I. INTRODUCTION

After the discovery of the Higgs boson, the next task is to determine its couplings to as many Standard Model (SM) particles as possible. Only by doing so can the true nature of electroweak symmetry breaking be determined. It is particularly important to measure the parameters of the scalar potential, which entails measuring double Higgs production [1–3]. In the SM, this rate is small at the LHC [4–9], but may be significantly enhanced in models with new physics. One simple extension of the SM is to add a scalar, S, which is a singlet under all the gauge symmetries [10–13]. After electroweak symmetry breaking, S can mix with the SM Higgs boson, leading to a modification of Higgs couplings to SM particles and to the parameters of the scalar potential. In such models, there can be an enhancement of the di-Higgs rate due to the resonant production of the new scalar [14–16].

Models with a Higgs singlet are highly motivated by Higgs portal models [17–19]. In such models, S is the only particle which couples to a dark matter sector. Couplings of the dark matter to the known particles occur only through the mixing of S with the SM Higgs boson. If the Higgs singlet model possesses a $Z_2$ symmetry, the scalar singlet itself could be a dark matter candidate. Without a $Z_2$ symmetry, cubic and linear self-coupling terms are allowed in the scalar potential and a strong first order electroweak phase transition is allowed. Motivated by the possibility of explaining electroweak baryogenesis [20–22], we examine enhanced double Higgs production in a model with a scalar singlet and no $Z_2$ symmetry. The requirement that the electroweak minimum be a global minimum provides stringent restrictions on the allowed parameter space.

Attempts to increase the di-Higgs production rate by adding new particles which contribute to double Higgs production from gluon fusion have generally not found increases of more than a factor of 2–3 over the SM rate [23–25]. More successful has been the study of resonant enhancements, where increases up to a factor of $\sim 50$ relative to the SM prediction for double Higgs production have been found in 2 Higgs doublet models and the MSSM [26–30]. We determine the maximum allowed enhancement from resonant di-Higgs production in the singlet model without a $Z_2$ symmetry [31], such that the parameters correspond to a global electroweak minimum [21]. This case has a number of novel features in comparison with the well studied $Z_2$ symmetric singlet model [10].

In Sec. II, we review the Higgs singlet model and the minimization of the potential. Our results for the maximum allowed enhancement of the di-Higgs cross section, subject to the restriction that the electroweak minimum be a global minimum, are in Sec. III. Experimental constraints and theoretical restrictions on the parameters are given in Sec. IV. We include 2 appendices: Appendix A has the complete set of cubic and quartic Higgs self-couplings and Appendix B includes a description of the vacuum with $v = 0$.

II. MODEL

We consider a model containing the SM Higgs doublet, $H$, and an additional Higgs singlet, S. The most general scalar potential is

$$V(H, S) = V_H(H) + V_{HS}(H, S) + V_S(S),$$

with

$$V_H(H) = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

and

$$V_{HS}(H, S) = \frac{a_1}{2} H^\dagger H S + \frac{a_2}{2} H^\dagger H S^2$$

and

$$V_S(S) = b_1 S + \frac{b_2}{2} S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4.$$
We do not assume a $Z_2$ symmetry which would prohibit $a_1$, $b_1$ and $b_3$. The neutral component of the doublet $H$ is denoted by $\phi_0 = (h + v)/\sqrt{2}$, where the vacuum expectation value (vev) is $\langle \phi_0 \rangle = \frac{v}{\sqrt{2}}$. Similarly, the vev of $S$ is defined as $x$.

The extrema of the potential are obtained by requiring $\partial V(v, x)/\partial v = 0$ and $\partial V(v, x)/\partial x = 0$.

\[
\frac{v}{2} (-2\mu^2 + 2\lambda v^2 + a_1 x + a_2 x^2) = 0, \quad (5)
\]

\[
x (b_2 + b_3 x + b_4 x^2 + \frac{v^2}{2} a_2) + b_1 + \frac{v^2}{4} a_1 = 0. \quad (6)
\]

Solving Eqs. (5) and (6) produce many possible extrema of the potential. We require that one of these extrema correspond to the electroweak symmetry breaking (EWSB) minimum, $v = v_{\text{EW}} = 246$ GeV. It is important to note that a shift of the singlet field by $S \to S + \Delta S$ is just a redefinition of the parameters of Eq. (4) and does not change the physics. Hence, we are free to choose our EWSB minimum as $(v, x) = (v_{\text{EW}}, 0)$, since changing $x$ would correspond to shifting the singlet field.

With this criteria, solving Eqs. (5) and (6) produces,

\[
\mu^2 = \lambda v_{\text{EW}}^2, \quad b_1 = -\frac{v_{\text{EW}}^2}{4} a_1. \quad (7)
\]

The mass eigenstates are

\[
\begin{pmatrix}
 h_1 \\
 h_2
\end{pmatrix} = \begin{pmatrix}
 \cos \theta & \sin \theta \\
 -\sin \theta & \cos \theta
\end{pmatrix} \begin{pmatrix}
 h \\
 S
\end{pmatrix}. \quad (12)
\]

The physical masses of $h_1$ and $h_2$ are $m_1^2$ and $m_2^2$, respectively:

\[
m_{1,2}^2 = \frac{1}{2} \left( M_{11}^2 + M_{22}^2 + \sqrt{(M_{11}^2 - M_{22}^2)^2 + 4 M_{12}^4} \right). \quad (13)
\]

Note that the range of the mixing angle is $-\pi/4 < \theta < \pi/4$. We take $h_1$ to be the SM-like Higgs boson with $m_1 = 126$ GeV.

As mentioned earlier, we are interested in the scenario where $(v, x) = (v_{\text{EW}}, 0)$ is the global minimum of the potential. Hence, we require that the correct masses and mixing of the Higgs bosons are reproduced at this minimum:

\[
\det M^2|_{v=v_{\text{EW}}} = m_1^2 m_2^2, \quad \text{Tr}M^2|_{v=v_{\text{EW}}} = m_1^2 + m_2^2, \quad \text{and} \quad \frac{2M_{12}^2}{m_1^2 - m_2^2} = \sin 2\theta. \quad (14)
\]

From inspection, using Eq. (7) and $x = 0$, the mass matrix only depends on three combinations of parameters. These can be solved for:

There are two solutions. We choose this solution by using the further constraint that $\lambda$ obtains the SM value, $\lambda = m_1^2/2v_{\text{EW}}$, in the limit $\theta \to 0$. 

Using these solutions, the potential can be written in a more suggestive form, in terms of the neutral component of the Higgs field:

\[
V(\phi_0, S) = \lambda \left( \phi_0^2 - \frac{v_{\text{EW}}^2}{2} \right)^2 + \frac{a_1}{2} \left( \phi_0^2 - \frac{v_{\text{EW}}^2}{2} \right) S + \frac{a_2}{2} \left( \phi_0^2 - \frac{v_{\text{EW}}^2}{2} \right)^2 S^2 + \frac{1}{4} (2b_2 + a_2 v_{\text{EW}}^2) S^2 + \frac{b_3}{3} S^3 + \frac{b_4}{4} S^4. \quad (8)
\]

where an arbitrary constant factor has been dropped. Then $v = v_{\text{EW}}$ and $x = 0$ is a minimum by construction.

**A. Scalar masses and mixing**

The scalar mass matrix is,

\[
V_{\text{mass}} = \frac{1}{2} U M^2 U^T, \quad (9)
\]

where

\[
U = \begin{pmatrix}
 h \\
 S
\end{pmatrix}, \quad (10)
\]

\[
M^2 = \begin{pmatrix}
 M_{11}^2 & M_{12}^2 \\
 M_{21}^2 & M_{22}^2
\end{pmatrix} = \begin{pmatrix}
 3\lambda v^2 - \mu^2 + x(a_1 + a_2 x)/2 & a_1 v/2 + a_2 vx \\
 a_1 v/2 + a_2 vx & b_2 + a_2 v^2/2 + x(2b_3 + 3b_4 x)
\end{pmatrix}. \quad (11)
\]
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\[ a_1 = \frac{m_1^2 - m_2^2}{v_{\text{EW}}} \sin 2\theta, \]
\[ b_2 + \frac{a_2}{2} v_{\text{EW}}^2 = m_1^2 \sin^2 \theta + m_2^2 \cos^2 \theta, \]
\[ \lambda = \frac{m_1^2 \sin^2 \theta + m_2^2 \cos^2 \theta}{2 v_{\text{EW}}^2}. \] (15)

Our free parameters are then:
\[ m_1 = 126 \text{ GeV}, m_2, \theta, \]
\[ v_{\text{EW}} = 246 \text{ GeV}, \]
\[ x = 0, a_2, b_3, b_4. \] (16)

Note that once we choose the masses, mixing, and vevs, there is little choice in the free parameters. That is, all parameters are fully determined except \( a_2, b_2, b_3, \) and \( b_4, \) and there is a relation between \( b_2 \) and \( a_2. \)

Since the singlet Higgs does not couple to the SM fermions and vector bosons, the couplings of \( h_1 \) and \( h_2 \) are determined by those of the neutral component, \( h, \) of the Higgs doublet. From Eq. (12), one can see that the coupling of \( h_1 \) to the SM fermions and vector bosons, normalized to the SM values, is suppressed by a factor \( \cos \theta, \) while the coupling of \( h_2 \) is suppressed by \( -\sin \theta. \)

The self-interactions of the Higgs bosons in the basis of mass eigenstates \( h_1 \) and \( h_2 \) are
\[ V_{\text{self}} \supset \frac{\lambda^{111}}{3!} h_1^3 + \frac{\lambda^{211}}{2!} h_2 h_1^2 + \frac{\lambda^{221}}{2!} h_2^2 h_1 + \frac{\lambda^{222}}{3!} h_2^3 \]
\[ + \frac{\lambda^{1111}}{4!} h_1^4 + \frac{\lambda^{1211}}{3!} h_2 h_1^3 + \frac{\lambda^{1221}}{3!} h_2^2 h_1^2 \]
\[ + \frac{\lambda^{1222}}{3!} h_2 h_1^3 + \frac{\lambda^{2222}}{4!} h_2^4. \] (17)

The cubic and quartic couplings are listed in Appendix A.

The partial width of \( h_2 \to h_1 h_1 \) is then
\[ \Gamma(h_2 \to h_1 h_1) = \frac{\lambda^{111}}{32 \pi m_2^2} \sqrt{1 - \frac{4 m_2^2}{m_1^2}}. \] (18)

Since the coupling of \( h_2 \) to other SM particles is suppressed by \( \sin \theta \) we can write the total width\(^3\)
\[ \Gamma(h_2) = \sin^2 \theta \Gamma^{\text{SM}}|_{m_2} + \Gamma(h_2 \to h_1 h_1), \] (19)

where \( \Gamma^{\text{SM}}|_{m_2} \) is the SM Higgs total width evaluated at mass \( m_2. \) In future calculations we use the results in Ref. [32] to calculate \( \Gamma^{\text{SM}}. \)

\(^3\)We neglect the partial width \( h_2 \to h_1 h_1 h_1 \) since this is additionally suppressed by three body phase space.

**B. Vacuum structure**

Vacuum stability requires that the scalar potential must be positive definite as \( \phi_0 \) and \( S \) become large. The behavior of the potential at large values of the fields is governed by the quartic interactions,
\[ 4 \lambda \phi_0^4 + 2 a_2 \phi_0^2 S^2 + b_4 S^4 > 0. \] (20)

We know that \( \lambda \) and \( b_4 \) must both be positive since the potential needs to be stable along the axes \( S = 0 \) or \( \phi_0 = 0. \) Also, for \( a_2 > 0 \) the potential is clearly stable. For \( a_2 < 0, \) rewrite Eq. (20) as,
\[ \lambda \left( 2 \phi_0^2 + \frac{a_2}{2 \lambda} S^2 \right) + \left( b_4 - \frac{a_2^2}{4 \lambda} \right) S^4 > 0. \] (21)

Since the first term is positive definite, we obtain the stability bound
\[ -2 \sqrt{\lambda b_4} \leq a_2. \] (22)

Following the methods of Ref. [21], the extrema of Eq. (8) for which \( v \neq 0 \) can be found:
\[ (v, x) = (v_{\text{EW}}, 0), \quad \text{and} \quad (v, x) = (v_\pm, x_\pm) \] (23)

where
\[ x_\pm = \frac{v_{\text{EW}} (3 a_1 a_2 - 8 b_3 \lambda) \pm 8 \sqrt{\Delta}}{4 v_{\text{EW}} (4 b_4 \lambda - a_2^2)}, \]
\[ v_\pm = \frac{1}{2 \lambda} (a_1 x_\pm + a_2 x_\pm^2), \]
\[ \Delta = \frac{v_{\text{EW}}^2}{64} (8 b_3 \lambda - 3 a_1 a_2)^2 - \frac{m_1^2 m_2^2}{2} (4 b_4 \lambda - a_2^2). \] (24)

For three real solutions to exist, we need \( \Delta > 0 \) and \( v_\pm > 0. \) There are also solutions for \( v = 0, \) which we include in the appendix.

First, we analyze the \( v^2 \neq 0 \) solutions. For the global minimum to be \( v = v_{\text{EW}} \) and \( x = 0, \) the potential of Eq. (8) must satisfy
\[ V(v_{\text{EW}}, 0) < V(v_\pm, x_\pm). \] (25)

It can be shown that this occurs for
\[ v_{\text{EW}} |8 b_3 a_2 - 3 a_1 a_2| < 6 m_1 m_2 \sqrt{4 b_4 \lambda - a_2^2}, \quad \text{or} \quad 4 b_4 \lambda < a_2^2. \] (26)

The vacuum structure of \( v^2 \neq 0 \) is shown in Fig. 1 with \( m_2 = 370 \text{ GeV}, \) \( \cos \theta = \sqrt{2}/2, \) and \( b_4 = 1. \) The region with \( a_2 \lesssim -1 \) does not satisfy the stability bound of Eq. (22).

The white region is where the \( (v, x) = (v_{\text{EW}}, 0) \) solution is
Since we require that the global minimum be real, we can always reject solutions for which \( v^2 \neq 0 \), as given in Eq. (26). The shaded areas show \( b_3, a_2 \) values where \( V(v_-,x_-) < V(v_{\text{EW}}, 0) \) with \( v_0^2 < 0 \) (red horizontal lines) and \( v_0^2 > 0 \) (blue squares), and \( V(v_+,x_+) < V(v_{\text{EW}}, 0) \) with \( v_0^2 < 0 \) (green vertical lines) and \( v_0^2 > 0 \) (maroon hatched lines). All three solutions are never simultaneously minima.

It can be shown that \( (v_{\text{EW}}, 0) \) always corresponds to a minimum. Hence, this exhausts the possibilities for \( v^2 \neq 0 \). Since we require that the global minimum be real, we can also reject solutions for which \( v_0^2 < 0 \). Hence, \( v = v_{\text{EW}} \) and \( x = 0 \) is the lowest lying real minimum with \( v^2 \neq 0 \) in the red-lined, green-lined, and white regions. However, we must consider also the case \( v = 0 \), which is discussed in the appendix.

The final results for the allowed \((b_3, a_2)\) region with a global minimum at \((v, x) = (v_{\text{EW}}, 0)\) are shown in Fig. 2. This includes the analysis of the \( v = 0 \) minima. Inside the contours \((v, x) = (v_{\text{EW}}, 0)\) is the global minimum. Figure 2(a) shows the dependence on the heavy scalar mass \( m_2 \), and Fig. 2(b) shows the dependence on \( b_4 \). Increasing \( b_4 \) and \( m_2 \) increases the upper bounds on \( a_2 \) slightly. The difference in allowed regions between Figs. 1 and 2(a) corresponds to the case where the \( v = 0 \) minimum is the global minimum.

In Fig. 2(a), there is an interesting point on the contours that appears to be independent of \( m_2 \). From Eq. (26), this section of the contour arises from the inequality

\[
b_3^{\text{min}} = \frac{3}{8 \lambda v_{\text{EW}}} \left( a_1 a_2 v_{\text{EW}} - 2 m_1 m_2 \sqrt{4 b_4 \lambda - a_2^2} \right) < b_3.
\]

(27)

The stationary points on this line can be found by solving \( \partial b_3^{\text{min}} / \partial m_2 = 0 \) for \( a_2 \). Assuming \( \sin \theta > 0 \), one of these solutions corresponds to

\[
a_2 = -\sqrt{2 b_4 \cos \theta} \frac{m_1}{v_{\text{EW}}}, \quad \text{and} \quad b_3 = \frac{3}{2} \sqrt{2 b_4 \sin \theta} m_1,
\]

(28)

which is independent of \( m_2 \). This exactly corresponds to the degenerate point on the contours in Fig. 2(a).

It is clear from these results that both \( a_2 \) and \( b_3 \) are bounded for fixed masses, mixing, and \( b_4 \). As we will see in Sec. IV, requiring perturbative unitarity bounds \( b_4 \). Hence, all parameters are either determined by the masses and

\[
\begin{align*}
\text{FIG. 1 (color online). Structure of the } v^2 \neq 0 \text{ vacua in the } b_3 \text{ vs } a_2 \text{ plane for } m_2 = 370 \text{ GeV, } b_4 = 1, \text{ and } \cos \theta = \sqrt{0.88}. \text{ The different regions are where the } (v, x) = (v_{\text{EW}}, 0) \text{ minimum is the lowest lying (white region), } (v_-, x_-) \text{ is the lowest lying minimum with } v_0^2 < 0 \text{ (red horizontal lines) and } v_0^2 > 0 \text{ (blue squares), and } (v_+, x_+) \text{ is the lowest lying minimum with } v_0^2 < 0 \text{ (green vertical lines), and } v_0^2 > 0 \text{ (maroon hatched region).}
\end{align*}
\]

\[
\begin{align*}
\text{FIG. 2 (color online). Constraints on the } (b_3, a_2) \text{ parameter space obtained by requiring that the global minimum is at } (v, x) = (v_{\text{EW}} = 246 \text{ GeV}, 0). \text{ Regions enclosed by the lines are allowed. Figure 2(a) shows the allowed regions with various values of } m_2 \text{ for } b_4 = 1. \text{ The solid (red), dashed (blue), and dash-dotted (black) represent } m_2 = 270, 370, \text{ and } 500 \text{ GeV, respectively. Figure 2(b) shows the allowed regions with } b_4 = 1 \text{ (blue dashed) and } b_4 = 3 \text{ (black solid) for } m_2 = 370 \text{ GeV. The parameters used are } m_1 = 126 \text{ GeV and } \cos \theta = 0.94.
\end{align*}
\]
mixings of the Higgs sector or are bounded by theoretical considerations. This will have a direct influence on the phenomenology of the singlet model at the LHC.

III. RESONANT DI-HIGGS PRODUCTION

A. Results without a $Z_2$ symmetry

We turn now to the results for di-Higgs production obtained by imposing the parameter restrictions described above to find the maximum enhancement possible in the $gg \rightarrow h_1h_1$ channel relative to the SM rate. Di-Higgs production proceeds through the diagrams shown in Fig. 3. For $m_2 \gtrsim 2m_1$, it is possible to have a large resonant enhancement from the diagram of Fig. 3(c). Our numerical results use CT12NLO PDFs with $\mu = M_{h_1h_1}$. We normalize many of our plots to the LO SM predictions, $\sigma(gg \rightarrow h_1h_1)|_{SM} = 15$ fb (0.6 pb) at $\sqrt{s} = 14$ TeV (100 TeV).\textsuperscript{4}

From the mass matrix in Eq. (11), we know that varying $b_3$ does not change $m_1$, $m_2$ and the mixing angle $\theta$. In contrast, one can observe that $\lambda_{211}$ in Eq. (A1) is a function of $b_3$. In Fig. 4, we show the dependence on $b_3$ of the branching ratio of the heavier Higgs, $h_2$, into the SM-like Higgs, $h_1$, For $b_3$ small, the branching ratio has little dependence on $m_2$, while for large $b_3$, the branching ratio can be large and depends significantly on $b_3$. The dotted curves represent regions where the parameters do not correspond to a global electroweak minimum. We see then that for a given mass this constraint corresponds to an upper limit on the branching ratio $\text{Br}(h_2 \rightarrow h_1h_1)$.

To understand the features of Fig. 4, use the solutions in Eq. (15) to rewrite

$$\lambda_{211} = \sin \theta \left[ - \frac{2m_1^2 + m_2^2}{v_{EW}^2} \cos^2 \theta - a_2 v_{EW} (1 - 3 \cos^2 \theta) + b_3 \sin(2\theta) \right].$$

From this we see that $b_3 \sin(2\theta)$ and $m_2$ make opposite sign contributions to $\lambda_{211}$. Hence, for $b_3 \sin(2\theta) < 0$, they constructively contribute to $\lambda_{211}$. The major feature of this region in Fig. 4 is then understood by noting that the partial widths of $h_2$ into $h_1$, $W$s, and $Z$s scale like

$$\Gamma(h_2 \rightarrow h_1h_1) \propto \sin^2 \theta m_2, \quad \text{and} \quad \Gamma(h_2 \rightarrow W^+W^-/ZZ) \propto \sin^2 \theta m_2.$$ 

\textsuperscript{4}Radiative corrections in the SM are large, typically a factor of \sim 2 enhancement \cite{7-9}, and are not included here since they are simply an overall normalization factor to the results we present.
Hence, as the mass of $h_2$ increases the partial widths into $W$s and $Z$s grow much more quickly than the partial width into $h_1h_1$. The branching ratio $\text{Br}(h_2 \to h_1h_1)$ therefore decreases with mass.

The region for $b_3 \sin(2\theta) > 0$ is slightly more involved. Using Eq. (29), the triple coupling $\lambda_{211}$ goes to zero when

$$b_3 \sin(2\theta) = \frac{2m_1^2 + m_2^2}{v_{\text{EW}}} \cos^2 \theta + a_2 v_{\text{EW}} (1 - 3 \cos^2 \theta).$$

(31)

We see that for smaller $m_2$ the zero corresponds to smaller $b_3 \sin(2\theta)$. As $b_3 \sin(2\theta)$ goes from negative to positive, the smaller $m_2$ values turn over and approach zero more quickly than the larger $m_2$. This is the behavior we see in Fig. 4. Note that for our representative parameters, we have $\theta > 0$, so the sign of $b_3 \sin(2\theta)$ is the same as $b_3$.

In Fig. 5, we plot the dependence of the ratio of the di-Higgs production cross section in the singlet model to that in the SM at (a) $\sqrt{S} = 14$ TeV and (b) $\sqrt{S} = 100$ TeV as a function of $b_3$. The parameters used are $m_1 = 126$ GeV, $\cos \theta = 0.94$, $a_2 = 0$, $v_{\text{EW}} = 246$ GeV, and $b_4 = 1$. The solid (dashed) lines stand for regions that are allowed (excluded) by the requirement of EW stability.

Hence, we would expect this dependence to be a straight line, as seen for $m_2 = 270$ and 420 GeV. However, we see that this is not the case for $m_2 = 1000$ GeV. In Fig. 7 we show the ratio of the total width of $h_2$ and $m_2$ as a function of the branching ratio of $h_2 \to h_1h_1$. As can be seen for $m_2 = 1000$ GeV, the width is always large and the narrow width approximation holds and the production cross section $h_2$ is sufficiently larger than the SM di-Higgs rate, we have

$$\sigma(pp \to h_1h_1) \approx \sigma(pp \to h_2) \text{Br}(h_2 \to h_1h_1).$$

(32)

$\sqrt{S} = 100$ TeV and $b_3 < 0$ [Fig. 5(b)], the cross section for $m_2 = 270$ GeV drops below that of $m_2 = 370$ GeV. As to be discussed later, this is due to specific properties of di-Higgs production.

In Fig. 6 we show the enhanced di-Higgs ratio as a function of the $h_2 \to h_1h_1$ branching ratio. If the narrow width approximation holds and the production cross section $h_2$ is sufficiently larger than the SM di-Higgs rate, we have

$$\sigma(pp \to h_1h_1) \approx \sigma(pp \to h_2) \text{Br}(h_2 \to h_1h_1).$$

Hence, we would expect this dependence to be a straight line, as seen for $m_2 = 270$ and 420 GeV. However, we see that this is not the case for $m_2 = 1000$ GeV. In Fig. 7 we show the ratio of the total width of $h_2$ and $m_2$ as a function of the branching ratio of $h_2 \to h_1h_1$. As can be seen for $m_2 = 1000$ GeV, the width is always large and the narrow width approximation is poor. This explains why the $m_2 = 1000$ GeV line in Fig. 6 is not straight. Also, as the branching ratio of $h_2 \to h_1h_1$ increases, the total width become larger. This is due to the partial width $h_2 \to h_1h_1$ becoming large, since the partial widths into $W$ and $Z$ bosons is fixed by the mass $m_2$ and mixing angle $\theta$.

In Fig. 6, it is interesting to note that the enhancement for $m_2 = 420$ GeV is larger than that for 270 GeV at $\sqrt{S} = 100$ TeV. This can be understood from the parton luminosity plot of Fig. 8(a), where we show the gluon-gluon parton luminosity (normalized to that at 2$m_t$). The $\sqrt{S} = 14$ TeV luminosity falls much more quickly as a function of invariant mass than does the corresponding luminosity at $\sqrt{S} = 100$ TeV. We compare this with the resonant production of $gg \to h_2$ in Fig. 8(b) and observe that at $\sqrt{S} = 100$ TeV the resonant enhancement at the $t\bar{t}$ threshold is more important than at $\sqrt{S} = 14$ TeV. Finally,
we show the dependence on $m_2$ of the full cross section for $gg \rightarrow h_1 h_1$ in Fig. 9. The resonant structure near $2m_t$ is clearly visible.

**B. The $Z_2$ limit**

It may be necessary in certain models to impose a $Z_2$ symmetry on the potential under which $S$ is odd and $H$ is even. This may be motivated from a dark matter perspective, where $S$ is a dark matter particle, or the point of view of a complex hidden sector. The potential for this case can be obtained in the limit $a_1, b_1, b_3 \rightarrow 0$. If the $Z_2$ remains unbroken, there is no resonance enhancement in di-Higgs production, since the $S \rightarrow hh$ decay breaks the $Z_2$ symmetry and there is no mixing between $S$ and $h$. We ignore this case. However, the $Z_2$ symmetry may be broken by a vev of $S$. Unlike the case outlined above, the vev of $S$ is then physically meaningful and we cannot set $\langle S \rangle = x = 0$ arbitrarily. The $Z_2$ symmetric potential is

\[
V(H, S) = -u^2 H^zh^z + \lambda(H^zh^z)^2 + \frac{a_2}{2}H^zHS^2 + \frac{b_2}{2} S^2 + \frac{b_4}{4} S^4. \tag{33}
\]

We shift the fields in the usual manner to find the $h_2 h_1$ coupling in the $Z_2$ symmetric limit [10],

\[
\lambda_{2111}^Z = a_2 [v s (2c^2 - s^2) - x c (2s^2 - c^2)] - 6\lambda v^2 s + 6b_4 x c s^2. \tag{34}
\]

In the limit $x = 0$ and $a_1, b_1, b_3 = 0$, Eq. (34) is in agreement with Eq. (A1). We impose the conditions of positivity of the potential, $\lambda > 0, b_3 > 0$ and $4\lambda b_4 - a_2^2 > 0$ [Eq. (20)] and require the couplings to be perturbative, $a_2, b_4, \lambda < 4\pi$.

The physical parameters are taken as

\[
m_1, m_2, \cos \theta \equiv c, v_{EW}, x. \tag{35}
\]

Using Eqs. (34) and (18), the branching ratio for $h_2 \rightarrow h_1 h_1$ can be found and is shown in Fig. 10. Comparing with Fig. 6, it is apparent that the branching ratios are similar in the models with and without the $Z_2$ symmetry for large values of $x/v_{EW}$, where the branching ratio asymptotes to around $\text{BR}(h_2 \rightarrow h_1 h_1) \sim 0.3$. The branching ratio $h_2 \rightarrow h_1 h_1$ appears to have little discriminating power between the $Z_2$ symmetric and nonsymmetric potentials.
There are a number of well-known experimental and theoretical limits on the Higgs singlet model, which we briefly review in this section.

A. Experimental limits

From the direct measurements of the Higgs coupling strengths, ATLAS [33] places a constraint on the mixing angle, $\theta$, of the singlet model, where $\cos^2 \theta \leq 0.88$ has been excluded at 95% CL. This limit assumes that there is no branching ratio to invisible particles. Here we take the upper limit of $\sin^2 \theta \leq 0.0\text{.}88$ as a representative point. Direct searches for the heavy Higgs ($h_2$) decaying into $W^+ W^-$ and $ZZ$ from ATLAS and CMS [34,35] can also give bounds on $\sin^2 \theta$ with $\sin^2 \theta \lesssim 0.2$ for $m_2 \sim 200–400$ GeV and $\sin^2 \theta \lesssim 0.4$ for $m_2 \sim 600$ GeV. However, these constraints are not as strong as the ATLAS limit from the Higgs coupling strengths.

The existence of a Higgs singlet which mixes with the SM Higgs boson is also restricted by electroweak precision observables. A fit to the oblique parameters, $S$ and $T$ (fixing $U$ to be 0), is shown in Fig. 11 [20,36]. We see that limits from the oblique parameters are not competitive with the ATLAS limit from the Higgs coupling strengths.

ATLAS and CMS have obtained upper bounds on the cross section for the resonant production of SM Higgs bosons pairs through the process $pp \rightarrow h_2^+ h_2^-$ in the $\gamma \gamma b \bar{b}$ [37,38] and $b\bar{b} b\bar{b}$ [39] channels at a center-of-mass energy of $\sqrt{s} = 8$ TeV with an integrated luminosity of 20 fb$^{-1}$ as summarized in Fig. 12. In the low mass region the $\gamma \gamma b\bar{b}$ channel gives a stronger bound as opposed to a weaker bound obtained in the $b\bar{b} b\bar{b}$ channel due to the
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**FIG. 11.** Constraints on the mixing angle, $\sin \theta$, as a function of the mass of the heavier Higgs scalar, $m_2$, from fits to the oblique parameters, $S$ and $T$.

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**Fig. 12 (color online).** Observed 95% CL upper limits at $\sqrt{S} = 8$ TeV with an integrated luminosity of 20 fb$^{-1}$ on the resonant di-Higgs production cross section from ATLAS in the $\gamma\gamma b\bar{b}$ channel (black solid), CMS in the $\gamma\gamma b\bar{b}$ channel (blue dashed) and CMS in the $b\bar{b}b\bar{b}$ channel (red dot-dashed), normalized to the leading order cross section predicted by the SM, and the regions allowed by the requirement that the electroweak minimum be a global minimum for $(b_4, a_2) = (3, 0)$ (green solid) and $(b_4, a_2) = (1, -1)$ (magenta solid).

---

large QCD background. However, the limit from the $b\bar{b}b\bar{b}$ channel becomes more constraining above $m_2 \sim 400$ GeV.

We compare the experimental upper limits on the production cross sections for resonant di-Higgs production with $m_2$ between 270 GeV and 1 TeV, normalized to the leading order cross section predicted by the SM, with the range of allowed cross sections consistent with the requirement that the parameters correspond to a global electroweak minimum. (The allowed region is between the curves). Two sets of parameter points $(b_4, a_2) = (3, 0)$ and $(b_4, a_2) = (1, -1)$ are considered. The former has a larger value of $b_4$ and hence the bound is less stringent as illustrated in Fig. 2(b). The lower limit of the allowed region on $m_2$, which starts at $m_2 \sim 370$ GeV, for $(b_4, a_2) = (1, -1)$ can be explained by Eq. (22) as due to the vacuum stability constraint. Plugging in $\lambda$ defined in Eq. (15), one can obtain the lower limit for $m_2^2$ for a given $b_4$ and negative $a_2$,

$$m_2^2 \geq \frac{1}{\sin^2 \theta} \left( \frac{a_2^2}{2b_4^2} \frac{m_{EW}^2}{m_1^2 \cos^2 \theta} \right).$$

Throughout the $m_2 < 1$ TeV mass range, the constraints derived from the global electroweak minimum requirement are always stronger than those currently available from the LHC experiments at $\sqrt{S} = 8$ TeV. We make naive projections for the expected constraints at the LHC at $\sqrt{S} = 14$ TeV with an integrated luminosity of 300 fb$^{-1}$ by rescaling the expected 95% CL upper limits at $\sqrt{S} = 8$ TeV with an integrated luminosity of 20 fb$^{-1}$, using the ratios of gluon-gluon luminosities (evaluated at the scale $2m_1$) given in Ref. [40]. As shown in Fig. 13, the projected bounds from the CMS $\gamma\gamma b\bar{b}$ channel can rule out the entire parameter space where the electroweak minimum is a global minimum for $(b_4, a_2) = (1, -1)$ and can exclude much of the allowed region for $(b_4, a_2) = (3, 0)$. Moreover, the projected limits from the CMS $b\bar{b}b\bar{b}$ channel can potentially exclude the entire parameter space allowed by the electroweak minimum requirement for $(b_4, a_2) = (1, -1)$ and rule out two thirds of the allowed region in the high mass range for $(b_4, a_2) = (3, 0)$.

---

**Fig. 13 (color online).** Projected 95% CL upper limits at $\sqrt{S} = 14$ TeV with an integrated luminosity of 300 fb$^{-1}$ on the production cross section from the ATLAS $\gamma\gamma b\bar{b}$ channel (black solid), CMS $\gamma\gamma b\bar{b}$ (blue dashed) and CMS $b\bar{b}b\bar{b}$ (red dot-dashed), normalized to the leading order cross section predicted by the SM, and the regions allowed by the requirement that the electroweak minimum be a global minimum for $(b_4, a_2) = (3, 0)$ (green solid) and $(b_4, a_2) = (1, -1)$ (magenta solid).
B. Unitarity

The coefficients of the potential cannot be too large or perturbative unitarity will be violated in the $h_i h_j$ scattering processes [41]. The simplest limit comes from the high energy scattering of $h_2 h_2 \to h_2 h_2$, where the $J = 0$ partial wave is

$$a_0(h_2 h_2 \to h_2 h_2) \to_{s \to m_1^2} \frac{3b_4}{8\pi}.$$  

(37)

Requiring $|a_0| < \frac{1}{2}$ yields $|b_4| \leq 4.2$. Limits from a coupled channel analysis of $h_i h_j$ scattering show that for small $\theta$, multi-TeV scale masses are allowed for $m_2$ [10].

Similarly, we can consider the $h_1 h_1 \to h_1 h_1$ scattering to find the $J = 0$ partial wave.

$$a_0(h_1 h_1 \to h_1 h_1) \to_{s \to m_1^2} \frac{3\lambda}{8\pi}.$$  

(38)

Then using Eq. (15) and $|a_0| < \frac{1}{2}$, an upper limit on $m_2$ can be found:

$$m_2^2 < \frac{1}{3\sin^2\theta}(8\pi v_{EW}^2 - 3m_1^2\cos^2\theta).$$  

(39)

For $\cos^2\theta = 0.88$ and $m_1 = 126$ GeV, this limit is $m_2 \lesssim 2$ TeV.

V. DISCUSSION AND CONCLUSIONS

We studied resonance enhancement of di-Higgs production in a generic singlet extended Standard Model. By imposing conditions on the masses, mixing, and vacuum expectation values of the bosons we were able to identify the three parameters that are left free. These three parameters were then bounded by unitarity constraints and the requirement that the electroweak symmetry breaking minimum be the global minimum. With these constraints, $\text{Br}(h_2 \to h_1 h_1)$ is bounded from above. Hence, we found that theoretical considerations bound the di-Higgs production in this model and that the theoretical constraints are more stringent than the current limits from direct searches for $h_1 h_1$. We then provided predictions for the cross sections and branching ratios for $\sigma(pp \to h_2 \to h_1 h_1)$ at both the 14 TeV LHC and a 100 TeV collider. The di-Higgs production enhancement can be as large as a factor of $\sim 18(13)$ for $m_2 = 270(420)$ GeV relative to the SM rate at 14 TeV for parameters corresponding to a global EW minimum.

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We now evaluate the extrema of the potential with $v = 0$. These are found by evaluating the extrema of Eq. (8). The solutions for $(S)$ are

\[
\begin{align*}
x_1^0 &= \frac{(2b_3 - \kappa^{1/3})^2 - 12b_2 b_4}{6b_2^{1/3}} + b_3 \frac{1}{3b_4}, \\
x_2^0 &= \frac{(2b_3 - e^{2i\pi/3} \kappa^{1/3})^2 - 12b_2 b_4}{6b_2 e^{2i\pi/3} \kappa^{1/3}} + b_3 \frac{1}{3b_4}, \\
x_3^0 &= \frac{(2b_3 - e^{4i\pi/3} \kappa^{1/3})^2 - 12b_2 b_4}{6b_2 e^{4i\pi/3} \kappa^{1/3}} + b_3 \frac{1}{3b_4},
\end{align*}
\]

where we have defined,

\[
\begin{align*}
\kappa &= -4b_3(2b_3^2 - 9b_2 b_4) + 27a_1 b_4^2 v_{EW}^2 + 3b_4 \sqrt{3} \Delta^0, \\
\Delta^0 &= -16b_2(2b_3^2 - 4b_2 b_4) - 8a_1 b_3 v_{EW}^2(2b_3^2 - 9b_2 b_4) \\
&\quad + 27a_1 b_4^2 v_{EW}^4.
\end{align*}
\]

In Fig. 14, we show the vacuum structure of the $\langle \phi_0 \rangle = 0$ minima compared to the $(v, x) = (v_{EW}, 0)$ minima. The white region corresponds to where the EWSB minima lies below the $v = 0$ minima, the red lined region to where $(v, x) = (0, x_1^0)$ lies below $(v_{EW}, 0)$, the blue squares to where $(0, x_2^0)$ lies below $(v_{EW}, 0)$, and the green hashed region is where both $(0, x_1^0)$ and $(0, x_0^0)$ lie below $(v_{EW}, 0)$. We do not find any region where $V(0, \langle S \rangle = x_0^0)$ is below the EWSB minima. Combining the results of Figs. 1 and 14 we can understand the contour in fig. 2.