Using Student Discourse to Support Multiplication Fact Strategy Instruction

By

Deborah Marie Thompson

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A. Susan Gay, Chair

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Barbara Bradley

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Reva Friedman

____
Kelli Thomas

____
Bruce Frey

Date Defended: August 24, 2017

The dissertation committee for Deborah Marie Thompson
certifies that this is the approved version of the following dissertation:

Using Student Discourse to Support Multiplication Fact Strategy Instruction

Chairperson A. Susan Gay

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Abstract

Basic multiplication fact fluency is an expected outcome of most third grade classrooms throughout the United States. Students need to know their multiplication facts. This statement does not seem to be under question; it is more about *how* will students learn their multiplication facts.

The purpose of this study was to investigate the use of student discourse as part of third grade teachers' fact strategy instruction and students' use of fact strategies while learning basic multiplication facts. Quantitative and qualitative data were used to generate a picture of how student discourse supports fact strategy instruction.

The semester-long study gathered data from participants using observations, interviews and written assessments. Four third-grade classroom teachers were observed on three separate occasions so information could be collected concerning the frequency of the various levels of questions asked during fact strategy lessons. Interviews were conducted at the end of the study to gather their insights into how discourse impacted their instruction and their students' use of fact strategies. All students from the four classrooms were given pre- and post-tests targeting use of fact strategies. After the post-test, 22 students were selected to be interviewed in order to collect more detailed information about fact strategy implementation.

This study used a descriptive, nonexperimental design utilizing a mixed methods approach. The descriptive statistics from the assessments and results from the interviews indicated that using student discourse to support fact strategy instruction was positive for the students and the teachers. During the semester, the teachers increased their use of higher-level questions in lessons about multiplication fact strategies. There was a significant increase of multiplication fact strategy use by the students over the course of the semester. During the

interview, more students used fact strategies to solve multiplication fact problems; this increase could be attributed to the amount of discourse done in the classroom since the oral exchange in the interview was similar to classroom discussions. The teachers also reported they believed that student discourse positively impacted multiplication fact strategy implementation through increased number sense and reasoning skills.

Acknowledgements

First, I want to express my gratitude to my advisor, Dr. Susan Gay. Her combination of being a critical consultant and an optimistic mentor was invaluable as I was confronted with several obstacles and unforeseen life happenings during the completion of this dissertation. I am truly grateful for her steadfast encouragement.

I would like to express my thanks and appreciation to the participants in my study. The principal, the instructional coach, the teachers, and the students willingly gave of their time and energy so I could thoroughly examine and investigate questions regarding mathematics instruction. Thank you to Linda B., Linda C., Casandra, Cherylyn, Chris, Jerret, and all the third grade students.

My thanks are also extended to the members of my committee. I sincerely appreciated the opportunity of being able to work with each person, Dr. Barbara Bradley, Dr. Bruce Frey, Dr. Reva Friedman, and Dr. Kelli Thomas. The questions and directions given by these mentors enabled me to gain focus for the research and to strengthen the connections between research and practicing educators.

There were many friends and colleagues who provided support and assistance throughout this dissertation, and throughout my career as an educator. Without their thoughtful insights and tough questions, I would not have asked my own questions which prompted me to pursue an advanced degree.

Finally, I wish to extend my love and thanks to my family. Their unwavering support and encouragement provided me the confidence to persevere. First, my parents, David and Joyce, provided constant reassurance so I could keep moving forward. My children, Andrew and Linnea, were my indispensable cheerleaders who kept me on track during times that were a struggle. My in-laws, Ed and Sharon, gave support not only to me but also to my husband as he had to live with a sometimes stressed graduate student. This leads me to my husband without whose untiring assistance I could have never completed this research. He was my foundation and my energy. Thank you, Bill, for your infinite love and support.

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CHAPTER 1

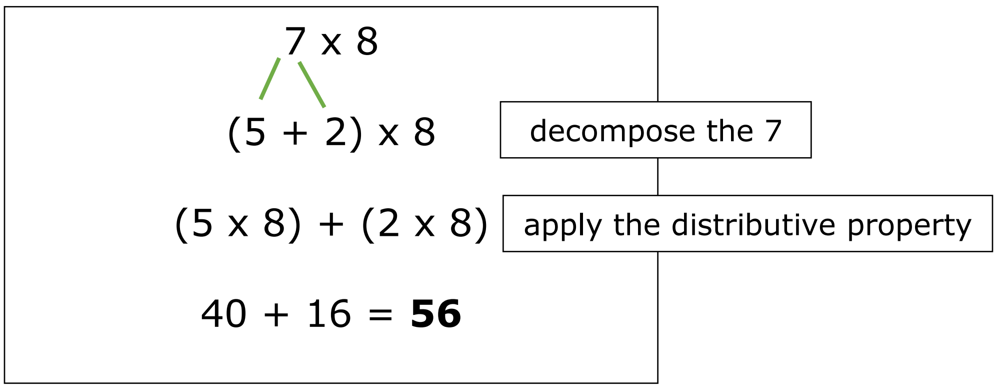
THE RESEARCH PROBLEM

Introduction

While basic multiplication facts instruction has been a staple in elementary mathematics curriculum for decades, there is also evidence of multiplication fact tables being used centuries ago. Swetz (1995) described one of the oldest multiplication fact tables which was approximately 4000 years old from the Old Babylonian period. He referred to a variety of mathematical visuals with one of those visuals being described as organized multiplication tables, or Pythagorean tables as they have been called.

Instruction about multiplication facts has been and continues to be a part of mathematics curriculum. Learning basic facts occurred primarily through rote memorization until Brownell and Chazal (1935) described “learning by insight” (p. 656) and challenged educators to instruct for meaning, not just memorization. The search for meaning and increased understanding of facts using strategies seemed to reach its peak in the 1980s (Baroody & Gannon, 1984; Carnine & Stein, 1981; Carpenter, 1985; Cook & Dossey, 1982; Heege, 1985; Steinberg, 1985; Thornton, 1978), but beginning in the 2000s, there was a resurgence in exploring the use of fact strategies to increase fact retention and efficiency (Boaler, Williams, & Confer, 2015; Flowers & Rubenstein, 2010/2011; Kling & Bay-Williams, 2015; Sherin & Fuson, 2005; Wallace & Gurganus, 2005). The National Council of Teachers of Mathematics (NCTM, 2014) has identified research findings from the cognitive sciences and from mathematics education that support active learning in mathematics so each student can build understanding. This type of learning for understanding includes foundational skills in mathematics, and basic facts are one of those foundational skills.

Fact strategies are basic number combinations that use the idea of equivalence which "involve[s] breaking the calculation apart into an equivalent representation that uses known facts to figure out the unknown fact" (Charles, 2005, p. 10). For example, 7 x 8 is a basic multiplication fact that is frequently "remembered" incorrectly and causes concern for many students. This fact can be decomposed into an equivalent representation for easier mental computation. The strategy is shown below:



The *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and *Kansas College and Career Ready Standards for Mathematics* (Kansas State Department of Education, 2010) place an emphasis on learning addition and multiplication facts through the use of strategies. Beginning in first grade, students will "use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., 'making tens')" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 13). The properties of addition and multiplication referenced in the standards include the Commutative Property, the Associative Property, and the Distributive Property. In first grade, students are working with basic facts in addition and subtraction. Beginning in third grade, students learn to "use properties of operations to calculate products of whole numbers … involving single-digit factors" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 21). Third grade students are expected to use and compare different strategies. By the end of third grade, students should "know from memory all products of two one-digit numbers" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 23).

Since the first mathematics content standards were produced, communication in the classroom has also been a focus (NCTM, 1989). Discourse in mathematics classrooms should encompass the students' and teachers' ways of communicating, representing, talking, supporting, and disagreeing about mathematical ideas (NCTM, 2011). Students and teachers are encouraged to "talk mathematics" (NCTM, 1989, p. 26) and to listen to each other. "Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (NCTM, 2014, p. 29). This idea of student discourse hinges on the instructor's ability to "listen to students' thinking and ask probing questions to help students find their own ways through the problems and honor their struggles" (Clark et al., 2012, para. 12). Once a safe learning environment is established then students come to "care about each other's learning. They learn that in trying to understand the thinking of others, they understand the mathematics at a deeper level themselves" (Clark et al., 2012, para. 18). Effective student discourse depends on the teacher facilitating the conversations and students listening to each other to make sense of mathematics.

Statement of the Problem

The purpose of this study was to investigate the use of student discourse as part of third grade teachers' fact strategy instruction and students' use of fact strategies while learning basic multiplication facts. A group of third grade teachers received training on how to facilitate student discourse along with a review session on fact strategies before beginning their classroom instruction on multiplication facts. Quantitative and qualitative data were collected from teachers and students to generate a picture of how student discourse can support fact strategy instruction.

Research Questions

This study will explore the following research questions.

- What insights do teachers have about the multiplication fact strategies used by their students and the use of student discourse while teaching multiplication fact strategies?
- How did the frequency of different categories of questions used by teachers change during the semester's instruction on fact strategies?
- How did the number of students at higher levels of multiplication fact strategy implementation change as a result of instruction that included student discourse?

Information related to these questions was gathered through classroom observations, student and teacher interviews, and student pre- and post-tests on fact strategy use. The study occurred in the fall semester of 2016 during which the curriculum provided two separate units concentrating on building conceptual understanding of multiplication and the use of fact strategies. Student pre-tests were administered in September and post-tests given in December. Student interviews were completed within a week after the post-tests so students could more easily recall the strategies used. Teacher interviews were conducted the first week in January so they could provide information regarding the entire previous semester.

Rationale for the Study

Professional Experiences

The use of fact strategies to instruct basic facts came to my attention early in my career from an experienced mentor teacher, Mary Krehbiel-Shau. Through my work with her and in working with my students, I became interested in discovering why some of my students were able to learn their basic facts with relative ease while others seemed to have more difficulty in learning them. These struggling students were relying on finger counting well past the time when this should have been an abandoned practice. The search for answers led me to the work of mathematics education experts, such as Van de Walle (2001), Carpenter, Fennema, Franke, Levi, and Empson (1999) and the recommendations from the *Adding It Up* report by the National Research Council (2001). These experts and researchers explained that the use of fact strategies builds on conceptual understanding of operations and the properties of operations. My instruction began to include many of these strategies as I investigated ways to reach all of my students.

Support in National and State Standards

Fact strategy instructional support using student discourse is beginning to be examined critically within mathematics education. One reason this is happening is because of the increased focus on fact fluency and the emphasis on learning addition and multiplication facts through the use of strategies as described in the *Common Core State Standards for Mathematics (CCSSM)* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In Kansas, the *Kansas College and Career Ready Standards for Mathematics* (Kansas State Department of Education, 2010) aligned with the CCSSM in promoting the use of fact strategies.

The CCSSM (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and the Kansas Standards (Kansas State Department of Education, 2010) include eight Standards for Mathematical Practice. Two of these mathematical practice standards highlight the importance of student discussions. The third mathematical practice, “Construct viable arguments and critique the reasoning of others” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, pp. 6-7) states an expectation that students are able to listen to the justifications and arguments of others and are able to communicate their own reasoning and thinking. The sixth mathematical practice, “Attend to precision” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 7) conveys the expectation for precision not only with symbols and procedures but also in vocabulary and concepts when communicating with others in both written and verbal forms.

Using Student Discourse to Improve Mathematics Knowledge

Student discourse positively impacts students’ understanding and retention of mathematical concepts and skills (Carpenter et al., 1999; Chazan & Ball, 1999; Cobb, Boufi, McClain, & Whitenack, 1997; Franke et al., 2009; Kazemi & Stipek, 2001; Khisty & Chaval, 2002; NCTM, 2014; Stein, Engle, Smith, & Hughes, 2008; Van de Walle, Lovin, Karp, & Bay-Williams, 2014). NCTM (2014) states that learners must have experiences that allow them to “construct knowledge socially, through discourse” (p. 9) in order to build their own mathematical knowledge. Fiori, Boaler, Cleare, DiBrienza, and Sengupta (2004) found that when students were able to discuss their thinking with others then misconceptions and misunderstandings were more easily identified by the teacher than using written work alone. This thinking was mirrored in the work by Burns (2005). She discovered that once students were able to voice their reasoning during a classroom discussion then she could uncover their misconceptions and assist them in building understanding.

Other research done in elementary schools using classroom discourse in mathematics showed promising results. Cobb, Boufi, McClain, and Whitenack (1997) and Fennema, Carpenter, Franke, Levi, Jacobs, and Empson (1996) had studies that extended for over a year in elementary schools. Both groups found a relationship between increased student conversations in mathematics and increased mathematical understanding. The researchers also discovered that the teachers were more focused in their support to students and uncovering misconceptions which contributed to the increase in mathematical understanding by the students.

Pilot Study

In the spring of 2016, a pilot study was conducted to test measurement instruments and the structure of the data collection process that would eventually be used in this dissertation research. All third grade teachers in the district where the dissertation study would occur had already completed instruction of fact strategies in the fall of 2015, so an alternate group of teachers and students needed to be located. After several discussions with district-level leadership, a plan was formed to run the pilot study with a small population of high school special education teachers.

This group of high school special education teachers instructed students that were operating at the academic levels of elementary students. The teachers had been concerned about their students' understanding and level of achievement with the basic facts for several years and were searching for instructional resources and techniques to enhance their lessons. District leadership proposed providing professional learning about fact strategies for the spring semester and this provided the foundation necessary to conduct the pilot study.

For the pilot study, 12 high school special education teachers received a half-day professional development workshop by the researcher on fact strategies and student discourse in February 2016. The workshop started with an exploration of growth mindset and its impact on student achievement. A short video clip, from former North Carolina special education teacher

Valerie Faulkner describing how fact strategies can positively impact student learning, was then used to provide the group of high school teachers a level of safety in understanding that the professional learning workshop was not something being "done to them" but was actually part of a larger national conversation.

The next part of the workshop explored nine multiplication fact strategies. It was necessary for the participants to experience each one and talk about the strategies with their peers to uncover misconceptions or misunderstandings. The teachers worked with manipulatives and talked about the needs of their students. Many of the participants were amazed at the versatility of the strategies and how these strategies then linked to more complex mathematics, especially the connections to algebra. One teacher, who had attended an addition fact strategies training the previous fall conducted by the researcher, shared that she had seen positive growth in her students by using the shared addition strategies. She explained that students who had not been able to remember their facts for years were now able to access the information and be successful. Being able to hear a success story from one of their peers was valuable.

During the last part of the workshop, the teachers examined a student discourse planning tool. Training on the tool was focused on helping them increase student talk during their lessons with the fact strategies.

After the workshop, three teachers were randomly selected from the group of 12 that finished the professional learning to form the final pilot study sample group. These three teachers were each observed one time in April and then interviewed the first week in May. The students in the three classrooms took a fact strategy pre-assessment in March and a post-assessment in May.

The data collected from this pilot study did not provide many productive insights into fact strategy instruction supported by student discourse, but did assist in fine-tuning the measurement tools and in clarifying the process to be used in the dissertation study. Of the three teachers, only one completed all components of the pilot study: pre- and post-tests for all students, observation of a fact strategy lesson with student discourse, and the teacher interview. Another teacher completed all components of the pilot study, except the observed lesson did not contain fact strategy instruction with student discourse. The third teacher was not able to have students complete the post-test, and the observed lesson did not contain fact strategy instruction with student discourse. The benefits from this pilot study were the learnings for the researcher in determining more effective procedures and processes for the final dissertation research.

The findings from the pilot were two-fold. In analyzing the data from the interviews, all three teachers felt that student discourse benefited their students in learning fact strategies more easily. They also felt that the discussion allowed the students to see other strategies that students used as valid and to try those strategies out. Even with the small number of students that were tested, the data showed that approximately 50% moved from using a slower strategy (e.g., using fingers) to a more efficient strategy.

Assumptions

During the course of this research, several assumptions had to be made about student participants. It was assumed that all students would choose their best strategy to answer the multiplication fact when completing the pre- and post-tests. It was also assumed that students did their best to effectively communicate their thinking on the assessment using words, pictures, diagrams, or equations.

Assumptions had to be made about the teachers in the study. It was assumed that all teachers instructed with fact strategies supported by student discourse during all fact strategy lessons throughout the semester. An assumption was made that these teachers provided the best instruction possible when working with their students during fact strategy lessons. When the pre- and post-tests were administered, it was assumed that all teachers followed directions from the researcher to remind students to do their best thinking and to show the explanations the best they could.

The researcher developed all measurement tools for this study and it is assumed that these tools were constructed to accurately measure the intended outcomes. The researcher was the only observer for the classroom observations and it was assumed that the researcher was consistent in rating the levels of questions teachers were asking. It is also assumed that the researcher was consistent in evaluating the students' pre- and post-tests when categorizing the students' levels of fact strategy acquisition.

Limitations

Students may have anxiety when taking an assessment even when every effort is made to help them be more comfortable, so this could limit their performance on the assessments in this study. There could also be limitations to the abilities of third grade students in accurately writing down their thought processes when solving multiplication fact problems. The students could also be influenced in selecting a strategy because that strategy was just presented during instruction instead of the one that works best for them or is best for that specific fact.

The school selected for this study was not randomly selected, so the results are not generalizable. The teachers in this school were chosen based on the grade level they taught, so their experiences and background in teaching are varied making comparisons among the teachers limited. The classroom management styles of the teachers are different making the social climate within each classroom different creating another limitation in the study.

The researcher was the only observer and interviewer for the study. During observations the researcher may have made inferences that were biased to the study when listening to the questions from the teachers and responses from the students. The researcher may have interpreted the student and/or teacher interviews to show more favorable results for the study. Efforts were made to counter these limitations but it is important to acknowledge that limitations were present.

Since the teachers received professional development concerning student discourse at the beginning of the semester and were informed that the focus of the study was about student discourse, this could have resulted in an increase of higher-level questions being asked by the teachers during instruction. This effect of their awareness must be recognized as a limitation of the study.

Overview

The first chapter provides an introduction to the research problem and a rationale to support the study as well as a short discussion about the assumptions and limitations. The second chapter presents a review of the literature relevant to this study. Topics include the meaning of multiplication, instruction on multiplication, and the role of discourse in the classroom. The third chapter offers a discussion of the methodology used in the study. The structure of the study is explained with details provided about the instruments and the method of data collection and analysis. The fourth chapter gives the results of the study starting first with the teachers and then the students. The fifth chapter provides the summary of the study and the conclusions drawn from the research. This chapter also offers recommendations for elementary educators and for future research.

CHAPTER 2

REVIEW OF LITERATURE

Introduction

Basic multiplication fact fluency is an expected outcome of most third grade classrooms throughout the United States. This expectation is significant enough that the National Research Council (2001) outlined recommendations concerning the instruction of single-digit number combinations.

- "Children should learn single-digit number combinations with understanding.
- Instructional materials and classroom teaching should help students learn increasingly abbreviated procedures for producing number combinations rapidly and accurately without always having to refer to tables and other aids" (p. 413).

The topics in this chapter provide background about multiplication and classroom discourse. The sections concerning multiplication focus on the meaning of multiplication, which includes the four structures of multiplication, and on the teaching and learning of multiplication facts. The sections about discourse in the mathematics classroom give information about the roles of teachers and students and the benefits and concerns about classroom discourse.

Meaning of Multiplication

Students' understanding of multiplication is initially built from their understanding of direct counting and additive thinking (Anghileri, 1989; Jacob & Willis, 2001; Kouba, 1989; Mulligan & Mitchelmore, 1997; Van de Walle, Karp, Lovin, & Bay-Williams, 2014). Clark and Kamii (1996) found that children have difficulties with multiplicative thinking if they are not assisted in moving beyond their additive thinking model. Educators need to involve students in multiplicative thinking that incorporates all structures of multiplication. The most common structures identified in research include: equal groups, multiplicative comparison, array/area model, and combinations/Cartesian product model (Lannin, Chval, & Jones, 2013; Mulligan & Mitchelmore, 1997; Van de Walle, Karp, Lovin, & Bay-Williams, 2014).

Additive and Multiplicative Thinking

The difference between additive and multiplicative thinking is the difference between unary and binary operational thinking. Additive thinking is a unary operation. It involves only one level of abstraction and one level of thinking about inclusion relations (Clark & Kamii, 1996). These relations build on the idea of hierarchal inclusion that is necessary for students in understanding how each number is built on the previous number (Clements & Sarama, 2009). An example of a student with additive thinking using the inclusion relation is shown in Figure 1.

Asked to solve “4 times 3” or “4 groups of 3,” the student first thinks of the unit as three ones. Then the student includes each unit successively so there is the first unit of 3 ones added on to the next unit of 3 ones (3 + 3), added onto the next unit of 3 ones (6 + 3), then the last unit of 3 ones (9 + 3).

Figure 1. Example of additive thinking using the inclusion relation (Clark & Kamii, 1996, p. 42).

Multiplicative thinking, on the other hand, is a binary operation. It “involves the making of two kinds of relations not required in addition: (a) the many-to-one correspondence . . .; and (b) the composition of inclusion relations on more than one level” (Clark & Kamii, 1996, p. 43). An example of a student with multiplicative thinking is shown in Figure 2. Building an understanding of multiplicative thinking is critical in order for students to be able to reason and solve multiplication problems (Lannin, Chval, & Jones, 2013). “Students who reason

Asked to solve "4 times 3" or "4 groups of 3," the student makes four units of one into one unit of four. There are now two inclusion relations which are shown horizontally and vertically in this diagram. The unit of 3 ones is evident at the bottom. Then the four units of three are shown to make the final product.

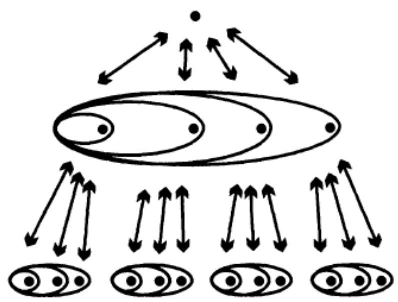


Figure 2. Example of multiplicative thinking using the binary operational thinking (Clark & Kamii, 1996, p. 42).

multiplicatively can 'see' a multiplicative unit and create multiple copies of it. An initial view of multiplication should involve understanding what it means to create 1, 2, 3, 5, or more copies of a given unit" (Lannin, Chval, & Jones, 2013, p. 13). The necessity of having an understanding that multiplication is a binary operation has proven to be significant in building meaning for multiplication (Anghileri, 1989; Barmby, Harries, Higgins, & Suggate, 2009; Clark & Kamii, 1996).

The Four Structures of Multiplication

Multiplication problems can be grouped into four different structures. Some researchers identify five or six structures but these four structures are most frequently identified. They are: equal groups, multiplicative comparison, array/area, and combinations/Cartesian product (Lannin, Chval, & Jones, 2013; Mulligan & Mitchelmore, 1997; Van de Walle, Karp, Lovin, & Bay-Williams, 2014). The equal groups structure is generally the first one introduced to students and is made up of three components: the size of the group(s), the number of groups, and the total.

The comparison structure "is based on one group being a particular multiple of the other (multiple copies)" (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p. 109). The array/area structure takes the form of a rectangular diagram so there are rows and columns instead of the number of groups and the size of the groups. The last structure, combinations/Cartesian product, involves "counting the number of possible pairings that can be made between two or more sets" (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p. 110).

Models, diagrams, and visual representations for instructing the equal groups structure.

Using a set of discrete objects to model equal groups is an effective first strategy, especially for students still tied to additive thinking (Clark & Kamii, 1996). Lannin, Chval, and Jones (2013) showed how equal groups can be modeled using physical objects, such as candy, so the number of groups (number of multiplicative units) and the amount of each group (size of the multiplicative units) are available for students to manipulate; see Figure 3. This type of model closely relates to most situations that students will first encounter in multiplication, but does not necessarily lead to thinking beyond the additive level.

Equal groups with discrete objects (can also be drawn for a pictorial representation):

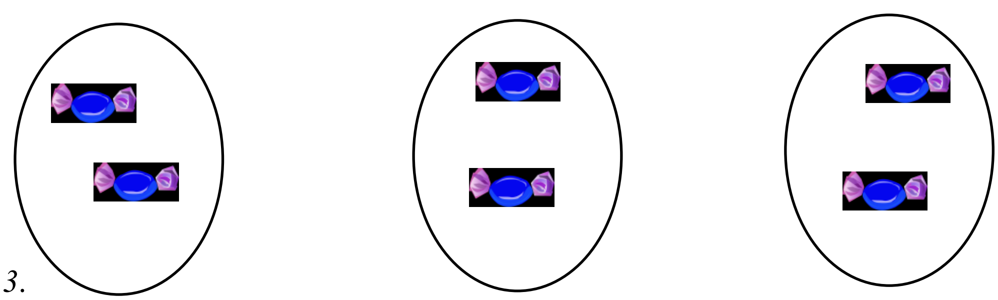


Figure 3.

The number line model can be used to represent equal groups. The multiplicative unit is the distance on the number line (representing the size of the groups) and the number of hops on the number line with that distance is the number of groups; see Figure 4. The model begins to make connections to number patterns more visible for students. The number line model is also mentioned frequently within the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Equal groups with the number line model:

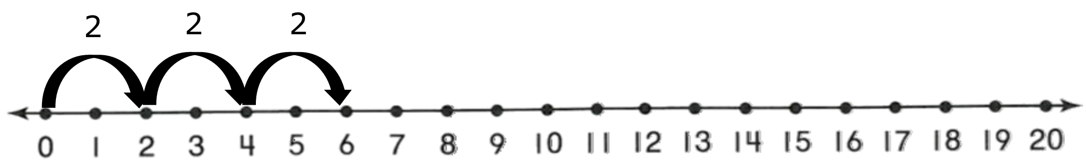


Figure 4. Example of the number line model (Lannin, Chval, & Jones, 2013).

Diagrams that organize the three parts of the multiplication problem can be used effectively with most students. The Wichita Public Schools (2014) use an organizer called the "Group-Group Size-Total" thinking tool that mirrors the description from the work of McCallum et al. (2011). The thinking tool, shown in Figure 5, labels each of the parts of the multiplication problem and the students determine what is known and unknown. Students can use post-it notes

Group	Group Size	Total

Figure 5. Example of Wichita Public Schools organizer for multiplication

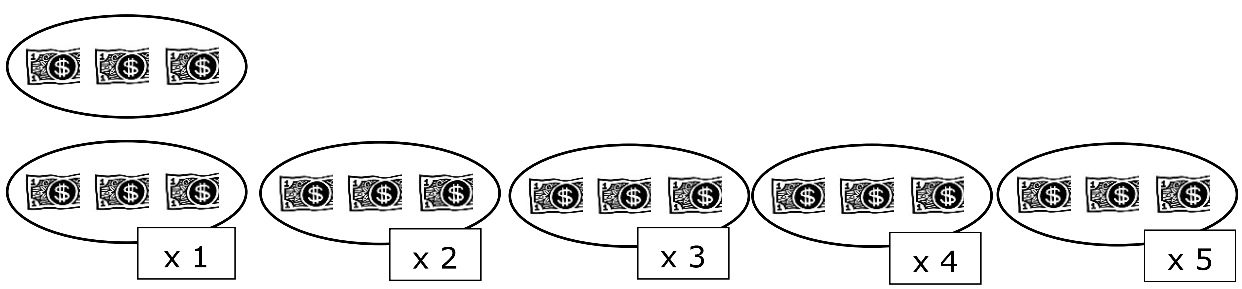
to fill in the required information or the thinking tool can be laminated so students are able to write directly on the tool to identify which quantity is the unknown. Once the unknown is determined, students are able to use multiplication or division to solve the problem and write the appropriate equation to represent the situation.

Models, diagrams, and visual representations for instructing the multiplicative comparison structure.

Comparison problems are more difficult for students to represent than equal groups problems (Lannin, Chval, & Jones, 2013; Van de Walle, Karp, Lovin, & Bay-Williams, 2014). Understanding how something can be 5 times as much is at a higher cognitive level than understanding the ideas in an equal groups situation (Lannin, Chval, & Jones, 2013).

Using physical objects with the multiplicative comparison structure is an appropriate first step for this model with students (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). In this problem, George has $3, but Ling has 5 times more than George, a student would create a group that represents the size of the multiplicative unit, $3, then make copies of that unit in order to complete the 5 times the $3; see Figure 6. When the students are comfortable with the physical

George's amount (multiplicative unit)



Ling's amount (copies of the multiplicative unit)

Figure 6. Example of multiplicative units in a comparison structure

model, they can then represent their thinking with this model in pictorial form. The pictures would be shapes, dots, or x's to represent the discrete objects in the physical model.

A tape diagram requires more representational thinking by students. McCallum et al. (2011) show how this tool assists in comparative understanding. The students make a rectangle which is the size of the multiplicative unit identified inside or above it; a question mark or other variable can be used if it is the unknown in the problem. Then directly below or above the multiplicative unit, they draw another rectangle that is the length of the number of copies of that multiplicative unit, or a long rectangle with the total number, if the number of copies is the unknown; see Figure 7. Using this relational thinking they can solve for the unknown.

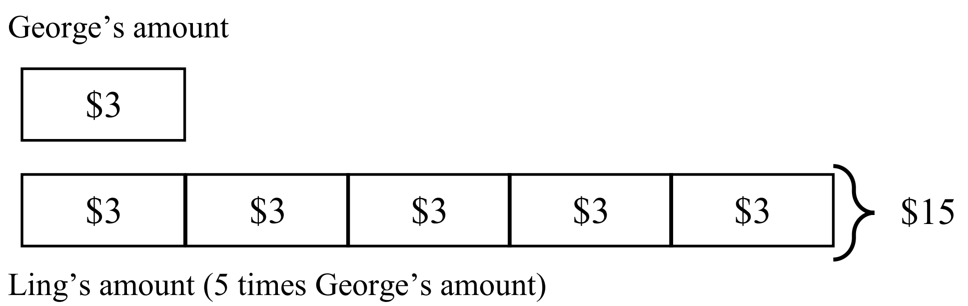


Figure 7. Example of tape diagram for comparison problems in multiplication.

Models, diagrams, and visual representations for instructing the array/area structure.

Frequently textbooks and supplemental resources use the array model after a short amount of instruction time on the equal groups structure. Aligning with the expectations of the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), the array model is being used more often as a connection to the area model (McCallum et al., 2011). Figure 8 presents the progression from equal groups to an array model to the area model.

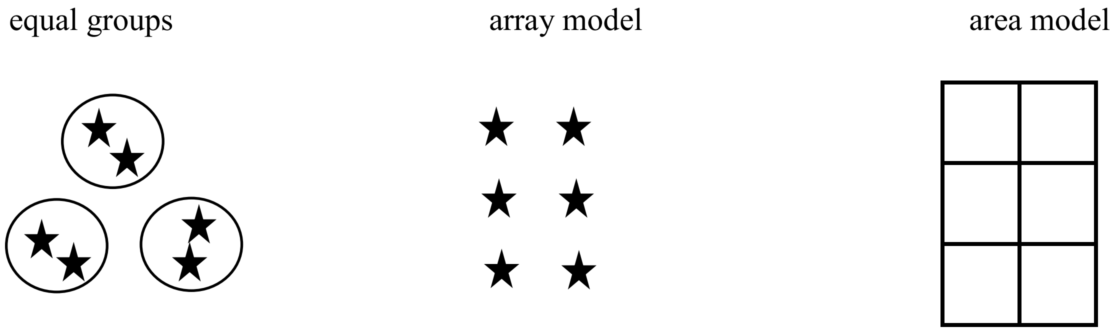


Figure 8. Example of the progression from equal groups to area model.

The form of the array and area models allows for a deeper understanding about the properties of operations than the models for equal groups (Barmby et al., 2009; Lannin, Chval, & Jones, 2013; McCallum et al., 2011; Van de Walle, Karp, Lovin, & Bay-Williams, 2014). Arrays and area structures support student understanding in moving from additive thinking to multiplicative thinking (Day & Hurrell, 2015). Jacob and Mulligan (2014) found that “[a]rrays provide a vehicle for teachers to focus students’ attention on the nature of the quantities involved, the associated language, the relationship between multiplication and division, and commutativity” (p. 39). Figure 9 contains a visual model that provides a picture to students showing how the commutative property works.

3 x 2 area representation – rotate the area model to show the commutative property (**2 x 3**)

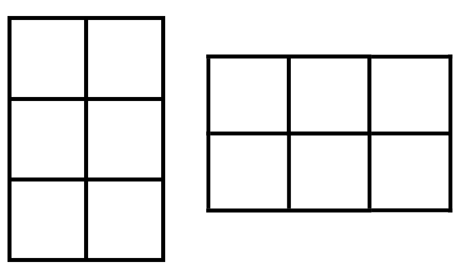


Figure 9. Example of commutative property using the area model.

An important concept about the area model that needs to be made explicit for students is that the product in the area model becomes a different unit from the unit given in the two factors

(Lannin, Chval, & Jones, 2013). In Figure 9 the area model on the left is made up of 3 units along the left side and 2 units along the top. When multiplied together the total area is 6 *square* units. The units have changed to a new unit.

Models, diagrams, and visual representations for instructing the combination/ Cartesian product structure.

Combination/Cartesian product structures "involve counting the number of possible pairings that can be made between two or more sets" (Van de Walle, Karp, Lovin, & Bay-Williams, 2014, p. 110). This structure is usually associated with probability which is not

A costume designer needs to know how many different outfits can be made from the set of pants and shirts he has available for the next play.

		Pants		
		black	brown	blue
Shirts	striped			
	plaid			
	solid			
	checked			

Figure 10. Example of a matrix for the combination structure.

expected until middle school grades (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), but students can be introduced to this concept as another form of multiplication in elementary school without expectation of mastery. A matrix is the most effective model for this structure. Students are familiar with the form since it is similar to the array/area model. The array or area model is constructed, but for the combination structure, each unit is labeled with the name of a specific object in a set (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). Figure 10 shows an example of the visual model for this structure. The array in the matrix represents how many different combinations can be made from the two sets. When students create an ordered pair in the intersecting box (for example; the student can write {striped shirt, brown pants} in the box where the striped shirt row and the brown pants column intersect), this begins to create the Cartesian product model of this structure.

The combinations structure can help students gain a better understanding of the partial-products algorithm, the traditional algorithm, and ultimately, in future courses, to an algebraic understanding of multiplying polynomials (Lannin, Chval, & Jones, 2013; Van de Walle, Karp, Lovin, & Bay-Williams, 2014). The structure allows the students to see that each term within a

partial-products method

34 x 127

	100	20	7
30	3000	600	210
4	400	80	28

3000 + 600 + 400 + 210 + 80 + 28 = 4,318

multiplying polynomials

$(2x + 4)(x^2 + 3x + 5)$

	x^2	$3x$	5
$2x$	$2x^3$	$6x^2$	$10x$
4	$4x^2$	$12x$	20

$2x^3 + 10x^2 + 22x + 20$

Figure 11. Example of matrices using the combinations structure to solve multi-digit multiplication and multiplying polynomials.

set needs to be multiplied by each term in the other set. The matrices in Figure 11 show these connections to the combinations structure of multiplication.

Teaching and Learning Multiplication Facts

Students need to know their multiplication facts. This statement does not seem to be under question. It is more about *how* will students learn their multiplication facts. Since Brownell and Chazal's (1935) and Brownell's (1944) research on learning facts, there have been ongoing debates about the most appropriate methods for fact instruction. Brownell and Chazal (1935) questioned the focus of basic fact instruction. They proposed a difference between "*how well* one performs, . . . and *how* one performs" (p. 19). As an example, these researchers went on to describe how two particular students performed on the same multiplication fact task. Both students were able to get a correct answer, in approximately the same amount of time, but one student used the strategy of counting and the other used a known fact to assist in finding the answer to the unknown fact. The second student used more sophisticated thinking strategies, but there was no distinction between the two on the task since it was about speed. Brownell and Chazal (1935) urged educators to focus on instruction that is based "on children's thought processes" (p. 20) and stated that "effective drill must be preceded by sound instruction" (p. 26).

Baroody's (2006) work identified three phases that children progress through as they master their basic facts. Phase two of his developmental progression is based on "reasoning strategies – using known information (e.g. known facts and relationships) to logically determine (deduce) the answer of an unknown combination" (p. 22). Kling and Bay-Williams (2015) expanded on his work noting, "Research tells us that students must deliberately progress through these phases, with explicit development on reasoning strategies, which helps students master the facts and gives them a way to regenerate a fact if they have forgotten it" (p. 551).

The research that is available for single-digit multiplication is much less than for single-digit addition (National Research Council, 2001), but there are some studies that have provided insight into how students learn and the methods that should be used to instruct multiplication facts. Lampert (1986) suggested that teachers need to question students so they can think about how the problems are solved. She advised that students "need to be treated like sense-makers rather than rememberers and forgetters" (Lampert, 1986, p. 340). "They need to learn to do computation competently and efficiently without losing sight of the meaning of what they are doing and its relation to solving real problems" (Lampert, 1986, p. 340). Many researchers have found that teaching for understanding and emphasizing thinking strategies is most productive for students in learning and retaining their mathematics facts (Chapin & Johnson, 2000; Heege, 1985; Kling & Bay-Williams, 2015; Sousa, 2015; Van de Walle, Karp, Lovin, & Bay-Williams, 2014; Wallace & Gurganus, 2005).

There is a progression for most learners in acquiring their multiplication facts (Anghileri, 1989; Baroody, 1985; Cooney, Swanson, & Ladd, 1988; Kouba, 1989; Lemaire & Siegler, 1995; Mulligan & Mitchelmore, 1997). In its simplest form, the progression generally starts with counting strategies (e.g., direct object counting and rhythmic counting), moves to various forms of repeated addition strategies (e.g., skip counting and strings of addition equations), and finally advances to multiplicative strategies (e.g., derived fact strategies and recalled facts). Kling and Bay-Williams (2015) recommend that educators build a conceptual foundation for multiplication and then work on the foundational facts (2s, 5s, 10s, multiplication squares, 1s, and 0s). After students show understanding and knowledge of these foundational facts, then instruction should move to using derived fact strategies with the 3s, 4s, 6s, and 9s. McCallum et al. (2011) shared a tri-level system for multiplication and division problems in their learning progression. Level 1 –

“making and counting all of the quantities involved in a multiplication” problem (p. 25). Level 2 – “repeated counting on by a given number, such as 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30” (p. 25). Level 3 – “use the associative property or the distributive property to compose and decompose” (p. 26). Since their work is so closely tied to the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), thought and attention is being given to their level system by teachers in the classroom.

Multiplication Fact Strategies

Specific fact strategies can help teachers focus their basic fact instruction on thinking and understanding (Flowers & Rubenstein, 2010/2011; Kling & Bay-Williams, 2015; Van de Walle, Karp, Lovin, & Bay-Williams, 2014). These strategies are often grouped into three categories based on the underlying math concepts being used in that group of strategies. The categories are: using counting all, using counting on or repeated addition, and using the properties of operations.

Strategies that use counting all.

Counting all strategies use concrete objects or pictorial representations of objects which are placed into equal groups based on the structure of the problem; e.g., 3 x 4 is solved using 3 groups of 4 objects or drawings and then each individual object or picture is counted (Anghileri, 1989; Mulligan & Mitchelmore, 1997, Sherin & Fuson, 2005; Van de Walle, Karp, Lovin, & Bay-Williams, 2014). Students using these strategies are beginning to make meaning of the multiplicative structure but have not internalized the structure enough to move beyond the counting all strategies.

Mulligan and Mitchelmore (1997) and Sherin and Fuson (2005) shared how students continue to utilize counting all strategies by direct counting or rhythmic counting. When using direct counting, students place the objects into equal groups, or draw dots into equal groups, following the structure of the problem and then count all objects or dots. Rhythmic counting follows the structure of the problem by emphasizing the last number counted within each equal group (Anghileri, 1989; Mulligan & Mitchelmore, 1997; Sherin & Fuson, 2005). A student using rhythmic counting when solving 3 x 4 would emphasize the fourth number in each of the three sets while counting out loud, e.g., 1, 2, 3, **4**, 5, 6, 7, **8**, 9, 10, 11, **12**. Rhythmic counting requires the students to keep track of the number of groups while counting all objects or pictures within each of the groups.

Strategies that use counting on or repeated addition.

Skip counting, repeated addition, and "collapse groups and add" (Sherin & Fuson, 2005, p. 358) are strategies used by students as they begin to implement multiplicative understanding in knowing the number of groups and the size of the groups (Anghileri, 1989; Barmby, Harries, Higgins, & Suggate, 2009; Clark & Kamii, 1996; Lannin, Chval, & Jones, 2013; Lemaire & Siegler, 1995; Mulligan & Mitchelmore, 1997; Sherin & Fuson, 2005). To implement these strategies students choose one of the factors in the problem as the group size and then count that quantity a determined number of times given by the other factor; e.g., 3 x 4 could be solved by skip counting by 3s or 4s, by using repeated addition with 3s (3 + 3 + 3 + 3) or 4s (4 + 4 + 4) or by adding groups (usually tally marks) into a resulting sum. Students are required to keep track of the size of the group and the subsequent subtotals when using these strategies. Sherin and Fuson (2005) called this set of strategies additive calculation and noted that students were beginning to learn patterns that were "particularly important in multiplication" (p. 354).

The Doubles strategy is another strategy in this group that is considered foundational to learning many other strategies in multiplication (Cook & Dossey, 1982; Flowers & Rubenstein, 2010/2011; Heege, 1985; Kling & Bay-Williams, 2015; Van de Walle, Karp, Lovin, & Bay-

Williams, 2014). This strategy links directly to students' understanding of addition and serves as a springboard for other strategies. The Doubles strategy requires one of the factors to be a two, and builds on the students' understanding from knowing their doubles in addition as well as their understanding of repeated addition. Making the connection to the multiplicative unit (size of the group) and the number of those units (number of groups) supports students in moving from additive thinking to multiplicative thinking; see Figure 12.

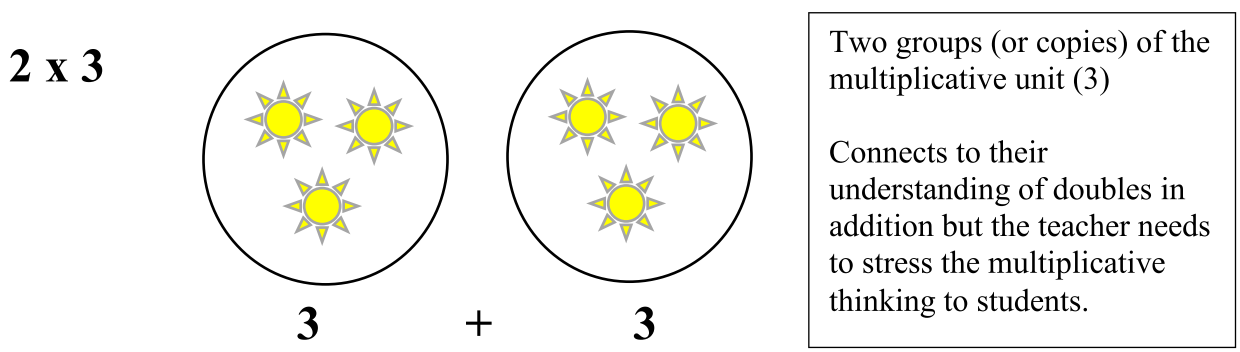


Figure 12. Example of repeated addition connecting to multiplicative thinking.

Derived fact strategies.

Once students have the foundational fact strategies (Doubles [x 2], Zeros, Ones, Fives, and Tens), instruction moves to the derived fact strategies. Derived fact strategies use the properties of operations along with the foundational fact strategies to solve multiplication problems. An example of a derived fact strategy is the Double and Double Again strategy (x 4) (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). This strategy, illustrated in Figure 13, requires the user to understand decomposition of numbers in order to obtain multiplication facts that are already known by the user. The user who understands the foundational Doubles strategy can decompose the four into two plus two and use the distributive property to create two doubles facts which can then be added together to give the final product for the original x 4 problem.

$$3 \times 4$$

$$3 \times (2 + 2)$$ decomposition of 4

$$(3 \times 2) + (3 \times 2)$$ distributive property

$$6 + 6 = 12$$

Figure 13. Example to show how the distributive property is used for the Double and Double Again strategy.

Double and One More Set is a strategy used to solve problems when one of the factors is a 3 (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). Again students are required to understand how numbers can be decomposed to create facts that are already known. The factor of 3 is decomposed into two plus one so the students can use the Identity Property of One and the Doubles fact to solve the original problem. See Figure 14 for an illustrated example of this strategy.

$$6 \times 3$$

$$6 \times (2 + 1)$$ decomposition of 3

$$(6 \times 2) + (6 \times 1)$$ distributive property

$$12 + 6 = 18$$

Figure 14. Example to show the Double and One More set strategy.

Another derived fact strategy is the Half then Double strategy (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). This strategy, shown in Figure 15, builds on the understanding of the multiplication structure of arrays/area. Usually facts that have a factor of 6 or 8 are used with this strategy, but any fact that has one factor that is even can be easily solved using this strategy. The even factor is decomposed into two equal parts. One of those parts is doubled to produce the final product. The array or area model can be manipulated so students understand how this

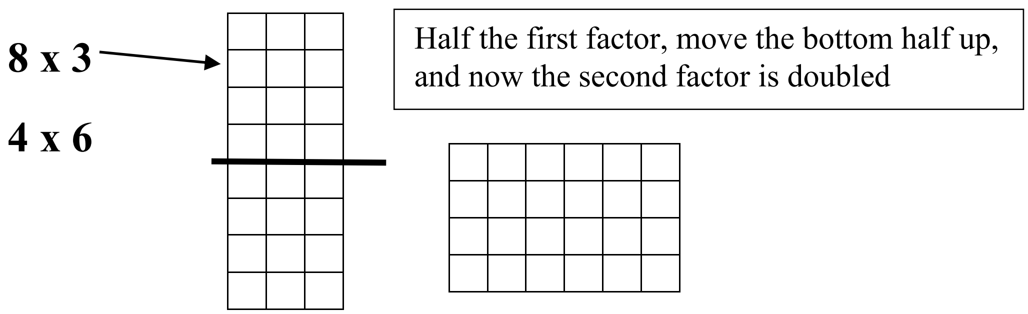


Figure 15. The students use the array/area structure to decompose and recompose the factors of the multiplication fact.

strategy works. For the multiplication fact shown in Figure 15, 8 x 3, the strategy directs the students to halve the 8 and double the 3. This creates an equivalent fact, 4 x 6. To model this pictorially, the students would begin with an array of 8 by 3. The students then cut the side with 8 squares in half (halving the factor of 8). The two halves are then joined so that the other factor is seen as doubled. Once this strategy is mastered it can be used for numbers larger than just the basic multiplication facts.

When students are familiar with the various strategies then those strategies can be combined to solve one multiplication problem. For example, 7 x 8 can be decomposed into an equivalent representation using two foundational fact strategies (Fives and Doubles) for easier mental computation. The strategy is shown in Figure 16.

$$7 \times 8$$

$$(5 + 2) \times 8$$ decompose the 7

$$(5 \times 8) + (2 \times 8)$$ apply the distributive property

$$40 + 16 = \mathbf{56}$$

Figure 16. Decomposition and the distributive property being used to solve a basic fact.

The method in Figure 16, using decomposition and the distributive property, can be applied with larger numbers and even when multiplying polynomials; see Figures 17 and 18.

$$24 \times 32$$

$$(20 + 4) \times (30 + 2)$$

$$(20 \times 30) + (20 \times 2) + (4 \times 30) + (4 \times 2)$$

$$600 + 40 + 120 + 8 = \mathbf{768}$$

Figure 17. Decomposition and distributive property with larger numbers.

$$(x + 4)(x + 2)$$

$$(x \cdot x) + (x \cdot 2) + (4 \cdot x) + (4 \cdot 2)$$

$$X^2 + 2x + 4x + 8$$

$$\mathbf{X^2 + 6x + 8}$$

Figure 18. Distributive property to multiply two binominals.

Concerns About Fact Strategy Instruction

Some educators believe fact strategy instruction takes too long to teach and students just need to memorize the facts. Baroody (1985, 2006) and Baroody, Bajwa and Eiland (2009) shared that some teachers and researchers consider strategies to be a hindrance when memorizing number facts. These opponents of strategy instruction believe students take too long in thinking about the relationships between the numbers and become especially difficult when the numbers become larger. Today some teachers and parents believe the best way to learn number facts is through drill instruction because the use of counting and reasoning strategies is "immature" (Baroody, Bajwa, & Eiland, 2009, p. 70).

Other researchers are concerned about students who have difficulty in learning mathematics. Dowker (2009) found that students with mathematical difficulties and who ranked at the lower section of Addition Performance Levels struggled to use fact strategies. She indicated that this could be due to their reliance on counting strategies and their difficulty in understanding counting concepts which is essential in order to move on to strategies that focus more on relationships. She also discovered that struggling students who were at the higher sections of Addition Performance Levels "used just as many derived fact strategies as children without such difficulties" (p. 407). Dowker (2009) suggested that working memory may play a part in struggling students' infrequent use of strategies. Strategy use requires students to have several known facts in their memory while working out the strategy needed to complete the process.

Support for Fact Strategy Instruction

Research that focuses on cognitive science addresses some of the concerns educators have about teaching fact strategies. Sousa (2015) presented an example that demonstrates how rote memorization of facts without meaning and understanding is taxing to the brain. He explained that the brain is a strong pattern seeker and seeks to create associations. Associative memory is usually a strength, but rote memorization of basic facts becomes difficult since we need "various pieces of information [to not interfere] with one another" (p. 39). Sousa's (2015) example was to try to memorize "three names and addresses:

- Carl Dennis lives on Allen Brian Avenue
- Carl Gary lives on Brian Allen Avenue
- Gary Edward lives on Carl Edward Avenue" (p. 40).

These would certainly be difficult to memorize since they have similar names and phrasings which could easily be confused and jumbled. Sousa continues,

> Let the names . . . represent the digits . . . and replace the phrase "lives on" with the equal sign. That yields three multiplications:

- 3 x 4 = 12
- 3 x 7 = 21
- 7 x 5 = 35

> From this perspective, we can now understand why the multiplication tables present such difficulty when children first encounter them. Patterns interfere with one another and cause problems. (Sousa, 2015, p. 40)

Memorizing without connections to the relationships between numbers relies on too many discrete pieces of information within the brain. Ideas need to be connected to other ideas to form relationships which will lead to increased understanding (Van de Walle, Karp, Lovin, & Bay-Williams, 2014).

Other cognitive researchers, such as Jost, Beinhogg, Hennighausen, and Rosler (2004) and LeFevre, Bisanz, Daley, Buffone, Greenham, and Sadesky (1996), found evidence from their research that fact strategies are used in conjunction with retrieval when solving single-digit multiplication problems. Multiple methods of solving multiplication facts are necessary and "[f]uture models of arithmetic need to include multiple routes to solution of even the simplest problems" (LeFevre et al., 1996, p. 303).

Mathematics education researchers from the 1980s (Cook & Dossey, 1982; Heege, 1985; Steinberg, 1985; Thornton, 1978; Thornton & Smith, 1988) found fact strategy instruction supported students in number fact understanding and in becoming more fluent in recalling basic number facts. Cook and Dossey (1982) and Thornton (1978) theorized that teaching strategies before drill work would improve student retention of basic facts. Heege (1985) proposed that instruction utilizing fact strategies would lead to more flexibility in student thinking and lead to a greater potential for application of these strategies with multiplication of larger numbers.

As research continued into the 2000s more findings continued to support fact strategy instruction (Baroody, 2006; Baroody, Bajwa & Eiland, 2009; Boaler, Williams, & Confer, 2015; Flowers & Rubenstein, 2010/2011; Gersten & Chard, 1999; Imbo & Vandierendonck, 2008; Kling & Bay-Williams, 2015; Sherin & Fuson, 2005; Van de Walle, Karp, Lovin, & Bay-Williams, 2014; Woodward, 2006). The connection between number sense and fact strategy instruction was identified as one of the most beneficial aspects of this type of instruction (Baroody, 2006; Baroody, Bajwa & Eiland, 2009; Boaler, Williams, & Confer, 2015; Gersten & Chard, 1999; Kling & Bay-Williams, 2015; Sherin & Fuson, 2005; Van de Walle, Karp, Lovin, & Bay-Williams, 2014). These researchers discovered that fact strategies (also called reasoning strategies or thinking strategies) created foundations in seeing patterns and relationships that will lead to essential understandings in later mathematics concepts, such as solving multi-digit problems using the properties of operations and multiplying polynomials. Baroody (2006) stated that this strategic thinking will help students with not only computational fluency but in "conceptual understanding, strategic mathematical thinking, and a productive disposition" in mathematics (p. 30).

Discourse in the Mathematics Classroom

The educational dialogue concerning recitation versus discussion methods has been ongoing for several years (Chapin & O'Connor, 2007; Chapin, O'Connor, & Anderson, 2013; Chazan & Ball, 1999; Clark et al., 2012; Cobb, Yackel, & Wood, 1992; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Forman, McCormick, & Donato, 1998; Franke & Kazemi, 2001; Franke, Webb, Chan, Ing, Fruend, & Battey, 2009; Hattie & Yates, 2014; Hufferd-Ackles, Fuson, & Sherin, 2004; Kazemi & Hintz, 2014; Khisty & Chval, 2002; Lampert, 1990; Nathan & Knuth, 2003; NCTM, 2000, 2014; Parrish, 2010; Smith, 1996; Smith & Stein, 2011; Stein, Engle, Smith, & Hughes, 2008; Van de Walle, 2001). Hattie and Yates (2014) investigated the persistence of the recitation method in spite of strong evidence to support well-structured student talk. They found that recitation produced mainly low-level questions, low student engagement, and sterile, uninspiring tasks. In contrast, discussions, if purposefully structured, can produce powerful effects. NCTM (2014) advocated for mathematical discussions with students and stated that "[e]ffective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (p. 29).

NCTM (2000, 2011, 2014) continued to advance the use of classroom discourse through their work and publications. In 1991 they promoted an increase in classroom discourse with the publication of *Professional Standards for Teaching Mathematics* which addressed teachers' and students' roles in discourse and tools to enhance that discourse. Their next major publication, *Principles and Standards for School Mathematics* (2000), explained The Teaching Principle and how student discourse needs to be nurtured to create an environment that is conducive to learning mathematics which is relevant and reachable for all.

This idea of student discourse hinges on the instructor's ability to "listen to students' thinking and ask probing questions to help students find their own ways through the problems and honor their struggles" (Clark, Cochran, Dominick, Fulmore, Mayer, Parrish, Lofgren, Parker, Mullins, & Spieler, 2012, para. 12). Once a safe learning environment is established then students come to "care about each other's learning. They learn that in trying to understand the thinking of others, they understand the mathematics at a deeper level themselves" (Clark et al., 2012, para. 18). Much of this thinking is built on Vygotsky's work of students' learning from social interactions with their peers and the need to give voice to their thinking (Vygotsky, 1978/1997). Vygotsky (1978/1997) found that a child learned when "interacting with people in his environment and in cooperation with his peers" (p. 90).

Recommendations for Implementing Classroom Discourse

In order to effectively implement classroom discourse certain parameters and conditions are necessary. Current researchers (Chapin & O'Connor, 2007; Chapin, O'Connor, & Anderson, 2013; Kazemi & Hintz, 2014; Khisty & Chval, 2002; Parrish, 2011) have written about the necessity of creating a community that encourages and fosters the sharing of ideas and strategies. Educators must communicate to the students that their ideas are valued and are worth discussing. When ideas are readily shared and discussed then students can engage in conversations that assist them in developing concepts and skills in mathematics.

Even the sharing of wrong answers during classroom discourse is helpful in understanding mathematics and uncovering misconceptions and misunderstandings (Chapin & O'Connor, 2007; Chapin, O'Connor, & Anderson, 2013; Kazemi & Hintz, 2014; Khisty & Chval, 2002; Nathan & Knuth, 2003; Parrish, 2011). Parrish (2011) explained that "[r]ich mathematical discussions cannot occur if this expectation is not in place. We must remember that wrong answers are often rooted in misconceptions, and unless these ideas are allowed to be brought to the forefront, we cannot help students confront their thinking. Students who are in a safe learning environment are willing to risk sharing an incorrect answer with their peers to grow mathematically" (p. 202). Ball (1993) referred to the classroom environment in her research from the 1990s, when she wrote that teachers need to explore "ways to construct classroom discourse such that the students learn to rely on themselves and on mathematical argument for making mathematical sense" (p. 388).

Research from Chapin and O'Connor (2007) and Chapin, O'Connor, and Anderson (2013) found there are certain conditions that must be present in the classroom to support academically productive talk. "Foremost is the requirement that teachers establish conditions for *respectful* discourse. Discourse is respectful when each person's ideas are taken seriously and no one is ridiculed or insulted" and "when no one is ignored" (Chapin & O'Connor, 2007, p. 124). These researchers outlined steps teachers can take to begin productive talk in their classroom which incorporated *talk moves*. Talk moves are "strategic ways of asking questions and inviting participation in classroom conversations" (Chapin, O'Connor, & Anderson, 2013, p. 11). Some examples of talk moves are described in the following.

- Turn and Talk – The teacher can use this talk move when students are not responding to a question. Students are directed to talk to a shoulder partner about the question that was asked. This allows the students the opportunity to discuss their initial thoughts and feel more confident in providing an answer to the whole group.

- Revoicing – The teacher uses this talk move when the response is unclear. The teacher "tries to repeat some or all of what the student has said, and then *asks the student to verify* whether or not the teacher's revoicing is correct" (Chapin, O'Connor, & Anderson, 2013, p. 18). The important differentiation between revoicing and repeating is that the teacher is asking the student what was meant for clarification not to just restate what the student said without verification.
- Say More – This talk move is used when the student is not elaborating enough on an idea in order to be understood by all listeners. Often the teacher can just ask the students to tell more about what they were thinking or to provide an example of what they were talking about to the class. This "sends the message that the teacher wants to understand the student's thinking" (Chapin, O'Connor, & Anderson, 2013, p. 17) and gives the teacher more insight into the mathematical reasoning used.

Implementing these talk moves in a safe learning environment allows students to be able to access mathematics that is sometimes difficult to reach (Fiori, Boaler, Cleare, DiBreienza, & Sengupta, 2004).

At the root of all classroom discussions there must be a clear mathematical goal (Chapin, O'Connor, & Anderson, 2013; Kazemi & Hintz, 2014; Smith & Stein, 2011; Van de Walle, Lovin, Karp, & Bay-Williams, 2014). Engaging students in tasks that stretch their minds and encourages different ideas will deepen their reasoning skills. Different tasks require different discussions with students based on the goals of the mathematics (Kazemi & Hintz, 2014; Smith & Stein, 2011). Some discussions need to be open to gather as many different strategies as possible to broaden students' thinking and reasoning strategies. Other discussions will be more focused so the students can deepen their thinking about a particular method or so a misconception can be clarified. Keeping the mathematical goal at the forefront of the discussions is essential.

Roles of Teachers and Students During Classroom Discourse

Traditional teaching is generally teacher-centered so the teacher is viewed as the ultimate authority for all learning in the classroom. To fully implement effective classroom discourse the teacher must focus on purposeful questioning and on creating a student-centered environment (Chapin & O'Connor, 2007; Chapin, O'Connor, & Anderson, 2013; Cobb, Yakel, & Wood, 1992; Franke et al., 2009; Khisty & Chval, 2002; Nathan & Knuth, 2003; Smith, 1996). This does not mean that the teacher takes a backseat to the interactions in the classroom. The teacher plays a critical part in participating in order to solidify ideas and connections, but not eliminating the learning from the students by providing too much information.

As teachers question students about the mathematics during instruction, care must be taken that the questions are connected with students' ideas and thoughts that are focused on critical mathematics content (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Chapin & O'Connor, 2007; Chazan & Ball, 1999; Franke et al., 2009; Kazemi & Hintz, 2014; Lampert, 1990; Nathan & Knuth, 2003; Smith & Stein, 2011). Franke et al. (2009) found that "teachers' questions can position the student thinking in relation to the mathematics in ways that support student understanding" (p. 391). Increasing student discourse without connecting the talk to essential mathematics does not impact students' understanding. Discussion without connections is just sharing and sharing is not learning (Chazan & Ball, 1999).

Effective student discourse depends on the teacher facilitating the conversations and students listening to each other to make sense of mathematics. Kilpatrick, Swafford, and Findell (2001) made it clear that it is the teacher's responsibility "for moving the mathematics along while affording student opportunities to offer solutions, make claims, answer questions, and provide explanations to their colleagues. The point of classroom discourse is to develop students' understanding of key ideas," so "discourse needs to be planned with these goals in mind, not merely as a 'checking for understanding' form of recitation" (pp. 345-346). One of the most important ideas about student discourse is that educators must engage students in discussions that move beyond superficial implementation of the standards (Kazemi & Stipek, 2001; NCTM, 2014). Teachers must be willing to "engage students in genuine mathematical inquiry and push them beyond what might come easily for them" (Kazemi & Stipek, 2001, p. 60). This is not just any kind of student talk. "[S]tudents must be precise and explicit in their talk, especially providing enough detail and making referents clear so that the teacher and classmates can understand their ideas" (Franke, Webb, Chan, Ing, Freund, & Battey, 2009, p. 381).

Classroom discussions will reveal disagreements, misconceptions, and discrepant viewpoints; these are "inevitable – and moreover, essential to students' learning" (Chazan & Ball, 1999, p. 7). It is critical that the discussions move beyond mere sharing of ideas. Teachers must monitor and manage classroom discussions to maximize student benefit from interactions with their peers (Chazan & Ball, 1999). Through thoughtful guidance from a skilled educator, students can "actively construct their mathematical understandings as they participate in classroom social processes" (Cobb, Boufi, McClain, & Whitenack, 1997, p. 264).

Benefits of Classroom Discourse

There is evidence that student discourse works for all levels of students if the questioning is directed towards the mathematics in the problems. In a case study conducted by Hufferd-Ackles, Fuson, and Sherin (2004), the authors described an essential element as moving from a "focus on *questioning to find answers* to a focus on *questioning to uncover the mathematical thinking behind the answers*" (p. 92). Thus, students see themselves as contributors to their own learning of mathematics instead of empty receptacles being filled by someone else (Carpenter et al., 1999; Fennema et al., 1996; Reinhart, 2000).

Student attitudes are positively impacted by intentional and purposeful classroom discussions. Chapin, O'Connor, and Anderson (2013) and Kazemi and Hintz (2014) pointed out that when students feel that they are involved in a community of learners then their attitude about mathematics is positively enhanced. In an *Education Week* blog, high school freshman, Kyna Airriess (2016), reflected on her experiences in mathematics classes. She described how she decided in third grade that math was not for her. She and her fellow classmates were constantly being timed on multiplication facts and she began to feel inadequate. She wrote off being a mathematician until her freshman year in high school. She took a course with a math teacher who encouraged the students to focus on what they know and to talk about it. The students were encouraged to lead discussions and share their thoughts without judgement and for the purpose of learning from one another. Kyna now sees herself as a mathematician. She shared that

> anyone can do math at high levels, and your math brain depends on your willingness to exercise it. Your brain grows the more you use it. Now more than ever I believe I am living proof that this is true, and that anyone who wants to do math can find joy in it. (Airriess, 2016, para. 8)

Teachers that effectively implement classroom discourse actively promote student authority and student accountability (Stein, Engle, Smith, & Hughes, 2008). When students are authors of their own ideas and are recognized as being mathematicians, the positive attitudes about their abilities in mathematics increase. As students are encouraged to be accountable to the mathematics within their ideas, they learn that their ideas are valued but they must be backed up by evidence and supported through foundational mathematics. This benefits the teacher in encouraging the students to take ownership of their ideas and to explore the basis of their reasoning to determine if that reasoning is accurate or flawed.

A research project, called Project Challenge, was implemented from 1998 to 2002 and was the basis for the book *Classroom Discussions in Math* (Chapin, O'Connor, & Anderson, 2013). The researchers for this project "hoped that by combining a solid curriculum, instruction based on mathematical understanding, and a heavy emphasis on talk and communication about mathematics, [they] would be able to help . . . students become robust learners of mathematics" (Chapin, O'Connor, & Anderson, 2013, p. 319). Chapin, O'Connor and Anderson (2013) shared their results from the Test of Mathematical Abilities, Second Edition (TOMA 2). At the beginning of the project

> [o]nly 4 percent [of the students in the project] were rated as "Superior" or "Very Superior" (which indicates a "high probability of giftedness in mathematics") and only 23 percent were rated as "Above Average." The remaining 73 percent were rated as "Average" or "Below Average" in their mathematical abilities. (Chapin, O'Connor, & Anderson, 2013, p. 321)

Two years later the researchers administered the TOMA 2 again and found that

> 41 percent were now “Superior” or “Very Superior,” and 36 percent more were “Above Average.” Only 23 percent of [their] first cohort were classified as having “Average” ability in mathematics after two years in the program, and none were “Below Average.” (Chapin, O’Connor, & Anderson, 2013, p. 321)

Chapin, O’Connor and Anderson (2013) also examined student achievement data from the California Achievement Test (CAT) to identify where the research participants scored in relation to a national sample. After two years in the project, students “scored better, on average, than 91 percent of a national sample” (p. 322) using the results from the CAT. When the authors examined Massachusetts state assessment data on students who had participated in the project for three years, they found “90 percent of the one hundred sixth graders in [the] first cohort placed in the top two categories . . ., a rate greater than that of nearby affluent suburbs” (p. 322).

In addition, Marzano, Pickering and Pollock (2001) conducted meta-analyses of research studies to find strategies that would improve student achievement. They reported that cues and questions by classroom teachers to engage students in conversation were one of their top nine strategies. These researchers found nine studies dating from 1976 to 1993 showing that cues and/or questions gave anywhere from a 10 to 39 percentile gain in student achievement.

Walshaw and Anthony (2008) conducted a review of recent research concerning classroom discourse in mathematics classrooms. They described how they deepened their understanding of mathematics discourse by examining the various articles and studies. Some of their positive conclusions concerning discourse were:

- “Facilitating respectful and patterned interaction in the classroom contributes to the enhancement of students’ aspirations” (p. 542).

- "Teachers who set up conditions that are conducive to classroom discussion come to understand their students better" (p. 542).
- Students enrich their knowledge of mathematics through "listening respectfully to other students' ideas" and "arguing and defending their own positions" (p. 542).

Discourse that engages teachers and students in mathematical thinking and sense-making is important when thinking about innovation and reform in mathematics instruction.

Concerns About Classroom Discourse

Student discourse as a strategy in education has its critics. Some researchers have indicated that teacher efficacy could be a potential obstruction to the use of student discourse within the classroom (Chazan & Ball, 1999; Hufferd-Ackles, Fuson, & Sherin, 2004; Smith, 1996; Stein, Engle, Smith, & Hughes, 2008). If student talk reveals mathematics that is unfamiliar to the teacher then that teacher believes his/her authority will be undermined or his/her level of competency will be questioned.

One of the most common roadblocks to increasing effective classroom discourse is the fact that most practicing teachers were not taught with this method nor were they trained this way within their colleges and universities (Clark et al., 2012; Franke et al., 2009; Kazemi & Stipek, 2001; Lampert, 1990; Nathan & Knuth, 2003). Without having the theoretical background or the practical knowledge gathered from using classroom discourse, teachers will abandon this practice since the benefits are not immediately seen. To leave the "safe practice" of being the provider of all mathematical information by following exactly what is in the textbook is difficult and takes an educator who is very efficacious and reflective.

Also many teachers worry that instruction including student discourse will not be seen as *real* teaching (Chapin, O'Connor & Anderson, 2013; Kazemi & Hintz, 2014). In the past several years there has been an emphasis on teacher evaluations so many teachers are concerned about going outside of the norm to include classroom discussions. Many administrators are worried about producing positive test scores and most often their background in mathematics instruction did not include much discourse, so the administrator may be uneasy when observing lessons that include this type of instruction. In Smith and Stein's (2011) work they described a small group of teachers' journey in utilizing classroom discourse and how important it was to include the principal early in the process so there were no surprises. The principal in this setting realized the students were learning at a deeper level than he had originally thought after being included in the process with his teachers.

Using Student Discourse to Support Multiplication Fact Strategy Instruction

Marilyn Burns' publishing company, Math Solutions, was one of the first companies to publish a resource for teachers that included a DVD showing teachers how student discourse can be used when instructing with specific number and fact strategies. This resource, titled *Number Talks* by Sherry Parrish (2010), explains how strategies, such as *Making Tens*, *Doubles*, and *Near Doubles*, can be taught incorporating student talk that emphasizes conceptual understanding. Teachers are to probe for student thinking by asking strategic questions based on the mathematics within the problem. In this particular resource, students are presented with a problem, asked to solve that problem, and finally to think about how a strategy was used to solve the problem. Students are expected to explain their thinking and reasoning to a partner and potentially to the entire class. This usually leads to deep discussions and sometimes even a break-through in understanding, but only if the teacher is very purposeful and intentional in the types of questions being asked of the students. Parrish explained that "[c]lassroom conversations and discussions around purposefully crafted computation problems are at the very core of number talks. These are opportunities for the class to come together to share their mathematical thinking. The problems in a number talk are designed to elicit specific strategies that focus on number relationships and number theory" (p. 5).

Additional advocates for this type of instruction, Boaler, Williams and Confer (2015), supported the number talk discussion technique on the website, *youcubed*. They explained, "One of the best methods for teaching number sense and math facts at the same time is a teaching strategy called 'number talks', . . ." (Boaler, Williams, & Confer, 2015, p. 6). O'Connell and SanGiovanni (2011) also described using student discourse and fact strategies; they explained that giving students more opportunities to engage in meaningful discussions builds their efficacy in fact fluency.

Woodward (2006) completed a study that compared two types of multiplication fact instruction; one type used timed practice drills only and the other type, called the integrated approach, used fact strategy instruction which included student discourse and timed practice drills. He specifically looked at the difference between identified learning disabled (LD) students and non-LD students in their attainment and retention of facts. His conclusions showed that both groups improved in solving basic multiplication facts with both types of instruction, but there was a significant difference in using the integrated fact instruction instead of just the timed practice drills for both groups of students when solving more complex problems and developing number sense. "[S]tudents had the opportunity to see and discuss connections between basic facts, extended facts, the partial algorithm, and methods for approximating answers to multiplication problems" (pp. 285-286). The use of student discourse was a secondary support for fact strategy instruction but Woodward indicated that the discussion was important to see the connections of the mathematics that underlie the fact strategies.

CHAPTER 3

METHODS

Introduction

The purpose of this study was to investigate the use of student discourse as part of third grade teachers' fact strategy instruction and students' use of fact strategies while learning basic multiplication facts. The study was designed to answer the following research questions:

- What insights do teachers have about the multiplication fact strategies used by their students and the use of student discourse while teaching multiplication fact strategies?
- How did the frequency of different categories of questions used by teachers change during the semester's instruction on fact strategies?
- How did the number of students at higher levels of multiplication fact strategy implementation change as a result of instruction that included student discourse?

Research Framework

This research is situated within the pragmatic paradigm. This paradigm is formed by the research problem and the results needed to fully investigate that particular research problem (Chen, 1997; Greene & Caracelli, 1997; Marshall & Rossman, 2016; Mertens, 2015). It flows logically from the research questions to the end results (Marshall & Rossman, 2016) and its framework emphasizes the "importance of commonsense and practical thinking" (Mertens, 2015, p. 35). The objective of the pragmatic paradigm is to study what is of value to the researcher and to determine what can bring about positive changes by making the most of the range and amount of information gathered and analyzed to reinforce the end results (Greene & Caracelli, 1997; Lukenchuk & Kolich, 2013; Marshall & Rossman, 2016; Patton, 2002).

According to Greene and Caracelli (1997), researching social scientific problems is complex and "what will work best is often a combination of different methods" (p. 8). The various methods within a pragmatic paradigm offer practical solutions "to the tensions created in the research community concerning the use of quantitative or qualitative methods" (Mertens, 2015, p. 38). When uncovering what is practical and useful in educational settings, the pragmatic paradigm is often the best paradigm to frame the research.

Within the pragmatic paradigm, a mixed-methods design can be utilized so both qualitative and quantitative data are applied in order to effectively answer research questions. The data are used to tell a more complete story since it can be compared and contrasted between the various types of methods and data analyses (Greene & Caracelli, 1997; Mertens, 2015). A better understanding of many types of social and educational phenomenon can be reached through a pragmatic mixed-methods design. Greene and Caracelli (1997) explained that various combinations of methods are best to analyze complex social phenomenon. The combinations can resolve inconsistencies in data and resolve some issues related to bias that can occur if researchers use only one method for gathering information (Chen, 1997; Greene & Caracelli, 1997).

The mixed-methods design establishes a foundation based on triangulation of data. This triangulation strengthens the evaluation of a study and can assist in establishing validity (Caracelli & Greene, 1997; Chen, 1997; Marshall & Rossman, 2016; Mertens, 2015; Patton, 2002). It tests for consistency in results and involves looking for a "deeper insight into the relationship between inquiry approach and the phenomenon under study" (Patton, 2002, p. 248). The multiple methods within a mixed-methods design enable the researcher to gather several different types of data for one investigation to create this triangulation of data (Yin, 2003).

According to Stake (1995) "we have ethical obligations to minimize misrepresentation and misunderstanding" (p. 108) in inquiries which is assisted by having multiple methods of collecting and analyzing data.

When mixing methods, there are limitations that must be acknowledged (Marshall & Rossman, 2016; Mertens, 2015). The very structure of the mixed-methods design can be a weakness if the researcher is not careful in the construction (Chen, 1997; Marshall & Rossman, 2016; Mertens, 2015). "[M]ixed-methods evaluation needs to meet standards of rigor for both quantitative and qualitative methods" (Chen, 1997, p. 70). This potential for compromised rigor can "undermine the soundness and strength of their inferences" (Chen, 1997, p. 70) if not given careful thought as to the needs and requirements of both types.

The research design used in this dissertation study is classified as descriptive, nonexperimental research and utilizes both quantitative and qualitative data to bring a more complete picture of how student discourse supports fact strategy instruction. Descriptive, non-experimental research provides a description or report of the characteristics of a current or past situation (Gall, Gall, & Borg, 2005; Johnson & Christensen, 2000; McMillan, 2004; Punch, 1998). Since this type of research does not include random assignment nor the manipulation of an independent variable, the "researcher must study the world as it naturally occurs" (Johnson & Christensen, 2000, p. 282). Johnson (2001) explained that "nonexperimental research is frequently an important and appropriate mode of research in education" (p. 3). Slavin (2002) and Johnson (2001) considered descriptive research to be of great value in the education setting since experiments are difficult to accomplish in a live classroom. Researchers, Cook and Cook (2008), believed that because of the large amount of time and resources needed to conduct an experimental study, descriptive and correlational studies should be completed first so groundwork can be laid to determine the necessity for further research.

Setting of the Study

The study took place in a diverse, Midwestern, urban school district. At the time of the study, there were approximately 51,000 students within this district categorized in the following ethnic groups: Caucasian – 34%, Hispanic – 34%, African-American – 19%, Multi-Racial – 8%, Asian – 4%, and Native American – 1%. About 75% of all students came from homes of poverty and 13% of the total student population received special education services.

One elementary school was selected based on level of commitment from the administration and level of instructional support that was available within the building. The third grade team of teachers was asked to participate in this study. This team consisted of four teachers described in further detail in the section below, along with more information about their school and students.

Participants

Third grade was chosen as the targeted grade level since the curriculum indicated that students would begin working on multiplication facts at the beginning of the school year. Also, third grade students are generally mature enough to be able to articulate responses to teacher questions and explain their thinking which was required during the interview portion of data collection. This was based on Piaget's (1971) research where he stated that "genuine argument and collaboration in abstract thought constitute a stage of development which only intervenes after the age of 7" (p. 73) and that "before the age of 7 or 8 children have no conversation bearing upon logical or causal relations" (p. 75). According to the *Kansas College and Career Ready Standards for Mathematics* (Kansas State Department of Education, 2010), third graders are expected to know all their multiplication facts from memory by the time they leave their grade, so instruction on multiplication facts takes up a large portion of instruction in mathematics classrooms.

Information About the Teacher Group

Four teachers formed the teacher group in this study. Two were male and two were female. The number of years as an educator ranged from one to twenty-one years, with half of the teachers having 21 years of experience. All of them have a BA or BS with 50% having completed an English for Speakers of Other Languages (ESOL) certification program. Two of the teachers have experience at other grade levels; one taught at the Prekindergarten level and another taught at the fourth and fifth grade levels. All were committed to improving their students' fluency with multiplication facts and were interested in being a part of this research study that could potentially impact their students' understanding and use of multiplication facts.

Information About the Student Group

Data about the students attending this elementary school were gathered from the state's website which allows public access to demographic information concerning all schools across the state. The school in this study had a student enrollment of approximately 560. It serviced prekindergarten through fifth grade. They offered Special Education (SPED) programs for those students with disabilities, which accounted for about 13% of their students, and services to their English Language Learners (ELL) which were about 27% of their student population.

The number of males and females was almost equal with 51% being male and 49% being female. The ethnic diversity showed the majority of students were Hispanic (33%) with White (30%), African-Americans (20%), and Other (17%) following in decreasing percentages. Since

85% of the students were economically disadvantaged, the school was labeled Title I and received extra support to educate their students.

The students in the third grade classrooms mirrored the school's makeup. The majority of the students were economically disadvantaged and had an ethnic background that was not white. Students with these characteristics need extra support in order to be successful in school (Eamon, 2002; Hill & Sandfort, 1995; Taylor, 2005).

Professional Development Workshop

In this district all experienced teachers have received professional learning sessions on instruction using the fact strategies. One of the four participating teachers was new to teaching but had received instruction in her college mathematics methods course about fact strategies, so the decision was made to provide a refresher workshop about the fact strategies and then to provide information about instruction using student discourse with the fact strategies at the same workshop. The agenda for the workshop is provided in Appendix A and shows the sequence of learning from the fact strategies portion to the student discourse portion of the workshop.

The four teachers and the instructional coach at the building were participants in the workshop. To begin, the researcher provided background about the study and allowed time for participants to ask questions and sign the teacher participation consent forms. The researcher then concentrated on a review about the purposes of fact strategy instruction: building number sense, assisting in fluency and efficiency, and building a foundation in understanding number relationships that are critical in learning more complex mathematics. After that point, the participants determined which strategies they had questions about and those were discussed as a group. Finally the subject of student discourse was brought in to the workshop so the participants understood that the purpose was not to just get students to talk, but that the discourse was to provide a window into the students' thinking and allow students to use language to process information. A planning tool (Appendix B) was provided but was not required to be used. Instead, the teachers could use any planning tool to assist them in using student discourse to support their fact strategy instruction.

Instruments

Teacher Group Instruments

Multiplication fact strategies workshop assessment.

An assessment targeting fact strategies was given to participants after the professional learning workshop. The assessment focused on fact strategies so the researcher could evaluate teachers' knowledge concerning appropriate use of those strategies. The researcher needed to determine that all teachers had knowledge of the content they were to instruct before the study began. If any teachers had shown a need for extra support, then the researcher would have provided the necessary training in addition to the workshop already given.

The assessment first asked the teachers to name and explain different strategies that could be used to solve 7 x 8; see Appendix C. The teachers were then asked to categorize the strategies they used into levels of basic, intermediate, or advanced, so the researcher could determine if the participants understood the varying levels of the strategies. Next they were asked to identify a strategy, or strategies, that would not be appropriate to use to solve that particular fact. This was necessary so the researcher could determine if the teachers understood that not all strategies are appropriate for all problems when looking at efficiency and fluency. Finally, they were asked to share any concerns they had about teaching fact strategies so those worries could be addressed before the study began.

Information chart for classroom observations.

For each classroom observation, information was collected on the researcher-developed <u>Data Collection Chart by Teacher</u>; see Appendix D. The dates and times of the observations were noted for each teacher as well as information about the type of lesson (e.g., introduction, practice, or review) taught. The name of the fact strategy taught was logged into the document and any other pertinent information that was deemed important by the researcher during the course of the observations in the classrooms was recorded.

Categorizing fact strategy instruction questions during classroom discussions.

A researcher-developed measurement tool was used for collecting data regarding the teacher observations; see Appendix E. This tool was developed using aspects of the *Cognitive Rigor Matrix* (Hess, 2009) and the *Depth of Knowledge Question Stems* (Collins, n.d.). By combining these two resources, the researcher produced a measurement tool that aided the researcher in categorizing the teachers' questions into four different levels. This tool, <u>Categorizing Fact Strategy Instruction Questions During Classroom Discussions</u>, utilized a 1 through 4 rating scale where 1 indicated a low-level question and 4 indicated a high-level question.

The categorization tool gave a description of each level, sample general questions, and example strategy questions specific to fact strategy instruction appropriate for that level. Level 1 indicated "Recall and Reproduction" questions. This level of questioning expected students to recall information (facts, definitions, terms, or simple procedures) and to perform simple algorithms or apply formulas (one-step, well-defined, straight-forward procedures). Level 2 indicated "Skill and Concept" questions. These questions expected students to make some decisions about how to approach the task and be able to discuss some similarities and differences, but the explanations may have been missing some details. Level 3 indicated

"Strategic Thinking and Reasoning" questions. Level 3 questions expected students to reason, plan, and use evidence in their solutions; use complex and abstract thinking; and justify that thinking. Level 4 has "Extended Thinking" questions. These questions expected students to use complex reasoning, planning, and thinking; make connections to other ideas; select an effective approach among many; and justify their reasoning for that approach or strategy. Only questions focused on multiplication facts content were noted and questions related to behavior or classroom management were ignored.

Teacher interview protocol.

The interview questions used for the teacher interview protocol were designed by the researcher; see Appendix G. The questions were organized under the following topics: *Instruction of Fact Strategies Supported by Student Discourse*, *Student Understanding and Use of Multiplication Fact Strategies*, *Challenges with this Instruction*, and *Benefits of this Instruction*. The instrument allowed for a brief "setting the stage" question at the beginning of the interview so teachers would feel comfortable before the questions in each topic were introduced. At the conclusion of each interview, the researcher asked if the interviewee had more information to provide that was not included in the interview, so any important information from the perspective of the teacher would not be overlooked.

The questions in the *Instruction of Fact Strategies Supported by Student Discourse* section of the interview concentrated on instruction with fact strategies and student discourse. The participants were encouraged to think about the first lesson and the last lesson that was observed and explain if the instruction provided to the students had improved or not. This was followed by an inquiry into whether there was an increase in higher-order thinking questions to the students compared to instruction in the past. A question concerning the kinds of support needed to increase the number of higher-order thinking questions during their instruction was also asked.

For the *Student Understanding and Use of Multiplication Fact Strategies* section, the questions asked focused on the teachers' opinions about their students' reactions and understandings about the strategies. The teachers were instructed to think about their students now and at the beginning of the semester to determine if there was a deeper understanding about the strategies, and if they felt their students considered themselves successful and fluent with the strategies. Teachers were also asked if they noticed the students using more of these strategies outside of the fact strategy lessons. The last question focused on whether the teachers felt that the students responded to the higher-order thinking questions during classroom discourse in a way that promoted deeper thinking.

The *Challenges* and *Benefits with this Instruction* portions were brief. In the *Challenges with this Instruction* section of the interview, the teachers were asked to explain some of the challenges and explain the top two weaknesses with this type of instruction. And finally in the *Benefits with this Instruction* section, they were to describe the benefits of this type of instruction and then give the top two strengths with this type of instruction.

Student Group Instruments

Multiplication fact strategy measurement tool (pre- and post-assessment).

The <u>Fact Strategy Measurement Tool</u> used to collect data concerning students' use of strategies when solving multiplication facts was created by the researcher; see Appendix H. Each teacher was directed to administer the assessment to all students but information from the assessments was collected only from those students who had signed consent forms. The teachers asked their students to explain in writing, using words, pictures and diagrams, how they would answer the problem 8 x 6. This tool was given as a pre-test and a post-test to provide data about students' use of fact strategies from the beginning of the study to the end of the study.

Students' answers from the pre-test and the post-test were collected and organized using the Levels of Multiplication Fact Strategy Implementation for Data Collection chart; Appendix I. This chart was constructed using research from Baroody (2006), Carpenter et al. (1999), Kling and Bay-Williams (2015), and the National Research Council (2001). These researchers established a system of levels that students generally progress through as they work toward fluency with fact strategies. After reviewing the work from these researchers, a chart was constructed to identify the level of fact strategy implementation starting at the foundational level of constructing meaning for multiplication, which usually requires the students to count all. Subsequent levels are characterized by decomposing numbers and utilizing the properties of operations, to finally fluent recall of facts.

Multiplication fact strategy student interviews.

The researcher interviewed five to six students from each classroom who had correct answers on the written post-test. An interview was determined to be the best strategy to provide insight into students' thinking and use of fact strategies when solving multiplication facts. Researchers Allsop, Kyger, Lovin, Gerretson, Carson, and Ray (2008), Crespo and Nicol (2003), and Fiori, Boaler, Cleare, DiBrienza, and Sengupta (2004) advocated for interviewing students by explaining that students provide more details and clarity when giving verbal explanations than when relying only on written work.

The researcher-developed Fact Strategy Student Interview Tool was used for collecting data during the student interviews; see Appendix J. In this tool, two multiplication facts were selected by the researcher because a variety of strategies can be used to solve each problem. The first multiplication fact was 8 x 6 and the second fact was 6 x 7. Each multiplication fact could be solved by at least three different derived fact strategies based on the properties of operations and number. For example, 8 x 6 can be solved by decomposing the 8 into 4 + 4 and then using the distributive property so the two products of 24 are added together to get 48. This fact can also be solved by decomposing the 6 into 3 + 3, multiplying 8 x 3 to get a product of 24 and then adding the two products together to create the problem of 24 + 24 to get the answer of 48. This fact can also be solved by decomposing the 6 into 5 + 1 in order to create two easier facts of 5 x 8 and 1 x 8; 5 x 8 is 40 and then adding one more set of 8 results in the final solution of 48.

During the interview, the student was shown a card with the multiplication fact 8 x 6 and asked to say how he/she would solve the problem. Appendix J also displays the recording sheet which shows each multiplication fact and some of the likely strategies the student may give as a way to determine the answer. The interviewer marked the strategy described by the student or completed the section for "Other." Notes were taken to provide details as students explained their thinking.

Procedures

Data Collection of Teacher Information

It was an expectation that all teachers in this district had received professional learning on fact strategy instruction. However a refresher workshop was provided to all study participant teachers in order to make sure everyone had the same background knowledge for fact strategy instruction; see Appendix A. The teachers then participated in learning how to plan and incorporate student discourse in their instruction. They received information about a student discourse planning tool (Appendix B) and how this could assist in their instruction as they increase their level of questioning for students during fact strategy lessons. This was not a required tool, but the teachers were expected to think about how to improve student discourse during their instruction and to increase their level of questions. At the end of the workshop, all teachers were administered a short assessment to determine content knowledge about multiplication fact strategies; see Appendix C. If any teacher had proven a need for extra support to understand the strategies then additional professional learning sessions would have been provided for those teachers. In this instance all teachers were sufficient in their understanding of the strategies.

All four teachers were observed by the researcher and data were collected using the Categorizing Fact Strategy Instruction Questions During Classroom Discussions tool and the Data Collection Chart; see Appendices E and D. Three observations of each teacher were scheduled; the first occurred at the beginning of multiplication fact strategy instruction, another in the middle of the semester, and the last at the end of the semester. Three different mathematics units were taught to the students during this time frame. The first and last mathematics units had outcomes that were linked to fact strategies and building fact fluency. The unit taught during the second set of observations concerned measurement standards and outcomes. Each observation was video-recorded and transcribed by the researcher.

All four teachers were interviewed after all instruction on multiplication facts and after the post-test and student interviews. Information was collected from them about their use of student discourse and their perception of student accessibility to the fact strategies; see Appendix G.

Data Collection of Student Information

All students were administered the Fact Strategy Measurement Tool as a pre- and post-test; see Appendix H. The pre-test was given to students shortly after fact strategy instruction began in order to determine initial use of fact strategies for each student. The post-test was administered at the end of the semester to determine the level of fact strategy use after instruction on all fact strategies. The assessment was given in the students' regular classroom by the regular classroom teacher under normal classroom conditions. The Levels of Multiplication Fact Strategy Implementation for Data Collection chart was used to assign each student a score for the written work on the pre-test and post-test.

The researcher intended to select six students from each classroom so a total of 24 interviews could be conducted, but because of the low number of students who returned consent forms in two of the classrooms only five students were able to be selected from those student groups. Students that were selected for the interview were chosen because they had written a correct numerical response of 48 to the 8 x 6 problem on the Fact Strategy Measurement Tool post-test, did not have a lower category rating on the post-test when compared to the pre-test, and had returned consent forms.

At the interview each selected student was shown the multiplication fact problem, 8 x 6, from the written task completed in class a few days earlier; see Appendix J. Each student explained how he/she figured out the correct answer and/or was asked to share how he/she would explain getting a correct answer to someone who did not know the answer. Next each student was shown a card with the multiplication problem 6 x 7 printed on it. The student was asked to give the answer to the fact and then explain how he/she solved that problem. The highest level of strategy implementation for either problem was taken as the score. The interviews were video-recorded so the researcher could verify the data that were taken during the interview process.

The Levels of Multiplication Fact Strategy Implementation for Data Collection chart was used to assign each student a score. Students received the highest score of a 3 for that item if they were able to use a derived fact in order to find the solution for either problem. If students used a derived fact strategy incorrectly or used a counting strategy, they received a score of 2. If students used a count all strategy, they received a score of 1. Finally, if students answered incorrectly and could not provide a coherent strategy of any type, they received a score of 0.

Data Analysis

Data From Teachers

Observation data.

All four teachers were observed three separate times. The observations were set so the teachers had at least one week's notice before each visit. The first observation was at the beginning of the semester, the second observation was in the middle of the semester and the final observation was at the end of the semester. The researcher video-recorded each session so the questions asked during instruction could be marked appropriately during later viewing of the video. Only those questions that pertained to fact strategy instruction were marked on the tool. Those questions that dealt with classroom management were ignored. The marks for each type of question on the Categorizing Fact Strategy Instruction Questions During Classroom Discussions tool were counted and a percent from the total number of questions was found for each of the categories. An excerpt from one observation is presented in Appendix F to illustrate the questions used by the teacher and students' responses. Information about the level assigned to the questions is also included.

The data were arranged so the researcher could examine increases and/or decreases by observation dates and by categories. The second observation for Teacher TC had unusable data since there was no instruction concerning fact strategies for that date. All other dates and categories were complete.

Interview data.

Each teacher was interviewed on the same date. The interviews were video-recorded and then transcribed by the researcher. The transcribed interviews were summarized for each teacher and analyzed for commonalities and differences based on the following topics: *Instruction of Fact Strategies Supported by Student Discourse, Student Understanding and Use of Multiplication Fact Strategies, Challenges with this Instruction*, and *Benefits of this Instruction*.

Data From Students

Pre-test and post-test data.

Forty-nine students' data from the pre-test and the post-test were collected and organized so each student had a document displaying all scores; six students did not have pre-test data. The common document allowed for quick analysis of student movement from one category to another. The raw scores for each student were then compiled into a table. Those students who participated in the interview had their interview scores added to the table. The data were then condensed into the different category levels showing frequency and percent for the pre-test and the post-test. Descriptive statistics were calculated in order to obtain the mean, median, and mode scores, and the standard deviation for both the pre-test and the post-test. These statistics were analyzed to discover movement from level to level within the categories.

Further statistical analyses were conducted on the pre- and post-test data. An exact McNemar's test was used to determine if the difference in the number of students implementing Category 3 strategies between the pre-test and the post-test was statistically significant. The level of significance used for the analyses was set at $\alpha = .05$.

Interview data.

Twenty-two students were interviewed at the end of the semester in order to gather specific details concerning the strategies they used to find the correct answer on their post-test and the strategies used to solve another multiplication problem, 6 x 7. The researcher used the Fact Strategy Student Interview Tool to collect students' strategy use for 8 x 6 and 6 x 7. Almost all students solved each fact using more than one strategy. They received a score based on the highest level strategy they used during the interview. These scores were recorded along with their pre- and post-test scores. Descriptive statistics were run on the interview data. An excerpt from two student interviews is presented in Appendix K to illustrate student responses. An explanation is provided to support the level assigned to each response.

Further statistical analyses were conducted to compare the pre-test, post-test, and interview scores of the 17 students who had completed all three assessments. A Friedman test was used to determine if the differences in the students' outcomes were statistically significant. The level of significance used for the analyses was set at $\alpha = .05$.

CHAPTER 4

RESULTS

Introduction

The purpose of this study was to investigate the use of student discourse as part of third grade teachers' fact strategy instruction and students' use of fact strategies while learning basic multiplication facts. This study was designed to answer the following research questions:

- What insights do teachers have about the multiplication fact strategies used by their students and the use of student discourse while teaching multiplication fact strategies?
- How did the frequency of different categories of questions used by teachers change during the semester's instruction on fact strategies?
- How did the number of students at higher levels of multiplication fact strategy implementation change as a result of instruction that included student discourse?

This chapter summarizes the data analysis. The SPSS statistical package (version 23) was used to analyze the student assessment and interview data. Qualitative and quantitative methods were used to analyze the teacher data.

The group that was studied included four third grade teachers and their students. All students participated in the instruction and the assessment activities but data were collected only from students who had submitted consent forms signed by their parent or guardian. The first classroom had 18 consent forms signed (37% of final group), the second had six signed consent forms (12%), the third had 15 signed consent forms (31%), and the last had 10 consent forms signed (20%). The student group selected for the final student interview was comprised of six students from the first classroom, five from the second classroom, six from the third classroom, and five from the fourth classroom.

Teacher Data

Data for the study were generated from classroom observations and interviews. Three observations occurred during the fall semester which also coincided with three different mathematics units being taught to the students. The first and last mathematics units had outcomes that were linked to fact strategies and building fact fluency. The unit taught during the second set of observations concerned measurement standards and outcomes. The teacher interviews were conducted the first week after winter break.

Observation Information and Analysis

During the observations, data were collected by the researcher using a researcher-developed tool, Categorizing Fact Strategy Instruction Questions During Classroom Discussions. This categorization tool allowed the researcher to identify levels of questions asked by the teachers.

The researcher video-recorded each of the observations in order to carefully categorize each question using the categorization tool following the observation. Questions pertaining to instruction were tallied. Those questions that did not impact instruction, but were intended for classroom management, were not counted. The number of questions for each individual observation was totaled and percents were found for each category level for each observation; see Table 1.

Table 1

Questions in Each Category for Each Teacher's Three Observations

Teacher ID	1st Observation category 1	1st Observation category 2	1st Observation category 3	2nd Observation category 1	2nd Observation category 2	2nd Observation category 3	3rd Observation category 1	3rd Observation category 2	3rd Observation category 3
TA	52 (66%)	14 (18%)	13 (16%)	17 (68%)	8 (32%)	0 (0%)	31 (62%)	9 (18%)	10 (20%)
TB	14 (100%)	0 (0%)	0 (0%)	25 (100%)	0 (0%)	0 (0%)	16 (64%)	1 (4%)	8 (32%)
TC	32 (84%)	3 (8%)	3 (8%)				23 (64%)	3 (8%)	10 (28%)
TD	2 (100%)	0 (0%)	0 (0%)	22 (100%)	0 (0%)	0 (0%)	7 (64%)	1 (9%)	3 (27%)

Note. Gray portions of table indicate data that were unusable since instruction did not pertain to fact strategies.

The percents indicate an increase in higher-level questions from the first observation to the third for all teachers. Category 1 level questions were recall and reproduction questions. Category 3 level questions required students to reason and use evidence to support their thinking.

A comparison of the data from the first observation to the second was made. Teacher TA decreased the number of higher-level questions (category 3) asked during the second observation from the first. Teacher TC's second lesson did not include fact strategy instruction so data could not be used. Teachers TB and TD had no change in the types of questions they asked their students from observation one to observation two. When considering the data from the Data Collection Chart by Teacher and the video recordings, all teachers during the second observation were focused on practicing the strategies without much discussion beyond the basics of how the strategies work (with the exception of Teacher TC who did not teach fact strategies during that observation).

Further analysis of the data from the Data Collection Chart by Teacher and the video recordings comparing the first and third observations revealed that the teachers at all levels were intentional in setting up a classroom discussion about the different strategies for the third observation so students could explain their thinking and reasoning to the teacher and their classmates. They were intentional in laying out the goals for the classroom discourse and what the target was concerning the discussion. For all teachers there was an increase in the number of higher-level questions (category 3) asked during the third observation.

When reviewing Teacher TA's data from Table 1, the increase in higher-level questions was small compared to the other teachers. This teacher was intentional in setting up classroom discourse for his first observation as well as his last. This is reflected in the percents in Table 1. When he was observed for the second observation, he was focused more on practice than on understanding student thinking and reasoning which also shows in the percents. Since Teacher TA specifically laid the groundwork early in the semester with his students his percents do not show much of an increase in higher-level questions for the third observation as it does for the other three teachers.

Teachers TB and TD had a large increase in the percent of higher-level questions asked. Both teachers were focused more on fluency practice during observations 1 and 2 than on understanding the fact strategy. The teachers were having students quiz each other or were having fact races which limited the opportunities for higher-level questions. For the final observation, Teachers TB and TD focused on getting into student understanding and reasoning with the fact strategies, so their percents of higher-level questions increased. The students were providing in-depth responses to teacher questions instead of just numeric answers.

Teacher TC had a definite increase from Category 1 to Category 3 from the first observation to the third observation. The outline of the lessons was similar. The teacher began with a problem solving component and then led the students to the fact strategy targeted for the day's instruction. The main difference between the lessons was the intentional use of a structure to provide student discussion opportunities for the third observation. This structure was not in place for the first observation so the resulting opportunities for increased student discourse in the third observation were noticeable.

Overview of Teacher Interviews

The teacher interviews were comprised of questions that fit under the following topics: *Instruction of Fact Strategies Supported by Student Discourse*, *Student Understanding and Use of Multiplication Fact Strategies*, *Challenges with this Instruction*, and *Benefits of this Instruction*. All interviews were conducted on one day at the beginning of the spring semester. Notes were taken during the interviews and digital video-recordings were taken so the researcher could complete transcriptions of the interviews in detail.

In the following paragraphs, summaries of each teacher's responses are provided. These summaries offer a picture of each teacher's thoughts and feelings concerning instruction and students' learning. After the individual summaries, commonalities and differences are explored among the four teachers to find themes from their experiences.

Summary of Teacher TA's Interview

Teacher TA noticed an improvement when he started utilizing the strategies even though he admitted that these strategies were difficult for him to teach since he had been taught through only memorization. He observed a marked improvement in his students' universal screener scores in computation. He attributed this improvement, in part, to the instructional focus on fact strategies and student discussions.

When thinking about his instruction, Teacher TA believed there was growth over the semester in his students' use of fact strategies and in his own ability to ask higher-order thinking questions. He shared how his students learned to attack problems using various strategies that made sense to them and how they created easier methods for solving problems. He mentioned that all his students were not accurate all the time, but he saw they were thinking about actual solution paths instead of just making random guesses. He saw a definite need for more support from other teachers or resource books to increase the amount and quality of higher levels of questions when the numbers become more complex for his students. He wondered if manipulatives or more hands-on work would help struggling students as they worked with those complex numbers. When preparing his instruction, he tried to make sure he presented a variety of multiplication facts. He found that if he presented a couple of facts with one of the factors remaining the same, then he and his students could have conversations about the relationship between the facts. An example he provided was 8 x 4 and 6 x 4; they would discuss the similarities and differences between the two facts and talk about the relationships.

When thinking about student understanding and the use of fact strategies, Teacher TA noticed how his students used two particular strategies often and with great success: the Doubles strategy and the Double and Double Again strategy. The Double and Double Again strategy was one that his students used with great success, but the students had to frequently explain how they were using this strategy because they were using it in ways that he had not thought of before using fact strategies in his instruction. He stated that his students were "kind of teaching [him] different ways too." He remembered that he was taught to just memorize and that "there really was no explanation of why. You just had to know them." He believed the majority of his students felt they were successful and fluent with their multiplication facts and the fact strategies. He also thought that at least half of his students are able to successfully reason using the strategies. There were still a few students working on some of the more basic strategies, but he was working with them and taking time to get them to see the relationships in the strategies.

The two biggest challenges with this type of instruction are the availability of time and students' lack of confidence. He believed during the classroom discussions that the students were engaged and enjoyed the sessions, but the amount of time that was available for having in-depth classroom discussions was limited. This put a strain on completing the sessions in a manner that was beneficial to the students. Then the students' lack of confidence in themselves was another challenge, especially those that have difficulty in learning their facts and using the strategies. He mentioned again that the universal screener computation scores were much higher than they were the previous year and he believed the fact strategies made that impact.

Student engagement and results are the two biggest benefits with this type of instruction. Teacher TA firmly believed that kids needed to enjoy the work and should have fun learning the fact strategies. The students liked to share their thinking with others and basically show off what they know. The teacher found that even when the strategies became more difficult, the kids would "stick with it" so it made it even more disappointing when their schedule made them move on to another subject or topic.

At the conclusion of the interview, Teacher TA explained that he would like to learn even more about the fact strategies and become more familiar and comfortable with them. He stated that it can be hard to teach something differently. Even after the semester of the study, every so often, he would catch himself falling back into teaching the way he was taught, through rote memorization, because teaching through strategies is difficult. He shared how attending professional learning sessions just this past year had impacted how he thinks about numbers and their relationships. During one professional learning session about the fact strategies he remembered thinking that "many of these are easier, why didn't I think of that." He remembered learning by doing worksheets each night and getting very proficient at counting on his fingers instead of reasoning about the numbers, the operations, and their relationships.

Summary of Teacher TB's Interview

Teacher TB believed most of the lessons went well. Some were confusing to his students but for the most part he believed the lessons went well. He had no questions before the interview began.

When thinking about instruction with the fact strategies supported by student discourse, Teacher TB felt he had to do quite a bit of instruction at the beginning of the year in getting the students to listen before he could actually do any teaching of content. Once he began instruction with the strategies, he felt the students benefitted from a better understanding of number sense. He thought teaching the strategies developed number sense at a deeper level with his students than just memorization of the facts. He believed his instruction included more higher-order thinking questions this semester because in the past he focused only on skip counting which is a skill at the lower levels of thinking compared to fact strategies. The strategies taught foundational ideas in number sense and included decomposing and the distributive property. The process was slow at first but finally his students were able to share their thinking and reasoning with others.

When questioned more about his instruction, Teacher TB thought having more conversations and planning opportunities with other teachers about fact strategy instruction would be most beneficial in increasing higher level questioning during his classroom discussions. He is able to take ideas from others and work it in to his own style of teaching. When preparing for his instruction, he tried to find problems that were just right for his students. He explained that these needed to be problems that were challenging, but would not lead to confusion. He also talked about how he had to have a "general idea of where [he wanted] things to go" but also knew he had to be prepared for anything because sometimes the course of the conversation would go a different direction. To him this was the fun part of teaching; the "freedom to just go ahead and go with some of those learning moments."

When Teacher TB responded to questions concerning student understanding and use of fact strategies, he felt that the fact strategies led to a better understanding of number sense for his students. He believes number sense is the foundation to understanding the fact strategies and will eventually transfer to more complex thinking. His students were using these strategies outside of the lessons, but not as consistently as he had hoped. He believed it takes the students extra effort to use the strategies and he is working on ways to get the students to use them more often during their problem solving lessons. He thought the strategies helped his students feel successful but noted that fluency with the strategies is taking longer. He noticed that the strategies provided students a different way to think about solving problems that they would normally have given up on in the past. He believed it has promoted deeper thinking in the majority of his students and the discussions have been very beneficial in allowing the students to think differently.

Teacher TB found the biggest challenge was in adapting the lessons for the various levels of mathematical understanding in his students. Some students are able to understand and acquire the strategies easily and others are still working on understanding the concept of equal groups. Finding the middle ground to have classroom discourse with all levels can be difficult.

As for the biggest benefits with this type of instruction, Teacher TB noted two that rose to the top in his classroom. The greatest benefit he discovered was the increased understanding of number sense for his students. He mentioned that specific topic frequently throughout the interview. The next benefit for his students was that it provided them with another tool for solving multiplication facts. If his students could not solve the problem one way then there was another strategy that could be used.

In wrapping up the interview, Teacher TB explained that he would not worry about the time constraints in the school schedule. Next year, he determined that he will take more time to instruct a fact strategy well and not worry about the time it takes. The greatest benefactor will be his students.

Summary of Teacher TC's Interview

Teacher TC believed the fact strategy lessons went much better this year than the previous year because she was motivated to focus on teaching each strategy since she was being observed. Her students used the strategies more effectively this year also. One strategy she noted that the students used particularly well was the Double and One More Set strategy which students used when multiplying by three.

When reflecting on her instruction, Teacher TC found that there was definite improvement between the first lesson and the last lesson. She thought it was primarily due to the fact that she was going to be observed and wanted to make sure she was prepared. Ultimately, she believed this made her a better teacher but confessed that the original motivation to improve was due to the scheduled observations. She did ask higher-level questions in her teaching because she expected students to solve in many different ways, instead of just one or two ways. Having several strategies available to solve problems was important to her. She indicated that observing another teacher implementing classroom discussions with the fact strategies would assist her in increasing the number of higher-order questions. For assistance in planning her lessons, she relied on discussions with her peers to better understand how she could teach the fact strategy lessons.

Teacher TC felt that her students were starting to get a better understanding of the fact strategies but said it was dependent on their academic levels in mathematics. The struggling students were getting confused with some of the strategies, but she believed her stronger students were pushed to think differently. She believed about a fourth of her students were highly successful with the strategies, and those students were using the strategies in other situations. She believed the practice she incorporated into her lessons helped her students implement the strategies and see the patterns and relationships more easily. About a handful of her students are thinking at a deeper level. She noted one student in particular felt very successful with the topic of multiplication because of the discussions embedded within the fact strategy instruction.

The challenges with this type of instruction for Teacher TC are dealing with the different levels of students and communicating expectations of this instruction with another educator that teaches in the room with her. The first challenge was the various levels of mathematical understanding in her classroom. Her group consisted of students that were identified as having learning disabilities along with her general education students. Since learning disabled students were in her room, she was assisted during mathematics instruction by another teacher. Often that teacher instructed the learning disabled students differently instead of using the fact strategies. Communicating with another teaching adult in the classroom concerning a new type of thinking and instruction was challenging.

Teacher TC believed one of the best benefits of this instruction was having the discussions separate from the core curriculum. She indicated that having this separation was critical for her students and allowed them to engage in the strategies without having to process other mathematical ideas. She believed it “helped build a foundation for when they were going into word problems.”

In conclusion, Teacher TC explained that next year she will make some slight changes so she can focus on a strategy for about a week and then move on to another strategy. She believed she was stressing too much about the students not understanding each fact strategy and took too much time on each one. She has determined that if a particular strategy does not work for them then another one will.

Summary of Teacher TD's Interview

Teacher TD began the interview by explaining how challenging it is to start anything new. As she reflected she noted that being consistent in her expectations helped the students. She made sure she provided time in practicing each strategy so they were prepared before moving on to the next one.

When asked about her instruction during the semester, Teacher TD stated that her instruction "improved dramatically" from the beginning to the end. She believed this was due to the higher levels of questioning in her instruction this year when compared to her past teaching. She also felt that her students learned not only from her questioning but from each other during their classroom discussions. The discussions allowed the students to build on each other's ideas as well as their own.

To improve her instruction even more, Teacher TD would like to have peers observe her teaching and provide direct feedback. She described a time when the instructional coach in the building happened to have been in the room during one of her math classroom discussions and how the coach was able to provide real-time coaching. This helped her to understand that she has to have questions well-thought out beforehand to target the mathematical ideas during discussions, but she also needed be ready to "go with the flow of the students" to push them further in their thinking when they were ready.

Teacher TD explained how this has impacted her students' understanding and use of fact strategies. She said her students were able to identify specific strategies used when solving problems now. The biggest contributor to their increased understanding was the amount of time she provided to practice those strategies. She generally provided at least a week for each strategy but increased the time for those strategies that proved more difficult to master. She noted students were seeing themselves as successful and fluent with these strategies so they were able to use them during their core math lessons. Some students needed more help and assistance, but the increased expectation for classroom discourse has been beneficial for them. For her academically advanced students, it provided a challenge. For her struggling students, it allowed them more time to think and to process. The discussions also provided students an opportunity to share their thinking and have that thinking confirmed or clarified by another student.

When asked about the challenges with this type of instruction, Teacher TD listed time and student engagement. She believed the time constraints of the school schedule were a hindrance when implementing this type of instruction. She frequently felt rushed and wished there was more time to dig deeper with the students' thinking. The challenge with student engagement is ensuring that all students are actively engaged and not just being compliant. One solution she adopted was to periodically have the discussions take place in smaller groups. This allowed more opportunities for each student to share thoughts and ideas.

According to Teacher TD, the top two benefits of this instruction were that the students learned to be patient and they learned from each other. She believes it teaches the students "how to be patient" with each other when they are listening to the different solution paths. The students found that sometimes another person can put their ideas into words or they discovered a new approach to a strategy. Teacher TD was also surprised that she has learned many new ways of thinking from her students.

In completing the interview, Teacher TD explained how she adjusted her instruction after just the first few lessons. She found she needed to limit the number of students that were able to talk during a classroom discussion because of the time constraints mentioned earlier. She found that using classroom sticks (tongue depressors with a student's name on each one) was helpful because she could draw a specific number out for the discussion that day and keep within the specified school schedule.

Commonalities and Differences Found in Teachers' Interview Data

Several similarities arose from the analysis of the interviews. The first common thread was that each teacher felt there was definite improvement in instruction when comparing their first lesson with their last lesson. Along with the improvement in teaching, three of the teachers specifically stated that their students were using various solution paths to find answers instead of relying on just one way to solve a problem. Two of the four teachers also mentioned that the students were learning from each other as they shared their reasoning and thinking about the strategies. Two mentioned the idea of having a plan but then being flexible enough to follow the students in their thinking to get to the mathematics. Finally, three of the teachers indicated that observing others and planning with others would assist them in being more successful in their instruction.

When examining the data concerning student understanding and use of strategies, three teachers explained that most or some of their students felt successful in using strategies and two of the teachers noted their students felt more fluent. Three of the four teachers also discovered that their students were able to share their reasoning with the various strategies and were able to show a deeper level of thinking when using the strategies.

The teachers identified challenges with this type of instruction. All teachers mentioned time as a challenge. The school schedule is determined by a building leadership team so there is limited availability for setting up purposeful student discourse. Another challenge with this type of instruction noted by two of the educators was the variety of academic levels within their classrooms. Both struggled with how to create a meaningful learning environment for their struggling learners and their high achieving students.

The data provided few commonalities about benefits of this instruction. Only two of the teachers shared a common idea that the students were learning from each other. Both identified this as a strength of having student discussions embedded in their instruction.

There were several ideas that came from each teacher which were specific to their individual experiences in using student discourse to support fact strategy instruction. One teacher reflected that this type of instruction was difficult because it was very different from his own instruction in school. Another teacher found difficulties in communicating expectations with a second educator in the classroom when that second educator had different ideas about fact strategy instruction. One teacher indicated that student engagement was a benefit because the levels of student participation were high, whereas another teacher said student engagement was a challenge because she knew some of her students were just compliant during the classroom discussions and not really cognitively engaged. When thinking about student understanding one teacher stated that this instruction increased an understanding of number sense with his students and another teacher liked that her students increased their level of patience with each other.

Student Data

The student data were collected from a population of 49 students in four third grade classrooms. The number of students eligible to participate in providing data varied by classroom. The researcher collected pre- and post-test data for all students and then conducted individual interviews with 22 of those students.

Pre-test and Post-test Information

All third grade students were given a pre-test and a post-test that consisted of one multiplication fact, 8 x 6. Students were instructed to explain how they would answer the problem, not just provide a numerical answer. They were told that they could use words, pictures, diagrams, etc. to explain how they would answer the problem. See Appendix H to view the pre- and post-test.

The students' answers were collected and organized using the Levels of Multiplication Fact Strategy Implementation for Data Collection chart; see Appendix I. The levels on the chart are 1 to 4. Level 1 included students that were still making sense of multiplication by creating equal groups but needed to count all in order to reach a solution. Level 2 comprised students that were skip counting or using repeated addition to find the total with the number of groups. Level 3 consisted of students that were decomposing and recomposing factors in order to use known facts to find the answer. Level 4 contained students that were able to provide answers to the facts efficiently and accurately. It was not possible to determine if any students were at the final level of fact strategy implementation since the students were required to explain how they solved the problems instead of just providing a numerical answer. A level 0 was given to those students who could not demonstrate an understanding of equal groups which is critical in understanding multiplication.

Pre-test and post-test data were collected from students. The raw data showing each student's scores for the pre-test and post-test, and the interview score for the few students that were interviewed are provided in Appendix L. The pre-test data from 43 students had a mean score of 1.26, a median score of 1.00, a mode score of 1, and a standard deviation of .581. The pre-test was the only test where students had a level 0 score indicating that they were not able to construct meaning with the multiplication fact given. The post-test data from 49 students had a mean score of 2.12, a median score of 2.00, a mode score of 2, and a standard deviation of .726. Tables 2 and 3 present information about the frequency and percent for each level for the pre-test and the post-test.

Table 2

Pre-test Data for Levels of Multiplication Fact Strategy Implementation

Level	Frequency	Percent
0	2	4.7
1	29	67.4
2	11	25.6
3	1	2.3

Table 3

Post-test Data for Levels of Multiplication Fact Strategy Implementation

Level	Frequency	Percent
1	10	20.4
2	23	46.9
3	17	32.7

As represented in Tables 2 and 3, the data show movement from no use or low levels of fact strategy application to higher levels of fact strategy use. On the pre-test only 28% of the students were at Level 2 or higher but after completion of the post-test those same levels accounted for almost 80% of the students.

Further statistical analyses were conducted with the pre-test and post-test scores from the 43 students. An exact McNemar test determined that there was a statistically significant difference (p = .001) in the number of students implementing Category 3 strategies after participating in the opportunities provided in the classrooms.

Table 4

McNemar Test Crosstabulation Results for Students with Pre-Test and Post-Test Scores

	Post-test Score	
Pre-test Score	Below Category 3	Category 3
Below Category 3	31	11
Category 3	0	1

Analysis of Student Interviews

Twenty-two students were interviewed within one week after taking their post-test. The post-tests were evaluated and students with correct answers that showed use of fact strategy application were chosen to be interviewed. The interviews were conducted by the researcher with responses recorded in written form and in video format to allow the researcher to verify notes taken during the interview.

Students were shown a card displaying the multiplication fact 8 x 6 and asked to explain how they solved this fact. This is the same fact that was given on the pre-test and the post-test. Students were then shown a card with the multiplication fact 6 x 7 and asked to explain how they would solve this fact to determine if they used similar strategies with a fact that was not on the assessment. All students had the same score for both problems except for student TCS5. This student scored a 3 on the second problem so a score of 3 was recorded as the interview score.

Table 5 presents data on the students that were interviewed, listing their pre-test, post-test, and interview scores showing levels of fact strategy implementation. The pre-test data from 17 students had a mean score of 1.35, a median score of 1.00, a mode score of 1, and a standard deviation of .606. The post-test data from 22 students had a mean score of 2.32, a median score of 2.00, a mode score of 2, and a standard deviation of .646. The interview data from the 22 students had a mean score of 2.64, a median score of 3.00, a mode score of 3, and a standard deviation of .492.

Table 5

Levels of Multiplication Fact Strategy Implementation for Students Who Were Interviewed

Student ID	Pre-test	Post-test	Interview
TAS1	2	3	3
TAS2	0	3	3
TAS3		3	3
TAS4	1	3	3
TAS5	1	3	3
TAS6	1	3	3
TBS1		3	3
TBS2	1	2	2
TBS3	1	3	3
TBS4		3	3
TBS5	1	1	2
TCS1	1	2	2
TCS2	2	2	2
TCS3	2	2	2
TCS4	2	2	2
TCS5	2	2	3
TCS6		1	2
TDS1	2	2	2
TDS2	2	2	3
TDS3	1	2	3
TDS4	1	2	3
TDS5		2	3

The post-test and interview scores were collected within one week of each other. The median and mode scores were one level higher for the interview scores than for the post-test scores with just one week separating the activities. On the post-test, nine students scored at level 3, but on the interview 14 students scored at level 3.

Seventeen of the twenty-two students had scores for the pre-test, the post-test, and the interview. A non-parametric Friedman test of differences among repeated measures was conducted and rendered a Chi-square value of 21.415 which was significant ($p < .001$). For this group of students, there was a significant increase in scores across time.

CHAPTER 5

SUMMARY, CONCLUSIONS, and RECOMMENDATIONS

Summary

Instruction about multiplication facts has been and continues to be a part of mathematics curriculum. Learning basic facts occurred primarily through rote memorization until Brownell and Chazal (1935) described "learning by insight" (p. 656) and challenged educators to instruct for meaning, not just memorization. The search for meaning and increased understanding of facts using strategies seemed to reach its peak in the 1980s (Baroody & Gannon, 1984; Carnine & Stein, 1981; Carpenter, 1985; Cook & Dossey, 1982; Heege, 1985; Steinberg, 1985; Thornton, 1978), but beginning in the 2000s, there was a resurgence in exploring the use of fact strategies to increase fact retention and efficiency (Boaler, Williams, & Confer, 2015; Flowers & Rubenstein, 2010/2011; Kling & Bay-Williams, 2015; Sherin & Fuson, 2005; Wallace & Gurganus, 2005). The National Council of Teachers of Mathematics (NCTM, 2014) has identified research findings from the cognitive sciences and from mathematics education that support active learning in mathematics so each student can build understanding. This type of learning for understanding includes foundational skills in mathematics, and basic facts are one of those foundational skills.

Since Brownell and Chazal's (1935) and Brownell's (1944) research on learning basic facts, there have been ongoing debates about the most appropriate methods for fact instruction. Brownell and Chazal (1935) questioned the focus of basic fact instruction. They proposed a difference between "*how well* one performs, . . . and *how* one performs" (p. 19). The *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and *Kansas College and Career*

Ready Standards for Mathematics (Kansas State Department of Education, 2010) placed an emphasis on learning addition and multiplication facts through the use of strategies. These fact strategies are basic number combinations that use the idea of equivalence which "involve[s] breaking the calculation apart into an equivalent representation that uses known facts to figure out the unknown fact" (Charles, 2005, p. 10).

Baroody (2006) identified three phases that children progress through as they master their basic facts. Phase two of his developmental progression is based on "reasoning strategies – using known information (e.g. known facts and relationships) to logically determine (deduce) the answer of an unknown combination" (p. 22). Kling and Bay-Williams (2015) expanded on his work noting, "Research tells us that students must deliberately progress through these phases, with explicit development on reasoning strategies, which helps students master the facts and gives them a way to regenerate a fact if they have forgotten it" (p. 551). Specific fact strategies can help teachers focus their basic fact instruction on thinking and understanding (Flowers & Rubenstein, 2010/2011; Kling & Bay-Williams, 2015; Van de Walle, Karp, Lovin, & Bay-Williams, 2014). These strategies are often grouped into three categories based on the underlying math concepts being used in that group of strategies. The categories are: using counting all, using counting on or repeated addition, and using the properties of operations.

Mathematics education researchers from the 1980s (Cook & Dossey, 1982; Heege, 1985; Steinberg, 1985; Thornton, 1978; Thornton & Smith, 1988) found fact strategy instruction supported students in number fact understanding and in becoming more fluent in recalling basic number facts. Cook and Dossey (1982) and Thornton (1978) theorized that teaching strategies before drill work would improve student retention of basic facts. Heege (1985) proposed that instruction utilizing fact strategies would lead to more flexibility in student thinking and lead to a greater potential for application of these strategies with multiplication of larger numbers. As research continued into the 2000s more findings continued to support fact strategy instruction (Baroody, 2006; Baroody, Bajwa & Eiland, 2009; Boaler, Williams, & Confer, 2015; Flowers & Rubenstein, 2010/2011; Gersten & Chard, 1999; Imbo & Vandierendonck, 2008; Kling & Bay-Williams, 2015; Sherin & Fuson, 2005; Van de Walle, Karp, Lovin, & Bay-Williams, 2014; Woodward, 2006). The connection between number sense and fact strategy instruction was identified as one of the most beneficial aspects of this type of instruction (Baroody, 2006; Baroody, Bajwa & Eiland, 2009; Boaler, 2015; Gersten & Chard, 1999; Kling & Bay-Williams, 2015; Sherin & Fuson, 2005; Van de Walle, Karp, Lovin, & Bay-Williams, 2014).

Since the first mathematics content standards were produced, communication in the classroom has also been a focus (NCTM, 1989). Discourse in mathematics classrooms should encompass the students' and teachers' ways of communicating, representing, talking, supporting, and disagreeing about mathematical ideas (NCTM, 2011). Students and teachers are encouraged to "talk mathematics" (NCTM, 1989, p. 26) and to listen to each other. "Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (NCTM, 2014, p. 29). Effective student discourse depends on the teacher facilitating the conversations and students listening to each other to make sense of mathematics.

Student discourse positively impacts students' understanding and retention of mathematical concepts and skills (Carpenter et al., 1999; Chazan & Ball, 1999; Cobb, Boufi, McClain, & Whitenack, 1997; Franke et al., 2009; Kazemi & Stipek, 2001; Khisty & Chaval, 2002; NCTM, 2014; Stein, Engle, Smith, & Hughes, 2008; Van de Walle, Lovin, Karp, & Bay-Williams, 2014). NCTM (2014) stated that learners must have experiences that allow them to "construct knowledge socially, through discourse" (p. 9) in order to build their own mathematical knowledge. Fiori, Boaler, Cleare, DiBrienza, and Sengupta (2004) found that when students were able to discuss their thinking with others then misconceptions and misunderstandings were more easily identified by the teacher than using written work alone. This thinking was mirrored in the work by Burns (2005). She discovered that once students were able to voice their reasoning during a classroom discussion then she could uncover their misconceptions and assist them in building understanding. A clear mathematical goal must be at the center of all classroom discussions (Chapin, O'Connor, & Anderson, 2013; Kazemi & Hintz, 2014; Smith & Stein, 2011; Van de Walle, Lovin, Karp, & Bay-Williams, 2014).

Traditional teaching is generally teacher-centered so the teacher is viewed as the ultimate authority for all learning in the classroom. To fully implement effective classroom discourse the teacher must focus on purposeful questioning and on creating a student-centered environment (Chapin & O'Connor, 2007; Chapin, O'Connor, & Anderson, 2013; Cobb, Yakel, & Wood, 1992; Franke et al., 2009; Khisty & Chval, 2002; Nathan & Knuth, 2003; Smith, 1996). In moving to more student-centered instruction, the teacher must keep in mind that increasing student discourse without connecting the talk to essential mathematics does not impact students' understanding. Discussion without connections is just sharing and sharing is not learning (Chazan & Ball, 1999).

Hattie and Yates (2014) investigated the persistence of the recitation method in spite of strong evidence to support well-structured student talk. They found that recitation produced mainly low-level questions, low student engagement, and sterile, uninspiring tasks. In contrast, discussions, if purposefully structured, can produce powerful effects. NCTM (2014) advocated for mathematical discussions with students and stated that "[e]ffective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (p. 29).

The purpose of this study was to investigate the use of student discourse as part of third grade teachers' fact strategy instruction and students' use of fact strategies while learning basic multiplication facts. The study was designed to answer the following research questions:

- What insights do teachers have about the multiplication fact strategies used by their students and the use of student discourse while teaching multiplication fact strategies?
- How did the frequency of different categories of questions used by teachers change during the semester's instruction on fact strategies?
- How did the number of students at higher levels of multiplication fact strategy implementation change as a result of instruction that included student discourse?

To answer the research questions, this study used descriptive, nonexperimental research and utilized both quantitative and qualitative data to present a picture of how student discourse supports fact strategy instruction in selected classrooms.

The study took place in a diverse, Midwestern, urban school district. One elementary school was selected based on level of commitment from the administration and level of instructional support that was available within the building. Third grade was chosen as the targeted grade level since the curriculum required that students will begin working on multiplication facts at the beginning of the school year. Also, third grade students are generally mature enough to be able to articulate responses to teacher questions and explain their thinking which was required during the interview portion of data collection. This was based on Piaget's (1971) research where he stated that "genuine argument and collaboration in abstract thought constitute a stage of development which only intervenes after the age of 7" (p. 73) and that "before the age of 7 or 8 children have no conversation bearing upon logical or causal relations" (p. 75).

The four third grade teachers at this elementary school participated in a workshop that provided information about fact strategy instruction and the role of student discourse. They were required to take a workshop assessment so the researcher could identify any additional professional learning needs concerning fact strategies. The teachers were each visited three times throughout the semester so observation data could be collected. Each teacher was also interviewed at the end of the semester to gather data concerning their insights about instruction of fact strategies supported by student discourse.

All students in the four classrooms were administered a pre- and post-test about fact strategy use. Based on post-test results, five to six students from each classroom were identified to participate in an interview conducted by the researcher to explain how they solved the problem on the assessment and then to solve one more multiplication fact, explaining how they came to their solution.

Analyses were conducted on the data collected. From the teacher observations, the percents indicated an increase in higher-level questions from the first observation to the third for all teachers. When examining the information gathered on the Data Collection Chart by Teacher and the video-recordings, the teachers were intentional in setting up a classroom discussion about the different strategies for the third observation so students could explain their thinking and reasoning to the teacher and their classmates. They were intentional in laying out the goals for the classroom discourse and what the target was concerning the discussion. This was different from the first two observations where three of the four teachers were doing more of the talking instead of questioning the students to gather insight into their thinking and reasoning.

Analysis of the interviews found several similarities. All teachers felt there were definite improvements in instruction when comparing their first observed lesson with their last observed lesson. Three of the teachers specifically stated that their students were using various solution paths to find answers instead of relying on just one way to solve a problem. Two of the four teachers mentioned the students were learning from each other as they shared their reasoning and thinking about the strategies. Two mentioned the idea of having a plan but then being flexible enough to follow the students in their thinking to get to the mathematics. Finally, three of the teachers indicated that observing others and planning with others would assist them in being more successful in their instruction.

Student data included pre-test and post-test scores for all students and interview scores for those that were selected based on the correct responses on their post-tests. The pre-test data from 43 students had a mean score of 1.26, a median score of 1.00, a mode score of 1, and a standard deviation of .581. These scores indicate that most students were at low levels of fact strategy implementation, using counting all as a strategy. The pre-test was the only test where students had a level 0 score indicating that they were not able to construct meaning with the multiplication fact given. The post-test data from 49 students had a mean score of 2.12, a median score of 2.00, a mode score of 2, and a standard deviation of .726. These scores show students moving from additive thinking toward multiplicative thinking. The interview data from the 22 students that were selected to be interviewed had a mean score of 2.64, a median score of 3.00, a mode score of 3, and a standard deviation of .492. The data show movement from no use or low levels of fact strategy implementation to higher levels of fact strategy use. On the pre-test only 28% of the students were at Level 2 or higher but after completion of the post-test those same levels accounted for almost 80% of the students. The McNemar Test conducted on the 43 students that had scores for both the pre-tests and post-tests showed that there was a statistically significant difference in performance after implementation of fact strategies instruction.

Twenty-two students were interviewed within one week after taking their post-test. The selected students had correct answers on the post-test and showed use of fact strategy application. Students were shown a card displaying the multiplication fact 8 x 6 and asked to explain how they solved this fact. This is the same fact that was given on the pre-test and the post-test. The pre-test data from 17 interviewees had a mean score of 1.35, a median score of 1.00, a mode score of 1, and a standard deviation of .606. The post-test data from 22 interviewees had a mean score of 2.32, a median score of 2.00, a mode score of 2, and a standard deviation of .646. The interview data from the 22 interviewees had a mean score of 2.64, a median score of 3.00, a mode score of 3, and a standard deviation of .492. The median and mode scores were one level higher for the interview scores than for the post-test scores with just one week separating the activities. On the post-test, nine students scored at level 3, but on the interview 14 students scored at level 3. The Friedman test conducted on the 17 students who had scores for the pre-test, the post-test, and the interview showed a statistically significant difference from the beginning of the semester to the end of semester in their use of fact strategies from Category 3.

Conclusions

The following conclusions address the research questions for this study.

1. Teachers within this study indicated through their interviews that multiplication fact strategies positively impacted their students' learning of multiplication facts. Almost all the teachers reported that their students felt successful in using the strategies and fluent in providing correct answers to multiplication facts. Three of the teachers reported that their

students were using more solution paths based on their understanding of numbers instead of relying on just one way to solve the problem. The ability of their students to be able to think deeply and reason through the problems when using strategies was important to them. The reinforcement of number sense and decomposing and recomposing numbers was viewed as a strength of fact strategy instruction. This finding supports conclusions drawn by other researchers (e.g., Baroody, 2006; Baroody, Bajwa & Eiland, 2009; Boaler, Williams & Confer, 2015; Cook & Dossey, 1982; Heege, 1985; Kling & Bay-Williams, 2015; Sherin & Fuson, 2005; Steinberg, 1985; Thornton, 1978; Thornton & Smith, 1988; Van de Walle, Karp, Lovin, & Bay-Williams, 2014).

Most of the teachers indicated that the embedded discourse improved their students' reasoning and understanding of the multiplication fact strategies. Two of the teachers strongly stated that their students were positively impacted by listening to each other as they described their thinking of each of the strategies. This finding is similar to that of other researchers (e.g., Chapin & O'Connor, 2007; Chapin, O'Connor, & Anderson, 2013; Clark et al., 2012; Hattie & Yates, 2014; NCTM, 2014; Parrish, 2011)

2. The time to use student discourse was identified as a challenge by all teachers. Teachers wanted more time on the days when students were actively engaged in discussing their mathematical ideas. Pressure to adhere to a daily time schedule for each subject that was determined by administrators and to adhere to a set instructional guide was disconcerting when teachers felt the need to go a little longer or take another day or two for specific strategy work.
3. The frequency of questions asked in Category 3 increased for all teachers from the first observation to the last. Category 3 questions required students to reason and use evidence

to support their thinking. For the last observation almost all teachers asked Category 3 questions for at least a quarter of their total number of questions. This was a large increase since half of them had no questions for that category during the first observation.

4. The number of students at the higher levels of multiplication fact strategy use increased significantly from the pre-test to the post-test. Only a little over a quarter of the students were at a Level 2 or higher on the Levels of Multiplication Fact Strategy Implementation for Data Collection chart for the pre-test. This increased to over three-quarters of the students on the post-test data.

    When examining the interview and post-test scores for the 22 students that were selected to be interviewed, the percent of students scoring at a Level 3 rose from 41% to 64%. This notable increase could be attributed to the amount of discourse the students were doing in the classroom; the oral exchange during the interview was similar to the experiences they had during classroom discussions when they were describing their thinking to their peers and the teacher. The results showed that, for this group of students, the opportunities provided in the classroom made a significant difference on assessment outcomes, as indicated by the analyses from the McNemar and Friedman statistical tests.

Recommendations

The relationship between multiplication fact strategy instruction and student discourse in the field of mathematics education research has not been explored in much depth. This study makes a contribution to provoke further conversations and exploration about this specific topic. Continuing the search for productive instruction and finding links between research and actual application within the classroom is extremely important to educators. Some suggestions for future research studies and use of the present study's findings include the following.

1. Further research should endeavor to explore this topic using a quasi-experimental study comparing a group of teachers that use classroom discourse during fact strategy instruction to a group of teachers that does not use classroom discourse during fact strategy instruction. Such a study would be a next step because the present study was a non-experimental, descriptive study and was not intended to support generalizations.
2. Investigating the usefulness of a planning tool to assist teachers' implementation of higher-levels of questioning during instruction and classroom discussions is recommended. This current study did not require the use of a planning tool, but the data indicated more use of higher-level questions for the last observation when the teachers were more intentional in planning the discourse for the lesson.
3. Further study into the differences between students writing out their strategy explanations and orally providing their strategy explanations would give valuable information to educators. The results of the post-test and the interview scores showed a noticeable difference between the two modes of strategy explanation in this group of students.
4. Additional research exploring how fact strategy instruction benefits students' mathematical understanding of other concepts is suggested. Feedback from the teachers during their interviews indicated that fact strategy instruction impacted other foundational concepts, such as number sense.
5. District leaders, instructional coaches, and classroom teachers are encouraged to take steps in implementing fact strategy instruction with student discourse in elementary school classrooms when teaching multiplication facts and to investigate further with their own action research. The evidence from this study indicated immediate positive results for a large number of students and positive feedback from practicing teachers.

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Appendices

Using Student Discourse to Support Multiplication Fact Strategy Instruction

Appendix A

Agenda for Multiplication Fact Strategies and Student Discourse Workshop

Agenda for Multiplication Fact Strategies and Student Discourse Workshop

Prior to the workshop teachers will be asked to bring their Fact Strategies booklets produced by the district. If they are unable to find their copy, a replacement copy will be provided at the training.

- Introduction:
  - ✓ All teachers will introduce themselves.
  - ✓ Explanation of the study will be provided.
  - ✓ Consent forms will be handed out and explained – both teacher participant and parent consent form for the students.
  - ✓ Questions will be addressed about the study.

- Overview of purpose of fact strategies:
  - ✓ Use derived facts from known facts
  - ✓ Use properties of operations
  - ✓ Use the relationships between the numbers
  - ✓ Build number sense - flexibility and efficiency
  - ✓ Fact strategies in and of themselves are not the end goal – it is to assist students in seeing patterns, relationships, and be able to extend critical thinking beyond the facts to more complex operations and situations.

- Review the various fact strategies:
  - ✓ Allow participants to reflect on the ones they already know and explain how they work
  - ✓ Provide examples and situations for the strategies that are not as well known or used by the teachers. (Most likely the following: Double and Double Again [x4], Half and then Double [x6 and x8], Double and One More Set [x3]).
  - ✓ Understanding of the strategies will be assisted by using the same models and representations that the students will use: arrays, area models, group/group size/total

- Purposeful practice opportunities will be shared and discussed:
  - ✓ Using timed drills before students are ready has proved to be ineffective and even detrimental – will visit that research and discuss
  - ✓ More effective tools will be shared and we will discuss how these can be used with students (Practice Partner Pages, games, triangle flashcards)

- Overview of purpose of student discourse:
  - ✓ Language is how we process information and remember
  - ✓ Talk is the representation of thinking
  - ✓ Thinking is our goal in mathematics and so talking needs to be encouraged
  - ✓ Student discourse for our purposes is not used for students to try to figure out what they need to say to please the teacher but to allow student thinking to become visible to help the teacher help the student

- Discourse Planning Tool:

- ✓ A tool to assist in planning discourse will be shared. It will also be explained that this tool is not required. It is being shared to provide a structure that could be used to make student discourse more prominent in the classroom for this study. If this is not used, they may use any other type of planning process to increase student discourse during their fact strategy instruction.
- ✓ Each section of the tool will be explained as to the purpose and the process (Anticipate, Monitor, Select, Sequence, Connect)
- ✓ As we progress through the sections, adaptations to each will be discussed to make it user friendly for the teachers.

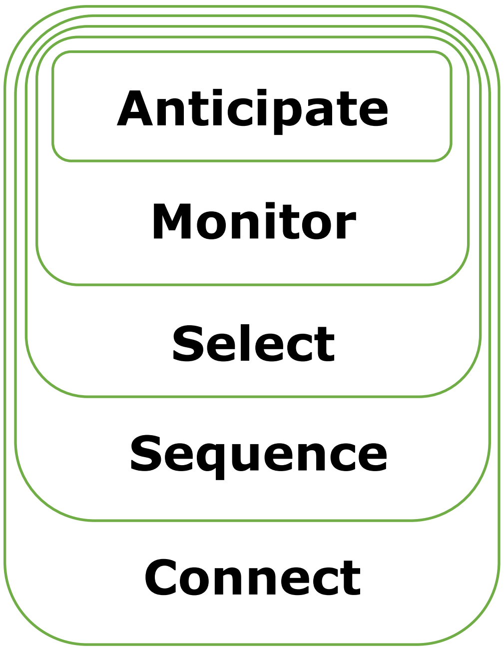
- ➢ Thoughts about the uses of this tool, or other strategies, to improve student discourse:
  - ✓ Challenges they are worried about during instruction will be shared and discussed
  - ✓ Opportunities that they see will be valuable for their students and for themselves will be shared and discussed

- ➢ Closing:
  - ✓ Review purpose and goals of the workshop
  - ✓ Answer questions
  - ✓ Administer post-test to all teachers
  - ✓ Set up schedule for observation dates and times
  - ✓ Provide copies of parent consent forms and have teachers sign and return their own participant consent forms.

Appendix B

Planning Tool for Classroom Discourse

Planning Tool for Classroom Discourse



This graphic from Stein, Engle, Smith, and Hughes (2008, p. 322) shows the different layers of decisions as the classroom teacher plans for classroom discourse. The authors called these the five practices for facilitating mathematics discussions. This sequence provided the foundation for the planning tool the classroom teacher could use as they plan their fact strategy discussions during the research study.

Each layer of this tool represents different decisions that the teacher must make in order to effectively plan and engage students in a classroom discussion that is purposeful and based in worthwhile mathematics. In order to make the planning more streamlined, this tool was supplied to the teacher in a format that allowed the teacher to have each component of the tool easily available with the critical questions listed on cards that were ringed together.

The five practices from Stein et al. (2008) are explained and the information on the card is listed:

Anticipate

Explanation – "Anticipating students' responses involved developing considered expectations about how students might mathematically interpret a problem, the array of strategies – both correct and incorrect – they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn" (Stein et al., 2008, pp. 322-323).

Process information –

1. Work out and solve the task before the students are asked to complete the problem. Working collaboratively with other teachers is best.
2. Actively think about various solution methods students might use as they work on the task and try all of them out.
3. Think about the misunderstandings and misconceptions students may bring to this task.
4. Refer to other reliable resources (print and electronic) that can provide extra information about the task or topic.

Monitor

Explanation – "The goal of monitoring is to identify the mathematical learning potential of particular strategies or representations used by the students, thereby honing in on which student responses would be important to share with the class as a whole during the discussion phase" (Stein et al., 2008, p. 326)

Process information –

1. Note the mathematical ideas that students are using.
2. Is the mathematics valid?
3. If students have a misconception or misunderstanding, what is the mathematical basis in their thinking and where does it seem that they got off course?
4. Ask questions to probe in understanding the students' thinking.

Select

Explanation – The teacher is in "control of which students present their strategies, and therefore what the mathematical content of the discussion will likely be" (Stein et al., 2008, p. 328). The teacher makes sure the important mathematical ideas are discussed and that the common misconceptions and misunderstandings are investigated.

Process information –

1. Purposeful selection of groups/individuals needs to be focused on important mathematics.
2. Let students or groups of students know that they will be sharing their thinking with the whole class.
3. All strategies and techniques do not need to be shared, but make sure you are making decisions based on mathematics and not how uncomfortable you are with a particular strategy.

Sequence

Explanation – "By making purposeful choices about the order in which students' work is shared, teachers can maximize the chance that their mathematical goals for the discussion will be achieved" (Stein et al., 2008, p. 329).

Process information –

1. Think about the mathematics involved and begin the sequence with one of the following situations:
    a. Start with a common strategy used by most groups.
    b. Begin with a strategy that is easy for everyone to understand.

c. Open with a group presentation that uses a common misconception so that can be handled first.

2. Have groups that have related or contrasting methods present one after the other so comparison is easier.
3. Not all strategies need to be presented to the whole class, but make sure there are no mathematically sound strategies that are being avoided because they are uncomfortable for you.

Connect

Explanation – "[T]he goal is to have student presentations build on each other to develop powerful mathematical ideas" (Stein et al., 2008, p. 330) and allow teachers to "help students draw connections between the mathematical ideas that are reflected in the strategies and presentations" (Stein et al., 2008, p. 330).

Process information –

1. Ask students to make judgements about which strategies may be more efficient or effective and then support their reasoning.
2. Question students so they begin noticing the patterns in mathematics.
3. Ask students what is similar or different about particular strategies or methods. Allow students to see the structure of mathematics.

Cards for teacher use on following pages. They will be copied on Bristol paper, cut and ringed for easy use. First page = front of cards; second page = back of cards.

Anticipate

Explanation:

"Anticipating students' responses involved developing considered expectations about how students might mathematically interpret a problem, the array of strategies – both correct and incorrect – they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn" (Stein et al., 2008, pp. 322-323).

Monitor

Explanation:

"The goal of monitoring is to identify the mathematical learning potential of particular strategies or representations used by the students, thereby honing in on which student responses would be important to share with the class as a whole during the discussion phase" (Stein et al., 2008, p. 326).

Anticipate

Process:

1. Work out and solve the task before the students are asked to complete the problem. Working collaboratively with other teachers is best.
2. Actively think about various solution methods students might use as they work on the task and try all of them out.
3. Think about the misunderstandings and misconceptions students may bring to this task.
4. Refer to other reliable resources (print and electronic) that can provide extra information about the task or topic.

Monitor

Process:

1. Note the mathematical ideas that students are using.
2. Is the mathematics valid?
3. If students have a misconception or misunderstanding, what is the mathematical basis in their thinking and where does it seem that they got off course?
4. Ask questions to probe in understanding the students' thinking.

Select

Explanation:

The teacher is in "control of which students present their strategies, and therefore what the mathematical content of the discussion will likely be" (Stein et al., 2008, p. 328). The teacher makes sure the important mathematical ideas are discussed and that the common misconceptions and misunderstandings are investigated.

Sequence

Explanation:

"By making purposeful choices about the order in which students' work is shared, teachers can maximize the chance that their mathematical goals for the discussion will be achieved" (Stein et al., 2008, p. 329).

Select

Process:

1. Purposeful selection of groups/individuals needs to be focused on important mathematics.
2. Let students or groups of students know that they will be sharing their thinking with the whole class.
3. All strategies and techniques do not need to be shared, but make sure you are making decisions based on mathematics and not how uncomfortable you are with a particular strategy.

Sequence

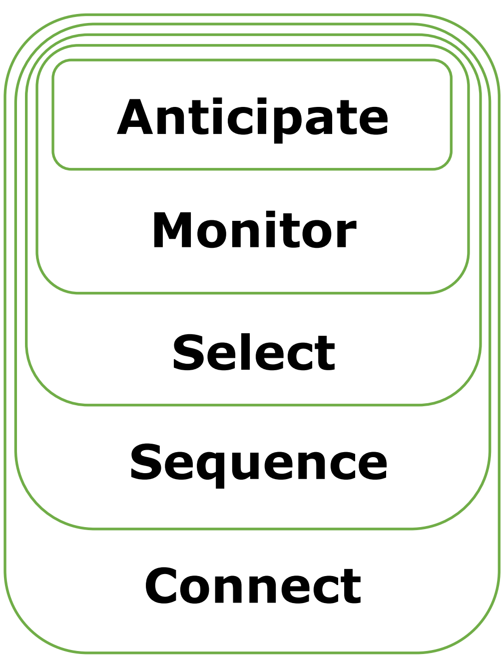
Process:

1. Think about the mathematics involved and begin the sequence with one of the following situations:
    a. Start with a common strategy used by most groups.
    b. Begin with a strategy that is easy for everyone to understand.
    c. Open with a group presentation that uses a common misconception so that can be handled first.
2. Have groups that have related or contrasting methods present one after the other so comparison is easier.
3. Not all strategies need to be presented to the whole class, but make sure there are no mathematically sound strategies that are being avoided because they are uncomfortable for you.

Connect

Explanation:

"[T]he goal is to have student presentations build on each other to develop powerful mathematical ideas" (Stein et al., 2008, p. 330) and allow teachers to "help students draw connections between the mathematical ideas that are reflected in the strategies and presentations" (p. 330).



Teacher Cards for Planning Classroom Discourse

Connect

Process:

1. Ask students to make judgements about which strategies may be more efficient or effective and then support their reasoning.
2. Question students so they begin noticing the patterns in mathematics.
3. Ask students what is similar or different about particular strategies or methods. Allow students to see the structure of mathematics.

Appendix C

Multiplication Fact Strategies Workshop Assessment

Name ___________________________ Date ____________

Multiplication Fact Strategies Workshop Assessment

This assessment will not be shared with anyone who evaluates your teaching. It will be used only to provide data to the researcher concerning the outcomes of the professional learning workshop essential to the dissertation study.

What are the different fact strategies that could be used to solve 7 x 8? Please name and explain each one thoroughly.

Thinking about the strategies that you explained for the previous problem, group them into the following categories:		
Basic	Intermediate	Advanced
Is there a strategy (or more than one) that is not appropriate to be used when solving 7 x 8? Explain why or why not.		

What concerns you most about teaching multiplication fact strategies?

Appendix D

Data Collection Chart by Teacher

Data Collection Chart by Teacher

Teacher A		
Date of 1st Observation / /	Date of 2nd Observation / /	Date of Final Observation / /
Start time/End time	Start time/End time	Start time/End time
Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*	Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*	Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*
Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*	Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*	Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*
Fact Strategy Taught	Fact Strategy Taught	Fact Strategy Taught
Comments:	Comments:	Comments:

Teacher B		
Date of 1st Observation / /	Date of 2nd Observation / /	Date of Final Observation / /
Start time/End time	Start time/End time	Start time/End time
Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*	Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*	Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*
Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*	Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*	Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*
Fact Strategy Taught	Fact Strategy Taught	Fact Strategy Taught
Comments:	Comments:	Comments:

Teacher C		
Date of 1st Observation / /	Date of 2nd Observation / /	Date of Final Observation / /
Start time/End time	Start time/End time	Start time/End time
Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*	Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*	Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*
Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*	Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*	Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*
Fact Strategy Taught	Fact Strategy Taught	Fact Strategy Taught
Comments:	Comments:	Comments:

Teacher D		
Date of 1st Observation / /	Date of 2nd Observation / /	Date of Final Observation / /
Start time/End time	Start time/End time	Start time/End time
Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*	Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*	Lesson Structure: ☐ *Introduction Lesson* ☐ *Practice Lesson* ☐ *Review Lesson*
Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*	Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*	Grouping of Students: ☐ *Whole Class* ☐ *Small Group* ☐ *Individual*
Fact Strategy Taught	Fact Strategy Taught	Fact Strategy Taught
Comments:	Comments:	Comments:

Appendix E

Categorizing Fact Strategy Instruction Questions During Classroom Discussions

Categorizing Fact Strategy Instruction Questions during Classroom Discussions

Constructed using the Cognitive Rigor Matrix and DOK Question Stems

1 - Recall and Reproduction	2 – Skills and Concepts	3 – Strategic Thinking and Reasoning	4 – Extended Thinking
Expect students to: recall information (fact, definition, term, or simple procedure); perform simple algorithms or apply formulas (one-step, well-defined, straight-forward procedures) *Bloom's* at this level: Remember, Understand, Apply, Analyze	Expect students to: make some decisions about how to approach the task; more than one-step problems; discuss similarities and differences but may not be able to explain why *Bloom's* at this level: Understand, Apply, Analyze	Expect students to: reason, plan, and use evidence; explain their thinking; use complex and abstract thinking; solve problems with more than one solution path and justify their thinking *Bloom's* at this level: Understand, Apply, Analyze, Evaluate, Create	Expect students to: use complex reasoning, planning, and thinking, usually over a period of time; make connections to other ideas; select an approach among many and justify reasoning *Bloom's* at this level: Understand, Apply, Analyze, Evaluate, Create
• Can you calculate? • Can you identify a pattern with this strategy? • How can you recognize this strategy? • Can you identify this strategy? • How would you describe this strategy?	• How are these two strategies alike and different? • Can you use a strategy that is based in place value understanding? • How would you classify this type of strategy? • How would you summarize your strategy? • Can you extend the pattern in the strategy to something besides a basic fact?	• Can you predict the outcome if we changed a number in the fact? • Can you explain more about your thinking with the strategy? • How does this connect to another strategy? • Why do you think your solution is reasonable? • Can you show a number model to represent your thinking with this strategy? • What evidence supports your thinking?	• What is the best strategy? Why? • Can you use this strategy all the time? Why or why not? • This problem can be solved with more than one strategy. Which one would you choose and why?
Example strategy questions: • How do you know this is the *Double and Double Again* strategy? • How would you describe the *Double and One More Set* strategy?	**Example** strategy questions: • How is the *Half then Double* strategy similar to the *Double and Double Again* strategy? • Will the *Half then Double* strategy work with another problem that is not a basic fact?	**Example** strategy questions: • Why did you half this specific number in the *Half then Double* strategy? • Can you explain why you used the *Fives* and *Doubles* strategies for 6 x 7?	**Example** strategy questions: • Why do you think the *Half then Double* strategy is the best for this problem? • The problem 4 x 8 can be solved using different strategies. Which one would you choose and why?

Appendix F

Excerpt From an Observation to Illustrate Teacher's Questions and Students' Responses

Excerpt From an Observation to Illustrate Teacher's Questions and Students' Responses

Teacher writes the problem, **8 x 6**, at the top of a tablet of chart paper and asks students to tell how they solved this problem from the assessment. Several students raise their hands.

T: Damarian*, how did you solve it? (**Level 1** – asking student to recall solution from assessment.)

S: Double and Double Again.

T: So you're thinking back almost a month and a half to two months ago. When you doubled and doubled again, tell me, what did you do? (**Level 2** – asking students to explain the strategy.)

S: I put the 6 with the 4 and the 6 with the 4.

T: Okay. What number are you doubling there? (**Level 1** – recall)

S: You're adding . . . you're doubling the 6.

T: So if I double that 6 . . .

S: No – 6 and 4 and . . .

Several students saying one of the following: No! Decompose! That's decompose!

T: So let's listen to Damarian, but if you have a comment raise your hand.

T: So Damarian if you double and double again, what you could do . . . . I have feeling that this is your favorite one, so turn around and look where it says "Double and Double Again" or says "Damarian" over there on the door [*teacher is referring to a chart the students worked on in the past with strategies and their names listed next to various strategies on the chart*]. Okay? So what did you do? (**Level 2** – explaining the strategy)

S: I . . . what I did is . . .

T: But notice what that fact is. That fact is 4 x 6. Okay, now we're at 8 x 6.

S: I . . . put it . . . put the 8 into 4 and 4 and then I did the 6.

T: Okay – so hold on. *[Teacher begins to write down what the student is saying on a large chart tablet at the front of the class.]* So what you are basically saying is . . .? (**Level 3** – strategic thinking and reasoning by asking student to provide reasoning behind their solution)

S: Decompose.

T: So you're basically decomposing, Damarian. The 8 right? (**Level 1** – recall)

S: Yes.

T: Now I can kind of see what you are talking about. So you took the 8 and made it into 4 and 4. *[Writes the decomposition of the 8 into 4 + 4.]* Correct? [*Refers to the decomposition of the 8 into two sets of parentheses – (4 x) + (4 x).]* Is this what you are saying you did? (**Level 1** – recall)

S: Yes.

T: Okay and then what did you do? So inside of each set of parentheses what did you multiply by? (**Level 1** – recall)

S: 6.

T: Okay so, Damarian . . . what is 4 x 6, buddy? (**Level 1** – recall)

S: 4 x 6 is 24.

T: And, Damarian, if we add 24 and 24, what do we get? (**Level 1** – recall)

S: 48.

T: Who has another way to share? (**Level 3** since there is an expectation that this problem has more than one solution path.)

*The student name is a pseudonym.

Appendix G

Teacher Interview Questions

Teacher Interview Questions

Participating teachers will be asked the following questions which are grouped according to topic:

Setting the stage for the interview –
I want to remind you that at any time you can decline to answer any question and you may even terminate this interview with no repercussions from me or any administration within the Wichita Public School system.

- Do you have any questions before we begin?

- How do you think your fact strategy lessons went?

- Was there anything you wanted to say about the lessons before I ask some of these other questions?

Questions concerning comparisons about their instruction –

- Thinking about this last lesson I observed and the first lesson I observed, do you think your instruction with fact strategies has improved?
    - *If the answer is yes* - what do you think has made the greatest impact in this improvement?
    - *If the answer is no* – why do you think there has been no improvement?

- As you think about the questions you asked during these fact strategy lessons, do you believe the level of questioning has included more of the higher-order thinking questions than when you have instructed fact strategies in the past?
    - *If the answer is yes* – why do you think you have been able to ask more of these types of questions?
    - *If the answer is no* – do you feel that you already ask enough of these questions during fact strategy instruction?
        - *AND/OR* – do you believe higher-order thinking questions are appropriate for your students?

- What kind of supports do you think you would need to increase the number of higher-order thinking questions in your instruction?

- What type of preparation did you do when thinking about the types of questions you would ask your students during instruction?

Questions concerning comparisons about student understanding and use of multiplication fact strategies –

- Do you think your students have a better understanding about fact strategies now than they did at the beginning of the semester?

    - *If the answer is yes* – what do you think is the biggest contributor to their understanding?
    - *If the answer is no* – what do you think has prevented them from being successful?

- Do you notice your students using more of these strategies outside of the fact strategy lessons?
    - *If the answer is yes* – when are they using them? How successful are they when using them?
    - *If the answer is no* – why do you think this is not transferring to use outside of the lessons?

- Do you feel that the students see themselves as successful and fluent with the strategies that they have learned?
    - *If the answer is yes* – what do you think helped them feel this success?
    - *If the answer is no* – why do you think the strategy work did not help them feel successful?

- Did you feel that your students began responding to your higher-order thinking questions in a way that promoted deeper thinking for them?
    - *If the answer is yes* – do you remember specific instances that you could tell me about?
    - *If the answer is no* – why do you think they were not thinking more deeply?

Challenges with this instruction –

- What are some of the challenges you found with using this type of instruction?

- What do you believe to be the top 2 weaknesses with this type of instruction?

Benefits of this instruction –

- What are some of the benefits using this type of instruction?

- What would you list as the top 2 strengths with this type of instruction?

Wrapping up the interview –

- Is there anything you would change about how you have instructed your students using the fact strategies?

- Is there anything you would like to tell me that I haven’t asked you about?

Thank you! Please let me know if you have any questions or concerns later.

Appendix H

Fact Strategy Measurement Tool

Fact Strategy Measurement Tool

Explain how you would solve the problem below:

$$\begin{array}{r} 8 \\ \underline{\times\ 6} \end{array}$$

Appendix I

Levels of Multiplication Fact Strategy Implementation for Data Collection

Levels of Multiplication Fact Strategy Implementation for Data Collection

Level	Name	Description
1	**Constructing Meaning & Counting All**	Students are making sense of equal groups and the individual unit within each group.
2	**Counting Strategies** (includes skip counting)	Students understand equal groups. They create skip-count lists for each multiplier. These students may need to use their fingers to keep track of the number of groups.
3	**Derived Facts**	Students are taking unknown facts and decomposing and recomposing factors into known facts using the properties of operations. These students may need to write or verbally explain the derived facts being used as they work on flexibility and efficiency.
4	**Mastery** (derived facts may be used but application is quick and efficient)	Students are able to provide answers to single-digit multiplication problems efficiently.

Level 4 was not evaluated in this study since students were required to explain their thinking.

Research used to construct sequence of multiplication fact strategy implementation

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Appendix J

Fact Strategy Student Interview Tool

Fact Strategy Student Interview Tool

This tool is to be completed in an interview format with select students based on their correct response on the written task completed in class. Read the "Assent Form for Student Fact Strategy Interview" before beginning the interview.

My name is Mrs. Debbie Thompson, and I am learning about how children think about solving multiplication facts. I would like you to answer some questions that will take less than 10 minutes. I would like you to explain how you solved the multiplication fact problem you worked on earlier, and then I will show you one more problem to answer and explain your solution. If you don't feel like talking with me, you don't have to. You can stop at any time and that will be all right. Do you want to answer these problems for me?

Each selected student will be shown the multiplication fact problem from the written task done in class a few days earlier. The student will be asked to explain how he/she figured out the correct answer and/or asked to share how he/she would explain getting a correct answer to someone who didn't know the answer.

Next the student will be given a card with the multiplication problem shown below. The student will be asked to give the answer to the fact and then explain how he/she solved that problem.

$$\begin{array}{r} 6 \\ \underline{\times\ 7} \end{array}$$

Fact Strategy Student Interview Tool ~ recording page

This page to be completed by researcher to note each student's strategies.

ID: ____________________________ Date & Time: ______________________

$$\begin{array}{r} 8 \\ \underline{\times\ 6} \end{array}$$

- I split the 6 into 5 + 1. Then 5 x 8 = 40 and 1 x 8 = 8, so 40 + 8 = 48.
- I split the 8 into 4 + 4. Then 4 x 6 = 24 and 24 + 24 = 48.
- I split the 6 into 3 + 3. Then knew that 8 x 3 = 24, so 24 + 24 = 48.
- Other: ____

____

____

$$\begin{array}{r} 6 \\ \underline{\times\ 7} \end{array}$$

- I split the 6 into 3 + 3. I know that 7 x 3 is double plus one more set of 7, so 14 + 7 = 21. 21 + 21 = 42.
- I split the 6 into 5 + 1. Then 5 x 7 = 35 and 1 x 7 = 7, so 35 + 7 = 42.
- I split the 7 into 5 + 2. Then 5 x 6 = 30 and 2 x 6 = 12, so 30 + 12 = 42.
- Other: ____

____

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Appendix K

Excerpts From Student Interviews to Illustrate Students' Responses

Excerpts From Student Interviews to Illustrate Students' Responses

The researcher (R) explained to each student (S) that he/she will be asked to explain how the problem from the assessment and one extra problem would be solved.

The following student **scored a 3** by decomposing both problems shown by the researcher. This excerpt is showing the solution for the 8 x 6 problem.

S: I can decompose the 8 into 3, I mean into 5 and 3. And then I can . . . and then I can 5 times 6 and then 3 times 6. [*Student pauses*.]

R: And then what would you get? So you said 5 times 6, you would get . . . ?

S: 30.

R: And then . . . ?

S: 18.

R: And so then what do you do with those numbers?

S: I add them together and I get 48.

The following student **scored a 2** by skip counting to solve 8 x 6.

R: [*Researcher displays the problem 8 x 6*.] What is the easiest way for you to solve that problem?

S: Counting by 6s.

R: Counting by 6s. Can you do that for me?

S: Yeah.

R: Okay, go ahead and do that for me.

S: 6, 12, 18, 24, 30, 36, 42, . . . 48. Is that all?

R: Is that it?

S: [*Student shakes head yes*.]

R: Okay. So you counted by 6 how many times?

S: 8 times

Appendix L

Raw Data From Students' Pre-tests, Post-tests, and Interviews

Raw Data From Students' Pre-tests, Post-tests, and Interviews

Student ID Number	Pre-test Score	Post-test Score	Interview Score
TAS1	2	3	3
TAS2	0	3	3
TAS3		3	3
TAS4	1	3	3
TAS5	1	3	3
TAS6	1	3	3
TAS7	1	2	
TAS8		3	
TAS9	1	3	
TAS10	1	2	
TAS11	1	2	
TAS12	1	1	
TAS13	1	2	
TAS14	1	2	
TAS15	1	3	
TAS16	1	2	
TAS17	3	3	
TAS18	1	3	
TBS1		3	3
TBS2	1	2	2
TBS3	1	3	3
TBS4		3	3
TBS5	1	1	2
TBS6	1	1	
TCS1	1	2	2
TCS2	2	2	2
TCS3	2	2	2
TCS4	2	2	2
TCS5	2	2	3
TCS6		1	2
TCS7	1	1	
TCS8	2	2	
TCS9	1	2	
TCS10	2	2	
TCS11	0	2	
TCS12	1	2	
TCS13	2	2	
TCS14	2	3	
TCS15	1	3	
TDS1	2	2	2
TDS2	2	2	3
TDS3	1	2	3
TDS4	1	2	3
TDS5		2	3
TDS6	1	1	
TDS7	1	1	
TDS8	1	1	
TDS9	1	1	
TDS10	1	1	