REGIONAL FIRM SIZE DIFFERENTIALS IN ANTEBELLUM UNITED STATES MANUFACTURING

by

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TABLE OF CONTENTS

I

INTRODUCTION

Various writers have asserted that inadequate demand and insufficient supplies of capital and labor prevented ante-bellum Southern manufacturing firms from taking advantage of economies of scale. The purpose of this study is to determine the validity of these assertions. Recent evidence on the profitability of plantation agriculture and the rate of growth of Southern per capita income has suggested that the smaller aggregate size of the Southern industrial sector was neither unwarranted nor detrimental. (*8*; *15*; *14*) However, this evidence does not explain why firms within the Southern industrial sector should have been any smaller than in other regions.

Regional manufacturing comparisons for 1860 are presented in Table 1. Whether size is defined by output value, value added, capitalization, or employment, the average Southern manufacturing firm was at best only one-half the size of the average Northern firm. However, the South compares quite favorably with the West. Despite the West's greater overall level of industrialization, the average size characteristics of Western firms were very similar to those in the South.

One could argue that this West-South comparison is the most relevant since both were predominantly agricultural regions with seminal manufacturing sectors relative to the North. However, the firm size issue is not traditionally framed in this way, and this is what makes the Southern case interesting. While ante-bellum Western firms were smaller than Northern ones, this is regarded as an expected characteristic of the industrial sector in a developing region. In such an economy the activities selected from the industrial spectrum would

Table 1

Regional Manufacturing Comparisons, 1860

Region	Number of Firms	Capital ($000)	Employees (000)	Total Output ($000)	Total Value Added ($000)	Capital Per Firm ($)	Employment Per Firm	Output Per Firm ($)	Value Added Per Firm ($)
Northern States	73,958	693	938	1,270,938	581,288	9,364	12.7	17,185	7,860
Western States	32,884	173	187	341,711	140,070	5,249	5.7	10,391	4,260
Southern States	24,081	116	132	193,463	84,624	4,827	5.5	8,034	3,514

Source: U.S. Census of Manufactures, 1860.

Northern States: Connecticut, Delaware, District of Columbia, Maine, Maryland, Massachusetts, New Hampshire, New Jersey, New York, Pennsylvania, Rhode Island, Vermont.

Western States: Illinois, Indiana, Iowa, Michigan, Minnesota, Missouri, Ohio, Wisconsin.

Southern States: Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, Tennessee, Texas, Virginia.

naturally be those in which efficiency could be achieved at a relatively small scale. Many writers do not accept this as a "natural" course of events for the South; instead, they regard the South as a case of stymied development. North-South comparisons pervade the literature, as if the size characteristics of Northern industry should have been realized, and would have been, were it not for a host of peculiarly Southern inhibiting factors.

The basic theory of the firm permits us to group these factors according to their influence on either the output or various input markets, since only market constraints would prevent a firm from taking advantage of economies of scale. If all markets, output and input, are perfectly competitive, the profit maximizing entrepreneur would never operate at output levels characterized by increasing returns to scale. Facing competitive factor and product markets, the firm can buy all its inputs and sell all its output at constant prices. Where increasing returns prevail, the firm can increase inputs by some multiple and thereby increase output by more than that multiple. At constant prices this would obviously bring an increase in profits, so the firm would expand output.

The situation described is one of falling average costs -- at constant input prices the multiple increase in inputs increases costs by the same multiple, but output is increased by an even greater multiple.[1] In terms of the usual graphical depiction of the determination of production levels, the U-shaped cost curve, derived under conditions of competitive input markets, has its minimum at the point where returns to scale are one. At lower output levels production is characterized by increasing returns, at higher output by decreasing returns. The firm facing a perfectly elastic demand curve will operate, if it does so at all, only at or to the right of the minimum, in the range of non-increasing returns.

An entrepreneur would choose to operate at a point of increasing returns only if less than competitive conditions in one or more of his markets justifies this choice. If a firm could sell more output only at lower prices or buy more inputs only at higher ones, it might find its profit maximum in the range of increasing returns. It would be lamentable for the regional economy if firms in any industry were in this situation. The very same resources already employed, distributed among fewer firms so they could take advantage of their economies of scale, would produce even more output. Or, alternatively, the existing output could be produced with fewer resources, efficiently allocated.

The historical literature contains abundant speculation concerning the problems Southern industrialists faced in their output, capital, and labor markets. An extensive, but probably not exhaustive, review of these is presented in Chapter II. As is frequently the case in historical literature, most of the arguments are descriptions of Southern economic conditions, rather than rigorously formulated hypotheses. A lament on the smaller size of Southern firms may not even mention returns to scale, but the inability to realize these is the reason that smaller firm size is injurious to the economy. [2] An author may blame some peculiarity in either the Southern output, capital or labor market for constraining firm size without specifically arguing that firms faced less than perfectly elastic conditions in that market. But only if firms were unable to sell output or obtain inputs at constant prices would they be prevented from taking advantage of economies of scale. For example, when Eugene Genovese says that limited "purchasing power" prevented "large-scale operations," he must mean that negatively sloped demand curves forced operation in the range of increasing returns. (25, p. 437)

More generally, some of the arguments assembled in Chapter II were not advanced in the context of the issue of firm size, but were meant to refer to the aggregate level of industrial activity. Douglass North cites the distribution of Southern income as accounting for the "lack of locally oriented industry." (35, pp. 94-95) On the other hand, Eugene Genovese blames this same factor for both a smaller sector and smaller firms. It is only the size of the industrial enterprise with which this thesis is concerned. However, the arguments in these chapters, even those originally advanced primarily in reference to aggregates, together present a picture of market restrictions so pervasive as to have influenced the firm's output decisiton as well as the regional economy's resource allocation decision.

In Chapter III a model is developed to investigate the issue of regional firm size. It has two distinguishing features which are fundamental to this issue in its Southern context. The production function exhibits returns to scale that vary over the level of output. If it did not, the notion of taking advantage of economies of scale by increasing firm size would have no meaning. And the model incorporates market conditions that allow for, but do not require, imperfections in the output, capital, and labor markets. If it did not, the model would not be internally consistent with a finding that Southern firms were operating in the range of increasing returns. If, indeed, Southern firms were operating predominantly in the range of increasing returns some market imperfection must have constrained firms to this output range. On the other hand, if Southern firms are discovered to have been operating in the range of non-increasing returns, then any market restrictions that did exist were not severe enough to have had the alleged effect. In any case, if market constraints are indicated I shall attempt to identify the particular market or markets responsible - the output, capital, or labor market.[3] Unfortunately, the model does not allow any more specific identification of the source of market imperfections -- for example, whether the distribution of Southern income was indeed responsible for an imperfection in the Southern output market, if this is discovered. It is hoped that subsequent work and the research of others will identify the particular sources of the constraint.

Chapter IV discusses the measurement of variables used in the model, and Chapter V presents the results for three industries -- lumber, flour and meal, and cotton and woolen textiles. The estimates are based on individual firm data collected from the 1860 manuscript census records. The lumber and grain milling industries are studied for all three regions -- South, North, and West, and the textile industry is studied for the South and North. The importance of these industries is revealed in Table 2, which ranks by value added the ten leading branches of manufacture in the United States and in each region. They were the three largest industries in value of output in the United States in 1860. By value added, textiles ranked first, lumber second, and flour and meal fifth. Their total value added constituted just over 20 per cent of the value added by all manufacturing. Most significantly, in the South these industries contributed 30 per cent of the value added by all manufacturing. Lumber ranked first, flour and meal third, and textiles fourth. In the North they contributed 18 per cent of the value added, and in the West the two industries studied there, lumber and flour and meal, ranked first and second, respectively, and contributed 24 per cent of Western value added. Thus, the industries studied represent a major portion of the manufacturing activity of each region, particularly in the South. The results should go far in addressing the issue of regional firm size differentials.

Table 2

The Leading Branches of Manufacture in 1860, By Value Added

A. United States

Industry	Capital ($000)	Employees	Value of Products ($000)	Value Added ($000)
Cotton and Woolen Goods	126,561	157,930	171,724	80,962
Lumber	76,643	75,595	104,928	53,570
Boots and Shoes	23,258	123,026	91,889	49,161
Clothing	28,668	120,539	88,012	40,539
Flour and Meal	84,585	27,682	248,580	40,083
Iron*	47,711	48,975	73,175	35,689
Machinery	36,031	41,233	52,010	32,566
Leather	38,908	26,145	75,318	25,784
Carriages and Wagons	18,724	37,102	35,553	23,655
Liquors	30,249	12,706	56,588	21,666
All Manufacturing	1,009,865	1,311,246	1,885,862	854,257

* cast, forged, rolled, and wrought.

B. The Southern States

Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, Tennessee, Texas, Virginia.

Industry	Capital ($000)	Employees	Value of Products ($000)	Value Added ($000)
Lumber	15,391	17,789	22,633	13,477
Tobacco, manufactured	7,532	15,827	18,020	7,127
Flour and Meal	16,044	6,849	44,788	7,021
Cotton and Woolen Goods	11,117	12,392	11,228	4,695
Machinery	5,848	5,120	6,791	4,587
Carriages and Wagons	3,611	6,842	6,282	4,498
Iron	4,459	4,435	6,241	3,064
Boots and Shoes	1,643	5,042	4,637	2,735
Leather	4,191	3,584	5,602	2,499
Turpentine, distilled	4,007	4,167	6,423	2,103
All Manufacturing	116,231	131,979	193,463	84,624

C. The Northern States

Connecticut, Delaware, District of Columbia, Maine, Maryland, Massachusetts, New Hampshire, New Jersey, New York, Pennsylvania, Rhode Island, Vermont.

Industry	Capital ($000)	Employees	Value of Products ($000)	Value Added ($000)
Cotton and Woolen Goods	112,870	142,230	156,736	74,504
Boots and Shoes	18,395	106,934	77,793	40,912
Clothing	22,740	99,074	71,142	32,261
Iron	36,634	37,529	54,926	26,351
Leather	30,500	19,245	62,844	20,350
Machinery	21,624	27,293	32,930	20,236
Lumber	30,618	28,288	42,028	17,610
Flour and Meal	34,785	10,156	95,649	14,294
Printing	15,659	14,863	24,682	14,070
Wagons and Carriages	10,778	21,819	21,038	13,527
All Manufacturing	692,540	938,079	1,270,938	581,288

D. The Western States

Illinois, Indiana, Iowa, Michigan, Minnesota, Missouri, Ohio, Wisconsin.

Industry	Capital ($000)	Employees	Value of Products ($000)	Value Added ($000)
Lumber	26,396	25,683	32,263	17,071
Flour and Meal	31,446	9,991	101,112	16,737
Liquors	12,159	5,159	24,173	9,555
Machinery	7,937	8,316	10,445	6,692
Agricultural Implements	5,474	6,541	8,194	5,806
Iron	6,194	6,650	10,933	5,774
Boots and Shoes	3,221	10,831	9,161	5,315
Clothing	3,440	14,543	10,554	5,164
Furniture	4,162	7,775	6,865	5,043
Carriages and Wagons	3,894	7,890	7,088	4,918
All Manufacturing	172,604	186,580	341,711	140,070

Source: <u>U. S. Census of Manufactures</u>, 1860.

NOTES

Chapter I

1. This particular statement is in general true only if the production function is homothetic. A transition has been made here from a returns to scale concept involving production to one involving costs. The former involves the relative increase in output that occurs as all input quantities are increased proportionately. The latter involves the increase in output relative to the increase in costs for variations along the expansion path (the set of cost minimizing input combinations at the constant input prices.) Hanoch (27) has shown that the two concepts yield equal measures of returns to scale at every minimum cost point in the input space. But he has also shown that the change in returns to scale with output is generally different for the two concepts "since expansion paths do not coincide with rays, unless the production function is homothetic."

It is the change in returns to scale along the expansion path which determines the shape of the average cost function. Returns to scale decreasing through a value of one along rays of equiproportional changes in inputs does not insure a U-shaped average cost function. The average cost function is U-shaped (and has a minimum where returns to scale are one) if and only if returns to scale decrease through one along the expansion path. Therefore, the remaining discussion in this paragraph in the text, concerning average costs and returns to scale, is entirely accurate and does not depend on homotheticity of the production function.

The fundamental point is that under perfectly competitive conditions, the profit maximizing firm would never operate in the range of increasing returns. On all these points, see Hanoch. (27)

2. This refers to the remark above that more resources than necessary would be used to produce the existing output. If there were "external economies" that would have accrued to the region from having larger firms, then of course these would be lost. This paper deals only with the technical issue of production.

3. This may not be possible since the conditions of three markets are described in the model by only two parameters. This will be discussed in detail in Chapter III.

II

THE ECONOMICS OF THE HISTORICAL LITERATURE ON SOUTHERN OUTPUT AND FACTOR MARKETS

THE OUTPUT MARKET

The arguments concerning the stifling of demand for industrial output are a lower Southern per capita income, the inequitable distribution of that income, a plantation oriented transport network and a lack of urbanization.

The Level of Per Capita Income

The low level of per capita income in the South is blamed for a low aggregate demand for manufactures. The South did have a per capita income below that in the Northeast, but if slaves are counted as intermediate goods rather than as consumers, it was equal to the national average. Engerman's reworking of Easterlin's income figures shows Southern per capita income in 1860 to have been either six or eight-tenths that of the Northeast, depending on how slaves are counted. However, regardless of the counting procedure, Southern per capita income exceeded that of the "North Central" region (the West). (*14*, pp. 85-87) Thus, relative income levels could explain a lesser demand for manufactures in the South than in the Northeast, but not relative to the West, nor possibly to the nation.

The Distribution of Income

Eugene Genovese and William Parker contend that the inequitable distribution of income resulting from slavery had effects on the size of Southern firms. Genovese emphasizes the inadequacy of demand in general and suggests an across the board effect on firm size. Parker focuses on the composition of demand, which he claims discouraged formation of medium-size firms.

Genovese argues that retardation of the home market "in itself provides an adequate explanation of the slave South's inability to industrialize." Further, "the root of the insufficient demand must be sought in the poverty of the rural majority composed of slaves, subsistence farmers, and poor whites." (*25*, pp. 423, 427) Fabian Linden makes this same point: "Adding to the difficulties of Southern industrialists was the inadequacy of the home market.... The slave system failed to produce a significant buying public." (*30*, p. 330) Genovese concludes that "plantation slavery, then, so limited the purchasing power of the South that it could not sustain much industry. That industry which could be raised usually lacked a home market of sufficient scope to permit large-scale operation; the resultant cost of production was often too high for success in competition with Northern firms drawing on much wider markets." (*25*, p. 437)

In his article on "The Specification Problem in Economic History," Robert Fogel has attempted to formulate Genovese's arguments as well-defined hypotheses. He states that the assertion that demand was insufficient "to permit large scale operations" implies that the supply functions of Southern industries had negative slopes. (*21*, pp. 283-289) This specification is still not entirely accurate. Since Genovese implies that demand was not perfectly elastic, a supply function per se has no meaning. Presumably, Fogel was referring to the slope of the cost curves, but his formulation is unnecessarily strong. These need not be everywhere negatively sloped. Genovese only implies that inadequate demand trapped Southern firms at output levels where average cost is negatively sloped -- where increasing returns to scale prevail. Decreasing returns may characterize higher output levels, but demand was not sufficiently elastic to allow Southern firms to attain these levels.

Figure 1 graphically depicts Genovese's argument concerning the influence of demand on firm size. Rather than simply depicting a Southern firm in isolation, a Northern firm is introduced to show how the "threat of Northern competition" might influence the Southern firm's price and output decision. The Northern firm is assumed to operate under perfectly competitive conditions and stands ready to sell its output in any market at price P_n plus unit distribution costs. The Southern firm operates as a local monopolist, facing a downward sloping demand in its local market. The "wider" Northern market is not available to the Southern producer.[1] Since Genovese considers demand an "adequate explanation" of smaller Southern firm size, we may assume input markets competitive, allowing the construction of cost curves. These curves are drawn identically for South and North, implying the same production function and input prices. We could relax these assumptions, but the possible outcomes would remain the same.

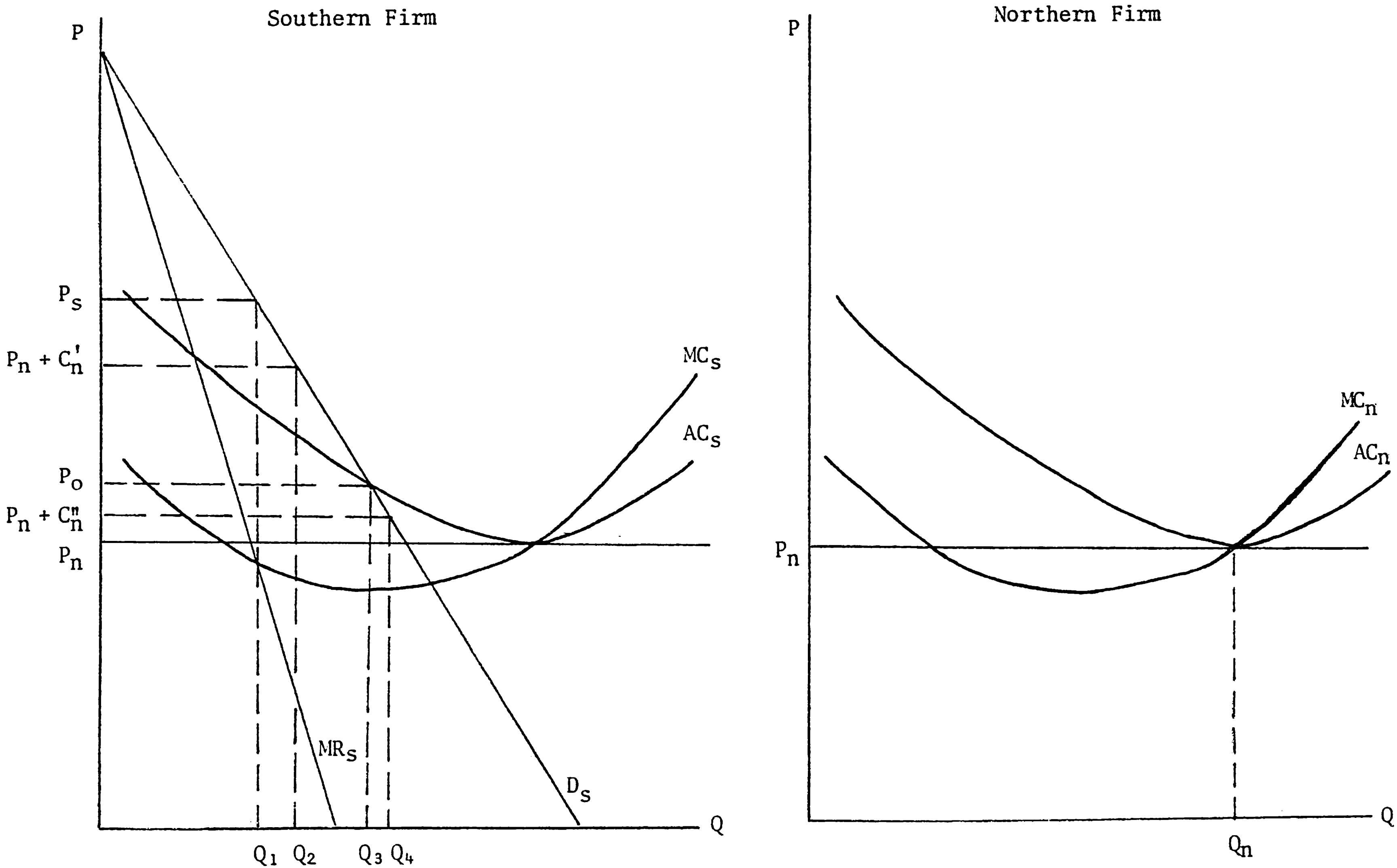
Let C_n represent the distribution cost for the Northern firm selling a unit of output in the South. Let Q_1 denote the Southern firm's profit maximizing output level "in isolation." Q_1 is determined, as usual, by the intersection of marginal revenue, MR_s, and marginal cost, MC_s, and can be sold at the price P_s. Let P_o and Q_3 denote the price and output, respectively, as the intersection of the Southern firm's demand and average cost curves, D_s and AC_s. If

(i) $P_n + C_n > P_s$, the Southern market belongs entirely to the Southern firm, which sells output Q_1 at price P_s.

(ii) $P_o < P_n + C_n < P_s$, the market will still be captured by the Southern firm since it will be willing to meet (or just beat) the price $P_n + C_n$. The effect of the Northern firm's willingness to sell at, for example, $P_n + C_n'$, is to face the Southern firm with kinked demand and marginal revenue curves. Up to and including output Q_2 these curves are both given by the horizontal line at $P_n + C_n'$, since an attempt to exact any higher price would invite Northern competition. Thereafter, the demand curve resumes its normal downward slope, and the marginal revenue curve continues with the downward portion of the normal MR_s curve to the right of Q_2. The Southern firm sells Q_2 (or just above) at price $P_n + C_n'$ (or just below) and still profits since $P_n + C_n' > AC_s(Q_2)$.

(iii) $P_n + C_n < P_o$, the Southern firm will be unable to compete. The Southern market is willing to absorb output exceeding Q_3 only at prices below average cost. This is Genovese's case in which "the [Southern] cost of production was ... too high for success in competition with Northern firms drawing on much wider markets." "Northern penetration of the Southern market [was possible] despite the costs of transportation." (25, p. 430) The Northern firm will sell output Q_4 at price $P_n + C_n''$ in the Southern market and will sell its remaining output, $Q_n - Q_4$, at price P_n in the

Figure 1


Southern Firm
P
P_s
$P_n + C_n'$
P_o
$P_n + C_n''$
P_n
MC_s
AC_s
MR_s
D_s
Q
Q_1 Q_2 Q_3 Q_4
Northern Firm
P
P_n
MC_n
AC_n
Q
Q_n

North. This firm's wider market allows it to achieve an output level (Q_n) at which price equals average cost.

Genovese alleges that the inability of Southern firms to compete, as described by case (iii), occurred "often." He specifically mentions agricultural implements and machinery, cotton textiles and boots and shoes. He claims the Southern market for sophisticated agricultural implements, quality boots and shoes, and fine textiles was simply too "narrow." In other words, the range of vulnerability to extinction by Northern competition, as represented by $P_o - P_n$ in Figure 1, was quite broad.

Even for those products for which Southern demand was large -- "crude agricultural implements, the cheapest and coarsest kind of cloth, and cheap shoes," the Southern market was penetrated by Northern firms. The North had what the South lacked: "the capital and skilled labor for fairly large-scale production." (25, pp. 430-432) If this greater factor scarcity meant only that Southern factor prices were higher, but still available in perfectly elastic supply, the cost curves illustrated in Figure 1 would be higher for the South. Again the range of vulnerability, $P_o - P_n$, is increased. So, too, is the chance that the imperfect output market would actually result in operation in the range of increasing returns. (In other words, the marginal revenue curve cuts the marginal curve cost curve more to the left.) However, by mentioning factor scarcity in conjunction with the scale of production, Genovese seems to posit that Southern factor markets, as well as the output market, were imperfect and played a role in restricting firms there to the range of increasing returns. Since factor prices would not then be constant over output levels the cost curve depiction of Figure 1 is not available. However, the effect is obvious: increased Southern vulnerability.

It has also been suggested that Southerners' lack of experience with manufacturing led to greater organizational inefficiency. (7, pp. 323-324) In production function analysis this could be reflected in a lower neutral technical coefficient for the South: the inability to transform the same inputs into as much output as the North. This would uniformly raise the cost curve of Figure 1. The range P_o - P_n would be larger, increasing the vulnerability to Northern competition. Also, at any given level of demand, output would be smaller than otherwise, increasing the chance that the profit maximizing output would indeed be in the range of increasing returns.

The present study cannot address or distinguish case (iii) in which Southern firms were excluded from a market. If Southern firms were unable to compete in an entire industry, no observations on firms would exist. Or, if they were unable to compete in accessible areas that afforded low enough transport costs to grant the market to Northern imports, those firms that did exist would be in either situation (i) or (ii). Industrial output was considerably more varied in the North than in the South and the per capita number of firms greater, and Genovese's argument is a conjecture as to the reason. We are interested in cases (i) and (ii) in which we should find, if Genovese is correct, smaller Southern firms trapped in the range of increasing returns.

William Parker argues that the Southern income distribution produced a "peculiar structure of demand" that discouraged formation of medium-sized firms, denying the Southern economy the externalities attending a broadly-based industrial sector.

> The structure of demand produced in fact a very mass market for simple items of plantation consumption -- notably coarse textiles, apparel and simple implements. At the other end of the spectrum, it produced the demand for many consumer goods of high quality. But in the early nineteenth century these were items in which a plantation could be self-sufficient (like the simpler forms of food processing and carpentry), which required higher degrees of skill and the prestige of foreign workmanship, or which became subject to factory methods on a large scale quite early in the period.
>
> What was missing from the Southern market was the demand for manufactures for which the technology permitted production on a modest local scale, in small shops scattered across the countryside. This was exactly the sort of demand which was present in the North -- first in the North East and the Middle Colonies, then in the Old Northwest -- and the sort of demand that would have been present in the South under a different form of agricultural organization and system of labor. (36, p. 117)

Using average firm figures from the published Census data, Parker classifies firms as small, medium, or large by a capital-employment definition. Comparing the number of firms in each category per 100,000 population, he concludes that the West (Ohio, Indiana, Michigan, Wisconsin, Illinois) had over three times the number of medium size firms per capita as the South (Georgia, Alabama, Mississippi, South Carolina). (36, p. 124)

Bateman and Weiss, using individual firm data underlying the published Census material, have demonstrated that Parker's use of averages seriously distorts the relative firm size distributions. Their results, presented in Table 3, show the West to have had only 60 per cent, 53 per cent, and 68 per cent more medium size firms per capita, depending on whether output, capital, or employment is used to define size. Futhermore, when a simple percentage distribution of firms among size categories is constructed, the Southern distribution is virtually identical to the Western, regardless of the classification criterion. They conclude that the West's "apparent advantage in the per capita measures resulted entirely from the greater absolute number of firms present in the area. ...If the South had had in total as many firms per capita it would have had about as many medium-size firms per 100,000 people as did the West, and in fact about as many as the East." (*4*, pp. 192-193)

While these latest results question Parker's evidence and hypothesis, it is a fact that the South (and the West) had more small and fewer large firms than the East. On average Southern firms were smaller, and Genovese's claim that too many Southern firms were trapped in an "uneconomic" output range remains to be investigated. Of course, some Western and Eastern firms were trapped there as well. And it remains true that Southern industrial output was considerably less varied than Eastern or Western.

Transportation and Urbanization

Various authors assert that a lack of urbanized areas and an inferior transportation network resulted in dispersed markets incapable

Table 3

Regional Size Distribution of Firms, 1860

A. Firms Per 100,000 Population

Manuscript Census Classifications

Firm Size Category	Output Definition			Capital Definition			Employment Definition		
	East	West	South	East	West	South	East	West	South
Small	64.2	61.0	45.5	151.7	133.5	85.9	216.0	198.3	136.4
Medium	322.7	251.7	156.6	326.9	213.5	141.4	221.8	148.7	88.4
Large	186.8	68.6	50.6	105.1	34.4	25.4	145.9	34.3	27.9

Published Census Classifications

(Parker Capital-Employment Definition)

	West	South
Small	20.7	53.9
Medium	344.0	102.8
Large	17.8	15.9

B. Percentage Distribution of Firms Among Size Categories

Manuscript Census Classifications

Firm Size Category	Output Definition			Capital Definition			Employment Definition		
	East	West	South	East	West	South	East	West	South
Small	11	16	18	26	35	34	37	52	54
Medium	57	66	62	56	56	56	38	39	35
Large	32	18	20	18	9	10	25	9	11

East: Delaware, Maryland, Massachusetts, New York.
West: Illinois, Indiana, Ohio, Wisconsin.
South: Alabama, Arkansas, Florida, Mississippi, North Carolina, South Carolina, Tennessee, Texas, Virginia.

Source: Manuscript Census Classification: Data from Fred Bateman and Thomas Weiss, "Comparative Regional Development in Antebellum Manufacturing," Explorations in Economic History, Vol.12, No. 3 (July, 1975), pp. 211-231. Percentage distribution derived using random samples from census manuscripts, firms per 100,000 population calculated by applying percentage distribution to aggregate number of firms as reported in published census as adjusted for population in region. Published census classification: Data from William N. Parker, "Slavery and Economic Development: An Hypothesis and Some Evidence," Agricultural History, Vol. XLIV, No. 1 (January, 1970), pp. 115-125. Size categories as follows: "Small" firms are those producing less than $1,000 annual output with less than $1,000 capital investment or with fewer than 5 employees. "Medium-sized" firms produced between $1,000 and $10,000 annual output, had $1,000 to $10,000 investment or between 5 and 10 employees. "Large" firms produced more than $10,000 output, had more than $10,000 invested capital, or more than 10 employees.

of supporting efficient size firms. Genovese observes that in 1860 the urban population of the lower South was only seven per cent of the total. This compares with "thirty-seven per cent in New England, thirty-five per cent in the Middle Atlantic states, including Ohio, and perhaps most significantly, with fourteen per cent in Indiana, Illinois, Michigan, and Wisconsin." (25, p. 436) Fishbaum and Rubin lay blame for the lack of urban areas squarely on slavery: "...slavery made plantations possible, these tended to concentrate in river valleys and to trade long-distance -- usually directly with the port cities -- consequently interior town development was inhibited." (16, p. 124) Genovese argues further that this inhibition was not only a matter of fact, but also a matter of design. Tied to the ports, planters saw little need for a unifying transport network. "The planters controlled the state legislatures in an era in which state participation was decisive in railroad construction and generally refused to assume the tax burden necessary to open the back country and thereby encourage and strengthen politically suspect farmers." (25, p. 429) Failure to absorb these non-staple producers into the market economy locked in the unequal distribution of income, leaving the South incapable of generating an adequate rural market.

The Debate over the Extent of the Southern Market

Even among historians not inclined to blame slavery for every ill, there are those who argue that inequality in the distribution of income, inferiority of the transport network, and a lack of urbanization were sufficient to stifle manufacturing in the South. In one of the early revisionist approaches to the slavery question, Robert Russel dismisses most of the slavery-based explanations for what he still accepts as Southern "industrial backwardness." He writes that

> ...population was comparatively sparse in the South, distances were great, and means of transportation poor. The poorer whites afforded little demand for manufactured goods. Neither did the slaves, but the masters, who exploited their labor, presumably compensated for them in this regard. So markets were too dispersed and inadequate to encourage large-scale manufacturing. (*40*, p. 47)

Many more recent historians continue to ascribe to this view. In both his familiar economic history texts, Douglass North emphasizes the port-connected transport mechanism and the "obviously very unequal" distribution of income as responsible for "the lack of urbanization and locally oriented industry." (*35*, p. 95; *34*, pp. 130-133)

Standing somewhat alone, Stanley Engerman has taken issue with each of the supposed causes of insufficient demand -- arguing that the distribution of income may not have been severely skewed, that the transport network was not as poor as suggested, and that economic growth should not be equated with urbanization. Using Genovese's own estimates of the purchasing power of slaves and free whites, he argues that the implied income distribution was no more skewed than that in Milwaukee in 1929, and suggests that notions of what constitutes a "severely skewed" distribution may need reappraisal. On the transport network, he observes that in 1860 "the South had 31 per cent of the nation's railroad mileage with per capita mileage only slightly below the national average.... While the track to area ratio was lower in the South than elsewhere, the southern economy was favored by a transport network based upon navigable streams and rivers." (*14*, pp. 88-90)

Engerman's discussion contrasts interestingly with Genovese's. In cautioning that the rate of growth of Southern income implies that growth should not be equated with urbanization, he notes that while the South compares "unfavorably" with the rest of the nation, "comparisons with the agricultural states of the North Central region, however, are less unfavorable to the South." As evidence, he cites the very same percentages of population in urban areas -- fourteen per cent in the North Central states, seven per cent in the South -- that Genovese found most telling of the South's failings in this regard. (*14*, p. 87; *25*, p. 436)

What was the actual extent of the Southern market? What did the South do with its export earnings, which totalled about one quarter of regional income? North quotes an ante-bellum observer who writes:

> The planters supply themselves with their own necessaries and luxuries of life directly through agents in the large towns, and comparatively little of the money drawn for the cotton crop is spent in the Southern States. Many of the planters spend their incomes by travelling with their families in the Northern States or in Europe during the summer, and a large sum is required to pay the hog-raiser in Ohio, the mule breeder in Kentucky, and above all the northern capitalists, who have vast sums of money on mortgage over the estates. (*34*, pp. 131-132)

This observation fits nicely with North's view of the South as an engine of growth for the entire country -- export earnings from cotton spent on Western foodstuffs and Northern manufactures.

This perception ignited much discussion of ante-bellum interregional trade flows, particularly the extent of the Southern demand for Western foodstuffs. Work by Albert Fishlow on observed trade flows and by Robert Gallman on the ability of Southern grain and meat production to meet requirements suggest that the importation of foodstuffs was not in fact a significant outlet of funds. (*17; 18; 19; 20; 24*) Engerman points out that the major potential outlets of export earnings were "1) purchases of foodstuffs; 2) purchases of manufactures; 3) capital exports to the North and Europe (either for new investments or to repay previous capital imports); and 4) payments for land purchases and for taxes to the federal government." He notes that "most indications are that (3) and (4) were quite small, if not negative and a source of inflow funds." (*13*, p. 129) If Fishlow and Gallman are correct about (1), this leaves primarily the purchase of manufactures.

Many pro-industrial Southerners echoed the lament of Hinton Helper:

> The North is the Mecca of our merchants, and to it they must and do make two pilgramages per annum -- one in the spring and one in the fall. All our commercial, mechanical, manufactural, and literary supplies come from there. We want Bibles, brooms, buckets and books, and we go to the North; we want pens, ink, paper, wafers and envelopes, and we go to the North; we want shoes, hats, handkerchiefs, umbrellas, and pocket knives, and we go to the North; we want furniture, crockery, glassware, and pianos, and we go to the North; we want toys, primers, school books, fashionable apparel, machinery, medicines, tombstones, and a thousand other things, and we go to the North for them all. Instead of keeping our money in circulation at home, by patronizing our own merchants, manufacturers, and laborers, we send it all away to the North, and there it remanins; it never falls into our hands again. (*28*, p. 22)

Engerman says, "We should remember that imports of manufactures from the North and Europe were frequently commented upon, and the suggestion of an adequate southern market, though one supplied by outsiders and not southerners, is frequently found. Many of the contemporary arguments about the establishment of manufactures in the South made sense only if the presence of adequate demand was presumed." (13, p. 129)

Summary -- The Output Market

The level of Southern per capita income and the supposed inequitable distribution of that income would certainly effect the aggregate level of industrial activity and the composition of demand. But these forces alone could hardly result in the individual firm facing a less than perfectly elastic demand. If the movement of goods were costless, efficient size firms could have been supported as long as the range of increasing returns was not so large nor the level of demand so small to prevent it. However, coupled with the issues of transportation and urbanization, these forces could become very real influences on firm size. Firms face the reality of the dispersion of demand across both persons and space. Genovese observes that "A country that is sparsely settled, in absolute terms, may have a high population density, in economic terms, if its system of transportation and commodity production are well developed and integrated." However, he argues that this was not true of the South. In fact, "the superiority of Northern transportation and economic integration" relative to the South, leads him to conclude that the inadequacy of Southern markets was even worse than that suggested by lower Southern population density figures. It is with this in mind that he argues that "that industry which could be raised usually lacked a home market of sufficient scope to permit large scale operation...." (25, pp. 428-429, 437) The counter arguments could be carried on further. Engerman might well argue that, just as the sale of Northern manufactures in the South constitutes evidence of an adequate market, the distribution of those imports constitutes evidence of adequate access to markets. The point, as far as we are concerned, is that the arguments of an imperfect output market restricting Southern firms to uneconomically small scale operation are of broad historical interest, have theoretical merit and require testing empirically.

THE CAPITAL MARKET

It has been claimed that securing capital for industry was as great a problem for Southern industrialists as was finding markets. Russel observes that whereas the North could rely on the profits of merchandising and shipping, as well as manufacturing, to generate capital for industry, the South could count only on its small industrial sector. The profits of agriculture were absorbed in its expansion. (*40*, pp. 48-49) Linden claims that "the planters reinvested profits in land and labor..., " leaving Southern industry dependent "for financial backing almost solely upon the initiative of scattered individuals." Echoing Genovese's claim of inefficient size firms, Linden argues that "the size, as well as the number of industrial enterprises, was limited by the scarcity of investors' capital. The scattered and unconcentrated quality of Southern industry handicapped it badly in competition with the North. To each establishment it meant concretely a relative increase in the cost of production." (*30*, p. 329)

Of course, if the reinvestment of funds in agricultural expansion was a rational course for the South, then the capital shortage perceived by pro-industrialists was not a serious problem. In Russel's view, "if the [industrial] sector had offered exceptional opportunities, capital and labor would have been diverted from agriculture or would have flowed from outside, but such was not the case." (*40*, p. 49) However, many writers disagree. Lance Davis says: "Much of the [South's] original accumulations had been in agriculture but despite declining returns in the 'Old South' after the 1830's, capital did not flow into more lucrative activities. Some agricultural capital did move into the 'West' (Alabama, Mississippi, and east Texas) but usually only when it was accompanied by its owner. Moreover, within the 'Old South' there was practically no transfer from agriculture to industry. Despite locational advantages, textiles were unable to attract external finance; instead funds continued to be reinvested in agriculture." (9, p. 93)

Was capital sufficiently mobile, as Russel suggests, to justify the idea that reinvestment in agriculture was evidence of that sector's greater profitability? Some writers have inferred this from the evidence on the profitability of slavery. Conrad and Meyer's original work showed that slave investments earned rates of return at least as high as selected alternative investments. For these comparisons they used the rates of return on commercial paper and municipal and railroad bonds. (8, pp. 101-103) An alternative they ignored and perhaps the one which should be used is the rate of return in manufacturing. A study by Bateman, Foust and Weiss found rates of return in Southern manufacturing in 1850 and 1860 to have been far above any other alternatives, slave agriculture included. (3; 5) While this evidence certainly suggests some capital immobility within the South, it is possible that other factors such as imperfect market conditions, inadequate information and greater risk account for some of the differential.

Suppose that the earlier assertions about the output market are true. The industrialist was limited by an inferior transport network to a local market in which he faced downward sloping demand due to the low level and inequitable distribution of income. The firm is a localized monopoly, making excess profit, which accounts for the authors' finding of a high rate of return. If the market limitation was severe enough, the entry of another firm in this market might drive the rate of return down precipitously -- below the market rate. Entry would then be uneconomic, and the differential would persist until the constraints were removed.

Bateman, Foust and Weiss dismiss this scenario as extremely unlikely.

> While this argument is possible it is not a very plausible one, at least not for all southern industries. Some southern industries were small, and the optimum size plant relatively large so that new entry may have appeared irrational, but many industries were large relative to the average size firm. In fact, in every southern industry the average output of a firm was equal to less than ½ of one percent (and in most cases less than 1/10 of one percent) of output. Clearly, the average industry could have absorbed a new entrant with no significant effect on industry supply, prices, or rates of return. (3, p. 228)

We cannot accept this dismissal since it could amount to a rejection of Genovese's assertions. The relation of average firm output to industry output is not conclusive evidence of opportunity for entry. In fact, if the localized market argument accurately characterizes conditions in an industry we would expect to find the very thing the authors suggest refutes that argument -- an industry composed of small producers.

Risk and risk preference undoubtedly accounted for some of the differential. Linden claims that both Southerners and, perhaps most unfortunately, Eastern business men attached a high risk element to the establishment of manufacturing firms in the South. (30, p. 328)

There are also several comments in the literature about the difficulty of infant industry in the South facing unrestricted competition from entrenched Northern concerns. (*25*, p. 430; *40*, p. 49) While risk may explain part, it is unlikely to explain all the sizeable differential between manufacturing and agricultural returns.

Market conditions, actual risk, and risk preference by lenders may each explain some or perhaps even all of the high rate of return in Southern manufacturing. However, that finding may also have been the result, in whole or in part, of capital immobility. Davis says such immobility will result if "savers are unwilling or unable to make their accumulations available to capital users whose activities yield the highest economic return." (*9*, p. 88) Historians have offered several reasons why Southern savers were both unwilling and unable to invest in manufacturing: agriculture was more familiar, the capital market was inefficient, and planters politically opposed industrialization. In addition to being immobile, it has also been argued that capital was unusually scarce in the South because certain characteristics of plantation agriculture, viz. capitalization of the labor supply, indebtedness, and the distribution of income -- militated against capital accumulation.

The Familiarity of Agriculture

Linden argues that "Planters, rather than look to new and precarious forms of investment, turned instinctively to the improvement and expansion of their plantations." (*30*, p. 328) Undoubtedly inertia is a very real economic force. Davis somewhat equivocally adds that the attraction of agriculture "almost certainly in part reflects the close ties that existed between land ownership and social position." (9, p. 97)

Inefficiency of the Capital Market

There is no doubt that the ante-bellum capital market was less developed in the South than in the Northeast and probably the West. In the latter nineteenth century, Southern and Western interest rates were higher than Eastern, and the regional gap closed more quickly for the West than for the South. (11) Innovative institutional changes and capital transfer mechanisms were slow to appear in the South. Davis notes that before the war most Southern states failed to adopt the free banking laws common in the North and West. He adds that "...the banks that were chartered tended to be dominated by the local landed gentry. Given the directors' personal biases and the absence of effective competition, it is not surprising that they tended to discriminate against industrial loans." (9, p. 100) Commercial paper made its first appearance in the East in the 1840's and had spread to encompass the mid-West by the 1870's, but it was the early twentieth century before this vehicle became common in the South. (12, pp. 351-352)

Planters' Political Opposition

Linden has charged that the planters, who molded public opinion, wielded political power, and controlled most of the available funds, regarded industrialization as inimical to the very slave system on which their livelihood was based. Consequently, many refused to invest in industry, presumably despite the incentive of greater returns, and carried their opposition into the political arena to keep the more profit-motivated in check. Linden says that state legislatures frequently denied articles of incorporation and the Southern journalistic establishment reflected planters' views. Immigration, unionization and protective tariffs were feared as inevitable results of industrialization. The European immigrants with their anti-slavery traditions would swell the abolitionists' ranks. Immigrant and native workers were sure to unionize and agitate against slavery since the exploitation of slaves represented competition at depressed wages. Linden points out that in North Carolina the Raleigh Workingmen's Association challenged an old revenue law which taxed mechanics' tools more heavily than slave property. Finally, the planters feared that Southern industrialists would join Northerners in demands for protective tariffs. (*30*, pp. 324-326)

The Capitalization of Labor

Ulrich Phillips was convinced that slavery, by capitalizing the labor supply, spelled doom for the Southern economy. He argued that investment in slaves absorbed precious funds, reducing other types of capital formation. Profits, "as fast as made," went into the purchase of additional labor. The planter "invested all his profits in a fictitious form of wealth and never accumulated a surplus for any other sort of investment." (*38*, p. 272) As Engerman and others have noted, "the crude version of this hypothesis is clearly wrong." After the cessation of the external slave trade in 1807, the purchase of slaves merely represented a transfer of funds among persons within the South. (*22*,

p. 336; *40*, p. 52; *8*, pp. 120-121)

However, John Moes has breathed some new life into this old argument. He points out that expenditures on children are considered consumption in a free society, but are considered investment in a slave society. Therefore, if the two societies have identical savings-income ratios, the slave society will necessarily invest less in non-human capital. (*31*) The critical question, of course, is whether Northern and Southern savings-income ratios were similar. Savings-income ratios are typically much higher for upper income groups, so the more skewed income distribution of a slave society might still generate greater capital formation. Engerman also points out that the ownership of slaves, by providing the collateral of a marketable asset, may have actually increased capital formation by increasing the ability to borrow. (*14*, p. 88)

Phillips had actually acknowleged that the flow of capital outside the South ceased with the external slave trade. But he argued that the "drain of wealth from the lower South was not checked at all, but merely diverted from England and New England to the upper tier of southern states; and there it did little but demoralize industry and postpone to a later generation the agricultural revival." (*38*, pp. 272-273) No evidence is offered to support this point, and it is very difficult to see how an infusion of funds could leave any region worse off. Moreover, the authors of two recent articles have suggested that the Old South did not gain, but may well have lost wealth as a result of Southern territorial expansion and the accompanying shift of slave population. (29, 37)

Phillips also argued that the capitalization of labor denied industry funds by locking capital in agriculture. In good times profits were invested in agricultural expansion. And in bad, the investment could not be liquidated. "With large amounts of capital invested in slaves, the system would be maintained even in times of depression, when planttations were running at something of a loss; for just as in a factory, the capital was fixed, and operations could not be stopped without still greater loss." In any event there was no one to sell to, for the planter's "neighbors were involved in the same difficulties which embarrassed him, and when he would sell they could not buy." Capital stayed put; good times followed bad, and the resources devoted to agriculture "tended always to increase." (38, pp. 259, 273-274)

However, there is certainly nothing special about Southern agriculture in its ownership of fixed factors and the difficulty of their liquidation in depressed times. It is indeed "just as in the factory." And there was nothing inevitable about capital remaining invested in slaves. There were certainly plenty of good times in which a planter could find a ready market for his slaves if he wished. Phillips merely disguises the claim that too many resources were devoted to plantation agriculture in a specious argument.

Indebtedness

Both Phillips and Russel, despite their conflicting views on slavery, bemoan the diminishment of planters' savings "by the almost universal practice of living and operating not upon the income from the preceding crop but upon the anticipated income from the next crop...." Supplies were bought on credit extended "in the last analysis by Northern or British firms," either by the cotton factor directly or local merchants who in turn had credit from their Northern suppliers. The payment of interest drained much needed funds from the region. (*40*, pp. 50-51; *38*, pp. 271-272) Engerman takes a different view. While it would certainly have been preferable for the interest payments to remain within the South, he says these are merely "the costs to be paid for profitable borrowing and speculation." Presumably such borrowing would raise, rather than lower, the growth rate of the Southern economy. (*14*, p. 89)

In a related point, Linden argues that this indebtedness inhibited industrialization because the liquidity required to support this purchase of goods on credit was often too great for capital-strapped Southern firms. (*30*, p. 329)

The Distribution of Income

The distribution of income resulting from plantation agriculture is also said to have retarded capital formation. The lion's share filled the pockets of profligate planters with a taste for the luxurious. The impoverished hoard at the bottom could provide no funds. And, according to Linden, "the South, unlike the North, had but a small middle class to which to sell its stock." (*30*, p. 328)

In argument, Engerman again points out that we have no suitable information on the actual distribution of income and that it may not have been "that" inequitable. In reference to planters' penchant for consuming, he notes that "large consumption expenditures do not imply low savings." In 1925, "the ratio of consumption per capita of the upper one percent to that of the rest of the population was 9:1; yet the upper income group had a savings-income ratio of 42.9 per cent, and accounted for fifty one per cent of personal savings." Finally, he raises the point, common in development literature, that savings may actually have been higher than they would have been with a less skewed income distribution. (*14*, pp. 88-89, 96)

Summary -- The Capital Market

It seems clear that capital was not perfectly mobile, either among regions of the country or between sectors of the Southern economy. Postbellum regional interest rate differentials and antebellum sectoral rate of return differentials are evidence of this. The inertial tendency of capital to remain in agriculture, an inefficient capital market, and the political opposition of planters are all possible causes of this immobility. The result was a shorter supply of capital for industry. The other factors which supposedly aggravated this shortage -- the capitalization of labor, planter indebtedness and the distribution of income -- could arguably have increased, rather than decreased, the supply of capital.

Even if all these factors are accepted as having caused capital to be in shorter supply, it does not necessarily follow that firms faced an imperfect capital market. Scarcity simply assures a non-zero price, perhaps higher in the South than elsewhere, and higher still for Southern industry, but it does not necessarily imply that firms had to pay prices that rose as they demanded more capital. A higher price of capital is not capable by itself (i.e. in the absence of imperfect conditions in other markets) of preventing firms from taking advantage of economies of scale. Only an imperfect market is. And it is the failure to take advantage of economies of scale that is the heart of the issue of firm size since this constitutes a waste of resources for the regional economy. Several historians obviously perceive a shortage of capital in the South that was serious and pervasive -- constraining firm size to the detriment of the Southern economy. To do justice to their arguments we must certainly allow for imperfection in the capital market.

It has been argued that the industrial labor supply of the South was constricted and of low quality for several reasons. It has been claimed that slave labor itself was unsuited to manufacturing. Where slavery was employed, it has been argued that a social stigma was attached to that employment and whites were discouraged from such pursuits. It has also been advanced that planter political opposition to manufacturing discouraged immigration and urbanization and that slavery discouraged investment in education for both black and white.

The Unsuitability of Slaves for Manufacturing

Of various arguments concerning the unsuitability of slave labor for manufacturing, the least sophisticated was that Negroes were inherently incompetent. Phillips says, "A slave among the Greeks or Romans was generally a relatively civilized person.... But the negro slave was a negro first, last, and always, and a slave incidentally." (*38*, p. 270) This remark was directed at the work of J. E. Cairnes, who writing over 75 years previously, had advanced a more enlightened view. Cairnes did conclude that a slave was so unskilled that it was "quite impossible that he should take part with efficiency in the difficult and delicate operations which most manufacturing and mechanical processes involve." But he attributed this to the begrudging manner in which slave labor was naturally given and to the low educational status to which slaves were condemned. (*6*, pp. 44-46, 70-72)

Cairnes also argued that slavery was not adaptable to the urban environment in which manufacturing increasingly located. The congregation of slaves in towns afforded too great a potential for insurrection. (*6*, pp. 70-71) Linden claims that slavery was too expensive and inflexible for manufacturing. To purchase slaves, the aspiring industrialist would face the need for a large, immediate capital outlay, "in the face of a constant insufficiency of funds," and such ownership of labor "would freeze Southern industry at the start, denying it the capacity to expand and contract with relative ease." And unfortunately slaves were only available for hire between the planting and picking seasons, thereby failing to provide a continuous labor supply and preventing the accumulation of industrial skill. (*30*, pp. 326-327)

More recently, Stanley Engerman and Robert Fogel claim these charges were untrue: "The course of slavery in the cities does not prove that slavery was incompatible with an industrial system or that slaves were unable to cope with an industrial regimen." On slave competence, they state that "slaves employed in industry compared favorably with free workers in diligence and efficiency," and they point to the use of slaves as artisans on plantations and in the hire market. They claim a healthy urban demand for slaves existed and emphasize Goldin's finding that this urban demand for slaves was elastic, in contrast with an inelastic rural demand for slaves. This they interpret as implying that slave and free labor were good subtitutes in an urban context. Finally, they assert that "the dictum ... that the ownership of men was incompatible with the shifting labor requirements of capitalist society, is without warrant in fact." As evidence they cite:

1) the shift in slave population from the Old to the New South in response to the latter's greater profitability in the production of cotton; 2) a vigorous hire market in slaves which found some 31 per cent of urban slave workers on hire during 1860; and 3) the shifting allocation of slave population between city and country in response to fluctuations in slave prices. Unfortunately, this evidence does not relate to the industrial experience with enough specificity to be convincing. (*23*, pp. 38-43, 56-58, 97-102, 235)

Fogel and Engerman also do not deal explicitly with Linden's claim that slave hire could not provide a steady labor supply to factories. In fact, they note that "hire contracts rarely ran for more than a year, and many were for substantially shorter periods of time." These contracts were suitable to "industrial firms which experienced sharp cyclical or seasonal fluctuations in business." (*23*, pp. 56-57) Robert Evans observes that while the characteristics of hired slave employment cannot be quantified, "the practice of yearly re-hire suggests that there may have been a high turnover rate of individual slaves among employers." (*15*, pp. 192-193) If asked, Fogel and Engerman would probably argue that Linden's claim is nonsense, since slaves would have been hired for the entire year or longer if it were profitable to have done so.

The Supply of Free Labor

Linden argues that the institution of slavery worked to inhibit the supply of free workers. The lack of opportunity had reduced the "poor whites" to a "backward, sickly people, unskilled in any craft and difficult to train." Slavery lowered the standard of living for all Southern labor and a social stigma was attached to wage employment and even to physical labor itself. "Non-slaveholding farmers ... were reluctant to give up independence to sink to the level of 'hired' help." Immigrants, too, were "as reluctant to accept social degradation in a backward system as they were to accept a standard of living set for slaves." The result was a shortage of skilled labor. (*30*, pp. 327-328)

Russel takes issue with the supposed Southern contempt for physical labor. While slavery did cause Southern whites to shun menial tasks, "there was no stigma attached ... to the performance of manual labor." He claims that whites, free Negroes, and slaves worked side by side, with only an evident distinction in tasks required to content white workers. (*40*, pp. 38-39)

Russel also devotes considerable attention to the question of whether or not slavery discouraged emigration to the South. He concludes that "No doubt thousands of individuals were deterred from going South by race prejudice, dislike of slavery, or a disinclination to compete with slaves for jobs. But, since so many others were undeterred by such motives and considerations, it is reasonable to suppose that, if economic opportunities had been great enough, people would have come in greater numbers from the North and from Europe to seize upon them." (*40*, p. 44)

Planter Political Opposition

Planters' hostility to manufacturing, discussed above in connection with the capital market, is also blamed for restricting the labor supply. The dearth of manufacturing discouraged urbanization with its attendant concentration of an available labor force.

Education

Finally, slavery, as an institution, is blamed for discouraging investment in education for both slave and free workers. Whatever the cause, it is true that the South did not compare favorably with the rest of the nation. The education of slaves was forbidden by law. (*14*, p. 91) And in 1840 the ratio of pupils to free white population was 18.41 per cent in non-slaveholding states, but a meager 5.72 per cent in slave states. If Maryland, Delaware, Kentucky, and Missouri had been excluded from the latter, the comparison would be even less favorable. (*34*, p. 133) Relative to its population, the South would not have been able to marshall as large a formally educated labor supply as other regions. Whether 19th Century education improved the skill level of workers is uncertain, and if it did, whether the low level of education in the South was an effective constraint on manufacturing is an open question.

Summary -- The Labor Market

The adaptability of slaves to manufacturing and to an urban environment has nothing itself to do with whether industrial firms could obtain labor in a perfectly elastic supply. The firms would simply have to draw from the free labor pool, as Northern and Western firms did. The relevance of slavery to the supply of labor depends on whether slavery discouraged native population growth, immigration, or the willingness of free persons to enter industrial employment. The less urbanized character of the South certainly disadvantaged the South relative to other regions in securing whatever gains in labor market efficiency accrue from the concentration of a labor supply. Also, educated workers were certainly relatively more scarce in the South. If there were industries in which education was an 'if and only if' requirement, this shortage may have been serious. In others, if it meant anything at all, it may only have meant that Southern workers were less productive, but not that they were not perfectly elastic in supply.

As with the observations on the capital market, these concerning the labor market argue that labor was more scarce in the South than in other regions. Again, whether this scarcity was pervasive enough to effect an individual firm is the question. The available industrial labor force in the South was certainly smaller than in other regions. The South was less urbanized, less densely populated even in rural areas, and more engaged agriculturally. If the market for any factor of production is localized, certainly it is the labor market. Inducing the movement of persons may have been more difficult than inducing the movement of capital.

NOTES

Chapter II

1. Genovese argues that, "The Northeast had the capital and skilled labor for fairly large-scale production and had established its control over existing markets in the North and West. Southern manufacturers could not hope to compete with Northern outside the South...." (*25*, p. 430)

2. Russel also admits there were times when "labor and capital might more profitably have been directed into other channels." But these times were the consequence of a fall in revenue when large crops met inelastic demand. To Russel this was quite simply "the speculative character of commercial agriculture.... Occasional overproduction was preferable to consistent underproduction." (*40*, pp. 52-53)

III

THE MODEL

THE MODEL

A model of production capable of investigating the charge that Southern firms were prevented from taking advantage of economies of scale because of their small size must have two very fundamental characteristics: 1) a production function exhibiting returns to scale that vary over output, and 2) profit maximizing conditions that allow for market imperfections.

Homogeneous production functions, such as the familiar Cobb-Douglas and CES functions, are analytically useless in this case. They dictate that returns to scale are the same at all output levels, so they would not reveal whether or not advantages of economies of scale would accompany an increase in firm size. They would show that one and only one of either decreasing, constant, or increasing returns to scale existed at all output levels. If we were to estimate a homogeneous function, we could not claim that a finding of decreasing returns to scale refutes the hypothesis. Increasing returns may still have actually existed at some output levels, but our chosen form would be incapable of revealing this. A proper test of the hypothesis requires that we determine whether returns to scale varied with output and whether existing firms actually were so small that they were operating in the range of increasing returns. This is not to say that we must force non-homogeneity on the model -- only that we must not force homogeneity. The production function I have chosen to estimate allows returns to scale to either vary over output or remain the same at all outputs, depending on which best describes production.

The model must also allow for imperfections in the product and factor markets. Otherwise, it would be incapable of explaining why a rational, profit-maximizing firm would produce at a point of increasing returns. Operation at output levels characterized by increasing returns is fundamentally inconsistent with a competitive market structure. (27, p. 495) This represents an additional drawback to the use of homogeneous functions where increasing returns are suspected. If they are discovered and the production function is homogeneous, the implication must be that a competitive structure for that industry cannot be achieved without a change in technology. On the other hand, if the production function is non-homogeneous, profit maximizing firms could operate with increasing returns to scale, and yet removal of the market imperfection would allow firms to move into the non-increasing returns range.

Zellner and Revankar have shown that a generalized production function exhibiting variable returns to scale can be derived from any of the familiar homogeneous forms. (44) One such function they derive, and which we shall use, is a generalized form of the Cobb-Douglas function:

$$Ye^{\theta Y} = \Gamma K^{\alpha_K} L^{\alpha_L}$$

where

Y: output

K: capital

L: labor

and $\Gamma > 0$, $\alpha_K > 0$, $\alpha_L > 0$, $-\frac{1}{Y} < \theta < \infty$

These parameter restrictions are necessary to insure that the marginal products are positive. The marginal products are:

$$MP_K = \frac{\alpha_K Y}{K(1+\theta Y)} \qquad \text{and} \qquad MP_L = \frac{\alpha_L Y}{L(1+\theta Y)}$$

The returns to scale function, which we will denote $\alpha(Y)$, is defined as the elasticity of output with respect to proportionate changes in the inputs, capital and labor.

$$\alpha(Y) = \left.\frac{\partial Y}{\partial \lambda}\frac{\lambda}{Y}\right|_{\lambda=1}$$ where λ denotes the factor by which inputs are changed.

The function $\alpha(Y)$ satisfies what is sometimes called the generalized Euler equation:

$$K\frac{\partial Y}{\partial K} + L\frac{\partial Y}{\partial L} = \alpha(Y)Y. \qquad (27, \text{ p. } 493)$$

For our production function we find $\frac{\partial Y}{\partial \lambda}$ by taking the differential of

$$Ye^{\theta Y} = \Gamma(\lambda K)^{\alpha_K}(\lambda L)^{\alpha_L} .$$

We have:

$$(1+\theta Y)e^{\theta Y}dY = (\alpha_K + \alpha_L)\lambda^{\alpha_K+\alpha_L-1}\,\Gamma K^{\alpha_K} L^{\alpha_L}\, d\lambda.$$

Since

$$\Gamma K^{\alpha_K} L^{\alpha_L} = Ye^{\theta Y}\lambda^{-(\alpha_K+\alpha_L)} ,$$

we have

$$(1+\theta Y)e^{\theta Y}dY = (\alpha_K + \alpha_L)\lambda^{-1}Ye^{\theta Y}d\lambda$$

$$\frac{\partial Y}{\partial \lambda} = \frac{(\alpha_K + \alpha_L)Y}{(1 + \theta Y)\lambda} .$$

Therefore the returns to scale function is

$$\alpha(Y) = \left. \frac{\partial Y}{\partial \lambda} \frac{\lambda}{Y} \right|_{\lambda=1} = \frac{\alpha_K + \alpha_L}{1 + \theta Y} .$$

The behavior of returns to scale with respect to output depends on the parameter θ.

a) If $\theta > 0$, returns to scale fall from $\alpha_K + \alpha_L$ at zero output, to zero as $Y \to \infty$. Whether or not there actually is a range of increasing returns depends on whether $\alpha_K + \alpha_L > 1$ or $\alpha_K + \alpha_L \leqq 1$.

b) If $\theta = 0$, the function reduces to the familiar homogeneous Cobb-Douglas function:
$$Y = \Gamma K^{\alpha_K} L^{\alpha_L}$$
Returns to scale are $\alpha_K + \alpha_L$ at all output levels.

c) If $\theta < 0$, returns to scale increase from $\alpha_K + \alpha_L$ at $Y = 0$ to approach infinity as $Y \to -1/\theta$. This behavior of the returns to scale function is contrary to accepted notions of production technologies, but it is a possible statistical outcome since $\theta \geqq 0$ is not imposed in the estimation procedure.[1]

In conformance with the hypothesis, the function allows variable returns to scale to emerge, if they exist, but does not insist on it. If they do vary over output, there still may or may not be a range of increasing returns. Therefore, the functional form is biased neither for nor against the hypothesis, but is completely neutral with respect to its assertions.

With the subscript i added to indicate the observations on individual firms, and random disturbances introduced and assumed independent over firms, the production function is:

$$1)\quad Y_i e^{\theta Y_i} = \Gamma K_i^{\alpha_K} L_i^{\alpha_L} u_{i0}^* \qquad i = 1, \ldots , n$$

where $u_{i0} = \ell n\ u_{i0}^*$; $i = 1, \ldots , n$

are independently distributed $N(0, \sigma_{00})$.

The manner in which the profit maximizing conditions enter the model depends on the interpretation of the randomness in production represented by u_{i0}^*. Because of the variablility of machine performance, the weather, management expertise and the like, the output realized by the firms was not an exact function of inputs. One alternative is to assume that each entrepreneur knew precisely how these various factors influenced his production, (i.e. he knew u_{i0}^* exactly), and would employ this information in choosing his profit maximizing input combination. This is referred to as the maximization of actual profits. Another alternative is to assume that the entrepreneur did not know how these things affected his output, and decided it was best to choose inputs as if he could hit production squarely on target. This view, referred to as maximization of anticipated profits, was suggested by Mundlack and Hoch. (32) [2] It is based on two defined properties of the anticipated operator, A: 1) $A(u_{i0}^*) = 1$ and 2) $A[f(u_{i0}^*)] = f[A(u_{i0}^*)]$.

Reality, as usual, probably falls somewhere in between, since many factors are involved in u^*_{io}. But, of the two alternatives, the latter seems more realistic, so we assume that firms maximized anticipated profits, $A(\pi_i)$:

$$A(\pi_i) = A(P_{iY}Y_i - P_{iK}K_i - P_{iL}L_i)$$

Since the inputs, K and L, are choice variables, there is no "anticipation" involved in their values, and

$$A(\pi_i) = A(P_{iY}Y_i) - P_{iK}K_i - P_{iL}L_i$$

Employing the properties of the A operator, anticipated output, $A(Y_i)$, is given by

$$A(Y_i e^{\theta Y_i}) = A(\Gamma K_i^{\alpha_K} L_i^{\alpha_L} u^*_{io})$$

$$A(Y_i)e^{\theta A(Y_i)} = \Gamma K_i^{\alpha_K} L_i^{\alpha_L} A(u^*_{io})$$

2) $$A(Y_i)e^{\theta A(Y_i)} = \Gamma K_i^{\alpha_K} L_i^{\alpha_L}$$

Although production is in fact stochastic, u^*_{io} does not enter the input choice decision, since entrepreneurs base that decision on anticipated output defined by the known, non-stochastic part of the production function.

Before we actually derive the profit maximizing conditions, we must allow for market imperfections. Frequently in the production function literature, the profit maximizing conditions for a production function $Y = f(K,L)$ are written:

$$\frac{\partial f}{\partial K} = R_K \frac{P_K}{P_Y}$$

$$\frac{\partial f}{\partial L} = R_L \frac{P_L}{P_Y}$$

where P_Y, P_K and P_L denote the prices of output, capital and labor, respectively.

Marc Nerlove has shown that if certain output demand and input supply functions prevail (given directly below), then R_K and R_L may be interpreted as indicators of market conditions. (33, pp. 8-11)

Assume that the output demand and factor supply functions depend only on own-price and are log-linear of the form:

$$Y_i = b'_Y P_{iY}^{\varepsilon_Y} \quad \text{output demand}$$

3) $$K_i = b'_K P_{iK}^{\varepsilon_K} \quad \text{capital supply}$$

$$L_i = b'_L P_{iL}^{\varepsilon_L} \quad \text{labor supply}$$

These are constant elasticity functions, where ε_Y, ε_K, and ε_L are the elasticities of demand, capital supply and labor supply, respectively.

From equations 3) :

$$P_{iY} = \left[\frac{1}{b'_Y} Y_i\right]^{1/\varepsilon_Y}$$

$$P_{iK} = \left[\frac{1}{b'_K} K_i\right]^{1/\varepsilon_K}$$

$$P_{iL} = \left[\frac{1}{b'_L} L_i\right]^{1/\varepsilon_L}$$

Therefore

$$P_{iY}Y_i = \left[\frac{1}{b'_Y}\right]^{1/\varepsilon_Y} Y_i^{\,1+1/\varepsilon_Y}$$

$$P_{iK}K_i = \left[\frac{1}{b'_K}\right]^{1/\varepsilon_K} K_i^{\,1+1/\varepsilon_K}$$

$$P_{iL}L_i = \left[\frac{1}{b'_L}\right]^{1/\varepsilon_L} L_i^{\,1+1/\varepsilon_L}$$

Simplifying the notation and introducing disturbance terms we have:

4)

a) $$P_{iY}Y_i = b_Y Y_i^{\beta_Y} v^*_{i0}$$

b) $$P_{iK}K_i = b_K K_i^{\beta_K} v^*_{i1}$$

c) $$P_{iL}L_i = b_L L_i^{\beta_L} v^*_{i2}$$

where $b_Y = \left[\frac{1}{b'_Y}\right]^{1/\varepsilon_Y}$ $\quad b_K = \left[\frac{1}{b'_K}\right]^{1/\varepsilon_K}$ $\quad b_L = \left[\frac{1}{b'_L}\right]^{1/\varepsilon_L}$

$$\beta_Y = 1 + \frac{1}{\varepsilon_Y} \qquad \beta_K = 1 + \frac{1}{\varepsilon_K} \qquad \beta_L = 1 + \frac{1}{\varepsilon_L}$$

Note that if the output market was competitive, the elasticity of demand, ε_Y, is infinite and $\beta_Y = 1$. Similarly, if the input markets were competitive, the supply elasticities are infinite and $\beta_K = 1$, $\beta_L = 1$.

The logs of v^*_{i0}, v^*_{i1}, v^*_{i2} are denoted v_{i0}, v_{i1}, v_{i2} and are assumed to be normally distributed with means zero, finite variances, and independent of u_{i0}. The introduction of disturbances in this manner means we assume the levels of the output demand and factor supply curves differed from firm to firm, but the elasticities (and hence β_Y, β_K, β_L) are assumed not to have differed. (33, pp. 20-21) The independence of these market disturbances from u_{i0} simply says that those factors which cause output demand and input supply to differ among firms (e.g. locational differences in population density, income levels, tastes, labor force competition, financial infrastructure) are not related to those factors which cause variability in the conversion of inputs into output (e.g. machine performance, management expertise).

The entrepreneur is assumed to have maximized anticipated profit

$$A(\pi_i) = A(P_{iY}Y_i) - P_{iK}K_i - P_{iL}L_i$$

which, from (4), is

$$A(\pi_i) = b_Y[\bar{A}(Y_i)]^{\beta_Y} v^*_{i0} - b_K K_i^{\beta_K} v^*_{i1} - b_L L_i^{\beta_L} v^*_{i2}$$

subject to the production constraint, (2).

It is assumed, as evidenced by the appearance of v^*_{i0}, v^*_{i1}, and v^*_{i2} in the anticipated profit function, that each firm knew the particular output demand and input supply conditions it faced.

The Lagrangian is:

$$Z = b_Y[\bar{A}(Y_i)]^{\beta_Y} v^*_{i0} - b_K K_i^{\beta_K} v^*_{i1} - b_L L_i^{\beta_L} v^*_{i2} + \lambda[\bar{A}(Y_i)e^{\theta A(Y_i)} - \Gamma K_i^{\alpha_K} L_i^{\alpha_L}]$$

The necessary conditions for an interior maximum are:

$$\text{5)}\quad \begin{aligned}
&\text{a)}\quad \frac{\partial Z}{\partial \lambda} = A(Y_i)e^{\theta A(Y_i)} - \Gamma K_i^{\alpha_K} L_i^{\alpha_L} = 0\\
&\text{b)}\quad \frac{\partial Z}{\partial Y_i} = b_Y\beta_Y[A(Y_i)]^{\beta_Y-1} v_{i0}^{*} + \lambda e^{\theta A(Y_i)}[1 + \theta A(Y_i)] = 0\\
&\text{c)}\quad \frac{\partial Z}{\partial K_i} = -b_K\beta_K K_i^{\beta_K-1} v_{i1}^{*} - \lambda\alpha_K \Gamma K_i^{\alpha_K-1} L_i^{\alpha_L} = 0\\
&\text{d)}\quad \frac{\partial Z}{\partial L_i} = -b_L\beta_L L_i^{\beta_L-1} v_{i2}^{*} - \lambda\alpha_L \Gamma K_i^{\alpha_K} L_i^{\alpha_L-1} = 0
\end{aligned}$$

The second order condition for an anticipated profit maximum is

$$[\alpha_K\beta_L + \alpha_L\beta_K]\{\beta_Y[1+\theta A(Y_i)] - \theta A(Y_i)\} < \beta_K\beta_L[1+\theta A(Y_i)]^2 .$$ [3]

From 5a) we have

$$\text{5a')}\quad \Gamma K_i^{\alpha_K-1} L_i^{\alpha_L} = \frac{A(Y_i)e^{\theta A(Y_i)}}{K_i}$$

and

$$\Gamma K_i^{\alpha_K} L_i^{\alpha_L-1} = \frac{A(Y_i)e^{\theta A(Y_i)}}{L_i}$$

Solving 5b) for λ yields:

$$\text{5b')}\quad \lambda = \frac{-b_Y\beta_Y[A(Y_i)]^{\beta_Y-1} v_{i0}^{*}}{e^{\theta A(Y_i)}[1+\theta A(Y_i)]}$$

Using 5a'), 5b'), 4b), and 4c) we may re-write 5c) and 5d) as:

$$\text{c')}\quad \beta_K P_{iK} K_i - \frac{\alpha_K \beta_Y b_Y [A(Y_i)]^{\beta_Y}}{[1+\theta A(Y_i)]} v^*_{i0} = 0$$

5)

$$\text{d')}\quad \beta_L P_{iL} L_i - \frac{\alpha_L \beta_Y b_Y [A(Y_i)]^{\beta_Y}}{[1+\theta A(Y_i)]} v^*_{i0} = 0$$

Note that these are simply the familiar profit maximizing conditions equating the marginal cost of an input to its marginal revenue product. For example, consider the capital input, K. Its marginal cost, MC_K, is obtained from 4b) :

$$MC_K = \frac{\partial(P_{iK}K_i)}{\partial K_i} = \beta_K b_K K_i^{\beta_K - 1} v^*_{i1}$$

The marginal product of capital at the anticipated output is:

$$MP_K = \frac{\partial A(Y_i)}{\partial K_i} = \frac{\alpha_K A(Y_i)}{K_i [1+\theta A(Y_i)]}$$

Marginal revenue, MR, at the anticipated output level is obtained from 4a) :

$$MR = \frac{\partial A(P_{iY}Y_i)}{\partial A(Y_i)} = \beta_Y b_Y [A(Y_i)]^{\beta_Y - 1} v^*_{i0}$$

Equating $MC_K = (MP_K)(MR)$ yields:

$$\beta_K b_K K_i^{\beta_K - 1} v^*_{i1} = \frac{\alpha_K \beta_Y b_Y [A(Y_i)]^{\beta_Y} v^*_{i0}}{K_i [1+\theta A(Y_i)]}$$

or

$$\beta_K b_K K_i^{\beta_K} v_{i1}^* = \frac{\alpha_K \beta_Y b_Y [A(Y_i)]^{\beta_Y}}{[1+\theta A(Y_i)]} v_{i0}^*$$

Since $P_{iK}K_i = b_K K_i^{\beta_K} v_{i1}^*$, we have

$$\beta_K P_{iK} K_i = \frac{\alpha_K \beta_Y b_Y [A(Y_i)]^{\beta_Y}}{[1+\theta A(Y_i)]} v_{i0}^*$$

This is condition 5c'). The same can be shown for the labor input, L.

Returning to 5c') and 5d'), we substitute $A(P_{iY}Y_i) = b_Y A(Y_i)^{\beta_Y} v_{i0}^*$ from 4a) and introduce the stochastic terms u_{i1}^* and u_{i2}^*. We have

6)

$$\frac{\alpha_K A(P_{iY}Y_i)}{P_{iK}K_i[1+\theta A(Y_i)]} = R_K u_{i1}^*$$

$$\frac{\alpha_L A(P_{iY}Y_i)}{P_{iL}L_i[1+\theta A(Y_i)]} = R_L u_{i2}^*$$

where $R_K = \frac{\beta_K}{\beta_Y}$

$R_L = \frac{\beta_L}{\beta_Y}$

The disturbance terms u_{i1}^* and u_{i2}^* are introduced to account for the fact that the profit maximizing conditions will not be exactly met. Their logs, denoted u_{i1}, u_{i2} are assumed normally distributed, independent over firms, with means zero and positive definite covariance matrix Σ_{11}. Furthermore, it is assumed that u_{i1} and u_{i2} are independent of u_{i0}.

By virtue of the relationship between the β's and their respective elasticities set out in (4), R_K and R_L can be interpreted as indicators of market conditions. If all markets are competitive, the respective elasticities are infinite and $\beta_Y = 1$, $\beta_K = 1$, and $\beta_L = 1$. Therefore, $R_K = 1$ and $R_L = 1$. Imperfections are indicated by the divergence of R_K and/or R_L from the value of 1.

The model does imply that this divergence must be in the direction of R_K and R_L exceeding 1. Since all other elements -- parameter and observed variables -- in the profit maximizing conditions, (6), are positive we must have R_K and R_L positive to preserve the relationship. First, we surely expect $\beta_K \geqq 1$ and $\beta_L \geqq 1$ since the factor supply elasticities ε_K and ε_L are certainly positive and $\beta_K = 1 + \frac{1}{\varepsilon_K}$, $\beta_L = 1 + \frac{1}{\varepsilon_L}$. Second, we expect the elasticity of demand, ε_Y, to be negative. But if, in addition, $|\varepsilon_Y| < 1$ then $\beta_Y = 1 + \frac{1}{\varepsilon_Y} < 0$ and we would have $R_K < 0$, $R_L < 0$. To have $R_K > 0$, $R_L > 0$, the hypothesized constant elasticity demand function must be elastic, i.e. $|\varepsilon_Y| > 1$. If $|\varepsilon_Y| < 1$, no positive profit maximizing output exists. Of course, if it is known that demand for the product under consideration was not elastic, then the model is not applicable. (33, p. 16)

To summarize:

$$\varepsilon_K > 0 \text{ implies } \beta_K \geqq 1$$

$$\varepsilon_L > 0 \text{ implies } \beta_L \geqq 1$$

$$\varepsilon_Y < 0 \text{ and } |\varepsilon_Y| > 1 \text{ implies } 0 < \beta_Y \leqq 1$$

Therefore, it follows from the definitions of R_K and R_L that

$$R_K \geqq 1, \quad R_L \geqq 1.$$

Market conditions that diverge from competitiveness would be revealed by $R_K > 1$ and/or $R_L > 1$. Of course, what we would most like to know are not the values of R_K and R_L, but the values of the three market parameters β_Y, β_K and β_L. We would then have direct evidence on which, if any, of the markets were imperfect. There is no assurance that R_K and R_L will provide this evidence since they are combinations of β_Y, β_K and β_L. However, there are three situations in which R_K and R_L would reveal conditions in the separate markets.

a) $R_K = 1$, $R_L > 1$. In this case $R_K = 1$ cannot imply $\beta_Y = \beta_K \neq 1$ since $\beta_K \geqq 1$ and $0 < \beta_Y \leqq 1$. It must imply that $\beta_K = 1$, $\beta_Y = 1$. Therefore, the labor market must have been the source of the imperfection causing $R_L > 1$ (i.e. $\beta_L > 1$), since all other markets were competitive.

b) Similarly, $R_K > 1$, $R_L = 1$ implies $\beta_Y = 1$, $\beta_L = 1$, but $\beta_K > 1$. The ouput and labor markets were competitive but the capital market was not.

c) Of course, $R_K = 1$, $R_L = 1$ implies that all markets were competitive.

If neither R_K nor R_L equals one, their values will provide no information about particular markets. The most we could conclude is whether the capital market was more or less competitive than the labor market. The estimation and testing of R_K and R_L are discussed in the remaining sections of this chapter.

Summary

The model consists of the production function, the profit maximizing conditions and the output and input price functions:

$$
7)\quad
\begin{aligned}
&a)\quad Y_i e^{\theta Y_i} = \Gamma K_i^{\alpha_K} L_i^{\alpha_L} u_{i0}^{*} \\
&b)\quad \frac{\alpha_K A(P_{iY}Y_i)}{P_{iK}K_i\left[1+\theta A(Y_i)\right]} = R_K u_{i1}^{*} \qquad\qquad i = 1,\ldots,n. \\
&c)\quad \frac{\alpha_L A(P_{iY}Y_i)}{P_{iL}L_i\left[1+\theta A(Y_i)\right]} = R_L u_{i2}^{*} \\
&d)\quad P_{iY} = b_Y Y_i^{\beta_Y-1} v_{i0}^{*} \\
&e)\quad P_{iK} = b_K K_i^{\beta_K-1} v_{i1}^{*} \\
&f)\quad P_{iL} = b_L L_i^{\beta_L-1} v_{i2}^{*}
\end{aligned}
$$

The restricted parameter space is

$$\Gamma > 0,\ \alpha_K > 0,\ \alpha_L > 0,\ \theta \geqq 0,\ R_K \geqq 1,\ R_L \geqq 1$$

Let

$$u_{i0} = \ln u^*_{i0}; \qquad u_{i1} = \ln u^*_{i1}; \qquad u_{i2} = \ln u^*_{i2}$$

$$v_{i0} = \ln v^*_{i0}; \qquad v_{i1} = \ln v^*_{i1}; \qquad v_{i2} = \ln v^*_{i2}$$

The assumptions on the disturbance terms are

$u_{i0} \sim N(0, \sigma_{00})$ independent over i

$(u_{i1}, u_{i2}) \sim N(0, \Sigma_{11})$ independent over i

where

$$\Sigma_{11} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$$

$\sigma_{00} > 0$, Σ_{11} positive definite

Furthermore, it is assumed that u_{i1} and u_{i2} are independent of u_{i0} for all i. Therefore $(u_{i0}, u_{i1}, u_{i2}) \sim N(0,\Sigma)$ independent over i

where

$$\Sigma = \begin{bmatrix} \sigma_{00} & 0' \\ 0 & \Sigma_{11} \end{bmatrix}$$

Finally, it is assumed that (v_{i0}, v_{i1}, v_{i2}) are normally distributed with mean vector zero, positive definite covariance matrix and independent of u_{i0} for all i.

The estimation and testing of the parameters of this model are considered in the remaining sections of this chapter.

Estimation

The assumption that firms maximize anticipated profits implies that $A(Y_i)$ in the profit maximizing conditions, (7b) and (7c), is non-stochastic. The production disturbance, u^*_{i0}, is therefore not transmitted to the input choice functions. In conjunction with the assumption of independence between u_{i0} and (u_{i1}, u_{i2}), this assures that K_i and L_i are themselves independent of u^*_{i0}.[4]

The log of the production function, (7a), is

$$8) \quad \ell n Y_i + \theta Y_i = \ell n \Gamma + \alpha_K \ell n K_i + \alpha_L \ell n L_i + u_{i0}$$

Although this equation is non-linear for θ unknown, note that for a given θ, say θ^*, it simply constitutes a classical linear equation for the regression of $\ell n Y_i + \theta^* Y_i$ on $\ell n K_i$ and $\ell n L_i$. Because $\ell n K_i$ and $\ell n L_i$ are independent of u_{i0} we are assured that the regression, under the normality assumption on u_{i0}, will yield the maximum likelihood estimates of α_K, α_L, and $\ell n \Gamma$, for that particular θ^*. (*41*, p. 126) This fact affords a relatively straightforward search procedure, described in detail below, for finding the maximum likelihood estimates of all the production function parameters. (*44*, pp. 246-247) The resulting estimates of θ, α_K, α_L, $\ell n \Gamma$, and σ_{00} are not actually the full information maximum likelihood estimates for the model. They are derived not from the joint distribution of Y_i, $\ell n K_i$, $\ell n L_i$ and the prices, but from the conditional distribution of Y_i given $\ell n K_i$ and $\ell n L_i$. Therefore, they should be referred to as the limited information or conditional maximum likelihood estimates, but in order not to be tedious this qualification is dropped in the text.

By virture of the normality assumption on u_{i0} and the independence of $(\ell n K_i, \ell n L_i)$ and u_{i0}, the density function of Y_i, given $\ell n K_i$ and $\ell n L_i$, can be derived from (8) by the simple transformation from u_{i0} to Y_i.

$$H_i = |J_i| \left(\frac{1}{2\pi}\right)^{\frac{1}{2}} \left(\frac{1}{\sigma_{00}}\right)^{\frac{1}{2}} \exp\left\{- \frac{1}{2\sigma_{00}} [Z_i(\theta) - \ell n \Gamma - \alpha_K \ell n K_i - \alpha_L \ell n L_i]^2\right\}$$

where

$$Z_i(\theta) = \ell n Y_i + \theta Y_i$$

and J_i is the Jacobian of the transformation:

$$|J_i| = \left|\frac{\partial u_{i0}}{\partial Y_i}\right| = \frac{1+\theta Y_i}{Y_i}$$

The observations are assumed independent over i (firms), so the likelihood function is:

$$9)\ H = \prod_{i=1}^{n} H_i = J\left(\frac{1}{2\pi}\right)^{n/2} \left(\frac{1}{\sigma_{00}}\right)^{n/2} \exp\left\{- \frac{1}{2\sigma_{00}} \sum_{i=1}^{n} [Z_i(\theta)-\ell n\Gamma-\alpha_K \ell n K_i-\alpha_L \ell n L_i]^2\right\}$$

where

$$J = \prod_{i=1}^{n} J_i = \prod_{i=1}^{n} \left[\frac{1+\theta Y_i}{Y_i}\right]$$

Substituting for J and taking the log:

$$\ell n H = C - \frac{n}{2}\ \ell n \sigma_{00} + \sum_{i=1}^{n} \ell n(1+\theta Y_i) - \frac{1}{2\sigma_{00}} \sum_{i=1}^{n} \left[Z_i(\theta) - \ell n\Gamma - \alpha_K \ell n K_i - \alpha_L \ell n L_i\right]^2$$

where $C = -\frac{n}{2}\ \ell n 2\pi - \sum_{i=1}^{n} \ell n Y_i$ is a constant term.

Differentiating with respect to σ_{00} and setting the derivative equal to zero yields the estimate of σ_{00}:

10) $$\hat{\sigma}_{00} = \frac{1}{n} \sum_{i=1}^{n} \left[Z_i(\theta) - \ell n\Gamma - \alpha_K \ell n K_i - \alpha_L \ell n L_i\right]^2$$

Substituting this back into $\ell n H$ yields:

$$\ell n H^* = C^* - \frac{n}{2}\ \ell n \sum_{i=1}^{n} \left[Z_i(\theta) - \ell n\Gamma - \alpha_K \ell n K_i - \alpha_L \ell n L_i\right]^2 + \sum_{i=1}^{n} \ell n(1+\theta Y_i)$$

where

$$C^* = C - \frac{n}{2}$$

For a particular θ, say θ^*, $\ell n H^*$ is maximized by the values of $\ell n\Gamma$, α_K and α_L which minimize

$$\sum_{i=1}^{n} \left[Z_i(\theta) - \ell n\Gamma - \alpha_K \ell n K_i - \alpha_L \ell n L_i\right]^2$$

These estimates are obtained by regressing $Z_i(\theta^*)$ on $\ell n K_i$ and $\ell n L_i$. By repeating this procedure for various values of θ, evaluating $\ell n H^*$ each time, the maximum of $\ell n H^*$ over all parameters can be located. The values $\ell n\ \hat{\Gamma}$, $\hat{\alpha}_K$, $\hat{\alpha}_L$, and $\hat{\theta}$ which produce this maximum are the maximum likelihood estimates. These values are associated with the global maximum of $\ell n H^*$ and are therefore unique. (*44*, p. 247) Substituting these into the expression for $\hat{\sigma}_{00}$, (10), yields the maximum likelihood estimate of σ_{00}.

The parameters remaining to be estimated are R_K and R_L in the profit maximizing conditions (7b) and (7c). Unfortunately, with the estimation techniques employed in this paper these parameters cannot be consistently estimated separately.[5] In order to consistently estimate R_K and R_L we require consistent estimates of 1) α_K, α_L, and θ, which we have, 2) $A(Y_i)$, which we could obtain by convergence routine solution of $A(Y_i)e^{\hat{\theta}A(Y_i)} - \hat{\Gamma}K_i^{\hat{\alpha}_K} L_i^{\hat{\alpha}_L} = 0$ for each i, and 3) $A(P_{iY}Y_i)$, a consistent estimate of which cannot be obtained with the estimation techniques employed here. The reason for this is that

$$A(P_{iY}Y_i) = b_Y A(Y_i)^{\beta_Y} v_{i0}^*$$

and we cannot obtain consistent estimates of b_Y and β_Y.

However, some information on market imperfections can still be obtained. When (7b) is divided by (7c) the output related terms cancel to yield

$$\frac{\alpha_K P_{iL} L_i}{\alpha_L P_{iK} K_i} = R_{KL} \frac{u_{i1}^*}{u_{i2}^*}$$

where

$$R_{KL} = \frac{R_K}{R_L} = \frac{\beta_K}{\beta_L} \qquad\qquad 0 < R_{KL} < \infty$$

Taking the log we have

$$u_{i1} - u_{i2} = -\ln R_{KL} + \ln\left[\frac{\alpha_K P_{iL} L_i}{\alpha_L P_{iK} K_i}\right]$$

Since $u_{i1} - u_{i2}$ is $N(0, \sigma_{11} - 2\sigma_{12} + \sigma_{22})$ and independent over i, we have the consistent estimate

$$11)\quad \ln\hat{R}_{KL} = \ln\left[\frac{\hat{\alpha}_K}{\hat{\alpha}_L}\right] + \frac{1}{n}\sum_{i=1}^{n} \ln\left[\frac{P_{iL}L_i}{P_{iK}K_i}\right] .$$

Of course, $\hat{R}_{KL} = e^{\ln\hat{R}_{KL}}$. [6]

Since the two parameters R_K and R_L have now been subsumed into one, R_{KL}, some information may have been lost. Since $R_{KL} = \beta_K/\beta_L$ it can only tell us about the relative competitiveness of the capital and labor markets and cannot reveal conditions in particular markets. First, it is clear that all potential information on the output market is lost since β_Y is no longer involved. Second, we cannot conclude that $R_{KL} = 1$ implies competitive capital and labor markets ($\beta_K = 1$, $\beta_L = 1$) since it is possible that $\beta_K = \beta_L \neq 1$.

It is impossible to say whether any real loss of information is actually incurred in estimating only R_{KL}. This depends on whether separate estimates of R_K and R_L would have provided evidence on conditions in each market. It was pointed out above that there is no assurance of this since R_K and R_L are themselves combinations of β_Y, β_K and β_L. Only if one or both of R_K and R_L equals one would we have direct evidence on particular markets. If neither equals one, the most we could conclude is whether the capital market was more or less competitive than the labor market -- precisely the information provided by R_{KL}.

It will be shown later in this chapter that the hypothesis $\theta = 0$ is testable and that if we are willing to accept this hypothesis, estimates of R_K and R_L can be derived.

Hypothesis Testing

There are three historical issues we would like to consider using tests of the parameters of the model:

(a) Did the technology of an industry exhibit returns to scale that varied with the level of output?

That is, is θ significantly different from zero?

(b) Did production have a range of increasing returns to scale?

That is, is $\alpha_K + \alpha_L$ significantly greater than one?

(c) Were markets imperfect?

That is, are R_K and/or R_L significantly greater than one?

Unfortunately, the fact that the finite sample distribution of the estimator $\hat{\theta}$ is unknown prevents us from conducting exact tests on θ or on any of the other parameters. Their estimators all depend on $\hat{\theta}$, so their exact distributions are unknown. However, since the production function parameter estimators are (limited information) maximum likelihood estimators they are asymptotically efficient. Large sample tests of these parameters can therefore be based on the inverse of the estimated information matrix.

The estimation method employed in this paper does not allow us to completely address the issue of the competitiveness of markets when $\theta \neq 0$. We do not have estimates of R_K and R_L; only of their ratio R_{KL}. At most we might hope to ask whether the capital and labor markets differed in their "degree of competitiveness" (i.e. their elasticities). In other words, is R_{KL} significantly different from one? As pointed out above, the distribution of $\hat{R}_{KL}$ depends on $\hat{\theta}$, so its exact distribution is unknown. ($\hat{\theta}$ does not appear directly in the expression for $\hat{R}_{KL}$, but $\hat{\alpha}_K$ and $\hat{\alpha}_L$ do, and they depend on $\hat{\theta}$.) Furthermore, $\hat{R}_{KL}$ is not a maximum likelihood estimator, so a large sample test is also not available. When $\theta \neq 0$ we must rely on point estimation to judge the relative competitiveness of the capital and labor markets. In the next section of this chapter we will show that when $\theta = 0$ limited information maximum likelihood estimators of R_K and R_L are available. If the test on θ supports the hypothesis that $\theta = 0$ we can then estimate and test the market parameters.

Let the n x 1 vectors of observations on Y_i, $\ell n K_i$, and $\ell n L_i$ be denoted $\underline{\bar{Y}}$, $\ell n\underline{\bar{K}}$, and $\ell n\underline{\bar{L}}$. The density function of $\underline{\bar{Y}}$ given $\ell n\underline{\bar{K}}$ and $\ell n\underline{\bar{L}}$, given by (9) above, can be denoted

$$H\ (\ \underline{\bar{Y}}\ /\ \ell n\underline{\bar{K}},\ \ell n\underline{\bar{L}}:\ \phi\ )$$

where ϕ is the parameter vector, $\phi = (\ell n\Gamma \quad \alpha_K \quad \alpha_L \quad \theta \quad \sigma_{00})$.

The information matrix is

$$I(\phi) \quad = \quad -E\left[\frac{\partial^2\ \ell n\ H(\ \underline{\bar{Y}}\ /\ \ell n\underline{\bar{K}},\ \ell n\underline{\bar{L}};\ \phi\ )}{\partial\phi\ \partial\phi'}\right]$$

The maximum likelihood estimator $\hat{\phi}$ is a consistent estimate of the true parameter vector ϕ, and $n^{\frac{1}{2}}(\hat{\phi}-\phi)$ is asymptotically normally distributed with mean vector 0 and covariance matrix $T(\phi) = \lim_{n\to\infty}\left[\frac{1}{n}\ I(\phi)\right]^{-1}$.

A consistent estimator of this matrix is

$$S\ (\hat{\phi}) \quad = \quad \left[\frac{1}{n}\quad I(\hat{\phi})\right]^{-1} \qquad . \qquad (41,\ \text{pp. } 395\text{-}396)$$

The matrix $I(\hat{\phi})$ is given in Appendix A. Our hypothesis tests will be based on these properties of the maximum likelihood estimator $\hat{\phi}$.

A Test on the Existence of Variable Returns to Scale

The answer to whether or not the technology of an industry exhibited returns to scale that varied over output lies in the true value of the parameter θ. The returns to scale function is $\alpha(Y) = \dfrac{\alpha_K + \alpha_L}{1 + \theta Y}$.

If θ is indeed greater than zero then returns to scale do vary over output, but if $\theta = 0$ they are $\alpha_K + \alpha_L$ everywhere. The test we will conduct for each sample is:

$$H_0: \ \theta = 0 \qquad \text{against} \qquad H_1: \ \theta > 0$$

The test is conducted as a one-tail rather than two-tail test (i.e. H_1: $\theta \neq 0$) since we have a priori reason to expect that $\theta \geqq 0$ even though this restriction is not imposed in the estimation.

The random variable $n^{\frac{1}{2}}(\hat{\theta}-\theta)$ is asymptotically normally distributed with mean 0 and variance υ_θ^2, the diagonal element corresponding to θ is the asymptotic covariance matrix $T(\phi)$. Since $S(\hat{\phi})$ is a consistent estimator of $T(\phi)$, s_θ^2 converges in probability to υ_θ^2 ($plim\ s_\theta^2 = \upsilon_\theta^2$), where s_θ^2 is the diagonal element corresponding to θ in $S(\hat{\phi})$. Therefore, $n^{\frac{1}{2}}(\hat{\theta}-\theta)/s_\theta$ is asymptotically distributed $N(0,1)$. Therefore we may base our test of H_0: $\theta = 0$ on the random variable $\vec{\Lambda} = n^{\frac{1}{2}}\ \theta/s_\theta$ which is asymptotically distributed $N(0,1)$. Throughout this paper we will employ $\vec{\Lambda}$ to denote an asymptotically distributed $N(0,1)$ random variable employed as a test statistic, λ to denote the actual or realized value of $\vec{\Lambda}$, and Λ to denote the $N(0,1)$ random variable.

Typically we would next select some significance level at which the test is to be conducted, and corresponding to that level would be a value λ_0 from the tables of the standard normal distribution. Since large values of λ tend to refute H_0, we would reject H_0 if $\lambda \geqq \lambda_0$.

(The significance level would tell us the probability of falsely rejecting H_0: Pr $\{\Lambda \geqq \lambda_0; H_0\}$). If $\lambda < \lambda_0$ we would accept H_0.

The test results in this paper are reported in somewhat different fashion. The result will be reported as the largest significance level (i.e. the largest probability of falsely rejecting H_0) at which the test can be conducted and still accept H_0. This level is denoted δ, and

$$\delta = \Pr \{\Lambda \geqq \lambda; H_0\}.$$

Obviously, the larger the value of δ, the stronger is the case in favor of H_0 and against the alternative hypothesis.

All of the tests in this paper are formulated so that the alternative hypothesis, H_1, represents the traditional historical view. In the present case, Genovese's contention that smaller Southern firms were unable to take advantage of returns to scale would certainly seem to imply that he believes returns to scale varied with output, i.e. that $\theta > 0$. Also, he clearly believes that a range of increasing returns existed, and this is represented by the alternative hypothesis H_1: $\alpha_K + \alpha_L > 1$ in the test on $\alpha_K + \alpha_L$, discussed below. [7] The method of presenting test results employed in this paper makes clear just how strong the case against the alternative hypothesis is. A decision to either accept or reject H_0 must still be made, of course, but the presentation will permit each person to make this decision. For those who wish to use the commonly accepted significance levels of five or ten per cent, a value of δ greater than .05 or .10 means that they should not reject the null hypothesis.

A Test on the Existence of Increasing Returns to Scale

The true value of $\alpha_K + \alpha_L$ determines whether or not there were any increasing returns available to firms at all. For each sample the following test is conducted:

$$H_0: \alpha_K + \alpha_L \leqq 1 \qquad \text{against} \qquad H_1: \alpha_K + \alpha_L > 1$$

The random variable

$$\frac{n^{\frac{1}{2}} \left[(\hat{\alpha}_K + \hat{\alpha}_L) - (\alpha_K + \alpha_L) \right]}{\left[s^2_{\alpha K} + 2s_{\alpha KL} + s^2_{\alpha L} \right]^{\frac{1}{2}}} \text{ is asymptotically distributed } N(0,1)$$

where

$s^2_{\alpha K}$: estimated asymptotic variance of $n^{\frac{1}{2}} (\hat{\alpha}_K - \alpha_K)$ from $S(\hat{\phi})$

$s^2_{\alpha L}$: estimated asymptotic variance of $n^{\frac{1}{2}} (\hat{\alpha}_L - \alpha_L)$ from $S(\hat{\phi})$

$s_{\alpha KL}$: estimated asymptotic covariance of $n^{\frac{1}{2}} (\hat{\alpha}_K - \alpha_K)$ and $n^{\frac{1}{2}} (\hat{\alpha}_L - \alpha_L)$ from $S(\hat{\phi})$

We may therefore base the test on the random variable

$$\vec{\Lambda} = \frac{n^{\frac{1}{2}} \{ (\hat{\alpha}_K + \hat{\alpha}_L) - \left[(\alpha_K + \alpha_L); H_0 \right] \}}{\left[s^2_{\alpha K} + 2s_{\alpha KL} + s^2_{\alpha L} \right]^{\frac{1}{2}}}$$

where

$$[(\alpha_K + \alpha_L);\ H_0]$$ indicates the value of $\alpha_K + \alpha_L$ under H_0.

A value of $\alpha_K + \alpha_L$ must be selected from the composite hypothesis H_0: $\alpha_K + \alpha_L \leqq 1$. As with our test on θ, we wish to report the test result as the largest significance level, δ, at which H_0 can be accepted. A small test statistic, λ, tends to confirm H_0; a large λ tends to refute H_0. The smaller the value of $\alpha_K + \alpha_L$ we select in H_0, the larger is λ, and therefore the smaller the significance level. The maximum over H_0 of the significance level, δ, at which H_0 is just accepted occurs at $\alpha_K + \alpha_L = 1$. Therefore,

$$\vec{\Lambda} = \frac{(\hat{\alpha}_K + \hat{\alpha}_L) - 1}{\left[s^2_{\alpha K} + 2s_{\alpha KL} + s^2_{\alpha L}\right]^{\frac{1}{2}}}$$

and

$$\delta = \Pr\{\Lambda > \lambda\}$$

THE CASE OF $\theta = 0$

The problem of not being able to estimate R_K and R_L and conduct tests on their values is absent if $\theta = 0$ and thus the true production function is the homogeneous Cobb-Douglas function. It is shown below that in this case limited information maximum likelihood estimators of R_K and R_L can be found and hypothesis tests can be based on the asymptotic efficiency property of such estimators.

Previously, with $\theta \neq 0$, the exponential term in the production relationship made it impossible to find an explicit relationship between actual and anticipated output. However, when $\theta = 0$, $Y_i = A(Y_i)u^*_{io}$. This relationship can be used to find the relationship between actual and anticipated revenue, enabling us to eliminate the anticipated revenue term from the profit maximizing conditions. Therefore, we no longer need an estimate of $A(P_{iY}Y_i)$, and hence of b_Y and β_Y, to estimate R_K and R_L. We have:

$$A(P_{iY}Y_i) = b_Y \, A(Y_i)^{\beta_Y} \, v^*_{io}$$

$$= b_Y \, Y_i^{\beta_Y} \, \frac{v^*_{io}}{u^{*\,\beta_Y}_{io}}$$

$$= \frac{P_{iY} \, Y_i}{u^{*\,\beta_Y}_{io}} \qquad \text{since} \qquad P_{iY} \, Y_i = b_Y \, Y_i^{\beta_Y} \, v^*_{io}$$

Therefore, we may write the profit maximizing conditions, (7b) and (7c), as:

12)

$$\text{a)} \quad \frac{\alpha_K P_{iY} Y_i}{P_{iK} K_i} = R_K u^*_{i1} u^{*\,\beta_Y}_{i0}$$

$$\text{b)} \quad \frac{\alpha_L P_{iY} Y_i}{P_{iL} L_i} = R_L u^*_{i2} u^{*\,\beta_Y}_{i0}$$

There is an important difference between the present model, which allows imperfect markets, and the more usual models derived under competitive market conditions. In the latter, prices are either constant or, if random, are assumed to be distributed independently of output and inputs. (*43*, pp. 787-788) Therefore, models consisting solely of a production function and profit maximizing conditions contain all the information necessary to derive full information maximum likelihood estimates of the parameters. Under imperfect markets, prices are functionally related to output and inputs and are decidedly not independent.

In the present case, with $\theta = 0$, it is theoretically possible to formulate a six equation linear simultaneous equations model in the six endogenous variables output, capital, labor, and their respective prices. The model would consist of the three market equations relating prices with output and inputs, the production function, and the two profit maximizing conditions. However, there are two insurmountable problems associated with the use of individual firm prices -- the lack of data on each firm's capital price and the problem of pricing the "output" of multi-product firms. Therefore, the model is re-formulated to avoid the need for firm prices. The complete six equation system is used to derive a smaller, three equation system in the variables output, capital and labor. The estimates of the production function parameters will again, as when $\theta \neq 0$, depend on inputs and output alone. And the estimates of R_K and R_L will require only the means of the prices of output, capital and labor, or, alternatively, the means of value added, capital expenditures and labor expenditures.

The problem of pricing capital is the most obvious reason for avoiding individual firm prices. Each firm's "value of capital invested" is the only reported datum on capital. There is, quite simply, no information on each firm's "price." In the next chapter a measure of capital expenditure is proposed that is based on data or estimates of depreciation, repair, insurance, tax and interest rates. Each of these rates, with the exception of the very minor component of the state tax rate, is taken to be the same for all firms. In essence, then, the only information available on the price of capital is a construct which we intend to represent its mean. The parameter R_K can be consistently estimated using this information.

The price of each firm's output or outputs was not explicitly reported, but is implied by the data on quantities and values of products. However, only in the lumber industry was there a single output and hence a single price of output. Grain and textiles were multi-product industries. Of course, since the model is derived from profit maximization conditions as if there were only one "output," any deviation from this condition is an abstraction. However, production in these industries consisted of transforming a simple raw material into some output by a process that had little to do with the particular output being produced (e.g. wheat to wheat flour, rye to rye flour). The concept of "output" seems justified and is measured by value added. However, it is impossible to construct a meaningful "price" for that "output." The proposed estimates of R_K and R_L will require only the available information on value added.

Finally, even in the lumber industry where a single price of output does exist, there is good reason to question which of the implied output price variations are meaningful since the quantity and value data were subjected to rounding when reported. Hopefully in the present approach this rounding averages out since only mean value added will be involved in estimating R_K and R_L. For all these reasons, then, the model will be formulated to avoid reliance on individual firm prices in the estimation of parameters.

The re-formulated, three equation system is derived below. It consists of the production function and profit maximizing conditions with prices eliminated from the latter by substituting the reduced form expression for prices from the larger model. The complete model consists of 1) the logs of the price equations, (7d), (7e) and (7f), and 2) the log of the production function, (7a), with $\theta = 0$, and 3) the logs of the profit maximizing conditions (12a) and (12b).

We have:

$$\ell n\ P_{iY} + (1-\beta_Y)\ \ell n Y_i = \ell n b_Y + v_{i0}$$

$$\ell n\ P_{iK} + (1-\beta_K)\ \ell n K_i = \ell n b_K + v_{i1}$$

$$\ell n\ P_{iL} + (1-\beta_L)\ \ell n L_i = \ell n b_L + v_{i2}$$

$$\ell n\ Y_i - \alpha_K\ \ell n K_i - \alpha_L\ \ell n L_i = \ell n \Gamma + u_{i0}$$

$$\ell n\ P_{i0} - \ell n P_{iK} + \ell n Y_i - \ell n K_i = \ell n(\beta_K/\beta_Y\alpha_K) + \beta_Y\ u_{i0} + u_{i1}$$

$$\ell n\ P_{i0} - \ell n P_{iL} + \ell n Y_i - \ell n L_i = \ell n(\beta_L/\beta_Y\alpha_L) + \beta_Y\ u_{i0} + u_{i2}$$

This system can be set up in convenient matrix form. Let

$$\mathcal{B} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & (1-\beta_Y) & 0 & 0 \\ 0 & 1 & 0 & 0 & (1-\beta_K) & 0 \\ 0 & 0 & 1 & 0 & 0 & (1-\beta_L) \\ \hline 0 & 0 & 0 & 1 & -\alpha_K & -\alpha_L \\ 1 & -1 & 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 1 & 0 & -1 \end{array}\right] = \begin{bmatrix} I & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$
C = \begin{bmatrix} \ln b_Y \\ \ln b_K \\ \ln b_L \\ \hdashline \ln \Gamma \\ \ln (\beta_K/\beta_Y\alpha_K) \\ \ln (\beta_L/\beta_Y\alpha_L) \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}
$$

$$
H = \begin{bmatrix} I_{(3\times 3)} & 0 \\ 0 & H_{22} \end{bmatrix} \quad \text{where } H_{22} = \begin{bmatrix} 1 & 0 & 0 \\ \beta_Y & 1 & 0 \\ \beta_Y & 0 & 1 \end{bmatrix}
$$

$$
X_i = \begin{bmatrix} \ln P_{iY} \\ \ln P_{iK} \\ \ln P_{iL} \\ \hdashline \ln Y_i \\ \ln K_i \\ \ln L_i \end{bmatrix} = \begin{bmatrix} P_i^* \\ X_i^* \end{bmatrix}
$$

$$W_i = \begin{bmatrix} v_{i0} \\ v_{i1} \\ v_{i2} \\ u_{i0} \\ u_{i1} \\ u_{i2} \end{bmatrix} = \begin{bmatrix} V_i \\ U_i \end{bmatrix}$$

We may write the system as

$$BX_i = C + HW_i$$

The reduced form solution is

$$X_i = B^{-1} C + B^{-1} H W_i$$

Since W_i is normally distributed with mean 0 and finite covariance matrix, X_i is normally distributed with mean $B^{-1} C$ which we denote

$$B^{-1} C = M_X = \begin{bmatrix} M_P^* \\ M_X^* \end{bmatrix}$$

The reduced form solution for the logs of prices, P_i^*, is

13) $P_i^* = M_P^* + V_i + B_{12} \left[B_{22} - B_{21} B_{12}\right]^{-1} \left[B_{21} V_i - H_{22} U_i\right]$

The production function and profit maximizing conditions are

$$14) \quad B_{22} X_i^* = C_2 - B_{21} P_i^* + H_{22} U_i$$

By substituting (13) into (14) we can eliminate individual firm prices from the profit maximizing conditions and generate the following system:

$$\ell n Y_i - \alpha_K \ell n K_i - \alpha_L \ell n L_i = \ell n \Gamma + u_{i0}$$

$$15) \quad \ell n Y_i - \ell n K_i = \ell n(R_K/\alpha_K) + \mu_{PK}^* - \mu_{PY}^* + u_{i0} + \xi_{i1}$$

$$\ell n Y_i - \ell n L_i = \ell n(R_L/\alpha_L) + \mu_{PL}^* - \mu_{PY}^* + u_{i0} + \xi_{i2}$$

where

$R_K = \beta_K/\beta_Y$

$R_L = \beta_L/\beta_Y$

$\mu_{PY}^*, \mu_{PK}^*, \mu_{PL}^*$ denote the elements of M_P^* -- the means of the logs of the prices of output, capital, and labor, respectively

$$\xi_{i1} = \frac{1}{\beta_K \beta_L - \alpha_K \beta_Y \beta_L - \alpha_L \beta_Y \beta_K} \Big[(\alpha_L \beta_K + \alpha_K \beta_L - \beta_L) v_{i0}$$

$$+ (\beta_L - \alpha_L \beta_Y - \alpha_K \beta_L)(v_{i1} + u_{i1}) + (\alpha_L \beta_Y - \alpha_L \beta_K)(v_{i2} + u_{i2}) \Big]$$

$$\xi_{i2} = \frac{1}{\beta_K \beta_L - \alpha_K \beta_Y \beta_L - \alpha_L \beta_Y \beta_K} \Big[(\alpha_L \beta_K + \alpha_K \beta_L - \beta_K) v_{i0}$$

$$+ (\alpha_K \beta_Y - \alpha_K \beta_L)(v_{i1} + u_{i1}) + (\beta_K - \alpha_K \beta_Y - \alpha_L \beta_K)(v_{i2} + u_{i2}) \Big]$$

From the distributional assumptions on the disturbances we see that $E(\xi_{i1}) = E(\xi_{i2}) = 0$ and that ξ_{i1} and ξ_{i2} are independent of u_{i0}, since each of v_{i0}, v_{i1}, v_{i2}, u_{i1}, and u_{i2} were assumed independent of u_{i0}. The covariance matrix of ξ_{i1} and ξ_{i2} is a function of the covariance matrix of the five disturbance terms which compose them. It serves no purpose here to state this relationship explicitly so we simply let

$$E(\xi_{i1}^2) = \sigma_{11}^* \qquad E(\xi_{i2}^2) = \sigma_{22}^* \qquad E(\xi_{i1}\,\xi_{i2}) = \sigma_{12}^*$$

Letting

$$W_i^* = \begin{bmatrix} u_{i0} \\ \xi_{i1} \\ \xi_{i2} \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

We may write the three equation system, (15), in matrix form:

$$B_{22} X_i^* = C_2 - B_{21} M_P^* + T W_i^*$$

$$W_i^* \sim N(0, \Sigma^*) \text{ independent over } i$$

where

$$\Sigma^* = \begin{bmatrix} \sigma_{00} & 0 & 0 \\ 0 & \sigma_{11}^* & \sigma_{12}^* \\ 0 & \sigma_{12}^* & \sigma_{22}^* \end{bmatrix} = \begin{bmatrix} \sigma_{00} & 0' \\ 0 & \Sigma_{11}^* \end{bmatrix}$$

The reduced form is

$$X_i^* = B_{22}^{-1} \left[C_2 - B_{21} M_P^* \right] + B_{22}^{-1} T W_i^*$$

So we have that

$$X_i^* \sim N(M_X^*, \Omega) \text{ independent over } i$$

where

16) a) $M_X^* = B_{22}^{-1} \left[C_2 - B_{21} M_P^* \right]$

b) $\Omega = B_{22}^{-1} T \Sigma^* T' B_{22}^{-1}{}'$

Estimation

This is the system which we will use to estimate the set of structural parameters:

17) $\phi^* = \{\Gamma, \alpha_K, \alpha_L, R_K, R_L, \sigma_{00}, \sigma^*_{11}, \sigma^*_{12}, \sigma^*_{22}\}$

The maximum likelihood estimates of the reduced form parameters M^*_X and Ω are, of course, the sample mean and covariance matrices. Maximum likelihood estimates of the structural parameters are derived from the relationships between structural and reduced form parameters given by (16). However, it is important to note that the resulting estimates are not full information maximum likelihood estimates. These could only come from the joint likelihood function of output, capital, labor and their prices. By not using that likelihood function some information has been lost. In estimating ϕ^* the mean of P^*_i, M^*_P, is used, but the covariance matrices of P^*_i and between P^*_i and X^*_i are not.

Therefore the resulting estimate of ϕ^* is only a limited information maximum likelihood estimate, denoted LIML. But since it maximizes the likelihood function of the X^*_i's for a given value of M^*_P, it has all the usual properties of maximum likelihood estimators. This is important since we will rely on the asymptotic efficiency property of such estimators to conduct large-sample hypothesis tests of R_K and R_L.

The LIML estimators of α_K and α_L can be derived from the inverse transformation of (16b). Partition the matrices B_{22}, T, and Ω:

$$B_{22} = \begin{bmatrix} 1 & -\alpha' \\ \iota_2 & -I \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0' \\ \iota_2 & I \end{bmatrix} \qquad \Omega = \begin{bmatrix} \omega_{00} & \Omega_{01}' \\ \Omega_{10} & \Omega_{11} \end{bmatrix}$$

where

$$\iota_2' = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\alpha' = \begin{bmatrix} \alpha_K & \alpha_L \end{bmatrix}$$

From the inverse transformation $\Sigma^* = T^{-1} B_{22} \Omega B_{22}' T^{-1'}$, the zero covariances between ξ_{i1} and u_{i0} and ξ_{i2} and u_{i0} result in the matrix equation

$$(\iota_2\alpha' - I)\ (\Omega_{10} - \Omega_{11}\alpha) = 0_{(2\times 1)}.$$

Therefore,

$$\alpha = \Omega_{11}^{-1}\ \Omega_{10}$$

The maximum likelihood estimator of Ω is the sample covariance matrix. Let the vector X_i^* be partitioned

$$X_i^* = \begin{bmatrix} Y_i^* \\ X_{2i}^* \end{bmatrix}$$

where

$$Y_i^* = \ell n\ Y_i \quad \text{and} \quad X_{2i}^* = \begin{bmatrix} \ell n K_i \\ \ell n L_i \end{bmatrix}$$

Also, let $\underline{\overline{X}}^*$ denote the $3 \times n$ matrix whose columns are the observation vectors X_i^* :

$$\underline{\overline{X}}^* = \begin{bmatrix} X_1^* \, X_2^* \cdots X_n^* \end{bmatrix} = \begin{bmatrix} Y_1^* & Y_2^* & \cdots & Y_n^* \\ X_{21}^* & X_{22}^* & \cdots & X_{2n}^* \end{bmatrix} = \begin{bmatrix} \underline{\overline{Y}}^* \\ \underline{\overline{X}}_2^* \end{bmatrix}$$

The estimator of Ω is

$$\hat{\Omega} = (1/n) \left[\underline{\overline{X}}^* \underline{\overline{X}}^{*\prime} - (1/n) \underline{\overline{X}}^* \iota\iota' \underline{\overline{X}}^{*\prime} \right]$$

where ι is an $n \times 1$ vector of 1's.

Partitioning this matrix:

$$\hat{\Omega} = \begin{bmatrix} \hat{\omega}_{00} & \hat{\Omega}_{01}' \\ \hat{\Omega}_{10} & \hat{\Omega}_{11} \end{bmatrix} = \frac{1}{n} \begin{bmatrix} \underline{\overline{Y}}^*\underline{\overline{Y}}^{*\prime} - (1/n)\underline{\overline{Y}}^*\iota\iota'\underline{\overline{Y}}^{*\prime} & \underline{\overline{Y}}^*\underline{\overline{X}}_2^{*\prime} - (1/n)\underline{\overline{Y}}^*\iota\iota'\underline{\overline{X}}_2^{*\prime} \\ \underline{\overline{X}}_2^*\underline{\overline{Y}}^{*\prime} - (1/n)\underline{\overline{X}}_2^*\iota\iota'\underline{\overline{Y}}^{*\prime} & \underline{\overline{X}}_2^*\underline{\overline{X}}_2^{*\prime} - (1/n)\underline{\overline{X}}_2^*\iota\iota'\underline{\overline{X}}_2^{*\prime} \end{bmatrix}$$

Therefore, the LIML estimator of α is

$$\hat{\alpha} = \hat{\Omega}_{11}^{-1} \hat{\Omega}_{10} = \left[n \, \underline{\overline{X}}_2^* \underline{\overline{X}}_2^{*\prime} - \underline{\overline{X}}_2^* \iota\iota' \underline{\overline{X}}_2^{*\prime} \right]^{-1} \left[n \, \underline{\overline{X}}_2^* \underline{\overline{Y}}^{*\prime} - \underline{\overline{X}}_2^* \iota\iota' \underline{\overline{Y}}^* \right]$$

It can be shown that this is the least squares estimator of α from the production function regression. Let

18) $F = \left[\iota \; \underline{\overline{X}}_2^{*\prime} \right]$.

The ordinary least squares estimator of $\begin{bmatrix} \ell n\Gamma \\ \alpha \end{bmatrix}$ is

$$\begin{bmatrix} \ell n\tilde{\Gamma} \\ \tilde{\alpha} \end{bmatrix} = (F'F)^{-1} F' \underline{\overline{Y}^*}$$

By carrying out the partitioned inversion of $F'F$ it can be shown that $\tilde{\alpha} = \hat{\alpha}$.

Now that we have the estimator of α, the LIML estimator of Σ^* can be derived from (16b).

$$\hat{\sigma}_{00} = \hat{\omega}_{00} - 2\hat{\alpha}' \hat{\Omega}_{10} + \hat{\alpha}' \hat{\Omega}_{11} \hat{\alpha}$$

$$\hat{\Sigma}^*_{11} = (\iota_2 \hat{\alpha}' - I) \hat{\Omega}_{11} (\hat{\alpha} \iota_2' - I)$$

where

$$\iota_2' = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

Finally, the estimators of Γ, R_K and R_L can be derived from (16a). We have:

$$C_2 - B_{21} M_P^* = B_{22} M_X^*$$

Since the maximum likelihood estimator of M_X^*, denoted $\hat{M}_X^*$, is the sample mean vector, the LIML estimators of $\ell n\Gamma$, ℓnR_K and ℓnR_L are

$$19) \quad \begin{aligned} &a)\quad \ln \hat{\Gamma} = \hat{\mu}^*_Y - \hat{\alpha}_K \hat{\mu}^*_K - \hat{\alpha}_L \hat{\mu}^*_L \\ &b)\quad \ln \hat{R}_K = \ln \hat{\alpha}_K + \hat{\mu}^*_Y + \mu^*_{PY} - \hat{\mu}^*_K - \mu^*_{PK} \\ &c)\quad \ln \hat{R}_L = \ln \hat{\alpha}_L + \hat{\mu}^*_Y + \mu^*_{PY} - \hat{\mu}^*_L - \mu^*_{PL} \end{aligned}$$

where

$\hat{\mu}^*_Y$, $\hat{\mu}^*_K$, $\hat{\mu}^*_L$ are the elements of $\hat{M}^*_X$ -- the sample means of $\ln Y_i$, $\ln K_i$ and $\ln L_i$, respectively.

μ^*_{PY}, μ^*_{PK}, μ^*_{PL} are the elements of M^*_P -- the means of the logs of P_{iY}, P_{iK}, and P_{iL}, respectively.

Of course,

$$\hat{\Gamma} = \exp \{ \ln \hat{\Gamma} \}$$

$$\hat{R}_K = \exp \{ \ln \hat{R}_K \}$$

$$\hat{R}_L = \exp \{ \ln \hat{R}_L \}$$

As they now stand, the estimators of $\ln R_K$ and $\ln R_L$ require that we know the true or population means of (the logs of) prices. Unfortunately, this information is not available. But consistent estimators of $\ln R_K$ and $\ln R_L$ can obviously be obtained by replacing μ^*_{PY}, μ^*_{PK} and μ^*_{PL} with the sample means $\hat{\mu}^*_{PY}$, $\hat{\mu}^*_{PK}$ and $\hat{\mu}^*_{PL}$. The resulting estimators are

$$\text{19')}\quad \begin{aligned} &\text{b')}\ \ell n\,\tilde{R}_K = \ell n\,\hat{\alpha}_K + \frac{1}{n}\sum_{i=1}^{n} \ell n\,(P_{iY}Y_i) - \frac{1}{n}\sum_{i=1}^{n} \ell n\,(P_{iK}K_i) \\ &\text{c')}\ \ell n\,\tilde{R}_L = \ell n\,\hat{\alpha}_L + \frac{1}{n}\sum_{i=1}^{n} \ell n\,(P_{iY}Y_i) - \frac{1}{n}\sum_{i=1}^{n} \ell n\,(P_{iL}L_i) \end{aligned}$$

These estimators, although consistent, are technically no longer maximum likelihood estimators. The likelihood function of X_i^* is characterized by the parameters M_X^* and Ω, and the sample mean vector $\hat{M}_X^*$ and covariance matrix $\hat{\Omega}$ maximize the likelihood function with respect to these parameters. But M_P^* does not appear as a parameter of this likelihood function. It appears as a known value in the relationship between M_X^* and the structural parameters $\ell n R_K$ and $\ell n R_L$ given by (16a). Therefore, it cannot be said that replacing M_P^* with $\hat{M}_P^*$ yields maximum likelihood estimators of $\ell n R_K$ and $\ell n R_L$. At first, this might appear to be a serious loss since we intended to use our knowledge of the asymptotic distribution of maximum likelihood estimators to conduct tests of R_K and R_L. However, it will be shown below that it is possible to derive the asymptotic distributions of $n^{\frac{1}{2}}(\ell n\tilde{R}_K - \ell n R_K)$ and $n^{\frac{1}{2}}(\ell n\tilde{R}_L - \ell n R_L)$.

Tests for the Existence of Market Imperfections

The relationship between the values of R_K and R_L and competitive conditions in the output, capital and labor markets was discussed in detail earlier in this chapter. To review:

i) $R_K = 1$, $R_L = 1$ implies that all markets were perfectly competitive.

ii) $R_K = 1$, $R_L > 1$ implies that the output and capital markets were competitive, while the labor market was not.

iii) $R_K > 1$, $R_L = 1$ implies that the output and labor markets were competitive, while the capital market was not.

iv) $R_K > 1$, $R_L > 1$ implies that imperfect market conditions prevailed in at least two, and possibly in all three, of the markets.

For those industries in which the production function was homogeneous ($\theta = 0$), estimates of R_K and R_L can be derived. We would then like to conduct the following tests for the existence of market imperfections:

$$
20)\qquad \begin{array}{llll} H_0: & R_K = 1 & \text{against} & H_1: \; R_K > 1 \\ H_0: & R_L = 1 & \text{against} & H_1: \; R_L > 1 \end{array}
$$

One-sided tests are proposed since our a priori knowledge indicates that $R_K \geqq 1$, $R_L \geqq 1$.

To conduct such tests we require information on the distributions of our estimators. It was noted above that the maximum likelihood estimators, $\hat{R}_K$ and $\hat{R}_L$, are not known. The estimators $\tilde{R}_K$ and $\tilde{R}_L$ are used instead. However, the information on the asymptotic distributions of the maximum likelihood estimators can be used to derive the asymptotic distributions of $n^{\frac{1}{2}}(\ell n\tilde{R}_K - \ell nR_K)$ and $n^{\frac{1}{2}}(\ell n\tilde{R}_L - \ell nR_L)$. Large sample hypothesis tests can then be based on this information.

Since it will be necessary to work with $\ell n R_K$ and $\ell n R_L$ rather than R_K and R_L, the hypothesis tests in (19) can be equivalently expressed as:

$$
21) \quad \begin{array}{llll} H_0: \ell n\ R_K = 0 & \text{against} & H_1: & \ell n\ R_K > 0 \\ H_0: \ell n\ R_L = 0 & \text{against} & H_1: & \ell n\ R_L > 0 \end{array}
$$

The derivations are analogous so we consider only $\ell n\tilde{R}_K$ in detail.

From (19) and (19') we have:

$$\ell n\ \hat{R}_K = \ell n\ \hat{\alpha}_K + \hat{\mu}^*_Y + \mu^*_{PY} - \hat{\mu}^*_K - \mu^*_{PK}$$

and

$$\ell n\ \tilde{R}_K = \ell n\ \hat{\alpha}_K + \hat{\mu}^*_Y + \hat{\mu}^*_{PY} - \hat{\mu}^*_K - \hat{\mu}^*_{PK}$$

Subtracting the equation for $\ell n\hat{R}_K$ from that for $\ell n\tilde{R}_K$ yields

$$\ell n\ \tilde{R}_K = \ell n\ \hat{R}_K + (\hat{\mu}^*_{PY} - \mu^*_{PY}) - (\hat{\mu}^*_{PK} - \mu^*_{PK}).$$

Subtracting $\ell n R_K$ from both sides and multiplying by $n^{\frac{1}{2}}$, we have

$$22) \quad n^{\frac{1}{2}}(\ell n\tilde{R}_K - \ell n R_K) = n^{\frac{1}{2}}(\ell n\hat{R}_K - \ell n R_K) + n^{\frac{1}{2}}(\hat{\mu}^*_{PY} - \mu^*_{PY}) - n^{\frac{1}{2}}(\hat{\mu}^*_{PK} - \mu^*_{PK}).$$

Since $\ell n\hat{R}_K$ is the maximum likelihood estimator of $\ell n R_K$, we know that

$$n^{\frac{1}{2}}\ (\ell n\ \hat{R}_K - \ell n\ R_K) \to N(0,\varepsilon^2_K)$$

and

$$plim\ \ e^2_K = \varepsilon^2_K$$

where

e^2_K denotes the estimator of the asymptotic variance, ε^2_K, of $n^{\frac{1}{2}}(\ell n\ \hat{R}_K - \ell n\ R_K)$.[8]

$\rightarrow$ is read "is asymptotically distributed,"

$plim\ e_K^2 = \varepsilon_K^2$ is read "e_K^2 converges in probability to ε_K^2."

Therefore,

$$\frac{n^{\frac{1}{2}}(\ell n\ \hat{R}_K - \ell n\ R_K)}{e_K} \rightarrow N(0,1).$$

Let

$$N_1 = \frac{n^{\frac{1}{2}}(\ell n\ \hat{R}_K - \ell n\ R_K)}{e_K}$$

$$N_2 = \frac{n^{\frac{1}{2}}(\hat{\mu}^*_{PY} - \mu^*_{PY})}{e_K}$$

$$N_3 = \frac{n^{\frac{1}{2}}(\hat{\mu}^*_{PK} - \mu^*_{PK})}{e_K}$$

Dividing (22) by e_K, we may write

$$\frac{n^{\frac{1}{2}}(\ell n\tilde{R}_K - \ell n\ R_K)}{e_K} = N_1 + N_2 - N_3.$$

The asymptotic distribution of $\left[n^{\frac{1}{2}}(\ell n\, \tilde{R}_K - \ell n\, R_K)\right] / e_K$ can be established by determining the asymptotic distribution of $N_1 + N_2 - N_3$. The asymptotic distribution of N_1 has just been shown to be N(0,1). That of $N_2 - N_3$ is considered next.

From the reduced form expression for the logs of prices, given by (13) above, it can be seen that $\ell n P_{iY}$ and $\ell n P_{iK}$ are linear functions of the normally distributed components of the disturbance vector W_i. Therefore, $\ell n P_{iY}$ and $\ell n P_{iK}$ and their sample means $\hat{\mu}^*_{PY}$ and $\hat{\mu}^*_{PK}$ are also normally distributed. We have

$$\hat{\mu}^*_{PY} \sim N\left[\mu^*_{PY};\ \frac{var\,(\ell n\, P_{iY})}{n}\right]$$

$$\hat{\mu}^*_{PK} \sim N\left[\mu^*_{PK};\ \frac{var\,(\ell n\, P_{iK})}{n}\right].$$

where $var\,(\ell n\, P_{iY})$ and $var\,(\ell n\, P_{iK})$ denote the variances of $\ell n P_{iY}$ and $\ell n P_{iK}$, respectively.

Therefore,

$$N_2 \rightarrow N\left[0;\ \frac{var\,(\ell n\, P_{iY})}{\varepsilon^2_K}\right]$$

$$N_3 \rightarrow N\left[0;\ \frac{var\,(\ell n\, P_{iK})}{\varepsilon^2_K}\right]$$

and

$$N_2 - N_3 \to N\left[0;\ \frac{var\,(\ell n\ P_{iY}) + var\,(\ell n\ P_{iK}) - 2\ cov(\ell n\ P_{iY},\ \ell n\ P_{iK})}{\varepsilon_K^2}\right]$$

where $cov(\ell n\ P_{iY},\ \ell n\ P_{iK})$ denotes the covariance of $\ell n\ P_{iY}$ and $\ell n\ P_{iK}$.

Finally, since competitive markets imply the independence of prices and quantities, N_1 and $N_2 - N_3$ are independent under the null hypothesis H_0: $\ell n\ R_K = 0$. Therefore, under H_0

$$\frac{n^{\frac{1}{2}}(\ell n\tilde{R}_K - \ell n R_K)}{e_K} \to N\left[0;\ 1 + \frac{var(\ell n P_{iY}) + var(\ell n P_{iK}) - 2cov(\ell n P_{iY},\ \ell n P_{iK})}{\varepsilon_K^2}\right]$$

and

$$23)\quad \frac{n^{\frac{1}{2}}\ (\ell n\ \tilde{R}_K - \ell n\ R_K)}{e_K^*} \to N(0,1)$$

where

$$e_K^* = \left[e_K^2 + \hat{var}(\ell n\ P_{iY}) + \hat{var}(\ell n\ P_{iK}) - 2\hat{cov}(\ell n\ P_{iY},\ \ell n\ P_{iK})\right]^{\frac{1}{2}}$$

and

$\hat{var}$ and $\hat{cov}$ denote the sample variances and covariances.

A test of H_0: $\ell n\ R_K = 0$ can be based on the statistic

$$\frac{n^{\frac{1}{2}}\ \ell n\ \tilde{R}_K}{e_K^*}\ .$$

The completely analogous result for $\ell n\ \tilde{R}_L$ is

$$24)\quad \frac{n^{\frac{1}{2}}(\ell n\ \tilde{R}_L - \ell n\ R_L)}{e_L^*} \rightarrow N(0,1)$$

where

$$e_L^* = \left[e_L^2 + \hat{var}(\ell n\ P_{iY}) + \hat{var}(\ell n P_{iL}) - 2\hat{cov}(\ell n\ P_{iY}, \ell n\ P_{iL}) \right]^{\frac{1}{2}}$$

and

e_L^2 denotes the estimated asymptotic variance of $n^{\frac{1}{2}}(\ell n\ \hat{R}_L - \ell n\ R_L)$.

The test of H_0: $\ell n\ R_L = 0$ can be based on the statistic

$$\frac{n^{\frac{1}{2}}\ \ell n\ \tilde{R}_L}{e_L^*}\ .$$

After all this, there still remains a serious problem which prevents our applying these tests in all of the industries under consideration. In order to compute the necessary sample variances and covariances of the logs of prices we must, of course, have price observations. Labor price data exist in all cases. But it was noted previously that no capital price observations exist at all and that no single output price exists for the multi-product grain and textile firms.

We can "overcome" the capital price problem. The next chapter proposes a price construct composed of depreciation, repair, insurance, tax and interest rates. Since this is intended to represent the mean price we can simply assume that μ_{PK}^* is known. Then the only difference between $\ell n\tilde{R}_K$ and $\ell n\hat{R}_K$ is that the former uses $\hat{\mu}_{PY}^*$ instead of μ_{PY}^*.

The N_3 term is dropped, and

$$25)\quad e_K^* = \left[e_K^2 + v\hat{a}r(\ell n\ P_{iY}) \right]^{\frac{1}{2}}.$$

Since the mean price of capital is "known," no observations on individual firm capital prices are required to conduct the test on R_K.

However, since no single output price, P_{iY}, exists for the grain and textile industry firms, $v\hat{a}r(\ell n\ P_{iY})$ cannot be found. Since this term is required for both e_K^* and e_L^* neither test can be conducted as outlined above. If we are to test R_K and R_L at all, the best we can do is to treat $\tilde{R}_K$ and $\tilde{R}_L$ as if they were the maximum likelihood estimators. We simply take $e_K^* = e_K$ and $e_L^* = e_L$. Because this would actually underestimate the asymptotic variances of $n^{\frac{1}{2}}(\ell n\tilde{R}_K - \ell nR_K)$ and $n^{\frac{1}{2}}(\ell n\tilde{R}_L - \ell nR_L)$, the tests would be biased against H_0. However, certain results for the lumber industry, discussed below, suggest that this bias is insignificant.

For the lumber industry the "correct" tests can be conducted since output price observations exist. These were conducted with e_K^* given by (25) and e_L^* given by (24). The results, expressed as the largest significance level at which H_0 would still be accepted, were compared with the results obtained when $\ell n\tilde{R}_K$ and $\ell n\tilde{R}_L$ were simply treated as maximum likelihood estimators ($e_K^* = e_K$ and $e_L^* = e_L$). In no case was the difference in this significance level even four tenths of one per cent. None of the conclusions about the true values of R_K and R_L would have been altered. Therefore, it was decided to report the test results for all three industries on a comparable basis -- treating $\tilde{R}_K$ and $\tilde{R}_L$ as if they were indeed the maximum likelihood estimators. Hereafter the distinction between $\tilde{R}_K$, $\tilde{R}_L$ and $\hat{R}_K$, $\hat{R}_L$ is dropped. Also, since we are now no longer concerned with the distributions of $\hat{\mu}^*_{PY}$, $\hat{\mu}^*_{PK}$ and $\hat{\mu}^*_{PL}$ we can abandon the log format of (21) and employ the more direct testing format of (20).

Let $G(\underline{\overline{X}}^*; \phi^*)$ denote the joint likelihood function of the n observations on $X_i^{*\prime} = \left[\ell n Y_i \quad \ell n K_i \quad \ell n L_i\right]$. $G(\underline{\overline{X}}^*; \phi^*)$ is a normal distribution with mean M_X^* and covariance matrix Ω, given by (16), and parameter vector ϕ^* given by (17). The information matrix is

$$I(\phi^*) = -E\left[\frac{\partial^2 \ell n\, G(\underline{\overline{X}}^*; \phi^*)}{\partial\phi^*\partial\phi^{*\prime}}\right].$$

The maximum likelihood estimator $\hat{\phi}^*$ is a consistent estimator of the true parameter vector ϕ^*, and $n^{\frac{1}{2}}(\hat{\phi}^* - \phi^*)$ is asymptotically normally distributed with mean vector 0 and covariance matrix

$$T^*(\phi^*) = \lim_{n\to\infty}\left[\frac{1}{n} I(\phi^*)\right]^{-1}.$$

A consistent estimator of this matrix is

$$S^*(\hat{\phi}^*) = \left[\frac{1}{n} I(\hat{\phi}^*)\right]^{-1}.$$

The matrix $I(\hat{\phi}^*)$ is given in Appendix B.

The random variable

$$\frac{n^{\frac{1}{2}}(\hat{R}_K - R_K)}{s_K^*} \text{ is asymptotically distributed } N(0,1)$$

where s_K^{*2} is the diagonal element corresponding to R_K in $S^*(\hat{\phi}^*)$.

The test of H_0: $R_K = 1$ may be based on the random variable

$$\vec{\Lambda} = \frac{n^{\frac{1}{2}}(\hat{R}_K - 1)}{s_K^*}$$

If the test statistic, λ, is small it tends to confirm H_0; if λ is large it tends to refute H_0. Again the result is reported as the largest significance level, δ, at which H_0 would still be accepted,

$$\delta = \Pr\left[\Lambda \geqq \lambda\right] .$$

Similarly, the test on R_L is based on the statistic

$$\frac{\hat{R}_L - 1}{s_L^*}$$

where s_L^{*2} is the diagonal element corresponding to R_L in $S^*(\hat{\phi}^*)$.

An Exact Test on the Existence of Increasing Returns to Scale

When $\theta = 0$ we can conduct an exact test on the value of the returns to scale factor $\alpha_K + \alpha_L$. The (limited information) maximum likelihood estimators for the production function parameters are simply the least squares estimators. Since these no longer depend on θ their exact distribution is known.

We know that

$$\begin{bmatrix} \ell n\, \hat{\Gamma} & \hat{\alpha}' \end{bmatrix} \sim N\left\{ \begin{bmatrix} \ell n\, \Gamma & \alpha' \end{bmatrix}, (F'\,F)^{-1}\,\sigma_{00} \right\}$$

where $\alpha' = \begin{bmatrix} \alpha_K & \alpha_L \end{bmatrix}$

F is given by (18).

Let $L' = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$.

Then,

$$L'\begin{bmatrix} \ell n\, \hat{\Gamma} & \hat{\alpha}' \end{bmatrix} = (\hat{\alpha}_K + \hat{\alpha}_L) \sim N\left[(\alpha_K + \alpha_L),\ L'(F'F)^{-1}\, L\,\sigma_{00} \right]$$

and

$$\frac{(\hat{\alpha}_K + \hat{\alpha}_L) - (\alpha_K + \alpha_L)}{\left[L'(F'F)^{-1}\, L\,\sigma_{00}\right]^{\frac{1}{2}}} \sim N(0,1).$$

also

$$\frac{n\,\hat{\sigma}_{00}}{\sigma_{00}} \sim \chi^2\,(n - 3).$$

Therefore, the random variable

$$\frac{(\hat{\alpha}_K + \hat{\alpha}_L) - (\alpha_K + \alpha_L)}{\left[\dfrac{n\ L'(F'F)^{-1}\ L\ \hat{\sigma}_{00}}{n - 3}\right]^{\frac{1}{2}}} \sim T\ (n - 3)$$

The test conducted is

$$H_0:\ \alpha_K + \alpha_L \leqq 1 \quad \text{against} \quad H_1:\ \alpha_K + \alpha_L > 1.$$

This test may be based on the random variable

$$T = \frac{(\hat{\alpha}_K + \hat{\alpha}_L) - 1}{\left[\dfrac{n\ L'(F'F)^{-1}\ L\ \sigma_{00}}{n - 3}\right]^{\frac{1}{2}}}$$

The test result is again reported as the maximum significance level at which H_0 will still be accepted:

$$\delta = \Pr\left[T \geqq t\right]$$

where t denotes the test statistic.

NOTES

Chapter III

1. Generally we expect returns to scale to either decrease or remain constant as output expands, as in (a) or (b), not to increase. If θ is negative the average and marginal cost functions for our production function are either decreasing throughout or first increasing, then decreasing as output expands. This unorthodox behavior of these cost functions and its implications are discussed below in the context of the competitive model.

The average and marginal cost functions for our production function, derived under the competitive assumption of constant input prices, are:

$$AC(Y) = (\alpha_K + \alpha_L)\, A\, g(Y)$$

$$MC(Y) = (1 + \theta Y)\, A\, g(Y)$$

where

$$g(Y) = \left[Y^{1-(\alpha_K + \alpha_L)}\, e^{\theta Y} \right]^{1/(\alpha_K + \alpha_L)}$$

$$A = \left[(1/\Gamma)\; (P_K/\alpha_K)^{\alpha_K}\; (P_L/\alpha_L)^{\alpha_L} \right]^{1/(\alpha_K + \alpha_L)}$$

P_K, P_L denote the prices of capital and labor, respectively.

The first derivatives with respect to output are:

$$AC'(Y) = A \left[(1 + \theta Y) - (\alpha_K + \alpha_L) \right] \frac{g(Y)}{Y}$$

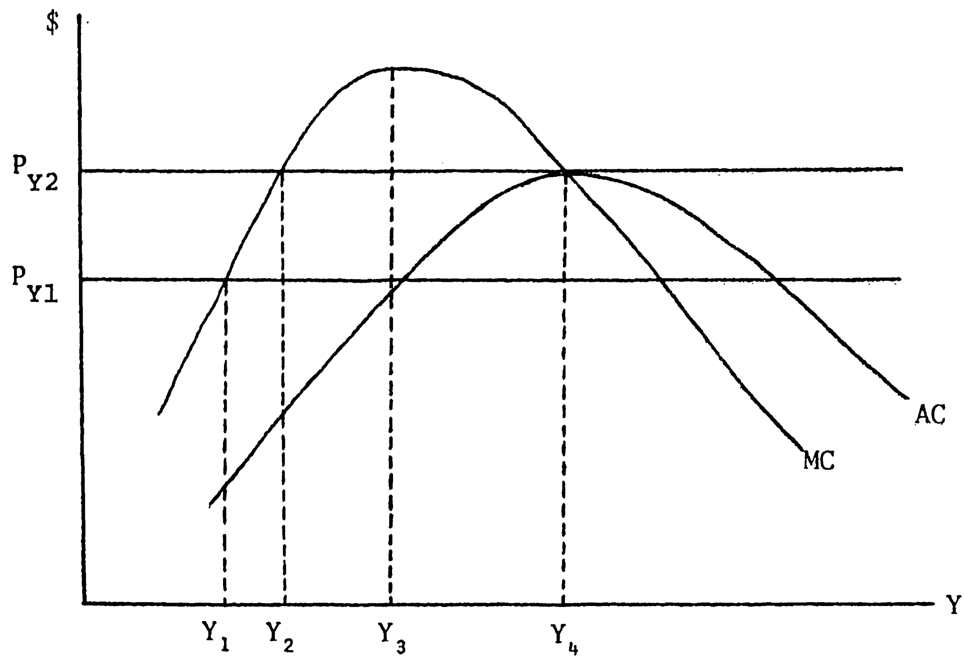
$$MC'(Y) = \frac{A}{(\alpha_K + \alpha_L)} \left[(1 + \theta Y)^2 - (\alpha_K + \alpha_L)\right] \frac{g(Y)}{Y}$$

The behavior of average and marginal cost depends on the sign of θ and the value of $\alpha_K + \alpha_L$. When $\theta < 0$ and

(i) $\alpha_K + \alpha_L \geqq 1$, average and marginal cost are continuously decreasing. Since $\theta < 0$, $1 + \theta Y < 1$ for all $Y > 0$. Therefore, $(1 + \theta Y) < (\alpha_K + \alpha_L)$ and $(1 + \theta Y)^2 < (\alpha_K + \alpha_L)$. Since the second order condition for an interior profit maximum for the competitive firm requires that marginal cost be increasing, it clearly cannot be satisfied.

(ii) $\alpha_K + \alpha_L < 1$, average and marginal cost first increase, then decrease as output expands. Average cost has an extremum where $\alpha_K + \alpha_L = 1 + \theta Y$, and a check of the second derivative will reveal that it attains a maximum there. Since $\alpha(Y) = (\alpha_K + \alpha_L)/(1 + \theta Y)$, this is where returns to scale are one. Where decreasing returns prevail, average cost is increasing, and where increasing returns prevail, average cost is decreasing. Marginal cost attains a maximum where $(1 + \theta Y)^2 = (\alpha_K + \alpha_L)$, which of course is at a lower output than the maximum of average cost and is therefore in the range where decreasing returns prevail. The average and marginal curves for case (ii) are depicted in Figure 2.

Figure 2



The unorthodox cost behavior depicted in Figure 2 has quite unusual implications for firm equilibrium under competitive conditions. At the firm's profit maximizing output we must have price equal to marginal cost and marginal cost increasing. In Figure 2, the output Y_2 satisfies these conditions for the price P_{Y2}. The output Y_4 [note that $\alpha(Y_4)=1$] does not satisfy the second order condition since marginal cost is decreasing there. Note that even though there is a local profit maximum at Y_2, it may not be a global maximum, in sharp contrast to the local maximum which will also be a global maximum when cost curves have the conventional shape.

See Hanoch (27) for a more general discussion of returns to scale, average cost and competitive equilibrium.

2. A third alternative is the maximization of expected profits, suggested by Zellner, Kmenta and Drèze. (43)

3. This condition simply requires that marginal cost be either increasing or, if decreasing, then decreasing less rapidly than marginal revenue.

If $\theta = 0$, in other words, if the production function is the homogeneous Cobb-Douglas function, the condition reduces to:

$$\alpha_K \frac{\beta_Y}{\beta_K} + \alpha_L \frac{\beta_Y}{\beta_L} < 1, \quad (33, \text{p.}10)$$

If, in addition, all markets are competitive, then $\beta_Y = \beta_K = \beta_L = 1$ and the condition further reduces to:

$$\alpha_K + \alpha_L < 1.$$

This is the familiar result that the Cobb-Douglas function must be characterized by decreasing returns to scale for a valid profit maximum to exist.

If $\theta \neq 0$, but all markets are competitive the condition reduces to that given in Note 1 to this chapter, directly above, requiring that marginal cost be increasing at the anticipated output:

$$[1 + \theta\, A(Y_i)]^2 > \alpha_K + \alpha_L$$

4. Consider K_i. From (7b) it can be seen that K_i is a function of $A(Y_i)$, $A(P_{iY}Y_i)$, P_{iK} and u^*_{i1}. $A(Y_i)$ is non-stochastic, $A(P_{iY}Y_i)$ is a function of v^*_{i0}, and P_{iK} is a function of v^*_{i1}. Therefore, K_i is a function of the disturbance terms v^*_{i0}, v^*_{i1} and u^*_{i1}. Since these are all assumed to be independent of u^*_{i0}, K_i itself is independent of u^*_{i0}. A similar argument can be made for the independence of L_i and u^*_{i0}.

5. A considerably more sophisticated estimation technique employing equations (7a) through (7f) in a non-linear simultaneous equations routine might generate consistent estimates.

6. While it is not of any real use, we can also obtain an estimate of $g = \sigma_{11} - 2\sigma_{12} + \sigma_{22}$. Substituting $\hat{R}_{KL}$ into the expression for $(u_{i1} - u_{i2})$ yields $(\hat{u}_{i1} - \hat{u}_{i2})$ and

$$\hat{g} = \frac{1}{n} \sum_{i=1}^{n} (\hat{u}_{i1} - \hat{u}_{i2})^2$$

7. In terms of our model the best description of Genovese's contention is clearly $\theta > 0$, $\alpha_K + \alpha_L > 1$. However, he may have meant that increasing returns prevailed throughout: $\theta = 0$, $\alpha_K + \alpha_L > 1$. Of course, since we can test both θ and $\alpha_K + \alpha_L$, both representations are testable.

8. The estimated information matrix, $I(\hat{\phi}^*)$, associated with the

parameter vector

$$\phi^* = [\,\Gamma \quad \alpha_K \quad \alpha_L \quad R_K \quad R_L \quad \sigma_{00} \quad \sigma^*_{11} \quad \sigma^*_{12} \quad \sigma^*_{22}\,]$$

is given in Appendix B. It was most convenient to obtain e_K^2 by expressing this matrix in terms of $\ell n\hat{R}_K$ and $\ell n\hat{R}_L$ instead of $\hat{R}_K$ and $\hat{R}_L$. This can be accomplished by

i) multiplying the $R_K, R_K|$ and $R_L, R_L|$ terms by $\hat{R}_K^2$ and $\hat{R}_L^2$, respectively

ii) multiplying the $R_K, R_L|$ term by $\hat{R}_K\,\hat{R}_L$

iii) multiplying all other cross partial terms with R_K by $\hat{R}_K$ and all other cross partial terms with R_L by $\hat{R}_L$.

When this is done it can be seen that none of the matrix entries contain any $\ell n\hat{R}_K$ or $\ell n\hat{R}_L$ terms. Furthermore, none of the other parameter estimates depend on $\hat{R}_K$ or $\hat{R}_L$. (This can be verified by checking their expressions in the text.) Therefore, the estimated asymptotic variances of $n^{\frac{1}{2}}(\ell n\hat{R}_K - \ell nR_K)$ and $n^{\frac{1}{2}}(\ell n\hat{R}_L - \ell nR_L)$ do not depend on $\ell n\hat{R}_K$ or $\ell n\hat{R}_L$.

IV

THE MEASUREMENT OF VARIABLES

THE DATA

The data for this study are part of a much larger body of data collected by Fred Bateman, James Foust and Thomas Weiss. A more detailed discussion of the data than is presented here can be obtained from them (2).

The data for the present study consists of random samples of firms drawn from the census manuscripts of manufactures for the census year 1860. The sampling procedure generated approximately 200 firms from each state and contained the following information used in this study:

1. Type of business or product.
2. State location of the firm.
3. Capital invested in the business.
4. Kinds, quantities, and values of raw materials used.
5. Average number of male hands employed and average monthly cost of male labor.
6. Average number of female hands employed and average monthly cost of female labor.
7. Kinds, quantitites, and values of products produced annually.
8. Kinds of motive power or machinery.

THE MEASUREMENT OF VARIABLES

Output, Y_i, *and Revenue,* $P_{iY}Y_i$

The production function is estimated for three industries -- lumber, grain, and cotton and woolen textiles. The varied output of the grain milling and textile industries necessitates the use of value added as a measure of output. However, the output of the lumber industry is homogeneous and measurable in physical units -- board feet of sawed lumber. Also, from the Southern sample of water powered grain milling firms it was possible to cull a reasonably large sample of firms producing only corn meal. The parameters for this sample were estimated for comparison with the value added results.

When $\theta \neq 0$ no measure of $P_Y Y$ is required to calculate $\hat{R}_{KL}$ in (11) since all output related terms canceled when the profit maximizing conditions were divided by each other. However, when $\theta = 0$ a measure of $P_Y Y$ is needed to calculate $\hat{R}_K$ and $\hat{R}_L$ by (19'). Intermediate inputs (e.g., unsawn timber, wheat, raw cotton or wool, fuel) will be handled by the net price approach. It is assumed that J intermediate inputs, M_j, bear a fixed relationship to output:

$$M_j = c_j Y \qquad j = 1, \ldots, J.$$

The profit function can therefore be written:

$$\pi = P_Y Y - \sum_{j=1}^{J} P_{mj} M_j - P_K K - P_L L$$

$$= \left[P_Y - \sum_{j=1}^{J} c_j P_{mj} \right] Y - P_K K - P_L L$$

where P_{mj} denotes the price of M_j.

The maximization of this function subject to the production constraint yields the results described in Chapter III, except that the output price is now re-defined as the net price

$$P_{YN} = P_Y - \sum_{j=1}^{J} c_j P_{mj}.$$

The product of net price and output is

$$P_{YN} Y = P_Y Y - \sum_{j=1}^{J} P_{mj} M_j$$

since $c_j Y = M_j$. Note that this is simply value added. It can be conveniently calculated from the revenue and expenditure data reported, without directly computing the net price.

An adjustment was required for some of the steam powered firms in the lumber and grain industries since less than a third of those firms that reported steam as their source of power actually reported their expenditure on fuel. Therefore, those firms that did report fuel (overwhelmingly wood) were used to estimate expenses for those that failed to do so. In the lumber industry, firms failing to report were charged by multiplying their physical output (in thousands of board feet) by the average ratio of fuel expenditure to physical output of reporting firms. These ratios, followed in parentheses by the number of firms used to compute them, are: South, 0.87, (31); West, 0.94, (24). In the grain industry firms failing to report were charged by multiplying their total revenue by the average ratio of fuel expenditure to total revenue of reporting firms. These ratios and the number of firms on which they are based are: South, 0.023, (16), West 0.03, (22). (No adjustment figures are given here for the North since in both industries the entire sample of steam powered firms was too small to justify carrying out the estimation. No results will be presented for steam powered lumber or grain milling in the North.)

The output price is also further re-defined to take account of interest charges on working capital. It is assumed that the generation of output requires a fixed relationship between the value of working capital, W, and the gross value of output, $P_Y Y$, (i.e., $W = wP_Y Y$). An interest charge, r, is levied against W, so that the net price P_{YN} is now $P_{YN} = [1-rw]\, P_Y - \sum_{j=1}^{J} c_j P_j$. The regional interest rates are discussed below in the section on capital. The fraction, w, of working capital to gross value of output is taken to be the regional average of the state ratios of live assets to value of products for the particular industry as reported in the 1890 Census of Manufactures. These are given in Table 4 below. These "live assets" included the inventories of

Table 4

Ratio of Live Assets to Value of Products

	South	North	West
Lumber	.35	.37	.40
Grain	.09	.19	.13
Textiles	.46	.48	N.A.

N.A.: Not Applicable (No results will be presented for Western textile mills.)

Sources: U.S. Census of Manufacturing, 1890.

Lumber: Part III, Selected Industries, "Forest Industries," Table 2, p. 614.

Grain: Part I, Totals for States and Industries, "Flouring and Grist Mill Products," Table 8, p. 696.

Textiles: Part III, Selected Industries, "Woolen Mills," Table 5, pp. 98-99, "Cotton Mills," Table 2, pp. 194-195. (The figures above are weighted averages based on the regional sample composition.)

finished products and raw materials, cash, accounts receivable, and "all sundries not elsewhere reported." Since these items do not really represent a physical input into the production process, they are treated as influencing the net price of output, not as capital per se. Expenditures on working capital reflect the cost of delay -- for example, of having value tied up in unsold inventory. This is a necessary part of doing business and must be accounted for in the profit maximization process, but does not represent a distinct physical input.

Under the assumed relationship between intermediate inputs and output the product $P_{YN}Y$ is still a measure of value added:

$$P_{YN}Y = (1\text{-}rw)\ P_Y Y - \sum_{j=1}^{J} P_{mj}\ M_j .$$

Gross output revenue is simply "deflated" by (1-rw) before raw material expenditures are deducted. For the lumber industry output is measured in physical units and $P_{YN}Y$ is employed in the profit maximizing conditions where $P_Y Y$ appears. However, since no physical measure of output exists for grain and textile firms, this "deflated" value added, $P_{YN}Y$, is used not only in the profit maximizing conditions but also as a proxy for output, Y, in the production function.

Note that $\hat{\alpha}_K$, $\hat{\alpha}_L$, and $\hat{\theta}Y$ are insensitive to change in the units of measurement of Y. Hence, the returns to scale function $\alpha(Y)$ and the market parameter(s) $\hat{R}_{KL}$ (or $\hat{R}_K$ and $\hat{R}_L$ if $\theta = 0$) are also insensitive. If value added represented simply a change in the units of measure, its use as a proxy for output would produce no errors. If output and intermediate input prices were the same for all firms, this would be true if a single output were produced under the assumed relationship between raw materials and output.

The existence of multiple outputs in the grain and textile industries makes it impossible to strictly interpret value added in this way, even if output and intermediate input prices had been identical, since no single item called "output" exists. However, it is certainly much easier to conceive of the notion of "output" for firms in these industries than it is for modern multi-product conglomerates. Flour industry products consisted of wheat flour, buck wheat flour, rye flour, corn meal, oat meal, rye meal, etc. Textile output consisted of cotton goods, cotton cloth, cotton sheeting, cotton duck, woolen goods, flannel, cassimeres, etc. These are certainly industries in which production consisted of transforming one or another raw material, probably in fixed or very nearly fixed proportion, into one of various but similar outputs by a production process that varied little, if at all, as the particular output being produced changed. One simply ground wheat or rye or corn and produced wheat flour, rye flour, or corn meal. The notion of "output" as a single entity seems clearly justified and value added is certainly the best available measure of it. In the only instance in which it was possible to get a sufficiently large sample -- Southern water powered grain milling -- the parameters for a single output sample (corn meal) were estimated for comparison with the results for the larger value added sample.

Capital, K_i*, and Expenditures,* $P_{iK}K_i$

The "capital invested" data are without a doubt the least satisfactory of the observations. The definition and valuation of capital as reported are unclear. Bateman, Foust, and Weiss observe that "The respondent was to report the value of real and personal estate invested in the business, but no explicit instructions were given as to how broadly the estate was to be defined, nor how it was to be valued . . .. The consensus currently is that the figures are gross book value of invested capital, valued at original cost and excluding the value of working capital" (2, p. 5). Furthermore, such a value is not ideally suited for production function estimation since a measure of the physical flow of services from capital is desired. However, were it not for the fact that we have gross rather than net value, this particular problem would be no worse with this 1860 data than with 1960 data. Even the use of gross value may not cast much shadow on this early historical data, since it is by no means clear that contemporary net figures bear any relationship to the actual physical capacity of capital goods.

The capital figure, as reported, is used as K_i in the production function estimation. For this to produce no estimation errors, it must be that the flow of physical services from capital is proportional to this figure for all firms. Then only Γ is effected, since α_K, α_L, and θY, and hence the returns to scale function $\alpha(Y)$, are insensitive to changes in the units of measurement. If the physical flow, K_i^*, is proportional to the reported value K_i

$$K_i^* = kK_i \text{ for all } i$$

then, if we wish to estimate

$$Y_i e^{\theta Y_i} = \Gamma K_i^{*\,\alpha_K} L_i^{\alpha_L}$$

we may use

$$Y_i e^{\theta Y_i} = \Gamma (kK_i)^{\alpha_K} L_i^{\alpha_L}$$

$$= \Gamma^* K_i^{\alpha_K} L_i^{\alpha_L}$$

where $$\Gamma^* = k^{\alpha_K}\Gamma.$$

This measure of capital is certainly not the only one which could be employed. One alternative was originally used by Zvi Griliches and defined as "the sum of insurance premium, rental payments, property taxes paid, depreciation and depletion and .06 (six per cent) of gross book value . . ." (*26*, p. 280). Zellner and Revankar employ an identical measure, except that six per cent of the net stock is used, in their article on generalized production functions (*44*, p. 249). Estimating CES production functions, Paul Zarembka uses net capital stock, but comments that

"Use of Griliches' service flow concept of capital would have been preferable, but the required data are not available . . ." (*42*, p. 51). The required data are also not available for this 1860 study on a firm by firm basis. Some good approximations can be constructed from various sources -- Bateman, Foust, and Weiss have estimated depreciation rates, data on 1860 property tax rates exist, and the 1890 Census recorded much data on miscellaneous expenditures, including insurance payments. These will be used to estimate expenditures in the profit maximizing conditions. In practice, using these as a measure of capital would produce virtually identical results as using stock, since, except for taxes which vary over states, each will be taken as the same fraction of the reported stock for all firms. Nevertheless, it is not at all clear that Griliches' measure would be "preferable" even if we had the expenditure data for each firm. In fact, it seems to have earned a reverence that even he never had for it. In the article in which it was used, a footnote relates that "Besides the capital service variable described above, two other capital measurements were also tried: gross book value and a capital services concept as above except that six per cent was taken of the net stock of capital." He rejected the first on goodness of fit grounds and the second because results were essentially equivalent to the measure used (*26*, p. 280). That alternatives were tried and judgments based on something as mundane as goodness of fit certainly suggests that the actual measure had no exclusive appeal for Griliches.

The only defense actually offered is: "The flow formulation whenever it is not proportional to the stock, is the more relevant measure of capital services. The variable I use approximates the idea of capital services -- capital stock times ($\delta + r$), where δ is the depreciation rate and r is the interest rate" (*26*, p. 281). The first sentence is an attempt at legerdemain, since it is merely a tautology. It says that if the (physical) flow is not proportional to the stock, then the flow is the more relevant measure of the flow. The problem is not knowing the physical flow, and the issue is what is the best measure of it. Many of the components of the suggested measure -- insurance, taxes, and interest -- are expenditures on capital, the product of prices and quantitites. If prices are the same for all firms, then certainly higher expenditures imply greater physical services. However, it would seem that by their nature these expenditures would be incurred by the firm by applying the insurance rate, tax rate, and interest rate to the value of its (net) stock. Since these prices are identical, the flow measure will be proportional to the stock, and we might just as well use the latter. If these various components of the "use" price of capital differ over firms, how can we be sure, short of knowing both prices and "quantities," that higher expenditures reflect greater physical services and not simply higher prices.

If demand for the capital input is inelastic, the firm facing a higher price will use less input, but will actually expend more. Even if expenditure were less, there is no reason that relative expenditures reflect relative physical usage accurately. In practice then, as a measure of the physical usage of capital, the value of the stock seems at least as good, if not better than the expenditure measure.

Expenditures incurred in the use of capital are considered in the estimation of R_{KL} by (11) or R_K by (19b'), where the term $P_{iK}K_i$ appears. The items considered are depreciation, repairs, insurance, taxes and interest. No data on any of these items were reported in the 1860 Census. The first extensive collection of capital and expenditure data for our three industries appears in the 1890 Census. Depreciation, repairs, and insurance estimates for 1860 were constructed from the 1890 data as described below. There is, of course, the question of whether this is appropriate, but it is certainly preferable to ignoring these expenses completely. Regional averages were used rather than assuming that the state-by-state differences of 1890 also applied in 1860. Tax rate data for 1860 were available, so the state-by-state differences were maintained.

Depreciation rates were estimated following the procedure used by Bateman, Foust and Weiss. (2) The annual depreciation rate of each type of asset was taken to be constant at the rates:

land, 0.0; buildings, 0.02; machinery, 0.05. The fractions of capital (excluding live assets) devoted to land, buildings, and machinery for each industry and state were computed from the reported valuation of these components in the 1890 Census. From these the regional averages were computed. The depreciation rate for a particular industry and region was computed by summing the products of the asset rate and the fraction of capital devoted to that asset. The fractional composition of capital and the resulting depreciation rates for each industry and region are given in Table 5. Depreciation expense for the firm was taken to be the relevant depreciation rate times its reported value of capital.

The estimated depreciation rates do not take account of repair expenditure. The "repair rate" for each industry and region was estimated as the regional average of the state ratios of repair expenditure to value of capital (excluding live assets) for each industry, as reported in the 1890 Census.[1] These repair rates are given in Table 5. Repair expenditure was taken as the product of the repair rate and the firm's reported value of capital. The insurance rate is similarly estimated from the 1890 data, based on the regional average of state ratios of insurance expenditure to value of capital (excluding live assets) for each industry. These rates are also given in Table 5. Insurance expense was taken as the product of the insurance rate and the firm's reported value of capital.

Table 5

Depreciation, Repair, and Insurance Rates

Industry	Region	Composition of Capital: Land	Buildings	Machinery	Depreciation Rate	Insurance Rate	Repair Rate
Lumber	South	.17	.20	.63	.035	.010	.020
	North	.20	.27	.53	.032	.008	.018
	West	.21	.21	.58	.033	.010	.020
Grain	South	.17	.31	.51	.032	.008	.018
	North	.21	.36	.43	.029	.013	.020
	West	.15	.28	.56	.034	.018	.022
Textiles	South	.07	.27	.66	.038	.009	.013
	North	.10	.31	.59	.036	.007	.020

Sources: The composition of capital, insurance rates, and repair rates were computed (as described in the text) from figures in the following tables of the U.S. Census of Manufacturing, 1890:

Lumber: Part III, Selected Industries, "Forest Industries," Table 2, pp. 613-614.

Grain: Part I, Totals for States and Industries, "Flouring and Grist Mill Products," Table 8, pp. 696-697.

Textiles: Part III, Selected Industries, "Woolen Mills," Table 5, pp. 98-99, "Cotton Mills," Table 2, pp. 194-195. (The figures above are weighted averages based on the regional sample composition.)

The depreciation rates were computed following Bateman, Foust and Weiss (2), as described in the text.

Taxes are estimated from 1860 data reported in the 1870 and 1890 Census. The 1870 Census reports both the assessed and true values of real and personal estates by state for 1860. From this the ratio of assessed value to true value is computed for each state. The 1890 Census reports for each state the 1860 ad valorem rate per $100 of total assessed value. This is converted to the rate per $1, multiplied by the ratio of assessed to true value, and the result multiplied by the firm's reported capital figure to estimate taxes. Because this data is for 1860 itself, the state by state differences are certainly meaningful and are maintained. The tax rates per $1 of capital for the various states are given in Table 6.

Finally, an interest charge is levied against the reported value of capital. There is, of course, some difficulty in determining what the rate for each region should be. Conrad and Meyer present several interest rate series -- 1831-1860 rates on prime commercial paper in New York and Boston, 1857-1865 New England bond yields, 1857-1865 call money rates at the New York Stock Exchange, and 1857-1865 railroad bond yields. These rates ranged from six to eight per cent, but unfortunately pertain almost exclusively to Northern rates, are generally short term, or represent more secure returns than the literature suggests were applicable at least to Southern manufacturing. Conrad and Meyer do say, however, that "in the contemporary chronicles southerners and northerners alike considered six to eight per cent . . . a reasonable asking price for loans"

Table 6

Tax Rate Per $1 of Capital

	Ratio of Assessed to True Value[a]	Ad Valorem Rate Per $100 of Assessed Value[b]	Tax Rate Per $1 of Capital
The South			
Alabama	.87	.20	.0017
Arkansas	.82	.35	.0029
Florida	.94	.23	.0022
Kentucky	.80	.41	.0033
Mississippi	.84	.19	.0016
North Carolina	.81	.36	.0029
South Carolina	.89	.26	.0023
Tennessee	.77	.29	.0022
Texas	.73	.20	.0015
Virginia	.83	.56	.0046
The North			
Connecticut	.76	.30	.0023
Delaware	.86	.52	.0045
Maine	.81	1.46	.0118
Maryland	.79	.73	.0058
Massachusetts	.95	.96	.0091
New Hampshire	.79	1.02	.0081
New Jersey	.63	.49	.0031
New York	.75	1.10	.0083
The West			
Illinois	.45	1.57	.0070
Indiana	.78	.90	.0070
Iowa	.83	1.16	.0096
Kansas	.72	.87	.0063
Missouri	.53	1.54	.0082
Ohio	.80	1.00	.0080
Wisconsin	.68	1.25	.0085

Sources: a) Ninth Census (1870), Vol. II, The Statistics of Wealth and Industry, Table 1, "Wealth, Taxation and Public Indebtedness of the U.S.," p. 10.

b) Eleventh Census (1890), Report on Wealth, Debt, and Taxation, Part II, Valuation and Taxation, Table 3, p. 61.

c) Column a x Column b x .01.

(*8*, pp. 101-103). Davis presents 1880-1889 mortgage interest rates by region, and a rate such as this is probably more akin to the rate on industrial loans that we desire. Over this period, these rates averaged 5.8 per cent for the North and Mid-Atlantic states, 7.9 per cent for the South, and 7.3 per cent for the Mid-West (*10*, p. 291). It certainly seems reasonable to use these figures as a floor for the 1860 rates. Also, they certainly reflect the minimum rate differentials that existed among regions in 1860. Inter-regional long term rate differentials were even slower to disappear over the course of the late nineteenth and early twentieth centuries than were short term differentials (*10*, pp. 290-292). If it is feared that the rate for the South may still reflect the ravages of the war, the 7.3 per cent figure for the Mid-West in the 1880's should relieve this anxiety. The interest rates employed in this paper are 6.0 per cent for the North, 7.5 per cent for the West, and 8.0 per cent for the South.

Table 7 summarizes the various charges against capital discussed in this section. It gives, by industry and by state, the factor by which a firm's reported "value of capital invested" was multiplied to obtain the firm's capital expenditure. For brevity these are termed "capital expenditure factors." The figures are the sum of the regional depreciation, insurance and repair rates for the industry, the regional interest rate and the state tax rate.

Table 7

Capital Expenditure Factors

	Lumber	Grain	Textiles
The South			
Alabama	.1467	.1397	.1417
Arkansas	.1479	.1409	N.A.
Florida	.1471	N.A.	N.A.
Kentucky	.1483	.1413	.1433
Mississippi	.1466	.1396	.1416
North Carolina	.1479	.1409	.1429
South Carolina	.1473	.1403	.1423
Tennessee	.1472	.1402	N.A.
Texas	.1465	.1395	.1415
Virginia	.1496	.1426	.1446
The North			
Connecticut	.1203	.1243	.1253
Delaware	.1225	.1265	.1275
Maine	.1298	.1338	.1348
Maryland	N.A.	.1278	.1288
Massachusetts	.1271	.1311	.1321
New Hampshire	.1261	.1301	.1311
New Jersey	.1211	.1251	.1261
New York	.1263	.1303	.1313
The West			N.R.
Illinois	.1450	.1560	
Indiana	.1450	.1560	
Iowa	.1476	.1586	
Kansas	N.A.	.1553	
Missouri	.1462	.1572	
Ohio	.1460	.1570	
Wisconsin	.1465	.1575	

N.A.: Not Applicable - the sample contains no firms from the state.

N.R.: No results will be presented for Western textile mills.

Labor, L_i, *and Wages*, $P_{iL}L_i$

The reported "average number of hands employed" is taken as the labor variable. In lumber milling and grain milling there were so few firms employing female labor that only those employing exclusively male hands were used. In the textile industry, male and female labor are aggregated, females being weighted by the relative wage rate from the sample. This figure was 0.61 in the North and 0.54 in the South. Labor expenses are taken as the product of the reported data on monthly wages and the estimated number of months employed, discussed below.

Bateman, Foust, and Weiss mention several possible problems concerning the labor related data: "the extent to which 'hands employed' reflects full-time versus part-time work, whether the labor figure includes supervisory personnel, clerical help or the owner-manager, (and) which 'average' the monthly cost of labor represents" (*2*, p. 4). There is very little that can be done about these problems, but they are worth noting. The wage rates implied by dividing the reported monthly cost by reported hands employed were inspected for obvious errors, but few observations were rejected from the samples on this ground.

The Operating Period

It was common practice, particularly in the lumber and grain industries, to operate for less than a full year, but the 1860 Census did not record months operated for each firm. Since the labor expense was reported as monthly cost and not as an operating period figure coincident with the time frame of the reported revenue figure we need an estimate of the number of months firms operated.

The 1890 Census presents state data for each of the three industries on the average number of weeks employed for various classes of workers. The average number of weeks employed for "operatives and skilled males above 16 years" was used to compute the regional average operating period (converted to months), which was taken as the 1860 operating period. (In the textile industry there was no need to consider differing terms of employment for males and females, since the regional averages of weeks employed for "operatives and skilled, females above 16 years" differed by only a fraction of a week from the male term of employment.) The operating periods for each industry and region are given in Table 8 below.

An attempt was made to deduce more specific information on operating periods for the lumber industry. The hypothesis was that the differences in the state average operating periods in 1890 were due to the differences in the porportion of water and steam

Table 8

Operating Period (Months)

	South	North	West
Lumber	8.6	8.3	7.7
Grain	6.7	10.3	9.6
Textiles	11.0	11.0	N.A.

N.A.: Not Applicable (No results will be presented for Western textile mills.)

Sources: U.S. Census of Manufacturing, 1890

Lumber: Part III, Selected Industries, "Forest Industries," Table 3, p. 625.

Grain: Part I, Totals for States and Industries, "Flouring and Grist Mill Products," Table 8, p. 698.

Textiles: Part III, Selected Industries, "Woolen Mills," Table 11, p. 135, "Cotton Mills," Table 3, p. 207. (The figures above are weighted averages based on the regional sample composition.)

powered mills in each state. The underlying hypothesis was that rainfall and freezing would effect water powered mills, but not steam mills. The 1890 Census published sufficient data to test this hypothesis. An assumption was made that if the hypothesis was upheld, then we could reasonably expect 1860 operating periods to have been similarly determined by power type.

The model consisted basically of two variables -- a regional dummy and the fraction of steam powered mills in a state. The regional groupings of states were based, as nearly as possible considering the arbitrary nature of states as geographical areas, on climatological data on average rainfall and average number of days below freezing. Several alternative groupings were tried.

The results showed the hypothesis to be completely false. Furthermore, very broad regional groupings largely unrelated to weather data -- such as South, North, and Mid-West -- were only very marginally inferior to more detailed regional formulations. For these reasons it was decided that there was no justification for allowing even the state by state differences in operating periods reported in 1890 to manifest themselves in 1860. The 1860 operating period was taken as the regional average of the 1890 state operating periods.

AN ADJUSTMENT TO THE SAMPLE

Some of the problems with the 1860 Census have already been noted, perhaps the most serious being the question of the definition and valuation of capital. Another problem was also mentioned above -- the lack of data on individual firm operating periods. Some firms in our sample, for one reason or another, may have operated only briefly during the year and are in danger of appearing quite inefficient under the assumed operating period. In some instances an owner did report to the census taker that he had just started operations, or that a fire had forced him to close, but not every owner reported these aberrations. Other problems are simply that the census is known to have made errors, and bookkeeping methods were crude. For all these various reasons, it was felt that the sample should be examined for firms for which the data appear unreasonable.

The difficulty, of course, is that it is hardly fair to base such an examination on personal judgements of what seems reasonable, so an impersonal rule is required. An obvious one is suggested by the theory of the firm -- the elimination of firms making losses in excess of their fixed costs, since these firms violated the profit maximization assumption. By eliminating those firms we are increasing the confidence we may place in that assumption and therefore increasing the confidence we may place in the estimates. Fixed costs were assumed to consist only of interest expenditures and taxes; all other costs were assumed variable.

NOTES

Chapter IV

1. It was noted earlier in the text that our capital data for 1860 most probably did not include the value of working capital. Therefore the measure of capital used from the 1890 Census was that termed "value of plant" -- the total value of land, buildings and machinery. "Live assets" were not included.

V

THE RESULTS:

THE LUMBER, GRAIN, AND TEXTILE INDUSTRIES

LUMBER MILLING

The lumber milling firms in the sample were divided into water powered and steam powered samples, and the parameters were estimated separately since water and steam may well have represented distinct technologies. The estimation of parameters for water powered mills is carried out for all three regions -- the South, the North and the West. Only the South and the West are considered for steam powered milling, as the Bateman, Foust and Weiss sample included only eleven steam powered firms in the North, a sample size too small to justify carrying out the estimation.[1]

The lumber mills included in the sample are only those whose output was recorded as board feet of lumber. The sample was so restricted in order that output would be measureable in physical units, which meant the elimination of firms producing more complex wood products -- staves, shingles, lath, broom handles, shoe boxes, cribs, soil casks and chair stock. Of these various outputs, Southern and Western firms produced only lath and shingles. All the more exotic products were found in the Northern sample. The numbers of firms eliminated from the samples are: 1) the South -- 2 water powered firms; 2) the North -- 22 water powered firms; 3) the West -- 3 water powered and 2 steam powered firms.

The average firm's output, capital, and labor figures for the three regions and their respective states are presented in Table 9 for water powered mills and Table 10 for steam powered mills. Southern water powered firms clearly suffer in size comparisons with firms in other regions. The average Southern firm in the sample produced only three-fifths the output of the average Northern firm and less than half the output of the average Western firm. However, Table 10 reveals a strikingly different comparison for steam powered mills. Southern firms, on the average, produced over twice the output of Western firms.

Table 11 presents the regional percentage distributions of firms by output for the water and steam powered samples. For water powered mills two facts about the South stand out: 1) the conspicuous lack of large firms; only 9 per cent of Southern firms produced more than 300,000 board feet, compared with some 19 and 32 per cent for the North and West, respectively; and 2) the much greater prevalence of small firms producing under 100,000 board feet. In the medium size category, from 100,000 to 300,000 board feet, the South is well represented. For the steam powered mills the comparison of the South and West is very nearly the reverse of the water powered sample. The West is the region over-represented (by comparison) in small output categories and under-represented in large ones. Only 8.6 per cent of the Western firms produced more than 700,000 board feet compared with 46.4 per cent of Southern firms.

Table 9

Water Powered Lumber Milling: State and Regional Averages, 1860

State	Firms	Output (1000 board feet)	Capital ($)	Labor (males)
Alabama	22	173.2	1655	2.6
Arkansas	3	123.3	1700	3.7
Florida	4	222.5	2500	3.5
Kentucky	15	107.0	1173	1.5
Mississippi	4	212.5	1375	2.8
North Carolina	17	108.2	1018	1.8
South Carolina	41	166.1	1686	2.3
Tennessee	5	114.0	1170	1.4
Texas	3	92.0	2833	2.3
Virginia	10	110.4	1750	1.2
The South	124	146.2	1733	2.2
Connecticut	4	117.8	800	1.0
Delaware	9	143.3	2334	1.7
Maine	7	722.1	4491	4.3
Massachusetts	4	146.9	1200	1.0
New Hampshire	11	188.4	1564	1.6
New Jersey	5	174.0	1840	1.2
New York	17	210.2	2056	1.8
The North	57	244.2	2137	1.9
Indiana	19	173.7	1316	1.6
Iowa	11	195.1	1836	1.6
Missouri	2	350.0	3000	3.0
Ohio	4	160.8	1625	1.8
Wisconsin	14	676.4	6700	4.4
The West	50	325.2	3030	2.5

Source: Bateman, Foust, Weiss Manuscript Census Sample

Table 10

Steam Powered Lumber Milling: State and Regional Averages, 1860

State	Firms	Output (1000 board feet)	Capital ($)	Labor (males)
Alabama	12	688.3	5879	6.6
Arkansas	14	790.7	4164	7.4
Florida	14	1258.0	7700	11.9
Kentucky	14	587.9	2750	5.7
Mississippi	19	768.8	5711	8.4
North Carolina	9	1149.3	8856	8.9
South Carolina	5	590.0	4100	6.6
Tennessee	10	366.4	1950	4.7
Texas	18	786.9	5558	6.8
Virginia	10	503.4	3693	7.7
The South	125	767.5	5123	7.6
Illinois	17	265.8	2441	3.9
Indiana	33	333.6	1815	3.1
Iowa	23	215.1	2280	3.3
Missouri	7	609.1	3350	6.1
Ohio	7	425.7	1914	2.9
Wisconsin	6	833.3	4083	5.2
The West	93	351.8	2314	3.6

Source: Bateman, Foust, Weiss Manuscript Census Sample

Table 11

Percentage Distribution of Firms by Output, by Region, 1860

	Water Powered Lumber Mills			Steam Powered Lumber Mills	
Output (1000 board feet)	South	North	West	South	West
0.0 - 99.9	35.5	15.8	16.0	0.0	8.6
100.0 - 299.9	55.6	64.9	52.0	14.4	39.8
300.0 - 699.9	8.1	14.0	26.0	39.2	43.0
700.0 -	0.8	5.3	6.0	46.4	8.6

Source: Bateman, Foust, Weiss Manuscript Census Sample

The Production Technology -- Water Powered Mills

The production function and market parameters for water powered lumber milling are presented in Table 12. The estimated returns to scale function[2] for the South is

$$\hat{\alpha}\ (Y) = \frac{\hat{\alpha}_K + \hat{\alpha}_L}{1 + \hat{\theta}Y} = \frac{0.87}{1 + .0003Y}$$

where Y is output in thousands of board feet.

Despite their smaller size, no Southern firms were trapped in the range of increasing returns, since no such range existed. Returns to scale fell from 0.87 at zero output to zero as output approached infinity. For the firms in the sample, returns to scale ranged from 0.86 for the smallest firm (Y = 20) to 0.71 for the largest (Y = 720). At the average output level of 146,200 board feet returns to scale were 0.83. This absence of a range of increasing returns is strongly confirmed by the results of the hypothesis test H_0: $\alpha_K + \alpha_L \leq 1$ vs. H_1: $\alpha_K + \alpha_L > 1$, presented in the bottom panel of Table 12.[3] The value of δ reveals that H_0 would be accepted even if the test is subjected to an 85 per cent chance of falsely rejecting H_0. In addition, the value of .33 for δ in the test of $\theta = 0$ suggests that returns to scale did not actually vary with the level of output at all.

The situation of water powered Western firms was very similar to that of Southern firms. Returns to scale descended from 1.0 as output increased. Over the sample, returns to scale fell from 1.00 for the smallest firm ($Y = 40$) to 0.85 for the largest ($Y = 4500$). The variation over the sample is even less than this range indicates. The second largest firm produced 1,200,000 board feet, and $\hat{\alpha}(1200) = 0.96$. In other words, 49 of the 50 firms had returns to scale between 1.0 and 0.96. At the average output level of 325,000 board feet, returns to scale were 0.99. As with the Southern firms, the test on $\alpha_K + \alpha_L$ confirms that there was no range of increasing returns available to Western firms. Also the test on θ indicates that $\theta = 0$ can be confidently accepted.

The results for the North present an interesting outcome, namely $\hat{\theta} < 0$. This implies that returns to scale actually increased, rather than decreased, as output increased. In this case, the firm's average and marginal cost curves will be hump-shaped rather than U-shaped -- each attains a maximum and then decreases. (See Note 1 of Chapter III for a demonstration of this point.) Such unorthodox cost behavior is, of course, contrary to the conventional wisdom of economic theory.

However, the restriction $\theta \geqq 0$ was not actually imposed in the estimation procedure. $\hat{\theta}$ is a random variable and may take on negative values as it has done in this case. One way to regard this result is simply to conclude that $\hat{\theta} < 0$ implies that the true

θ is indeed zero. Of course the regular hypothesis test we are using -- of whether θ is zero or positive, will support the conclusion that $\theta = 0$ even more strongly than it does for the South and West. However, the negative estimate of θ could also indicate some fundamental source of model failure. If the true θ is negative its conflict with our a priori notions of production would force the conclusion that either 1) the model is inappropriate for the North -- perhaps the production function is wrong or the profit maximization assumption invalid or perhaps small and large firms used different technologies, or 2) there are some serious problems with the data. Therefore the following test on θ was conducted for the North to determine if θ is in fact significantly negative:

$$H_0: \ \theta = 0 \quad \text{vs. } H_1: \ \theta < 0$$

The test statistic remains -0.61. Even if this test is subjected to a 27% chance of falsely rejecting H_0 (i.e. $\delta = .27$), we would still accept H_0. This result certainly seems to indicate that θ is not negative, so we accept $\theta = 0$.

Since no region in the water powered group displays returns to scale that varied significantly with the level of output, tests of the degree of homogeneity and market conditions were carried out by imposing $\theta = 0$. These results are presented in Table 13. Since the true production functions are homogeneous, returns to scale are the same regardless of firm size -- 0.83 in the South, 0.98 in the North, and 0.97 in the West. The tests on $\alpha_K + \alpha_L$, which are now exact rather than large sample tests, indicate that there were no increasing returns to scale in any region.

The Production Technology -- Steam Powered Mills

The results for steam powered lumber mills are presented in Table 14. Returns to scale fell from 1.05 in the South and from 1.04 in the West as output increased. In the South, returns to scale for the sample firms ranged from 1.04 for the smallest (Y = 120) to 0.72 for the largest (Y = 5000), with 97% of the firms in the range from 1.04 to 0.88. At the average output of 767,500 board feet returns to scale were 0.985. In the West the range was from 1.04 (Y = 50) to 1.01 (Y = 2000), and at the average output of 351,800, $\hat{\alpha}(Y) = 1.03$. The tests on $\alpha_K + \alpha_L$ for both regions indicate that we may reject the existence of any significant increasing returns. Furthermore, as with the water powered group, the tests on θ indicate that returns to scale did not actually vary significantly with the level of output. This conclusion is on more solid ground for the West than the South (as indicated by the larger value of δ), but even for the South the test stands up to nearly a 1 in 5 chance of falsely rejecting $\theta = 0$. The results with $\theta = 0$ imposed are presented in Table 15. Returns to scale are 0.97 in the South and 1.04 in the West for all firms, with no significant increasing returns in either region.

Steam power appears to have been a technological boon for Southern firms but not for Western firms. (All references to parameter values in this discussion refer to the results for the homogeneous cases in Tables 13 and 15, since $\theta = 0$ has been accepted for both power groups in both regions.) A comparison of steam and water technologies in the South indicates that steam powered firms had the advantage of a neutral technological coefficient, Γ, that was nearly double (19.43 vs. 10.44) that of water powered mills and of a labor coefficient some 36% higher (0.72 vs. 0.53). This advantage is clearly reflected by the fact that some 85% of the Southern steam powered mills in the sample produced 300,000 board feet or more, compared with a mere 9% of the water powered firms (see Table 11).

Steam power, however, did not confer any such advantage on Western mills. Steam and water power production technologies were virtually identical there, as the estimates in Tables 13 and 15 reveal. This technological similarity is reflected in the similar sizes of the average firm in the two groups. The average water powered mills in our sample produced some 325,200 board feet, the average steam powered mill 351,800 board feet. In comparison, the average steam powered Southern mill produced over twice the output (767,500 board feet) of its Western counterpart. Our evidence points to a clear regional technical superiority, not in the West as one might expect from the tone of the historical literature, but in the South. The difference is in a technical coefficient, Γ, of 19.43 in the South versus 8.25 in the West. The capital coefficient was actually lower in the South (0.25 vs. 0.35) and the labor coefficient only insignificantly greater (0.72 vs. 0.68). But, given the same quantities of capital and labor to work with, Southern steam powered mills could produce over twice the output of their Western counterparts.

Whatever gave rise to this regional technical superiority in 1860, it did not persist. By 1890, when detailed industry data on power sources were first compiled for the published Census reports, steam was the source of 94 per cent of the total horsepower of lumber mills in the Southern states in our sample and the source of 92 per cent in the Western states. By this time the average Western lumber mill generated a value added of $9024 compared with $6641 for the average Southern mill.[4]

Market Conditions

Since the hypothesis $\theta = 0$ has been accepted in all cases, more information on market conditions can be secured from the homogeneous rather than the non-homogeneous cases -- in other words from $\hat{R}_K$ and $\hat{R}_L$ rather than $\hat{R}_{KL}$. The estimates of R_{KL} are less than one (see Tables 12 and 14) in every region (and in both power groups for the South and West). Since $R_{KL} = \beta_K/\beta_L$ these values indicate that in each region the capital market was more competitive than the labor market (i.e., the elasticity of supply was greater).

In the water powered group, relative factor market conditions appear to have been virtually identical in the South and West ($\hat{R}_{KL}$ = 0.79 for the South, 0.71 for the West), but in the North the labor market was even less competitive relative to the capital market there than in other regions ($\hat{R}_{KL}$ = 0.47). In the South the estimate of R_{KL} for the steam group (0.74) is little different from the water power result. However, in the West the relative non-competitiveness of the labor market appears to have been less severe for steam powered firms ($\hat{R}_{KL}$ = 0.92). Unfortunately, since our estimator of R_{KL} is not a maximum likelihood estimator we have no hypothesis tests to guide us in judging the results. However, the deviations from 1 in all cases except steam powered milling in the West certainly appear substantial. Furthermore, since there is little reason, in the West, to expect the value of R_{KL} to have any relation to mode of power, we could reasonably argue that the difference there is probably entirely random.

The estimates of R_K and R_L in Tables 13 and 15 provide much more information on market conditions. For the South and West we have estimates on R_K and R_L from both the water and steam powered groups. Superficially it might seem that we should expect these estimates to be the same in a region, except for some random statistical variation. However, there is a reason why this might not be true. If the specification that the output demand and input supply functions possessed a constant elasticity throughout does not actually hold over all levels of output demanded or factors supplied and if water and steam powered firms differed substantially in output sold or factors purchased we might find that R_K and/or R_L differ. The figures in Tables 9 and 10 reveal that the average Western water and steam powered lumber mills employed similar amounts of labor and capital to produce similar outputs. We would therefore expect that the estimates of R_K and R_L from these groups would not differ significantly, and a glance at the standard errors verifies this. The values of R_K -- 1.44 and 1.60 -- do not seem to have differed significantly from 1 since the hypothesis tests result in values of .23 and .16 for δ. On the other hand, R_L is clearly not 1 in either case. The hypothesis $R_L = 1$ would be accepted only at significance levels of 4% or less for the water powered group and only at 2% or less for the steam group. If it is accepted that the hypothesis tests on R_K indicate $R_K = 1$ and hence $\beta_Y = 1$, then the source of deviation of $R_L = \beta_L/\beta_Y$ from 1 must be a less than competitive labor market.

Unlike the Western firms, Southern water and steam powered lumber mills were clearly different breeds. The average Southern steam powered firm produced 5.2 times the output and employed 3 times the capital and 3.5 times the labor of its water powered counterpart. Still, the estimates of R_K -- 1.24 and 1.59 -- are not significantly different. The acceptance of the null hypothesis $R_K = 1$ is supported more strongly in the water powered case where $\delta = .20$ than in the steam case where δ = .10. However, the significance level of 10% (δ = .10) is a commonly employed, and presumably a commonly accepted, level for conducting such tests. Some readers will be content with the conclusion that the capital and output markets were perfectly competitive (as implied by $R_K = 1$) over the entire range of capital employed and output produced encompassed by the two groups of firms. For those that might object it is still clear that if either market was less competitive at the higher levels of the steam group, it was not particularly serious. The fact that steam technology was accompanied by a five-fold increase in average firm size must certainly indicate a very accommodating demand structure.

The estimates of R_L for the two groups of Southern firms are another matter. The change from 1.55 to 2.26 is quite dramatic (note that the standard errors are nearly identical). In both cases the hypothesis $R_L = 1$ is firmly rejected. If we accept the conclusion above that the output market was competitive, both the divergence of R_L from 1 and the change in R_L between groups must have been caused by the labor market. The supply of labor was significantly less than perfectly elastic overall and became even less elastic as more labor was demanded.

For the Northern mills the value of 0.80 for $\hat{R}_K$ violates the parameter restriction $R_K \geq 1$. As with $\hat{\theta}$, $\hat{R}_K$ is a random variable and may take on any value since $R_K \geq 1$ was not imposed in the estimation. However, R_K does not differ significantly from 1. A test of H_0: $R_K = 1$ vs. H_1: $R_K < 1$ would reject H_0 only at significance levels above .23, a value which most would feel certainly affirms $R_K = 1$. Thus, in all regions the capital and output markets were competitive. And, as in the other regions, the labor market in the North was not. $\hat{R}_L = 1.69$ and differs significantly from one.

Table 12

Water Powered Lumber Milling

Production Function and Market Parameter Estimates*

Region (n)	Γ	α_K	α_L	θ ($\times 10^{-4}$)	R_{KL}	$\bar{R}^2 \mid \theta = \hat{\theta}$
	Parameter					
The South (124)	9.66	0.31	0.56	2.99	0.79	0.49
	(4.78)	(0.075)	(0.109)	(6.70)		
The North (57)	24.79	0.22	0.69	-1.64	0.47	0.62
	(16.18)	(0.093)	(0.135)	(2.68)		
The West (50)	9.69	0.35	0.65	0.39	0.71	0.61
	(9.70)	(0.145)	(0.195)	(1.65)		

* Large sample standard errors appear in ( ).

Hypothesis Tests

Test / Region	$H_0: \theta = 0$; $H_1: \theta > 0$		$H_0: \alpha_K + \alpha_L \leqq 1$; $H_1: \alpha_K + \alpha_L > 1$	
	λ	δ	λ	δ
The South	0.45	.33	-1.05	.85
The North	-0.61	.73	-0.62	.73
The West	0.24	.40	0.02	.49

Table 13

Water Powered Lumber Milling

Production Function and Market Parameter Estimates Under $\theta = 0$ *

Region (n)	Γ	α_K	α_L	R_K	R_L	R^2
	Parameter					
The South (124)	10.44 (4.64)	0.30 {0.065}	0.53 {0.082}	1.24 (0.283)	1.55 (0.253)	0.48
The North (57)	22.46 (14.87)	0.24 {0.094}	0.74 {0.119}	0.80 (0.318)	1.69 (0.291)	0.63
The West (50)	10.45 (9.80)	0.34 {0.136}	0.63 {0.177}	1.44 (0.585)	2.03 (0.593)	0.60

* Large sample standard errors appear in ( ), exact standard errors in { }.

Hypothesis Tests

Test / Region	$H_0: R_K = 1$; $H_1: R_K > 1$		$H_0: R_L = 1$; $H_1: R_L > 1$		$H_0: \alpha_K + \alpha_L \leq 1$; $H_1: \alpha_K + \alpha_L > 1$	
	λ	δ	λ	δ	t	δ
The South	0.84	.20	2.18	.015	-2.30	.99
The North	-0.64	.74	2.37	.01	-0.24	.59
The West	0.75	.23	1.74	.04	-0.21	.58

Table 14

Steam Powered Lumber Milling

Production Function and Market Parameter Estimates*

Region (n)	Γ	α_K	α_L	θ $(x10^{-4})$	R_{KL}	$R^2 \mid_{\theta = \hat{\theta}}$
The South (125)	14.68 (9.17)	0.28 (0.083)	0.77 (0.104)	0.943 (1.043)	0.74	0.63
The West (93)	8.18 (7.87)	0.36 (0.138)	0.68 (0.151)	0.108 (2.677)	0.92	0.45

* Large sample standard errors appear in ().

Hypothesis Tests

Test / Region	$H_0: \theta = 0$; $H_1: \theta > 0$ — λ	δ	$H_0: \alpha_K + \alpha_L \leqq 1$; $H_1: \alpha_K + \alpha_L > 1$ — λ	δ
The South	0.90	.18	0.48	.32
The West	0.40	.48	0.23	.41

Table 15

Steam Powered Lumber Milling

Production Function and Market Parameter Estimates Under $\theta = 0$*

Region (n)	Γ	α_K	α_L	R_K	R_L	R^2
	Parameter					
The South (125)	19.43	0.25	0.72	1.59	2.26	0.62
	(9.88)	{0.071}	{0.082}	(0.454)	(0.273)	
The West (93)	8.25	0.35	0.68	1.60	1.73	0.45
	(7.72)	{0.135}	{0.136}	(0.617)	(0.363)	

* Large sample standard errors appear in ( ), exact standard errors in { }.

Hypothesis Tests

Test / Region	$H_0: R_K = 1$, $H_1: R_K > 1$		$H_0: R_L = 1$, $H_1: R_L > 1$		$H_0: \alpha_K + \alpha_L \leqq 1$, $H_1: \alpha_K + \alpha_L > 1$	
	λ	δ	λ	δ	t	δ
The South	1.30	.10	4.62	<.001	-0.39	.65
The West	0.98	.16	2.0	.02	-0.26	.60

GRAIN MILLING

As with the lumber mills, the grain mills were divided into water and steam powered samples. Again, too, there were not enough steam powered mills in the North to justify estimating the parameters. As discussed in Chapter IV, in general (net) value added was the measure of output used in estimating the production function because of the multiple products of the industry -- flour, meal, and feed from the various grains. However, for a sub-sample of Southern water powered firms it was possible to estimate the parameters using physical output. The 46 firms in this sub-sample produced only corn meal.

The average firm's output, capital, and labor figures for the regions and their respective states are presented in Table 16 for water powered mills and Table 17 for steam powered mills. The average Southern water powered firm produced only seven-tenths the output of the average Northern firm and barely over four-tenths the output of the average Western firm. While the average Southern steam powered mill compares more favorably with the West -- producing nearly eight-tenths the output -- steam certainly appears not to have conferred anywhere near the benefits on Southern grain mills as it did in the lumber industry.

Table 18 presents the regional distributions of water and steam powered firms by value added. The South is over-represented in

Table 16

Water Powered Grain Milling: State and Regional Averages, 1860

State	Firms	Value Added ($)	Capital ($)	Labor (males)
Alabama	27	1362	2109	1.4
Arkansas	8	3224	3469	1.8
Kentucky	21	2083	3790	1.6
Mississippi	10	1351	2354	1.5
North Carolina	38	1008	2482	1.4
South Carolina	25	1236	3134	1.5
Tennessee	8	1687	4938	1.5
Texas	8	2503	1775	1.1
Virginia	41	3090	6336	2.6
The South	186	1878	3623	1.7
Connecticut	3	1180	2667	1.0
Delaware	15	1510	2823	1.5
Maine	1	330	1200	1.0
Maryland	13	4022	6077	1.6
Massachusetts	1	1795	4000	1.0
New Hampshire	5	2369	4480	1.2
New Jersey	13	2175	5423	1.6
New York	12	3896	6642	1.9
The North	63	2658	4875	1.6
Illinois	6	4839	10000	2.0
Indiana	17	4474	8697	2.5
Iowa	13	3795	5808	1.9
Kansas	1	5709	12000	5.0
Missouri	2	2526	5250	1.5
Ohio	3	5553	10200	3.3
Wisconsin	17	4606	8382	2.7
The West	59	4409	8118	2.4

Source: Bateman, Foust, Weiss Manuscript Census Sample

Table 17

Steam Powered Grain Milling: State and Regional Averages, 1860

State	Firms	Value Added ($)	Capital ($)	Labor (males)
Alabama	1	474	2750	2.0
Arkansas	3	2055	5000	2.7
Kentucky	10	4358	7655	2.5
Mississippi	6	2920	4133	1.7
North Carolina	4	894	2175	1.8
Tennessee	5	3702	2790	2.0
Texas	28	5400	4834	3.3
Virginia	1	45752	35000	10.0
The South	58	4944	5381	2.8
Indiana	12	7605	12525	4.1
Illinois	6	3957	14167	3.8
Iowa	8	9112	9750	5.0
Kansas	3	1236	2500	2.0
Missouri	4	4595	12250	4.0
Ohio	3	3745	5833	3.0
Wisconsin	1	11610	20000	4.0
The West	37	6292	11008	4.0

Source: Bateman, Foust, Weiss Manuscript Census Sample

Table 18

Percentage Distribution of Firms by Value Added, by Region, 1860

Value Added ($)	Water Powered Grain Mills			Steam Powered Grain Mills	
	South	North	West	South	West
0 - 499	29.6	15.9	3.4	22.4	0.0
500 - 999	25.3	25.4	8.5	20.7	10.8
1000 - 1999	25.3	28.6	15.2	20.7	18.9
2000 - 9999	18.8	23.8	66.1	20.7	45.9
10000 -	1.1	6.4	6.8	15.5	24.4

Source: Bateman, Foust, Weiss Manuscript Census Sample

small output categories and under-represented in large ones, relative to the other regions. The comparison with the North is not really very harsh, but that with the West certainly is.

The Production Technology -- Water Powered Mills

Table 19 presents the results for water powered mills. (See page 167) For the South we have two sets of parameter estimates. For the large Southern sample, in which value added was used in the production function, the variation in returns to scale as value added increased is barely perceptible. In fact, no differences in returns to scale among the firms are noticeable at all unless $\hat{\alpha}(Y)$ is expressed with more than two decimal places. Returns to scale were 1.1799 for the smallest firm, with a value added of \$96, and were 1.1783 for the largest, with a value added of \$13,272. This constancy of $\hat{\alpha}(Y)$ is reflected in the hypothesis test on θ -- H_0: $\theta = 0$ would be accepted even if the test is subjected to a 50% chance of falsely rejecting H_0. The evidence from this sample clearly indicates a homogeneous production function.

For the physical output sub-sample, returns to scale fell from 1.28 as output increased, ranging over the sample from 1.25 at the smallest output of 500 bushels of corn meal to 0.67 at the largest, 20,000 bushels. However, except for a few relatively large firms, most firms operated within a much narrower range. The second largest firm produced 12,000 bushels ($\hat{\alpha}(Y) = .83$), and 42 of the 46 firms had an output of 7200 bushels or less. Thus, returns to scale for most firms were between 1.25 and .97. At the average output of 4295 bushels returns to scale were 1.07.

The estimates of $\alpha_K + \alpha_L$ from the two samples are nearly the same - 1.18 and 1.28. The noticeable difference is in the behavior of returns to scale, $\alpha(Y)$. This, of course, is a result of a difference in the significance of θ, and is reflected by the difference in the maximum significance levels to which the test on θ can be subjected and still accept $\theta = 0$ -- 50% in the value added case, 20% in the physical output case.[5] While the latter result still suggests that homogeneity should be accepted, it is worth noting that under certain conditions this kind of difference in the behavior of returns to scale may result from the use of value added as a measure of output.

Suppose that imperfect conditions actually prevailed in the output market. Then a rise in output would have been accompanied by a fall in price. Thus, if inputs were increased by some multiple, the consequent increase in revenue, and hence in value added, would have been less than the increase in physical output.[6] The variation in value added over a sample of firms would be flatter than the variation in physical output. We might expect this to depress the value of $\hat{\theta}$ when value added is used in the production function. There is not much that can be done about this problem, except perhaps to recognize it and be more cautious about accepting $\theta = 0$. We must allow for output market imperfection in order to conduct an internally consistent inquiry into Genovese's hypothesis. And there is no other reasonable alternative to the use of value added in multi-product industries. Its use here for the large sample of Southern grain mills may have suppressed some variation in returns to scale, since, as we will argue below, the market parameter estimates suggest that these firms did indeed face imperfect conditions in their output market. Nevertheless, since $\theta = 0$ is also supported by the results from our sub-sample where physical output is employed, we can confidently accept that the production function was homogeneous.

The results for the homogeneous cases are presented in Table 20. The Southern value added sample had returns to scale everywhere of 1.18, and the hypothesis test confirms the existence of increasing returns ($\delta = .04$). The physical output sample had returns to scale of 1.07. The hypothesis test in this case does not support $\alpha_K + \alpha_L > 1$. However, the production function regression in this case is quite poor. The R^2 is less than half that of the value added regression (0.25 vs. 0.54), and the standard errors of $\hat{\alpha}_K$ and $\hat{\alpha}_L$ are over two and three times as large, respectively, despite the nearness of their actual values. It was decided, therefore, to accept the value added result as indicative of Southern conditions.

The results for Northern firms reveal that returns to scale fell from 1.68 as output increased. Over the sample the range was from 1.67 at Y = \$244 to 1.37 at Y = \$18,632. At the average value added of \$2658, $\hat{\alpha}(Y) = 1.62$. The test of $\theta = 0$ supports its acceptance ($\delta = .34$), and the results in Table 20 show returns to scale of 1.61

everywhere, which the hypothesis test confirms as significantly greater than one ($\delta < .001$). The most striking thing about the Northern results is the magnitude of the labor coefficient. Labor appears to have been more productive in the North than in the rest of the country. It is interesting that the same pattern -- a higher labor and lower capital coefficient, was also true in lumber milling, although it was not as pronounced.

Water powered grain production in the West was not characterized by the increasing returns to scale that prevailed in other regions. Returns to scale fell from 1.13, ranging from 1.12 (Y = \$179) to 0.59 (Y = \$16,278) over the sample. At the average value added of \$4409, $\hat{\alpha}(Y) = 0.90$. The existence of a non-zero θ is supported more strongly for the West than for other regions, but it still seems that returns to scale did not vary significantly with output ($\delta = .12$). The result for $\theta = 0$ shows $\hat{\alpha}_K + \hat{\alpha}_L = 0.96$, and the hypothesis test strongly supports $\alpha_K + \alpha_L \leqq 1$.

The Production Technology -- Steam Powered Mills

The Southern and Western results for steam powered mills are presented in Table 21. In the South returns to scale fell from 1.37, narrowly ranging from 1.36 (Y = \$183) to 1.35 (Y = \$77,473) over the entire sample, with a value of 1.36 at the average value added, \$4944. In the West, returns to scale fell from 1.24, varying from 1.22 (Y = \$512) to 0.93 (Y = \$30,794) over the sample. At the average value added of \$6292, $\hat{\alpha}(Y) = 1.15$. In neither region is θ

significantly different from 0. The $\theta = 0$ results in Table 22 show returns to scale of 1.37 in the South and 1.18 in the West. The hypothesis tests confirm the existence of significant increasing returns in the South, but not in the West.

Market Conditions

Since $\theta = 0$ has been accepted in all cases $\hat{R}_K$ and $\hat{R}_L$ can be used to analyze market conditions. These of course confirm the information of $\hat{R}_{KL}$ -- that in all regions the labor market was less competitive than the capital market. This is exactly the situation that was found in the lumber industry.

For the South the estimates of R_K are 1.73 and 2.80 for the water and steam powered groups, respectively. The hypothesis tests clearly indicate that R_K differed significantly from one in both cases ($\delta < .001$, $\delta = .002$, respectively).[7] Either the capital or output market, or both, that faced Southern grain mills must have been less than perfectly competitive. This contrasts sharply with the conclusion for the lumber industry that $R_K = 1$. If the firms in both these industries participated in the same capital market, which was competitive according to the lumber industry result, then the divergence of R_K from 1 in grain milling implies a less than perfectly elastic demand for grain in the South.

There are several points that support this conclusion. First, the distributions of the sample firms by state are very similar in both industries - no state's representation varies by more than eight per cent. There would seem little chance that the different results could be attributable to any differences that might have existed in capital markets at the state level. Second, the average capital invested was similar in both industries - \$3435 for lumber mills, \$4041 for grain mills. Grain mills were not operating in the upper reaches of the capital supply function where we might find capital available in less elastic supply. Furthermore, given the similarity in location by state, the general dispersion of both activities within states and the similarity in capital requirements, it seems much more likely that firms were using the same, rather than distinct, sources of supply. Finally, there is no apparent reason to believe that suppliers of capital would have had any relative prejudice against grain mills. While Bateman, Foust and Weiss found that lumber milling offered the highest rate of return among the most prevalent Southern manufacturing activities, flour milling offered the second highest (3). Between 1850 and 1860 the value of capital invested per firm grew by nearly forty per cent in grain milling, compared with just over twenty-three per cent in lumber milling. For these various reasons it seems likely that lumber and grain mills faced similar conditions of capital supply, and we therefore conclude that an imperfect output market explains the divergence of R_K from 1 for the grain industry.

This important conclusion can be further supported by a comparison of the estimates of R_L for the two industries. For the lumber industry we estimated R_L to be 1.55 for water powered firms and 2.26 for steam powered firms. Since we concluded that the demand for lumber was perfectly elastic, the fact that R_L significantly exceeded one led us to the further conclusion that the labor market was imperfect. If both industries participated in the same labor market and if the demand for grain products were also perfectly elastic, we would expect to find comparable values of R_L in both industries. However, our estimates of R_L for the grain industry are 3.75 and 3.38 for water and steam powered firms, respectively. If labor market conditions were similar these much higher values can only be explained by an imperfect grain output market.

As with the capital market, there are several reasons for believing that the industries did indeed participate in the same labor market. Again, the similarity in the location of our sample firms by state and the general dispersion of both activities within states argue for this conclusion. So too does the similarity in the amount of labor employed. Finally, the kindred production processes of lumber and grain milling surely did not require labor with distinctly different skills.

If the labor markets were the same, the grain industry estimates of R_L should be compared with the 1.55 estimate for water powered lumber mills. Average employment there was 2.2 persons and was 1.7

and 2.8 persons in water and steam powered grain milling. Recall that steam powered lumber mills employed 7.6 persons on average and that we attributed the higher estimate of R_L in that instance to a lower elasticity at this higher rate of supply. When compared with 1.55, the grain industry estimates of 3.75 and 3.38 provide even more dramatic evidence of an imperfect output market for grain in the South.

It was mentioned in Chapter II that there has been vigorous debate in the literature on the extent of Southern demand for Western foodstuffs. At issue is the nature of growth in the antebellum United States -- whether Southern demand for Western grain fueled the Western expansion (*17, 18, 19, 20, 24, 34*). Our results provide support for the Fishlow and Gallman position that Southern importation of grain was not significant. Southern demand for imports could not have been extensive if even the individual Southern producers in this unconcentrated industry would have had to lower prices to expand output.

The difference between the estimates of R_L (3.75 and 3.38) for Southern water and steam powered grain mills is obviously entirely random. It is not, however, so obvious that the difference between the estimates of R_K - 1.73 and 2.80 - is just random. If it is not, then either the capital or output market, or both, facing steam powered firms was less competitive. However, even though the average "output" of steam firms was 2.6 times that of water powered mills, there is no evidence from our separate estimates of R_L that demand was less elastic at higher output levels. $\hat{R}_L$ is not greater, but in fact slightly lower, for steam mills. Then, was the capital market responsible? Steam mills on average did employ more capital, but the difference is not dramatic - \$5381 compared with \$3623. The \$5381 of capital employed by steam powered grain mills was quite similar to the average of \$5123 for steam powered lumber mills. There was only scant evidence from the results there that the capital market became less competitive at these levels of supply. ($\hat{R}_K$ was 1.24 for water mills and only 1.59 for steam mills despite a tripling of capital employed.) It is possible then that there was some slight difference in the elasticity of supply of capital for steam grain mills, but certainly not enough to account for the 1.07 difference in $\hat{R}_K$. Since there is no apparent market-based explanation for this difference, most of it must be random. The 90% (large sample) confidence intervals on R_K are $(1.34 < R_K < 2.12)$ for water mills and $(1.76 < R_K < 3.84)$ for steam mills, with an overlap in the range from 1.76 to 2.12.

In the North our estimate of R_K is 1.31. Since the hypothesis test supports $R_K = 1$ ($\delta = .22$), we conclude, as we did in the lumber industry, that both the capital and output markets were competitive. Therefore the significant divergence of R_L from one ($\hat{R}_L = 5.13$, $\delta < .001$) indicates again that the labor market was not. However, the difference between the present estimate of R_L, 5.13, and the estimate 1.69 for the lumber industry is staggering and very difficult to understand. Grain mills actually employed less labor on average than lumber mills -- 1.6 persons compared with 1.9. There is no apparent reason why the labor market facing grain mills should have been dramatically less competitive, but the difference in the estimates is certainly not just random. There are some significant differences between the samples in the distribution of firms, output and employment by state. Most noticeable is that there are no lumber firms from Maryland in that sample, but 21% of the grain mills, employing 21% of the labor and producing 31 % of the value added, were in Maryland. On the other hand, only one of the sixty-three grain mills was in Maine, compared with seven of the fifty-seven lumber mills. And Maine lumber mills were generally much larger than those in the other states. Accounting for only 12% of the sample, Maine firms employed 28% of the labor and produced 36% of the output. If there were differences in state labor market conditions, the sample composition would of course effect our estimate of R_L. Still, it would seem that any market differences would have to have been quite dramatic to account for our results. Since we have no evidence on whether such differences existed, we can only beg rather than answer the question.

In the West the estimates of R_K, 1.64 and 1.46, support the conclusion that both the capital and output markets were competitive. The hypothesis test supports this more strongly for the steam group

(δ = .25 vs. δ = .10). Since capital employed and output sold were higher for this group we would expect, if anything, that R_K would be higher for steam mills. It is not, so the stronger confirmation there of $R_K = 1$ is accepted as indicative of true conditions. R_L is much greater than 1 in both groups -- 2.57 for water, 3.31 for steam. However, because of a large standard error the hypothesis $R_L > 1$ is not strongly supported in the water powered case even though $\hat{R}_L = 2.57$. Still, the collective evidence of the two estimates certainly indicates $R_L > 1$, implying imperfect conditions in the Western labor market.

Table 19

Water Powered Grain Milling

Production Function and Market Parameter Estimates*

Region (n)	Γ	α_K	α_L	θ ($\times 10^{-5}$)	R_{KL}	$R^2 \big\vert_{\theta=\hat{\theta}}$
The South (186)	12.35 (6.07)	0.54 (0.067)	0.64 (0.134)	0.0106 (0.917)	0.46	0.54
The South** (46)	70.66 (93.55)	0.56 (0.201)	0.72 (0.526)	4.52 (5.38)	0.51	0.25
The North (63)	40.73 (37.23)	0.40 (0.117)	1.28 (0.231)	1.18 (2.90)	0.26	0.65
The West (59)	13.10 (20.10)	0.61 (0.197)	0.52 (0.286)	5.64 (4.86)	0.66	0.41

* Large sample standard errors appear in ( ).

Hypothesis Tests

Test / Region	$H_0: \theta = 0$; $H_1: \theta > 0$		$H_0: \alpha_K + \alpha_L \leqq 1$; $H_1: \alpha_K + \alpha_L > 1$	
	λ	δ	λ	δ
The South	0.01	.50	1.57	.06
The South**	0.84	.20	0.51	.31
The North	0.41	.34	2.94	.002
The West	1.16	.12	0.50	.31

** Physical output sample

Table 20

Water Powered Grain Milling

Production Function and Market Parameter Estimates Under $\theta = 0$ *

Region (n)	Γ	α_K	α_L	R_K	R_L	R^2
	Parameter					
The South (186)	12.36 (6.02)	0.54 {0.066}	0.64 {0.126}	1.73 (0.235)	3.75 (0.773)	0.54
The South** (46)	107.33 (112.18)	0.48 {0.148}	0.59 {0.426}	1.50 (0.494)	2.80 (2.06)	0.25
The North (63)	45.50 (38.56)	0.38 {0.108}	1.23 {0.188}	1.31 (0.393)	5.13 (0.915)	0.65
The West (59)	26.78 (31.06)	0.51 {0.144}	0.45 {0.230}	1.64 (0.494)	2.57 (1.349)	0.42

* Large sample standard errors appear in ( ), exact standard errors in {}.

Hypothesis Tests

Test / Region	$H_0: R_K = 1$ $H_1: R_K > 1$ λ	 δ	$H_0: R_L = 1$ $H_1: R_L > 1$ λ	 δ	$H_0: \alpha_K + \alpha_L \leq 1$ $H_1: \alpha_K + \alpha_L > 1$ t	 δ
The South	3.12	<.001	3.56	<.001	1.77	.04
The South**	1.01	.16	0.87	.19	0.14	.44
The North	0.79	.22	4.52	<.001	3.79	<.001
The West	1.29	.10	1.17	.12	-0.26	.60

** Physical output sample

Table 21

Steam Powered Grain Milling

Production Function and Market Parameter Estiamtes*

Region (n)	Γ	α_K	α_L	θ $(\times 10^{-5})$	R_{KL}	$R^2 \big\vert_{\theta=\hat{\theta}}$
The South (58)	5.18 (4.72)	0.66 (0.132)	0.71 (0.257)	0.0075 (0.841)	0.83	0.71
The West (37)	19.78 (36.26)	0.50 (0.237)	0.74 (0.345)	1.07 (2.72)	0.45	0.48

* Large sample standard errors appear in ( ).

Hypothesis Tests

Test / Region	$H_0: \theta = 0$; $H_1: \theta > 0$ λ	δ	$H_0: \alpha_K + \alpha_L \leqq 1$; $H_1: \alpha_K + \alpha_L > 1$ λ	δ
The South	0.01	.50	1.90	.03
The West	0.39	.35	0.86	.20

Table 22

Steam Powered Grain Milling

Production Function and Market Parameter Estimates Under $\theta = 0$ *

Region (n)	Γ	α_K	α_L	R_K	R_L	R^2
The South (58)	5.18 (4.69)	0.66 {0.132}	0.71 {0.239}	2.80 (0.633)	3.38 (1.226)	.71
The West (37)	24.36 (40.24)	0.47 {0.213}	0.71 {0.319}	1.46 (0.694)	3.31 (1.551)	.49

* Large sample standard errors appear in ( ), exact standard errors in {}.

Hypothesis Tests

Test / Region	$H_0: R_K = 1$, $H_1: R_K > 1$ λ	δ	$H_0: R_L = 1$, $H_1: R_L > 1$ λ	δ	$H_0: \alpha_K + \alpha_L \leqq 1$, $H_1: \alpha_K + \alpha_L > 1$ t	δ
The South	2.84	.002	1.95	.03	2.19	.014
The West	0.67	.25	1.49	.07	0.78	.22

TEXTILES

The textile industry parameters are estimated for the North and South. Both samples include cotton and woolen mills. This aggregation was necessary to maintain a reasonable sample size, particularly in the South, and is certainly justified by the kindred nature of the production processes. The state and regional averages of value added, capital, and employees are presented in Table 23, the percentage distributions of firms by value added in Table 24. The average Southern textile firm was only one-half the size of the average Northern firm. In the distribution of firms the South is over-represented in small value added classes, under-represented in large ones, but well represented in the broad middle range from $10,000 to $50,000 where most Northern firms operated.

The Production Technology

The parameter estimates are presented in Table 25. In the South returns to scale fell from 1.32 as output increased. In this case θ is significantly different from zero (δ = .016), and the test on $\alpha_K + \alpha_L$ indicates that there clearly was a range of increasing returns (δ = .05). However, only the smallest firms in the sample operated there. Returns to scale ranged from 1.30 for the smallest firm with $398 in value added to 0.41 for the

Table 23

Textiles: State and Regional Averages, 1860

State	Firms	Value Added ($)	Capital ($)	Labor (males)	Labor (females)	Labor (weighted)
Alabama	2	17944	22000	19.5	35.0	38.3
Kentucky	2	9330	4000	7.0	2.5	8.3
Mississippi	1	46833	108000	47.0	40.0	68.5
North Carolina	3	7798	11667	7.7	19.0	17.9
South Carolina	6	20164	42413	29.0	43.5	52.4
Texas	1	8224	10000	6.0	7.0	9.7
Virginia	5	11870	20450	15.0	19.6	25.5
The South	20	15672	28086	18.9	26.9	33.4
Connecticut	10	14164	22800	14.5	15.4	23.8
Delaware	5	36660	56300	40.6	42.8	66.5
Maine	1	24795	25000	12.0	14.0	20.5
Maryland	2	41224	100500	64.0	77.5	110.9
Massachusetts	7	60858	112286	52.9	100.1	113.4
New Hampshire	4	45115	65250	60.3	42.5	86.0
New Jersey	1	4547	1600	15.0	0.0	15.0
New York	3	17902	34667	36.7	35.3	58.0
The North	33	33239	57215	37.1	45.9	64.9

Source: Bateman, Foust, Weiss Manuscript Census Sample

Table 24

Percentage Distribution of Textile Firms

by Value Added, by Region, 1860

Value Added ($)	Percentage	
	South	North
0 - 4999	30.0	18.2
5000 - 9999	20.0	12.1
10000 - 49999	45.0	51.5
50000 - 99999	5.0	9.1
100000 -	0.0	9.1

Source: Bateman, Foust, Weiss Manuscript Census Sample

largest with \$61,536 in value added. At the average value added of \$15,672 returns to scale were 0.85. Increasing returns ceased to prevail at a value added of \$9000, which from the distributions of Table 24 clearly represents a rather small textile mill. Fully 70% of the Southern firms operated in the range from 1.045 to 0.41. The remaining 30% (6 firms) were in a range from 1.30 to 1.21. Since θ differs significantly from zero, the homogeneous case for the South is not estimated. In the North there was no range of increasing returns. Returns to scale fell from 0.92, ranging between 0.92 for the smallest firm (Y = \$1439) to 0.79 for the largest (Y = \$193,620). At the average value added of \$33,239 returns to scale were 0.90. θ is not significantly different from zero, so the homogeneous case is estimated for the North and presented in Table 26. Returns to scale were 0.90 for all firms. The existence of increasing returns is overwhelmingly rejected by the test on $\alpha_K + \alpha_L$.

Why the production technology differed so dramatically in the North and South is still a puzzle at this point, but our results are in accord with much historical evidence indicating that it did. The R^2s for the production function regressions are nearly identical for both regions and are the highest of any of the cases in this study. Also, the results of the tests on θ are unequivocal in both regions: $\theta = 0$ is strongly rejected for the South, but strongly accepted for the North.

Market Conditions

Since homogeneity is clearly rejected for the South we have only $\hat{R}_{KL}$ as an indicator of market conditions. As was true in the lumber and grain industries, the labor market was less competitive than the capital market. Unfortunately, $\hat{R}_{KL}$ provides us with no information on the individual markets, and it would be hazardous to generalize any of the more specific conclusions from the other industries since the amounts of capital and labor employed in textiles were so much greater than in lumber or grain. Still, it is most interesting, in light of the rather lopsided attention to capital in the literature, that capital was available in more elastic supply than labor to every industry studied here.

The market parameter results for the North with $\theta = 0$ imposed are surprising at first glance. (See Table 26) In the other industries the labor market consistently showed less competitive conditions than the capital market, but the ratio of $\hat{R}_K$ to $\hat{R}_L$ reveals just the opposite in this case. The estimate of R_L, 1.11, is not significantly different from 1, implying that both the labor and output markets were competitive. However, $\hat{R}_K = 2.26$ and since $R_K = 1$ is roundly rejected we must conclude that the capital market was imperfect.

While this conclusion that the capital market was imperfect contrasts with previous conclusions, it is not difficult to explain. The average amount of capital employed in the textile industry in the North was 21.5 times that employed in grain milling and 27 times that in lumber milling. The capital market for textile firms was simply not the same market that faced grain and lumber mills.

It is the conclusion that the labor market facing textile firms was perfectly competitive that is interesting. We concluded, with very strong support, that the market facing lumber and grain firms was imperfect, even when the average employment was just 2 persons per firm. Now, when the average textile mill employs 37 males and 46 females, we conclude that the market was perfect. One possible explanation concerns the distribution of firms by state in the various industries. Only fourteen per cent of the lumber mills and six and one half per cent of the grain mills in our samples were located in Connecticut and Massachusetts, compared with fifty-one and one half per cent of the textile mills. Together these two states accounted for fifty per cent of the total textile industry employment in the sample (Massachusetts alone for thirty-nine per cent), compared with only seven and four-tenths per cent and four per cent of lumber and grain industry employment, respectively. If labor market conditions varied significantly among the states this difference in sample composition could certainly explain some part of the difference in our regional estimates. Another intriguing possibility concerns the other striking difference between our industry samples -- the presence, in fact predominance, of women in textile industry employment. Perhaps the existence of women in the labor force moderated the market.[8]

Table 25

Textiles

Production Function and Market Parameter Estimates*

Region (n)	Γ	α_K	α_L	θ ($\times 10^{-5}$)	R_{KL}	$R^2\big\vert_{\theta=\hat{\theta}}$
The South (20)	18.78 (36.43)	0.45 (0.274)	0.87 (0.286)	3.58 (1.66)	0.83	0.92
The North (33)	62.02 (35.66)	0.37 (0.091)	0.55 (0.128)	0.0896 (0.217)	1.98	0.93

* Large sample standard errors are given in ( ).

Hypothesis Tests

Test / Region	$H_0: \theta = 0$, $H_1: \theta > 0$: λ	δ	$H_0: \alpha_K + \alpha_L \leq 1$, $H_1: \alpha_K + \alpha_L > 1$: λ	δ
The South	2.16	.016	1.66	.05
The North	.41	.34	-1.03	.85

Table 26

Textiles

Production Function and Market Parameter Estimates Under $\theta = 0$ *

Region (n)	Γ	α_K	α_L	R_K	R_L	R^2
	Parameter					
The North (33)	66.39	0.37	0.53	2.26	1.11	0.92
	(35.50)	{0.088}	{0.116}	(0.584)	(0.254)	

* Large sample standard errors appear in ( ), exact standard errors in { }.

Hypothesis Tests

Test / Region	$H_0: R_K = 1$, $H_1: R_K > 1$		$H_0: R_L = 1$, $H_1: R_L > 1$		$H_0: \alpha_K + \alpha_L \leqq 1$, $H_1: \alpha_K + \alpha_L > 1$	
	λ	δ	λ	δ	t	δ
The North	2.15	.016	0.43	.33	-1.94	.97

SUMMARY

The evidence for the lumber industry supports the same general conclusions about the technology and the market conditions in all three regions. The production technology was homogeneous and characterized by decreasing returns to scale. The demand for lumber and the supply of capital facing firms in each region were perfectly elastic, but the supply of labor was not.

This evidence clearly refutes the hypothesis that Southern firms in this industry were trapped in a range of increasing returns. There was quite simply no such range in the technology; returns to scale were 0.83 for water powered mills and 0.97 for steam powered mills. As we have seen, the concern about smaller Southern firms does not really apply to steam powered mills, since Southern firms were actually much larger than in other regions. Even for the smaller Southern water powered mills there is no evidence that they were forced by imperfect market conditions to operate at uneconomically small output levels relative to the other regions. Product demand and capital supply were perfectly elastic. The supply of labor was not, and Southern firms were of course smaller than they would otherwise have been had this not been true. However, this labor supply constraint was not unique to the South since labor markets were also imperfect in the North and West. Smaller Southern firm size in water powered lumber milling seems to have been primarily a consequence of the technology. Still, there is no sense in which these firms were "uneconomically small" since production was everywhere characterized by decreasing returns.

In the grain milling industry the only common ground among the regions was the homogeneity of the production technology. In the South there were significantly increasing returns to scale, an imperfect output market, a perfectly competitive capital market, and an imperfect labor market. In the North and West the capital markets were also competitive and the labor markets imperfect, but the output markets in both regions were competitive. In the North there were significantly increasing returns, but not in the West.

Southern water powered firms were smaller than both Northern and Western firms, and Southern steam powered firms, in contrast to the situation in the lumber industry, were smaller than Western firms. In both groups Southern firms operated under conditions of increasing returns -- 1.18 for water powered firms and 1.37 for steam powered firms. The unfavorable comparison for the South was with the West, where water powered firms faced returns to scale of 0.96 and steam powered firms faced 1.18, which was judged not significantly above 1.0. Northern water powered firms actually operated under greater increasing returns, 1.61, than other regions.

Despite the existence of increasing returns in the South, the evidence does not support the composite hypothesis that imperfect markets forced operations there. Operation under conditions of increasing returns was a consequence of the homogeneous production technology, not of market conditions. It is however true that product demand was not perfectly elastic in the South, a condition which did not prevail in the North or West and which therefore constituted a relative constraint on Southern firm size.

Southern textile milling presents the single clear instance in this study of a non-homogeneous production technology. Returns to scale fell from 1.32 as output increased, but increasing returns were actually rapidly exhausted. Returns to scale were one at a value added of $9000, a fairly small firm size. Seventy per cent of the Southern firms operated in a range of returns to scale below 1.04. The average Southern firm, despite the fact that it was only half as big as the average Northern firm, was well into the range of decreasing returns.[9] The only market information we have in this case shows that the labor market was less competitive than the capital market, a condition that was also true for both the previous industries.

The production function for Northern textile firms was homogeneous and characterized by decreasing returns of 0.90. Why the regional technologies should have differed so distinctly is not known. The rapidity with which decreasing returns became pronounced in the South did constitute a constraint on firm size, relative to the Northern technology. By itself, however, this should not have directly constrained total industry output. On the other hand, it indirectly placed a heavy demand on entrepreneurship in the South.

With the existing technology many more Southern firms would have been required to achieve a particular industry output than in the North. There is much comment in the literature about a lack of entrepreneurial initiative and talent for Southern manufacturing. Perhaps these results cast some light on the perennial question of why the Southern textile industry was not larger than it was.

Some recent evidence on these industries using the same Bateman-Foust-Weiss data and the same production function employed in this study has appeared in a paper by Jeremy Atack entitled "Returns to Scale in Antebellum United States Manufacturing." (1) The fundamental difference between the present study and Atack's work is the use here of an internally consistent model. As has been pointed out, there is no valid profit maximizing output in the range of increasing returns for a firm facing perfectly competitive markets. Thus any model which purports to investigate the hypothesis that Southern firms were trapped in a range of increasing returns must allow for imperfect markets. Even if this were not a theoretical requirement, it would be necessary to do justice to the numerous comments of authors such as Genovese, Russel and Linden on Southern market conditions.

Although the estimation method in Atack's production function study follows Zellner as this work does, the results are not strictly comparable. First, the measurement of certain input variables differs, especially the items of intermediate costs. Second, I have separated water and steam powered firms in constructing estimates for the lumber and grain industries. Third, in two instances I have produced estimates using physical measures of output. All of these changes should improve the estimates and thereby extend Atack's work. Comparisons of the two sets of results are still informative. The Atack study concludes that there were no significant variable returns to scale in any of the three industries in any region. This agrees with our results with the single exception of Southern textiles mills where θ is clearly greater than zero. On the subject of returns to scale the two studies disagree on the lumber industry. Atack found significantly increasing returns, where we do not. Both studies show increasing returns in grain milling and constant returns for Northern textile mills. For Southern textile mills Atack found homogeneously decreasing returns, while we have found that although most firms were operating in the range of decreasing returns, there was a range of increasing returns in which very small firms operated.

NOTES

Chapter V

1. The Bateman, Foust and Weiss sampling procedure was designed to generate a sample representing the industrial composition of manufacturing in each state, not to generate any minimum size sample for each industry. The lesser importance of lumber milling in the North than in other regions and the predominance of water as a power source there combined to produce a steam powered sample which is simply too small to warrant carrying out the estimation of the parameters.

2. Recall that the "returns to scale function," $\alpha(Y)$, is the elasticity of output with respect to proportionate changes in the inputs capital and labor. $\alpha(Y)$ is therefore frequently referred to as the elasticity of scale. Throughout this chapter we refer to $\alpha(Y)$ simply as "returns to scale."

3. Throughout this chapter the test results will be presented in terms of the notation developed in Chapter III. The symbol λ denotes the realized value of the test statistic for all hypothesis tests based on random variables (which we denoted $\vec{\Lambda}$) that are asymptotically distributed N(0,1). This includes all our tests except that on the value of $\alpha_K + \alpha_L$ when $\theta = 0$. Recall that that particular test is an exact test based on a random variable (which

we denoted T) that is distributed T(n-3). For this test, the realized value of the test statistic is denoted t. For all tests δ denotes the largest probability of falsely rejecting H_0 to which the test can be subjected and still accept H_0.

4. Both the power source and average value added figures are from the U.S. Census of Manufacturing, 1860. The power source figures are from Part I, Totals for States and Industries, Table 6, "Motive Power Used in Manufactures," p. 693. The value added data are from Part III, Selected Industries, "Forest Industries," Table 2, pp. 612-623.

5. The actual values of $\hat{\theta}$ are not comparable because the units of measure of Y_i differ.

6. Note that total revenue should increase, not decrease. It was noted in Chapter III that profit maximizing firms will not operate at output levels where demand is inelastic.

7. The figures here that refer to water powered milling are the results from the value added sample. Of the two sets of estimates for water powered mills, these results are more indicative of true market conditions for two reasons. First, the model fit is much better. Second, and most importantly, no measure of output per se

is required to estimate R_K and R_L (see equations (19') and the discussion in Chapter IV), so the larger sample is the better indicator.

8. An inquiry into this issue is in progress -- Howard Reese, "The Competitive Edge: Women in the Ante-Bellum Labor Force."

9. All the discussion of "too small" Southern firms tends to leave the impression that because increasing returns are bad, decreasing returns are good. This is simply not so. Operation of firms in the range of decreasing returns constitutes a misallocation of resources just as much as operation in the range of increasing returns. When it is possible, as it is if a non-homogeneous technology possesses both ranges, all firms should ideally operate at the point where returns to scale are one. Only in this configuration is industry output maximized for some given resource usage. Given the estimated production technology for Southern textile firms we could argue that they should have been smaller, not larger, than they were since returns to scale are one at a value added of $9,000.

APPENDIX A

THE ESTIMATOR OF THE INFORMATION MATRIX FOR THE PARAMETER VECTOR ϕ

The matrix given in this appendix is

$$\mathcal{I}(\hat{\phi}) = - E \left[\frac{\partial^2 \ell n H}{\partial \phi \, \partial \phi'} \right]_{\phi=\hat{\phi}}$$

where

$$H = H\,(\underline{Y} \,/\, \ell n \underline{\bar{K}},\ \ell n \underline{\bar{L}};\ \phi) \text{ is given by (9)}$$

and

$$\phi = \begin{bmatrix} \Gamma & \alpha_K & \alpha_L & \theta & \sigma_{00} \end{bmatrix}$$

The matrix is symmetric so the expression for cross partial terms is given only once. Two conventions are adopted to simplify the notation: 1) Each matrix entry is denoted by the parameters to which it refers, separated by a comma and followed by a slash. For example,

$$- E \left[\frac{\partial^2 \ell n H}{\partial \Gamma \, \partial \alpha_K} \right]_{\phi=\hat{\phi}} \quad \text{is denoted by} \quad \Gamma,\ \alpha_K \ |$$

When the expectation was difficult to take, the expected value was replaced by its consistent estimator.

2) The summation $\sum_{i=1}^{n}$ is written simply as Σ. (In order that this symbol not be confused with any covariance matrix notation, note that the only variance term appearing in this matrix is σ_{00}.)

The matrix entries are:

$$\Gamma, \Gamma \mid : \quad \frac{n}{\hat{\Gamma}^2 \hat{\sigma}_{00}}$$

$$\Gamma, \alpha_K \mid : \quad \frac{\Sigma \ln K_i}{\hat{\Gamma} \hat{\sigma}_{00}}$$

$$\Gamma, \alpha_L \mid : \quad \frac{\Sigma \ln L_i}{\hat{\Gamma} \hat{\sigma}_{00}}$$

$$\Gamma, \theta \mid : \quad \frac{-\Sigma Y_i}{\hat{\Gamma} \hat{\sigma}_{00}}$$

$$\Gamma, \sigma_{00} \mid : \quad 0$$

$$\alpha_K, \alpha_K \mid : \quad \frac{\Sigma (\ln K_i)^2}{\hat{\sigma}_{00}}$$

$$\alpha_K, \alpha_L \mid : \quad \frac{\Sigma \ln K_i \ln L_i}{\hat{\sigma}_{00}}$$

$$\alpha_K, \theta \mid : \quad \frac{-\Sigma\, Y_i\, \ell n K_i}{\hat{\sigma}_{00}}$$

$$\alpha_K, \sigma_{00} \mid : \quad 0$$

$$\alpha_L, \alpha_L \mid : \quad \frac{\Sigma\, (\ell n L_i)^2}{\hat{\sigma}_{00}}$$

$$\alpha_L, \theta \mid : \quad \frac{\Sigma\, Y_i\, \ell n L_i}{\hat{\sigma}_{00}}$$

$$\alpha_L, \sigma_{00} \mid : \quad 0$$

$$\theta, \theta \mid : \quad \Sigma \left[Y_i \,/\, (1+\hat{\theta} Y_i) \right]^2 + \frac{\Sigma\, Y_i^{\,2}}{\hat{\sigma}_{00}}$$

$$\theta, \sigma_{00} \mid : \quad \frac{-\Sigma\, Y_i\, \hat{u}_{i0}}{\hat{\sigma}_{00}^{\,2}}$$

$$\sigma_{00}, \sigma_{00} \mid : \quad \frac{n}{2\, \hat{\sigma}_{00}^{\,2}}$$

APPENDIX B

THE ESTIMATOR OF THE INFORMATION MATRIX FOR THE PARAMETER VECTOR ϕ^*

The matrix given in this appendix is

$$I\ (\hat{\phi}^*) = -E\left[\frac{\partial^2\ \ell n G}{\partial\phi^*\ \partial\phi^{*'}}\right]_{\phi^*=\hat{\phi}^*}$$

where

$G = G(\ \underline{X}^*;\ \phi^*\ )$ denotes the joint likelihood function of the n observations on $X_i^* = \left[\ \ell n Y_i \quad \ell n K_i \quad \ell n L_i\ \right]$.

G is a normal distribution with mean M_X^* and covariance matrix Ω, given by (16).

and

$$\phi^* = \left[\ \Gamma \quad \alpha_K \quad \alpha_L \quad R_K \quad R_L \quad \sigma_{00} \quad \sigma_{11}^* \quad \sigma_{12}^* \quad \sigma_{22}^*\ \right]$$

The matrix is symmetric so the expression for cross partial terms is given only once. Two conventions are adopted to simplify the notation:

1) Each matrix entry is denoted by the parameters to which it refers, separated by a comma and followed by a slash. For example,

$$-E\left[\frac{\partial^2\ \ell n G}{\partial\Gamma\ \partial\alpha_K}\right]_{\phi^*=\hat{\phi}^*} \qquad \text{is denoted by} \quad \Gamma,\ \alpha_K\ \Big|$$

When the expectation was difficult to take, the expected value was replaced by its consistent estimator.

2) The summation $\sum_{i=1}^{n}$ is written simply as Σ. (In order that this symbol not be confused with any covariance matrix notation, note that

the only variance-covariance terms appearing in this matrix are σ_{00} and Σ_{11}^* and its elements σ_{11}^*, σ_{12}^*, and σ_{22}^*.)

$$\text{Let} \quad \hat{d} = \frac{|\hat{\Sigma}_{11}^*|}{\hat{\sigma}_{00}} + \hat{\sigma}_{11}^* + \hat{\sigma}_{22}^* - 2\,\hat{\sigma}_{12}^*$$

The matrix entries are:

$$\Gamma,\Gamma \;\Big| \;:\; \frac{n\,\hat{d}}{\hat{\Gamma}^2 |\hat{\Sigma}_{11}^*|}$$

$$\Gamma,\ \alpha_K \;\Big| \;:\; \frac{1}{\hat{\Gamma}\ |\hat{\Sigma}_{11}^*|} \quad \{(n/\hat{\alpha}_K)\ (\hat{\sigma}_{22}^* - \hat{\sigma}_{12}^*) + \hat{d}\ \Sigma\ \ell n K_i\}$$

$$\Gamma,\alpha_L \;\Big| \;:\; \frac{1}{\hat{\Gamma}\ |\hat{\Sigma}_{11}^*|} \quad \{(n/\hat{\alpha}_L)\ (\hat{\sigma}_{11}^* - \hat{\sigma}_{12}^*) + \hat{d}\ \Sigma\ \ell n L_i\}$$

$$\Gamma,\ R_K \;\Big| \;:\; \frac{n\ (\hat{\sigma}_{12}^* - \hat{\sigma}_{22}^*)}{\hat{R}_K\ \hat{\Gamma}\ |\hat{\Sigma}_{11}^*|}$$

$$\Gamma,\ R_L \;\Big| \;:\; \frac{n\ (\hat{\sigma}_{12}^* - \hat{\sigma}_{11}^*)}{\hat{R}_L\ \hat{\Gamma}\ |\hat{\Sigma}_{11}^*|}$$

$$\Gamma,\ \sigma_{00}\ \Big| \ = \ \Gamma,\ \sigma^*_{11}\ \Big| \ = \ \Gamma,\ \sigma^*_{12}\ \Big| \ = \ \Gamma,\ \sigma^*_{22}\ \Big| : \quad 0$$

$$\alpha_K,\ \alpha_K\ \Big| : \ \frac{n}{(1-\hat{\alpha}_K-\hat{\alpha}_L)^2} + \frac{1}{|\hat{\Sigma}^*_{11}|}\{\hat{d}\Sigma(\ell nK_i)^2+(2/\hat{\alpha}_K)(\hat{\sigma}^*_{22}-\hat{\sigma}^*_{12})\ \Sigma\ell nK_i$$

$$+ n\ \hat{\sigma}^*_{22}/\hat{\alpha}_K{}^2\}$$

$$\alpha_K,\ \alpha_L\ \Big| : \ \frac{n}{(1-\hat{\alpha}_K-\hat{\alpha}_L)^2} + \frac{1}{|\hat{\Sigma}^*_{11}|}\{\hat{d}\Sigma(\ell nK_i)(\ell nL_i)+(1/\hat{\alpha}_K)(\hat{\sigma}^*_{22}-\sigma^*_{12})\ \Sigma\ell nL_i$$

$$+ (1/\hat{\alpha}_L)(\hat{\sigma}^*_{11}-\hat{\sigma}^*_{12})\ \Sigma\ell nK_i - n\ \hat{\sigma}^*_{12}/\hat{\alpha}_K\hat{\alpha}_L\}$$

$$\alpha_K,\ R_K\ \Big| : \ \frac{1}{\hat{R}_K|\hat{\Sigma}^*_{11}|}\ \{(\hat{\sigma}^*_{12}-\hat{\sigma}^*_{22})\ \Sigma\ell nK_i - n\ \hat{\sigma}^*_{22}/\hat{\alpha}_K\}$$

$$\alpha_K,\ R_L\ \Big| : \ \frac{1}{\hat{R}_L|\hat{\Sigma}^*_{11}|}\ \{(\hat{\sigma}^*_{12}-\hat{\sigma}^*_{11})\ \Sigma\ell nK_i + n\ \hat{\sigma}^*_{12}/\hat{\alpha}_K\}$$

$$\alpha_K,\ \sigma^*_{00}\ \Big| : \quad 0$$

$$\alpha_K,\ \sigma^*_{11}\ \Big| : \ \frac{1}{|\hat{\Sigma}^*_{11}|^2}\ \Sigma\{\Big[(\hat{\sigma}^*_{12}\ \hat{\sigma}^*_{22}-\hat{\sigma}^{*\,2}_{22})\hat{\xi}_{i1} + (\hat{\sigma}^*_{12}\ \hat{\sigma}^*_{22}-\hat{\sigma}^{*\,2}_{12})\ \hat{\xi}_{i2}\Big]\ell nK_i\}$$

$$\alpha_K,\ \sigma_{12}^*\ |\ :\quad \frac{1}{|\hat{\Sigma}_{11}^*|^2}\,\Sigma\{\left[(2\,\hat{\sigma}_{22}^*\,\hat{\sigma}_{12}^* - \hat{\sigma}_{11}^*\,\hat{\sigma}_{22}^* - \hat{\sigma}_{12}^{*\,2})\quad \hat{\xi}_{i1}\right.$$

$$\left. + \ (2\,\hat{\sigma}_{11}^*\,\hat{\sigma}_{12}^* - \hat{\sigma}_{11}^*\,\hat{\sigma}_{22}^* - \hat{\sigma}_{12}^{*\,2})\quad \hat{\xi}_{i2}\right]\ \ell n K_i\}$$

$$\alpha_K,\ \sigma_{22}^*\ |\ :\quad \frac{1}{|\hat{\Sigma}_{11}^*|^2}\,\Sigma\{\left[(\hat{\sigma}_{12}^*\,\hat{\sigma}_{11}^* - \hat{\sigma}_{12}^{*\,2})\ \hat{\xi}_{i1} + (\hat{\sigma}_{12}^*\,\hat{\sigma}_{11}^* - \hat{\sigma}_{11}^{*\,2})\ \hat{\xi}_{i2}\right]\ell n K_i\}$$

$$\alpha_L,\ \alpha_L\ |\ :\quad \frac{n}{(1-\hat{\alpha}_K-\hat{\alpha}_L)^2} + \frac{1}{|\hat{\Sigma}_{11}^*|}\ \{\hat{d}\Sigma(\ell n L_i)^2 + (2/\hat{\alpha}_L)(\hat{\sigma}_{11}^* - \hat{\sigma}_{12}^*)\ \Sigma\ \ell n L_i$$

$$+ \ n\ \hat{\sigma}_{11}^*/\hat{\alpha}_L^{\ 2}\}$$

$$\alpha_L,\ R_K\ |\ :\quad \frac{1}{\hat{R}_K|\hat{\Sigma}_{11}^*|}\ \{(\hat{\sigma}_{12}^* - \hat{\sigma}_{22}^*)\ \Sigma\ell n L_i + n\ \hat{\sigma}_{12}^*/\hat{\alpha}_L\ \}$$

$$\alpha_L,\ R_L\ |\ :\quad \frac{1}{\hat{R}_L|\hat{\Sigma}_{11}^*|}\ \{(\hat{\sigma}_{12}^* - \hat{\sigma}_{11}^*)\ \Sigma\ell n L_i - n\ \hat{\sigma}_{11}^*/\hat{\alpha}_L\ \}$$

$$\alpha_L,\ \sigma_{00}\quad :\quad 0$$

$$\alpha_L,\ \sigma_{11}^* \mid : \quad \frac{1}{|\hat{\Sigma}_{11}^*|^2}\,\Sigma\{[(\hat{\sigma}_{12}^*\ \hat{\sigma}_{22}^* - \hat{\sigma}_{22}^{*2})\ \hat{\xi}_{i1} + (\hat{\sigma}_{12}^*\ \hat{\sigma}_{22}^* - \hat{\sigma}_{12}^{*2})\hat{\xi}_{i2}]\ell nL_i\}$$

$$\alpha_L,\ \sigma_{12}^* \mid : \quad \frac{1}{|\hat{\Sigma}_{11}^*|^2}\,\Sigma\{[(2\ \hat{\sigma}_{22}^*\ \hat{\sigma}_{12}^* - \hat{\sigma}_{11}^*\ \hat{\sigma}_{22}^* - \hat{\sigma}_{12}^{*2})\ \hat{\xi}_{i1}$$

$$+ (2\ \hat{\sigma}_{11}^*\ \hat{\sigma}_{12}^* - \hat{\sigma}_{11}^*\ \hat{\sigma}_{22}^* - \hat{\sigma}_{12}^{*2})\ \hat{\xi}_{i2}]\ \ell nL_i\}$$

$$\alpha_L,\ \sigma_{22}^* \mid : \quad \frac{1}{|\hat{\Sigma}_{11}^*|^2}\,\Sigma\{[(\hat{\sigma}_{12}^*\ \hat{\sigma}_{11}^* - \hat{\sigma}_{12}^{*2})\hat{\xi}_{i1} + (\hat{\sigma}_{12}^*\ \hat{\sigma}_{11}^* - \hat{\sigma}_{11}^{*2})\ \hat{\xi}_{i2}]\ell nL_i\}$$

$$R_K,\ R_K \mid : \quad \frac{n\ \hat{\sigma}_{22}^*}{\hat{R}_K^{\,2}|\hat{\Sigma}_{11}^*|}$$

$$R_K,\ R_L \mid : \quad \frac{-n\ \hat{\sigma}_{12}^*}{\hat{R}_K\ \hat{R}_L\ \ |\hat{\Sigma}_{11}^*|}$$

$$R_K,\ \sigma_{00} \mid = R_K,\ \sigma_{11}^* \mid = R_K,\ \sigma_{12}^* \mid = R_K,\ \sigma_{22}^* \mid : \quad 0$$

$$R_L,\ R_L \mid : \quad \frac{n\ \hat{\sigma}_{11}^*}{\hat{R}_L^{\,2}|\hat{\Sigma}_{11}^*|}$$

$$R_L, \sigma_{00} \;|\; = \; R_L, \sigma_{11}^* \;|\; = \; R_L, \sigma_{12}^* \;|\; = \; R_L, \sigma_{22}^* \;|\; : \quad 0$$

$$\sigma_{00}, \sigma_{00} \;|\; : \quad \frac{n}{2\,\hat{\sigma}_{00}^{\,2}}$$

$$\sigma_{00}, \sigma_{11}^* \;|\; = \; \sigma_{00}, \sigma_{12}^* \;|\; = \; \sigma_{00}, \sigma_{22}^* \;|\; : \quad 0$$

$$\sigma_{11}^*, \sigma_{11}^* \;|\; : \quad \frac{n\,\hat{\sigma}_{22}^{*\,2}}{2|\hat{\Sigma}_{11}^*|^2}$$

$$\sigma_{11}^*, \sigma_{12}^* \;|\; : \quad \frac{-n\,\hat{\sigma}_{22}^*\,\hat{\sigma}_{12}^*}{|\hat{\Sigma}_{11}^*|^2}$$

$$\sigma_{11}^*, \sigma_{22}^* \;|\; : \quad \frac{n\,\hat{\sigma}_{12}^{*\,2}}{2|\hat{\Sigma}_{11}^*|^2}$$

$$\sigma_{12}^*, \sigma_{12}^* \;|\; : \quad \frac{n\,(\hat{\sigma}_{11}^*\,\hat{\sigma}_{22}^* + \hat{\sigma}_{12}^{*\,2})}{|\hat{\Sigma}_{11}^*|^2}$$

$$\sigma_{12}^*, \sigma_{22}^* \;|\; : \quad \frac{-n\,\hat{\sigma}_{11}^*\,\hat{\sigma}_{12}^*}{|\hat{\Sigma}_{11}^*|^2}$$

$$\sigma_{22}^{*},\ \sigma_{22}^{*}\ \Big| :\quad \frac{n\,\hat{\sigma}_{11}^{*\,2}}{2|\hat{\Sigma}_{11}^{*}|^{2}}$$

APPENDIX C

COVARIANCE PARAMETER ESTIMATES

Water Powered Lumber Milling, $\theta \neq 0$**

Region \ Parameter	σ_{00}	$\sigma_{11} - 2\sigma_{12} + \sigma_{22}$
The South	0.242 (0.054)	0.466
The North	0.167 (0.040)	0.373
The West	0.295 (0.666)	0.406

Water Powered Lumber Milling, $\theta = 0$**

Region \ Parameter	σ_{00}	σ_{11}^*	σ_{12}^*	σ_{22}^*
The South	0.223 (0.028)	0.213 (0.048)	-0.098 (0.019)	0.067 (0.024)
The North	0.183 (0.034)	0.221 (0.068)	-0.072 (0.023)	0.024 (0.018)
The West	0.288 (0.057)	0.135 (0.062)	-0.070 (0.019)	0.037 (0.030)

** Large sample standard errors are given in ( ).

Steam Powered Lumber Milling, $\theta \neq 0$**

Region \ Parameter	σ_{00}	$\sigma_{11} - 2\sigma_{12} + \sigma_{22}$
The South	0.221 (0.042)	0.382
The West	0.307 (0.073)	0.276

Steam Powered Lumber Milling, $\theta = 0$**

Region \ Parameter	σ_{00}	σ_{11}^*	σ_{12}^*	σ_{22}^*
The South	0.194 (0.025)	0.187 (0.041)	-0.066 (0.014)	0.024 (0.012)
The West	0.304 (0.045)	0.097 (0.038)	-0.049 (0.011)	0.026 (0.019)

**Large sample standard errors are given in ( ).

Water Powered Grain Milling, $\theta \neq 0$**

Region \ Parameter	σ_{00}	$\sigma_{11} - 2\sigma_{12} + \sigma_{22}$
The South	0.501 (0.055)	0.602
The South***	0.594 (0.247)	0.601
The North	0.382 (0.087)	0.496
The West	0.685 (0.248)	0.413

Water Powered Grain Milling, $\theta = 0$**

Region \ Parameter	σ_{00}	σ_{11}^*	σ_{12}^*	σ_{22}^*
The South	0.501 (0.052)	0.134 (0.040)	-0.144 (0.019)	0.195 (0.050)
The South***	0.432 (0.090)	0.120 (0.072)	-0.108 (0.023)	0.097 (0.064)
The North	0.361 (0.064)	0.280 (0.094)	-0.029 (0.054)	0.155 (0.066)
The West	0.466 (0.086)	0.093 (0.057)	-0.094 (0.018)	0.097 (0.058)

**Large sample standard errors are given in ( ).
***Physical output sample

Steam Powered Grain Milling, $\theta \neq 0$**

Region \ Parameter	σ_{00}	$\sigma_{11} - 2\sigma_{12} + \sigma_{22}$
The South	0.549	0.531
	(0.112)	
The West	0.661	0.432
	(0.253)	

Steam Powered Grain Milling, $\theta = 0$**

Region \ Parameter	σ_{00}	$\sigma_{11}{}^*$	$\sigma_{12}{}^*$	$\sigma_{22}{}^*$
The South	0.548	0.085	-0.066	0.396
	(0.102)	(0.059)	(0.063)	(0.143)
The West	0.585	0.105	-0.077	0.092
	(0.136)	(0.085)	(0.033)	(0.079)

**Large sample standard errors are given in ( ).

Textiles, $\theta \neq 0$**

Region \ Parameter	σ_{00}	$\sigma_{11} - 2\sigma_{12} + \sigma_{22}$
The South	0.274 (0.113)	0.221
The North	0.101 (0.028)	0.400

Textiles, $\theta = 0$**

Region \ Parameter	σ_{00}	σ_{11}*	σ_{12}*	σ_{22}*
The North	0.096 (0.024)	0.226 (0.076)	-0.080 (0.030)	0.052 (0.028)

**Large sample standard errors are given in ( ).

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