CORRELATION BETWEEN THE MASTERY OF THE SKILLS IN THE FUNDAMENTALS AND REASONING ABILITY IN ARITHMETIC.

by

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Chapter I.

Introductory.

The purpose of this study is to assertain to what extent the ability to solve reasoning problems in arithmetic is correlated with efficiency in the fundamentals. Does the pupil's ability to reason in general arithmetic have a constant ratio to his ability to perform accurately the specific processes of the fundamentals in arithmetic, is a problem with which every school administrator is confronted. If we find that the pupils ability to solve reasoning problems correlates with his mastery of the fundamentals, it is imperative that pupils should not be advanced in arithmetic until they have thoroughly mastered a prepondering number of the specific processes in the four fundamentals. If on the other hand we do not find a fairly close relationship between the two we might advance the pupil and expect him to get a further mastery of the fundamentals, by their use, in reasoning problems in his class recitation and in the preparation of his lessons. In fact this usage might stimulate an interest in a correct application of the fundamental processes and thus cause the pupil to master them better than he would in a purely mechanical process of drilling for no other apparent reason

than accuracy.

Accuracy in the performance of the specific mechanical skills in the fundamental processes in arithmetic has no particular value in itself. The skills in the fundamentals are merely tools to be used in solving practical problems arising in everyday activities of life. Thus, it is important that the individual perform these skills accurately in order to secure the correct answers to his problems, but if he did not, in life's activity, have to solve these problems which require reasoning for a correct application of principles by which they are solved and in which the mechanical skills are to be applied, he would have no practical use for the skills. There is no excuse nor reason for taking the child's time to teach him these skills except that he needs to use them in solving problems. The reason for teaching arithmetic is mainly to teach the child to solve correctly, problems which arise out of the business activities of life.

Chapter II.

General Problem.

In this study it is the aim to find how best the problem solving ability can be increased, and if in any way, to what extent this ability correlates with a mastery of the fundamentals, ability in silent reading, special training in the reasoning process, enlarged experience on the part of the pupil, clearness of language in the problem, native ability, or any other related phase that may be found in this study. Before an arithmetic instructor can render his best service he needs to know and understand the pupils difficulty. He must know the seat of the trouble to cure or remedy the defect. This study aims to help instructors discover the cause of pupils inability to solve reasoning problems and endeavor to find the relative importance of the element, mentioned above, in training the pupils to reason more effectively in problem solving.

Chapter III.

Sources of Data.

Data for this study were gathered from three hundred seventy-eight pupils selected from the Newkirk and Ponca City schools. The tests were given to three hundred fifty pupils in the high school at Newkirk. After giving the first test to the Newkirk pupils it was decided that this number was not enough on which to base the study, so it was arranged to go to Ponca City where permission was secured from Supt. John Hefley to give the tests to one hundred fifty of his pupils in the Freshman and Sophomore classes, selecting seventy-five from each class.

The following tests were given in the survey,

1. Courtis Standard Research Test in the Four Fundamentals.

2. Woody-McCall Mixed Fundamentals, Test 1.

3. Woody-McCall Mixed Fundamentals, Test 2.

4. Stanford Achievement Test, Form A. in Arithmetic Computation.

5. Stanford Achievement Test, Form A. in Arithmetic Reasoning.

The date of taking these data was the fall of 1927 during the months of October and November. This may seem to make the data a little old for a thesis, but taking into consideration

the nature of this study it will be readily seen that the facts in this case would be the same twenty months ago as they would be today. The elapsed time has only given additional opportunity to observe pupils' mathematical work in the school and reflect over the results obtained from these tests, given some time before. If the delay has in any way effected this study its effects have been beneficial and not hurtful.

The pupils taking this test were not told that their papers were to be used in a special study. They were left under the impression that the grades they made would effect their class grades and standing only. The reason for saying nothing about this special study to the pupils was to keep them as nearly as possible under normal conditions doing normal work. To further remove all unnatural conditions these tests were given in the Algebra and Geometry classes, leaving the impression that it was some of the regular work. By giving the tests in these classes practically all the Freshmen and Sophomores and many of the Juniors and Seniors were included. With the care that has been given to securing the data in this survey it may be assumed that a fairly average group of pupils, working under as nearly normal conditions as it is possible to get, has been secured.

From ten days to two weeks was allowed between tests, thus giving the pupils time for relaxation from the aforegoing test and to prevent them from becoming test wise or test trained by having them follow in close succession. This process, however, drew the test out over a period of two months.

In this survey it was the intention to get about four hundred fifty or five hundred cases, but by scattering the tests out over a period of eight weeks, it was found that quite a number of the pupils had failed to get all the tests. Some had moved to other towns, some were sick on the day of a test, and others were out for various minor reasons. At best they were not present at school and thus missed one or more of the tests. This caused the papers of a goodly number of the cases to be discarded. The final hope to still retain four hundred cases was shattered when it was found that only three hundred seventy-eight had taken all the tests in a satisfactory manner so as to produce fairly normal results. A few were discarded because the pupil was detected, during some of the test, in employing unfair means in securing answers to the problems. Still a few others were discarded because of the erotic condition of the pupil. When it was found that a certain pupil had made a good score in one test and an exceedingly low score in another test of the same difficulty and the same type, the pupil's school record was investigated. If found that the pupil under consideration was given to such unusual variations in his other school work and his record showed nervousness, his papers were discarded from this study. It seems this was the fair thing to do in order to get an honest and fair conclusion. An erotic pupil who is likely to make a high or a very low score in the same test on two different days, due to his neural condition and not to his knowledge of the

subject, can add nothing to a study of this kind. To find a fair degree of accuracy between pupils' mastery of the fundamentals in arithmetic and his ability to solve reasoning problems we need to deal with normal pupils only. However, it may be gratifying to know that only six papers were discarded due to the erotic condition of the pupil.

Chapter IV.

Related Literature.

There is considerable literature on teaching arithmetic, including methods and time allotted to the various divisions but most of the literature in this field makes no attempt to find the existing relation between the mastery of the fundamental operations and the ability to reason in problem solving. Much of it, however, does deal with the ability to read and problem solving. Some of the material along this line will be included here because it vitally effects the ability of problem solving in arithmetic, a phase of this thesis.

In the last decade such scholars as Judd, Buswell, Osburn, Banting, Harlan, Winch, Bonser, Hackler, Wilson, Terry, Thorndike, and Klapper whose works are available, and others whose works are not available for this study have, by means of various surveys, contributed much valuable information to this field of research. In their work many other phases of arithmetic were included in their studies, nevertheless, they throw much light on the subject of this thesis.

Judd and Buswell, whose work is published in the "Research Bulletin of the N.E.A., Vol. 3. No. 5." have compiled considerable data on findings in arithmetic surveys. They attempted to learn the best prodedure for building up ability to

handle concrete or reasoning problems. In their research they took the grades one to eight, in forty-nine large cities. They found that the average time devoted to arithmetic in the grades in these cities was one hundred eighty-one minutes per week, or twelve and two-tenths per cent of the school time for the eight grades. Since the time of their survey the tendency has been to decrease the time allotted to arithmetic in the schools. They further found that most of our school text books contain exercises in the fundamentals having considerable more digits in the addends, multipliers, multiplicands, divisors etc., than are used in business procedure. Less than three per cent of the business problems involve as many digits as we find in many of our text book exercises. In extending their research into the field of reasoning they find some of the causes why pupils reason incorrectly in solving problems. They outline the following reasons.

1. Lack of general ability in silent reading.
2. Lack of familiarity with technical terms in arithmetic.
3. Carelessness in reading.
4. Lack of experience necessary to understand the setting of the problem.
5. Inadaquate skill in computing.
6. Lack of knowledge of such essential facts as tables of weights and measures.
7. Inability to see the relation in the problem so as to choose the proper operation.
8. Inability to do reflective thinking, low intelligence.

This is quite an array of causes, but for all that, they do not mention, lack of ability to perform correctly the separate skills in the fundamentals, as one of the causes. On first reading this report it was a surprise to find this cause omitted from the

list. However, on the finding of so low a correlation, in the survey of this study, between the skills in the fundamentals and reasoning in arithmetic it was not difficult to understand why this reason is omitted in the above outline.

In his publication, "Corrective Arithmetic", published by Houghton Mifflin and Company, W.J. Osborn has some valuable information on the, "Diagnostic and Remedial Treatment of Errors in Arithmetic Reasoning", He found that thirty thousand errors made by six thousand children on the Buckingham Problem Test, Form 1, about sixth per cent of the wrong answers were due to failure to understand what was to be done in the very beginning of the problem. With this lack of ability to know how to proceed with the solution of a problem we can readily understand that it would be of little value to teach and drill the pupil in the mastery of the fundamentals without teaching them how to apply and use these fundamental processes in their proper relation in the solving of problems such as arise in the everyday business affairs of life.

The important question for our schools is how to increase the ability of pupils to solve arithmetic problems. In this field we are not in unexplored territory. Tests have been made to show that the problem solving ability can be increased.

1. By teaching pupils to analyse text book problems.

2. By solving problems containing data from actual life situations.

3. By solving problems without numbers or actual comput-

ation.

4. By studying the different words that appear in text book problems.

Tha above method to improve the pupils problem solving ability must not be construed to mean that all pupils ability can be equally improved. The amount of increase in ability will, in this training, depend much more upon the pupils intelligence than it will in teaching him to perform the skills required in the fundamentals. By drill the dull pupil may be taught that six times seven equals forty-two Etc. but no amount of drill can teach this dull pupil an existing relation slightly different than any he has and before unless he has enough intelligence to see the new situation and comprehend the new relations. Judd in the "Research Bulletin of the N.E.A." mentions studies that have been made which justify the conclusion that there is a closer relationship between reasoning ability and general intelligence than there is between reasoning ability and skill in computation.

In 1922 O.G. Banting in the schools of Waukesha, Wisconsin made a study of pupils difficulty in reasoning. His findings are published in the Third Year Book of the Department of Superintendents. He found the causes of failure or inability to reason correctly were practically the same as those mentioned above under the studies of Buswell.

In this same periodical Judd and Buswell give percentages on a similar list of causes for failure to solve reasoning

problems. Their list as given is as follows.

1. Total failure to comprehend the problem. ---30 %.

2. Procedure partly correct but with the omission of one or two essential elements. ----------20 %.

3. Ignorance of fundamental quantitative relations. ----------10 %.

4. Errors in Fundamentals. ----------20 %.

5. Miscellaneous errors. ---------- 2 %.

6. Errors whose causes could not be discovered-18 %.

Total 100 %

The Buckingham Scale was used in the above survey.

It will be noticed that all the aforegoing surveys show a close similarity in their findings.

In further evidence of the superior reasoning ability of the pupils with high I.Q.s Charles L. Harlan in an article in the "Journal of Educational Research", Sept. 1923, and partially reprinted in the "Survey of Educational Investigation Relating to Arithmetic", Feb. 1925, by Buswell, has shown that, grade for grade, the accelerated pupils make better scores in arithmetic reasoning than do pupils who are retarded. That is, pupils who have been in school more than the normal number of years do not do so well as the pupils who have been in school less than the normal number of years.

W. H. Winch in his survey, "The Reorganization of Mathematics in Secondary Schools", reported in the "Bureau of Education

Bulletin", Number 32, studied the extent to which accuracy in number work was transferred to accuracy in arithmetic reasoning. He found that a substantial improvement in accuracy in arithmetic computation seemed to have produced no improvement whatever in the accuracy of arithmetic reasoning. Nevertheless, he claims to have secured a high correlation between numerical computation and arithmetic reasoning. No other case was found where a high correlation was found between these.

In 1910 Bonser carried out a series of experiments in reasoning with the fourth, fifth and sixth grades. He found that in many cases the reasoning ability of the younger pupils surpassed that of the older pupils. As a result of this he believes native ability a large factor in arithmetic reasoning.

John M. Harlan in his Masters Thesis, Educational Department, Chicago University, and Wilson in a survey, "Improving the Ability to Read Arithmetic Problems", printed in the Elementary School Journal", January 1922, each made a survey on reading ability and arithmetic reasoning. Both found a very low correlation between these two, reading ability and ability to solve arithmetic problems. This finding is somewhat different from those found by Thorndike, Osborn and others. This may be somewhat accounted for by the fact that they used reading tests in Geography and History.

However, a much more satisfactory answer may be found in a study by Paul W. Terry in, "The Reading Problems in Arith-

metic", published in the Journal of Educational Psychology, October 1921, Pages 365-377. He shows that when reading numerals one divides the material being read into much smaller units than when reading words. He finds that readers very seldom apprehend more than two numbers at a time. He says, "The child who is reading a problem with numerals, is as a result, stopped in his progress through the problem every time he comes to a number. Not only so, but it requires an entirely different type of memory to keep numbers in mind, from that which is required to recall the logically coherent words and phrases which make up the verbal part of the problem".

He further explains that an adult when confronted by the two different intellectual demands described, adjusts himself to the difficulty by reading the problem twice. The first time he gives the numbers only casual attention, glancing at them but neglects all details. During his first reading he gets the story of the problem and discovers the arithmetical operation necessary to reach a solution. He then goes back and looks at the numbers a second time. This time he notes the exact digits of which the numbers are composed. He is now ready to proceed with the solution of the problem.

Buswell thinks this finding by Terry very important for successful teaching in arithmetic, especially for remedial purposes. He considers the analysis made by Terry of vital importance. In substance he says. It shows why pupils are often greatly confused in reading arithmetic problems. It also becomes evident

that pupils must be given much opportunity in cultivating effective reading habits if they are to escape this confusion. They should be trained in the reading of numerals and also in the method of interpreting the verbal part of arithmetic. This means that the teacher should give careful attention to the vocabulary of arithmetic, a careful explanation of many terms which are too often taken for granted, and give careful attention to the grouping of numbers.

E. L. Thorndike in the "American Journal of Psychology", July 1910 says in regard to reading in arithmetic. "The first test is not so much in arithmetic as in language; "the difficulty is in the words, 'already', 'too', 'some one', 'would', 'in all'." In one of his later works, "The New Method in Arithmetic", Rand McNally and Co., Chicago, the same author presents an analysis of textbooks, showing how difficulties in language are carelessly tolerated by authors. Charles D. Dawson in "The Journal of Educational Research", October 1920, call attention to same situation, carelessness by authors of text books.

Paul Klapper in his book, "The Teaching of Arithmetic", published by, Appleton and Company, devotes some time to the reading and reasoning process in arithmetic. He places much stress on the teachers part in teaching the pupil the process of reasoning. He says, "Teaching with a preconceived purpose leads to mental development, without it to mental wondering. The auto-

mobile that is wrecked in a collision is not analyzed, it is smashed. Nor is the act of gathering up the broken parts one of synthesis."

By this statement he evidently means to say that much which passes for analysis and reasoning is not that at all. He explains further that a problem must be set forth in language that is clear, simple, and attractive. To word a problem correctly requires, among other things, a knowledge of the extent of the pupils vocabulary, a ready control of a sufficient number of words used in business and practical selling and a fluency of expression. Formulating problems is, therefore, an exacting composition exercise which many teachers can not perform during an arithmetic lesson. This is primarily the task of the text book writer.

Chapter V.

Specific Problem.

The tests completed, the papers collected, many of them scored and others to be scored the tedious task of finding the relation that exists, if such relation does exist, between the pupils ability to perform correctly the specific fundamental operations in arithmetic and their ability to solve reasoning problems, is the problem for solution. The specific process by which this must be done is to place all the scores on a scatter diagram and find the correlation between the various tests. This process should give us some desired information on the question. Did the pupil who made a low score in computation also score low in reasoning problems and the pupils who scored high in computation also make high scores in the reasoning test? In checking over the papers some glaring differences were discovered between some of the pupils grades in computation and reasoning. However, it was found that in the two Woody-McCall Tests, one and two, the entire group of pupils with very few exceptions, made practically the same in test one and test two. With this result in view it was decided to use only Form 2. in comparing the results with the reasoning test. There could be no additional information attained by correlating both these tests with the reasoning test since the result would be practically

the same.

The correlation found between the Courtis Standard Research Test and the Stanford Achievement Test is also omitted in this because it showed to be of no particular significance as to their findings. This is mostly due to the arrangement of the Courtis Test, since the four fundamental processes addition, subtraction, multiplication, and division are each grouped separately. They are also arranged so as to score every pupil separately in each of the four skills. In grading the papers it was found that many of the pupils would score high in one or more of these skills and drop quite low in others. This caused so great a variation in the grades of these tests that the correlation was so low it had no significance.

With the above tests eliminated, the Woody-McCall Test, Mixed Fundamentals Form II.; Stanford Achievement Test in Computation; and Stanford Achievement Test in Reasoning; in connection with the Arithmetic Age of the pupils are used for the correlations in this study.

Chapter VI.

Presentation of Data.

In diagram one ~~of~~ the scores made in the Stanford Computation Test and in the Woody-McCall Mixed Fundamentals are placed on the scatter diagram. It will be readily seen that a high correlation can not be secured from this scatter diagram. The co-efficient of correlation equals .665. ±.019 One should naturally expect a high correlation between these two tests since both are in fundamentals, but somehow the difference in arrangement of the problems in the two tests and the processes involved in their solution has caused quite a number of pupils to show wide variations in their grades.

It is seen that the scores in the Stanford Test average about four points higher than do those in the Woody-McCall Test but this has no significance. This is due to the greater numbers of problems offered for solution in the Stanford test, and as a result more short easy solutions at the beginning of the test. Woody-McCall has only thirty-four whereas, the Standard has thirty-seven. In a large number of cases the pupils who ranked high or low in one of these tests did likewise in the other one. It is by far the minority number of pupils who were clear out of line and caused a rather low correlation.

Attention may be called here to a few outstanding pupils. They will be found in each scatter diagram on the extremes of the scores, three at the bottom and three at the top. These same pupils will be checked through all the tests and a comparison of their scores made later in this discourse. Since they were all consistant in scoring either low or high in all the tests they may be referred to as typical cases in this survey. In tabulating their scores they will be referred to as pupils A.B. C. D. E. and F. and on diagram one they may be located by the following scores.

Figure I.

Pupil.	Computation.	Mixed Fundamentals.
A.	18	19.
B.	19	26.
C.	21	27.
D.	31	45.
E.	33	44.
F.	34	45.

Diagram I.

Computation in Stanford Achievement Test IV.

Woody-McCall — Mixed Fundamentals

Score	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	f
34																												1	1
33											1			1	1				1		1	2	3	3			1		14
32																		2	1	3	1	5	4	1					19
31												2	3		2			2	2	3	3	3	2	2				1	25
30							1	1	1			3	2		2		3	2	5	6	3	4	2	1					36
29								2	2		4	2	2		5	1	4	5	3	3	2	1	1						37
28						1	1	4	3		3	5	4	3	4	4	8	5	1	1		2	1		1				51
27					1		1	2	6		3	2	6	3	7	3	4	5		1									44
26						2	1	2	4	1	1	4	5	5	2	3	5	3											38
25						2	2	3	4	1	4	5	6	4	3	2	1	1											38
24				1		1	2	1	5	2	3	4	5	2			1												27
23				2		1	1	2	2	2	3	5	1	2	1														22
22					1	1	2	2	0	0	2	2	1	1	4		1												17
21					1				1	1					1														4
20										1		1																	2
19	1								1		1																		3
18																													/
f	1	0	0	3	2	9	11	19	29	8	25	35	35	21	32	13	27	25	15	17	10	17	12	10	1	0	1	2	378.

r = .665 ± P.E. = .019 K = .746

The next correlation is shown between Woody-McCall Mixed Fundamentals and Stanford Achievement in reasoning, see Diagram two. Here there is some surprise because there is no correlation of any significance to show any relation. The coefficient of correlation equals only .492 ±.025 This shows plainly that there was very little uniformity among the pupils in regard to their ranks in the two tests. We may call this step number one in showing that the ability to solve reasoning problems is not materially aided by ability in the fundamental skills.

In getting the coefficient of alienation the results are no more assuring, K equaling .871. Interpreting the value of K. one would conclude that it can not be predicted with any degree of certainty, that a pupil who has mastered the skills in the fundamentals will do relatively well in the reasoning process of arithmetic.

In this diagram the six typical pupils have the following score.

Pupil.	Woody-McCall Fundamentals.	Stanford A Reasoning.
A.	19	16.
B.	19	16.
C.	21	16.
D.	31	37.
E.	33	32.
F.	34	28.

Diagram II.

Stanford Achievement in Reasoning.

Woody-McCalls - Mixed Fundamentals

Score	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	f
34													1										1
33							1	2		4	3	1	3										14
32						2		2	2	3	3	3	3				1						19
31				1	1		2	2	5	2	2	3	4	2								1	25
30				1	1	2	5	4	5	3	4	3	3	4				1					36
29			1	1	1	2	4	4	5	5	3	3	3	3			2						37
28			2	3	1	4	3	6	3	9	4	6	4	2	2	2							51
27			2	4	3	5	4	6	8	5	3	2	2										44
26			2	2	4	5	7	6	5	2	3	[illegible]	2										38
25	1	2		3	3	5	8	8	4	2	2												38
24	1	2	2	4	3	3	2	3	4	2				1									27
23		2	3	3	2	3	2	2	3	1		1											22
22		2	2		3		3	2	2	2	1												17
21					2				1	1													4
20	1									1													2
19	2	1																					3
18																							
f	5	9	14	22	24	31	41	47	47	42	28	22	25	12	2	2	3	1	0	0	0	1	378.

r = .492 ± P. E. = .025 K = .871

Diagram three shows the pupils arithmetic age in relation to their reasoning ability. In this the coefficient of correlation is higher than it is in diagram two. It seems to show a much higher existing relation between the pupils arithmetic age and their reasoning ability, than was found to exist between their skill in the fundamentals and their reasoning ability. This may be an indication that reasoning ability in arithmetic is more dependent on the general intelligence of the pupil than it is upon the training he may have received in the performance of the specific skills in the fundamentals. By taking the I. Q. of the pupils in place of the arithmetic age a still higher correlation may be expected than was secured by using the arithmetic age. The Arithmetic Age being based partly on the ability to perform correctly the separate fundamental skills it would tend to vary somewhat from the pupils I. Q. which would to some extent effect the coefficient of correlation.

Also in finding K, the coefficient of alienation, we get a lower figure than we do in diagram two. In diagram two K equaled .871 whereas in this diagram K equals only .653 or more than .2 less. This difference, no doubt, is sufficiently large to predict a higher relation between the arithmetic age and reasoning in arithmetic than there is between the ability to perform the skills in the fundamentals of arithmetic and ability to solve reasoning problems.

Checking the scores of the three typical cases in this diagram we find them as follows.

Figure III.

Pupil.	Arithmetic Age. Yrs.	Months.	Stanford Reasoning.
A.	11	8	16.
B.	12	7	16.
C.	12	6	16.
D.	18	6	37.
E.	18	1	32.
F.	19	0	28.

In figure three it will be noticed that the three pupils A. B. and C. who have low arithmetic ages have corresponding low scores in their reasoning test, whereas, the other three D. E. and F. whose arithmetic age is much higher also have a much higher score in their reasoning test. If these six cases were taken as averages for the entire group of pupils used in this study the result would be a very high correlation between arithmetic age and reasoning ability in arithmetic as measured by the Stanford Achievement Test in Reasoning. Although the cases in figure three are not averages, the result obtained may be of some significance.

Diagram III.

Stanford Achievement Test V for Reasoning.

Arithmetic Age.

Stanford Achievement Test V for Reasoning: Score	10-10.9	11-11.9	12-12.9	13-13.9	14-14.9	15-15.9	16-16.9	17-17.9	18-18.9	19-19.9	20-20.9	f
37										1		1
36												0
35												0
34												0
33										1		1
32							1	2				3
31									2			2
30							2					2
29							4	6	2			12
28							11	10	4			25
27						4	4	11	2	1		22
26				1	1	5	9	7	5			28
25					5	13	12	10	2			42
24			2	6	15	19	3	2				47
23				7	18	15	2	5				47
22				13	17	8	3					41
21				7	18	4	2					31
20				15	6	3						24
19			10	6	4	2						22
18			3	5	3	3						14
17			3	4	1	1						9
16	1	1										5
	1	1	18	78	88	77	53	53	18	3		378

$r = .757 \pm$ P.E. = .015 K = .653

Diagram four shows the relation existing between the arithmetic age of the pupils and their ability in computation according to the Stanford Achievement Test. Here we find a higher correlation, the coefficient equaling .857±.009 This may not be high enough to indicate any close existing relation, nevertheless, the coefficient is enough higher to indicate a closer relation between these than was found in abilities in the fundamentals and ability to solve reasoning problems. It may also indicate that there is a higher correlation between pupils intelligence and their arithmetic skills than there is between the different abilities within the field of arithmetic.

In this diagram K equals .515, a lower figure than is secured in diagram II, which may indicate that it is rather safe to predict that pupils with a high arithmetic age will have a tendency to perform more quickly and accurately the processes required in computation than will pupils with lower arithmetic ages. That the correlation between arithmetic age and ability to perform the skills is higher than the correlation between the ability to perform the skills and reasoning in arithmetic.

The scores of the six typical cases on this diagram are as follows.

Figure IV.

Pupil.	Arithmetic Age. Yrs.	Months.	Stanford Computation.
A.	11	8	18.
B.	12	7	26.
C.	12	6	27.
D.	18	6	45.
E.	18	1	44.
F.	19	0	45.

Diagram IV.

Computation in Stanford Achievement Test.

Arithmetic Age

Computation in Stanford Achievement Test	10-10.9	11-11.9	12-12.9	13-13.9	14-14.9	15-15.9	16-16.9	17-17.9	18-18.9	19-19.9	20-20.9	f
45										2		2
44									1			1
43												0
42									1			1
41							2	3	5			10
40							3	6	3			12
39						2	3	6	6			17
38							5	4	1			10
37						3	3	10		1		17
36							6	7				13
35				3		5	10	7				25
34				6	3	7	6	5				27
33					1	5	4	3				13
32				3	1	20	6	2				32
31					8	10	3					21
30			1		17	15	2					35
29			1	6	18	10						35
28			1	7	17							25
27				4	4							8
26			1	14	14							29
25			2	13	6							19
24			4	6	1							11
23			6	3								9
22				2								2
21			2	1								3
20												
19												
18		1										1
		1	18	68	88	77	53	53	17	3		378

r = .857 ± P. E. = .009 K = .515

Diagram five is the scatter diagram of the pupils ages and their score in the Woody-McCall Fundamentals. The coefficient .653 ±.02 is again quite low but considerably higher than that in diagram two where the pupils skill in the fundamentals and their ability in reasoning are compared. No further comment is made concerning this diagram since the findings simply further verify the results found in the former diagrams.

The scores of the six typical cases are as follows.

Figure V.

Pupil.	Arithmetic Age. Yrs.	Months.	Woody-McCall Fundamentals.
A.	11	8	19.
B.	12	7	19.
C.	12	6	21.
D.	18	6	31.
E.	18	1	33.
F.	19	0	34.

Diagram V.

Arithmetic Age

Woody-McCall Mixed Fundamentals — Score	10-10.9	11-11.9	12-12.9	13-13.9	14-14.9	15-15.9	16-16.9	17-17.9	18-18.9	19-19.9	20-20.9	f
35												
34									1			1
33						4	5	5	1			14
32						6	1	6	6			19
31			1		2	2	4	10	4	2		25
30				2	4	5	6	16	2	1		36
29				2	8	7	11	8	1			37
28				8	13	11	9	8	2			51
27				7	17	12	8					44
26				10	15	9	4					38
25			1	9	13	11	4					38
24			4	11	7	3	2					27
23			7	9	3	3						22
22			2	10	4	1						17
21			1		2	1						4
20			1			1						2
19		1	1			1						3
18												
		1	18	68	88	77	53	53	17	3		378

$r = .653 \pm$ P. E. = .02 K = .757

As a whole these five diagrams cause one to look further to find the reasons for pupils inability to solve reasoning problems. It does not appear that the inefficiency or high efficiency of the pupil to perform the fundamental processes in arithmetic materially effect his ability to solve the reasoning problems. If this study has in any way pointed out the direction toward which we may look for the cause of the inability to reason in arithmetic, it seems to point to the lack of native intelligence as one of the basic causes, barring of course a total lack of training.

The scores of the six typical pupils may here be compiled, in figure VI, for a further review. This group of six pupils, three from either end of the scores, was selected not because of their chronological age nor their arithmetic age, but because they were quite consistant in their scores. The three A. B. and C. scoring very low in every test, whereas, D. E. and F. scored relatively high in all the tests.

Figure VI.

Pupil.	Chronological Age. Yrs.	Chronological Age. Months.	Arithmetic Age. Yrs.	Arithmetic Age. Months.	I.Q.	Mixed Funda.	Computation	Problem Reasoning
A.	16	1	11	8	71	19	18	16.
B.	17	9	12	7	79	19	26	16.
C.	17	8	12	6	81	21	27	16.
D.	14	4	18	6	130	31	45	37.
E.	15	9	18	1	118	33	44	32.
F.	14	10	19	0	119	34	45	28.

In figure six we have six pupils, three selected from either extreme of the scores. The I.Qs. in this figure are based on the Otis Group Intelligence Test. It will be noticed that the chronological ages of A. B. and C. are rather high and that of D. E. and F. much lower, whereas, their intelligence quotients and arithmetic ages are just the reverse. The scores made by these pupils in arithmetic tests also accord with their I.Q. and with their arithmetic age, but not at all with their chronological age. True the number of cases taken for this comparison is too small to establish any general principle, it is nevertheless, worthy of consideration. If the entire group of three hundred seventy-eight pupils were checked in the same manner a similar result would no doubt be obtained, except that the variations would increase as we approach the center of the scatter diagram. A small number do break away quite radically from the general rule, which has helped to cause the low correlations in these diagrams. So far as the six cases are concerned it does show that pupils with a low I.Q. do not do well in solving reasoning problems and that pupils with a high I.Q. do much better in this respect regardless of their chronological ages. It will be noticed that pupils A. B. and C. are older pupils than are D. E. and F. These particular ages were not selected with any forethought. They were selected because of the low and high scores they made. The fact that the first three of these pupils all scored consist-

antly low in all the tests and the last three consistantly high is perhaps due to being extreme cases. With average pupils similar results, although less pronounced, may be expected.

Chapter VII.

Causes of Failures in Reasoning Problems.

To discover further causes for the low correlations secured in this survey one needs to review the examination papers from some other standpoint than merely the scores made by the pupils. In order to find other causes and get a more satisfactory result from these tests the papers were checked with respect to the causes of errors and low scores in the reasoning test. In doing so the following results were obtained.

In the Stanford Achievement Test in Reasoning fifty pupils made a score of less than twenty. This score is based on the number of problems solved and is not multiplied by four as directed by instruction with the test. These fifty pupils attempted 402 problems in which they did not get the correct answer. In checking over these papers carefully and enumerating the probable causes for failure in the problems the following results or findings were obtained.

Scores and Percentages.

	Number	Per Cent.
1. Lack of reasoning ability, not knowing the process by which to proceed in solving the problem.	216	54
2. Not understanding the meaning of the problem. Poor silent reading.	57	14.2
3. Errors in the fundamentals.	56	13.8
4. Misreading the problem.	47	12
5. Carelessness in their figures, misplacing figures or decimals.	17	4

6. Confusion in use of fractions. ---------- 9 --- 2.

Total ---------- 402 -- 100

Of the fifty pupils making this low score there were five who did not attempt a sufficient number of problems to score well in the test. They attempted less than twenty problems, nevertheless, made a fairly good score in the fundamentals, hence, their lack of attempt may be ascribed to not knowing how to proceed in the solution of the problems.

Four of the pupils skipped around among the problems from the first to the last instead of taking them somewhat in consecutive order. These four have many answers wrong, from which it may be assumed that their skipping around was due to their inability to know how to proceed with those problems they omitted.

In order to get a sampling of the pupils at the other end of the score 46 pupils who made the highest scores, 28 or better, were selected. These forty-six pupils attempted only seventy-two problems in which they secured the wrong answers. Their probable causes for failure to get the correct answers were as follows.

Scores and Percentages.	Number of Problems.	Per Cent.
1. Lack of reasoning ability, not knowing the process by which to proceed in solving the proceed.	39	54
2. Misreading the problem	24	33.4
3. Errors in fundamentals	8	11.2
4. Misplacing the decimal points.	1	1.4
Total	72	100

It will be noticed that the causes for failure to get the correct answers are very nearly the same as those of the pupils at the other end of the score. They seem to have a better understanding of the meaning of the problems, that is, they appear to be better silent readers.

The above may also be something of an explanation as to why we get such low correlations between the pupils efficiency in the fundamentals and his ability to solve reasoning problems. These figures were compiled before any research had been made in the related literature and were compiled entirely independent from the findings of any other survey. This gives these findings more significance since they are quite similar to former findings. However, the reasons have not been divided into so many causes for the reason that in grading these papers only the answers, and not the process of solution, were available. Reason number one is given in case the answer is entirely wrong and has no relation to the correct answer. This might be due to lack of experience necessary to understand the setting of the problem, inability to see the relation in the problem so as to choose the proper operation, or inability to do reflective thinking, as enumerated by Boswell. Reason number two is given to problems where the pupil found an answer different in nature than the one called for in the problem. Number three is self explanitory. Number four is given to problems where the answers were given in different terms than called for in the problem. For example giving the

an

answer in cents when the answer requires per cent, or days for weeks, minutes for hours, etc. These are all cases where the meaning of the word is quite obvious but simply misread.

Chapter VIII.

Conclusion.

The evidence in this survey fails to show any definite existing relation between the pupils skill to perform correctly the fundamental operations in arithmetic and his ability to reason, or solve reasoning problems. All the evidence in this survey, as well as the related literature, confirm this conclusion. It appears from the findings in this survey that in arithmetic the reasoning ability needs to be trained and developed independently from the skill processes. It appears further that training in silent reading would materially aid, in that reading the problem intelligently, would give the pupil a better conception of what is required in the problem, and thus aid him in finding the correct procedure for its solution. Also that training the pupil to read more carefully so as to get the correct meaning of the words in the problem and their relation one to another within the sentence. To increase the pupil's vocabulary and his knowledge of the meaning of technical terms in arithmetic would be very helpful to him in interpreting the problem, and lastly a broadening of his general experience to enable him to understand the setting of the problem are essential factors to develop reasoning ability in solving arithmetic problems.

Bibliography

1. Buswell and Judd Survey of Educational Investigations Relating to Arithmetic.
Research Bulletin of the N.E.A. Vol. 3; No.5.

2. Osborn W. J. Corrective Arithmetic.
Houghton Mifflin and Company.

3. Banting O.G. Research in Waukesha, Wis.
Second Year Book of the Department of Elementary School Principles.

4. Harlan Charles L. Journal of Educational Research. September 1923.

5. Winch W. H. The Reorganization of Mathematics in Secondary Schools. Bureau of Education Bulletin, No. 32.

6. Hackler John M. Masters Thesis in Department of Education, Chicago University.

7. Terry Paul W. Reading Problems in Arithmetic.
Journal of Educational Psychology.Oct. 1921.

8. Thorndike E. L. American Journal of Psychology, July 1910.

9. Klapper Paul, The Teaching of Arithmetic.
Appleton and Company.

10. Wilson Improving the Ability to read in Arithmetic.
Elementary School Journal. Jan. 1922.

Measure the efficiency of the entire school, not the individual ability of the few.

COURTIS STANDARD RESEARCH TESTS

Arithmetic. Test No. 1. Addition

Series B Form 1

SCORE
No. Attempted .....
No. Right .....

You will be given eight minutes to find the answers to as many of these addition examples as possible. Write the answers on this paper directly underneath the examples. You are not expected to be able to do them all. You will be marked for both speed and accuracy, but it is more important to have your answers right than to try a great many examples.

927	297	136	486	384	176	277	837
379	925	340	765	477	783	445	882
756	473	988	524	881	697	682	959
837	983	386	140	266	200	594	603
924	315	353	812	679	366	481	118
110	661	904	466	241	851	778	781
854	794	547	355	796	535	849	756
965	177	192	834	850	323	157	222
344	124	439	567	733	229	953	525

537	664	634	572	226	351	428	862
695	278	168	253	880	788	975	159
471	345	717	948	663	705	450	383
913	921	142	529	819	174	194	451
564	787	449	936	779	426	666	938
932	646	453	223	123	649	742	433
559	433	924	358	338	755	295	599
106	464	659	676	996	140	187	172
228	449	432	122	303	246	281	152

677	223	186	275	432	634	547	588
464	878	478	521	876	327	197	256
234	682	927	854	571	327	685	719
718	399	516	939	917	394	678	524
838	904	923	582	749	807	456	969
293	353	553	566	495	169	393	761
423	419	216	936	250	491	525	113
955	756	669	472	833	885	240	449
519	314	409	264	318	403	152	122

Name....., ..., Age last birthday.....
Boy or Girl

School..... Grade..... Room.....

City..... State..... Date.....

15565

Measure the efficiency of the entire school, not the individual ability of the few.

COURTIS STANDARD RESEARCH TESTS

Arithmetic. Test No. 2. Subtraction

Series B Form 1

SCORE
No. Attempted .....
No. Right .....

You will be given four minutes to find the answers to as many of these subtraction examples as possible. Write the answers on this paper directly underneath the examples. You are not expected to be able to do them all. You will be marked for both speed and accuracy, but it is more important to have your answers right than to try a great many examples.

107795491	75088824	91500053	87939983
77197029	57406394	19901563	72207316
160620971	51274387	117359208	47222970
80361837	25842708	36955523	17504943
115364741	67298125	92057352	113380936
80195261	29346861	42689037	42556840
64547329	121961783	109514632	125778972
48813139	90492726	81268615	30393060
92971900	104339409	60472960	119811864
62207032	74835938	50196521	34379846
137769153	144694835	123822790	80836465
70176835	74199225	40568814	49178036

Measure the efficiency of the entire school, not the individual ability of the few.

COURTIS STANDARD RESEARCH TESTS

Arithmetic. Test No. 3. Multiplication

Series B Form 1

SCORE
No. Attempted .....
No. Right .....

You will be given six minutes to work as many of these multiplication examples as possible. You are not expected to be able to do them all. Do your work directly on this paper; use no other. You will be marked for both speed and accuracy, but it is more important to have your answers right than to try a great many examples.

8246 29	3597 73	5739 85	2648 46	9537 92
4268 37	7593 640	6428 58	8563 207	2947 63
5368 95	4792 84	7942 72	3586 36	9742 59
6385 48	8736 502	5942 39	6837 680	4952 47
3876 93	9245 86	7368 74	2594 25	6495 19

Measure the efficiency of the entire school, not the individual ability of the few.

COURTIS STANDARD RESEARCH TESTS

Arithmetic. Test No. 4. Division

Series B **Form 1**

SCORE
No. Attempted .....
No. Right .....

You will be given eight minutes to work as many of these division examples as possible. You are not expected to be able to do them all. Do your work directly on this paper; use no other. You will be marked for both speed and accuracy, but it is more important to have your answers right than to try a great many examples.

25)6775	94)85352	37)9990	86)80066
73)58765	49)31409	68)43520	52)44252
37)14467	86)60372	94)67774	25)9750
68)39508	49)28420	52)21112	73)33653
28)23548	54)48708	39)32760	67)61707
45)33795	76)57000	93)28458	82)29602

WOODY-McCALL MIXED FUNDAMENTALS: FORM I

Name..... *Age*..... *Grade*..... *Building*..... *City*.....

Get the right answer to as many examples as you can in 20 minutes. Do all work on the front or back of this sheet.

(1) Add
2
3

(2) $2 \times 3 =$

(3) $3 \overline{)\,6}$

(4) Subtract
2
1

(5) Multiply
23
3

(6) Subtract
13
8

(7) Add
17
2

(8) $3 + 1 =$

(9) Subtract
16
9

(10) Multiply
254
6

(11) $4 \div 2 =$

(12) Add
23
25
16

(13) Subtract
393
178

(14) $2 \overline{)\,13}$

(15) Add
9
24
12
15
19

(16) Multiply
5096
6

(17) $2\frac{3}{4} - 1 =$

(18) Add
$12.50
16.75
15.75

(19) Multiply
7898
9

(20) $\frac{1}{4}$ of 128 =

(21) Add
547
197
685
678
456
393
525
240
152

(22) Multiply
287
.05

(23) $248 \div 7 =$

(24) Subtract
27
$12\frac{5}{8}$

(25) Add
4.0125
1.5907
4.1000
8.6730

(26) Multiply
9742
59

(27) $\frac{7}{8}$ of 624 =

(28) Add
.49
.28
.63
.95
1.69
.22
.33
.36
1.01
.56
.88
.75
.56
1.10
.18
.56

(29) $\frac{1}{8} \times 2 =$

(30) Multiply
$987\frac{3}{4}$
25

(31) $\frac{3}{4} \div 5 =$

(32) $7.3 - 3.00081 =$

(33) $9 \overline{)\,69 \text{ lb. } 9 \text{ oz.}}$

(34) Multiply
$.0963\frac{1}{8}$
.084

(35) $25.091 + 100.4 + 25 + 98.28 + 19.3614 =$

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WOODY-McCALL MIXED FUNDAMENTALS: FORM II

Name..... *Age*..... *Grade*..... *Building*..... *City*.....

Get the right answer to as many examples as you can in 20 minutes. Do all the work on the front or back of this sheet.

(1) Add
2
4
3

(2) $3 \times 7 =$

(3) $9 \overline{)\,27}$

(4) Subtract
8
5

(5) $4 \times 8 =$

(6) Subtract
11
7

(7) Add
72
26

(8) Add
21
33
35

(9) Subtract
78
37

(10) Multiply
50
3

(11) $9 \overline{)\,0}$

(12) $25 + 42 =$

(13) Subtract
50
25

(14) $8 \overline{)\,5856}$

(15) Multiply
1036
8

(16) Subtract
567482
106493

(17) Add
$ .75
1.25
.49

(18) Multiply
8754
8

(19) $50 \div 7 =$

(20) Add
$8.00
5.75
2.33
4.16
.94
6.32

(21) Multiply
24
234

(22) $23 \overline{)\,469}$

(23) Subtract
$8\frac{7}{8}$
$5\frac{3}{4}$

(24) $\frac{1}{3} + \frac{1}{3} =$

(25) Multiply
16
$2\frac{5}{8}$

(26) $.003 \overline{)\,.0936}$

(27) Add
$2\frac{1}{2}$
$6\frac{3}{8}$
$3\frac{3}{4}$

(28) Multiply
6.25
3.2

(29) $62.50 \div 1\frac{1}{4} =$

(30) $3\frac{7}{8} - 1\frac{5}{8} =$

(31) $2\frac{1}{2} \times 3\frac{1}{2} =$

(32) Subtract
5 yd. 1 ft. 4 in.
2 yd. 2 ft. 8 in.

(33) Add
2 yr. 5 mo.
3 yr. 6 mo.
4 yr. 9 mo.
5 yr. 2 mo.
6 yr. 7 mo.

(34) $2\frac{1}{4} \times 4\frac{1}{2} \times 1\frac{1}{2} =$

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Stanford Achievement Test

By TRUMAN L. KELLEY, GILES M. RUCH, and LEWIS M. TERMAN

ARITHMETIC EXAMINATION: FORM A

FOR GRADES 2–8

Name.....Grade.....Boy or girl.....

Age.....When is your next birthday?.....How old will you be then?.....

Name of school.....Date.....

TEST	SCORE
4. Arithmetic: Computation	
5. Arithmetic: Reasoning	
TOTAL ARITHMETIC SCORE	
ARITHMETIC AGE	

Note. This examination contains all the problems of the Arithmetic Tests (Tests 4 and 5) of the Primary and Advanced examinations. See page 5 of the Manual of Directions for the Stanford Achievement Test.

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 SAT : AE : A–3

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To the Examiner: Do not administer this test without first reading carefully the Manual of Directions. The Manual must be ordered extra.

TEST 4. ARITHMETIC: COMPUTATION

Get the answers to these examples as quickly as you can without making mistakes. Look carefully at each example to see what you are to do.

Begin here.

(1)

$3 + 2 =$

(2)

$3 + 4 =$

(3)
Add
$$\begin{array}{r} 2 \\ \underline{5} \end{array}$$

(4)
Add
$$\begin{array}{r} 7 \\ \underline{4} \end{array}$$

(5)
Add
$$\begin{array}{r} 13 \\ \underline{2} \end{array}$$

(6)
Add
$$\begin{array}{r} 17 \\ \underline{2} \end{array}$$

(7)
Subtract
$$\begin{array}{r} 4 \\ \underline{2} \end{array}$$

(8)
Subtract
$$\begin{array}{r} 7 \\ \underline{4} \end{array}$$

(9)

$2 \times 3 =$

(10)
Add
$$\begin{array}{r} 16 \\ 53 \\ \underline{32} \end{array}$$

(11)
Subtract
$$\begin{array}{r} 16 \\ \underline{5} \end{array}$$

(12)
Subtract
$$\begin{array}{r} 96 \\ \underline{25} \end{array}$$

(13)
Subtract
$$\begin{array}{r} 13 \\ \underline{5} \end{array}$$

(14)
Subtract
$$\begin{array}{r} 765 \\ \underline{327} \end{array}$$

(15)
Multiply
$$\begin{array}{r} 26 \\ \underline{2} \end{array}$$

(16)
Multiply
$$\begin{array}{r} 253 \\ \underline{6} \end{array}$$

(17)
Divide

$2\overline{)6}$

(18)
Divide

$4\overline{)8}$

(19)
Add
$$\begin{array}{r} 684876542 \\ 791654220 \\ \underline{587339364} \end{array}$$

(20)

$6 \div 3 =$

(21)
Add
$$\begin{array}{r} 24 \\ \underline{12\frac{4}{5}} \end{array}$$

(22)
Multiply
$$\begin{array}{r} 6389 \\ \underline{7} \end{array}$$

(23)
Multiply
$$\begin{array}{r} 4679 \\ \underline{68} \end{array}$$

(24)

$2\overline{)15.8}$

(25)

$2\frac{7}{8} - 1 =$

Go right on to next page.

TEST 4, CONTINUED

(26)

$\frac{1}{4}$ of 8 2 8 =

(27)

$9\frac{3}{5} - 4\frac{1}{5} =$

(28)

Subtract

$$\begin{array}{r} 7\ 9 \\ \underline{1\ 6\frac{3}{8}} \end{array}$$

(29)

$\frac{1}{7} \times 2 =$

(30)

$.4\ 5\overline{)27.90}$

(31)

$3\frac{6}{7} \div 1\frac{1}{2} =$

(32)

Multiply

$$\begin{array}{r} 9.7\ 2 \\ \underline{2\ 1.9} \end{array}$$

(33)

Multiply

$$\begin{array}{r} 6\ 9\ 7\frac{1}{2} \\ \underline{1\ 8} \end{array}$$

(34)

$\frac{2\ 7}{2\ 8} \div \frac{6}{7} =$

(35)

4.40 + .00044 + 4400 + .04 =

(36)

$48.76 - 4\frac{9}{10} =$

(37)

$\frac{1}{2} + \frac{3}{4} + \frac{1}{6} + \frac{2}{3} + \frac{7}{8} =$

(38)

$27.34 + 2\frac{1}{4} + 89.2 + 4\frac{3}{4} =$

(39)

$3\frac{1}{4} \times 5\frac{1}{2} \times 3\frac{1}{2} =$

(40)

$1\frac{3}{4} + 25.2 + 4\frac{1}{5} + 48.961 =$

(41)

$\sqrt{4\ 5\ 3\ 6\ 9} =$

(42)

$(4)^3 =$

(43)

Subtract

$$\begin{array}{r} 8 \text{ yd. } 1 \text{ ft. } 3 \text{ in.} \\ \underline{6 \text{ yd. } 3 \text{ ft. } 9 \text{ in.}} \end{array}$$

(44)

Add

$$\begin{array}{r} 5 \text{ yr. } 9 \text{ mo.} \\ 6 \text{ yr. } 7 \text{ mo.} \\ \underline{8 \text{ yr. } 2 \text{ mo.}} \end{array}$$

(45)

$6\ 7.3\ 6 \div \frac{2}{3} =$

(46)

Multiply

$$\begin{array}{r} 4 \text{ gals. } 3 \text{ qts. } 1 \text{ pt.} \\ \underline{4} \end{array}$$

(47)

Express as a decimal to three places

$\frac{2\ 9}{6\ 4} =$

Test 4. Number right ..... × *4 = Score* .....

TEST 5. ARITHMETIC: REASONING

Find all the answers as quickly as you can.
Write the answers on the dotted lines.
Use the blank sheets of paper to figure on.

Begin here.

1 How many are 3 eggs and 2 eggs? *Answer*.....
2 Mary is 7 years old. How old will she be in 3 years? *Answer*.....
3 A hen had 9 chicks and 3 of them died. How many were left? *Answer*.....
4 Milk costs 8 cents a pint and the milkman is going to raise the price 2 cents. What will it then cost? *Answer*.....
5 If you buy a pencil for 4 cents and pay for it with a dime, how much change should you get? *Answer*.....

6 How many dimes are there in a dollar? *Answer*.....
7 How many eggs are there in 7 nests if each nest has 3 eggs? *Answer*.....
8 How many cents will 8 oranges cost at 3 cents each? *Answer*.....
9 David earned \$3.50 in June, \$2.25 in July, and \$1.50 in August. How much did he earn in all? *Answer*.....
10 Frank bought 3 two-cent postage stamps and 13 one-cent stamps. How much did he pay for all? *Answer*.....

11 Five girls buy a present costing 25 cents. How many cents does each pay? *Answer*.....
12 If a train goes 60 miles in three hours, how far does it go in one hour? *Answer*.....
13 John has saved \$3.75. How many dollars more does he need to buy a pony which costs \$45.75? *Answer*.....
14 A man pays the street-car fare for himself and two friends. If the fare is 7¢, how much change should he receive from a half dollar? *Answer*.....
15 A train which was due at 2 P.M. was $3\frac{1}{2}$ hours late. When did it arrive? *Answer*.....

16 What is the cost of 10 oranges at 2 for 5 cents? *Answer*.....
17 Edward has \$1.67 in the bank and takes out 2 quarters, a dime, and a cent. How much does he have left in the bank? *Answer*.....
18 What is the cost of a $4\frac{3}{4}$-pound roast at 40 cents a pound? *Answer*.....
19 A boy saved 5 cents a day for two weeks, and 10 cents a day for the next four weeks. How much money does he then have? *Answer*.....
20 A gallon is equal to 231 cubic inches. How many gallons are there in a tank $6 \times 7 \times 11$ inches? *Answer*.....

21 The tax rate in an Eastern city has varied as follows: 1910, 21¢ on each \$100; 1911, 17¢ on each \$100; 1912, 27¢ on each \$100; 1913, 26¢ on each \$100; 1914, 34¢ on each \$100; 1915, 33¢ on each \$100. The highest rate was how many times as great as the lowest? *Answer*.....

Go right on to next page.

TEST 5, CONTINUED

22 Henry was marked 87 in geography the first month, 91 the second, and 93 the third month. What was his average grade? *Answer*.....

23 If the butcher's scales read one ounce too much on each weighing, how much is a customer overcharged on a pound of steak at 48¢ a pound? *Answer*.....

24 At $1.00 a bushel for potatoes and $30.00 a car for freight, how much will a 400-bushel carload of potatoes cost? *Answer*.....

25 Tom has just 4 weeks' vacation and wishes to spend it in a city which it takes two days to reach by train. How many days can he spend in the city? *Answer*.....

26 If a fence rail is 10 feet long, how many rails will it take to reach a mile? *Answer*.....

27 Sound travels about 1100 ft. a second. If you see the flash of a cannon and 12 seconds later the sound reaches you, how far away is the cannon? *Answer*.....

28 A man had $5000, from which he received 6 per cent income each year. In addition he earned $1500 in business. What was his total income for the year? *Answer*.....

29 Frank and George buy 300 marbles for 50 cents. Frank pays 35 cents and George 15 cents. How many marbles should George receive? *Answer*.....

30 If a watch gains 20 seconds in 24 hours, what fraction of a minute will it gain between noon and 6 P.M.? *Answer*.....

31 The heights of 4 boys in a class are 5 feet 10 inches, 5 feet 9 inches, 5 feet 7 inches, and 5 feet 6 inches. What is the average height? *Answer*.....

32 An article which formerly sold at 12 cents was raised to 18 cents. What per cent was the price advanced? *Answer*.....

33 A broker charges $25 commission on every sale plus 5 per cent on all over $200. What would be his commission on a $500 sale? *Answer*.....

34 If 72 per cent of potatoes is water, how many pounds of solid material are there in a ton of potatoes? *Answer*.....

35 A man invested $1000 in each of 3 different bonds. The first paid 8 per cent dividend and the second 6 per cent, but on the third he lost $5 on each hundred dollars invested. What was his net yearly gain on the three investments? *Answer*.....

36 If the circumference of a circle is 12.5664 feet, what is its diameter? *Answer*.....

37 The regular price of a certain piece of linen is $4 per yard. A remnant $1\frac{1}{4}$ yards long is offered at $2.50. What per cent reduction is made? *Answer*.....

38 A man six feet tall casts a shadow 8 feet long at 9 A.M. A telephone pole casts a shadow 100 feet long at the same time. How high is the pole? *Answer*.....

39 It costs 43 cents to send a 10-pound parcel post package from New Orleans to Dallas. What will it cost to send an 8-pound package if the cost is 3 cents more on the first pound than on additional pounds? *Answer*.....

40 If the hour hand of a clock is 3 inches long and the minute hand is 4 inches long, how far apart are the tips of the two hands at 9 A.M.? *Answer*.....

Test 5. Number right ..... × 4 = *Score* .....