A PROPOSED METHOD FOR THE MEASUREMENT OF THE LOSS
IN VELOCITY OF ELECTRONS IN GASES
AT REDUCED PRESSURES

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Introduction

If a layer of matter is placed in front of a stream of electrons, the emerging stream differs from the incident stream in two respects. In the first place the number of electrons in the emergent beam is less than in the incident beam; in the second place, the energies of the electrons in the emergent beam are less than the energies of the incident electrons. The theoretical expression for the loss of energy of the electrons in passing through a layer of matter was developed by Sir J.J. Thomson. He assumed that the electrons lose energy upon collision with the electrons contained in the atoms of matter. From kinetic energy considerations, based upon this assumption, he showed that:

\[ K.E._0^2 - K.E.}_x^2 = C \times \]

where \( K.E.}_o \) is the initial kinetic energy of the electrons, \( K.E.}_x \) is the kinetic energy of the electrons after having gone a distance \( x \) through the material and \( C \) is a proportionality constant. Expressing the kinetic energies in terms of mass

and velocity, and assuming that the velocity is small enough that the mass may be considered constant, we have:

\[ \left( \frac{1}{2} m u_o^2 \right)^2 - \left( \frac{1}{2} m u_x^2 \right)^2 = C x \]

or

\[ u_o^4 - u_x^4 = K x \]

where \( K \) is a new constant.

Whiddington\(^2\) has verified this law for metals in the form of very thin foils. He caused electrons of known velocity to penetrate these foils and measured their emergent velocity by deflecting them in a magnetic field. Whiddington\(^2\) also verified the law using air as the absorber. He caused high speed electrons to penetrate air at atmospheric pressure, and found that Thomson's law was obeyed within the limits of experimental error.

The importance of this law was brought to attention by some experimental work of Charlotte T. Perry and E.L. Chaffee\(^3\). They attempted to attribute the discrepancy in measurements of the ratio \( e/m_0 \) to this loss of velocity of the electron. The methods for determining \( e/m \) may be divided into two classes. The one class includes those determinations made with free electrons, as cathode rays, photo-electrons, or beta particles. The other class is composed of spectroscopic determinations and

deals only with the electrons within atoms. The values obtained for $e/m_0$ by the two classes of experiments are not in satisfactory agreement. After a critical survey of the literature, Birge gives as the most probable result of experiments on free electrons the value $(1.769 \pm 0.002) \times 10^7$ e.m.u./gr. He gives as the most probable value from spectroscopic evidence $(1.761 \pm 0.001) \times 10^7$ e.m.u./gr. The difference between these values is larger than the probable experimental error. Perry and Chaffee attempted to explain this discrepancy by the fact that some energy is lost by the free electrons upon collision with the air molecules. Such being the case, the correct value of the electron velocity has not been used in the free electron calculations of the ratio $e/m_0$. Perry and Chaffee made an attempt to eliminate this error by use of very good vacuum and rather high speed electrons; and they obtained a value of $(1.761 \pm 0.001) \times 10^7$ for $e/m_0$ of free electrons, a value in close agreement with that obtained from spectroscopic data.

In order to correct for the error due to the decreased electron velocity, it is necessary to know the proportionality constant corresponding to the pressure used in any particular determination. Whiddington determined the constant for air at atmospheric pressure. However, none of the $e/m$ determinations have been carried out at such high pressures. Perry and Chaffee

attempted to make the corrections by assuming that $K$ varies directly as the gas pressure. However, there is no experimental evidence to substantiate this manner of variation. Also, Whiddington's determination was made for electron velocities of the order of $10^{10}$ cm./sec., while velocities used in most $e/m$ determinations are much smaller than this. And there are no experimental data showing whether the Thomson expression, with a fixed value of $K$, accurately describes the facts over any great range of original electron velocity, even for a fixed pressure. In order to correct properly the value of $e/m_0$ for the error introduced by loss of electron velocity in the residual gas, the factor $K$ should be known for the residual gas pressure and for the original electron velocity used in the particular determination of $e/m$. It was because of the apparent need for such data that the present work on the development of a suitable method was undertaken.
Basis for the Proposed Method

It was thought advisable, from the standpoint of accuracy and convenience, to eliminate the magnetic field in the determination of the electron velocities, and to substitute for this a retarding electric field. The electrons emitted from a hot filament can be accelerated by an electric field. They may then pass through air at a known pressure, after which they can be stopped by a retarding electric field. Then by knowing the accelerating potential $V_a$, the initial velocity of the electrons can be found from the relation $V_a e = \frac{1}{2} m u_o^2$.

From the retarding potential $V_r$ necessary to stop the electrons, the velocity of the electrons after having traveled a distance $x$ through the air can be found from a similar relation $V_r e = \frac{1}{2} m u_x^2$. From these two relations and the distance through which the electron traveled, the factor $K$ of Thomson's law can be found for any fixed pressure. By finding values of $K$ for various pressures, the manner of variation of $K$ with pressure can be determined. By using electrons of various speeds, the validity of the Thomson expression can be tested in those regions of pressure and velocity commonly used in $e/m$ determinations.
Design of Apparatus

The experimental work done in connection with this problem was carried out from the point of view of elimination of difficulties, some of which were found to be very troublesome, and of finally arriving at a satisfactory design of apparatus and method of procedure. The general design of the apparatus is quite simple; it is shown in Fig. 1. The electrodes $F$, $A$, $B$, and $R$ were sealed into a long glass tube approximately 3 cm. in diameter. The filament $F$ was sealed into the removable half of a ground glass joint so that it was easily accessible for adjustments and changes in design. The anode $A$ has a hole through its center through which a stream of electrons might pass. The electrode $B$, held at essentially the same potential as $A$, consisted of a lattice work of wires through which electrons might pass after having come through the air between $A$ and $B$. Electrode $R$ consisted of a solid brass plate, held at a variable potential considerably lower than that of $B$, for the final reception of the retarded electrons. The distance between $A$ and $B$ was made very large, relatively, so that practically all
collisions made by electrons in going from F to R would occur within this region. Much effort was necessary in developing a satisfactory design of the first two of these elements, F and A.

The Filament: The electron source was an electrically heated oxide coated filament. Several filaments were used which had been taken from radio tubes. Other filaments were made in the laboratory. Carbonates of both barium and strontium were mixed in melted paraffin. Thin coats of this mixture were placed on a metal filament core. Both nickel and platinum cores were found to be good. After each coating, the filament was heated. The paraffin melted and vaporized, leaving the barium and strontium compounds on the metal core. Four or five coatings were found to be sufficient for good electron emission. These filaments were capable of producing an electron current of several milliamperes, even though a very small length of heated filament was used.

Somewhat to the author's surprise, the design of the filament and filament holder was found to be highly important from the standpoint of stable and otherwise satisfactory operation. The first filament holder constructed is illustrated in Fig. 2. The length of the
active filament was here approximately 6 mm. This filament was tried and found to have two very serious disadvantages. The first and perhaps more serious of these was the effect of rather large space charges built up in the regions near the filament. Well defined and quite brilliant luminous streams emanating from the hot filament were visible. Deflection of these streams by small magnetic fields showed that they were made up of electrons coming from the hot filament. These electron streams seemed to start out from the filament in almost any direction, sometimes toward the glass wall of the tube and sometimes directly away from the anode. The fact that the number, the intensity, and the directions of these electron streams shifted around continuously, made the use of such a filament impossible. No adjustments that could be made in the spacing between filament and anode, or in the centering of the filament wire over the anode hole, seemed to change appreciably this behavior. Even changes in the design of the anode, which will be mentioned later, did not prove beneficial. The second disadvantage of this type of filament was that it did not provide a sufficiently localized source of electrons. A well localized source is essential in providing a well defined stream of electrons through the hole in the anode. Because of these disadvantages, this type of filament holder was discarded.

Observations made on filaments similar to the above seemed to dictate two requirements for a satisfactory filament. First, the emitting surface must be quite small, both to provide a localized
source of electrons and to provide a point source about which symmetry of the electric field could be attained. The second type of filament and holder was therefore designed to meet these requirements. This second type is shown in Fig. 3.

The two brass tubes were made as small in diameter (approximately 1.5 mm.) and as thin-walled as stability would permit. They were tapered and made extremely thin at the lower end.

The set-screws which held the filament in place were placed at the upper part of the holder in order not to distort the electric field near the filament itself. It was possible to place the tubes extremely close together. Then by drawing the filament tightly between the ends of the tubes, only a small filament surface became hot enough to emit electrons. Hence, this arrangement provided an approximate point source.

The entire length of filament outside the brass tubes constituting the holder was not more than 2 mm. The part heated sufficiently for appreciable electronic emission was less than this. Furthermore, this type of filament and holder, tapering down as it did at the end, allowed a practically symmetrical electric field distribution between filament and anode plate. Its use completely eliminated the serious effects of space charges about the filament; no glow was ever observed back of or at the sides of
this filament. The design seemed to be entirely satisfactory. The only difficulty encountered with it was a tendency for the oxide coating to crack off the filament, due to the rather sharp bending necessary in mounting.

The Other Electrodes: Of the three electrodes other than the filament, only the anode gave any difficulty. Considerable difficulty was experienced in finding an anode which would allow an appreciable number of electrons to pass through an opening at its center and still serve to define moderately well the stream of electrons getting through. An anode was required such that a potential difference between it and the filament could be applied, and yet allow a large number of electrons to pass through the electrode. In some previous work on another problem a flat brass disc had been used for such an electrode, a small hole having been drilled in the plate to allow the passage of electrons. A \( \frac{1}{16} \) inch hole was therefore drilled in a thin brass disc and this used at first for the anode A.

A potential difference of several hundred volts was then applied between the hot filament and the anode. Upon investigation in a partial vacuum, it was found that very few electrons traveled a sufficient distance in the tube, that is to electrodes B and R. There are two possible explanations for this failure. It is possible that only a small fraction of the emitted electrons passed through the hole, or it is possible that a large per cent of the emitted electrons passed through the hole,
but the beam was not well defined and the electrons passed to the walls of the tube. It appeared probable that there was both a scarcity of electrons coming through the hole and poor definition of the stream that did get through.

In order better to define the electron stream, another electrode was constructed. A $\frac{1}{16}$ inch hole was drilled in a $\frac{1}{8}$ inch diameter brass rod. This rod was tapered at one end, and a section $\frac{5}{8}$ inches long was soldered into a hole in a brass disc. A sketch of this electrode is given in Fig. 4. It was found that a very small per cent of the emitted electrons passed through this electrode. Those that did get through constituted a very well defined beam. However, the intensity of this beam was too small to allow the desired measurements to be made conveniently. Changes in the design of the electrode at the very top of the taper did not increase appreciably the number of electrons getting through the opening.

Another disc electrode was then made. This time a $\frac{1}{8}$ inch hole was drilled in the disc to allow the passage of many electrons. The disc was then completely covered with a piece of screen wire soldered to it. The screen was used to provide for the necessary uniform field distribution. With this type of electrode a large per cent of the emitted electrons passed through
the electrode. Owing to the large hole in the electrode, the electron stream was not well defined. However, it was made fairly well defined by moving the electron point source to a suitable distance (approximately 6 mm.) from the electrode. It is the author's opinion that the fine mesh screen over the anode hole is important. The number of electrons getting through the opening is far greater with the screen present, and the definition of the stream is undoubtedly better. These facts are believed due to the more nearly uniform electric field distribution near the edges of the anode hole and, with the screen mesh present, even over the area represented by the opening itself. This more uniform field distribution undoubtedly directs a greater fraction of the emitted electrons toward the center of the anode opening. It is felt that the smaller anode hole first used might perhaps have proven satisfactory had it been covered with a fine mesh screen. At least it would have been greatly improved.

Another pair of electrodes, B and R, was necessary in order to provide for the retarding potential. The first of these electrodes was composed of a brass screen wire mounted on a narrow rim of solid brass. A potential could be applied to this electrode and yet electrons could pass rather freely through the screen. The last electrode was a solid brass disc placed parallel to and 3 mm. from the screen electrode. This electrode stops all electrons which pass through the screen. The distance between these last two electrodes could well be decreased
somewhat. Such a decrease would lessen the error resulting from the fact that some collisions take place within this region, an error for which a correction must be made in the final calculation of results.

Shielding of the Region Between Electrodes A and B:— The region between electrodes A and B should be essentially an equipotential space. It is here that the electrons coming through the anode opening lose velocity by collisions with gaseous molecules. It is desired to have as many of these electrons as possible finally reach the electrodes B and R. On the other hand, any electrons which are deviated in collisions sufficiently to strike the walls of the tube should be prohibited from reaching the final electrodes; these electrons have lost energy not only in collisions with gas molecules but also in their encounters with the walls.

In order to provide an equipotential region, to eliminate the accumulation of charge on the glass walls, and to trap a goodly share of the electrons which do strike the walls, a cylindrical shield of screen wire was fitted inside the glass tube. This conducting screen was connected to the anode A. It extended to within about 1 cm. of electrode B. At the end near B there was soldered over the end of the screen shield a brass plate having a large hole approximately 1.6 cm. in diameter in the center. This opening was slightly smaller than the central mesh wire section of electrode B. It served to fix definitely the
solid angle subtended by the effective part of electrode B at the point represented by the opening in the anode.

**Vacuum System:** The apparatus proper, including the mercury vapor pumps and the McLeod gauge, was made entirely of pyrex glass. At no place was rubber tubing used. A vapor trap which could be cooled with liquid air or carbon dioxide snow was included in the system. The vacuum was produced by a mercury vapor pump, backed by a good oil pump. For some of the work two mercury vapor pumps were used in series, though it seemed that little was gained by use of the second pump. The residual gas pressure was measured by a McLeod gauge. The smallest pressure which the particular gauge used would indicate was 0.1 micron.

When the system was operating properly it was possible to maintain, without the use of a liquid air trap, a vacuum of 2 microns, even though four ground glass joints were used in the system. One of these was a large joint, unsealed with mercury, for convenient removal of the filament; one was a three-way mercury sealed stop-cock used for pressure regulation; two were mercury sealed joints on the McLeod gauge. Should it prove advisable to go to still lower pressures when the method herein developed is used, it will probably be necessary to eliminate all of these ground glass joints. It would also be necessary to use a more sensitive gauge than the one available to the author, either a lower range McLeod gauge or an ionization gauge. It is the writer's opinion, however, that it will not be at all necessary to.
go to lower pressures than those here available to obtain all the information desired.

**A Focusing Magnetic Field:** In order to obtain a larger electron current to the electrode B, a focusing magnetic field may be applied. The electrons, after leaving the filament and passing through the electrode A, tend to diverge. A large fraction of these electrons never reach the electrode B, but strike the walls of the tube or the shield. The velocity of such an electron may be resolved into two components, one parallel to the axis of the tube, the other perpendicular to this axis. Now if a magnetic field be applied in a direction parallel to the axis of the tube, the velocity component parallel to the axis will not be affected by the field. The velocity component perpendicular to the field gives rise to a force always perpendicular to the velocity component. Then if the magnetic field is uniform, the electron path will be that of a right circular helix. The magnitude of the magnetic field may be so adjusted that the electron makes one circle in going the distance between the filament and the electrode B. It can be shown that all electrons, regardless of divergence, make one complete circle in going the same longitudinal distance along the tube, provided only that the longitudinal velocity be the same for all. Since this condition is nearly fulfilled, all electrons will be brought to an approximate focus at electrode B.

The necessary uniform magnetic field may be obtained by wrapping a wire uniformly around the experimental tube and sending
a current through this solenoid. The strength of the magnetic field can be controlled readily by changing the current through the solenoid.

By using the longitudinal focusing magnetic field, the electron current to the final electrode can be made sufficiently large that it can probably be measured satisfactorily with a moving coil type of galvanometer. The measurement of this current requires a rather sensitive instrument, for it is desired to measure, not the current to electrode B, but the way in which this current approaches zero as the retarding potential between electrodes B and R is increased. Because of the smallness of the electron current to electrode R near the limiting conditions, it may still be necessary to measure it with an electroscope. If an electroscope is used, the current to the electroscope can best be evaluated by a balancing method similar to that used by Dempster in his positive ray analysis. The use of an electroscope would also necessitate the addition of another electrode to the experimental tube. This additional electrode would be required to eliminate any accelerating effect due to the potential to which the electroscope leaf is charged.

5. F.W. Aston, Isotopes, p. 90.
Experimental Procedure

The electrons could be expected to lose only a very small part of their velocity in passing through air at low pressures. Therefore, the accelerating and retarding potentials must be nearly equal. If the two nearly equal potentials are measured separately, a small percentage error in either measurement will result in a large percentage error in the difference between the two. Then there would be a large error in the loss of velocity measurements. This error in the experimentally determined loss of velocity can be eliminated by measuring directly the difference between the accelerating and retarding potentials; this difference can be measured accurately, and used to calculate the loss of energy. The accelerating potential must be measured also, to enable calculation of the original velocity with which the electrons start through the gas.

In order to supply the accelerating and retarding potentials, two separate sources of potential might be used. However, if there is any variation in the potential of either source, the difference between the accelerating and retarding potentials varies greatly. Since this difference is the important factor in these determinations, such a variation would not allow accurate observations. To eliminate this variation in the difference between accelerating and retarding potentials, the same source should be used to supply both potentials. A diagram of such an arrangement is shown in Fig. 5.
The source, in this case a rectifier, supplies both accelerating and retarding potentials. Dry cells placed in opposition to the source supply the necessary decrease in potential corresponding to the equivalent loss of electron velocity. With this arrangement, any variation in the source potential causes no change whatever in the difference between the accelerating and retarding potentials. Hence the adjustment for the loss of velocity, the critical quantity to measure, is not made difficult by variations in rectifier potential. These variations can easily be kept sufficiently small that they will produce no serious effect upon the original velocity given to the electron.

The theoretical law deduced by J.J. Thomson indicates that the loss of electron velocity is a function of the distance traveled
through the material as well as of the initial velocity. A change in the length of the electron path would necessitate a shift in the position of the electrodes. This shift is not easily brought about. However, the actual distance between electrodes may be kept fixed and an equivalent change introduced by increasing or decreasing the gas pressure. The resulting observation will give us information as to the manner in which the factor K varies with gas pressure.

There are several variations in procedure that might be followed in evaluating experimentally the loss in electron velocity occasioned by gas at different pressures. For a fixed accelerating potential and a fixed gas pressure, the retarding potential might be made just large enough so that none of the electrons reach the plate R, Fig. 1. Then by measuring the accelerating potential and the difference between the accelerating and retarding potentials, the proportionality constant K could then be calculated for that particular gas pressure. Then by obtaining similar sets of data for various fixed pressures, the theoretical expression could be tested to see whether the constant K is proportional to the gas pressure. This procedure has the advantage of not necessitating maintenance of constant pressure over any appreciable length of time; the pressure would have to be maintained constant only long enough for the one adjustment of retarding potential. The method suffers, however, from the fact that it is rather difficult to locate directly that retarding
potential for which the current to electrode R becomes zero. This last difficulty might be avoided by maintaining the pressure constant at some desired value and observing actual currents to electrode R for various retarding potentials. The retarding potential just necessary to stop all electrons could then be obtained more accurately by extrapolating the resulting curve of observed current versus retarding potential to the zero current axis; the potential intercept is the desired retarding potential. This procedure necessitates holding the pressure constant for considerable length of time. While this is somewhat difficult, it is entirely feasible.

In view of the desirability of eliminating the necessity of maintaining a constant pressure over a considerable length of time, a more desirable procedure is to maintain a constant accelerating potential and a constant retarding potential, and to observe the currents to electrode R for various gas pressures. These observed currents can be plotted against corresponding pressures. Extrapolation of the curve to zero current gives an intercept which represents that pressure for which the fixed retarding potential used is just sufficient to stop all electrons. A value of K can be calculated from these corresponding values of retarding potential and pressure. Values of K corresponding to other gas pressures can be determined in a similar way, using other fixed values of retarding potential. This method has the great advantage that the pressure need not be kept constant for long, nor
even adjusted to definite values; the author feels that it should prove rather satisfactory from an experimental point of view.

Since the electron current to or through electrode B (to electrodes B and R) will not remain constant during the observations, this variation must be considered in plotting the curves for purposes of extrapolation. The current to B, as well as to R, must be observed. Then the ratio of the current to R to that to R and B together should be used for plotting against retarding potential or gas pressure. The resulting curve will then be independent of current variations in the tube. This procedure will be necessary with either method outlined above.

A third method, based upon the distribution of velocities of the electrons emitted from a hot filament, may be used. The theoretical expression for the velocity distribution was developed by Maxwell. The number of electrons emitted that have a velocity between \( u \) and \( (u + du) \) is given by the expression:

\[
N_{du} = 4\pi N \left( \frac{m}{2\pi K T} \right)^{3/2} \frac{u^2}{e^{\frac{mu^2}{2KT}}} du,
\]

where:

- \( N \) is the total number of electrons emitted.
- \( K \) is the Boltzmann constant.
- \( m \) is the mass of the electron.
- \( u \) is the velocity of the electron.
- \( T \) is the absolute temperature of the filament.

If \( \frac{N du}{N} \) be plotted versus \( u \), a bell-shaped curve results. The area under this curve is a measure of the electron current. If a small retarding potential be applied, the electrons which were emitted with only a small velocity will be stopped first before reaching the negative electrode. As the retarding potential is increased, the electrons of higher velocities are stopped. At the peak of the distribution curve the rate of decrease in current with respect to velocity is a maximum. Since the electron velocity is proportional to the square root of the potential, the curve of current versus the retarding potential also has a point of maximum rate of decrease of current with respect to the retarding potential. Curves of electron current versus retarding potential can be plotted for various fixed gas pressures. The position of maximum rate of decrease can be found for each pressure, and the value of retarding potential corresponding to this used in the calculations. It will be necessary to keep the temperature of the filament approximately constant, since the position of the peak of the distribution curve shifts with changes in filament temperature. This can be accomplished by maintaining the emission current at a constant value. Also, since the total electron current to electrodes B and R will vary with changes in pressure, it will be necessary to measure this current and always use the fraction of this current which reaches R as the basis for the current versus potential curves. This method of course requires the maintenance of constant pressure over
considerable periods of time.

Only experiment can tell which of the three methods suggested will yield more definite values of retarding potential. The third method avoids the necessity of extrapolating to a zero current intercept. This method will be found profitable only if the position of the maximum rate of decrease of current is better defined than is the extrapolated zero current intercept. The author feels that the second method will prove far the least troublesome from an experimental point of view, chiefly because it avoids holding gas pressures fixed at particular values. Any one of the methods will furnish the data desired, namely, values of \( K \) for various gas pressures and a fixed value of original velocity of electron. Data regarding the constancy of \( K \) for a fixed pressure but for different original electron velocities is also made possible by any one of the three methods.
Calculation of Results

The factor $K$ can be calculated from the Thomson law,

$$u_o^4 - u_x^4 = K x,$$

for various gas pressures. The initial velocity $u_o$ is to be calculated from the relation

$$V_a e = \frac{1}{2} m u_o^2,$$

and the velocity of the electron after having passed a distance $x$ is to be calculated from the relation

$$V_r e = \frac{1}{2} m u_x^2.$$  

Substituting these velocity values and the value of $x$ into the Thomson equation, the value of $K$ can be found corresponding to a known pressure. By a series of such calculations, at various pressures, the manner of variation of $K$ with pressure can be found.

The value of $K$ calculated as outlined above will be slightly in error. The electrons make some collisions in passing between $F$ and $A$, Fig. 1; hence, they do not possess as large a velocity upon reaching $A$ as the velocity calculated from

$$V_a e = \frac{1}{2} m u_o^2.$$  

Likewise, the electrons make some collisions in passing between $B$ and $R$; therefore, the velocity calculated from the relation $V_r e = \frac{1}{2} m u_x^2$ is too small. These errors may be quite readily corrected.

The approximate value of $K$ is given by the relation

$$u_a^4 - u_x^4 = K x,$$

where $u_a$ and $u_x$ are uncorrected values. Having found an approximate value of $K$, this can be used to find more accurate values of $u_a$ and $u_x$. Let $u_{ao}$ be the correct value of velocity of the electron upon
reaching \( A \). Then \( u_a^4 - u_{as0}^4 = K x_1 \). From this relation the value of \( u_{as0}^4 \) can be calculated. In a similar manner the corrected value of the electron velocity at \( B \) may be calculated. Let this corrected value be \( u_{xo}^4 \). Then
\[
\frac{u_{xo}^4}{u_x^4} = K x_2.
\]
From this relation the value of \( u_{xo}^4 \) can be calculated. Then using these corrected values of velocities a better value of \( K \) may be calculated from
\[
\frac{u_{as0}^4}{u_{xo}^4} = K_0 x.
\]
If accuracy of experimental data warranted it, the process might be repeated and a still more accurate value of \( K \) obtained. In view of the fact that the distance \( x \) is quite large as compared to either \( x_1 \) or \( x_2 \), however, the author is confident that the one approximation will yield a value as free from this error as the data can possibly warrant.

In conclusion the author wishes to express his sincere thanks to Dr. J.D. Stranathan for his continued assistance during this work.