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Shear Strength of Lightly Reinforced T-Beams



by Michael N. Palaskas, Emmanuel K. Attiogbe, and David Darwin

Fifteen lightly reinforced concrete T-beams, 11 with stirrups and four without stirrups, were tested to failure. The major variables in the study were the amounts of flexural and shear reinforcement. The flexural steel varied from 0.5 to 1 percent, and the shear reinforcement varied from 0 to 110 psi (0.75 MPa). The test results are analyzed and compared with the shear design provisions of "Building Code Requirements for Reinforced Concrete (ACI 318-77)" and the recommendations of ACI Committee 426, Shear and Diagonal Tension.

The test results confirmed the findings of other investigators, that the present ACI equations for the shear cracking load are unconservative for beams without stirrups, having a longitudinal reinforcing ratio less than 1 percent. However, for beams with stirrups the web reinforcement was 1.5 times as effective as predicted by ACI 318-77 and compensated for the lower shear strength of the concrete.

It is recommended that the shear design provisions of ACI 318-77 be retained in their current form.

Keywords: beams (supports); cracking (fracturing); diagonal tension; flexural strength; loads (forces); reinforced concrete; reinforcing steels; research; shear strength; stirrups; structural design; T-beams; web reinforcement.

During the past 20 years, a number of investigators¹⁻³ have conducted shear tests on reinforced concrete beams with low ratios of longitudinal reinforcement. Mathey and Watstein² have pointed out that although the use of a higher grade of reinforcement can result in considerable savings in flexural steel, it can lead to a progressive loss of shear strength with a decreasing reinforcement ratio. MacGregor and Gergely, in a paper⁶ prepared in conjunction with the work of ACI-ASCE Committee 426, Shear and Diagonal Tension,⁷ have suggested that a beam with a longitudinal reinforcement ratio less than 1 percent and having minimum web reinforcement can be under-strength in shear, especially in regions away from points of maximum moment, where some of the longitudinal reinforcement has been terminated. Tests by Krefeld and Thurston,¹ Kani,³ and Rajagopalan and Ferguson⁴ have shown that the ACI 381-77⁸ provisions for shear strength appear to be unduly optimistic for beams without stirrups having a longitudinal reinforcement ratio less than 1 percent.

On the other hand, current procedures used for stirrup design appear to be overconservative. According

to the findings of Bresler and Scordelis⁹ and Haddadin, Hong, and Mattock,¹⁰ the effect of low to moderate amounts of shear reinforcement on the shear strength of beams is about 75 percent higher than the strength calculated using the provisions of ACI 318-77.⁸ Beams in these two studies, however, had flexural reinforcement ratios in excess of 1.8 percent.

A better understanding of the shear behavior of beams with low ratios of longitudinal reinforcement, both with and without stirrups, is important since beams of this type are used widely in practice, and in many parts of the world, flexural reinforcement ratios less than 1 percent offer the most economical reinforced concrete sections.

The main objectives of this study are to experimentally investigate the shear behavior of reinforced concrete beams with low amounts of both flexural and shear reinforcement and to use the results to formulate design recommendations. The test results are compared with equations developed to predict shear cracking,^{4,5,7,10-13} and a linear regression analysis is used to determine the effectiveness of web reinforcement, as measured by the increase over the cracking load. The details of the investigation are presented in References 14 and 15.

PREDICTIVE EQUATIONS

The nominal shear strength of a reinforced concrete beam, V_n , is normally expressed as the sum of the individual contributions of the concrete V_c and the shear reinforcing V_s ⁸

$$V_n = V_c + V_s \quad (1)$$

For the purposes of comparing analytical predictions with experimental results, it is somewhat more useful to express Eq. (1) in terms of shear stress

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ACI member **Michael N. Palaskas** received his BSCE, MSCE, and PhD degrees from the University of Kansas. He is a professional engineer specializing in bridge design with 10 years experience in the United States. He is currently a structural engineer with ACE, Athens, Greece.

ACI member **Emmanuel K. Attlogbe** received his BSCE from the University of Science and Technology in Ghana and his MSCE from the University of Kansas. He is currently pursuing a PhD at the University of Kansas.

ACI member **David Darwin** is an associate professor of civil engineering at the University of Kansas. He is an active researcher in the fields of plain and reinforced concrete. He is chairman of ACI Committee 224, Cracking; a member of ACI-ASCE Committee 445, Shear and Torsion; and is past president of the ACI Kansas Chapter.

$$v_n = V_n/b_w d = v_c + v_s \quad (2)$$

in which b_w = width of the web and d = effective depth of the beam.

Concrete capacity

A number of equations have been proposed for the shear stress carried by concrete, v_c .

ACI-ASCE Committee 326¹¹ used cracking loads determined by various investigators to obtain the equation which forms the basis for shear design of reinforced concrete members in the ACI Building Code⁸

$$v_c = V_c/b_w d = 1.9\sqrt{f'_c} + 2500 \rho_w V_u d/M_u \leq 3.5\sqrt{f'_c}, \text{ psi} \quad (3)$$

in which f'_c = concrete compressive strength, ρ_w = web reinforcing ratio ($A_s/b_w d$), A_s = area of flexural reinforcement, V_u = factored shear force at section, and M_u = factored bending moment at section. The committee also proposed the simpler equation

$$v_c = 2\sqrt{f'_c}, \text{ psi} \quad (4)$$

to be used in place of Eq. (3). These equations have been shown to be unconservative for beams without stirrups having web reinforcement ratios ρ_w less than 1 percent^{2-6,16} and overconservative for beams with higher values of ρ_w .^{9,10}

Zsutty¹² combined test results from several different laboratory sources. He carried out a combination of dimensional and statistical regression analyses and derived an equation which is currently the most accurate empirical formulation available

$$v_c = 59(f'_c \rho_w d/a)^{0.5}, \text{ psi} \quad (5)$$

in which a = length of the shear span.

Placas and Regan¹³ derived the equation

$$v_c = 8(f'_c 100\rho_w)^{0.3}, \text{ psi} \quad (6)$$

Eq. (5) and (6) give the same prediction for v_c when a/d , the shear-span to depth ratio, is about 4.

Rajagopalan and Ferguson⁴ studied beams without web reinforcement, having ρ_w less than 1 percent. From

their results and those of other investigators,^{1,3,17,18} they derived the equation

$$v_c = (0.8 + 100 \rho_w)\sqrt{f'_c} \leq 2\sqrt{f'_c}, \text{ psi} \quad (7)$$

to represent the lower bound of the test results. ACI-ASCE Committee 426⁷ has proposed an expression for normal weight concrete, which is similar to Eq. (7)

$$\sqrt{f'_c} \leq v_b = (0.8 + 120 \rho_w) \sqrt{f'_c} \leq 2.3\sqrt{f'_c}, \text{ psi} \quad (8)$$

in which v_b is known as the basic shear stress (shear stress carried by concrete). This equation is currently under consideration for adoption in the ACI Building Code.

Batchelor and Kwun,⁵ from a study aimed at assessing the feasibility of using Eq. (8), have proposed an alternative expression for normal weight concrete

$$\sqrt{f'_c} \leq v_b = (0.60 + 110 \rho_w)\sqrt{f'_c} \leq 2.25\sqrt{f'_c}, \text{ psi} \quad (9)$$

Steel capacity

The contribution of the shear reinforcement is normally based on the modified truss analogy, assuming a horizontal projection of the critical shear crack to be equal to the effective depth of the section. Using this analogy, the contribution of vertical stirrups is expressed in ACI 318-77⁸ as

$$V_s = \frac{A_v f_{vy} d}{s} \quad (10)$$

or in terms of stress

$$v_s = \frac{A_v f_{vy}}{b_w s} = \rho_v f_{vy} \quad (11)$$

in which A_v = area of stirrups, f_{vy} = yield strength of stirrups, s = stirrup spacing, and $\rho_v = A_v/b_w s$. Previous work^{9,10} has shown Eq. (10) and (11) to be conservative.

Of special importance in this study is the determination of how much the conservatism of Eq. (10) and (11) compensates for the unconservative nature of Eq. (3) and (4) for beams with low values of ρ_w .

EXPERIMENTAL INVESTIGATION

The test specimens in this study were designed to fail in shear, and care was taken to eliminate other possible modes of failure. A description of the materials and the procedures used is presented below.

Test specimens

The test specimens (Fig. 1 and 2) consisted of 15 concrete T-beams, 11 with stirrups and four without. The geometry of these specimens was selected to closely resemble members in actual structures. All beams had the same cross section: web width = 7.5 in. (191 mm); total depth = 18 in. (457 mm); flange width = 24 in. (610 mm); and flange thickness = 4 in. (102 mm). The span of the beams was 13 ft 2 in. (4.01 m), and the length was 20 ft 0 in. (6.10 m). The 3 ft-5 in.

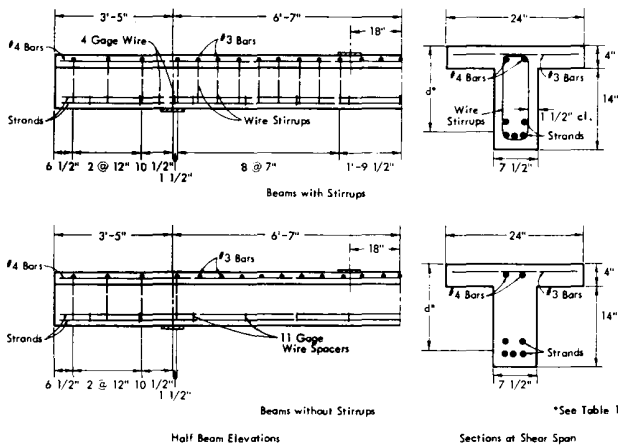


Fig. 1 — Beam details

(1.04 m) overhangs at the ends of the beams increased the embedment and prevented slippage of the reinforcing steel. Nonprestressed, prestressing strands were selected for the longitudinal reinforcement to prevent flexural failures in the test specimens. The use of the high strength steel also allowed high strains to be obtained in the flexural steel, as would occur in continuous reinforced concrete beams undergoing moment redistribution following the formation of one or more plastic hinges.

The beams were divided into three series as a function of the quantity of the longitudinal reinforcement. Five strands were used in each beam to eliminate the effect of the arrangement of the longitudinal reinforcement on the shear capacity of the beams. The stirrups were spaced at about half the effective depth of the beam. The amount of longitudinal steel was controlled using different diameter strands, 1/2 in., 7/16 in., and 0.6 in. (12.7 mm, 11.1 mm, and 15.2 mm) for Series A, B, and C, respectively. The amount of shear rein-

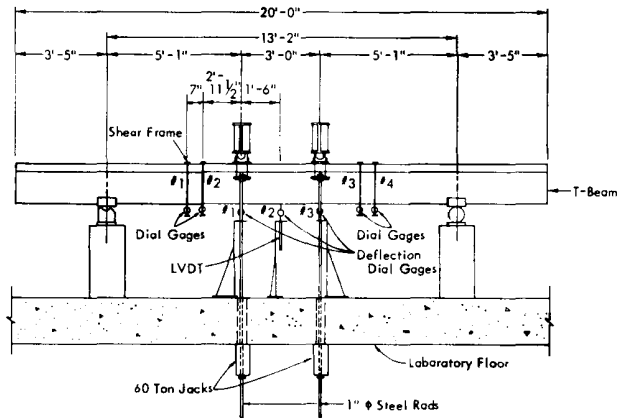


Fig. 2 — Test setup

forcement $\rho_v f_v$, varied from 0 to about 110 psi (0.75 MPa) using different sizes of smooth wire. The flange reinforcement in all beams consisted of two #4 longitudinal bars and #3 transverse bars, spaced as shown in Fig. 1.

Material properties

Concrete: Air-entrained concrete was supplied by a local ready-mixed concrete plant. Type 1 cement and 3/4-in. (19-mm) aggregate were used. Compressive strengths and moduli of rupture are presented in Table 1.

Steel: Three different types of reinforcement were used in the test specimens: prestressing strands, deformed bars, and smooth wire.

The flexural steel in all beams was nonprestressed ASTM A 416 Grade 270 seven-wire stress-relieved strand. The strands were stored outside the laboratory to obtain a uniform coat of rust, which improved the

Table 1 — Beam properties and test results

Beam	d, in.	$\rho_w = \frac{A_s}{b_w d}$, percent	$\rho_v f_{vy}$, psi	f'_c , [†] psi	f_m , [‡] psi	a/d	v_c , psi	v_n , psi
#2	14.72	0.693	0.0	4750	437	4.14	—	147
A00	15.54	0.656	0.0	4740	667	3.92	111	125
A25	15.38	0.663	31.8	4720	396	3.97	112	167
A25a	15.26	0.668	31.8	4790	664	4.00	114	182
A50	15.42	0.661	74.0	3810	380	3.96	116	224
A50a	15.49	0.658	75.0	4060	512	3.94	114	212
A75	15.56	0.655	97.0	4670	550	3.92	110	274
#1	14.60	0.699	110.2	5520	717	4.18	—	275
B00	15.70	0.488	0.0	4640	567	3.88	89	136
B25	15.52	0.494	32.4	4470	525	3.93	93	152
B50	15.39	0.498	76.2	4390	585	3.96	98	208
C00	15.41	0.943	0.0	4270	604	3.96	96	115
C25	15.33	0.948	32.4	4100	462	3.98	114	162
C50	15.47	0.939	76.2	4300	650	3.94	115	260
C75	15.57	0.933	103.0	4260	585	3.92	116	266

All beams — $b_w = 7.5$ in., $a = 61$ in., and $s = 7$ in. (= 7.5 in. for #1)

Group A: A_s — five 1/2 in. diameter strands* = 0.765 in.²

Group B: A_s — five 7/16 in. diameter strands* = 0.575 in.²

Group C: A_s — five 0.6 in. diameter strands* = 1.09 in.²

1000 psi = 6.8948 MPa.

1 in. = 25.4 cm.

*Grade 270.

†Compressive strength from 6 x 12-in. cylinders.

‡Modulus of rupture from 6 x 6-in. beams loaded at the third points.

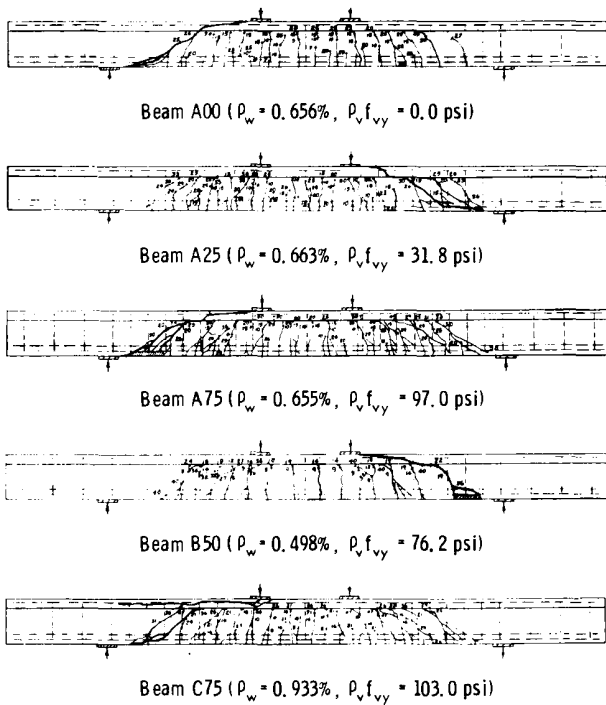


Fig. 3 — Typical crack patterns

bond and prevented slippage of the strands during the tests. The flange reinforcement consisted of ASTM A 615 Grade 60 deformed billet steel bars. The stirrups were low carbon, smooth wires with diameters of 0.132, 0.186, 0.229, or 0.244 in. (3.35, 4.72, 5.82, or 6.20 mm). The wires were annealed to obtain a yield stress close to the design yield stress obtained with deformed reinforcing bars. Information on the reinforcing steel is summarized in Table 1.

Test procedure

Four techniques were used to determine the shear cracking loads. These techniques used data obtained from the cracking patterns, the stirrup strain gages, the concrete strain gages, and four 0.0001 in. (2.54 μ m) scale dial gages installed on specially constructed shear cracking frames to measure the increase in beam depth.

One dial gage and a linear variable differential transformer were placed at the center of the span. Two other dial gages were placed at the load points, shown in Fig. 2. The four shear cracking frames were secured at the locations of the third and fourth stirrups from the center line of the span, on both sides of the beam.

To check the equipment, all beams were loaded to about one-third of the calculated flexural cracking load and unloaded. The readings of all strain and dial gages were then recorded for zero load. The load was applied incrementally. The size of the load increments was reduced around the calculated flexural cracking, shear cracking, and failure loads. The smallest increments in total load* were about 1250 lb (5.5 kN), and the largest were about 5000 lb (22 kN). At each increment, strain and dial gage readings were recorded and cracks were marked while the applied load was held constant. Loading continued until the beam failed.

The average time for a test, from initial loading to failure, was about 1 hr and 45 min.

Results and observations

A summary of the properties of the test specimens and the test results is presented in Table 1.

Using photographs which were taken during and after the tests, the complete crack pattern for each beam was reproduced. Typical crack patterns are shown in Fig. 3. The heavy crack line represents the failure crack. The numbers represent the total load, to the nearest kip, at which the crack formed.

The beams behaved in a manner that was quite similar to beams with higher reinforcement ratios. In the first stages of loading, the beams were free of cracks. Since the stresses were very small and the full section participated in carrying the load, the deflection was small and proportional to the applied load. At a load close to the calculated flexural cracking load, the first cracks were observed, accompanied by a considerable increase in deflection. These cracks always initiated in the region of pure flexure and extended vertically up to the neutral axis of the beam. With increasing load, more flexural cracks developed in both the center and the shear span regions of the beam. Cracks in the two regions propagated differently. Cracks in the pure moment section were vertical, while cracks in the shear span curved toward the point of the applied load as soon as they entered the area between the level of the tension reinforcement and the neutral axis.

The trend of cracking was almost the same in all beams, except that the number and the size of the cracks seemed to depend on the amount of the flexural reinforcement. Compared to the other series, the beams in Series B ($\rho_w = 0.5$ percent) exhibited fewer and wider cracks. In contrast, the other series ($\rho_w = 0.66$ and 0.94 percent) exhibited a greater number of cracks of smaller width. The cracks in the Series C beams ($\rho_w = 0.94$ percent) were so narrow that additional light was required to locate them.

The trend of flexural cracking continued until shear cracking began. Shear cracking was accompanied by an increase in stirrup strain and initiated close to the midheight of the beam. The shear cracks propagated at an inclination flatter than 45 deg in two directions, toward the flange and toward the flexural reinforcement. When the bottom end of the crack reached the flexural reinforcement, it continued to propagate with increasing load along the reinforcement for a distance equal to at least one stirrup spacing. The other end of the crack propagated until it reached the bottom of the flange. From there on, two possible crack paths were observed. For most beams, the crack extended horizontally along the junction of the flange and the web. In a few cases, the crack remained fairly stable after it reached the bottom of the flange until failure occurred. For beams with the first type of crack path, the crack entered the flange at failure or at a load stage

*The loading rods shown in Fig. 2 were strain gaged and calibrated to act as load cells.

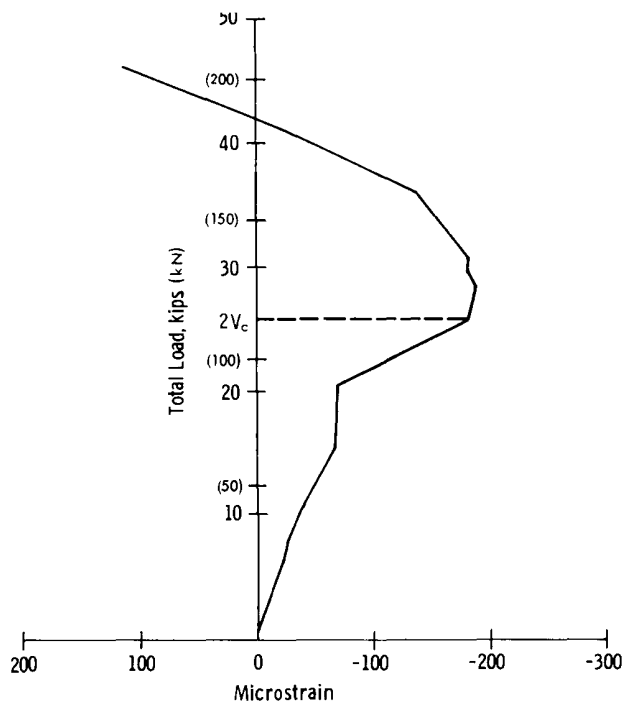


Fig. 4 — Total load versus concrete strain in compression flange

prior to failure. In both cases, failure occurred with a sudden extension of the crack toward the point of loading. The only exception to this failure mode was Beam C75. In this beam, the failure crack in the flange was a horizontal crack extending along the total length of the shear span of the beam (Fig. 3). Beams without stirrups failed shortly after the initiation of shear cracking, while the beams with stirrups exhibited additional cracking and were able to carry additional load.

EVALUATION OF EXPERIMENTAL RESULTS

Shear cracking

Four methods were used to determine the shear cracking load. These approaches were based on the stirrup strains, the concrete strain in the compression flange, the increase in beam depth following crack formation, and the cracking patterns. The four methods were used to provide detailed information on beam behavior and to compare procedures for defining shear cracking.

The shear cracking load is considered to be the load at which significant changes in the load-carrying mechanisms occur, resulting in the redistribution of stresses within the beam. Using this criterion, the objective is to determine the load at which this change occurs.

Plots of load versus concrete strain (similar to Fig. 4) indicate that an appreciable reduction in the compressive strain occurs in the extreme compression fiber within the shear-span at a load coinciding with the formation of diagonal cracks. This load is defined as the shear cracking load.

Shear cracking loads are obtained from stirrup strain and depth increase using figures similar to Fig. 5.

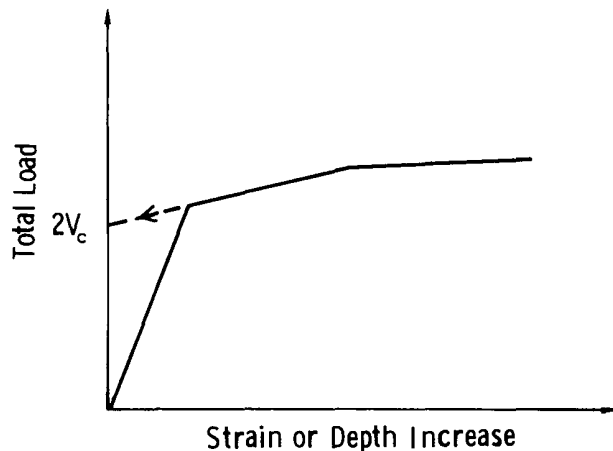


Fig. 5 — Schematic of total load versus stirrup strain or depth increase

Load-strain and load-depth increase curves show essentially no reading up to a load of 1.5 to 2 times the flexural cracking load. However, in most beams, small readings were recorded before the formation of the first shear cracks, due to inclined flexural cracks within the shear-span. To obtain the cracking load from these figures, the portion of the graph which shows a marked increase in strain or depth is extended back until it intersects the load axis, as illustrated in Fig. 5. The point of intersection is defined as the shear cracking load.

The shear cracking load obtained from the crack patterns (Fig. 3) is taken as the load at which a crack forms at the level of the neutral axis (the centroid of the uncracked transformed cross section) at an angle of 45 deg or less to the longitudinal axis of the beam. This is similar to the procedure used by Haddadin et al.¹⁰

The four methods were compared in detail in Reference 14. The shear cracking loads obtained using the concrete and stirrup strains matched each other quite well. The values obtained from the depth increase data were equal to or greater than the values obtained from the stirrup and concrete strains. The shear cracking loads obtained from the crack patterns did not show a consistent relationship to the loads obtained from the stirrup and concrete strains. The cracking stresses obtained using stirrup and concrete strains also exhibited less scatter than the cracking stresses obtained using either depth increase or cracking patterns. Both the stirrup and the concrete strains appear to be more sensitive to the change in the load-carrying mechanisms at shear cracking than do the other two procedures.

Overall, the procedures utilizing the concrete and stirrup strains appear to be the most reliable. The values based on concrete strains are used in this paper.

A study of the data offers at least a partial explanation of these observations. Unlike flexural cracking, shear cracking does not represent the formation of a discrete crack or cracks. Instead, shear cracking represents a change in the way that the beam carries load within the shear-span. Stresses are redistributed and

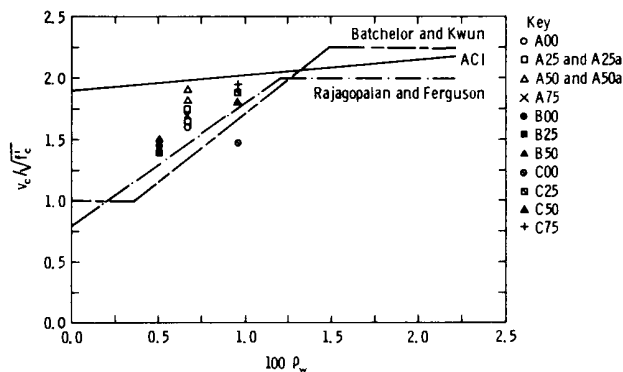


Fig. 6 — Stress at diagonal cracking v_c versus reinforcing ratio ρ_w .

shear deflection increases markedly. The most reliable measures of this change in behavior in this study, concrete and stirrup strains, may be reliable because these strain readings reflect the behavior of a sizable portion of the beam. The concrete gages pick up the softening due to diagonal cracks. The stirrups pick up load as the load-carrying mechanisms change, and are thus sensitive to the change in behavior. Since depth increase is a function of stirrup stiffness as well as the concrete strength, it may not be as sensitive as the stirrup gages. Crack patterns may be somewhat random, and the local angle of a crack does not appear to be of great interest. Therefore, using crack patterns alone may be a somewhat risky way to establish the shear cracking load.

A comparison is made between the observed shear cracking stresses v_c and the calculated shear cracking stresses obtained using Eq. (3) through (9) in Fig. 6 and 7 and Table 2.* To help reduce the effect of the variable concrete strength, the results are normalized with respect to $(f'_c)^{3/4}$ and $(f'_c)^{3/8}$ in the figures. For the beams in this study ($a/d \approx 4$), the predictions of Eq. (5) and (6) are the same. It is observed in Table 2 that

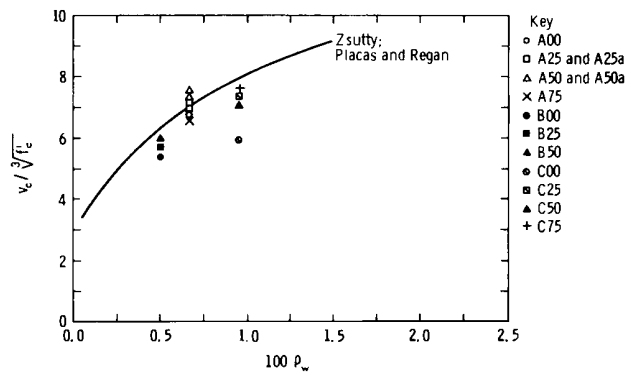


Fig. 7 — Stress at diagonal cracking v_c versus reinforcing ratio ρ_w .

the test results are generally higher than the predictions of Eq. (7) and (9), while lower than Eq. (3) through (6). These trends are also shown in Fig. 6 and 7. The unconservative nature of Eq. (3) (ACI 318-77) for ρ_w less than 1 percent is borne out in Fig. 6. The fact that the test results are consistently lower than the prediction of Eq. (5) by Zsutty¹² (normally an accurate predictor of v_c) may be because most of the test results he used for his analysis were for beams with high ratios of longitudinal reinforcement. The nature of the bond strength between the flexural steel and the concrete in the current tests may have also had an effect. Eq. (9) by Batchelor and Kwun⁵ is more conservative than Eq. (7) by Rajagopalan and Ferguson.⁴ Both equations are safe lower bounds. The results are also compared with Eq. (8) proposed by ACI-ASCE Committee 426.⁷ Eq. (8) is adequate for the beams in Series A and B, but unconservative for the beams in Series C.

Stirrup effectiveness

The increase in shear stress above the cracking stress v_c to the nominal shear stress v_n is used as a measure

Table 2 — Comparison of measured and calculated shear cracking stresses

Beam	V_c (test), kips	v_c (test), psi	v_c (test)				
			$\frac{v_c \text{ (test)}}{[v_c \text{ Eq. (3)]}}$	$\frac{v_c \text{ (test)}}{[v_c \text{ Eq. (5)]}}$	$\frac{v_c \text{ (test)}}{[v_c \text{ Eq. (7)]}}$	$\frac{v_c \text{ (test)}}{[v_c \text{ Eq. (8)]}}$	$\frac{v_c \text{ (test)}}{[v_c \text{ Eq. (9)]}}$
A00	12.9	111	0.82	0.94	1.10	1.01	1.22
A25	12.9	112	0.83	0.95	1.12	1.02	1.23
A25a	13.1	114	0.84	0.97	1.13	1.03	1.24
A50	13.5	116	0.95	1.06	1.29	1.18	1.41
A50a	13.2	114	0.91	1.02	1.23	1.12	1.36
A75	12.9	110	0.82	0.94	1.10	1.01	1.22
B00	10.5	89	0.67	0.90	1.00	0.94	1.16
B25	10.8	93	0.72	0.89	1.07	1.00	1.22
B50	11.3	98	0.76	0.94	1.14	1.06	1.29
C00	10.9	96	0.74	0.75	0.84	0.76	0.89
C25	13.2	114	0.89	0.90	1.02	0.92	1.09
C50	13.1	115	0.88	0.90	1.00	0.91	1.06
C75	13.4	116	0.89	0.91	1.02	0.93	1.08
Mean			0.82	0.93	1.08	0.99	1.19
Standard deviation σ			0.082	0.073	0.113	0.105	0.137
Coefficient of variation V_c , %			10.0	7.8	10.4	10.6	11.5

1 kip = 4.448 kN.
1000 psi = 6.8948 MPa.

*Results are not available for Beams 1 and 2, since these beams were not fully instrumented.

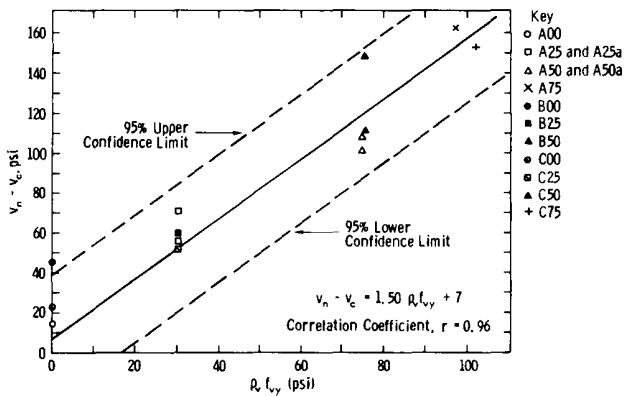


Fig. 8 — Effectiveness of web reinforcement $v_n - v_c$

of the effectiveness of the web reinforcement. This is in line with the philosophy of ACI 318-77.⁸ This increment of stress ($v_n - v_c$) is plotted against the nominal shear stress resisted by stirrups, $\rho_s f_{vy}$, in Fig. 8. A linear regression analysis (correlation coefficient $r = 0.96$) indicates strongly that the results can be represented by a straight line with a slope of 1.5. The web reinforcement is therefore 1.5 times as effective as predicted by the modified truss analogy with a horizontal crack projection equal to the effective depth of the beam. The average results of this study can be expressed as

$$v_n - v_c = 1.5 \rho_s f_{vy} + C \quad (10)$$

with $C = 2-8$ psi. The value of C depends on the method used to obtain v_c .¹⁴

Bresler and Scordelis⁹ and Haddadin, Hong, and Mattock¹⁰ found that the contribution of web reinforcement to nominal strength was 1.8 and 1.75 times $\rho_s f_{vy}$, respectively. These higher values may be due to the higher longitudinal reinforcement ratios used in those studies.

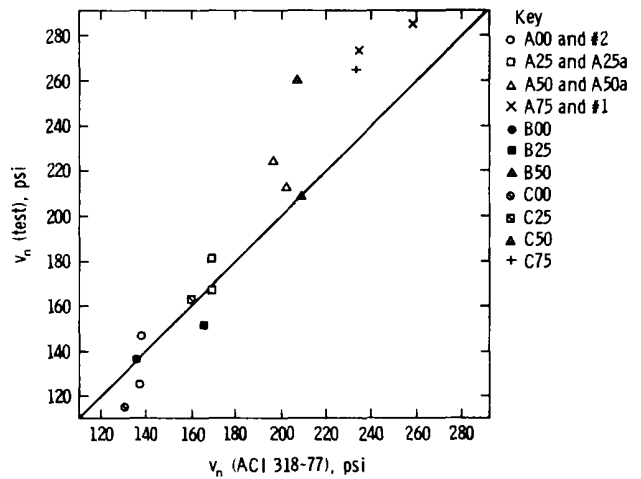


Fig. 9 — Comparison of predicted and experimental nominal shear stresses

The measured nominal shear stresses are compared with the nominal stress values predicted by ACI 318-77⁸ in Table 3 and Fig. 9. This comparison shows that using Eq. (2) with either Eq. (3) or (4) results in slightly unconservative predictions of the nominal shear strength for only two beams with stirrups, A25 and B50 are, respectively, 0.4 and 1.5 percent stronger than the predicted nominal shear strength, while Beams A75 and C75 are 19 and 14 percent stronger, respectively. These trends indicate that although the concrete contribution to shear strength is less than predicted by ACI 318-77, the lower concrete strength is compensated by the higher effectiveness of the stirrups.

A study of the crack patterns shows that further cracking occurs after initial shear cracking for all beams with stirrups. During the test, additional load was carried in every case, demonstrating that the stir-

Table 3 — Comparison of measured nominal shear stresses with values predicted by ACI 318-77

Beam	$\rho_s f_{vy}$, psi	V_n (test), kips	v_n (test), psi	$[v_n \text{ Eq. (2)}]$ $[v_n \text{ Eq. (3)}]$	$[v_n \text{ Eq. (2)}]$ $[v_n \text{ Eq. (4)}]$	v_n (test)	
						$[v_n \text{ Eq. (2)}]$ $[v_n \text{ Eq. (3)}]$	$[v_n \text{ Eq. (2)}]$ $[v_n \text{ Eq. (4)}]$
#2	0.0	16.2	147	135.1	137.8	1.089	1.067
A00	0.0	14.6	125	135.0	137.7	0.925	0.907
A25	31.8	19.3	167	166.5	169.2	1.004	0.987
A25a	31.8	20.8	182	167.5	170.2	1.084	1.066
A50	74.0	26.0	224	195.4	197.3	1.148	1.137
A50a	75.0	24.7	212	200.3	202.4	1.060	1.049
A75	97.0	32.0	274	231.0	233.8	1.186	1.172
#1	110.2	31.3	286	255.3	258.8	1.119	1.104
B00	0.0	16.0	136	132.6	136.2	1.026	0.999
B25	32.4	17.7	152	162.5	166.1	0.934	0.914
B50	76.2	24.1	208	205.3	208.7	1.015	0.999
C00	0.0	13.3	115	130.1	130.7	0.882	0.878
C25	32.4	18.7	162	160.0	160.5	1.014	1.011
C50	76.2	30.2	260	206.8	207.3	1.256	1.253
C75	103.0	31.0	266	233.0	233.5	1.140	1.137

$$v_n = v_c + \rho_s f_{vy} \quad (2)$$

$$v_c = 1.9\sqrt{f'_c} + 2500 \rho_w \frac{V_u d}{M_u} \quad (3)$$

$$v_c = 2\sqrt{f'_c} \quad (4)$$

1 kip = 4.448 kN.
1000 psi = 6.8948 MPa.

rups improved the ductility of the beams, thus providing a warning of impending failure [even for $\rho_v f_y < 50$ psi (0.34 MPa)]. Based on the data analyzed in this investigation, therefore, the equations for nominal shear strength in ACI 318-77 appear to provide reasonable predictions of the strength of beams with values of $\rho_v f_y$ greater than 32 psi (0.22 MPa) and with ρ_v greater than 0.5 percent.

Other considerations

Haddadin, Hong, and Mattock¹⁰ found that the increase in nominal shear strength of beams with low shear-span to depth ratios (a/d) over the strength of beams with high a/d ratios was larger than predicted by Eq. (3). A significant effect of the a/d ratio on shear strength over a wide range of longitudinal reinforcement ratios is also reported by Batchelor and Kwun.⁵ The influence of the a/d (or M/Vd) ratio implies that in low moment regions (where longitudinal reinforcement may be terminated), the contribution of concrete to shear strength is increased. While the present provisions of ACI 318-77 underestimate this increase, Eq. (7), (8), and (9) do not account for it at all.

To insure adequate ductility, ACI 318-77 requires that a minimum amount of shear reinforcement [$\rho_v f_y = 50$ psi (0.34 MPa)] be provided where the factored shear stress v_u exceeds one-half of ϕv_c , in which ϕ is the strength reduction factor (=0.85 for shear). Fig. 6 shows that $v_c = \sqrt{f'_c}$ (i.e., one-half of $v_c = 2\sqrt{f'_c}$) is a very safe lower bound for the results of this study. Thus, this requirement for minimum shear reinforcement appears to be adequate for the design of beams without stirrups. Even though Eq. (7), (8), and (9) provide conservative estimates for v_c , their use as the basis for requiring minimum shear reinforcement may not be justified in view of the fact that they do not reflect the effect of the a/d ratio on shear strength.

RECOMMENDATIONS AND CONCLUSIONS

Beams with stirrups

In this study of reinforced concrete beams with low ratios of longitudinal reinforcement, the effectiveness of stirrups in resisting shear stresses was 50 percent higher than predicted by ACI 318-77, while the shear stress carried by concrete was lower. Further, the lower strength of the concrete was compensated by the higher effectiveness of the stirrups. Values of web reinforcement $\rho_v f_y$, as low as 32 psi (0.22 MPa) effectively increased the strength and ductility of the test beams following shear cracking. Proposed equations by Rajagopalan and Ferguson⁴ [Eq. (7)], and Batchelor and Kwun⁵ [Eq. (9)] are safe lower bounds for the measured shear cracking loads. Eq. (8) by ACI-ASCE Committee 426⁷ is slightly unconservative in predicting the shear cracking loads for the beams in Series C. However, for beams with stirrups, these equations are overconservative for determining nominal shear strength unless the strength of the stirrups is utilized. It is, therefore, recommended that the present procedures in ACI 318-77 for determining nominal shear

strength should be retained for beams with stirrups [$\rho_v f_y \geq 50$ psi (0.34 MPa), $\rho_v \geq 0.5$ percent], until such time as the full strength of stirrups is utilized in design.

Beams without stirrups

The test results clearly indicate that the equations for v_c in ACI 318-77 are unconservative for ρ_v less than 1 percent. This was a special concern of ACI Committee 426 when it recommended the use of Eq. (8). MacGregor and Gergely⁶ specifically cited regions where a portion of the longitudinal reinforcement is terminated. The code, however, requires that for beams without stirrups, the shear stress carried by the concrete must be no larger than one-half of ϕv_c . This requirement gives a safe lower bound prediction for the results of this and other studies.^{4,5} Eq. (7), (8), and (9) do not take advantage of the effect of the a/d ratio, which tends to increase v_c at locations of low moment (e.g., where longitudinal reinforcement may be terminated). It is, therefore, recommended that the present ACI Building Code provisions be retained for beams without stirrups.

Future work

Additional tests should be carried out on beams in which the longitudinal reinforcement is terminated to determine the effect of ρ_v on their behavior in shear. Only simply supported beams have been tested in this investigation. Additional tests, therefore, also need to be conducted on continuous beams to give a better understanding of the behavior of actual structures.

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