A STUDY OF DISTRIBUTION CIRCUITS OF SINGLE PHASE RAILWAY ELECTRIFICATION WITH SPECIAL REFERENCE TO INDUCTIVE EFFECTS AND REGULATION

By

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PREFACE

This thesis is intended to cover in a broad way the determination of inductive effects in communication circuits caused by single phase railway electrifications and the inter-relation between these inductive effects and the regulation in the railway circuits themselves.

The studies on which the paper is based were carried on at several different places and times. Original investigation of inductive effects and factors affecting them was carried on in May, 1906, on the circuits of the Indiana and Cincinnati Traction Company near Rushville, Indiana. The study was continued on the interurban electrification between Derry and Latrobe, Pennsylvania, later in the same year.

The major investigations, however, were made over a period of eighteen months in 1907, on the New York, New Haven and Hartford main line electrification and over a period of three years in 1915-19 on the Paoli and the Chestnut Hill electrifications of the Pennsylvania Railroad at Philadelphia.

It was during the last period that the regulation factor came to the fore and that the inter-relation between regulation and inductive effects became evident. The methods of calculation of regulation shown here were then developed, greatly simplifying means previously in use.

The report covers the status of electrifications in the year 1920. Since that time further alternating current single phase electrifications have been installed and older systems have been largely extended, the plans utilizing the considerations set forth here. Communication systems are operating successfully in the immediate vicinity of the railway systems.

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Introduction

Single phase railway electrification initiated in the early years of this century, met with strong opposition from advocates of other systems and one of the great objections urged against it was the interference caused by the flow of single phase trolley and ground current in the circuits of communication companies which might be located on the railway right-of-way or within a moderate distance of it. Early investigation in 1906 on the interurban Indianapolis and Cincinnati Traction Company was carried on with a view to anticipation of inductive effects and their mitigation on the main line of the New York, New Haven and Hartford Railroad between Mount Vernon, New York, and Stamford, Connecticut, which was then under construction. Certain fundamentals were determined during this investigation and it was made possible to calculate the approximate voltages that might be expected in the Western Union Telegraph circuits on the railroad right-of-way. Means were devised for overcoming in large measure these voltages which were so high as to make communication impossible in the state of the art then existing.

On later electrifications the study of the inductive effects was taken up early enough so that the distribution circuits could be laid out to give better conditions. It is now well recognized that the two factors, inductive effects and system regulation, are inter-related and that a change in the system that affects one will generally affect the other. These two factors must therefore be considered together.

Regulation - Impedance of the Trolley-Rail Circuit.

The first element to be considered in working out the regulation of the system is the impedance of the circuit composed of the trolley wire and return path. This impedance consists of the resistance of the trolley wires and the rails and the reactance due to the magnetic field between them. The resistance of the trolley wire is readily obtained as, for the railroad frequency, of 25 cycles, it is practically the same as for direct-current. The rails, however, have higher resistance to alternating than to direct-current. The ratio between the 25 cycle alternating-current and the direct-current resistance has been experimentally determined as about 2.5 to 1. This value was arrived at after many experiments on the 100 pound rails of the New York, New Haven and Hartford Railroad by measuring voltage drop along a given length of rail with the potential conductor in several locations relative to the rail. It was necessary to calculate the effect of the magnetic field set up between the rail and the potential wire and to eliminate the voltage resulting from this which appeared in the voltage readings, an item previously overlooked in published data. The reactance per mile due to the trolley wire is given by the formula, -

$$X = 0.116 \log_{10} \frac{d}{0.78r} \text{ for 25 cycles.}$$

where d is the distance between the rails and the trolley and r the radius of the trolley. Likewise the reactance per mile due to the track is, -

$$X' = I' \, 0.116 \log \frac{d}{r'} \text{ for 25 cycles.}$$

where I' is the proportion of trolley current which returns by rail, and r' is the effective radius of the two rails, or the geometric mean distance between the rails, which is equal to

$$\sqrt[4]{0.78\, r_a\, d_{ab} \times 0.78\, r_b\, d_{ab}}$$

where r_a and r_b are the equivalent radii of the two rails and d_{ab} the distance between them.

In the case of two wires being used as trolley conductors the same method applies for obtaining the proper value of 0.78r for calculation of reactance. Likewise in the case of double or four track the geometric mean distance of the wires from themselves must be used in place of 0.78r and the geometric mean distance of the wires from the track in place of d.

The radius for a single rail has been determined empirically so that the effect of the internal field in the steel may be neglected in calculations. For a 100 pound rail, the value used is 0.35 inches. Considerable variation of this value makes very little difference in the total reactance calculated and a high degree of accuracy for the effective radius of one rail is not necessary.

Proportion of Earth Current

As mentioned above the proportion of current returning by rails must be known in order to have complete impedance data. This proportion has been determined by experiment in several cases and has also been calculated by approximate methods. The agreement between the results of tests and the calculations have been good. Local conditions affect the results to a certain extent but with the tracks well bonded and with ordinary leakage conditions, the values can be approximately stated and are as follows:-

Single track - rail current = 40 per cent of trolley current.

Double track - rail current = 60 per cent of trolley current.

Four track - rail current = 70 per cent of trolley current.

The above values are such as would be measured in the middle of a section four or five miles long or more. For shorter sections, or for positions within two miles from the point of power supply, the values should be higher. The results of some of the first tests made on single-phase roads for the determination of earth and rail currents are shown in Fig. 1. The tests were made on the 25 cycle, single-phase line of the Indianapolis and Cincinnati Traction Company in 1906, which was then operating from Rushville, Indiana, to Indianapolis, a distance of 40 miles. For the tests, current was fed by trolley from the power house at Rushville to various points where the trolley was connected to the rails. The curves show how little effect the length of feed has on the division of the current between rails and earth. Further tests of a similar nature were made later on other roads, notably the New York, New Haven and Hartford Railroad, which confirmed the results obtained at Rushville.

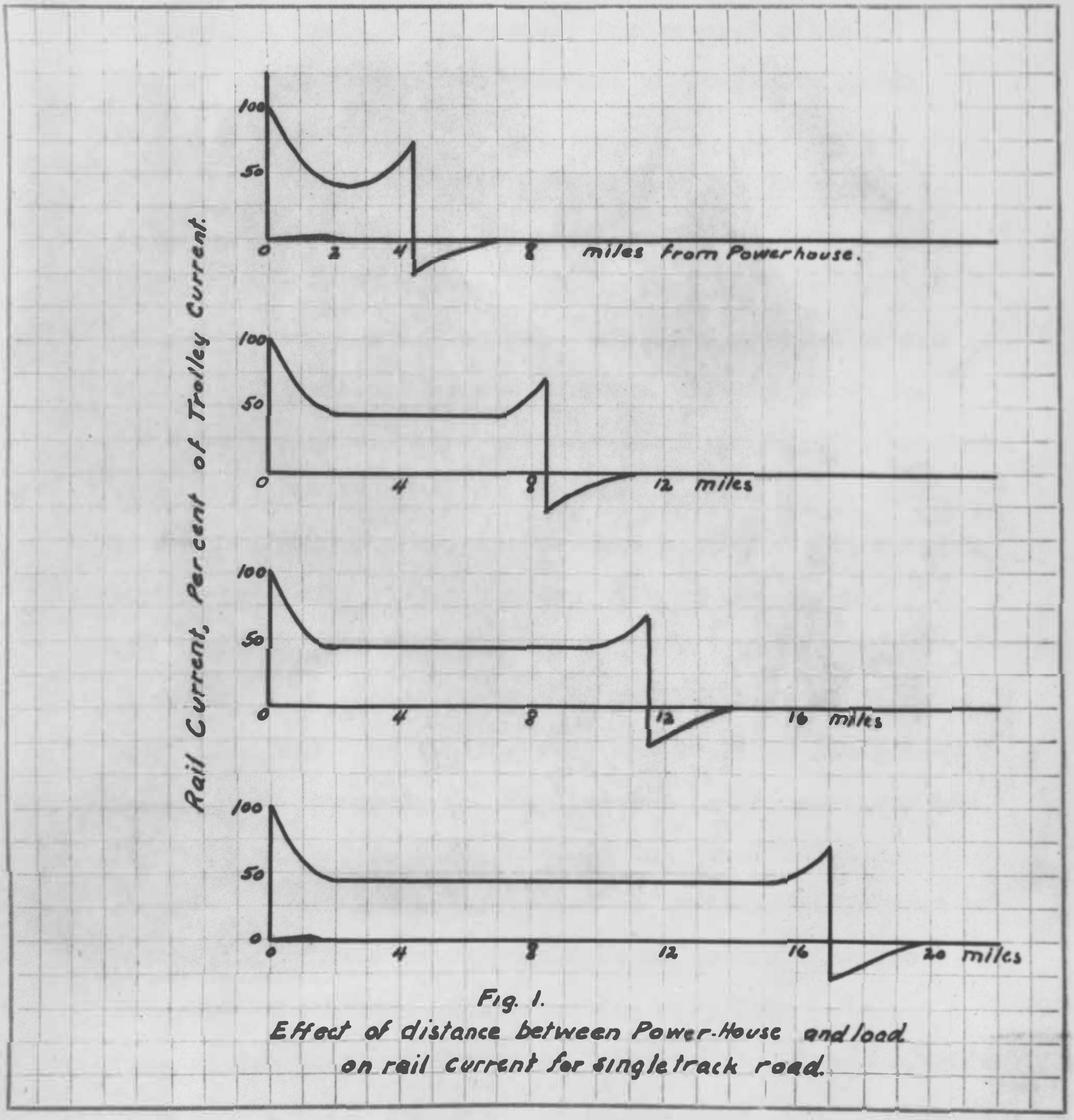

Fig. 1.
Effect of distance between Power-House and load on rail current for single track road.

With the value of rail current determined, the impedance of the trolley-rail circuit may readily be calculated. A table of impedances for several sizes of trolley wire and different numbers of tracks is given in the appendix.

Total Impedance From Substation to Load

The vector difference of potential from a substation to a point fed by a single trolley from the substation is obtained by multiplying the impedance of the trolley-rail circuit by the current. If the point is fed from two substations between which two or more trolley wires run, the calculation can also be made quite simply. If the two stations are at the same potential the currant to the load will divide between them in inverse ratio to the distance from them and the drop will be $mI_c Z_{bc} = (1 - m) I_c Z_{ac}$. If the two substations are at different potentials, the load current can still be considered as dividing inversely as the distance to the stations and, in addition, a drop will be added due to circulating current between the stations caused by their difference of potential. This drop is proportional to the distance from the substation. The potential drop from a to c, Fig. 2, due to the difference of potential between a and b is $m (E_a - E_b)$. The drop due to the load I_c at c is as above, the total being $m (E_a - E_b) + (1 - m) I_c Z_{ac}$.

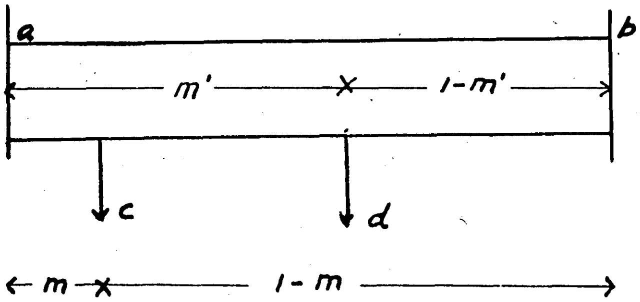

Fig. 2.
Two loads on same section between Substations.

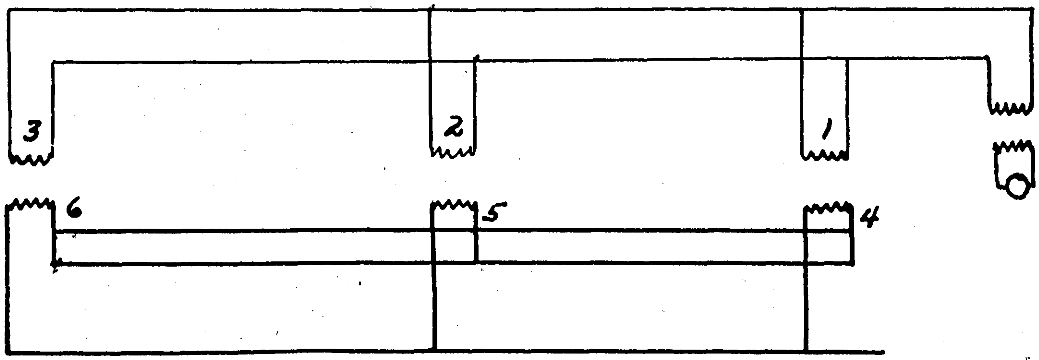

Fig. 3.
Generator, Step-up Transformers, Transmission Line, Step-down Transformers and Trolley Circuit.

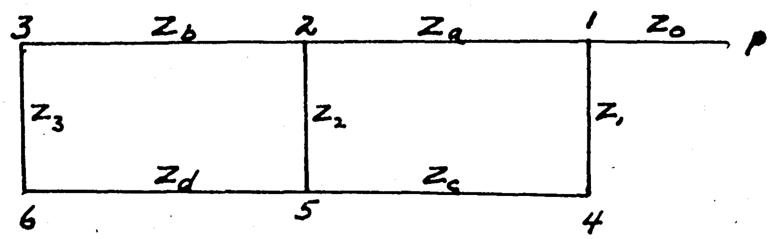

Fig. 4.
Equivalent Network of Figure 3.

For a second load d in the same section distance m' from a and 1 — m' from b, the drop from a to c becomes m $(E_a - E_b) + \left[(1 - m) I_c + (1 - m') I_d\right] Z_{ac}$. This may be continued for any number of loads.

The Equivalent Network for Complete System

The foregoing gives the electromotive force drops from substations to loads. The drop from the power house to the low-tension side of the substations can also be reduced to quite a simple calculation by considering the system, high-tension, low-tension and transformers, as a network. Fig. 3 shows such a simple system, comprising a generating unit P with a step-up transformer and transmission line and three step-down transformer stations which are connected to the trolley on the low tension side. The equivalent network is shown in Fig. 4. Z_o = impedance of step-up transformer and high-tension line as far as 1, reduced to trolley potential, i.e., the high tension impedance divided by the square of the step-down transformer ratios. Z_1, Z_2, and Z_3 = impedance of transformers 1, 2 and 3 at trolley voltage. Z_a and Z_b = the impedance of transmission line from 1 to 2 and from 2 to 3 respectively, reduced to trolley potential. Z_c and Z_d = the impedance of trolley-rail circuit from 4 to 5 and from 5 6o 6 respectively.

Distribution Currents - By the application of Kirchoff's laws, it is a very simple matter to find the currents which will flow in each transformer for load at any one on the low tension side. A chart can then be prepared as shown in Fig. 5, which shows the proportion of current supplied by each substation to a load at any point on the line. A load of one ampere at point X will be fed with 0.18 amperes from transformer station 1, 0.34 amperes from transformer station 2, and 0.48 amperes from transformer station 3. A study of the chart shows that any load, such as that at X may be replaced by two loads at the adjacent substations inversely proportional in magnitude to the distance of the point of application of the load from them. The sum of the two equivalent loads must be equal to the original load. Such replacement or substitution may be made without affecting the transformer loads, and therefore, the e.m.f. drops to the low-tension side of the transformers are unchanged.

Self and Mutual Impedance - The values of self and mutual impedance which will determine the drops to the transformer stations can be calculated from the above data. For simplicity these impedances are designated as follows: *

Z_{4-4} = self impedance to 4 or voltage drop from P to 4 for a load of 1 ampere at 4.

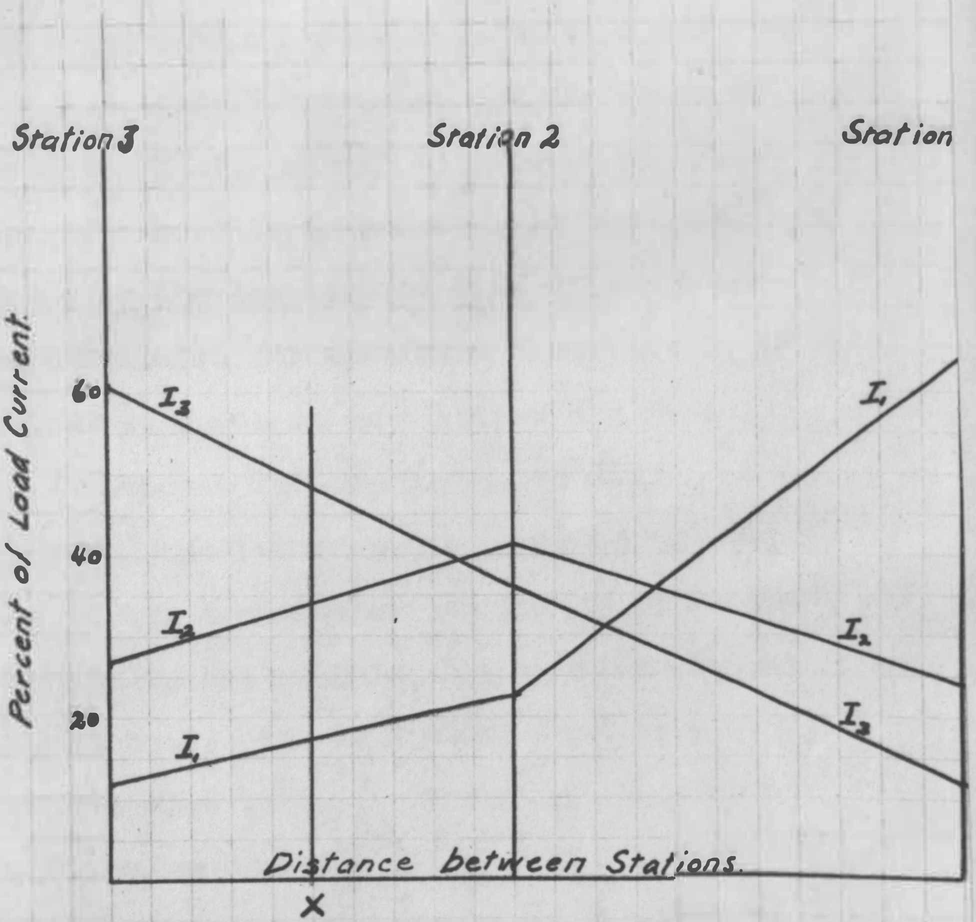

Fig. 5.
Proportion of current supplied by each Substation to a load at any point on the line.

Z_{5-5}, Z_{6-6} = self impedance to 5 and 6 respectively.

Z_{4-5} = mutual impedance between 4 and 5 or the voltage drop from P to 5 for load of 1 ampere at 4.

Z_{5-4} = voltage drop from P to 4 for load of 1 ampere at 5 = Z_{4-5}.

$Z_{4-6} = Z_{6-4}$ = mutual impedance between 4 and 6.

$Z_{5-6} = Z_{6-5}$ = mutual impedance between 5 and 6.

After the self and mutual impedances at transformer stations or other tie-in points have been obtained, the voltage drops to the low tension side of the transformers can be calculated for any fixed distribution of load. The equivalent loads at substations are first found for each individual load as explained above. These loads are then added and the system considered as one, with loads only at the transformer stations. To obtain the voltage drop to 4, for example, the equivalent load at 4 is multiplied by Z_{44} the equivalent load at 5 by Z_{45}, the equivalent load at 6 by $Z_{4\ 6}$. The sum of the three results gives the value desired. The e.m.f. drop to any particular load may then be obtained by application of the formula given in an earlier paragraph on the trolley-rail impedance.

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* The simple and direct method by which these values may be obtained is illustrated in the appendix.

System Fed from Two Sources of Power - A special case occurs when power is fed from two points. If the amount of current fed from the second point is known, it may be considered simply as a negative load and calculations are made in exactly the same way as for positive loads. If the amount of current fed from the second source is not known, it can be calculated by algebraic methods quite simply by considering it as an unknown negative load which maintains the voltage at its point of feeding, to the value for which the second source regulates.

Short-Circuit Currents - As the question of short-circuit currents will come up later when inductive interference is discussed, the methods for determining them will be briefly stated. The self impedance of the point on the system at which the short-circuit occurs determines the current, when the voltage and the impedance of the power plant are known. Any internal impedance in the power plant is added to the self impedance of the point of short-circuit and the generated voltage divided by this value gives the symmetrical r.m.s. value of the current.

In case there are two or more sources of feed, the calculations are a little more involved, but when the second source of power is considered as a negative load the calculation is somewhat simplified. Only the formulae will be given here and the mathematical proof and an example will be found in appendix A.

I_x = the current in the short circuit at X,

$$= \frac{E_o (Z_{ss} + Z_s) - Z_{sx} (E_o - E_s)}{Z_{xx} (Z_{ss} + Z_s) - Z^2_{sx}}$$

I_s = the current from secondary source of power,

$$= \frac{E_o Z_{xs} - Z_{xx} (E_o - E_s)}{Z_{xx} (Z_{ss} + Z_s) - Z^2_{sx}}$$

Where E_o and E_s are the generated voltages at the main and second sources of power respectively, Z_s = the impedance of the second source of power, Z_{ss} = self-impedance to the point of application of second source of power, Z_{xx} = self-impedance to point of short circuit and Z_{sx} = mutual impedance between point of application of second source of power and point of short-circuit.

Formulae for obtaining the short-circuit currant for any number of power stations can be worked out, although for more than two there is considerable complication. If, however, the generated voltages at the power stations are taken as equal to each other, which is generally an allowable assumption, the work is much simplified.

A special case of network is presented by the three wire distribution as applied to the New York, New

Haven and Hartford Railroad electrification. This system utilizes the trolley line as one side and the feeder line as the other side of a 22,000 volt, single-phase transmission. Each line is 11,000 volts above the rail or ground potential and balancer or auto-transformers are connected at intervals between the trolley and feeder with the middle points connected to the rail, as shown in Fig. 6. The calculations for distribution of current, regulation, etc., are made in a manner similar to that for the plain two-wire system. In fact, an equivalent network exactly like that for the two-wire can be constructed and used for current distribution calculations. Impedance calculations to the low-tension sides of the transformers involve a little more work, as the mutual impedance between the trolley-track and the feeder-track circuits must be considered. Calculations of the voltage drop in the trolley-rail circuit of any single section is made in exactly the same manner as that for the two-wire system. In the appendix will be found a brief calculation showing the construction of the network.

Inductive Interference

Two electric circuits paralleling each other have an effect one upon the other, due to the potential of and the current carried by the wires. A voltage on the wires of one circuit will induce a voltage on the

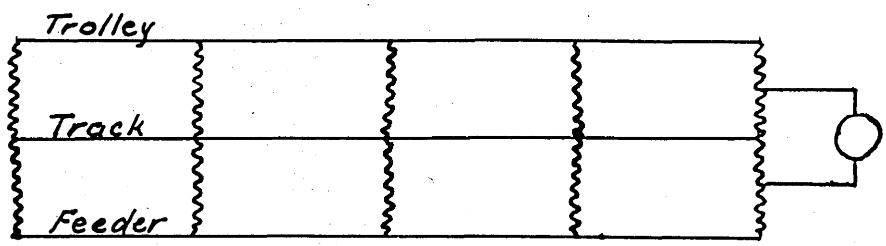


Fig. 6.
Auto-transformers Connected between Trolley and Feeder.

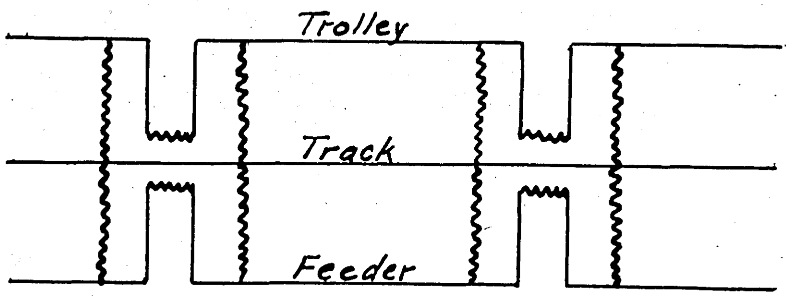


Fig. 7.
Equalizer Transformers Connected between Trolley and Feeder.

wires of the other circuit. A change in the current flowing in one circuit will induce through the changing electro-magnetic field a voltage in the second circuit. The induced voltage from either cause is dependent upon the relative positions of the circuits. For a given position the electrostatically induced voltage is proportional to the potential of the inducing circuit and the electromagnetically induced voltage is proportional to the rate of change of current in the inducing circuit, or for alternating-current of a given frequency is proportional to the magnitude of the current.

<u>Electrostatic Induction</u> - A railway circuit consisting of a trolley wire and return in rails, earth or ground feeder and a parallel telegraph or telephone circuit consisting of a wire with earth or wire return present a case with possibility of induction from the causes mentioned. Considered in regard to the electric induction, a trolley wire is a conductor at a potential above ground with an electrostatic capacity to ground. The trolley wire also has a capacity to a paralleling telegraph or telephone wire which in turn has a capacity to ground. If the telephone wire were perfectly insulated from ground the potential would rise to a value determined by the relative capacity of the wire to the trolley wire and to earth. If the telephone wire is connected

to ground either directly or through other apparatus, it will act as one plate of a condenser, the trolley wire being the other plate. Charging current will then flow from ground into the telephone wire, which will be proportional in magnitude to the capacities between it and the trolley wire. That interference from this cause is not probable is shown by experience on alternating-current railways which have been installed. Generally there is more than one wire on a telegraph or telephone pole line and the charging current is divided among these wires so that the amount in any one is insignificant. If one wire should be entirely insulated from the ground the others which are grounded cause its capacity to ground to be so much in excess of its capacity to the trolley wire that a high voltage is improbable.

Electromagnetic Induction - In working with the problem of electromagnetic induction, the trolley wire and its return may be considered as the primary winding of a large air-core transformer, the telegraph or telephone wire and its return being the secondary winding. The induced electromotive force in a wire paralleling a circuit carrying current is proportional to the logarithm of the ratio of the distances between the wire and the two sides of the power circuit. If a telephone circuit is being considered, in which case there are two wires in which voltage is induced, the e.m.f. may be calculated for each, and the difference will be the voltage which will appear in the circuit. Such resultant induced voltage is caused by "transverse" induction or is due to the magnetic field between the two wires. The voltage calculated for a single wire is that which appears between the wire and the return at infinite distance, at which position the ground return can be assumed, and is caused by the magnetic flux linking with such a circuit, or the "longitudinal" induction. If the wire is grounded at one end, the calculated voltage will appear between the other end and ground. If it is not grounded at all and is uniformly insulated, the middle of the line will be at ground potential, and each end at one-half the calculated voltage from ground--one end being positive and the other negative. If the wire is grounded at both ends a current will flow through it proportional to the voltage and inversely proportional to the impedance of the circuit.

The transverse field can be made very small by so transposing the two wires of the telephone circuit as to cause them to have the same average position with respect to the inducing source. Perfect transposition, such as obtained by twisted pair or cable circuits, reduces the transverse induction to zero. The longitudinal induction, however, cannot be reduced by transposition of the telephone wires. This induction is the cause of voltages between the telephone line and ground and those appearing in grounded telegraph circuits. Longitudinal electromagnetic induction from transmission lines can be neutralized by transposition of the power wires. Transmission lines for railway circuits are no different in this respect from power lines used for general power transmission. Induction arising from the current flow in single phase trolley and rail circuits, however, presents a different situation on account of the impossibility of transposition of the two sides of the power circuit. The magnitude of the electromagnetically induced voltage can assume rather large proportions with such a circuit, as compared with even an untransposed power circuit of oridinary dimensions, because of the distance between the two conductors, the trolley wire on one side and the return circuit made up of rails and earth on the other side.

<u>Position of Earth Current</u> - If all current returns by rail the induced voltage at 25 cycles per mile per ampere is $0.116 \log \frac{D}{D'}$, where D and D' are the distances of the trolley and rail respectively from the telegraph wire. It is easily seen that at a distance of a few hundred feet D and D' would become practically equal

and the value of the voltage, therefore, would be zero. If part of the return current flows in the earth the induced voltage becomes $0.116\ I_t \log \frac{D}{D'} + 0.116\ I_e \log \frac{D''}{D}$ where I_t and I_e are the track and earth currents and D" the distance between the telegraph wire and the path of the earth current. This latter is an unknown quantity and can only be estimated. Tests made in various places have given quite different results. The determination of the effective position of the current for any given case is quite easily made, providing measurements have been obtained of the values of I_t, I_e, D and D'. On the single track line of the Indianapolis & Cincinnati Traction Company, tests and calculations showed the effective position of the earth current to be about 400 feet below the surface. On the New York, New Haven and Hartford Railroad and on the Pennsylvania Railroad, which are four-track roads, the effective position of the current as calculated from the inductive effects is 5000 feet below the surface. Tests on the Norfolk and Western Railread, a two-track road, agreed more nearly with Indianapolis and Cincinnati Traction Company results, but these tests were not as extensively carried out as at the other places.

There is another method of obtaining the effective position of the earth current which does not involve telegraph line voltage measurements. This uses the distribution of current between the earth and track in connection with the known characteristics of the track circuit. Assuming that the earth current is distributed uniformly in a cylindrical conductor tangent to the surface at the tracks and having zero resistance, the reactance may be calculated, and from the reactance the diameter of the conductor may be deducted. Calculations of this kind have shown the effective center of the earth current to be from 200 to 600 feet below the surface. While many of the calculations seem to favor a position of earth current about 500 feet below the surface, it is not considered that this value is a safe one to use in calculating the induced voltages in communication circuits when laying out a system. It is probable that the earth current is not uniformly distributed and possibly it does not penetrate into the earth to any great distance, but is spread out in a layer a few feet or a few hundreds of feet thick and a mile or so wide. Up to the present no single method for calculating the position of the current has proven itself correct for even a few cases for which tests have been made. It is recommended therefore that on new projects where it is impossible to obtain actual measurement of induced voltage per ampere, the value of 5000 feet be used, as it will probably give results within a reasonable degree of accuracy and at least will err on the safe side. For calculating induction in conductors placed within 50 ft. of the trolley it is safe to use a lower value, say 1000 ft. for the depth of the current.

Total Value of Induced Voltage--When the induced voltage per ampere is known for each section of the line it is an easy matter to calculate the total voltage for any given load conditions, by calculating the voltage for each part of the line in which the current is uniform and adding or subtracting the voltages of these sections according to the flow of current. It will be noticed at once that in a layout such as shown in Fig. 2, there will be a neutralizing action for any load between transformer stations, as current will flow in opposite directions on either side of the load. A stub end feed in which all the current comes from one direction is much more severe for induction. If a load is at one end of the network, however, there is the condition of feed in one direction from all substations except the one at which the load is applied. It is necessary to keep such through feed currents to a minimum in order to restrict induced voltages to a low value.

The limiting condition for induction is generally not the hormal load condition, which can usually be kept so distributed in a system fed by several transformer stations, as to cause a good deal of neutralizing action.

Short circuit conditions, however, always have the effect of concentrated loads, and if the short-circuit occurs at the end of a system the through feed is most harmful.

Effects of Induction in Telegraph and Telephone Lines

Telegraph Lines - The effects of the induction can be stated quite simply for telegraph lines. If the induced voltage rises above certain rather definite values, the telegraph instruments chatter and cannot be used. It has been found experimentally that ordinary simple Morse circuits can stand a voltage of about 40 percent of the amount of direct-current battery used. Duplex circuits can stand about 80 percent, while quadruplex is more sensitive and can stand only 15 or 20 percent of the direct-current battery potential under fair operating conditions. Short-circuits are generally not as serious as continued loads, as the disturbance passes quickly and repetition of a word or two of the message is usually the only inconvenience caused. There is a possibility of cases, however, where such a disturbance would simply distort the signals so as to allow a wrong interpretation of the message. So far as is known there have never been any cases of the kind detected.

Telephone Lines - The effects in telephone lines onthe other hand are most seriously felt during short-circuit conditions. Metallic telephone circuits are protected at the ends by carbon block protector gaps between each wire and ground, these gaps being designed for a breakdown of 400 volts or thereabouts, but maintained in certain instances to stand a 200 volt direct-current test. Unless these break downs or other grounds occur on the lines, telephones are seldom disturbed by induction from single-phase railway currents as they are two-wire circuits and usually transposed and balanced sufficiently to keep noise from the operation of trains to a permissible value. Cable lines are particularly good in this respect, whereas open wire lines are more susceptible to trouble. If the two wires of the telephone circuit are not so transposed as to give equal induced voltages in them, noise will result, also even if the wires are well transposed there is liability of noise due to unbalance in capacity or leakage resistance to earth. The noise is not caused directly by 25 cycle current but by ripples on the main frequency wave which are present in the trolley current. Most of the telephone troubles have been found to result from a breakdown of the protector gaps. Such breakdown is dependent upon the magnitude of the induced potential under the short-circuit conditions, which depends upon the absolute rate of change of railway current flow.

Among the troubles caused by gap breakdown, the principal ones are bell-ringing, loading coil trouble, and acoustic shock. If the gap fails there is the danger of cable or exchange wiring insulation breakdown and of electrical shock, although there are no instances of this on record for lines subject to railway induction. If the insulation on the board breaks down there is added a fire hazard. There is also a fire hazard due to voltage which does not approach the breakdown value of the protector gap providing such voltages are sustained for several seconds or minutes. Sustained voltages will be found during train operation, but generally speaking, if the voltages occurring during short-circuit are not high enough to cause hazard from the causes mentioned above, the normal sustained operating voltage will not be high enough to cause fire hazard. The various troubles should probably be explained more fully.

<u>Bell Ringing</u> - This is simply a ringing of the subscribers' bell or the operating of the drop at the central office which, while not of a very serious nature, is an annoyance which should be reduced to a minimum.

<u>Loading Coil Trouble</u> - This will occur in loaded trunk lines. The two wires of the telephone line pass through the loading coil windings and if the protector gap on one line breaks down, more current will flow thru one winding than thru the other, sometimes permanently

changing the permeability of the iron and rendering the coil inefficient. This effect is produced by the magnitude of the current, independent of its frequency.

<u>Acoustic Shock</u> - This is caused by loud noise in the receiver in case a gap breaks down with any heavy discharge following. It is experienced chiefly by operators and is considered a serious condition.

<u>Electrical Shock</u> - Operators and switchboard or line repairmen will be subject to this hazard.

<u>Fire Hazard</u> - Any breakdown of insulation on the board or in the exchange wiring is liable to cause fire, provided enough current flows. Telephone engineers have stated that a breakdown of insulation at 200 volts with one-half ampere flowing for two seconds is considered as a possible cause of fire.

With the above points in view it can be said that if the maximum induced voltage for a period of two seconds is not over 200 volts the chances for trouble in the telephone circuits would be very small. For a longer time, covering several minutes, an induced voltage in the neighborhood of 100 volts or less is probably safe and for a shorter time than two seconds it would appear that higher voltages than 200 can be tolerated. In many cases voltages of several times 200 volts have been induced with no bad results aside from bell-ringing even when continued for one or two seconds. Just where the safe limit of voltage is for a duration of time represented by a short circuit (less than one-half second) cannot be stated. For equal fire hazard the allowable voltage should increase inversely as the square root of the time. None of the above limits can be considered as hard and fast, but represent opinions gained after a study of the records of telephone disturbances and general conditions obtained in the communication circuits.

Systems of Feed Which Tend to Reduce Disturbance

In laying out the transmission line and transformer stations very much can be done which will tend to make the system harmless as far as telephone induction is concerned. Some of the points which can well be considered have already been suggested.

Location of Transformer Stations - Probably the most harmful railway system from the viewpoint of the telephone or telegraph engineer is one in which the trolley is fed for several miles from one end only. If any telegraph or telephone lines parallel such a system, they are pretty certain to be disturbed. Feeding by a transformer station at the end opposite from the first feed-in point will reduce induced voltages to about 30 percent and sometimes to considerably less than this. A liberal use of transformer stations is one of the best cures for the induction evil. The characteristics of transformer stations, the impedance of the trolley-rail circuit, the transmission line impedance, all can be adjusted so as to reduce the through feed factor, and the impedance of step-up transformers can also be varied to advantage. Each particular case must be treated individually and no set rulecan be given to eliminat inductive troubles. In general, through feed is reduced by increasing the trolley-rail impedance and decreasing the impedance of step-down transformers and transmission lines. The trolley-rail impedance is determined by the size of the trolley wire, which in turn should be determined by the carrying capacity of the wire. This will give the highest impedance permissible for the trolley-rail circuit. Step-down transformers should then be made of as low reactance as is compatible with cost and the transmission line impedance kept low in comparison with that of the trolley rail. With these factors determined and the regulation desired fixed, whatever reactance is still allowable should be put in the step-up transformers and generators so that short circuit currents will be as small as possible.

Sectionalized System - In cases where telephone wire exposure is very severe and through feed conditions must be reduced beyond the amount given by the above methods, good results can be obtained by sectionalizing the transformer stations, i.e. using two transformers at each station with the high-tension sides paralleled, but with the low-tension sides separate, one transformer feeding each direction. This impairs the regulation of the system somewhat, unless the load is well distributed.

If these points are carefully considered in laying out the system, it will be found that the addition of any special devices, such as will be spoken of later, may be in a large measure, if not altogether, obviated.

Three-Wire System - The three-wire scheme as applied on the New York, New Haven and Hartford Railroad is a special form of multiple transformer station system. This system has all the advantages of the plain two-wire system which has been mentioned above, with the added advantage of using auto-transformers which, for equal capacity, are smaller than two coil transformers, and of using the copper trolley as part of the transmission circuit. Transformer stations can, therefore, be placed a little closer together for the same cost, and induction will be reduced if this is done. The transmission voltage is limited, however, to a value of double the trolley voltage. The calculation of the induction in telephone and telegraph lines is made in exactly the same way as described for the simple two-wire system.

Sectionalized Three-Wire System - Several schemes for the sectionalizing of a three-wire system at the transformer stations have been worked out. Some of them have been tried in the laboratory with very good results, but none have been applied in practice. The simplest of these schemes is shown in Fig. 7. Equalizer transformers with one to one ratio are connected between trolley and feeder lines at the transformer stations so that the current in the trolley and feeder windings must be equal. The equalizer must have a current capacity equal to the total line current at the point at which it is installed and a voltage rating equal to the e.m.f. drop in the auto-transformers at the station where it is installed. If short-circuits are to be cared for without any through feed this means an equalizer transformer rating of full trolley voltage (11,000 volts), but if a small amount of through feed is permissible under severest short-circuit conditions, a lower rating can be used.

Special Devices for Introduction Into Railway Circuits to Reduce Induction

What has been said covers generally the whole system of distribution on the railroad. There are, however, certain minor changes which can be made which have the effect of reducing induction. The object of such devices is to reduce the earth current.

<u>Ground Feeders</u> - Overhead return feeders will allow a portion of the return current to flow in them and so reduce the rail and earth currents. Calculations have been made which indicate that the induction may be reduced in the neighborhood of 30 to 50 percent by using a 4/0 wire as return feeder, paralleling each pair of rails, if the feeder is placed within a few feet of the trolley wire.

If a return feeder is placed close to the disturbed telegraph or telephone line, the reduction in induced voltage for that particular line is greater. The current carried by such a feeder will depend upon the voltage impressed upon it and its impedance. If the feeder circuit impedance were entirely reactance, the current flow would be of the correct phase and approximate amount to neutralize the voltage appearing in the telephone line, with the feeder wire simply connected to earth. But even if the wires were large, the resistance of ground connections would be so high, ordinarily, that the current would be too small for anything like complete neutralization. Connection to the rails gives a circuit of lower impedance. In order to get a current in the feeder of the desired amplitude and phase, however, it is necessary to use some such device as a booster transformer between it and the trolley wire. With such an arrangement, distribution of the transformers and conditions which might cause saturation must be carefully considered, as brought out in the following paragraphs on track and feeder boosters. This scheme has never been applied on account of the many practical difficulties against it.

Booster Transformers - The device which has been used more than any other for accomplishing the reduction of induced voltage is the booster transformer for track or for a feeder close to the trolley wire. This is a transformer whose primary winding is connected in series with the trolley circuit and whose secondary is connected in series either with the track or a return feeder, the primary and secondary ratio being approximately one to one. The use of a return feeder overhead in connection with booster transformers is objectionable first from the standpoint of cost. The feeders are used instead of the track for the return path for the current, and in order to keep the regulation of the line within reasonable limits they must be of large section. Two 100 lb. rails have a resistance to 25 cycles of approximately 0.08 ohms per mile, which is equivalent to about three 4/0 copper wires. Smaller feeders, of course, may be used at the expense of regulation and an increased size of the booster transformers. The voltage rating of the transformer must be the voltage drop in the section of feeder to which it is connected. The advantage of the use of the return feeder rather than the track, lies in the greater reduction of induced voltage in telegraph and telephone lines running very close to the railroad. The use of booster transformers connected to the track utilizes the low impedance return circuit which the rails provide and saves the cost of the copper feeders. The impedance of the trolley-rail circuit is increased a little over the value when no boosters are used, on account of the current being forced to return entirely by rail, and on account of the impedances of transformer windings. The increase is shown in the tables given in the appendix.

Booster transformers, when of sufficient size and used in sufficient numbers, give very good results in actual service. There are practical limitations, however, both as to size and number. The limitation as to size appears in the voltage across the secondary. This is limited, especially in the case of the track booster, as it is not advisable to introduce high voltage between two adjacent sections of track either under normal operating conditions or during periods of short-circuit. The maximum voltage allowable under short-circuit conditions can probably be taken as the limiting value. Once this is set the number of transformers necessary is easily determined by spacing them at such intervals that the maximum voltage drop in the sections of track will be less than the limiting value of the voltage. It will be found on the larger electrifications that quite a close spacing will generally be necessary, i.e., one, two or even more per mile being required. The objection to close spacing lies in the number of insulating joints or section breaks in the trolley or track to be installed and maintained. With a return feeder, spacing closer than a mile apart may not be necessary but the voltage of the individual transformer must be correspondingly raised.

When transformers are not installed in sufficient numbers complications result, especially under short-circuit conditions. When the short-circuit occurs the transformer iron becomes saturated and the voltages across them take on a decidedly peaked wave shape which is hard on insulating joints and on short telephone lines with a terminal close to a booster transformer. Moreover, the induced voltages in all telegraph and telephone lines is peaked and in some cases will rise to values considerably in excess of the values which it would attain were the boosters not in the circuit. The latter phenomenon is due to the fact that with boosters there is always a residual current in the earth, this current being the magnetizing current of the transformer. Under saturated conditions the magnetizing current is not only large but is of distorted wave shape. As far as short-circuit conditions are concerned it is better not to have any transformers in service than to have so few that saturation will occur. Track boosters are here at somewhat of a disadvantage as against feeder boosters, as the former are apt to be short-circuited by trains on the tracks at insulating joints and so possibly many of them would be out of commission at the time they are most needed.

Conditions of Railway Operation

It is possible so to operate a well laid out road, as far as induction is concerned, as to cause severe inductive effects in telephone and telegraph lines as well as to give poor regulation. A few simple precautions, however, if observed, will generally prevent trouble from these causes.

One way feed to a load gives best opportunity for indictive troubles to appear. Therefore the circuit breakers should be closed on both ends of a section of trolley on which there is any considerable load. On a multiple transformer station system, precautions should be taken against operation with an end station out of commission and with considerable load in the end section. This condition gives a stub end feed on all tracks from the next transformer station. Provision should be made in the end station to minimize the possibility of such operation becoming necessary. If circuit breakers are to be cut out for inspection or repair, such work should be done when the section is not loaded, or spare apparatus should be available so that the feed from the end station will not be interrupted.

On a sectionalized system, it is necessary to see that train loads do not get spaced so that they are one substation apart. It is conceivable that such a condition might be obtained, although it is unlikely. If all the trains were leaving or all approaching substations, in relatively the same positions, under such a condition, the current feed in the trolley would in effect be entirely in one direction and therefore inductive interference might become severe.

Under short-circuit conditions, the circuit breakers should operate quickly as well as selectively. A short-circuit is a heavy concentrated load. If it be in the middle of a section the current feeds in the two directions and will tend to neutralize their individual effects but if the short-circuit is at the end of a section - expecially in the case of an end section - the inductive effect can become very high. It is therefore desirable that circuit breakers be set to operate so quickly that the induced voltage will not last long enough to cause trouble. High speed under such conditions is not incompatible with selective action, as the high speed is necessary only when the current is so high that there is no doubt that the circuit breaker is feeding directly into the short-circuited section, and the trip can be set to operate under this condition irrespective of the time relay which controls it under lower values of overload. Thus selectivity is obtained on heavy short-circuits on account of the fact that only circuit breakers feeding directly to the short-circuit can have enough current to operate the trip directly, and on high resistance short-circuits because of the time limit relays.

Special Devices for Introduction Into Telephone and Telegraph Lines

Certain schemes which have been used involve the introduction of apparatus directly into telephone or telegraph lines in order to neutralize the induced voltage or to keep it from causing disturbance. Much progress has been made along this line and several schemes have been proposed, many of which have been tried out experimentally, and found satisfactory up to certain limits, and there are others which have actually been put into regular service.

The Neutralizing Transformer - First among these is the neutralizing transformer. It was proposed for use on the New York, New Haven & Hartford Railroad before the electrical operation of the road was begun and one pole line in the worst exposure was equipped in time to take care of the heavy induction which ensued when trains began to operate. Under normal operating conditions the induced voltage on the telegraph line paralleling the road frequently rose as high as one thousand volts, yet the wires were worked as usual without serious interruption. Other lines were later equipped with the device.

The neutralizing transformer is simply a multiple secondary, one to one ratio (approximately) transformer, whose primary is connected into circuit with a wire running among the wires to be protected and grounded at the ends of the electrification, and between the transformers if more than one is used. Each telegraph wire is connected to a separate secondary winding so that the voltage of the windings opposes the induced voltages in the wire. The residual voltage in the telegraph or telephone lines is limited by the resistance of the neutralizing or pilot wire and the magnetizing current in the transformer and under normal conditions is equal to the product of these two. The objection to its use is mainly in the transmission loss added to the telegraph or telephone line. While loss in transmission efficiency is not great, it is considered objectionable when it is considered what extreme measures are taken by telephone companies to improve the transmission of their lines. It is not economical when used on long lines when the induction is only moderate in amount. On the New York, New Haven and Hartford Railroad the amount of neutralization obtained on the main telegraph pole line was approximately 95 percent, the residual voltage being five percent of the total induced voltage.

Increase of Resistance of Telegraph Wires - For induced voltages of moderate value it is sometimes possible to insert resistance in a telegraph circuit and raise the direct-current working voltage accordingly, so that the ratio of induced voltage to direct-current potential does not exceed the allowable amount. Impedance added also tends to reduce the disturbance although it slows up slightly the working of the telegraph circuit.

Resonant Shunts - have been tried and are now being used in several telegraph lines. One type of shunt consists of a parallel resonant circuit for 25 cycles, and so is in effect a very high resistance to this frequency. For direct current, however, it introduces very little resistance. When it is connected in series with the telegraph instrument, the telegraph current, which has a relatively steep wave front, passes through the condenser at the beginning of the signal, and the sustaining or the following current flows through the inductive reactance with a small voltage drop due solely to the resistance of the coil. This kind of a shunt may be used directly in a Morse circuit or in the relay circuits of a duplex or composite outfit.

A circuit in series resonance for 25 cycles can be placed in parallel with the telegraph relay and such an arrangement will cause the alternating current to flow through the resonant circuit rather than through the relay. The device has been successfully applied and has worked well with an induced potential ten times the maximum allowable without protection.

Variations of these two devices constitute most of the other schemes proposed with the same end in view. Some of these schemes have been experimented with in Europe and several apparently have considerable merit. Those which have been tested in telegraph lines along the Pennsylvania Railroad Philadelphia-Paoli electrification are shown in appendix B.

Telephone Circuits - There is some objection to the interposition of corrective devices in telephone lines on account of the high transmission efficiency which must be maintained and the more delicate nature of telephone instruments when compared with telegraph instruments.

The neutralizing transformer has been used for telephone circuits, but it must be wound with each secondary composed of two wires wound side by side so as to prevent the introduction of impedance directly into the talking circuit. While they are more expensive than the transformers made for telegraph circuits, still the principal objection to their use is the loss in transmis sion efficiency.

Telephone trunk lines have been operated in the neighborhood of railway circuits which induce potentials in them, with reactance coils bridged across the telephone line with the middle points grounded. With a device of this kind at each end of the trunk circuit no 25 cycle voltage can appear on the switchboard, but the transmission loss again appears as an objection.

The main safeguard against telephone trouble seems to be in the maintenance of the line in good condition and depending upon the protector gaps to remove hazardous potentials.

<u>Lead Covered Cables</u> - The use of lead covered cables for either telephone or telegraph lines has an advantage besides the easier maintenance, in that the lead sheath acts as a return feeder close to the telephone wires. There are objections to connecting the lead sheath to the rails and so the full value of the return is not felt, but experience indicates that the

lead sheath on a large cable reduces the induction by about 10 to 30 percent.

Insulating Transformers - Telephone instruments may be connected to the telephone line through repeating transformers and thus prevent any induced voltage on the line from being transmitted to the instrument. This scheme is particularly applicable to private telephone lines, such as those from substations to power-house, etc. It is often used for lines running on the same poles as high-tension transmission wires. Transverse induction is not protected against by the transformer, and therefore there is no reduction in noise resulting from imperfect transposition or protector breakdowns. Transmission loss is involved in the transformer, but this is generally not serious on a private line, providing not too many transformers are bridged across one pair of wires.

Inductive Interference From Existing Single-Phase Railways

The first roads to adopt the single-phase, alternating-current system were interurban trolley lines. The traffic was light compared with later installations, but the voltage was lower and therefore the current was high enough to give as much electromagnetic induction as a higher voltage, high powered line. The first study of inductive interference was made on the Derry and Latrobe line, a short line in Pennsylvania run at a trolley voltage of 1000 volts. The test made there gave enough information to enable a more comprehensive set of tests to be made on the 3300 volt Indianapolis and Cincinnati Traction Company line running from Indianapolis to Rushville, Indiana, a distance of 40 miles. It was during these tests, in 1906, that an idea of the division of return current in rail and earth was first obtained with any degree of definiteness. The neutralizing transformer for use in telephone and telegraph lines was also first tried out during these tests, with the result that it was considered advisable to depend upon this scheme for taking care of the inductive interference problems of the New Haven electrification.

The Indianapolis and Cincinnati electric line did not cause serious interference in neighboring circuits, principally because it was broken into sections of about ten miles in length, each section being fed from a separate transformer station. The line was later extended, using the same feeding system, and inductive interference trouble has not occurred.

The New Haven electrification was first laid out with only one point of feed, at Cos Cob, near one end of the line. Induced voltages in the railroad telegraph line running on the right of way were calculated from the data obtained on the Indianapolis and Cincinnati line and the values indicated were checked later by test and found to be substantially correct. The neutralizing transformer scheme was adopted and the transformers were put in position and connected before operation began. The result was that the railroad telegraph lines gave successful service and were operative even when the induced voltage was as high as 1000 volts. The transformers neutralized this enough to keep the residual below 50 volts. When short-circuits occurred on the line, the transformers probably were saturated and the residual must have been in the neighborhood of 1000 volts and possibly more, but no damage was caused. The relays would rattle for a second, or until the short-circuit was cleared, and this was the only result.

The American Telegraph and Telephone Company had lines paralleling the New Haven at a distance of a few hundred feet. These lines were worked as telephone lines during the first months of the electrification with no inconvenience, other than some noise in the open wire circuits. The voltage under normal operation reached values of about 200 volts, and under short-circuit conditions it must have reached several times this amount, although no measurements to determine the mqximum values were made. The lines could not be used for composite telephone and telegraph service, however, and for this reason neutralizing transformers, especially built so as to be suitable for composite circuits were installed.

The increase and extension of the New Haven electric service necessitated a better system of feed, on account of voltage regulation and also because of the increased induced voltages. The three-wire system was adopted and the entire line was changed to this scheme. The result was very beneficial to telegraph operation and it has been found possible to operate the railroad telegraph circuits with the neutralizing transformers cut out of service with very little disturbance, although the length of the electrification, and therefore the exposure of the lines, is three times as great as when the road began operation under the old system of feed.

The scheme of using track booster transformers was given a test in the spring of 1914 on the New Canaan branch of the New Haven Railroad, which is a seven mile stub end road branching from the main line at Stamford, Conn. The tests showed that this scheme was workable and it was installed on the branch, clearing up the inductive interference there.

The same scheme was applied on the Norfolk and Western Railroad and has given successful results. It was brought out in tests at this place that the transformer scheme was not giving as low an induced voltage as indicated by the New Canaan experiments. This was found to be due to abnormal leakage around insulating joints, on account of the kind of ballast and a water piping system close to the rails. However, the induction is not such as to cause interference at present. The distribution system is on the multiple substation plan.

The Philadelphia-Paoli electrification employed booster transformers at the start, on account of the railroad telegraph circuits which, without boosters, would have between one and two hundred volts induced during normal operation. Telegraph operation was perfect, but considerable bell ringing trouble at times of short-circuits was caused in the Telephone Company lines and investigation traced the cause to the saturation of the boosters. The resonant shunt for telegraph lines was developed and connected into the telegraph circuits in order to allow the removal of the boosters, with the object of getting rid of bell ringing. Boosters on one half the line have been removed and resonant shunts installed in the telegraph lines with very favorable results. The telegraph business of the railroad is carried on as duplex composite, four wires making up a set of four duplex telegraph circuits, two direct telephone circuits and one phantom telephone circuit. Bell ringing in telephone lines has been practically eliminated.

On the next single-phase electrification made in the United States, which is that of the Chestnut Hill branch of the Pennsylvania Railroad at Philadelphia, a careful study of the substation layout was made prior to the installation, with the idea of keeping inductive interference to a minimum. The transformer stations were located with this in view, and the result has been that it has been unnecessary to install any corrective devices, either in the railway circuits or the communication lines. This line went into service in 1918 and no case of bell ringing or other disturbance has been reported.

Other single-phase electrifications have been installed with multiple substation systems, and the inductive interference has been practically negligible. The special cases mentioned above are those in which special investigations and tests seemed desirable, and are therefore those on which the most data is available.

Conclusion

In planning for an electrification which is paralleled by telephone and telegraph circuits, it is essential that induction be taken into account from the start. The introduction of special devices into either the telegraph or the railway circuits is annoying and sometimes quite expensive. By carefully laying out the transformer stations and determining the probable induced voltages beforehand, very little extra expense should be involved and a system can be obtained which is free from most, if not all, of the extra devices. If the stations are so laid out that the short-circuit conditions on the railroad do not cause dangerous voltages in the telephone lines, normal operating conditions will, as a general rule, take care of themselves. If, after all reasonable steps have been taken in the railroad circuit, the induced voltage is still above the working limits and yet does not cause a physical hazard, the use of some such device as the resonant shunt in the telegraph lines will probably take care of whatever voltage remains. If there is a short distance where the exposure is very severe, and the spacing of transformer stations to take care of this condition would be so close as to cause considerable expense, such a short stretch could be protected by some of the special devices considered above, without affecting the rest of the system.

TABLE I—TROLLEY RAIL IMPEDANCES

Trolley Rail Circuit	TROLLEY	TRACKS	TROLLEY	RETURN CIRCUIT		TOTAL IMPEDANCE	
				With Boosters	Without Boosters	With Boosters*	Without Boosters
3/0 Phono 1/0 Copper 100 lb. Rails	1	1	0.316 + j0.306	0.092 + j0.211	0.037 + j0.084	0.422 + j0.534 = 0.680	0.353 + j0.390 = 0.526
	1	2	0.316 + j0.310	0.046 + j0.122	0.028 + j0.073	0.376 + j0.449 = 0.585	0.344 + j0.383 = 0.514
	1	4	0.316 + j0.322	0.023 + j0.073	0.016 + j0.051	0.353 + j0.412 = 0.541	0.332 + j0.373 = 0.498
	2	2	0.158 + j0.170	0.046 + j0.122	0.028 + j0.073	0.218 + j0.309 = 0.377	0.186 + j0.243 = 0.305
	2	4	0.158 + j0.180	0.023 + j0.070	0.016 + j0.049	0.195 + j0.267 = 0.330	0.174 + j0.229 = 0.289
	4	4	0.079 + j0.094	0.023 + j0.070	0.016 + j0.049	0.109 + j0.173 = 0.204	0.095 + j0.143 = 0.171
3/0 Phono 2/0 Copper 100 lb. Rails	1	1	0.270 + j0.309	0.092 + j0.211	0.037 + j0.084	0.376 + j0.537 = 0.654	0.307 + j0.393 = 0.498
	1	2	0.270 + j0.313	0.046 + j0.122	0.028 + j0.073	0.330 + j0.452 = 0.560	0.298 + j0.386 = 0.488
	1	4	0.270 + j0.325	0.023 + j0.073	0.016 + j0.051	0.307 + j0.415 = 0.515	0.286 + j0.376 = 0.472
	2	2	0.135 + j0.172	0.046 + j0.122	0.028 + j0.073	0.195 + j0.311 = 0.366	0.163 + j0.245 = 0.294
	2	4	0.135 + j0.182	0.023 + j0.070	0.016 + j0.049	0.172 + j0.269 = 0.318	0.151 + j0.231 = 0.275
	4	4	0.068 + j0.095	0.023 + j0.070	0.016 + j0.049	0.098 + j0.174 = 0.199	0.084 + j0.144 = 0.166
3/0 Phono 3/0 Copper 100 lb. Rails	1	1	0.232 + j0.312	0.092 + j0.211	0.037 + j0.084	0.338 + j0.540 = 0.634	0.269 + j0.396 = 0.478
	1	2	0.232 + j0.316	0.046 + j0.122	0.028 + j0.073	0.292 + j0.455 = 0.538	0.260 + j0.389 = 0.467
	1	4	0.232 + j0.327	0.023 + j0.073	0.016 + j0.051	0.269 + j0.417 = 0.494	0.248 + j0.378 = 0.450
	2	2	0.116 + j0.173	0.046 + j0.122	0.028 + j0.073	0.176 + j0.312 = 0.358	0.144 + j0.246 = 0.282
	2	4	0.116 + j0.183	0.023 + j0.070	0.016 + j0.049	0.153 + j0.270 = 0.310	0.132 + j0.232 = 0.266
	4	4	0.058 + j0.096	0.023 + j0.070	0.016 + j0.049	0.088 + j0.175 = 0.195	0.074 + j0.145 = 0.162
4/0 Phono 4/0 Copper 100 lb. Rails	1	1	0.189 + j0.309	0.092 + j0.211	0.037 + j0.084	0.295 + j0.537 = 0.610	0.226 + j0.393 = 0.452
	1	2	0.189 + j0.313	0.046 + j0.122	0.028 + j0.073	0.249 + j0.452 = 0.515	0.217 + j0.386 = 0.442
	1	4	0.189 + j0.325	0.023 + j0.073	0.016 + j0.051	0.226 + j0.415 = 0.471	0.205 + j0.376 = 0.428
	2	2	0.094 + j0.172	0.046 + j0.122	0.028 + j0.073	0.154 + j0.311 = 0.346	0.122 + j0.245 = 0.273
	2	4	0.094 + j0.182	0.023 + j0.070	0.016 + j0.049	0.131 + j0.269 = 0.298	0.110 + j0.231 = 0.256
	4	4	0.047 + j0.095	0.023 + j0.070	0.016 + j0.049	0.077 + j0.174 = 0.189	0.063 + j0.144 = 0.157
4/0 Copper 100 lb. Rails	1	1	0.260 + j0.368	0.092 + j0.211	0.037 + j0.084	0.366 + j0.596 = 0.696	0.297 + j0.452 = 0.540
	1	2	0.260 + j0.372	0.046 + j0.122	0.028 + j0.073	0.320 + j0.511 = 0.600	0.288 + j0.445 = 0.528
	1	4	0.260 + j0.384	0.023 + j0.073	0.016 + j0.051	0.297 + j0.474 = 0.558	0.276 + j0.435 = 0.513
	2	2	0.130 + j0.200	0.046 + j0.122	0.028 + j0.073	0.190 + j0.339 = 0.387	0.158 + j0.273 = 0.315
	2	4	0.130 + j0.210	0.023 + j0.070	0.016 + j0.049	0.167 + j0.297 = 0.340	0.146 + j0.250 = 0.296
	4	4	0.065 + j0.109	0.023 + j0.070	0.016 + j0.049	0.095 + j0.188 = 0.211	0.081 + j0.158 = 0.177

Booster Impedance = 0.014 + j0.017 except 4-Trolleys, 4-Track = 0.007 + j0.009.

APPENDIX A

TWO-WIRE NETWORK

To construct the equivalent network, reduce all impedances to a base voltage (trolley voltage is preferable as base). For example, in Fig. 8,

Z_a, Z_b = impedance of feeders.
Z_c, Z_d = impedance of trolley-track.
Z_1, Z_2, Z_3 = impedance of step down transformers.
Z_0 = impedance of step up transformers and feeder to *1*.

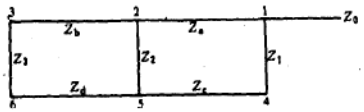


FIG. 8—EQUIVALENT TWO-WIRE NETWORK

First consider mesh *2-3* separately and as fed from *2* with one ampere load at *5*, Fig. 8.

$$I_3 = \frac{Z_2}{Z_b + Z_3 + Z_d + Z_2} = m$$

$$I_2 = \frac{Z_b + Z_3 + Z_d}{Z_b + Z_3 + Z_d + Z_2} = n$$

$I_2 Z_2 = Z_{235}$ = impedance of network 2-3 from 2 to 5.

Consider mesh *1-2* with side *2-5* made up of mesh *2-3*. For a load at *4*,

$$I_1 = \frac{Z_a + Z_c + Z_{235}}{Z_a + Z_c + Z_1 + Z_{235}} = K$$

$$I_{235} = \frac{Z_1}{Z_a + Z_c + Z_1 + Z_{235}} = L$$

$I_1 Z_1 = Z_{44}$ = self impedance to *4*.
$I_2 = nL \qquad I_3 = mL$
$Z_{45} = Z_{44} - LZ_c$
$Z_{46} = Z_{45} - mL\ Z_d$

For load at *5*,—

$$I_1 = \frac{Z_a + Z_{235}}{Z_a + Z_c + Z_1 + Z_{235}} = K'$$

$$I_{235} = \frac{Z_1 + Z_c}{Z_a + Z_c + Z_1 + Z_{235}} = L'$$

$I_2 = nL' \qquad I_3 = mL'$
$Z_{54} = K'\ Z_1$
$Z_{55} = K'\ (Z_1 + Z_c)$
$Z_{56} = K'\ (Z_1 + Z_c) - mL'\ Z_d$

To obtain the constants for load at *6* consider the network as supplying three loads, a positive load at *6*, a positive load at *5* and a negative load at *5*, all loads being numerically equal. First consider a load at *6* fed from *5*. To solve this, mesh *2-1-5* must first be solved alone, giving the value of the impedance from *5* to *2* = Z_{512}. Then, in mesh *5-1-2* with load at *2* fed from *5*,—

$$I_1 = \frac{Z_2}{Z_c + Z_1 + Z_a + Z_2} = m'$$

$$I_2 = \frac{Z_c + Z_1 + Z_a}{Z_c + Z_1 + Z_a + Z_2} = n'$$

$Z_{512} = n'\ Z_2$

Consider mesh *5-3-2* with side *2-5* made up of mesh *5-1-2*. For load at *6* fed from *5*,—

$$I_d = \frac{Z_3 + Z_b + Z_{512}}{Z_3 + Z_b + Z_{512} + Z_d}$$

$$I_{512} = I_3 = \frac{Z_2}{Z_3 + Z_b + Z_{512} + Z_d} = m''$$

$I_2 = n'\ m''$
$I_1 = m'\ m''$

Superimpose these values on those obtained for load at *5* and the result is obtained for a load at *6* fed from *4*.

$I_1 = K' - m'\ m''$
$I_2 = nL' - n'\ m''$
$I_3 = mL' + m''$
$Z_{64} = (K' - m'\ m'')\ Z_1$

$Z_{65} = Z_{64} + (K' - m'\ m'')\ Z_c$
$Z_{66} = Z_{65} + (K' - m'\ m'' + nL' - n'\ m'')\ Z_d$

The results must satisfy the following equations:—

$Z_{45} = Z_{54}$
$Z_{46} = Z_{64}$
$Z_{56} = Z_{65}$

The above results give self and mutual impedances from *1*. To change these to the values from the source of power add Z_0 to each.

SHORT CIRCUIT CURRENT IN NETWORK FED FROM TWO SOURCES OF POWER

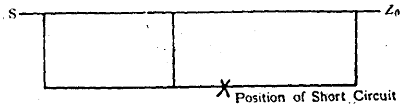


FIG. 9—SHORT-CIRCUIT IN NETWORK FED FROM TWO SOURCES

In Fig. 9—

Z_{xx} = Impedance of point x from primary source of power
Z_{ss} = Impedance of point x from secondary source of power
Z_{sx} = Mutual impedance of points x and s
Z_s = Internal impedance of secondary source of power
E_o = Generated voltage of primary source of power
E_s = Generated voltage of secondary source of power
I_x = Current at x
I_s = Current from secondary sources

E.m.f. drop from primary source to $x = I_x Z_{xx} - I_s Z_{sx} = E_o$
E.m.f. drop from primary source to $s = I_x Z_{sx} - I_s Z_{ss}$
E.m.f. at $s = E_o - I_x Z_{sx} + I_s Z_{ss} = E_s - I_s Z_s$
$= I_x Z_{sx} - I_s (Z_s + Z_{ss}), = E_o - E_s$

$$I_s = \frac{Z_{xx}(E_o - E_s) - E_o Z_{sx}}{-Z_{xx}(Z_s + Z_{ss}) + Z_{sx}^2} = \frac{E_o Z_{sx} - Z_{xx}(E_o - E_s)}{Z_{xx}(Z_{ss} + Z_s) - Z_{sx}^2}$$

$$I_x = \frac{-E_o (Z_s + Z_{ss}) + Z_{sx}(E_o - E_s)}{-Z_{xx}(Z_s + Z_{ss}) + Z_{sx}^2} = \frac{E_o (Z_{ss} + Z_s) - Z_{sx}(E_o - E_s)}{Z_{xx}(Z_{ss} + Z_s) - Z_{sx}^2}$$

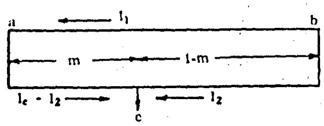


FIG. 10—EQUIVALENT NETWORK OF TROLLEY-TRACK CIRCUIT
For one track of a two track system.

In Fig. 10—

Z_1 = Impedance of trolley-track circuit for one trolley from a to b.

Z_m = Mutual impedance between trolley wires from a to b

(1) $I_1 Z_1 + (I_c - I_2) mZ_1 - I_2 (1\text{-}m) Z_1 - I_1 Z_m - (I_c - I_2)(m Z_m) + I_2 (1 - m) Z_m = 0$

(2) $I_1 Z_1 + I_2 (1\text{-}m) Z_m - (I_c - I_2) m Z_m = E_b - E_a$

(3) $E_a - E_c = (I_c - I_2) mZ - I_1 m Z_m$

From (1) $I_1 = I_2 - m I_c$

Substituting this value of I_1 in (2) and (3) and solving gives

$$E_a - E_c = m (E_a - E_b) + (1\text{-}m) I_c mZ$$
$$= m (E_a - E_b) + (1 - m) I_c Z_{ac}$$

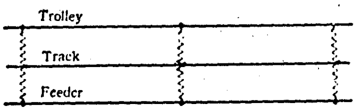


FIG. 11—THREE-WIRE SYSTEM

IMPEDANCE FROM SUBSTATION TO LOAD ON ONE TRACK OF A TWO-TRACK SYSTEM

Consider the network in Fig. 11 as made up of two circuits with self and mutual impedance.

1—Trolley-feeder circuit at double voltage.

2—Trolley-rail circuit at base voltage.

Reduce all impedances to base voltage and let

Z_1, Z_2, etc. be the impedances of the auto-transformers.

Z_{12}, Z_{23}, etc. be the impedances of the trolley-feeder circuit = one-fourth actual impedances.

Z_{52}, Z_{56}, etc. be the impedances of the trolley-rail circuit.

Z_{m1}, Z_{m2}, etc. be the mutual impedances between the two circuits.

The network can then be constructed in the same manner as for the two-wire system except that the mutual impedances must be taken into account.

In Fig. 12, consider mesh 3-7 with unity load at 7 fed from 3.

(1) $I_4 (Z_{34} + Z_4) + I_3 \frac{Z_{m3}}{2} = I_3 (Z_3 + Z_{67}) + I_4 \frac{Z_{m3}}{2}$

(2) $I_3 + I_4 = 1$

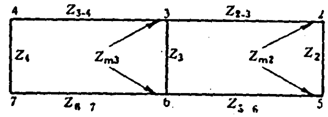


FIG. 12—EQUIVALENT NETWORK FOR THREE-WIRE SYSTEM
THREE-WIRE SYSTEM—EQUIVALENT NETWORK

From which

$$I_3 = \frac{Z_{34} Z_4 - \frac{Z_{m3}}{2}}{Z_3 + Z_4 + Z_{34} + Z_{67} - Z_{m3}}$$

$$I_4 = \frac{Z_{67} Z_3 - \frac{Z_{m3}}{2}}{Z_3 + Z_4 + Z_{34} + Z_{67} - Z_{m3}}$$

Consider load at 6 fed from 3.

(1) $I_4 (Z_{34} + Z_4 Z_{67} - \frac{Z_{m3}}{2} - \frac{Z_{m3}}{2}) = I_3 Z_3$

(2) $I_3 + I_4 = 1$

$$I_3 = \frac{Z_{34} + Z_4 + Z_{67} - Z_{m3}}{Z_3 + Z_4 + Z_{34} + Z_{67} - Z_{m3}}$$

$$I_4 = \frac{Z_3}{Z_3 + Z_4 + Z_{34} + Z_{67} + Z_{m3}}$$

The values of I_3 and I_4 are exactly the same as if the network were as shown in Fig. 13.

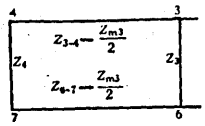


FIG. 13—SIMPLE TWO-WIRE NETWORK

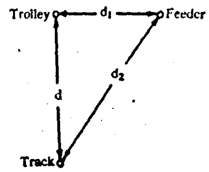


FIG. 14—RELATIVE LOCATIONS OF TROLLEY, TRACK AND FEEDER

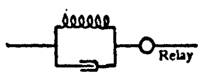
This is a simple two wire network. For calculation of self and mutual impedances, the network with mutual impedance should be used, but for calculation of distribution currents the simple network is applicable.

In the above Z_m is calculated as equal to the resistance of the trolley + j (Reactance of trolley with respect to track + mutual reactance of feeder with respect to trolley-track.) Reactance of trolley with respect to track = 0.116 $\log \frac{d}{0.78r}$ for 25 cycles. Mutual reactance of feeder with respect to trolley-track = 0.116 $\log \frac{d_1}{d_2}$ for 25 cycles, where the distances, d, d_1 and d_2 are as represented in Fig. 14.

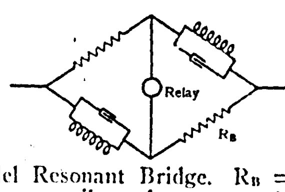
APPENDIX B

RESONANT SHUNTS FOR TELEGRAPH CIRCUITS

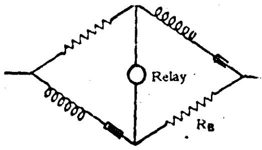
I. Circuit in series resonance shunted around relay. This forms a low resistance path for the induced voltage of railway frequency. By the use of a commercial retardation coil and 6 m. f. of condensers very good telegraph operation has been obtained with an alternating current voltage of four times the direct current voltage.



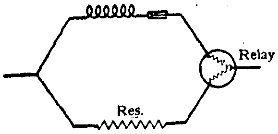
II. Circuit in parallel resonance in series with relay with same coils and condenser as used for I; this has been effective with an alternating current voltage equal to the direct current voltage.



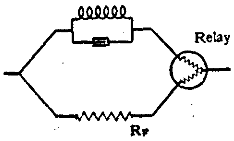
III. Parallel Resonant Bridge. R_B = Resistance of resonant bridge arm at railway frequency. An improvement over II but has not proved to be as good as I.



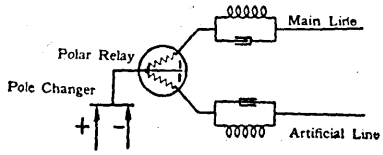
IV. Series Resonant Bridge. R_B = Resistance of resonant bridge arm at railway frequency. Results show an improvement over I.



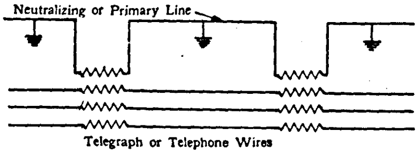
V. Differential Relay. Special double winding relay. Alternating current divides evenly in windings. The direct current flows only in the resistance side.



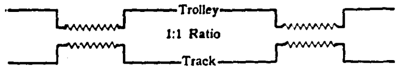
VI. Differential Relay with parallel resonance. R_F = Resistance of resonant circuit at railway frequency.



VII. Paralleled Resonant Circuit for duplex working.

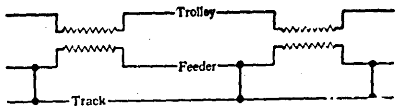


VIII. Neutralizing Transformer System. Neutralizing wire runs among tel. wires and is subject to the same induction.



IX. Track Booster Transformers.

Current in track = current in trolley — magnetizing current of transformer. E. M. F. across transformer = impedance drop in rail section in which it is connected.



X. Feeder Booster Transformer.

Bibliography

A.H. Babcock	1908	Discussion, From Steam to Electricity on a Single Track Road. Trans. A.I.E.E., Vol. 27, P 1170.
J.C. Barclay	1909	Discussion, Telegraph and Telephone Systems as Affected by Alternating Current Lines. Trans. A.I.E.E., Vol. 28, P 1234.
A.W. Copley	1906	Electro-Motive Forces Induced in Parallel Circuits. Electric Journal, Vol. 3, P 437.
	1908	Constants of Single Phase Railway Circuits. Electric Journal, Vol. 5, P 631.
	1908	Discussion, From Steam to Electricity on a Single Track Road. Trans. A.I.E.E., Vol 27, P 1171.
	1909	Discussion, Telegraph and Telephone Systems as Affected by Alternating Current Lines. Trans. A.I.E.E., Vol. 28, P 1221.
	1920	Regulation and Inductive Effects in Single Phase Railway Circuits. Electric Journal, Vol. 17, P 326.
F.F. Fowle	1909	Discussion, Telegraph and Telephone Systems as Affected by Alternating Current Lines. Trans., A.I.E.E., Vol. 28, P 1239.
Griswold and Mastick	1916	Inductive Interference as a Practical Problem. Trans., A.I.E.E., Vol. 35, P 1051.
Marius Latour	1918	Discussion, Inductive Effects of Alternating Current Railroads on Communication Circuits. Trans., A.I.E.E., Vol. 37, P 533.
P.M. Lincoln	1904	Transmission and Distribution Problems Peculiar tothe Single Phase Railway. Trans. International Electrical Congress, Vol. 3, P. 194.
W.S. Murray	1908	The Log of the New Haven Electrification. Trans. A.I.E.E., Vol. 27, P 1615.

W.S. Murray 1909 Discussion, Telegraph and Telephone Circuits as Affected by Alternating Current Lines.
Trans. A.I.E.E., Vol. 28, P 1225.

F. Niethammer 1904 Alternating vs. Direct Current Traction.
Trans. International Electrical Congress, Vol. 3, P 194.

L.C. Nicholson 1908 Discussion, Telegraph and Telephone Circuits as Affected by Alternating Current Lines.
Trans. A.I.E.E., Vol. 28, P 1232.

Parshall and Hobart 1907 Electric Railway Engineering PP 280-287.

C.F. Scott 1906 Induction in Transmission Circuits.
Electric Journal, Vol. 3, P 81.

1906 The Calculation of Electro-Motive-Forces Induced in Transmission Circuits.
Electric Journal, Vol. 3, P 334.

1909 Discussion, Telegraph and Telephone Circuits as Affected by Alternating Current Lines.
Trans. A.I.E.E., Vol. 28, P 1217.

C.P. Steinmetz 1909 Discussion, Telegraph and Telephone Circuits as Affected by Alternating Current Lines.
Trans. A.I.E.E., Vol. 28, P 1228.

J.B. Taylor 1909 Telegraph and Telephone Circuits as Affected by Alternating Current Lines.
Trans. A.I.E.E., Vol. 28, P 1169.

H.S. Warren 1918 Inductive Effects of Alternating Current Railroads on Communication Circuits.
Trans. A.I.E.E., Vol. 37, P 503.

J.B. Whitehead 1908 From Steam to Electricity on a Single Track Road.
Trans. A.I.E.E., Vol. 27, P 1139.

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