies:

Observation 1: *The quality reputation r must be lower than $2/v$, and the production cost c must be higher than certain threshold values for the feasibility of each of the two strategies F and H.*

For the optimal interior solutions to be meaningful, constraints are required on the optimal qualities, as the working product probabilities, to be less or equal to one. Similarly, consumer quality perceptions has the same restriction. Similar constraints have been made in Savaskan et al. (2004) and Savaskan and Van Wassenhove (2006). The conditions that satisfy those constraints are as follows. Note that the second order conditions for the maximization problems are satisfied under these conditions as well.

Strategy F: $r < 2/v$ and $c \geq \max\{\frac{(1+2a)rv+1}{8r}, \frac{arv+1}{4r(2-rv)}\}$;

Strategy H: $r < 2/v$ and $c \geq \max\{\frac{5rv+1}{10r}, \frac{1+4rv-2r^2v^2}{r(2-rv)(8-3rv)}\}$.

Please refer to Appendix A for the derivation of the above conditions. Observation 1 has interesting implications. The upper bound on quality reputation ($r < 2/v$) indicates the upper limit that a consumer is willing to deduce quality from warranty information. Innis and Unnava (1991) presented evidence that, when the manufacturer's quality reputation is high enough, warranties hardly affect consumer's perceptions on new product quality. The lower bound on production cost ensures that the production is costly enough, so that the manufacturer will not be able to offer such a high quality product that the product never fails.

We answer our first set of research questions regarding the optimality of various warranty policies by comparing the optimal warranty payments and the corresponding optimal prices. We then compare manufacturer's optimal qualities and consumer's quality perceptions in each period to examine the quality signaling functions of warranty and quality reputation. Lastly, we compare the optimal prices and warranties in each period to explore how they change over time. We also compare the optimal profits between the first and the second period in each strategy, so as to further understand the two market coverage strategies.

There are various warranty policies on the market. The industry-standard warranty policy usually has monetary warranty compensation less than the price paid. The above-industry-standard warranty policy can be viewed as when the monetary warranty is higher or equals to the price paid. To investigate the manufacturer's optimal warranty policy, we compare the optimal warranty payment with its corresponding optimal price in each strategy and present the optimality market conditions of each policy in the following proposition.

Proposition 1: *There exists such a threshold that in each strategy, when* $c \geq \tilde{c}_i^S(r)$, *the optimal warranty payment is lower than the optimal price, i.e.,* $w_i^{S*} \leq p_i^{S*}$, *where* $i = 1$ *or* 2 *and S = F or*

H. In addition, when $c \in [\bar{c}^S(r), \tilde{c}_i^S(r)]$, the reverse relations hold, where $\bar{c}^S(r)$ is the cost lower bound in each strategy.

The proofs of all results are included in Appendix A. The values of $\tilde{c}_i^S(r)$ and $\bar{c}^S(r)$ in each strategy can be found in Table 3 in Appendix A.

Figure 1 schematically depicts the relations in Proposition 1. The solid line shows the values of $\tilde{c}_i^S(r)$ and the dotted line is the cost lower bound $\bar{c}^S(r)$. Region A is the area where $c \geq \tilde{c}_i^S(r)$, representing the region that the optimal warranty payment is lower than the optimal price. Region B is the small shaded area in the lower left corner between the line of $\tilde{c}_i^S(r)$ and the cost lower bound $\bar{c}^S(r)$. Region B depicts the area where the optimal warranty payment is higher than the optimal price.

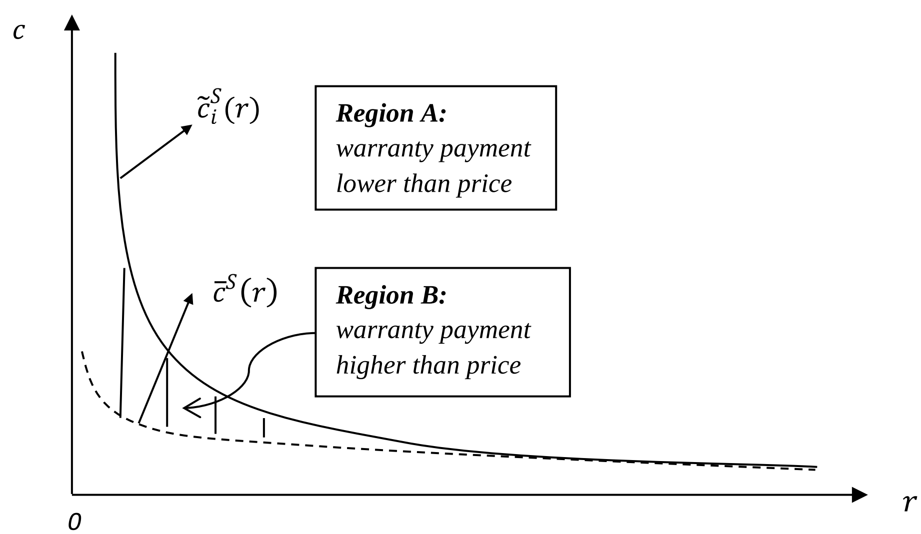

Figure 1: Comparison of Optimal Warranty and Price

As illustrated in Figure 1, region B is relatively small and corresponds to market conditions of either low production costs or low quality reputations. Low cost enables the manufacturer to offer a product with high quality. To signal the high quality at a low quality reputation, it is optimal for the manufacturer to provide a warranty compensation that is higher than the retail price, or to provide an above industry standards warranty. Many new firms without established reputation often promote their quality products with higher-than-industry-standard warranties. Memory foam mattress introduced by Tempur-Pedic is an example of such. In the early 1990s, the newly founded company Tempur-Pedic introduced the first-generation memory foam mattress to the U.S. market. The company promoted this new type of mattress with warranties far beyond the industry standard, which generally has a coverage of 10 years. The warranty Tempur-Pedic provided then was an impressive 25 years.

Region A represents the area where it is optimal for the manufacturer to offer an industry standard warranty, or limited warranty coverage. As observed in practice, most firms follow this practice. Companies suitable for this strategy have either high quality reputation, or low quality reputation with low quality product due to high production cost involved, as illustrated in Figure 1. An example of such is Apple iPod. This revolutionary product came with a one-year standard industry warranty, when it was introduced by the highly reputable Apple in 2001.

The manufacturer's warranty compensation equals to the product price at the boundary between regions A and B. It is easy to verify that the optimal quality equals to the consumer quality perception here. For well-established and well-known brands, consumers could form quality perception that are very close to the true quality of the product. Thus, it is optimal for these companies to offer warranty compensation equals to the price paid.

Besides cost and quality reputation, the optimal regions are also affected by other market conditions in some strategies, such as consumer valuation v and the valuation discount factor a. The area of region B decreases (or region A increases), when consumer valuation increases. It is intuitive that when consumers have higher regards towards the product, the manufacturer does not need to provide a larger warranty to attract consumers anymore. Therefore, the area of region B decreases. The valuation discount factor has the opposite effect. The area of region B increases when the consumer valuation becomes smaller due to a larger discount.

We next compare manufacturer's optimal qualities and consumer's quality perceptions in each period and present the results as follows.

Proposition 2: *Under the two market coverage strategies,*

(a) in Period I, when $c \geq \tilde{c}_1^S(r)$, the optimal qualities are lower than the consumer quality perceptions, i.e., $q_1^{S*} \leq q_1^{eS*}$; otherwise, when $c \in [\bar{c}^S(r), \tilde{c}_1^S(r)]$, the reverse relations hold, where $\bar{c}^S(r)$ is the cost lower bound in each strategy, S = F or H.

(b) in Period II, the optimal qualities are equal to the consumer quality perceptions, i.e.,$q_2^{S*} = q_2^{eS*}$, where S = F or H.

Note that the values of $\tilde{c}_i^S(r)$ and $\bar{c}^S(r)$ in Proposition 2(a) are the same as those in Proposition 1. Therefore, region B in Figure 1 corresponds to the area where the optimal qualities are higher than the consumer quality perceptions, while region A represents the reverse relationship. We use region B as an example to illustrate the relation. The explanation for region A is similar. Recall that the manufacturer offers generous warranties to signal the good product quality in region B. However, due to his relatively low quality reputation, the consumers generally believe the quality is not as good as signaled, and therefore form a low quality perception at the time of purchase. Consequently, we have the relations $q_1^{S*} > q_1^{eS*}$.

Proposition 2(b) shows that, in Period II, it is optimal for the manufacturer to offer a quality that matches consumer's quality perception. We assume consumers update their quality perception in Period II by their quality learning in Period I, i.e., $q_2^e = q_1$. Hence, warranty in Period II and manufacturer's quality reputation have no impact on how consumers evaluate the product in the second period. Therefore, in Period II, the manufacturer has no incentive to either enhance the quality (i.e., $q_2 > q_1$) as this would increase the cost of production; nor to reduce the quality (i.e., $q_2 < q_1$) as this would increase the probability of product failure and incur higher warranty cost. In Section 5 of the paper, we will explore an alternative form of consumer quality perception update in Period II, and derive insights on how warranty, quality reputation, and the realized quality in Period I affect the manufacturer's choice on the optimal quality in Period II .

We next examine how the optimal quality and warranty change in the parameters related to quality and quality perceptions. This would provide insights on the relation among quality, warranty and manufacturer's quality reputation. We conduct sensitivity analysis by analyzing the first derivatives with respect to quality cost and quality reputation, and summarize the results in the corollary below.

Corollary 1: *The optimal quality and warranty decrase in the cost of quality and in the quality reputation.*

Corollary 1 shows interesting interactions among quality, warranty and quality reputation. When the cost of quality increases, both the optimal quality and warranty decrease as shown in the proof. It is intuitive that the manufacturer lowers quality in response to an increased quality cost. The warranty coverage is decreased accordingly to avoid excess warranty cost associated with a lower quality. Corollary 1 also indicates that, when quality reputation decreases, both the optimal quality and warranty increase. The manufacturer needs to increase warranty to maintain consumers' quality perceptions when reputation decreases. A higher quality level is needed for the increased warranty to avoid excess warranty cost.

We next compare the warranty and price decision over two periods for each of the strategies. The results are illustrated in the following proposition.

Proposition 3: *Under the two market coverage strategies,*

(a) there exists such a condition that in each strategy, when $c \geq \hat{c}_w^S(r)$, *the optimal warranty payment in Period I is lower than the optimal warranty payment in Period II, i.e.,* $w_1^{S*} \leq w_2^{S*}$, *where S = F or H. In addition, when* $c \in [\bar{c}^S(r), \hat{c}_w^S(r)]$, *the reverse relations hold, where* $\bar{c}^S(r)$ *is the cost lower bound in each strategy.*

(b) there exists such a condition that in each strategy, when $c \geq \hat{c}_p^S(r)$, *the optimal price in Period I is lower than the optimal price in Period II, i.e.,* $p_1^{S*} \leq p_2^{S*}$, *where S = F or H. In addition, when* $c \in [\bar{c}^S(r), \hat{c}_p^S(r)]$, *the reverse relations hold, where* $\bar{c}^S(r)$ *is the cost lower bound in each strategy.*

The expressions for the parameters $\hat{c}_w^S(r)$ and $\hat{c}_p^S(r)$ in each strategy can be found in Table 3 in Appendix A.

We use the relations $w_1^{S*} > w_2^{S*}$ and $p_1^{S*} > p_2^{S*}$ as an example to explain the proposition. These relations occur in the region where quality reputation is low, and quality level is high due to a small quality cost. Under such conditions, according to Proposition 2, the manufacturer offers quality that is higher than consumer's quality perception in Period I and the same quality as consumer's quality perception in Period II. This explains the higher warranty and the corresponding higher price in Period I.

Observation 2: Under each strategy, *the first period profit is greater than that in the second period, i.e.,* $\pi_1^{S*} \geq \pi_2^{S*}$, *where S= F or H.*

Observation 2 demonstrates the value of information. At the beginning of each period, consumers have no knowledge of the product quality. Instead, they use quality perception to make purchase decisions. In Period I, quality perception is based on warranty information and the manufacturer's quality reputation. In Period II, consumers have experienced the product quality in Period I, and choose to use it as their quality perception for Period II. As stated in Proposition 2(b), the optimal quality equals to consumer's quality perception in Period II, which is the realized quality in Period I. Consumers happen to have full information about the product quality and product failure rate in Period II, but not in Period I. Therefore, the manufacturer is able to extract more profit in Period I than in Period II, due to the fact that he has more knowledge about the product than the consumers.

We next compare between the two market coverage strategies to identify the best strategy for the manufacturer and to investigate their characteristics.

4.2 Analysis of Market Coverage Strategies

In this subsection, we first compare between the two market coverage strategies (F and H) in terms of total optimal profit to identify the optimal strategy for the manufacturer. We next investigate which strategy would be more beneficial to the consumers in terms of qualities and warranties. We further explore consumer welfare to understand which strategy would be better for the consumers in a comprehensive manner.

We compare the total optimal profits of the two market coverage strategies and conclude the following proposition.

Proposition 4: *When consumer valuation discount factor is greater than a threshold value, i.e.,*$a \geq \bar{a}_\pi$*, strategy F is more profitable for the manufacturer than strategy H, i.e.,* $\pi_t^{F*} \geq \pi_t^{H*}$. *Otherwise, the reverse relation holds.*

The expression for $\bar{a}_\pi$ can be found in the proof in Appendix A. The optimal profit of strategy F increases in the valuation discount factor as shown in the proof. Therefore, the manufacturer enjoys higher profit in strategy F when the discount factor is larger. This proposition shows either strategy F or strategy H can be an optimal market coverage strategy depending on the market conditions. Specifically, it is more profitable for the manufacturer to serve the total market in Period II, if the consumers do not discount the product valuation too much (a relative large value of a). When the discount is deep, it is optimal for the manufacturer to serve only the high market. The expression of the threshold $\bar{a}_\pi$ is quite complicated. We next numerically illustrate its properties. We wish to learn how its value changes with the production cost, the quality reputation, and the consumer valuation. Figure 2(a) shows the relation between the threshold value $\bar{a}_\pi$ and the manufacturer's quality reputation r. We set $c = 1$ and $v = 1$, vary the value of r from 0.4 to 1, and obtain the line $a = \bar{a}_\pi^r$. The area above (below) this line is where strategy F (strategy H) is optimal. Similarly, line $a = \bar{a}_\pi^c$ in Figure 2(b) shows the influence of production cost c on the threshold value and is plotted by setting $v = 1$, $r = 0.8$ and varying the value of c from 0.4 to 1. Line $a = \bar{a}_\pi^v$ in Figure 2(c) explores the effect of consumer valuation with $c = 1$, $r = 0.8$ and the values of v from 0.4 to 1. Note that these values satisfy the model constrains we imposed. It illustrates that the threshold value $\bar{a}_{\pi}$ increases in consumer valuation, and decreases in the production cost and the quality reputation of the manufacturer.

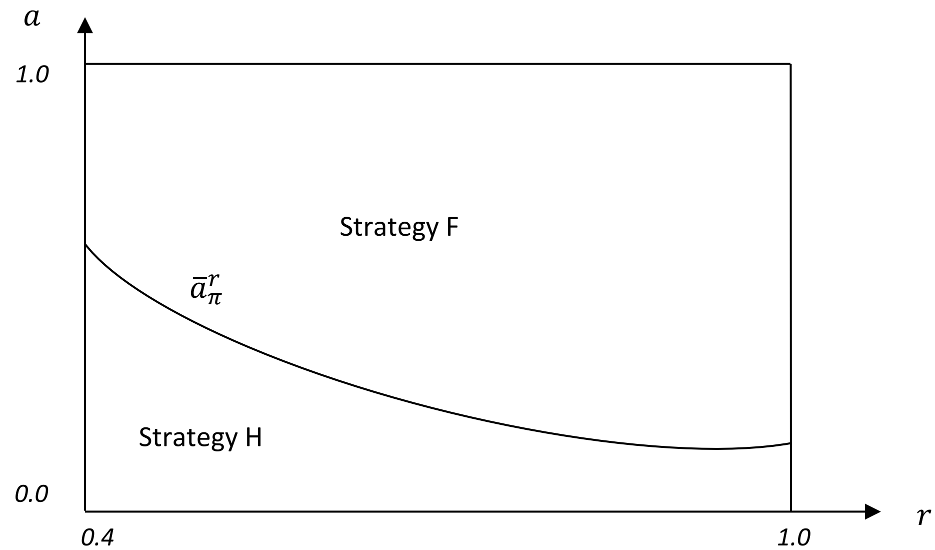

Figure 2(a): Optimal strategy for Manufacturer (with $\mathbf{c = 1}$, $\mathbf{v = 1}$, $\mathbf{r \in (0.4, 1)}$)

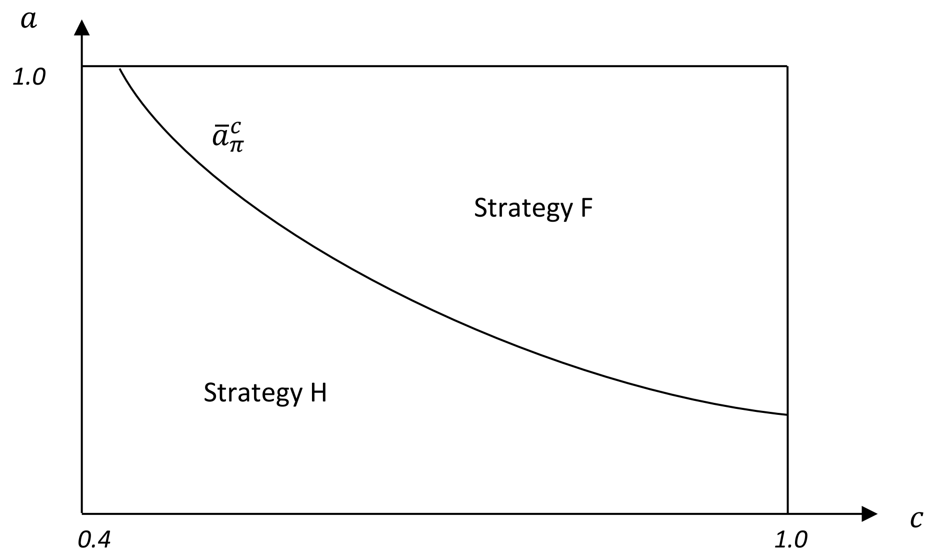

Figure 2(b): Optimal strategy for Manufacturer (with $\boldsymbol{r = 0.4}$, $\mathbf{v = 1}$, $\mathbf{c \in (0.4, 1)}$)

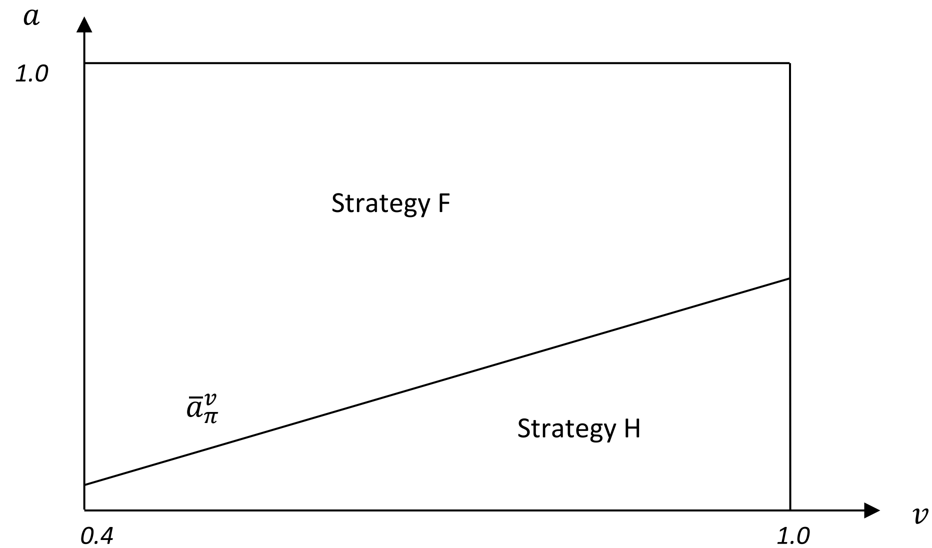

Figure 2(c): Optimal strategy for Manufacturer ($c = 1$, $r = 0.8$, $v \in (0.4, 1)$)

The condition $a \geq \bar{a}_\pi$ is easier to satisfy when the threshold value is small. Thus, the proposition indicates that the full market coverage strategy is more likely to be optimal when the manufacturer's quality reputation is high, the production cost is high, or the consumer valuation is low. This is reflected in Figures 2(a), 2(b) and 2(c), as the optimal region for strategy F gets larger when consumer valuation becomes smaller, or the production cost and the quality reputation become larger.

When consumer valuation is smaller, same discount factor decreases the valuation in a smaller magnitude. Consequently, the optimal region of strategy F decreases less. This explains why the optimal region of strategy F actually increases when consumer valuation becomes smaller. When the production cost increases, both profits decrease. However, the total profit in strategy H decreases at a faster rate. Therefore, the optimal region for strategy H becomes smaller and that of strategy F gets larger. The explanation of quality reputation is similar with that of production cost.

We next identify and investigate the strategy that consumers would be better off in terms of higher quality and better warranty.

Proposition 5: *When consumer valuation discount factor is greater than a threshold value, specifically, when* $a \geq \bar{a}_w$ *, consumers enjoy higher quality and better warranty in strategy F than in strategy H in each period. Otherwise, the reverse relation holds.*

The expression for $\bar{a}_w$ can be found in the proof. The optimal qualities and warranties of strategy F increase in the valuation discount factor. Therefore, when the discount factor gets larger, the qualities and warranties in strategy F become higher than those of strategy H.

We next compare the two threshold values ($\bar{a}_\pi$ and $\bar{a}_w$) in Propositions 4 and 5. We are interested in investigating the influence of manufacturer's optimal strategy on the qualities and warranties. We numerically plot the two threshold values under different market parameters. In Figure 3, we set $c = 2$, $v = 0.6$ and vary the value of r from 0.4 to 1 to explore the influence of quality reputation on the two threshold values. Note that the area above line $a = \bar{a}_\pi$ is where strategy F generates higher profit for the manufacturer; and the area above line $a = \bar{a}_w$ is where consumers enjoy higher qualities and warranties in strategy F. We see that there exist an area between line $a = \bar{a}_\pi$ and line $a = \bar{a}_w$ where manufacturer would adopt strategy F for higher profit; however consumers would suffer from lower qualities and warranties. Which strategy would be better overall for the consumers, when quality, warranty, and price are comprehensively considered? We answer this question by looking into consumer welfare next.

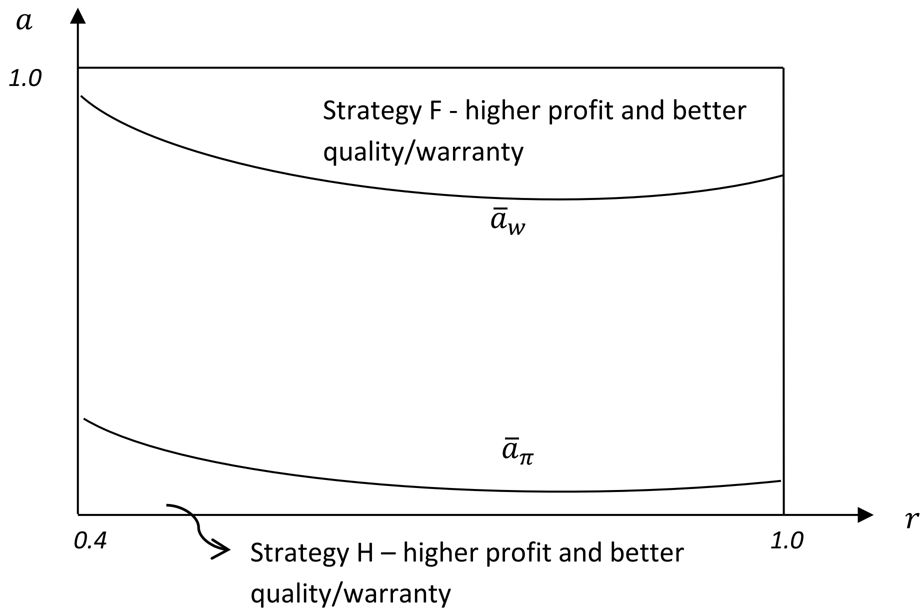

Figure 3: Strategies beneficial to manufacturer and consumer (with better quality/warranty) ($c = 1$, $\mathbf{v} = \mathbf{1}$, $\mathbf{r} \in (\mathbf{0.4}, \mathbf{1})$)

Consumer Welfare

The surplus of an individual consumer is the difference between consumer utility and the price paid. In Period I, the consumer surplus (CS) is $CS_1^S = vq_1^{S*} + (1 - q_1^{S*})w_1^{S*} - p_1^{S*}$, where $S = F$ *or H*. In strategy F, the consumer surplus in Period II is $CS_2^F = vq_2^{F*} + (1 - q_2^{F*})w_2^{F*} - p_2^{F*}$ if the consumer did not experience product failure in Period I, or $\widetilde{CS}_2^F = avq_2^{F*} + \left(1 - q_2^{F*}\right)w_2^{F*} - p_2^{F*}$ if the consumer did experience product failure in Period I. In strategy H, consumer surplus in Period II is $CS_2^H = vq_2^{H*} + \left(1 - q_2^{H*}\right)w_2^{H*} - p_2^{H*}$.

Thus, the consumer welfare (CW) for strategy F is

$$CW^F = CS_1^F M + CS_2^F q_2^{F*} M + \widetilde{CS}_2^F (1 - q_2^{F*}) M$$

The consumer welfare for strategy H is

$$CW^H = CS_1^H M + CS_2^H q_2^{H*} M$$

The expression of the consumer welfare can be found in the proof of Proposition 6.

Proposition 6: *When consumer valuation discount factor is greater than a threshold value, specifically,when* $a \geq \bar{a}_{cw}$ *, consumer welfare is higher in strategy F than in strategy H. Otherwise, the reverse relation holds.*

The value of $\bar{a}_{cw}$ is the value of valuation discount factor that solves the equation $CW^F = CW^H$. The expression of $\bar{a}_{cw}$ is rather complicated. Thus, we solve for it numerically and compare it to the value of $\bar{a}_{\pi}$ in Proposition 4, so as to investigate the relation between consumer welfare and the manufacturer's optimal strategy. The following figures illustrate the two thresholds under various market conditions. In Figure 4(a), we set $c = 0.7$, $v = 1$ and vary the value of r from 0.4 to 1 to explore the influence of quality reputation on the two threshold values. Obviously, the two threshold values are different. This creates a region between line $a = \bar{a}_{cw}$ and line $a = \bar{a}_{\pi}$, where the strategy that has the higher consumer surplus is different from the strategy that yields higher profit for the manufacturer. The area between the two lines indicates the size of this interest conflicting region. On the other hand, the remaining regions have a win-win situation, where the consumer welfare is higher in the manufacturer's optimal strategy and the interests of the consumers and the manufacturer are aligned together. The win-win region gets larger when the conflicting region becomes smaller.

Figure 4(b) is obtained by increasing the production cost value to $c = 0.8$, while consumer valuation stays at $v = 1$. Similarly, Figure 4(c) is when the consumer valuation increases to $v = 1.2$ and production cost remains at $c = 0.7$. When production cost increases, we see that the win-win region increases for strategy F, and decreases for strategy H. When consumer valuation increases, we see that the opposite effect: the win-win region decreases for strategy F, and increases for strategy H.

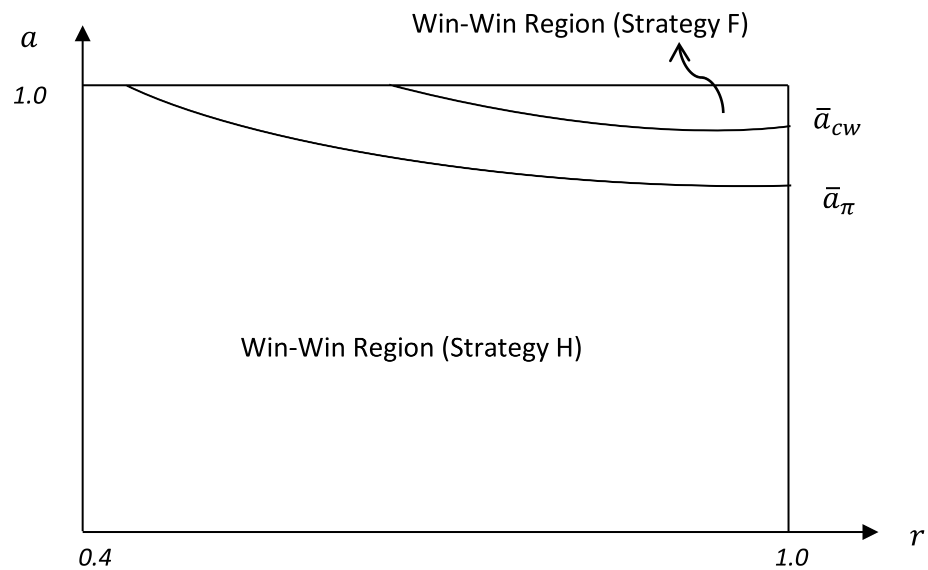

Figure 4(a):Win-win regions with higher manufacturer profit and higher consumer welfare ($c = 0.7, v = 1$)

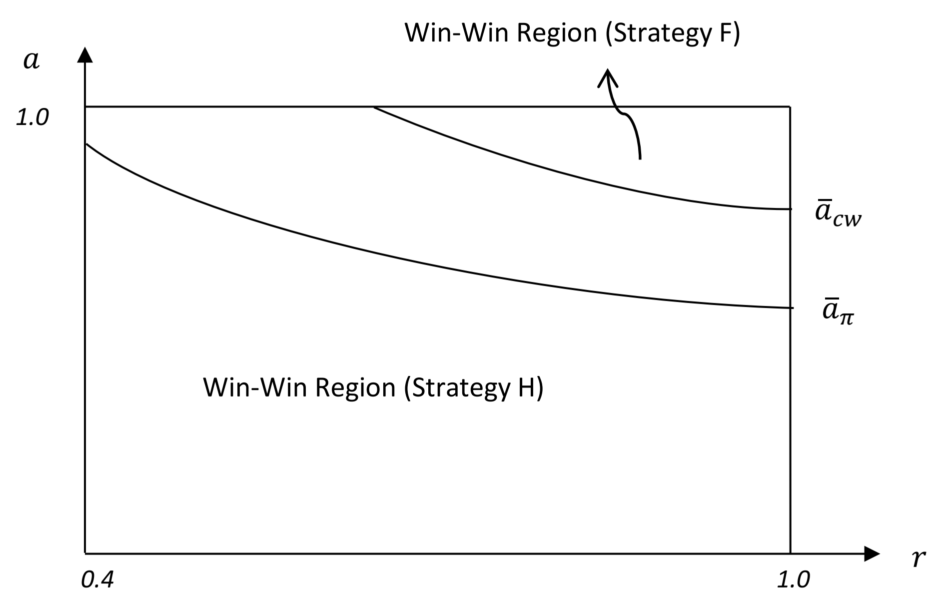

Figure 4(b): Win-win regions with higher manufacturer profit and higher consumer welfare ($c = 0.8, v = 1$)

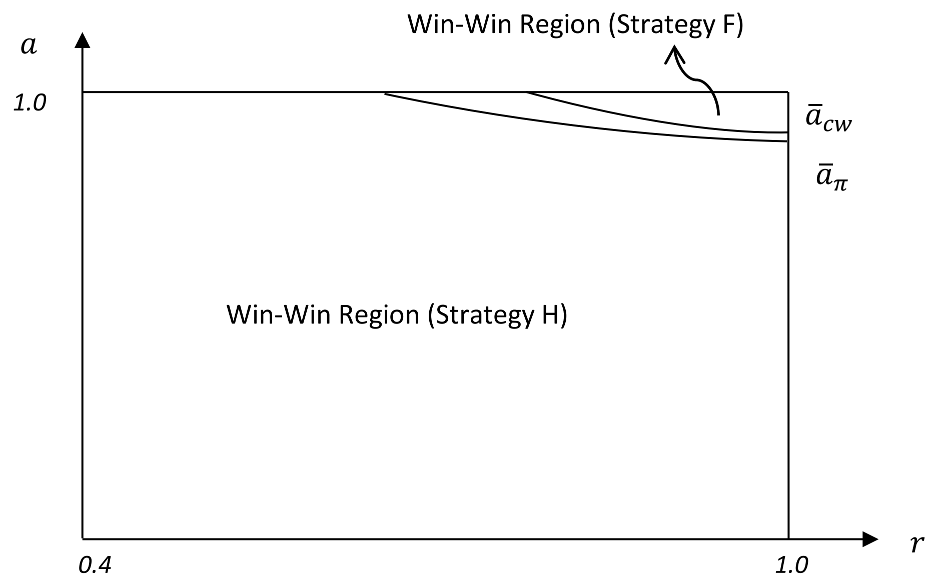

Figure 4(c): Win-win regions with higher manufacturer profit and higher consumer welfare ($c = 0.7$, $v = 1.2$)

We conclude the following observation regarding the win-win region.

Observation 3:

- *When $a \geq \bar{a}_{cw}$, strategy F becomes win-win strategy for both the manufacturer and the consumer. This win-win region becomes larger either when production cost increases, or consumer valuation decreases.*
- *When $a \leq \bar{a}_{\pi}$, strategy H becomes win-win strategy for both the manufacturer and the consumer. This win-win region becomes larger either when production cost decreases, or consumer valuation increases.*

The implication of Observation 3 is interesting. From Observation 3 and Figures 4 (a)-(c), we can find that (i) when both reputation and valuation discount factor are high, total market coverage is a better strategy for both the manufacturer and consumers; (ii) when both discount factor of valuation and reputation are low, only high segment coverage is a better strategy for both the manufacturer and consumers. In case (i), when consumer valuation is not affected much by the product failure experience, and when the consumers are aware of high reputation and hence hold high quality perception, the manufacturer should be able to generate higher profit by targeting both of the high and low consumer segments. However, when consumer valuation is sensitive to the product failure, the low segment holds a much lower valuation in the Period II, which would hence discount the price significantly and reduce the profit. Therefore, in case (ii), the manufacturer should only cover the high consumer segment to maximize the profit.

An additional finding is that, by decreasing the cost coefficient, or increasing the consumer valuation, the win-win region of strategy H will expand, and the win-win region of strategy F will shrink. In many countries, governments subsidize manufacturers by covering a portion of production cost, which would hence reduce the production cost and make strategy H more favorable to the manufacturer. Such a subsidy program will also reduce the region of conflict where the manufacturer adopts strategy F but consumers suffer from lower consumer warfare. Another way to reduce the region of conflict is through increasing consumer valuation. Many firms have initiated practices to increase consumer valuation. For example, Magnolia Home theater store, a specialized store at Best Buy geared toward high-end electronic home theater items, has employed various practices to improve consumer valuation of their product. These practices include improving the ambience of the store to enhance the presentation of product category, and hiring well-trained experts as salespeople to explain product characteristics (Perdikaki and Swaminatha, 2013).

5. Extension: A Generalization of Quality Perception Update

For the tractability of model results in Section 3 and Section 4, we assumed the consumer quality perception in Period II equals to the quality in Period I, i.e., $q_2^e = q_1$. We now relax this assumption to explore a more general expression for q_2^e. This section assumes that consumers form quality perceptions from two sources in Period II: (i) the realized first period quality q_1, and (ii) the second period warranty w_2 together with the manufacturer's long-term quality reputation r. Reputation is associated with the brand name (Li et al., 2011), whereas the brand name encapsulates a firm's long quality history (Cabral, 2008) and barely changes in a short period. Therefore, we assume the reputation parameter r stays the same over two periods.

When consumers soley use the realized first period quality to update their quality perceptions, we have $q_2^e = q_1$. When consumers soley rely on manufacturer's quality reputation and warranty to form their quality preceptions, we have $q_2^e = rw_2$. Combining these two sources, we express the quality perception in Period II as a weighted average of the two sources: $q_2^e = \beta q_1 + (1 - \beta)\, rw_2$, where $\beta \epsilon [0,1]$, depending on the weight consumers put on the two sources of quality perception. Consumers choose to put more emphasis on the realized quality, when the weigth on the realized quality, β, is high. Similar weighted average expression can be found in Kwark et al. (2014), where shopper's expected quality difference is modeled from two information sources.

Therefore, the expected consumer utility in Period I is

$$EU_1 = \ q_1^e(v - p_1) + (1 - q_1^e)(w_1 - p_1),$$

where $q_1^e = rw_1$. The expected consumer utilities of the two segments of consumers in Period II are

$$EU_2^h = q_2^e(v - p_2) + (1 - q_2^e)(w_2 - p_2),$$

$$EU_2^l = q_2^e(av - p_2) + (1 - q_2^e)(w_2 - p_2),$$

where, $q_2^e = \beta\, q_1 + (1 - \beta)\, rw_2$, $\beta \in [0,1]$.

The model setup and solution procedures are the same as in the original models for the two market coverage strategies (Strategy F and Strategy H). Since finding the optimal solution is not tractable, we conduct numerical analyses and find that our major results are qualitatively similar to those with the original models. We use the full market coverage strategy as an example to illustrate the model results. The results from high market coverage strategy are similar.

We set $r = 0.5$, $\alpha = 0.9$, $\beta = 0.8$, $v = 1$ and $M = 1$, vary the value of c from 0.8 to 1.7 in steps of 0.1, and numerically solve the model. The graphs below illustrate the updated model results corresponding to those in Propositions 1, 2 and 3 in the main paper.

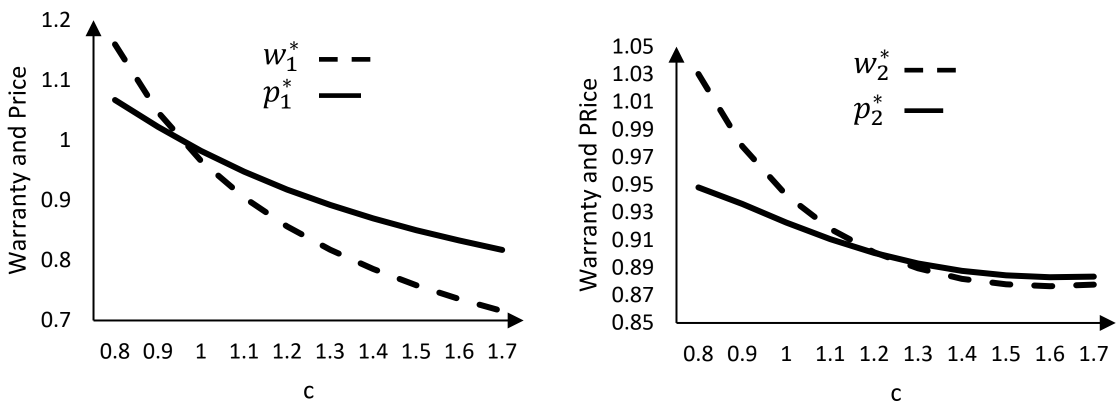

Figure 5: Updated Model Results Corresponding to Proposition 1

Figure 5 illustrates that there exists a threshold value c such that, when c is larger than that value, the optimal warranty payment is lower than the optimal price, i.e., $w_i^* \leq p_i^*$, where $i = 1\ or\ 2$. When c is smaller than the threshold value, the reverse results hold. These findings are qualitatively similar to those in Proposition 1.

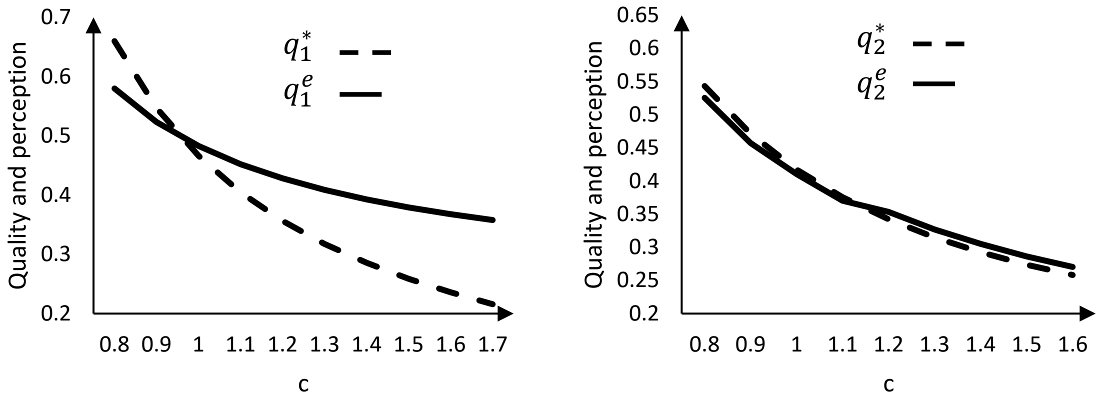

Figure 6: Updated Model Results Corresponding to Proposition 2

We next compare the optimal quality and the quality perception in Period I and find similar results as in Proposition 2(a), as illustrated in the left graph in Figure 6. That is, there is a threshold value for the unit production cost c such that when c is larger than that value, the optimal quality smaller than the quality perception. However, for Period II, the results of our current model differ from those in Proposition 2(b). As the right graph in Figure 6 illustrates, the relationship between q_2^* and q_2^e varies and is now dependent on the production cost c (note that we obtained $q_2^* = q_2^e$ in Proposition 2(b)). This difference is due to the impact of warranty and quality reputation on the quality perception $q_2^e = \beta q_1 + (1-\beta)\, rw_2$. The explanation is similar as in Propostion 2(a). In the $q_2^* > q_2^e$ region, both production cost c and quality reputation r are low. The low production cost enables the manufacturer to offer high quality. The manufacturer also offers generous warranties to signal the high quality. However, due to the relatively low quality reputation, the consumers generally believe the quality is not as high as signaled, and therefore form a low quality perception.

Our numerical results also show that, as β increases (i.e., as consumer puts a heavier weight on q_1), the discrepancy between q_2^* and q_2^e becomes smaller. When β increases to its maximum value of 1, this discrepancy decreases to zero, i.e., q_2^* becomes equal to q_2^e. This is identical to our finding in in Proposition 2(b). Our model in Section 3 is a special case of this general form of quality perception when $\beta = 1$, where consumers solely use q_1 to form their quality perceptions in Period II. In Section 3, the warranty and the manufacturer's quality reputation have no impact on consumer's quality perception in Period II. Therefore, the manufacturer chooses to offer the same quality as consumer's quality perception, as any deviation would increase his cost (either production cost or warranty cost) as discussed in Proposition 2(b).

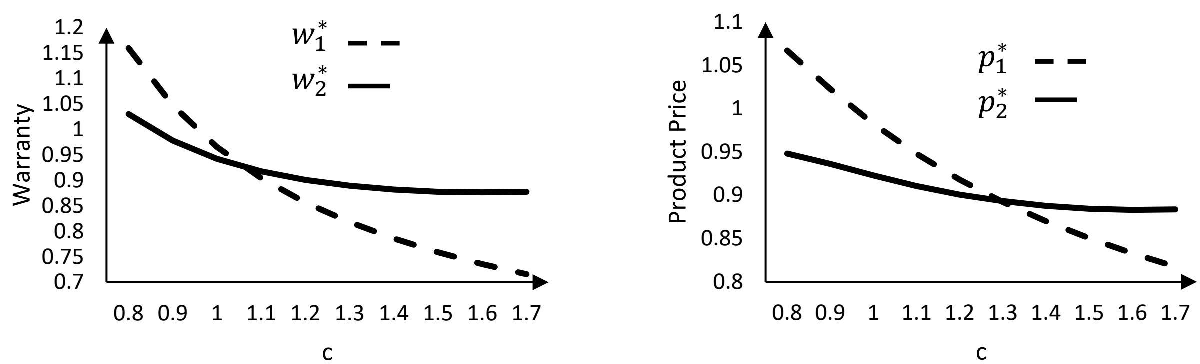

Figure 7: Updated Model Results Corresponding to Proposition 3

Figure 7 compares warranty and price decisions between the two periods. Again, we find similar result as in Proposition 3. That is, there exist a threshold value for c, when c is greater than that value, Period II warranty (price) will be higher than Period I warranty (price).

The following graph (Figure 8) illustrates the profit relationship in the updated model between strategies F and H. The numerical profit values are obtained by setting $r = 0.3$, $c = 2$, $\beta = 0.9$, $v = 1$ and $M = 1$, and varying the value of a from 0.5 to 0.6. Again, the result is qualitatively similar to that in Proposition 4. Specifically, the graph illustrates that there exists a threshold value such that, when a is larger than that value, the optimal profit of strategy F becomes higher than that of strategy H.

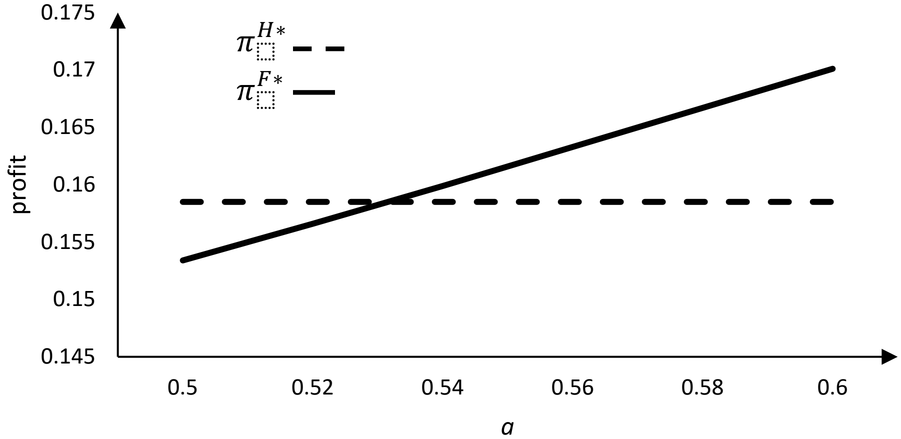

Figure 8: Updated Model Results Corresponding to Proposition 4

6. Conclusion

Product failures affect not only a manufacturer's quality and warranty decisions, but also a consumer's purchase decision. A failure increases the manufacturer's warranty cost and decreases a consumer's valuation towards the product. In this paper, we developed a two-period model and explored the manufacturer's optimal market strategies for a durable product. The quality of the product is unobservable to consumers and is directly related to the probability of a product failure. Consumers' purchase decisions are influenced by her quality perception, her valuation for the product, as well as the manufacturer's price and warranty terms.

Our model results yielded interesting implications about the relationship of the optimal quality and the consumer quality perception. We answered our first research question regarding optimal warranty strategies by analyzing the relation between the optimal warranty and the optimal price.

We identify the market conditions for various warranty policies to be optimal. We further investigated our second research questions regarding the optimal market strategy of the manufacturer by comparing his two possible strategies. We showed that either strategy F or strategy H can be optimal depending on the discount factor of the consumer valuation. Lastly, our examination of consumer welfare illustrated a win-win region where both the manufacturer and the consumers would be better off with higher profit and higher consumer welfare respectively.

Like any other model in operations management literature, our model is based on a set of assumptions. We studied the manufacturer's market strategy under consumer quality perception and valuation change in a monopolistic setting. An important extension of our model would be to include competition. We modeled the change of consumer valuation as the result of consumer's own experience of product failure. Blog sharing and product reviews are common today. A consumer's purchasing decision might be influenced by other people's good and bad experiences towards the product. To consider the influence of other people towards a consumer's valuation, as well as her valuation change would also be an interesting extension.

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Appendix A

Table 2: Optimal Solutions for the Market Coverage Strategies

	Strategy F
p_1^*	$\frac{v(4cr+a)[8cr-1+rv(4cr-1-a)]}{(8cr-1)^2}$
p_2^*	$\frac{rv(1+2a)[2c(8cr-1)+av(4cr-1)-2crv]}{(8cr-1)^2}$
q_1^*	$\frac{rv(1+2a)}{8cr-1}$
q_2^*	$\frac{rv(1+2a)}{8cr-1}$
w_1^*	$\frac{v(4cr+a)}{8cr-1}$
w_2^*	$\frac{2crv(1+2a)}{8cr-1}$
π_1^*	$\frac{Mr[4acr+a^2(4cr-1)+cr(16cr-1)]v^2}{(8cr-1)^2}$
π_2^*	$\frac{(1+2a)Mr[a(6cr-1)-cr]v^2}{(8cr-1)^2}$
π_t^*	$\frac{Mr(a+a^2+2cr)v^2}{8cr-1}$

Optimal Solutions for Strategy H:

$$p_1^* = \frac{4cr(5+17rv)+8c^2r^2(18rv+9r^2v^2-8)-(1+4rv)^2+(16cr-1-4rv)\sqrt{16c^2r^2-8cr(1+rv)+(1+4rv)^2}}{288c^2r^3}$$

$$p_2^* = \frac{2cr(7+16rv)-1-40c^2r^2-5rv-4r^2v^2+(10cr-1-rv)\sqrt{16c^2r^2-8cr(1+rv)+(1+4rv)^2}}{36cr^2}$$

$$q_1^* = \frac{1-4cr+4rv+\sqrt{24cr^2v+(1-4cr+4rv)^2}}{12cr}$$

$$q_2^* = \frac{1-4cr+4rv+\sqrt{24cr^2v+(1-4cr+4rv)^2}}{12cr}$$

$$w_1^* = \frac{1+4rv+4cr(3rv-1)+\sqrt{24cr^2v+(1-4cr+4rv)^2}}{24cr^2}$$

$$w_2^* = \frac{1-4cr+4rv+\sqrt{24cr^2v+(1-4cr+4rv)^2}}{6r}$$

$$\pi_1^* = \frac{(1+4rv)^2 - 4cr(3+10rv+4r^2v^2) + 8c^2r^2(6+4rv+9r^2v^2) - 64c^3r^3 + [1+16c^2r^2+4rv-4cr(2+rv)]\sqrt{16c^2r^2-8cr(1+rv)+(1+4rv)^2}}{288c^2r^3/M}$$

$$\pi_2^* = \frac{64c^3r^3 - 24c^2r^2(2+rv) + (2rv-1)(1+4rv)^2 + 6cr(2+5rv) - [1+16c^2r^2+2rv-8r^2v^2-2cr(4+rv)]\sqrt{16c^2r^2-8cr(1+rv)+(1+4rv)^2}}{432c^2r^3/M}$$

$$\pi_t^* = \frac{(1+4rv)^3 - 12cr(1+5rv+4r^2v^2) + 24c^2r^2(2+2rv+9r^2v^2) - 64c^3r^3 + [16c^2r^2-8cr(1+rv)+(1+4rv)^2]^{3/2}}{864c^2r^3/M}$$

Table 3: Conditions in Propositions 1, 2 and 3

	Strategy F	**Strategy H**
Cost lower bound $\bar{c}^S(r)$	$\max\{\frac{(1+2a)rv+1}{8r}, \frac{rva+1}{4r(2-vr)}\}$	$\max\{\frac{5rv+1}{10r}, \frac{1+4rv-2r^2v^2}{r(16-14rv+3r^2v^2)}\}$.
Value of $\tilde{c}_1^S(r)$ in Period I	$\frac{1+a}{4r}$	$\frac{1+2rv}{r(2+3rv)}$
Value of $\tilde{c}_2^S(r)$ in Period II	$\frac{a}{2r(2a-1)}$ *Here needs* $a > 0.5$	$\frac{1+rv}{2r}$
$\hat{c}_w^S(r)$	$\frac{a}{2r(2a-1)}$ *Here needs a>0.5*	$\frac{2}{5r}$
$\hat{c}_p^S(r)$	$\frac{a(arv-1)}{2r(1-2a+rv)}$	*2nd root of G(c)**

*Where $G(c) = 8cr\left(-1+16cr+2rv-\sqrt{A}\right)\left(1-4cr+4rv+\sqrt{A}\right) + (1+4rv-4cr+12cr^2v+\sqrt{A})(1+4rv-28cr-12cr^2v+\sqrt{A})$ and $A = 4\,c\,r^2v + (1-4cr+4rv)^2$.

Proof of Observation 1.

For strategy F:

$q_1^{F*} \le 1 \Leftrightarrow c \ge \frac{(1+2a)rv+1}{8r}$; $rw_1^{F*} \le 1 \Leftrightarrow c \ge \frac{arv+1}{4r(2-rv)}$. Condition $r < 2/v$ is needed to ensure this condition to be positive.

Therefore, the condition is $r < 2/v$ and $c \ge \max\{\frac{(1+2a)rv+1}{8r}, \frac{arv+1}{4r(2-rv)}\}$

For strategy H:

$q_1^{H*} \le 1 \Leftrightarrow c \ge \frac{5rv+1}{10r}$;

$rw_1^{H*} \leq 1 \Leftrightarrow c \geq \frac{1+4rv-2r^2v^2}{r(2-rv)(8-3rv)}$. Condition $r < 2/v$ is needed to ensure this condition to be positive.

Therefore, the condition is $r < 2/v$ and c $\geq$ max$\{\frac{5rv+1}{10r}, \frac{1+4rv-2r^2v^2}{r(2-rv)(8-3rv)}\}$.

Proof of Proposition 1.

The proof of Proposition 1 is straightforward. Table 3 presents the values of $\tilde{c}_i^S(r)$ and $\bar{c}^S(r)$ that are obtained by comparing the optimal price and optimal warranty in each period within each strategy.

Proof of Proposition 2.

The comparison in Proposition 2(a) between the optimal qualities and the consumer quality perceptions, i.e., $q_1^{S*}$and rw_1^{S*}, is straightforward. The results are listed in Table 3. Proposition 2(b) is obvious.

Proof of Corollary 1.

Strategy H:

$\frac{\partial q_i}{\partial c} = \frac{-8vr^2(1+2a)}{(8cr-1)^2}$; $\frac{\partial w_1}{\partial c} = \frac{-4vr(1+2a)}{(8cr-1)^2}$; $\frac{\partial w_2}{\partial c} = \frac{-2vr(1+2a)}{(8cr-1)^2}$;

$\frac{\partial q_i}{\partial r} = \frac{-v(1+2a)}{(8cr-1)^2}$; $\frac{\partial w_1}{\partial r} = \frac{-4cv(1+2a)}{(8cr-1)^2}$; $\frac{\partial w_2}{\partial r} = \frac{-2cv(1+2a)}{(8cr-1)^2}$.

All the above values are negative.

Stategy F:

$\frac{\partial q_i}{\partial c} = \frac{4cr(1+rv)-(1+4rv)(1+4rv+\sqrt{16c^2r^2-8cr(1+rv)+(1+4rv)^2})}{12rc^2\sqrt{24cvr^2+(1-4cr+4rv)^2}}$, which is smaller than a negative value $\frac{-rv}{c\sqrt{24cvr^2+(1-4cr+4rv)^2}}$.

$\frac{\partial w_1}{\partial c} = \frac{4cr(1+rv)-(1+4rv)(1+4rv+\sqrt{16c^2r^2-8cr(1+rv)+(1+4rv)^2})}{24r^2c^2\sqrt{24cvr^2+(1-4cr+4rv)^2}}$, which is smaller than a negative value $\frac{-rv}{2c\sqrt{24cvr^2+(1-4cr+4rv)^2}}$.

$\frac{\partial w_2}{\partial c} = \frac{2}{3}(-1 + \frac{-1+4cr-rv}{\sqrt{24cr^2v+(1-4cr+4rv)^2}})$, which is negative.

$\frac{\partial q_i}{\partial r} = \frac{4cr-1-4rv-\sqrt{24cvr^2+(1-4cr+4rv)^2}}{12rc^2\sqrt{24cvr^2+(1-4cr+4rv)^2}}$, which is negative.

$\frac{\partial w_1}{\partial r} = \frac{-8c^2r^2+2cr(3+2rv+\sqrt{16c^2r^2-8cr(1+rv)+(1+4rv)^2})-(1+2rv)(1+4rv+\sqrt{16c^2r^2-8cr(1+rv)+(1+4rv)^2})}{12cr^3\sqrt{24cr^2v+(1-4cr+4rv)^2}}$,

which is smaller than a negative value $\frac{-v}{r\sqrt{24cvr^2+(1-4cr+4rv)^2}}$.

$\frac{\partial w_2}{\partial r} = \frac{-1+4cr-4rv-\sqrt{24cr^2v+(1-4cr+4rv)^2}}{6r^2\sqrt{24cr^2v+(1-4cr+4rv)^2}}$, which is negative.

Proof of Proposition 3.

The comparison in Proposition 3 between the optimal prices and warranties over the two periods is straightforward. The results are listed in Table 3.

Proof of Proposition 4.

$\pi_t^{F*} \geq \pi_t^{H*}$ when $a \geq \bar{a}_\pi$, *where*

$$\bar{a}_\pi = \frac{\sqrt{(8cr-1)\{-64c^3r^3+48c^2r^2(1+rv)+(1+4rv)^3-12cr(1+5rv+4r^2v^2)+[16c^2r^2-8cr(1+rv)+(1+4rv)^2]^{3/2}\}}}{12\sqrt{6}cr^2v} - \frac{1}{2}.$$

$\frac{\partial \pi_t^{F*}}{\partial a} = \frac{(1+2a)Mrv^2}{8cr-1}$, which is positive. Thus, π_t^{F*} increases in the valuation discount factor a.

Proof of Proposition 5.

$\pi_{q_i}^{F*} \geq \pi_{q_i}^{H*}$ when $a \geq \bar{a}_w$, and $\pi_{w_i}^{F*} \geq \pi_{w_i}^{H*}$ when $a \geq \bar{a}_w$, where $i = 1$ or 2 and $\bar{a}_w = \frac{(8cr-1)[1-4cr+4rv+\sqrt{24cr^2v+(1-4cr+4rv)^2}]}{24cr^2v} - \frac{1}{2}$.

Proof of Proposition 6.

$$CW^F = \frac{rv^2[(1-a)(1+2a)^2rv-(1+a-4cr)^2]}{(8cr-1)^2}.$$

$$CW^H = \frac{8cr\left(1+4rv+6r^2v^2\right)-(1+4rv)^2-8c^2r^2\left(2+6rv+9r^2v^2\right)+[4cr(1+3rv)-1-4rv]\sqrt{16c^2r^2-8cr(1+rv)+(1+4rv)^2}}{288c^2r^3}.$$

Appendix B

Extension: Heterogeneous Consumers

Our main manuscript assumes consumers are homogenous. We relax this assumption to account for heterogeneous consumers in this Appendix. We note that this extension is independent of the extension described in Section 5 of the manuscript, implying our extension to heterogeneous consumers does not assume the quality perception in the second period is depend on the second period warranty. Similar to Moorthy and Png (1992) and Lutz and Padmanabhan (1995, 1998), we model the heterogeneity in consumer valuation by considering two segments of consumers in Period I: the high segment and the low segment. The consumer valuation for the high and the low segments are denoted by v_h and v_l respectively, while the corresponding market sizes are n_h and n_l.

In Period I, given the price and warranty information, consumes derive quality perception of the product based on the warranty and the manufacturer's quality reputation. The expected utility for the high segment is:

$$EU_h = rw_1(v_h - p_1) + (1 - rw_1)(w_1 - p_1) \quad \text{(B1)}$$

The expected utility for the low segment is:

$$EU_l = rw_1(v_l - p_1) + (1 - rw_1)(w_1 - p_1) \quad \text{(B2)}$$

The firm, once again, could choose to either cover only the high segment market or cover the full market with both the high and low segments.

In Period II, consumers update their quality perception with q_1. The consumers who experienced product failure during Period I will discount their valuation to av_h or av_l. We then enumerate all the strategies of market coverage through two periods.

- If the manufacturer covers full market in Period I. Among high segment consumers, $q_1 n_h$ of them have not experienced product failure, and would hold the same valuation v_h in Period II. The rest, $(1 - q_1)n_h$, would discount their valuations to av_h. Among the low segment consumers, $q_1 n_l$ of them have not experienced product failure, and would hold the same valuation v_l in Period II. The rest, $(1 - q_1)n_l$, would discount their valuations to av_l. In Period II, the manufacturer has the following market coverage options:
  - The manufacturer sets the price to cover all the consumers, i.e., the high and the low segments. We call this strategy *TT*.
  - The manufacturer sets the price to cover the entire high segment including those have experienced product failure and those who have not, and the low segment who have not experience product failure. We call this strategy *TM2*.
  - The manufacturer sets the price to cover only the high segment consumers, including those who have experienced failure and those who have not. We call this strategy *TM1*.

- The manufacturer sets the price to cover only the high segment consumers who have not experienced failure. We call this strategy *TH*.

- If the manufacturer covers only the high segment in Period I. Among those consumers, $q_1 n_h$ of them have not experienced product failure would have the same valuation v_h in Period II. The rest, $(1 - q_1)n_h$, would discount their valuations to av_h. In Period II, the manufacturer has the following market coverage options:
  - The manufacturer sets the price to cover all the consumers. We call this strategy *HT*.
  - The manufacturer sets the price to cover the high segment consumer, including those have experienced failure and those have not. We call this strategy *HM*.
  - The manufacturer sets the price to cover only the high segment consumers who have not experienced failure. We call this strategy *HH*.

We next describe the maximization problem of the manufacturer subject to self-selection constraints under each market coverage strategy.

<u>Strategy *TT*</u>

$$\underset{p_1,p_2,q_1,q_2,w_1,w_2}{\text{Max}} \pi^{TT} = [p_1 - (1 - q_1)w_1 - cq_1^2](n_h + n_l) + [p_2 - (1 - q_2)w_2 - cq_2^2](n_h + n_l)$$

Subject to:

$$EU_1^l = rw_1(v_l - p_1) + (1 - rw_1)(w_1 - p_1)$$

$$EU_2^{al} = q_1(av_l - p_2) + (1 - q_1)(w_2 - p_2)$$

<u>Strategy *TM2*</u>

$$\underset{p_1,p_2,q_1,q_2,w_1,w_2}{\text{Max}} \pi^{TM2}$$

$$= [p_1 - (1 - q_1)w_1 - cq_1^2](n_h + n_l) + [p_2 - (1 - q_2)w_2 - cq_2^2](n_h + q_1 n_l)$$

Subject to:

$$EU_1^l = rw_1(v_l - p_1) + (1 - rw_1)(w_1 - p_1)$$

$$EU_2^l = q_1(v_l - p_2) + (1 - q_1)(w_2 - p_2)$$

<u>Strategy *TM1*</u>

$$\underset{p_1,p_2,q_1,q_2,w_1,w_2}{\text{Max}} \pi^{TM1} = [p_1 - (1 - q_1)w_1 - cq_1^2](n_h + n_l) + [p_2 - (1 - q_2)w_2 - cq_2^2]n_h$$

Subject to:

$$EU_1^l = rw_1(v_l - p_1) + (1 - rw_1)(w_1 - p_1)$$

$$EU_2^{ah} = q_1(av_h - p_2) + (1 - q_1)(w_2 - p_2)$$

<u>Strategy *TH*</u>

$$\underset{p_1,p_2,q_1,q_2,w_1,w_2}{\text{Max}} \pi^{TH} = [p_1 - (1 - q_1)w_1 - cq_1^2](n_h + n_l) + [p_2 - (1 - q_2)w_2 - cq_2^2](q_1 n_h)$$

Subject to:

$$EU_1^l = rw_1(v_l - p_1) + (1 - rw_1)(w_1 - p_1)$$

$$EU_2^h = q_1(v_h - p_2) + (1 - q_1)(w_2 - p_2)$$

<u>Strategy *HT*</u>

$$\underset{p_1,p_2,q_1,q_2,w_1,w_2}{\text{Max}} \pi^{HT} = [p_1 - (1 - q_1)w_1 - cq_1^2]n_h + [p_2 - (1 - q_2)w_2 - cq_2^2](n_h + n_l)$$

Subject to:

$$EU_1^h = rw_1(v_h - p_1) + (1 - rw_1)(w_1 - p_1)$$

$$EU_2^l = q_1(v_l - p_2) + (1 - q_1)(w_2 - p_2)$$

<u>Strategy *HM*</u>

$$\underset{p_1,p_2,q_1,q_2,w_1,w_2}{\text{Max}} \pi^{HM} = [p_1 - (1 - q_1)w_1 - cq_1^2]n_h + [p_2 - (1 - q_2)w_2 - cq_2^2]n_h$$

Subject to:

$$EU_1^h = rw_1(v_h - p_1) + (1 - rw_1)(w_1 - p_1)$$

$$EU_2^{ah} = q_1(av_h - p_2) + (1 - q_1)(w_2 - p_2)$$

<u>Strategy *HT*</u>

$$\underset{p_1,p_2,q_1,q_2,w_1,w_2}{\text{Max}} \pi^{HH} = [p_1 - (1 - q_1)w_1 - cq_1^2]n_h + [p_2 - (1 - q_2)w_2 - cq_2^2](q_1 n_h)$$

Subject to:

$$EU_1^h = rw_1(v_h - p_1) + (1 - rw_1)(w_1 - p_1)$$

$$EU_2^h = q_1(v_h - p_2) + (1 - q_1)(w_2 - p_2)$$

The Mathematica files containing the detailed solving procedure for each of the seven cases are available upon request.

To compare the heterogeneous consumer setting with the homogeneous one in the manuscript, we check for the validity of Propositions 1, 2 and 3 under heterogenous consumers.

We first compare the optimal warranty payment with the optimal price under heterogeneous consumer setting, and for all the strategies. We find the same results as in Proposition 1. That is, there exists a threshold value in cost parameter ($\tilde{c}$), such that when $c \geq \tilde{c}$, the optimal warranty payment is lower or equal to the optimal price.

We next compare the manufacturer's optimal quality and consumer quality perception in both Period I and Period II. For all the strategies under heterogeneous consumer setting, we find the same results as in Proposition 2. That is, there exists a threshold value in cost parameter ($\tilde{c}$), such that when $c \geq \tilde{c}$, the quality is lower than the consumer quality perception.

We then compare the optimal warranties and prices over two periods for each of the strategies. We find the same results under heterogeneous consumer setting as in Proposition 3.

To explore how market coverage affects the profitability of the manufacturer, we compare the profit under total market coverage in Period II with that under partial market coverage in Period

II. Specifically, we compare the profits between strategies HT and HM, and between strategies TT and TM1. In both of the comparisons, we find that, when the discount factor of consumer valuation is larger than a threshold value, total market coverage leads to higher profit. This result is consistent with Proposition 4.

To conclude, the heterogeneity in consumer valuation does not affect the major results in our model.