Technicolor-assisted leptogenesis with an ultra-heavy Higgs doublet

Hooman Davoudiasl* and Ian Lewis†

*Department of Physics, Brookhaven National Laboratory, Upton, New York 11973, USA
†Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794, USA

(Received 13 December 2011; published 23 July 2012)

The mechanism for electroweak symmetry breaking (EWSB) is a question of great fundamental importance and remains a mystery. While the standard model (SM) picture based on a single Higgs doublet can accommodate all the relevant phenomenology, it could very well be the case that nature realizes EWSB in a more complicated way. For example, multiple sectors could contribute different pieces of the observed effects at low energy, with some mainly providing $W^\pm$ and $Z$ boson masses, while others are responsible for the masses of the fermions. In fact, some theoretical considerations lead to such a scenario. In particular, the apparent hierarchy between the weak scale $\sim 100$ GeV and other potentially large scales of physics motivates one to consider a dynamical mechanism based on condensation of fermion pairs with quantum numbers of the SM Higgs, such as those in technicolor models [1,2].

However, while a dynamical mechanism can naturally endow $W^\pm$ and $Z$ with their observed masses $m_W \sim m_Z \sim 100$ GeV, generation of fermion masses is a challenge in this framework [3–5].

The above considerations have provided motivation for a hybrid proposal, namely the bosonic technicolor scenario [6–8], where fermions obtain their masses through Yukawa couplings with a Higgs doublet $\Phi$, as in the SM. To see how this works in a bit more detail, let us assume that $\Psi_L$ and $\Psi_R$ are a left-handed $SU(2)_L$ doublet and a right-handed singlet, respectively, endowed with the appropriate $U(1)_Y$ hypercharge quantum numbers to couple to $\Phi$. The Higgs potential is then given by

$$ V_\Phi = m_\Phi^2 \Phi^\dagger \Phi - \lambda_\Phi \Phi \Psi_L^\dagger \Psi_R - \lambda_f \Phi F_L f_R + \cdots, $$

where $F_L$ and $f_R$ are SM weak doublet and singlet fermions, respectively. Upon EWSB through $\langle \Phi \rangle \neq 0$, quite generally a vacuum expectation value $\langle \Phi \rangle \neq 0$ is induced for $\Phi$, given by

$$ \langle \Phi \rangle = \lambda_\Phi \frac{\langle \Psi_L^\dagger \Psi_R \rangle}{m_\Phi^2}. $$

Now we have two sources of electroweak symmetry breaking: $\langle \Phi \rangle$ and $\langle \Psi_L^\dagger \Psi_R \rangle = 4\pi f^2_{TC}$, where $f_{TC}$ is the technipion decay constant and $\langle \Phi \rangle^2 + f_{TC}^2 = (246 \text{ GeV})^2$. For reasonable values of Yukawa couplings, say $\lambda_i = 2$ and $\lambda_\Phi = 1$, we see that the Higgs doublet responsible for the top mass $m_t \approx 172$ GeV can easily have a mass of a few hundred GeV to a TeV. However, for somewhat heavier Higgs fields $\langle \Phi \rangle \ll m_W$, and one does not need very small Yukawa couplings to obtain the lighter fermions’ masses.

To avoid reintroducing the hierarchy problem through ultraviolet quadratic quantum corrections, this Higgs must be assumed to be composite, or else protected by a symmetry, such as supersymmetry [8]. Here, we mainly assume the former possibility, but the nature of this doublet does not enter our discussion in a crucial way. If the Higgs field in bosonic technicolor models is to be composite, we may expect $m_\Phi \lesssim 1$ TeV, and some small Yukawa couplings become necessary.

Here, we make the simple observation that the extreme smallness of neutrino masses, compared to other mass scales of the SM, motivates one to treat them somewhat differently. That is, if $m_\nu$ is set by compositeness for all Higgs fields, neutrino masses require very suppressed Yukawa couplings $\lambda_\nu$. Instead, we will consider a Higgs doublet $H$ that, like other SM fields, is an elementary degree of freedom and interacts with neutrinos through $O(1)$ Yukawa couplings. This elementary Higgs particle is then subject to large quadratic quantum corrections to its mass and is generally expected to be very heavy. For $m_\nu \sim 0.1$ eV, and assuming $\lambda_\nu \sim 1$, we need $\langle H \rangle \sim 0.1$ eV.

As before, we can have interactions of the form

$$ V_H = m_H^2 H^\dagger H - \lambda_\chi H \bar{X}_L \chi_R - \lambda_\nu H^\dagger \nu_R + \cdots, $$

where $X_L$ and $\chi_R$ are technifermions coupled to $H$, in analogy to $\Psi_L$ and $\Psi_R$ coupled to $\Phi$ in Eq. (1). $L$ is a lepton doublet in the SM, and $\nu_R$ is a singlet right-handed neutrino. We will assume $\lambda_\chi \sim \lambda_\nu \sim 1$. Let us take
\[ \langle X_L X_R \rangle \sim (100 \text{ GeV})^3. \] Equation (2), applied to \( H \), then yields \( m_H \sim 10^8 \text{ GeV} \). We see that the requisite mass for \( H \) is quite large. However, as mentioned before, this is a typical expectation for an elementary Higgs, which is the origin of the hierarchy problem in the SM. Here, assuming a typical loop suppression, we may infer a cutoff scale of order \( \Lambda \sim 10^{10} \text{ GeV} \), relevant for the SM sector. A similar-in-spirit but distinct scenario in which neutrinos acquire small Dirac masses via heavy messenger couplings has previously been explored [9].

From the above discussion, we see that with fermion condensation as a main source of EWSB, an ultra-heavy Higgs doublet can provide a seesaw mechanism for neutrino masses, in a natural way. However, one may worry that neutrino couplings to a TeV scale composite doublet \( \Phi \), required to give masses to other SM fermions, can spoil this picture. Here, we simply assume that such couplings are forbidden by a \( U(1) \) Peccei-Quinn (PQ) symmetry [10] under which \( H, X_L, X_R, \) and \( \nu_R \) are charged. This implies that the strongly interacting sector responsible for EWSB via fermion condensation includes other technifermions, hereafter denoted by \( \Psi_L \) and \( \psi_R \), that are not charged under the PQ symmetry. These fermions can then provide masses to the rest of the SM leptons and quarks through couplings with a composite doublet \( \Phi \), as in Eq. (1). To avoid problems from an unwanted light axion we will assume that the PQ symmetry is not spontaneously broken at high scales. We note that generation of mass for fermions other than neutrinos could be realized in other ways, and the assumption of a weak scale \( \Phi \) is not a necessary ingredient of our scenario; we only demand technifermion condensation and neutrino mass generation via ultra-heavy Higgs interactions.

The induced technifermion mass from Eq. (3) is of the order of the neutrino mass. Such small masses would lead to unacceptable light technipions and a light axion asso-
ciated with the techniquark sector, such as in extended technicolor models [4]. We note that the tree-level SM flavor changing neutral currents that plague extended technicolor models can be avoided, since the new chiral symmetry breaking interactions are only required to act on technifermions (given that here the charged leptons and quarks get their masses form the TeV-scale doublet). Hence, we expect the resulting technifermions and axion to be part of the electroweak-scale technihadron spectrum. For technicolor models with technifermions charged under \( SU(3)_C \) strong interactions, Ref. [12] used current LHC diphoton and ditau Higgs constraints to exclude technifermion masses from 110 GeV to around twice the top quark mass for a variety of models. However, since our proposal does not contain color-charged technifermions, we expect single production via gluon fusion to be suppressed and these bounds to be considerably weaker. Also, LEP bounds on the anomalous couplings of technipions to neutral gauge bosons can be quite constraining for technipions with masses below \( \sim 160 \text{ GeV} \) [13]. These bounds are quite model dependent and, for masses above 160 GeV, almost nonexistent, since the LEP energy reach was saturated.

As mentioned before, the generation of neutrino masses \( m_\nu \) in the picture presented above is analogous to the usual seesaw mechanism, except that the smallness of \( m_\nu \) is due to the ultra-heavy Higgs instead of the ultra-heavy right-handed neutrino in conventional models [14]. We will show that this analogy can be extended to the possibility of leptogenesis, where it is typically assumed that out-of-equilibrium decays of heavy right-handed neutrinos lead to the generation of a \( B-L \) charge [15] that electro-
weak sphaleron processes turn into a baryon asymmetry [16–18]. Here, we will see that a similar mechanism can realize a new kind of Dirac leptogenesis [19,20], where the out-of-equilibrium decays of \( H \) lead to \( \delta(B - L) \neq 0 \).

Given that the couplings \( \lambda_{a\nu} \) in Eq. (3) are generally complex, the decays \( H \to X_L X_R \) and \( H \to \bar{L}_\nu R \) are CP violating and would lead to the generation of an asymmetry in the fermions numbers. In particular, if these decays are out of equilibrium, a \( \delta(B - L) \neq 0 \) asymmetry produced by \( H \) decays will be processed into \( \delta B \neq 0 \) and \( \delta L \neq 0 \), as long as sphalerons are in thermal equilibrium, requiring a reheating temperature \( T_{RH} \gtrsim 100 \text{ GeV} \). Nonetheless, complex couplings are not sufficient for the generation of the asymmetry through CP violation; this also requires interfer-
ence between processes at leading and subleading orders that involve physical phases. This can be arranged if there is another Higgs particle that can contribute the necessary phase. Note that in the absence of a Majorana mass, the one-loop vertex corrections do not contribute to the decay process of interest. Hence, in order to provide a mechanism for leptogenesis, we enlarge the content of the model and assume that there are two elementary and ultra-heavy Higgs doublets, \( H_1 \) and \( H_2 \), leading to a simple generalization of Eq. (3):

\[
V_H = m_a^2 H_d^\dagger H_d - \lambda_{a\nu}^2 H_d \tilde{X}_L \tilde{X}_R^D - \lambda_{a\nu}^2 H_d^* \tilde{X}_L \tilde{X}_R^{lU} - \lambda_{a\nu}^2 H_u^* \bar{L}_\nu R + \cdots, \tag{4}
\]

with \( a = 1, 2 \), \( m_a \) the mass of \( H_a \), and we have explicitly shown the couplings of up- and down-type \( \chi \) techniquarks. We will assume that \( m_2 > m_1 \) and that the initial popula-
tion of particles is dominated by the symmetric produc-
tion of \( H_1 H_1^* \). Hence, the effects of \( H_2 \) are only important through their virtual contributions to \( H_1 \) decays. To prevent the asymmetries from getting washed out, we must ensure that inverse decay processes are decoupled from the thermal bath after \( H_1 \) decays have taken place. This amounts to decoupling processes of the sort \( \tilde{X}_L \tilde{X}_R \to \bar{L}_\nu R \).
At temperatures $T < m_1$, such processes are mediated by a dimension-six operator suppressed by $m_1^2$, and we must demand that their rate be smaller than the Hubble rate $H(T_{RH}) = 1.7g^{1/2}T^2_{RH}/M_P$ at $T = T_{RH}$, where $g_*$ is the number of relativistic degrees of freedom and $M_P = 1.2 \times 10^{19}$ GeV. This implies that $T_{RH}$ should satisfy

$$T_{RH} \leq g_*^{1/6}(m_1/M_P)^{1/3}m_1.$$  \hspace{1cm} (5)

We then see that for $g_* \sim 100$ and $m_1 \sim 10^8$ GeV, we get $T_{RH} \lesssim 5 \times 10^4$ GeV, which is well above the electroweak phase transition temperature, but well below $m_\nu$. Thus, one must assume that some nonthermal process, such as inflation, gives rise to a population of $H_1^0$ particles and a relatively low-reheat initial plasma (such requirements are shared by a variety of other models; see for example Ref. [21]). The details of the nonthermal process are not very crucial, as long as the above general features can be obtained from it.

For a simple estimate, let us assume a modulus field $\rho$ that couples universally with gravitational strength; for example, it couples to the heavy Higgs fields through $(\rho/M_P) \times (\partial^\mu H^\dagger \partial_\mu H)$. The width of $\rho$ is roughly estimated by

$$\Gamma_\rho \sim g_* m_\rho^3/16\pi M_P.$$  \hspace{1cm} (6)

We assume that the Universe was at some early stage in a matter-dominated era due to the oscillations of $\rho$. These oscillations get damped by the decay of $\rho$, leading to a radiation-dominated era at a reheat temperature estimated by

$$T_{RH} \sim (g_*^{1/2}m_\rho^3/M_P)^{1/2}.$$  \hspace{1cm} (7)

Upon the decay of $\rho$ and the subsequent prompt decays of the heavy Higgs fields, the SM and the technisectors come to thermal equilibrium, at $T = T_{RH}$, via their gauge interactions. Requiring $T_{RH} \lesssim 5 \times 10^4$ GeV yields $m_\rho \lesssim 2 \times 10^9$ GeV, which easily allows for $\rho$ to decay into $H_1$ fields of mass $\sim 10^9$ GeV.

We will parametrize the asymmetry generated in the $H_1$ decays by

$$\epsilon \equiv \frac{\Gamma(H_1 \rightarrow \bar{L} L_L) - \Gamma(H_1^\dagger \rightarrow \bar{L} \bar{L}_R)}{2\Gamma(H_1)},$$  \hspace{1cm} (8)

where $\Gamma(H_1)$ is the total width of $H_1$. For $\lambda_L \sim \lambda_\nu$, we expect that $\epsilon \sim 1/(16\pi^2)$, given by the interference between the tree-level and the 1-loop amplitude for $H_1$ decay into the leptons. For example, assuming that diagonal couplings of $H_1$ to lepton flavors are dominant, $\epsilon$ is mostly given by the contribution of diagrams of the type in Fig. 1.

With the technifermions in the fundamental representation of a $SU(N_{TC})$ technicolor gauge group, we find

$$\epsilon \simeq \frac{N_{TC}}{8\pi} \frac{m_1^2}{m_L^2 - m_t^2} \sum_i \text{Im}[\lambda_i^L \tilde{\lambda}_i^L + \lambda_i^L \tilde{\lambda}_i^T - \lambda_i^\dagger \tilde{\lambda}_i^L - \lambda_i^T \tilde{\lambda}_i^\dagger]$$

$$\times \frac{1}{N_{TC}(|\lambda_i^L|^2 + |\tilde{\lambda}_i^L|^2) + \sum_j |\lambda_j|^2}.$$  \hspace{1cm} (9)

As expected, $\epsilon$ is of order $10^{-2}$ for $m_2 = 2m_1$ and order-1 couplings of $H_1$ to leptons and technifermions in Eq. (4).

Let us now estimate the size of baryon asymmetry of the Universe (BAU) in our scenario. After a period of inflation, we assume that the Universe gets reheated to $T_{RH}$, through the decay of the inflaton into $H_1$ and the massless degrees of freedom in the theory. The prompt decays of the $H_1$ population contribute to the reheating. However, since we would like to maintain a low reheat temperature; that is, $T_{RH} \ll m_1$, we must require the ratio

$$r = \frac{m_1 m_1}{g_* T^4_{RH}}$$  \hspace{1cm} (10)

of the energy densities in $H_1$ and radiation to be smaller than unity; here $n_1$ is the $H_1$ number density. We can then estimate the abundance of $H_1$ by

$$Y_1 = (T_{RH}/m_1)r.$$  \hspace{1cm} (11)

As usual, we will give the BAU in terms of the ratio

$$\eta = \frac{n_B}{s},$$  \hspace{1cm} (12)

where $n_B$ is the baryon number density and $s \simeq g_* T^3$ is the entropy density. Cosmological observations have yielded $\eta \approx 9 \times 10^{-11}$ [22]. The asymmetry $\epsilon$ generated in $H_1$ decays will get processed by the various interactions that are in thermal equilibrium in the plasma. In particular, electroweak sphaleron processes will distribute an initial asymmetry in $B-L$ (which does not get violated by any of the thermal interactions assumed here) and provide various other asymmetries. We will outline the derivation of general formulas for such asymmetries that are relevant in our framework in the Appendix. However, let us assume a minimal setup with one generation of $(X_L, X_R)$ and $(\Psi_L, \Psi_R)$ technifermions each. $N_X = N_\psi = 1$, charged under a technicolor group $SU(2)$ (i.e. $N_{TC} = 2$), and only one light Higgs doublet $\Phi$ near the weak scale. One can then show from the results in the Appendix that

$$B = \frac{13}{67}(B-L).$$  \hspace{1cm} (13)

In the above equation, $B-L$ is given by the amount of lepton asymmetry produced in the $H_1$ decays. It is also assumed that at $T_{RH}$ the weak scale Higgs $\Phi$ behaves as an elementary particle (it is not resolved into its constituents). For this minimal setup, we then get an estimate for $\eta$ given by
\[ \eta \sim \frac{13}{67} e Y_1. \]  

Assuming \( r \sim 0.1, \sigma_{RH} \sim 10^4 \text{ GeV}, \ m_1 \sim 10^8 \text{ GeV}, \ m_2 = 2m_1 \) and adopting \( O(1) \) couplings for \( H_1 \), we find \( \eta \sim 10^{-8} \) which is about 2 orders of magnitude larger than the observed value. Hence, our leptogenesis model can easily account for the BAU, say, for somewhat smaller values of couplings or slightly larger values of \( m_2 \).

With the minimal parameters used for Eq. (13) and the results presented in the Appendix, we also find

\[ B_\phi = \frac{13}{201} (B-L), \]  

where \( B_\phi \) refers to the total technibaryon number from a single generation of \((\Psi_L, \psi_R)\) fermions. Let us assume that these fermions form the lightest technibaryon \( S = \Psi u \psi d \) with zero electric charge. If we also assume that all the interactions that would violate \( B_\phi \) are sufficiently suppressed, in analogy with the SM proton decay operators, the associated \( S \)-baryons are cosmologically stable. The above result [Eq. (15)] then suggests that such a particle made of \((\Psi_L, \psi_R)\) could be a good dark matter (DM) candidate.

Since the energy density in DM is about 5 times larger than that in ordinary baryons, Eqs. (13) and (15) imply that with a mass \( m_b \sim 15 \text{ GeV} \), \( S \) could be a good DM candidate. However, most likely, \( m_b \sim 1 \text{ TeV} \), given that we expect \( \langle \Psi_L, \psi_R \rangle \sim (100 \text{ GeV})^3 \). This seems to suggest that a suppression of \( O(10^{-2}) \) in \( B_\phi \) is necessary, so that \( S \) can have the required cosmological energy density. Remarkably, given a reasonable value for \( T_c \sim 200 \text{ GeV} \), the sphalerons will typically lead to a suppression of order \( (m_b/T_c)^{3/2} e^{-m_b/T_c} \sim 10^{-2} \) [23,24]. Hence, we see that our leptogenesis mechanism can, in principle, naturally lead to a good asymmetric DM candidate \( S \) [25]. In any event, the viability of the DM candidate in our scenario depends on the details of its specific implementation, which is outside the main scope of the current work.

It may also be possible that technibaryon number is violated by higher dimensional operators, and technibaryons are unstable. In such a case, the decay of the primordial technibaryons into light SM particles will cause a large increase in the entropy of the early universe. If this decay occurs during or after big bang nucleosynthesis, the increase in entropy will strongly perturb the abundances of the light elements. Hence, the technibaryons must either decay before big bang nucleosynthesis, i.e., \( \tau_{TB} \ll 1 \) second, where \( \tau_{TB} \) is the technibaryon lifetime, or be long-lived on cosmological time scales. Assuming that technibaryon number is violated by a dimension-six operator, the first scenario leads to the condition

\[ \frac{m_b^5}{M^4} \geq 10^{-24} \text{ GeV}. \]  

Hence, for \( m_b \sim 1 \text{ TeV} \), the cutoff for the technibaryon violating process is \( M \sim 10^{10} \text{ GeV} \), close to the cutoff for the SM sector. In the second, “long-lived” scenario, agreement with observation requires \( \tau_{TB} \geq 10^{36} \text{ sec} \). Such a case would lead to the interesting possibility of decaying DM [26]. For \( m_b \sim 1 \text{ TeV} \), Eq. (16) implies the cutoff is then \( M \sim 10^{16} \text{ GeV} \), near the Grand Unified scale.

Although the mechanism for neutrino mass generation is far out of the reach of present experimental searches, the model presented here is still falsifiable and may have some signatures at the LHC. First, this scenario generates Dirac neutrino masses. Hence, if neutrinos are determined to be Majorana; for example, through observation of neutrinoless double \( \beta \)-decay [27], our model will be ruled out.

Since technicolor is the main source of EWSB, we would expect to see TeV-scale technihadrons at the LHC. In the scenario presented here technicolor was paired with a composite Higgs. For this specific realization, Higgs-like scalars may also be accessible at the LHC. As mentioned earlier, for reasonable values of \( \lambda_t \) and \( \lambda_s \), the composite Higgs scalars have masses on the order of several hundred GeV to a TeV. Compared to the SM, the composite Higgs has a suppressed coupling to \( W/Z \); hence, traditional searches [28] for a high-mass Higgs boson may need to be modified. It is also possible for the Higgs-like scalars to have masses near the LEP Higgs mass bound 144 GeV [29]. In that case, using a holographic approach, it has been shown that in a bosonic technicolor model similar to ours, LEP data and EW precision constraints bound the technihadrons to have masses above \( \sim 2 \text{ TeV} \) and technipion decay constant \( f_{TC} \leq 100 \text{ GeV} \) [30]. The Higgs-like scalar may then have couplings to SM particles similar to those of the SM Higgs boson and, hence, may have similar signatures to the SM Higgs at the LHC. However, we stress that this neutrino mass scenario does not rely on the mechanism through which the other fermions gain mass; i.e., it can be paired with any viable technicolor model.

We examined the possibility that dynamical electroweak symmetry breaking, as in technicolor models, could provide Dirac masses for neutrinos via an ultra-heavy Higgs doublet of mass \( \sim 10^2 \text{ GeV} \), with couplings of order unity. The hierarchic mass scale of this doublet suggests it should be considered an elementary degree of freedom, far above the weak scale. Adopting the bosonic technicolor framework for illustrative purposes, we showed that the \( CP \)-violating decays of the ultra-heavy Higgs scalar can provide a novel mechanism for leptogenesis. Typical parameters in our setup can yield the correct cosmological baryon number. This setup, under some conditions, can also lead to a viable asymmetric dark matter density made up of technibaryons. Our model implies the emergence of technihadrons at the TeV scale. In a bosonic technicolor framework, one would also expect the appearance of composite Higgs-like scalars at the weak scale, but with non-SM-like interactions, which could be studied at the LHC. Quite generally, the observation of neutrinoless double \( \beta \)-decay can rule out the scenario introduced here.
We thank David Morrissey for comments. This work is supported in part by U.S. DOE Grant No. DE-AC02-98CH10886.

APPENDIX: BARYON NUMBER CALCULATION

The asymmetry between particle and antiparticle density is proportional to the particle’s chemical potential, \( \mu_i \). Hence, only relationships between chemical potentials need to be calculated. Here we comment on properties peculiar to our scenario. Generic details of the calculation can be found in Ref. [18].

As noted previously, sphaleron processes are expected to contribute to rapid fermion number violation at temperatures \( T > T_c \). These interactions will create \( N_f \) baryons and leptons, and \( N_\phi \) and \( N_\chi \) technibaryons of \( \psi \) and \( \chi \) type, respectively. When interactions are in thermal equilibrium the sum of the chemical potentials of the incoming particles is equal to the sum of the outgoing. Hence, for \( T > T_c \) sphaleron processes imply

\[
0 = \frac{N_{TC}}{2} \left( \mu_{\chi_L} + \mu_{\chi_R} \right) + \frac{N_{TC}N_\phi}{2} \left( \mu_{\phi_L} + \mu_{\phi_R} \right) + N_f \left( 2\mu_{dL} + \mu_{uL} \right) + \sum_i \mu_{v_L}.
\]

(Flavor changing Yukawa interactions equalize the chemical potentials of the \( \psi^U \), \( \psi^D \) and quark generations, and we use one chemical potential for each particle type. At the reheat temperatures we are interested in, the flavor changing interactions of neutrinos, \( \chi_L \) and \( \chi_R \), are out of equilibrium; hence, the generational chemical potentials are kept distinct.

Following the usual arguments for \( B-L \) conservation, we find that \( N_f \chi_L - N_f B_{X_L} \) and \( N_\phi \phi_\nu - \nu_L \) are also conserved, where \( L \) is the charged lepton and \( \nu_L \) number, and \( B_{X_L} \) is the \( X_L \) technibaryon number. We expect \( \chi_R \) and \( \nu_R \) numbers to be separately conserved since the reheat temperature is below the energy at which interactions mixing \( \chi_R \) or \( \nu_R \) with other species are in thermal equilibrium. Finally, we note that if in Eq. (4) \( \lambda_\chi^* = \lambda_\chi^\ast \), then

\[
B-L = \frac{N_f}{N_\chi} B_{X_L} - L = \frac{N_f}{N_\phi} \phi_\nu - L = B_{\chi_R}^D - B_{\chi_R}^U = -L_{\text{init}},
\]

where \( L_{\text{init}} \) is the initial lepton number injected by \( H_i \) decays. Once the algebra is accomplished, one obtains Eqs. (13) and (15).


