

The Resistivity of Liquid Alloys
of Mercury and Tin

by

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RESISTIVITY OF MERCURY-TIN ALLOYS

INTRODUCTION : The way in which a quantity of electricity is passed along a conductor or through a medium appears to differ with the medium. In one class of conductors, metals, carbon, etc., no change in the conductor is found other than a rise in temperature.

In another class, namely electrolytes, the current is carried by the dissociated parts, or ions, of the acid, base, or salt, the positively charged ions migrating in the direction of the positive current, and the negatively charged ions in the opposite direction. That is, there are two oppositely charged streams of particles, traveling in opposite directions. This results in changes in concentration of the electrolyte about the points where the current enters and leaves.

In gases a still different mechanism of conduction appears to exist. A free electron moving in an electric field acquires sufficient velocity to free other electrons on impact with a gas atom. This repeated, soon results in a large number of free electrons, which drifting in the direction of the field, constitute a negative current, while the slower, heavier positive atoms moving in the opposite direction, constitute a positive current. A short time after the field is removed, the gas regains its non conducting properties.

When a metal passes into the liquid state, its conducting properties change considerably. Its resistivity is increased, and its temperature-resistance coefficient is very small in comparison with the coefficient for the solid state. Its conductivity is still very large in comparison with electrolytes, but as no change occurs in the liquid metal with the passage of the current, it is still in the first named class. Once in the liquid state, its previous history can have no influence on its conducting properties, and this along with the fact that exactly similar conditions can be easily reproduced simplifies somewhat the study of the conducting properties of liquid metals.

From the optical data of C.V.Kent, Physical Review, N.S., Vol. XIV, No. 6, December 1919, it would seem that the number of free electrons and the frequency of electron impact both vary linearly with the atomic concentration in the alloys

investigated in which no compounds are formed. The following work is an attempt to apply Drude's free electron theory of conduction to a liquid alloy of mercury and tin, and to compare the results with those in the article cited.

II. DRUDE'S THEORY OF CONDUCTION : * The current carriers are considered to be free electrons of charge e and mass m moving with a mean velocity v in inter atomic spaces. Their drift in the direction of the field E constitutes the current. This drift velocity is very small, however, and is assumed negligible in comparison with v . The average time between impacts of a free electron and an atom is t .

Under a field E the electrons are accelerated with a force Ee and their acceleration between impacts is

$$a = \frac{\text{force}}{\text{mass}} = \frac{Ee}{m}$$

The velocity acquired in time t then is

$$u = at = \frac{Ee}{m} \cdot t, \quad \text{and the average drift velocity is}$$

$$\bar{u} = 1/2 u = \frac{Ee}{2m} \cdot t.$$

This drift velocity is supposed on the average to be lost on each impact.

With N free electrons per cubic centimeter the current density is the total quantity per cm times its mean velocity, or

$$Ne\bar{u} = Ne \frac{Ee}{2m} t = \frac{Ne^2 Et}{2m}$$

If we let R represent the resistivity of the conductor, and i the current density,

$$R = \frac{E}{i} = \frac{E}{\frac{Ne^2 Et}{2m}} = \frac{2m}{Ne^2 t}$$

Replacing t , the average time between impacts, by $\frac{1}{W}$

where W is the number of impacts per second per electron we may write

$$R = \frac{2mW}{Ne^2}, \quad \text{or the conductivity may be similarly}$$

expressed as

$$C = \frac{Ne^2}{2mW}$$

* Richardson, The Electron Theory of Matter, p 409.

III. DRUDE'S THEORY APPLIED TO LIQUID ALLOYS:

A. Alloys in which no compounds are formed. On this basis if we assume a simple mixture of two liquid metals of N_0 and N_1 free electrons per centimeter cube, and corresponding impact frequencies W_0 and W_1 respectively, we may express the number of free electrons in the mixture as

$$N_e = N_0(1 - \theta) + \theta N_1,$$

in which θ is the volumetric or atomic percent of the second metal. The impact frequencies may be expressed similarly as

$$W_e = W_0(1 - \theta) + \theta W_1,$$

and the resistivity of the alloy may then be expressed as

$$R_e = \frac{2mW_e}{e^2 N_e} = \frac{2m}{e^2} \frac{W_0(1 - \theta) + W_1\theta}{N_0(1 - \theta) + N_1\theta} \quad (1)$$

This equation may be expressed in terms of the resistivity of the first metal as follows:

$$\begin{aligned} R_e &= \frac{2m(W_0 - W_0\theta + W_1\theta)}{e^2(N_0 - N_0\theta + N_1\theta)} \\ &= \frac{2m}{e^2} \frac{W_0 - \theta(W_0 - W_1)}{N_0 - \theta(N_0 - N_1)} \\ &\quad 1 + \theta \left(\frac{W_0 - W_1}{W_0} \right) \\ &= \frac{2mW_0}{e^2 N_0} \frac{1 + \theta \left(\frac{W_0 - W_1}{W_0} \right)}{1 + \theta \left(\frac{N_0 - N_1}{N_0} \right)} \end{aligned}$$

But $\frac{2mW_0}{e^2 N_0} = R_0$, and if we write w for $\frac{W_0 - W_1}{W_0}$, and n for $\frac{N_0 - N_1}{N_0}$, we have

$$R_e = R_0 \frac{1 + \theta w}{1 + \theta n}, \quad \text{in which } R_0, w, \text{ and } n, \text{ are (2)}$$

constants.

Moreover this equation may be shown to be a rectangular hyperbola between θ , and R_e . Writing the general form for the equation of a rectangular hyperbola, using θ , and R_e , we have

$$(R_e + A)(\theta + B) = C^2$$

or $R_0\theta + R_0B + A\theta + AB - C^2 = 0$

Dividing by B

$$\frac{1}{B} R_0\theta + R_0 + \frac{A}{B} \theta + A - \frac{C^2}{B} = 0$$

Clearing equation (2) of fractions we have

$$R_0 + n R_0\theta - R_0 - w R_0\theta = 0$$

These two equations are seen to be of the same form and equating the coefficients, we have

$$n = \frac{1}{B}, \quad -wR_0 = \frac{A}{B}, \quad \text{and} \quad -R_0 = A - \frac{C^2}{B}$$

Therefore from the concentration-resistance curve of an alloy we should be able to determine of w and n on finding the origin of the hyperbola and the values of the constants A , B , and C .

A straight line relationship may be had from (2) as follows:

$$(1 + n\theta) R_0 = R_0 (1 + w\theta)$$

$$R_0 + R_0n\theta = R_0 + R_0w\theta$$

$$R_0 - R_0 = R_0n\theta - R_0w\theta$$

$$\frac{R_0 - R_0}{\theta} = R_0n - R_0w \quad (3)$$

This equation represents a linear relationship between $\frac{R_0 - R_0}{\theta}$ as one variable and R_0 as the other. The slope of the line is

$$\frac{d \left(\frac{R_0 - R_0}{\theta} \right)}{d R_0} = n$$

With no change in the constants w and n this relationship should hold.

The intercept of the straight line on the $\frac{R_0 - R_0}{\theta}$ axis

is $-R_0w$. Both w and n can therefore be found and equation (3) should be a test for the presence of a compound

or any change in w or n .

If θ is expressed in percent, $\frac{R_0 - R_e}{\theta}$ will be 100

times smaller. The equation will still hold, however, if w and n are correspondingly the relative change in W_0 and N_0 respectively for a change in concentration of one percent, since $R_0 n - R_0 w$ is likewise 100 times smaller. Similarly, any other unit might be chosen for θ and the equation still hold if w and n are the relative changes for this amount. *

B. COMPOUNDS : When a compound is formed, N_1 and W_1 may now represent the free electrons and frequency of impact of the compound. θ however, the atomic concentration of the original components, could not represent the concentration of the compound, as usually considered. The concentration of the compound expressed in terms of the original components would depend on the number of atoms of each of the components uniting to form the new molecule.

If, however, the second component merely robbed the first, or caused it to give up more free electrons above the number found in the original components, then some abrupt change might be found at some definite concentration, and a new N_2 would appear which would be the free electrons of the second original component.

$$w_1 = \frac{W_0 - W_1}{W_0}, \quad n_1 = \frac{N_0 - N_1}{N_0}$$

$$w_2 = \frac{W_1 - W_2}{W_1}, \quad n_2 = \frac{N_1 - N_2}{N_1}$$

Thus if θ remains the atomic concentration of the original components, anything other than a simple addition of electron densities should show a change in the slope of the line represented by equation (3).

IV. THE PURPOSE OF THIS WORK was to determine experimentally the resistivities of alloys of tin and mercury at various concentrations and temperatures and with the data,

(a) examine equations (2) and (3) for possible changes in w and n , or possible compounds, and

* This graphical method of testing the hypothesis is due C.V.Kent.

(b) determine the effect of change in temperature on w and n , hence on W and N .

V. APPARATUS AND METHODS :

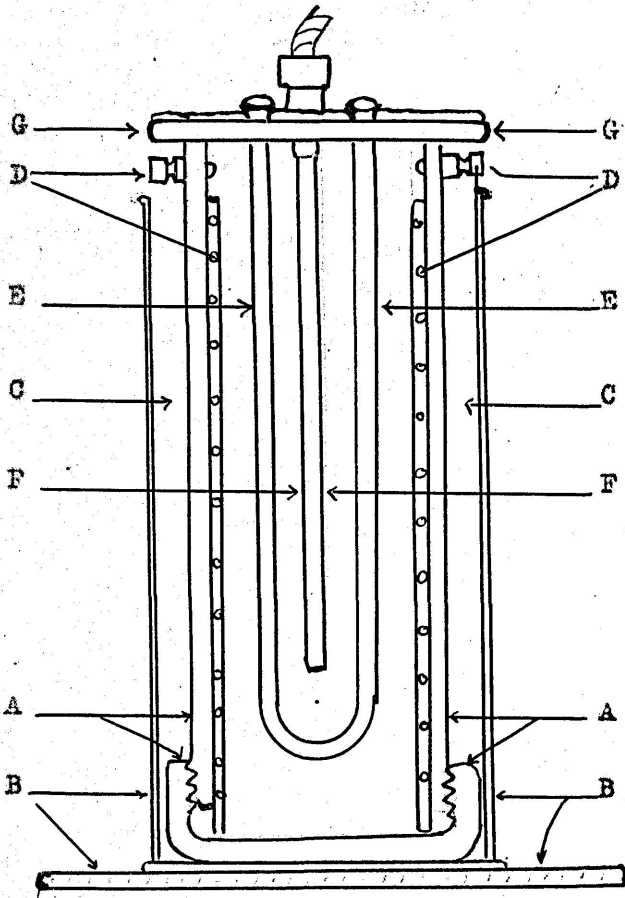


Figure 1

- A. Cast Iron pipe, 5 in. diam. x 14 in. and cap
- B. Outer sheet iron container and asbestos pad.
- C. Asbestos packing.
- D. Binding posts and heating element of furnace.
- E. Pyrex tube for alloy.
- F. Platinum thermometer.
- G. Transit, or asbestos board cover.

The furnace, (figure 1) consisted of a piece of 5 inch cast iron pipe, jacketed with asbestos wool and an outer case of sheet iron, the whole resting on an asbestos pad. A heating element of No.18 nichrome wire was wound on a sheet iron cylinder, insulated with asbestos. It fit snugly within the heavy iron pipe.

The container for the liquid alloy was a pyrex tube of about 70 cm. length, bent into a U shape. The ends were slightly flared for ease in filling, and to support the tube in the furnace. It was 9 mm tubing, (outside) with about 1 mm walls, leaving a 7 mm bore.

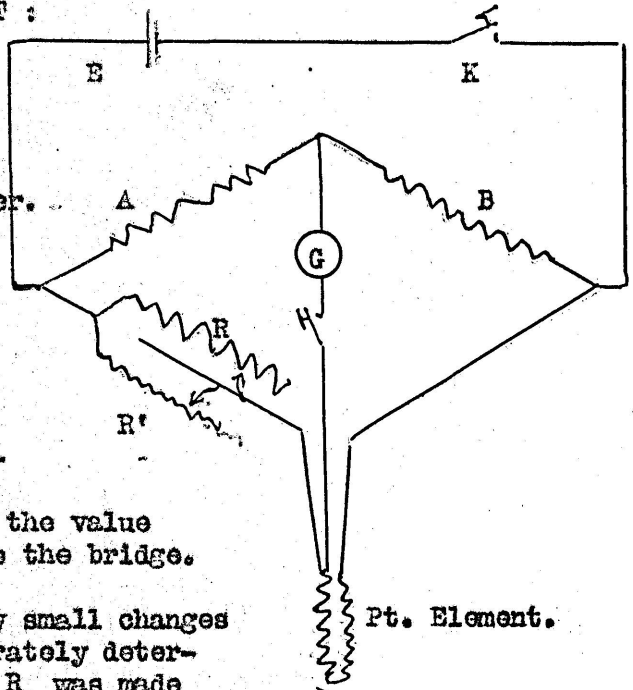
A platinum resistance thermometer was set well down in the center of the furnace.

The top of the heater consisted of two pieces of transite, or asbestos board, separating easily to allow the pyrex tube to be removed

TEMPERATURE MEASUREMENT :

The platinum thermometer element had three leads, one for each arm and one for the galvanometer. Any change then in the thermometer leads was shared equally by the two arms of the bridge. With a one to one ratio in A and B, any change in the resistance of the thermometer coil only was accurately measured by the value of R needed to balance the bridge.

In order that very small changes of resistances be accurately determined, the resistance R was made up of two standard boxes in parallel, one whose resistance was always slightly greater than that of the thermometer, and varied in steps of 0.1 ohm, the other of 200 to 999 ohms, varying by single ohms. The low resistance was an Otto Wolf, Berlin, standard of great accuracy. (No.5149) The other was a three dial Leeds and Northrup box, (No.26016). The setting on



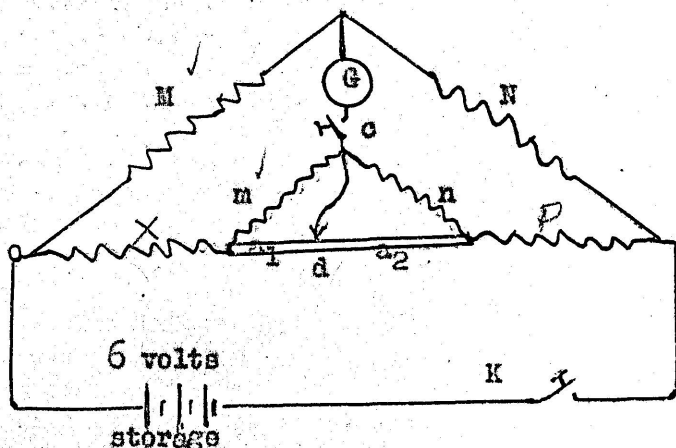
this higher resistance was usually in the order of 250 - 600 ohms. As a change of one ohm was noticeable in balancing, it was possible to determine the resistance of the thermometer to within about 0.001 ohm, when measuring 20 ohms.

$$R_1 - R_2 = \frac{20 \times 501}{20 + 501} = \frac{20 \times 500}{20 + 500} = 0.001 \text{ ohm.}$$

The ratio coils were also Otto Wolf, Berlin resistances, and were used 10 to 10, as the platinum element was about 10 ohms.

The galvanometer was a L. and N. instrument of 525 ohms resistance, and a sensitivity of 2241 mehoohms. (No. 3936)

RESISTANCE OF ALLOY : To determine this low resistance accurately, a Kelvin bridge was used, in which the standard resistance changed was of a high rather than a low value. The diagram and theory of the Kelvin bridge follows :



The arms of the bridge are M , N , $X + \frac{ma_1}{mta_1}$, and $P + \frac{na_2}{nta_2}$,

With the bridge balanced we may assume some point d on the heavy connector a_1a_2 to be at the same potential as c , and that

$$M : N :: X + \frac{ma_1}{mta_1} : P + \frac{na_2}{nta_2} .$$

As c and d are at the same potential,

$$\frac{m}{a_1} = \frac{n}{a_2} , \text{ or } \frac{m}{mta_1} = \frac{n}{nta_2}$$

Substituting $\frac{n}{n+a_2}$ for its equal, $\frac{m}{m+a_1}$ we have

$$\begin{aligned} \frac{M}{N} &= \frac{X + \left(\frac{m}{m+a_1}\right) a_1}{P + \left(\frac{m}{m+a_1}\right) a_2} \\ X &= \frac{M}{N} \left(P + \frac{m}{m+a_1} a_2\right) - \frac{m}{m+a_1} a_1 \\ &= \frac{M}{N} P + \left(\frac{M}{N} \left(\frac{m}{m+a_1}\right) a_2 - \frac{m a_1}{m+a_1}\right) \\ &= \frac{M}{N} P + \frac{m}{m+a_1} \left(\frac{M}{N} a_2 - a_1\right) \\ &= \frac{M_p}{N} + \frac{m a_2}{m+a_1} \left(\frac{M}{N} - \frac{a_1}{a_2}\right) \end{aligned}$$

Substituting $\frac{m}{n}$ for its equal $\frac{a_1}{a_2}$

$$X = \frac{M}{N} P + \frac{m a_2}{m+a_1} \left(\frac{M}{N} - \frac{m}{n}\right)$$

If therefore the ratio of M/N and m/n is the same, the second term vanished and

$$X = \frac{M}{N} P .$$

Keeping the values of M , N , m , n , relatively large, the resistance of their external connections introduce no appreciable error, while the current leads to the low resistances, P and X , lie entirely outside the points between which resistances are to be measured.

The bridge connections to the alloy were of considerable importance. In order to avoid possible contamination of the alloy by the leads, the current was lead into the mercury through large iron wires which did not amalgamate with the mercury. Platinum wires were found necessary for the potential leads to secure consistent results. Small platinum wire tips were welded to the ends of the copper leads which terminated within a small capillary glass tube. (See figure) The potential connection was therefore at the

tip of the capillary tube P. This tube was always fixed carefully in the same position, not varying more than 0.2 mm from the same position.

Different materials used for the tip of the copper wire gave surprisingly different results. Iron wire was tried, but while the resistances M and m were quite large, 1000 ohms and more, the results were very inconsistent. Copper tips gave good results, as did platinum, but the latter was chosen to avoid any possible impurity which might come from the copper. Contact resistance may have been the difficulty with the iron.

No trouble was experienced with thermo electric forces. A variation of one or two millimeters in the zero point of the galvanometer was observed, but was not considered of sufficient importance to warrant the insertion of a counter e.m.f.

The standard resistance bar of the bridge was somewhat worn, so a positive contact was soldered to it in place of the usual slider, and the two resistances M and m were varied instead. This was done by inserting two supplementary resistances in the potential leads to the alloy. Keeping the supplementary resistances equal, and the low standard constant, permitted a variation in small and positive increments and made possible a much greater accuracy than could have been obtained, had the sliding contact on the low resistance bar been used, first on account of the difficulty in reading the scale divisions sufficiently close, and secondly on account of worn places on the bar.

The two additional standards were 4 dial Leeds and Northrup boxes, checked against each other and the Otto Wolf standards and the resistances in the bridge itself.

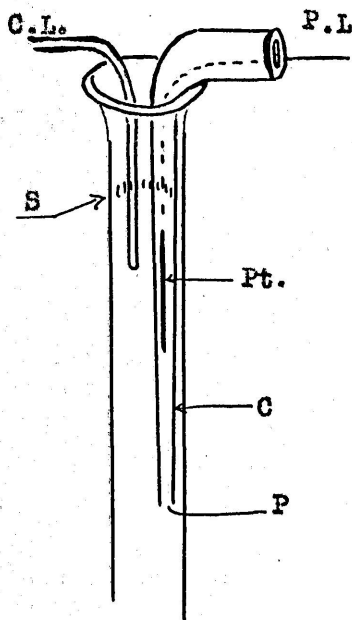


Figure 4

- C.L. Current lead.
- P.L. Potential lead.
- Pt. Platinum tip.
- P. Capillary tube.
- P. Point of potential contact.
- S. Surface of alloy.

The portion of the bar used was taken as 0.01021 ohms and the resistance of the alloy read

$$R_0 = 0.01021 \frac{M}{N} \text{ ohms.}$$

PREPARATION OF THE ALLOY : The mercury was freshly distilled. The tin, secured from the chemistry stock room, was highly pure, a grade used in analytical work. The quantities desired for a given concentration were carefully weighed and placed in a pyrex beaker with a small piece of paraffine wax. The wax successfully prevented oxidation of the alloy after melting.

The U shaped tube was placed in the furnace and heated to a point somewhere near the temperature of the prepared alloy.

When the tin-mercury alloy was thoroughly mixed by stirring, the paraffine wax was largely removed by means of porous paper (toweling), and the liquid alloy poured through a paper funnel into the U tube. In this way the tube was easily filled with clean alloy, and at no time did the wax appear to get in the tube or give any trouble. Its use assured a definite concentration of tin, without loss by oxidation, which is otherwise quite troublesome at high temperatures.

EQUILIBRIUM TEMPERATURES AND READINGS : With the current and potential leads in place, the rheostat in series with the heater was set to allow a current to flow which would produce a suitable maximum, somewhat under the boiling point of the alloy. About 3 or 4 hours were required to get a reasonably steady state of temperature so that we could safely assume the alloy and thermometer to be within a few tenths of a degree of the same temperature. As the thermal capacity of the thermometer was much the smaller, and consequently always ahead of the alloy in following the temperature of the heater, its resistance was read first. Both readings were usually made within the same minute, as an approximate adjustment was always made previous to the final balance and reading.

CALIBRATION OF THE PLATINUM THERMOMETER : The resistance of the platinum was determined for three temperatures, ice, steam,

and the melting point of tin. A large quantity of finely cracked ice was used, and a very steady reading obtained. For the steam point, a steam jacketed steam chamber at the barometric pressure was used, and the temperature taken from Kaye and Laby's Chemical and Physical Constants.

A cooling curve through the freezing point of tin was obtained, and the middle of the horizontal resistance-time curve used as the resistance of the thermometer at 232° C.

These three points were plotted on a large graph of 5 inches per 100° and 1 inch per ohm, and a smooth curve drawn through them, extending to 400° C.

The tin point checked within two degrees with the formula,* (Callendar's difference formula) for changing platinum thermometer readings to the gas scale, namely

$$t - pt = d \left(\frac{t}{100} - 1 \right) \frac{t}{100}$$

in which t is the gas thermometer temperature, d a constant, approximately 1.5 for pure platinum, pt , the platinum temperature from

$$pt = 100 \frac{R_t - R_0}{R_1 - R_0}$$

in which R_0 = ice resistance, R_1 = steam resistance, and R_t = any other resistance.

* G.W.C. Kaye and T.H. Laby, Physical and Chemical Constants, fourth edition, page 46.

DATA :

Composition % Wt. % At.	Platinum Res. r x r'	ohms	Temp.	Kelvin K/N	Bridge Alloy R.
0 0	20x600	19.35	330	1.747	.01885
	18x282	16.90	241	1.598	.01724
	14.5x659	14.20	152	1.457	.01570
	10.6x521	10.40	32.5	1.298	.01400
0.1 0.168	20.5x483	19.65	337	1.740	.01880
	18x660	17.52	261	1.608	.01735
	15x471	14.53	162.5	1.453	.01570
1.502 2.5	20.5x510	19.73	339.5	1.453	.01570
	19.5x385	18.58	298	1.402	.01513

Composition		Platinum Res.		Temp.	Kelvin	Bridge
% Wt.	% At.	r x r'	ohms		H/N	Alloy R.
1.502	2.5	17.2x620	16.75	235	1.331	.01438
		14.2x494	13.80	139	1.227	.01324
2.57	4.25	19.2x670	18.67	301	1.298	.01400
		18.5x890	18.12	282.5	1.280	.01380
		16.5x840	16.20	216.5	1.217	.01312
		13.5x540	13.18	119	1.133	.01223
5.135	8.35	21x728	20.40	363	1.267	.01293
		18x568	17.45	259.5	1.183	.01210
		15x530	14.59	164.5	1.111	.01138
		13x480	12.66	102	1.079	.01102
9.9	15.6	21x569	20.27	359	1.111	.01135
		19x309	17.90	275	1.061	.01085
		16.2x537	15.73	201.3	1.018	.01040
		15x316	14.32	155.5	0.998	.01019
20	29.6	20x475	19.2	320	.922	.00995
		18.5x600	17.95	276	.904	.00975
		16x580	15.58	195.5	.869	.00937
37.3	50.0	21x730	20.40	363	.856	.00924
		19x590	18.40	292	.828	.00893
		16x320	15.70	200	.793	.00855
38.89	52.8	21.5x850	20.95	383	.860	.00927
		19x489	18.30	289	.812	.00887
		16.5x310	15.67	199	.778	.00838
		15x254	14.18	151	.759	.00819
59.83	71.5	22x470	21.04	386	.7825	.00884
		20.2x941	19.76	340.5	.768	.00828
		19x122	18.18	284.5	.749	.00807
		17x460	16.40	223.5	.729	.00786
		16x430	15.43	191	.718	.00775
79.2	86.5	21x960	19.56	334	.7315	.00788
		19.5x520	18.80	305.5	.711	.007665
		17x900	16.68	233	.687	.00741
100	100	25x780	24.23	510	.742	.00801
		21.2x608	20.50	367	.693	.00748
		17.5x756	17.10	247.5	.6115	.00660

Composition Sn. in Hg. % atomic	R ₄₀₀ x 10 ⁻⁵	$\frac{R_0 - R_\infty}{\theta}$ x 10 ⁻⁴	R ₃₀₀ x 10 ⁻⁵	$\frac{R_0 - R_\infty}{\theta}$ x 10 ⁻⁴	R ₂₀₀ x 10 ⁻⁵	$\frac{R_0 - R_\infty}{\theta}$ x 10 ⁻⁴
0.0	2005	15	1832		1655	
1.66	1740	1595	1597	1415	1458	1186
2.5	1635	1480	1515	1270	1397	1031
4.25	1513	1157	1400	1016	1295	847
8.35	1325	814	1243	705	1163	589
15.6	1160	541	1100	469	1038	395.5
29.6	1035	327.5	987	285	938	242
50.0	942	212.0	895	187.4	848	161.4
71.5	850	161.6	815	142.3	778	122.5
86.5	802	139.1	765	123.5	730*	106.9
100.0	761	124.4	725	110.7	690*	96.5

* Extrapolated.

Hg. in Sn.	x 10 ⁻⁵	x 10 ⁻⁵	x 10 ⁻⁵
0.0	761	725	690*
13.5	802	765	730*
28.5	850	815	778
50.0	942	895	848
70.4	1035	987	938
84.4	1100	1100	1038
91.65	1325	1243	1163
95.75	1513	1400	1295
97.5	1635	1515	1397
98.34	1740	1597	1458
100.	2005	1832	1655

CONDUCTIVITY OF ALLOY 300° C.

Referred to pure mercury
% Sn. Atomic Conductivity

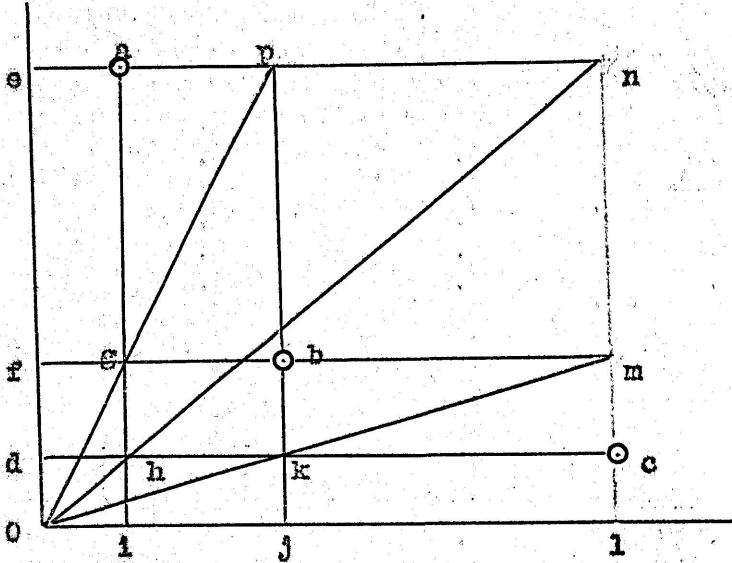
0.0	1.000
1.661	1.148
2.5	1.21
4.25	1.308
8.35	1.475
15.6	1.605
29.6	1.858
50.0	2.048
71.5	2.250
86.5	2.390
100.0	2.530

Referred to pure tin
% Hg. Atom. Conductivity

0.0	1.000
13.5	.9475
28.5	.890
50.0	.810
70.4	.735
84.4	.658
91.65	.583
95.75	.5175
97.5	.478
98.34	.453
100.0	.396

GRAPHICAL RESULTS :

A. The Concentration - Conductance Curve.



Three points lie on a rectangular hyperbola if the diagonals of the rectangles formed on the points, sides parallel, intersect in a common point.

Let a , b , and c be the points. Construct the rectangles $apbg$, $amch$, and $bnck$. Draw the diagonals pg and mk , and let them intersect at O . Extend ap to e and n , gb to f and m , and ck to hd , etc.

The rectangle $oeal$ equals rectangle $ofbj$. For

$$\begin{aligned} \text{tri. } Oep &= \text{tri. } Opj \\ \text{gap} &= \text{gpb} \\ \text{Ofg} &= \text{Ogi} \end{aligned}$$

The diagonal of a rectangle divides it into two equal similar triangles.

$$\begin{aligned} \text{Rect. } Oeal &= \text{tri. } Oep - \text{tri. } gap + \text{tri. } Ogi \\ \text{Ofbj} &= \text{tri. } Opj - \text{tri. } gpb + \text{tri. } Ogi \end{aligned}$$

But the triangles being equal as indicated above, the rectangles are also equal.

A similar proof holds for rectangles $Ofbj$ and $Odcl$. As the area of each rectangle is the product of the x and y values of the point it contains, $xy = \text{constant}$ for each point. Moreover, the diagonal of a third rectangle, $anch$, may be shown to intersect at the same point O .

The rectangle $Oenl$ is bisected by On . h is the intersection of rect. $Oeah$ and rect. $Odcl$ and lies on the diagonal On , for subtracting already proven equals $OeahO$ from the tri. Oen , and $OhclO$ from tri. Onl leaves equal remainders. This being the case, the line hn must bisect rect. $hanc$ and be its diagonal.

This test was applied to a portion of the concentration-conductivity curve, plate III. A few points between 5 and 29.6% were used to locate the common point of intersection of the diagonals. From this point, other points on a constructed hyperbola were obtained, and placed on the graph. In every case the points by construction lay on, or very close to the curve drawn from the actual data, falling slightly inside at the lower concentrations, probably within the limits of error for the low concentrations.

From 0 - 29.6% Sn., then, the concentration-conductivity curve satisfies equation (2) in the conductivity form

$$C_e = C_0 \frac{1 + n\theta}{T F w \theta} \quad (2')$$

From 29.6 to 100% Sn., the curve is practically a straight line. Plate II. While it is conceivable that both n and w should both vary to produce this effect, it is much simpler to assume $w = 0$ for this part of the line, i.e. equation (2') becomes a straight line when $w = 0$. At 29.6 Sn., then, the impact frequency no longer changes with the concentration, and W_1 , the impact frequency of the 29.6 alloy is the same as for pure tin.

Plate IV. A similar curve plotted from the tin end ~~end~~, shows the same general curvature, w constant, or nearly so at concentrations of mercury up to 70.4%, then increasing rapidly to 100% Hg.

Plate 5. The resistance-% by weight curve was constructed to compare with similar curves on page 399 of Kraus' Properties of Electrical Conducting Systems, and Graetz' Handbuch der Elektrizität und des Magnetismus, page 652. These latter, were for 200° C however, while the graph included is for 300°

The straight line relationship of equation (3) expressed in curves I and II, plate VI also indicate a definite change in the values of the constants w and n . Curve I departs from the straight portion at, or near, 29.6 % Sn., and the same point, 70.4 % Hg., reckoned from the tin end, lies on the short straight portion 0 - 70 %. This point is then common to both lines.

We can examine changes in w and n very readily from these curves. The intercept of the portion of the curve in which w and n are constant is $-R_0 w$. From curve I the intercept is estimated to be

$$-1650 + 310 = -1340$$

$$R_0 = .01832$$

$$\therefore w = \frac{-1340 \times 10^{-4}}{-.01832} = \underline{7.31} \text{ or } .0731 \text{ for } 1\%$$

For 29.6 % $29.6 \times .0731 = 2.16$, or $w_{29.6} = 3.16 w_0$

Beyond this point, reckoned from the pure tin end, curve II, w is found to be nearly zero. The intercept is at -40×10^{-5} , $R_0 = .00725$, and

$$w = \frac{-40 \times 10^{-5}}{-.00725} = .0552 \text{ or } .000552 \text{ for } 1\%$$

For 70.4 % Hg., $70.4 \times .000552 = .0388$ total fractional change in w .

This value is positive reckoned from the tin end, is the impact frequency is increasing with addition of Hg. From 29.6 to 100 % Sn., however, it is decreasing instead, but very slightly. In either case it is nearly zero.

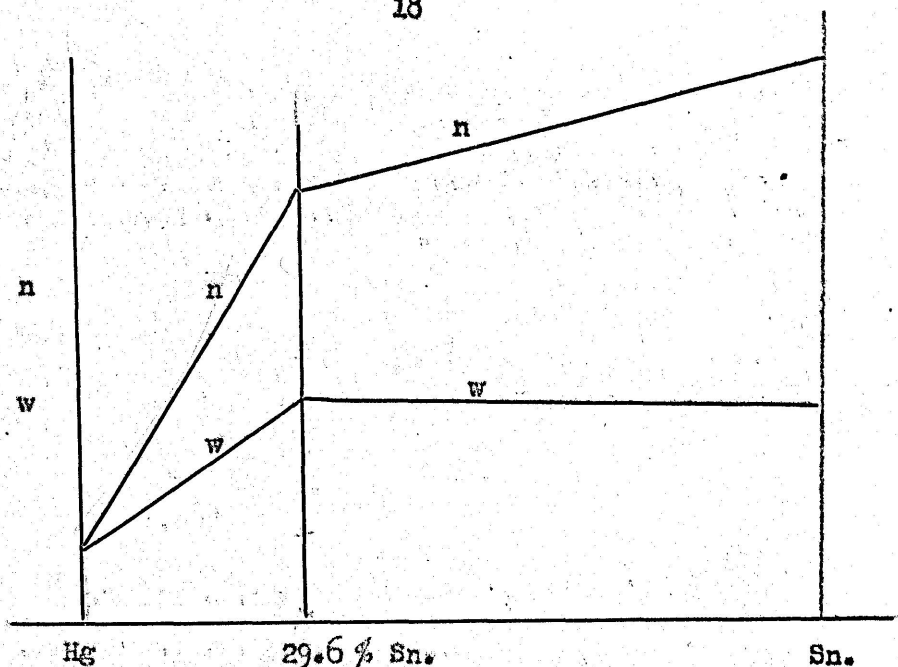
This agrees very well with the hyperbola which indicated a zero value for w , the fractional change in impact frequency above 29.6 % Sn.

The slope, n , representing the change in free electron density is, for the 0 - 29.6 % Sn. portion of the curve (I)

$$\frac{330 \times 10^{-4}}{.002} = 16.5 \text{ or for } 1\% \text{ is } .165$$

$$29.6 \times .165 = 4.83 \text{ or } n_{29.6} = 5.83 n_0$$

For the 29.6 - 100 % Sn., n was taken from curve II.



Using positive increments,

$$n = \frac{69 \times 10^{-5}}{-.002} = -.345 \text{ per } 100 \% \text{ or}$$

$$\frac{70.4}{100} \times -.345 = -.243.$$

Therefore n is decreasing from the tin end .243 of its pure tin value. From 29.6 - 100 Sn it is increasing.

n and w both increase rapidly at first, n the more so, however, causing a rapid rise in the conducting property, or

$$\frac{C_{29.6}}{C_0} = 1.855$$

This checks very closely with the specific-conductance curve.

From 29.6% n continues to rise, but less rapidly. Impact frequency remains constant as w is approximately zero.

Curves drawn through the points for 200° and 400° C have the same slope as the one for 300° C. From this it may be deduced, n is constant, and only w , or impact frequency changes with rise in temperature. As w alone changes with increase in temperature, and the resistance changes likewise, the temperature coefficient for w and

R should be the same. Both are linear.

Plotting the resistance-temperature coefficient against θ , it is seen to be very small and practically constant over that range where w is very small, or zero.

The melting point curve of mercury tin alloys in Landolt, Börnstein Physical Chemistry tables, 3rd edition, 1905, shows a very rapid rise in the small percent of tin, with a dip in the curve at 20% by weight. This is 29.6% atomic. Undoubtedly some abrupt change in w and n must take place at this point with the change in its physical properties.

At the very small concentration of 0.2% Sn. by weight also, a change is indicated. While some data was taken near this point, it was not considered sufficiently accurate to use. The resistance of the pure mercury was used for R_0 and this may have introduced some error in the results. Its effect is perhaps negligible when using the larger values of θ . It is possible this point might be investigated from the tin end.

CONCLUSIONS :

1. The resistance of mercury tin alloys has been experimentally determined for varying concentrations from 200° to 400° C.

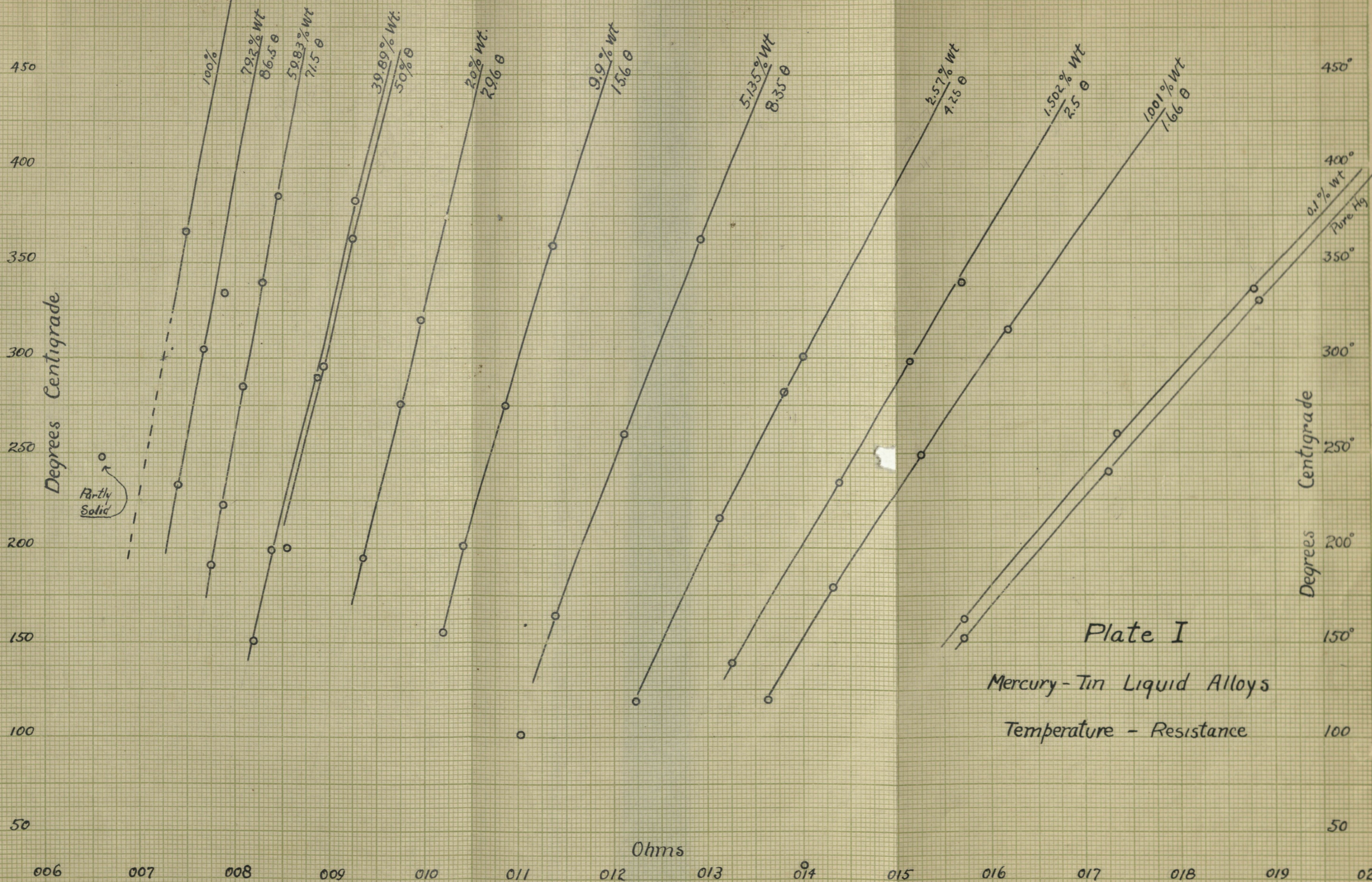
2. The resistance at a given temperature is of the form

$$R_{\theta} = R_0 \frac{1 + w\theta}{1 + n\theta}$$
 where w and n are constants having one set of values between 0.2% and 29.6% tin, and another set of values between 29.6 and 100% tin

3. n is independent of temperature. w varies linearly with the temperature.

4. The data are consistent with the simple electron theory of resistance developed by Drude, with the exception of his assumptions as to temperature.

In concluding this report the writer wishes to express his thanks to Dr. C.V. Kent for his constant interest and assistance in the work, and to Dr. F.E. Kester and staff for assistance in securing the necessary apparatus and supplies.



Mercury - Tin Liquid Alloy Curve

Specific Conductance Referred

to Pure Hg = % Sn. Atomic

$T = 300^{\circ}C$

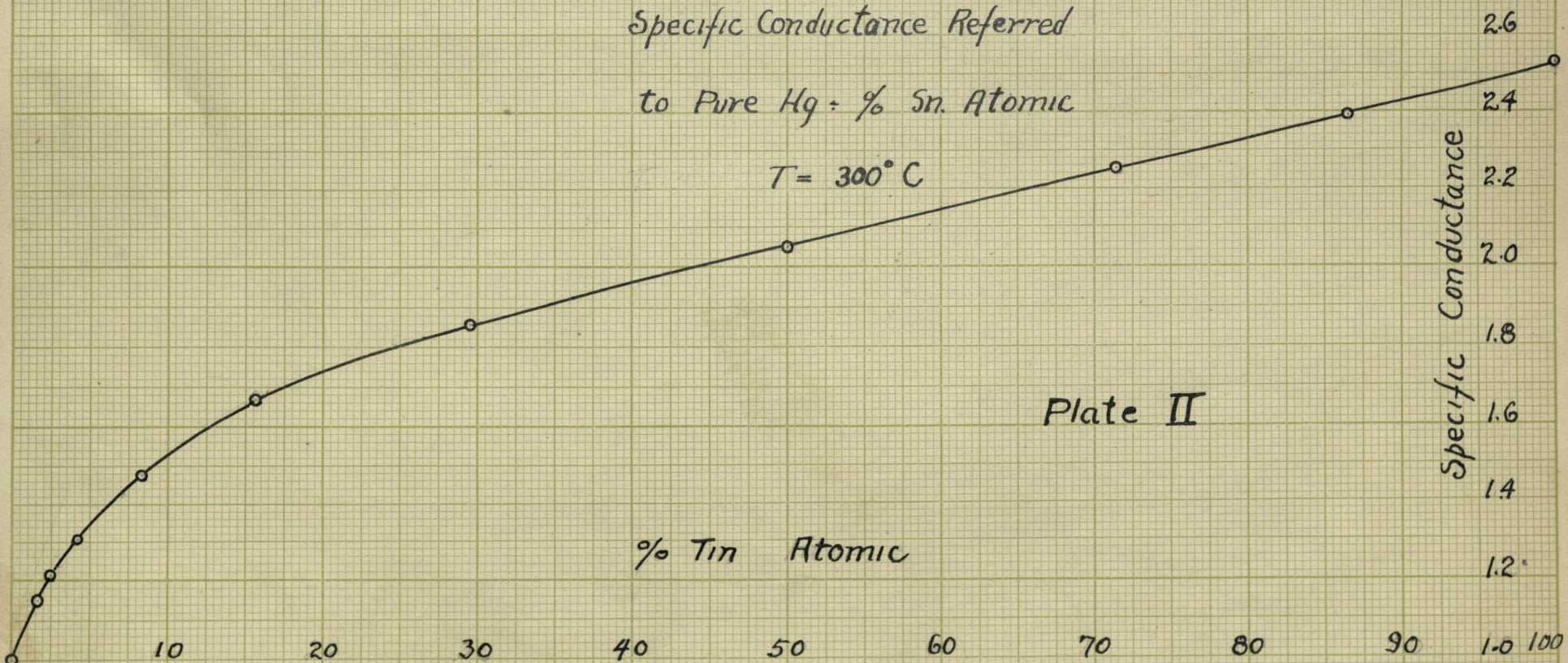


Plate II

0 5 10 15 20 25 30

Atomic % Tin

Mercury-Tin Liquid Alloys

300°C

2.0

1.8

1.6

1.4

1.2

Specific Conductance

Plate III

Hyperbola by Construction

Curve Compared with

Specific Conductance -

Atomic % Curve. $T=300^{\circ}\text{C}$

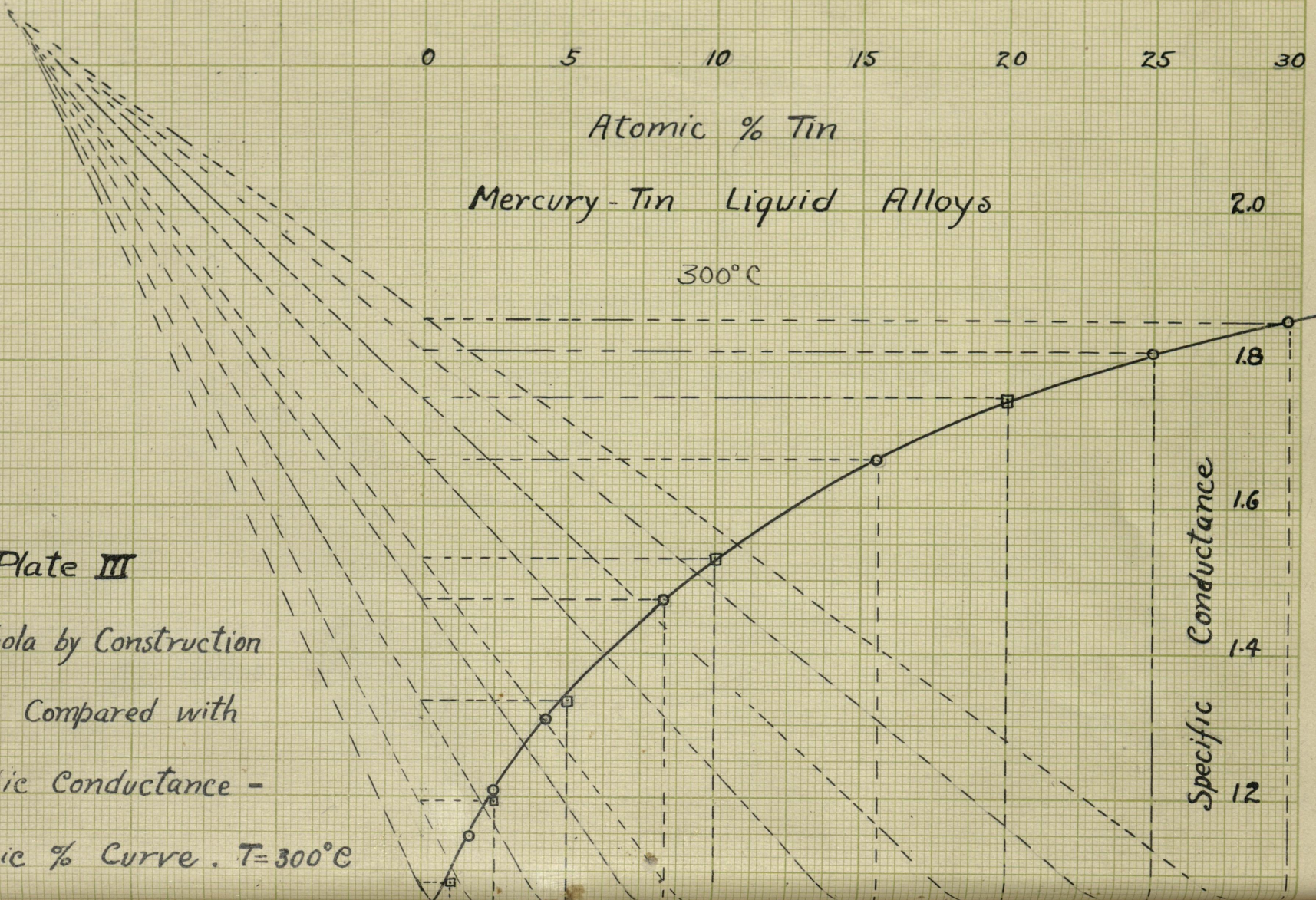


Plate IV

Tin - Mercury Liquid Alloy Curve

Specific Conductance referred

to Pure Tin $T=300^{\circ}\text{C}$

Atomic % Hg. in Sn.

Specific Conductance

4

5

6

7

8

9

10

10

20

30

40

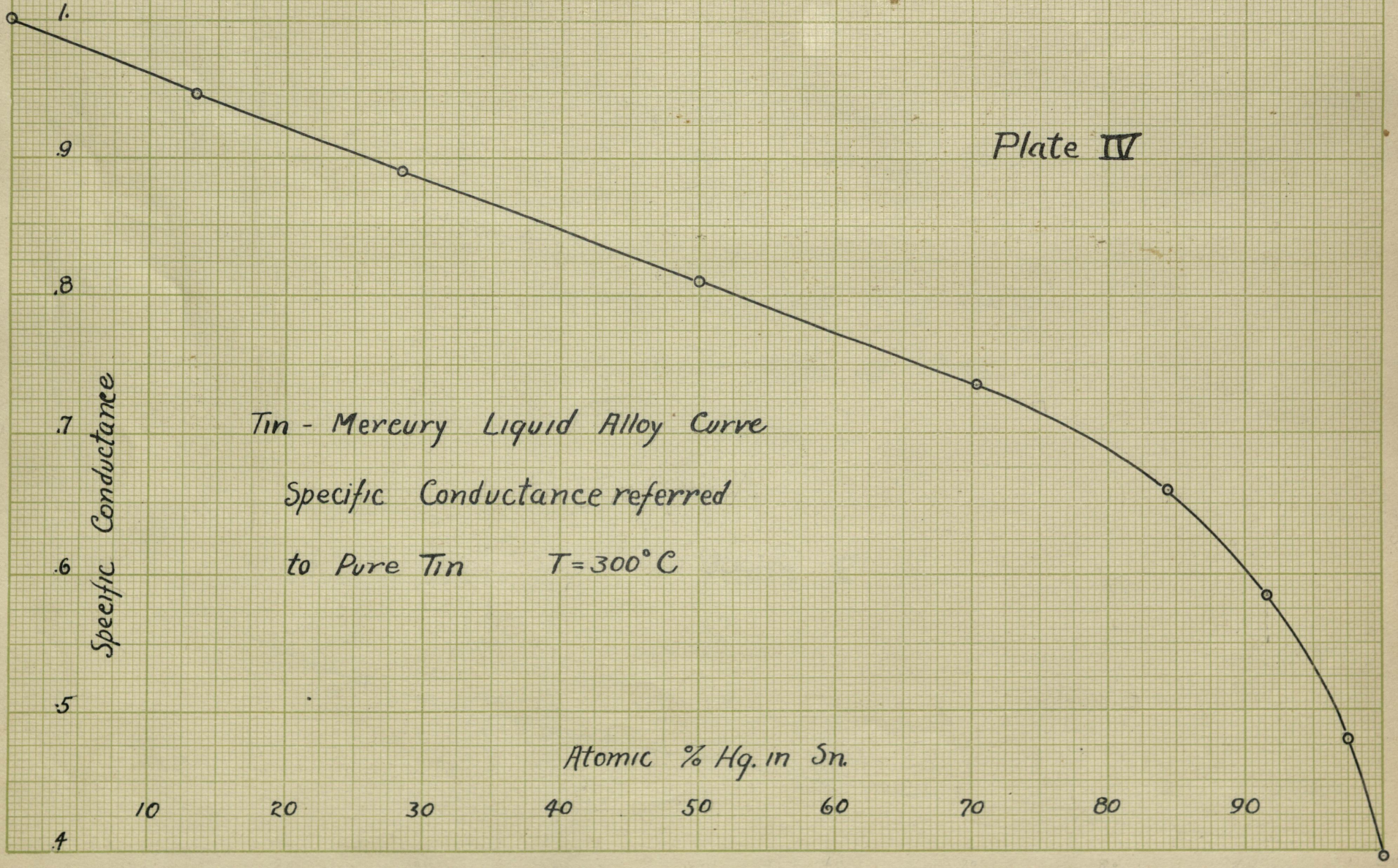
50

60

70

80

90



% Tin by Weight

.00018

10 20 30 40 50 60 70 80 90 100

.00016

Mercury Tin Liquid Alloy Curve

Resistance % Tin by Weight

.00014

T = 300° C

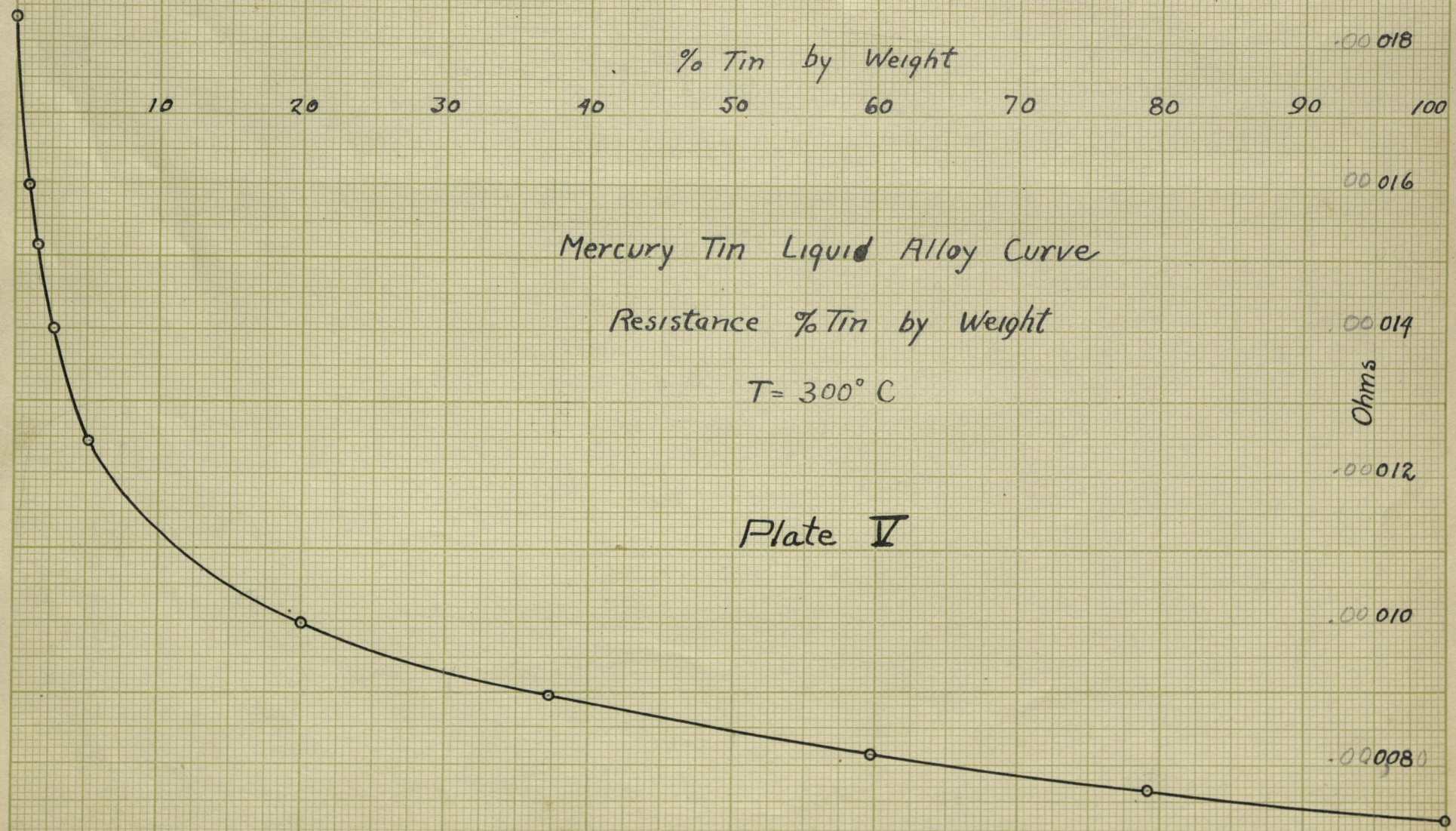
Ohms

.00012

Plate V

.00010

.000080



Temperature-Resistance Coefficient of Mercury-Tin Alloys at 300°C

Plate VII

Composition Atomic %		Coefficient
Hg	Sn	
100	0	9.83×10^{-4}
98.34	1.66	8.9 ..
97.5	2.5	8.0 ..
95.75	4.25	7.57 ..
91.65	8.35	6.35 ..
84.4	15.6	5.45 ..
70.4	29.6	4.76 ..
50.0	50.0	5.58 ..
29.6	71.5	4.3 ..
13.5	86.5	4.84 ..
0	100	5.10 ..

Coefficient $\times 10^{-4}$

9

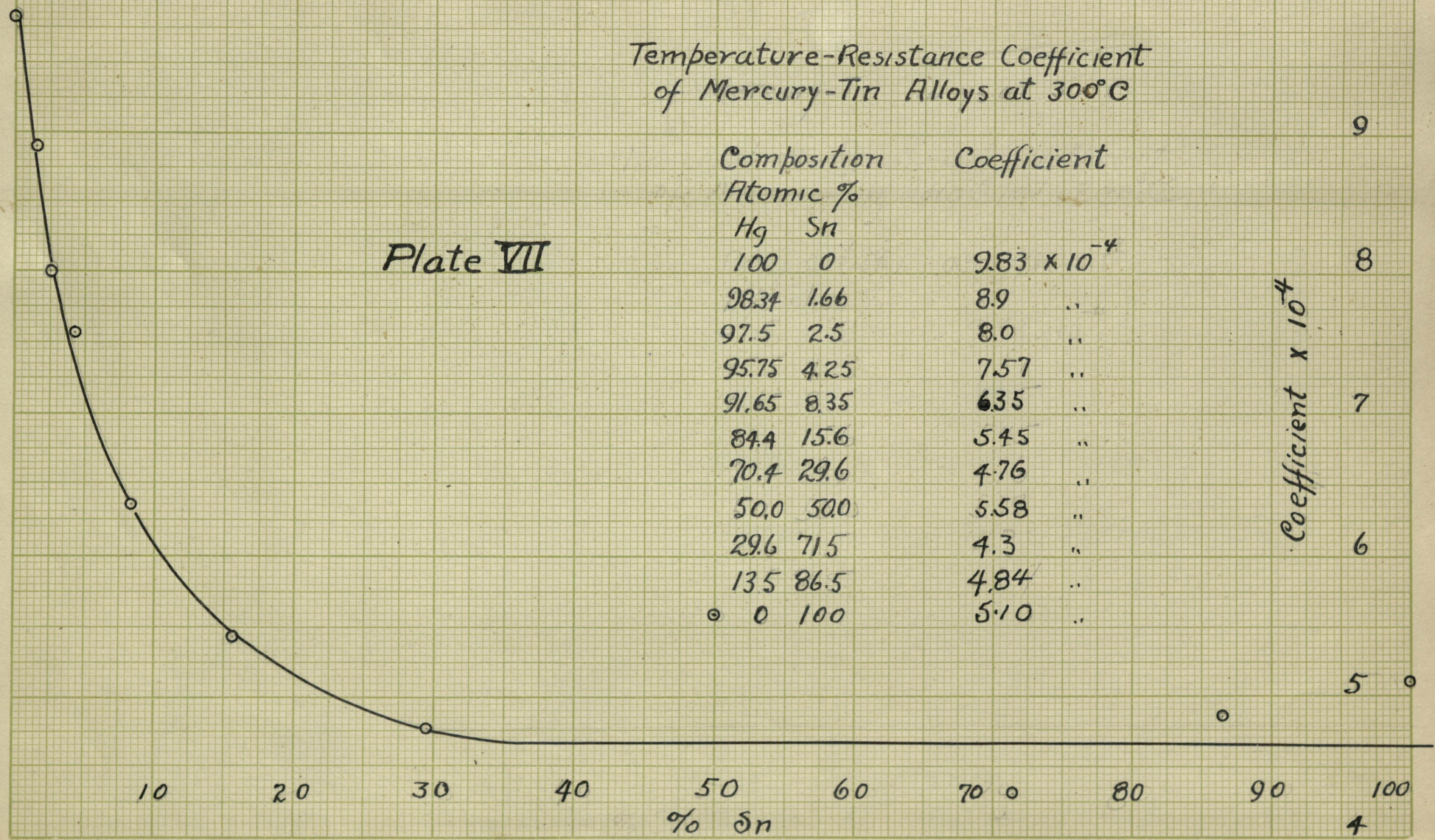
8

7

6

5

4



10

20

30

40

50

60

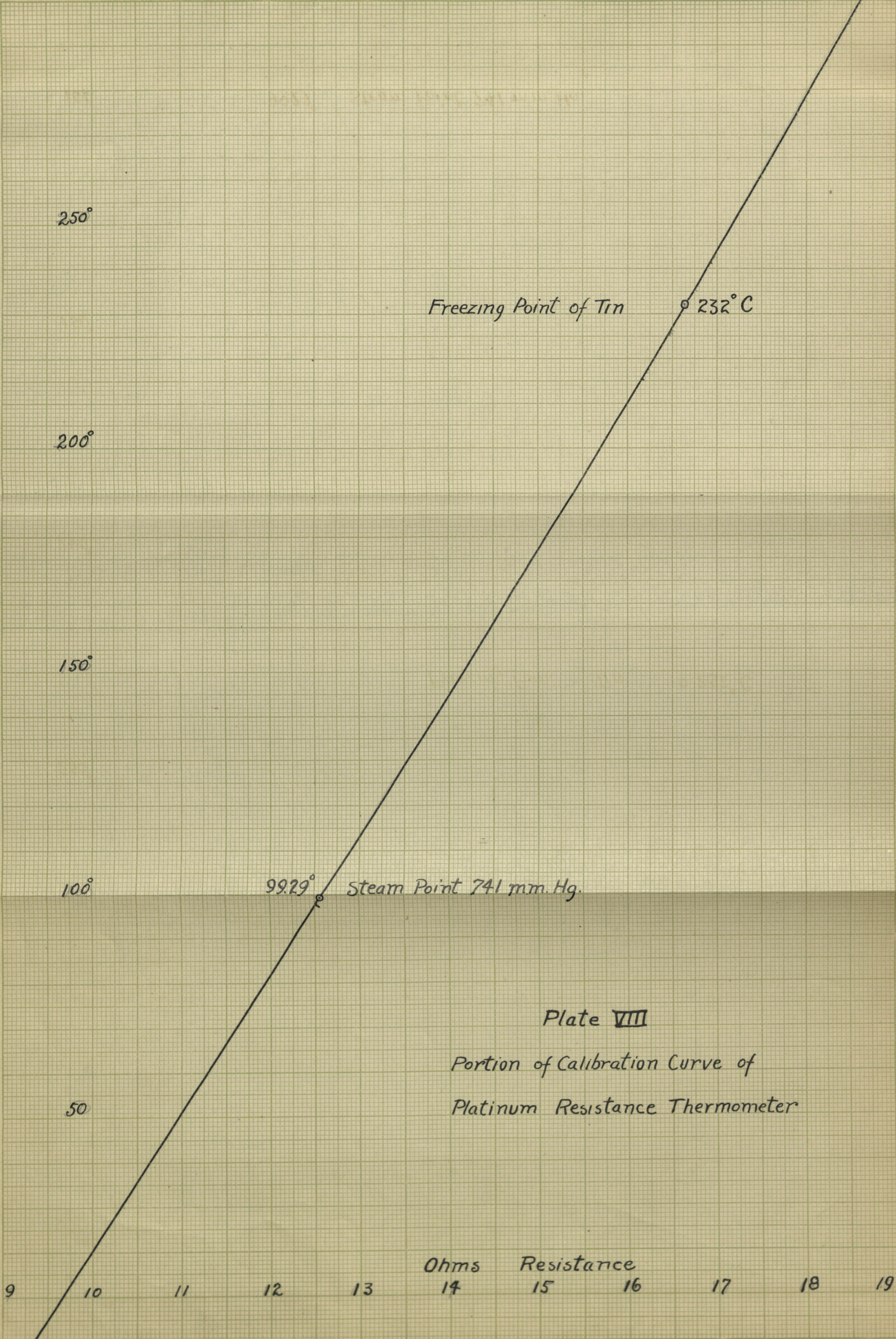
70

80

90

100

% Sn



250°

Freezing Point of Trn 232° C

200°

150°

100°

99.29° Steam Point 741 mm. Hg.

50

Plate VIII

Portion of Calibration Curve of
Platinum Resistance Thermometer

Ohms Resistance

9 10 11 12 13 14 15 16 17 18 19