A PSYCHOLOGICAL STUDY OF CURRICULA IN BEGINNING ARITHMETIC

by

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PREFACE

The writer's request to make an experimental study of the arithmetic curriculum met with an enthusiastic response from the supervisors and the teachers of the Lawrence Public Schools. Several important innovations, which reflected a progressive point of view on the part of the school officials, had already been considered for improving the course of study in arithmetic. The situation, therefore, was quite favorable for a co-operative enterprise which aimed to discover second grade children's methods of solving mathematical problems.

The application of established principles of organismic psychology to a specific subject field in a specific classroom proved to be no little task. The writer at the beginning of the school year lacked an extensive knowledge of child life and child reasoning at the seven year level. Many problems encountered in the teaching procedure were discussed with Ursula Henley and Marcia W. Carter who gave liberally of their time for ironing out every difficulty. The latter, with the exception of a few days, assisted directly in the classroom activities. The writer wishes to acknowledge a debt to these persons who graciously accepted this additional responsibility to a heavy teaching assignment.

The testing of the control groups was made possible by the co-operation of the following teachers: Marian Lane,

Tillie Oberle, Mildred Kirkham, and Catherine Beckwith. Individual intelligence test results were obtained for a majority of all the pupils reported in the investigation. This testing was in charge of Byron Sarvis, head of the University of Kansas Psychological Clinic.

Several assistants in the department of psychology have aided the writer by the preparation of materials for the experiment. He wishes to acknowledge the work of Edgar Robinson, Tom Palen, William Orbison, and Charles Manley. The difficult task of drawing 25 of the tables for the thesis was performed by Edward Hampton and Peggy Perkins. The writer wishes to express his appreciation to these persons and also to Charles Dreher, Gladys Irvine, and Oscar Hall for the typing of the manuscript.

Many suggestions regarding the experimental procedure were obtained from the staff members of the psychology department. The writer desires to acknowledge their contributions and their continued interest in the study. To F. Theodore Perkins, he is very grateful for the capable supervision of the class teaching and also for the constructive criticisms offered during the writing of the thesis.

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I. INTRODUCTION

Arithmetic a Difficult Subject. For many years arithmetic has been recognized as the chief stumbling block to elementary school children. Investigators in the field of education have sought diligently to discover why children do not readily learn this subject, but at the present date no thoroughgoing solution to the difficulty has been forthcoming. The nature of child learning with respect to this subject is not understood and the goal of an adequate arithmetic curriculum is not yet formulated. The reader of the educational literature can note at once a feeling that the writers appear baffled by this problem. One authority believes that arithmetic has been little improved since the appearance of Warren Colburn's textbook in 1821. (18, p. 582) Another authority questions the assumptions underlying our present day teaching methods in beginning arithmetic. (19, 235) Still another raises the question, Is arithmetic, especially in some of its aspects as generally administered, an appropriate subject for the lower grades? (7, p. 339) At best there are only hypothetical answers to these problems. The belief is still held, nevertheless, that children should be introduced to arithmetical concepts as early as possible, for mathematics is considered one of the greatest tools which society has developed and refined. Mathematics is a method of thinking which enables man to to communicate his quantitative experience. There is hope that the future must disclose an adequate teaching procedure. As Judd states it, "Some day we shall see arithmetic made into an interesting chapter in a new curriculum". (18, p. 582)

Trend in Research on the Learning of Arithmetic. Until 1930 interest was centered primarily upon an analysis of the subject matter to be used in textbooks. Thorndike's investigations in the early 1920's revealed that much useless and meaningless material was being taught in the grades and, as a consequence, a general attack was made on the arithmetic curriculum in an attempt to analyze out fundamental facts. Since the transfer-of-training hypothesis had fallen into disrepute, investigators sought to uncover all the specific facts which would have to be taught more or less independently. The market became flooded with standardized teaching devices which were essentially drill exercises. Drill became the most important aspect of the teaching program; drill was needed for skill-building and skill-holding. It was an attempt on a grand scale to put the theory of bond psychology into practice. Certain statements from the teacher's manual of a typical arithmetic text, although taken out of their setting, show in no uncertain fashion these theoretical implications in the writer's thinking.

"It therefore follows that the teaching technique (in arithmetic) which most nearly eliminates, from the start, any dependence upon 'reasoning' or counting, or any other clumsy method, is the best." (35, p. 20)

"Real learning is sometimes slow and requires constant practice, but mastery of the combinations is always the result of <u>knowing</u> and never <u>thinking</u>." (35, p. 25)

The research studies dominated by this psychology reached a climax in 1929-1930 with the publication "Twenty-ninth Yearbook of the National Society for the Study of Education" (39). By this time the relative difficulty of the discrete facts in all four mathematical processes had been determined; the grade placement of arithmetic materials had been decided upon, the best method of teaching subtraction had been discovered; the question of mixed *versus* isolated drill had been determined; and the ability of young children to manipulate specific number facts without training had been investigated. The mastery of arithmetic and the efficiency of computation was reduced to a consideration of methods of drill. For obtaining 100 per cent efficiency it was necessary, according to the theory, to obtain a balanced drill program.

After 1930, however, a change in the type of arithmetic investigation becomes apparent. Between 1933 and 1935, it has been recognized, even by many members of the old school, that something was fundamentally wrong with the arithmetic curriculum. It did not produce the desired results. Drill exercises on discrete number facts did not stimulate real learning. Brownell's evaluation of the arithmetic curriculum is important in this connection. (5, pp. 1-31) Buckingham, also, has an important discussion of this topic and sums up the issue as follows:

"Almost at once after. . .counting of objects the pupils are required to learn the number facts in their abstract form. The rest is drill. This is the program which proves so unsatisfactory." (7, p. 340)

Brownell and Chazal, who report an experiment dealing with this question, give one finding which is important:

". . .drill (in the first two grades). . . failed to set up the proposed associations, and it left the pupils (at the beginning of grade III) equipped with the most immature ways of thinking of numbers." (6, p. 23)

A great deal of emphasis since 1933 has been placed upon a socialized arithmetic program; education is now generally defined in terms of child activity not simply receptivity. (14, p. 293) The success of some experimental schools utilizing activity programs has been quite marked in the subjects: reading, writing, spelling, music, and art. Isaacs draws the conclusion that some of her pupils may have been below standard in reading and writing when they left her school, but that this handicap was more or less temporary in character. The gain in pupil enthusiasm and adaptability outweighed by far any temporary losses. (16, p. 155) Shaw's finger painting involves something quite new in the development of children's appreciation of art. In this field she has discovered a new world of child expression just as a new world for children had been discovered previously in reading. (31) At the Ohio State University experimental school striking advances have been made in dealing with the subjects reading, writing, spelling, and music. In defining the objectives of their program, the research workers emphasize that a school is no longer a place where specific skills are drummed into the pupil, but that a school must aid the child to master the fundamentals as significant material in a social situation. The fundamental facts are 'parts' determined by a 'social pattern'. Reading, writing, and spelling, for example, are studied as acts in the process of communication. Each child learns to share books and ideas, to express appreciation for services and courtesies, and to accomplish creative individual work. In brief, the individual is led to evolve his own standards of action as a member of a social group. The development of two things, openmindedness of attitude and responsibility for action, occur together. True individual freedom and true social control do not conflict, but are treated as complementary attributes of one learning process. Judd undoubtedly was speaking for objectives such as these when he said:

"I am not arguing. . .for an abandonment of systematic, intensive training in the arts of civilization. . .(but). . . for a reimportation into these arts of that social significance which has often been lost because teachers and pupils are microscopic in their vision, looking only at the task itself and failing to see that each subject in the school is merely the mature stage of one of the fundamental treads of civilization." (18, p. 585)

Emphasis on child activity in a social situation means, in turn, that consideration must be given to problems pertaining to the child's goals. Research workers are now attempting to discover what goals of learning children can set up for themselves and to discover how they may achieve these goals. The readiness with which pupils grasp and understand problems becomes most significant. Hilgard and Jersild found that maturity of the pupil was important in this connection. The attempt to teach certain materials too soon gave unsatisfactory results. Children will not learn certain abstract materials when they are immature. (15) (17) The more recent publications by Knight and Buckingham give typical examples in the learning of arithmetic. The latter writer says:

> "Meet the child where he is. Don't meet him where you think he is. Know where he is. . . . It is important that there should be no mistake about the interests and abilities of the pupil whom we are taking in hand." (7, p. 341)

Knight insists that we should find exactly what process of quantitative thinking a child actually employs in a problem-situation. (19, p. 236) He implies that we have had too much arm-chair analysis by adults who do not consider fully the nature of child reasoning. Judd insists that arithmetic is a method of thinking. (18, p. 583) This view makes necessary an understanding of child thought.* The learning of arithmetic is not just a matter of memory or simple mastery of discrete skills. Brownell and Chazal, through an extensive study of classroom performances in

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* Brownell's development of a "meaning" theory in connection with the learning of arithmetic indicates how important this point appears to research workers at the present time. (5, pp. 19-20, 28)

arithmetic, have demonstrated beyond reasonable doubt the validity of this conclusion. (6) Knight says:

"A sense of truth and reasonableness should permeate all learning in the first and second grade arithmetic curriculum. This is not done at present." (19, p. 238)

"Passivity in learning must give way to activity. The fundamental pedagogy in teaching the one hundred addition and the one hundred subtraction facts should be setting the stage for an experience of discovery and of verification instead of accepting information as a devine dispensation." (19, p. 239)

The crux of the whole problem is contained in one statement by this author, namely, "We must increase the permanence of first learning." (19, p. 234)

There has been more and more recognition given to the emotional factors which are intimately connected with all learning. Pupils in many instances have gained a dislike of arithmetic which in turn results in a dislike for mathematics in general. It is now admitted that the mastery of abstract, technical facts should not be insisted upon if the child is not ready for such mastery and if such learning comes at the cost of too great emotional strain. (19) For it is certainly true that any learning, based on the principle of 'discovery and verification' can be accomplished only in that situation which guarantees a satisfactory type of emotional adjustment.

<u>The Present Day Arithmetic Curriculum.</u> With the increasing dissatisfaction for established curricula in arithmetic, has come considerable confusion among teachers concerning the time to begin instruction in formal arithmetic. Em-

phasis since 1930 has been placed largely on informal or incidental instruction, especially in grade one. The tendency is to push arithmetic up the grade scale and to postpone the teaching of the addition and subtraction facts until grade three. The teachers are expected to go as far as possible with informal activities and then to use drill whenever mastery is incomplete. This is an attempt to make two incompatible procedures work together. The first is based on the principle of freedom for the child and the second is founded on a mechanical, bond psychology.

A Criticism of Present Practice. The notion is still held by most teachers that a drill procedure is absolutely essential if a skill is to be gained by a pupil. They assume that any specific achievement comes about only by mechanical practice or repetition of response which forms bonds in the nervous system of the individual. This idea has been exploded as Stoddard and Wellman have carefully pointed out. (34, p. 228) If a pupil memorizes the number facts in a drill situation, as it has been demonstrated that he can, it is not because he has formed certain, specific connections. The repetition of problems serves, instead, as a stimulating device. This stimulation provides usable energy for the organism which will result, when expended under favorable circumstances, in a mastery of the specific number facts. Evidence for this interpretation can be found by observing the great number of young children who 'go to pieces' or who work under too great tension when drill is introduced into the classroom. Further evidence is given by Sneddy who found that learning under these conditions was temporary and unstable in character. When the highly stimulating conditions disappear, the highly specialized learning disappears. (33)

The teacher ~~too~~ often loses sight of the fact that learning must be goal-activity not just activity. The action, furthermore, must be controlled and directed principally by the child, not the adult. The adult must find what goals the children can attain, but in the last analysis the child must himself perceive the goals if he is to reach them. Investigators in the field of education are giving implicit recognition to this point when they attempt to discover 'where the child is' and to determine what sort of thinking he employs.

The problem of learning arithmetic would be largely resolved if the notion that organization is something derived from the combination of discrete parts were completely discarded. McLaughlin, for example, assumes that for the child to perceive five similar objects as a group (cardinal concept) requires an abstraction of a higher order than to perceive the individual objects as such. (23, p.351) This interpretation has meaning only from the point of view of an adult's differentiated perception of a group as composed of separate objects. But children readily deal with groups as wholes. Five as a group of blocks placed in an irregular pattern is readily distinguished from a group of seven. Comparison or contrast of groups similar to these is possible at a very early age. It is only when the child attempts to differentiate a group by counting or by the formation of sub-groups that the dominating idea of the large group may be broken down. It should be pointed out that when this occurs, the pupils become confused and the counting of objects is usually very inaccurate. It should be remembered, also, that a unity (whole) which has subordinate parts (members) may be as readily perceived by the child as by the adult, but the whole and the parts of the same pattern may be different for the two observers. (24) The failure to recognize the whole-part problem has lead to the mistaken theory that all children's notions of unity, form, and rhythm are derived from elemental parts. In reality, the child in the first grade already knows a great deal about systematization and organization of objects, about accuracy and precision of calculation in the concrete situation. It is the duty of the teacher to aid the child in expanding and differentiating this nucleus of mathematical ideas already present and not simply to make order, symmetry, and precision a part of his thinking. Many current arithmetic curricula attempt to force a ready-made, abstract organization upon the child. Instead, we should permit the child to have the opportunity of utilizing and expanding that organization which he already possesses.

II. THE NATURE OF THE PRESENT STUDY

Preliminary Experiment. A general investigation of number concepts in children, four to eight years of age, already has been made. (30) It was found in this study that counting was a relatively advanced stage in the development of the number concept. Counting of objects up to ten was imperfect in average children, seven and eight years of age. The cardinal and the ordinal concepts were found to differentiate together as members of a unitary number concept. The results indicated that addition and subtraction problems would probably be too abstract for first grade children. It seemed unlikely, although memorization of the easier arithmetical facts is possible, that the meaning of addition and subtraction could be fully grasped by children of this age.

The preliminary study demonstrated clearly that there was a great need for thorough study of the learning of beginning arithmetic under actual class conditions. How do children in the beginning classes actually solve arithmetical problems? What concepts of mathematics can be taught during the first year of formal arithmetic instruction? In brief, these are the problems of the present investigation.

Experimental Procedure. The writer was given permission to conduct an experiment in the Lawrence, Kansas Public Schools during the year 1934-1935, and it was decided that he should teach the beginning arithmetic daily in one class. Since formal arithmetic instruction begins with the second grade in this school system, an average second grade was selected. Several pupils in the class were advanced and several were very slow. Two repeaters of the first grade were included and, also, during the second semester, two outstanding first grade pupils. A total of 32 different pupils were in the group at one time or another, although the daily average for the year lay between 20 and 25. (Additional information concerning the pupils in the experimental group will be given in part III.)

The year's course was not absolutely predetermined. Conferences were arranged once, twice, or even three times a week with Dr. F. T. Perkins, the advisor, to go over the data and to arrange new teaching material. This enabled the experimenter to meet specific difficulties as they arose. In order to maintain a situation comparable to the usual school environment, the class was taught as a group. Individual aid was limited for the most part to that amount which could be given during the regular period of 20 minutes. Occasionally a pupil was kept after class to finish a piece of work or to receive individual aid. Several tests were given the poorer pupils during the latter part of the experiment in order to diagnose difficulties.

Emphasis was placed on class demonstrations; both blackboard demonstrations and object demonstrations were employed. Pupil participation was the essential feature in the success of these procedures. The exercises which utilized group responses as well as individual responses proved most satisfactory. The method used in illustrating the process of multiplication (discussed in part III) provides an outstanding example. Demonstrations were used in connection with the following work: (1) Writing of numbers, (2) Making equalities, (3) Finding differences, and (4) Learning the four processes and the interrelation of these processes. Critical problems or exercises worked individually were always used during the same period as a check on the demonstration.

A great deal of emphasis was placed also upon accuracy of calculation and at all times speed was discouraged. Drill methods were avoided completely. Some problem situations were repeated, but this repetition occurred under widely varying conditions. The children were encouraged to find their own method of solving problems. Mastery of a number combination was accomplished through verification by the child and not through simple memorization. As a result, the child was not encouraged to take a number fact for granted and accept it on the basis of insufficient evidence. Even the best pupils were extremely slow in calculating the addition and subtraction combinations. A month and a half before the close of school these pupils required approximately 60 minutes total time to work 228 addition and subtraction problems. These pupils worked accurately, however, and this mastery indicates undoubtedly that the process of learning was <u>not</u> simple or mechanical in character.

A record of all methods of procedure was filed along with a record of the results. Wherever possible the experimenter calculated for each day the percentage of correct performance for each pupil; the basis for calculation was the number of items completed by the pupil. These data appear in the Appendix. Immediately after each period, notes were made concerning the general reactions of the pupils, understanding of directions, unwillingness to co-operate, effects of success, failure, and confusion. The pupils' own methods of solution were recorded whenever the experimenter happened to discover how the subjects attacked a given problem. It was particularly difficult in many instances to ascertain just what type of thinking and what method of procedure the children did employ. Other qualitative notes dealt with the effects of the group on the individual, the effects of the teacher's personality on the group, and on the individual, and finally, the results of motivation devices such as class demonstrations, returning of graded papers, and correction of papers in class.

<u>Summary of the Objectives of the Experiment.</u> The present investigation was undertaken:

1. to motivate the learning of arithmetic in a class situation according to the principle of expansion-differentia-

tion.*

a. To develop quantitative thinking simultaneously with the growth and appreciation of qualitative experience.

b. To allow the child to discover his own solutions to arithmetic problems; (to permit the child to discover a means for reaching the mathematical goal); to aid the child to discover a need for verification of his computation.

c. To pace the class as a group.

d. To avoid drill and memorization.

2. To obtain qualitative data on the effects of success or failure, the effects of the group on the pupil, and the effects of the adult personality on the group and the pupil.

3. To determine, qualitatively, the limitations of child thought (reasoning) in dealing with arithmetic problems.

4. To contrast the progress of pupils taught by an expansion-differentiation method with pupils taught by the current arithmetic programs.

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* According to this principle, learning is described as a mental development or growth which provides the necessary background upon which details may emerge already integrated. (40, pp. 8, 23-25, 140-143, 375-376, 479 ff.)

III. RESULTS OF THE INVESTIGATION

Daily Procedure and Quantitative Results. The experiment has been divided arbitrarily into six weeks periods beginning Monday, October 1. Since this work began several weeks later than the public school term, complete data for only 33 weeks are recorded. A quantitative measure of the pupils' work was calculated for a total of 126 days. The six weeks periods will be designated by capital letters. The weeks within each of these periods will be designated by Arabic numerals. The pupils will be referred to as S1, S2, . . . S32.

A. First Six Weeks

1. Monday, October 1.

The material already prepared by the second grade teacher, Miss Marcia Carter, was used. This consisted of hectographed sheets containing pictures of various objects, for example, seven chairs, eight blocks, ten clowns, eight cats, and six chickens. The children were instructed (1) to count the numbers of these various objects and (2) to take their pencils and draw a ring around a certain number of objects. Some examples are as follows: "Draw a ring around six chairs"; "Draw a ring around four blocks".

The children could follow these instructions, although the writing of numbers up to ten proved difficult. No quantitative scores were recorded.

Tuesday, October 2.

(1) The children were asked to count (rote) to 30; the slow pupils experienced difficulty.
(2) The second task was to write the numbers 1 to 10. Six of the numbers were made incorrectly.
(3) The meaning of a 'number picture' was explained. An illustration was made on the blackboard with the following: o o o
o o o
The children were requested to draw a number picture of 8 and then of 15.

(4) The following was put on the board for number identification:

o o o
o o o
o o o

(5) The experimenter drew seven 'hats' on the board; the children were instructed to count the 'hats'.

Successful completion of the five items was used as the basis to calculate scores; the average was 78.40 per cent correct.

Wednesday, October 3.
(1) The experimenter wrote the numbers 3,2,7, 8,9 on the board and had the children copy them. The numbers were written very large so that the children could observe carefully how each movement was made.
(2) Another demonstration of number pictures was given. Number pictures in the following arrangement were then placed on the board:

4	5
9	7
8	10
15	17

The children were asked to write down the number of the larger picture in each pair. There was considerable miscounting; seven was written instead of nine many times in the second combination. No individual counted 15 or 17 correctly.
(3) A number picture of nine was placed on the board; the experimenter drew squares in this instance instead of circles. He then instructed the pupils to "Draw as many marbles as there are blocks." Nearly every member of the class got the number correctly, although many drew 'blocks' instead of 'marbles.'

Grades were assigned on the basis of ten items (five on part 1, four on part 2, and one on part 3); the average was 72.59 per cent correct.

Thursday, October 4.
(1) Further work was given on the writing of 3, 9, and 10. The numbers 11 and 12 also were demonstrated. The following task was given: "Can you write thirteen?"
(2) The pupils were called to the board one at a time to pick out the smaller or the larger of one of the following pairs of numbers placed on

the board in the form of number pictures:

4 5
4 3
2 1
6 7
8 10

(3) Three large dots (called marbles) were drawn on the board: o o
o

The class was instructed: "You draw a number picture that has one more marble in it."

The responses on this day were on the whole successful; no quantitative scores were calculated.

Friday, October 5.

(1) Because of a disturbance outside of the classroom at the beginning of the period, the following task was not solved by the pupils: "Copy this number picture": o o
o

"Make a number picture that has one more than this one": o o
o o o

(2) The figures 11 and 12 were made correctly when copied from the board; 8 and 15 were made incorrectly.

No quantitative scores were recorded.

2. Monday, October 8.

(1) "Copy the following": o o o   o o o o o
o o o o   o o o o o

(2) "Draw some blocks on your papers so that you will have the same number of blocks as marbles." (The 'marbles' had been drawn on their papers; see part 1.)

(3) "Copy the following": o o o "Draw another number picture that has one more marble."

(4) "Write the numbers from 1 to 12." The results on (4) indicated that the children had failed to master the writing of numbers.

Total items 18; average score 76.77 per cent correct. (In this case and those which follow the score of a given pupil is calculated as per cent correct of the items attempted and not on the basis of the total items in the day's work.)

Tuesday, October 9.

A hectographed sheet was prepared containing 8 pairs of chicks.

(1) "Count all the chicks." The experimenter had to show many pupils how to write 15. Several who knew what number to write did not know how to write it.

(2) "Draw a ring around 9 chicks." This exercise proved to be a good one since copying from one's neighbor was difficult. Almost all pupils responded correctly.
(3) "Draw seven hats."
(4) "Copy this number picture": o o o
o o o
o o o
(5) "Write the numbers from 1 to 12." Considerable difficulty was encountered and the experimenter again demonstrated carefully how 6, 8, and 12 were written.

Total items 20; the average score raised to 85.38 per cent correct.

Wednesday, October 10.
(1) "Draw 9 marbles at the top of your page." "Draw 3 boxes in the middle of the page". "Draw 5 hats at the bottom of the page."
(2) The experimenter demonstrated at the board how one ring could be drawn around 4 marbles, 1 box, and 2 hats so that these seven objects would be included within the 'ring'. The following instructions were given: (a) "You draw one ring around 3 marbles, 1 box, and 2 hats." (b) "Draw one ring around 2 marbles, 2 boxes, and 1 hat." The results indicated that it was difficult for a child to remember all the factors in these instructions. The instructions were repeated several times, but confusion was apparent.
(3) The writing of the numbers 1 to 13 inclusive was demonstrated and the following given: "Write all the numbers from 1 to 15."

The total items 20; average score 83.80 per cent correct.

Thursday, October 11.

Mimeographed material was used for the first time. (Tables I and II.)
(1) The experimenter demonstrated at the board the 'biggest' number picture of a pair and also how it could be marked by drawing a ring around it. When each member of the pair contained the same number, no marking was done. The larger number was also counted and the number written within the ring drawn by the experimenter. The children were then asked to imitate this procedure on the mimeographed material.
(2) The writing of the numbers 4, 7, 9 was again demonstrated, after which the pupils were asked to write the numbers 1 to 12.

TABLE I

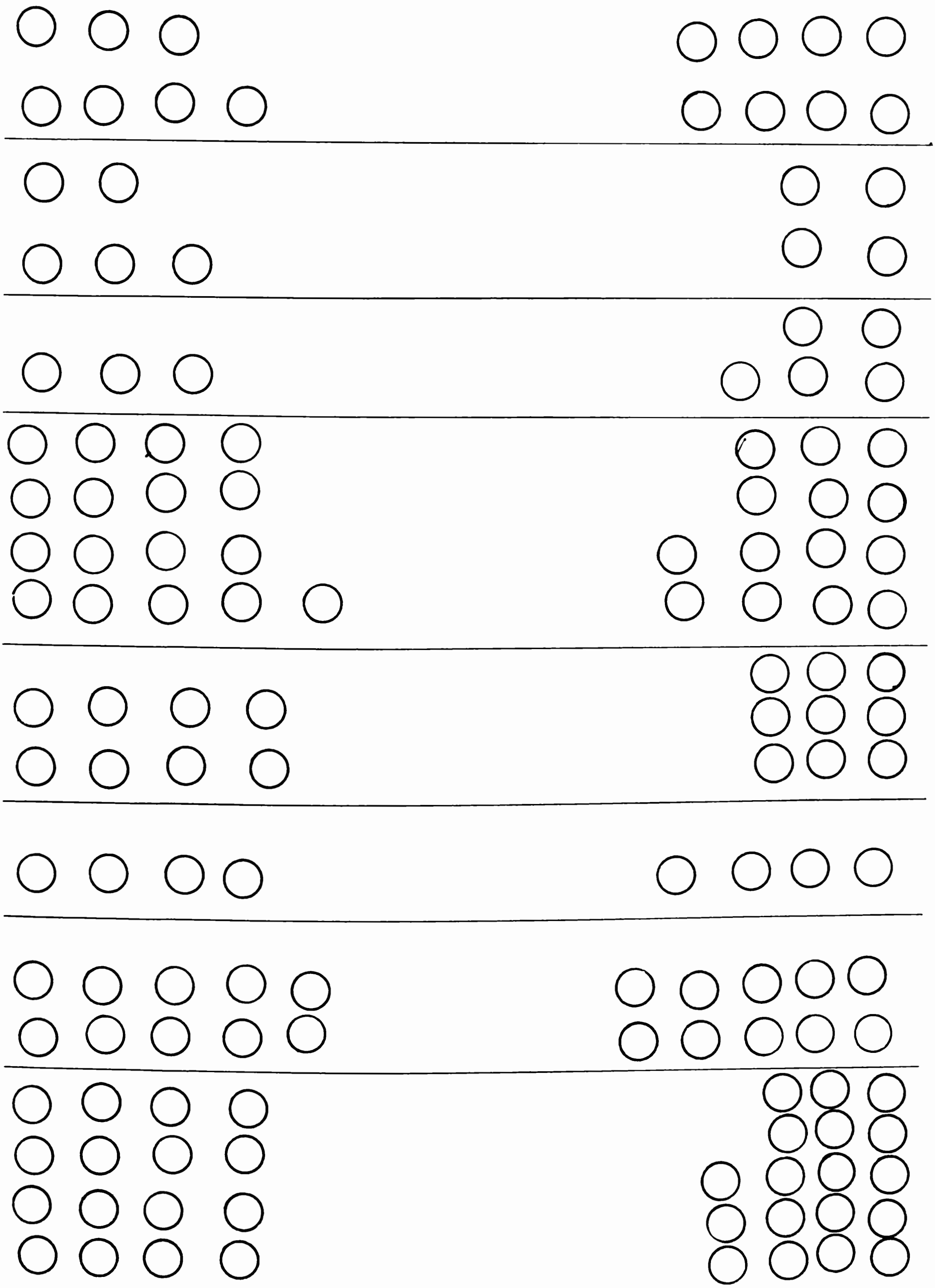
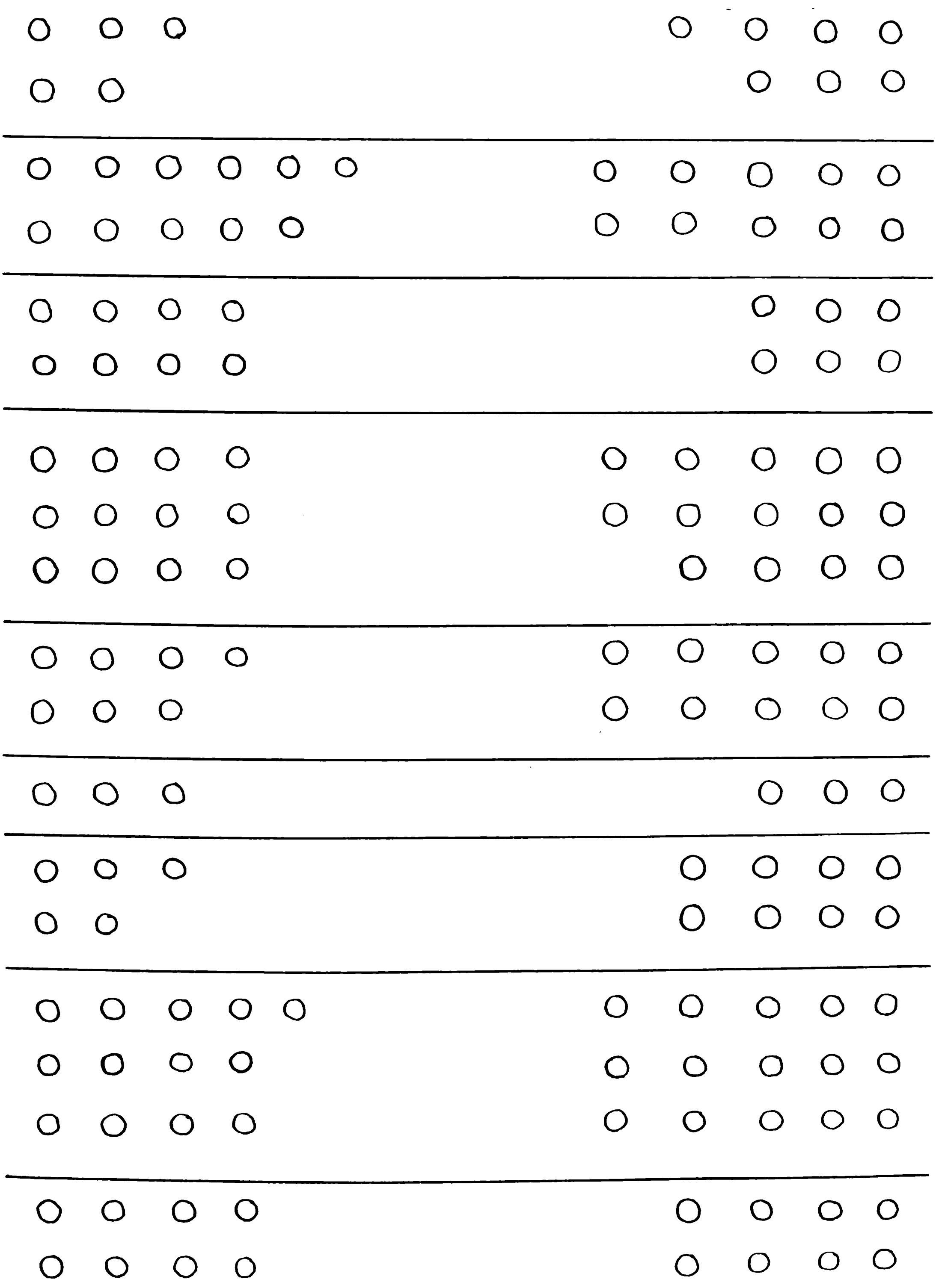


TABLE II



(3) The pupils were instructed to draw a number picture of 5 and 6, writing the number underneath each picture.

Total items 34; average score 80.64 per cent correct.

Friday, October 12.

Thursday's work with the mimeographed material was repeated with a new sheet shown in Table III. The experimenter again carefully demonstrated: (a) How to find the larger number picture of each pair, (b) How to draw a ring around this larger picture, (c) Where to write the number of the larger picture, and (d) How to treat those combinations which contained equal number pictures.

Total items 16; average score 78.00 per cent correct.

3. Monday, October 15.

Another blackboard demonstration was given similar to that of Friday. The mimeographed materials appear in Tables II and III.

Difficulties in counting were obvious by the amount of lip movements and by the large number of total body movements.

Total items 34; average score 86.54 per cent correct.

Tuesday, October 16.

(1) The children were requested to check the papers of Monday, October 15. The introduction of this new task was confusing. Very few actually accomplished a check of their own papers.

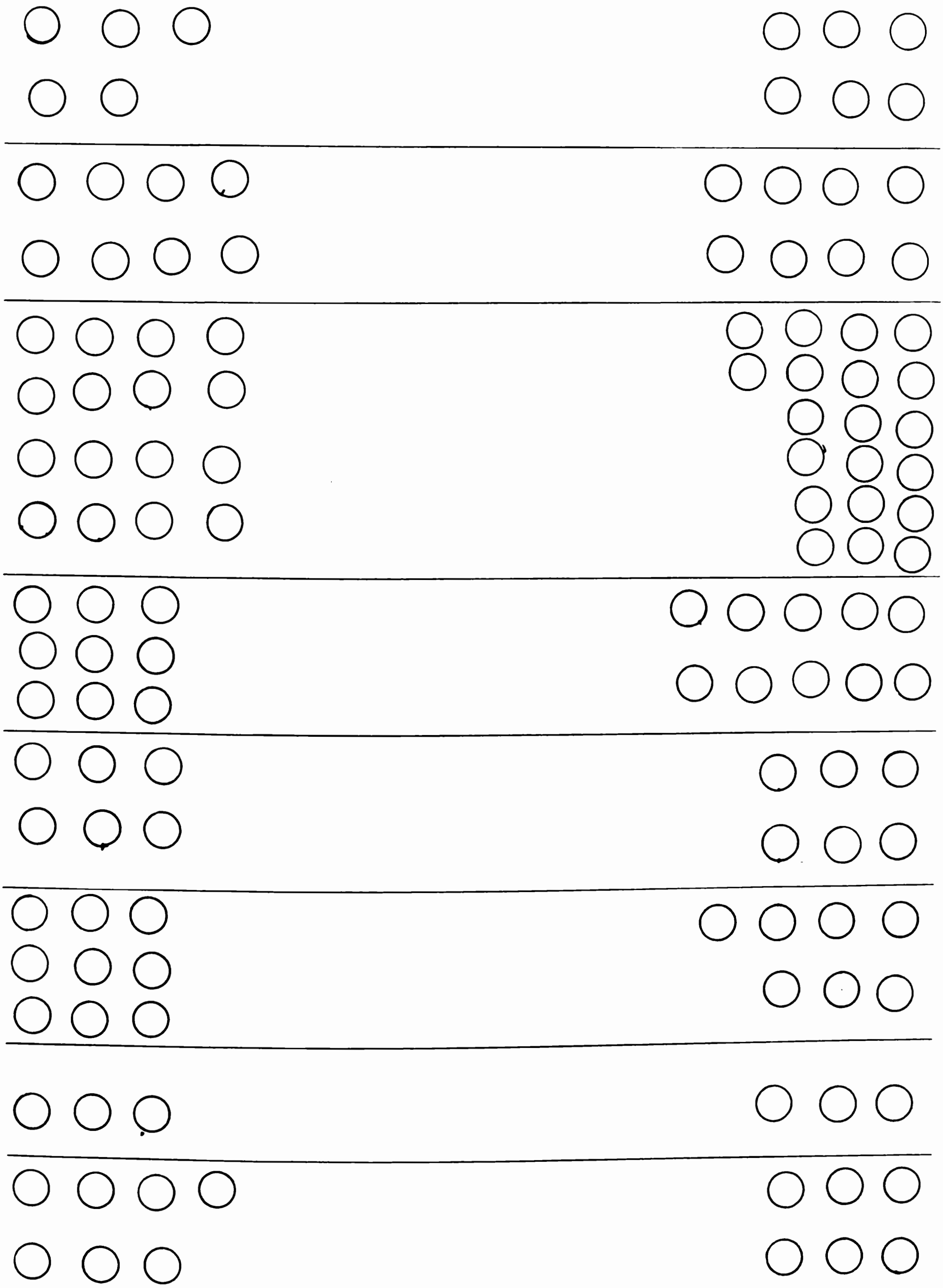
(2) The pupils were given one page of number pictures and were requested to make pictures of a pair equal in number. The actual instructions were: "Make the pictures the same." Many of the children made similar patterns and, consequently, number pictures above seven were not equated correctly and accurately.

Total items 8; average score 62.14 per cent correct.

Wednesday, October 17.

The children purchased Arithmetic Work Pad I by Lennes (22) at the beginning of the school year and some of this material was utilized in the present experiment. No quantitative scores of the results with this material were recorded. The children could not read the instructions for the majority of the exercises and thus the ex-

TABLE III



perimenter usually explained what type of work was required. The principal exercises used in this pad were as follows: Counting of objects, recognizing number symbols, comparing and contrasting the size of number pictures, comparing various objects, and solving the easier addition and subtraction problems.

On Wednesday, October 17, pages 11, 12, and 13 of the pad were used (comparing sizes of objects, recognizing the number of objects, recognition of the number names, and recognizing the number symbols). The instructions for the three parts of the exercise on page 13 were given all at once which resulted in some confusion.

Thursday, October 18.

The class worked the exercise on page 14 of Lennes Pad I (recognizing the number of objects, the number names, and the number symbols).

A demonstration of how to equate two number pictures was given. The children encountered considerable difficulty following the experimenter's procedure. No quantitative scores were recorded.

Friday, October 19.

The pupils were given three sheets of mimeographed material (Tables I, II, and III). They were required to draw enough circles to make the number pictures of each pair equal.

Numbers above 6 were difficult to equate in this fashion. Both #20 and S31 became 'panic stricken' on these higher numbers; with encouragement, both managed to work the easier problems satisfactorily.

Maximum items completed 18; average score 69.15 per cent correct.

4. Monday, October 22.

A new set of number pictures were given the children and they were requested to make the pairs equal by drawing circles wherever necessary. (See Tables IV and V. No numbers above ten appear in Table IV.)

S20 and S31 improved; the former was given special instruction. There was much counting and recounting by these subjects (as many as six or seven times) on the harder problems.

The slower pupils completed only one sheet of problems (10 items); the others completed two sheets (20 items). Average score 73.75 per cent correct.

TABLE IV

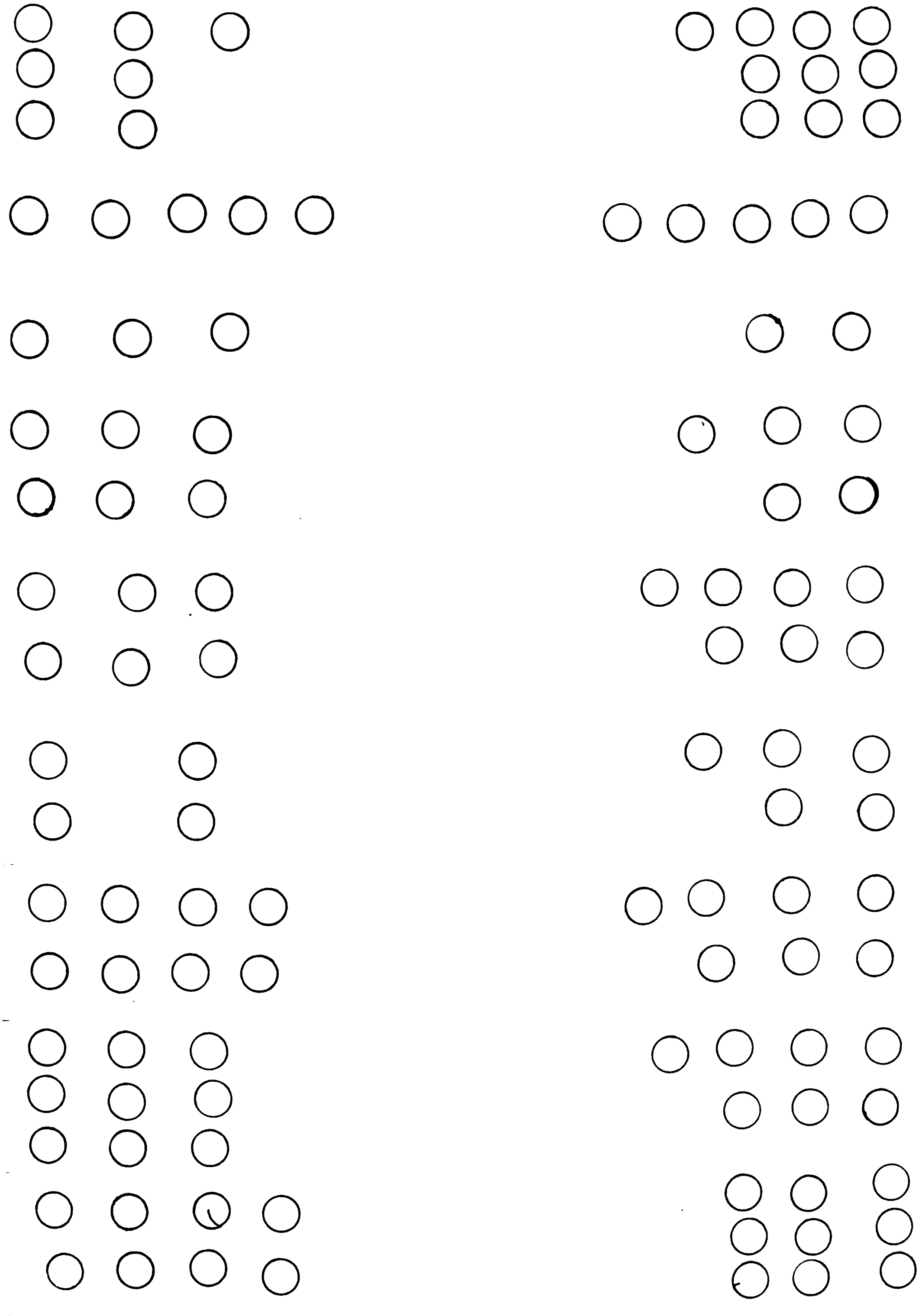
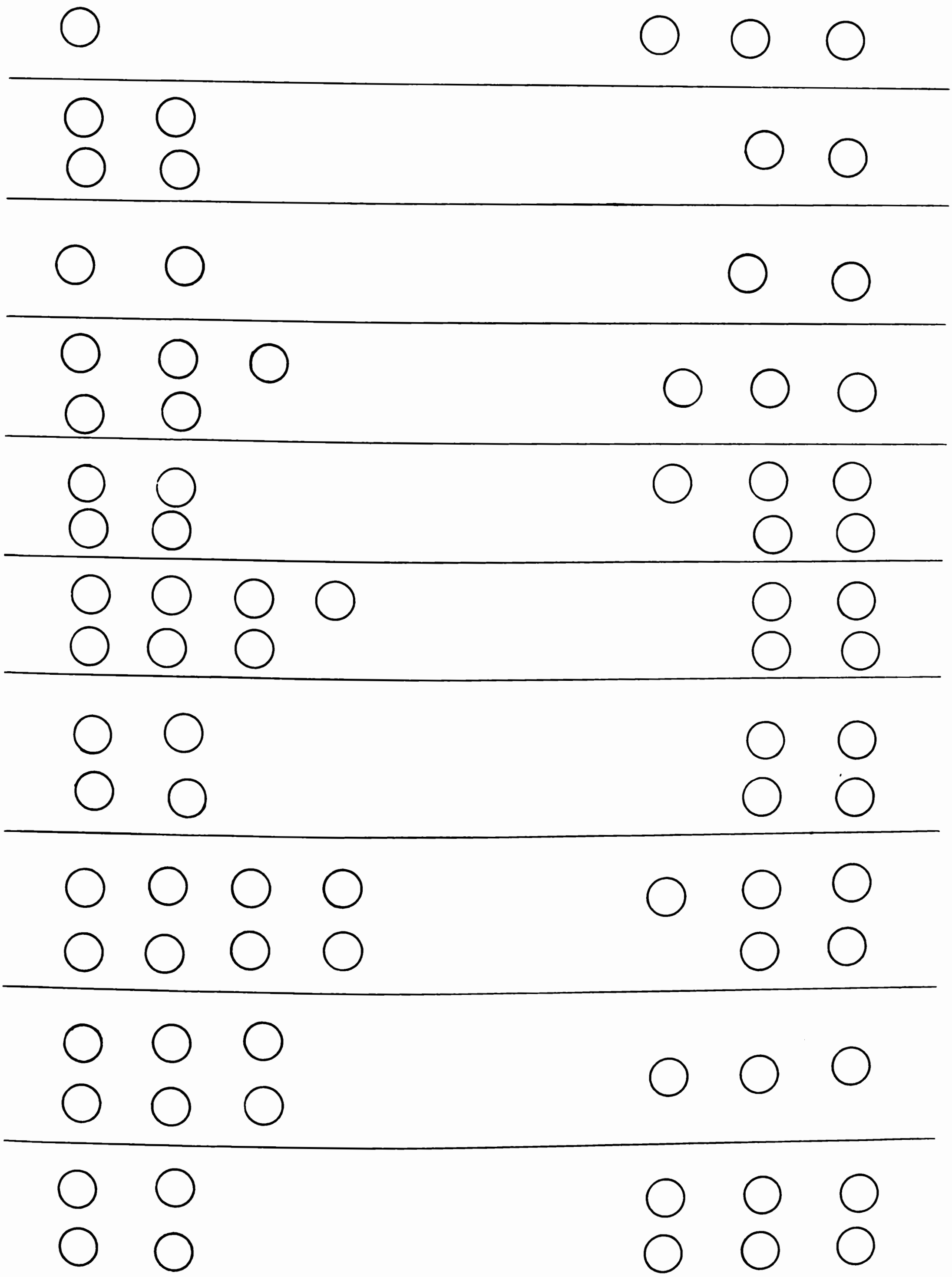


TABLE V



Tuesday, October 23.

The pupils were asked to write the numbers 1 to 10. The figures 5, 6, and 7 offered difficulty. The results showed that no improvement had taken place since this exercise was last used approximately a week previously.

The work of Monday, October 22, was repeated. The class showed improvement; all but five members of the class made at least one perfect paper. The papers with no errors were returned to the pupils.

Maximum items completed 20; average score 77.00 per cent correct.

Wednesday, October 24.

Hectographed papers were provided containing pictures of 15 hens, 2 apples, 2 dolls, 6 chairs, and 11 milk bottles.

(1) "Count the hens, then the dolls, (and so on)."
(2) "Draw the same number of marbles as there are chairs."
(3) "Draw two more boxes than there are apples."
(4) "Draw three more hats than there are dolls."

S30 was given special aid. The experimenter had him make equal various pairs of number pictures. This subject had been unable to keep up with the class and in his attempt to do so had adopted undesirable methods of counting -- skipping around more or less at random. When numbers were once comprehended, the problem of making a pair of number pictures equal proved easy.

Total items 15; average score 84.96 per cent correct.

Thursday, October 25.

The children were given a mimeographed sheet (Table VI). The resemblance of the large circle to a pie and the parts to pieces of a pie was drawn to the pupils' attention. The pupils were shown how the pieces of pie were to be counted and labeled.

(1) "Count the pieces of pie and write the number beside the pie."
(2) "Draw lines in the blank circles of each pair so that they have the same number of 'pieces of pie' in them."
(3) Three pupils (S14, S17, S26) were given another opportunity to work the exercises of Tuesday, October 23.

The advanced pupils used inspection on the simpler diagrams and recounted the more difficult diagrams rather than making use of the number symbols in performing instruction (2).

Total items 13; average score 77.23 per cent correct.

Friday, October 26.

The materials of Thursday, October 25, were given again. The pupils still failed to utilize the short cut (making use of the number symbol) in performing the division of the blank circles into parts.

Total items 13; average score 82.04 per cent correct.

5. Monday, October 29.

Mimeographed sheets containing 12 pairs of circles each divided into parts (up to ten) were given the pupils (Table VII).

(1) "Count the parts of each circle and write the number above the circle."

(2) "Underline the circle of each pair which has the greater number of parts."

(3) "Make a cross above a circle which has 7 parts."
"Make a dot above a circle which has 5 parts."
"Make a line above a circle which has 10 parts."
"Make two dots above another circle which has 5 parts."

No quantitative scores were recorded; the papers were returned to the pupils.

Tuesday, October 30.

The mimeographed materials shown in Tables VIII and IX were given. (The difference in each pair is one.)

The instructions required the pupils to count the parts of each circle, to write the number above the circle, and then to underline the circle of each pair having the greater number of parts.

Total items 24; average score 86.43 per cent correct.

Wednesday, October 31.

(1) The class was given material shown in Table VI. The pupils were asked to write the number of parts of the divided circle in the blank circle. This task was mastered so easily that it was not used in calculating the quantitative score.

(2) The class also was given material shown in Table V. The instructions were changed, however, so that the pupils were expected to write the number of circles which should be added. A board demonstration of this procedure was given. Only eight pupils (one-third of the class) grapsed the problem.

Errors were of four types: (a) Placing the number symbol (which was to indicate the number of circles to be added to the number picture) on the wrong number picture, (b) indicating by number symbol how large the smaller of the pair should be, (c) writing the actual numbers of all the pictures, and (d) making equality by drawing circles

TABLE VI

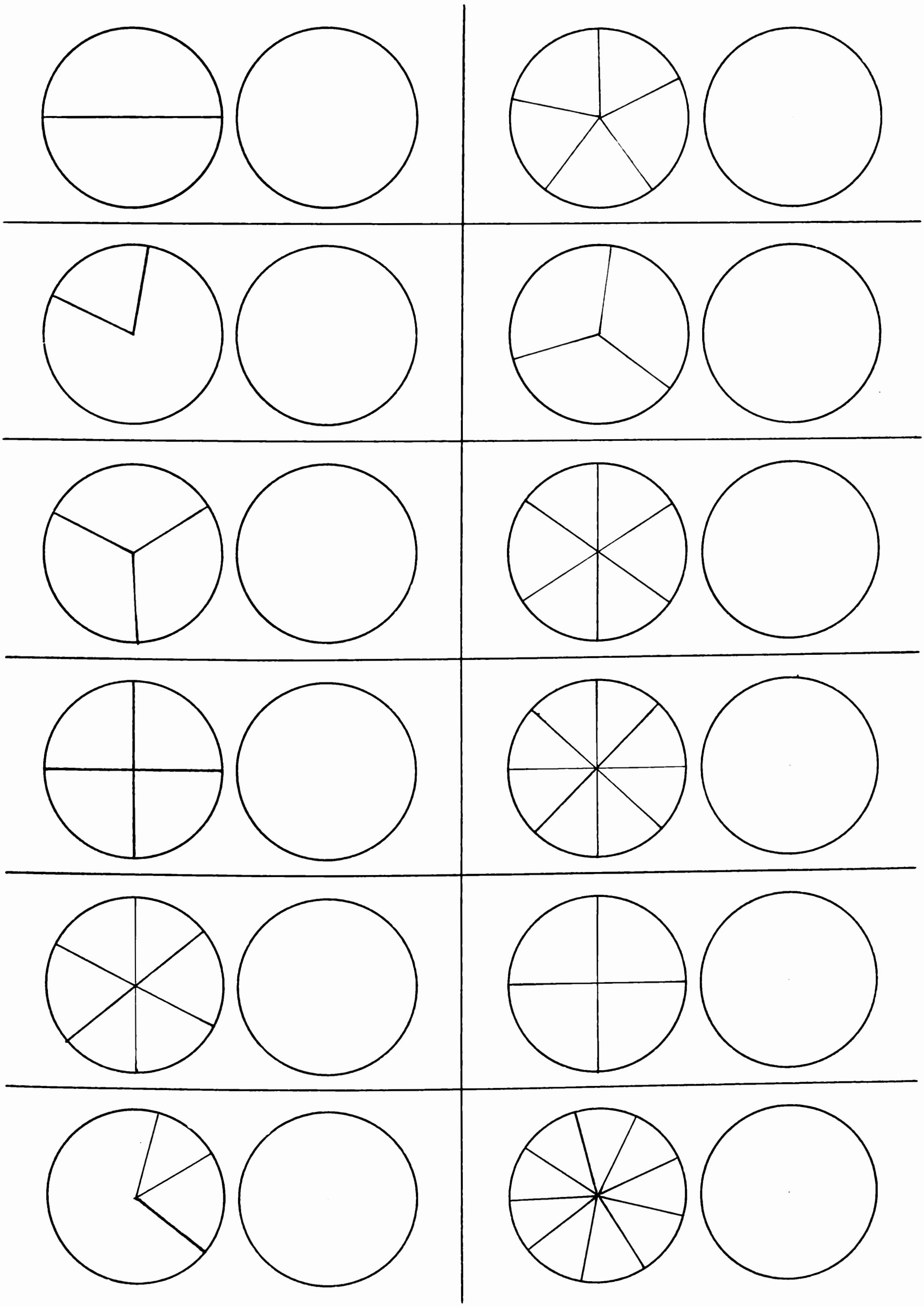


TABLE VII

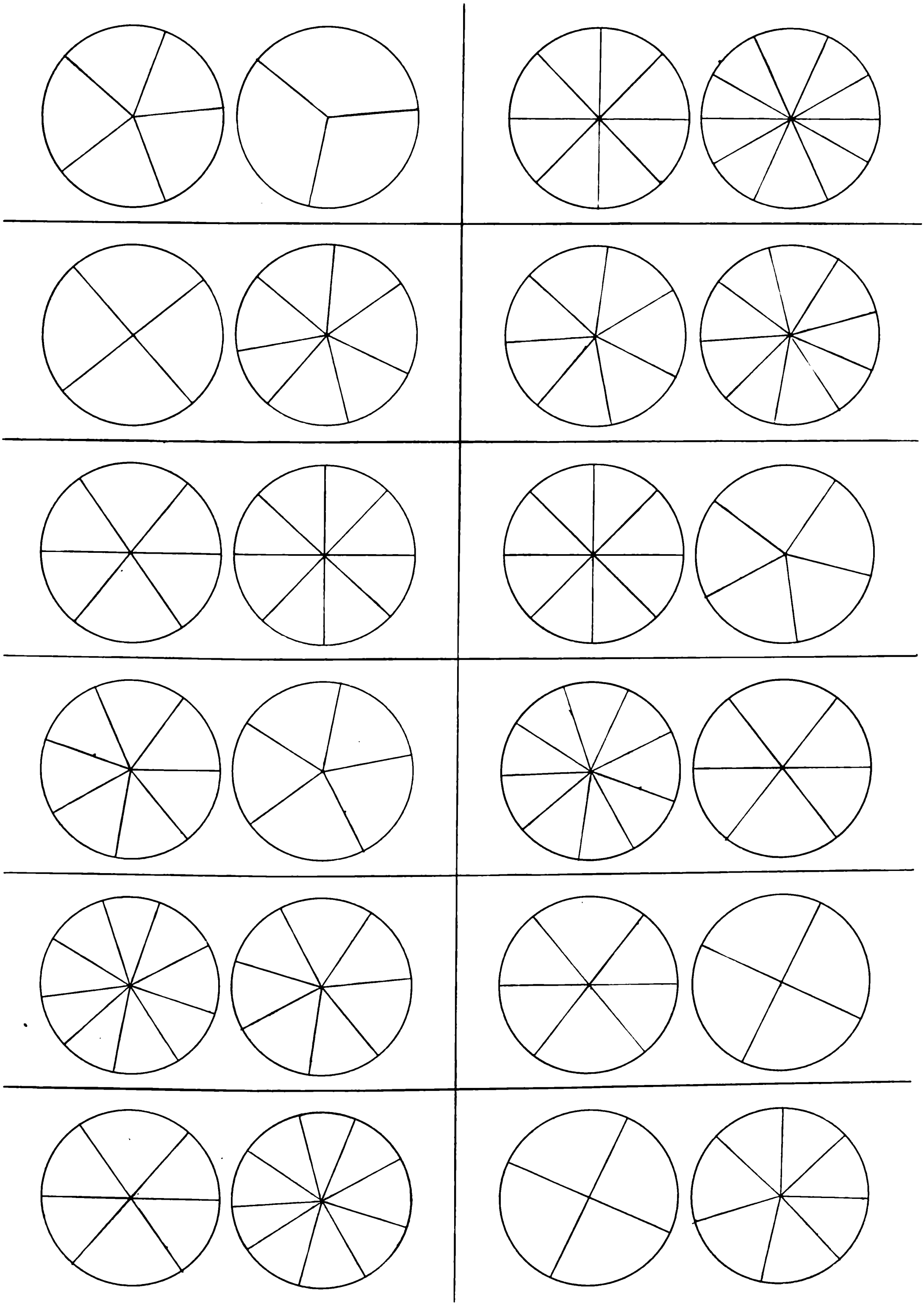


TABLE VIII

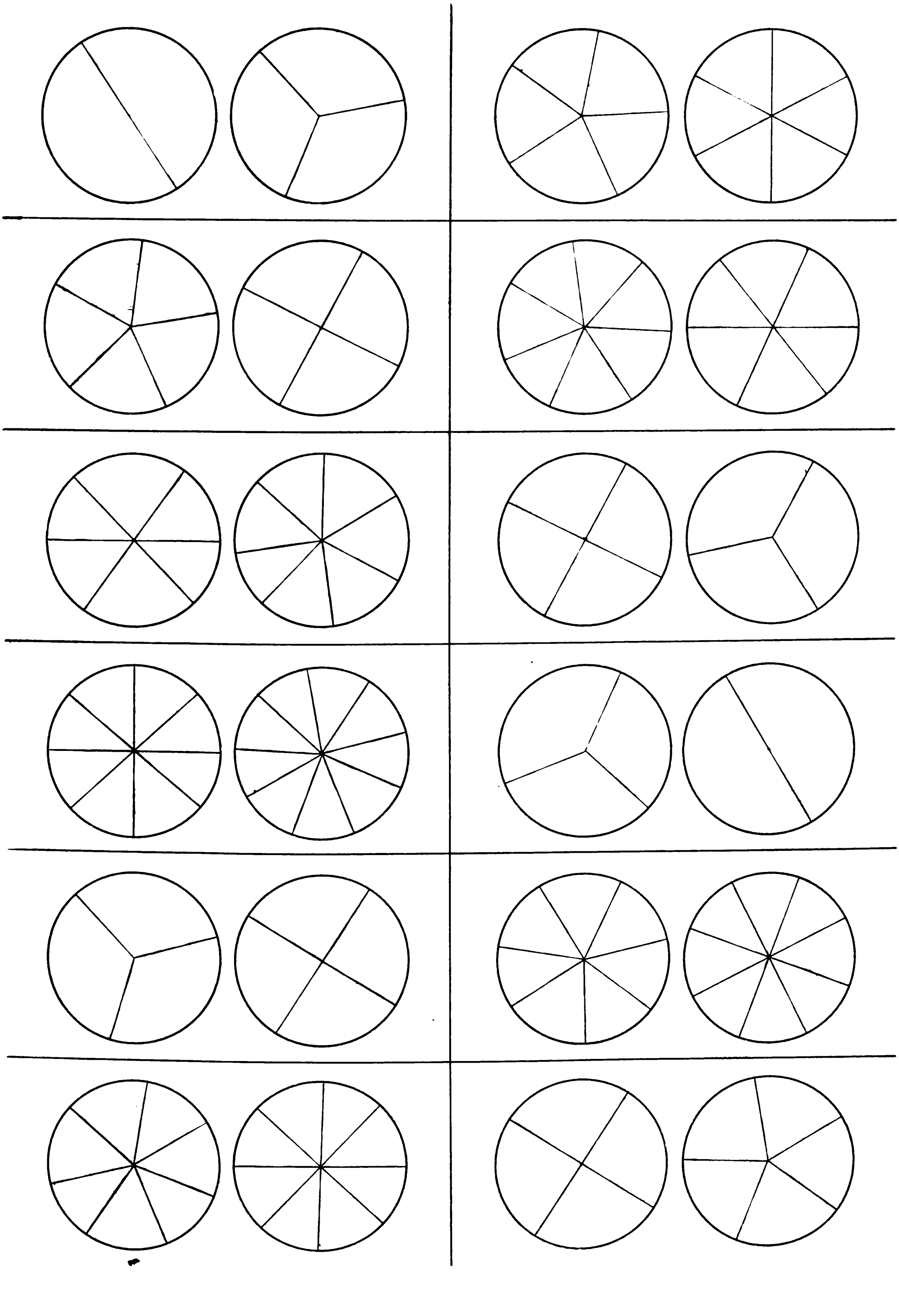
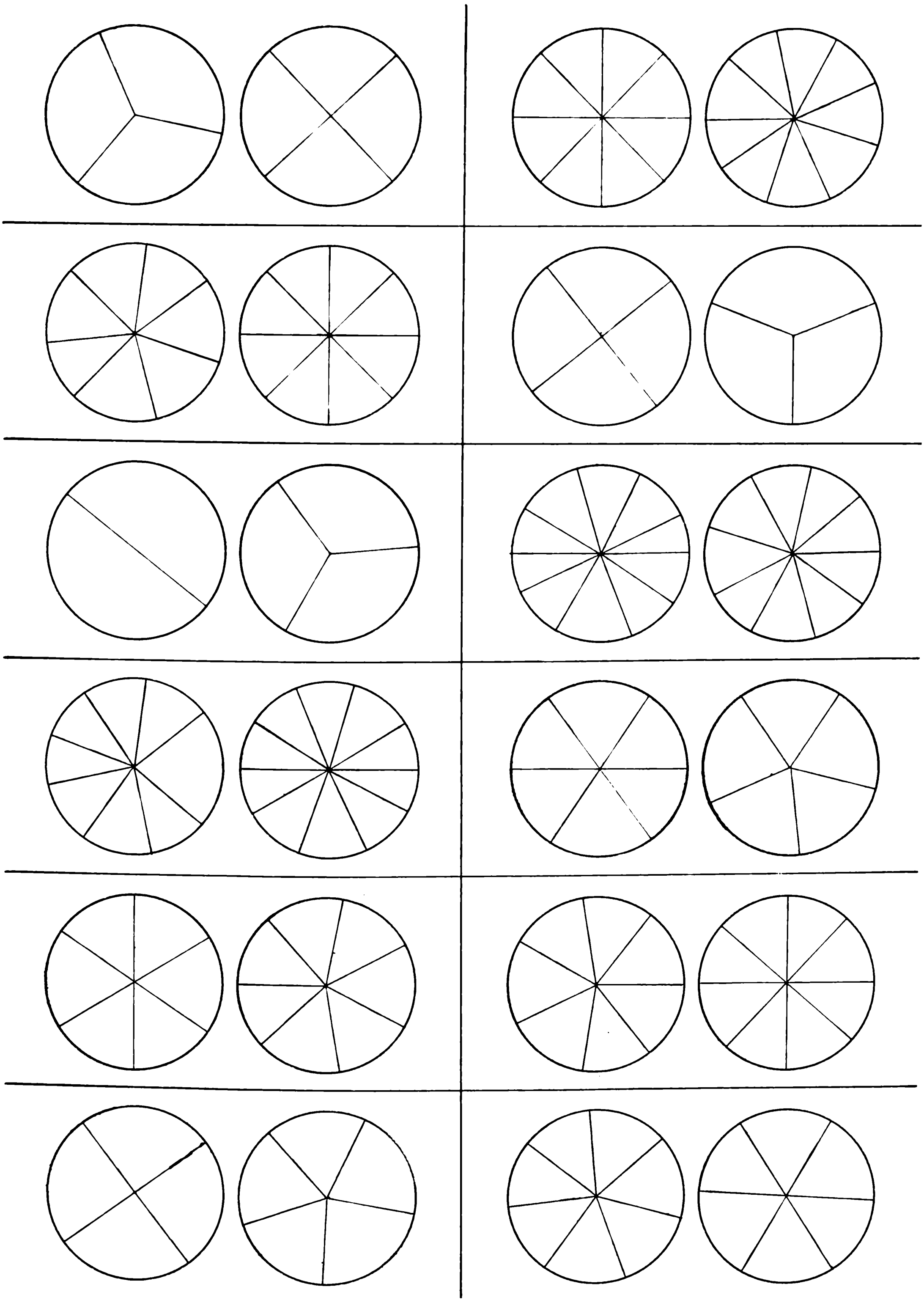


TABLE IX



on the picture having the smaller number.

Although considerable confusion resulted, some real thinking was stimulated. S3 and S16 asked the significant question in connection with the combination 5-5, "How do you make nothing?" Sensing the need for writing a symbol for zero indicated clearly that these pupils had grasped the meaning of addition and subtraction involved in this exercise.

Total items 10; average score 51.25 per cent correct.

Thursday, November 1.

The class was given pages 16 and 19 of the Lennes Work Pad I (recognizing the greatest and the least number of objects, also recognizing 'first' and 'second'; drawing a designated number of objects). No quantitative scores were recorded.

6. Monday, November 5.

The instructions of Wednesday, October 31 were repeated with new material (Table X). The types of errors were similar to those previously mentioned.

Total items 12; average score 54.99 per cent correct.

Tuesday, November 6.

The 'difference' problem used on Wednesday, October 31 and Monday, November 5, was given with another sheet of mimeographed material (Table XI).

It was discovered that S23 had been making number pictures simply symmetrical in pattern and not necessarily equal in number. The more recent instructions, therefore, which called for the use of number symbols in making two pictures equal had no meaning for the subject. A few problems with number pictures (not over 5) were given to aid in straightening out this difficulty.

Total items 12; average score 71.19 per cent correct.

Wednesday, November 7.

The materials were in the Lennes Pad I, pages 20 and 23 (drawing a designated number of objects; reading and writing of number symbols). No quantitative scores were recorded.

Thursday, November 8.

The pupils were given mimeographed material (Tables VIII and IX) and were instructed simply to count the parts of the circles and to write the number above the circle.

TABLE X

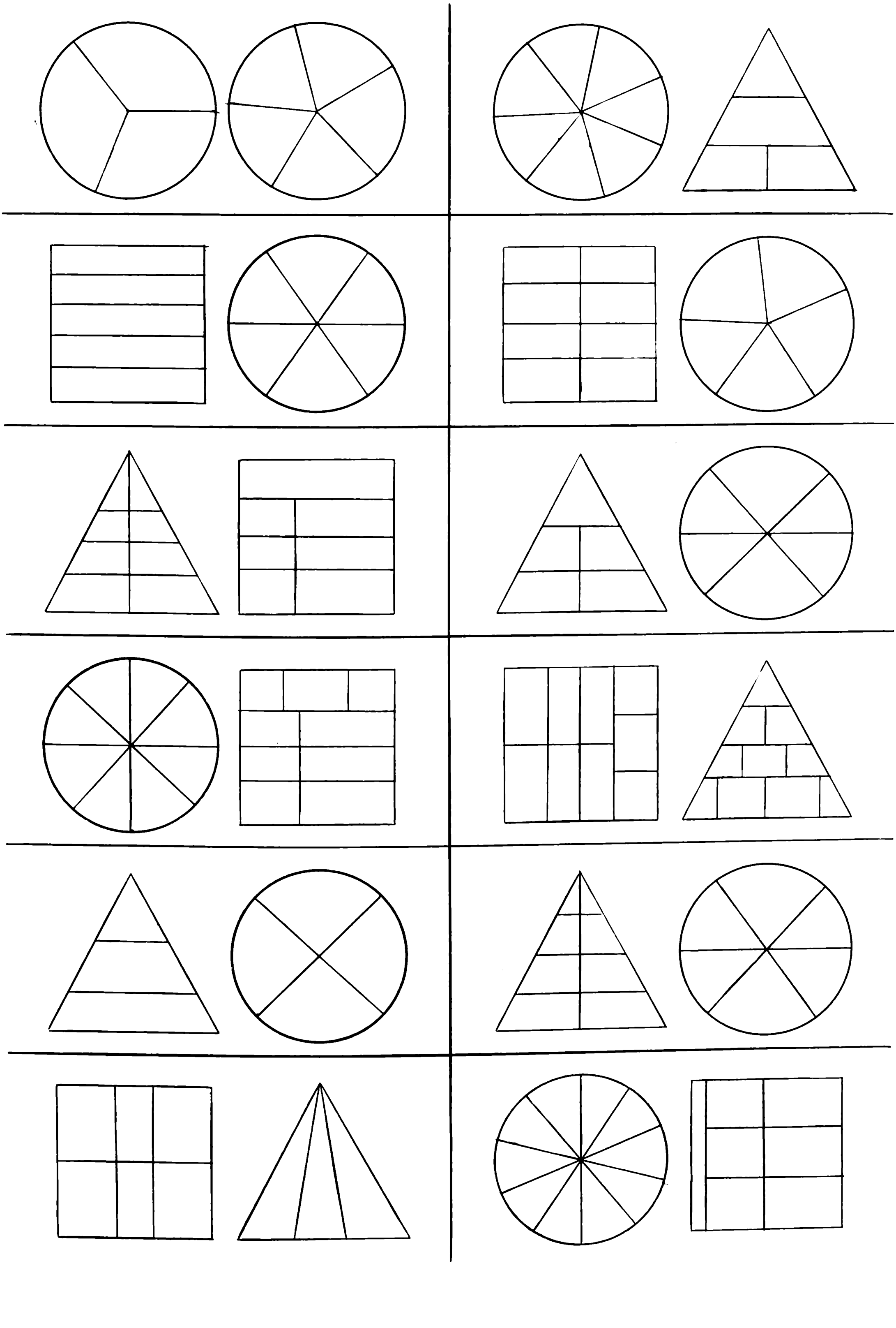
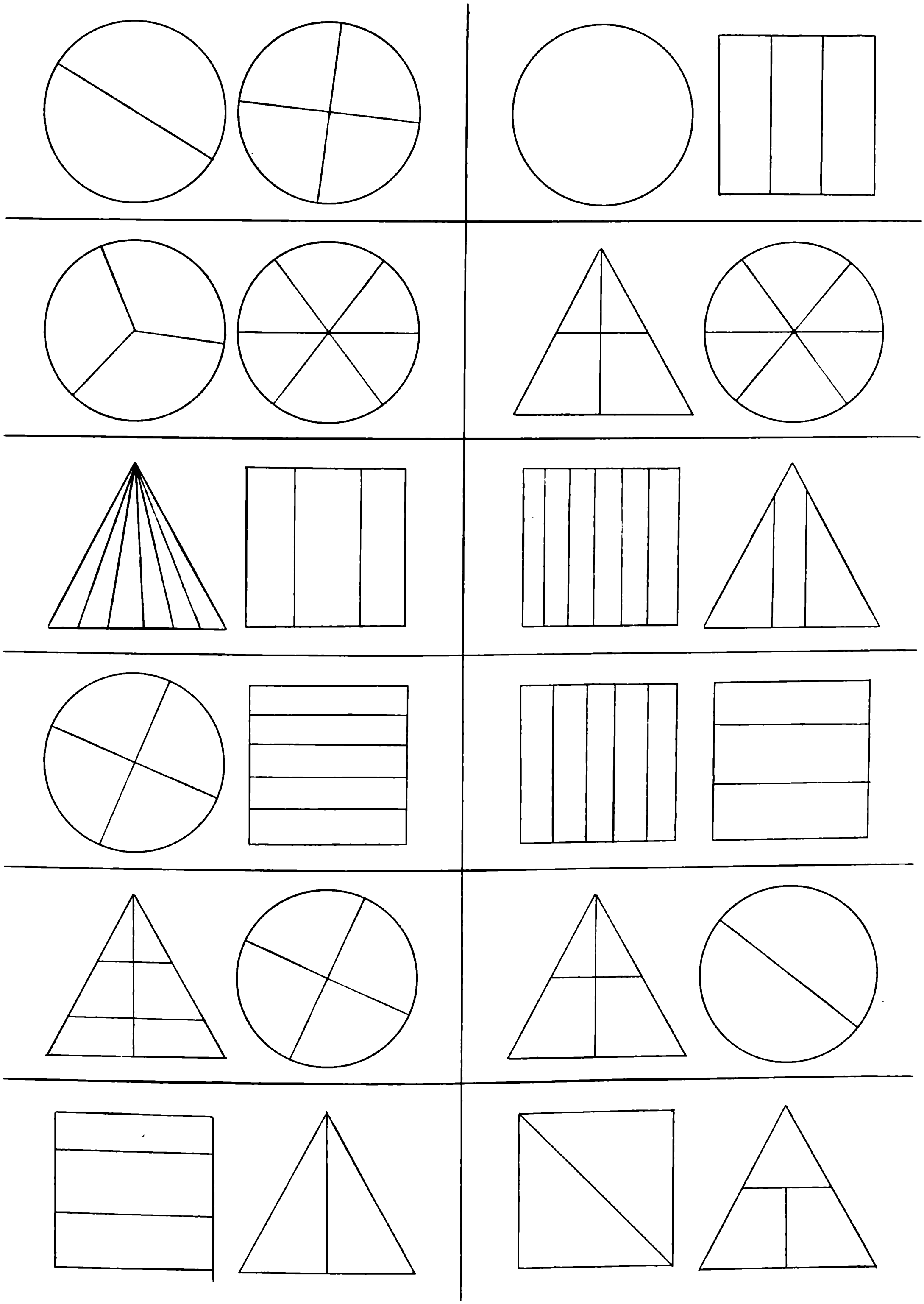


TABLE XI



Maximum items completed 48; average score 95.71 per cent correct.

Friday, November 9.

The instructions and materials were identical with those of Thursday, November 8. All correct papers were returned to the pupils. One-third of the class counted 48 items correctly.

Maximum items completed 48; average score 94.19 per cent correct.

B. Second Six Weeks

1. Tuesday, November 13.

(1) The pupils were given an opportunity to count and label the parts of the circles on two out of three papers (Tables VII, IX, and XII). (2) S19, S21, S16, S13, S14, S32, and S26 remained after class to work a few 'difference' problems with number pictures. They were required to write the number of parts to be added to the smaller of a pair of number pictures in order to obtain equality. S26 was the only pupil to make a perfect response. S16 failed altogether.

Maximum items attempted 48; average score 90.00 per cent correct (part 1).

Wednesday, November 14.

Mimeographed sheets (Table XII) were given the pupils with instructions (1) to count and label the number of parts in each circle and (2) to write the number below each pair which would indicate the parts that should be added to make the circles equal. Two examples were given on the board.

Six pupils failed to respond satisfactorily. No quantitative scores were recorded.

Thursday, November 15.

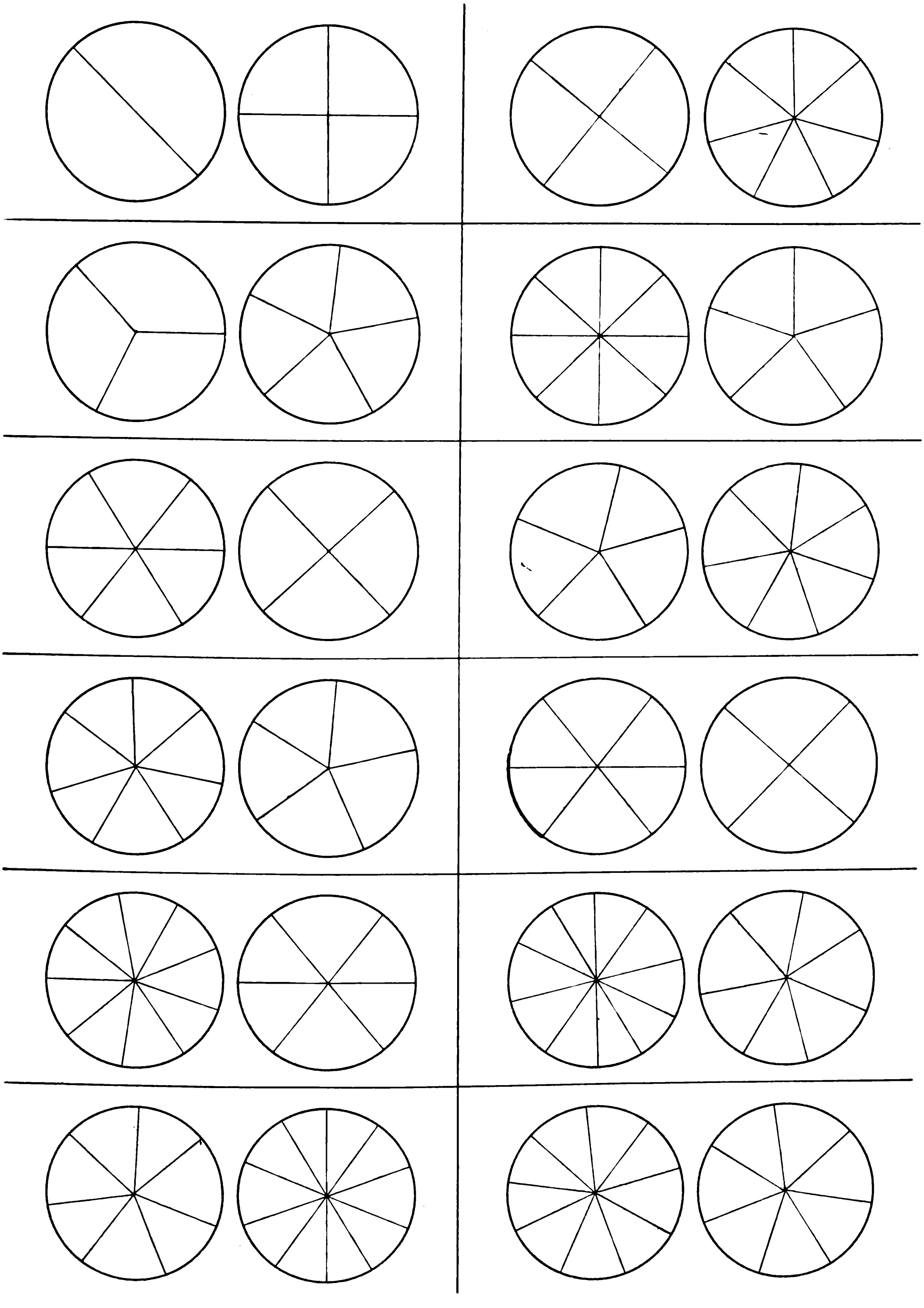
The work of Wednesday, November 14 was repeated with a different set of materials (Table VII). The class as a group appeared to grasp better the meaning of the problem. S26 had the only perfect paper. No quantitative scores were recorded.

Friday, November 16.

The 'difference' problem was given again. The material is shown in Table XI; only the numbers up to six appear in this material.

Approximately one-half of the class solved the problem in a satisfactory manner. S12, S20,

TABLE XII



S23, and S31 failed to discover what was demanded of them.

Total items 12; average score 61.04 per cent correct.

2. Monday, November 19.

(1) The children were given pages 17 and 18 of the Lennes Pad I. This work was a matching exercise involving addition. The children were required to draw lines connecting the 'equal numbers'. The following example illustrates this type of exercise:

o and o	o o o
o o and o	o o
o and o	o o o o
	o o

Although the instructions were made as clear as possible, many of the pupils did not understand what was required of them. It was necessary to give individual aid to most of the children. No scores were recorded.

(2) S23 remained after class to work the problems missed Tuesday, November 16. She learned to employ a counting method in solving a 'difference' problem on as difficult a combination as 7-4. The solutions were verified also by counting.

Tuesday, November 20.

It was decided that a set of number pictures should be made in which the pattern was irregular and in which the parts themselves were different in size and shape. The material in Table XIII was devised to meet these qualifications.

(1) "Count and label each number picture."

(2) "Write the numbers from 1 to 12." The writing of the numbers from memory was not satisfactory; exactly one-half of the class made some error in writing the symbols.

Total items 24 (part 1); average score 89.67 per cent correct.

Wednesday, November 21.

The 'difference' problem described Wednesday, November 14 was given the class (Table X).

Total items 12; average score 66.18 per cent correct.

Thursday, November 22.

The pupils were requested to count the parts of the circle (Table VI) and then to write the number in the accompanying blank circle.

Total items 13; average score 95.55 per cent correct.

TABLE XIII

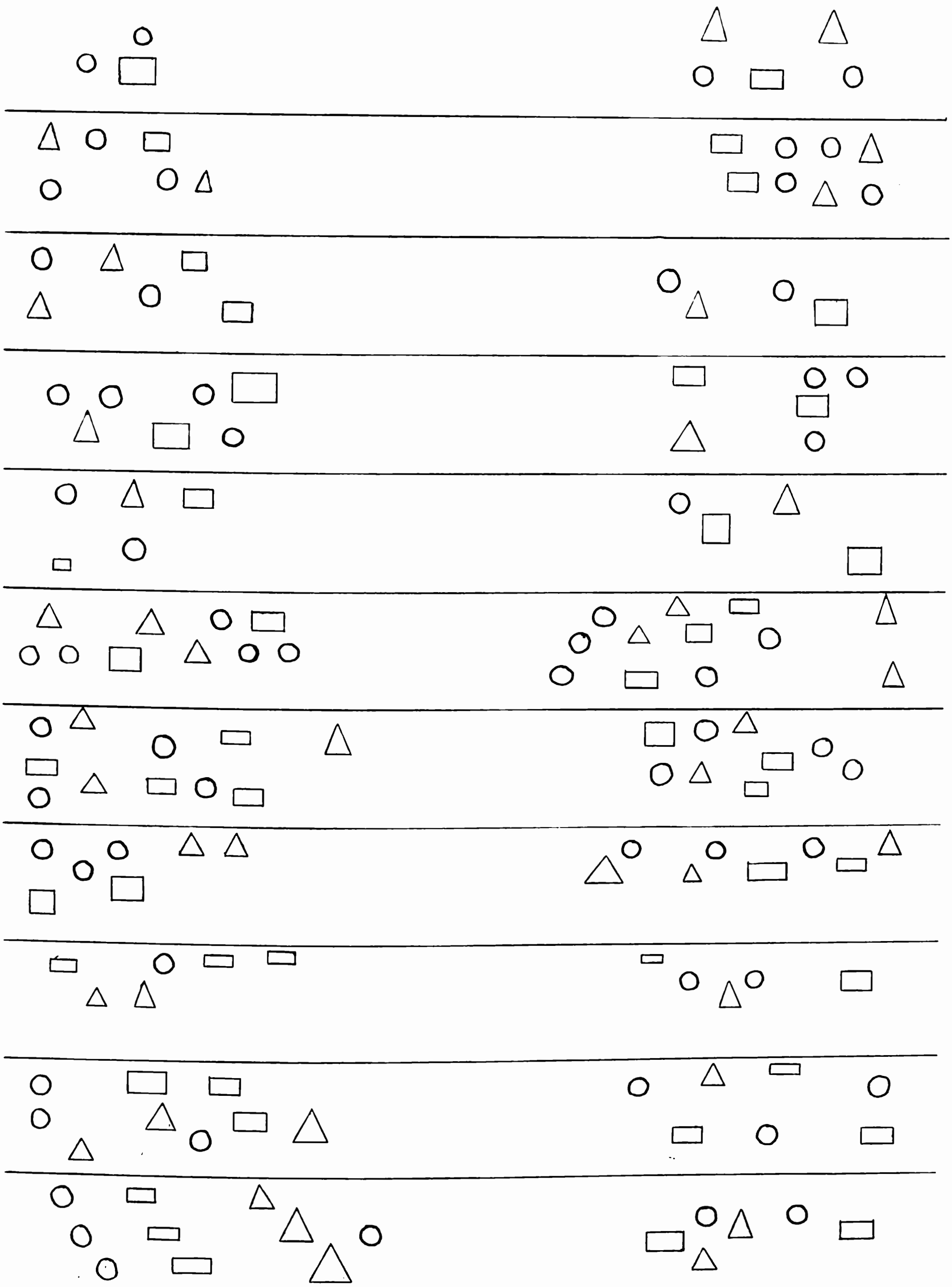


TABLE XIV

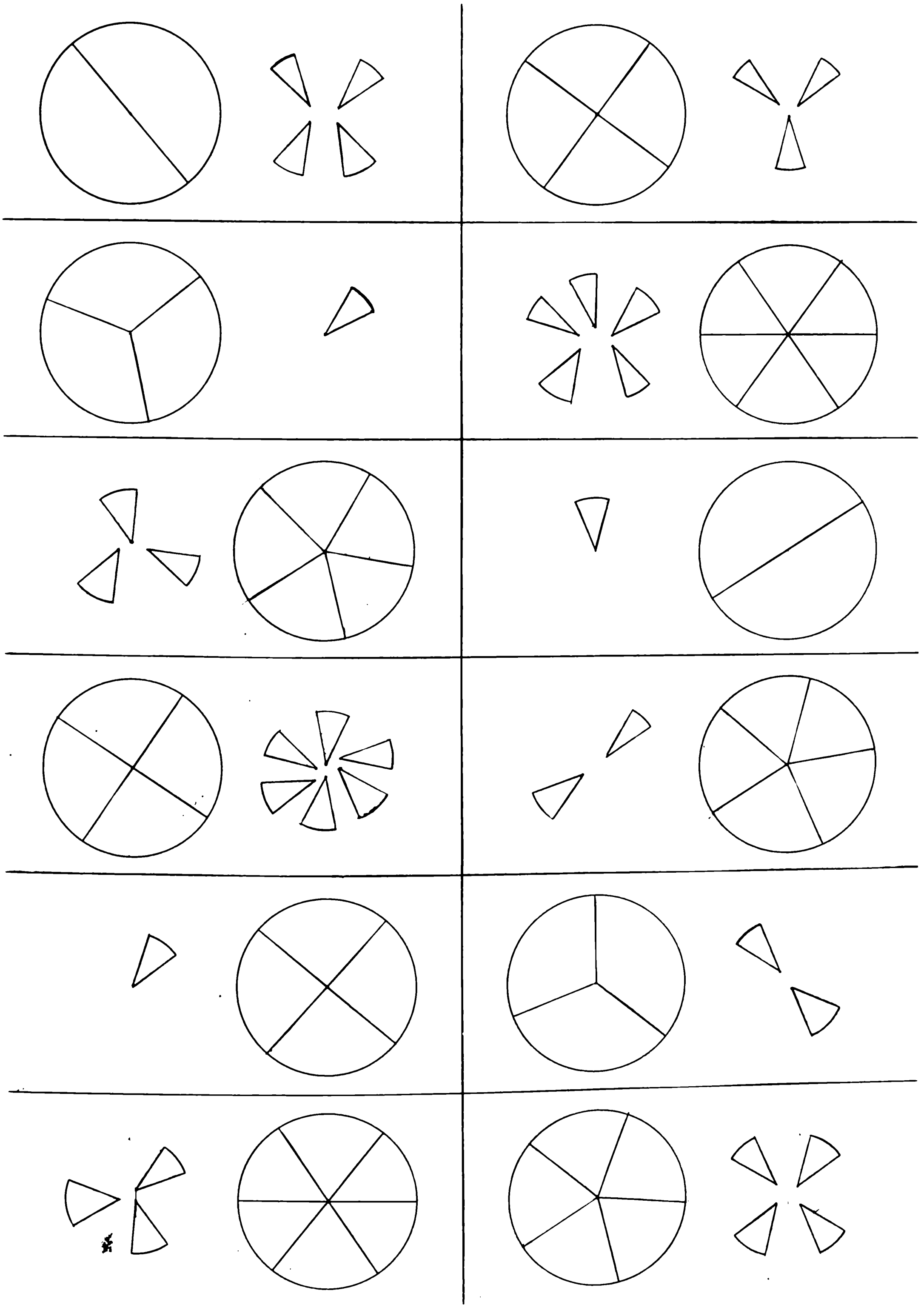
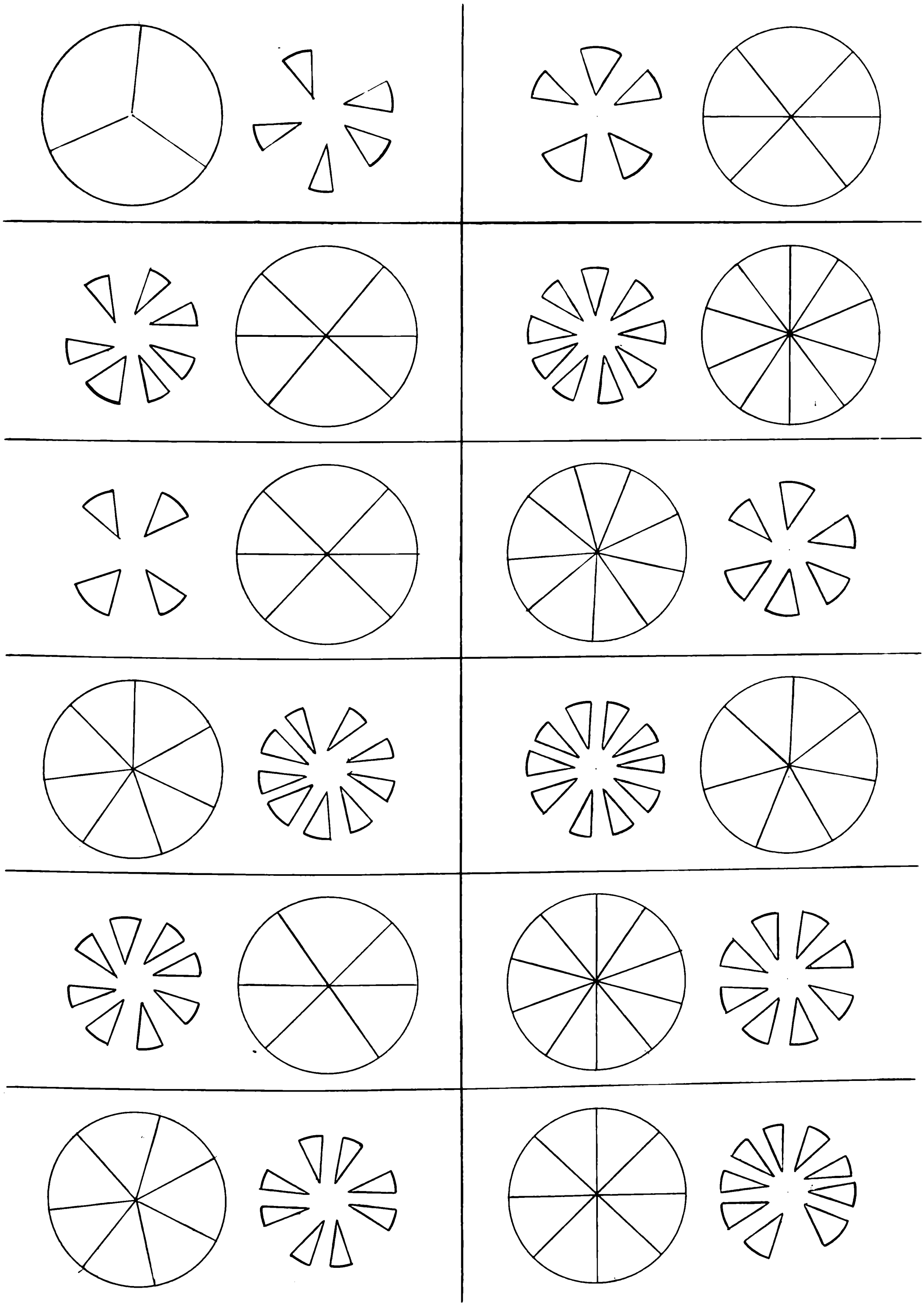


TABLE XV



Friday, November 23.

The class was given pages 21 and 22 of the Lennes Pad I (matching exercise involving addition). A considerable number of errors occurred. No quantitative scores were recorded.

3. Monday, November 26.

The instructions were similar to those of Wednesday, November 14. New materials were used (Tables XIV and XV). The pupils first counted the parts of each combination and wrote the number on the circle or the group of parts. They next found how many more parts were needed on the smaller member of the combination in order to obtain equality. This number, with a circle drawn around it, was written below the smaller member of the combination.

Three types of errors occurred: (1) The wrong number of parts for a member of a combination was indicated, but the difference between the members was marked correctly (indicating very likely that the pupil had recounted to solve the 'difference' problem). (2) The difference was incorrect, but the parts were counted correctly. (3) Both the counting and the difference were incorrect.

Total items 12; average score 80.23 per cent correct.

Tuesday, November 27.

The work of Monday, November 26 was given again and a closer check was kept on the responses of the pupils. The results were: (1) Counting proved to be inaccurate. S6 was given special aid and improved considerably when asked to recount a series (Table XIV). (2) Some of the children failed to grasp the demonstrated procedure for finding differences. (3) S21, S26, S23, and S9 appeared to use the number symbols to some extent in establishing differences between quantities.

All scores were calculated on the basis of 12 items; average score 54.43 per cent correct.

Wednesday, November 28.

The materials are shown in Tables XIV and XV. The pupils were requested to count the parts of every combination and to write down the numbers. A time limit of six minutes was set for this work.

Items completed varied from 32 to 48; average score 92.38 per cent correct.

4. Monday, December 3.

Materials were prepared to demonstrate how parts of a whole (circle) could be re-grouped

TABLE XVI

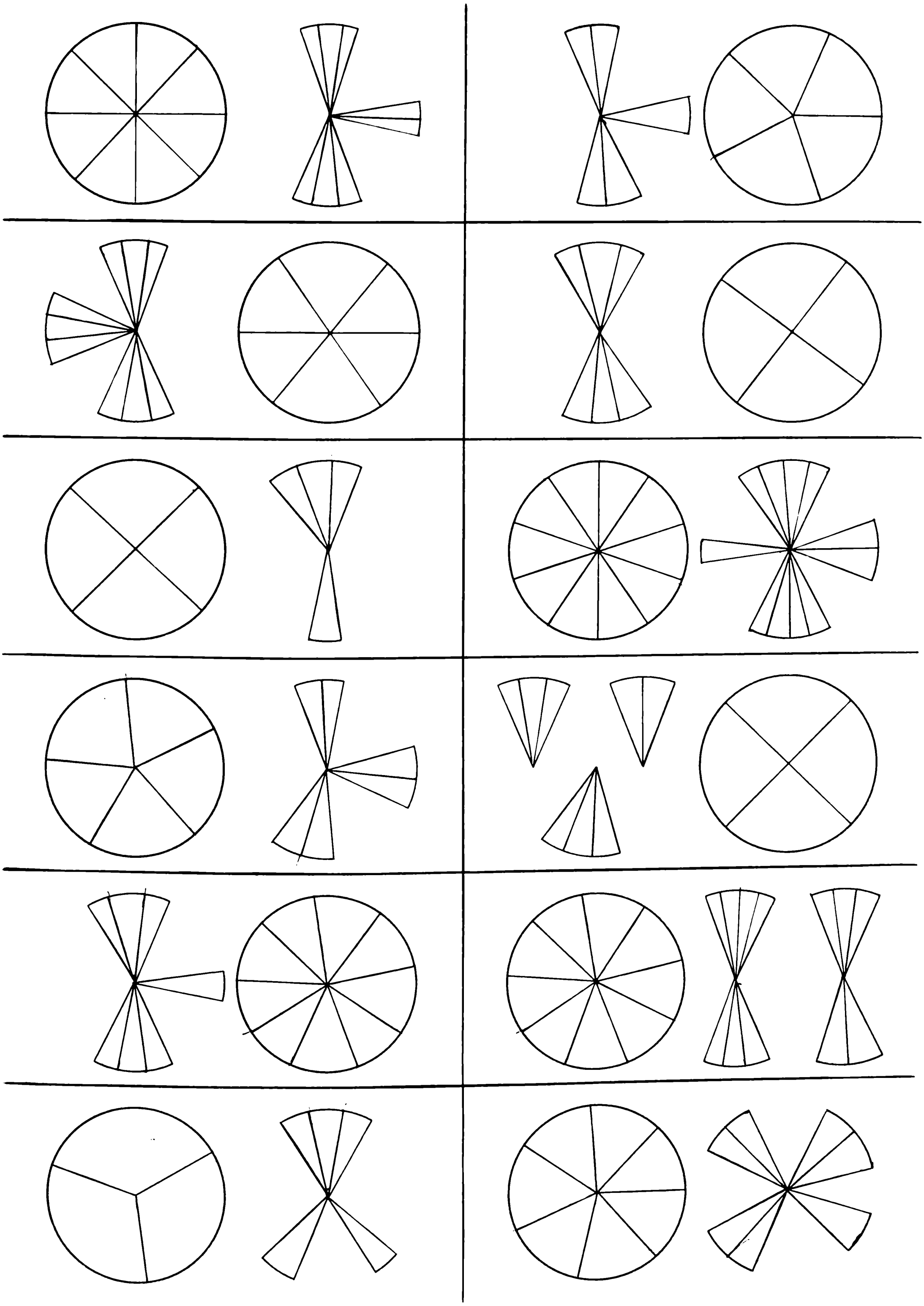


TABLE XVII

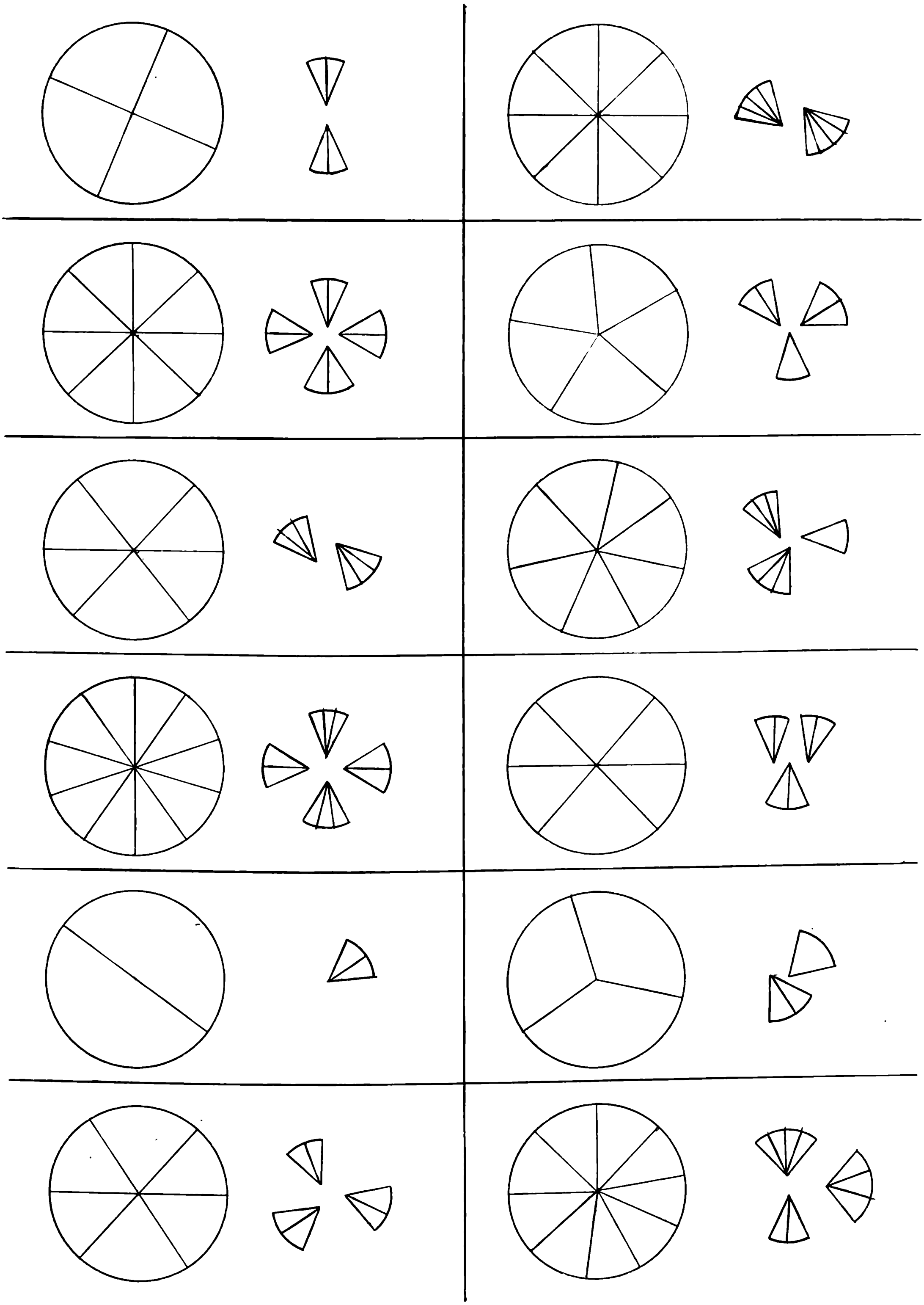
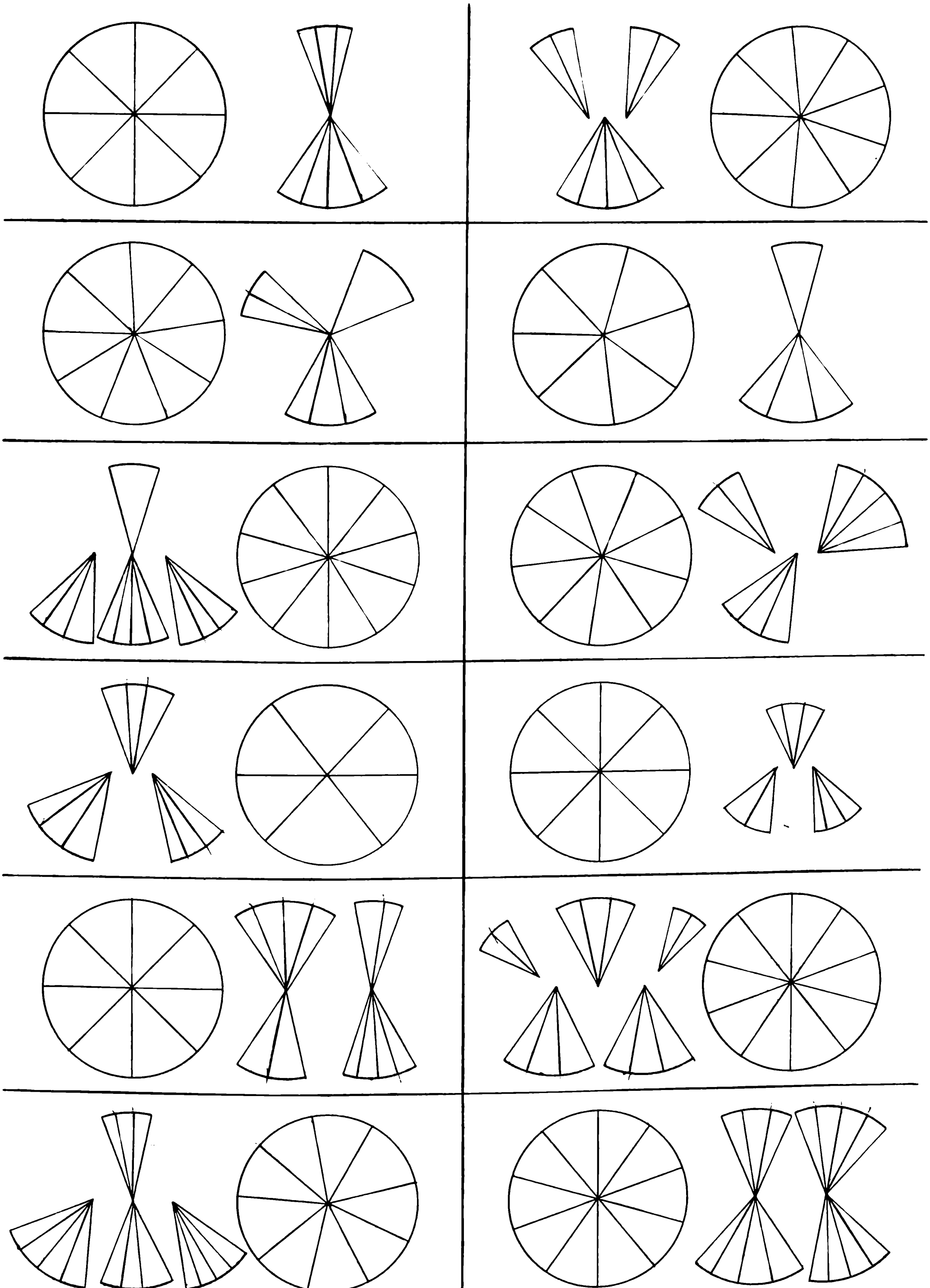


TABLE XVIII



(Tables XVI, XVII, and XVIII). An eight, for example, might become two 3's and a 2; a six might become three 2's or two 3's. The class was instructed to count and write the number of parts of every 'group.'

Even under these new conditions, the pupils counted up to twelve with a high degree of accuracy.

Total items 24; average score 98.05 per cent correct.

Tuesday, December 4.

The pupils were given the materials shown in Tables XVI and XVII. They were instructed to count the parts of all the groups.

Total items 24; average score 94.77 per cent correct.

Wednesday, December 5.

The instructions to count and label the number of parts of all the groups (Table XVIII) were given the pupils.

Total items 24; average score 90.44 per cent correct.

Thursday, December 6.

The instructions of the previous day were repeated.

Total items 24; average score 92.42 per cent correct.

Friday, December 7.

The class was given the 'difference' problem (Tables XIV , XV, and XVIII). The pupils were requested to write the number of parts needed on the smaller group of a combination. The best paper of three attempted was used as a basis for scoring.

Total items 12; average score 77.89 per cent correct.

5. Monday, December 10.

The problem on Friday, December 7, was given with new material (Tables XIX and XX). The results on 12 items (Table XIX) were used as a basis for calculating scores.

Total items 12; average score 62.06 per cent correct.

6. Monday, December 17.

The 'difference' problem was given a few of the pupils with new material (Table XXI). The materials in Tables XIX and XX also were used for part of the class.

TABLE XIX

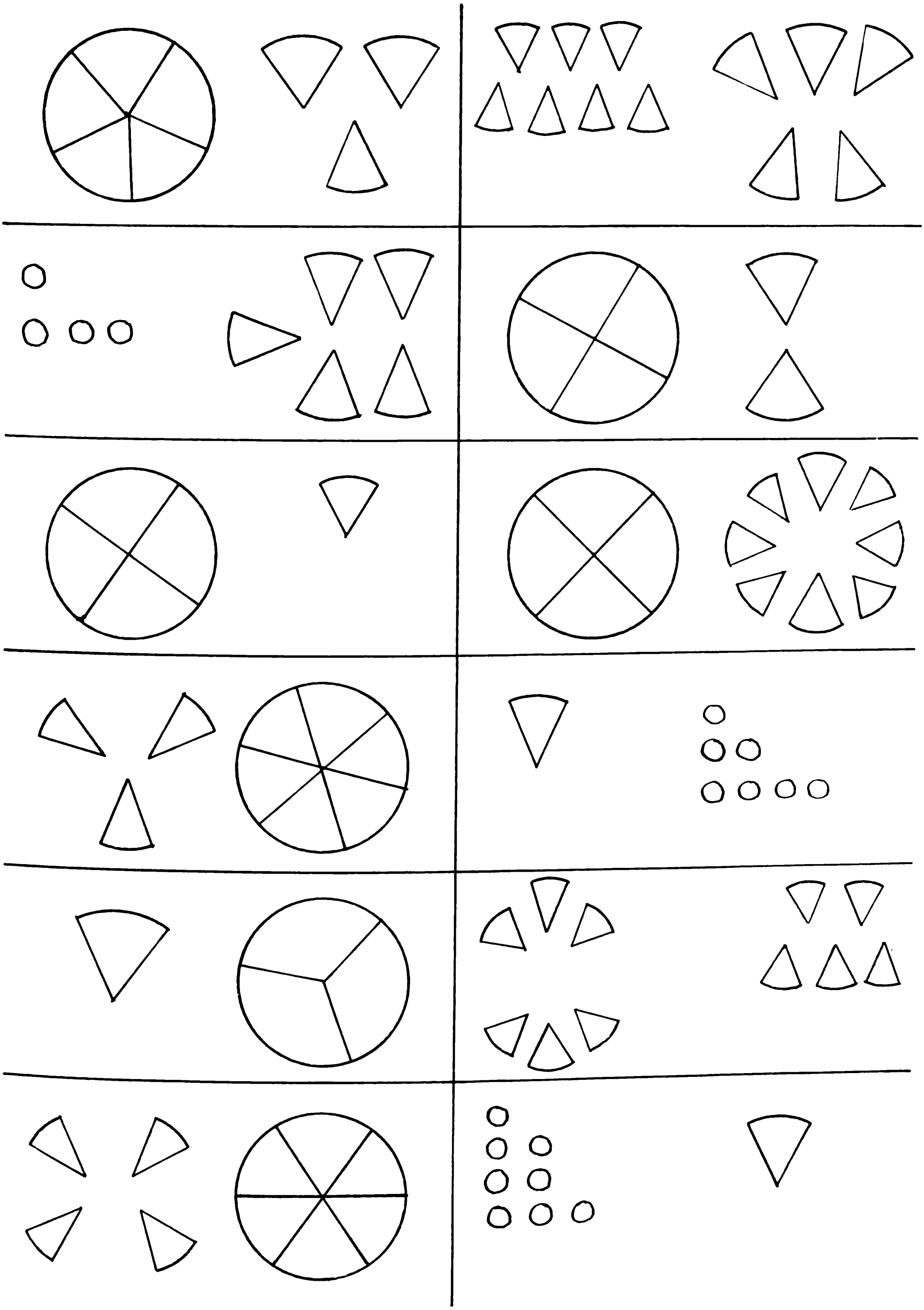


TABLE XX

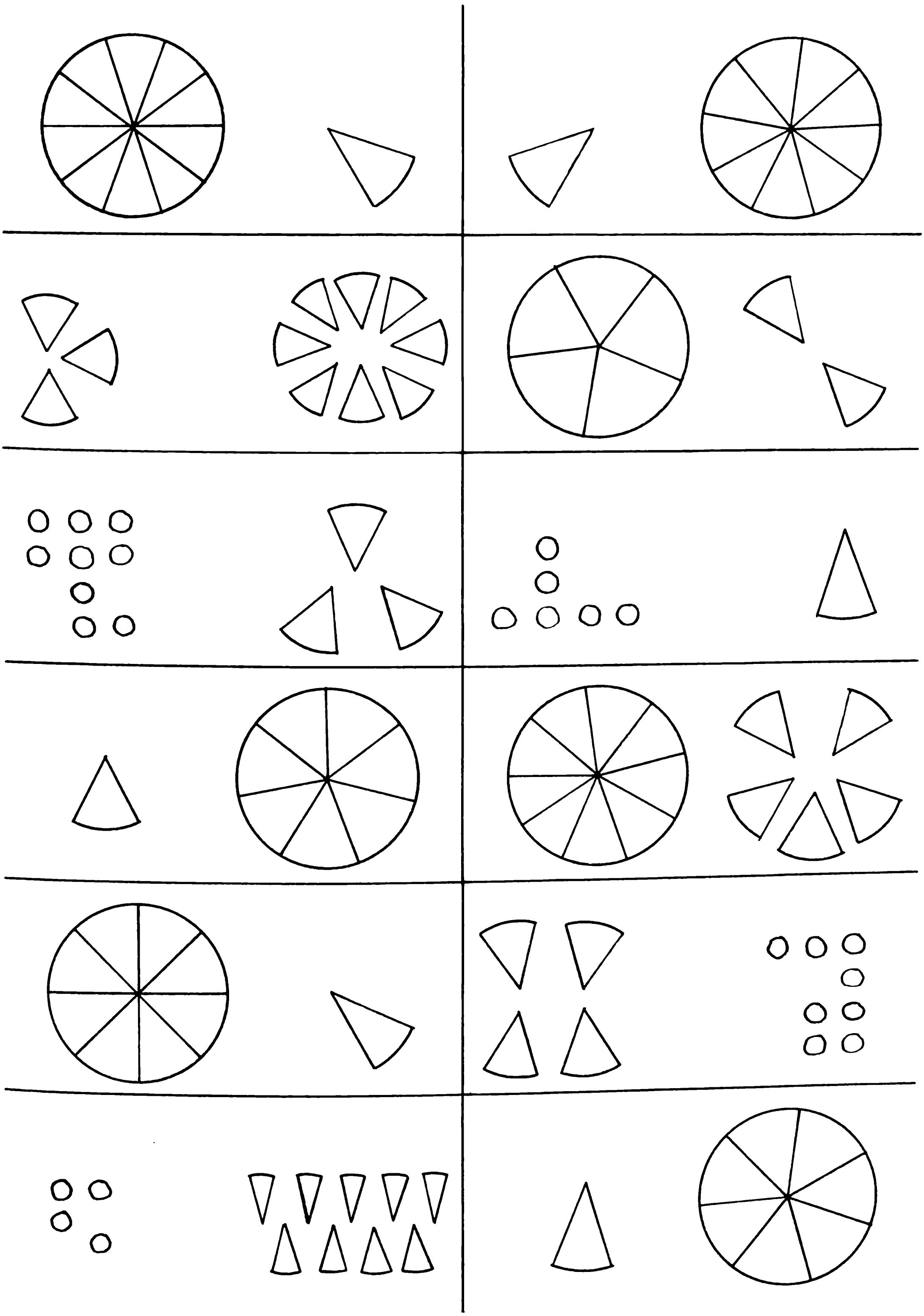
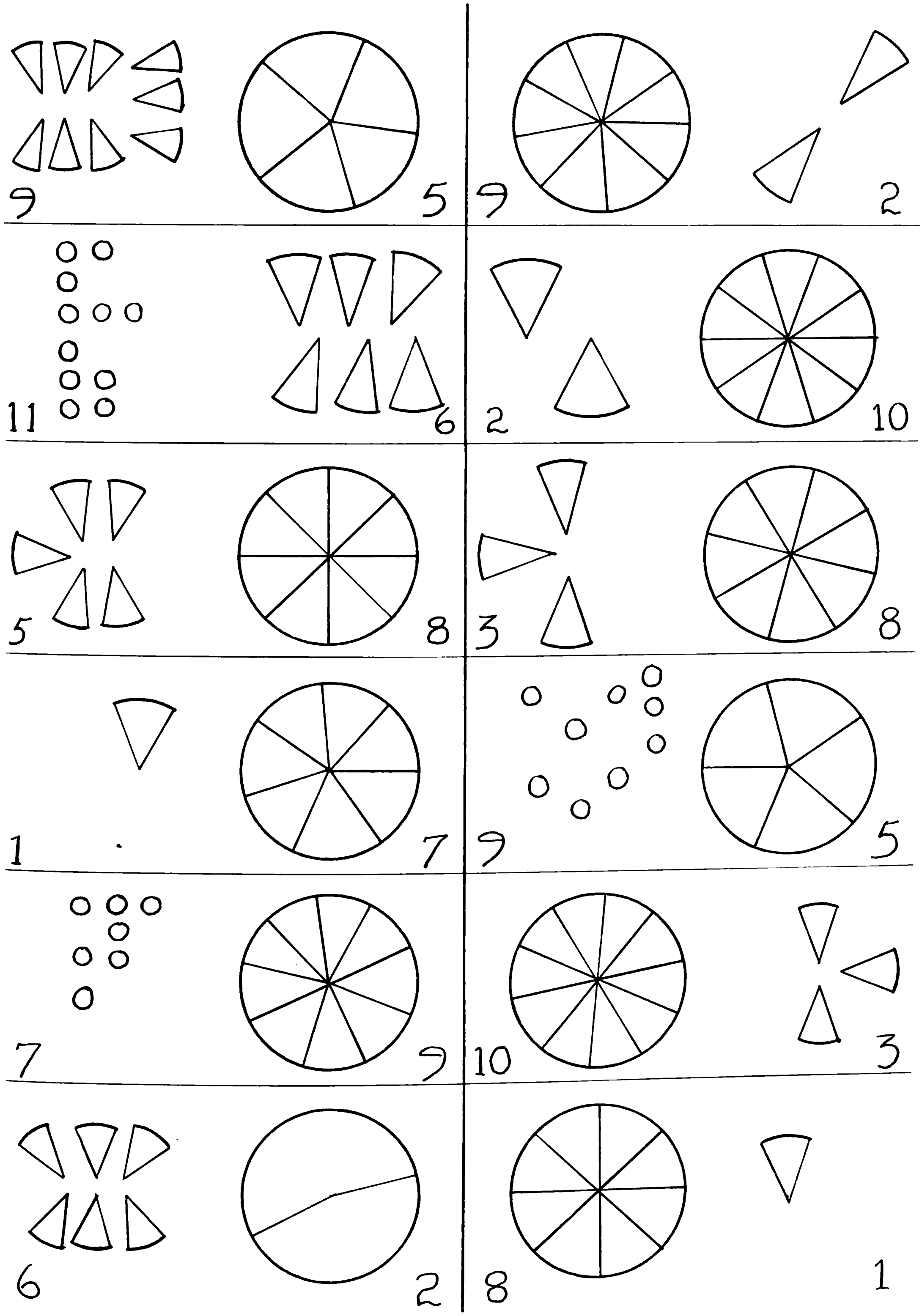


TABLE XXI



Maximum items completed 12; average score 49.81 per cent correct.

Tuesday, December 18.

The class was divided so that two-thirds received the material in Table XXI and one-third the material in Table XIX.

Total items 12; average score 75.09 per cent correct.

Wednesday, December 19.

The 'difference' problem was again presented (Tables XX and XXI).

S17 was given individual aid. Number pictures (not over five) were used to illustrate for him what the correct procedure should be. He was then given two problems with number pictures to work out for himself, one of which was the combination 5-8. These were solved correctly. S28, S30, and S31 who had had difficulty with this type of problem since it was first introduced, simply went through the motions on this day's performance without really attacking the problems. An analysis of the performances of S7, S9, S15, and S24 showed that they responded in a hit-or-miss fashion from the experimenter's point of view. Errors were made on easy problems and yet some rather difficult problems were solved correctly. It seemed impossible to give these pupils a type of work on which 100 per cent accuracy could be achieved.

Total items 24; average score 61.93 per cent correct.

Thursday, December 20.

(1) The 'difference' problem was given with the materials in Table XXI.

(2) The meaning of the phrase, "6 is the same as 3 and ___," was illustrated on the blackboard. This was an attempt to put the principles of addition and subtraction into language form. A mimeographed paper (Table XXII) was then given the pupils and they were assisted in solving the first two problems. The last four problems were then solved individually. Five pupils made a correct response to the four critical problems; one pupil made one error; two pupils made more than one error; five pupils did not complete the problems.

Total items 12 (part 1); average score 74.32 per cent correct. (Two papers were so illegible that they could not be graded.)

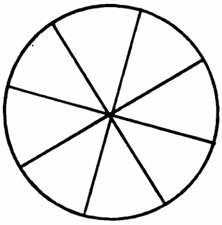
C. Third Six Weeks

1. Monday, January 7.
(1) The 'difference' problem was given again (Table XXI). No scores were recorded.
(2) The instructor demonstrated the addition problem first introduced Thursday, December 20 (Table XXIII). Two pupils, S21 and S32, made no errors.
Total items 29 (part 2); average score 41.55 per cent correct.

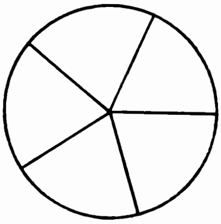
Tuesday, January 8.
Considerable time was given to a demonstration of addition problems. Three of these problems were as follows:
(1) The experimenter held four books in one hand and then picked up four in the other. These two groups were then called 'the same'. Three books from the left hand were replaced on the table. The question was asked the class, "How many more books do I need in my left hand (the left was held up) in order to have the same number as I have in my right hand?" The problem was easily solved.
(2) The experimenter held up four books. "I want six books. How many more do I need?" S15 immediately gave the correct response. S20, who had experienced difficulty previously, also did well on this type of problem.
(3) The experimenter called four girls to the front of the class. S9, a boy, was then asked to come to the front of the class. He was asked, "How many more boys would you need to call in order to have the same number of boys as girls?" He gave the correct response.
The phrase, "8 is the same as 5 and ___", was then printed on the board by Miss Carter, the second grade teacher. (This class had had difficulty with reading and the printed form, she suggested, would be more desirable than the script or long-hand style which had been used previously by the experimenter.) Several members of the class were given an opportunity to fill in the 'blank' at the blackboard. No scores were recorded.

Thursday, January 10.
The addition problems (Table XXIII) were given the class. Five pupils made no errors and seven, about one-third of the class, showed marked improvement over the performance of Monday, January 7.

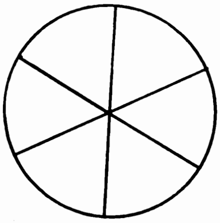
TABLE XXII



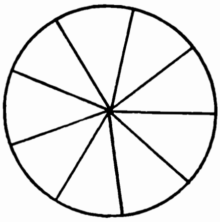
8 IS THE SAME AS 4 AND



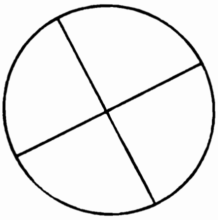
5 IS THE SAME AS 2 AND



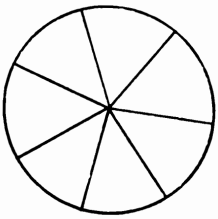
6 IS THE SAME AS 4 AND



9 IS THE SAME AS 6 AND



4 IS THE SAME AS 3 AND



7 IS THE SAME AS 1 AND

TABLE XXIII

5 IS THE SAME AS 3 AND 2	7 IS THE SAME AS 2 AND 5
5 IS THE SAME AS 4 AND 1	4 IS THE SAME AS 3 AND 1
4 IS THE SAME AS 1 AND 3	7 IS THE SAME AS 6 AND ___
4 IS THE SAME AS 3 AND ___	10 IS THE SAME AS 5 AND ___
4 IS THE SAME AS 2 AND ___	8 IS THE SAME AS 4 AND ___
6 IS THE SAME AS 4 AND ___	6 IS THE SAME AS 3 AND ___
6 IS THE SAME AS 2 AND ___	4 IS THE SAME AS 2 AND ___
6 IS THE SAME AS 1 AND ___	2 IS THE SAME AS 1 AND ___
6 IS THE SAME AS 3 AND ___	9 IS THE SAME AS 7 AND ___
3 IS THE SAME AS 1 AND ___	9 IS THE SAME AS 5 AND ___
3 IS THE SAME AS 2 AND 1	9 IS THE SAME AS 6 AND ___
7 IS THE SAME AS 4 AND 3	9 IS THE SAME AS 2 AND ___
7 IS THE SAME AS 6 AND ___	8 IS THE SAME AS 6 AND ___
7 IS THE SAME AS 5 AND ___	8 IS THE SAME AS 4 AND ___
7 IS THE SAME AS 3 AND ___	8 IS THE SAME AS 7 AND ___
2 IS THE SAME AS 1 AND ___	8 IS THE SAME AS 5 AND ___
8 IS THE SAME AS 4 AND ___	6 IS THE SAME AS 2 AND ___
9 IS THE SAME AS 6 AND ___	7 IS THE SAME AS 6 AND ___

The following did not attempt to work the problems: S30, S31, and S23.

Total items 29; average score 62.47 per cent correct.

2. Monday, January 14.

The materials on pages 27 and 31 of the Lennes Pad I were employed (recognizing the numbers of various objects, recognizing the number symbols and the number names; solving ten addition problems requiring the use of numbers up to four). Since the pupils could not read well enough to follow the instructions, considerable aid was necessary. The majority solved the addition problems easily. No quantitative scores were recorded.

Tuesday, January 15.

A demonstration was given of addition problems in the form: "2 is the same as 1 and ___." The pupils filled in the blanks of Table XXIV. Seven papers were satisfactory (90 per cent correct or above); five papers were unsatisfactory (44 per cent correct or below).

Total items 36; average score 66.67 per cent correct.

Wednesday, January 16.

The exercises on pages 25 and 26 of the Lennes Pad I were employed. These consisted of matching, for example, '2 and 1' with '3'. The following illustration was used by the experimenter to demonstrate the procedure:

2 and 1	2
1 and 2	3
1 and 1	4

S23, S27, and S28 failed to discover that '4 and 2' was in any way related to '2 and 4' in this particular situation. No scores were recorded.

Thursday, January 17.

Another demonstration of the addition problem was given. The material in Table XXIV was then presented.

The class was divided into four sections according to accuracy of performance: (1) S21, S5, and S25. (2) S7, S14, and S19. (3) S27, S9, S16, and S26. (4) S13, S24, S30, S20, S15, and S28. S17 copied, therefore, his score was not included. S23 was given aid and her score was not included; she refused to give up, but the work was very dif-

TABLE XXIV

2 IS THE SAME AS 1 AND ___

~~3 IS THE SAME AS 2 AND~~ ___

3 IS THE SAME AS 1 AND ___

~~4 IS THE SAME AS 2 AND~~ ___

4 IS THE SAME AS 1 AND ___

4 IS THE SAME AS 3 AND ___

~~5 IS THE SAME AS 2 AND~~ ___

5 IS THE SAME AS 4 AND ___

5 IS THE SAME AS 3 AND ___

5 IS THE SAME AS 1 AND ___

~~6 IS THE SAME AS 3 AND~~ ___

6 IS THE SAME AS 4 AND ___

6 IS THE SAME AS 1 AND ___

6 IS THE SAME AS 2 AND ___

6 IS THE SAME AS 5 AND ___

~~7 IS THE SAME AS 4 AND~~ ___

7 IS THE SAME AS 5 AND ___

7 IS THE SAME AS 3 AND ___

7 IS THE SAME AS 6 AND ___

7 IS THE SAME AS 2 AND ___

7 IS THE SAME AS 1 AND ___

~~8 IS THE SAME AS 5 AND~~ ___

8 IS THE SAME AS 4 AND ___

8 IS THE SAME AS 6 AND ___

8 IS THE SAME AS 2 AND ___

8 IS THE SAME AS 3 AND ___

8 IS THE SAME AS 7 AND ___

8 IS THE SAME AS 1 AND ___

~~9 IS THE SAME AS 6 AND~~ ___

9 IS THE SAME AS 7 AND ___

9 IS THE SAME AS 4 AND ___

9 IS THE SAME AS 3 AND ___

9 IS THE SAME AS 5 AND ___

9 IS THE SAME AS 2 AND ___

9 IS THE SAME AS 8 AND ___

9 IS THE SAME AS 1 AND ___

ficult for her. S23 and S20 stayed after class to receive help. The former appeared to gain some insight into the problem.

Total items 36; average score 58.56 per cent correct.

Friday, January 18.

The work of Thursday, January 17 was repeated (Table XXIV). The members of the class were seated as far apart as possible to avoid copying.

The division of the class according to accuracy of results was as follows: (1) S21, S5, and S14. (2) S19, S25, S13, and S7. (3) S9, and S26. (4) S15, S17, S20, S28, S27, S24, and S16. Both S23 and S30 received aid and their scores were not included with the group.

Total items 36; average score 60.13 per cent correct.

3. Monday, January 21.

(1) The experimenter demonstrated with books as materials that 4 objects could be divided into 3 and 1. The symbolic expression $\begin{array}{r}1\\3\\\hline 4\end{array}$ was placed on the board and copied. Similarly, $\begin{array}{r}2\\2\\\hline 4\end{array}$ was demonstrated and copied, and also $\begin{array}{r}1\\1\\\hline 2\end{array}$ and $\begin{array}{r}4\\2\\\hline 6\end{array}$.

(2) The following problem was explained and was copied by the class, "1 and 3 are the same as 4."

(3) Four test problems were given: $\begin{array}{r}4\\1\\\hline\end{array}$ $\begin{array}{r}2\\2\\\hline\end{array}$ $\begin{array}{r}3\\1\\\hline\end{array}$ $\begin{array}{r}3\\3\\\hline\end{array}$

Total items 9; average score 96.85 per cent correct.

Tuesday, January 22.

The demonstration of the previous day was repeated, except that the phrase, "1 and 3 are the same as 4," was not written out and copied. Subtraction also was demonstrated by the statement, "Three take away 1 gives 2." The symbolic statement for subtraction was written on the board. The following problems were then introduced:

$$\begin{array}{r}1\\1\\\hline\end{array}\quad\begin{array}{r}2\\-1\\\hline\end{array}\qquad\begin{array}{r}2\\1\\\hline\end{array}\quad\begin{array}{r}3\\-2\\\hline\end{array}\qquad\begin{array}{r}4\\1\\\hline\end{array}\quad\begin{array}{r}5\\-4\\\hline\end{array}$$

The demonstration was only fairly successful. No record of the scores was made. On two critical problems, 2+2 and 4-2 (placed on the board in the vertical form), the following pupils gave correct answers: S26, S7, S27, S14, S19, S13, S5, and S21.

Wednesday, January 23.

The first section (S26, S7, S14, S19, S13, S27, and S21) was given a sheet of mimeographed material containing both addition and subtraction problems (Table XXV).

The second section of the class worked also on addition and subtraction problems. (1) The following were written on the board (in the vertical form) and were worked out with books as counters: 2+1, 2-1, 3+1, 2+2. (2) The experimenter erased these problems and then wrote the following: 4-2, 3+1, and 2-1. The children were asked to copy and work the problems. The experimenter then asked for the answers from the members of the class. A demonstration with books was made to verify the answers. (3) The pupils worked three problems by themselves: 3-2, 2+2, 4-3. Those pupils giving a satisfactory performance on this test were: S25, S20, S23, S6, S17, and S32.

Only the scores of the pupils in the first section were recorded. Total items 36; average score 77.71 per cent correct.

Thursday, January 24.

The Lennes Work Pad I, pages 29 and 30, was used. This consisted of matching addition problems with the correct answer, for example:

1+2	4
1+4	3
2+2	6
5+1	5

The class did well on the whole. No scores were recorded.

Friday, January 25.

The class again was divided. (It should be noted that both the better and the poorer students were represented in the two sections and that the sections were not exactly the same as Wednesday, January 23.) The first section worked the problems in Table XXV. Scores were recorded for S6, S7, S13, S14, S20, and S32. Since S19 and S25 failed to work any of the 'written' problems on this paper, their scores were not included. The pupils of this section were given aid on the first three 'written' problems of Table XXVI. The unaided responses on the next four problems were unsatisfactory.

The second section worked several problems aided by the experimenter and then solved the following: 1+1, 1+2, 1+3, 1+4; 2-1, 3-2, 4-3; 1+1, 2-1; 2+2, 4-2; 2+1, 3-2; 4+1, 5-1 (written

TABLE XXV

2 IS THE SAME AS 1 AND ___ 1 AND 1 ARE THE SAME AS ___	1 1	2 -1
3 IS THE SAME AS 2 AND ___ 2 AND 1 ARE THE SAME AS ___	2 1	3 -2
3 IS THE SAME AS 1 AND ___ 1 AND 2 ARE THE SAME AS ___	1 2	3 -1
4 IS THE SAME AS 2 AND ___ 2 AND 2 ARE THE SAME AS ___	2 2	4 -2
4 IS THE SAME AS 1 AND ___ 1 AND 3 ARE THE SAME AS ___	1 3	4 -1
4 IS THE SAME AS 3 AND ___ 3 AND 1 ARE THE SAME AS ___	3 1	4 -3
5 IS THE SAME AS 2 AND ___ 2 AND 3 ARE THE SAME AS ___	2 3	5 -2
5 IS THE SAME AS 3 AND ___ 3 AND 2 ARE THE SAME AS ___	3 2	5 -3
5 IS THE SAME AS 4 AND ___ 4 AND 1 ARE THE SAME AS ___	4 1	5 -4

in the vertical form on the board). S16, S21, and S26 worked rapidly and were given harder problems with sums up to ten. No scores of this section were recorded.

Total items 36 (section one, Table XXV); average score 78.00 per cent correct.

4. Monday, January 28.

The class was divided into three sections. The first section (S26, S6, S21, S13, and S14) worked the following ten problems: 2+4, 6-2, 8-5, 6+1, 5+4, 3+7, 8+5, 7+6, 5+6, 8-6.

The second section (S28, S16, S31, S15, S30, S24 and S9) worked the 18 problems with number symbols in Table XXV.

The third section (S20, S19, S32, S17, S25, S7, S27) worked the following twelve problems: 2+2, 2-1, 4+4, 3+2, 3+3, 4+1, 2+3, 4-2, 3+3, 6-3, 4+2, 6-4.

Maximum items completed 18; average score 82.21 per cent correct.

Tuesday, January 29.

The pupils were given two papers (Tables XXV and XXVI) and were instructed to work only the addition and subtraction problems with the number symbols.

Total items 36; average score 74.39 per cent correct.

Wednesday, January 30.

Two papers with addition and subtraction problems were given the class (Tables XXVI and XXVII). The harder problems (Table XXVII) were not worked accurately. The subtraction seemed particularly difficult. On the easier set of problems, S24, S9, S7, and S15 made all their errors on subtraction. S7 made the least number of errors (5) and S9 made the greatest number (10).

Total items 36; average score 62.06 per cent correct.

Thursday, January 31.

The Lennes Work Pad I was utilized to offer a change in materials and instructions. The exercises on pages 34 and 35 were employed (solving addition problems; recognizing the numbers of various objects and recognizing the number names). No quantitative scores were recorded.

Friday, February 1.

A test was given on 36 problems with number symbols (Tables XXVI and XXVII). The pupils were

TABLE XXVI

5 IS THE SAME AS 1 AND ___ 1 AND 4 ARE THE SAME AS ___	1 4	5 -1
6 IS THE SAME AS 3 AND ___ 3 AND 3 ARE THE SAME AS ___	3 3	6 -3
6 IS THE SAME AS 4 AND ___ 4 AND 2 ARE THE SAME AS ___	4 2	6 -4
6 IS THE SAME AS 1 AND ___ 3 AND 3 ARE THE SAME AS ___	1 5	6 -1
6 IS THE SAME AS 2 AND ___ 2 AND 4 ARE THE SAME AS ___	2 4	6 -2
6 IS THE SAME AS 5 AND ___ 5 AND 1 ARE THE SAME AS ___	5 1	6 -5
7 IS THE SAME AS 4 AND ___ 4 AND 3 ARE THE SAME AS ___	4 3	7 -4
7 IS THE SAME AS 5 AND ___ 5 AND 2 ARE THE SAME AS ___	5 2	7 -5
7 IS THE SAME AS 3 AND ___ 3 AND 4 ARE THE SAME AS ___	3 4	7 -3
7 IS THE SAME AS 6 AND ___ 6 AND 1 ARE THE SAME AS ___	6 1	7 -6

TABLE XXVII

7 IS THE SAME AS 2 AND ___ 6 AND 1 ARE THE SAME AS ___	2 5	7 -2
7 IS THE SAME AS 1 AND ___ 1 AND 6 ARE THE SAME AS ___	1 6	7 -1
8 IS THE SAME AS 5 AND ___ 5 AND 3 ARE THE SAME AS ___	5 3	8 -5
8 IS THE SAME AS 4 AND ___ 4 AND 4 ARE THE SAME AS ___	4 4	8 -4
8 IS THE SAME AS 6 AND ___ 6 AND 2 ARE THE SAME AS ___	6 2	8 -6
8 IS THE SAME AS 2 AND ___ 2 AND 6 ARE THE SAME AS ___	2 6	8 -2
8 IS THE SAME AS 3 AND ___ 5 AND 3 ARE THE SAME AS ___	8 -5	5 3
8 IS THE SAME AS 7 AND ___ 7 AND 1 ARE THE SAME AS ___	8 -7	7 1
9 IS THE SAME AS 6 AND ___ 6 AND 3 ARE THE SAME AS ___	9 -6	6 3

to solve the 'written' problems on these papers if time permitted. The subtraction problems proved most difficult. Eight of twelve children, who made errors on the symbol problems, made all their errors in subtraction.

Total items 36; average score 35.81 per cent correct.

5. Monday, February 4.

A new set of materials was devised to aid the pupils with addition and subtraction. Each row of problems was arranged so that some 'pattern' of answers appeared (Table XXVIII). The problems 10-9 and 13+2, for example, were easily solved at the end of a series of problems. There was evidence, also, that the children would attempt to fit their own pattern of answers to the problems; this effort proved valuable whenever a check was made by the student himself.

Maxmimum items completed 120; average score 79.86 per cent correct.

Tuesday, February 5.

The list of problems given the pupils Monday, February 4, unfortunately, was too long. The papers were returned to the pupils during this work period to be completed. (Two pupils who had completed their papers on Monday were given new sheets.) The poorer students made the attempt to accomplish as much as possible and failed to work accurately. These persons were instructed to work more slowly. The responses, in general, improved over the previous day's work.

Maximum items completed 77; average score 83.33 per cent correct.

Wednesday, February 6.

Reading problems were formulated especially for this class (Table XXIX). The attempt was made (1) to state the addition problems in such a manner that the 'meaning' of addition could be clearly grasped, (2) to motivate the children by use of their own names in story situations, and (3) to use terms already studied by the pupils.

The class as a group enjoyed the novelty and worked ten minutes overtime without noticing the fact. The results indicated, on the other hand, that problems stated in this fashion were not readily grasped by the pupils. The reading, futhermore, was difficult in spite of the fact that all the terms employed had been used in other contexts.

Total items 15; average score 69.93 per cent correct.

TABLE XXVIII

2	3	4	5	6	7	8	9	10	11	12	13	
2	2	2	2	2	2	2	2	2	2	2	2	
2	3	4	5	6	7	8	9	10	11	12	13	
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
1	2	3	4	5	6	7	2	3	4	5	6	7
1	2	3	4	5	6	7	1	2	3	4	5	6
10	9	8	7	6	5	4	3	2	13	14	15	
-9	-8	-7	-6	-5	-4	-3	-2	-1	-12	-13	-14	
3	8	4	9	2	7	1	6	10	11	12	13	
2	-3	1	-4	3	-2	4	-1	-5	-6	-7	-8	
2	1	6	1	9	3	4	4	3	7	5	4	
-1	1	-3	3	-4	3	2	1	1	-4	-3	-3	
5	7	2	4	1	4	2	4	1	7	5	6	
-4	-5	1	-3	1	-1	-1	-2	2	-6	-3	-3	
2	3	5	1	10	11	6	5	8	7	6	5	
2	1	-1	3	-6	-7	-2	-1	-4	-3	-2	-1	
3	4	2	2	1	2	4	2	3	6	7	8	
3	1	2	1	1	-1	2	3	1	-3	-5	-7	
4	4	4	4	4	4	4	4	4	4	4	4	
2	3	4	5	6	7	8	9	10	11	12	13	

TABLE XXIX

2 boys and 1 boy are the same as ______ boys.

4 girls are the same as 1 girl and ______ girls.

4 girls are the same as 3 girls and ______ girls.

Wallace had 1 apple and Charles had 2 apples. They had ______ apples.

Jack, Jimmy, and Roland are in the same row. Shirley is in the same row. ______ children are in the same row.

Thomas found 6 nuts. Jeanne found 3 nuts. They found ______ nuts.

Betty had 2 cats. Peggy had 2 cats. They had ______ cats.

Robert saw 4 bears. Marilee saw 2 bears. They saw ______ bears.

3 balls and 1 ball are the same as ______ balls.

6 lambs and 3 lambs are the same as ______ lambs.

Eugene ate 5 cookies. Marian ate 2 cookies. They ate ______ cookies.

Billy T. saw 3 pigs. Sonny saw 5 pigs. They saw ______ pigs.

Marjorie had 4 pennies. Bob had 5 pennies. They had ______ pennies.

Wilfred saw 7 cows. Howard saw 1 cow. They saw ______ cows.

Elizabeth had 2 toys. Willard had 3 toys. Billy H. had 2 toys. They all had ______ toys.

Thursday, February 7.

Another paper was devised similar to the series of problems in Table XXVIII (Table XXX).

Maximum items completed 120; average score 80.95 per cent correct.

Friday, February 8.

The same materials of Thursday, February 7 were employed (Table XXX). The class can be divided into four sections according to excellence of scores: (1) S21, S26, S3, and S14. (2) S32, S19, S6, and S7. (3) S30, S23, S17, and S25. (4) S15, S24, S20, S31, S28, and S13.

Maximum items completed 120; average score 80.83 per cent correct.

6. Monday, February 11.

Table XXIX, which was first presented Wednesday, February 6, was again employed. The work in this period was limited, however, to the ten problems in which the children's own names appeared.

The majority of the pupils needed aid on many of the words and it was necessary, finally, for the experimenter to read the problems aloud to the class. The poorer children were aided by this procedure since the answers in several instances were given aloud by the brighter pupils. (It was the usual procedure to get a class response when problems were read to the pupils and it was difficult for the enthusiastic members of the group to remain silent.)

Total items 10; average score 83.24 per cent correct.

Tuesday, February 12.

The material in Table XXVIII was employed for the majority of the class. Since these problems were too numerous and too difficult for some pupils, a similar paper was devised for S28, S23, and S17 which contained only 36 easy problems. S31 was given another especially prepared paper of the same sort which contained 48 problems.

Although S23 needed considerable aid, she made no errors. S17 also did good work, making only one error.

Maximum items completed 120; average score 87.92 per cent correct.

Wednesday, February 13.

The majority of the class was given Table XXX. Two especially prepared papers containing

TABLE XXX

4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1
4	5	6	7	8	9	10	11	12	13	14	15
2	[illegible]	2	2	2	2	2	2	2	2	2	2
4	5	6	7	8	9	10	11	12	13	14	15
3	3	3	3	3	3	3	3	3	3	3	3
2	4	6	2	3	4	3	5	2	8	4	3
-1	-2	-3	2	2	2	-2	-3	1	-4	1	3
3	4	5	6	7	8	9	10	11	12	13	14
-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
4	5	2	3	6	8	7	9	6	4	2	8
-3	-4	-1	-2	-5	-7	-6	-8	-5	-3	-1	-7
5	1	6	2	4	1	3	6	7	6	1	2
-4	1	-3	-1	-2	2	-2	-4	-4	-5	1	1
2	6	1	2	8	6	4	3	3	1	7	8
2	-3	1	-1	-4	-3	-2	-2	1	2	-5	-7
1	1	1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	8	7	6	9	10	11
2	3	4	5	6	7	-5	-4	-3	-6	-7	-8

36 and 48 easy problems of this same type were given to S23, S24, S31, S28, and S20.

An attempt was made, by examining previous work of the pupils, to ascertain how much each pupil could accomplish in 12 to 15 minutes. The experimenter placed a mark on each paper indicating the number of problems to be worked and when this amount was completed and corrected, the pupil was dismissed from class. Although a comparatively small amount of work was accomplished, the work was quite accurate.

Maximum items completed 94; average score 89.81 per cent correct.

Friday, February 15.

The same procedure and the same materials were employed as on Wednesday, February 13. S28, S20, S23, and S31 were given a special paper containing either 36 or 48 easy problems. S23 did much better work than usual. S20 and S31 also did well. S28 still failed to apply himself; this case appeared rather seriously maladjusted.

Maximum items completed 120; average score 87.63 per cent correct.

D. Fourth Six Weeks

1. Monday, February 18.

A demonstration was given of the zero difficulty both in subtraction and addition.

(1) 'Two take away two' was demonstrated and then a pupil was asked to come to the board and write the answer to 2-2 (vertical form). A similar procedure was followed in connection with 1-1.

(2) The following were copied from the board and then solved: 1-1, 2+2, 3-3, and 4+0. After all had obtained answers, the problems were checked with counters.

(3) Six problems were given as a final check: 2+1, 3-2, 3-3, 4+0, 5-5, and 8-8. Those making perfect scores were: S27, S5, S17, S26, S15, S32, S14, S19, S16, S13, S21, and S23. S20 failed altogether to understand the concepts involved; her score was not included.

Total items 6 (part 3); average score 90.81 per cent correct.

Tuesday, February 19.

A demonstration of problems involving zero again was given. The pupils were given Tables XXXI and XXXII. Each of these papers contained 32 problems. The best paper of each pupil was selected for scoring.

TABLE XXXI

5	6	7	8	6	7	8	9
4	3	2	1	5	4	3	2
7	7	8	8	6	6	9	9
3	4	4	5	4	5	3	4
7	8	8	9	8	9	8	8
7	7	8	8	6	6	8	9
9	8	9	8	5	6	8	9
5	7	7	9	9	9	8	8

TABLE XXXII

2	1	2	3	3	4	1	2
-2	1	1	-3	-2	-2	-1	2
4	5	6	3	2	4	3	2
-4	-5	-6	1	3	2	-3	-2
1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0
1	2	3	4	7	3	5	4
1	1	-3	-2	0	0	-[illegible]	-4

TABLE XXXIII

3	2	4	1	3	2	1	4
-3	-1	-2	2	1	3	5	3
5	1	1	2	6	4	6	1
-5	-1	3	2	-6	-4	-3	2
2	4	2	3	3	1	1	4
-2	-3	0	0	-3	0	1	-1
4	5	2	1	4	6	6	7
2	-1	0	-1	-2	-6	-2	-7

TABLE XXXIV

2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7
5	5	6	6	7	7	8	8
4	5	4	5	4	5	4	5
8	6	6	7	11	6	3	1
0	3	4	4	0	4	6	7
7	8	9	8	9	8	9	9
6	6	5	7	6	8	7	8

Total items 32; average score 91.39 per cent correct.

Wednesday, February 20.

Section 1 (S26, S16, S21, S13, S19, S6, S32, S14, and S5) worked the problems shown in Table XXXIV. Section 2 (S20, S12, S31, S28, S15, S3, S30, S7, S27, S23, and S17) was given the material in Table XXXIII. It will be noted that the papers handed the pupils in the two sections were quite similar in external appearance.

Maximum items used as a basis for scores 32; average score 90.10 per cent correct.

Thursday, February 21.

Some members of the class still had difficulty with the symbol problems and an attempt to secure accuracy was made by limiting the number of problems worked by each individual. One section of the class (S31, S3, S30, S27, S23, S7, S15, S28, S17, S26, S12, and S16) was given Table XXXII. The other section (S5, S14, S13, S32, and S6) was given Table XXXV.

Maximum items completed 32; average score 92.82 per cent correct.

Friday, February 22.

The addition and subtraction problems in Table XXXIII were given. The errors were checked by the experimenter and the papers were returned to the pupils for correction. The faster pupils worked an extra set of problems (Table XXXI).

Maximum items completed 64; average score 93.87 per cent correct.

2. Monday, February 25.

The pupils who had worked more rapidly on the previous day received the difficult problems (Table XXXIV). The slower pupils were given the easier problems (Table XXXIII). The faster pupils also had an opportunity to work the easier set of problems.

Maximum items completed 64; average score 90.38 per cent correct.

Tuesday, February 26.

(1) Five problems were written on the board. The pupils were required to copy the numbers, filling in those which were missing.

(a) 1 2 3 4 5---7 8 9 10 11 12---14 15.
(b) 6 7 8 9 10---12 13 14 15---17 18 19 20.
(c) 4 5 6---8 9 10 11 12 13---15 16 17 18.
(d) 5 6 7 8---10 11 12 13 14 15---17 18 19.

TABLE XXXV

6	7	6	7	6	7	6	7
4	3	5	4	6	5	7	6
8	7	9	8	8	9	8	9
3	5	4	6	7	7	9	9
4	6	3	9	7	6	5	4
2	0	3	-3	-1	0	1	2
5	5	2	5	6	9	6	7
3	2	4	0	4	0	2	0

TABLE XXXVI

4	5	6	4	7	6	3
2	2	1	-1	-4	2	3
5	2	3	9	6	5	1
1	4	5	-6	0	-2	6
3	5	10	6	4	8	6
4	3	-7	-3	4	-4	-2
7	10	9	1	6	6	7
1	-3	-4	8	2	-1	-2
4	7	1	9	7	8	6
5	2	7	-5	-3	1	-4
1	6	10	10	5	3	10
5	1	-8	-7	5	7	-6
1	2	5	9	7	6	8
9	7	3	-4	3	4	2

(e) 18 19 20 21 22---24 25 26 27.
(2) The following problems were put on the board to be copied and solved: 3-3, 2-2, 4-4, 6+5, 7+4, and 9+4.

There was a lack of co-operation on the part of the group. The pupils seemed to anticipate something 'hard' and failed to concentrate on the problems. The copying proved especially difficult.

Total items 15; average score 67.69 per cent correct.

Wednesday, February 27.
(1) The class was given Tables XXXI and XXXIII. The poorer students were given the easier set of problems first (Table XXXIII). The sectioning was not made according to any fixed plan. The students who finished the first paper were given the second one.
(2) S27 and S30 were given special aid by the experimenter a few minutes before the regular class period. They were given 'oral' drill with flash cards on the problems 9+6, 9+7, and 8+8. After six minutes of this work, the boys were asked to solve these combinations and six new problems. Both could not solve the critical problems.

Maximum items completed 64 (part 1); average score 93.60 per cent correct.

Thursday, February 28.

The materials in Tables XXXII and XXXIV were used. The pupils on the whole did not attack the difficult problems. (S22 joined the class.)

Maximum items completed 64; average score 92.61 per cent correct.

Friday, March 1.
(1) Table XXXV was given the class; five minutes were allowed to work these problems. The following completed all the items: S31, S15 (copied), S19, S16, S7, and S32.
(2) For the remainder of the period the children worked on one or two or three sets of materials (Tables XXXII, XXXIV, and XXXV). The experimenter promised to hand back all those papers which contained no errors. S21, S15, S14, S24, S7, S17, S28, and S30 made no errors.

Maximum items completed 32 (part 1); average score 88.65 per cent correct.

3. Monday, March 4.
(1) A time test (4 minutes) was given on Table XXXIII. S14 finished his paper in approximately one minute. S20, S28, and S17 failed to complete the problems on their papers; S25 made 14

TABLE XXXVII

1	6	6	8	7	2	7
7	3	4	0	3	9	5
4	5	9	1	6	10	12
4	3	-1	7	2	-2	-4
8	7	2	12	5	9	3
3	4	9	-1	6	2	8
3	5	7	9	8	12	11
9	7	4	0	3	-2	-1
4	5	6	4	10	8	9
8	5	2	2	-2	2	3
7	5	8	11	9	8	4
4	4	-1	-6	-2	1	7
1	6	2	7	5	5	6
7	3	8	0	3	4	4

TABLE XXXVIII

1	6	7	9	7	7	9
9	6	7	0	3	5	5
7	9	12	8	9	10	12
-4	-5	-7	-2	-2	-2	-3
10	8	11	13	10	14	11
-7	-4	-6	-7	-3	-6	-2
12	11	13	14	13	16	13
-9	-7	-8	-8	-6	-8	-4
11	12	10	15	14	15	14
-8	-8	-5	-9	-7	-7	-5
9	7	6	5	4	5	8
3	5	7	8	9	9	6
6	7	7	9	6	7	8
8	7	8	6	9	9	8

errors.
(2) The pupils were given an opportunity to work the problems in Tables XXXII and XXXI. The following handed in perfect papers: S16, S17, S28, S30, S27, S26, and S12.
Total items 32 (part 1); average score 92.61 per cent correct.

Tuesday, March 5.
A different set of materials was prepared (Tables XXXVI and XXXVIII). 49 problems appear on each page; they were mimeographed on blue paper which proved quite attractive to the pupils. No individual attempted all the 98 problems on both papers; the class had difficulty with these harder addition and subtraction 'facts.'
Maximum items completed 70; average score 73.53 per cent correct.

Wednesday, March 6.
Table XXXVI was used. 21 pupils completed the 49 items before the end of the period and 10 of these had time enough to try a different set of subtraction problems (Table XL, mimeographed on colored paper). The answers on this set were arranged so that on the first row the answers were all 3's, on the second row 4's, the third 5's, and so on. S26 was the only pupil who clearly grasped this pattern.
Total items 49 (Table XXXVI); average score 86.91 per cent correct.

Thursday, March 7.
(1) The problems (Table XL), which were first introduced Wednesday, March 6, were given to the whole class. Several members of the class caught the pattern idea and finished the 49 problems quickly. Pupils who made no errors were: S32, S6, S14, S15, S28, S7, S5, S13, S19, S26, and S21.
(2) Pupils who completed part 1 were allowed to work the problems in Table XXXVIII. The responses to these problems were unsatisfactory.
Total items 49 (part 1); average score 91.24 per cent correct.

Friday, March 8.
The assignment of Thursday, March 7, was repeated. In calculating the scores, however, the work on both sets of material (Tables XXXVIII and XL) was taken into consideration.
Total items 98; average score 87.45 per cent correct.

4. Monday, March 11.

(1) The material in Table XXXVI was employed. (A new pupil, S10, joined the class.)
(2) The pupils were allowed to try the problems in Table XXXIX (mimeographed on colored paper).

Total items 49 (part 1); average score 89.38 per cent correct.

Tuesday, March 12.

The class was divided. The first section (S26, S10, S6, S24, S16, S25, S14, S9, S7, and S13) worked on Table XXXIX. The remainder of the class (S30, S15, S19, S20, S32, S31, S28, S12, S17, S3, and S5) was given Table XXXVI. Those pupils completing their problems were given the other paper.

Maximum items completed 63; average score 82.23 per cent correct.

Wednesday, March 13.

In the work of Monday, March 11, and Tuesday, March 12, too few problems were solved, although the pupils worked fairly accurately. Therefore, easier material was introduced during the present class period (Table XXXV). Table XL also was used later in the period, but the results were not used in calculating the pupils' scores.

Total items 32; average score 96.05 per cent correct.

Thursday, March 14.

The material appears in Table XXXVIII. Three pupils, S26, S31, and S9, worked extra problems in Tables XXXV and XL; these results were not included in calculating the scores.

Total items 49; average score 75.23 per cent correct.

Friday, March 15.

Subtraction problems were presented (Table XLII, mimeographed on colored paper). Since the problems were quite difficult, the pattern of answers was not grasped by the pupils.

Total items 49; average score 81.00 per cent correct.

5. Monday, March 18.

Since one page of 49 problems was evidently too large a unit for some members of the class, the experimenter divided some of the mimeographed sheets into two parts; the upper part of the sheets contained 21 and the lower part 28 problems.

TABLE XXXIX

9	8	7	12	7	8	9
9	8	7	-9	6	7	8
14	16	6	13	7	12	13
-7	-9	9	-9	8	-3	-4
15	17	7	14	8	13	14
-7	-9	9	-9	8	-5	-6
16	18	8	15	9	14	15
-7	-9	9	-9	8	-7	-8
4	6	9	16	7	9	9
9	8	6	-9	9	8	9
8	5	7	17	8	8	9
5	9	8	-9	8	9	9
4	6	7	18	10	9	9
4	4	5	-9	6	7	9

TABLE XL

6	7	8	9	10	11	12
-3	-4	-5	-6	-7	-8	-9
7	8	9	10	11	12	13
-3	-4	-5	-6	-7	-8	-9
8	9	10	11	12	13	14
-3	-4	-5	-6	-7	-8	-9
9	10	11	12	13	14	15
-3	-4	-5	-6	-7	-8	-9
10	11	12	13	14	15	16
-3	-4	-5	-6	-7	-8	-9
11	12	13	14	15	16	17
-3	-4	-5	-6	-7	-8	-9
12	13	14	15	16	17	18
-3	-4	-5	-6	-7	-8	-9

During this work period the pupils were given an opportunity to work the upper and lower parts of two papers (Tables XXXVII and XLII). Each unit, when completed, was graded by the experimenter and then corrected by the pupil before another unit was attempted.

Maximum items completed 70; average score 85.52 per cent correct.

Tuesday, March 19.

(1) The class was given part of Table XLI (21 problems mimeographed on colored paper).

(2) S14 and S21 also completed the 21 problems on the upper part of Table XXXIX.

The pattern of answers to the problems in Tables XXXIV and XLI was not grasped; the children simply worked from one problem to the next without looking for a significant relationship among the problems.

Total items 21 (part 1); average score 74.27 per cent correct.

Wednesday, March 20.

The class, with the exception of S31, was given the second part of Table XXXIX (28 problems). S31 was given the upper part of this sheet (21 problems). Three pupils, S26, S14, and S17, who had finished the first unit, attempted the upper part of Table XLI. (Two new pupils, S11 and S29, joined the class.)

Total items 28 (Table XXXIX); average score 77.88 per cent correct.

Thursday, March 21.

The pupils worked the easy problems in Table XXXV. As an effect of introducing an easy set of materials after a long period of difficult work, the class appeared to have a surplus of energy. The majority of the pupils had time to work part of the problems in Table XXXVI.

Total items 32 (Table XXXV); average score 93.19 per cent correct.

Friday, March 22.

(1) The pupils were given Table XXXIV.

(2) Those pupils who completed the assigned material were given either problems (21) in Table XLII or problems (28) in Table XXXVII.

Total items 32 (part 1); average score 89.42 per cent correct.

6. Monday, March 25.

(1) The lower part of Table XLII was presented.

TABLE XLI

10 -1	16 -8	13 -6	12 -6	15 -8	17 -9	16 -7
18 -9	15 -7	14 -7	14 -8	12 -5	14 -6	14 -5
17 -8	12 -4	16 -9	11 -5	10 -3	9 -1	12 -3
9 -5	10 -4	11 -2	14 -7	9 -4	9 -1	12 -8
7 -1	13 -4	11 -4	10 -5	14 -6	11 -7	8 -2
14 -5	12 -5	7 -2	16 -8	13 -9	15 -9	18 -9
15 -8	13 -8	16 -8	7 -3	11 -5	17 -5	12 -5

TABLE XLII

8 -4	7 -4	10 -4	13 -4	8 -4	12 -4	11 -4
6 -4	10 -5	8 -5	11 -5	14 -5	9 -5	13 -5
12 -5	7 -5	11 -6	9 -6	12 -6	15 -6	10 -6
14 -6	13 -6	8 -6	12 -7	10 -7	13 -7	16 -7
11 -7	15 -7	14 -7	9 -7	13 -8	11 -8	14 -8
17 -8	12 -8	16 -8	15 -8	10 -8	14 -9	12 -9
15 -9	18 -9	13 -9	17 -9	16 -9	11 -9	8 -3

Many pupils could not complete these 28 problems during the regular class period. Three pupils, S29, S26, and S14, had perfect papers. S13 made 7 errors, but corrected all of them. These four pupils received their papers back.
(2) S14, S3, and S24 worked 21 extra problems (Table XXXVII) without making a single error. S26 made three errors on these problems.
Total items 28 (part 1); average score 75.54 per cent correct.

Tuesday, March 26.
The class was divided into four groups: (1) Table XXXVIII; perfect papers: S14, S7, S32, S25. (2) Table XXXVI; no pupil made a perfect score. (3) Table XXXI; no perfect scores were made. This result was rather surprising as the problems were comparatively easy. S22 and S3 made only one error; S19 made three errors. (4) Table XXXV; perfect papers: S14, S9, S24, S10, S25, S29, S21.
Maximum items completed 49; average score 84.64 per cent correct.

Wednesday, March 27.
(1) The group was given 28 problems (Table XLI).
(2) Those pupils who completed part 1 were given Table XXXIX or Table XXXVII.
Total items 28 (part 1); average score 82.92 per cent correct.

Thursday, March 28.
Twelve minutes were allowed to work 49 problems (Table XXXIX). S24, S3, S22, S13, S29, S31, and S14 completed the test. Only S14 worked all these problems without an error.
Total items 49; average score 80.46 per cent correct.

Friday, March 29.
(1) The pupils who did not finish the papers begun Thursday, March 28 (Table XXXIX0, completed these problems.
(2) The other pupils worked a fresh sheet of these problems (Table XXXIX). Two pupils, S21 and S22, also worked 21 extra problems in Table XLII.
Total items 49 (Table XXXIX); average score 81.48 per cent correct.

E. Fifth Six Weeks

1. Monday, April 1.
Tables XLIII to XLVII, inclusive, form a test over the more important addition and subtrac-

TABLE XLIII
TEST A-1

2 3	2 1	3 1	1 1	3 2	3 3	1 4
2 1	2 8	4 1	2 6	3 4	1 2	4 5
2 5	1 3	7 1	3 7	6 3	4 5	2 4
1 6	5 4	8 1	1 5	6 2	4 4	5 2
7 2	5 3	4 2	2 7	4 3	5 1	6 1
1 9	6 7	4 8	2 9	6 5	5 5	3 6
		1 8	4 6	1 7		

TABLE XLIV
TEST A-2

7 7	6 4	4 9	6 6	8 7	2 0	9 2
7 9	5 0	9 0	6 8	4 7	8 9	6 9
3 8	9 8	5 8	6 0	5 9	7 0	8 0
8 8	7 6	9 5	7 3	1 0	8 2	7 8
3 0	9 6	7 4	8 6	9 3	8 5	7 5
4 0	3 9	9 1	9 7	5 7	9 9	8 3
		8 4	5 6	9 4		

tion 'facts'. Tables XLIII and XLIV each contain 45 ad ition problems. These two tests will be referred to as AI and AII, respectively. In Table XLVII there are 25 subtraction and 23 addition problems; this material will be referred to as Test M. Tables XLV and XLVI each contain 45 subtraction problems. These tests will be referred to as SI and SII, respectively. The number of items on the whole test is 228.

The testing of the class began during this work period (Tests AI and AII). The mimeographed sheets were halved so that a pupil had no more than 27 problems in any one unit. No time limit was set for completion of the test; the pupils were allowed to work at their usual rate of speed. The time taken by each pupil, however, was recorded. The procedure was the one ordinarily adopted except that the experimenter gave no aid to the pupils and graded no papers in class. (A new pupil, S38, joined the group.)

Maximum items completed 90; average score 91.63 per cent correct.

Tuesday, April 2.

The test was continued from Monday, April 1. The fastest pupils worked 66 subtraction problems in 20 minutes (Tests SI and SII). The subtraction again proved to be harder than addition.

Maximum items completed 66; average score 86.12 per cent correct.

Wednesday, April 3.

One subject, S14, finished the remaining 72 problems of the test in 20 minutes. His total time for the 228 problems was 60 minutes. (S29 also finished in the same length of time, but due to absences did not complete the test during this work period.) The members of the class varied markedly in speed. Seven of the ten units were employed by different pupils during the day's work. S14 finished all of Test M (last unit) while S31 continued on Test AII (fourth unit).

Maximum items completed 72; average score 78.54 per cent correct.

Thursday, April 4.

The test was continued. Several more pupils finished all the units and then repeated Tests SI and SII. All these results were taken into consideration in calculating the average daily score, but repetitions were not included in the general summary of test results.

Maximum items completed 111; average score 73.35 per cent correct.

TABLE XLV
TEST S-1

2 1	8 -3	5 -1	11 -9	10 -3	11 -7	3 -1
7 2	9 -2	6 -1	5 -3	10 -7	8 -4	6 -4
9 1	3 -2	11 -5	12 -8	4 -2	8 -2	5 -4
9 4	7 -5	6 -3	6 -5	12 -7	7 -3	8 -6
12 -6	6 -2	12 -3	11 -8	8 -1	4 -3	7 -4
11 -3	10 -5	7 -7	10 -2	9 -7	13 -7	13 -9
		10 -1	10 -8	11 -6		

TABLE XLVI
TEST S-2

4 -1	11 -4	7 -1	10 -4	15 -2	16 -9	10 -9
14 -7	1 -1	9 -5	8 -7	14 -5	15 -9	13 -6
2 -2	15 -6	15 -7	6 -6	9 -3	4 -4	9 -6
3 -3	12 -5	12 -4	8 -5	9 -9	9 -8	14 -9
16 -7	14 -8	5 -5	12 -9	18 -9	15 -8	17 -8
11 -2	7 -7	16 -8	8 -8	17 -9	13 -4	13 -5
		10 -6	14 -6	13 -8		

TABLE XLVII
TEST M

8	8	10	7	10	6	16
9	8	-1	9	-2	9	-8
9	11	7	5	14	9	11
9	-3	8	9	-7	-1	-4
8	9	7	15	9	8	14
5	-4	6	-6	8	7	-5
9	15	12	14	6	13	12
3	-8	-3	-6	7	-4	-4
9	11	6	18	13	12	13
5	-2	8	-9	-5	-5	-6
5	9	4	8	16	8	17
8	6	9	6	-9	4	-8
17	6	16	7	7	15	
-9	6	-7	5	7	-7	

Friday, April 5.

A majority of the members of the group completed all parts of the test. Due to absence or other reasons the following were tested later: S5, S7, S28, S32, S3, S27, S6, S12, S20, S17, S15, S25, and S30.

Maximum items completed 72; average score 73.96 per cent correct.

2. Monday, April 8.

(1) The pupils who had completed the test were given Table XLII. This set of problems was divided into an upper part (21 problems) and a lower part (28 problems). After completing the first unit, the pupil was given the second.

(2) Twelve pupils (S3, S7, S28, S5, S20, S17, S12, S25, S6, S15, S32, and S30) continued the test begun Monday, April 1.

Maximum items completed 98; average score 62.81 per cent correct.

Tuesday, April 9.

(1) The following still worked on the test: S12, S17, S20, S15, S5, and S25.

(2) S14 worked the problems in Table XLI; S14, S26, S10, S19, S13, S21, and S22 worked the problems in Table XL. The remainder of the class was given Table XLII.

Maximum items completed 94; average score 83.55 per cent correct.

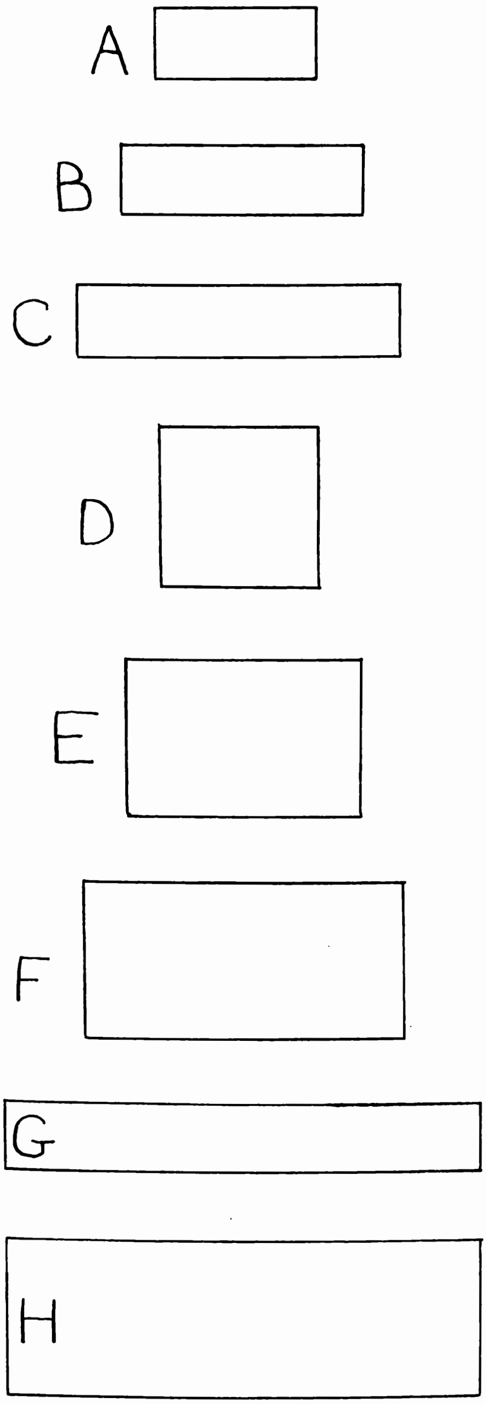
Wednesday, April 10.

A demonstration of the multiplication process was given. The materials consisted of colored inch cubes (blocks) and some especially prepared boxes. Two of these boxes were capable of holding just two cubes, two others held three cubes, two held four cubes, two held six cubes, two held eight cubes, and two held twelve cubes. One box held nine and the last box held ten cubes.

The pupils were given Table XLVIII. Before proceeding to problems involving multiplication, it was necessary to acquaint the pupils with the materials and to get them accustomed to the new instructions.

(1) "Place a cross in box A on your papers." Practically all the pupils immediately found the proper box in which to make the cross. They were then shown how to fold their papers so that box A was hidden from view. The pupils usually replied together whenever the experimenter gave them a problem verbally and it was necessary to counteract this tendency. They were required to listen to the problem stated by the experimenter, write their responses in the designated box, and then

TABLE XLVIII



fold their papers so that no one could see what they had written.
(2) "Copy this number in Box B." (The experimenter wrote '6' on the board.) "Fold box B under as you did box A."
(3) The experimenter held up three books. "Place the number of books in box C."
(4) The box containing two cubes was held up for inspection. The experimenter had several pupils count the cubes. The number was written in box D and the papers were folded as before.
(5) The experimenter held up the box used in the previous exercise and then stated the following problem: "If I had two boxes just like this one, how many blocks would I have altogether?" The pupils worked the problem individually, put their answers in box E. S29, who ranked highest on the test over addition and subtraction, was the only pupil to put down an incorrect answer.
(6) "If I had three boxes full of blocks, how many blocks would I have altogether?" The answers were placed in box F.
(7) Two counting exercises were introduced and the answers written by the pupils in boxes G and H, respectively.

The co-operation of the class was readily secured. The problem in instruction (6) proved rather difficult. S31 gave his answer aloud, contrary to instruction, and nine pupils followed his suggestion that five was the correct answer. The following ignored the suggestion of S31 and wrote the correct response: S26, S22, S19, S21, S7, S28, S16, S14, S25, S6, S8, and S24.

Total items 8; average score 93.46 per cent correct.

Thursday, April 11.
(1) S25, S15, and S17 continued the test over addition and subtraction; S25 completed the test. (S27, S30, S20, and S12 never did complete the test. S27 and S30 did not have sufficient time because of absences. The material was too difficult for S20 and S12 to complete.)
(2) The class as a whole worked on the problems in Tests AI, AII, SI, SII, and M. The first 21 problems in Table XLII were also used.

Maximum items completed 66; average score 91.16 per cent correct.

Friday, April 12.
(1) S15 and S17 completed the test.
(2) The other members of the class were given the 45 problems in Table XLV (Test SI). S14, S16,

S22, S10, and S26 also worked the 45 problems in Table XLIII (Test A1).

Total items 90; average score 91.39 per cent correct.

The complete results on Tests AI, AII, SI, SII, and M are shown in Table XLIX. It will be noted that for the 23 pupils the median number of errors was 23; the median in minutes was 80. This time was extremely slow, about three problems per minute. The fastest pupils (S14 and S29) worked only four problems per minute. S29, a first grade pupil who had worked regularly with the class for 16 days, was the only one to exhibit a perfect score on the test. The teacher's report on this case and also that of the mother showed that no previous formal training in arithmetic had been given. The case demonstrates that drill is not always essential to attain accuracy.

The last subjects to complete the test, S15, S17, and S25, did so under a great pressure. S17 benefited considerably by the extra amount of time. He attained a rank error score of nine. The score of S25 was the median score (rank twelve). S15 made an error score of 66; his rank was twenty. These cases will be referred to later in the discussion of comparative scores and retests.

3. Monday, April 15.

The materials described for Wednesday, April 10 were again presented (Table XLVIII). The demonstration was similar to that employed on the earlier date; the problem of division, however, was introduced.

(1) The experimenter placed cubes in the 'ten' box. S27 was asked to come to the front of the room and count the cubes. Each pupil was requested to place the number in box A printed on their papers.

(2) A box containing two cubes was held up. "Put the number of blocks in B on your papers."

(3) "How many of these boxes (two boxes holding 2 cubes each were held up) do we need to hold 10 blocks?" "Put your answer in C." S26 gave the only correct response.

(4) "If we had four boxes like this one (a box holding 2 cubes), how many blocks would we have altogether?" "Write your answer in D."

(5) The class worked out a problem together and then placed the answer in E. (The number to be written was four).

(6) A box containing 3 cubes was held up. "How many blocks would we have altogether if we had two boxes like this one?" "Put your answer in F."

TABLE XLIX

Pupil	Total Errors (228 problems)	Time (Estimated Min.)
S9	42	80
S31	134	120
S21	10	80
S10	18	80
S24	10	80
S14	2	60
S29	0	60
S19	34	80
S22	5	110
S1	70	80
S26	5	80
S13	25	80
S16	25	80
S8	47	80
S28	100	130
S17	16	165
S6	15	122
S32	11	90
S3	21	90
S15	66	170
S5	26	100
S25	23	160
S7	39	122
Total pupils 23	Median 23 (Range 0-134) S.D. 33.56 S.E. 6.997	Median 80 (Range 60-170)
	AV. 32.30 S.D. 31.42 S.E. 6.551	AV. 99.96 S.D. 31.27 S.E. 6.52

(7) A group of eight cubes was counted for the class. "How many of these boxes (holding two blocks each) would we need to hold all the blocks?" "Put your answer in G."
(8) The class worked another problem and the answer (6) was written in H.

There was some confusion due to the fact that the experimenter did not make it clear when the class as a whole was to respond and when the individuals were to work independently. S28, S31, S15, and S19 persisted in 'giving away' the answers when the instructor desired the pupils to keep the answers to themselves.

Total items 8; average score 76.27 per cent correct.

Tuesday, April 16.

The class was given 49 problems (Table XXXVII). S19, S9, S16, S8, S24, and S14 finished these problems and worked some or all of the problems in Table XLI. The following subjects, who also finished the first list of problems, were given Table XLVII: S21, S26, S10, S15, and S7.

The pupils who did accurate work had their papers returned to them: S24, S1, S14, S6, S8, S22, S19, S10, S9, S16, S21, S29, S26, S15, and S7.

Maximum items completed 98; average score 84.71 per cent correct.

Wednesday, April 17.

Multiplication and division were again demonstrated. The instructions were made carefully in order that the pupils understood when to reply together and when to work individually. Sample problems are as follows:
(1) A box containing 2 cubes was held up. A duplicate, empty box was then picked up in the other hand. "How many blocks would we have if both boxes were filled?"
(2) A box containing 2 cubes was held up. "How many blocks would we have altogether if we had three boxes like this one?"
(3) A box with 3 cubes was picked up. A duplicate, empty box was picked up with the other hand. "How many blocks would we have if both boxes were filled?"
(4) A group of six cubes was shown and counted. An empty box which could hold 2 cubes was picked up by the experimenter. "How many boxes like this one would we need to hold all these blocks?" (The members of the class solved this problem together.)
(5) A group of eight cubes was counted. "How many boxes like this one (holding 2 cubes) would

TABLE L

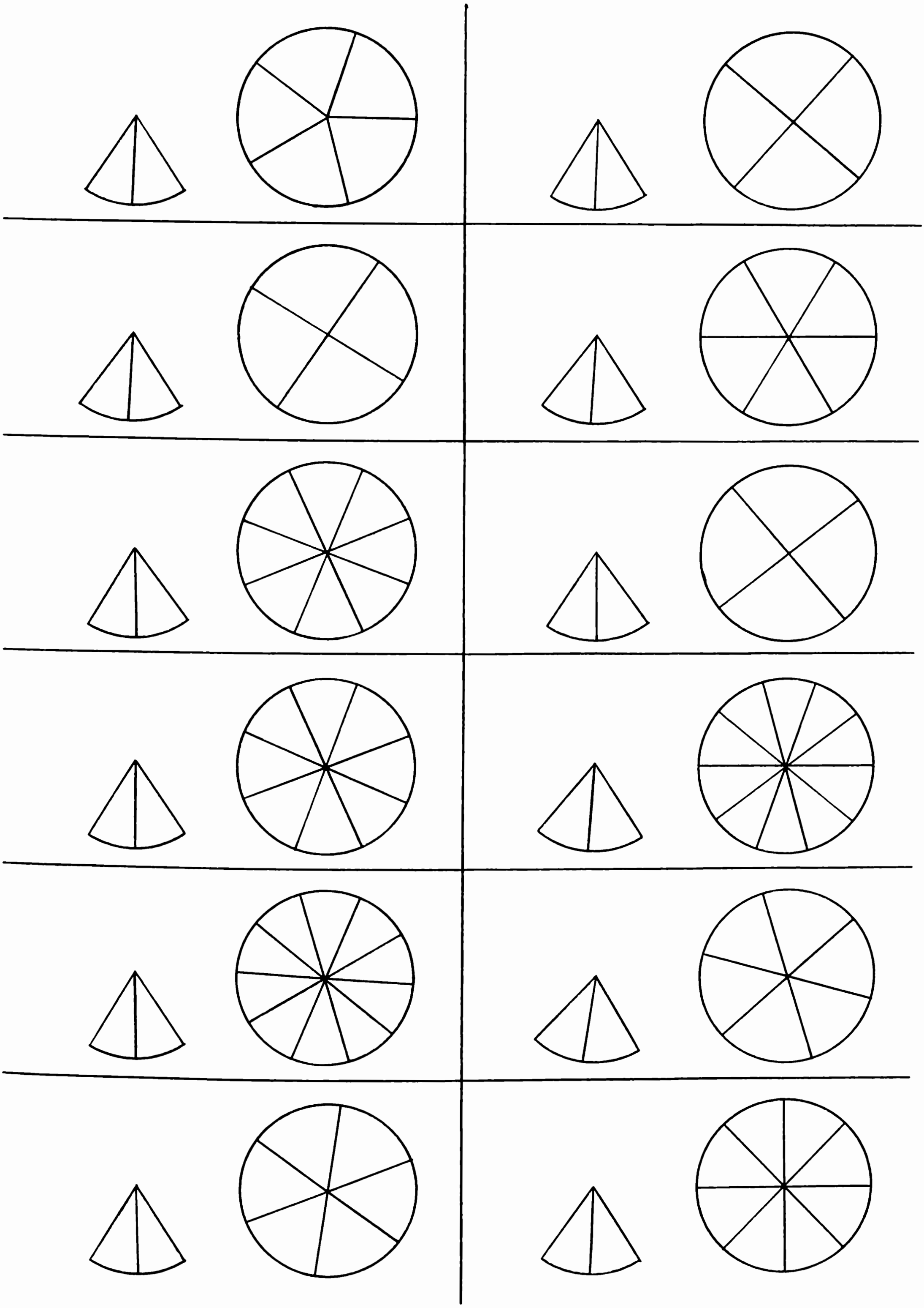
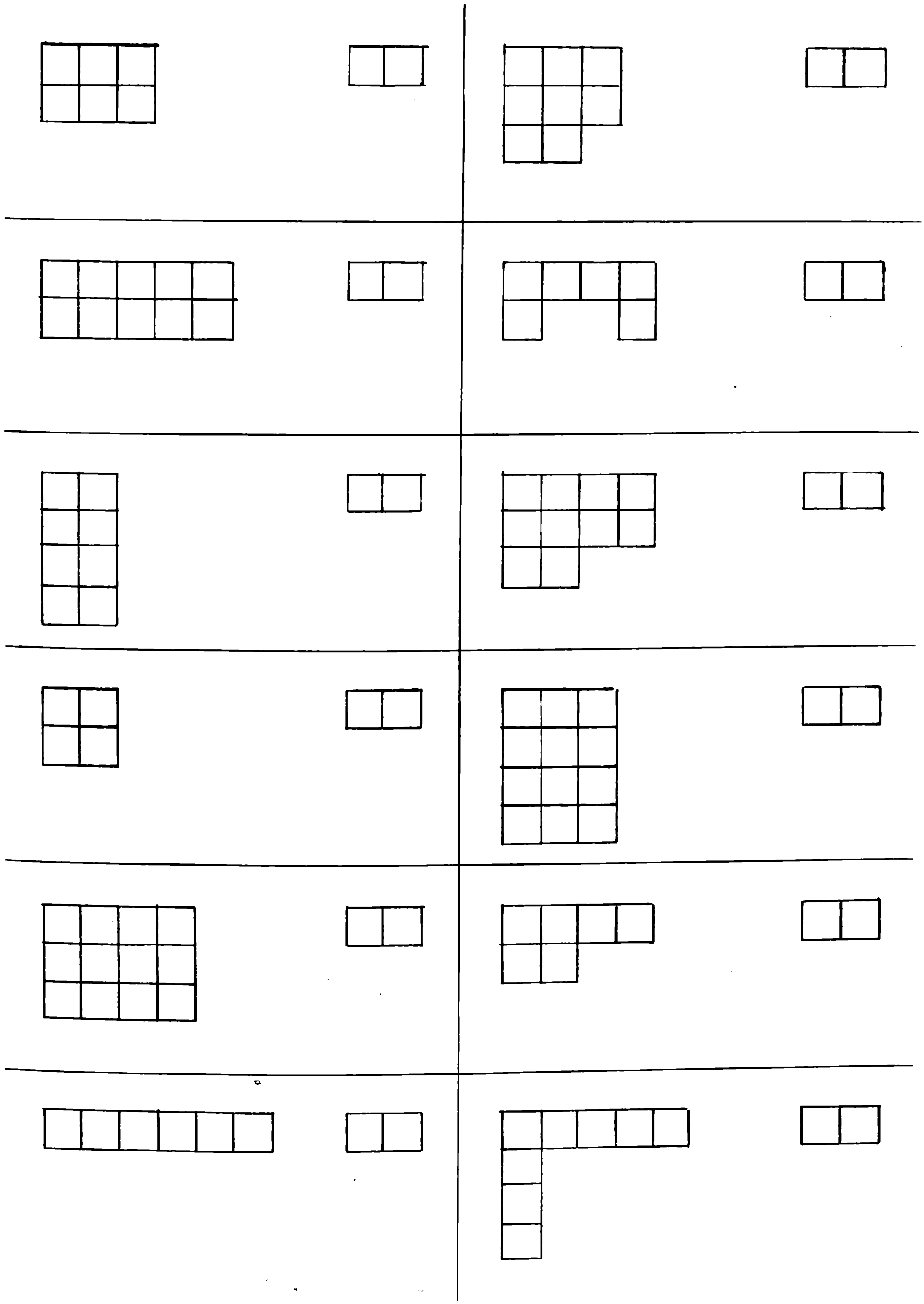


TABLE LI



we need to hold all these blocks?"

Total items 8; average score 95.00 per cent correct.

Thursday, April 18.

Table XLI was given. S14, S29, and S26 worked the 49 problems without errors. The following finished this paper and also worked part of Table XXXVI; S14, S19, S26, S6, S28, and S29. All papers were returned to the pupils.

Maximum items completed 98; average score 82.50 per cent correct.

4. Tuesday, April 23.

Problems on multiplication and division were given. Table XLVIII was used by the pupils to record their responses. On most of the problems the class worked as a group; the answers were given aloud, verified, and then copied by the pupils in the boxes appearing on their papers. There were three critical problems to be calculated individually. The poorest students obtained suggestions, however, from S32 who failed to comply with the instructions and gave his answers aloud. The three problems were: (1) "Here are eight blocks. How many boxes like this one, which holds two blocks, would I need to hold all these blocks?" (2) "Here is a box which holds two blocks. If I had three boxes like this one, how many blocks would I have altogether?" (3) "There are twelve blocks in this group. How many boxes, each holding two blocks, would I need to hold all these blocks?"

Total items 8; average score 88.71 per cent correct.

Wednesday, April 24.

Materials in addition and subtraction were employed. (1) The following pupils received Table XLI: S17, S10, S30, S27, S32, S25, S15, S5, S3, S22, S26, S14, and S29. S14 and S26 also worked the problems in Table XL.
(2) S19 and S13 were given Table XLII. S13 also worked 21 problems in Table XXXIX.
(3) S31 worked 28 problems in Table XXXVI.
(4) The following were given Table XLVII: S9, S24, S7, S12, S20, S6, S16, and S28.

S24, S16, S25, S22, S19, S26, S14, and S29 solved at least one page of problems without errors; these pupils received their papers back.

Maximum items completed 98; average score 85.33 per cent correct.

Thursday, April 25.

(1) The experimenter attempted to define 'pair'

TABLE LII

2 X 2 = ____

3 X 2 = ____

4 X 2 = ____

5 X 2 = ____

6 X 2 = ____

3 X 3 = ____

2 X 3 = ____

2 X 4 = ____

2 X 3 = ____

3 X 2 = ____

3 X 3 = ____

3 X 4 = ____

5 X 2 = ____

2 X 2 = ____

TABLE LIII

6 ÷ 2 = ____

4 ÷ 2 = ____

8 ÷ 2 = ____

4 ÷ 2 = ____

10 ÷ 2 = ____

8 ÷ 2 = ____

6 ÷ 2 = ____

6 ÷ 3 = ____

9 ÷ 3 = ____

12 ÷ 3 = ____

8 ÷ 4 = ____

6 ÷ 2 = ____

10 ÷ 2 = ____

8 ÷ 2 = ____

for the class. The first examples involved books, erasers, and cubes; the box holding two cubes was used. "It holds just one pair of blocks." S27 was the only member of the class who could define the meaning of 'pair'; he understood that a pair meant 'two of a kind'. After the concept had been explained, S21 said, "A pair would have to be two because when you buy a pair of shoes you get two shoes."
(2) A demonstration was made to show how the parts of a circle could be divided into 'pairs'. For an illustration the experimenter had obtained a large circle of cardboard on which segments had been drawn. Each 'pair' of segments was made a different color. Several children were called to the front of the room to count the 'pairs' in this large circle; the problem apparently offered little or no difficulty.
(3) Papers with circles divided into parts were given (Table L). The members of the class were instructed to find the 'pairs' and to write the number below each circle. The instruction proved quite difficult to follow. S21, who usually grasped a problem quickly, did not understand how to proceed. S13, S26, and S27 counted the number of parts of the circles instead of calculating the number of pairs.
Total items 12 (part 3); average score 55.96 per cent correct.

Friday, April 26.
(1) The papers used to record answers to multiplication and division problems were given the pupils (Table XLVIII). The attempt was made to clarify the meaning of the word 'pair'. The class solved several problems together, e.g., two pairs of blocks would be 4 blocks and three pairs of blocks would be 6 blocks. The following were given the group to be worked individually: (a) "Four pairs of blocks would be how many blocks?" (b) The experimenter showed a large circle divided into ten parts; every pair was a different color. "There are five pairs of parts. How many parts are there?"
(2) Problems similar to those of Thursday, April 25, were employed (Table LI).
Total items 12 (part 2); average score 77.00 per cent correct.

5. Monday, April 29.
(1) S20, S24, S30, S12, and S9, who did not appear to grasp completely the idea of 'pair', were asked to work several problems in class. Instructions similar to the following were given: "Bring

me one pair of erasers"; "How many pairs of blocks do I have in my hand (four or six cubes?"; "Bring me three pairs of books." S20 picked without trouble two 'pairs' from a group of six books. None of these pupils had difficulty with the concept as long as numbers above eight were not involved and as long as the concrete materials were used.
(2) The whole class wrote the number of 'pairs' on two papers (Tables L and LI). A few pupils repeated the problems in Table L.

Total items 24 (part 2); average score 74.79 per cent correct.

Tuesday, April 30.
(1) The experimenter attempted to demonstrate 'three-ness'. Phrases such as "Two 3's" and "Three 3's" were employed. The symbol form was explained: 3x3=9, 2x3=6. The problems 4x2=8, 2x4=8, 3x2=6, and 2x2=4 were worked out and written on the board in this form. S21 and S2? were called upon to verify the problems by counting cubes.
(2) The class was given Table LI and instructed to write the number of 'pairs'.

Total items 12 (part 2); average score 88.96 per cent correct.

Wednesday, May 1.
(1) Six multiplication problems were worked by the class. Blocks were used to aid the pupils. Whenever a problem had been solved correctly, a pupil was permitted to write the problem on the board. The problems were: 2x3, 2x2, 3x2, 2x4, 3x3, and 2x3.
(2) The class also worked addition and subtraction problems. S9, S15, and S30 worked 28 problems in Table XXXIX. The upper part (21 problems) of this table was given to S31, S12, and S28. S1, S32, S24, and S25 worked 21 problems in Table XLII. The lower part of this table (28 problems) was given to S13, S6, S27, S21, S29, S3, S22, S26, and S14. S17 worked 28 problems in Table XXXVII and S7 worked 21 problems in Table XLVII.

Maximum items completed 28 (part 2); average score 73.10 per cent correct.

Thursday, May 2.
(1) The class worked on multiplication. Boxes and cubes were employed to verify the answers. S14 was the first to solve 3x4; he then wrote the problem on the blackboard. Other problems worked by the class were: 2x4, 2x2, 2x4, 3x2, 3x3, and 4x3.

(2) The pupils worked the following problems individually: 2x5, 3x3, 2x6, 2x2, 2x3, 2x4, and 4x4. Problems 2x5, 2x3, 2x6, and 4x4 were critical problems inasmuch as they were not dealt with in the demonstration (part 1). S21, S14, and S13 solved all these problems without making an error. S3 and S31 made the largest number of errors.

Total items 7 (part 2); average score 65.40 per cent correct.

Friday, May 3.

(1) The material on pages 36 and 37 of the Lennes Work Pad I was employed (reading problems; recognizing numbers of various objects, the number names, the number symbols, and working seven addition problems). The class showed marked improvement in reading and understanding of the instructions. No quantitative scores were recorded.

(2) Special instruction on addition was given to S1 and S27 (ten to fifteen minutes per day for five preceeding days). (a) Problems were given orally; one pupil and then the other was required to give the answer. If the answers were not immediately forthcoming, an error was recorded.

(b) Addition problems were written on the subjects' paper. The doubles, e.g., 1-1, 2-2, were learned quickly; a series usually began with one of these problems. The following is an example:

$$\begin{array}{r}4\\ \underline{4}\end{array}\quad \begin{array}{r}4\\ \underline{5}\end{array}\quad \begin{array}{r}4\\ \underline{6}\end{array}\qquad\qquad \begin{array}{r}7\\ \underline{7}\end{array}\quad \begin{array}{r}7\\ \underline{8}\end{array}$$

$$\begin{array}{r}8\\ \underline{8}\end{array}\quad \begin{array}{r}8\\ \underline{9}\end{array}$$

$$\begin{array}{r}6\\ \underline{6}\end{array}\quad \begin{array}{r}6\\ \underline{7}\end{array}\quad \begin{array}{r}6\\ \underline{8}\end{array}$$

The work was not mechanical drill. A goal of accuracy was stressed, but the pupil was allowed to find the answers by his own method. The experimenter arranged the problems to make it possible for the subject to discover a relationship. Both S1 and S27 finally grasped the notion of relationship and learned to examine the list for 'connections'. Earlier they had begun by counting on each problem.

(3) S30 and S17 were given special instruction. The procedure was the same as that used with S1 and S27. S30 and S17 counted on their fingers for the simplest addition problems, 4-2, 3-2, and 2-1. Problems such as 9-3 and 11-2 were so difficult to solve by this method that an excessive time was necessary to calculate the answers.

Errors were common in dealing with these higher numbers. These pupils always began a problem by counting from one. They had not learned to count with the first number of a combination as a base, e.g., "2,3,4", on the combination 2-2.

6. Monday, May 6.
(1) The experimenter had some of the children count. The pupils had received no training in class in counting the higher numbers or in counting by 2's, or 5's, or 10's. They, however, had been required to recognize the numbers above 100 in order to locate the pages assigned in their readers. S28 counted by 1's to 50 without difficulty. S7 was asked to count from 50 to 100. He was not certain of himself at 70, 80, and 90. S21 counted by 5's to 50. S13 and S26 counted readily by 10's to 100.
(2) The class solved the following four problems: 2x4, 2x3, 3x4, and 3x2. The answers were written on the board by four different pupils and then copied by the group.
(3) The experimenter gave the class a paper containing 14 multiplication problems (Table LII). About one-fourth of the class appeared to grasp the essential idea of these problems (S26, S29, S22, S14, S7, and S21).
Total items 14 (part 3); average score 69.70 per cent correct.

Tuesday, May 7.
(1) The work on counting was continued. Several pupils counted by 2's to 20; S25 reached 38 after much effort. S14 and S1 counted by 3's to 9; S13 reached 12; S29 counted to 15. The experimenter had all members of the class count by 5's to 20; this was done in written form. S26, S31, and S12 were the only pupils to make errors.
(2) The experimenter placed the number 10 on the blackboard. He then asked the question, "If I had 10 blocks, how many pairs would I have?" The pupils also solved the number of pairs in 12 blocks.
(3) The pupils received Table LII. (No aid was given.) S14, S3, S7, S21, S26, and S9 made 90 per cent correct or above (about one-third of the class).
Total items 14 (part 3); average score 61.32 per cent correct.

Wednesday, May 8.
(1) The class was given the reading problems on addition, page 38 of the Lennes Work Pad I. Several pupils made a large number of errors. No quantitative scores were recorded.

(2) The explanation of division was repeated. S27 was asked how many 'pairs' there were in six cubes. S7 found the number of 'pairs' in eight. The experimenter demonstrated that 8 cubes could be divided into four 2's. The symbolic form, 8÷2=4, was placed on the board. The class and the experimenter together worked out the problems 6÷2 and 10÷2. The answers were copied from the board.
(3) The pupils were asked to turn over their papers. The problem, 6÷2, was then placed on the board. Fourteen pupils gave the correct answer: S13, S27, S29, S31, S7, S21, S3, S9, S24, S15, S28, S14, S1, and S26. Two pupils, S16 and S31, failed to turn over their papers, but got the correct answer. Only S17 and S30 failed on the problem.

Thursday, May 9.
(1) The material on page 40 of the Lennes Work Pad I (reading problems) was employed. Several errors were made; only two pupils worked twelve problems correctly. No scores were recorded.
(2) The experimenter gave another demonstration of division. Cubes and boxes were used. The experimenter started with two as a divisor: "How many pairs can I make from ten blocks? Six blocks? Eight blocks?" (The box holding 2 cubes was used in the demonstration.) As the answers were given, the experimenter wrote the problems with the answers on the board in the symbol form. These were copied by the pupils. Three also was introduced as a divisor. The pupils were then asked to work two problems copying them from the board: 12÷2 and 9÷3. S3, S14, S7, S21, and S29 solved these two problems correctly. S27, S17, S12, and S28 also obtained the correct answers, but received aid from other students who knew how to work the problems. S31 missed both problems; they were not copied correctly. (The following incorrect answers were given: 12÷2=4; 9÷3=5; 9÷3=6; 9÷3=8.)

Total items 2 (part 2); average score 70.00 per cent correct.

F. Last Three Weeks

1. Monday, May 13.
(1) The pupils worked page 44 of the Lennes Work Pad I (seven addition problems; three reading problems). The arithmetic problems were easily solved whereas the reading matter proved difficult.
(2) A demonstration of division again was made. Two problems not used in the demonstration, 8÷2 and 9÷3, were given as a test. The following gave correct responses: S21, S3, S25, S26, S7, S22, S1, S5, S14, S29, and S16. S12 copied her work. The only pupil to miscalculate both problems was S24. Those making one error were: S6, S30, S15, S32, S27, S13, S28, S31, and S9.

Total items 2 (part 2); average score 75.00 per cent correct.

Wednesday, May 15.
A test over division was given. The problems (Table LIII) were mimeographed on colored paper. This test contained eight different problems, several being repeated. All scores were based on the total of fourteen items. The following made no errors: S21 and S22. The following made one or two errors: S13, S26, S3, and S27. The following made three to nine errors: S9, S14, S29, S25, S24, S30, S1, S5, S6, S15, S32, and S7. The following failed to work a single problem correctly: S16, S28, S17, S12, and S31.
Total items 14; average score 57.65 per cent correct.

Thursday, May 16.
(1) The process of division was again demonstrated. Boxes and cubes were used to illustrate the following: 6÷2, 6÷3, 9÷3, 8÷2, 8÷4, and 10÷2. As long as the concrete materials were employed, many pupils were able to 'see through' the process. S12 and S8, who had been absent for some time, gained the most from the demonstration.
(2) Three test problems were given: 8÷2, 10÷2, and 12÷4. (Two of these problems were given in part 1.) Fourteen pupils made no errors: S28, S26, S9, S22, S21, S3, S12, S6, S7, S29, S14, S13, S1, and S25. Six pupils missed 12÷4: S8, S17, S32, S24, S30, and S27. Three pupils had no conception of division: S15, S16, and S31.
Total items 3 (part 2); average score 78.86 per cent correct.

Friday, May 17.
(1) The test over division (Table LIII) was given. The following made no errors: S22, S21, S3, S8, S25, and S26. Four pupils made only one error: S13, S1, S29, and S6. An examination of the poorer papers revealed that many of the pupils multiplied instead of divided, several added, and one or two simply miscalculated. One-fourth of the class made 100 per cent correct scores. One-half of the class made 86 per cent correct or above.
(2) The class also worked the problems on pages 47 and 48 of the Lennes Work Pad I. The pupils were expected to read the instructions on page 47 which contained items similar to the following: "Write the numbers from one to ten." "What number comes before 6? After 3? Between 4 and 6?" These reading problems proved too difficult for the group. On page 48 the pupils were required to recognize the numbers of various objects.
(3) S14, who missed 11 problems on the division test, was kept a few minutes after class and given another test sheet. It was found that he was neglecting the sign indicating division and was performing multiplication instead. His answers showed that the relation of multiplication to addition had been grasped (12÷3=33, 6÷3=18, and 1÷4=31). The answers to two problems, 10÷2

and 9÷3, had been memorized. This raised a question, namely, do the better pupils solve these division problems by relying chiefly on memory? The experimenter was able to observe S7, S29, and S26 on later occasions and it was evident that the answers on the test sheet were obtained by means of calculation. The experimenter always found after a demonstration that the better pupils had learned enough about the process of division to work critical problems that had not been used during the demonstration. This evidence showed that the process was grasped, at least, temporarily. The poorer students, of course, could not 'see through' the process and failed to memorize any answers to the problems during the demonstrations.

Total items 14 (part 1); average score 67.29 per cent correct.

2. Monday, May 20; Tuesday, May 21; and Wednesday, May 22.

Miss Carter, the second grade teacher, gave the pupils work in addition and subtraction for the first three days of this week. The material is shown in Tables XLIII, XLIV, and XLV (Tests AI, AII, SI). The performance of each pupil has been summed up in one score for Monday, May 2. (Miss Carter also used number readers during these three periods.)

Maximum items completed 135; average score 83.60 per cent correct.

Thursday, May 23.

Part of the class was given the addition problems in Table XLIII (Test AI) and the rest of the class was given the addition problems in Table XLIV (Test AII).

Total items 45; average score 90.48 per cent correct.

Friday, May 24.

A test over addition and subtraction was begun. The same materials and the same procedure were employed as on the previous test (Monday, April 1) except that the subtraction problems were given first (Table XLV, Test SI).

Total items 45; average score 83.75 per cent correct.

3. Monday, May 27.

(1) The test was continued (Table XLIV; Test AII).

(2) The experimenter also tested the pupils individually outside of the usual class period. At no time had the members of the class been drilled and it was decided that a flash card technique would reveal whether or not the responses to any addition and subtraction problems had been memorized. The class was tested on 45 addition problems which were printed on flash cards; the pupils were allowed a maximum

of three seconds to each problem. A recording sheet was mimeographed so that the childrens' responses could be checked off quickly; it was the original intention to expose each card for the required amount of time and then expose the next card whether or not the pupil had succeeded in getting the answer. The new device was strange to these pupils, however, and a modification of the technique had to be made. If a pupil hesitated longer than three seconds, he was allowed to continue on the problem, but the answer was not recorded. By this procedure it was possible to prevent the pupil from lagging behind. The results of this test appear in Table LIV under the heading, "Flash card test, addition". The 45 problems were as follows: 2+3, 3+3, 3+8, 3+4, 3+6, 7+1, 2+5, 6+3, 4+5, 3+7, 2+4, 6+2, 4+4, 5+2, 7+2, 4+2, 2+7, 4+3, 6+7, 4+8, 2+8, 6+5, 5+5, 3+9, 4+6, 7+7, 4+9, 6+6, 8+7, 7+9, 6+8, 8+9, 6+9, 3+8, 5+8, 5+9, 8+8, 7+8, 9+6, 7+4, 9+3, 7+5, 9+7, 9+9, and 8+3.

Total items 45 (part 1); average score 79.64 per cent correct.

Wednesday, May 29.

(1) The class continued the test (Tables XLIII and XLVI, Tests AI and SII). The summary of the results on Tests AI, AII, SI, and SII appear in Table LIV under the heading, "Addition-subtraction test, 180 facts (May 29)". (During the year the following pupils left the class: S11, October 9; S4, December 4; S18, December 20; S2, December 28; S23, February 21; S10, April 24.)

Maximum items completed 90; average score 83.50 per cent correct.

(2) The flash card technique was repeated with 45 subtraction facts. The results appear in Table LIV under the heading, "Flash card test, subtraction". The 45 problems were: 8-3, 11-9, 5-1, 10-3, 11-7, 9-2, 6-1, 10-7, 8-4, 11-5, 8-2, 4-2, 5-4, 7-5, 9-4, 7-3, 8-6, 12-6, 6-2, 11-8, 8-1, 7-6, 4-1, 5-2, 14-7, 9-5, 8-7, 15-7, 9-3, 9-6, 12-5, 8-5, 14-9, 14-8, 18-9, 15-8, 11-2, 16-8, 17-9, 13-4, 10-6, 14-6, 13-8, 7-3, and 9-1.

The results on both flash card tests (Table LIV) showed the superiority of S16 which was unexpected. This situation which required concentrated effort for a short period of time proved favorable for this student. Throughout the year she had worked satisfactorily only in spurts; prolonged effort seemingly was impossible for her. She did well on easy work, as a rule, and failed to attack the more difficult problems. These results might be interpreted as showing a necessity for 'holding some pupils to the job'. S29, on the other hand, did very poorly on this flash card test. She was ordinarily the most consistent worker of the whole group, but almost 'went to pieces' in this test situation. Lack of social maturity together with a knowledge of failure made the situation unfavorable for an adequate arithmetic response. These two extreme cases show how varied are the circumstances which call forth the best effort of different pupils.

The group had had considerable work on addition and subtraction as well as multiplication and division since the first addition-subtraction test was completed, April 12. (A summary of these results (180 problems) is given in Table LIV.) It might be expected that the class would improve after a month of work on arithmetic. A comparison of results will show, however, that the class did not improve. Certain individuals made less satisfactory scores, although several improved a great deal. The score of S1 raised appreciably. S21, S19, S5, and S7 also showed improvement. S28, on the other hand, apparently became badly adjusted at the end of the school year, making a much poorer score on the second test. S22, S13, and S8 also failed to equal their previous records. A consideration of individual case histories and the evidence of retests will be given later in an attempt to make clear the significance of these results.

TABLE LIV

Flash Card Test, Addition

Pupil	Errors
S16	5
S22	6
S14	9
S7	11
S29	12
S8	12
S21	12
S3	13
S13	15
S19	17
S6	18
S1	19
S31	21
S15	28
S27	31
S30	35
S28	37
S17	38
S12	43
Med. 17	

Flash Card Test, Subtraction

Pupil	Errors
S16	3
S6	7
S13	8
S22	10
S14	15
S21	15
S15	21
S5	22
S29	23
S7	24
S8	27
S27	29
S3	29
S1	31
S31	33
S19	33
S28	35
S30	39
S12	41
S17	42
S20	43
Med.27	

Addition-Subtraction Test, 180 Facts.

Pupil	Errors Apr. 12	Errors May 29
S9	31	abs.
S31	93	inc.
S21	7	1
S10	14	abs.
S24	6	abs.
S14	2	1
S29	0	0
S19	27	23
S22	2	6
S1	51	6
S26	4	abs.
S13	8	19
S16	18	17
S8	28	34
S28	65	99
S17	11	12
S6	6	5
S32	3	abs.
S3	5	abs.
S15	41	inc.
S5	15	4
S25	7	abs.
S7	22	13
S27	abs.*	54
S30	abs.	inc.
S20	inc.**	inc.
S12	inc.	inc.
	Med. 11	Med. 12

* absent.
** incomplete.

Qualitative Results. (a) Reasoning of the Second Grade Pupil. (1) Counting of objects and writing of number symbols. The results of the preliminary experiment indicated that enumeration and comparison of quantities were difficult for first grade children. These simple tasks proved difficult also for second grade children at the beginning of the year. Enumeration of objects above eight or ten was very inaccurate. At the close of the first six weeks period (November 9), counting and labeling of objects up to ten was satisfactory (90 per cent correct) for a majority. The writing of number symbols from memory, however, was not satisfactory.

(2) Comprehension of number at the perceptual level. A lack of comprehension of abstract number is illustrated by the results of October 16. The class was instructed to equate pairs of number pictures whereas many simply made the pictures similar in pattern. Although the pupils had been trained to count each picture before making a judgment, the numbers as such were neglected; equality and difference were estimated perceptually and not mathematically.

After considerable experimentation with comparisons of number pictures, a new type of problem was devised to encourage the use of symbols (October 25). The parts of a circle were to be counted and labeled and then the pupil was expected to draw the same number of parts in a blank circle. No pupil consistently employed the number symbols in solving this task. The better pupils inspected the circles with few parts to ascertain the number and re-counted when there were four or more parts.

Much of the work for the first two six weeks periods, ending December 20, had been arranged to show the pupils, in a general fashion, the relations of quantities within quantities. Materials of considerable difficulty were prepared in order that a comparison of two groups could be made only by considering sub-groups. The class could solve problems involving these intricate relationships as long as the process was kept on the perceptual level, i.e., perceptual estimation of concrete materials. All pupils, for example, could add parts which completed the pattern of a number picture. Most of them could not write the number designating the amount to be added to make one picture identical with another.

(3) Finding sameness or difference with the use of abstract concepts. A few pupils immediately grasped the essential idea of equating a pair of number pictures. On the first day this type of problem was presented , October 31, both S3 and S16 asked the question about the combination 5-5, "How do you make nothing?" It was quite evident in these cases that comparisons were being made at a mathematical level. A close check was kept on November 27. Four pupils used the number symbols in establishing differences between quantities.

(4) Addition and subtraction in language form. Before introducing addition and subtraction problems in sym-

bol form, the experimenter attempted to make clear the whole-part relations by 'telling the whole story' about the problem. Six objects, for example, were divided into three and three. The language used was, "Six is the same as three and three." The pupils were then expected to 'tell the whole story' about a combination given them: "Six is the same as three and three" or "Six is the same as four and two." These facts were then stated in problem form: "6 is the same as 3 and ___." Test problems in this form (January 15 and 17) proved too difficult for the class. S21, one of the best students, grasped the process from the class demonstration, but could not remember from one day to the next how to proceed. S14, another capable student, also tried to understand the directions, but could not master the process. In Tables XXV, XXVI, and XXVII the following types of problems were used: "2 is the same as 1 and ___."; "1 and 1 are the same as ___." Both forms appearing on one paper resulted in complete confusion for all pupils.

The experimenter made an attempt (February 66 and 11) to interest the pupils in reading problems. Stories were written containing the pupils own names. This reading material only complicated the situation, however, and made the arithmetic too difficult. It should be noted that the pupils in this class had considerable difficulty with all reading materials.

(5) Demonstration of addition and subtraction in symbol form. The easier addition problems were solved with-

out difficulty whenever concrete objects were employed (January 8). Ten addition problems of the following type with sums to four were also easily solved January 14:

0   oo
1 and 2 are ___.

The experimenter demonstrated with books several addition problems and also explain them in the vertical form: 1   (January 21).
            3
            4

The class solved four critical problems accurately. Subtraction was introduced along with addition (January 22) in order that the pupils could learn the corresponding number facts together. The difficulty with the language problem was not surmounted entirely by this method as the class scores indicated. Subtraction proved especially hard. This result shows, we believe, that the pupils were learning merely to obtain an answer rather than learning to think mathematically. Most of the answers were gained by counting; since subtraction involved <u>counting</u>, <u>taking away</u>, and <u>counting the remainder</u>, the respective relations of whole to part were more difficult to keep straight.

<u>(6) Learning of addition and subtraction 'patterns'</u>. The materials in Tables XXVIII and XXX were devised to encourage the pupils to discover relationships among problems. The answers in every row represented a 'pattern'. The pupils discovered the pattern of answers in most cases, although many did not learn to look for the pattern until the material had been presented several times. These papers

contained too many problems and also contained problems which were too difficult. The material used later in Tables XXXI to XXXV and Table XL gave much more satisfactory results (90 per cent correct or above). It was impossible to arrange material that was well graded for the entire class. The patterns proved either too easy or too difficult in many instances and the experimenter accordingly lost several opportunities to follow up a suitable page of materials with another one equally well adapted to the situation.

Although the easier addition and subtraction problems were finally mastered, the harder facts proved very difficult for the group (March 5 - April 12). The addition-subtraction test, which was concluded April 12, revealed that too much time was taken even by the pupils who worked accurately.

(7) Mastery of multiplication and division. Since the pupils understood the process of addition and subtraction it was decided to test their knowledge of multiplication and division (April 10, 15, 17, 23, 26). The pupils, when working with concrete materials, easily solved simple problems with numbers as high as 10 or 12. The experimenter was able to demonstrate 'pair' and 'three' taken as multipliers and as divisors (April 25 and 30). The class succeeded well enough to justify a demonstration of these processes using the symbol form (May 6, 7, 8, 15, 17). About one-fourth of the class showed a mastery of multiplication and about the same number showed a mastery of

division. A number of pupils failed completely on these tests.

(8) Reasoning of the slow pupil. Some individual attention was given to four slow students (reported with quantitative results, May 3) in order to determine how they solved addition problems. It was found that these pupils counted in nearly every case. The counting was done ordinarily on the fingers. This process was very tedious and, therefore, was quite inaccurate. These slow pupils appeared to lose out gradually as far as arithmetical thinking was concerned. Under classroom conditions, they did not take time to obtain an overview of the problem on which they worked; estimation and comparison of quantities were seldom made. Arithmetic was reduced to a process of verification (counting). Under these conditions, the goal of accuracy became meaningless.

The experimenter made a special study of S25. (This investigation was conducted October 18, 1935, in connection with a retest of the experimental group. S25 failed to complete the test. When he was offered aid, however, he was quite willing to make another attempt to complete the work.) The subject's motor co-ordination was poor and it was not unusual for him to lose track of the problem on which he was working. All writing was difficult. The paper was moved accidentally several times and this upset the orientation so that the problem being attempted had to be re-discovered. Re-orientation had to occur, also, each time he looked away from the problem to count on his fingers or to use some other method of calculating. His methods were as follows: (a') When a number fact was not known, he attempted first of all to find what the answer should be. For example, 13-5 was unknown. He glanced over the page and found that he had written the answer to 14-5. He then went back to the first problem, 13-5, and wrote 10. The experimenter had him correct this answer. Later, he wrote the wrong answer to 12-5. The experimenter interrupted him and pointed to 14-5=9 and 13-5=8; he then pointed to 12-5 and asked S25 to estimate the answer. The subject wrote 6. It is important to note that the subject, under these conditions which gave stability to the performance, compared different problems of his own accord in order to find what the answers should be and that he estimated, within a reasonable degree of error. (b') S25 explained how he solved 13-6. "Seven", he said, looking up to the experimenter. "Why?" he was asked. "Because 7 and 6 are 13." This solution is significant since he had never been taught specifically to use the 'additive' procedure in solving subtraction problems (c') S25 many times had to resort to counting on his fingers. On 13-9, the larger number, 13, was counted first using the fingers of both hands. The smaller number in the combination was then counted and the fingers 'left over' gave the answer. The subject ordinarily became lost on a problem of this difficulty. After several trials, the answer to 14-5 was calculated correctly. (d') The experimenter, during the 1934-1935 school year, had taught the slower pupils to write marks on their papers which would serve as counters. On the problem 7-5, for example, seven marks were to be made; five then would be crossed out (~~/////~~//) and the marks left over gave the answer. The subject employed this process to advantage when asked to verify a problem. Due to poor motor co-ordination, however, errors often were made even with this method.

(9) Methods of attacking problems: individual differences. It is clear that in many instances difficulties in arithmetic may be reduced to difficulties of writing or calculating rather than to difficulties of 'reasoning'. It is also true that no two pupils will attack problems exactly alike. Four general types of students found in the study are as follows: (a') Logical type -- verified calculations of own accord. S14, S26, and S29 were methodical workers and profited by the procedure which minimized the factor of speed. These pupils were satisfied to work accurately on a few problems or to work over time in order to complete a given unit. (b') Memory type -- succeeded on oral work; proof of number facts taken largely for granted; no verification. S13 worked rapidly on the written material without applying himself fully and never mastered the harder combinations. But he did well on the oral work for which no scores were given. S16 also did well on oral work. She proved most efficient on the flash

card test (May 29) which called for quick calculations. It may be mentioned again that this test situation was most unfavorable for S29, a slow, accurate worker. (c') Visual type -- number range remained relatively undifferentiated. S6, S20, and S23 worked best with concrete materials and appeared to have difficulty with abstract concepts. These pupils accomplished a great deal when the experimenter made a slow transition from the concrete to the abstract fields. After a careful demonstration on multiplication, for example, S7 still was confused. "What does times mean?" he asked. The experimenter again went through the demonstration making certain that he followed each step. S7 mastered the idea of 'times' temporarily. S23 had particular difficulty equating number pictures. She was shown (with small numbers) that similarity of pattern was different in some cases from equality of number. She was a persistent worker and in a period of 15 minutes learned to base comparisons on number. (d') Reading type-- worked rapidly. S21, an excellent reader, enjoyed working on story problems. She willingly spent considerable time untangling a story problem, but disliked prolonged calculations. Subtraction was her particular 'bugbear'. It is likely that arithmetic and reading can be worked in together very profitably for a pupil who has mastered the latter process. Simple problems regarding the seasons of the year, the days of the week, and telling time can be made into interesting reading-arithmetic studies. (36) (37) (38)

This brief account of the types of reasoning will show to some extent how many different kinds of procedure the teacher must employ if she is to meet adequately all individual differences.

(b) Understanding of Instructions.

(1) The mathematical vocabulary of the second grade pupil. At the beginning of the school year, the pupils already understood the terms number, count, bigger, and larger. In the preliminary study (30) it was found that understanding of the superlative degree preceded the comparative degree. The pupils used same to describe similarity of perceptual patterns and also understood add in the sense of 'placing some objects with a group of objects'. They grasped phrases such as 'one more than' or 'two more than'. But the instruction, "Draw the same number of blocks as there are marbles", proved very difficult. The term 'number picture' was strange at first, although it offered no particular difficulty.

(2) Understanding of complicated oral instructions. An oral instruction which included several tasks was not comprehended by the group. The following, which was given October 10, will serve as an example: "You draw one ring around 3 marbles, 2 boxes, and 1 hat." The experimenter illustrated the procedure, but only one or two could keep straight the details of the problem. On October 12, the instructions contained four parts, namely, counting num-

ber pictures, drawing a ring around the larger in each pair, writing the number of the larger picture, and leaving equal pictures unmarked. These more or less related tasks were grasped by the majority after a detailed demonstration. As has been pointed out, the abstract meaning of arithmetical terms were not comprehended by many. Instructions to make number pictures 'the same' did not mean to several pupils that these pictures were to be equated (October 16).

(3) Understanding of exercises in the Lennes Work Pad. When the Lennes Work Pad I was first introduced (October 17), the material proved difficult even when the experimenter read the instructions aloud. Matching exercises (November 19 and 23), calling for a knowledge of addition, were not understood, although the children later solved this type of problem when symbols were employed (January 24). It may be mentioned again that reading problems were never mastered. The simplest directions in the work pad had to be read to the class and often supplementary explanations of the assignment were necessary. The following rather simple problem given late in the school year (May 9) was one of twelve which proved troublesome: "One hat and 7 hats are ___ hats."

(4) The method of class demonstrations. The experimenter relied a great deal on class demonstrations to explain mathematical processes and concepts. It was out of the question to work individually with each pupil during

every class period and the demonstration was chosen as the best, alternative procedure. The experimenter employed materials such as books, blocks, pencils, or erasers. Every problem (e.g., comparison of groups) was worked out with the pupils. Then, the experimenter placed a problem on the board making drawings similar to those used later on the pupils' papers. These papers given after the demonstration provided a check on the pupils' comprehension of the work. This test proved most important. It was found that the slower pupils invariably became confused when they attempted to apply the principles employed during the demonstration. Additional explanation to these individuals quite often enabled them to grasp the problem. S7, as was mentioned above, failed to master the multiplication process until given individual attention. There were occasions when a majority of the class became confused during a demonstration. Miss Carter's assistance with these pupils was invaluable; the experimenter and the teacher together could give some time to practically all of those experiencing difficulty.

The demonstration of April 10 undoubtedly was the most successful type used in the experiment. Since pupil-participation proved essential for obtaining the best results from a demonstration, the situation was arranged for the pupils to give both oral (group) and written (individual) responses. The written responses enabled the instructor to check how every step of the procedure influenced the individual. Multiplication and division were successfully explained by an adaptation of this method (April 17, 23, 26).

(c) The Re-Presentation of Problems.
(1) The effect of repetition on the better student.
In the class situation, the slow pupils tended to lag behind and failed to give accurate responses on the small amount of material they covered. During the year, therefore, certain procedures were repeated a number of times with the intention of clarifying the instructions. Outstanding reactions were given by the faster pupils, however, as well as by the slower ones. When there was nothing at all new or different to be accomplished, the best pupils lost interest. Accuracy did not diminish appreciably, but the quantity of work completed diminished sharply. S14 and S26, for example, quickly mastered the problem of comparing number pictures and on October 23 it was quite evident that they had lost interest in this type of exercise. The situation no longer was demanding that they apply themselves fully.

(2) The effect of repetition on the mentally immature student. On February 27, the experimenter gave a flash card exercise to S27 and S30. Three different addition problems were considered. These boys then were asked to write the answers to nine addition problems which contained the three drilled upon. Both students failed to recall the answers to the critical combinations. This result,

which by itself is perhaps not significant, was typical of several experiences during the school year. Repetition here served only to complicate the situation when the task had little or no 'mathematical meaning'.

(3) A case of irradiation. A striking case of irradiation was found March 15-20. The whole class encountered difficulty with the harder subtraction facts and the patterns worked out by the experimenter were not discovered by any of the pupils. Under these conditions, the pupils simply counted the answer to each problem, without regard to the other problems in the series. Forcing students to work when the goal of accuracy was meaningless to them resulted clearly in irradiation. S31 replied at random, and refused to 'get down to work'; his dislike of the whole situation became very noticeable. He realized that he was behind the group and resented being pushed too rapidly. With encouragement and aid on difficult problems, his error score improved. S21, although an outstanding pupil, lost interest entirely in this work on subtraction. She failed to accomplish even one unit (49 problems) during a 20 minute period. On March 21, a change was made from difficult subtraction to easy addition and subtraction problems. The whole group realized quickly that the work could be accomplished accurately and a surplus of 'pupil-energy' suddenly became available; several (S9, S13, S16, and S17), who had been lagging behind, eagerly completed two units within a fifteen minute period. During

previous periods, the class protested when they were given extra work; under the changed circumstances, many members of the class requested more problems.

(4) The influence of re-presentation of problems in varied situations. One-fourth of the pupils failed at the first of the year to count accurately ten or more objects and also failed to write accurately the number symbols to 10. In spite of repetitions and in spite of individual aid to these slow pupils, accurate responses could not be gained. The experimenter finally left these pupils behind, figuratively speaking, and gave the class tasks with number pictures and later tasks in addition and subtraction. According to the scores, these slower pupils were not left hopelessly behind, but continued to give about as accurate responses as on the earlier work. This can be explained, perhaps, by the fact that the new problems called chiefly for counting and for writing of number symbols in various situations not just one, specific situation. At the end of the first semester, it may be noted, counting and the writing of symbols had improved considerably. At the end of the school year, very few errors were made on these tasks.

One-fourth of the class also failed to master the easier addition and subtraction facts when these problems were first presented. Nevertheless, the slow pupils were required to work on the harder facts when these were given the group. At the end of the year, the whole class was found to have mastered the easier facts. Both speed and accuracy had been attained on the easy addition problems.

The conclusion can be drawn, we believe, that even though a goal of 100 per cent accuracy is meaningless for some individuals, it is still possible to stimulate growth in arithmetical thinking. When various types of stimulus situations are presented, these pupils will extend their knowledge about addition and subtraction. The outstanding case of S29 shows, furthermore, that if a pupil is mature enough to comprehend a goal of 100 per cent accuracy, the learning of addition and subtraction will occur rapidly.

(d) The Problem of Pacing.

(1) Use of daily scores as a criterion for determining adequacy of pacing. A pupil has been paced, generally speaking, if mental growth has proceeded at an optimum rate. In the present study, the difficulty of the work was increased gradually and the daily scores, therefore, were taken as an indication whether or not pacing had been adequate. Table LV, which gives a summary of these accuracy scores, shows that the upper one-fourth of the group had an average score of 90 or above; the lower one-fourth had an average score below 70; only two pupils had scores below 50. If one should expect a pupil to work absolutely accurately on whatever he attempts, it is evident at once that only a few in the experimental situation were adequately paced. Even though a less strict criterion be adopted, the fact remains that some pupils were not adjusted

to the class situation. S12 and S24 were cases who fell down especially toward the close of the school year. S20, S28, and S31 gave consistently low scores throughout the experiment. These five pupils, it should be noted, received a low rank in every school activity. S28 presented the most baffling case with which the experimenter had to deal. This subject made an increasingly large number of errors on the three addition-subtraction tests given in April, May, and October. Instead of holding his own over a period of six months, he consistently lost ground. Needless to say, he still represents a problem in the classroom.

(2) The goals of the poor students: devices for encouraging accurate work. The goals adopted by the poor students after a short period of time were not the goals of learning which the experimenter desired the pupils to establish. S17, a typical case, failed on December 19 to attack his problems as something he could solve. It was evident that he was simply 'going through the motions' in order to obtain a written response acceptable to the teacher. He had not achieved any appreciable success for a long period of time and, finally, the desire to succeed vanished altogether. Some measures to stimulate learning proved adequate to meet circumstances such as these, but no cure-all was discovered by means of which one could handle every difficult case. (a') It was possible, occasionally, to aid a pupil in solving an easy type of problem and then follow up immediately with similar, but more

difficult problems. The case of S15 illustrates this point. He fell far behind the class soon after addition and subtraction were presented and he then preferred to play some game of his own during the arithmetic period; several times he brought marbles and tin soldiers to school and played by himself, working intermittently at his problems. An effort was made to show him that he could do arithmetic and on February 22 these efforts were partially rewarded. S15 completed one entire unit and ran to the teacher exclaiming, "See, I've done a whole page." The best students also reacted enthusiastically to success. S21 quickly caught the idea of the 'difference' problem (December 18) and of her own accord came to the experimenter for more work. S26, who failed to succeed on this material, disliked the task intensely. On a later occasion (May 2), however, S26 grasped the notion of multiplication and after class made up several problems of her own which were solved correctly. (b') A change of material seldom failed to arouse pupil interest. In many instances, it was possible to gain concentrated effort even though the pupils had not succeeded on the previous work. The new material gave everybody a new chance. Sometimes the change was merely the substitution of easy work for hard work (March 21). The experimenter introduced colored papers during the second semester and each change of color aroused a great deal of interest. It was rather easy, on the other hand, to change material or to change the general procedure so radically that confusion resulted. During several periods (e.g., February 26) the experimenter failed to demonstrate the task in a clear enough fashion and the class became confused on a relatively simple task. (c') With Miss Carter's aid, all the completed papers could be graded and corrected during the class period and this procedure, more than any other, stimulated accurate work. An attempt was made (February 5) to have the pupils correct errors made on the previous day, but no interest in the earlier performance could be aroused. On the other hand, when papers were checked immediately, the pupils were anxious to correct their mistakes. (d') Some papers were returned to the pupils and it was intended that these papers were to be taken home. The poor students, however, failed to take their papers home. The perfect papers were given back to the members of the class occasionally; this returning of papers was done very few times since it discriminated openly between the best and the poorest students.

(3) Learning and the emotional attitudes of the pupils. The problem of pacing must take into consideration the attitudes of the pupils. These traits vary greatly from day to day and methods of instruction must be altered to meet the specific occasion. Loud, unusual noises outside the room may disturb the class and make it impossible to carry out a demonstration. S13 scattered some paper over the floor (November 23) and the teacher informed him that he would have to stay after class in order to help the janitor clean

the room. It was impossible for this pupil to concentrate during the arithmetic period and, as a result, he disturbed the whole class. As a disciplinary measure on several occasions, the pupils were required to remain after class, to complete their work. A dislike for the task invariably developed. The only positive gain from such a procedure was made by the poor student who discovered that <u>the task could be done</u>.

(4) <u>Learning and the length of the work period</u>. The problem of pacing also must take into consideration the question of length of the work period and length of a unit of work. In the normal play environment, the second grade pupil can concentrate on one task for only a short period of time. The best pupils, under classroom conditions, could not work steadily on one type of task for an extended period. The optimum time for difficult work on addition and subtraction did not exceed fifteen minutes. An average time for the class probably would not exceed seven to ten minutes; however great variations did occur with changes in the stimulus situation. During the tests over addition and subtraction (April 12 and May 23), a great deal of pressure was brought to bear on the slow students to complete at least two units (45 to 49 problems per unit) within a single work period. Some of these pupils were forced to spend twenty-five to thirty minutes on this material. This period obviously was too long. The conclusion is drawn here that if we are to stimulate a maximum of learning without pro-

ducing harmful emotional strain, the average work period of thirty minutes must be shortened. When pupil interest is once thoroughly aroused, it may be added, learning will take place rapidly if it takes place at all.

(e) The Social Group and the Individual's Goal.

(1) Learning stimulated by the social group: importance of the child's contact with the adult world. Many educators have emphasized that subject matter ought to be adapted to the individual, but only recently have they emphasized at all that the material should be considered for 'the individual as a member of a group'. Two questions immediately arise: How important for learning is the teacher? How important is the social group of which the student is a member? Conant, who studied the origin of number concepts, has some evidence which bears indirectly on these problems. He states that African savages have only a few number names and that counting to ten is impossible for most of them. Nevertheless, these primitive people usually develop number systems to a remarkable degree provided that their life is one which favors trade and barter with their neighbors (10, pp. 31-33). The contact made with the number systems of the outside world undoubtedly results also in a rapid acquisition of the higher number names. (10, pp. 23, 27, 58) So-called intelligent behavior, in other words, is acquired quickly once the social environment becomes favorable to this development. In a very similar fashion, children quickly master the number names from adults, either in school or outside of school. Without this stimu-

lation, children's growth of number ideas would be slow indeed and it is very likely that <u>many</u> systems of number not just <u>one</u> system would be developed. The child's contact with the adult world, therefore, is an important one.

<u>(2) Adult leadership and the child's goal.</u> Real leadership, as Shaw (31) has pointed out, makes room for child discovery. In her work with finger painting, the children mastered their techniques and skills independently. A child's own efforts, furthermore, carried him farther and farther afield. This goal of learning has been realized to some extent in reading. Most children soon learn to like reading and make efforts to get new materials without specific stimulation in this direction on the part of the teacher. This accomplishment hardly has been realized as yet in the teaching of arithmetic. We have not been able to give the pupils a start on something which they can develop and carry through to completion of their own accord.

There were several occasions in the present experiment, however, when the originality of a student did express itself. S26, who was shown how to employ the multiplication process, found several problems of her own and proceeded to solve them. The interest in this original work, of course, was only temporary. The whole class became interested in the use of the concept 'pair' and several members discovered problems of their own in school and even outside of school. S13, it was reported, used his knowledge of multiplication at home and learned to find products as high as 15 or 20.

(3) <u>The function of the teacher in the present-day classroom.</u> The elementary school teacher always dominates the class situation. Even when she relaxes vigilance for a moment, the social group begins at once to disintegrate. It is hardly an exaggeration to say that a second grade teacher spends 50 per cent of her efforts securing and maintaining discipline. Some definite contributions by the teacher, on the other hand, are not to be disparaged. The pupils learn to depend upon the security and upon the stimulation which an adult can give them. In the present experiment, for example, it was noted that, if the teacher followed the work of a pupil and simply verified the correctness or incorrectness of his processing, the child's performance improved considerably. The poor students especially made a great deal of progress with this aid; often they gained enough confidence in themselves to work accurately for several succeeding periods.

It is evident from a study of the present classroom procedures, however, that the teacher cannot give the same degree of motivation to each of 20 or 25 pupils. The teacher's attitude, determining the kind of work expected of a pupil, is certain to be reflected in the child's attitude toward his assignment. After a few weeks of the experiment had elapsed, for example, the instructor did not <u>expect the best work to be done by S6, S15, S16, S17, S28, and S31.</u>

But on different occasions all of these pupils (the cases of S15 and S16 were outstanding) showed spurts during which some excellent scores were made. There is no doubt but that is was possible for these pupils to do much better work on the average.

(4) Influence of the group on the capable students. Investigators have hardly begun to discover what influences one child exerts upon another or what influences a group of children exert upon the individual. Observations made in the present study show that children are keenly aware of the opinions and of the accomplishments of each other. Although outward appearance may not show it, children are quick to perceive and then to copy a teacher's evaluation of class work. Teachers, especially more recently, are very careful about forming sub-groups in the class and attempt to change the pupils from one group to another. Every child, nevertheless, knows who is doing poorly and who is doing well and this knowledge influences his behavior markedly. Those pupils in the present study who ranked highest made the greatest effort to maintain their class position. S21 (February 4) insisted that she be given an opportunity to correct her errors on the day's work. She ordinarily ranked highest in the class and made every effort to keep up with S14 who also worked accurately. S26 on this same day failed to complete her paper on time. The experimenter gave the instruction to turn over the papers at the end of the work period; S26 turned her's over, and

then after a moment started again to work on it. This pupil worked slowly, but insisted upon finishing a given unit of work which enabled her to keep up with the class. Similar incidents occurred frequently throughout the experiment. It was usually true that the average members of the class hurried through their work after the experimenter gave a second sheet of problems to the fast pupils. On many occasions, pupils came to the experimenter and asked for the blue papers or the red papers which had been given earlier to the faster workers.

(5) Influence of the group on the poor student. Those pupils having the lowest rank were influenced 'negatively' by their class position. In general, they grew careless and simply made an appearance of working. The experimenter's instructions were difficult for them, but since the class situation called for a response of some kind, they either established goals of their own or, which was easier, copied the answers from a neighbor. S17 and S31 were typical students in this group. The class was instructed (December 10) to find a difference between two groups of printed objects; these two subjects, however, added the numbers of each pair instead of finding a difference. During an earlier period, it was found that these pupils, together with S13 and S7, merely wrote the numbers of the group of objects instead of comparing the groups. But this task enabled the pupils to make the appearance of working with the class.

The effort on the part of slow pupils to keep pace with the class produced a great deal of emotional strain and always resulted in a distorted conception of arithmetic. Individual work with S30 (October 24) revealed that his hurried efforts to keep up with the group were the cause of his many errors in counting number pictures. Although the circles appeared in rows on the papers, S30 tallied them in random fashion. He therefore lost track of the circles he had enumerated. He was encouraged to work more carefully, counting each row in a systematic manner. This suggestion aided him temporarily and the experimenter found that the 'difference' problem offered no difficulty when the pupil once learned the numbers of the combinations. But his slowness in counting proved too big a handicap and he never succeeded in keeping up with the class. This failure, which is a typical case, presents a serious problem to the public school teacher. The conclusion is made here that as long as a class of 25 to 35 pupils is maintained for a period of a whole year, there is little possibility that this type of pupil can ever be stimulated to do his best work.

TABLE LV

Attendance and Scores by Six Weeks Periods

Number of Days Present							
Pupil	I	II	III	IV	V	VI	Total
S1	—	—	—	7	23	10	40
S2	20	7	—	—	—	—	27
S3	21	15	10	27	20	9	102
S4	19	6	—	—	—	—	25
S5	20	15	10	26	16	8	95
S6	20	17	15	28	20	10	110
S7	20	14	17	30	25	10	116
S8	—	—	—	—	14	7	21
S9	20	14	13	20	23	6	96
S10	—	—	—	15	15	—	30
S11	4	—	—	—	—	—	4
S12	21	10	5	30	19	10	95
S13	18	15	19	24	20	10	106
S14	21	10	16	30	24	10	111
S15	20	13	14	28	23	10	108
S16	18	15	10	29	23	10	105
S17	16	15	12	27	22	8	100
S18	19	10	—	—	—	—	29
S19	17	18	18	22	18	5	98
S20	19	17	15	23	12	5	91
S21	19	15	20	24	24	10	112
S22	—	—	—	14	21	10	45
S23	20	14	10	4	—	—	48
S24	19	17	19	24	25	6	110
S25	16	13	15	25	23	6	98
S26	20	15	15	25	25	5	105
S27	20	19	13	17	16	10	95
S28	21	17	12	28	24	10	112
S29	—	—	—	8	23	9	40
S30	21	14	13	24	13	9	94
S31	19	12	16	30	23	10	110
S32	17	12	17	28	16	9	99

Average per cent Correct Score							
Pupil	I	II	III	IV	V	VI	Av. for year
S1	——	——	——	74.71	69.87	88.50	77.69
S2	69.15	59.71	——	——	——	——	64.43
S3	85.38	91.20	93.70	91.37	88.15	94.11	90.65
S4	92.21	89.00	——	——	——	——	90.61
S5	73.75	87.53	82.40	89.88	91.94	80.75	84.38
S6	86.50	84.41	93.40	90.32	92.45	86.80	88.98
S7	77.85	79.21	71.53	90.07	86.20	80.30	80.86

Table LV (Cont'd)

Attendance and Scores by Six Weeks Periods

	Average per cent Correct Score						
Pupil	I	II	III	IV	V	VI	Av. for year
S8	-----	-----	-----	-----	66.36	83.86	75.11
S9	85.18	72.21	60.62	85.35	73.22	73.50	75.00
S10	-----	-----	-----	88.67	91.67	-----	90.17
S11	84.50	-----	-----	-----	-----	-----	84.50
S12	64.81	73.00	66.40	69.63	64.74	44.30	63.81
S13	89.00	90.27	58.37	88.38	74.35	87.10	81.25
S14	91.24	89.60	92.13	95.17	97.79	86.50	92.07
S15	70.65	74.54	50.43	82.86	67.48	64.90	68.48
S16	87.83	83.47	76.00	82.38	81.43	57.10	78.04
S17	74.06	66.67	80.83	88.00	80.68	70.63	76.81
S18	77.74	55.30	-----	-----	-----	-----	66.52
S19	92.12	91.78	83.22	86.95	78.00	85.60	86.28
S20	60.37	40.71	39.40	73.43	29.67	46.40	48.33
S21	96.53	98.80	94.45	95.92	93.67	99.90	96.55
S22	-----	-----	-----	91.29	94.38	95.50	93.72
S23	80.25	84.00	85.60	97.00	-----	-----	86.71
S24	70.58	82.06	51.21	83.58	75.72	46.17	68.22
S25	83.13	88.54	91.73	87.24	86.52	77.33	85.75
S26	87.25	97.80	86.47	95.36	94.64	98.60	93.35
S27	70.70	77.64	76.54	93.41	84.50	78.50	80.21
S28	53.52	56.47	48.92	85.07	66.58	55.50	61.01
S29	-----	-----	-----	98.38	95.87	96.00	96.75
S30	66.38	63.50	57.15	87.04	45.77	49.67	61.59
S31	43.63	35.67	50.56	56.40	45.00	41.60	45.48
S32	82.65	97.42	71.12	91.58	86.06	71.78	83.45
Av. no. pupils in class	24.09	19.42	15.43	20.57	22.00	22.20	21
Av. score (class)	77.25	77.86	73.33	85.78	78.99	75.05	78.04

Control Studies and Retests. (a) Results on the objective addition-subtraction tests. Through the co-operation of the Lawrence Public School administrators, the writer was able to test two control groups at the second grade level and thereby obtain a check on the performance of the experimental class. These groups will be referred to as Control I and Control II. The class taught by the writer will be referred to as the Experimental Group. The teaching procedure employed for this last group already has been discussed. The method adopted for Control I at the beginning of the school year was that of Studebaker, Knight, and Findley (35). The teacher, however, did not use the drill techniques as outlined in this course of study; some flash card drills were used, but little attempt was made to force the pupils to memorize the addition and subtraction facts. Considerable emphasis was placed upon speed. The material in the Lennes Work Pad I (22) was utilized to a great extent for Control II. The teacher supplemented this work with written, speed drills.

It should be mentioned that the Experimental Group contained seven very slow pupils including two first grade repeaters. It also contained three very capable students. The experimenter, during the second semester, added two first grade pupils with outstanding achievement records. Control I was a selected group with respect to reading. These pupils as a class also ranked high in the other school subjects. Control II was an unselected group; the pupils' scores spread widely on the Stanford Revision of the Binet-Simon Intelligence Test. The number of pupils having relatively low I.Q's outnumber by far those with high I.Q's. The intelligence test scores are given for all the groups in Table LVI.

The results on the first addition and subtraction test are shown for all groups in Table LVII. The test contained 228 problems and has been described in the "Daily procedure and quantitative results" for April 1.

The superiority of Control I is very evident both on error and time scores. The difference between the scores of this group and the Experimental Group are quite reliable. (The chances are 99 plus in 100 that a difference in error in the same direction would occur in future samples.) The difference in average time between Control I and Control II is fairly reliable. (The chances are 94 out of 100 that the true difference is greater than zero.)

The Experimental Group showed a superiority to Control II on errors; the opposite was true with respect to the average time scores. The teaching procedure in the first instance stressed accuracy of response and the procedure in the second case stressed speed. The results indicated that the nature of instructions influences markedly the pupils' approach to arithmetic problems.

The addition and subtraction test was repeated for the Experimental Group and Control I on October 16, 1935. The pupils had had approximately a month of school after the summer vacation. No systematic drill procedures in arithmetic were employed during this period. The results on the test are shown in Table LVIII.

A superiority of Control I was found again on both time and error scores. The teachers' estimates which placed this group above the Experimental Group in achievement were completely in accord with the results on this objective test. A comparison of the records on October 16 of the two respective groups with their previous records, however, reveals some facts regarding their performances that should be mentioned. Neither the Control I nor the Experimental Group made any improvement in the error scores. The averages in the fall test, instead, are slightly higher for both groups. The increase in errors for Control I is negligible and the increase for the Experimental Group is not greatly significant. (The chances are 58 out of 100 in the one case and 64 out of 100 in the other that the differences are greater than zero.) On the time scores, on the other hand, the Experimental Group showed a definite improvement. The average time for completing the test dropped sixteen minutes. The Control I failed to show a similar improvement. It may be mentioned again that three subjects in the Experimental Group, S15, S17, and S25, failed to complete the October test. When these pupils' scores are eliminated from the results of both tests, the drop in time for the Experimental Group becomes fairly reliable. The difference in average scores divided by the standard error of the difference gives a ratio of 1.761 which means that

TABLE LVI

INTELLIGENCE TEST SCORES*

Experimental Group Pupil	I.Q.	Control I Pupil	I.Q.	Control II Pupil	I.Q.
S1	109	1.	103	1.	109
S2	97**	2.	110**	2.	80
S3	108	3.	113	3.	86
S4	----	4.	104	4.	75
S5	100	5.	110	5.	92
S6	146	6.	102	6.	93
S7	114	7.	122	7.	88
S8	136	8.	113	8.	129
S9	101	9.	121	9.	102
S10	113	10.	105	10.	98
S11	125	11.	116	11.	98
S12	----	12.	114	12.	107
S13	96	13.	109	13.	77
S14	101**	14.	114**	14.	118
S15	106**	15.	117**	15.	128
S16	96**	16.	103	16.	117
S17	107	17.	124**	17.	79
S18	----	18.	100**	18.	102
S19	132***	19.	105**		
S20	----	20.	119		
S21	138**	21.	109		
S22	122***	22.	107		
S23	----	23.	123		
S24	103**	24.	128		
S25	102**				
S26	118**				
S27	102				
S28	94				
S29	148***				
S30	105**				
S31	85				
S32	----				
Av.	111.69	Av.	112.12	Av.	98.77

* The I.Q. scores were obtained according to the Stanford Revision of the Binet-Simon Intelligence Test. Unless otherwise indicated, the tests were given during the 1935-1936 school year.

** These scores were obtained from tests given during the 1933-1934 school year.

*** These scores were obtained during the 1934-1935 school year.

TABLE LVII

FIRST ADDITION-SUBTRACTION TEST *

	Exper. Group (Completed Apr. 12)	Control I (Completed Apr. 12)	Control II (Completed May 9)
		ERRORS	
No. of pupils	23	19	26
Av. score	32.30	13.73	87.96
S.D.	31.42	9.12	56.44
S.E.	6.551	2.090	11.069
		TIME (Min.)	
Av. Score	99.96	47.61	56.87
S.D.	31.27	21.39	15.57
S.E.	6.520	4.907	3.054

COMPARISON OF GROUPS

S.E. Diff. (Exper.--CI) --- 6.87 (Errors) Diff./S.E. Diff.--2.718

S.E. Diff. (Exper.--CII) --12.86 (Errors) Diff./S.E. Diff.--4.329

S.E. Diff. (CI -- CII) ---- 5.77 (Time) Diff./S.E. Diff.--1.6

S.E. Diff. (Exper.--CII) -- 7.2 (Time) Diff./S.E. Diff.--5.9

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* It should be pointed out that the distribution curves founded on the error and time scores are skewed. Because of this fact a statistical treatment of the data can not be expected to give reliable results. Interpretations based on the quantitative data, therefore, are given in the text only as supplementary information to the qualitative results. The material in Tables LVIII and LXII are interpreted in the text in a similar fashion.

TABLE LVIII

SECOND ADDITION-SUBTRACTION TEST

	Exper. Group (Completed Oct. 16,'35)	Control I (Completed Oct. 16,'35)
	ERRORS	
No. of pupils	23.00	20.00
Av. score (errors)	36.95	14.85
S.D.	36.62	23.97
S.E.	7.636	5.360
	TIME (Min.)	
Av. Score (time)	83.66	43.58
S.D.	40.00	14.54
S.E.	8.34	3.251

COMPARISON OF GROUPS

S.E. Diff. (Exper.--CI) (Errors) ------ 9.33 Diff./S.E. Diff.-- 2.277

S.E. Diff. (Exper. Apr.12- Exper. Oct.16) (Time) -- 10.580 Diff./S.E. Diff.-- 1.561

S.E. Diff. (CI--Apr. 12- CI--Apr. 16) (Time) -- 5.87 Diff./S.E. Diff.-- 0.704

r* (Exper. Apr.12- Exper. Oct.16) (Errors) -------- + 0.846 S.E. (r) --- 0.064

r* (Exper. Apr.12- Exper. Oct.16) (Time) -------- + 0.698 S.E. (r) --- 0.114

r* (CI - Apr. 12- CI - Oct. 16) (Errors) --------- + 0.065 S.E. (r) --- 0.270

r* (CI - Apr. 12- CI - Oct. 16) (Time) --------- + 0.565 S.E. (r) --- 0.175

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* The coefficients of correlation have been calculated by the Pearson product-moment formula from the standard scores.

96 times in 100 one would expect the difference to be greater than zero. This finding becomes more striking when the raw time scores are considered for the five highest ranking pupils of both groups. In the Control I the five highest pupils in October required an average of two minutes more than the corresponding April pupils to complete the test. In the Experimental Group the five highest ranking pupils in October required an average of 27.5 minutes less than the corresponding group in April. This evidence points clearly to the conclusion that a majority in the Experimental Group (not all the pupils) maturated with respect to the speed of processing. This conclusion, we believe, justifies a method of procedure which allows the pupil to work at his own rate of speed. One may well expect greater rapidity of calculation to develop as maturity proceeds. One may expect, futhermore, a relatively stable performance in those instances where the pupils are not pushed too rapidly. The additional evidence of the correlations of the first and second tests (Table LVIII) shows that the pupils of the Experimental Group maintained approximately the same respective class positions in October as in April. The correlations of error and time scores, respectively, for Control I indicates that these pupils, on the other hand, varied greatly on the two tests.

(b) Comparisons of groups based on a flash card test, on 'series' problems, and on reading problems. Tables LIX and LX show a comparison of results of the Experimental

TABLE LIX

FLASH CARD TEST

Experimental Group Pupil	45 add. (errors)	Pupil	45 subt. (errors)	Control I Pupil	45 add. (errors)	Pupil	45 subt. (errors)
S16	5	S16	3	1.	0	1.	0
S22	6	S6	7	2.	1	2.	4
S14	9	S13	8	3.	1	3.	4
S7	11	S22	10	4.	2	4.	5
S29	12	S14	15	5.	2	5.	6
S8	12	S21	15	6.	3	6.	7
S21	12	S15	21	7.	4	7.	7
S3	13	S5	22	8.	5	8.	8
S13	15	S29	23	9.	6	9.	8
S19	17	S7	24	10.	6	10.	9
S6	18	S8	27	11.	6	11.	9
S1	19	S27	29	12.	7	12.	11
S31	21	S3	29	13.	7	13.	11
S15	28	S1	31	14.	7	14.	11
S27	31	S31	33	15.	9	15.	12
S30	35	S19	33	16.	9	16.	12
S28	37	S28	35	17.	9	17.	13
S17	38	S30	39	18.	9	18.	15
S12	43	S12	41	19.	11	19.	17
		S17	42	20.	11	20.	18
		S20	43	21.	12	21.	19
				22.	14	22.	19
				23.	18	23.	23
				24.	19	24.	22
				25.	21	25.	25
Med.	17.00	Med.	27.00	Med.	7.0	Med.	11.0
Av.	20.11	Av.	25.24	Av.	8.0	Av.	11.8

TABLE LX

TEST ON 'SERIES' PROBLEMS
(121 problems in Table XXVIII)

Experimental Group (Feb.4)			Control I (Feb.20)		
Pupil	Prob. Attempted	Per Cent Correct	Pupil	Prob. Attempted	Per Cent Correct
S20	17	100	1.	73	100
S6	37	100	2.	88	97
S23	29	100	3.	109	96
S21	49	100	4.	119	96
S26	49	100	5.	48	95
S13	64	98	6.	90	94
S25	42	97	7.	52	94
S17	41	97	8.	58	94
S7	42	92	9.	121	93
S19	49	89	10.	93	93
S32	37	83	11.	121	93
S24	120	57	12.	38	92
S31	30	56	13.	48	91
S15	31	54	14.	77	88
S9	40	53	15.	59	84
S30	120	37	16.	57	80
			17.	73	77
			18.	65	55
			19.	121	54
			20.	70	45
Range.	17-120		Range.	38-121	
Av.	50.38	Av. 82.06	Av.	79.00	Av. 85.55

Per Cent Correct Score	Per Cent of Class	Per Cent Correct Score	Per Cent of Class
95-100	50.00	95-100	25.00
90-94	6.25	90-94	40.00
80-89	12.50	80-89	15.00
70-79	00.00	70-79	5.00
-69	31.25	-69	15.00

TABLE LXI

READING PROBLEMS: CONTROL I
(Feb. 22)

	'Mixed' Problems (Table XXVII)		Story Problems (Table XXIX)	
Pupil	Attempted	Per Cent Correct	Attempted	Per Cent Correct
1.	34	100	15	100
2.	34	100	15	100
3.	34	97	14	100
4.	34	97	4	100
5.	34	97	5	60
6.	34	94	15	100
7.	34	94	3	33
8.	34	91	15	86
9.	34	91	14	78
10.	33	93	15	73
11.	34	88	15	60
12.	33	84	15	72
13.	34	79	4	75
14.	28	71	—	—
15.	34	67	14	85
16.	34	67	14	71
17.	26	61	—	—
18.	28	60	12	16
19.	32	50	15	80
20.	34	41	15	86
21.	33	36	12	41

Max. Prob.	35
Av. Attempted	32.81
Av. Score	78.95

Per Cent Correct Score	Per Cent of Class
90-100	47.7
80-89	9.5
70-79	9.5
-69	33.3

Max. Prob. 15

(Thirteen out of nineteen pupils completed either 14 or 15 problems. Average score of these thirteen = 83.92.)

Group and Control I on a flash card test and a set of 'series' problems. Table LXI contains the scores made by Control I on two sets of reading problems. As one might predict, the selected group represented in the Control I did excellently on the flash card test in which speed was the most important factor (Table LIX). On the 'series' problems (see Table XXVIII for an example of this type of problem) the pupils in Control I were not given any special instructions to work accurately nor were they given any clue as to the pattern of answers. The results indicate that more pupils in the Experimental Group made perfect scores than did the pupils of Control I. The number of problems completed by the latter group, however, was much higher than that completed by the Experimental Group. Control I was given Table XXX on a later date as a 'follow up' to this particular 'series' test; on this later occasion the pupils worked as accurately as those of the Experimental Group. The experimenter gave a demonstration of reading problems on Wednesday, February 20, 1935 to Control I. Books were used to show that "4 is the same as 3 and 1." This statement was written on the board and was copied by the class. Several other problems likewise were demonstrated. On the following day, February 21, a demonstration of two statements "4 is the same as 3 and 1" and "2 and 2 are the same as 4" were given. Both were copied by the children. Members of the class were called upon to give answers to problems similar to the following: "3 and 3 are the same as ____"; "5 is the same as 3 and ____." On Friday, February 22, Table XXVII was given to this class. Table XXIX also was given, although no demonstration of this latter type of problem was made. The class was instructed simply to work all the problems. The results are shown in Table LXI. This class was definitely superior to the Experimental Group on these reading problems, but the average scores indicate that these problems were not completely mastered. The conclusion seems justified, however, that those pupils who read well also may be expected to solve reading problems.

(e) Individual differences recorded on the tests. The discussion of average scores and the performance of total groups leaves out of account the individual. On the second addition-subtraction test some pupils improved considerably on time and errors. Others improved on time or on errors, still others failed to equal their earlier performances. In order to give consideration to these individual differences, the standard scores have been calculated for the Experimental group and for Control I. These scores on the two addition-subtraction tests appear in Table LXII. In the Experimental Group, two of the best students S14 and S29, made slightly less satisfactory time and error scores on the second test. S21 and S10, on the other hand, who also were in the upper one-third of this class, improved slightly on the tests.

A rather unexpected result was the remarkable improve-

TABLE LXII
STANDARD SCORES: ADDITION-SUBTRACTION TESTS*

Pupil	Errors Apr.12	Errors Oct.16	Time Apr.12	Time Oct.16
		Experimental Group		
S31	+3.20	+2.13	+0.64	-0.43
S1	+1.20	-0.08	-0.63	-0.05
S28	+2.15	+2.67	+0.96	-0.25
S17	-0.51	+0.13	+[illegible].09	+0.93
S6	-0.61	+0.21	+0.73	+0.23
S32	-0.67	+0.27	-0.31	+1.88
S15	+0.75	+1.69	+2.24	+1.55
S8	+0.46	-0.43	-0.63	-0.89
S7	+0.21	-0.27	+0.70	-0.54
S9	+0.34	+0.14	-0.63	-0.60
S21	-0.71	-0.79	-0.63	-1.00
S24	-0.71	-0.39	-0.63	-0.71
S26	-0.86	-0.87	-0.63	-0.61
S13	-0.23	-0.27	-0.63	-0.91
S3	-0.36	-0.41	-0.31	+1.34
S[illegible]	-0.29	-0.11	+1.92	+2.66
S22	-0.87	-0.66	+0.03	-0.70
S5	-0.20	-0.55	0.00	+0.26
S10	-0.45	-0.85	-0.63	-1.09
S14	-0.96	-0.93	-1.28	-0.99
S29	-1.03	-0.82	-1.28	-0.43
		Control I		
1.	-0.03	-0.20	+1.28	+0.13
2.	-1.18	-0.62	-1.36	-1.14
3.	+1.34	+0.05	+1.74	+0.54
4.	-0.85	-0.58	-1.15	-1.17
5.	-0.02	+0.06	-0.26	+2.23
6.	-0.96	-0.16	-0.92	-0.45
7.	-0.03	+4.17	+1.96	-0.2[illegible]
8.	+0.58	-0.20	-0.45	-0.31
9.	-0.41	-0.49	-0.12	-0.93
10.	+0.03	-0.54	-0.82	-1.35
11.	+2.66	-0.41	-0.57	-0.35
12.	-0.85	-0.37	-0.08	+0.17
13.	-0.29	-0.54	+1.12	+1.39
14.	+1.24	+0.17	+0.84	+1.06

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* A positive score (+) indicates a large error or a large time score. A negative score (-) indicates a low error or a short time score.

ment of S31. This pupil ranked lowest on daily scores and yet made as much improvement as any pupil in the Experimental Group. Three students who gave more or less average performances throughout the year also showed improvement on both time and errors. These were S7, S8, and S13. S1, who improved considerably on errors, also may be mentioned in this group. (S13 and S8, in May, 1935, made worse error scores on 180 problems than in April, 1935. The October score, on the other hand, shows improvement on both time and errors. It is quite difficult to ascertain what factors might have influenced the lack of achievement recorded on the May test. It seems probable that both students became upset to a considerable degree by the difficult work introduced between the April and May tests. The lowest semester scores recorded for these pupils, at any rate, are found during this period.)

There were three pupils, S25, S32, and S6, who failed to equal their record on the first test in either time or errors. From the daily scores, one would suppose that these pupils were average or above average and that they should improve rather than fall behind in a retest situation. S25 along with S17 and S15, however, failed to complete the retest, although with encouragement these pupils had completed the test on the earlier occasion.

The results of individual students in Control I give a similar picture of the learning situation. The pupil in this class who ranked highest required more time for the second test. Another student who was about average improved considerably on time. On the basis of the standard scores for both time and errors, the first pupil showed no improvement, the second improved considerably.

Several students who were below average showed remarkable improvement on error scores while another student who was below average made a much higher error score. This last case worked rapidly on the last test, but obviously quite superficially.

How are these individual differences to be explained? It is evident that pupil variation is great and that, on the surface, it is almost impossible to predict what a particular student will do after the summer vacation. Certain qualitative data, however, may serve to make these scores meaningful. In the Experimental Group, the two best students were stimulated to work at approximately their greatest speed. Unknowingly, these pupils were being pushed to the limit and irradiation resulted. On the basis of the maturation hypothesis, it would be expected that children who are developing physically at a rapid rate ought to improve on school work after a vacation period. (It may be recalled that some time elapsed after school began in the fall before the second test was administered. This permitted a re-adjustment to the school environment.) It is to be inferred from this same hypothesis that failure to improve indicates over-stimulation with the result of irradiation.

The factor of over-stimulation may have entered into the picture for a number of poor students as well as good students. S6, S25, and S32 made worse scores on both time and errors on the second test. The fact that S6 has a high I.Q. only serves to complicate the situation. The qualitative data show that these pupils were not well adjusted to the class situation. S6 proved to be a discipline problem at the first of the school year (1934-1935). S25 was a repeater in first grade and for the year 1935-1936 was placed with the slow group of the second grade. This subject has been reported in the section, "Reasoning of the slow pupil." S32 started out well in the experimental class, but failed to keep up with the group.

S7 and S8, who improved on both time and errors, were quite different in disposition and attitude from S6 and S25. Although they were slow pupils in class, S7 and S8 followed instructions carefully and made an effort to master their assignments. S7 was an adopted child whose home environment was very satisfactory. He was encouraged to do his best, but never was pushed to reach impossible goals. This case demonstrated, without any question, that a slow pupil can make an adjustment to a stimulating school environment provided adequate co-operation between the home and the school is maintained.

In summarizing the data on individual scores, the following may be pointed out: (a) The above-average students, if overstimulated, fell behind on retests. This result indicates a lack of pupil maturation with respect to the specific learning situation and presents a serious problem to the teacher. (b) The scores of those students who ranked above average and who had not been subjected to overstimulation, did not vary a great deal. (c) The average or slightly below average students made advances on the retest when conditions permitted a satisfactory adjustment to the school environment. The very slow pupils, under these favorable circumstances, made remarkable progress on the retest. (d) The poor student, whose maladjustment was marked, failed to co-operate on the second test and, consequently, the scores indicated considerable loss of learning.

IV. DISCUSSION: THE PSYCHOLOGY OF NUMBER

Fundamental Assumptions: A theory dealing with the mastery of the first number concepts already has been considered (30). This theory was based upon the laws of Configuration, Field Genesis, and Individuation (40). It was demonstrated that the individual's response to a situation calling for a judgment of more or less was configurational in character. The child's insight into the problem occurred suddenly; the relations of wholes and their respective parts offered no inherent difficulty to him unless structural analysis of groups, through the counting process, complicated the situation. The serial (ordinal) and the quantity (cardinal) concepts of number were found to evolve simultaneously as members of a single mathematical concept. Additional evidence will be considered under a later topic which further substantiates this interpretation. It was found that, with the maturation of the individual, concepts of parts within a group or whole underwent a change described as structurization or differentiation. Sub-groups or members of a group at first were not equal in perceptual form to the child; the spatial relations between the members of a group assumed at first more significance than the size and form of the members themselves. At a later stage the parts attained a homogeneity of form, i.e., a homogeneity in the sense that every part was given a value equal to every other part. This stage may be said to represent a primitive conception of the universality of number.

On the basis of the principle of Individuation, we will consider how still more abstract mathematical concepts, which are concerned in addition and subtraction, are developed. It will be assumed at the beginning that this learning follows the laws of growth, which growth is continuous in character. The consequences of this assumption are far reaching. The learning, for example, cannot be saltatory in nature, but must follow a sequence, every part of which bears a definite relation to every other part. Until the present time, no theorist has taken a clear stand on this question and it has been generally assumed that when the child proceeded from concrete number concepts to notions of abstract addition and subtraction, he necessarily had to bridge a distinct gap between these two rather unrelated ideas. The evidence from other fields, however, would indicate that the hypothesis making growth a continuously expanding process is more fruitful than this older notion. There is no longer any doubt in the physical realm, for example, that the growth hypothesis has been substantiated. The experiments of Arnold Gesell (13) and Mary Shirley (32) have demonstrated conclusively that behavior follows a definite sequence which can be accounted for only in terms of maturation and differentiation of the individual.

In the field of perception, likewise, important evidence has been discovered which points toward a growth theory.

This is of great importance since perception plays so large a role in the development of number ideas. Purdy's recent articles on "The Structure of the Visual World" (27) (28) represent a significant advance in harmonizing the perceptual processes of the child with those of the adult. Purdy implies quite definitely that the differences between the two are differences of degree rather than differences in kind. This view has been made possible by the assumption that perception follows a principle of differentiation of wholes in contrast to the older assumption which held that wholes were to be synthesized from discrete parts. The inference may be made that exactly the same advantage which Purdy found would be gained by the application of this same principle to the development of arithmetical ideas. A quotation will illustrate the significance of Purdy's assumption:

"Disjunction, then is not self-explanatory, indeed the primary problem, at the starting-point of a theory of perception, is not to explain coherence, but to explain <u>disjunction</u>." (27, p. 411)

". . . the young child's impressions tend to be more coherent than those of the adult. This difference, according to our theory, is based upon a difference in the properties of the two motor systems, or, as we shall say, upon a difference of motor 'attunement.'

"We can describe the attunement in the case of the child by saying that the motor system has a pronounced tendency to attain <u>equilibrium</u> between the impulses due to the different elements of a visual figure. With the adult this equilibrium-tendency is weaker.

"In order to experience a figure as disjoined, the child has to exert a special effort. That is, he must voluntarily bring an influence into play which reinforces the weak rivalry-tendency, and which opposes the strong combination-tendency.

"The motor system of the adult, however, has undergone a process of training which has changed its original attunement." (28, p. 77)

On the basis of the evidence thus far given, the members of some schools of psychology doubt the assumption that the laws applying to growth on a 'physiological level' will apply as well to development on a 'mental level'. Although the Gestalt psychologist holds rigidly to the principle of isomorphism (21, pp. 56-57) in this connection, it seems likely that the results reported by Snoddy (33) should dispel any doubts for those who are inclined to raise this problem. Snoddy gave his subjects the task of tracing a star pattern in the mirror; his chief goal was to discover the effects of repetition on the learning behavior. This experimenter discovered what he terms a primary growth and a secondary growth which were direct functions of time and of repetition, respectively. The point of greatest significance here was the discovery of the primary growth which proved to be a continuous function of time. This growth preceded the secondary development and was a necessary basis for the later development. Any attempt to secure a specific skill before a sufficient base of primary growth had been established resulted in a 'conflict' which was recorded as a plateau on the learning curve.

In conclusion, there is every reason to believe that the assumption, making learning of arithmetic a continuously expanding growth process, is a correct one. We can proceed, therefore, to establish an hypothesis for explaining how mathematical ideas develop in the child. We wish to consider first, however, additional data concerning the development of the series and the quantity ideas of number.

Does number have two independent roots? Many teachers as well as experimenters in educational psychology make the assumption that the counting of the child indicates a mastery of the series idea of number as opposed to the quantity concept of number. It is supposed, also, by some that the cardinal notion develops out of the ordinal idea of number, i.e., the units analyzed through counting are supposed to be synthesized into groups representing abstract quantity concepts. (23, p. 351) An argument against this hypothesis was given above in the "Introduction". It was shown that organization was not a derived property in child thought. Another authority, Karl Buehler, assumes that the series idea remains independent of the quantity idea and that the two notions represent entirely different roots of number. He states in his conclusion that "the child has subsequently to learn how to connect these two independent functions of words (the ordinals and the cardinals) which signify number." (9, p. 84) This writer, unfortunately, does not indicate the manner in which children are supposed to accomplish the connection of these discrete number concepts.

The basis for Buehler's theory was the finding of Wertheimer that primitive people often use different numerals when they wish to express 'collections of things.' It has already been pointed out (30) that children have no difficulty at any stage in the learning process with the cardinal idea of number and that the finding of Wertheimer undoubtedly indicates that number has two aspects which differentiate together from a more generalized notion of number.

Conant, who has compiled a considerable amount of data on the number development of primitives, gives a much more complete account of the origin of the cardinal number names than is given by Buehler or even by Koffka (20), who holds a similar view to that of Buehler. Although the actual meaning of names have not always been accurately traced by investigators, enough material has been collected to demonstrate the point we wish to make here. There can be no doubt that the first counting is a tallying -- some primitives use the fingers, others use seeds, pebbles, shells, notches, or knots -- which implies a development of the series idea. In nearly every instance with the most primitive types of scales, however, there is either a clear reference to the total group of counters which is being differentiated by the counting process or there is evidence that a given number is a compound of two smaller numbers. The scales in Table LXIII are introduced as examples. Conant makes the following statements bearing on this problem:

"Collecting together and comparing with one another the great mass of terms by which we find any number expressed in different languages, . . . we observe certain resemblances which were not at first supposed to exist. . . . a careful examination of the first decade warrants the assertion that the probable meaning of any one of the units will be found in the list given below. The words selected are intended merely to serve as indications of the thought underlying the savage's choice, and not necessarily as the exact term by means of which he describes his number:

1. = existence, piece, group, beginning
2. = repetition, division, natural pair
3. = collection, many, two-one
4. = two twos
5. = hand, group, division
6. = five-one, two threes, second one." (10, pp. 98-99)

Further examples might be taken from primitive binary systems. In these cases the important root is two and the odd numbers are designated by adding one to an even number of pairs. Three, for example, becomes 2-1 and five becomes 2 pairs plus 1. (10, pp. 110-111)

It may be pointed out that the names used for counting in some languages have their origin in a cardinal idea of number rather than in a distinct ordinal concept. For example, four equals 'toes of an ostrich'; five equals 'a five colored spotted hide' and again 'all the fingers on one hand'; six equals 'three (fingers) from each side'; and seven equals 'there are still three of them (fingers)'.

There are isolated cases where a finger gives a name to the numeral with which it is associated in counting; this fact has been taken by some authorities to illustrate the development of an ordinal idea apart from the cardinal function. In the Zulu language, for example, the word for thumb, tàtisitupa, becomes the numeral for six. Then the word Komba meaning 'to point' which refers to the forefinger or the 'pointer' makes the next numeral, seven. The uses of this numeral shows that a cardinal as well as the ordi-

TABLE LXIII

PRIMITIVE NUMBER NAMES

I*

1.	topinte	taken to start with.
2.	kwilli	put down together with.
3.	hai	the equally dividing finger.
4.	awite	all the fingers all but done with.
5.	opte	the notched off.
6.	topalik'ya	another brought to add to the done with.
7.	kwillilik'ya	two brought to and held up with the rest.

II**

1.	inl'are	the end is bent.
2.	nak'e	another is bent.
3.	t'are	the middle is bent
4.	dinri	there are no more except this.
5.	se-sunla-re	the row on the hand.
6.	elkke-t'are	3 from each side.
7.	t'a-ye-oyertan	there are still 3 of them.
	inl'as dinri	on one side there are 4 of them.
8.	elk-ke-dinri	4 on each side
9.	inl'a-ye-oyert'an	there is still 1 more.

III***

1.	initara	1 alone.
2.	inoaka	
3.	inoaka yekaini	2 and 1.
4.	geyenknate	toes of an ostrich.
5.	neehalek	a five colored, spotted hide.
	hanambegen	fingers of 1 hand.

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* (10, pp. 48-49)
** (10, p. 53)
*** (10, p. 71)

nal idea, however, is involved. The Zulu, for example, would say, "amahasi akombile," (the horses have pointed) meaning quite a different thing by this statement, namely, "There were seven horses." (10, pp. 62-63) Words derived in this manner from names of the fingers have been used in some scales for compounding, which would seem to indicate a similar inter-relation of the cardinal and the ordinal concepts.

Conant gives no instance where counting has developed in primitives apart from a process of differentiating quantities. From his data one would assume that the cardinal and ordinal ideas are really interchangeable concepts, i.e., a simple number name suffices for both meanings in the first stages of development.

Although children, under favorable conditions, can quickly master the number names to ten, there is no analogous learning among primitive people who lack this contact with a highly developed number system. Most of these people can deal with quantities up to ten or higher, but it is difficult for them to count above five.

> "Beyond 5 primitive man often proceeds with the greatest difficulty. Most savages, even those of the tribes just mentioned, can really count above here, even though they have no words with which to express their thought. But they do it with reluctance, and as they go on they quickly lose all sense of accuracy." (10, p. 76)

The learning of abstract number ideas (a) Perception of relations. The first quantitative notions of the child involve differences in size, shape, and distance. Judgments of more or less appear early because the child soon learns to reach in the right direction and at about the correct distance in order to obtain various objects. The visual perception undoubtedly plays the largest role in making these adjustments. The language of the child soon reflects these ideas. The child between two and three years of age can grasp the questions, "Which is bigger (of two objects)," "Which is biggest?" "Which is larger?" "Which is largest?" The word littlest can also be understood and used. It seems very probable that expressions of difference or contrast are mastered before words indicating sameness or equality are fully comprehended. Accurate discriminations between large concrete quantities are found in six year old children even though the ideas of number above five or six are very inaccurate and incomplete. This evidence is further indication of the great importance of visual perception in the development of the first number ideas. The data concerning the number systems of primitives shows a similar growth. These people can deal with fairly large quantities, but accurate concepts very seldom develop beyond five. (10, p. 76)

(b) Numbers as 'names'. In the first attempts to develop a language for expressing the idea of number, the mathematical notion is closely related to concrete objects or groups of concrete objects. If number names have been developed chiefly through the process of counting or tallying, numbers are given a name from the counters employed or the particular sign (e.g., hand away, hand off, hand high, hand up) with which a given number has been connected. (10, p. 59) There are perhaps more names to express two than for any other number. Practically any name which indicates that two things belong together or are found together has been used to designate a 'pair'. In Sanskrit two can be expressed by any of the following: hand, arm, wing, and eye. In other languages the word for two may denote separation or distinction and it may denote 'putting to' or 'putting with'. The meaning of one may be 'taken to start with' which refers specifically to the finger that is used to begin tallying. The number one can also mean 'a very small thing' or 'the least', which idea is somewhat more abstract than those usually found for the other numerals. (10, pp. 92-93) Four may refer to the 'toes of an ostrich' and in one language the word meaning four is 'knot' which refers to the practice of tying breadfruit into knots of four. (10, p. 93) Five in one primitive language refers to a colored, spotted skin; very often, however, five refers to 'hand'. This same tendency is observed in the names adopted for higher numbers. Ten, for example, may be 'two hands', 'half a man', or simply 'man'. A common name for twenty likewise is 'man'.

The enumeration of objects by children follows a similar trend. In the report of Decroly and Degand (12), the first distinctions between the members of a group on the basis of number was as follows: "One, One"; "One, another one"; "One, another one, still another one." The number names in these instances were very closely connected with the particular objects by pointing to or by placing the finger on every object as the name was spoken. The child quickly memorizes the number names to ten, but at first he cannot apply them correctly even when he tallies objects by pointing. After the child has mastered 'two' and 'three' and can always identify these numbers correctly, he still tends to apply the wrong number name when tallying beyond three. Any name suffices for the fifth, sixth, and seventh object, for example, just as long as it is a number name.

<u>(c) The first addition: compounding.</u>* As the primitive refines his notions of <u>more</u>, <u>less</u>, <u>many</u> and as he begins to develop number names to express these ideas, he employs a simple form of addition. This is shown by the use of compound language forms such as 3 = 2 and 1; 4 = two 2's; 4 = 3 and 1; 6 = 5 and 1; and 7 = 5 and 2. In the better developed number systems a definite number base

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* It is maintained here that all phases in the growth of number ideas can be described as a differentiation of the quantity idea. Both the savage and the child perceive quantities as wholes, i.e., wholes which have parts. The relations of whole to part, furthermore, must be clearly defined in order for growth to occur. If a child loses his notion of a particular whole because of structural analysis (through counting, for example) de-differentiation occurs and the child is thrown back to a more primitive level, which in this case means that the judgments of <u>more</u> and <u>less</u> become very difficult for him. In order to describe some of these processes of the child, on the other hand, it seems expedient to make use of the terms compounding, aggregate (29, p. 140), transduction (25, p. 198), and juxtaposition (25, p. 59) which are meaningful from an adult's point of view.

is employed which may be any numeral from one to five or even ten in those tribes which have reached a high level of achievement. Without this base a system can hardly be said to exist. The names which serve to mark the successive steps in the expansion of number ideas enables the savage to keep track of his progress in the same manner that we refer to a milestone to mark progress when making a journey. In the beginning, a number which cannot be shown by the fingers is described as many, or heap. When the savage must be more precise, progress beyond his first range is made easier by indicating the distance travelled from his <u>basic number</u> and not the distance from his original starting point. When 11, for example, is to be expressed, he calculates 'all the fingers and one more' or 'both hands and one more.' (10, pp. 100-101) A common system, although only one among many, is based on five or 'hand'. Six and seven are designated as 'hand with one more' and 'hand with two,' respectively. Eight and nine may then be signified in a similar fashion. (But the terms 'four from each side' and 'there is only one' for eight and nine, respectively, may be employed instead.) For the word ten we generally find 'hand hand'. (10, pp. 37-99)

Does this evidence demonstrate that addition is a primitive mode of thought? If such is the case, why should children in first and second grade, who have learned a great deal about a ready-made number system, experience difficulty in performing this process? There can be no doubt that a knowledge of addition exists very early, but the investigator needs to distinguish carefully in this respect between the child and adult types of thought.

When the child first differentiates between the size and shape of two objects, he is capable at the same time of observing quantitative changes which a single object undergoes. At 18 months of age changes in a group of objects which belong together are quickly noticed. Decroly and Degand point out that one child at 14 months observed the disappearance of one of three weights with which she was playing. Experiences of eating and of play furnish many opportunities for the development of concepts relating to quantitative changes. At approximately two years of age the child will say, "Give me more," or "Still more." The word more or again appears likewise when the child wishes an act or a song repeated (12). The development which occurs here may be described from the adult's point of view as simple expansion or contraction of aggregates. (By an aggregate we mean a group whose parts "have no direct connection *inter se* but only the indirect connection involved in being parts of one and the same whole." (29, p. 140) The term compounding may be used here in order to distinguish this simple expansion and contraction from the abstract addition found at the adult level.)

The examples of addition among savages which are given in this section are, in reality, illustrations of compounding. When the primitive says, "Hand and one," he is simply expanding the unit with which he started, but without full regard for the changes among all the parts which of necessity must be comprehended in abstract addition. The idea expresses clearly and precisely that the new group is greater in size and does so in such a manner that it can be verified. That is all in which the savage is interested. From the civilized adult's point of view, the words (number ideas) employed by the savage are placed in juxtaposition, as Piaget uses the term (25, p. 59). There is, as he points out, an absence of specific relations among details or parts of the whole.

These ideas of number exist in children at six or seven years of age. Addition for them may be described as compounding for they still deal with aggregates rather than with adult number concepts. The resemblance in form of children's thought and primitive thought is striking in this respect. The savage probably learns numbers readily only to five, which can be described as the 'easy limit' as far as dealing with aggregates is concerned. (Visual perception of wholes is the limiting factor involved.) Accuracy beyond five diminishes sharply and beyond ten all primitive systems become very complicated. Children in first grade were found to be accurate, likewise, in manipulating groups of blocks as high as five to seven in number. Estimates above this figure were not precise. Brilliant second grade children quickly mastered addition and subtraction combinations below ten and memorized the harder combinations as well. But the procedure for solving these harder facts was tedious and was never absolutely accurate. The pupils' compounding, it may be concluded, is a principle which is not easy to transpose to the higher numbers. The awkward primitive name for the highest numbers, 20, 40, 100, and 1000 indicate the great difficulty involved.

(d) Transposition of number names. The numbers which apply in the beginning to specific objects enable the savage to compare directly two groups of objects, but his language does not enable him to express precisely the numbers of various groups of objects. The transference of these names to groups, other than the original, calls for a new effort and a new development in thought. This situation may be illustrated as follows: The savage calls five 'hand.' If he is making a trade, he is able to designate by means of signs that the 'objects to be traded' have a number equal to the fingers on one hand. Suppose the occasion arises, however, when he wishes to state the number of objects not immediately present. Some development of language quite definitely is essential to express the notion that 'hand' and the 'animals to be referred to', for example, are to be connected quantitatively. As might be expected, a juxtaposition (from the civilized adult's point of view) occurs in the first attempts to make this step. The Zulu, as was mentioned above, said, "The horses have pointed," when he wanted to express the idea that there were seven horses. The development of number among certain tribes illustrates a unique method of solving this difficulty. It offers clear-cut evidence that an important stage exists between that of 'number as a name' and 'number as a symbol'. In these languages the number word undergoes a change, sometimes a radical change, when various types of groups (men, canoes, and round objects) are referred to. Conant describes this development as follows:

"One noteworthy and interesting fact relating to numerical nomenclature is the variation in form which words of this class undergo when applied to different classes of objects. To one accustomed as we are to absolute and unvarying forms for numerals, this seems at first a novel and almost unaccountable linguistic freak. But it is not uncommon among uncivilized races, and is extensively employed by so highly enlightened a people, even, as the Japanese. This variation in form is in no way analogous to that produced by inflectional changes, such as occur in Hebrew, Greek, Latin, etc. It is sufficient in many cases to produce almost an entire change in the form of the word, or to result in compounds which require close scrutiny for the detection of the original root. For example, in Carrier, one of the Dene dialects of western Canada, the word <u>tha</u> means three things; <u>thane</u>, 3 persons; <u>that</u>, 3 times; <u>thatoen</u>, in 3 places; <u>thauh</u>, in 3 ways; <u>thailtoh</u>, all of the 3 things; <u>thahoeltoh</u>, all of the 3 persons; and <u>thahultoh</u>, all of the 3 times. In the Tsimshian language of British Columbia, we find seven distinct sets of numerals which are used in various classes of objects that are counted . . . ." (10, p. 86)

An example of the variations found in the first and second numerals of the Tsimshian language are shown in Table LXIV.

TABLE LXIV *

Number	Counting	Flat Objects	Round Objects	Long Objects	Canoes	Measures	Men
1.	gyak	gak	g'erel	k'awutskan	k'amaet	k'al	k'al
2.	t'epqat	t'epqat	goupel	gaopskan	g'alpeeltk	gulbel	t'epqadal

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* (10, p. 87)

Many children undoubtedly have not passed this stage, at least not very far, by the time they enter school. They have had the advantage of using a specialized number system and the advantage of stimulation by adults which prevents any development of language to express this precise notion of transposition. They learn to say, "Six," whenever six books, six erasers, or six pieces of paper are given to them. But the idea of <u>six-ness</u> grasped by the adult is not present. Six is simply a name for various groups of objects. S20, who experienced considerable difficulty mastering the material in the experimental class, could grasp 'adding' or 'taking away' as long as the concrete objects were before her. But it was impossible for her 'to think' the answers to problems involving comparisons of quantities. S23 had difficulty in divorcing a number above four or five from the specific pattern the experimenter presented to her.

<u>(e) Use of number symbols: reference to aggregates.</u> It is not until a relatively late stage in social evolution that the number names gain the meaning of number symbols. The number is recognized at the level without special reference to the object or objects from which the name was derived. The number symbol <u>transfers</u> to various groups. The number, however, still refers to <u>a certain number of objects</u> and is not yet an independent idea. The primitive's conception of a group at this stage, therefore, may still be described as an aggregate since a number cannot be con-

ceived except in conjunction with the notion of some specific aggregate of objects. It is possible that for one, two, and perhaps three, this stage is reached quite early.*

The majority of the children in the second grade experimental class had reached this point in their growth of number ideas. The numbers, one, two, three, and four especially, were well differentiated. The difficulty experienced by some children at this point is hardly appreciated, however, by adults. The trouble these children encounter in mastering and in writing the symbols, 1, 2, . . . 10, is unexpected, but it is, nevertheless, quite real. Pupils who did excellent work in making comparisons, and in counting, occasionally failed on this task. When these children had to write the symbols to ten from memory they found it necessary to repeat the series time after time in order to make certain which number should come next. The instructor found that symbols not only were commonly misplaced as to order, but many times they were written incorrectly. These errors were not caused altogether by faulty

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* There is no reason for believing that all the numbers to ten or beyond follow each of the steps thus far enumerated at identically the same time. For the higher numbers, a lag in transposition is bound to occur; it is likely that the development of the smaller numbers continually sets the stage for making a new conception of a higher number possible. For example, the notions of 1001, 1002, 1003, and so on undoubtedly have no meaning except as abstract concepts; it is impossible to conceive them except as one transfers from abstract notions of number farther down the scale.

perception and by unco-ordinated writing movements even though these factors were involved. The children simply did not have an idea of number which was completely differentiated from the notion of one definite, concrete group. Any attempt to introduce work with symbols, which implies that the name for a quantity will transfer to all particular groups, resulted in pupil difficulty. For these cases the use of the symbol must always be accompanied by a definite quantity. The symbol itself does not as yet readily 'call forth' the notion of a specific group.

(f) The transition to abstract number ideas. (1) Illustrations of the principle of expansion. It was emphasized above that number development is a continuous process and that no gap between the concrete and the abstract realms is to be found. We have observed in this connection that whole, part, addition, subtraction, multiplication, division (fractions) are notions which exist from the very beginning in the growth of mathematical ideas. We may think of this growth as a continuous series which undergoes expansion and differentiation according to the law of Reciprocal Change (40, p. 465n).*

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* The notion of whole and part, e.g., are not the same in form at one point as at a later point arbitrarily chosen from this series. An example of the changes which parts of a perceptual pattern undergo for the child has been given by Binet (Stanford Revision of the Binet-Simon Intelligence Test); he lists these stages as: (a) enumeration, (b) description, and (c) interpretation. This same type of change can be found in the growth of number concepts, e.g., (a) notion of size (quantity), (b) notion of aggregates, and (c) notion of abstract number.

It is convenient, on the other hand, to regard the growth of an individual as 'occupying' certain stages which are chosen at various points along the continuum. The advantage is lost, however, if in performing this abstraction it is assumed that stage '2' represents only an extension or refinement of certain concepts encountered at stage '1'. It is true that refinement of stage '1' does occur, but entirely new processes are evident at stage '2' as well. It is for this reason that the law of Reciprocal Change assumes an important role in this connection.

The general theory maintained here supposes that growth of abstract concepts involves, on the part of the individual, the use of functional analyses in dealing with the variables of a system. For the infant a system is the universe; everything is related to everything else. This system differentiates into members which may be designated as dependent-systems which in turn are differentiable. A system, therefore, is any whole; this means any whole which has parts. The child's differentiation of a system involves the implicit use of functional analysis inasmuch as the whole is not reduced to its parts. The child simply discovers the conditions under which a given event within the system may take place. In the development of abstract reasoning, more and more conditions are brought under control (40, p. 241).

Before the ideas or hypotheses of the child reach the abstract level, two important stages may be distinguished. First, the child's ideas are influenced by the relations of those ideas to his own being. Piaget points out that, at five years of age, the child's ideas of <u>right</u> and <u>left</u> refer only to himself (25, p. 107).

Second, concepts appear which pertain to systems in which the variable of 'self' is constant. The child at seven or eight years of age, for example, can designate <u>right</u> and <u>left</u> of a person opposite himself (25, p. 107). The other important variables concerned in an hypothesis, however, are not yet under control; thus, the concepts of a child are still influenced by the social or moral implications of the ideas. The notion of necessity is purely moral at this age and physical determinism is not separated from the notion of social obligation (26, pp. 273ff.). Another illustration from Piaget may clarify our meaning. For a child a pebble is heavy or light depending upon 'recent' or 'the present' circumstances.

> ". . . a boy tells us that large-sized or 'big' bodies are heavier than small ones; yet a moment later he declares that a small pebble is heavier than a large cork. But he does not, for that matter, give up his first affirmation, he only declares that the stone is heavier than the cork 'because some stones are bigger than corks.' Thus the character 'big' has not at all the same meaning as for us. It does not define a class, it is transmitted by syncretistic communication to analogous objects: since there are big stones, little stones participate in their bigness, and thus acquire weight. . . . the concept 'heavy' is one that qualifies absolutely: a thing is either heavy or not heavy. It is not a relation, such that one object is always heavier than another, and less heavy than a third." (26, pp. 294-295)

The child's concepts thus appear 'substantial' rather than relative in character from an adult's point of view. Piaget describes the child's reasoning as transduction: ". . . (it) is a combination of elementary relations, but without reciprocity of the relations amongst each other, and consequently without the element of necessity that would lead to generalization." (25, p. 198) In other words, the child, who makes two comparisons of an object under two different circumstances, cannot at first conceive any disadvantage to the fact that he applies both the terms heavy and light to the same object.

Finally, at the last stage, the child is capable of holding constant (and in this sense eliminating) the influence of all the extraneous factors with which an idea is related; it enables him to consider an hypothesis 'as such', i.e., to admit of premises from which he can infer abstract relationships. In the terminology used above, we would state that the child at this level can deal with the variables within a system which exists 'outside' of himself. This is realized rather late, at approximately 10 or 11 years of age. As an example of the child's ideas at this level, he gives to right and left the meanings which the adult possesses. When three objects, e.g., a coin, a pin, and a key, are placed in a line before the subject, he designates right and left in a relative fashion (25, pp. 110-112). Piaget states that in making judgments of weight the child also uses relative concepts which are based on the principle defined in adult language as specific gravity (26, pp. 156-157) (25, p. 192). One may ask, does the child accomplish this abstract idea by simply <u>refining</u> or by <u>extending</u> his idea of weight found at the earlier, transductive level? Does he actually <u>eliminate</u> any of the variables concerned in this earlier process? The transition involves instead a new conception of the situation. In the earlier instance the child reacted to the properties of the objects, weight and volume, according to the way in which they appeared together in the specific situation. Piaget would say that weight and volume were variables which were in juxtaposition in the child's thought.* At the last stage, however, the child has become able to hold constant one of these variables, e.g., volume, which permits him to compare the weight of objects in a relative manner.**

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* At the transductive level described here as the second stage in reasoning, we find ". . . the child has discovered that bulky objects are not necessarily the heaviest. Henceforward, he distinguishes between weight and volume. But he has not yet discovered the specific weight of bodies: all he does is to regard different objects as made of more or less condensed (or rarefied) material." (26, pp. 160-161)

** It should be mentioned that a child will employ a principle correctly before he can explain logically what the principle is. The order of conscious realization has been found to be the opposite to the order of actual construction (26, p. 300). Piaget reports an experiment in which children predicted the rise of water in a vessel when objects were immersed. Children were found to be capable of predicting the rise in water level on the basis of volume of the body immersed and yet their own explanations of the process were based solely on weight (26, pp. 298-299).

(2) <u>Application of the principle of expansion to the growth of mathematical ideas.</u> The principle discussed in the above section applies to the known facts concerning number development to the eight year level. Number in primitives and in children was found at first to be an expression of specific quantitative relations between the individual and his environment. At the next important stage, found in a majority of second grade children, a number was discovered as a concept (symbol) that could be treated independently of the particular quantity from which the number name was derived. The number was a name which was transferable to <u>any</u> aggregate, although this name still referred to <u>a certain number of objects</u> and was not yet a completely differentiated idea.

There are no data from the present study to demonstrate how the final stage, the abstract number concept, actually is achieved. This aspect of the theory, therefore, remains to be verified. Theoretically, the child would gain the abstract notion of number at the same time at which he is capable of formulating an hypothesis, i.e., reasoning from a premise. This development probably would occur at approximately ten to eleven years of age.

For our purpose we may assume that abstract number is an independently differentiated hypothesis. It can be expected, therefore, that is would develop as an abstract concept develops, and that it would reveal the whole-part properties found at this level of logical thought. The impertance of this generalization regarding the growth of number is the fact that it enables us to obtain a new view of addition and subtraction. How this process fits into the picture will be illustrated shortly.

Piaget describes the achievement of the logical thought in the child as a realization (or as an assumption) of specific inter-relations among the parts of a proposition. He uses the term reciprocal relations in this connection. In the development of formal thought, to give an example, a child can very early give judgments about the whole group of his brothers and sisters, but it is only at a late stage that their reciprocal relations can be taken into account. At first, a brother need not be, necessarily, a brother of somebody (25, p. 131). Along with this development of inter-relations is the growth of the notion of reversibility of relations which leads to the notion of causality.

If we interpret Piaget's data according to the principles governing 'the whole and its parts', the ideas involved are greatly simplified. At the abstract level a whole '5' is no longer an aggregate for the individual; the relations among the parts 'three plus two' are well defined. Each part has a particular 'place' in the whole and in turn contributes to the whole. That is to say, there is in a sense a reversibility or a reciprocal relation among the parts and between the part and the whole. The whole determines the parts and any change in the former must be reflected in the latter. Likewise, a change in one of the parts changes the remainder of the parts and also the whole.*

The evidence for the knowledge of abstract thought in arithmetic, therefore, would be the ability on the part of the child to perceive these reciprocal changes between the whole '5' and the parts '3 plus 2', i.e., to predict with absolute accuracy what changes will occur in the parts when a certain change is encountered in the whole. In other words, the principle of addition should transpose rapidly and easily to all combinations in the number system. This learning, it was found, did not occur at the second grade level. The children would estimate answers to hard problems with fair accuracy, but at this stage the transposition of addition, in the sense of the term used here, did not occur. Numbers above ten could not be comprehended in an abstract fashion.

A fact regarding our number system should be mentioned in this connection. The system has been so devised that a change (quantitative) in a whole results in an identical change to a part or rather to the parts of the whole (5 (+1) = 3+2 (+1)). The reversibility, therefore, between the whole and the parts is 'absolutely' predictable. Thus,

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* It may be mentioned here that the changes in all propositions do not of necessity occur in a one to one ratio on the phenomenological level. This principle can be shown to apply to the perceptual as well as the logical aspects of behavior. In a picture, for example, the details do not have equal value in contributing to the pattern of the picture.

we have a perfect tool for expanding number ideas and for transferring to new problems. With the knowledge of this reversibility, it is possible to give meaning to the statement 112+216 = 328. Without this knowledge, the statement is meaningless.

This reversibility should mean that arithmetic can become a most important and useful device for the development of logical ideas in children. As soon as they are capable of abstract thought, they can employ this tool to test for themselves the correctness or incorrectness of their steps in reasoning. Arithmetic should become a worthwhile game for the child in the same sense that reading becomes a game.

(3) Criticisms of former theories explaining the development of abstract thought. The theory, which holds that the notion of reversibility accompanies the development of abstract thought, furnishes the basis for demonstrating conclusively that counting in and of itself has nothing to do with the development of abstract ideas regarding addition. With the use of counting it is possible to verify whether or not the parts actually are quantitatively equal to a whole, but the counting act does not demonstrate for the child either the idea of a whole or the idea of whole-part relations. Since the true notion of addition, according to our hypothesis, cannot be achieved apart from these 'reversible' relations of the whole and the part, the act of counting in itself is not, fundamentally, a beginning of the process of abstract addition. The error alluded to here

has been made, however, by the majority of the writers of texts in arithmetic. The authors of one text state: "Addition is fundamentally an extension of the principle of counting." These authors go on as follows: "The danger in this existing relationship is evident. . . . This tendency (to count by ones) must not be allowed to develop, for it will handicap the child in his use of addition." (35, p. 46) The fallacy of this last conclusion has been acknowledged by one of these authorities. He asserts in a later article that counting may be expected to cease when there is no longer a need for it (19).

Brownell (4) likewise relates addition to counting and supposes that addition is mastered because the child _expands_ his notion of counting to include larger and larger groups; counting by ones is, according to his theory, gradually eliminated. It is sufficient to reiterate an earlier conclusion to the effect that methods of teaching based on this principle have not proved adequate to instill abstract mathematical notions in children (30).

These theorists have made the error referred to above of attempting to explain a later stage in the learning behavior by supposing that the notions actually found in the earlier stage undergo a _simple change_ -- a process of extention or of refinement. We believe that, instead, counting may be found both before and after abstract notions have developed and that these authorities have succeeded only in giving us a picture which _reverses_ the natural order of construction with respect to number concepts.

(4) Conclusion. One of the most striking things to be found in the classes of our public schools is the extent of the individual variation. Pupils develop at greatly different rates of speed conditioned principally by their background or home environment. It is difficult for this reason to state arbitrarily, without further experimentation, at what ages children will pass through the stages in reasoning enumerated above. There can be no doubt, on the other hand, that all of the second grade pupils tested by the experimenter failed to show a knowledge of abstract number concepts. If we may rely on Piaget's conclusions regarding the development of logical explanations in children, we could not expect the average child's ideas to function abstractly at least until some time in the fourth grade. It is likely that this occurs at a still later date (fifth grade) for the majority. This raises the problem concerning the teaching of multiplication and division. Where do these processes enter into the picture? It is impossible at the present time to answer this question. One can state with some assurance, however, that expansion of number ideas to include multiplication and division at the abstract level is not a simple extension of the ideas of addition and subtraction. Addition and subtraction can function as a device for verifying these higher processes in much the same fashion that simple counting can serve to verify problems in addition and subtraction. The

investigator must not lose sight of the fact that multiplication involves a differentiation of an abstract unity '25' into its members 'five 5's' in the same manner that addition involves a differentiation of a unity '5' into its members '3 plus 2'. Unless we start with unity at the very beginning, we have no means for achieving unity at the end of the process.

Although the foregoing theoretical discussion has been concerned chiefly with a description of quantitative concepts, it must be emphasized again that for the child these quantitative notions do not develop independently of the qualitative aspects of experience. Long before the growth of abstract ideas the child gives evidence of his appreciation of symmetry and form. The scientific demands for precision and orderliness go hand in hand with aesthetic demands for perfection and harmony. There is every reason for believing that mathematics can and should function continually as a fundamental part of everyday living.

The teacher of arithmetic, also, must not lose sight of the fact that the learning of the pupil occurs in a social environment. It may be re-emphasized that the need for proof and the need for logical demonstration are essentially a social product. The child is satisfied to invent erroneous explanations of natural events until he is faced with the necessity for convincing his fellows that his theories are correct.

We can agree with Buckingham that formal arithmetic, as usually applied in our present-day school system, should be dropped from the elementary curriculum. The child already knows a great deal about number when he enters school and continues to apply his knowledge in situations analogous to those in which learning originally took place, namely, the natural play environment. The teacher can further stimulate this learning, instead of hindering it, if only she can be permitted to work with the individual as a member of a natural, social group. Perhaps this objective for class study can be realized in the near future. When teachers themselves discover that the principles of growth can be applied even with the limited equipment now available, this goal will be quickly achieved.

V. DISCUSSION: APPLICATIONS TO THE TEACHING OF ARITHMETIC

Methods Adapted to the Concrete Level of the Second Grade Pupil. Some general suggestions made by Studebaker, Knight, and Findley (35) concerning methods of procedure may be considered here with regard to the problem of learning at the second grade level. First, these authors recommend that the teacher use an abundance of illustrations and keep her work largely on the concrete level. Second, the teacher should attempt to develop only one idea at a time. It is necessary to proceed slowly in order to avoid confusion. Third, the response of the pupil should be immediate. He should not be allowed to count the answers to addition and subtraction facts; this process slows up the individual and may lead to habits that may have to be broken at a later time.

These suggestions to the teacher were based on actual classroom experience which reflect quite well, on the whole, the type of learning found in the first and second grade. While it is now generally admitted that counting of answers probably does not have the unfortunate consequences stated by these authors, it is important that responses to situations be immediate and not delayed. It was found in the present study, for example, that children were quite willing to correct errors during the period in which they were made. When the checking was delayed one day, however, no interest in their errors could be aroused. This evidence along with other qualitative data indicated that the child's period of concentration on arithmetic material was very short. No arbitrary limit to the work period can be set, although ten minutes on highly abstract problems should be sufficient for most pupils. The teacher, as has been emphasized, can easily expect too much of the class. The result of this overstimulation was found to be harmful especially on the poor students. In the present study it was found that the confusion and lack of a desirable goal on the part of these pupils persisted throughout the year and that the performance of several students became noticeably worse on a retest over addition and subtraction. In the light of these findings, it is concluded that several of the pupils in the class would have profited by a continuation of the work on comparison and contrast of groups as was presented in a concrete fashion during the first and second six weeks periods. The delay of a semester in presenting addition and subtraction was still not long enough for these pupils.

Since the social group exerts considerable influence on the types of arithmetic goals adopted by the pupils, it seems likely that the groups should be small in order to allow the slow pupil the advantages of this stimulation. At any rate, a large group of 25 to 35 pupils results in tremendous effort on the part of the slow pupil to keep pace. The accompanying emotional strain places too great a hardship on the individual. Arithmetic classes with a maximum of ten or twelve permit each pupil to participate fully in demonstrations or games and, furthermore, permit a closer teacher-pupil relationship which is essential to optimum achievement of the child.

Objectives of the teacher. The authors mentioned above (35) maintain that a statement of topics should be clear from the point of view of the teacher and that the order of topics to be learned should be well defined. In spite of the fact that many courses of study in beginning arithmetic are available, it is a safe conclusion that the average elementary school teacher knows neither the general objectives of teaching arithmetic nor the specific objectives in the form of well defined topics. Although one would expect different teachers to employ slightly different procedures to fit the needs of particular classes, the differences show, instead, a lack of knowledge regarding methodology. In most instances the responsibility for this state of affairs can be placed upon the authors of the courses of study rather than upon a lack of training on the part of the teachers. Some of the recent criticisms of the arithmetic curriculum by leading writers (7), (19), (5), corroborate this conclusion. The teacher of 35 pupils in second grade, for example, who employs the Standard Service Arithmetics (35), finds individual differences among the pupils which are of such significance, after the first ten units have been taught, that further progress by the group as a whole is considerably hampered. She has no way of predicting what difficulties may appear in teaching the succeeding units as a result of failure to master the preliminary units. There is some indication that pupils may make fair progress on succeeding units of study without having mastered the preliminary material to the addition and subtraction problems.

These classroom results with standardized procedures indicate how complicated is the task of formulating an adequate course of study in this subject. At the present time, it is impossible to determine in an arbitrary fashion what objectives the teacher should formulate. Since the extent to which she may overturn the present practice in the public elementary school is limited, only three suggestions will be made here. First, no pupil must be forced to master addition or subtraction if processing is difficult and tedious to him. It is easier to overstimulate the pupil in arithmetic than it is to understimulate him. The student who can learn addition and subtraction will not only do so at a rapid rate, but he will grasp immediately the goal of 100 per cent accuracy. Second, class demonstrations, which show comparisons of groups (not more than 15 units in size) and ultimately show the numerical differences of groups, enable the work in arithmetic to be maintained at the concrete level. This demonstration method when check materials are employed will enable the teacher to discover readily those pupils who confuse number with particular patterns of objects. Third, most children, even though not capable of abstract thought, can understand addition or subtraction as the simple increase or decrease, respectively, in the size of aggregates. (This point was discussed under the topic "The psychology of number.") The demonstrations, therefore, which show how specific groups can be divided into sections and how groups may be equated, will be grasped by the majority. The danger which may occur for the slow pupil in this procedure is the emphasis which may be placed on symbols to represent the groups. To avoid difficulty, the teacher may employ small numbers and make certain the groups are presented the children in such a manner that all the groups and sub-groups can be perceived at once. This procedure in some instances may involve considerable work on the part of the teacher. But there is every indication to believe that this procedure is important, for the pupil will of his own accord refine his technique and employ abstract number concepts as early as possible in order to gain answers more quickly and also to verify his calculations more accurately.

<u>Testing</u>. The authors of the better known texts (8) (35) make an extensive provision for an inventory of the pupils' knowledge about arithmetic and also give examples of the more important records to be kept by the teacher. In many cases, also, the actual tests to be employed after each unit of study are supplied.

The data from the present investigation show that an immediate check of the daily work is important. If a pupil has not reached the goal desired by the teacher, immediate aid to the individual may accomplish a great deal. Experience in the class room indicates that a particular situation cannot be reconstructed by the child on a later occasion. Furthermore, an error in procedure or in calculation, if allowed to go uncorrected, will persist. It is important that the teacher have an abundance of materials with which to check the performances of her pupils. It is necessary here to distinguish between materials for teaching number facts and materials for testing mastery of arithmetic processes. Under an administration which encourages an incidental type of work in 'skill' subjects, the teacher is left to her own ingenuity to devise material for the class. Usually she discovers or originates materials for teaching a given number idea, but then lacks time to devise adequate test measures to determine whether or not the pupils can employ the principles in various situations. It is emphasized here that every teacher should have a supply of materials for the purpose of demonstrating number ideas and for checking the mastery of arithmetical principles. If the teacher must stop to devise and then to print her own materials, too much time is lost which could be spent more profitably on a consideration of individual performances. These materials, it may be added, should be simply arranged, i.e., the papers should not be crowded with illustrations or figures. They should require only a minimum of reading and of writing on the part of the pupil.

It is essential for the teacher to give extensive tests at various periods as a second check on pupil performances. The addition-subtraction tests and the flash card tests reported in Section III give examples of this type of survey. What general interpretations can be drawn from the results of tests such as these? Are group results reliable? The reliability coefficients of the tests in addition and subtraction reported in Section III were low. Certain individuals varied remarkably, although many pupils gave consistent performances. Illness and inevitable changes of pupils in the class are two factors which affect to the greatest extent the reliability of group results. Over a reasonably long period of time it is almost impossible to maintain a group of 20 to 30 pupils. A reliable experiment, nevertheless, should extend over a considerable period in order to obtain an adequate sampling of the individual's learning behavior. The tests themselves should be extensive enough to permit a consideration of every pupil's performance as well as the performance of the group.

Many studies dealing with the difficulty of addition and subtraction are to be criticized because the investigators have failed to consider these points; conclusions have been based only on an average of group results and in many instances the duration of the experiments has been very short. It may be pointed out that even the present study, which continued for an entire school year, has not been of sufficient length to permit a check of the learning of number with respect to a complete development of mathematical ideas. Only a still more extensive investigation of maturation throughout the elementary grades will enable the educator to determine an adequate curriculum in arithmetic.

When shall we teach arithmetic? The grade placement of various topics in arithmetic has been a complicated problem. Washburne in 1931 (see discussion in reference 30) implied in this conclusions that children needed to attack the fundamental processes at an early age in order to gain the needed skills in arithmetic. Other investigators, on the other hand, found evidence that no harm was done by allowing children to go without 'drills' until grade three. That is to say, the performance of third grade pupils with one year of drill could equal that of the control groups which had been given three years of drill. Benezet in a recent series of articles makes a still more striking claim. In his school system an experimental group of sixth grade pupils equalled in one year the performances in the four fundamental processes of other groups which had been given formal training beginning with grade 3-A. In other words, one year of drill at the sixth grade level enabled pupils to attain the same level of achievement as three and a half years under the traditional system. Benezet's critique of our present arithmetic curriculum offers a challenge in no uncertain terms to the conventionally minded teacher.

"For some years I had noted that the effect of the early introduction of arithmetic had been to dull and almost chloroform the child's readoning faculties." (1, p. 242)

"If I had my way, I would omit arithmetic from the first six grades. I would allow the children to practice making change with imitation money, if you wish, but outside of making change, where does an eleven year-old child ever have to use arithmetic?" (1, p. 241)

The more conservative investigator, on the other hand is not ready to accept the broad interpretations such as those advanced by Benezet. Although admitting many of the criticisms leveled at the present curriculum, Buckingham concludes with the following statement:

"If the time to begin is when the child is ready and has use for the subject, then we should begin the teaching of arithmetic as soon as he comes to school." (7, p. 343)

The arguments of Buckingham to support his view have an important bearing for a genetic theory of learning which is neglected for the most part by Benezet. It is pointed out that first grade children not only possess considerable information about arithmetic, but they also make use of their knowledge inside and outside of school. Buckingham draws upon an experiment by Nila B. Smith to emphasize this point:

"They (the first grade children) counted, added, subtracted, used fractions and numerals; they measured and compared; they multiplied and divided. The more frequent activities in which they used these number ideas and processes were classified by Miss Smith as follows: transactions carried on in stores, games involving counting, reading Roman numerals on the clock, reading Arabic numerals in finding pages in books, dividing food with playmates and pets (fractions), playing store, depositing money in and drawing money from toy banks, measuring distance, using calendars, running errands, setting the table, buying and selling tickets, acting as newsboy, measuring in sewing.

"Anyone who knows children will recognize the authenticity of this list of activities and will grant the probable need for number in pursuing them." (7, p. 342)

There is no need to amplify the argument. There can be no doubt that young children know and employ arithmetic in their everyday living (30). There is every indication that children will continue to develop and use arithmetical concepts regardless of the program adopted by the school. But why is it that our curricula have thus far failed to meet the arithmetical needs of elementary school children? The answer which Buckingham quotes directly from Connor is appropriate here even though not particularly illuminating.

"When one considers, the ordinary teaching materials in arithmetic. . . , it would almost seem as if the course-of-study makers, textbook writers, and teachers of the last generation had conspired to retard by every means possible the natural growth of number concepts which takes place in little children from four to seven years of age." (7, p. 342)

Certain data and conclusions from the present study, we believe, supplement the findings and the interpretations of these investigators. First, the manipulation of symbols in a mechanical fashion (in addition, subtraction, multiplication, and division) requires an abstract conception of arithmetic that most elementary children simply do not possess. Investigators in this field might use to advantage the results of Piaget on child thought which corroborate this view. It is only in the second grade, perhaps, that most children begin to be aware of their own thought processes as such; they cannot yet reason, at this level of development, on the basis of a proposition independently of the personal, moral, or social connections of the proposition. Not until the pupil has reached the fourth or fifth grade can he explain logically the causal inter-relations of events. Under these circumstances, a purely formal system of mathematics is certain to be foreign to child thought in the first and second grades.

Second, the procedure of eliminating drill appears completely justified. In all probability no significant loss occurs to an individual if he fails to gain proficiency in processing before he enters Junior High School. At least, in the present study the factor of speed was relatively unimportant and appeared to increase with an increase of maturity in the pupils. It is undoubtedly more important that the pupils in the beginning grades spend their time differentiating and refining their concepts of comparison and contrast with regard to quantities. Both Benezet and Buckingham, it may be added, now place emphasis on a type of arithmetic during the first few years in school which calls for reasoning, namely, estimating of quantities and understanding of measurement. Problems should be only of such difficulty that the student can determine for himself <u>how to obtain a given answer and how to verify the accuracy of his processing.</u>

Third, the accomplishment of the students in the experimental group has indicated in no uncertain manner that children will refine their methods of dealing with quantities when the proper environment is provided. The best pupils quickly grasped addition and subtraction when small quantities were involved and even mastered simple tasks of multiplication and division in the concrete situation. It is significant also that these pupils of their own accord demanded more work and even, in some instances, discovered problems of their own which they proceeded to solve. No school system can afford to neglect these pupils who are capable of extending their knowledge of arithmetic under these conditions.

Fourth, we have some cue as to the nature of the objectives toward which the pupils must strive. Second grade pupils are rapidly becoming aware of the opinions and the ideas of others and they are beginning to feel the need for communicating their own ideas to one another. It is the teacher's task, therefore, to make quantitative reality, by means of which relations can be expressed, an integral part of the pupil's personality. The goals of accuracy and precision, as has been emphasized, should not be developed apart from the growth of standards of taste and appreciation. For the child the magnitude, position, and shape of objects are not attributes apart from the properties of unity, order, and symmetry. The idea of fundamental importance here is that the child should learn to express to others his quantitative world at the same time he learns to communicate his qualitative experiences.

An example of the proper beginning in this field has been given us by the teachers in the experimental school at the Ohio State University. They have found that when the pupils really experience reading, writing, spelling, music, and art as a part of their everyday lives, they express themselves through poetry, music, and painting. One discovery in this connection was the fact that primary children become adept at inventing their own rhythmic patterns in music and accompany themselves by beating out the patterns on percussion instruments. When the rhythms originate from the pupils instead of from a particular piece of music, "it becomes necessary for the teacher to improvise in order to fit the particular rhythm needed." (11, p. 135) Here is at least a beginning toward the fulfillment of the child's needs as stated by Knight: "As a child sees color. . . , living in a world of color and friends and the qualities of things, he should also live in a world interpreted . . . by quantitative thinking, even from the first grade." (19, p. 238)

Incidental procedures versus a pre-determined curriculum. The following questions may be asked, what is the relation of the teacher to the class and to the individual pupil? Is it her duty to pre-determine in a rigid fashion what the pupils will master in class? Or is it her duty simply to provide answers to questions as they arise in the pupils' thinking? In short, should the teacher rely on activities and on incidental methods of procedure, or should she plan in detail the course to be followed by the pupils?

The problem of incidental *versus* drill procedures was mentioned briefly in the "Introduction". It was pointed out that, from a theoretical point of view, drill was unsound as a teaching device and later it was pointed out on the basis of experimental data that drill in arithmetic was non-essential. Investigators have demonstrated conclusively that, at least until the pupil reaches Junior High School, drill can be eliminated completely without any ultimate loss of efficiency in processing.

There is a need for further discussion of the methods of the progressives and of the methods employed in the so-called activity schools in relation to Gestalt theory. The educators holding to the former views believe that learning is primarily a creative activity on the part of the pupil and that teaching methods should aid him to draw conclusions and generalizations from his own experience. Learning is a discovery of fundamental principles and ideas. The configurational theorist also holds to the notion that learning is a creative activity and that the independence of the learner must be assured (40, pp. 4, 13). At the same time, it is maintained that growth is a continuous development, the causes of which are external to the entire process. (40, p. 35) Mental development shows the properties of both spontaneity and dependence; while the growth "comes from within, necessary energy comes from without, whether the growth be physical or mental." (40, p. 13) The apparent contradiction between the freedom and the dependence of the individual has no logical foundation. It is sufficient neither to depend entirely upon an activity program nor upon an absolutely pre-determined, systematized curriculum. The present trend in education which relies chiefly on incidental learning for teaching arithmetic during the first school years neglects several points which have been mentioned in various parts of the present discussion, but need to be brought together under the following topics: First, the pupil works toward a remote goal. Although the goals of a child do not persist over a period of time comparable to those of the adult, the goals are, nevertheless, important. In the present study, the pupils grasped the goal of accuracy in arithmetic or else substituted with other goals, e.g., 'keeping up with the class' or 'satisfying the teacher'. At the second grade level, it is quite easy for the teacher to influence the pupil in such a manner that the goal adopted by him is one of 'satisfying the teacher.' Because of the large classes, perhaps, many teachers employ this device extensively for the purpose of maintaining discipline. There is little realization of the harm accomplished by this procedure. 'Artificial' goals cannot take the place of genuine learning objectives; in the present study, the cases of S28, S17, and S15 have been cited to illustrate how the lack of a suitable goal resulted in a loss of efficiency just at the period when one would expect a substantial increase in efficiency. It is hardly an exaggeration to state that teachers, as a whole, know little or nothing of the goals toward which their pupils are working. It is important that a teacher should measure what pupils learn in the light of what they are attempting to learn.

Second, learning is a growth process. Many references have been made throughout the previous sections to learning as a growth process. An attempt was made in Section IV to justify this assumption by showing specifically how it has been applied to related fields. It may be pointed out in this connection how certain advantages for the child and the teacher can be expected if this theory is applied to the class situation. At the same time, it may be shown that certain disadvantages have occurred because investigators have not yet made use of this hypothesis.

According to the principles governing physical growth, it has been shown that behavior patterns follow a definite sequence every part of which stands in a given relation to every other part. In the learning of number, therefore, we should expect that the development of reasoning and abstract mathematical concepts would follow a similar sequence. With this knowledge the teacher would have a scale by which she could determine where the child was and what his succeeding steps toward abstract thought should be. Without this knowledge it is impossible for her to guide the individual pupil; she has no means for discovering where the pupil's difficulty lies.

Textbook writers are to be criticised because they have failed to demonstrate whether or not their assumptions regarding the development of number ideas are verified in actual practice. There are some data which indicate that the present authorities have <u>reversed</u> the natural order of growth in their curricula as far as their methods of introducing addition and subtraction are concerned. At any rate, it is not to be denied that the steps outlined in the current courses of study do not follow a rigid sequence which is demanded by the laws of growth.

The problem of teaching arithmetic cannot be resolved, we believe, until the sequence in the steps of reasoning have been discovered. This point of attack, therefore, is the most important one at the present time. All subsequent studies on the teaching of this subject should begin with this fundamental question.

<u>Third, the school pupil must be considered as a member of a social group</u>. It is generally accepted that the social group determines the forms of behavior exhibited by the individual. Human nature is a product of a social environment and is found only in this environment (40, pp. 22-24). The actual part played by adult groups and the part played by play groups in the child's development, however, are almost unknown. Suffice it to say that the teacher makes extensive use of pupil competition in the classroom, but that she is unaware of the essential part which the social forces play in determining the concepts learned by the pupils. It is also not generally realize that the large groups in school are chiefly artificial, that is to say, they disintegrate with remarkable rapidity when the personality of the teacher is no longer brought to bear upon them.

There is a danger of making these groups unreal to the extent that the child cannot transpose his learned behavior to outside social life. The preliminary reports from the Ohio State Experimental School show that this problem is beginning to be faced by the educator (11). In this school the aim is to master the so-called skill subjects in relation to the social significance of these subjects.

The importance of the group in determining specifically the individual's differentiation of quantitative experience is clearly emphasized in a conclusion of Piaget.

> "We have on many occasions stressed the point that the need for checking and demonstration is not a spontaneous growth in the life of the individual;it is on the contrary a social product. Demonstration is the outcome of argument and the desire to convince. Thus the decline of ego-centrism and the growth of logical justification are part of the same process." (25, p. 15)

In the present experiment these facts were observed indirectly. It was noted primarily that the social group played an important part in determining the types of goals which the children set up for themselves. The good students were stimulated to maintain a high class rank and the poor students were forced either to keep pace with the class or to adopt other goals which would still give them acceptable group standing. Perhaps the results of greatest significance in this regard were those which demonstrated the effects of the teacher's personality on the individual-as-a-member-of-a-group. At times encouragement and verification of processing enabled the pupil to stabilize his performance. On the other hand, if the teacher for some reason did not expect good work of an individual, this was reflected in the child's own attitude toward his assignment. Several pupils who were not expected to keep up with the class worked only in spurts; on the basis of a few good performances it seems likely that, under more favorable circumstances, learning in these individuals would have improved.

By way of conclusion, it may be stated that children, similar to primitive people, acquire our present number system very rapidly when once the social environment becomes favorable to this development. Without stimulation of this kind, children could not possibly master in this rapid fashion the complicated number ideas which have been handed down from the past centuries. There is no doubt that the development of a particular number system as well as the development of a given language must depend entirely upon the specific demands of a particular social group. The teacher and parents, together therefore, must accept a responsibility for introducing the cultural heritage of society to child life. The process is one involving action and reaction of the adult and child worlds; children learn to depend upon both the security and upon the stimulation which their adult associates can give them. While the personality of the teacher is of great importance, especially at the beginning of any specific learning, the role of the successful teacher becomes less and less important as the learning process expands and differentiates. The energy of the organism released by the outside stimulation must carry out the learning. With a complete knowledge of the logical steps to be taken in a learning process, the teachers and the parents together can set up an environment favorable to that learning. But it is the growth and expansion from within that enables the pupil to attain the goal.

VI. SUMMARY

1. The objectives of the present study were:

A. To motivate the learning of arithmetic in a class situation according to the principle of expansion-differentiation.

(a) To develop quantitative thinking simultaneously with the growth and appreciation of qualitative experience.

(b) To allow the child to discover his own solution of arithmetic problems, i.e., to permit the child to discover a means for reaching the goal, and to permit the child to discover a need for verification of his computation.

(c) To pace the class as a group.

(d) To avoid drill and memorization.

B. To obtain qualitative data on the effects of success or failure, the effects of the group on the pupil, and the effects of the adult personality on the group and the pupil.

C. To determine, qualitatively, the limitations of child thought (reasoning) in dealing with arithmetic problems.

D. To contrast the progress of pupils taught by an expansion-differentiation method with pupils taught by the current arithmetic programs.

2. The writer taught the arithmetic in an average second

grade classroom of Cordley School, Lawrence, Kansas. The experiment began October 1, 1935 and continued for the remainder of the school year. A quantitative measure of the pupils' work was obtained for 126 days. There were 32 different pupils enrolled in the class; the daily average was 21.

For the first six weeks period, the work consisted of the following: A. Counting of objects; B. Writing of number symbols; C. Comparing and contrasting two number pictures or two groups of printed objects; D. Labeling of number pictures; E. Making number pictures equal by drawing parts on the smaller picture; F. Solving the 'difference' problem.

Second six weeks period: A. Solving the 'difference' problem with mimeographed material containing either pairs of number pictures or pairs of circles divided into parts; B. Writing of number symbols; C. Solving reading problems.

Third six weeks period: A. Solving the 'difference' problem; B. Solving reading problems; C. Solving the easy addition and subtraction problems in the symbol form.

Fourth six weeks period: A. Solving addition and subtraction problems with zero difficulty; B. Solving the harder addition and subtraction facts.

Fifth six weeks period: A. Tests over addition and subtraction (228 problems); B. Solving simple multiplication and division problems; C. Learning the meaning of pair and three taken as multipliers and divisors; D. Solving multiplication and division problems in the symbol form.

Last three weeks period: A. Solving simple reading problems; B. Solving addition and subtraction; C. Solving simple multiplication and division; D. Tests over addition and subtraction (180 problems).

3. The principal results were:

A. Enumeration of objects was a difficult task for the group. At the close of the first six weeks period, counting and labeling of objects to ten was satisfactory (90 per cent correct) for a majority. The writing of number symbols from memory, however, was not satisfactory.

B. Many in the class simply made number pictures similar in pattern when they were instructed to equate two pictures. The pupils could not write the symbol designating the amount to be added in order to make one picture identical with another.

C. The children could not solve reading problems. The reading material complicated the situation making the arithmetic too difficult.

D. The pupils encountered difficulty with addition and subtraction problems in the symbol form. They learned to obtain an answer rather than to think mathematically.

E. The pupils grasped simple problems of multiplication and division in the concrete situation. The experimenter succeeded in demonstrating 'pair' and 'three' taken as multipliers and divisors. About one-fourth of the group showed a mastery of multiplication (symbol problems) and about the same number showed a mastery of division.

F. The social group exerted considerable influence on both the capable and the poor student. The class was quick to perceive and then to copy a teacher's evaluation of a pupil's work. The effort of the slow pupil to keep pace with the class produced a great deal of emotional strain and always resulted in a distorted conception of arithmetic.

4. The achievement of the experimental class was compared with a selected (Control I) and also an unselected group (Control II):

A. On the first addition-subtraction test (228 problems), Control I was superior on both time and errors. A superiority of the Experimental Group over Control II was found on errors. The opposite was found with respect to time scores.

B. On the second addition-subtraction test (228 problems), Control I again proved its superiority. Neither the Experimental Group nor Control I improved upon the error scores made on the first test. The Experimental Group showed a definite improvement on time; no such improvement was recorded for Control I. This evidence indicated that a majority in the Experimental Group maturated with respect to the speed of processing.

5. A theory explaining the learning of number was based on the principle of individuation. It was assumed that this learning follows the laws of growth; this hypothesis is opposed to the notion that the child necessarily has to bridge a gap between concrete ideas and the abstract concepts of number.

Certain evidence from primitives was considered to illustrate how development of true counting (not simply memorization of number names) concerns the quantity as well as the series ideas of number.

Six stages in the development of number were considered: A. Perception of relations. The first quantitative notions of the child involve differences in size, shape and distance. B. Numbers as 'names'. A language for expressing a quantitative idea concerns at first a mathematical notion which is closely related to concrete objects. C. The first addition: compounding. The group of objects perceived by the child is an aggregate. He understands addition or subtraction as the simple expansion or contraction, respectively, of aggregates. Addition at this level, therefore, may be termed compounding. D. Transposition of number names. In some primitive languages, this development is illustrated by a change in the number words, sometimes a radical change, when various groups of objects are referred to. E. Use of number symbols: reference to aggregates. At a relatively late stage, the number name gains the meaning of a number symbol. A symbol transposes to various groups. F. The transition to abstract number ideas. An hypothesis was formulated to explain this growth. Three stages were enumerated: (a) The child's ideas are influenced by the relations of those ideas to his own being; (b) The child's concepts pertain to systems in which the variable of 'self' is constant; (c) The child is capable of holding constant (and in this sense eliminating) the influence of all the extraneous factors to which an idea is related; it enables him to consider an hypothesis 'as such', i.e., to admit of premises from which he can infer abstract relationships. In the language used above, the child at this stage can deal with the variables within a system which exists 'outside' of himself.

It is assumed that abstract number represents the third stage in reasoning mentioned above; abstract number is an independently differentiated hypothesis or premise from which the child can reason. Two points may be mentioned which appear to support the application of this theory to the learning of arithmetic. First, it was found that second grade children could estimate the answers to difficult addition and subtraction problems with fair accuracy, but that abstract comprehension of number in the sense of the term used here simply did not occur. Abstract addition and subtraction are to be found in average students only at a much higher level, probably in the fourth or fifth grade. Second, the theories previously advanced by psychologists to explain the growth of number concepts have not been applicable to the classroom situation.

6. Some applications to the teaching of arithmetic may be made as follows:

A. Drill procedures, it would seem, can be dispensed with at least until the pupils reach the sixth grade.

B. Arithmetic cannot be neglected during the first years in school. Many children will gain practical information about addition and subtraction as well as multiplication and division. A majority of second grade pupils will profit by a type of work which permits comparisons and contrasts of groups at the concrete level.

C. The pupil must be considered as a member of a social group. The goals of <u>accuracy</u> and <u>precision</u>, furthermore, should not be established apart from the development of standards of <u>taste</u> and <u>appreciation</u>.

D. The teacher must proceed slowly. It is easier to overstimulate than it is to understimulate the pupil in arithmetic. Classes should be reduced in average size to a maximum of ten or twelve in order to enable each pupil to participate fully in demonstrations and games.

E. An immediate check of a pupil's work is important. Extensive tests should be employed at intervals in order to obtain an adequate measure of each individual's performance.

VII. APPENDIX

DAILY SCORES *

I.

		S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
1.	T		100	100		60	100	100		80		80	80	80
	W		80	60	70	80	70	70		80		80	60	80
2.	M		61	83	89	78	89	89		78		83	56	100
	T		70	100	100	90	95	70		100		95	70	95
	W		90	100	100	70	100	90		95			75	100
	Th		68	85	94	79	88	76		88			79	100
	F		69	75	94	81	94	94		75			81	
3.	M		68	97	100	97	97	94		79			82	97
	T		75	38	88	38	63	75		88			63	65
	F		63	100	94	88	75	75		63			59	89
4.	M		90	90	90	80	90	75		75			90	
	T		95	95		75	95			80			55	95
	W		80	92	96	92		88		96			68	96
	Th		85	100	92	62	92	69		85			69	100
	F		85	77	92		77	100		92			85	
5.	T		88	96	96	75	96	96		100			92	92
	W		10	40	90	30	20	20		80			10	50
6.	M		08	75	67	58	100	50					08	67
	T		08	92	100	58	100	92		75			00	100
	Th			100	100	96	98	67		96			100	98
	F		90	98	100	88	96	67		98			[illegible]	100

II.

		S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
1.	T		83	100	100	100	94	79		98			81	83
	F		42	100	67	100	75	50		50			00	83
2.	T		58	100		75	96	92		92			96	88
	W		08	83		100	92	50		75			75	83
	Th		100	100		100	100	100					97	100
3.	M		58				75			67				92
	T		69	33	67	42	75	42		17			17	88
	W			95	100	95	80	94		98			89	90

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* Roman numerals refer to the six weeks periods. Arabic numerals refer to the weeks within each six weeks period.

II. (cont.)

		S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
4.	M			96	100	92							100	100
	T				100	100	100							
	W						92			88				96
	Th			100		100	88	88					92	100
	F			100		75	100	92					83	
5.	M			92		67	83	75		63				75
	T			83			92			83				92
6.	M			86			34	78		83				
	T			100		100		91		92				92
	W					92	67	86		50				92
	Th			100		75	92	92		75				

III.

		S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
1.	M			92		95	70	00		12				10
	Th			100		96		67		86			45	00
2.	T					97		86					14	
	Th					100		83		64				17
	F					100		86		72				100
3.	M					100	100	100		100				
	W							64						58
	F						94	76						64
4.	M						100	83		67				80
	T					92	100	72		64				94
	W					94	67	72		50				31
	F					00	95	00		50				16
5.	M						100	92		53				98
	T			100			97	88		45				53
	W			87			92	89		33				93
	Th			66			99	73		92				52
	F			100			98	85						28
6.	M			100		50							90	70
	T			99			94							100
	W			93			97						94	94
	F			100			98						89	51

IV.

		S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
1.	M			83		100	67	50					83	100
	T			94		97	91	97					94	81
	W			100		88	84	100					100	97
	Th			97		63	90	100					92	79
	F			97			94	92					95	91
2.	M			100		75	91	95					93	
	T			80			53	93					40	
	W			97			97	88					97	
	Th					97	97	98					97	
	F			96		69		100		100			67	
3.	M					91		100					94	97
	T					86	90	92		54			81	74
	W			94		92	100	94		88			92	96
	Th			100		100	100	100		65			84	100
	F			77		100	86	100		97			94	92
4.	M			79		100	96	98		96	98		100	
	T			90		98	92	94		84	76		95	93
	W			100		100	96	98		94	98		94	96
	Th			96		69	98	76		82	92		47	80
	F			81		95	100	82		70	86		26	80
5.	M			100		95	98	88		100	71		71	83
	T			90		86	76	86		90	52		10	81
	W	93		71		71	86	91		79	96		76	82
	Th	90		90		100	100	81		98	100		93	96
	F	81		97		90	97	97		91	97		50	91
6.	M			89		96	80	76		64	86		14	75
	T	68		94		93	97	100		86	92		50	81
	W	50		100		86	96	89		89	93		43	96
	Th	70		92		100	93	65		86	93		00	90
	F	71		83			84	82		94	100		17	90

V.

		S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
1.	M	93		98		96	98	93	91	94	99		93	98
	T	58		98		84	99	97	93	91	93		76	93
	W	27		96		87	53	82	62	80	78		52	89
	Th	60		76			90	67	60	71	90		67	52
	F	62		95			100	79	35	78	86		67	
2.	M	43		44		67	81	59	18	71	89		22	57
	T	47		100		96	89	90	32	67	100			78
	W	88		88		88	100	100	100	88	88			88
	Th	52		90		96	100	97	96	62				100
	F	96				100	100	84	98	91	100			

V. (cont.)

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
3. M	75				88	63	75	75	75	88			75
T	96				100	100	95	69	86	92			
W	100				88		88			100		75	
Th	93				96	100	86			92		60	
4. T	75		88		100	100	100	100	75	88		88	88
W			94		93	94	86		94	92		73	56
Th			50		92	92	33		08			67	92
F	100		100		100	100	100		58			100	100
5. M	21		100			96	75	00	67			83	96
T	67		100			100	100		100			100	100
W	90		96			94	90		93			24	25
Th	71		71				86		14			40	100
6. M	79		79				100		71			14	29
T	64		100				93		100			29	21
Th	50		100				100		50			100	50

VI.

	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
1. M	100		100		100	50	100		50			100	50
T	33		67		33	67	67		33			33	100
W	71		93		57	79	57		79			00	86
Th	100		100			100	100	67	100			100	100
F	93		100		71	93	00	100	86			07	93
2. M	96		93		97	97	96	74	93			41	71
Th	100		96		91	93	100	98				11	100
F	96		98			100	98	100				55	91
3. M	100		100		100	89	96	71				22	91
W	96				97	100	89	77				74	89
Av. (yr.)	78	64	91	91	84	89	81	75	75	90	85	64	81

I.

		S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25	S26
1.	T	80	80	80	80	80	100	60	100		100	60	40	80
	W	70	80	70	70	70	90	70	90		80	70	70	70
2.	M	83	50	83		83	66	61	100		83	78	66	66
	T	95	70	95		85	70	60	100		100	90	90	65
	W	100	55	80		75	75	85	100		100	85	100	90
	Th	82	62	82	82	71	91	56	100		97	85	91	97
	F	94	61	100	81	81	94	50	94		81	94	81	94
3.	M	100	72	94	97	88	91	68	100		94	91	88	88
	T	100	38		75	88		38	88		50			100
	F	75	63	69	59	65	94	53	94		50	56	88	83
4.	M	90	90		70	80		30			90	50		60
	T	90			60		95	60			85	80	90	80
	W	92	68	88	72	92	88	80	96		100	84	92	92
	Th	100	77	100	77	85	100	69	92		77	69	92	92
	F	92	92	85	100	92	100	85	100		62	77	100	92
5.	T	100	96	100		88	100	46	100		100	96		100
	W	90	60	80	30		90		80		80	10	50	100
6.	M	83	42	83		00	92	17	100		34	08		
	T	100	75	100	42	58	100	67	100		42	58	92	100
	Th	100	92	96	98	100	100		100		100	100		96
	F	100	90	96	92	96		92	100				100	100

II.

		S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25	S26
1.	T	100	79	98	72	92	92	50	100		100	94	97	100
	F	83	50	75	17	50	100	00	100		08	67	100	100
2.	T				96	100	100	97	100		100	88	96	100
	W			75	57	75	83	00	100		75	33	100	100
	Th			77	100		100	85	100		100	85	100	100
3.	M			82		20	100		100		88	83		100
	T		75	100	58	00	88	25	100		25	08	00	100
	W		86	100	98	91	96	98	98		100	98	98	100

II. (cont.)

		S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25	S26
4.	M	100	100	100	100		100	100	100		100	100	100	100
	T	100		96	96		100		100			96		100
	W		83				96	79	96					
	Th	83	88	96	100		92	96	96		96	100		100
	F	80	67	00	78		80	17	100		92	92	80	
5.	M		67	78	04		75	08				92	88	92
	T		92		58			08				92	92	
6.	M	75	00	83	08		83	17						75
	T	100	90	100	58	33	92	08	92		100	100	100	100
	W	83	92			42	75	04			92	92		
	Th	92		92		50	100	00	100		100	75	100	100

III.

		S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25	S26
1.	M	80	12	67	22		21	34	100			15	34	
	Th	83	07	66	96		10	38	100			00	100	100
2.	T	94	27	83	94		95	06	100			06	94	
	Th	86	33	78			94	11	97			08	100	71
	F	100	17	22	11		94	08	100			14	94	91
3.	M	100		100	100				100			89	100	67
	W	100					69		78					97
	F	92						61						
4.	M	70	73	67	100		100	92	100			61	100	90
	T	100	56				97	37	92			58	97	94
	W	97	50				100	17	86			42	92	94
	F	100	60				75	00	53		05	47		00
5.	M		54				89		100		100	57	97	100
	T	100	79				92		100		77	58	100	97
	W	73	82				87	55	87		88	33		
	Th	100	86		88			36	99		100	98	93	100
	F	99	70		70		92	73	100		100	70	80	100
6.	M			100	100		100		100		90	40	95	100
	T				97		99		98		100	78		
	W			92	100		100	44	99		96	100	100	
	F			85	92		84	79	100		100	99		96

IV.

		S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25	S26
1.	M	100	100	100	100		100		100		100			100
	T	88	79	81	83			94	100		94			100
	W	94	100	79	88		88	75	100		94			93
	Th	96	100	94	100						100			100
	F	98	100	95	100									98
2.	M	100	98	84	100							66	90	
	T	60		100	40							53	87	
	W	84	97	88	100							97	94	
	Th	100	100	56	95		97	88		83		100	74	
	F	100	94	78	100		75		100			96	100	
3.	M	100	100	100	100		84	88	97			97	56	100
	T	88	90				93	71	100			91	98	98
	W	88	92	88			98	75	94	96		98	94	100
	Th	100	100	66			100	86	100	100		98	100	100
	F	86	46	92	100		90	90	100	95		85	99	93
4.	M	98	66	69	98		90	70	100			96	45	100
	T	100	91	86	49		90	84	90			14	92	58
	W	100	100	92	96			92	100			100	100	94
	Th	92	81	92	92		71	73	100			67	90	96
	F	87	100	71	93		95	80	94			12	88	100
5.	M	96	81	93	70		98	25	90	95		96	86	100
	T	100	80	62	75		57	62	81	90			71	100
	W	100	32	61	96		71	32	96	82		93	93	96
	Th	100	78	100	100		100	72	100	100		100	98	96
	F	100	94	81	88		100	87	100	91		100	78	72
6.	M	100	43	46	90		61	46	88	86		93	76	100
	T	100	61	97	62		91	84	90	86		100	100	90
	W	100	64	82	82		93	75	100	100		93	86	100
	Th	100		74	82			60	96	92		69	95	100
	F	100	53	82	92		71	80	86	82		92	91	100

V.

		S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25	S26
1.	M	100	98	96	96		96	80	99	99		97	100	96
	T	97	80	93	91		87	13	95	98		100	96	96
	W	100	81	80	100		79	00	96	100		91	92	98
	Th		58	91	83		85	13	94	94		93	98	100
	F	98	67	89			48	17	100	94		88	90	96

V. (cont.)

		S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25	S26
2.	M	98	44	92	89		32	04	96	98		93	56	98
	T	100		94			84		100	97		93	[illegible]	100
	W	100	88	100	88		100	88	100	100		100	100	100
	Th	100	91	100			91		98	98		97		100
	F	98	67	87	95		96		98	90		93	100	99
3.	M	75	75	75	88		63		75	75		88	75	100
	T	100	46	63	87		67		100	98		53	96	99
	W	100	100	100	88		100		88			100	100	100
	Th	99	59		89		88		88			90	100	97
4.	T	100	75	88	88		88	63	100	100		100	75	88
	W	100	97	94	78		100	78		90		100	100	99
	Th	100	83	92	00		00	00	50	92		00	100	92
	F	100	00	100	100		100	00	83	92		00	100	100
5.	M	96	75	96	96			00	100	92		92	96	100
	T	100	50	100	100				100	100		100		100
	W	100	25		86				95	96		76	86	93
	Th	100		43	40				100	86		43	71	86
6.	M	93	93	29	64				100	93		14	64	79
	T	93	50	21	29				93			43	64	100
	Th	100	50	50	100				100			50	50	50

VI.

		S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25	S26
1.	M	100	50	100					100	100		00	100	100
	T	67	33	00	33		67		100	67		67	33	100
	W	79	64	00	00				100	100		36	43	93
	Th	100	00	00					100	100		67	100	100
	F	21	86	00	64				100	100		43	100	100
2.	M	100	85	97	93		96	47	100	98		64	88	
	Th	100	96	100	93		96	85	100	100				
	F	100	60	98	98			09	100	96				
3.	M	100	77	87	93		87	47	100	96				
	W	98	98	89	91		82	44	99	98				
Av. (yr.)		92	68	78	77	67	86	48	97	94	87	68	86	93

I.

		S27	S28	S29	S30	S31	S32
1.	T		80		60	20	80
	W	80	80		70	30	70
2.	M	72	72		72	61	94
	T	90	60		85	80	100
	W	90	20		70	55	95
	Th	68	72		83	24	97
	F	13	69		63	50	87
3.	M	88	66		82	32	100
	T	50	38		24	24	63
	F	50	18		50	31	94
4.	M	80	50		45	60	
	T	80	50		35	50	95
	W	88	56		72	64	92
	Th	62	23		62	54	23
	F	85	38		85	62	15
5.	T	88	05		96	42	
	W	30	10		30	40	100
6.	M	50	83		83	00	100
	T	50	42		50	50	100
	Th	100	96		77		
	F	100	96		100		

II.

		S27	S28	S29	S30	S31	S32
1.	T	88	81		98	83	98
	F	67	34			08	100
2.	T	96	79			38	96
	W	58	34		00		100
	Th	97	85		85		100
3.	M	78					100
	T	50	67		75		100
	W	95	79		79		91

II. (cont.)

		S27	S28	S29	S30	S31	S32
4.	M	96	92		100	83	100
	T	96	90			58	
	W	96	88				
	Th	100			100	41	
	F	92	83		100	67	
5.	M	83	00		67	08	
	T	75	40		75	42	100
6.	M	83	00		00	00	92
	T	29	83		00	00	92
	W	46	25		83	00	
	Th	50	00		27		

III.

		S27	S28	S29	S30	S31	S32
1.	M	38	00		29	00	100
	Th	83				10	100
2.	T	66				44	94
	Th	56	20		19		
	F	42	11				
3.	M	100	100		100		
	W	78					
	F						81
4.	M	92	80		67	65	75
	T	69	36		72	59	50
	W				26	61	14
	F				05	00	67
5.	M				37	56	83
	T					75	89
	W				63	00	87
	Th		86		73	78	38
	F		44		72	74	100
6.	M	90	60		80	60	90
	T	91	47		100	63	77
	W	92	47			89	100
	F	98	56			75	100

IV.

		S27	S28	S29	S30	S31	S32
1.	M	100	83			67	100
	T	92	96			88	91
	W	100	75		72	81	94
	Th	100	92		92	83	100
	F	91	84		100	78	95
2.	M	81	88		100	88	97
	T	87			87	00	100
	W	90	94		97	84	100
	Th	100	97		91	97	100
	F	100			92	42	98
3.	M	91	91		97	84	
	T	92	72		87	55	85
	W		27		94	27	93
	Th		100		93	24	100
	F		97		93	17	95
4.	M		93		100	87	98
	T		81		97	59	96
	W		92		91	84	100
	Th		74		22	00	65
	F		86		100	77	79
5.	M		95		86	60	90
	T		80		82	33	90
	W		58	93	92	14	93
	Th	94	93	98	90	62	94
	F	97	95	100	84	73	94
6.	M	89	92	100		92	31
	T	91	78	98		27	100
	W	93	84	100	50	29	
	Th		93	98		39	93
	F		92	100		41	91

V.

		S27	S28	S29	S30	S31	S32
1.	M		80			100	99
	T		58	100		67	100
	W		95			67	100
	Th		44	100		05	96
	F		42	100		20	76

V. (cont.)

		S27	S28	S29	S30	S31	S32
2.	M		26	100	56	11	89
	T		90	100		33	
	W		100	75	88	88	
	Th		73	100		92	
	F	96	93	98		40	
3.	M	75	75	75		50	
	T	89	69	94		80	
	W	100	88	100			
	Th	89	46	100		13	
4.	T	88	88	100	63	88	100
	W	81	43	100	28	82	96
	Th	100	00	100	08	00	92
	F	100	00	92	100	00	100
5.	M	96	88	100	13	21	96
	T	100	100	92	17	42	100
	W	46	14	100	43		90
	Th	57	86	86	71	00	57
6.	M	71	100	93	29	100	
	T	64		100	29	36	36
	Th	100	100	100	50	00	50

VI.

		S27	S28	S29	S30	S31	S32
1.	M	50	50	100	50	50	50
	T	67	33		33	00	33
	W	85	00	71	50	00	83
	Th	67	100	100	67	00	67
	F	79	79	93	07	71	29
2.	M	93	71	100	30	71	99
	Th	96	80	100		67	98
	F	98	53	100	76	49	100
3.	M	73	51	100	38	47	87
	W	77	38	100	76	61	
Av. (yr.)		80	61	97	62	45	83

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