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Shear Strength of Lightly Reinforced T-Beams in Negative Bending

by Carlos P. Rodrigues and David Darwin

Nine restrained T-beams were tested in a study of the negative moment region shear strength of lightly reinforced concrete T-beams. Variables included the longitudinal reinforcement ratio ρ_w (0.47 percent and 0.70 percent) and the nominal stirrup strength (0 to 84 psi [0 to 0.60 MPa]). The test results are compared with the shear provisions of "Building Code Requirements for Reinforced Concrete (ACI 318-83)."

Both the concrete and stirrup contributions to shear strength are lower in negative than in positive moment regions. In both moment regions, the cracking shear is lower and the stirrup contribution is higher than predicted by ACI 318. Current provisions are conservative for positive moment regions of beams with ρ_w to ≥ 0.5 percent. The provisions are also conservative for negative moment regions in which the factored shear V_u is less than or equal to the design shear strength of the concrete ϕV_c. However, ACI 318 appears to be unconservative for negative moment regions of beams with $\rho_w \leq 0.7$ percent and $V_u > \phi V_c$. The provisions may also be unconservative for joists with $\rho_w \leq 1$ *percent and* $\phi V_c/2 \leq V_u \leq \phi V_c$.

Keywords: beams (supports); cracking (fracturing); **diagonal tension;** flexural strength; loads (forces): reinforced concrete; reinforcing steels; research; **shear strength;** stirrups; structural engineering; **T-beams;** web reinforcement.

The effect of the longitudinal reinforcement ratio on the shear cracking load of reinforced concrete beams is not accurately incorporated in the design requirements of the ACI Building Code, ACI 318-83.[1] While the shear cracking loads in ACI 318-83 are conservative for beams having longitudinal reinforcement ratios ρ_w greater than 1 percent, they are unconservative for beams with ρ_w less than 1 percent.[2-9] In spite of this, the overall shear provisions have appeared to be satisfactory, since the Building Code underestimates the contribution of web reinforcement[9-11] and requires its use in beams with shears greater than one-half of the predicted cracking shear ($V_u \geq \phi V_c/2$).

Most of what is known about shear strength is based on tests of simply supported beams subjected to positive bending, even though most reinforced concrete beams are continuous. This tacitly assumes that continuity has no effect on shear strength. However, continuous beams behave differently. Most notably, the flanges crack in negative moment regions, giving a smaller effective section to carry shear than is available in positive moment regions. The flexural steel in the negative moment regions usually consists of top-cast bars, which have a lower bond strength than the bottom-cast bars in positive moment regions. This results in both wider flexural cracks and wider flexure-shear cracks in regions of negative moment. There is specific experimental evidence that the negative moment region has a lower shear strength than the positive moment region.[11]

Finally, as suggested by Ferguson,[12] continuous beams may have a reduced shear strength due to inclined flexure-shear cracks which result in an increased effective shear-span and a reduced concrete contribution to shear capacity.

Differences in the behavior of continuous and simple-span beams, along with the reduced concrete shear capacity in beams with $\rho_w < 1$ percent, makes the shear strength of continuous reinforced concrete beams with low values of longitudinal reinforcement a particular concern.

This paper presents the results of a limited experimental study of the negative moment region shear strength of lightly reinforced concrete T-beams. The primary variables in this investigation were the longitudinal reinforcement ratio and the nominal stirrup strength. The test results, along with test results for lightly reinforced simple-span beams,[9] are analyzed and compared with the ACI Building Code provisions.[1] The details of the investigation are presented in Reference 13.

RESEARCH SIGNIFICANCE

The current research is significant because it represents the first test data on the negative moment region shear strength of beams with low values of both flexural and shear reinforcement. These results are of par-

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ticular interest because they show that the shear provisions of ACI 318 may be significantly unconservative in negative moment regions of beams with ρ_w less than 0.7 percent and factored shears V_u greater than the design shear capacity of the concrete ϕV_c. The study also suggests that the shear provisions may be unconservative for joists with ρ_w less than 1 percent and $\phi V_c/2 \leqslant V_u \leqslant \phi V_c$.

EXPERIMENTAL INVESTIGATION

Test specimens

Nine restrained reinforced T-beams were tested to failure. The details and dimensions of the beams are shown in Fig. 1 and Table 1. The flange width b and thickness h_f were 24 in. and 4 in. (610 mm and 102 mm), respectively. The web width b_w was 7½ in. (191 mm), and the total beam depth was 18 in. (457 mm). The beams had a 15-ft (4.57-m) span, with a 5-ft (1.52-m) cantilever on one side. Extensions of 3½-ft (1.07-m) were added on each end of the beams to increase the embedment and prevent slippage of the flexural steel. The shear-span-to-depth ratio for the beams was approximately equal to 4, with the shear-span extending from the point of inflection to the maximum positive or negative moment sections. The moment and shear diagrams for the applied loads are shown in Fig. 2.

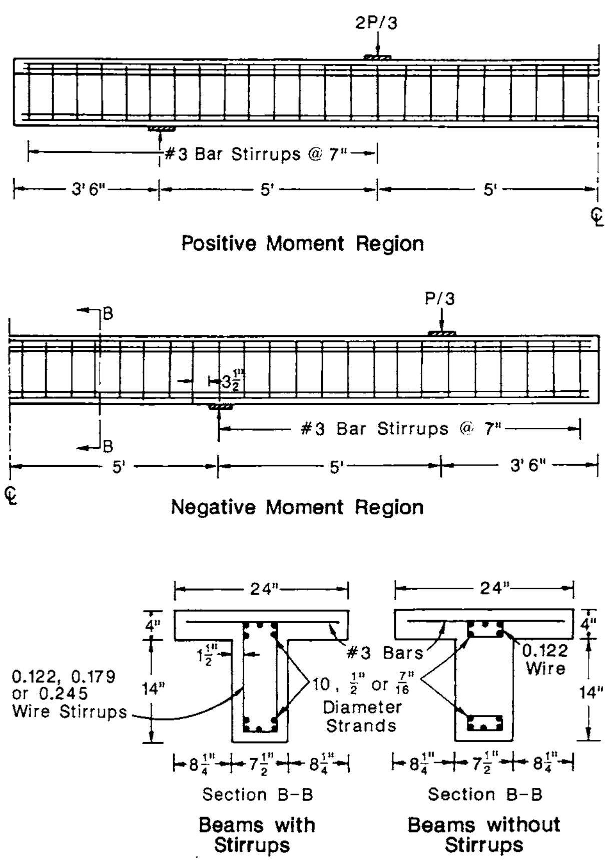

Fig. 1—Beam details (1 in. = 25.4 mm)

Two series, D and E, were tested, with longitudinal reinforcement ratios ρ_w (both top and bottom) equal to 0.70 percent and 0.47 percent, respectively. The longitudinal reinforcement consisted of non-prestressed, prestressing strands.

Strands were used to provide a low longitudinal reinforcement ratio and at the same time insure flexural safety. The specimens correspond to continuous beams with low values of ρ_w that have undergone local flexural failure but which remain intact due to moment redistribution; although the local flexural strength has been exceeded, the capacity is governed by the shear strength.

Web reinforcement consisted of smooth low-carbon wires. The nominal shear stress provided by the stirrups $v_s = \rho_v f_{vy}$ ranged from 0 to 84 psi (0 to 0.60 MPa) (ρ_v = shear reinforcement ratio = $A_v/b_w s$ in which A_v = stirrup area, s = stirrup spacing, and f_{vy} = stirrup yield strength). The wires were used in the test region, which consisted of one positive and one negative shear-span. Heavy web reinforcement was provided elsewhere.

Table 1 — Beam properties and test results

Beam	d, in.	b_w, in.	$\rho_w = \frac{A_s}{b_w d}$, percent	$\rho_v f_{vy}$, psi	f'_c,* psi	f'_r,† psi	a/d	v_c, psi	v_n, psi
Positive moment region									
D-80(1)	15.44	7.58	0.69	82.9	5380	445	3.89	103	238
D-80(2)	14.87	7.52	0.72	73.0	4070	435	4.03	139	—
D-40	14.69	7.50	0.73	37.0	4200	500	4.08	100	—
D-20	14.58	7.52	0.73	21.6	4290	515	4.11	145	—
D-0	14.69	7.53	0.73	0.0	4540	440	4.08	114	—
E-80	14.78	7.51	0.49	73.5	4010	500	4.06	114	—
E-40	15.14	7.50	0.48	36.8	4550	560	3.96	127	181
E-20	15.46	7.51	0.47	22.2	4210	475	3.88	89	—
E-0	15.43	7.50	0.47	0.0	4500	565	3.89	114	133
Negative moment region									
D-80(1)	15.18	7.52	0.71	83.6	5380	445	3.95	101	—
D-80(2)	15.32	7.51	0.70	73.1	4070	435	3.92	100	200
D-40	15.39	7.52	0.70	37.0	4200	500	3.90	111	146
D-20	15.21	7.51	0.71	21.6	4290	515	3.94	111	148
D-0	15.76	7.51	0.68	0.0	4540	440	3.81	119	138
E-80	15.04	7.51	0.48	73.5	4010	500	3.99	64	152
E-40	15.54	7.50	0.46	36.8	4550	560	3.86	—	—
E-20	15.42	7.50	0.47	22.2	4210	475	3.89	93	127
E-0	16.13	7.52	0.45	0.0	4500	565	3.72	68	—

All beams — b = 24 in., a = 60 in., s = 7 in.
Group D: A_s (5½ in. diameter strands, Grade 270, low relaxation) = 0.805 in.
Group E: A_s (5⁷⁄₁₆ in. diameter strands, Grade 270, low relaxation) = 0.540 in.
1000 psi = 6.895 MPa; 1 in. = 25.4 mm
*Compressive strength from 6 × 12-in. cylinders.
†Modulus of rupture from 6 × 6-in. beams, third point loading on an 18 in. span.

Material properties

Concrete — Type I portland cement and $\frac{3}{4}$ in. (19 mm) maximum size coarse aggregate were used. Air-entrained concrete was supplied by a ready-mix plant. Compressive strengths and moduli of rupture are summarized in Table 1.

Steel — ASTM A 416 Grade 270 (1860 MPa), $\frac{1}{2}$ in. (13 mm) diameter seven-wire low-relaxation strands and Grade 250 (1724 MPa), $\frac{7}{16}$ in. (11 mm) diameter seven-wire low-relaxation strands were used as the flexural steel in Series D and E, respectively. The strands were exposed to the weather and allowed to rust to improve bond and prevent slip during the tests. ASTM A 615 Grade 60 (414 MPa) #3 (9.5 mm) deformed billet steel bars were used as transverse flange reinforcement and as web reinforcement outside the test region. The low-carbon smooth wires used as stirrups within the test region had diameters of 0.122, 0.179, and 0.245 in. (3.10, 4.55, and 6.22 mm) for beams with $\rho_v f_{vy}$ of approximately 20, 40, and 80 psi (0.15, 0.30, and 0.60 MPa), respectively. Information on the reinforcing steel is summarized in Table 1.

Test procedure

Loads were applied to the beams at two points, using a longitudinal loading beam (Fig. 3), to develop equal maximum positive and negative moments and provide a constant value of applied shear throughout the length of the beam (Fig. 2).

The deflection at the load points was recorded. Stirrup strains were obtained with strain gages attached at midheight, while changes in the overall depth of the beam due to diagonal tension cracking were measured using dial gages attached to specially designed shear cracking frames.[9]

The beams were loaded incrementally until failure was obtained. Cracks were marked for each load increment. About 75 min were required for a test.

Results and observations

As the beams were subjected to increasing loads, flexural cracks were observed at or near the maximum moment sections. As the load increased, these cracks extended vertically to about the centroid of the uncracked section. Typical crack patterns are shown in Fig. 4. The cracks curved towards the point of applied load in the positive shear-span and towards the cantilever support in the negative shear-span.

The failure mode for all beams was diagonal tension. Six failed in the negative shear-span and three in the positive shear-span. In the positive moment region (Fig. 5), the critical shear crack appeared after the cut across earlier shear cracks, while in the negative moment region (Fig. 6), one of the initial shear cracks grew to become the critical shear crack.

In the negative shear-span, when the beams approached failure, a secondary crack developed and propagated along the intersection of the web and flange, usually cutting across two stirrups for the beams in Series D and one stirrup in Series E, as the bottom end of the critical shear crack neared the support. The bottom end of the critical shear crack extended until it reached the support.

The number and width of cracks depended on the amount of flexural reinforcement. The beams in Series D exhibited a greater number of cracks of narrower width than the beams in the more lightly reinforced Series E. No shear cracks were observed at or near the point of inflection.

The negative moment regions had fewer, more widely spaced cracks than the positive moment regions (Fig. 4). This difference in the crack patterns is in all likelihood due to the fact that the top-cast flexural reinforcement, which controls flexural cracking in the negative moment regions, has a lower bond strength than the bottom-cast flexural reinforcement, which controls flexural cracking in the positive moment regions. This difference in bond strength is commonly referred to as the top-bar effect.

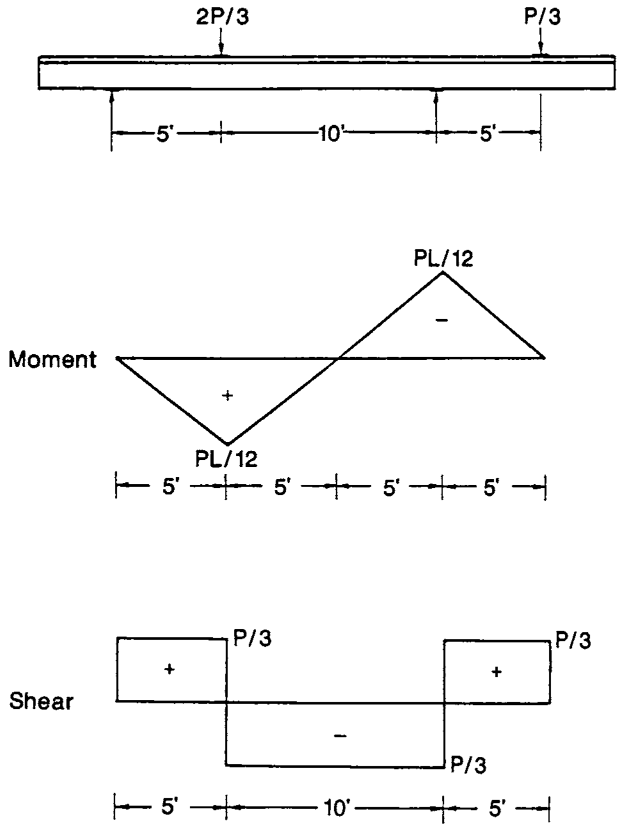

Fig. 2—Moment and shear diagrams due to loadings (1 ft = 305 mm)

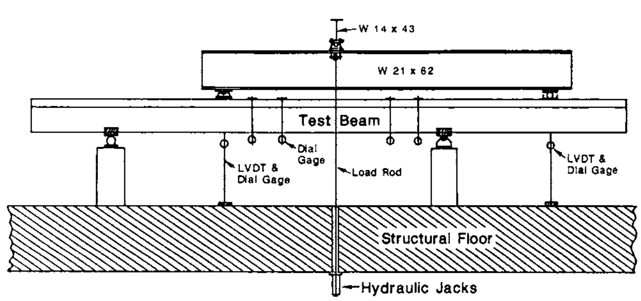

Fig. 3—Test setup

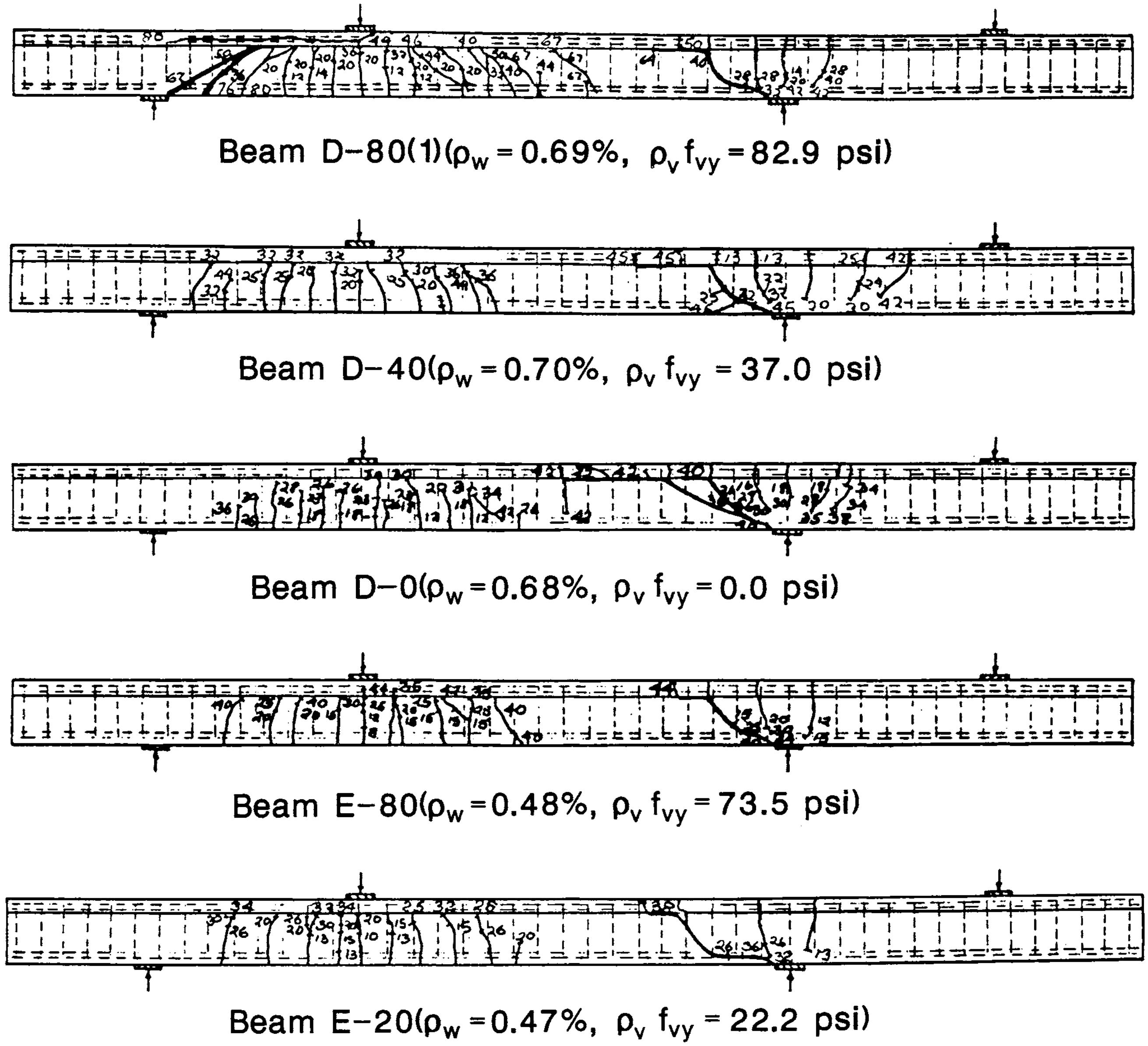

Fig. 4—Typical crack patterns

Stirrups intersected by a critical diagonal tension crack yielded prior to failure.

A summary of the shear cracking stresses (based on crack patterns) along with the nominal shear stresses is given in Table 1.

EVALUATION OF EXPERIMENTAL RESULTS

Shear cracking

For Series D and E, the shear cracking stress v_c is on the average 18 percent greater in the positive moment region than the negative moment region (Table 1). This should be expected since a T-beam is effectively a rectangular beam in the negative moment region. In the positive moment region, however, the compressive stresses are distributed over the area of the flange, which affects the total stress distribution. It is also likely that the lower bond strength of the top-cast reinforcement contributes to the lower relative shear strength of the negative moment region. The higher value of v_c in positive bending agrees with the results of Placas and Regan,[6] who found that beams with 12 in. or wider flanges had about 20 percent more shear strength than rectangular beams.

For beams without web reinforcement, the cracking shear is given by the following equation in ACI 318-83[1].

$$v_c = (1.9\sqrt{f'_c} + 2500\,\rho_w V_u d/M_u)b_w d \qquad \text{(1a)}$$
$$\leqslant 3.5\sqrt{f'_c}\,b_w d$$

in which f'_c = compressive strength of concrete, psi; V_u = factored shear force at section; M_u = factored bending moment at section; and d = effective depth.

Or more conservatively

$$V_c = 2\sqrt{f'_c}\,b_w d \qquad \text{(2a)}$$

In terms of shear stress v_c Eq. (1a) and (2a) can be rewritten as

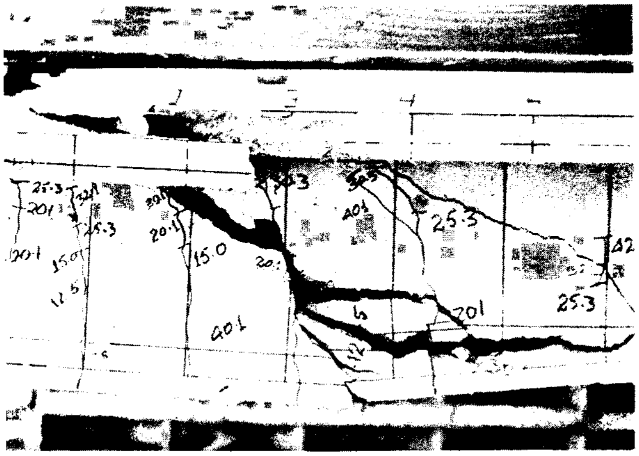

Fig. 5—Positive moment region shear failure, Beam E-40

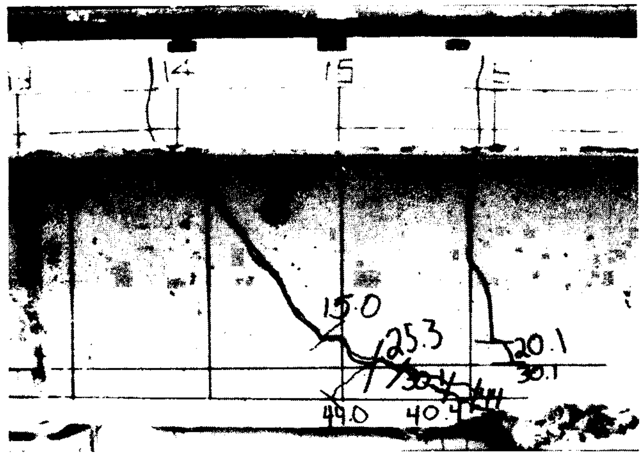

Fig. 6—Negative moment region shear failure, Beam E-80

$$v_c = V_c/b_w d \quad \text{(1b)}$$
$$= 1.9\sqrt{f'_c} + 2500\rho_w V_u d/M_u \leqslant 3.5\sqrt{f'_c}$$

$$v_c = V_c/b_w d = 2\sqrt{f'_c} \quad \text{(2b)}$$

For the beams tested in the current study, as well as those tested by Palaskas, Attiogbe, and Darwin,[9] the values of v_c obtained with Eq. (1) and (2) differ by less than 4 psi (0.03 MPa). For the balance of this paper, comparisons will be made using Eq. (2), the less conservative (for low ρ_w) but more commonly used expression.

For the positive moment region, the measured shear cracking stresses are on the average 13 percent lower than predicted by ACI 318, while for the negative moment region, the shear cracking stresses are on the average 29 percent lower than predicted by ACI 318 (Table 2).

Table 2 — Comparison of measured shear cracking stresses with values predicted by ACI 318-83

Beam	v_c (ACI),* psi	Positive moment region		Negative moment region	
		v_c (test), psi	v_c (test) / v_c (ACI)	v_c (test), psi	v_c (test) / v_c (ACI)
D-80(1)	146.7	103.4	0.70	100.8	0.69
D-80(2)	127.6	138.7	1.09	100.0	0.78
D-40	129.6	99.8	0.77	110.9	0.86
D-20	131.0	144.9	1.11	111.2	0.84
D-0	134.8	113.9	0.84	119.3	0.89
E-80	126.6	113.9	0.90	63.7	0.51
E-40	134.9	126.7	0.94	—	—
E-20	129.8	88.7	0.68	92.8	0.71
E-0	134.2	114.1	0.85	67.6	0.50
Mean			0.87		0.71
Standard deviation			0.15		0.15
Coefficient of variation			17.2		21.1

*$v_c = 2\sqrt{f'_c}$.
1000 psi = 6.895 MPa.

Stirrup effectiveness

Current ACI procedures for the shear design of beams with web reinforcement have appeared to be conservative,[9-11] especially in positive moment regions. The shear force resisted by the stirrups V_s is calculated assuming that the inclined crack has a horizontal projection equal to the effective depth of the beam d. For vertical stirrups, the nominal shear stress resisted by the stirrups v_s is expressed as

$$v_s = V_s/b_w d = A_v f_{vy}/b_w s = \rho_v f_{vy} \quad (3)$$

The increase in the shear stress $v_n - v_c$ above the shear cracking stress v_c is a measure of the effectiveness of the web reinforcement and can be compared to the predicted stirrup capacity, $\rho_v f_{vy}$. $v_n - v_c$ includes the shear carried by stirrups, as well as the shear carried by dowel action and aggregate interlock. The increment of stress $v_n - v_c$ is given in Table 3 for the current study and for the beams tested by Palaskas, Attiogbe, and Darwin.[9]

Postive moment region — The test results for beams with stirrups that failed in the positive moment region, D-80(1) and E-40, are combined with the results of the lightly reinforced, simply supported T-beams tested by Palaskas, Attiogbe, and Darwin.[9] The increment of shear stress $v_n - v_c$ is compared with $\rho_v f_{vy}$ in Fig. 7. Using a regression analysis, the following relationship is obtained

$$v_n - v_c = 1.59\rho_v f_{vy} + 1.4 \quad (4)$$
correlation coefficient, $r = 0.96$

Hence, the contribution of the web reinforcement in the positive moment region is on the average $1.59\rho_v f_{vy}$, which is 59 percent more effective than predicted by ACI 318-83[1] in Eq. (3). However, the average does not tell the whole story. For the beams tested by Palaskas, Attiogbe, and Darwin[9] with ρ_w = 0.95 percent, the contribution of web reinforcement to shear capacity matches the value of $1.8\rho_v f_{vy}$ obtained by Bresler and Scordelis[10] for beams with higher reinforcement ratios

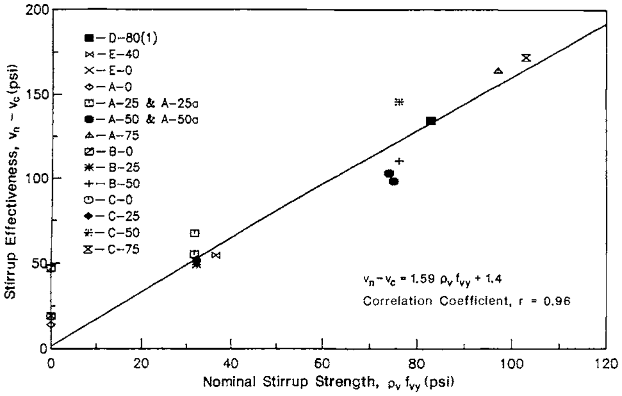

Fig. 7—Effectiveness of web reinforcement $v_n - v_c$ in positive moment regions (1000 psi = 6.895 MPa)

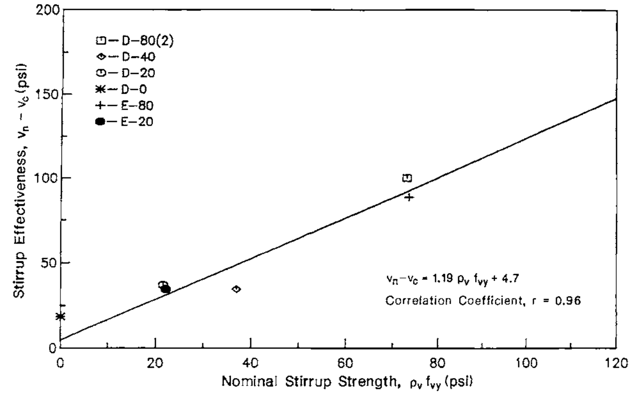

Fig. 8—Effectiveness of web reinforcement $v_n - v_c$ in negative moment regions (1000 psi = 6.895 MPa)

Table 3 — Stirrup effectiveness $v_n - v_c$ and comparison of measured nominal shear stresses with values predicted by ACI 318-83

Beam	$\rho_v f_{vy}$, psi	$v_n - v_c$, psi	v_n (test), psi	v_n (ACI),† psi	v_n (test) / v_n (ACI)
Positive Moment Region					
D-80(1)	82.9	135	238	230	1.03
E-40	36.8	55	181	172	1.05
E-0	0.0	19	133	134	1.00
#2*	0.0	—	147	138	1.10
A-00*	0.0	14	126	138	0.91
A-25*	31.8	55	167	169	0.99
A-25a*	31.8	68	182	170	1.10
A-50*	74.0	103	225	198	1.14
A-50(a)*	75.0	98	213	202	1.05
A-75*	97.0	164	275	234	1.18
#1*	110.2	—	286	259	1.10
B-00*	0.0	47	136	136	1.00
B-25*	32.4	49	153	166	0.92
B-50*	76.2	110	208	209	1.00
C-00*	0.0	19	115	131	0.88
C-25*	32.4	52	166	161	1.03
C-50*	76.2	146	261	207	1.26
C-75*	103.0	172	266	234	1.14
Mean					1.04
Coefficient of variation					9.3%
Mean (beams with stirrups)					1.07
Coefficient of variation					8.4%
Negative moment region					
D-80(2)	73.1	100	200	201	1.00
D-40	37.0	35	146	167	0.82
D-20	21.6	37	148	153	0.97
D-0	0.0	18	138	135	1.02
E-80	73.5	89	152	200	0.76
E-20	22.2	35	127	152	0.84
Mean					0.91
Coefficient of variation					8.4%
Mean (beams with stirrups)					0.89
Coefficient of variation					11.0%

*Test results of Palaksas, Attiogbe, and Darwin.[9]
†v_n (ACI) $= 2\sqrt{f'_c} + \rho_v f_{vy}$.
1000 psi = 6.895 MPa.

(ρ_w = 1.8 percent and 2.4 percent). Haddadin, Hong, and Mattock[11] found the contribution for beams with ρ_w = 3.8 percent to be at least $1.75\rho_v f_{vy}$ for values of $\rho_v f_{vy}$ less than 200 psi. For the beams with ρ_w = 0.50 percent and 0.70 percent the stirrup contribution is noticeably lower, at about $1.4\rho_v f_{vy}$ for both values of ρ_w.

Negative moment region — $v_n - v_c$ is compared to $\rho_v f_{vy}$ in the negative moment region in Fig. 8. Using a regression analysis, the following equation is obtained

$$v_n - v_c = 1.19\rho_v f_{vy} + 4.70 \quad (5)$$

correlation coefficient, r, = 0.96

On the average, the web reinforcement in the negative moment region is only 1.19 times as effective as predicted by Eq. (3), compared to about 1.4 for the beams with similar reinforcement ratios in the positive moment region.

Although Haddadin, Hong, and Mattock[11] had only limited data in the negative moment region, they also obtained a lower stirrup contribution to shear strength in the negative moment region than in the positive moment region at values of $\rho_v f_{vy}$ below 200 psi (1.4 MPa).

Horizontal crack projection

An explanation for the lower stirrup effectiveness in negative moment regions can be obtained by studying the horizontal crack projections. The horizontal projections of the critical shear cracks are noticeably greater for the beams which failed in positive moment regions than for the beams that failed in negative moment regions. In positive moment regions, horizontal projections averaged 1.7*d*, ranging from 1.4 to 2.2*d*, and while in the negative moment regions, they average 1.0*d*, ranging from 0.9 to 1.1*d*, with the exception of one beam, E-20, with a horizontal crack projection of 1.4*d* (Fig. 4).

Due to the longer horizontal crack projection in the positive moment region, the number of stirrups intercepted by the critical crack is larger. Hence, the shear taken by the stirrups is larger, explaining why $v_n - v_c$ is greater in the positive moment region.

The more heavily reinforced beams tested by Haddadin, Hong, and Mattock[11] also exhibited greater horizontal projections of critical shear cracks in positive moment regions (1.9 to 2.3*d*) than in negative moment regions (1.5 to 1.7*d*). These projections are on the average longer in both regions than obtained for the more lightly reinforced beams in this study.

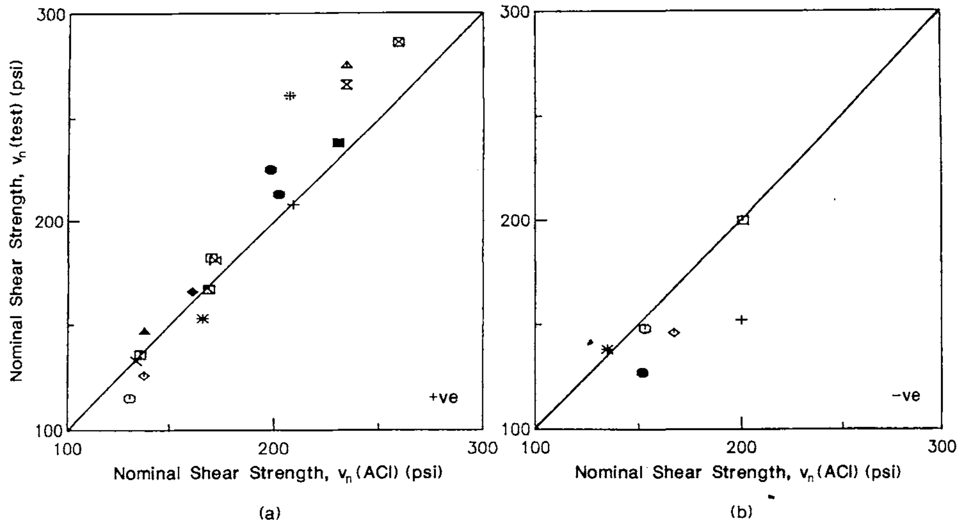

Fig. 9—Comparison of predicted and experimental nominal shear stresses: (a) positive moment region, and (b) negative moment region (1000 psi = 6.895 MPa)

Nominal shear stress

The measured nominal shear stresses are compared with the nominal shear stresses predicted by ACI 318-83[1] in Table 3 and Fig. 9 for the current series, as well as the 15 lightly reinforced beams tested by Palaskas, Attiogbe, and Darwin.[9]

For beams that failed in the positive moment regions, the ACI 318 provisions are conservative for 12 out of the 18 beams, both with and without stirrups. The average value of v_n (test)/v_n (ACI) in these 18 beams is 1.04, with extreme values of 1.26 and 0.88, and a coefficient of variation of 9.3 percent. The ACI provisions are conservative for 11 out of the 14 beams *with stirrups* that failed in the positive moment region. The average value of v_n (test)/v_n (ACI) for beams with stirrups is 1.07, with extreme values of 1.26 and 0.99, and a coefficient of variation of 8.4 percent.

For beams that failed in the negative moment region, the ACI provisions are unconservative for four out of the six beams, both with and without stirrups. The average value of v_n (test)/v_n (ACI) in these six beams is 0.91, with extreme values of 1.02 and 0.76, and a coefficient of variation of 8.4 percent. The ACI provisions are unconservative for four out of five beams *with stirrups* that failed in the negative moment region. The average value of v_n(test)/v_n(ACI) for the beams with stirrups is 0.89, with extreme values of 1.0 and 0.76, and a coefficient of variation of 11.0 percent.

On average, the nominal shear strength is 14.3 percent greater in the positive moment regions than in the negative moment regions. For beams with stirrups, the nominal shear strength is 20.2 percent greater in positive moment regions than in negative moment regions.

In the positive moment regions, the relatively small drop in the concrete contribution due to the low value of ρ_w, combined with the higher stirrup contribution to shear strength, makes ACI 318[1] conservative for nominal shear strength. Therefore, in positive moment regions, even though the concrete contribution to shear

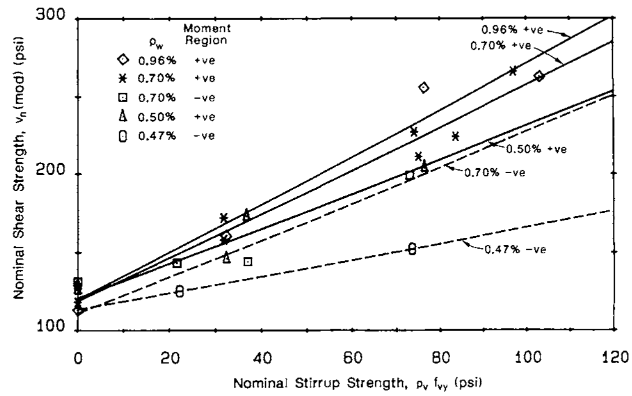

Fig. 10—Modified nominal shear stress v_n(mod) versus $\rho_v f_{vy}$ (1000 psi = 6.895 MPa)

strength is less than predicted by Eq. (1) and (2), this is more than compensated by the higher effectiveness of the stirrups.

However, in negative moment regions, although the stirrup contribution averages $1.19\rho_v f_{vy}$, it is not high enough to adequately compensate for the low concrete contribution to shear strength. Therefore, for low values of ρ_w, the nominal shear strength remains less than predicted by ACI 318.[1]

SAFETY OF CURRENT PROVISIONS

The major question that must be answered is: When are the current provisions safe, and when are they unsafe? The answer can be obtained by considering the combined effects of flexural reinforcement, moment region, and shear reinforcement.

In Fig. 10, v_n is plotted versus $\rho_v f_{vy}$ for each value of ρ_w and moment region. In this figure, v_n is modified to an equivalent strength, in terms of 4000 psi (27.6 MPa) concrete, in order to help eliminate the effect of variations in concrete strength

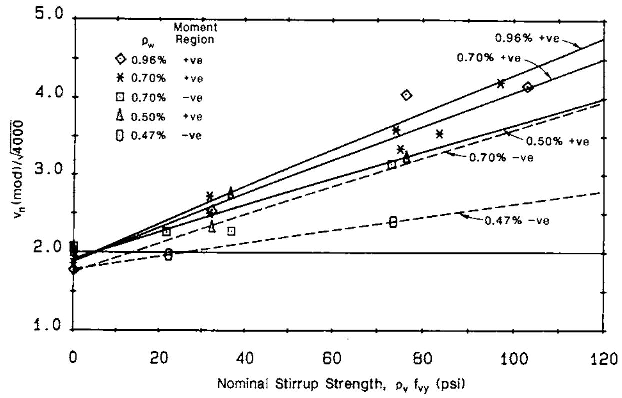

Fig. 11—Nominal shear stress normalized to concrete strength, $v_n(mod)/\sqrt{4000}$, versus $\rho_v f_{vy}$ (1000 psi = 6.895 MPa)

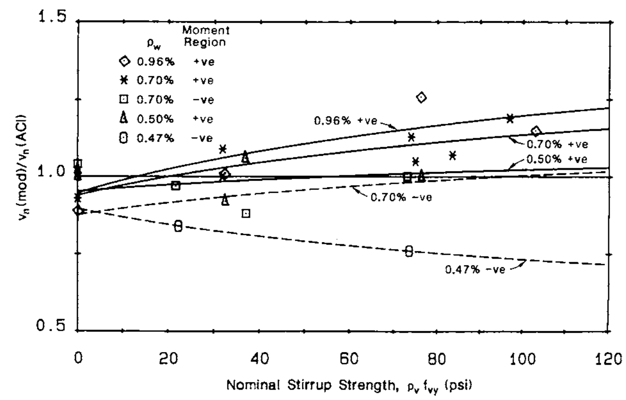

Fig. 12—Nominal shear stress normalized to predicted nominal shear stress, v_n (mod)/v_n(ACI), versus $\rho_v f_{vy}$ (1000 psi = 6.895 MPa)

$$v_n\,(\text{mod}) = v_c\,(\text{test})\,(\sqrt{4000/f'_c}) + [v_n\,(\text{test}) - v_c\,(\text{test})] \quad (6)$$

The lines are least-square fits of the data.

Fig. 10 illustrates that the nominal shear stress v_n and the stirrup effectiveness (slope of the lines) increase with ρ_w and that v_n and stirrup effectiveness are higher in positive moment regions than in negative moment regions.

The shear provisions in ACI 318[1] must be evaluated at two levels of factored load V_u: (1) values of $V_u \leqslant \phi V_c$, where minimum stirrups may be required, but need not be designed; and (2) values of $V_u > \phi V_c$, where the stirrups must be designed, i.e., $\phi V_s \geqslant V_u - \phi V_c$.

$V_u \leqslant \phi V_c$—For values of $V_u \leqslant \phi V_c/2$, no stirrups are required by ACI 318.[1] Lightly reinforced beams are safe at low values of V_u, since all tests indicate that the shear cracking stress v_c is in excess of $\sqrt{f'_c}$.[4,8,9,13]

For $\phi V_c/2 < V_u \leqslant \phi V_c$, ACI 318[1] requires minimum stirrups, except for joists and slabs.

Fig. 11 compares $v_n(\text{mod})/\sqrt{4000}$ versus $\rho_v f_{vy}$ to evaluate the safety of beams for which minimum shear reinforcement ($\rho_v f_{vy}$ = 50 psi [0.34 MPa]) is required. To insure safety, v_n(mod) must equal or exceed the code value of $v_c = 2\sqrt{f'_c}$ for beams with $\rho_v f_{vy} \geqslant 50$ psi (0.34 MPa) (i.e., $v_n\,(\text{mod})/\sqrt{4000}$ must exceed 2).

Fig. 11 illustrates that, in all cases, the use of the minimum web reinforcement allows the beams to develop a nominal shear stress v_n in excess of $2\sqrt{f'_c}$. As little as 25 psi (0.17 MPa) of web steel appears to be sufficient to raise v_n to $2\sqrt{f'_c}$ for beams with $\rho_w \geqslant 0.5$ percent, even in the negative moment region. Hence, if the minimum web reinforcement is used, the nominal shear capacity predicted by ACI 318,[1] $V_n = V_c$, is safe.

It is important to note that since joists are exempt from the minimum web reinforcement requirements,[1] they represent a widely used class of lightly reinforced member that may be unsafe.

$V_u > \phi V_c$—For $V_u > \phi V_c$, shear reinforcement is required. In this case, it is necessary to establish the ranges of ρ_w and $\rho_v f_{vy}$ over which the current provisions are conservative. This can be done by normalizing the modified data illustrated in Fig. 10 to the predicted nominal shear stress, $v_n(\text{ACI}) = 2\sqrt{f'_c} + \rho_v f_{vy}$. Fig. 12 shows that for positive moment regions, v_n is greater than v_n(ACI) if $\rho_v f_{vy}$ is greater than 50 psi. The shear capacity is actually about 0.3 percent less than predicted for beams with ρ_w = 0.5 percent and $\rho_v f_{vy}$ = 50 psi (0.34 MPa).

However, for negative moment regions, v_n is lower than v_n(ACI), even for values of $\rho_v f_{vy}$ much greater than 50 psi (0.34 MPa). For $\rho_v f_{vy}$ = 50 psi (0.34 MPa), the shear capacity is 5 and 21 percent less than predicted by ACI 318[1] for beams with ρ_w = 0.7 and 0.47 percent, respectively. For ρ_w = 0.7 percent, v_n only equals v_n(ACI) for values of $\rho_v f_{vy}$ greater than 95 psi (0.66 MPa). For ρ_w = 0.47 percent, the limited data suggests that v_n will not reach v_n (ACI) for any value of $\rho_v f_{vy}$.

Hence, while the minimum shear reinforcement provisions remain satisfactory, the safety of beams in negative moment regions with low amounts of flexural reinforcement is in doubt, if the strength of the stirrups is used to compute shear capacity. In this case, the minimum web reinforcement ($\rho_v f_{vy}$ = 50 psi [0.34 MPa]) will clearly not provide the predicted nominal shear capacity V_n.

CONCLUSIONS

The following conclusions are based on the test results and evaluation of the lightly reinforced T-beams described in this paper.

1. For the same longitudinal reinforcement ratio ρ_w, diagonal cracks form at a higher shear stress in positive moment regions than in negative moment regions of reinforced concrete T-beams. This lower relative negative moment region shear strength appears to be the result of a smaller effective concrete section due to cracking of the flanges, and a lower bond strength for the negative longitudinal reinforcement, due to the top-bar effect.

2. In both moment regions, the cracking shear is lower than predicted by ACI 318.[1]

3. Negative moment regions exhibit fewer cracks at a wider spacing than positive moment regions, also due to the top bar effect.

4. For lower values of shear reinforcement (up to about 200 psi [1.4 MPa]), the stirrup contribution to shear strength is greater in positive moment regions than in negative moment regions. This appears to be largely due to a greater horizontal crack projection, which results in a greater number of stirrups intersected by the critical shear crack.

5. In both moment regions, the stirrup contribution exceeds that predicted by ACI 318.[1]

6. The current shear provisions in ACI 318[1] appear to be conservative for the positive moment regions of beams with $\rho_w \leqslant 0.5$ percent.

7. The shear provisions[1] are also conservative for the negative moment regions of beams in which $V_u \leqslant \phi V_c$.

8. However, the shear provisions[1] appear to be unconservative for the negative moment regions of beams with $\rho_w \leqslant 0.70$ percent and $V_u > \phi V_c$.

9. The shear provisions may also be unconservative for joists with $\rho_w \leqslant 1.0$ percent and $\phi V_c/2 \leqslant V_u \leqslant \phi V_c$, since these members are not covered by the minimum web reinforcement provisions of ACI 318.[1]

FUTURE WORK

The current test series represents the only existing data for the negative moment region shear strength of beams with low values of both flexural and shear reinforcement. Clearly, the data is too limited to enable design provisions to be modified in a rational manner to account for the low shear strength of these members. Additional tests are needed, with a broader range of test variables.

These tests should include true continuous beams with different shear-span-to-depth ratios, concrete strengths, reinforcement ratios, mild reinforcement and perhaps deformed bars for stirrups. Reinforced concrete joist construction deserves special consideration, since it is currently exempt from the minimum shear reinforcement provisions and enjoys a 10 percent increase in the value of V_c.[1] Finally, the effect of reinforcement ratio on the shear capacity of beams in which longitudinal reinforcement is terminated remains a completely open question.

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