

Measurement of the Credit Card Augmented Monetary Service Flows in the Economy

By

Liting Su

M.A. University of Kansas, 2013
B.S., Renmin University of China, 2011

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of Doctor of Philosophy.

Chair: William A. Barnett

John Keating

Shigeru Iwata

Jianbo Zhang

Yaozhong Hu

Date Defended: 13 March 2017

The dissertation committee for Liting Su certifies that this is the approved
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Chair: William A. Barnett

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Dissertation Abstract

Under accounting conventions, monetary assets cannot be added to liabilities, for example, credit card balances, which are liabilities to consumers. However, from an innovative perspective proposed by Professor William A. Barnett, we perceive credit cards as transaction service providers, along with monetary assets, such as currency and demand deposit. Microeconomic aggregation theory and index number theory measure service flows and thereby provide a theoretical basis to aggregate jointly over credit card services and monetary services to produce our new Augmented Divisia Monetary Aggregates. Whether services are produced by assets or liabilities is not relevant to aggregation theory.

Following this micro-theoretic approach, my dissertation is organized in the following manners:

Chapter 1 documents detailed information on the data sources used in producing the new augmented Divisia monetary aggregates, together with other relevant sources that we extensively explored for availability of the needed credit card variables.¹

Chapter 2 contains the theoretical derivation needed to measure the joint services of credit cards and money. We provide and evaluate two such aggregate measures, having different objectives. We initially apply our new aggregates to NGDP nowcasting. Both aggregates are being

¹ This paper was invited by a special issue editor of the Elsevier journal *Research in International Business and Finance* and appeared in vol 39, Part B, January 2017, pp. 899-910. The special issue is the proceedings of a conference held in Paris on June 4-5, 2015.

implemented by the Center for Financial Stability, which will provide them to the public through monthly releases, as well as to Bloomberg Terminal users.²

Chapter 3 extends the above theory by removing the assumption of risk neutrality to permit risk aversion in the decision of the representative consumer.³

Chapter 4 investigates bivariate time series properties of Divisia money and nominal GDP to investigate the viability of recent proposals by authors who have advocated a role for a Divisia monetary aggregate in nominal GDP targeting.⁴

Chapter 5 provides theory needed to measure the supply of the joint services of credit cards and money by financial firms. The resulting model can be used to investigate the transmission mechanism of monetary policy and to measure inside money and value-added produced by banks. This measurement could also be helpful to economists working on the national accounts as well as to those investigating the growing role of shadow banking.⁵

² This paper has been submitted to the *Journal of Money, Credit, and Banking* and is currently in revise and resubmit status.

³ This paper has been invited and accepted for publication in “Macroeconomic Advances in Honor of Clifford Wymer,” a special issue of *Macroeconomic Dynamics*. The editors of the special issue are Giovanni De Bartolomeo, Daniela Federici, and Enrico Saltari. A short form of the theoretical results, without the proofs or discussion, has been published in the *Economics Bulletin*, vol 36, no 4, 2016, pp. A223-A234.

⁴ This paper has been invited and accepted for publication in *The International Journal of Business and Globalisation*, for a special issue containing selected papers from the May 2015 International Conference on Economic Recovery in the Post-Crisis Period in Skopje, Republic of Macedonia.

⁵ This paper has been submitted to the *Journal of Banking and Finance*, which has very high impact factor. We have not yet received referee reports or decision about that submission. We subsequently plan to begin econometric research using the theory in modeling bank behavior.

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Chapter 1

Data Sources for the Credit-Card Augmented Divisia Monetary Aggregates

William A. Barnett

University of Kansas and Center for Financial Stability⁶

Liting Su

University of Kansas and Center for Financial Stability⁷

March 20, 2016

⁶ University of Kansas, Lawrence, Kansas and Center for Financial Stability, New York City.

⁷ University of Kansas, Lawrence, Kansas and Center for Financial Stability, New York City.

Abstract

In 2013, the Center for Financial Stability (CFS) initiated its Divisia monetary aggregates database, maintained within the CFS program called Advances in Monetary and Financial Measurement (AMFM). The CFS will soon be making available Divisia monetary aggregates extended to include the transactions services of credit cards. The extended aggregates will be called the augmented Divisia monetary aggregates and will be available to the public in monthly releases. The new aggregates will also be available to Bloomberg terminal users. The theory on which the new aggregates is based is provided in Barnett and Su (2014).⁸ In this paper, we provide detailed information on the data sources used in producing the new augmented Divisia monetary aggregates.

Keywords: monetary aggregates; credit cards; aggregation theory; index number theory; data; Divisia index.

JEL Classifications: G, C8, E4.

⁸ A revised and updated version of that theory can be found in Barnett, Chauvet, Leiva-Leon, and Su (2016).

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This exhaustive data search has been supported by many people. They generously provided insight and expertise that assisted our research. We would like to express our special gratitude to Kate Flanagan at the Center for Financial Stability, Travis Nesmith at the Federal Reserve Board, Robert Hunt at the Philadelphia Federal Reserve Bank, Randall Olsen and Margaret Lowden at Ohio State University, Lee Andrew Smith at the Kansas City Federal Reserve Bank, Robert J. Tetlow at the Federal Reserve Board, Tamas Briglevics at the National Bank of Hungary, Robert DeYoung at the University of Kansas Business School, Ed Bachelder at Blueflame Consulting, David C. Stewart at McKinsey & Company, Geoffrey R. Gerdes and May X. Liu at the Federal Reserve Board, Jeff Sigmund at the American Bankers Association, Mark Wicks at the Federal Reserve Board, Yvonne St. Andrew and Monica Dalto at The Nilson Report, Lindsey Bauer at First Annapolis Consulting, Diana Popa at CardHub and WalletHub, and the Investor Relations Departments of Visa, MasterCard, American Express, and Discover.

1. Introduction

The Center for Financial Stability (CFS) will soon be making available Divisia monetary aggregates extended to include the transactions services of credit cards, in accordance with Barnett and Su (2014) and Barnett, Chauvet, Leiva-Leon, and Su (2016). To construct the resulting “augmented Divisia monetary aggregates,” we extensively explored relevant sources for the needed credit card variables. Since the credit card transactions volumes data are not publicly available from federal government agencies or the Federal Reserve System, our search took over a year. In this paper we make available the results of our search, not only to document the sources for the new CFS augmented Divisia monetary aggregates, but also for reference by future researchers who might want to work in this area and therefore be confronted with similar data search challenges. In this paper, we focus entirely on the credit card components of the new aggregates, since the sources for the other components have been documented by Barnett, Liu, Mattson, and Noort (2013), as was needed for construction of the original CFS aggregates.

The most difficult part of this search was the need to find data for credit card transactions volumes. To be consistent with the theory in Barnett and Su (2014) and Barnett, Chauvet, Leiva-Leon, and Su (2016), the credit card quantities to include in the augmented Divisia index formula are the monthly credit card transactions volumes, not the credit card balances. The balances include revolving debt used to pay for transactions in prior periods. To include those funds again in a subsequent period would produce double counting of transactions services. But only the total balances are available from governmental sources, such as the Federal Reserve. Those credit card balances can be used as related indicator variables within the Chow-Lin method to

interpolate the transactions volumes monthly. Those volumes are only available quarterly, while the total balances are available monthly from the Federal Reserve. But the credit card debt balances should not themselves be entered into the Divisia index formula to measure contemporaneous credit card transactions services.

The data search process was long and arduous. For the benefit of future researchers, who might be confronted with similar data needs, this paper not only documents our successful location of relevant sources but also makes public the many dead ends we encountered.

2. Adopted Data Sources

2.1 Data Sources for Credit Card Transactions Volumes

As observed by Barnett and Su (2014) and Barnett, Chauvet, Leiva-Leon, and Su (2016), implementing the theory using credit card transactions volumes has heavy data requirements. Numerous sources were extensively explored, as discussed in section 3. In this subsection, we introduce in detail where to find the ultimate sources we adopted and how to locate the specific data from the financial reports.

Our primary sources are the quarterly financial reports of the four credit card companies, Visa, MasterCard, Discover, and American Express. The total payment volume each period is summed over the four. According to trade jargon, the word “credit card” applies only to those four. Charge cards and store cards (called “private label” cards), are not credit cards. To be a credit card, it must be accepted for all goods and services not requiring cash-only payment. That rules out store cards, such as gasoline cards or department store cards. In addition, the card must

provide a line of credit. That rules out charge cards for which payment is required in full at the end of the period. To model consumer charge cards decisions, we would need to include an inequality constraint requiring credit card debt to be paid off at the end of the period. Our model does not have such a constraint. Our representative consumer model assumes that the representative consumer has access to a line of credit, if the debt is not paid off during the same period.⁹

For Visa cards, the quarterly reports can be found in their investor relations site:

<http://investor.visa.com/financial-information/quarterly-earnings/>

- ➔ Select the period of interest.
- ➔ Go to “Operational Data.”
- ➔ Page 1, first table.
- ➔ “Visa Credit Programs” section.
- ➔ Locate the row for “US.”
- ➔ Locate the column for “Payments Volume.”

For example, for the 3 months ending December 31, 2015, the Visa credit card transactions volume in the United States is 358 billion dollars. The release date each quarter is about 4 weeks

⁹ Historically, most American Express cards were charge cards, but in recent years even the former charge cards issued by American Express provide access to a line of credit. As a result, the distinction between charge cards and credit cards issued by American Express is not relevant to our model.

after the quarter ends.

Similarly, for MasterCard, the quarterly reports can be found in their investor relations site:

<http://investor.mastercard.com/investor-relations/financials-and-sec-filings/quarterly-results/default.aspx>

- ➔ Select the period of interest.
- ➔ Go to “Supplemental Materials.”
- ➔ Page 1, first table.
- ➔ “MasterCard Credit and Charge Programs” section.
- ➔ Locate the row for “United States.”
- ➔ Locate the column for “Purchase Volume.”

For example, for the 3 months ending December 31, 2015, MasterCard transactions volume in the United States was 174 billion dollars. The release date is about 4 weeks after the quarter ends.

For Discover Cards, the quarterly reports are researched by:

<http://investorrelations.discoverfinancial.com/phoenix.zhtml?c=204177&p=irol-quarterlyresults>

- ➔ Locate “Earnings” for the relevant quarter and year.
- ➔ Go to “Financial Supplement (PDF).”
- ➔ Search for the row for “Discover Card Sales Volume.”
- ➔ Locate the column for the relevant quarter and year.

For example, Discover credit card transactions volume in 2015 quarter 4 was 31.672 billion dollars. The release date is about 3 to 4 weeks after the quarter ends.

Lastly the quarterly reports for American Express can be found at:

<http://ir.americanexpress.com/Earnings-and-Events>

- ➔ Select the tab for “Past Events” and the period of interest.
- ➔ Go to “American Express Earnings Conference Call.”
- ➔ Go to “Financial Tables.”
- ➔ Search for “Card billed business (F).”
- ➔ Locate the row for United States,
- ➔ Locate the column for the relevant quarter and year.

For example, American Express credit card transactions volume in 2015 quarter 4 was 189.9 billion dollars. The release date is about 2 to 3 weeks after the quarter ends.

Another relevant source of credit card transactions volumes is a spreadsheet in the statistics site of PaymentsSource.com:

<http://www.paymentssource.com/statistics/>

- ➔ “US Quarterly Credit and Charge Card Payment Volumes: 3Q 2006 – Current.”

The spreadsheet contains payment volumes from 2006 Q2 to 2013 Q2. It confirmed that the transactions volumes found from the above-mentioned financial reports were correctly located in those reports. In addition, that spreadsheet improved the efficiency and accuracy of our collection of transactions volume series. As data up to the second quarter of 2013 were all included in the spreadsheet, we only needed to check the precision of those data and to update to the current period based on the four companies’ financial reports. From both of our sources, credit card transactions volumes are available only since 2006 quarter 3, which is when the firms

went public.¹⁰

The data from both sources are seasonally unadjusted. We adopted the latest Census X-13 ARIMA-SEATS program to adjust the level data of credit card transactions volumes.

A detailed description of the methods and theory of X-13 ARIMA-SEATS can be found at the US Census Bureau website.¹¹ In addition, its reference manual, theoretical background, and empirical applications are also available on the US Census Bureau website.¹²

2.2 Data Sources for Chow-Lin Interpolation

Since the credit card transactions volumes are released only on a quarterly basis, we need to interpolate the quarterly data monthly to permit monthly publication and release of the augmented Divisia monetary aggregates. For this purpose, we selected the well known and widely respected Chow-Lin (1971) procedure, which provides a unified approach to interpolation, distribution, and extrapolation.¹³ The procedure requires regression on a related

¹⁰ An exception is American Express, which has been a public company for a much longer time. But during the earlier years, American Express issued only charge cards, not credit cards.

¹¹ <https://www.census.gov/srd/www/x13as/>.

¹² <https://www.census.gov/srd/www/x13as/papers4newusers.html>.

¹³ According to Chow-Lin (1971), interpolation and distribution are respectively defined as follows. (a) Given the value of a time series at the beginning of each quarter for n quarters, and given the value of a related series at the beginning of each month for these $3n$ months, the problem of interpolation is to estimate the first series for the remaining $2n$ months. (b) Given the value of a series of flows during each quarter for n quarters, and the value of a related series for each month, the problem of distribution is to estimate the first series for the $3n$ months.

indicator series to obtain best linear unbiased estimates (BLUE) of the monthly series.

To implement the Chow-Lin procedure, at least one highly correlated series needs to be chosen as an indicator. Five potential candidates were selected for that purpose and their merits compared for the interpolation:

- Total consumer credit outstanding.

The Federal Reserve Board provides Total Consumer Credit Outstanding, with unique identifier “G19/CCOUT/DTCTL.M,” on a monthly basis through the G.19 survey by the Data Download Program.¹⁴ This seasonally adjusted series covers most credit extended to individuals, excluding loans secured by real estate. The release date is on the fifth business day of each month. Those data are also available in the St. Louis Federal Reserve Bank’s database, FRED, under the tag TOTALSL. This series is available beginning on January 1943.

- Revolving consumer credit outstanding.

This seasonally adjusted series, with unique identifier “G19/CCOUT/DTCTLR.M,” is from the same source as Total Consumer Credit Outstanding and is a component of it, while the other component is “non-revolving credit.” Credit card outstanding balance contains revolving consumer credit outstanding as a major component. Revolving Consumer Credit Outstanding is

¹⁴ <http://www.federalreserve.gov/datadownload/Choose.aspx?rel=G19>.

available beginning on January 1968.

- Credit card interest rate (all accounts).

This series is provided in the Federal Reserve Board’s G.19 release. The release provides two such interest rates. One is the interest rate on only those credit card accounts that pay interest to the bank issuing the account. The other interest rate, which is lower, includes those accounts that are not paying interest to the banks. The noninterest yielding accounts are paid off within the month. Our model is for the representative consumer, aggregated over both those consumers paying interest on credit card accounts and those that are not. Hence, the interest rate we use is the lower one, which accounts for the fact that not all credit card transactions volumes are being charged interest. This series is called Commercial Bank Interest Rate on Credit Card Plans, All Accounts, with unique identifier “G19/TERMS/RIFSPBICCC_N.M.” It is not seasonally adjusted, which is consistent with the convention at the Center for Financial Stability (CFS) and also at the Federal Reserve for interest rates. This series is available since 1994 Q4.

Note that this interest rate is also the choice used in the user cost formula for the credit card transactions services. At the present time in the United States, 58.7% of active credit card accounts pay interest.¹⁵ Since the interest rate paid on those accounts is high, the lower average credit card interest rate in the G.19 survey, averaged over both groups, is still much higher than

¹⁵ See the proportion of “revolvers,” “transactors,” and “dormants” on the following document provided by American Bankers Association: www.federalreserve.gov/datadownload/Choose.aspx?rel=G19.

our benchmark rate. As a result, the user cost is always positive – in fact very positive. Although the benchmark rate is higher than the interest rates paid to consumers on secured assets, the rate of interest on credit card debt is not on a consumer asset and is not secured to the issuing firms. For the issuing firms, those accounts are assets. Credit card debt is not secured and subject to fraud risk.¹⁶

- Nominal user cost of credit card services

The following formula for the nominal user cost of credit card services was derived in Barnett and Su (2014) and subsequently in Barnett, Chauvet, Leiva-Leon, and Su (2016):

$$\tilde{\pi}_{jt} = \frac{p_t^*(e_{jt} - R_t)}{1 + R_t},$$

where p_t^* is the true cost of living index, e_{jt} is credit card interest rate, and R_t is the yield on the benchmark asset during period t . We use the Labor Department's Consumer Price Index (CPI) to represent the true cost of living index, p_t^* , since the CPI is used as the “cost of living” in wage contracts. For the credit card interest rate, e_{jt} , we use the series discussed above. For the yield on

¹⁶ Even if credit card debt were secured and not subject to fraud risk, there would be no internal contradiction in assuming that the maximum interest rate available to one category of economic agents (consumers) is lower than that available to another category of economic agents (credit card companies), although the risk born by credit card companies is the primary reason for the high interest rate on credit card debt. The greatest source of risk is credit risk (called Net Credit Loss), but fraud risk along with high operating costs all play a role in the high interest rates on credit card debt.

the benchmark asset adopted by the CFS and used by us, see Barnett, Liu, Mattson, and van den Noort (2013). Restricted by the credit card interest rate's availability, the nominal user cost of credit card services is available since October 1994.

- Real user cost of credit card services.

The formula for the real user cost of credit card services is as follows:

$$\tilde{\pi}_{jt}^* = \frac{\tilde{\pi}_{jt}}{p_t^*} = \frac{e_{jt} - R_t}{1 + R_t}.$$

As with the nominal user cost of credit card services, the real user cost is available since October 1994.

To implement the Chow-Lin procedure, we used the statistical software, R. We used the temporal disaggregation package provided by R, and the descriptive links are below.

<https://cran.r-project.org/web/packages/tempdisagg/index.html> (download link)

<https://cran.r-project.org/web/packages/tempdisagg/tempdisagg.pdf> (manual)

<https://journal.r-project.org/archive/2013-2/sax-steiner.pdf> (tutorial article)

As the data are limited by the availability of the credit card interest rate, which is available only after 1994 Q4, we extrapolated and interpolated the data from October 1994 to the present, with all possible combinations of the above indicator series. The resulting table for the Chow-Lin procedure is in the Appendix table 1. Statistical significance tests determined that the best model with Chow-Lin uses only one indicator as a related series: total consumer credit outstanding.

Since credit card transactions volumes start in 2006 Q3, while all the indicator series start from October 1994, we investigate extrapolation backwards from 2006 Q3 to October 1994. We found that the backwards extrapolation of transactions volumes was highly nonrobust to the choice of indicators, since the extrapolation has no anchor in October 1994 without availability of transactions volumes data before 2006 Q3. As a result, we have forgone backwards extrapolation, and used Chow-Lin only for interpolation beginning in 2006 Q3.

To summarize all the adopted data sources to construct the augmented Divisia Index, we provided table 2 in the Appendix, following the tradition of Barnett, Liu, Mattson, and Van Den Noort (2013) and Anderson and Jones (2011). In addition, a graphical demonstration of the Chow-Lin interpolation is provided in Appendix Figure 1.

3. Other Potentially Relevant Sources

While searching for our chosen sources of credit card transactions volumes, we encountered numerous dead ends. We provide a summary for researchers interested in replicating our work or pursuing relevant extensions and alternatives to our approach.

3.1 Federal Reserve Board G.19 Release

The Federal Reserve Board G.19 release, “Consumer Credit,” reports outstanding credit extended to individuals for household, family, and other personal expenditures, excluding loans

secured by real estate.¹⁷ It was one of the first sources that we searched for credit card transactions volumes. Unfortunately, the Federal Reserve does not provide those transactions volumes. But this is where we acquired the total consumer credit outstanding, used as indicator for the Chow-Lin monthly volume interpolation procedure, and the credit card interest rate for all accounts to calculate the user costs of the credit card transactions services.

3.2 Consumer Credit Snapshot by Federal Reserve Bank of Philadelphia

This source provides updated statistics related to consumer credit and consumer payments.¹⁸ The most relevant data are at the “Consumer Debt” tab, which is a mirror of the G.19 statistics mentioned above.

3.3 Federal Reserve Payment Study, 2013

This document provides an overview of the aggregate trends in noncash payments in the United States.¹⁹ It does provide a few annual transactions volumes. For example, it reports that the total value of 2012 private label (store) card transactions in the U.S. was \$2.21 trillion, which is consistent with the data sources we mentioned above. However, it is far from a detailed and systematic source providing historical data at adequate frequency.

The Electronic Payments Study was performed by Blueflame Consulting. According to Ed

¹⁷ <http://www.federalreserve.gov/Releases/G19/current/>.

¹⁸ <http://www.philadelphiafed.org/consumer-credit-and-payments/statistics/>.

¹⁹ https://www.frbservices.org/files/communications/pdf/research/2013_payments_study_summary.pdf.

Bachelder, the Director of Research and Analytics at Blueflame Consulting, “the credit card data was collected on annual total basis, not monthly as is described in the methodology documents. It was also gathered on a confidential basis and cannot be shared beyond what was publicly released due to a number of legal restrictions.”

3.4 Credit Card Market Monitor by American Bankers Association

The Credit Card Market Monitor does not provide transactions volumes, but does provide an informative figure: Distribution of Accounts by Behavior Type.²⁰ This gives us information about how many credit card accounts are active. That data source, from the American Bankers’ Association, also provides information about how many active accounts are carrying credit card debt into the next period and are thereby paying credit card interest.

3.5 Call Reports Processed by FFIEC Central Data Repository

This file contains data from Call Reports received and processed by the FFIEC (Federal Financial Institutions Examination Council) Central Data Repository (CDR) as of 2016-01-14.²¹ The file is intended to provide an integrated view of financial data across those financial institutions filing Call Reports in a format that could facilitate analysis of such data by the public. The file does not necessarily provide the most recent Call Report and financial institutions data available in FFIEC CDR. In this source, we failed to find the credit card

²⁰ <http://www.aba.com/Press/Documents/12.16.14ABACreditCardMonitorFAQ.pdf>.

²¹ <https://cdr.ffiec.gov/public/PWS/DownloadBulkData.aspx>.

transactions volumes we need.

3.6 Creditcards.com

This site contains much informative data about credit card usage trends in the United States. For example, the transactors versus revolvers trend from 2009 to 2014 indicates that the percent of American households carrying rotating credit card debt from month to month (revolvers) have decreased from 44% in 2009 to 34% in 2014.²²

Meanwhile, the site's credit card market share statistics page provides some payment volume data, but only the 2013 and 2014 annual purchase volumes for each card network.²³ Although those volumes are inadequate for our use, the site's footnotes reveal the sources of purchase volumes: the financial reports for the four card companies. However, the footnote does not provide instructions on how to locate those data from within those financial reports.

3.7 The Nilson Report

The Nilson report purports to publish the US credit card purchase volumes quarterly and the global figures every six months. However, this requires a subscription to all the issues of the

²² <http://www.creditcards.com/credit-card-news/credit-card-debt-statistics-1276.php>. But the following statement is from www.motherjones.com/kevin-drum/2011/10/americans-are-clueless-about-their-credit-card-debt. "In the four working age categories, about 50% of households think they have outstanding credit card debt, but the credit card companies themselves think about 80% of households have outstanding balances." Since these percentages are of total households, including those having no credit cards, the percent of credit card holders paying interest can be even higher.

²³ <http://www.creditcards.com/credit-card-news/credit-card-market-share-statistics-1264.php>.

Nilson reports. The cost is currently \$1495 for each year (23 issues per year). New subscribers also receive a USB flash drive containing the last five years of issues. The cost for previous years extending back to 1997 is \$295 for each year, supplied on a CD ROM or flash drive, while the cost for the years of 1996-1990 is \$300 each year, supplied only in a hard copy format. Considering the cost versus the amount of information we need, and our inability to determine whether their reports provide exactly what we need, we decided instead to look further into the sources from which Nilson acquires those figures.

3.8 The 2015 Consumer Financial Literacy Survey

This survey was conducted online within the United States by Harris Poll on behalf of the NFCC (National Foundation for Credit Counseling) between March 11 and March 13, 2015 among 2017 adults age 18+.²⁴ Though it does not contain the credit card transactions volumes, it does provide an overview of the credit card expenditure trend in the US. For example, according to this report, one in three U.S. adults (33%) indicate their household carries rotating credit card debt from month to month, with about one in ten adults (11%) saying they roll over \$2500 or more in credit card debt each month.

3.9 SEC Filings of the Four Card Companies

We found the same figures for transactions volumes in the SEC filings of the four credit card

²⁴ https://www.nfcc.org/wpcontent/uploads/2015/04/NFCC_2015_Financial_Literacy_Survey_FINAL.pdf.

companies as in their financial reports. The SEC filings share the same release dates as their annual reports.

For Visa, the SEC filings can be found here:

[*http://investor.visa.com/sec-filings/*](http://investor.visa.com/sec-filings/)

- ➔ Select the “8-K” filing of the relevant period.
- ➔ Look for “Operational Performance Data” section.
- ➔ First table, under the title “Visa Credit Programs.”
- ➔ Locate the row of “US.”
- ➔ Locate the column of “Payments Volume.”

That 8-K Filing is usually released four weeks after the quarter ends.

The SEC filings for MasterCard are available from:

<http://investor.mastercard.com/investor-relations/financials-and-sec-filings/sec-filings/default.aspx>

- ➔ Select the “8-K” filing of the relevant period.
- ➔ “MasterCard Incorporated Operating Performance” table.
- ➔ “MasterCard Credit and Charge Programs” section.
- ➔ Locate the row for “United States.”
- ➔ Locate the column for “Purchase Volume.”

That 8-K Filing is usually released four weeks after the quarter ends.

For Discover cards, the SEC filings are posted at:

<http://investorrelations.discoverfinancial.com/phoenix.zhtml?c=204177&p=irol-sec>

- ➔ Select the “8-K” filing of the relevant period.
- ➔ Search for the row for “Discover Card Sales Volume.”
- ➔ Locate the column for the quarter and year.

That 8-K Filing is usually released about 3 to 4 weeks after the quarter ends.

Finally, the American Express SEC filings are posted on:

<http://ir.americanexpress.com/docs.aspx?iid=102700>

- ➔ Select the “8-K” filing of the relevant period.
- ➔ Search for “Card billed business (F).”
- ➔ Locate to the row for the United States.
- ➔ Locate the column for the quarter and year.

That 8-K Filing is usually released about 2 to 3 weeks after the quarter ends.

There are numerous types of other files apart from 8-K Filings on the SEC filings webpage. As a result, the needed files are very scattered. Moreover, there usually are several files called “8-K Filings” in a single period and only one of them contains the relevant spreadsheet. Therefore, we do not recommend acquiring the transactions volumes through this channel. Those data are more conveniently acquired from credit card companies’ annual reports.

3.10 Credit Bureaus

We have also looked into credit bureaus such as Equifax, Experian, and TransUnion for credit card transactions volumes data. However, providing this kind of data is not primarily what they do, they can be missing some relevant information. Researchers would need to acquire information from all of them, with some overlap. Following this path would be very time consuming, with possibly inadequate results.

3.11 FirstData

FirstData has a product called SpendTrend, which we originally thought could be helpful.

However, FirstData only has information about the card volumes processed through FirstData, so is missing a huge chunk of the relevant data. Furthermore, they would not provide any additional information they have privately.

3.12 First Annapolis

First Annapolis responded to our data requests by informing us of two other possible sources: the Federal Reserve Payment Study and the Philadelphia Federal Reserve Consumer Credit Snapshot. We acquired no positive results from those two sources.

3.13 CardHub

This website contains an annual purchase volume table, based on the SEC filings from Visa, MasterCard, American Express, and Discover. The table is provided only for the year 2014 and contains only annual data.²⁵ The Communications Manager of CardHub replied that they do not have any data other than those listed in that one report.

CardHub.com contains a table providing the Consumer Credit Card Debt from 2008 Q4.²⁶ That reported debt is the total outstanding credit card debt in each quarter, not the needed transactions

²⁵ <http://www.cardhub.com/edu/market-share-by-credit-card-network/>.

²⁶ <http://www.cardhub.com/edu/credit-card-debt-study/#data-and-graphs>.

volumes.

But these data help to confirm a comment mentioned by an expert on the Federal Reserve G.19 statistics team, when we contacted that team for relevant information. What he mentioned was that the G.19 statistics of revolving credit outstanding is mainly credit card debt outstanding, which comprises more than 90% of revolving credit. Comparing the two series in each quarter, we found this to be the case. This information was relevant to our choice of indicator variables in the Chow-Lin interpolation of transactions volumes from quarterly to monthly.

Another table on CardHub contains total credit card debt balance. The source is the Federal Reserve Bank of New York consumer credit panel.²⁷

3.14 Investor Relations Departments of Credit Card Companies

The Investor Relations Departments of Visa, MasterCard, American Express, and Discover were very helpful in our search to locate the transactions volumes, both via email and phone calls. The contact information is listed below:

A. Visa

Phone: 650-432-7644

Email: ir@visa.com

²⁷ <http://www.cardhub.com/edu/credit-card-debt/#card-debt>.

B. MasterCard

Phone: [914-249-4565](tel:914-249-4565)

Email: Investor_Relations@mastercard.com

C. Discover

Phone: 224-405-4555

Email: investorrelations@discover.com

D. American Express

Phone: 212-640-6348

Email: ir@aexp.com

3.15 Statista

This source claims to contain credit card purchase volume in the United States for the years 2000-2014 by type of credit card.²⁸ The report costs \$325. Upon contacting the support team, we were sent the report for free. However, the report only contains a snapshot of purchase volumes in the years of 2000, 2010, and 2014 by each credit card company.

3.16 Consumer Finance Monthly, Ohio State University

According to Professor Randall Olsen from the Ohio State University, they have stopped collecting the Consumer Finance Monthly survey, but are allowing people to access the past data they did collect. However, that survey did not include the amounts charged on credit cards. The

²⁸ <http://www.statista.com/study/12118/credit-cards-in-the-united-states-statista-dossier/>.

survey focused on stocks, including asset quantities, liabilities, and net worth, rather than spending flows.

3.17 CPRC Presentation

The Federal Reserve Bank of Boston publishes its Consumer Payment Research Center (CPRC) Events and Presentations. This site has preliminary monthly figures for average value of credit card purchases and average number of credit card purchases for U.S. adults.²⁹ One way to estimate the average purchase volume per month is to calculate the product of the two series and divide by the number of U.S. adults.

But if the number of transactions and the transactions values are highly correlated, then the product of the averages will not be an accurate estimate of the average of the products. As a result, we did not adopt that approach.

3.18 Diary of Consumer Payment Choice (DCPC)

This source offers consumers-only data. But the DCPC had not been released officially at the time we were looking for the data. We had signed up on the email list of the Consumer Payment Research Center to receive news about new data releases.³⁰ However, we have so far received

²⁹ <http://www.bostonfed.org/economic/cprc/presentations/index.htm>.

³⁰ <http://www.bostonfed.org/economic/cprc/contact/contact.htm>.

no helpful information from this source.

3.19 Federal Reserve Survey of Consumer Finances

This survey based report is available only every three years, with the most recent being for 2013.³¹ We have not found that source to be helpful.

3.20 Bankrate Monitor Survey

The Bankrate Monitor Survey provides fixed and variable credit card interest rates in its weekly report.³² The “fixed” column refers to fixed-rate credit cards, and “variable” column refers to variable-rate credit cards.³³ In fact, there are only five fixed-rate credit cards in Bankrate’s weekly survey or rates, including none from a major bank. As we are concerned with all the accounts rather than specific group categories, we did not adopt the data from Bankrate Monitor Survey.

4. Conclusion

To implement the theory originated by Barnett and Su (2015) and Barnett, Chauvet, Leiva-Leon, and Su (2016), we extensively explored relevant sources for the needed credit card variables. As the relevant credit card data are not available from governmental sources, the search for level and

³¹ <http://www.federalreserve.gov/econresdata/scf/scfindex.htm>.

³² <http://www.bankrate.com/finance/credit-cards/current-interest-rates.aspx>.

³³ <http://www.bankrate.com/finance/credit-cards/fixed-rate-cards-going-away.aspx>

rate data took over a year. We detail the results of this search in this paper as reference for future researchers confronted with a similar problem. Our focus in this paper is limited to the credit card data, since the other components for the CFS aggregates have been explained in Barnett, Liu, Mattson, and van den Noort (2013). The most difficult part of the search was to acquire credit card transactions volumes, as needed by the theory, since those volumes are not provided by any governmental sources. In our search, we encountered many “dead ends,” revealed in this paper for the benefit of future researchers on this subject. We primarily focus on our chosen best sources.

The theory and data have been integrated and applied by Barnett, Chauvet, Leiva-Leon, and Su (2016) to produce the new augmented Divisia monetary aggregates, which are to be made available to the public in regular monthly releases by the Center for Financial Stability in NY City and to Bloomberg terminal users. Barnett, Chauvet, Leiva-Leon, and Su (2016) have found the new aggregates to be highly informative. With the inclusion of credit card transactions services, the augmented aggregates have been found to lead conventional Divisia at all levels and to correlate better with nominal GDP. As indicators of nominal GDP, the new augmented Divisia monetary aggregates are found to have exceptional value in nowcasting. The construction of the augmented Divisia monetary aggregates has opened up a new branch and direction for our future research and for the research of others interested in the role of monetary services in the macroeconomy.

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Appendix

Table 1. Point estimates of the coefficients of indicators in the Chow-Lin procedure to interpolate and extrapolate credit card transactions volumes with combinations of indicator series (1994 Q4 - 2015 Q2).					
	Total credit	Revolving credit	Credit card interest rate	Nominal user cost	Real user cost
(1)	0.06039 (***) (3.907)				
(2)		0.1003 (*) (2.211)			
(3)			-2.301 (-1.123)		
(4)				7.79E-04 (0.002)	
(5)					-34.01 (-0.393)
(6)	0.11866 (*) (2.619)	-0.15811 (-1.398)			
(7)	0.05953 (***) (3.924)		-2.27779 (-1.174)		
(8)	0.06259 (***) (3.965)			-0.19457 (-0.504)	
(9)	0.06323 (***) (4.116)				-68.5469 (-0.832)
(10)		0.09995 (*) (2.221)	-2.28782 (-1.132)		
(11)		0.10110 (*) (2.177)		-0.06671 (-0.167)	
(12)		0.10213 (*) (2.215)			-45.29199 (-0.529)
(13)			-3.6347 (-1.409)	0.4256 (0.855)	
(14)			-2.939 (-1.108)		42.473 (0.384)

(15)				18.809 (***) (6.273)	-4073.196 (***) (-6.288)
(16)	0.11665 (*) (2.592)	-0.15555 (-1.383)	-2.20313 (-1.157)		
(17)	0.12425 (*) (2.683)	-0.16644 (-1.449)		-0.2519 (-0.661)	
(18)	0.12489 (**) (2.739)	-0.16754 (-1.473)			-77.80956 (-0.961)
(19)	0.05792 (**) (3.536)		-2.67904 (-1.074)	0.12967 (0.265)	
(20)	0.06014 (***) (3.777)		-2.08534 (-0.821)		-12.98218 (-0.122)
(21)	2.04E-02 (1.341)			16.97 (***) (4.925)	-3691 (***) (-4.995)
(22)		0.09577 (*) (2.08)	-3.30413 (-1.287)	0.32298 (0.648)	
(23)		0.09889 (*) (2.144)	-2.6484 (-1.007)		23.93598 (0.218)
(24)		0.03451 (0.978)		18.01 (***) (5.566)	-3906 (***) (-5.608)
(25)			-0.1693 (-0.086)	18.7412 (***) (5.94)	-4054.17 (***) (-5.821)
(26)	0.11587 (*) (2.447)	-0.15441 (-1.333)	-2.30039 (-0.932)	0.03107 (0.064)	
(27)	0.11954 (*) (2.561)	-0.15982 (-1.387)	-1.73876 (-0.694)		-30.99516 (-0.293)
(28)	0.04094 (0.996)	-0.05095 (-0.545)		16.41 (***) (4.525)	-3574 (***) (-4.608)
(29)	0.02037 (1.316)		-0.06286 (-0.032)	16.95 (***) (4.729)	-3684 (***) (-4.715)
(30)		0.03444 (0.96)	-0.1317 (-0.066)	17.95 (***) (5.299)	-3891 (***) (-5.233)
(31)	0.04092	-0.05091	-0.02285	16.40 (***)	-3572 (***)

(0.978)

(-0.535)

(-0.011)

(4.364)

(-4.376)

Notes: We use *** to denote significance at the 0.1% level, ** at the 1% level, and * at the 5%

level. The t-ratios are in parentheses below the point estimates.

Table 2. A summary of adopted data sources for the augmented Divisia Index:

Monetary Asset	Level Source	Sample Period	Rate	Rate Source
Credit Card Transactions Volumes	Financial Reports from Visa, MasterCard, American Express, and Discover	Since 2006 Q3 (quarterly data interpolated into monthly by Chow-Lin method)	Interest rates of Credit Card Plans – All Accounts	FRED / G.19

Indicator Series for Chow-Lin Interpolation	Level Source	Sample Period	Rate	Rate Source
Total Consumer Credit Outstanding	FRED / G.19	Since 2006.07	N.A.	N.A.

Figure 1a. Demonstration of Chow-Lin Interpolation, before interpolation.

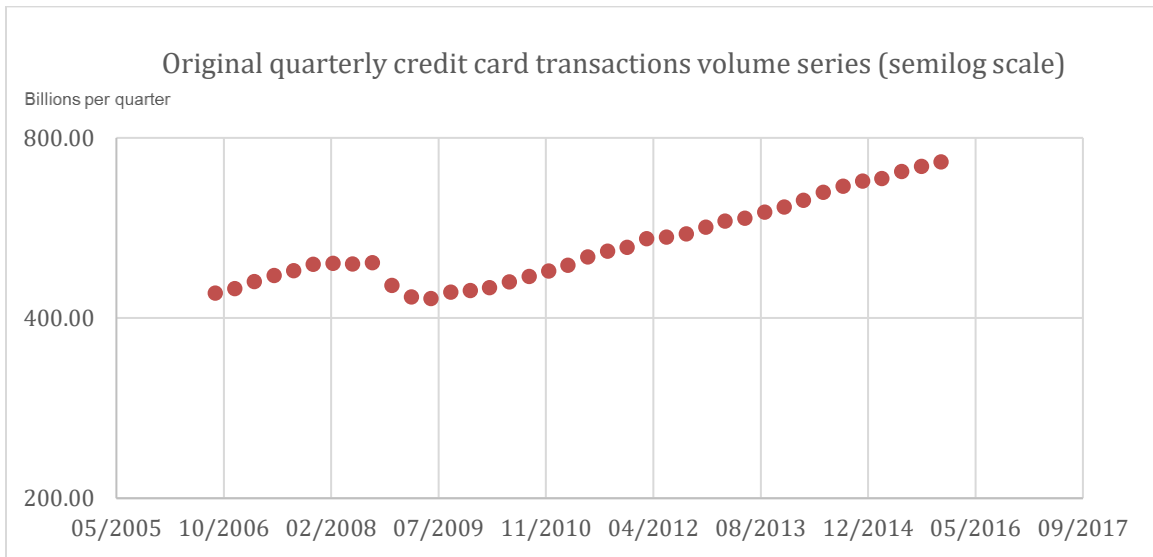
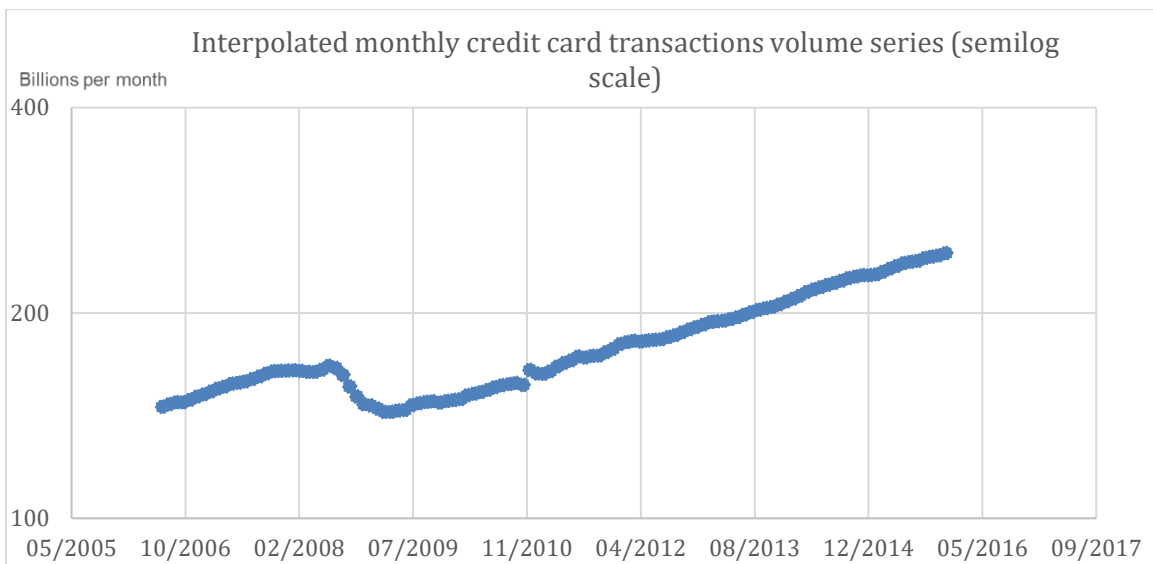


Figure 1b. Demonstration of Chow-Lin Interpolation, after interpolation.



Chapter 2

The Credit-Card-Services Augmented Divisia Monetary Aggregates

William A. Barnett

University of Kansas, Lawrence, and Center for Financial Stability, NY City

Marcelle Chauvet

University of California at Riverside

Danilo Leiva-Leon

Central Bank of Chile

Liting Su

University of Kansas, Lawrence, and Center for Financial Stability, NY City

August 11, 2016

Abstract

While credit cards provide transactions services, credit cards have never been included in measures of the money supply. The reason is accounting conventions, which do not permit adding liabilities to assets. However, index number theory measures service flows and is based on aggregation theory, not accounting. We derive theory needed to measure the joint services of credit cards and money. We provide and evaluate two such aggregate measures having different objectives. We initially apply to NGDP nowcasting. Both aggregates are being implemented by the Center for Financial Stability, which will provide them to the public monthly, along with Bloomberg Terminals.

Keywords: Credit Cards, Money, Credit, Aggregation Theory, Index Number Theory, Divisia Index, Risk, Asset Pricing, Nowcasting, Indicators.

JEL Classification: C43, C53, C58, E01, E3, E40, E41, E51, E52, E58, G17.

1. Introduction

Most models of the monetary policy transmission mechanism operate through interest rates, and often involve a monetary or credit channel, but not both. See, e.g., Bernanke and Blinder (1988) and Mishkin (1996). In addition, there are multiple versions of each mechanism, usually implying different roles for interest rates during the economy's adjustment to central bank policy actions. However, there is a more fundamental reason for separating money from credit. While money is an asset, credit is a liability. In accounting conventions, assets and liabilities cannot be added together. But aggregation theory and economic index number theory are based on microeconomic theory, not accounting conventions. Economic aggregates measure service flows. To the degree that money and some forms of credit produce joint services, those services can be aggregated.

A particularly conspicuous example is credit card services, which are directly involved in transactions and contribute to the economy's liquidity in ways not dissimilar to those of money.³⁴ While money is both an asset and part of wealth, credit cards are neither. Hence credit cards are not money. To the degree that monetary policy operates through a wealth effect (Pigou effect), as advocated by Milton Friedman, credit cards do not play a role. But to the degree that the flow

³⁴ We are indebted to Apostolos Serletis for his suggestion of this topic for research. His suggestion is contained in his presentation as discussant of Barnett's Presidential Address at the Inaugural Conference of the Society for Economic Measurement at the University of Chicago, August 18-20, 2014. The slides for Serletis's discussion can be found online at <http://sem.society.cmu.edu/conference1.html>.

of monetary services is relevant to the economy, as through the demand for monetary services or as an indicator of the state of the economy, the omission of credit card services from monetary services induces a loss of information. For example, Duca and Whitesell (1995) showed that a higher probability of credit card ownership was correlated with lower holdings of monetary transactions balances. Clearly credit card services are a substitute for the services of monetary transactions balances, perhaps to a much higher degree than the services of many of the assets included in traditional monetary aggregates, such as the services of nonnegotiable certificates of deposit.

In this seminal paper, we use strongly simplifying assumptions. We assume credit cards are used to purchase consumer goods. All purchases are made at the beginning of periods, and payments for purchases are either by credit cards or money. Credit card purchases are repaid to the credit card company at the end of the current period or at the end of a future period, plus interest charged by the credit card company. Stated more formally, all discrete time periods are closed on the left and open on the right. After aggregation over consumers, the expected interest rate paid by the “representative” credit card holder can be very high, despite the fact that about 20% of consumers pay no interest on credit card balances. Future research is planned to disaggregate to heterogeneous agents, including consumers who repay soon enough to owe no interest. In the current model, such consumers affect the results only by decreasing the average credit card interest rate aggregated over consumers.

To reflect the fact that money and credit cards provide services, such as liquidity and transactions services, money and credit are entered into a derived utility function, in accordance with Arrow

and Hahn's (1971) proof.³⁵ The derived utility function absorbs constraints reflecting the explicit motives for using money and credit card services. Since this paper is about measurement, we need only assume the existence of such motives. In the context of this research, we have no need to work backwards to reveal the explicit motives. As has been shown repeatedly, any of those motives, including the highly relevant transactions motive, are consistent with existence of a derived utility function absorbing the motive.³⁶

Based on our derived theory, we propose two measurements of the joint services of credit cards and money. These new Divisia monetary aggregates have different objectives. One is based on microeconomic structural aggregation theory, providing an aggregated variable within the macroeconomy. That aggregate is widely applicable to models and policies dependent upon a measure of monetary services within the structure of the macroeconomy. For example, that aggregate would be applicable to demand for money models or as possible intermediate targets

³⁵ Our research in this paper is not dependent upon the simple decision problem we use for derivation and illustration. In the case of monetary aggregation, Barnett (1987) proved that the same aggregator functions and index numbers apply, regardless of whether the initial model has money in the utility function or production function, so long as there is intertemporal separability of structure and separability of components over which aggregation occurs. That result is equally as applicable to our current results with augmented aggregation over monetary asset and credit card services. While this paper uses economic index number theory, it should be observed that there also exists a statistical approach to index number theory. That approach produces the same results, with the Divisia index interpreted to be the Divisia mean using expenditure shares as probability. See Barnett and Serletis (1990).

³⁶ The aggregator function is the derived function that always exists, if monetary and credit card services have positive value in equilibrium. See, e.g., Samuelson (1948), Arrow and Hahn (1971), Fischer (1974), Phelps and Spinnewyn (1982), Quirk and Saposnik (1968), and Poterba and Rotemberg (1987). Analogously, Feenstra (1986, p. 271) demonstrated "a functional equivalence between using real balances as an argument of the utility function and entering money into liquidity costs which appear in the budget constraints." The converse mapping from money and credit in the utility function back to the explicit motive is not unique. But in this paper we are not seeking to identify the explicit motives for holding money or credit card balances.

of policy. The relevant existence condition is weak separability within the structure of the economy.³⁷ The resulting structural aggregate is thereby directly factored out of the structure of the economy as a formal aggregator function. Because of the broad applicability of the structural aggregate, we leave its application to future research, as in replication of the extensive prior research using the Center for Financial Stability (CFS) Divisia monetary aggregates over monetary assets alone.

Our other credit-card-augmented aggregate is indicator optimized and is weakly separable within our optimal nominal GDP nowcasting equation. Hence that aggregate is directly derived from our nowcasting results as an aggregator function factored out of the nowcasting equation. Unlike the structural aggregate, which has broad potential applications, the indicator optimized aggregation is application specific and is the focus of our current empirical results provided in this paper. Relative to its objectives, each of the aggregates is uniquely derived from the relevant theory. We evaluate the ability of our indicator-optimized monetary services aggregate in nowcasting nominal GDP and as an indicator of the state of the economy. This objective is currently topical, given proposals for nominal GDP targeting, which requires monthly measures of nominal GDP. Both our structural credit-card augmented aggregates, based on the relevant theory in this paper, and our indicator optimized aggregates, derived and applied in this paper,

³⁷ Weak separability is the fundamental existence condition for quantity aggregation. See Barnett (1982). We do not empirically test the component clusterings. An important literature exists on testing for weakly separable functional structure and could contribute in major ways to further research in this area. A recent paper meriting serious consideration for future research is Hjertstrand, Swofford, and Whitney (2016).

will soon be available monthly from the CFS and to Bloomberg Terminal users.

Our nowcasts are estimated using only real time information, as available to policy makers at the time predictions are made. We use a multivariate state space model that takes into account asynchronous information --- the model proposed in Barnett, Chauvet, and Leiva-Leon (2016). The model considers real time information arriving at different frequencies and asynchronously, in addition to mixed frequencies, missing data, and ragged edges. The results indicate that the proposed model, containing information on real economic activity, inflation, the new Divisia monetary aggregates, and past information nominal GDP itself, produces the most accurate real time nowcasts of nominal GDP growth. In particular, we find that the inclusion of the new aggregates in our nowcasting model yields substantially smaller mean squared errors than inclusion of the previous Divisia monetary aggregates, which in turn had performed substantially better than the official simple sum monetary aggregates in prior research by Barnett, Chauvet, and Leiva-Leon (2016).

2. Intertemporal Allocation

We begin by defining the variables in the risk neutral case for the representative consumer:.

\mathbf{x}_s = vector of per capita (planned) consumptions of N goods and services (including those of durables) during period s .

\mathbf{p}_s = vector of goods and services expected prices, and of durable goods expected rental prices during period s .

m_{is} = planned per capita real balances of monetary asset i during period s ($i = 1, 2, \dots, n$).

c_{js} = planned per capita real expenditure with credit card type j for transactions during period s ($j = 1, 2, \dots, k$). In the jargon of the credit card industry, those contemporaneous expenditures are called “volumes.”

z_{js} = planned per capita rotating real balances in credit card type j during period s from transactions in previous periods ($j = 1, 2, \dots, k$).

$y_{js} = c_{js} + z_{js}$ = planned per capita total balances in credit type j during period s ($j = 1, 2, \dots, k$).

r_{is} = expected nominal holding period yield (including capital gains and losses) on monetary asset i during period s ($i = 1, 2, \dots, n$).

e_{js} = expected interest rate on c_{js} .

\bar{e}_{js} = expected interest rate on z_{js} .

A_s = planned per capita real holdings of the benchmark asset during period s .

R_s = expected (one-period holding) yield on the benchmark asset during period s .

L_s = per capita labor supply during period s .

w_s = expected wage rate during period s .

The benchmark asset is defined to provide no services other than its expected yield, R_s , which motivates holding of the asset solely as a means of accumulating wealth. As a result, R_s is the maximum expected holding period yield available to consumers in the economy in period s from holding a secured asset. The benchmark asset is held to transfer wealth by consumers between multiperiod planning horizons, rather than to provide liquidity or other services. In contrast, \bar{e}_{js} is not the interest rate on an asset and is not secured. It is the interest rate on an unsecured liability, subject to substantial default and fraud risk. Hence, \bar{e}_{js} can be higher than the

benchmark asset rate, and historically has always been much higher than the benchmark asset rate.³⁸

It is important to recognize that the decision problem we model is not of a single economic agent, but rather of the “representative consumer,” aggregated over all consumers. All quantities are therefore averaged over all consumers. Gorman’s assumptions for the existence of a representative consumer are implicitly accepted, as is common in almost all modern macroeconomic theory having microeconomic foundations. This modeling assumption is particularly important in understand the credit card quantities and interest rates used in our research. About 20% of credit card holders in the United States do not pay explicit interest on credit card balances, since those credit card transactions are paid off by the end of the period. But the 80% who do pay interest pay very high interest rates.³⁹ The Federal Reserve provides two interest rate series for credit card debt. One, \bar{e}_{js} , includes interest only on accounts that do pay interest to the credit card issuing banks, while the other series, e_{js} , includes the approximately

³⁸ We follow the Center for Financial Stability (CFS) and the Bank of Israel in using the short term bank loan rate as a proxy for the benchmark rate. That interest rate has always exceeded the interest rate paid by banks on deposit accounts and on all other monetary assets used in the CFS Divisia monetary aggregates, and has always been lower than the Federal Reserve’s reported average interest rate charged on credit card balances. For detailed information on CFS data sources, see Barnett, Liu, Mattson, and Noort (2013). For the additional data sources used by the CFS to extend to credit card services, see Barnett and Su (2016).

³⁹ The following statement is from www.motherjones.com/kevin-drum/2011/10/americans-are-clueless-about-their-credit-card-debt. "In the four working age categories, about 50% of households think they have outstanding credit card debt, but the credit card companies themselves think about 80% of households have outstanding balances." Since these percentages are of total households, including those having no credit cards, the percent of credit card holders paying interest might be even higher.

20% that do not pay interest. The latter interest rate is thereby lower, since it is averaged over interest paid on both categories of accounts. Since we are modeling the representative consumer, aggregated over all consumers, e_{js} is always less than \bar{e}_{js} for all j and s . The interest rate on rotating credit card balances, \bar{e}_{js} , is paid by all consumers who maintain rotating balances on credit cards. But e_{js} is averaged over both those consumers who maintain such rotating balances and hence pay interest on contemporaneous credit card transactions (volumes) and also those consumers who pay off such credit card transactions before the end of the period, and hence do not pay explicit interest on the credit card transactions. The Federal Reserve provides data on both \bar{e}_{js} and e_{js} . Although e_{js} is less than \bar{e}_{js} , e_{js} also has always been higher than the benchmark rate. This observation is a reflection of the so-called credit card debt puzzle.⁴⁰

We use the latter interest rate, e_{js} , in our augmented Divisia monetary aggregates formula, since the contemporaneous per capita transactions volumes in our model are averaged over both categories of credit card holders. We do not include rotating balances used for transactions in prior periods, since to do so would involve double counting of transactions services.

The expected interest rate, e_{js} , can be explicit or implicit, and applies to the aggregated representative consumer. For example, an implicit part of that interest rate could be in the form

⁴⁰See, e.g., Telyukova and Wright (2008), who view the puzzle as a special case of the rate dominance puzzle in monetary economics. The “credit card debt puzzle” asks why people do not pay down debt, when receiving low interest rates on deposits, while simultaneously paying higher interest rates on credit card debt.

of an increased price of the goods purchased or in the form of a periodic service fee or membership fee. But we use only the Federal Reserve's average explicit interest rate series, which is lower than the one that would include implicit interest. Nevertheless, that downward biased explicit rate of return to credit card companies, e_{js} , aggregated over consumers, tends to be very high, far exceeding R_s , even after substantial losses from fraud.

It is also important to recognize that we are using the credit card industry's definition of "credit card," which excludes "store cards" and "charge cards." According to the trade's definition, "store cards" are issued by businesses providing credit only for their own goods, such as gasoline company credit cards or department store cards. To be a "credit card" by the trade's definition, the card must be widely accepted for many goods and services purchases in the economy. "Charge cards" can be widely accepted for such purchases, but do not charge interest, since the debt must be paid off by the end of the period. To be a "credit card," the card must provide a line of credit to the card holder with interest charged on purchases not paid off by the end of the period. For example, American Express provides both charge cards and credit cards. The first credit card was provided by Bank of America. There now are four sources of credit card services in the United States: Visa, Mastercard, Discover, and American Express. From American Express, we use only their credit card account services, not their charge cards. We use data from only those four sources, in accordance with the credit card industry's conventional definition of "credit card."

We let u_t be the representative consumer's current intertemporal utility function at time t over the T -period planning horizon. We assume that u_t is weakly separable in each period's

consumption of goods and monetary assets, so that u_t can be written in the form

$$\begin{aligned}
u_t &= u_t(\mathbf{m}_t, \dots, \mathbf{m}_{t+T}; \mathbf{c}_t, \dots, \mathbf{c}_{t+T}; \mathbf{x}_t, \dots, \mathbf{x}_{t+T}; A_{t+T}) \\
&= U_t(v(\mathbf{m}_t, \mathbf{c}_t), v_{t+1}(\mathbf{m}_{t+1}, \mathbf{c}_{t+1}), \dots, v_{t+T}(\mathbf{m}_{t+T}, \mathbf{c}_{t+T}); \\
&\quad V(\mathbf{x}_t), V_{t+1}(\mathbf{x}_{t+1}), \dots, V_{t+T}(\mathbf{x}_{t+T}); A_{t+T}), \tag{1}
\end{aligned}$$

for some monotonically increasing, linearly homogeneous, strictly quasiconcave functions, $v, v_{t+1}, \dots, v_{t+T}, V, V_{t+1}, \dots, V_{t+T}$. The function U_t also is monotonically increasing, but not necessarily linearly homogeneous. Note that \mathbf{c}_t , not \mathbf{y}_t , is in the utility function. The reason is that \mathbf{y}_t includes rotating balances, \mathbf{z}_t , resulting from purchases in prior periods. To include \mathbf{y}_t in the utility function would introduce a form of double counting into our aggregation theory by counting prior transactions services more than once. Those carried forward balances provided transactions services in previous periods and were therefore in the utility function for that period. Keeping those balances in the utility function for the current period would imply existence of a different kind of services from the transactions and liquidity services we are seeking to measure.

Dual to the functions, V and $V_s (s = t + 1, \dots, t + T)$, there exist current and planned true cost of living indexes, $p_t^* = p(\mathbf{p}_t)$ and $p_s^* = p_s^*(\mathbf{p}_s) (s = t + 1, \dots, t + T)$. Those indexes, which are the consumer goods unit cost functions, will be used to deflate all nominal quantities to real quantities, as in the definitions of m_{is} , c_{js} , and A_s above.

Assuming replanning at each t , we write the consumer's decision problem during each period $s (t \leq s \leq t + T)$ within the planning horizon to be to choose

$$(\mathbf{m}_t, \dots, \mathbf{m}_{t+T}; \mathbf{c}_t, \dots, \mathbf{c}_{t+T}; \mathbf{x}_t, \dots, \mathbf{x}_{t+T}; A_{t+T}) \geq \mathbf{0} \text{ to}$$

$$\max u_t(\mathbf{m}_t, \dots, \mathbf{m}_{t+T}; \mathbf{c}_t, \dots, \mathbf{c}_{t+T}; \mathbf{x}_t, \dots, \mathbf{x}_{t+T}; A_{t+T}),$$

subject to

$$\begin{aligned} & \mathbf{p}'_s \mathbf{x}_s \\ &= w_s L_s \\ &+ \sum_{i=1}^n [(1 + r_{i,s-1}) p_{s-1}^* m_{i,s-1} - p_s^* m_{is}] \\ &+ \sum_{j=1}^k [p_s^* c_{js} - (1 + e_{j,s-1}) p_{s-1}^* c_{j,s-1}] \\ &+ \sum_{j=1}^k [p_s^* z_{js} - (1 + \bar{e}_{j,s-1}) p_{s-1}^* z_{j,s-1}] + [(1 + R_{s-1}) p_{s-1}^* A_{s-1} \\ &- p_s^* A_s]. \end{aligned} \tag{2}$$

Planned per capita total balances in credit type j during period s are then $y_{js} = c_{js} + z_{js}$.

Equation (2) is a flow of funds identity, with the right hand side being funds available to purchase consumer goods during period s . On the right hand side, the first term is labor income. The second term is funds absorbed or released by rolling over the monetary assets portfolio, as explained in Barnett (1980). The third term is particularly important to this paper. That term is the net change in credit card debt during period s from purchases of consumer goods, while the fourth term is the net change in rotating credit card debt. The fifth term is funds absorbed or

released by rolling over the stock of the benchmark asset, as explained in Barnett (1980). The third term on the right side is specific to current period credit card purchases, while the fourth term is not relevant to the rest of our results, since z_{js} is not in the utility function. Hence z_{js} does not appear in the user cost prices, conditional decisions, or aggregates in the rest of this paper.

Let

$$\rho_s = \begin{cases} 1, & \text{if } s = t, \\ \prod_{u=t}^{s-1} (1 + R_u), & \text{if } t + 1 \leq s \leq t + T. \end{cases} \quad (3)$$

We now derive the implied Fisherine discounted wealth constraint. The derivation procedure involves recursively substituting each flow of funds identity into the previous one, working backwards in time, as explained in Barnett (1980). The result is the following wealth constraint at time t :

$$\begin{aligned}
& \sum_{s=t}^{t+T} \left(\frac{\mathbf{p}'_s}{\rho_s} \right) \mathbf{x}_s + \sum_{s=t}^{t+T} \sum_{i=1}^n \left[\frac{p_s^*}{\rho_s} - \frac{p_s^*(1+r_{is})}{\rho_{s+1}} \right] m_{is} + \sum_{i=1}^n \frac{p_{t+T}^*(1+r_{i,t+T})}{\rho_{t+T+1}} m_{i,t+T} \\
& + \frac{p_{t+T}^*}{\rho_{t+T}} A_{t+T} + \sum_{s=t}^{t+T} \sum_{j=1}^k \left[\frac{p_s^*(1+e_{js})}{\rho_{s+1}} - \frac{p_s^*}{\rho_s} \right] c_{js} + \sum_{s=t}^{t+T} \sum_{j=1}^k \left[\frac{p_s^*(1+\bar{e}_{js})}{\rho_{s+1}} - \frac{p_s^*}{\rho_s} \right] z_{js} \\
& - \sum_{j=1}^k \frac{p_{t+T}^*(1+e_{j,t+T})}{\rho_{t+T+1}} c_{j,t+T} - \sum_{j=1}^k \frac{p_{t+T}^*(1+\bar{e}_{j,t+T})}{\rho_{t+T+1}} z_{j,t+T} \\
& = \sum_{s=t}^{t+T} \left(\frac{w_s}{\rho_s} \right) L_s + \sum_{i=1}^n (1+r_{i,t-1}) p_{t-1}^* m_{i,t-1} + (1+R_{t-1}) A_{t-1} p_{t-1}^* \\
& - \sum_{j=1}^k (1+e_{j,t-1}) p_{t-1}^* c_{j,t-1} - \sum_{j=1}^k (1+\bar{e}_{j,t-1}) p_{t-1}^* z_{j,t-1}.
\end{aligned} \tag{4}$$

It is important to understand that (4) is directly derived from (2) without any additional assumptions. As in Barnett (1978, 1980), we see immediately that the nominal user cost (equivalent rental price) of monetary asset holding m_{is} ($i = 1, 2, \dots, n$) is

$$\pi_{is} = \frac{p_s^*}{\rho_s} - \frac{p_s^*(1+r_{is})}{\rho_{s+1}}.$$

So the current nominal user cost price, π_{it} , of m_{it} reduces to

$$\pi_{it} = \frac{p_t^*(R_t - r_{it})}{1 + R_t}. \tag{5}$$

Likewise, the nominal user cost (equivalent rental price) of credit card transactions services, c_{js} ($j = 1, 2, \dots, k$), is

$$\tilde{\pi}_{jt} = \frac{p_s^*(1+e_{js})}{\rho_{s+1}} - \frac{p_s^*}{\rho_s}.$$

Finally, the current period nominal user cost, $\tilde{\pi}_{jt}$, of c_{jt} reduces to

$$\tilde{\pi}_{jt} = \frac{p_t^*(1 + e_{jt})}{1 + R_t} - p_t^* \quad (6)$$

$$= \frac{p_t^*(e_{jt} - R_t)}{1 + R_t}. \quad (7)$$

Equation (7) is a new result central to most that follows in this paper.⁴¹ The corresponding real user costs are

$$\pi_{js}^* = \frac{\pi_{is}}{p_s^*} \quad (8a)$$

and

$$\tilde{\pi}_{js}^* = \frac{\tilde{\pi}_{jt}}{p_s^*}. \quad (8b)$$

Equation (6) is particularly revealing. To consume the transactions services of credit card type j , the consumer borrows p_t^* dollars per unit of goods purchased at the start of the period during which the goods are consumed, but repays the credit card company $p_t^*(1 + e_{jt})$ dollars at the end of the period. The lender will not provide that one period loan to the consumer unless $e_{jt} > R_t$, because of the ability of the lender to earn R_t without making the unsecured credit card loan.

⁴¹ The same user cost formula applies in the infinite planning horizon case, but the derivation is different. The derivation applicable in that case is in the Appendix.

Consumers do not have access to higher expected yields on secured assets than the benchmark rate. Hence the user cost price in (7) is nonnegative.

Equivalently, equation (7) can be understood in terms of the delay between the goods purchase date and the date of repayment of the loan to the credit card company. Credit cards provide the opportunity for consumers to defer payment for consumer goods and services. During the one period delay, the consumer can invest the cost of the goods purchase at rate of return R_t . Hence the net real cost to the consumer of the credit card loan, per dollar borrowed, is $e_{jt} - R_t$.

Multiplication by the true cost of living index in the numerator of (7) converts to nominal dollars and division by $1 + R_t$ discounts to present value within the time period.

3. Conditional Current Period Allocation

We define J_t^* to be real, and J_t nominal, expenditure on augmented monetary services --- augmented to include the services of contemporaneous credit card transactions charges. The assumptions on homogeneous blockwise weak separability of the intertemporal utility function, (1), are sufficient for consistent two-stage budgeting. See Green (1964, theorem 4). In the first stage, the aggregated representative consumer selects real expenditure on augmented monetary services, J_t^* , and on aggregate consumer goods for each period within the planning horizon, along with terminal benchmark asset holdings, A_{t+T} .

In the second stage, J_t^* is allocated over demands for the current period services of monetary assets and credit cards. That decision is to select \mathbf{m}_t and \mathbf{c}_t to

$$\max v(\mathbf{m}_t, \mathbf{c}_t), \quad (9)$$

subject to

$$\boldsymbol{\pi}_t^{*'} \mathbf{m}_t + \tilde{\boldsymbol{\pi}}_t^{*'} \mathbf{c}_t = \mathcal{J}_t^*, \quad (10)$$

where \mathcal{J}_t^* is expenditure on augmented monetary services allocated to the current period in the consumer's first-stage decision.

The rotating balances, z_{js} , from previous periods, not used for transactions this period, add a flow of funds term to the constraints, (2), but do not appear in the utility function. As a result, z_{js} does not appear in the utility function, (9), or on the left side of equation (10), but does affect the right side of (10). To implement this theory empirically, we need data on total credit card transactions volumes each period, c_{js} , not just the total balances in the accounts, $c_{js} + z_{js}$. While those volumes are much more difficult to find than credit card balances, we have been able to acquire those current period volumes from the annual reports of the four credit card companies. For details on available sources, see Barnett and Su (2016).

4. Aggregation Theory

The exact quantity aggregate is the level of the indirect utility produced by solving problem ((9),(10)):

$$\begin{aligned} \mathcal{M}_t &= \max \{v(\mathbf{m}_t, \mathbf{c}_t): \boldsymbol{\pi}_t' \mathbf{m}_t + \tilde{\boldsymbol{\pi}}_t' \mathbf{c}_t = \mathcal{J}_t\} \\ &= \max \{v(\mathbf{m}_t, \mathbf{c}_t): \boldsymbol{\pi}_t^{*'} \mathbf{m}_t + \tilde{\boldsymbol{\pi}}_t^{*'} \mathbf{c}_t = \mathcal{J}_t^*\}, \end{aligned} \quad (11)$$

where we define $\mathcal{M}_t = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t) = v(\mathbf{m}_t, \mathbf{c}_t)$ to be the “structural augmented monetary aggregate” --- augmented to aggregate jointly over the contemporaneous services of money and credit cards. The category utility function, v , is the aggregator function we assume to be linearly homogeneous in this section. Dual to any exact quantity aggregate, there exists a unique price aggregate, aggregating over the prices of the goods or services. Hence there must exist an exact nominal price aggregate over the user costs $(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$. As shown in Barnett (1980,1987), the consumer behaves relative to the dual pair of exact monetary quantity and price aggregates as if they were the quantity and price of an elementary good. The same result applies to our augmented monetary quantity and dual user cost aggregates.

One of the properties that an exact dual pair of price and quantity aggregates satisfies is Fisher’s factor reversal test, which states that the product of an exact quantity aggregate and its dual exact price aggregate must equal actual expenditure on the components. Hence, if $\Pi(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$ is the exact user cost aggregate dual to \mathcal{M}_t , then $\Pi(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$ must satisfy

$$\Pi(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t) = \frac{J_t}{\mathcal{M}_t}. \quad (12)$$

Since (12) produces a unique solution for $\Pi(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$, we could use (12) to define the price dual to \mathcal{M}_t . In addition, if we replace \mathcal{M}_t by the indirect utility function defined by (11) and use the linear homogeneity of v , we can show that $\Pi = \Pi(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$, defined by (12), does indeed depend only upon $(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$, and not upon $(\mathbf{m}_t, \mathbf{c}_t)$ or J_t . See Barnett (1987) for a version of the proof in the case of monetary assets alone. The conclusion produced by that proof can be written in the form

$$\Pi(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t) = [\max_{(\mathbf{m}_t, \mathbf{c}_t)} \{v(\mathbf{m}_t, \mathbf{c}_t) : \boldsymbol{\pi}_t' \mathbf{m}_t + \tilde{\boldsymbol{\pi}}_t' \mathbf{c}_t = 1\}]^{-1}, \quad (13)$$

which clearly depends only upon $(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$.

Although (13) provides a valid definition of Π , there also exists a direct definition that is more informative and often more useful. The direct definition depends upon the cost function E , defined by

$$E(v_0, \boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t) = \min_{(\mathbf{m}_t, \mathbf{c}_t)} \{\boldsymbol{\pi}_t' \mathbf{m}_t + \tilde{\boldsymbol{\pi}}_t' \mathbf{c}_t : v(\mathbf{m}_t, \mathbf{c}_t) = v_0\},$$

which equivalently can be acquired by solving the indirect utility function equation (11) for \mathcal{J}_t as a function of $\mathcal{M}_t = v(\mathbf{m}_t, \mathbf{c}_t)$ and $(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$. Under our linear homogeneity assumption on v , it can be proved that

$$\Pi(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t) = E(1, \boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t) = \min_{(\mathbf{m}_t, \mathbf{c}_t)} \{\boldsymbol{\pi}_t' \mathbf{m}_t + \tilde{\boldsymbol{\pi}}_t' \mathbf{c}_t : v(\mathbf{m}_t, \mathbf{c}_t) = 1\}, \quad (14)$$

Which is often called the unit cost or price function.

The unit cost function is the minimum cost of attaining unit utility level for $v(\mathbf{m}_t, \mathbf{c}_t)$ at given user cost prices $(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$. Clearly, (14) depends only upon $(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$. Hence by (12) and (14), we see that $\Pi(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t) = \mathcal{J}_t / \mathcal{M}_t = E(1, \boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$.

5. Preference Structure over Financial Assets

5.1. Blocking of the Utility Function

While our primary objective is to provide the theory relevant to joint aggregation over monetary and credit card services, subaggregation separately over monetary asset services and credit card services can be nested consistently within the joint aggregates. The required assumption is blockwise weak separability of money and credit within the joint aggregator function. In particular, we would then assume the existence of functions \tilde{v} , g_1 , g_2 , such that

$$v(\mathbf{m}_t, \mathbf{c}_t) = \tilde{v}(g_1(\mathbf{m}_t), g_2(\mathbf{c}_t)), \quad (15)$$

with the functions g_1 and g_2 being linearly homogeneous, increasing, and quasiconcave.

We have now nested weakly separable blocks within weakly separable blocks to establish a fully nested utility tree. As a result, an internally consistent multi-stage budgeting procedure exists, such that the structured utility function defines the quantity aggregate at each stage, with duality theory defining the corresponding user cost price aggregates.

In the next section we elaborate on the multi-stage budgeting properties of decision ((9),(10)) and the implications for quantity and price aggregation.

5.2 Multi-stage Budgeting

Our assumptions on the properties of v are sufficient for a two-stage solution of the decision problem ((9),(10)), subsequent to the two-stage intertemporal solution that produced ((9),(10)). The subsequent two-stage decision is exactly nested within the former one.

Let $M_t = M(\mathbf{m}_t)$ be the exact aggregation-theoretic quantity aggregate over monetary assets alone, and let $C_t = C(\mathbf{c}_t)$ be the exact aggregation-theoretic quantity aggregate over credit card services. Let $\Pi_m^* = \Pi_m(\boldsymbol{\pi}_t^*)$ be the real user costs aggregate (unit cost function) dual to $M(\mathbf{m}_t)$, and let $\Pi_c^* = \Pi_c(\tilde{\boldsymbol{\pi}}_t^*)$ be the user costs aggregate dual to $C(\mathbf{c}_t)$. The first stage of the two-stage decision is to select M_t and C_t to solve

$$\max_{(\mathbf{m}_t, \mathbf{c}_t)} \tilde{u}(M_t, C_t) \quad (16)$$

subject to

$$\Pi_m^* M_t + \Pi_c^* C_t = \mathcal{J}_t^*.$$

From the solution to problem (16), the consumer determines aggregate real expenditure on monetary and credit card services, $\Pi_m^* M_t$ and $\Pi_c^* C_t$.

In the second stage, the consumer allocates $\Pi_m^* M_t$ over individual monetary assets, and allocates $\Pi_c^* C_t$ over services of individual types of credit cards. She does so by solving the decision problem:

$$\max_{\mathbf{m}_t} g_1(\mathbf{m}_t), \quad (17)$$

subject to

$$\boldsymbol{\pi}_t^{*'} \mathbf{m}_t = \Pi_m^* M_t.$$

Similarly, she solves

$$\max_{\mathbf{c}_t} g_2(\mathbf{c}_t), \quad (18)$$

subject to

$$\tilde{\pi}_t^{*'} \mathbf{c}_t = \Pi_c^* C_t.$$

The optimized value of decision (17)'s objective function, $g_1(\mathbf{m}_t)$, is then the monetary aggregate, $M_t = M(\mathbf{m}_t)$, while the optimized value of decision (18)'s objective function, $g_2(\mathbf{c}_t)$, is the credit card services aggregate, $C_t = C(\mathbf{c}_t)$.

Hence,

$$M_t = \max \{g_1(\mathbf{m}_t): \pi_t^{*'} \mathbf{m}_t = \Pi_m^* M_t\} \quad (19)$$

and

$$C_t = \max \{g_2(\mathbf{c}_t): \tilde{\pi}_t^{*'} \mathbf{c}_t = \Pi_c^* C_t\}. \quad (20)$$

It then follows from (11) and (15) that the optimized values of the monetary and credit card quantity aggregates are related to the joint aggregate in the following manner:

$$\mathcal{M}_t = \tilde{v}(M_t, C_t). \quad (21)$$

6. The Divisia Index

We advocate using the Divisia index, in its Törnqvist (1936) discrete time version, to track $\mathcal{M}_t = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$, as Barnett (1980) has previously advocated for tracking $M_t = M(\mathbf{m}_t)$. If there should be reason to track the credit card aggregate separately, the Törnqvist-Divisia index similarly could be used to track $C_t = C(\mathbf{c}_t)$. If there is reason to track all three individually, then after measuring M_t and C_t , the joint aggregate \mathcal{M}_t could be tracked as a two-good Törnqvist-Divisia index using (21), rather as an aggregate over the $n + k$ disaggregated components, $(\mathbf{m}_t, \mathbf{c}_t)$. The aggregation theoretic procedure for selecting the $n + m$ component assets is described in Barnett (1982).

6.1. The Linearly Homogeneous Case

It is important to understand that the Divisia index (1925,1926) in continuous time will track any aggregator function without error. To understand why, it is best to see the derivation. The following is a simplified version based on Barnett (2012, pp. 290-292), adapted for our augmented monetary aggregate, which aggregates jointly over money and credit card services. The derivation is equally as relevant to separate aggregation over monetary assets or credit cards, so long as the prices in the indexes are the corresponding user costs, ((5),(7)). Although Francois Divisia (1925, 1926) derived his consumer goods index as a line integral, the simplified approach below is mathematically equivalent to Divisia's original method.

At instant of continuous time, t , consider the quantity aggregator function, $\mathcal{M}_t = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t) = v(\mathbf{m}_t, \mathbf{c}_t)$, with components $(\mathbf{m}_t, \mathbf{c}_t)$, having user cost prices $(\boldsymbol{\pi}_t, \tilde{\boldsymbol{\pi}}_t)$. Let $\mathbf{m}_t^a = (\mathbf{m}_t', \mathbf{c}_t')'$ and $\boldsymbol{\pi}_t^a = (\boldsymbol{\pi}_t', \tilde{\boldsymbol{\pi}}_t')'$. Take the total differential of \mathcal{M} to get

$$d\mathcal{M}(\mathbf{m}_t^a) = \sum_{i=1}^{n+k} \frac{\partial \mathcal{M}}{\partial m_{it}^a} dm_{it}^a. \quad (22)$$

Since $\partial \mathcal{M} / \partial m_{it}$ contains the unknown parameters of the function \mathcal{M} , we replace each of those marginal utilities by $\lambda \pi_{it}^a = \partial \mathcal{M} / \partial m_{it}$, which is the first-order condition for expenditure constrained maximization of \mathcal{M} , where λ is the Lagrange multiplier, and π_{it}^a is the user-cost price of m_{it}^a at instant of time t .

We then get

$$\frac{d\mathcal{M}(\mathbf{m}_t^a)}{\lambda} = \sum_{i=1}^{n+k} \pi_{it}^a dm_{it}^a, \quad (23)$$

which has no unknown parameters on the right-hand side.

For a quantity aggregate to be useful, it must be linearly homogeneous. A case in which the correct growth rate of an aggregate is clearly obvious is the case in which all components are growing at the same rate. As required by linear homogeneity, we would expect the quantity aggregate would grow at that same rate. Hence we shall assume \mathcal{M} to be linearly homogeneous.

Define $\Pi^a(\boldsymbol{\pi}_t^a)$ to be the dual price index satisfying Fisher's factor reversal test,

$\Pi^a(\boldsymbol{\pi}_t^a) \mathcal{M}(\mathbf{m}_t^a) = \boldsymbol{\pi}_t^{a'} \mathbf{m}_t^a$. In other words, define $\Pi^a(\boldsymbol{\pi}_t^a)$ to equal $\boldsymbol{\pi}_t^{a'} \mathbf{m}_t^a / \mathcal{M}(\mathbf{m}_t^a)$, which can be shown to depend only upon $\boldsymbol{\pi}_t^a$, when \mathcal{M} is linearly homogeneous. Then the following lemma holds.

Lemma 1: Let λ be the Lagrange multiplier in the first order conditions for solving the constrained maximization ((9),(10)), and assume that v is linearly homogeneous. Then

$$\lambda = \frac{1}{\Pi^a(\boldsymbol{\pi}_t^a)}$$

Proof: See Barnett (2012, p. 291). ■

From Equation (23), we therefore find the following:

$$\Pi^a(\boldsymbol{\pi}_t^a)d\mathcal{M}(\mathbf{m}_t^a) = \sum_{i=1}^{n+k} \pi_i^a dm_i^a. \quad (24)$$

Manipulating Equation (24) algebraically to convert to growth rate (log change) form, we find that

$$d \log \mathcal{M}(\mathbf{m}_t^a) = \sum_{i=1}^{n+k} \omega_{it} d \log m_i^a, \quad (25)$$

where $\omega_{it} = \pi_i^a m_i^a / \boldsymbol{\pi}_t^{a'} \mathbf{m}_t^a$ is the value share of m_i^a in total expenditure on the services of \mathbf{m}_t^a .

Equation (25) is the Divisia index in growth rate form. In short, the growth rate of the Divisia index, $\mathcal{M}(\mathbf{m}_t^a)$, is the share weighted average of the growth rates of the components. Notice that there were no assumptions at all in the derivation about the functional form of \mathcal{M} , other than existence (*i.e.*, weak separability within the structure of the economy) and linear homogeneity of the aggregator function.

If Divisia aggregation was previously used to aggregate separately over money and credit card services, then equation (25) can be replaced by a two-goods Divisia index aggregating over the

two subaggregates, in accordance with equation (21).

6.2. The Nonlinearly Homogeneous Case

For expositional simplicity, we have presented the aggregation theory throughout this paper under the assumption that the category utility functions, v , g_1 , and g_2 , are linearly homogeneous. In the literature on aggregation theory, that assumption is called the “Santa Claus” hypothesis, since it equates the quantity aggregator function with the welfare function. If the category utility function is not linearly homogeneous, then the utility function, while still measuring welfare, is not the quantity aggregator function. The correct quantity aggregator function is then the distance function in microeconomic theory. While the utility function and the distance function both fully represent consumer preferences, the distance function, unlike the utility function, is always linearly homogenous. When normalized, the distance function is called the Malmquist index.

In the latter case, when welfare measurement and quantity aggregation are not equivalent, the Divisia index tracks the distance function, not the utility function, thereby continuing to measure the quantity aggregate, but not welfare. See Barnett (1987) and Caves, Christensen, and Diewert (1982). Hence the only substantive assumption in quantity aggregation is blockwise weak separability of components. Without that assumption there cannot exist an aggregate to track.

6.3. Discrete Time Approximation to the Divisia Index

If $(\mathbf{m}_t, \mathbf{c}_t)$ is acquired by maximizing (9) subject to (10) at instant of time t , then $v(\mathbf{m}_t, \mathbf{c}_t)$ is the

exact augmented monetary services aggregate, \mathcal{M}_t , as written in equation (11). In continuous time, $\mathcal{M}_t = v(\mathbf{m}_t, \mathbf{c}_t)$ can be tracked without error by the Divisia index, which provides \mathcal{M}_t as the solution to the differential equation

$$\frac{d \log \mathcal{M}_t}{dt} = \sum_{i=1}^n \omega_{it} \frac{d \log m_{it}}{dt} + \sum_{j=1}^k \tilde{\omega}_{jt} \frac{d \log c_{jt}}{dt}, \quad (26)$$

in accordance with equation (25). The share ω_{it} is the expenditure share of monetary asset i in the total services of monetary assets and credit cards at instant of time t ,

$$\omega_{it} = \pi_{it} m_{it} / (\boldsymbol{\pi}'_t \mathbf{m}_t + \tilde{\boldsymbol{\pi}}'_t \mathbf{c}_t),$$

while the share $\tilde{\omega}_{it}$ is the expenditure share of credit card services, i , in the total services of monetary assets and credit cards at instant of time t ,

$$\tilde{\omega}_{it} = \tilde{\pi}_{it} c_{it} / (\boldsymbol{\pi}'_t \mathbf{m}_t + \tilde{\boldsymbol{\pi}}'_t \mathbf{c}_t).$$

Note that the time path of $(\mathbf{m}_t, \mathbf{c}_t)$ must continually maximize (9) subject to (10), in order for (26) to hold.

In discrete time, however, many different approximations to (25) are possible, because ω_{it} and $\tilde{\omega}_{it}$ need not be constant during any given time interval. By far the most common discrete time approximations to the Divisia index is the Törnqvist-Theil approximation (often called the Törnqvist (1936) index or just the Divisia index in discrete time). That index can be viewed as the Simpson's rule approximation, where t is the discrete time period, rather than an instant of

time:

$$\begin{aligned}
& \log \mathcal{M}(\mathbf{m}_t^a) - \log \mathcal{M}(\mathbf{m}_{t-1}^a) \\
&= \sum_{i=1}^n \bar{\omega}_{it} (\log m_{it} - \log m_{i,t-1}) \\
&+ \sum_{i=1}^k \bar{\bar{\omega}}_{it} (\log c_{it} - \log c_{i,t-1}), \tag{27}
\end{aligned}$$

where $\bar{\omega}_{it} = (\omega_{it} + \omega_{i,t-1})/2$ and $\bar{\bar{\omega}}_{it} = (\bar{\omega}_{it} + \bar{\omega}_{i,t-1})/2$.

A compelling reason exists for using the Törnqvist index as the discrete time approximation to the Divisia index. Diewert (1976) has defined a class of index numbers, called “superlative” index numbers, which have particular appeal in producing discrete time approximations to aggregator functions. Diewert defines a superlative index number to be one that is exactly correct for some quadratic approximation to the aggregator function, and thereby provides a second order local approximation to the unknown aggregator function. In this case the aggregator function is $\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t) = v(\mathbf{m}_t, \mathbf{c}_t)$. The Törnqvist discrete time approximation to the continuous time Divisia index is in the superlative class, because it is exact for the translog specification for the aggregator function. The translog is quadratic in the logarithms. If the translog specification is not exactly correct, then the discrete Divisia index (27) has a third-order remainder term in the changes, since quadratic approximations possess third-order remainder terms.

With weekly or monthly monetary asset data, the Divisia monetary index, consisting of the first term on the right hand side of (27), has been shown by Barnett (1980) to be accurate to within

three decimal places in measuring log changes in $M_t = M(\mathbf{m}_t)$ in discrete time. That three decimal place error is smaller than the roundoff error in the Federal Reserve's component data. We can reasonably expect the same to be true for our augmented Divisia monetary index, (27), in measuring the log change of $\mathcal{M}_t = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$.

7. Data Sources

The credit card transactions services are measured by the transactions volumes summed over four sources: Visa, MasterCard, American Express, and Discover. Our theory does not apply to debit cards or to store cards or to charge cards not providing a line of credit. We acquired the volumes from their annual reports and seasonally adjusted them by the Census X-13ARIMA-SEATS program. The start date is the quarter during which those credit card firms went public and the annual reports became available. The contemporaneous transactions volumes do not include the carried forward rotating balances resulting from transactions during prior periods.⁴² The credit card interest rates imputed to the representative consumer are from the Federal Reserve Board's data on all commercial bank credit card accounts, including those not charged interest, since paid off within the month.⁴³ All other component quantities and interest rates are

⁴² Credit limits are not considered, since we do not have a way to untangle the effect of those constraints on contemporaneous transactions volumes from the effect on the carried forward rotating balances associated with previous period transactions.

⁴³ This interest rate includes those credit card accounts not assessed interest, and hence is lower than the Federal Reserve's supplied interest rates on accounts assessed interest. This imputation includes only explicit interest paid, averaged over all credit card accounts.

as used in the CFS Divisia monetary aggregates at
www.centerforfinancialstability.org/amfm.php.

Our extensive search for relevant sources of credit card data are provided in detail in Barnett and Su (2016), which documents our decisions about credit card data sources. All details about data sources and data decisions regarding monetary asset components and interest rates are provided in Barnett, Liu, Mattson, and van den Noort (2013). We use only sources available to the public.⁴⁴

The resulting augmented Divisia monetary services aggregates, $\mathcal{M}_t = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$, satisfy the existence conditions for a structural economic variable in a macroeconomic model. Hence those aggregates can be used as the quantity of monetary services in a demand for money equation, or as a monetary intermediate target or long run anchor in a monetary rule, or in any other econometric or policy application requiring a macroeconomic model containing the monetary service flow as a structural variable.

Alternatively, money can be used as an indicator of the state of the economy. For example, new-Keynesian nominal GDP targeting policies require monthly measures of nominal GDP, although

⁴⁴ The CFS sweep adjusts demand deposits. During periods when available from the Federal Reserve, the CFS uses the reported sweep adjustments. When not available, the CFS uses an econometric model to approximate the sweep adjustment. Although sweep adjustment is important at the M1 level of aggregation, the sweep adjustment has insignificant effect on the broader aggregates, since sweeps are largely internalized within those aggregates.

data on nominal GDP are available only quarterly. The usefulness of Divisia monetary aggregates in nowcasting monthly nominal GDP has been established by Barnett, Chauvet, and Leiva-Leon (2016). Indicator uses of monetary data are free from the controversies that have surrounded uses of money as a policy target. In the next section, we produce an indicator-optimized augmented monetary aggregate, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$. Since this aggregate is application specific, its existence condition is different from the one used above to produce the augmented structural Divisia monetary aggregates. Unlike the augmented structural aggregates, $\mathcal{M}_t = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$, which are statistical index numbers in the superlative index number class, the indicator optimized aggregates, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$, are econometrically estimated aggregator functions, not statistical index numbers. The estimated aggregator function is time dependent, because of the real time estimation used in the nowcasting.

In the near future, the CFS plans to add to its site our augmented Divisia structural monetary aggregates, $\mathcal{M}_t = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$, as defined in equations 11 and 21, including credit card services, along with our indicator optimized monetary aggregates, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$. Monthly updates will be provided to the public by the CFS through monthly releases. The monthly updates will also be provided by Bloomberg to its terminal users.

8. Nowcasting Nominal GDP

In this section we turn to the use of our data as indicators, rather than as policy targets or as structural variables in the macroeconomy. We find that the information contained in credit card transaction volumes is a valuable addition to the indicator set in formal nowcasting of nominal GDP. A consequence is a directly derived indicator-optimized augmented aggregator function

over monetary and credit card services. This aggregator function uniquely captures the contributions of monetary and credit card services as indicators of nominal GDP in the nowcasting.

An important contribution to the literature on nowcasting is Giannone, Reichlin, and Small (2008). Their approach, based on factor analysis, has proved to be very successful. Barnett, Chauvet, and Leiva-Leon (2016) propose an alternative methodology based on confirmatory factor analysis and find that Divisia monetary aggregates are particularly valuable indicators within the resulting set of optimal indicators. Barnett and Tang (2016) compared the factor analysis approach of Giannone, Reichlin, and Small (2008) and Barnett, Chauvet, and Leiva-Leon (2016) with alternative nowcasting approaches, and find that the factor analysis approaches are usually best and benefit substantially from inclusion of the CFS Divisia monetary aggregates among its indicators.

In this paper, we investigate the further gains from inclusion of credit card transactions volumes in the nowcasting. We also produce and explore the derived indicator optimized aggregates, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$.

8.1 The Model

In this paper we use data on credit card transaction volumes along with the optimal indicators found by Barnett, Chauvet, and Leiva-Leon (2016) to provide a model useful to yield accurate nowcasts of monthly Nominal GDP. Accordingly, as indicators we use growth rates of quarterly Nominal GDP, $y_{1,t}$, monthly Industrial Production, $y_{2,t}$, monthly Consumer Price Index, $y_{3,t}$, a monthly Divisia monetary aggregate measure, $y_{4,t}$, and a monthly credit card transaction volume,

$y_{5,t}$, to estimate the following Mixed Frequency Dynamic Factor model:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \\ y_{4,t} \\ y_{5,t} \end{bmatrix} = \begin{bmatrix} \gamma_1[(1/3)f_t + (2/3)f_{t-1} + f_{t-2} + (2/3)f_{t-3} + (1/3)f_{t-4}] \\ \gamma_2 f_t \\ \gamma_3 f_t \\ \gamma_4 f_t \\ \gamma_5 f_t \end{bmatrix} + \begin{bmatrix} (1/3)v_{1,t} + (2/3)v_{1,t-1} + v_{1,t-2} + (2/3)v_{1,t-3} + (1/3)v_{1,t-4} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \\ v_{5,t} \end{bmatrix}. \quad (28)$$

The model separates out, into the unobserved factor, f_t , the common cyclical fluctuations underlying the observed variables. The idiosyncratic movements are captured by the terms, $v_{i,t}$, for $i = 1, 2, \dots, 5$. The factor loadings, γ_i , measure the sensitivity of the common factor to the observed variables. The dynamics of the factor and idiosyncratic components are given by

$$f_t = \phi_1 f_{t-1} + \dots + \phi_p f_{t-p} + e_t, \quad e_t \sim N(0,1) \quad (29)$$

$$v_{i,t} = \varphi_{i1} v_{i,t-1} + \dots + \varphi_{iQ_i} v_{i,t-Q_i} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim N(0, \sigma_{\varepsilon_i}^2), \text{ for } i = 1, \dots, 5. \quad (30)$$

Following Stock and Watson (1989), the model assumes that f_t and $v_{i,t}$ are mutually independent at all leads and lags for all $n = 5$ variables.

The model in equations (28)-(30) can be cast into a measurement equation and transition equation yielding the following state-space representation

$$\mathbf{y}_t = \mathbf{H}\mathbf{F}_t + \boldsymbol{\xi}_t, \quad \boldsymbol{\xi}_t \sim i.i.d. N(\mathbf{0}, \mathbf{R}) \quad (31)$$

$$\mathbf{F}_t = \mathbf{G}\mathbf{F}_{t-1} + \boldsymbol{\zeta}_t, \quad \boldsymbol{\zeta}_t \sim i.i.d. N(\mathbf{0}, \mathbf{Q}). \quad (32)$$

We apply the Kalman filter to extract optimal inferences on the state vector, \mathbf{F}_t , which contains the common factor of interest, f_t , and the idiosyncratic terms, $v_{i,t}$.

Following Mariano and Murasawa (2003), we modify the state-space model to incorporate into the system missing observations, which are frequently present when performing nowcasts in real-time. The modification consists of substituting each missing observation with a random draw $\beta_t \sim N(0, \sigma_\beta^2)$. This substitution keeps the matrices conformable, without affecting the estimation of the model parameters, in accordance with the rule:

$$\begin{aligned} y_{i,t}^* &= \begin{cases} y_{i,t} & \text{if } y_{i,t} \text{ observed} \\ \beta_t & \text{otherwise} \end{cases}, & \mathbf{H}_{i,t}^* &= \begin{cases} \mathbf{H}_i & \text{if } y_{i,t} \text{ observed} \\ \mathbf{0}_{1k} & \text{otherwise} \end{cases} \\ \xi_{i,t}^* &= \begin{cases} 0 & \text{if } y_{i,t} \text{ observed} \\ \beta_t & \text{otherwise} \end{cases}, & R_{i,t}^* &= \begin{cases} 0 & \text{if } y_{i,t} \text{ observed} \\ \sigma_\beta^2 & \text{otherwise} \end{cases} \end{aligned}$$

where $\mathbf{H}_{i,t}^*$ is the i -th row of a matrix \mathbf{H}^* , which has k columns, and $\mathbf{0}_{1k}$ is a k row vector of zeros. Hence, the modified measurement equation of the state-space model remains as

$$\mathbf{y}_t^* = \mathbf{H}_t^* \mathbf{F}_t + \boldsymbol{\xi}_t^*, \quad \boldsymbol{\xi}_t^* \sim i.i.d. N(\mathbf{0}, \mathbf{R}_t^*). \quad (33)$$

The output is an optimal estimator of the dynamic factor, constructed using information available through time t . As new information becomes available, the filter is applied to update the state

vector on a real-time basis.

8.2 In-Sample Analysis

We empirically evaluate the predictive ability of the information contained in credit card volumes to produce the most accurate nowcasts of nominal GDP growth, when credit card transactions volumes are included into the optimal indicator set found by Barnett, Chauvet, and Leiva-Leon (2016). One of the indicators in that set is the current CFS Divisia monetary aggregates, unaugmented by inclusion of credit card data. We perform pairwise comparisons between models that include credit card information and models that do not. In the former case, the indicator set includes four variables, while in the latter case the indicator set includes five variables. Both sets include the same CFS unaugmented Divisia monetary aggregates, $M_t = M(\mathbf{m}_t)$, as defined in equation 19, among its optimal indicators. We first examine the predictive ability of both models, with and without credit card information as a fifth indicator, by performing an in-sample analysis. We consider the sample period from November 2003 until May 2015 as a result of the availability of the needed data. For the in-sample analysis, we estimate the model only once for the full sample. From November 2003 to June 2006, there are some missing observations of some variables, but this does not present a problem, since the nowcasting model allows dealing with missing observations using the Kalman filter. Regular data availability for all relevant variables begins in July 2006, when the credit card companies' data became available in annual reports.

The first two columns of Table 1 report the full sample Mean Square Errors (MSE) associated with the models containing each of the two indicator sets. The table shows that models

containing both CFS Divisia monetary aggregates and credit card transactions volumes produce lower MSE than models containing only Divisia monetary aggregates, $M_t = M(\mathbf{m}_t)$ among the other three indicators. This applies at any of the four levels of disaggregation, M1, M2, M3, and M4. Next, we compute the MSE only for the years associated with the Great Recession (2008-2009), reported in the last two columns of Table 1. The results show that the models including credit card information produce lower MSE than the models omitting such information in nowcasting of nominal GDP growth.

Table 1. In-Sample Mean Squared Errors

	FULL SAMPLE		GREAT RECESSION	
	CFS	Augmented	CFS	Augmented
DM1	0.16	0.17	0.33	0.30
DM2	0.18	0.17	0.36	0.31
DM3	0.16	0.15	0.32	0.26
DM4	0.18	0.15	0.39	0.25

Note. The table reports the mean squared errors associated with each model for the entire sample period, November 2003 - May 2015, and for the Great Recession years, January 2008 - December 2009. The CFS column includes the CFS Divisia monetary aggregates, $M_t = M(\mathbf{m}_t)$, among the Barnett, Chauvet, and Leiva-Leon (2016) optimal indicator set, but without inclusion of credit card transaction volumes, while the Augmented column includes credit card transactions volumes among the indicators as a fifth independent indicator.

To provide a deeper exploration about the role that each indicator plays in the construction of nominal GDP predictions, we follow the line of Banbura and Rustler (2007) and decompose each forecast into the relative contribution of each indicator, with emphasis on the Divisia monetary

aggregate, $M_t = M(\mathbf{m}_t)$, and credit card transactions volume. In doing so, we substitute the prediction error, $\boldsymbol{\xi}_{t|t-1}^*$, and the predicted state, $\mathbf{F}_{t|t-1}$, into the updating equation of the Kalman filter, yielding

$$\mathbf{F}_{t|t} = (\mathbf{I} - \mathbf{K}_t^* \mathbf{H}_t^*) \mathbf{G} \mathbf{F}_{t-1|t-1} + \mathbf{K}_t^* \mathbf{y}_t^*, \quad (34)$$

where the Kalman gain is denoted by $\mathbf{K}_t^* = \mathbf{P}_{t|t-1} (\mathbf{H}_t^{*'} (\mathbf{H}_t^* \mathbf{P}_{t|t-1} \mathbf{H}_t^{*'} + \mathbf{R}_t^*))^{-1}$, and the predicted variance of the state vector is given by $\mathbf{P}_{t|t-1} = \mathbf{G} \mathbf{P}_{t-1|t-1} \mathbf{G} + \mathbf{Q}$. When the Kalman filter approaches its steady state, the updated state vector can be decomposed into a weighted sum of observations

$$\mathbf{F}_{t|t} = \sum_{j=0}^{\infty} \mathbf{Z}_{jt}^* \mathbf{y}_{t-j}^*, \quad (35)$$

where $\mathbf{Z}_t^*(L) = (\mathbf{I} - (\mathbf{I} - \mathbf{K}_t^* \mathbf{H}_t^*) \mathbf{G} \mathbf{L})^{-1} \mathbf{K}_t^*$, and each element of the matrix $\mathbf{Z}_t^*(L)$ measures the effects of unit changes in the lags of individual observations on the inference of the state vector $\mathbf{F}_{t|t}$. Therefore, the matrix $\mathbf{Z}_t^*(1)$ contains the cumulative impacts of the individual observations in the inference of the state vector. For further details about this decomposition, see Banbura and Rustler (2007). Accordingly, the vector containing the cumulative impact of each indicator on the forecast of nominal GDP growth can be calculated as follows

$$\boldsymbol{\omega}_t = \mathbf{H}_1 \left(\frac{1}{3} \mathbf{z}_{1t}^* + \frac{2}{3} \mathbf{z}_{2t}^* + \mathbf{z}_{3t}^* + \frac{2}{3} \mathbf{z}_{4t}^* + \frac{1}{3} \mathbf{z}_{5t}^* \right) + \left(\frac{1}{3} \mathbf{z}_{7t}^* + \frac{2}{3} \mathbf{z}_{8t}^* + \mathbf{z}_{9t}^* + \frac{2}{3} \mathbf{z}_{10t}^* + \frac{1}{3} \mathbf{z}_{11t}^* \right), \quad (36)$$

where, \mathbf{z}_{1t}^* , is the i -th row of $\mathbf{Z}_t^*(1)$.

The average cumulative forecast weights, ω_t , associated with each indicator are reported in Table 2 for all the models under consideration. The results show that, on average, one third of the contribution is associated with previous releases of nominal quarterly GDP itself. Such information is primary in the model, but is only observed once per quarter. Regarding the monthly indicators, Industrial Production is the indicator that contributes the most to nominal GDP growth predictions, followed by the Divisia monetary aggregates. The indicator that provides the least contribution across models is often the Consumer Price Index, CPI. However, when credit card information is included, it shows a significantly greater forecast contribution than the unaugmented CFS Divisia monetary aggregates or the Consumer Price Index. This conclusion is independent of the aggregation level of the monetary measure. These results corroborate that the in sample predictive ability of the optimal combination, including both Divisia monetary aggregates and credit-card volumes, outperforms models that exclude credit card information.⁴⁵

⁴⁵ It should be observed that the weights in the CFS rows are not directly comparable to those in the Augmented rows, since the weights are relative and sum to one along the rows, with more indicators being weighted in the Augmented rows. Much of the weight on IP in the CFS rows is transferred to the credit card volumes in the Augmented rows, producing substantially better nowcasts. The weights on the Divisia monetary aggregates are consistent with the results in Barnett, Chauvet, and Leiva-Leon (2016), who found inclusion of the Divisia monetary aggregates to be highly statistically significant, in contrast with the many other indicators considered and rejected from the optimal indicator set.

Table 2. Cumulative Forecast Weight of Each Indicator

	NGDP	IP	CPI	DIVISIA	CREDIT
DM1 CFS	0.33	0.59	0.03	0.05	--
DM1 Augmented	0.33	0.34	0.05	0.03	0.25
DM2 CFS	0.33	0.58	0.03	0.06	--
DM2 Augmented	0.33	0.34	0.04	0.04	0.24
DM3 CFS	0.33	0.63	0.04	0.01	--
DM3 Augmented	0.33	0.35	0.05	0.01	0.26
DM4 CFS	0.33	0.60	0.03	0.03	--
DM4 Augmented	0.33	0.37	0.04	0.02	0.24

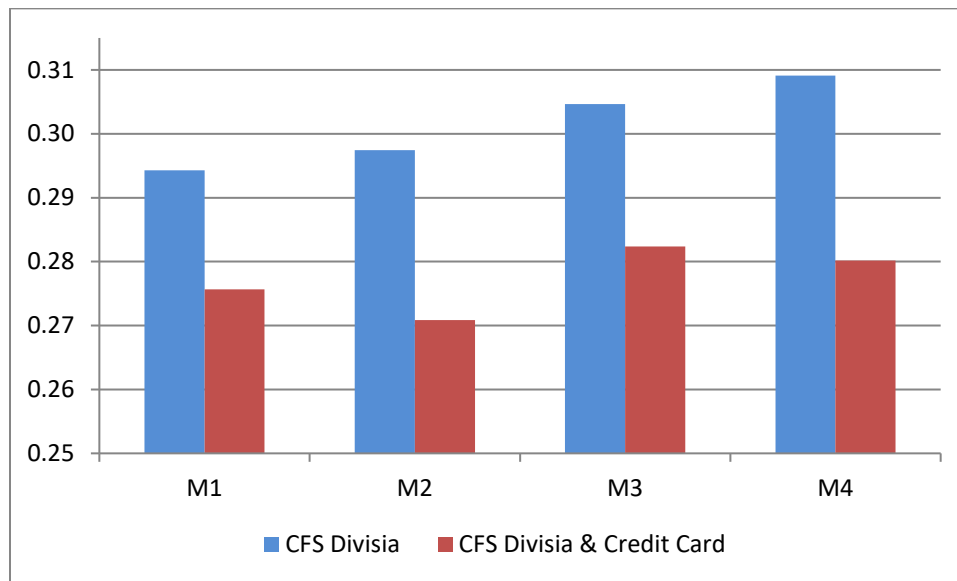
Note. The table reports the cumulative forecast weights, averaged over time, for the entire sample. As in table 1, the CFS rows include the CFS Divisia monetary aggregates among the Barnett, Chauvet, and Leiva-Leon (2016) optimal indicator set, but without inclusion of credit card transaction volumes, while the Augmented rows include credit card transactions volumes among the indicators as a fifth independent indicator. In both cases, the Divisia column is the CFS unaugmented Divisia monetary aggregate, $M_t = M(\mathbf{m}_t)$, defined in equation 19.

8.3 Real Time Analysis

For the initial estimation of the model in real time analysis, we use data from November 2003 to September 2007, yielding 47 observations. Hence, our nowcasting evaluation sample is the remaining observations from October 2007 to May 2015, yielding 92 observations. The samples have been chosen based on two criteria, (i) to guarantee that the estimation sample represents one third of the total available sample, and (ii) to incorporate the Great Recession episode in the

evaluation sample, since it is of particular interest.⁴⁶ For every month of the evaluation sample, we re-estimate the model parameters, compute the nowcast of the target variable, and compare it with the first release of nominal GDP to construct mean squared errors.

Figure 1: Mean Square Error Comparison (Full sample)



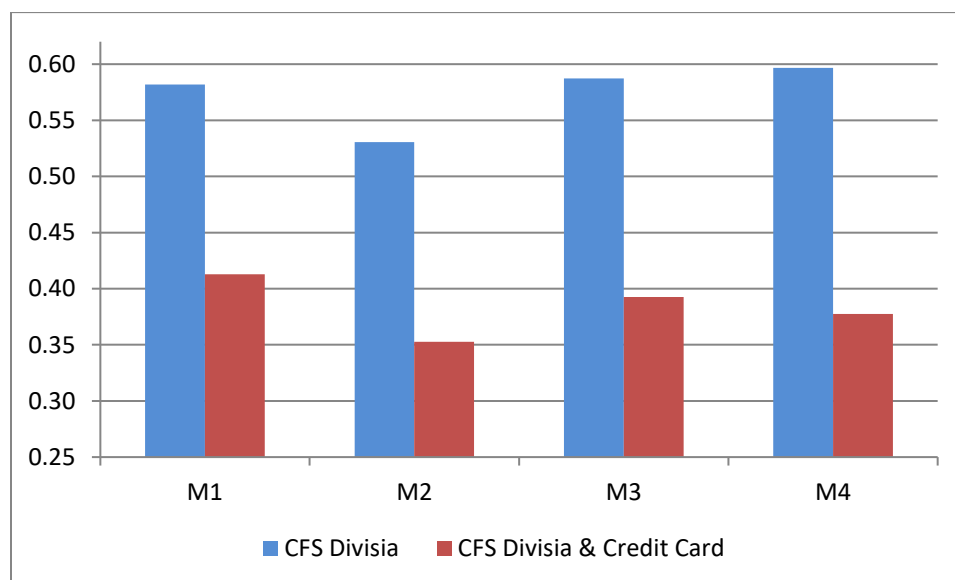
With each model, the MSE associated with the real-time nowcasts are shown in Figure 1 for the entire evaluation sample. The figure shows that models incorporating credit card information provide a significantly lower MSE than the models not incorporating such information. Optimal weighting between credit card transactions volumes and Divisia monetary aggregates improves the accuracy in producing real-time nowcasts of nominal GDP. The superiority of the extended

⁴⁶We also tried with different partitions of the sample, but the results remained qualitatively unchanged.

models, which include credit card information, over the un-extended models, omitting that information, can be observed at all four levels of aggregation and particularly for the M2 monetary aggregates.

Additionally, we perform the same evaluations, but only focusing on the subsample containing the years of the Great Recession. The motivation for doing this analysis relies on comparing the ability of the extended and un-extended models to track nominal GDP dynamics during recessionary periods, associated with macroeconomic instabilities and higher uncertainty. Figure 2 shows the mean squared errors associated with real-time nowcasts computed with each model for the evaluation sample, containing the years of 2008 and 2009. The results corroborate the significant superiority of the extended over unextended models in nowcasting nominal GDP during contractionary episodes.

Figure 2: Mean Square Error Comparison (Great Recession)



The model is re-estimated at every period of time during which new information is available, to simulate real-time conditions. We thereby investigate potential changes in the contemporaneous relationship between each indicator in the model and the extracted factor used to produce real-time nowcasts of nominal GDP growth. This information allows us to examine in detail the comovement between each indicator and the signals used to forecast nominal GDP during periods of instabilities, such as the Great Recession. In Table 1, the first row at each level of aggregation is for the four indicator model, while the second row is for the five indicator model.

The upper part of Table 3 reports the full sample average of the recursively estimated factor loadings for each indicator and for each model. The results show a positive and strong comovement between Industrial Production and the common factor, and a positive but weak comovement between Consumer Price Index and the common factor, with stronger comovement in the case of the five factor model. Regarding the CFS Divisia monetary aggregates, the results show relatively weak and sometimes negative comovement with the common factor. As the sample size grows in the future, we anticipate that the recursive loadings of the Divisia monetary aggregates on the common factor will remain small but will become consistently positive, as in Barnett, Chauvet, and Leiva-Leon (2016).⁴⁷ In the five factor models, credit card transactions

⁴⁷ The sample size in Barnett, Chauvet, and Leiva-Leon (2016) was much larger than in the current study, since the earlier research was not constrained by lack of availability of credit card volumes prior to the credit card firms going public. In the earlier study, the recursive loadings of the Divisia monetary aggregates in the common factor were always positive, but smaller than the loadings on the other optimal indicators. The sometimes negative out of sample average factor loadings on the Divisia monetary aggregates in the current study are associated with the smaller sample size, having a large percentage of observations during the Great Recession period of unusual

volumes show very strong comovement with the common factor, even stronger than the comovement of quarterly nominal GDP with the common factor. Clearly the four factor model is missing important indicator information.

To assess the comovements during the Great Recession period, we compute the average recursive loadings for the period January 2008 to December 2009 and report them in the lower part of Table 3. The comovement between each indicator and the common factor across models presents a similar pattern to the one obtained with the full sample averages, with one notable exception. With both the four indicator and the five indicator models, the Consumer Price Index experiences a negative relationship with the common factor, providing countercyclical signals to nowcasts of nominal GDP growth. Again the credit-card transactions volumes experience positive and strong comovement with the common factor, and hence show the ability to improve the accuracy of signals in nowcasting nominal GDP growth during periods of instability.

instability.

Table 3. Out of Sample Recursive Loadings

	<u>Full sample period</u>				
	NGDP	IP	CPI	DIVISIA	CREDIT
DM1 CFS	0.19	0.39	0.09	-0.10	--
DM1 CFS & CREDIT	0.22	0.42	0.15	-0.14	0.38
DM2 CFS	0.20	0.38	0.07	-0.13	--
DM2 CFS & CREDIT	0.22	0.41	0.14	-0.17	0.36
DM3 CFS	0.18	0.38	0.08	0.02	--
DM3 CFS & CREDIT	0.21	0.41	0.16	0.03	0.38
DM4 CFS	0.19	0.39	0.06	-0.11	--
DM4 CFS & CREDIT	0.21	0.41	0.14	-0.12	0.36
	<u>Great Recession period</u>				
DM1 CFS	0.21	0.43	-0.04	-0.05	--
DM1 CFS & CREDIT	0.24	0.48	0.00	-0.08	0.29
DM2 CFS	0.25	0.39	-0.08	-0.01	--
DM2 CFS & CREDIT	0.25	0.46	-0.03	-0.06	0.25
DM3 CFS	0.21	0.42	-0.05	0.00	--
DM3 CFS & CREDIT	0.23	0.48	-0.01	0.01	0.31
DM4 CFS	0.23	0.44	-0.09	-0.16	--
DM4 CFS & CREDIT	0.24	0.47	-0.01	-0.14	0.26

Note. The table reports the average out of sample recursively estimated factor loading. The upper part of the table focuses on the entire sample November 2003 - May 2015, while the lower part of the table focuses on the Great Recession years, January 2008 - December 2009.

9. Indicator Optimized Augmented Aggregate

As explained in the previous section, the nowcasts can be transformed into weighted averages of the indicators, with the weights being the vector ω_t provided in Table 2. The nowcasting-derived indicator-optimized aggregate, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$, is the weighted averages of the CFS Divisia monetary aggregate and the credit card transactions volume. The weights of those two components are in the fourth and fifth columns of Table 2, with those two weights renormalized

to sum to one. The estimated aggregator function, $\mathcal{M}_t^*(.)$, is time dependent, since the weights, ω_t , are time dependent.⁴⁸ The detailed procedure for computing the weights in Table 2 and the indicator optimized aggregate, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$, is provided in the appendix VI of Barnett, Chauvet, Leiva-Leon, and Su (2016).

It is important to observe that if the CFS Divisia monetary aggregate is replaced by \mathcal{M}_t^* computed in that manner, then all of the results in Tables 1, 2, and 3 for five indicators are equally and exactly applicable to the nowcasting with four indicators. As evident from those tables, replacing the CFS Divisia monetary aggregates, M_t , by \mathcal{M}_t^* produces very large gains in indicator information with four indicators in each case. No indicator information is lost by the aggregation, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$, since that optimized augmented indicator is uniquely nowcasting indicator exact.

All of the figures below display three graphs: (1) nominal quarterly measured GDP growth, (2) growth of the CFS Divisia monetary aggregates, $M_t = M(\mathbf{m}_t)$, and (3) growth of the indicator optimized augmented monetary aggregates, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$. Although the nowcasts and the monetary aggregates are available monthly, the plots below are quarterly, since GDP data are

⁴⁸ In principle, it might be possible to factor a non-time-dependent function solely of $(\mathbf{m}_t, \mathbf{c}_t)$ out of the nowcasting equation. But because of the deep nonlinearity of that equation in $(\mathbf{m}_t, \mathbf{c}_t)$ and the recursive real time nature of the nowcasting estimation, it would be impossible to solve for that aggregator function in algebraic closed form. The extreme difficulty of solving for that function numerically, if the function exists, would have no benefit, since $\mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$ is indicator optimal and loses no information in the nowcasting.

available only quarterly.

The following observations follow from the figures. The fluctuations in the credit-card augmented Divisia monetary aggregates lead the conventional Divisia monetary aggregates at all four levels of aggregation. The credit-card augmented Divisia monetary aggregates better correlate with nominal GDP than the conventional Divisia monetary aggregates do. The credit-card augmented Divisia monetary aggregates more accurately reflect the Great Recession time period than the conventional Divisia monetary aggregates do.

Although the broadest aggregates, DM3 and DM4, more accurately and completely measure the economy's flow of monetary services, the transmission of policy to the aggregates is somewhat slower for the distant substitutes for money than for the assets in DM1 and DM2.

It is evident from these results why, in Tables 1 and 2, the new credit-card augmented Divisia monetary aggregates improve so dramatically upon the performance of the nominal GDP nowcasting approach developed by Barnett, Chauvet, and Leiva-Leon (2016). That approach previously incorporated the conventional CFS Divisia monetary aggregates among its significant indicators, with improved performance compared with use of the official simple sum monetary aggregates in the same nowcasting procedure.

9.1. Average Quarterly Growth Rates

Figure 3: M1 Average Quarterly Growth Rates (2007Q4-2015Q1)

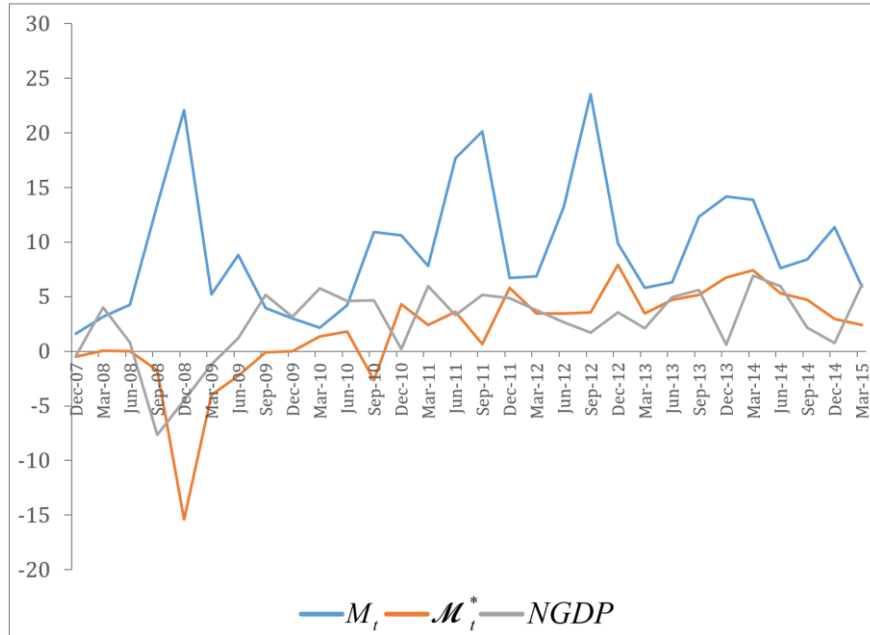


Figure 4: M2 Average Quarterly Growth Rates (2007Q4 – 2015Q1)

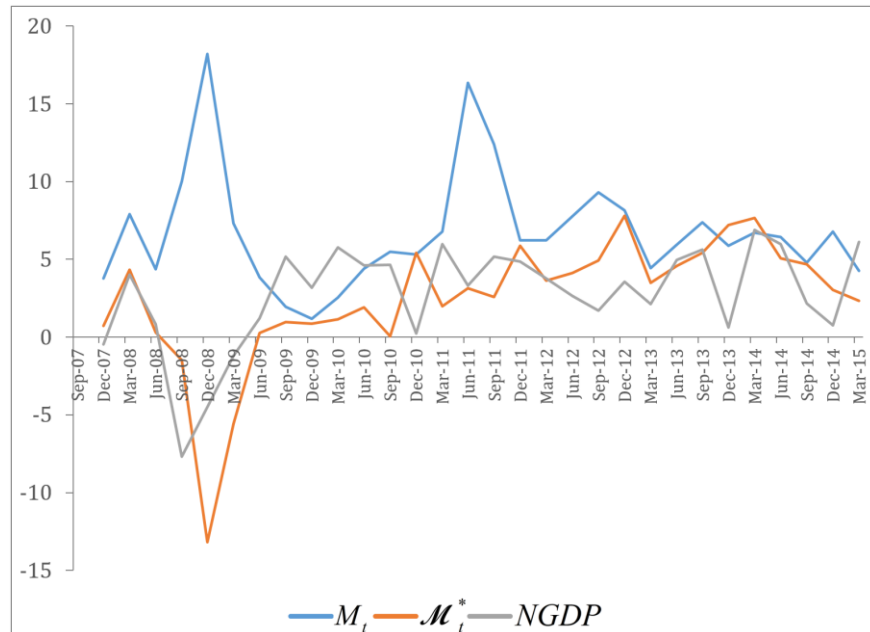


Figure 5: M3 Average Quarterly Growth Rates (2007Q4 – 2015Q1)

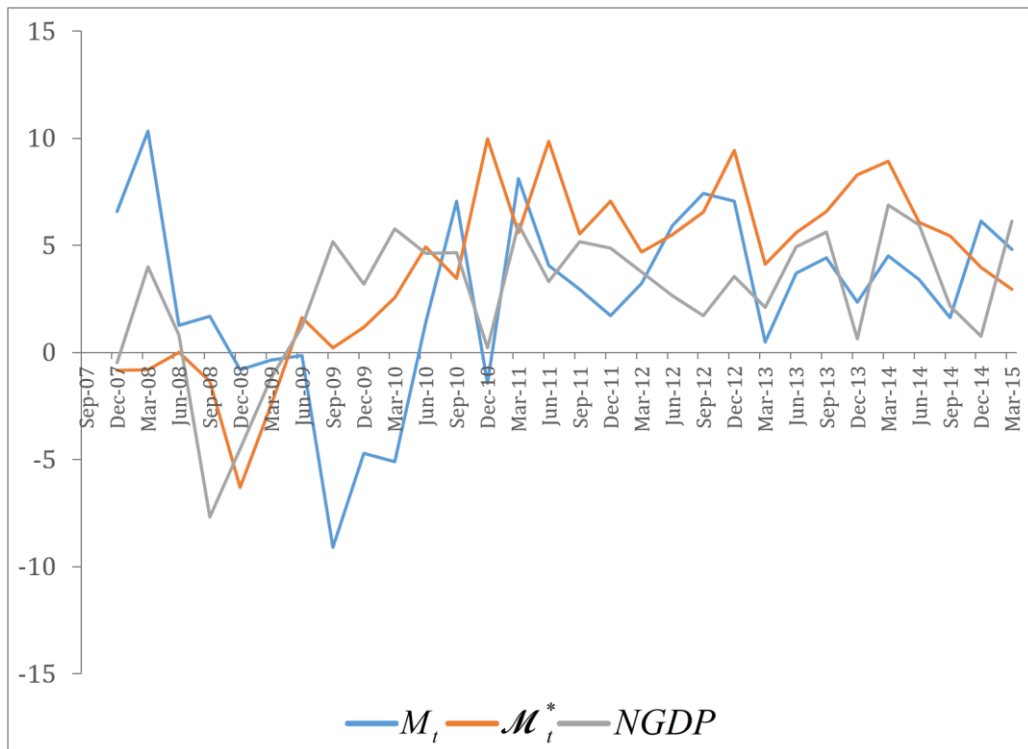
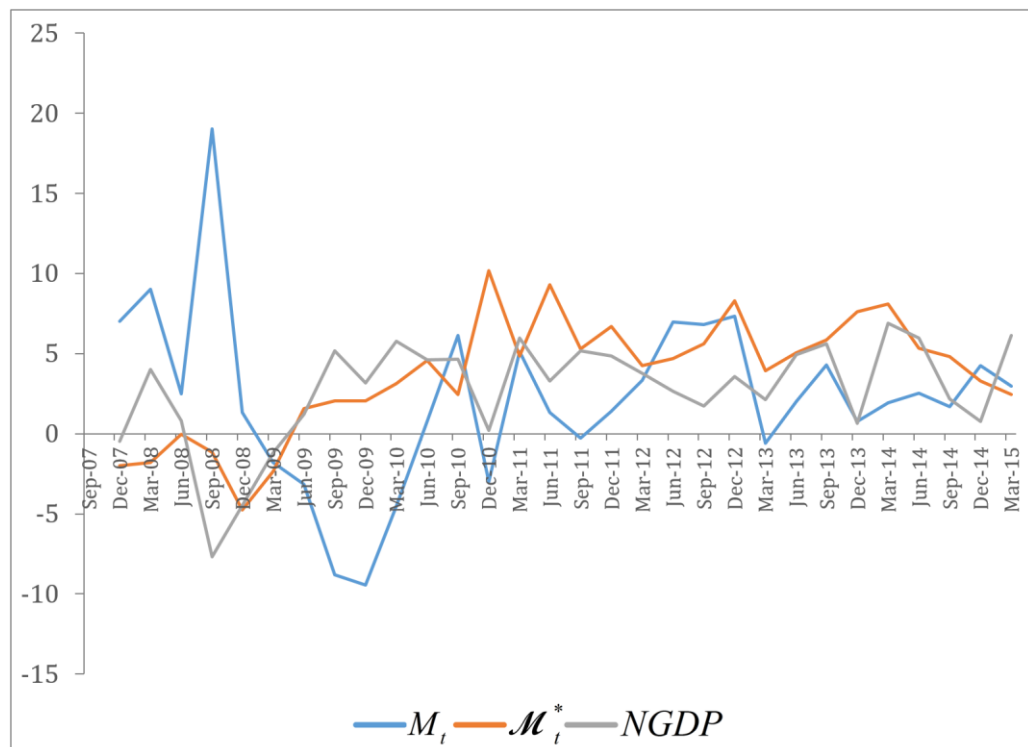


Figure 6: M4 Average Quarterly Growth Rates (2007Q4 – 2015Q1)



9.2. Quarterly Year-over-Year Growth Rates

Figure 7: M1 Quarterly Year-over-Year Growth Rates (2007Q4 – 2015Q1)

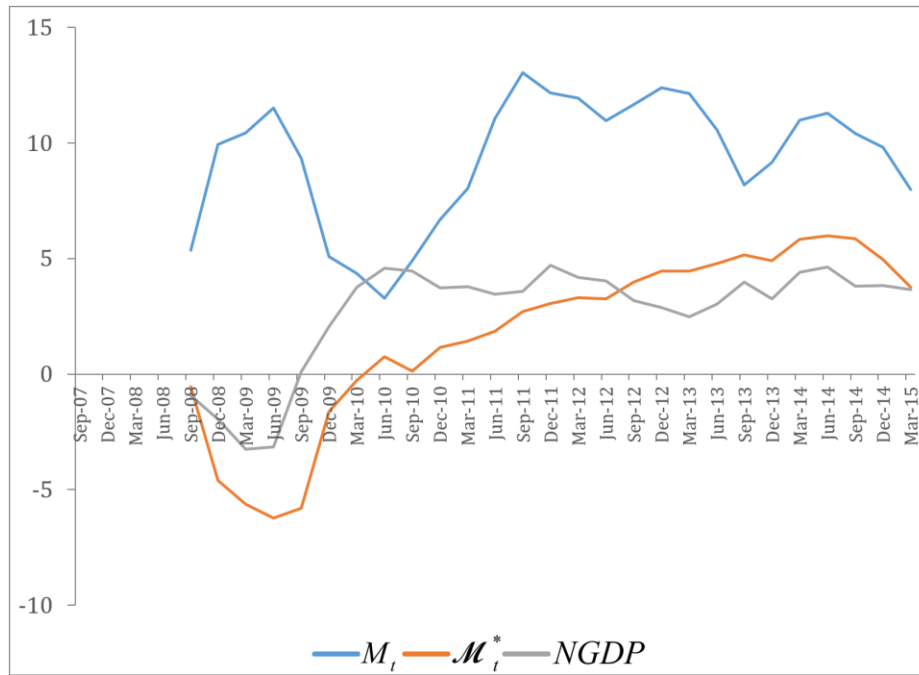


Figure 8: M2 Quarterly Year-over-Year Growth Rates (2007Q4 – 2015Q1)

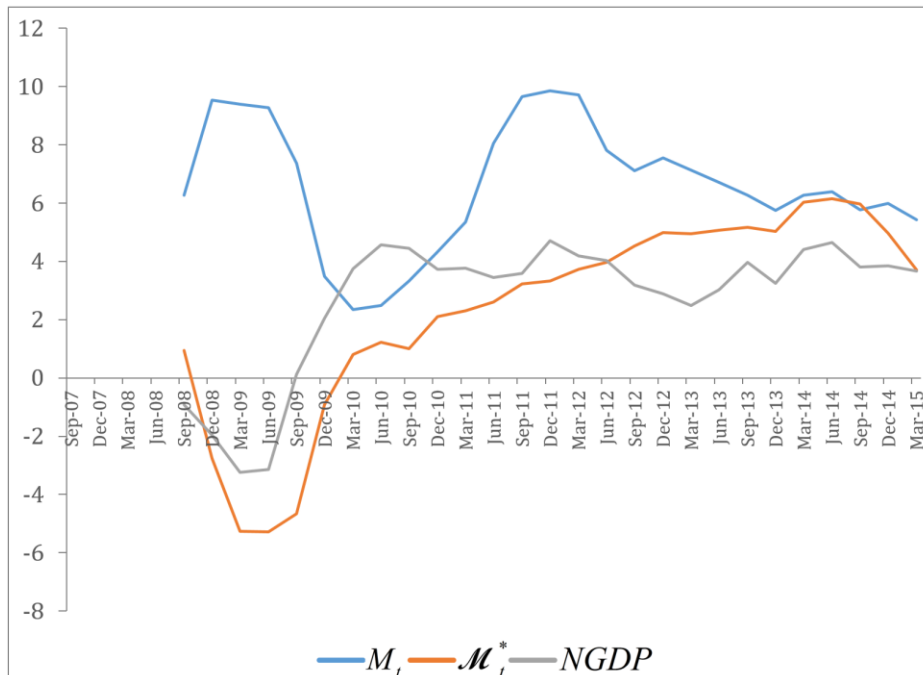


Figure 9: Quarterly M3 Year-over-Year Growth Rates (2007Q4 – 2015Q1)

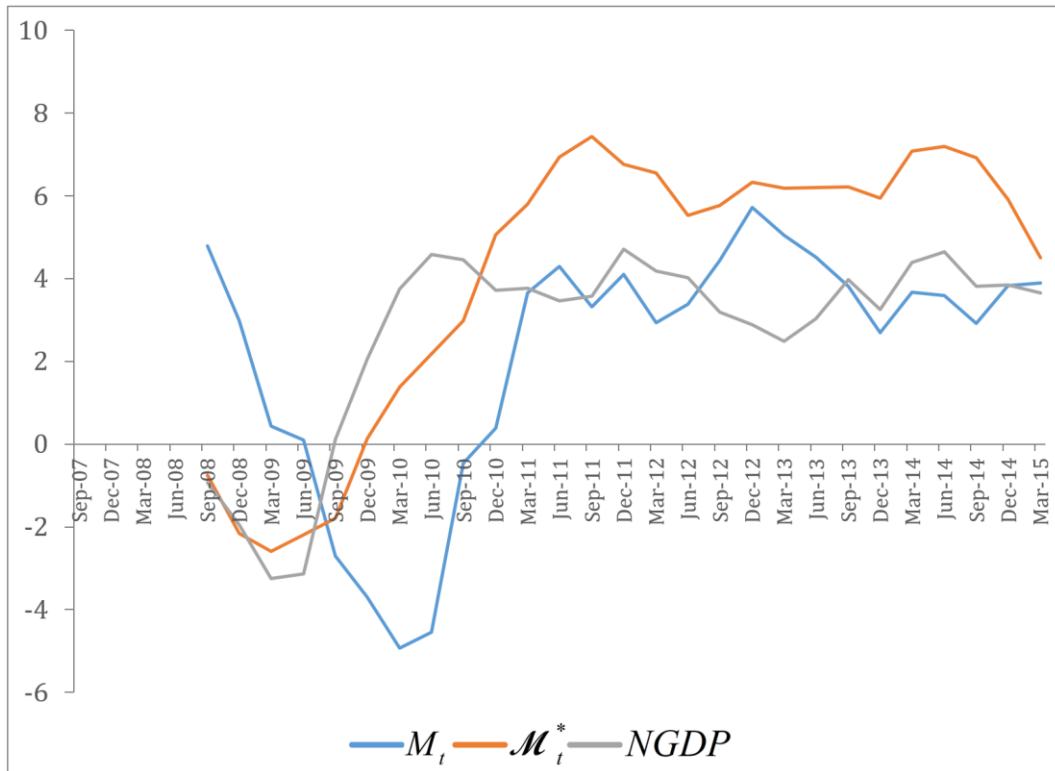
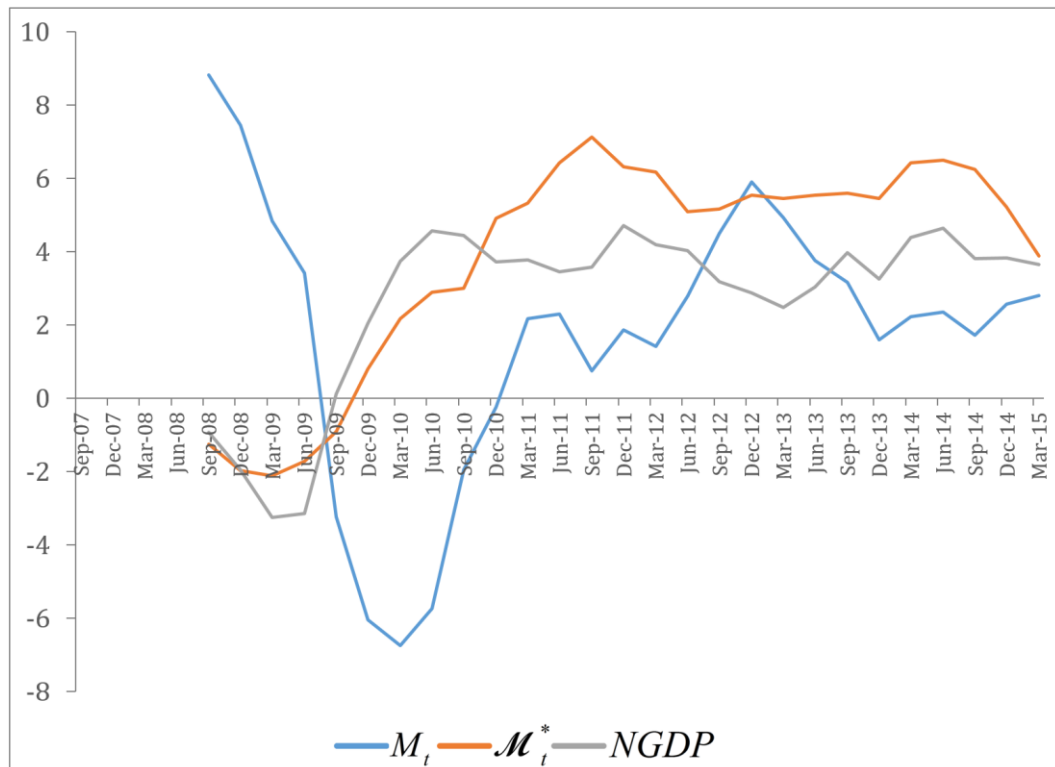


Figure 10: M4 Quarterly Year-over-Year Growth Rates (2007Q4 – 2015Q1)



10. Conclusions

Many economists have wondered how the transactions services of credit cards could be included in monetary aggregates. The conventional simple sum accounting approach precludes solving that problem, since accounting conventions do not permit adding liabilities to assets. But economic aggregation and index number theory measure service flows, independently of whether from assets or liabilities. We have provided theory solving that long overlooked problem both for use as a structural economic variable or as an indicator. Different theory is relevant to those two objectives, and hence we have provided two different aggregates. The aggregation-theoretic exact approach provides our credit card-augmented structural aggregate, $\mathcal{M}_t = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$, while the indicator optimized augmented aggregate, uniquely derived from our nowcasting model, produces our aggregate, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$. In the former case, the aggregate is defined to be weakly separable within the structure of the economy, while in the latter approach the aggregate is defined to be weakly separable within the nowcasting equation. The former approach is relevant to any application requiring a measure of monetary services within the structure of the economy, while the latter approach is application specific and only relevant for use as an indicator.

We have provided the solution under various levels of complexity in terms of theory, econometrics, and data availability. Both sets of new aggregates will be provided to the public in monthly releases by the Center for Financial Stability (CFS) and also to Bloomberg terminal users. The CFS is now providing the unaugmented aggregates, $M_t = M(\mathbf{m}_t)$, and will soon be providing both the structural augmented aggregates, $\mathcal{M}_t = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$, and indicator-optimized

augmented aggregates, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$.

In previous research, Barnett, Chauvet, and Leiva-Leon (2016) have found that the CFS Divisia monetary aggregate, $M_t = M(\mathbf{m}_t)$, is a valuable indicator in a four factor nowcasting model of nominal GDP. In this current research, we have found that our new augmented Divisia monetary aggregates, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$, provide substantially greater indicator value than $M_t = M(\mathbf{m}_t)$.

Although the greater indicator value is evident from our time series plots, we have displayed the formal nowcasting results to confirm the evidence from the plots. Among the potential applications of the indicator approach would be in nominal GDP targeting, requiring the existence of monthly nominal GDP nowcasts.

An extensive literature exists on policy relevance of the Divisia monetary aggregates.⁴⁹ Much of that literature could be strengthened further by use of the soon to be available credit-card-augmented CFS structural Divisia monetary aggregates, $\mathcal{M}_t = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$. We leave such empirical research with those aggregates to future applications, but we provide the supporting economic theory. It should be observed that \mathcal{M}_t and \mathcal{M}_t^* are not good substitutes for each other, having been derived from different existence conditions relevant to different objectives.⁵⁰ Our

⁴⁹ See, e. g., Barnett (2012), Belongia and Ireland (2006;2014; 2015a,b; 2016), Barnett and Chauvet (2011), Serletis and Rahman (2013), Barnett and Serletis (2000), and Serletis and Gogas (2014).

⁵⁰ A consequence is much higher weight on the credit card transactions volumes in the indicator optimized aggregator function, \mathcal{M}_t^* , than in the Divisia index, \mathcal{M}_t . A possible way to understand the different behaviors of \mathcal{M}_t^* and \mathcal{M}_t is relative to the transmission mechanism of monetary policy. As a potential intermediate target of policy, the growth of \mathcal{M}_t is strongly influenced by variations in the instruments of Central Bank policy as well as by private shadow banking activity. In contrast, \mathcal{M}_t^* is an indicator of a final target of monetary policy, nominal GDP,

empirical research in this paper focuses on the indicator optimized aggregates, $\mathcal{M}_t^* = \mathcal{M}_t^*(\mathbf{m}_t, \mathbf{c}_t)$.

A more challenging approach would introduce risk aversion in accordance with Barnett and Wu (2005).⁵¹ Adapting that advanced approach to our augmented aggregates remains another topic for future research, as does disaggregation to a heterogeneous agents approach.

and hence is much farther into the transmission of mechanism of monetary policy. As a result, \mathcal{M}_t^* might be more strongly influenced by factors unrelated to Central Bank policy, such as international energy price variations, and influenced by Central Bank policy with longer lags than \mathcal{M}_t . Since this paper does not model the transmission mechanism of monetary policy, these speculations are, at best, viewed as potential topics for future research.

⁵¹ Initial theoretical results in that direction are available in Barnett, Chauvet, Leiva-Leon, and Su (2016).

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APPENDIX

Derivation of the User Cost Formula for Credit Card Services, Equation (7), in the Infinite Lifetimes Case

From equation 2, the flow of funds identities, for $s = t, t+1, \dots, \infty$, are

$$\begin{aligned} \mathbf{p}_s' \mathbf{x}_s = & \omega_s L_s + \sum_{i=1}^n [(1+r_{i,s-1})p_{s-1}^* m_{i,s-1} - p_s^* m_{is}] + \sum_{j=1}^k [p_s^* c_{js} - (1+e_{j,s-1})p_{s-1}^* c_{j,s-1}] \\ & + \sum_{j=1}^k [p_s^* z_{js} - (1+\bar{e}_{j,s-1})p_{s-1}^* z_{j,s-1}] + [(1+R_{s-1})p_{s-1}^* A_{s-1} - p_s^* A_s]. \end{aligned} \quad (\text{A.1})$$

The intertemporal utility function

$$u(\mathbf{m}_t, \mathbf{c}_t, \mathbf{x}_t) + E_t \left[\sum_{s=t+1}^{\infty} \left(\frac{1}{1+\xi} \right)^{s-t} u(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s) \right]$$

under perfect certainty is

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+\xi} \right)^{s-t} u(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s). \quad (\text{A.2})$$

Let \mathfrak{Z} be the Lagrangian for maximizing intertemporal utility subject to the sequence of flow of funds identities for $s = t, \dots, \infty$, and let λ_t be the Lagrange multiplier for the t 'th constraint. Then the following are the first order conditions for maximizing (A.2) subject to the sequence of

constraints, (A.1).

$$\frac{\partial \mathfrak{S}}{\partial A_t} = -\lambda_{t+1}(1 + R_t)p_t^* + \lambda_t p_t^* = 0, \quad (\text{A.3})$$

$$\frac{\partial \mathfrak{S}}{\partial x_{it}} = \frac{\partial u}{\partial x_{it}} - \lambda_t p_{it} = 0, \quad (\text{A.4})$$

$$\frac{\partial \mathfrak{S}}{\partial m_{it}} = \frac{\partial u}{\partial m_{it}} - \lambda_t p_t^* + \lambda_{t+1}(1 + r_{it})p_t^* = 0, \quad (\text{A.5})$$

$$\frac{\partial \mathfrak{S}}{\partial c_{jt}} = \frac{\partial u}{\partial c_{jt}} + \lambda_t p_t^* - \lambda_{t+1}(1 + e_{jt})p_t^* = 0. \quad (\text{A.6})$$

From equation (A.3), we have

$$-\lambda_{t+1}(1 + R_t) + \lambda_t = 0. \quad (\text{A.7})$$

Substitute equation (A.7) into (A.6) to eliminate λ_{t+1} , we get

$$\frac{\partial u}{\partial c_{jt}} = -\lambda_t p_t^* + \frac{\lambda_t}{1 + R_t} p_t^* (1 + e_{jt}). \quad (\text{A.8})$$

Rearranging we get the first order condition that identifies $\tilde{\pi}_{jt}$ as the user cost price of credit card services:

$$\frac{\partial u}{\partial c_{jt}} = \lambda_t \tilde{\pi}_{jt}, \quad (\text{A.9})$$

where

$$\tilde{\pi}_{jt} = p_t^* \frac{e_{jt} - R_t}{1 + R_t}. \quad \blacksquare \quad (\text{A.10})$$

Chapter 3

Risk Adjustment of the Credit-Card Augmented Divisia Monetary Aggregates

William A. Barnett

University of Kansas, Lawrence, and Center for Financial Stability, NY City

Liting Su

University of Kansas, Lawrence, and Center for Financial Stability, NY City

August 27, 2016

Abstract

While credit cards provide transactions services, as do currency and demand deposits, credit cards have never been included in measures of the money supply. The reason is accounting conventions, which do not permit adding liabilities, such as credit card balances, to assets, such as money. However, economic aggregation theory and index number theory measure service flows and are based on microeconomic theory, not accounting. Barnett, Chauvet, Leiva-Leon, and Su (2016) derived the aggregation and index number theory needed to measure the joint services of credit cards and money. They derived and applied the theory under the assumption of risk neutrality. But since credit card interest rates are high and volatile, risk aversion may not be negligible. We extend the theory by removing the assumption of risk neutrality to permit risk aversion in the decision of the representative consumer.

Keywords: Credit Cards, Money, Credit, Aggregation, Monetary Aggregation, Index Number Theory, Divisia Index, Risk, Euler Equations, Asset Pricing.

JEL Classification: C43, C53, C58, E01, E3, E40, E41, E51, E52, E58, G17.

1. Introduction

While money is an asset, credit is a liability. In accounting conventions, assets and liabilities cannot be added together. But aggregation theory and economic index number theory are based on microeconomic theory, not accounting conventions. Economic aggregates measure service flows. To the degree that money and some forms of credit produce joint services, those services can be aggregated.

A particularly conspicuous example is credit card services, which are directly involved in transactions and contribute to the economy's liquidity in ways not dissimilar to those of money. While money is both an asset and part of wealth, credit cards are neither. Hence credit cards are not money. To the degree that monetary policy operates through a wealth effect (Pigou effect), as advocated by Milton Friedman, credit cards do not play a role. But to the degree that the flow of monetary services is relevant to the economy, as through the demand for monetary services or as an indicator measure, the omission of credit card services from "money" measures induces a loss of information.

Barnett, Chauvet, Leiva-Leon, and Su (2016) derived the aggregation and index number theory needed to aggregate jointly over the services of money and credit cards. The derivation uses strongly simplifying assumptions. They assume credit cards are used to purchase consumer goods. All purchases are made at the beginning of periods, and payments for purchases are either by credit cards or money. Credit card purchases are repaid to the credit card company at the end of the current period or at the end of a future period, plus interest charged by the credit card company. Stated more formally, all discrete time periods are closed on the left and open on

the right. After aggregation over consumers, the expected interest rate paid by the representative credit card holder can be very high, despite the fact that some consumers pay no interest on credit card balances.

The derivation in Barnett, Chauvet, Leiva-Leon, and Su (2016) assumes perfect certainty or risk neutrality. With monetary assets, having relatively low risk returns, risk aversion is not likely to have much effect on the behavior of aggregation theoretic monetary aggregates, such as the Divisia monetary aggregates. Studies have tended to show that weakening the assumption of risk neutrality in the derivation of the Divisia monetary aggregates has little effect on the behavior of the aggregates. See, e.g., Barnett, Liu, and Jensen (1997). But inclusion of credit card services introduces a high risk rate of return: the interest rate on credit card debt. As a result, extension of the aggregation theory to the case of risk neutrality might alter the behavior of the aggregate in a non-negligible manner. We extend the theory of Barnett, Chauvet, Leiva-Leon, and Su (2016) by removing the assumption of risk neutrality. The derivation is thereby altered by replacing the perfect certainty first order conditions with the relevant Euler equations.

To reflect the fact that money and credit cards provide services, such as liquidity and transactions services, money and credit are entered into a derived utility function, in accordance with Arrow and Hahn's (1971) proof.⁵² The derived utility function absorbs constraints reflecting the explicit

⁵² Our research in this paper is not dependent upon the simple decision problem we use for derivation and illustration. In the case of monetary aggregation, Barnett (1987) proved that the same aggregator functions and

motives for using money and credit card services. Since this paper is about measurement, we need only assume the existence of such motives. In the context of this research, we have no need to work backwards to reveal the explicit motives. As has been shown repeatedly, any of those motives, including the highly relevant transactions motive, are consistent with existence of a derived utility function absorbing the motive.⁵³

2. Intertemporal Allocation

We begin by defining the variables in the risk neutral case for the representative consumer:

\mathbf{x}_s = vector of per capita (planned) consumptions of N goods and services (including those of durables) during period s .

\mathbf{p}_s = vector of goods and services expected prices, and of durable goods expected rental prices during period s .

index numbers apply, regardless of whether the initial model has money in the utility function or production function, so long as there is intertemporal separability of structure and separability of components over which aggregation occurs. That result is equally as applicable to our current results with augmented aggregation over monetary asset and credit card services. While this paper uses economic index number theory, it should be observed that there also exists a statistical approach to index number theory. That approach produces the same results, with the Divisia index interpreted to be the Divisia mean using expenditure shares as probability. See Barnett and Serletis (1990).

⁵³ The aggregator function is the derived function that always exists, if monetary and credit card services have positive value in equilibrium. See, e.g., Samuelson (1948), Arrow and Hahn (1971), stockFischer (1974), Philips and Spinnewyn (1982), Quirk and Saposnik (1968), and Poterba and Rotemberg (1987). Analogously, Feenstra (1986, p. 271) demonstrated “a functional equivalence between using real balances as an argument of the utility function and entering money into liquidity costs which appear in the budget constraints.” The converse mapping from money and credit in the utility function back to the explicit motive is not unique. But in this paper we are not seeking to identify the explicit motives for holding money or credit card balances.

m_{is} = planned per capita real balances of monetary asset i during period s ($i = 1, 2, \dots, n$).

c_{js} = planned per capita real expenditure with credit card type j for transactions during period s ($j = 1, 2, \dots, k$). In the jargon of the credit card industry, those contemporaneous expenditures are called “volumes.”

z_{js} = planned per capita rotating real balances in credit card type j during period s from transactions in previous periods ($j = 1, 2, \dots, k$).

$y_{js} = c_{js} + z_{js}$ = planned per capita total balances in credit type j during period s ($j = 1, 2, \dots, k$).

r_{is} = expected nominal holding period yield (including capital gains and losses) on monetary asset i during period s ($i = 1, 2, \dots, n$).

e_{js} = expected interest rate on c_{js} .

\bar{e}_{js} = expected interest rate on z_{js} .

A_s = planned per capita real holdings of the benchmark asset during period s .

R_s = expected (one-period holding) yield on the benchmark asset during period s .

L_s = per capita labor supply during period s .

w_s = expected wage rate during period s .

$p_s^* = p_s^*(\mathbf{p}_s)$ is the true cost of living index, as defined in Barnett (1978, 1980).

The benchmark asset is defined to provide no services other than its expected yield, R_s , which motivates holding of the asset solely as a means of accumulating wealth. As a result, R_s is the maximum expected holding period yield available to consumers in the economy in period s from holding a secured asset. The benchmark asset is held to transfer wealth by consumers between multiperiod planning horizons, rather than to provide liquidity or other services. In contrast, \bar{e}_{js}

is not the interest rate on an asset and is not secured. It is the interest rate on an unsecured liability, subject to substantial default and fraud risk. Hence, \bar{e}_{js} can be higher than the benchmark asset rate, and historically has always been much higher than the benchmark asset rate.⁵⁴

It is important to recognize that the decision problem we model is not of a single economic agent, but rather of the “representative consumer,” aggregated over all consumers. All quantities are therefore averaged over all consumers. Gorman’s assumptions for the existence of a representative consumer are implicitly accepted, as is common in almost all modern macroeconomic theory having microeconomic foundations. This modeling assumption is particularly important in understand the credit card quantities and interest rates used in our research. About 20% of credit card holders in the United States do not pay explicit interest on credit card balances, since those credit card transactions are paid off by the end of the period. But the 80% who do pay interest pay very high interest rates.⁵⁵ The Federal Reserve provides two

⁵⁴ Barnett, Chauvet, Leiva-Leon, and Su (2016) follow the Center for Financial Stability (CFS) and the Bank of Israel in using the short term bank loan rate as a proxy for the benchmark rate. That interest rate has always exceeded the interest rate paid by banks on deposit accounts and on all other monetary assets used in the CFS Divisia monetary aggregates, and has always been lower than the Federal Reserve’s reported average interest rate charged on credit card balances. For detailed information on CFS data sources, see Barnett, Liu, Mattson, and Noort (2013). For the additional data sources used by the CFS to extend to credit card services, see Barnett and Su (2016).

⁵⁵ The following statement is from www.motherjones.com/kevin-drum/2011/10/americans-are-clueless-about-their-credit-card-debt. “In the four working age categories, about 50% of households think they have outstanding credit card debt, but the credit card companies themselves think about 80% of households have outstanding balances.” Since these percentages are of total households, including those having no credit cards, the percent of credit card holders paying interest might be even higher.

interest rate series for credit card debt. One, \bar{e}_{js} , includes interest only on accounts that do pay interest to the credit card issuing banks, while the other series, e_{js} , includes the approximately 20% that do not pay interest. The latter interest rate is thereby lower, since it is averaged over interest paid on both categories of accounts. Since we are modeling the representative consumer, aggregated over all consumers, e_{js} is always less than \bar{e}_{js} for all j and s . The interest rate on rotating credit card balances, \bar{e}_{js} , is paid by all consumers who maintain rotating balances in credit cards. But e_{js} is averaged over those consumers who maintain such rotating balances and hence pay interest on contemporaneous credit card transactions (volumes) and those consumers who pay off such credit card transactions before the end of the period, and hence do not pay explicit interest on the credit card transactions. The Federal Reserve provides data on both \bar{e}_{js} and e_{js} . Although e_{js} is less than \bar{e}_{js} , e_{js} also has always been higher than the benchmark rate. This observation is a reflection of the so-called credit card debt puzzle.⁵⁶

Barnett, Chauvet, Leiva-Leon, and Su (2016) use the latter interest rate, e_{js} , in their augmented Divisia monetary aggregates formula, since the contemporaneous per capita transactions volumes in our model are averaged over both categories of credit card holders. They do not include rotating balances used for transactions in prior periods, since to do so would involve

⁵⁶See, e.g., Telyukova and Wright (2008), who view the puzzle as a special case of the rate dominance puzzle in monetary economics. The “credit card debt puzzle” asks why people do not pay down debt, when receiving low interest rates on deposits, while simultaneously paying higher interest rates on credit card debt.

double counting of transactions services.

The expected interest rate, e_{js} , can be explicit or implicit, and applies to the aggregated representative consumer. For example, an implicit part of that interest rate could be in the form of an increased price of the goods purchased or in the form of a periodic service fee or membership fee. But we use only the Federal Reserve's average explicit interest rate series, which is lower than the one that would include implicit interest. Nevertheless, that downward biased explicit rate of return to credit card companies, e_{js} , aggregated over consumers, tends to be very high, far exceeding R_s , even after substantial losses from fraud.

We follow Barnett, Chauvet, Leiva-Leon, and Su (2016) in using the credit card industry's definition of "credit card," which excludes "store cards" and "charge cards." According to the trade's definition, "store cards" are issued by businesses providing credit only for their own goods, such as gasoline company credit cards or department store cards. To be a "credit card" by the trade's definition, the card must be widely accepted for many goods and services in the economy not constrained to cash-only sales. "Charge cards" can be widely accepted for goods purchases, but do not charge interest, since the debt must be paid off by the end of the period. To be a "credit card," the card must provide a line of credit to the card holder with interest charged on purchases not paid off by the end of the period. For example, American Express provides both charge cards and credit cards. The first credit card was provided by Bank of America. There now are four sources of credit card services in the United States: Visa, Mastercard, Discover, and American Express. From American Express, we use only their credit card account services, not their charge cards. We use data from only those four sources, in

accordance with the credit card industry's conventional definition of "credit card."

The resulting flow of funds identity for each period s is:

$$\begin{aligned}
& \mathbf{p}'_s \mathbf{x}_s \\
& = w_s L_s \\
& + \sum_{i=1}^n [(1 + r_{i,s-1}) p_{s-1}^* m_{i,s-1} - p_s^* m_{is}] \\
& + \sum_{j=1}^k [p_s^* c_{js} - (1 + e_{j,s-1}) p_{s-1}^* c_{j,s-1}] \\
& + \sum_{j=1}^k [p_s^* z_{js} - (1 + \bar{e}_{j,s-1}) p_{s-1}^* z_{j,s-1}] + [(1 + R_{s-1}) p_{s-1}^* A_{s-1} \\
& - p_s^* A_s].
\end{aligned} \tag{1}$$

Planned per capita total balances in credit type j during period s are then $y_{js} = c_{js} + z_{js}$.

Equation (1) is an accounting identity, with the right hand side being funds available to purchase consumer goods during period s . On the right hand side, the first term is labor income. The second term is funds absorbed or released by rolling over the monetary assets portfolio, as explained in Barnett (1980). The third term is particularly important to this paper. That term is the net change in credit card debt during period s from purchases of consumer goods, while the fourth term is the net change in rotating credit card debt. The fifth term is funds absorbed or released by rolling over the stock of the benchmark asset, as explained in Barnett (1980). The third term on the right side is specific to current period credit card purchases, while the fourth

term is not relevant to the rest of our results, since z_{js} is not in the utility function. Hence z_{js} is not relevant to the user cost prices, conditional decisions, or aggregates in the rest of this paper.

In the perfect certainty case, Barnett (1980) found that the current nominal user cost price, π_{it} , of m_{it} is

$$\pi_{it} = \frac{p_t^*(R_t - r_{it})}{1 + R_t}, \quad (2)$$

while Barnett, Chauvet, Leiva-Leon, and Su (2016) proved that the current period nominal user cost, $\tilde{\pi}_{jt}$, of c_{jt} is

$$\tilde{\pi}_{jt} = \frac{p_t^*(e_{jt} - R_t)}{1 + R_t}. \quad (3)$$

The corresponding real user costs are

$$\pi_{js}^* = \frac{\pi_{is}}{p_s^*} \quad (4a)$$

and

$$\tilde{\pi}_{js}^* = \frac{\tilde{\pi}_{jt}}{p_s^*}. \quad (4b)$$

Equation (3) can be understood in terms of the delay between the goods purchase date and the date of repayment of the loan to the credit card company. During the one period delay, the consumer can invest the cost of the goods purchased at rate of return R_t . Hence the net real cost to the consumer of the credit card loan, per dollar borrowed, is $e_{jt} - R_t$. Multiplication by the true cost of living index in the numerator of (3) converts to nominal dollars and division by $1 + R_t$ discounts to present value within the time period.

3. Risk Adjustment

In index number theory, it is known that uncertainty about future variables has no effect on contemporaneous aggregates or index numbers, if preferences are intertemporally separable. Only contemporaneous risk is relevant. See, e.g., Barnett (1995). Prior to Barnett, Liu, and Jensen (1997)), the literature on index number theory assumed that contemporaneous prices are known with certainty, as is reasonable for consumer goods. But Poterba and Rotemberg (1987) observed that contemporaneous user cost prices of monetary assets are not known with certainty, since interest rates are not paid in advance. As a result, the need existed to extend the field of index number theory to the case of contemporaneous risk.

For example, the derivation of the Divisia monetary index in Barnett (1980) uses the perfect certainty first-order conditions for expenditure constrained maximization of utility, in a manner

similar to Francois Divisia's (1925,1926) derivation of the Divisia index for consumer goods. But if the contemporaneous user costs are not known with certainty, those first order conditions become Euler equations. This observation motivated Barnett, Liu, and Jensen (1997)) to repeat the steps in the Barnett (1980) with the first order conditions replaced by Euler equations. In this section, we analogously derive an extended augmented Divisia index using the Euler equations that apply under risk, with utility assumed to be intertemporally strongly separable. The result is a Divisia index with the user costs adjusted for risk in a manner consistent with the CCAPM (consumption capital asset price model).⁵⁷

The approach to our derivation of the extended index closely parallels that in Barnett, Liu, and Jensen (1997), Barnett and Serletis (2000, ch. 12), and Barnett (2012, Appendix D) for monetary assets alone. But our results, including credit card services, are likely to result in substantially higher risk adjustments than the earlier results for monetary assets alone, since interest rates on credit card debt are much higher and much more volatile than on monetary assets.

3.1 The Decision

Define Y to be the consumer's survival set, assumed to be compact. The decision problem in this section will differ from the one in Barnett, Chauvet, Leiva-Leon, and Su (2016) not only by introducing risk, but also by adopting an infinite planning horizon. The consumption possibility

⁵⁷ Regarding CCAPM, see Lucas (1978), Breeden (1979), and Cochrane (2000).

set, $S(s)$, for period s is the set of survivable points, $(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s, A_s)$ satisfying equation (2).

The benchmark asset A_s provides no services other than its yield, R_s . As a result, the benchmark asset does not enter the consumer's contemporaneous utility function. The asset is held only as a means of accumulating wealth. The consumer's subjective rate of time preference, ξ , is assumed to be constant. The single-period utility function, $u(\mathbf{m}_t, \mathbf{c}_t, \mathbf{x}_t)$, is assumed to be increasing and strictly quasi-concave.

The consumer's decision problem is the following.

Problem 1. Choose the deterministic point $(\mathbf{m}_t, \mathbf{c}_t, \mathbf{x}_t, A_t)$ and the stochastic process $(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s, A_s)$, $s = t + 1, \dots, \infty$, to maximize

$$u(\mathbf{m}_t, \mathbf{c}_t, \mathbf{x}_t) + E_t \left[\sum_{s=t+1}^{\infty} \left(\frac{1}{1+\xi} \right)^{s-t} u(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s) \right], \quad (5)$$

subject to $(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s, A_s) \in S(s)$ for $s = t, t+1, \dots, \infty$, and also subject to the transversality condition

$$\lim_{s \rightarrow \infty} E_t \left(\frac{1}{1+\xi} \right)^{s-t} A_s = 0. \quad (6)$$

3.2 Existence of an Augmented Monetary Aggregate for the Consumer

We assume that the utility function, u , is blockwise weakly separable in $(\mathbf{m}_s, \mathbf{c}_s)$ and in \mathbf{x}_s .⁵⁸

Hence, there exists an augmented monetary aggregator function, \mathcal{M} , consumer goods aggregator function, X , and utility functions, F and H , such that

$$u(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s) = F[\mathcal{M}(\mathbf{m}_s, \mathbf{c}_s), X(\mathbf{x}_s)]. \quad (7)$$

We define the utility function V by $V(\mathbf{m}_s, \mathbf{c}_s, X_s) = F[\mathcal{M}(\mathbf{m}_s, \mathbf{c}_s), X_s]$, where aggregate consumption of goods is defined by $X_s = X(\mathbf{x}_s)$. It follows that the exact augmented monetary aggregate is

$$\mathcal{M}_s = \mathcal{M}(\mathbf{m}_s, \mathbf{c}_s). \quad (8)$$

The fact that blockwise weak separability is a necessary condition for exact aggregation is well known in the perfect-certainty case. If the resulting aggregator function also is linearly homogeneous, two-stage budgeting can be used to prove that the consumer behaves as if the exact aggregate were an elementary good. Although two-stage budgeting theory is not applicable under risk, $\mathcal{M}(\mathbf{m}_s, \mathbf{c}_s)$ remains the exact aggregation-theoretic quantity aggregate in a well-

⁵⁸ A long literature exists on testing the important assumption of blockwise weak separability of preferences. Recent contributions include Cherchye, Demuynck, Rock, and Hjerstrand (2015) and Hjerstrand, Swofford, and Whitney (2016).

defined sense, even under risk.⁵⁹

The Euler equations that will be of the most use to us below are those for monetary assets and credit card services. Those Euler equations are

$$E_s \left[\frac{\partial V}{\partial m_{is}} - \rho \frac{p_s^*(R_s - r_{is})}{p_{s+1}^*} \frac{\partial V}{\partial X_{s+1}} \right] = 0 \quad (9a)$$

and

$$E_s \left[\frac{\partial V}{\partial c_{js}} - \rho \frac{p_s^*(e_{js} - R_s)}{p_{s+1}^*} \frac{\partial V}{\partial X_{s+1}} \right] = 0 \quad (9b)$$

for all $s \geq t$, $i = 1, \dots, n$, and $j = 1, \dots, k$, where $\rho = 1/(1 + \xi)$ and where p_s^* is the exact price aggregate that is dual to the consumer goods quantity aggregate X_s .

Similarly, we can acquire the Euler equation for the consumer goods aggregate, X_s , rather than for each of its components. The resulting Euler equation for X_s is

$$E_s \left[\frac{\partial V}{\partial X_s} - \rho \frac{p_s^*(1 + R_s)}{p_{s+1}^*} \frac{\partial V}{\partial X_{s+1}} \right] = 0. \quad (9c)$$

For the two available approaches to derivation of the Euler equations, see the Appendix.

⁵⁹See Barnett (1995) and the appendix in Barnett, Liu, and Jensen (1997).

3.3 The Perfect-Certainty Case

In the perfect-certainty case with finite planning horizon, we have already shown in section 2 that the contemporaneous nominal user cost of the services of m_{it} is equation (2) and the contemporaneous nominal user cost of credit card services is equation (3). We have also shown in Barnett, Chauvet, Leiva-Leon, and Su (2016) that the solution value of the exact monetary aggregate, $\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t) = \mathcal{M}(\mathbf{m}_t^a)$, can be tracked without error in continuous time by the Divisia index.

The flawless tracking ability of the index in the perfect-certainty case holds regardless of the form of the unknown aggregator function, \mathcal{M} . Aggregation results derived with finite planning horizon also hold in the limit with infinite planning horizon. See Barnett (1987, section 2.2). Hence those results continue to apply. However, under risk, the ability of the Divisia index to track $\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$ is compromised.

3.4 New Generalized Augmented Divisia Index

3.4.1 User Cost Under Risk Aversion

We now find the formula for the user costs of monetary services and credit card services under risk.

Definition 1. The contemporaneous risk-adjusted real user cost price of the services of m_{it}^a is p_{it}^a , defined such that

$$p_{it}^a = \frac{\frac{\partial V}{\partial m_{it}^a}}{\frac{\partial V}{\partial X_t}}, i = 1, 2, \dots, n + k.$$

The above definition for the contemporaneous user cost states that the real user cost price of an augmented monetary asset is the marginal rate of substitution between that asset and consumer goods.

For notational convenience, we convert the nominal rates of return, r_{it} , e_{jt} and R_t , to real total rates, $1 + r_{it}^*$, $1 + e_{jt}^*$ and $1 + R_t^*$ such that

$$1 + r_{it}^* = \frac{p_t^*(1 + r_{it})}{p_{t+1}^*}, \quad (10a)$$

$$1 + e_{jt}^* = \frac{p_t^*(1 + e_{jt})}{p_{t+1}^*}, \quad (10b)$$

$$1 + R_t^* = \frac{p_t^*(1 + R_t)}{p_{t+1}^*}, \quad (10c)$$

where r_{it}^* , e_{jt}^* , and R_t^* are called the real rates of excess return. Under this change of variables and observing that current-period marginal utilities are known with certainty, Euler equations (9a), (9b), and (9c) become

$$\frac{\partial V}{\partial m_{it}} - \rho E_t \left[(R_t^* - r_{it}^*) \frac{\partial V}{\partial X_{t+1}} \right] = 0, \quad (11)$$

$$\frac{\partial V}{\partial c_{jt}} - \rho E_t \left[(e_{jt}^* - R_t^*) \frac{\partial V}{\partial X_{t+1}} \right] = 0, \quad (12)$$

and

$$\frac{\partial V}{\partial X_t} - \rho E_t \left[(1 + R_t^*) \frac{\partial V}{\partial X_{t+1}} \right] = 0. \quad (13)$$

We now can provide our user cost theorem under risk.

Theorem 1 (a). The risk adjusted real user cost of the services of monetary asset i under risk is

$\mathcal{P}_{it}^m = \pi_{it} + \psi_{it}$, where

$$\pi_{it} = \frac{E_t R_t^* - E_t r_{it}^*}{1 + E_t R_t} \quad (14)$$

and

$$\psi_{it} = \rho(1 - \pi_{it}) \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho \frac{\text{Cov}\left(r_{it}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}}. \quad (15)$$

(b). The risk adjusted real user cost of the services of credit card type j under risk is $\mathcal{P}_{jt}^c = \tilde{\pi}_{jt} +$

$\tilde{\psi}_{jt}$, where

$$\tilde{\pi}_{jt} = \frac{E_t e_{jt}^* - E_t R_t^*}{1 + E_t R_t} \quad (16)$$

and

$$\tilde{\psi}_{jt} = \rho \frac{\text{Cov}\left(e_{jt}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho(1 + \tilde{\pi}_{jt}) \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}}. \quad (17)$$

Proof. See the Appendix. ■

Under risk neutrality, the covariances in (16) and (17) would all be zero, because the utility function would be linear in consumption. Hence, the user cost of monetary assets and credit card services would reduce to $\pi_{i,t}$ and $\tilde{\pi}_{j,t}$ respectively, as defined in equation (14) and (16). The following corollary is immediate.

Corollary 1 to Theorem 1. Under risk neutrality, the user cost formulas are the same as equation (2) and (3) in the perfect-certainty case, but with all interest rates replaced by their expectations.

3.4.2 Generalized Augmented Divisia Index Under Risk Aversion

In the case of risk aversion, the first-order conditions are Euler equations. We now use those Euler equations to derive a generalized Divisia index, as follows.

Theorem 2. In the share equations, $\omega_{it} = \pi_{it}^a m_{it}^a / \pi_t^{a'} \mathbf{m}_t^a$, we replace the user costs, $\pi_t^a = (\pi_t', \tilde{\pi}_t')'$, defined by (2) and (3), by the risk-adjusted user costs, \mathcal{p}_{it}^a , defined by Definition 1, to produce the risk adjusted shares, $\mathcal{s}_{it} = \mathcal{p}_{it}^a m_{it}^a / \sum_{j=1}^{n+k} \mathcal{p}_{jt}^a m_{jt}^a$. Under our weak-separability assumption, $V(\mathbf{m}_s, \mathbf{c}_s, X_s) = F[\mathcal{M}(\mathbf{m}_s, \mathbf{c}_s), X_s]$, and our assumption that the monetary aggregator function, \mathcal{M} , is linearly homogeneous, the following generalized augmented Divisia

index is true under risk:

$$d\log \mathcal{M}_t = \sum_{i=1}^{n+k} s_{it} d\log m_{it}^a. \quad (18)$$

Proof. See the Appendix. ■

The exact tracking of the Divisia monetary index is not compromised by risk aversion, as long as the adjusted user costs, $\pi_{it} + \psi_{it}$ and $\tilde{\pi}_{jt} + \tilde{\psi}_{jt}$, are used in computing the index. The adjusted user costs reduce to the usual user costs in the case of perfect certainty, and our generalized Divisia index (18) reduces to the usual Divisia index. Similarly, the risk-neutral case is acquired as the special case with $\psi_{it} = \tilde{\psi}_{jt} = 0$, so that equations (14) and (16) serve as the user costs. In short, our generalized augmented Divisia index (18) is a true generalization, in the sense that the risk-neutral and perfect-certainty cases are strictly nested special cases. Formally, that conclusion is the following.

Corollary 1 to Theorem 2. Under risk neutrality, the generalized Divisia index (18) reduces to the perfect certainty Divisia index in Barnett, Chauvet, Leiva-Leon, and Su (2016), where the user costs in the formula are defined by (14) and (16).

3.5 CCAPM Special Case

As a means of illustrating the nature of the risk adjustments, $\psi_{i,t}$ and $\tilde{\psi}_{j,t}$, we consider a special case, based on the usual assumptions in CAPM theory of either quadratic utility or Gaussian stochastic processes. Direct empirical use of Theorems 1 and 2, without any CAPM simplifications, would require availability of prior econometric estimates of the parameters of the

utility function, V , and of the subjective rate of time discount. Under the usual CAPM assumptions, we show in this section that empirical use of Theorems 1 and 2 would require prior estimation of only one property of the utility function: the degree of risk aversion, on which a large body of published information is available.

Consider first the following case of utility that is quadratic in consumption of goods, conditionally on the level of monetary asset and credit card services.

Assumption 1. Let V have the form

$$V(\mathbf{m}_t, \mathbf{c}_t, X_t) = F[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t), X_t] = A[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)]X_t - \frac{1}{2}B[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)]X_t^2, \quad (19)$$

where A is a positive, increasing, concave function and B is a nonnegative, decreasing, convex function.

The alternative assumption is Guassianity, as follows:

Assumption 2. Let $(r_{it}^*, e_{jt}^*, X_{t+1})$ be a trivariate Gaussian process for each asset $i = 1, \dots, n$, and credit card service, $j = 1, \dots, k$.

We also make the following conventional CAPM assumption:

Assumption 3. The benchmark rate process is deterministic or already risk-adjusted, so that R_t^* is the risk-free rate.

Under this assumption, it follows that

$$\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right) = 0.$$

We define $H_{t+1} = H(\mathcal{M}_{t+1}, X_{t+1})$ to be the well-known Arrow-Pratt measure of absolute risk aversion,

$$H(\mathcal{M}_{t+1}, X_{t+1}) = -\frac{E_t[V'']}{E_t[V']}, \quad (20)$$

where $V' = \partial V(\mathbf{m}_{t+1}^a, X_{t+1})/\partial X_{t+1}$ and $V'' = \partial^2 V(\mathbf{m}_{t+1}^a, X_{t+1})/\partial X_{t+1}^2$. In this definition, risk aversion is measured relative to consumption risk, conditionally upon the level of augmented monetary services produced by $\mathcal{M}_{t+1} = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$. Under risk aversion, H_{t+1} is positive and increasing in the degree of absolute risk aversion. The following lemma is central to our Theorem 3.

Lemma 2. Under Assumption 3 and either Assumption 1 or Assumption 2, the user-cost risk adjustments, ψ_{it} and $\tilde{\psi}_{jt}$, defined by (15) and (17), reduce to

$$\psi_{it} = \frac{1}{1 + R_t^*} H_{t+1} \text{cov}(r_{it}^*, X_{t+1}) \quad (21a)$$

and

$$\tilde{\psi}_{jt} = -\frac{1}{1 + R_t^*} H_{t+1} \text{cov}(e_{jt}^*, X_{t+1}). \quad (21b)$$

Proof. See the Appendix. ■

The following theorem identifies the effect of the risk adjustment on the expected own interest rates in the user cost formulas.

Theorem 3. Let $\hat{H}_t = H_{t+1}X_t$. Under the assumptions of Lemma 2, we have the following for each asset $i = 1, \dots, n$, and credit card service, $j = 1, \dots, k$.

$$\mathcal{P}_{it}^m = \frac{E_t R_t^* - (E_t r_{it}^* - \phi_{it})}{1 + E_t R_t^*}, \quad (22)$$

where

$$\phi_{it} = \hat{H}_t \text{Cov}\left(r_{it}^*, \frac{X_{t+1}}{X_t}\right), \quad (23)$$

and

$$\mathcal{P}_{jt}^c = \frac{(E_t e_{jt}^* - \tilde{\phi}_{jt}) - E_t R_t^*}{1 + E_t R_t^*}, \quad (24)$$

where

$$\tilde{\phi}_{jt} = \hat{H}_t \text{Cov}\left(e_{jt}^*, \frac{X_{t+1}}{X_t}\right). \quad (25)$$

Proof. See the Appendix. ■

As defined, \hat{H}_t is a time shifted Arrow-Pratt relative risk aversion measure. Theorem 3 shows

that the risk adjustment on the own interest rate for a monetary asset or credit card service depends upon relative risk aversion, \hat{H}_t , and the covariance between the consumption growth path, X_{t+1}/X_t , and the real rate of excess return earned on a monetary asset, r_{it}^* , or paid on a credit card service, e_{jt}^* .

3.6 Magnitude of the Adjustment

In accordance with the large and growing literature on the equity premium puzzle, the CCAPM risk adjustment term is widely believed to be biased downward.⁶⁰ A promising explanation may be the customary assumption of intertemporal separability of utility, since response to a change in an interest rate may not be fully reflected in contemporaneous changes in consumption. Hence the contemporaneous covariance in the CCAPM “beta” correction may not take full account of the effect of an interest rate change on life style. An approach to risk adjustment without assumption of intertemporal separability was developed for monetary aggregation by Barnett and Wu (2005). We have not yet applied that more complicated approach to weaken our assumptions further. While we have removed the assumption of risk neutrality, we have assumed intertemporal separability in deriving the Euler equations on which our aggregation

⁶⁰See, e.g., Campbell and Cochrane (1999), Cochrane (2000), Kocherlakota (1996), Marshall (1997), Mehra and Prescott (1985).

theory is based. In later research, we plan to apply the approach of Barnett and Wu (2005) to further weaken the assumptions by removing the assumptions of intertemporal separability.

4. Conclusions

Many economists have wondered how the transactions services of credit cards could be included in monetary aggregates. The conventional simple sum accounting approach precludes solving that problem, since accounting conventions do not permit adding liabilities to assets. But economic aggregation and index number theory measure service flows, independently of whether from assets or liabilities. Barnett, Chauvet, Leiva-Leon, and Su (2016) provided the theory solving that long overlooked problem, but under the assumption of risk neutrality. The Center for Financial Stability (CFS) is now providing the unaugmented aggregates, $M_t = M(\mathbf{m}_t)$, and will soon be providing the credit-card-augmented aggregates $\mathcal{M}_t = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$, derived under the assumption of risk neutrality. The new aggregates will be provided to the public in monthly releases by the CFS and also to Bloomberg terminal users.

While excluding credit card services, the currently available CFS Divisia monetary aggregates have been found to be reasonably robust to introduction of risk, variations of the benchmark rate, introduction of taxation of interest rates, and other such refinements.⁶¹ But such simplifications

⁶¹ While those refinements slightly change the un-augmented Divisia monetary aggregates, those changes are negligible relative to the gap between the simple sum monetary aggregate path and the corresponding Divisia monetary aggregate path. See, e.g., the online library of relevant research and the Divisia monetary aggregates

might not be the case with the augmented monetary aggregates, because of the high and volatile interest rates on credit card balances. As a result, in this paper we have extended the theory to CCAPM risk adjustment under risk aversion. Empirical application of this theory remains a topic for future research.

An extensive literature exists on policy relevance of the Divisia monetary aggregates. See, e. g., Barnett (2012), Belongia and Ireland (2014; 2015a,b; 2016), Barnett and Chauvet (2011a,b), Serletis and Rahman (2013), and Serletis and Gogas (2014). Much of that literature could be strengthened further by use of the soon to be available credit-card augmented CFS Divisia monetary aggregates and perhaps further strengthened by removing the assumption of risk neutrality in accordance with the theory in this paper.

A more demanding approach would remove the CCAPM assumption of intertemporal separability, in accordance with Barnett and Wu (2005). Adapting that advanced approach to our augmented aggregates, including credit card services, remains a topic for future research.

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APPENDICES

Derivation of Euler Equations for Credit Card Services, Equation (12):

The following are the Euler equations provided in the paper as equations (11), (12), and (13):

$$\frac{\partial V}{\partial m_{it}} - \rho E_t \left[(R_t^* - r_{it}^*) \frac{\partial V}{\partial X_{t+1}} \right] = 0, \quad (\text{A. 1})$$

$$\frac{\partial V}{\partial c_{jt}} - \rho E_t \left[(e_{jt}^* - R_t^*) \frac{\partial V}{\partial X_{t+1}} \right] = 0, \quad (\text{A. 2})$$

$$\frac{\partial V}{\partial X_t} - \rho E_t \left[(1 + R_t^*) \frac{\partial V}{\partial X_{t+1}} \right] = 0. \quad (\text{A. 3})$$

for all $s \geq t$, $i = 1, \dots, n$, and $j = 1, \dots, k$, where $\rho = 1/(1 + \xi)$ and where p_s^* is the exact price aggregate that is dual to the consumer goods quantity aggregate X_s .

Equation (A.1) was derived in Barnett (1995, Sec 2.3) using Bellman's method. An alternative approach to that derivation using calculus of variations was provided by Poterba and Rotemberg (1987). Equation (A.2) follows by the same approach to derivation, using either Bellman's method or calculus of variations. We are not providing the lengthy derivation of (A.2) in this appendix, since the steps in the Bellman method approach for this class of models are provided in detail in Barnett and Serletis (2000, pp. 201-204).

Proof of Theorem 1

Theorem 1 (a). The risk adjusted real user cost of the services of monetary asset i under risk is

$p_{it}^m = \pi_{it} + \psi_{it}$, where

$$\pi_{it} = \frac{E_t R_t^* - E_t r_{it}^*}{1 + E_t R_t^*} \quad (\text{A.4})$$

and

$$\psi_{it} = \rho(1 - \pi_{it}) \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho \frac{\text{Cov}\left(r_{it}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}}. \quad (\text{A.5})$$

(b). The risk adjusted real user cost of the services of credit card type j under risk is $\wp_{jt}^c = \tilde{\pi}_{jt} +$

$\tilde{\psi}_{jt}$, where

$$\tilde{\pi}_{jt} = \frac{E_t e_{jt}^* - E_t R_t^*}{1 + E_t R_t^*} \quad (\text{A.6})$$

and

$$\tilde{\psi}_{jt} = \rho \frac{\text{Cov}\left(e_{jt}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho(1 + \tilde{\pi}_{jt}) \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}}. \quad (\text{A.7})$$

Proof. For the analogous proof in the case of monetary assets only, relevant to part (a), see Barnett, Liu, and Jensen (1997), Barnett and Serletis (2000, ch. 12), or Barnett (2012, Appendix D). We provide the proof of part (b) for the extended case including credit. There are two approaches to proving this important theorem, the direct approach and the indirect approach. We provide both approaches, beginning with the indirect approach.

By definition (1) in the paper, we have for the credit card services user cost price

$$\phi_{jt}^c = \frac{\frac{\partial V}{\partial c_{jt}}}{\frac{\partial V}{\partial X_t}}. \quad (\text{A.8})$$

Defining $\tilde{\psi}_{jt}$ to be $\tilde{\psi}_{jt} = \phi_{jt}^c - \tilde{\pi}_{jt}$, it follows that

$$\frac{\partial V}{\partial c_{jt}} = (\tilde{\pi}_{jt} + \tilde{\psi}_{jt}) \frac{\partial V}{\partial X_t}.$$

Substituting equations (A.2) and (A.3) into this equation, we get

$$\rho E_t \left[(e_{jt}^* - R_t^*) \frac{\partial V}{\partial X_{t+1}} \right] = (\tilde{\pi}_{jt} + \tilde{\psi}_{jt}) \rho E_t \left[(1 + R_t^*) \frac{\partial V}{\partial X_{t+1}} \right].$$

Using the expectation of the product of correlated random variables, we have

$$\begin{aligned} & E_t (e_{jt}^* - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + \text{Cov} \left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\ &= \left\{ \left[\frac{E_t e_{jt}^* - E_t R_t^*}{1 + E_t R_t^*} \right] + \tilde{\psi}_{jt} \right\} \left\{ E_t (1 + R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + \text{Cov} \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \right\}. \end{aligned}$$

Multiplying $(1 + E_t R_t^*)$ through on both sides of the equation, we get:

$$\begin{aligned}
& (1 + E_t R_t^*) E_t (e_{jt}^* - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + (1 + E_t R_t^*) \text{Cov} \left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\
& = \left[E_t (e_{jt}^* - R_t^*) + (1 + E_t R_t^*) \tilde{\psi}_{jt} \right] \left\{ E_t (1 + R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + \text{Cov} \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \right\}.
\end{aligned}$$

Manipulating the algebra, we have

$$\begin{aligned}
& E_t (e_{jt}^* - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + (E_t R_t^*) E_t (e_{jt}^* - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + \text{Cov} \left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\
& + (E_t R_t^*) \text{Cov} \left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\
& = \left[E_t (e_{jt}^* - R_t^*) + (1 + E_t R_t^*) \tilde{\psi}_{jt} \right] \left\{ E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + (E_t R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + \text{Cov} \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \right\},
\end{aligned}$$

and hence

$$\begin{aligned}
& E_t (e_{jt}^* - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + (E_t R_t^*) E_t (e_{jt}^* - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + \text{Cov} \left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\
& + (E_t R_t^*) \text{Cov} \left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\
& = E_t (e_{jt}^* - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + E_t (e_{jt}^* - R_t^*) (E_t R_t^*) \left(E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) \right) + E_t (e_{jt}^* - R_t^*) \text{Cov} \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\
& + (1 + E_t R_t^*) \tilde{\psi}_{jt} \left\{ E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + (E_t R_t^*) \left(E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) \right) + \text{Cov} \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \right\}.
\end{aligned}$$

Notice that by equation (A.3),

$$\begin{aligned}
\frac{\partial V}{\partial X_t} &= \rho E_t \left[(1 + R_t^*) \frac{\partial V}{\partial X_{t+1}} \right] \\
&= \rho \left\{ E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + (E_t R_t^*) \left(E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) \right) + Cov \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \right\}.
\end{aligned}$$

Substituting this back into the prior equation, we have

$$\begin{aligned}
&E_t (e_{jt}^* - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + (E_t R_t^*) E_t (e_{jt}^* - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + Cov \left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\
&+ (E_t R_t^*) Cov \left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\
&= E_t (e_{jt}^* - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + E_t (e_{jt}^* - R_t^*) (E_t R_t^*) \left(E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) \right) + E_t (e_{jt}^* - R_t^*) Cov \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\
&+ (1 + E_t R_t^*) \tilde{\psi}_{jt} \left(\frac{1}{\rho} \frac{\partial V}{\partial X_t} \right).
\end{aligned}$$

Simplifying the equation, we get

$$\begin{aligned}
&Cov \left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) + (E_t R_t^*) Cov \left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\
&= E_t (e_{jt}^* - R_t^*) Cov \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) + (1 + E_t R_t^*) \tilde{\psi}_{jt} \left(\frac{1}{\rho} \frac{\partial V}{\partial X_t} \right).
\end{aligned}$$

Recall that by equation (A.6),

$$\tilde{\pi}_{jt} = \frac{E_t e_{jt}^* - E_t R_t^*}{1 + E_t R_t^*}.$$

Substituting this equation back into the prior equation, we have

$$\begin{aligned} & Cov\left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}}\right) + (E_t R_t^*) Cov\left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}}\right) \\ &= \tilde{\pi}_{jt} (1 + E_t R_t^*) Cov\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right) + (1 + E_t R_t^*) \tilde{\psi}_{jt} \left(\frac{1}{\rho} \frac{\partial V}{\partial X_t}\right). \end{aligned}$$

Rearranging the equation, we have

$$(1 + E_t R_t^*) Cov\left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}}\right) = \tilde{\pi}_{jt} (1 + E_t R_t^*) Cov\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right) + (1 + E_t R_t^*) \tilde{\psi}_{jt} \left(\frac{1}{\rho} \frac{\partial V}{\partial X_t}\right),$$

so that

$$Cov\left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}}\right) = \tilde{\pi}_{jt} Cov\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right) + \tilde{\psi}_{jt} \left(\frac{1}{\rho} \frac{\partial V}{\partial X_t}\right).$$

Hence, it follows that

$$\begin{aligned}
\tilde{\psi}_{jt} &= \rho \frac{\text{Cov}\left(e_{jt}^* - R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho \tilde{\pi}_{jt} \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} \\
&= \rho \frac{\text{Cov}\left(e_{jt}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho \tilde{\pi}_{jt} \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} \\
&= \rho \frac{\text{Cov}\left(e_{jt}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho(1 + \tilde{\pi}_{jt}) \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}}.
\end{aligned}$$

The alternative direct approach to proof is the following.

By equation (A.3), we have

$$\begin{aligned}
\frac{\partial V}{\partial X_t} &= \rho E_t \left[\left(1 + R_t^*\right) \frac{\partial V}{\partial X_{t+1}} \right] \\
&= \rho(1 + E_t R_t^*) \left(E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) \right) + \rho \text{Cov} \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right).
\end{aligned}$$

Rearranging, we get

$$\rho(1 + E_t R_t^*) \left(E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) \right) = \frac{\partial V}{\partial X_t} - \rho \text{Cov} \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right),$$

and hence

$$\rho E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) = \frac{1}{1 + E_t R_t^*} \left[\frac{\partial V}{\partial X_t} - \rho Cov \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \right]. \quad (\text{A.9})$$

But from (A.12), we have

$$\frac{\partial V}{\partial c_{jt}} = \rho E_t \left[(e_{jt} - R_t^*) \frac{\partial V}{\partial X_{t+1}} \right].$$

From the expectation of the correlated product, we then have

$$\frac{\partial V}{\partial c_{jt}} = \rho E_t (e_{jt} - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + \rho Cov \left[(e_{jt} - R_t^*), \frac{\partial V}{\partial X_{t+1}} \right],$$

so that

$$\frac{\partial V}{\partial c_{jt}} = \rho E_t (e_{jt} - R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right) + \rho Cov \left(e_{jt}, \frac{\partial V}{\partial X_{t+1}} \right) - \rho Cov \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right). \quad (\text{A.10})$$

Now substitute equation (A.9) into equation (A.10), to acquire

$$\begin{aligned} \frac{\partial V}{\partial c_{jt}} &= \frac{E_t (e_{jt} - R_t^*)}{1 + E_t R_t^*} \left[\frac{\partial V}{\partial X_t} - \rho Cov \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \right] + \rho Cov \left(e_{jt}, \frac{\partial V}{\partial X_{t+1}} \right) - \rho Cov \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \\ &= \tilde{\pi}_{jt} \left[\frac{\partial V}{\partial X_t} - \rho Cov \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right) \right] + \rho Cov \left(e_{jt}, \frac{\partial V}{\partial X_{t+1}} \right) - \rho Cov \left(R_t^*, \frac{\partial V}{\partial X_{t+1}} \right). \end{aligned}$$

Multiplying and dividing the right side by $\frac{\partial V}{\partial X_t}$, we get

$$\begin{aligned}
\frac{\partial V}{\partial c_{jt}} &= \frac{\partial V}{\partial X_t} \left\{ \tilde{\pi}_{jt} - \rho \tilde{\pi}_{jt} \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} + \rho \frac{\text{Cov}\left(e_{jt}, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} \right\} \\
&= \frac{\partial V}{\partial X_t} \left\{ \tilde{\pi}_{jt} + \rho \frac{\text{Cov}\left(e_{jt}, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho(1 + \tilde{\pi}_{jt}) \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} \right\}.
\end{aligned}$$

Define $\tilde{\psi}_{jt}$ by

$$\tilde{\psi}_{jt} = \rho \frac{\text{Cov}\left(e_{jt}, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho(1 + \tilde{\pi}_{jt}) \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}}.$$

Then we have

$$\frac{\frac{\partial V}{\partial c_{jt}}}{\frac{\partial V}{\partial X_t}} = \tilde{\pi}_{jt} + \tilde{\psi}_{jt},$$

so that

$$\wp_{jt}^c = \tilde{\pi}_{jt} + \tilde{\psi}_{jt}. \quad \blacksquare$$

Proof of Lemma 2:

Assumption 1. Let V have the form

$$V(\mathbf{m}_t, \mathbf{c}_t, X_t) = F[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t), X_t] = A[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)]X_t - \frac{1}{2}B[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)]X_t^2, \quad (\text{A. 11})$$

Where A is a positive, increasing, concave function and B is a nonnegative, decreasing, convex function.

Assumption 2. Let $(r_{it}^*, e_{jt}^*, X_{t+1})$ be a trivariate Gaussian process for each asset $i = 1, \dots, n$, and credit card service, $j = 1, \dots, k$.

Assumption 3. The benchmark rate process is deterministic or already risk-adjusted, so that R_t^* is the risk-free rate.

Under this assumption, it follows that

$$\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right) = 0.$$

Define $H_{t+1} = H(\mathcal{M}_{t+1}, X_{t+1})$ to be the well-known Arrow-Pratt measure of absolute risk aversion,

$$H(\mathcal{M}_{t+1}, X_{t+1}) = -\frac{E_t[V'']}{E_t[V']}, \quad (\text{A. 12})$$

Where $V' = \partial V(\mathbf{m}_{t+1}^a, X_{t+1})/\partial X_{t+1}$ and $V'' = \partial^2 V(\mathbf{m}_{t+1}^a, X_{t+1})/\partial X_{t+1}^2$.

Lemma 2. Under Assumption 3 and either Assumption 1 or Assumption 2, the user-cost risk adjustments, ψ_{it} and $\tilde{\psi}_{jt}$, defined by (A.5) and (A.7), reduce to

$$\psi_{it} = \frac{1}{1 + R_t^*} H_{t+1} \text{cov}(r_{it}^*, X_{t+1}) \quad (\text{A. 13})$$

and

$$\tilde{\psi}_{jt} = -\frac{1}{1 + R_t^*} H_{t+1} \text{cov}(e_{jt}^*, X_{t+1}). \quad (\text{A. 14})$$

Proof. For the analogous proof in the case of monetary assets only, see Barnett, Liu, and Jensen (1997), Barnett and Serletis (2000, ch. 12), or Barnett (2012, Appendix D). We provide the proof of equation (A.14) for the extended case including credit.

Under Assumption 3, the benchmark asset is risk-free, so that

$$\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right) = 0.$$

By equation (A.7),

$$\begin{aligned}
\tilde{\psi}_{jt} &= \rho \frac{\text{Cov}\left(e_{jt}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho(1 + \tilde{\pi}_{jt}) \frac{\text{Cov}\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} \\
&= \rho \frac{\text{Cov}\left(e_{jt}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}}.
\end{aligned}$$

But by equation (A.3),

$$\frac{\partial V}{\partial X_t} = \rho E_t \left[(1 + R_t^*) \frac{\partial V}{\partial X_{t+1}} \right],$$

So

$$\begin{aligned}
\tilde{\psi}_{jt} &= \rho \frac{\text{Cov}\left(e_{jt}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\rho(1 + R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right)} \\
&= \frac{\text{Cov}\left(e_{jt}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{(1 + R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right)}. \tag{A.15}
\end{aligned}$$

Under Assumption 1,

$$\frac{\partial V}{\partial X_t} = A[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)] - B[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)] X_t$$

Hence,

$$\frac{\partial^2 V}{\partial X_t^2} = -B[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)]$$

Shifting one period forward, those two equations become

$$\frac{\partial V}{\partial X_{t+1}} = V' = A - BX_{t+1}$$

and

$$\frac{\partial^2 V}{\partial X_t^2} = V'' = -B$$

Substituting into equation (A.15), we get

$$\begin{aligned}
\tilde{\psi}_{jt} &= \frac{Cov(e_{jt}^*, A - BX_{t+1})}{(1 + R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right)} \\
&= \frac{-B}{1 + R_t^*} \frac{Cov(e_{jt}^*, X_{t+1})}{E_t(V')} \\
&= \frac{1}{1 + R_t^*} \frac{E(V'')}{E_t(V')} Cov(e_{jt}^*, X_{t+1}) \\
&= -\frac{1}{1 + R_t^*} H_{t+1} Cov(e_{jt}^*, X_{t+1}).
\end{aligned}$$

Alternatively, consider Assumption 2. We then can use Stein's lemma, which says the following.⁶² Suppose (X, Y) are multivariate normal. Then

$$Cov(g(X), Y) = E(g'(X))Cov(X, Y).$$

In that formula, let $g(X) = \frac{\partial V}{\partial X_{t+1}}$, $X = X_{t+1}$, and $Y = e_{jt}^*$. Then from Stein's lemma, we have

$$Cov\left(e_{jt}^*, \frac{\partial V}{\partial X_{t+1}}\right) = E_t\left(\frac{\partial^2 V}{\partial X_{t+1}^2}\right) Cov(X_{t+1}, e_{jt}^*).$$

Substituting into (A.15), we get

⁶² For Stein's lemma, see Stein (1973), Ingersoll (1987, p. 13, eq. 62) or Rubinstein (1976).

$$\tilde{\psi}_{jt} = \frac{E_t \left(\frac{\partial^2 V}{\partial X_{t+1}^2} \right) Cov(X_{t+1}, e_{jt}^*)}{(1 + R_t^*) E_t \left(\frac{\partial V}{\partial X_{t+1}} \right)}.$$

Using the definitions of V' , V'' , and H_{t+1} , we have

$$\tilde{\psi}_{jt} = -\frac{1}{1 + R_t^*} H_{t+1} Cov(e_{jt}^*, X_{t+1}).$$

■

Proof of Theorem 3:

Theorem 3. Let $\hat{H}_t = H_{t+1} X_t$. Under the assumptions of Lemma 2, we have the following for

each asset $i = 1, \dots, n$, and credit card service, $j = 1, \dots, k$,

$$\wp_{it}^m = \frac{E_t R_t^* - (E_t r_{it}^* - \phi_{it})}{1 + E_t R_t^*}, \quad (\text{A.16})$$

where

$$\phi_{it} = \hat{H}_t Cov\left(r_{it}^*, \frac{X_{t+1}}{X_t}\right), \quad (\text{A.17})$$

and

$$\wp_{jt}^c = \frac{(E_t e_{jt}^* - \tilde{\phi}_{jt}) - E_t R_t^*}{1 + E_t R_t^*}, \quad (\text{A.18})$$

where

$$\tilde{\phi}_{jt} = \hat{H}_t \text{Cov}\left(e_{jt}^*, \frac{X_{t+1}}{X_t}\right). \quad (\text{A.19})$$

Proof. For the proof in the case of monetary assets only, relevant to equations (A.16) and (A.17), see Barnett, Liu, and Jensen (1997), Barnett and Serletis (2000, ch. 12), or Barnett (2012, Appendix D). We here provide the proof of equations (A.18) and (A.19) for the extended case including credit.

From part b of Theorem 1,

$$\wp_{jt}^c = \frac{E_t e_{jt}^* - E_t R_t^*}{1 + E_t R_t} + \tilde{\psi}_{jt}.$$

Letting $\hat{H}_t = H_{t+1} X_t$ and using Lemma 2, we get

$$\begin{aligned} \wp_{jt}^c &= \frac{E_t e_{jt}^* - E_t R_t^*}{1 + E_t R_t} - \frac{H_{t+1} \text{Cov}\left(e_{jt}^*, X_{t+1}\right)}{1 + E_t R_t^*} \\ &= \frac{E_t e_{jt}^* - E_t R_t^*}{1 + E_t R_t} - \frac{H_{t+1} X_t \text{Cov}\left(e_{jt}^*, \frac{X_{t+1}}{X_t}\right)}{1 + E_t R_t^*} \\ &= \frac{E_t e_{jt}^* - E_t R_t^*}{1 + E_t R_t} - \frac{\hat{H}_t \text{Cov}\left(e_{jt}^*, \frac{X_{t+1}}{X_t}\right)}{1 + E_t R_t^*}. \end{aligned}$$

Define $\tilde{\phi}_{j,t} = \hat{H}_t \text{Cov}\left(e_{jt}^*, \frac{X_{t+1}}{X_t}\right)$ to get

$$\begin{aligned}
\wp_{jt}^c &= \frac{E_t e_{jt}^* - E_t R_t^*}{1 + E_t R_t} - \frac{\tilde{\phi}_{jt}}{1 + E_t R_t^*} \\
&= \frac{(E_t e_{jt}^* - \tilde{\phi}_{jt}) - E_t R_t^*}{1 + E_t R_t^*}.
\end{aligned}$$

■

Chapter 4

The Use of Divisia Monetary Aggregates in Nominal GDP Targeting

William A. Barnett

University of Kansas, Lawrence, and Center for Financial Stability, NY City

Liting Su

University of Kansas, Lawrence, and Center for Financial Stability, NY City

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Abstract

One of the hottest topics in monetary policy research has been the revival of the proposal for “nominal GDP targeting.” Recent research has emphasized the potential importance of the Divisia monetary aggregates in implementing that policy. We investigate bivariate time series properties of Divisia money and nominal GDP to investigate the viability of recent proposals by authors who advocate a role for a Divisia monetary aggregate in nominal GDP targeting.

There are two particularly relevant proposals: (1) the proposal by Barnett, Chauvet, and Leiva-Leon (2015) to use a Divisia monetary aggregate as an indicator in the monthly Nowcasting of nominal GDP, as needed in implementation of any nominal GDP targeting policy; and (2) the proposal by Belongia and Ireland (2015) to use a Divisia monetary aggregate as an intermediate target, with nominal GDP being the final target of policy.

We run well known diagnostic tests of bivariate time series properties of the Divisia M2 and nominal GDP stochastic processes. Those tests are for properties that are necessary, but not sufficient, for the conclusions of Belongia and Ireland (2014) and Barnett, Chauvet, and Leiva-Leon (2015). We find no time series properties that would contradict those implied by either of those two approaches.

Keywords: money, aggregation theory, index number theory, Divisia index, Divisia monetary aggregates, nominal GDP targeting.

JEL Classification Codes: C43, E01, E3, E40, E41, E51, E52, E58.

1. Introduction

The recent financial crisis has induced central banks to explore and undertake unconventional approaches to monetary policy. One of the hottest topics in monetary policy research has been the revival of the proposal for “nominal GDP targeting”, advocated by many leading monetary economists, including Michael Woodford, Christina Romer, and Paul Krugman. Proponents argue that nominal GDP targeting can stabilize the macroeconomy more effectively than inflation targeting. In particular, they argue that by committing to return nominal GDP to its pre-crisis trajectory, the Federal Reserve could improve confidence and expectations of future growth.

We take no position on whether nominal GDP should be adopted as the new monetary policy target, but we investigate the bivariate time series properties of Divisia money and nominal GDP that are relevant to recent results by authors who do advocate a role for a Divisia monetary aggregate in nominal GDP targeting. There are two such proposals. (1) The least controversial is the approach of Barnett, Chauvet, and Leiva-Leon (2015) to the use of Divisia money in Nowcasting of nominal GDP. Any approach to targeting nominal GDP requires availability of monthly measurements of nominal GDP. Monthly measurements of nominal GDP are needed regardless of the instrument of policy adopted to implement the targeting. But nominal GDP data are available only quarterly. Using an advanced dynamic factor analysis approach to Nowcasting, Barnett, Chauvet, and Leiva-Leon (2015) find that the most accurate available approach to Nowcasting nominal GDP would use a Divisia monetary aggregate as one of the relevant and highly significant associated variables, with the others being measures of real

economic activity and inflation dynamics. While Nowcasting does not imply unidirectional causation, Nowcasting approaches do require existence of strong bivariate time series associations among the interpolated variable and the associated variables. (2) The more controversial approach, suggesting a monetarist perspective, advocates the use of a Divisia monetary aggregate as an intermediate target in the procedure for targeting nominal GDP. Such an approach has been advocated by Belongia and Ireland (2015), while a new Keynesian approach has been proposed by the same authors in Belongia and Ireland (2014).

Early suggestions of the possible use of monetary aggregates in nominal GDP targeting include Feldstein and Stock (1993), who showed that the relation between M2 and nominal GDP is sufficiently strong to warrant further investigation into using M2 to influence nominal GDP, as would be relevant to the second approach described above. Since recent research has found Divisia monetary aggregates to be substantially superior to simple sum aggregates, we concentrate in this paper on Divisia M2. See, e.g., Barnett (2012,2015) and Barnett and Chauvet (2011) regarding the superiority of Divisia monetary aggregates over the now largely discredited simple sum monetary aggregates. But since our results are relevant to Nowcasting nominal GDP as well as intermediate targeting, our results are relevant even to proposals in which money is not used to influence nominal GDP, but only to interpolate the quarterly GDP data. In that case, our results need not be interpreted as having implications for the choice of instrument or intermediate targets in the policy rule.

Setting up a VAR model to indicate such relationship, we focus on $d(\ln NGDP)$ and $d(\ln M2)$, which are the growth rates of nominal GDP and Divisia M2. The estimated model indicates that

there is a bidirectional Granger Causality relation between the two. We can make predictions based on our estimated model and can investigate how growth rate of Divisia money supply is going to impact nominal GDP and vice versa. The primary objective of this research is to run well known diagnostic tests of bivariate time series properties of the Divisia M2 and nominal GDP stochastic processes. Those tests are for properties that are necessary, but not sufficient, for the conclusions of Belongia and Ireland (2014) and Barnett, Chauvet, and Leiva-Leon (2015).

2. Literature Review

A nominal GDP target was previously called a “nominal income target” by early supporters such as McCallum (2011,2013). This approach is often contrasted with inflation targeting. Under some proposals on nominal GDP targeting, the central bank would try to keep nominal GDP growing at a predetermined rate. A nominal GDP *level* target is similar, except that the central bank would recall any previous deviations of nominal GDP growth from target and seek to compensate in later years. Apart from Bennett McCallum, who advocates nominal GDP growth rate targeting, most of the current supporters of nominal GDP targeting favor nominal GDP level targeting, such as Woodford (2013), Belongia and Ireland (2015), and Sumner (2012).

Christina Romer (2011), then chair of the Council of Economic Advisers, has urged adopting nominal GDP targeting as the monetary policy rule. In Romer’s view, such a policy would be a powerful communication tool. By pledging to do whatever it takes to return nominal GDP to its pre-crisis trajectory, the Fed could improve confidence and expectations of future growth.

Because nominal GDP reflects the Fed's dual mandate, stable price level and maximum real output, Romer argues that nominal GDP targeting would have a better chance of reducing unemployment than any other monetary policy approach under discussion.

Woodford (2013) argues that long run inflation targeting does not need to be repudiated as a policy framework, but rather needs to be completed. He argues that the target path for nominal GDP could be chosen such that keeping nominal GDP on that path should ensure, over the medium run, an average inflation rate equal to the inflation target. In his view, nominal GDP targeting can complete inflation targeting without conflicting with it. He further maintains that nominal GDP targeting would reduce the tension between the goals of restraining risks to financial stability, on the one hand, and maintaining macroeconomic stability, on the other.⁶³

Sumner (2012), a persistent advocator of nominal GDP targeting and relentless blogger of "The Money Illusion," argues that the recent financial crisis exposed serious flaws with inflation targeting monetary policy regimes. In his view, GDP targeting would have greatly reduced the severity of the recession and also eliminated the need for fiscal stimulus. He also argues that nominal GDP targeting would make it easier for politicians to resist calls for bailouts of private sector firms, while assuring low inflation and reducing the severity of the business cycle. He also argues that nominal GDP targeting would make asset price bubbles less likely to occur. In

⁶³ Regarding inflation targeting, see, e.g., Bernanke and Mishkin (1997) and Svensson (1998).

summary, advocates of nominal GDP targeting believe it would provide the best environment for free-market policies to flourish.

On September 12, 2012, the Federal Reserve undertook policy initiatives influenced by Woodford (2003,2005,2012): an open-ended quantitative easing program, in which the amount of purchases depends on progress toward the policy goals. The Federal Reserve also announced it would maintain an easy money policy for some period after the economy has recovered. That announcement can be interpreted as an incremental move toward nominal GDP level targeting.

Nominal GDP targeting defines the final target of policy, but not the instrument, intermediate target, or rule used to implement the final target commitment. Many proposed approaches exist, including those that implement the final target for a new-Keynesian approach, a post-Keynesian approach, a monetarist approach, a classical approach, a new-classical approach, or an Austrian School approach. McCallum (1987) proposes a monetarist rule that uses the monetary base as instrument to target nominal GDP. He advocates targeting the growth rate of nominal GDP, rather than the level. His view is that if growth rates are on average equal to the target value over time, the policy would be unlikely to permit much departure from the planned path and should therefore be preferred. His rule employs a four-year moving average of past growth in monetary base velocity to forecast that velocity's growth in the coming quarter. Based on that forecast, the rule specifies the percentage of the gap between the targeted and actual levels of nominal GDP that the central bank should plan to close in the coming quarter.

In simulations, Dueker (1993) confronts McCallum's nominal GDP targeting rule with a world in which coefficients in the velocity equation for the monetary instrument are subject to

unpredictable stochastic change. His approach differs from McCallum's by using explanatory variables to help forecast velocity in a time-varying parameter model. By allowing for time-varying coefficients, Dueker's forecasting model is argued to be more stable over time than fixed-coefficient models. Dueker concludes that McCallum's approach to nominal GDP targeting is simple yet robust to velocity behavior. However, Dueker's forecast-based rule performed somewhat better in simulations in which velocity was generated from a time-varying parameter model.

Recent contributors to the literature on nominal GDP targeting also incorporate aggregation theoretic monetary aggregates. Belongia and Ireland (2015) derive an approach to targeting the level of nominal GDP using a framework first outlined by Working (1923) and used, with minor modifications, by Hallman, et al. (1991) in their P-Star model. Belongia and Ireland's framework is built on traditional quantity theoretic foundations and draws directly from Barnett's (1978,1980) economic approach to monetary aggregation. With any desired long-run trajectory for nominal GDP, the framework can find a consistent intermediate target path for Divisia money. The central bank can use the monetary base to control the intermediate target path for either a narrow or broad Divisia monetary aggregate and thereby keep nominal GDP growing along any desired long-run path.

Their innovation lies in employing Divisia monetary aggregates to establish a path for the intermediate target and uses a one-sided filtering algorithm to control for slow-moving trends in velocity. The merits of this approach are its transparency to outside observers, its forward-looking design, and its potentially straightforward implementation.

Barnett, Chauvet, and Leiva-Leon (2015) developed dynamic factor models to Nowcast nominal output growth, using information from the previous release of nominal GDP, Industrial Production, Consumer Price Index, and Divisia M3. Their model is useful in giving monthly assessment of the current nominal GDP quarterly growth. This ability plays an essential role in monitoring the effectiveness of nominal GDP targeting monetary policy, regardless of the approach to implementation. In fact any approach that uses monthly feedback in its nominal GDP targeting approach becomes undefined, and thereby not applicable, without access to monthly GDP Nowcasts.

3. The Bivariate Time Series Relationship between Divisia M2 and Nominal GDP

As explained above, the use of Divisia monetary aggregates has been proposed in two different potential roles in nominal GDP targeting. One role is as an indicator variable in Nowcasting of monthly nominal GDP, as needed in any implementation of nominal GDP targeting. The other roles is direct use as an intermediate target in the policy design. Both cases imply the existence of a bivariate time series relationship between a Divisia monetary aggregate and nominal GDP. In this paper, we explore the nature of that relationship.

The Divisia monetary aggregate we use is Divisia M2, as provided by the Federal Reserve Bank of St. Louis in its FRED database. We use those data since they are well known and have a long history in this literature. But in future research, we plan to use the broader Divisia monetary

aggregates, M3 and M4, supplied by the Center for Financial Stability in New York City.⁶⁴ The GDP data we use are supplied by the U.S. Bureau of Economic Analysis (BEA). Both series are seasonally adjusted. We eliminate heteroskedasticity by taking logarithms of the variables. We use $\ln NGDP$ and $\ln M2$ to denote the transformed data.

3.1. Unit Root Test

First we conduct a unit root test to examine stationarity of the series. If the series are non-stationary, regression could be spurious. We adopt the ADF (Augmented Dickey-Fuller) method for unit root test. The test results are displayed in the appendix as Table 1a.

The p values of both tests are greater than the 5% significance level, with 0.9951 for $\ln NGDP$ and 0.4876 for $\ln M2$ respectively. Hence, for each of the tests, we fail to reject the null hypothesis that the series has a unit root. Both $\ln NGDP$ and $\ln M2$ series are non-stationary.

To test for causality relationship between nominal GDP and Divisia M2 money supply, we need the series to be stationary. For that purpose, we first difference the series to produce two first order differenced series $d(\ln NGDP)$ and $d(\ln M2)$. We then again conduct the ADF test on each of those transformed series. The null hypotheses that $d(\ln NGDP)$ and $d(\ln M2)$ have unit roots are decisively rejected. The differenced time series are stationary processes. See Table 2a in the

⁶⁴ See Barnett, Liu, Mattson, and van den Noort (2013).

appendix.

3.2. Cointegration Test

Next we test cointegration between $\ln NGDP$ and $\ln M2$ to investigate whether there exists long run association between the two processes. If the two variables are not cointegrated, we could apply an unrestricted VAR model. If the variables are cointegrated, we should prefer a vector error correction model (VECM). We use Johansen's (1988,1991) methodology. The p values for unrestricted cointegration rank tests using trace and maximum eigenvalue are 0.0828 and 0.0646 respectively, both higher than 5% significance level. See Table 3a in the appendix. Hence we fail to reject the null hypothesis of no cointegration between $\ln NGDP$ and $\ln M2$. We use an unrestricted VAR model in the following step.

3.3. VAR Model

We begin with a preliminary unrestricted VAR(2) model, as shown in appendix table 4a. We use the Akaike Information Criterion (AIC) to determine the appropriate maximum lag length for the variables in the VAR. Since we are using quarterly data, we choose lag equal to 4, when conducting VAR lag order selection. As the following table 1 shows, lag equal to 3 gives us the lowest AIC value. Therefore, we revise our model to a VAR(3) and estimate its coefficients. Detailed results are in appendix table 5a.

Table 1: VAR Lag Order Selection Criteria

Endogenous variables: $d(\ln M2)$, $d(\ln NGDP)$

Sample: 1967Q1 - 2013Q4

Included observations: 183

Lag	Log L	LR	FPE	AIC	SC	HQ
0	1201.558	NA	6.94e-09	-13.10992	-13.07485	-13.09571
1	1259.317	113.6242	3.86e-09	-13.69745	-13.59222	-13.65480
2	1269.885	20.55929*	3.59e-09	-13.76924	-13.59386*	-13.69815*
3	1274.130	8.164230	3.58e-09*	-13.77191*	-13.52638	-13.67238
4	1277.989	7.338699	3.59e-09	-13.77037	-13.45468	-13.64241

* Identifies the lag order selected by the criterion in that column.

Log L: log likelihood

LR: sequential modified LR test statistic (each test at 5% level)

FPE: Final prediction error

AIC: Akaike information criterion

SC: Schwarz information criterion

HQ: Hannan-Quinn information criterion

Next we examine whether there exist autocorrelation problems among the disturbances. Using the Autocorrelation LM (Lagrange Multiplier) test with lag equal to 12, we acquire the following table 2 with most of the p values greater than the 5% significance level.

Table 2: VAR Residual Serial Correlation LM Tests

Null Hypothesis: no serial correlation at lag order

Sample: 1967Q1 - 2013Q4

Included observations: 184

Lags	LM-Statistic	P value*
1	8.170979	0.0855
2	10.45168	0.0335
3	6.668278	0.1545
4	6.192919	0.1852
5	10.20056	0.0372
6	7.367825	0.1177
7	2.768448	0.5973
8	4.482638	0.3446
9	9.023472	0.0605
10	1.994479	0.7368
11	12.65099	0.0131
12	5.147886	0.2725

*P value from chi-square with 4 degrees of freedom.

We fail to reject the null hypothesis of no serial correlation among the residuals of the VAR(3) model. The VAR(3) model is well-specified.

3.4. Granger Causality Test

We conducted Granger causality tests between $d(\ln NGDP)$ and $d(\ln M2)$. The results indicate that $d(\ln NGDP)$ Granger causes $d(\ln M2)$, and $d(\ln M2)$ also Granger Causes $d(\ln NGDP)$.

Listed below in table 3 are the Granger causality test results.

Table 3: VAR Granger Causality, Block Exogeneity Wald Tests

Sample: 1967Q1 2013Q4

Included observations: 184

Dependent variable: $d(\ln M2)$			
Excluded	Chi-sq	df	P value
$d(\ln NGDP)$	11.28757	3	0.0103
All	11.28757	3	0.0103
Dependent variable: $d(\ln NGDP)$			
Excluded	Chi-sq	df	P value
$d(\ln M2)$	11.67938	3	0.0086
All	11.67938	3	0.0086

The P value of the null hypothesis that $d(\ln NGDP)$ does not Granger cause $d(\ln M2)$ is 0.0103, which is smaller than the conventional critical value 0.05. We reject the null and therefore conclude that $d(\ln NGDP)$ does Granger cause $d(\ln M2)$. The P value of the null hypothesis that $d(\ln M2)$ does not Granger cause $d(\ln NGDP)$ is 0.0086, also smaller than the critical value 0.05. We reject the null hypothesis and therefore conclude that $d(\ln M2)$ does Granger cause $d(\ln NGDP)$. There exists a bidirectional Granger causality relationship between $d(\ln NGDP)$ and $d(\ln M2)$.

3.5. Estimation of the Final Bivariate VAR

We implemented the bidirectional Granger Causality relationship between $d(\ln NGDP)$ and $d(\ln M2)$ by estimating a bivariate VAR in those two stochastic processes with optimized lag lengths selected from the EViews program. The coefficients of the two equations are stacked into one vector having elements, $C(i), i = 1, \dots, 14$, as defined in table 6a in the appendix. The two equations we estimated in this VAR are defined in Table 6a. The coefficients of the first equation are $C(i), i = 1, \dots, 7$, while the coefficients of the second equation are $C(i), i = 8, \dots, 14$. See the table for the specification of those two equations and the estimates of their coefficients.

The p value for $C(1)$ is 0.0000, demonstrating that the coefficient of $d(\ln M2)_{t-1}$ is significant in the first equation. The growth rate of Divisia M2 money supply in the previous period has a significant impact on prediction of the current growth rate of Divisia M2. The corresponding p value of $C(2)$ is 0.9735, demonstrating that the second lag of the growth rate of M2 does not have significant predicting power for the current growth rate of M2. By eliminating the statistically insignificant coefficients, we acquire the following two estimated equations:

$$d(\ln M2)_t = 0.483728d(\ln M2)_{t-1} + 0.146457d(\ln M2)_{t-3} - 0.223671d(\ln NGDP)_{t-1} + 0.006672. \quad (1)$$

$$d(\ln GNP)_t = 0.223336d(\ln M2)_{t-1} + 0.318158d(\ln NGDP)_{t-1} + 0.288470d(\ln NGDP)_{t-2}. \quad (2)$$

Since $d(\ln M2)$ and $d(\ln NGDP)$ indicate the growth rates, the estimated equations can be interpreted as follows. The growth rate of Divisia M2 is affected by the growth rate of itself, lagged by 1 and 3 quarters, as well as by the growth rate of the previous quarter's nominal GDP.

Furthermore, holding other variables constant, we can reach the following conclusions. From the first equation, if the growth rate of Divisia M2 during the last quarter increases by 10%, then the growth rate of M2 this quarter will increase by 4.83728%. But if the nominal GDP growth rate of the previous quarter increases by 10%, the M2 growth rate this quarter will decrease by 2.23671%. If the M2 growth rate, lagged three quarters, reaches 10%, the current growth rate will increase by 1.46457%. Similar analysis applies to the second equation, where $d(\ln NGDP)_t$ is the dependent variable.

3.6. Prediction

Based on the estimation of equations (1) and (2), we can predict the growth rate of Divisia M2 and nominal GDP in 2014 Q1 using the quarterly data in our sample ending in 2013 Q4.⁶⁵

$$\begin{cases} d(\ln M2)_{2014Q1} = 0.483728d(\ln M2)_{2013Q4} + 0.146457d(\ln M2)_{2013Q2} - \\ \quad 0.223671d(\ln NGDP)_{2013Q4} + 0.006672 \\ d(\ln NGDP)_{2014Q1} = 0.223336d(\ln M2)_{2013Q4} + 0.318158d(\ln NGDP)_{2013Q4} \\ \quad + 0.288470d(\ln NGDP)_{2013Q2} \end{cases}$$

Substituting the measured values of the variables into the right hand sides, the predicted growth

⁶⁵ We could have used a longer sample period including more recent quarters by using data from the Center for Financial Stability (CFS) in New York City. But we limited this study to data made available by the Federal Reserve Bank of St. Louis, which has not updated its data as regularly as the CFS, which does update monthly.

rates are:

$$\begin{cases} d(\ln M2)_{2014Q1} = 0.014079 \\ d(\ln NGDP)_{2014Q1} = 0.012193 \end{cases}$$

The predicted growth rates can be used to predict the levels of M2 and NGDP in 2014Q1 by the following equations:

$$\begin{cases} M2_{2014Q1} = M2_{2013Q4} * (1 + d(\ln M2)_{2014Q1}) \\ NGDP_{2014Q1} = NGDP_{2013Q4} * (1 + d(\ln NGDP)_{2014Q1}) \end{cases}$$

Substituting into the right hand sides, we acquire:

$$\begin{cases} M2_{2014Q1} = 11758.8 \\ NGDP_{2014Q1} = 17286.5 \end{cases}$$

The 1.4% predicted growth rate of Divisia M2 money supply in 2014Q1 was inconsistent with the Federal Reserve's accommodative monetary policy. A consequence is reflected in the almost-non-growing 1.2% nominal GDP prediction in 2014Q1. In fact, the out of sample growth rate of 2014Q1 was -0.2%, according to the data released by Bureau of Economic Analysis (BEA).

4. Conclusion

In this paper we discuss the relationship between Divisia M2 money supply and nominal GDP. The primary objective of this research is to run well known diagnostic tests of bivariate time series properties of the Divisia M2 and nominal GDP stochastic processes that are necessary but

not sufficient for the conclusions of Belongia and Ireland (2014) and Barnett, Chauvet, and Leiva-Leon (2015). We find no evidence to contradict the conclusions of those two papers about the potential relevancy of Divisia monetary aggregates in targeting nominal GDP, either as an intermediate target or as an indicator. Our results are not specific to either of those approaches and hence cannot provide conclusions about which of those two approaches should be preferred. Since neither of those two approaches contradicts the other, one possibility would be to use both of those approaches simultaneously. In that case, Barnett, Chauvet, and Leiva-Leon (2015) could be used to interpolate the quarterly data to provide the needed Nowcast monthly nominal GDP data, while Belongia and Ireland (2014) would then be used to implement a policy design using a Divisia monetary aggregate as an intermediate target.

But if a different policy design were adopted without an intermediate target, Barnett, Chauvet, and Leiva-Leon (2015) would remain relevant to producing the monthly data necessary for any approach to nominal GDP targeting.

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Appendix

Table 1a. Unit Root Test Result for *lnNGDP* and *lnM2*

Null Hypothesis: *lnNGDP* has a unit root

	t-Statistic	P value*
Augmented Dickey-Fuller test statistic	-0.065053	0.9951
Test critical values: 1% level	-4.008154	
5% level	-3.434167	
10% level	-3.141001	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: *lnM2* has a unit root

	t-Statistic	P value*
Augmented Dickey-Fuller test statistic	-2.197872	0.4876
Test critical values: 1% level	-4.008154	
5% level	-3.434167	
10% level	-3.141001	

*MacKinnon (1996) one-sided p-values.

Table 2a. Unit Root Test Result for $d(\ln NGDP)$ and $d(\ln M2)$ Null Hypothesis: $d(\ln NGDP)$ has a unit root

	t-Statistic	P value*
Augmented Dickey-Fuller test statistic	-10.34110	0.0000
Test critical values:		
1% level	-4.008154	
5% level	-3.434167	
10% level	-3.141001	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: $d(\ln M2)$ has a unit root

	t-Statistic	P value*
Augmented Dickey-Fuller test statistic	-7.718251	0.0000
Test critical values:		
1% level	-4.008154	
5% level	-3.434167	
10% level	-3.141001	

*MacKinnon (1996) one-sided p-values.

Table 3a. Johansen Cointegration Test Between *lnNGDP* and *lnM2*

Sample (adjusted): 1968Q2 - 2013Q4

Included observations: 183

Series: *d(lnM2)*, *d(lnNGDP)*

Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	P value**
None	0.071362	14.00454	15.49471	0.0828
At most 1	0.002488	0.455880	3.841466	0.4996

Trace test indicates no cointegration at the 0.05 level

*Denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	P value**
None	0.071362	13.54866	14.26460	0.0646
At most 1	0.002488	0.455880	3.841466	0.4996

Max-eigenvalue test indicates no cointegration at the 0.05 level

*Denotes rejection of the hypothesis at the 0.05 level

**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegrating Coefficients:

<i>lnM2</i>	<i>lnNGDP</i>
1.852061	-3.068247
9.671655	-7.514269

Unrestricted Adjustment Coefficients (alpha):

<i>d(lnM2)</i>	0.000987	0.000297
<i>d(lnNGDP)</i>	0.001715	-0.000217

One Cointegrating Equation: Log likelihood 1284.763

Normalized cointegrating coefficients (standard error in parentheses)

<i>lnM2</i>	<i>lnNGDP</i>
1.000000	-1.656666 (0.23697)

Adjustment coefficients (standard error in parentheses)

<i>d(lnM2)</i>	0.001828 (0.00098)
<i>d(lnNGDP)</i>	0.003177 (0.00106)

Table 4a. VAR(2) Estimation

Vector Autoregression Estimates

Sample (adjusted): 1967Q4 - 2013Q4

Included observations: 185

Standard errors in () & t-statistics in []

	$d(\ln NGDP)$	$d(\ln M2)$
$d(\ln NGDP)_{t-1}$	0.343726 (0.07111) [4.83377]	-0.203535 (0.06393) [-3.18391]
$d(\ln NGDP)_{t-2}$	0.296876 (0.07117) [4.17157]	0.110804 (0.06398) [1.73192]
$d(\ln M2)_{t-1}$	0.230714 (0.08198) [2.81432]	0.502884 (0.07370) [6.82361]
$d(\ln M2)_{t-2}$	-0.018083 (0.08169) [-0.22137]	0.051093 (0.07344) [0.69573]
Constant intercept	0.002710 (0.00177) [1.53297]	0.007922 (0.00159) [4.98427]
R-squared	0.320419	0.300597
Adj. R-squared	0.305317	0.285055
Sum sq. residuals	0.011651	0.009416
S.E. equation	0.008045	0.007233
F-statistic	21.21725	19.34060
Log likelihood	632.2211	651.9214
Akaike AIC	-6.780769	-6.993745
Schwarz SC	-6.693732	-6.906708
Mean dependent	0.016113	0.014431
S.D. dependent	0.009653	0.008554
Determinant residual covariance (df adj)		3.38E-09
Determinant residual covariance		3.20E-09
Log likelihood		1284.335
Akaike information criterion		-13.77659
Schwarz criterion		-13.60252

Table 5a. VAR(3) Estimation

Sample (adjusted): 1968Q1 - 2013Q4

Included observations: 184

Standard errors in () and t-statistics in []

	$d(\ln NGDP)$	$d(\ln M2)$
$d(\ln NGDP)_{t-1}$	0.318158 (0.07460) [4.26460]	-0.223671 (0.06667) [-3.35472]
$d(\ln NGDP)_{t-2}$	0.288470 (0.07726) [3.73398]	0.062865 (0.06904) [0.91053]
$d(\ln NGDP)_{t-3}$	0.076208 (0.07535) [1.01134]	0.074424 (0.06734) [1.10515]
$d(\ln M2)_{t-1}$	0.223336 (0.08300) [2.69084]	0.483728 (0.07418) [6.52140]
$d(\ln M2)_{t-2}$	0.061580 (0.09397) [0.65531]	0.002791 (0.08398) [0.03323]
$d(\ln M2)_{t-3}$	-0.113475 (0.08194) [-1.38480]	0.146457 (0.07323) [1.99990]
Constant intercept	0.002610 (0.00190) [1.37251]	0.006672 (0.00170) [3.92561]
R-squared	0.331774	0.320114
Adj. R-squared	0.309123	0.297067
Sum sq. residuals	0.011451	0.009146
S.E. equation	0.008043	0.007188
F-statistic	14.64676	13.88960
Log likelihood	629.8995	650.5791
Akaike AIC	-6.770647	-6.995425
Schwarz SC	-6.648340	-6.873118
Mean dependent	0.016098	0.014412
S.D. dependent	0.009677	0.008574
Determinant residual covariance (df adj.)		3.34E-09
Determinant residual covariance		3.09E-09
Log likelihood		1280.607
Akaike information criterion		-13.76747
Schwarz criterion		-13.52286

Table 6a. Final VAR Coefficient Estimation

Estimation Software: EViews computer program

Sample: 1968Q1 - 2013Q4

Included observations: 184

Total system observations 368

	Coefficient	Std. Error	t Statistic	P Value
C(1)	0.483728	0.074176	6.521397	0.0000
C(2)	0.002791	0.083982	0.033235	0.9735
C(3)	0.146457	0.073232	1.999902	0.0463
C(4)	-0.223671	0.066674	-3.354718	0.0009
C(5)	0.062865	0.069043	0.910526	0.3632
C(6)	0.074424	0.067343	1.105148	0.2698
C(7)	0.006672	0.001700	3.925608	0.0001
C(8)	0.223336	0.082999	2.690835	0.0075
C(9)	0.061580	0.093971	0.655308	0.5127
C(10)	-0.113475	0.081943	-1.384797	0.1670
C(11)	0.318158	0.074604	4.264599	0.0000
C(12)	0.288470	0.077255	3.733981	0.0002
C(13)	0.076208	0.075353	1.011342	0.3125
C(14)	0.002610	0.001902	1.372509	0.1708

Equation: $d(\ln M2)_t = C(1)d(\ln M2)_{t-1} + C(2)d(\ln M2)_{t-2} + C(3)d(\ln M2)_{t-3} + C(4)d(\ln NGDP)_{t-1} + C(5)d(\ln NGDP)_{t-2} + C(6)d(\ln NGDP)_{t-3} + C(7)$

R squared	0.320114	Mean dependent var	0.014412
Adjusted R squared	0.297067	dependent var	0.008574
S.E. of regression	0.007188	Sum squared resid	0.009146
Durbin-Watson stat	1.977980		

Equation: $d(\ln NGDP)_t = C(8)d(\ln M2)_{t-1} + C(9)d(\ln M2)_{t-2} + C(10)d(\ln M2)_{t-3} + C(11)d(\ln NGDP)_{t-1} + C(12)d(\ln NGDP)_{t-2} + C(13)d(\ln NGDP)_{t-3} + C(14)$

R squared	0.331774	Mean dependent var	0.016098
Adjusted R-squared	0.309123	S.D. dependent var	0.009677
S.E. of regression	0.008043	Sum squared resid	0.011451
Durbin Watson stat	1.998140		

Chapter 5

Financial Firm Production of Monetary and Credit Card Services: An Aggregation Theoretic Approach⁶⁶

William A. Barnett

University of Kansas, Lawrence, and Center for Financial Stability, NY City

Liting Su

University of Kansas, Lawrence, and Center for Financial Stability, NY City

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⁶⁶ We have benefitted from constructive comments provided by Kimberley Zieschang.

Abstract

A monetary-production model of financial firms is employed to investigate supply-side monetary aggregation, augmented to include credit card transaction services. Financial firms are conceived to produce monetary and credit card transaction services as outputs through financial intermediation. While credit cards provide transactions services, credit cards have never been included into measures of the money supply. The reason is accounting conventions, which do not permit adding liabilities to assets. However, index number theory measures service flows and is based on aggregation theory, not accounting. Barnett, Chauvet, Leiva-Leon, and Su (2016) have derived and applied the relevant aggregation theory applicable to measuring the demand for the joint services of money and credit cards. But because of the existence of required reserves, there is a regulatory wedge between the demand and supply of monetary services. We derive theory needed to measure the supply of the joint services of credit cards and money. The resulting model can be used to investigate the transmission mechanism of monetary policy.

1. Introduction

Monetary policy is transmitted to the economy through banking firms and other financial intermediaries. Financial firms supply monetary assets through their financial intermediation between borrowers and lenders. These monetary assets play a central role in providing transaction services to the economy. In this context, rigorous microeconomic analysis of the optimal behavior of financial firms is essential to a clear understanding of the monetary transmission mechanism. We begin with the model of Barnett (1987) and generalize it to include production of credit card transactions services, using the approach developed initially for the demand side by Barnett, Chauvet, Leiva-Leon, and Su (2016).

The main objective of this study is to employ a production model of financial firms, which produce services through financial intermediation, and to investigate supply-side aggregation, when financial firms produce not only monetary services but also credit card transactions services. We derive the conditions under which a joint supply side aggregate over monetary and credit card transactions services exists, and we produce the resulting formula permitting Divisia monetary aggregation over those joint services. Empirically implementing the theory is a subject for future research.

As the first step in this direction, we make many simplifying assumptions, the strongest of which is perfect certainty or risk neutrality. As a result, we are implicitly assuming the existence of complete contingent claims perfect markets, so that decentralization by owners to managers is incentive compatible, when owners are risk averse but managers are risk neutral.

Generalizations under weaker assumptions are a subject for future research.

The ability to produce econometric models of financial intermediary production of transactions services could become a major source of future insights into the transmission mechanism of monetary policy. Since inside money and shadow banking have become major factors affecting monetary policy and are potentially directly measurable as value added in banking and shadow banking, we consider the theory produced in this paper to be a first step in research we expect to grow in importance in future years.

The theoretical foundation of the monetary production model is based on Barnett's (1987) monetary aggregation-theoretic approach, also consistent with Hancock's (1991) approach, but extended to include production of credit card transactions services. The role of produced credit card services has become far too important to overlook in modeling the output of financial firms and their contribution to transactions services in the economy. Financial firms are modeled as maximizing the discounted present value of variable profits, subject to given technology, while producing monetary assets and credit card services through financial intermediation. With the derivation of user-cost prices for monetary assets and credit card transaction services, the monetary production model can be transformed into the conventional neoclassical model of production by multiproduct firms. As a result, a neoclassical aggregate supply function on the production side can be constructed, using the existing literature on output aggregation.

The following section provides a general discussion of our model of the production of financial firms, based on Barnett's aggregation-theoretic approach, and describes the derivation of the user-cost prices for monetary assets and credit card services on the production side. The section also provides a discussion of aggregation theory relevant to our model formulation.

For a survey of the analogous results for monetary assets alone on the consumer demand side, see Barnett (2012).⁶⁷ For the results augmented to include credit card services on the demand side, see Barnett, Chauvet, Leiva-Leon, and Su (2016). Those publications survey the available results on demand for monetary assets and exact aggregation over those demands along with the extension to inclusion of credit card transactions services. The current paper produces results dealing with the supply of monetary and credit card services produced by financial intermediaries and the aggregation over those supplies.

2. The Model

First, we define the variables that are used in the financial intermediary's decision problem:

R_t = yield on the benchmark asset;

μ_t = real balances of monetary asset accounts serviced by the financial intermediary;

τ_t = vector of real expenditures "volumes," τ_{jt} , with credit card type j for transactions during period t ;

e_t = vector of expected interest rates, e_{jt} , on τ_t ;

ζ_t = vector of rotating real balances, ζ_{jt} , in credit card type j during period t from transactions

⁶⁷ Other relevant results on the demand side include Barnett and Chauvet (2011), Belongia and Ireland (2014), and Serletis and Gogas (2014).

in previous periods;

$\bar{\mathbf{e}}_t$ = vector of interest rates on ζ_t ;

c_t = real balances of excess reserves held by the intermediary during period t ;

\mathbf{L}_t = vector of labor quantities;

\mathbf{z}_t = quantities of other factors of production;

\mathbf{q}_t = prices of the factors, \mathbf{z}_t ;

\mathbf{k}_t = reserve requirements, where k_{it} is the reserve requirement applicable to μ_{it} and $0 \leq k_{it} \leq 1$

for all i ;

R_t^d = Federal Reserve discount rate;

$\bar{R}_t = \min\{R_t, R_t^d\}$;

\mathbf{p}_t = vector of yields paid by the firm on $\mathbf{\mu}_t$.

The yielding R_t on the “benchmark asset” is the yield on an investment that provides no services other than the yield itself. In classical economic theory under general equilibrium, R_t is “the interest rate” on pure capital and hence is secured by its ownership. In contrast, credit card loans are unsecured. The firm’s efficient production technology is defined by the transformation function $F(\mathbf{\mu}_t, \mathbf{\tau}_t, \mathbf{z}_t, \mathbf{L}_t, c_t; \mathbf{k}_t) = 0$, assumed to be strictly quasiconvex in $(\mathbf{\mu}_t, \mathbf{\tau}_t, \mathbf{z}_t, \mathbf{L}_t, c_t)$, strictly increasing in outputs $(\mathbf{\mu}_t, \mathbf{\tau}_t)$ and strictly decreasing in inputs $(\mathbf{z}_t, \mathbf{L}_t, c_t)$. Since the intermediary’s servicing of credit card transactions are during the current period, the firm’s production technology includes $\mathbf{\tau}_t$ but does not include ζ_t . The value added in servicing

transactions occurs during the period when the credit cards are used for transactions.⁶⁸ Hence the firm's optimization decision is conditional upon consumer choices of ζ_t , which convey no further services to consumers other than the unsecured rotating loan itself. The firm's technology can equivalently be defined by its efficient production set (production possibility set)

$$S(\mathbf{k}_t) = \{(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \mathbf{z}_t, \mathbf{L}_t, c_t) \geq \mathbf{0} : F(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \mathbf{z}_t, \mathbf{L}_t, c_t; \mathbf{k}_t) = 0\} \quad (1)$$

or by its production correspondence F , defined such that

$$G(\mathbf{z}_t, \mathbf{L}_t, c_t; \mathbf{k}_t) = \{(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t) \geq \mathbf{0} : (\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \mathbf{z}_t, \mathbf{L}_t, c_t) \in S(\mathbf{k}_t)\}. \quad (2)$$

We assume that required reserves are never borrowed from the Federal Reserve, but could be borrowed in the federal funds market.⁶⁹ Excess reserves can be borrowed from either source. In this initial model, we assume that the Federal Reserve does not pay interest on reserves, as has been the case during most of its history. Since we are assuming the existence of only one kind of

⁶⁸ The ability to borrow from banks and other lending institutions would exist, even if credit cards did not exist. Hence there is no value added in production from ζ_t in this model, for the same reason that the “benchmark” asset, having yield R_t , does not appear in the firm's technology. The value added from credit card servicing is the ability to buy goods with the card and defer payment. That service is provided at the time used to purchase the goods and is measured by credit card transactions “volumes,” τ_t . To be able to impute value added to other financial intermediary lending, we would need to impute asset management services to the financial intermediary. The current model does not include asset management as a service of the financial intermediary. Results relevant to inclusion of asset management services can be found in Fixler and Zieschang (2016a,b).

⁶⁹ This assumption of “perfect moral suasion,” could easily be weakened or removed.

primary market loan yielding R_t , it follows that the federal funds rate must always equal R_t . As a result, under our assumption of risk neutrality or perfect certainty, if $R_t^d < R_t$ then all excess reserves will be borrowed from the Federal Reserve and there are no free reserves. If $R_t^d > R_t$, then there is no borrowing from the Federal Reserve and free reserves equal excess reserves.

If $R_t \leq R_t^d$, then $\bar{R}_t = R_t$, and revenue from loans is

$$\sum_i (\mu_{it} p_t^* - \sum_i k_{it} \mu_{it} p_t^* - \sum_j p_t^* \tau_{jt} - \sum_j p_t^* \zeta_{jt} - c_t p_t^* - \mathbf{q}_t' \mathbf{z}_t) R_t + \sum_j e_{jt} p_t^* \tau_{jt} + \sum_j \bar{e}_{jt} p_t^* \zeta_{jt}. \quad (3)$$

If $R_t > R_t^d$, then $\bar{R}_t = R_t^d$, and revenue from loans is

$$\sum_i (\mu_{it} p_t^* - \sum_i k_{it} \mu_{it} p_t^* - \sum_j p_t^* \tau_{jt} - \sum_j p_t^* \zeta_{jt} - \mathbf{q}_t' \mathbf{z}_t) R_t - c_t p_t^* R_t^d + \sum_j e_{jt} p_t^* \tau_{jt} + \sum_j \bar{e}_{jt} p_t^* \zeta_{jt}. \quad (4)$$

Hence, in either case, revenue from loans is

$$[\sum_i (1 - k_{it}) \mu_{it} p_t^* - c_t p_t^* - \mathbf{q}_t' \mathbf{z}_t - \sum_j p_t^* \tau_{jt} - \sum_j p_t^* \zeta_{jt}] R_t + c_t p_t^* (R_t - \bar{R}_t) + \sum_j e_{jt} p_t^* \tau_{jt} + \sum_j \bar{e}_{jt} p_t^* \zeta_{jt}. \quad (5)$$

Variable cost, which must be paid out of revenue, is

$$\sum_i \mu_{it} p_t^* \rho_{it} + \mathbf{q}_t' \mathbf{z}_t + \mathbf{w}_t' \mathbf{L}_t. \quad (6)$$

At the end of period t , profit received is acquired by subtracting (6) from (5). Dividing by $1 + R_t$

to discount profits to the beginning of period t , we get the present value of period t profits to be

$$\begin{aligned} & P(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \zeta_t, \mathbf{z}_t, \mathbf{L}_t, c_t; p_t^*, \mathbf{q}_t, R_t, R_t^d, \mathbf{e}_t, \bar{\mathbf{e}}_t, \mathbf{p}_t, \mathbf{w}_t, \mathbf{k}_t) \\ &= \boldsymbol{\mu}_t' \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t' \tilde{\boldsymbol{\pi}}_t + \zeta_t' \boldsymbol{\sigma}_t - \mathbf{q}_t' \mathbf{z}_t - \mathbf{L}_t' \mathbf{w}_t / (1 + R_t) - \gamma_{ot} c_t, \end{aligned} \quad (7)$$

where the vector $\boldsymbol{\gamma}_t$ is defined such that the nominal user cost price for produced monetary asset

μ_{it} is $\gamma_{it} = p_t^* \frac{(1 - k_{it})R_{it} - \rho_{it}}{1 + R_t}$, the vector $\tilde{\boldsymbol{\pi}}_t$ is defined such that the nominal user cost price for

produced credit card service τ_{jt} is $\pi_{jt} = p_t^* \frac{e_{jt} - R_t}{1 + R_t}$, the vector $\boldsymbol{\sigma}_t$ is defined such that the

nominal user cost price for carried forward rotating credit card debt ζ_{jt} is $\sigma_{jt} = p_t^* \frac{\bar{e}_{jt} - R_t}{1 + R_t}$, and

the nominal user cost price of excess reserves, c_t , is $\gamma_{ot} = p_t^* \frac{\bar{R}_t}{1 + R_t}$. The corresponding real user

costs are $\frac{\gamma_t}{p_t^*}$, $\frac{\pi_t}{p_t^*}$, $\frac{\sigma_t}{p_t^*}$, and $\frac{\gamma_{ot}}{p_t^*}$.

If we write the vector of all variable factor quantities as $\mathbf{a}_t = (\mathbf{z}_t', \mathbf{L}_t', c_t)'$ and the vector of

corresponding factor prices as $\boldsymbol{\beta}_t = (\mathbf{q}_t', \mathbf{w}_t' / (1 + R_t), \gamma_{ot})'$, it becomes evident that profits take the

conventional form $\boldsymbol{\mu}_t' \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t' \tilde{\boldsymbol{\pi}}_t + \zeta_t' \boldsymbol{\sigma}_t - \mathbf{a}_t' \boldsymbol{\beta}_t$. But since the financial firm's decision is conditional

upon consumer choice of ζ_t , variable profits can be written as

$$P_t = \boldsymbol{\mu}_t' \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t' \tilde{\boldsymbol{\pi}}_t - \mathbf{a}_t' \boldsymbol{\beta}_t, \quad (8)$$

and the firm's variable profit maximization problem takes the conventional form of selecting $(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \mathbf{z}_t, \mathbf{L}_t, c_t) \in S(\mathbf{k}_t)$ to maximize (8). Hence the existing literature on output aggregation for multiproduct firms becomes immediately applicable to aggregation over the produced monetary services $(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t)$ and to measuring value added and technological change in financial intermediation.

3. Properties of the Model

Following Barnett (1987), variable revenue can be written in the form

$$\boldsymbol{\mu}_t' \boldsymbol{\gamma}_t = \boldsymbol{\mu}_t' \boldsymbol{\pi}_t^s - \frac{p_t^* R_t \mathbf{k}_t' \boldsymbol{\mu}_t}{1 + R_t}, \quad (9)$$

where

$$\pi_{it}^s = p_t^* \frac{R_t - \rho_{it}}{1 + R_t} \quad (10)$$

has the same form as the demand-side monetary-asset user-cost formula derived by Barnett (1978,1980) for consumers. Clearly π_{it}^s in (10) would equal γ_{it} if $\mathbf{k}_t = 0$, removing the regulatory wedge between the demand and supply side. For credit card transaction services, π_{jt} equals exactly the demand-side user cost of credit card services derived by Barnett, Chauvet, Leiva-Leon, and Su (2016) for consumers, since there is no regulatory wedge between the demand and supply side for credit card services.

The solution to the firm's variable profit-maximization problem is its factor demand functions for $\alpha = (\mathbf{z}_t', \mathbf{L}_t', c_t)'$ and its supply functions for its multiple products (μ_t, τ_t) conditionally upon consumers' choices of ζ_t . Derived demand is thereby produced for high-powered (base) money. That derived demand, in real terms, is

$$h_t = c_t + \sum_i k_{it} \mu_{it}. \quad (11)$$

The financial firm's nominal demand for high-powered money is $p_t^* h_t$.

4. Separability of Technology

Following Barnett (1987), we assume there exist functions f and H such that

$$F(\mu_t, \tau_t, \mathbf{z}_t, \mathbf{L}_t, c_t; \mathbf{k}_t) = H(f(\mu_t, \tau_t; \mathbf{k}_t), \mathbf{z}_t, \mathbf{L}_t, c_t).^{70} \quad (12)$$

Under the usual neoclassical assumptions on technology, there will exist a function g such that

$$f(\mu_t, \tau_t; \mathbf{k}_t) = g(\mathbf{z}_t, \mathbf{L}_t, c_t) \quad (13)$$

⁷⁰ The resulting functional structure is called blockwise weak separability. A large literature exists on testing weakly separable function structure, such as Cherchye, Demuynck, Rock, and Hjerstrand (2015) and Hjerstrand, Swofford, and Whitney (2016)

is the solution for $f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t)$ to

$$H(f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t), \mathbf{z}_t, \mathbf{L}_t, c_t) = 0. \quad (14)$$

The function $f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t)$ is called the factor requirements function, because it equals the right-hand side of (13), which is the minimum amount of aggregate input required to produce the vector $(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t)$. The function $g(\mathbf{z}_t, \mathbf{L}_t, c_t)$ is the production function, because it equals the left-hand side of (13), which is the maximum amount of aggregate output that can be produced from the inputs $(\mathbf{z}_t, \mathbf{L}_t, c_t)$. Hence f is both the factor requirements function and the outputs aggregator function, $M_t^s = f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t)$, while g is both the output production function and the inputs aggregator function.

We assume that f is convex and linearly homogeneous in $\boldsymbol{\mu}_t$ and $\boldsymbol{\tau}_t$. In addition, it follows from our assumptions on the neoclassical properties of the transformation function F , that g is monotonically increasing in all of its arguments and that f is monotonically increasing in $\boldsymbol{\mu}_t$ and $\boldsymbol{\tau}_t$. We assume that g is locally strictly concave in a neighborhood of the solution to the first-order conditions for variable profit maximization. In addition, it follows, from the strict quasiconvexity of the transformation function F , that g is globally strictly quasiconcave.

5. Financial Intermediary Aggregation Theory Under Homogeneity

In this section, we produce a two-stage decision for the financial intermediary. In the first stage, the firm solves for profit-maximizing factor demands and the profit-maximizing level of

aggregate financial services produced. In the second stage, the revenue-maximizing vector of individual financial service quantities supplied is determined at fixed aggregate financial service quantity supplied.

To display that decomposition of the firm's profit-maximization decision, we start by defining the relevant revenue functions. The financial firm's revenue function is

$$W(\mathbf{a}_t, \gamma_t, \boldsymbol{\pi}_t, \boldsymbol{\sigma}_t, \zeta_t; \mathbf{k}_t) = \max_{\{\boldsymbol{\mu}_t, \boldsymbol{\tau}_t\}} \{\boldsymbol{\mu}_t' \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t' \tilde{\boldsymbol{\pi}}_t + \zeta_t' \boldsymbol{\sigma}_t : f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t) = g(\mathbf{a}_t)\}.$$

Since the decision is conditional on consumer choice of ζ_t , the financial firm's variable revenue function can be written as

$$R^*(\mathbf{a}_t, \gamma_t, \boldsymbol{\pi}_t; \mathbf{k}_t) = \max_{\{\boldsymbol{\mu}_t, \boldsymbol{\tau}_t\}} \{\boldsymbol{\mu}_t' \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t' \tilde{\boldsymbol{\pi}}_t : f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t) = g(\mathbf{a}_t)\}, \quad (15)$$

where the firm selects \mathbf{a}_t to maximize variable profits

$$P_t = R^*(\mathbf{a}_t, \gamma_t, \boldsymbol{\pi}_t; \mathbf{k}_t) - \mathbf{a}_t' \boldsymbol{\beta}_t. \quad (16)$$

However, by Shephard's (1970, p. 251) Proposition 83, it follows that there exists a linearly homogeneous output price aggregator function Γ such that

$$R^*(\mathbf{a}_t, \gamma_t, \tilde{\boldsymbol{\pi}}_t; \mathbf{k}_t) = \Gamma(\gamma_t, \tilde{\boldsymbol{\pi}}_t) g(\mathbf{a}_t). \quad (17)$$

Hence the financial firm's variable profits can alternatively be written as

$$P_t = \Gamma(\gamma_t, \tilde{\pi}_t)g(\mathbf{a}_t) - \mathbf{a}_t' \boldsymbol{\beta}_t. \quad (18)$$

The firm's first-stage decision is to select \mathbf{a}_t^* to maximize (18). Substituting the optimized input vector \mathbf{a}_t^* into $g(\mathbf{a}_t)$, the firm can compute the optimum aggregate financial service quantity supplied, M_t^s , including both monetary services and credit card transaction services supplied. In stage two of the decentralized decision, M_t^s is substituted into (15) to replace $g(\mathbf{a}_t)$, and the maximization problem in (15) is solved to acquire the optimum vector of supplied monetary assets $\boldsymbol{\mu}_t$ and credit card transaction volumes $\boldsymbol{\tau}_t$, conditionally upon consumer's choices of carried-forward credit card debt, ζ_t . Observe that the intermediary's supply function for its output aggregate is produced from stage one alone.

Clearly, the exact economic output quantity aggregate for the financial firm is

$$M_t^s = f(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*; \mathbf{k}_t), \quad (19)$$

when $(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*)$ is the variable profit-maximizing vector of monetary assets and credit card transaction volumes produced. The corresponding variable output price aggregate is

$$\Gamma_t^s = \Gamma(\gamma_t, \tilde{\pi}_t). \quad (20)$$

Fisher's output reversal test states that $M_t^s \Gamma_t^s$ must equal actual revenue from production of $(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*)$. That condition is satisfied as a result of (15) and (17), and the fact that $f(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*; \mathbf{k}_t)$

must equal $g(\mathbf{a}_t)$ at $\mathbf{a}_t = \mathbf{a}_t^*$. Also observe from (15) and (17), with $g(\mathbf{a}_t)$ set equal to 1.0, that the variable output price aggregate is equal to

$$\Gamma(\gamma_t, \tilde{\pi}_t) = \max_{\{\mu_t, \tau_t\}} \{\mu_t' \gamma_t + \tau_t' \tilde{\pi}_t : f(\mu_t, \tau_t; \mathbf{k}_t) = 1\}, \quad (21)$$

which is the unit variable revenue function. The unit variable revenue function is the maximum variable revenue that can be acquired from the production of one unit of the output monetary aggregate, $M_t^s = f(\mu_t, \tau_t; \mathbf{k}_t)$. The linear homogeneity of Γ is clear from (21). In addition, the unit revenue function is convex and increasing in $(\gamma_t, \tilde{\pi}_t)$ and increasing in (σ_t, ζ_t) .

Instead of maximizing $\mu_t' \gamma_t + \tau_t' \tilde{\pi}_t$ subject to

$$f(\mu_t, \tau_t; \mathbf{k}_t) = g(\mathbf{a}_t^*)$$

to acquire the stage-two solution for (μ_t^*, τ_t^*) conditionally upon consumer choices of ζ_t^* , we could equivalently define the stage-two decision to be the selection of (μ_t^*, τ_t^*) to minimize the aggregate factor requirement $f(\mu_t, \tau_t; \mathbf{k}_t)$ subject to

$$\mu_t' \gamma_t + \tau_t' \tilde{\pi}_t = \Gamma(\gamma_t, \tilde{\pi}_t) g(\mathbf{a}_t^*).$$

As a result, we can rewrite (19) to obtain

$$M_t^s = \min_{\{\mu_t, \tau_t\}} \{f(\mu_t, \tau_t; \mathbf{k}_t) : \mu_t' \gamma_t + \tau_t' \tilde{\pi}_t = \Gamma(\gamma_t, \tilde{\pi}_t) g(\mathbf{a}_t^*)\}, \quad (22)$$

while our earlier statement of the stage-two decision produces the equivalent result that

$$M_t^s \Gamma(\gamma_t, \tilde{\pi}_t) = \max_{\{\mu_t, \tau_t\}} \{\mu_t' \gamma_t + \tau_t' \tilde{\pi}_t : f(\mu_t, \tau_t; \mathbf{k}_t) = g(\mathbf{a}_t^*)\}. \quad (23)$$

Comparing (21) and (22), we can see the clear duality between the decision problems. As usual, the exact quantity and price aggregates of economic theory are true duals.

Equation (21) defines the unit revenue (output price aggregator) function in terms of the factor requirement (output quantity aggregator) function. The converse is also possible as a result of the fact that

$$f(\mu_t^*, \tau_t^*; \mathbf{k}_t) = \left[\min_{\gamma_t \geq 0} \{ \Gamma(\gamma_t, \tilde{\pi}_t) : \mu_t^{*'} \gamma_t + \tau_t^{*'} \pi_t = 1 \} \right]^{-1},$$

using equation (3.2) in Diewert (1976).

6. Financial Intermediary Index Number Theory Under Homogeneity

Monetary output aggregation is produced by solving the financial intermediary's second-stage decision for $(\mu_t^{*'}, \tau_t^{*'})'$ and substituting it into f to acquire $M_t^s = f(\mu_t^*, \tau_t^*; \mathbf{k}_t)$. That second-stage decision is to select $(\mu_t^{*'}, \tau_t^{*'})'$ to

$$\max \mu_t' \gamma_t + \tau_t' \tilde{\pi}_t + \zeta_t' \sigma_t \quad \text{subject to } f(\mu_t, \tau_t; \mathbf{k}_t) = M_t^s. \quad (24a)$$

But since the decision is conditional on consumer choice of ζ_t , the decision is equivalent to

selecting $(\boldsymbol{\mu}_t^{*'}, \boldsymbol{\tau}_t^{*'})'$ to maximize variable revenue as follows:

$$\max \boldsymbol{\mu}_t' \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t' \tilde{\boldsymbol{\pi}}_t \quad \text{subject to } f(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t) = M_t^s. \quad (24b)$$

The following theorem proves that the Divisia index tracks M_t^s without error in continuous time, so long as $(\boldsymbol{\mu}_t^{*'}, \boldsymbol{\tau}_t^{*'})'$ is continually selected to solve (24b) at each instant, t .

Theorem 1. If $(\boldsymbol{\mu}_t^{*'}, \boldsymbol{\tau}_t^{*'})'$ solves (24b) continually at each instant $t \in T_0$, then for every $t \in T_0$

$$d \log M_t^s / dt = \sum_i s_{it} d \log \mu_{it}^* / dt + \sum_j u_{jt} d \log \tau_{jt}^* / dt, \quad (25)$$

where $s_{it} = \mu_{it}^* \gamma_{it} / (\boldsymbol{\mu}_t^{*'} \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t^{*'} \tilde{\boldsymbol{\pi}}_t)$ and $u_{jt} = \tau_{jt}^* \tilde{\pi}_{jt} / (\boldsymbol{\mu}_t^{*'} \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t^{*'} \tilde{\boldsymbol{\pi}}_t)$.

Proof: The first-order conditions for solution to (24b) are

$$\gamma_{it} = -\lambda \partial f / \partial \mu_{it}, \quad (26)$$

$$\tilde{\pi}_{jt} = -\lambda \partial f / \partial \tau_{jt}, \quad (27)$$

and $f(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*; \mathbf{k}_t) = M_t^s$, where λ are the Lagrange multipliers.

Compute the total differential of f to acquire

$$df(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \mathbf{k}_t) = \sum_i \frac{\partial f}{\partial \mu_{it}} d\mu_{it} + \sum_j \frac{\partial f}{\partial \tau_{jt}} d\tau_{jt}.$$

Substitute (26) and (27) to find, at $\boldsymbol{\mu}_t = \boldsymbol{\mu}_t^*$ and $\boldsymbol{\tau}_t = \boldsymbol{\tau}_t^*$, that

$$df(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*; \mathbf{k}_t) = -\frac{1}{\lambda} \sum_i \gamma_{it} d\mu_{it}^* - \frac{1}{\lambda} \sum_j \tilde{\pi}_{jt} d\tau_{jt}^*. \quad (28)$$

But by summing (26) over i and (27) over j , solving for λ , and substituting into (28), we obtain

$$\begin{aligned} d \log f(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*; \mathbf{k}_t) &= \frac{\boldsymbol{\mu}_t^{*'} \partial f / \partial \boldsymbol{\mu}_t + \boldsymbol{\tau}_t^{*'} \partial f / \partial \boldsymbol{\tau}_t}{f(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*; \mathbf{k}_t)} \sum_i \frac{\mu_{it}^* \gamma_{it}}{\boldsymbol{\mu}_t^{*'} \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t^{*'} \boldsymbol{\pi}_t} d \log \mu_{it}^* \\ &+ \frac{\boldsymbol{\mu}_t^{*'} \partial f / \partial \boldsymbol{\mu}_t + \boldsymbol{\tau}_t^{*'} \partial f / \partial \boldsymbol{\tau}_t}{f(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*; \mathbf{k}_t)} \sum_j \frac{\tau_{jt}^* \tilde{\pi}_{jt}}{\boldsymbol{\mu}_t^{*'} \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t^{*'} \boldsymbol{\pi}_t} d \log \tau_{jt}^*. \end{aligned} \quad (29)$$

But since f is linearly homogeneous in $\boldsymbol{\mu}_t$ and $\boldsymbol{\tau}_t$, we have from Euler's equation that

$$\boldsymbol{\mu}_t^{*'} \partial f / \partial \boldsymbol{\mu}_t + \boldsymbol{\tau}_t^{*'} \partial f / \partial \boldsymbol{\tau}_t = f(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*; \mathbf{k}_t) \quad (30)$$

Substituting (30) into (29), we obtain

$$d \log f(\boldsymbol{\mu}_t^*, \boldsymbol{\tau}_t^*; \mathbf{k}_t) / dt = \sum_i s_{it} d \log \mu_{it}^* / dt + \sum_j u_{jt} d \log \tau_{jt}^* / dt, \quad (31)$$

where $s_{it} = \mu_{it}^* \gamma_{it} / (\boldsymbol{\mu}_t^{*'} \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t^{*'} \boldsymbol{\pi}_t)$ and $u_{jt} = \tau_{jt}^* \tilde{\pi}_{jt} / (\boldsymbol{\mu}_t^{*'} \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t^{*'} \boldsymbol{\pi}_t)$.

□

Hence the Divisia index is equally as applicable to aggregating over the monetary services and credit card transaction services produced by the financial intermediary as over the monetary services and credit card transactions services by the consumers, as derived by Barnett, Chauvet, Leiva-Leon, and Su (2016). In addition, Simpson's rule produces the Törnqvist-Theil discrete time approximation

$$\log M_t^s - \log M_{t-1}^s = \sum_i \bar{s}_{it} (\log \mu_{it}^* - \log \mu_{i,t-1}^*) + \sum_j \bar{u}_{jt} (\log \tau_{jt}^* - \log \tau_{j,t-1}^*), \quad (32)$$

where $\bar{s}_{it} = \frac{1}{2}(s_{it} + s_{i,t-1})$ and $\bar{u}_{jt} = \frac{1}{2}(u_{jt} + u_{j,t-1})$. Furthermore, if the input requirement function f is translog, then the discrete Divisia index (32) is exact in discrete time [see Diewert (1976, p. 125)]. Hence (32) is a superlative index number.

Having produced the output quantity aggregate from the Divisia index, the dual price aggregate is produced from variable output reversal,

$$I_t = (\mathbf{\mu}_t^{*'} \boldsymbol{\gamma}_t + \boldsymbol{\tau}_t^{*'} \tilde{\boldsymbol{\pi}}_t) / M_t^s. \quad (33)$$

The user-cost price index produced in that manner is called the implicit Divisia price index. The resulting price index is superlative in the Diewert sense, as is easily shown from (33) and the fact that M_t^s is superlative.

7. Financial Intermediary Aggregation Without Homotheticity

Define the financial firm's output distance function implicitly to be the value of $D(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \boldsymbol{\alpha}_t; \mathbf{k}_t)$ that solves

$$f((\boldsymbol{\mu}_t, \boldsymbol{\tau}_t) / D(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \boldsymbol{\alpha}_t; \mathbf{k}_t); \mathbf{k}_t) = g(\boldsymbol{\alpha}_0), \quad (34)$$

for preselected reference input vector $\boldsymbol{\alpha}_0$. Then the exact monetary quantity output aggregate for the financial intermediary is

$$M^s(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t; \boldsymbol{\alpha}_0, \mathbf{k}_t) = D(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \boldsymbol{\alpha}_t; \mathbf{k}_t), \quad (35)$$

and the corresponding Malmquist economic output quantity index is

$$M^{ms}(\boldsymbol{\mu}_{t2}, \boldsymbol{\tau}_{t2}, \boldsymbol{\mu}_{t1}, \boldsymbol{\tau}_{t1}; \boldsymbol{\alpha}_0, \mathbf{k}_t) = D(\boldsymbol{\mu}_{t2}, \boldsymbol{\tau}_{t2}, \boldsymbol{\alpha}_0; \mathbf{k}_t) / D(\boldsymbol{\mu}_{t1}, \boldsymbol{\tau}_{t1}, \boldsymbol{\alpha}_0; \mathbf{k}_t). \quad (36)$$

The corresponding true output price aggregate is

$$\Gamma(\boldsymbol{\gamma}_t, \tilde{\boldsymbol{\pi}}_t; \boldsymbol{\alpha}_0, \mathbf{k}_t) = R^*(\boldsymbol{\alpha}_0, \boldsymbol{\gamma}_t, \tilde{\boldsymbol{\pi}}_t; \mathbf{k}_t), \quad (37)$$

and the corresponding Konüs true financial output price index is

$$\Gamma^k(\boldsymbol{\gamma}_{t2}, \tilde{\boldsymbol{\pi}}_{t2}, \boldsymbol{\gamma}_{t1}, \tilde{\boldsymbol{\pi}}_{t1}; \boldsymbol{\alpha}_0, \mathbf{k}_t) = R^*(\boldsymbol{\alpha}_0, \boldsymbol{\gamma}_{t2}, \tilde{\boldsymbol{\pi}}_{t2}; \mathbf{k}_t) / R^*(\boldsymbol{\alpha}_0, \boldsymbol{\gamma}_{t1}, \tilde{\boldsymbol{\pi}}_{t1}; \mathbf{k}_t). \quad (38)$$

The duality results are

$$D(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \boldsymbol{a}_0; \mathbf{k}) = \max_{\{\gamma_t, \tilde{\pi}_t\}} \{\gamma_t' \boldsymbol{\mu}_t + \tilde{\pi}_t' \boldsymbol{\tau}_t : R^*(\boldsymbol{a}_0, \gamma_t, \tilde{\pi}_t; \mathbf{k}_t) = 1\} \quad (39)$$

and

$$R^*(\boldsymbol{a}_0, \gamma_t, \tilde{\pi}_t; \mathbf{k}_t) = \max_{\{\boldsymbol{\mu}_t, \boldsymbol{\tau}_t\}} \{\gamma_t' \boldsymbol{\mu}_t + \tilde{\pi}_t' \boldsymbol{\tau}_t : D(\boldsymbol{\mu}_t, \boldsymbol{\tau}_t, \boldsymbol{a}_0; \mathbf{k}_t) = 1\}. \quad (40)$$

8. Value Added from Financial Intermediation

Partition the financial intermediary's input vector \boldsymbol{a}_t so that $\boldsymbol{a}_t = (\boldsymbol{a}_{1t}', \boldsymbol{a}_{2t}')'$, where \boldsymbol{a}_{1t} is the quantities of primary inputs to the financial intermediary, and \boldsymbol{a}_{2t} is quantities of intermediate inputs. Partition the factor-price vector correspondingly so that $\boldsymbol{\beta}_t = (\boldsymbol{\beta}_{1t}', \boldsymbol{\beta}_{2t}')'$. Then the financial intermediary's technology can be written as

$$M_t^s = g(\boldsymbol{a}_{1t}, \boldsymbol{a}_{2t}). \quad (41)$$

Let the firm's maximum variable profit level at given \boldsymbol{a}_{1t} be

$$V_t = V(\boldsymbol{a}_{1t}, \boldsymbol{\beta}_{2t}, \gamma_t, \tilde{\pi}_t), \quad (42)$$

which is the firm's variable profit function conditional upon \boldsymbol{a}_{1t} . As a function of \boldsymbol{a}_{1t} at fixed prices, V has all of the usual properties of a neoclassical production function. Sato (1975) calls

$$V_{t_0, t_1} = V(\boldsymbol{a}_{1t_0}, \boldsymbol{\beta}_2^*, \gamma^*, \tilde{\pi}_t^*) / V(\boldsymbol{a}_{1t_1}, \boldsymbol{\beta}_2^*, \gamma^*, \tilde{\pi}_t^*) \quad (43)$$

the true index of real value added, which depend upon the selection of the reference prices

$$(\beta_2^*, \gamma^*, \tilde{\pi}^*).$$

In order to provide a nonparametric (statistical) approximation to (43), assume constant returns to scale. Also assume that V is translog and select $(\beta_2^*, \gamma^*, \tilde{\pi}^*)$ to be the geometric means of those prices in periods t_0 and t_1 . Using Diewert (1980a, p. 459), it follows that (43) equals the discrete Divisia quantity index for aggregating over the primary inputs.

The need to select the reference prices $(\beta_2^*, \gamma^*, \tilde{\pi}^*)$ becomes unnecessary if and only if g is separable, so that (41) can be written

$$M_t^s = G(\varphi(\mathbf{a}_{1t}), \mathbf{a}_{2t}). \quad (44)$$

In that case, V can be written

$$V_t = V_1(\mathbf{a}_{1t})V_2(\beta_{2t}, \gamma_t, \tilde{\pi}_t). \quad (45)$$

So clearly

$$V_{t_0, t_1} = V_1(\mathbf{a}_{1t_0}) / V_1(\mathbf{a}_{1t_1}), \quad (46)$$

which does not depend on reference prices. The function V_1 has all of the properties of a conventional neoclassical production function. However, in this case $\varphi(\mathbf{a}_{1t})$ is itself a category subproduction function, so we can more directly define the value added index to be

$$V_{t_0, t_1}^* = \varphi(\mathbf{a}_{1t_0}) / \varphi(\mathbf{a}_{1t_1}). \quad (47)$$

If φ is translog, then the discrete Divisia index is exact for either (46) or (47), so the discrete Divisia index provides a second-order approximation for V_{t_0, t_1}^* or V_{t_0, t_1} for any φ . In continuous time, the Divisia index is always exact for $\varphi(\mathbf{a}_{1t})$, which is value added.

By accounting convention, “double deflation” requires the very restrictive assumption that (44) can be written in the form

$$M_t^s = \varphi_1(\mathbf{a}_{1t}) + \varphi_2(\mathbf{a}_{2t}). \quad (48)$$

Clearly $\varphi_1(\mathbf{a}_{1t})$ is value added, since it is added to $\varphi_2(\mathbf{a}_{2t})$ to get M_t^s . In that case, Sims (1969) has proved that value added is measured exactly by a Divisia index.

9. Data Sources

Although this initial theoretical paper does not include empirical application, the availability of the needed data sources is relevant to future use. Of particular importance is the availability of data on the benchmark interest rate, R_t , the vector, ζ_t , of rotating real balances, ζ_{jt} , in credit card type j , the vector, τ_t , of real expenditures “volumes,” τ_{jt} , with credit card type j ; the vector, \mathbf{e}_t , of expected interest rates, e_{jt} , on τ_t ; and the vector of interest rates, $\bar{\mathbf{e}}_t$, on ζ_t .

Complete details about those available data sources are documented in Barnett and Su (2016), as used on the demand side by Barnett, Chauvet, Leiva-Leon, and Su (2016).

The credit card transactions services can be measured by the transactions volumes summed over four sources: Visa, MasterCard, American Express, and Discover. Our theory does not apply to debit cards, or to store cards, or to charge cards not providing a line of credit. Barnett, Chauvet, Leiva-Leon, and Su (2016) acquired the volumes from the firms' annual reports and seasonally adjusted them by the Census X-13ARIMA-SEATS program. The start date is the quarter during which those credit card firms went public and the annual reports became available. The contemporaneous transactions volumes do not include the carried forward rotating balances resulting from transactions during prior periods.⁷¹ The credit card interest rates are available from the Federal Reserve Board's data on all commercial bank credit card accounts, including those not charged interest, since paid off within the month.⁷²

In classical economics, the benchmark asset is a secured pure investment. In contrast, \bar{e}_{js} is not the interest rate on a secured asset and is subject to substantial default and fraud risk. Hence, \bar{e}_{js} can be higher than the benchmark asset rate, and historically has always been much higher than the benchmark asset rate.⁷³

⁷¹ Credit limits are not explicitly considered in our current model, since we do not have a way to untangle the effect of those constraints on contemporaneous transactions volumes from the effect on the carried forward rotating balances associate with previous period's transactions.

⁷² This interest rate includes those credit card accounts not assessed interest, and hence is lower than the Federal Reserve's supplied interest rates on accounts assessed interest. This imputation includes only explicit interest paid, averaged over all credit card accounts.

⁷³ Barnett, Chauvet, Leiva-Leon, and Su (2016) follow the Center for Financial Stability (CFS) and the Bank of Israel in using the short term bank loan rate as a proxy for the benchmark rate. That interest rate has always

It is important to recognize that the decision problem we model is not of a single economic agent, but rather of a “representative bank” and a “representative consumer,” aggregated over all consumers and all banks. All quantities are therefore averaged over all consumers and banks. This modeling assumption is particularly important in understand the credit card quantities and interest rates relevant to this theory. About 20% of credit card holders in the United States do not pay explicit interest on credit card balances, since those credit card transactions are paid off by the end of the period. But the 80% who do pay interest pay very high interest rates.⁷⁴ The Federal Reserve provides two interest rate series for credit card debt. One, \bar{e}_{js} , includes interest only on accounts that do pay interest to the credit card issuing banks, while the other series, e_{js} , includes the approximately 20% that do not pay interest. The latter interest rate is thereby lower, since it is averaged over interest paid on both categories of accounts. Since the representative consumer is aggregated over all consumers, e_{js} is always less than \bar{e}_{js} for all j and s . The

exceeded the interest rate paid by banks on deposit accounts and on all other monetary assets used in the CFS Divisia monetary aggregates, and has always been lower than the Federal Reserve’s reported average interest rate charged on credit card balances. However, it is important to keep in mind that the benchmark rate in theory is the rate of return on an owned asset, pure capital. Since that asset is owned by its investors, it is fully secured. While short term bank loans are assets to banks, some are unsecured. For detailed information on CFS data sources, see Barnett, Liu, Mattson, and Noort (2013).

An alternative proxy for the benchmark interest rate has been proposed by Fixler and Zieschang (2016a,b). They advocate using the financial firm’s overall funding portfolio as the benchmark asset and the cost of funding rate as the benchmark rate. In macroeconomic research, we currently favor consistency with the CFS convention, the short term bank loan rate, which is easily available from the Federal Reserve. But we recognize that the Fixler and Zieschang (2016a,b) proposal is very reasonable.

⁷⁴ The following statement is from www.motherjones.com/kevin-drum/2011/10/americans-are-clueless-about-their-credit-card-debt. “In the four working age categories, about 50% of households think they have outstanding credit card debt, but the credit card companies themselves think about 80% of households have outstanding balances.” Since these percentages are of total households, including those having no credit cards, the percent of credit card holders paying interest might be even higher.

interest rate on rotating credit card balances, \bar{e}_{js} , is paid by all consumers who maintain rotating balances on credit cards. But e_{js} is averaged over both those consumers who maintain such rotating balances and hence pay interest on contemporaneous credit card transactions (volumes) and also over those consumers who pay off such credit card transactions before the end of the period, and hence do not pay explicit interest on the credit card transactions. The Federal Reserve provides data on both \bar{e}_{js} and e_{js} . Although e_{js} is less than \bar{e}_{js} , e_{js} also has always been higher than the benchmark rate.

The expected interest rate, e_{js} , can be explicit or implicit, and applies to the aggregated representative consumer. For example, an implicit part of that interest rate could be in the form of an increased price of the goods purchased or in the form of a periodic service fee or membership fee. But Barnett, Chauvet, Leiva-Leon, and Su (2016) use only the Federal Reserve's average explicit interest rate series, which is lower than the one that would include implicit interest. Nevertheless, that downward biased explicit rate of return to credit card companies, e_{js} , aggregated over consumers, tends to be very high, far exceeding R_s , even after substantial losses from fraud.

10. Conclusion

In this paper a monetary production model of financial firms is employed to investigate supply-side monetary aggregation augmented to include the credit card transactions services produced by those firms. Financial firms are viewed to produce monetary services and credit card transactions services as outputs through financial intermediation. The nature of financial firms'

outputs is related to their role in the transaction technology underlying the payment mechanism in the economy.

Much work remains to be done, including theoretical generalizations with weakened assumptions and empirical applications.⁷⁵ The most challenging generalizations could permit incomplete contingent claims markets and asymmetric information to explain the appearance of risk averse behavior by financial firms. But our initial theoretical results indicate the following tentative conclusions. Financial firm outputs of demand deposits, time deposit services, and credit card transactions services can be aggregated to produce an output aggregate, which then enters an aggregate services supply function for the financial firm. When all outputs are separable from inputs, there exists a single output aggregate, and hence the use of a single output aggregate can be justified in the formulation and estimation of the financial firm's production technology. The theory can be implemented to investigate the role of financial intermediaries in the production of inside money, which plays a role in the transmission mechanism of monetary policy. Further generalization could permit investigation of the role of shadow banking in central bank policy.

The theoretical and empirical problems previously associated with the inability to include credit card transactions services in financial intermediary output are solved.

⁷⁵ Empirical results in this tradition, but with credit card services omitted, can be found in Barnett and Hahm (1994), Barnett and Zhou (1994), Barnett, Kirova, and Pasupathy (1995), and Hancock (1991).

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