JOINT DESIGN FOR
REINFORCED CONCRETE BUILDINGS

by

Michael J. Pfeiffer
David Darwin

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Abstract

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This report discusses construction, contraction and expansion joints in reinforced concrete buildings. The report addresses the purpose of each type of joint and emphasizes the selection of joint locations and joint spacings. Some aspects of joint configuration and construction are also covered. Empirical and analytical design techniques are presented.

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INTRODUCTION

Volume changes caused by changes in moisture and temperature should be accounted for in the design of reinforced concrete buildings. The magnitude of the forces developed and the amount of movement caused by these volume changes is directly related to building length. Contraction and expansion joints limit the magnitude of forces and movements and cracking induced by moisture or temperature change by dividing buildings into individual segments. Joints can be planes of weakness to control the location of cracks (contraction joints), or lines of total separation between segments (expansion joints).

There is currently no universally accepted design approach to accommodate building movements caused by temperature or moisture changes. Many designers use "rules of thumb" that set limits on the maximum length between building joints.

Although widely used, rules of thumb have the drawback that they do not account for the many variables which control volume changes in reinforced concrete buildings. For example, variables which affect the amount of thermally induced movement include the percentage of reinforcement, which limits the amount of movement and cracking in the concrete; the restraint provided at the foundation, which limits the movement of the lower stories; the geometry of the structure, which can cause stress concentrations to develop, especially at abrupt changes in plan or elevation; and provisions for insulation, cooling, and heating, which affect the ability of a building to dampen the severity of outside temperature changes.

In addition to these variables, the amount of movement in a building is directly related to the type of aggregate, cement, mix proportions, admixtures, humidity, construction sequence, and curing procedures used. While these variables can be addressed quantitatively, their consideration is usually beyond the scope of a typical design sequence and will not be considered here. A number of these parameters are addressed by Mann (1970).

The purpose of this report is to provide guidance for the placement of contraction and expansion joints in reinforced concrete buildings. A section is included on construction joints. Isolation joints on slabs on grade within the buildings are not covered.

The following section provides a brief overview, outlining the need for joints. The next section is devoted to construction joints, reviewing current procedures for locating and detailing these joints. A section on contraction joints follows, reviewing current recommendations for contraction joint spacing. In the final section, three different approaches to expansion joint placement are presented: Martin and Acosta (1970), Varyani and Radhaji (1978), and the National Academy of Sciences (1974). Design examples illustrate the application of the three methods. For additional information, the reader is directed to an annotated bibliography by Gray and Darwin (1984).

THE NEED FOR JOINTS

Due to the low tensile capacity of concrete, some cracking in reinforced concrete is unavoidable. Contraction joints provide a weakened plane for cracks to form. Through the use of architectural details, these joints can be located so that cracks will occur in less conspicious locations within a building and possibly be eliminated from view.

Expansion joints allow thermally induced movements to occur with a minimum build-up of stress. The greater the spacing between joints, the greater the stresses. Typically, these joints isolate a frame into a series of segments with enough joint width to allow the building to expand with increasing temperature. By isolating the segments, expansion joints also provide relief from cracking due to contraction, and therefore act in a dual role.

Crack control in reinforced concrete buildings is needed for two reasons. The obvious reason is aesthetics. Where cast-in-place concrete is to be the finished product, cracks are unsightly. Cracks in major framing elements such as girders and columns tend to promote questions concerning the structural adequacy of the structure. They may, in fact, pose no structural problems, but to the average person without structural knowledge, they can be cause for alarm. Secondly, cracks of substantial width invite air and moisture into the framework of the structure, possibly having deleterious effects. Two examples illustrate the magnitude of potential cracks.

Lewerenz (1907) cites a plain concrete retaining wall located at the U.S. Navy Yard in Pudget Sound, Washington. This wall was built with expansion joints spaced every 70 ft (21.4 m). After being subjected to four complete cycles of summer-winter temperature changes [8 to 95 °F (-13 to 35 °C)], the joints had opened as much as 3/16 in. (4.8 mm).

Hunter (1953) describes a four story bakery, 200 ft (61 m) long by 50 ft (15 m) tall, built in 1937. An expansion joint placed at mid-length opened as much as 3/4 in. (19 mm) at the roof level. The width of the joint tapered to zero at the basement level. The magnitude of this movement is directly attributed to thermal strains caused by the heat generated by the ovens, coupled with outside temperature effects.

As demonstrated by these two examples, the need for crack control in reinforced concrete structures is real. The key questions are: How to control the amount of cracking (through the use of contraction joints), and how to limit stresses in members to an acceptable level (through the use of expansion joints)? In the sections that follow, recommendations are presented for contraction joint spacing, and specific procedures are presented for the placement of expansion joints.

Once joint locations are selected, the joint must be constructed so that it will act as intended. The weakened section at a contraction joint may be formed or sawed, either with no reinforcement or a portion of the total reinforcement passing through the joint. The expansion or isolation joint is a discontinuity in both reinforcement and concrete. Therefore, an expansion joint is effective for both shrinkage and temperature variations. Both joints can be used as construction joints, as described in the following section.

CONSTRUCTION JOINTS

Except for very small structures, it is impractical to place concrete in a continuous operation. Construction joints are needed in order to accommodate the construction sequence for placing the concrete. The amount of concrete that can be placed at one time is governed by batching and mixing capacity, crew size, and the amount of time allotted. Correctly sited and properly executed construction joints provide limits for successive concrete placements, without adversely affecting the structure.

For monolithic concrete, a good construction joint provides a well-bonded watertight surface, which allows for flexural and shear continuity through the joint. Without this continuity, a weakened region results, which may serve as a contraction or expansion joint. A contraction joint is formed by limiting the percentage of reinforcement through the joint, thus creating a plane of weakness. An expansion joint is formed by leaving a gap in the structure of sufficient width to remain open under extreme temperature conditions. If possible, construction joints should coincide with contraction or expansion joints, which are discussed in the following sections. The balance of this section is devoted to construction joints in regions of monolithic concrete.

Joint Construction

To achieve a well-bonded watertight joint, a few conditions must be met prior to placement of the fresh concrete. The hardened concrete must be clean and free of all laitance (ACI Committee 311 1981).

If only a few hours elapse between successive placements, a visual check is needed to be sure that all loose particles, dirt, and laitance are removed. The new concrete will be adequately bonded to the hardened green concrete, provided that the new concrete is vibrated thoroughly over the area.

Older joints need a little more surface preparation. Cleaning by means of an air-water jet or wire brooming can be done when the concrete is still soft enough that any laitance can be removed, but hard enough to prevent aggregate from loosening. Concrete that has set should be prepared using a wet sand blast or ultra-high pressure water jet (ACI Committee 311 1981).

ACI 318 states that existing concrete should be moistened thoroughly prior to placement of fresh concrete. Green concrete will not require any additional water, but concrete that has dried out may require saturation for a day or more. No pools of water should be left standing on the wetted surface at the time of placement.

Form construction plays an important role in the quality of a joint. It is essential to minimize the leakage of grout from under stop-end boards (Hunter 1953). If the placement is deeper than 6 in., the possibility of leakage is even greater due to the increase in the pressure head of the wet concrete. Grout which escapes under the form will form a thin wedge of material, which must be cut away prior to the next placement. If not removed, this wedge will not adhere to the fresh concrete, and under load, deflection in the element will cause this joint to open.

Joint Location

The final consideration is placing the construction joint in the right place. Assuming an adequate production capacity, construction joints should be located where they will least affect the structural integrity of the element under consideration, while at the same time being compatible with the building's appearance. Placement of joints varies, depending on the type of element under construction. For this reason, beams and slabs will be addressed separately from columns and walls.

Beams and Slabs--From the point of view of strength in beam and slab floor systems, desirable locations for joints placed perpendicular to the main reinforcement are at points of minimum shear or at points of contraflexure. Typically, joints are located at mid-span or in the middle third of the span, but locations should be verified by the engineer before placement is shown on the drawings. In beam and girder construction, where a beam intersects a girder at the point of minimum shear, ACI 318 states that the construction joint in the girder should be offset a distance equal to twice the width of the incident beam.

Horizontal construction joints in beams and girders are usually not recommended. Common practice is to place beams and girders monolithically with the slab. In the case of beam and girder construction where the members are of considerable depth, Hunter (1953) recommends placing concrete in the beam section up to the soffit of the slab, then placing the slab in a separate operation. The reasoning behind this is that cracking of the top surface may result due to vertical shrinkage in a deep member. With this procedure, there is a possibility that the two surfaces will slip due to horizontal shear in the member. In this case, adequate shear transfer must be provided (ACI 318).

Construction joints parallel to the slab span can be placed anywhere, except those locations in T-beam construction that rely on a portion of the slab to act with the beam in resisting flexure.

The main concern in joint placement is to provide adequate shear transfer and flexural continuity through the joint. Flexural continuity is achieved by continuing the reinforcement through the joint with enough length past the joint to insure an adequate splice length for the reinforcement. Shear transfer is provided by shear friction between the old and new concrete, and/or dowel action in the reinforcement through the joint. Shear keys are usually undesirable (Fintel 1974), since keyways are possible locations for spalling of the concrete. If proper concreting procedures are followed, the bond between the old and new concrete, plus the effect of the reinforcement crossing the joint, are adequate to provide the necessary shear transfer.

Columns and Walls--It is general practice to limit concrete placements to a height of one story. Construction joints in columns and bearing walls should be located at the undersides of floor slabs and beams, and at the top of floor slabs for columns continuing to the next floor. Column capitals, haunches, drop panels, and brackets, should be placed monolithically with the slab. Depending on the architecture of the structure, the construction joint may be used as an architectural detail, or located to blend in without being noticeable. Quality form construction is of paramount importance in order to provide the visual detail required (PCA 1982).

The placement of fresh concrete on a horizontal surface can affect the joint. Common practice has been to provide a bedding layer of mortar, of the same proportions as that in the concrete, prior to placement of new concrete above the joint. The ACI Manual of Concrete Inspection (ACI Committee 311 1981) recommends using a bedding layer of concrete with somewhat more cement, sand, and water than the design mix for the structure. Aggregate less than 3/4 in. can be left in the bedding layer, but all aggregate larger than 3/4 in. should be removed. This mix should be placed 4 to 6 in. deep and thoroughly vibrated with the regular mix placed above. To avoid settlement cracks in slabs and beams due to vertical shrinkage of previously placed columns and walls, the concrete in the columns and walls should be allowed to stand for at least two hours prior to placement of subsequent floors.

Placement of vertical construction joints in walls also needs to be compatible with the architectural flavor of the structure. Construction joints are often located near reentrant corners of walls, alongside columns, or other locations where they become an architectural feature of the structure. If the building architecture does not dictate where the joints should be placed, placement considerations, such as production capacity of the crew or whether or not one set of forms will be reused along the length of the pour may limit the length between joints. This criteria will usually limit the maximum horizontal length to 40 ft between joints in most buildings (PCA 1982). Due to the critical nature of building corners, it is best to avoid vertical construction joints at or near a corner, so that the corner will be tied together adequately.

Shear transfer and bending at joints in walls and columns should be addressed in much the same way it is for beams and slabs. The reinforcement should continue through the joint, with adequate length to insure a complete splice. If the lateral shears are high, the joint must be capable of transfering the load by shear friction or dowel action.

<u>Summary</u>

Construction joints are necessary in most reinforced concrete construction. Due to their critical nature, they should be located by the designer, and indicated on the design drawings to insure adequate force transfer and aesthetic acceptability at the joint. If concrete placement is stopped involuntarily for a time longer than the initial setting time of the concrete, the joint should be treated as a construction joint, with advance input from the designer as to any additional requirements needed to insure the structural integrity of the element being placed.

CONTRACTION JOINTS

Drying shrinkage and decreases in temperature cause tensile stresses in concrete, if the material is restrained. Cracks will occur when the tensile stress reaches the tensile strength of the concrete. Due to the relatively low tensile capacity of concrete ($f_{tc} = 4.0 - 7.5 \sqrt{f'_c}$ for normal weight concrete, f'_c and f_{tc} in psi), cracking is likely to occur. Contraction joints provide planes of weakness for cracks to form, without marring the appearance of a structure. Contraction joints are used primarily in walls and in slabs-on-grade.

The greater the distance between contraction joints, the greater will be the forces in a structure due to volume change. To resist these forces and minimize the amount of cracking in the concrete, greater amounts of reinforcement are required.

Joint Configuration

Contraction joints consist of a region with a reduced concrete cross section and reduced reinforcement. The concrete cross section should be reduced by a minimum of 25 percent to insure that the section is weak enough for a crack to form. In terms of reinforcement, there are two types of contraction joints currently in use, known as "full" and "partial" contraction joints (ACI 350R). Full contraction joints, preferred for most building construction, are constructed with a complete discontinuity in reinforcement at the joint. All reinforcement is terminated approximately 2 in. (51 mm) from the joint and a bond breaker placed between successive placements, if the joint is a construction joint. Partial contraction joints are constructed with not more than 50 percent of the reinforcement passing through the joint. Partial contraction joints are used in liquid containment structures. In both types of joint, waterstops may be used to insure watertightness.

Joint Location

Once the decision is made to use contraction joints, the question remains: What spacing is needed to limit the amount of cracking between the joints? As shown in Table 1, a number of recommendations are given for contraction joint spacing. Recommended spacings vary from 15 to 30 ft (4.6 to 9.2 m) and from one to three times the wall height. For sanitary structures, Rice (1984) prescribes contraction joint spacings for given reinforcement percentages (Table 2).

The limits prescribed by Rice in Table 2 are extensions of the limits recommended in ACI 350R, accounting for reinforcement grade and minimum bar size. A graphical representation of the same information is given by Gogate (1984). It should be noted that if a "partial" contraction joint is used, the joint spacing should be approximately 2/3 of the full contraction joint spacing (ACI 350R).

Wood (1981) suggests that any joint within a structure should go through the entire structure in one plane. If the joints are not aligned, movement at a joint may induce cracking in an unjointed portion of the structure until the crack intercepts another joint.

EXPANSION JOINTS

Temperature changes will induce stresses in a structure, if the structure is restrained. Without restraint, no stresses result. In practice, all buildings are restrained to some degree. Temperature induced stresses vary with the magnitude of the temperature change; large temperature variations can result in substantial stresses that must be accounted for in design, while low temperature changes may result in negligible stresses.

Temperature induced stresses are the direct result of volume changes within a structure between restrained points. A rough indication of the amount of elongation caused by temperature increases is obtained by multiplying the coefficient of expansion of concrete [about 5.5×10^{-6} in./in. per °F (9.9×10^{-6} mm/mm per °C)] by the length of the structure and the temperature change. A building 200 ft long subjected to a temperature increase of 25 °F (14 °C) will elongate about 3/8 in. (9.5 mm).

Expansion joints are used to limit member forces caused by thermally induced volume changes. Expansion joints permit separate segments of a building to expand or contract without adversely affecting structural integrity or serviceablity. Expansion joints should be wide enough to prevent portions of the building on either side of the joint from coming in contact, when the structure is subjected to the maximum expected temperature rise. Joints vary in width from 1 to 6 in. (25 to 152 mm) or more, with 2 in. (51 mm) being typical. The wider joints are used to accommodate additional differential building movement that may be caused by settlement or siesmic loading. Joints should pass through the entire structure above the level of the foundation. Expansion joints should be covered (Fig. 1) and may be empty or filled (Fig. 2). Filled joints are required for fire rated structures.

Expansion joint spacing is dictated by the amount of movement that can be tolerated, plus the allowable stresses and/or capacity of the members. As with contraction joints, rules of thumb have been developed (Table 3). These range from 30 to 400 ft (9 to 122 m) depending on the type of structure. In addition to the rules of thumb, a number of methods have been developed to calculate expansion joint spacing. This section presents three of these methods. The three methods are based on the work of Martin and Acosta (1970), Varyani and Radhaji (1978), and the National Academy of Sciences (1974).

Martin and Acosta (1970) present an expression for the maximum spacing of expansion joints in one story reinforced concrete buildings. The expression is developed based on the results of a study of frame structures subjected to temperature change. Joint spacing is a function of the length and stiffness of frame members, and seasonal temperature changes that occur at the building site. The design temperature change is based on the difference between the extreme values of the normal daily maximum and minimum temperatures plus an additional drop in temperature of about 30 °F (17 °C) to account for drying shrinkage in the concrete. Martin (1970) provides site specific values of shrinkage equivalent temperature drop. Because of the additional volume change due to drying shrinkage, joint spacing is governed by contraction instead of expansion.

Varyani and Radhaji (1978) present a method for calculating expansion joint spacing based on the design of corner columns in a symmetrical frame to resist a combination of gravity load plus thermally induced load. Joint spacing is a function of the extreme daily temperature changes, including an allowance for shrinkage, and the relative stiffnesses of the first floor beams and columns. The procedure is applicable to rectangular column layouts for single and multi-story construction. This method is somewhat more general but is similar in approach to the method presented by Martin and Acosta (1970).

The National Academy of Sciences (1974) present an empirical procedure for selecting expansion joint spacing. A graph relates joint spacing to yearly extremes in temperature at the building site. The procedure is applicable to single and multi-story frame structures, and accounts for variables such as building stiffness and configuration, the type of column connection to the foundation, and the use of heating and air conditioning systems.

Example calculations and a discussion of the relative merits of these methods are presented in Appendix B.

Single Story Buildings: Martin and Acosta

Martin and Acosta (1970) present a method for calculating the maximum spacing of expansion joints in one story beam-column frames with approximately equal spans. This method is based on the premise that with adequate joint spacing, the factor of safety for vertical loading will also provide an adequate safety factor for the effects of temperature change. By applying this premise to a number of frame structures, Martin and Acosta develop a single expression for expansion joint spacing.

To remove the need for specific temperature calculations in design, Martin and Acosta feel that a building should have an adequate factor of safety to withstand the applied lateral, vertical, and thermally induced loads. Due to the short-term nature of thermal loading, Martin and Acosta address this type of load in the same way as ACI 318-63 addressed wind loads. Accordingly, the structure should be designed to meet the following criteria:

Dead and live loads:

$$U = 1.5D + 1.8L \tag{1}$$

Dead, live, and temperature loads:

$$U = 1.25(D + L + T) \tag{2}$$

in which U = required strength of the element under consideration: D = dead load; L = live load, and T = thermal load.

Setting Eq. (1) equal to Eq. (2), yields an expression for the maximum temperature induced load, T, that is allowed in order for thermal effects to be omitted in the design calculations. This maximum equivalent load is represented by:

$$T = 0.2D + 0.44L \tag{3}$$

This means that temperature loads should not exceed the effect of 20 percent of the dead load plus 44 percent of the live load.

Beam-column frames with the geometries given in Table 4 were analyzed for temperature variations obtained at a number of geographical locations. The number of spans (and consequently the length of the buildings) were increased to model varying lengths between expansion joints.

The magnitude of the design temperature change, ΔT, is taken as two-thirds of the difference between the extreme values of the normal daily maximum and minimum temperatures, T_{max} and T_{min}, at these locations. Martin and Acosta arbitrarily chose the two-thirds factor to account for the fact that the temperature at which the building is completed would statistically not be at the maximum or minimum daily temperature, but somewhere between the two. They used a coefficient of thermal expansion, α, of 5.5×10^{-6} in./in. per °F (9.9×10^{-6} mm/mm per °C). To account for concrete shrinkage, Martin and Acosta used an additional equivalent temperature differential, which averages about 30°F, in the temperature analysis to obtain the total design temperature change.

$$\Delta T = \frac{2}{3}(T_{max} - T_{min}) + 30°F \qquad (4)$$

Although ΔT is used to proportion <u>expansion</u> joints, ΔT in Eq. (4) actually represents the combination of a temperature <u>drop</u> with shrinkage.

The shears and moments due to temperature and shrinkage were compared to the shears and moments due to vertical loads applied to the same frame. The loads considered in the vertical analysis were the self weight of the structure plus a roof live load of 30 psf (1.44 kPa), resulting in a uniformly distributed dead load of 2 kips per foot (29 kN/m), and a uniformly distributed live load of 0.5 kips per foot. The critical temperature effect for the longer beam spans (structures Type E through L of Table 4) was the bending moment in the exterior columns. Structures with shorter beam spans (Type A through D of Table 4) were governed by the beam moment at the exterior face of the first interior column.

Using the structural analyses and Eq. (3) as the basis to determine the maximum expansion joint spacing, Martin and Acosta developed an expression for maximum expansion joint spacing, L_j in ft.

$$L_j = \frac{112{,}000}{R\ \Delta T} \tag{5}$$

in which:

$$R = 144\frac{I_c}{h^2}\left(\frac{1 + r}{1 + 2r}\right) \tag{6}$$

r = ratio of stiffness factor of column to stiffness factor of beam = K_c/K_b; K_c = column stiffness factor = I_c/h, in.^{3}; K_b = beam stiffness factor = I_c/l, in.^{3}; h = column height, in.; l = beam length, in.; I_c = moment of inertia of the column, in.^{4}; I_b = moment of inertia of the beam, in.^{4} To calculate ΔT, T_{max} and T_{min} are obtained from the Environmental Data Service for a particular location (see Table 5 for a partial listing). The resulting length between joints, L_j, given by Eq. (5) for typical values of R is given in Fig. 3.

In order to avoid damage to exterior walls, the Martin and Acosta propose an additional criteria for L_j to limit the maximum allowable lateral deflection, δ, to 1/180 of the column height, h. The maximum lateral deflection imposed on a column is taken as

$$\delta = \frac{1}{2}\alpha L_j \Delta T \qquad (7)$$

This leads to the limitation on L_j of

$$L_j \leq \frac{2000h}{\Delta T} \text{ ; } \Delta T \text{ in } ^\circ F. \qquad (8)$$

Eq. (8) is based on the assumption that the lateral deflection of a floor system caused by a temperature change is not significantly restrained by the columns. This assumption is realistic since the in-plane stiffness of a floor system is generally much greater than the lateral stiffness of the supporting columns. Thus, the columns have little effect on δ.

Discussion--Martin and Acosta state that Eq. (5) yields conservative results in all cases studied, but is very conservative for very rigid structures. Due to changes in ACI 318 since 1963, Eq. (5) can be revised to account for current load factors (ACI 318-83). Eq. (1) and (2) become:

$$U = 1.4D + 1.7L \qquad (9)$$

$$U = 0.75(1.4D + 1.7L + 1.4T) \qquad (10)$$

Setting Eq. (9) equal to Eq. (10) yields the maximum allowable temperature induced load.

$$T = 0.33D + 0.41L \qquad (11)$$

In this case, temperature effects should not exceed the effect of 33 percent of the dead load plus 41 percent of the live load.

To obtain an updated version of Eq. (5) requires a reanalysis of the original data of Martin and Acosta. In lieu of a reanalysis, Eq. (5) remains as a conservative guideline for expansion joint spacing, since Eq. (3) is more conservative than Eq. (11) in practical applications.

Single and Multi-Story Buildings: Varyani and Radhaji

Varyani and Radhaji (1978) present a procedure to calculate the maximum distance between expansion joints for symmetrical, single and multi-story beam-column frames. Thermal loads are based on a temperature differential similar to that used by Martin and Acosta (1970), two-thirds of the difference between maximum and minimum daily temperatures, except Varyani and Radhaji select temperatures from the single day on which the difference between the maximum and minimum temperatures is the greatest. Moments due to thermal loading are calculated at the corner columns, accounting for maximum biaxial bending in the columns. Varyani and Radhaji find that only the first story columns and the beams supported by these columns are substantially affected by temperature change. Above the first story, the effect of temperature is dissipated. Therefore, only the first two stories of multistory buildings are assumed to be critical in the analysis.

The corner columns are designed for the factored axial load plus biaxial bending due to gravity load. Trial values of expansion joint spacing, L_j, are used to calculate temperature induced column moments. The final length between joints is selected so that the column design obtained under gravity load [Eq. (9)] is adequate under combined gravity and thermal loading [Eq. (10)]. Alternatively, column reinforcement can be increased to withstand increased moments for a selected value of L_j.

The thermally induced moments at the base and top of a corner column can be calculated using analysis techniques such as matrix analysis or moment distribution. Due to the tedious nature of these calculations for long structures, Varyani and Radhaji recommend the use of a substitute one-bay open frame, shown in Fig. 4. The one-bay frames replace the first interior column with a fixed point at the beam-column joint. Varyani and Radhaji compared the results obtained with this approximation to results obtained with more accurate methods (Reynolds 1960) for up to 4 bays, and compared the substitute bay analysis to frame analyses for buildings up to 10 bays. They considered the 0 to 10.5 percent margin of error to be satisfactory.

Varyani and Radhaji present expressions for the moment at the base of corner columns, M_g(base), in a number of different single bay frames under the effect of gravity. These are given here and in Fig. 5:

Single story, single bay:

$$M_g(\text{base}) = \frac{M_f}{(2 + r')} \tag{12}$$

Single story, multi-bay:

$$M_g(\text{base}) = \frac{M_f}{2(1 + r')} \tag{13}$$

Multi-story, single bay:

$$M_g(\text{base}) = \frac{M_f}{(4 + r')} \tag{14}$$

Multi-story, multi-bay:

$$M_g(\text{base}) = \frac{M_f}{2(2 + r')} \tag{15}$$

in which M_f = fixed-end moment at the beam-column joint due to gravity loading; and $r' = r^{-1} = K_b/K_c$

Varyani and Radhaji also present expressions for the base moment of corner columns in frames under the effect of temperature loading, M_T(base). These expressions are given here and in Fig. 6.

Single story, single bay:

$$M_T(\text{base}) = 6E_cK_c \frac{\delta}{h} \left(\frac{1 + r'}{2 + r'}\right) \tag{16}$$

Single story, multi-bay:

$$M_T(\text{base}) = 3E_cK_c \frac{\delta}{h} \left(\frac{1 + 2r'}{1 + r'}\right) \tag{17}$$

Multi-story, single bay:

$$M_T(\text{base}) = 6E_cK_c \frac{\delta}{h} \left(\frac{3 + r'}{4 + r'}\right) \tag{18}$$

Multi-story, multi-bay:

$$M_T(\text{base}) = 3E_cK_c \frac{\delta}{h} \left(\frac{3 + 2r'}{2 + r'}\right) \tag{19}$$

in which E_c = modulus of elasticity of concrete; and δ is given in Eq. (7).

Like Martin and Acosta (1970), Varyani and Radhaji use a 2/3 reduction factor for the maximum temperature change. The maximum fall in temperature (contraction) is adjusted by an equivalent temperature drop due to shrinkage of 15 °C (27 °F). The maximum rise in temperature (expansion) is adjusted by an equivalent temperature drop due to shrinkage of 8 °C (14 °F). Varyani and Radhaji recommend that these temperature changes be reduced by one-half to account for creep, duration of load, and loss of fixity due to soil movement.

$$\Delta T = \frac{1}{2}\left[-\frac{2}{3}(T_{max} - T_{min}) - 27\ °F\right] \tag{20a}$$

$$\Delta T = \frac{1}{2}\left[\frac{2}{3}(T_{max} - T_{min}) - 14\ °F\right] \tag{20b}$$

Substituting Eq. (7) for δ into Eq. (16) - (19) results in expressions for expansion joint spacing, L_j, in terms of $M_T = M_T$(base).

Single story, single bay:

$$L_j = \frac{M_T h}{3E_c K_c \alpha \Delta T}\left(\frac{2 + r'}{1 + r'}\right) \tag{21}$$

Single story, multi-bay:

$$L_j = \frac{2M_T h}{3E_c K_c \alpha \Delta T} \left(\frac{1 + r'}{1 + 2r'}\right) \tag{22}$$

Multi-story, single bay:

$$L_j = \frac{M_T h}{3E_c K_c \alpha \Delta T} \left(\frac{4 + r'}{3 + r'}\right) \tag{23}$$

Multi-story, multi-bay:

$$L_j = \frac{2M_T h}{3E_c K_c \alpha \Delta T} \left(\frac{2 + r'}{3 + 2r'}\right) \tag{24}$$

Discussion.--Although Varyani and Radhaji do not develop a "general" expression for L_j, such as Eq. (5) by Martin and Acosta, the two methods are similar in approach. Both methods base the expansion joint spacing on the ability of the first level beams and columns to resist the thermally induced loads.

Varyani and Radhaji equate the temperature induced moment at the base of a column with the moment due to gravity at the same point. However, a column design based on gravity loading is normally governed by the moment at the top of the column. Therefore, it makes more sense to compare the maximum combined factored moment at the top or bottom of a column [Eq. (10)] with the factored gravity moment [Eq. (9)]. This approach is demonstrated in the examples in Appendix B.

The following equations provide the values of M_g and M_T at the top of columns for the single bay frames shown in Fig. 5 and 6.

Single story, single bay:

$$M_g(\text{top}) = \frac{2M_f}{(2 + r')} \tag{25}$$

$$M_T(\text{top}) = 6E_cK_c \frac{\delta}{h} \left(\frac{r'}{2 + r'}\right) \tag{26}$$

Single story, multi-bay:

$$M_g(\text{top}) = \frac{M_f}{(1 + r')} \tag{27}$$

$$M_T(\text{top}) = 6E_cK_c \frac{\delta}{h} \left(\frac{r'}{1 + r'}\right) \tag{28}$$

Multi-story, single bay:

$$M_g(\text{top}) = \frac{2M_f}{(4 + r')} \tag{29}$$

$$M_T(\text{top}) = 6E_cK_c \frac{\delta}{h} \left(\frac{2 + r'}{4 + r'}\right) \tag{30}$$

Multi-story, multi-bay:

$$M_g(\text{top}) = \frac{M_f}{(2 + r')} \tag{31}$$

$$M_T(\text{top}) = 6E_cK_c \frac{\delta}{h} \left(\frac{1 + r'}{2 + r'}\right) \tag{32}$$

Varyani and Radhaji recommend the use of the maximum temperature differential for a single day, rather than the difference in the extreme value of normal maximum and minimum daily temperatures used by Martin and Acosta. However, logic suggests that seasonal changes in temperature are more appropriate for calculating dimensional changes of structures. For this reason, the temperature range suggested by Martin and Acosta should be applied with both methods.

Varyani and Radhaji do not specifically consider the effects of temperature change on interior beams and columns, and the results obtained with the method are very sensitive to the assumed beam and column stiffnesses. Although Varyani and Radhaji include a 50 percent reduction in ΔT [Eq. (20)] to account for creep and duration of load, the calculated expansion joint spacings obtained with this method are considerably less than expansion joint spacings which are used and have performed well in existing structures. This point is amply demonstrated in Appendix B. The results can be improved somewhat by using cracked, rather than uncracked section properties. Treating the members as cracked has relatively little effect on the gravity moments but reduces the calculated value of M_T. The result is an increased, and somewhat more realistic, value for L_j.

The relative complexity, time consuming nature, and overconservative results obtained with the method compared to the methods of Martin and Acosta (1970) and the National Academy of Sciences (1974) are likely to render this procedure unattractive to designers.

Single and Multi-Story Buildings: National Academy of Sciences

The lack of nationally accepted design procedures for locating expansion joints prompted the Federal Construction Council to undertake the task of developing more definitive criteria for locating expansion joints in federal construction. As a consequence, the Council directed its Standing Committee on Structural Engineering to develop a procedure for expansion joint design to be used by federal agencies in building design. The criteria formulated by the Committee was published by the National Academy of Sciences (1974) in the form of a graph (Fig. 7) which expresses the allowable building length as a function of a design temperature change. The relationships shown in Fig. 7 are directly applicable to beam-column frames, with columns hinged at the base and heated interiors. In order for the graph to be adaptable to a wide range of buildings, the Committee provided modification factors to reflect building stiffness and configuration, heating and cooling, and the type of column connection to the foundation.

Fig. 7 is based on the Committee's investigation of the procedures in use by federal agencies to select joint spacing, plus an analytical study comparing the theoretical effects of temperature change on two-dimensional elastic frames to the actual movements recorded during a one-year study by the Public Buildings Administration (1943-1944).

The Committee first examined procedures used by federal agencies to select expansion joint spacing. Although no significant quantitative data was found to support their criteria, most federal agencies relied on rules (Fig. 8) that provided maximum building dimensions for heated and unheated buildings as a function of the change in the exterior temperature. Temperature change is taken as the maximum difference between the mean temperature during the normal construction season and either the summer or winter extremes.

The criteria illustrated in Fig. 8 reflect two assumptions: that the maximum allowable building length between joints must be decreased as the maximum difference between the mean annual temperature and the maximum/minimum temperature increases; and that dimensions between joints can be increased for heated structures, which have the ability to dampen the severity of the outside temperature changes through temperature control in the building. The upper and lower bounds of 600 and 200 ft were felt to reflect a consensus within the engineering profession and have no experimental or theoretical justification.

Additional information was drawn from an unpublished report by structural engineers of the Public Buildings Administration (1943-1944). This report documents an investigation of expansion joint movement over a period of one year (September 1943 to August 1944) in nine federal buildings. Based on the report, the conclusions drawn by the Committee and implemented in their design recommendations are as follows:

> A considerable time lag (2 to 12 hours) exists between the maximum dimensional change and the peak temperature associated with this change. The investigators attributed this time lag to three factors: the temperature gradient between the outside and inside temperatures, the resistance to the heat transfer due to insulation, and the duration of the ambient temperature at its extreme levels.
>
> The coefficient of thermal expansion of the first floor level is approximately one-third to two-thirds that of the upper floors. The dimensional changes in the upper levels of the building correspond to a coefficient of thermal expansion between 2×10^{-6} and 5×10^{-6} in./in. per °F. The investigation seems to confirm that the upper levels of a building undergo dimensional changes corresponding to the coefficient of thermal expansion of the principal material of which each is constructed.

To interpret the observed dimensional changes reported by the Public Buildings Administration (1943-1944), and enhance their understanding of the distribution of stresses and resulting frame deformations, the Committee analytically studied the effects of a uniform temperature change on typical two-dimensional frames. This study was based on an analysis of nine beam-column frames subjected to a design temperature change, ΔT, of 100° F. In practice, ΔT, is calculated for a specific site as the larger of:

$$\Delta T = T_w - T_m \tag{33a}$$

$$\Delta T = T_m - T_c \tag{33b}$$

in which T_m = temperature during the normal construction season in the locality of the building, assumed to be the continuous period in a year during which the minimum daily temperature equals or exceeds 32 °F (0 °C); T_w = temperature exceeded, on average, only 1 percent of the time during the summer months of June through September; and T_c = temperature equaled or exceeded, on average, 99 percent of the time during the winter months of December, January, and February.

Observed values for T_m, T_w, and T_c for locations throughout the United States are given in Table 6 (ASHRAE Handbook of Fundamentals, 1972).

For the analysis, member forces, F, associated with ΔT = 100°F were applied at beam-column joints in buildings consisting of bay spacings of 25 ft (7.6 m), with the first story height assumed to be 13 ft (4 m) and the upper stories spaced at 10 ft (3 m). F is calculated according to the following:

$$F = \alpha A E_c \Delta T \tag{34}$$

in which F = the force resulting in a restrained member due to a temperature change ΔT; $\alpha = 6.0 \times 10^{-6}$ in./in. per °F (10.8×10^{-6} mm/mm per °C) (assumed in the Committee's study); and A = cross-sectional area of the member. The column sizes, beam sizes, number of stories, and type of connection to the foundation (hinged versus fixed) were varied to help determine the effects of temperature changes on a wide range of buildings.

The results of the analysis are given in Table 7 and summarized as follows:

> The intensity of the horizontal shear in the 1st story columns (Fig. 9) is greatest at the ends of the frame and approaches zero at the center.
>
> The beams near the center of the frame in Fig. 9 are subjected to maximum axial forces, while the columns at the ends of the frame are subjected to maximum bending moments and shears at the beam-column joint.
>
> Shears, axial forces, and bending moments at critical sections within the lowest story are almost twice as high for fixed-column buildings as the forces at corresponding locations in hinged-column buildings.
>
> The horizontal displacement of one side of the upper floors, δ, is approximately equal to the assumed displacement that would develop in an unrestrained frame if both ends of the frame are equally free to displace = $\frac{1}{2}\alpha L_j \Delta T$ [Eq.(7)].
>
> The horizontal displacement of a frame that is restricted from side displacement at one end (non-symmetrical stiffness, analysis M-1) results in a total horizontal displacement of the other end of approximately $\alpha L_j \Delta T$.

An increase in the relative cross-sectional area of the beams (not associated with a simultaneous increase in the moment of inertia of the beams), as illustrated in analysis A-1 versus A-2, results in a considerable increase in the controlling design forces. This occurs because the magnitude of the thermally induced force, F, is proportional to the cross-sectional area of the element [Eq. (34)].

Hinges placed at the top and bottom of the exterior columns of the frame result in a reduction of the maximum stresses that develop (analysis 1-1 versus M-2). These hinges, however, allow an increase in the horizontal expansion of the first floor.

As a result of the Committee's investigation, Fig. 7 was developed. This graphical representation of the maximum expansion joint spacing is based on the correlation of the observed dimensional changes given by the Public Buildings Administration (1943-1944) with the Committee's analytical study, plus the current practices of federal agencies (Fig. 8). The Committee rationalized that the step function of Fig. 8 could not represent the behavior of a physical phenomenon such as thermal effects and therefore assumed a linearly varying function for a 30 to 70 °F (17 to 39 °C) temperature change. The upper and lower bounds are based on Fig. 8.

The limits prescribed in Fig. 7 are directly applicable to buildings of beam-column construction (including structures with interior shear walls or perimeter base walls), hinged at the foundation, and heated. If these conditions are not met, the Committee recommends the following conservative modification factors which reflect the collective experience and judgement of the Committee.

If the building will be heated only and have hinged column bases, use the length specified.

If the building will be air-conditioned as well as heated, increase the allowable length by 15 percent.

If the building will be unheated, decrease the allowable length by 33 percent.

If the building will have fixed column bases, decrease the allowable length by 15 percent.

If the building will have substantially greater stiffness against lateral displacement at one end of the structure, decrease the allowable length by 25 percent.

When one or more of these conditions occur, the total modification factor is the algebraic sum of the individual adjustment factors that apply.

Discussion--The Federal Construction Council Committee does not recommend their procedure for all situations. Particularly, when a generic representation is not adequate for a unique structure or when the empirical approach provides a solution that professional judgement indicates is too conservative, a detailed analysis should be performed. This analysis should recognize the amount of lateral deformation that can be tolerated, and the structure should be designed so that this limit is not exceeded.

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Table 1 - Contraction joint spacings

Author	Contraction Joint Spacing
Merrill (1943)	20 ft for walls with frequent openings, 25 ft. in solid walls
Fintel (1974)	* 15 to 20 ft for walls and slabs on grade
Wood (1981)	20 to 30 ft for walls
PCA (1982)	20 to 25 ft for walls depending on number of openings
ACI 350R-83 (1983)	30 ft in sanitary structures
ACI 224R-80 (Revised 1984)	one to three times the height of the wall in solid walls

* Recommends joint placement at abrupt changes in plan and at changes in building height to account for potential stress concentrations.
1 ft = 0.305 m

Table 2 - Contraction joint spacings for sanitary engineering structures based on reinforcement percentage (Rice 1984)

Contraction joint spacing in ft.	Minimum percentage of shrinkage and temperature reinforcement*	
	Grade 40	Grade 60
less than 30	.30	.25
30 - 40	.40	.30
40 - 50	.50	.38
greater than 50	.60	.45

* minimum temperature and shrinkage reinforcement should be #5 (16 mm) bars or D-31 (200 mm²) wire, 12 in. (0.305 m) on center each face.
1 ft = 0.305 m; Grade 40 = 276 MPa; Grade 60 = 413 MPa.

Table 3-Expansion joint spacings

Author	Expansion Joint Spacing
Lewerenz (1907)	75 ft for walls
Hunter (1953)	80 ft for walls and insulated roofs 30 to 40 ft for uninsulated roofs
Billig (1960)	* 100 ft maximum building length without joints
Wood (1981)	100 to 120 ft for walls
Indian Standards (1964) Institution	45 m (approx. 148 ft) maximum building length between joints
PCA (1982)	200 ft maximum building length without joints
ACI 350R-83	120 ft in sanitary structures partially filled with liquid (closer) spacings required when no liquid present)

* Recommends joint placement at abrupt changes in plan and at changes in building height to account for potential stress concentrations.
1 ft = 0.305 m

Table 4-Beam-column frame geometry used for temperature and vertical load analysis (Martin and Acosta 1970)

	Beam			Square column	
Structure Type	Width in.	Depth in.	Span ft	Side in.	Height ft
A	14	20	20	14	12
B	14	20	20	14	20
C	14	20	20	18	12
D	14	20	20	18	20
E	14	20	30	14	12
F	14	20	30	14	20
G	14	20	30	18	12
H	14	20	30	18	20
I	14	30	30	14	12
J	14	30	30	14	20
K	14	30	30	18	12
L	14	30	30	18	20

1 in. = 25.4 mm; 1 ft = 0.305 m

Table 5-Maximum and minimum daily temperatures for given locations (Martin and Acosta 1970)

Location	Normal daily temperatures °F Maximum	Minimum
Anchorage, Alaska	66.0	4.3
Atlanta, Ga.	87.0	37.1
Boston, Mass.	81.9	23.0
Chicago, Ill.	84.1	19.0
Dallas, Tex.	95.0	36.0
Denver, Colo.	88.4	14.8
Detroit, Mich.	84.7	19.1
Honolulu, Hawaii	85.2	65.8
Jacksonville, Fla.	92.0	45.0
Los Angeles, Calif.	75.9	45.0
Miami, Fla.	89.7	57.9
Milwaukee, Wis.	78.9	12.8
New Orleans, La.	90.7	44.8
New York, N.Y.	85.3	26.4
Phoenix, Ariz.	104.6	35.3
Pittsburgh, Pa.	83.3	20.7
San Fransisco, Calif.	73.8	41.7
San Juan, P.R.	85.5	70.0
Seattle, Wash.	75.6	33.0
St. Louis, Mo.	89.2	23.5
Tulsa, Okla.	93.1	26.5

Temperature in C = $\frac{5}{9}$(Temperature in F-32)

Table 6-Temperatures used for calculation of ΔT (National Academy of Sciences 1974)

Location	Temperature °F T_w	T_m	T_c
Alabama			
Birmingham	97	63	19
Huntsville	97	61	13
Mobile	96	68	28
Montgomery	98	66	22
Alaska			
Anchorage	73	51	-25
Barrow	58	38	-45
Fairbanks	82	50	-53
Juneau	75	48	-7
Nome	66	45	-32
Arizona			
Flagstaff	84	58	0
Phoenix	108	70	31
Prescott	96	64	15
Tuscon	105	67	29
Winslow	97	67	9
Yuma	111	72	37
Arkansas			
Ft. Smith	101	65	15
Little Rock	99	65	19
Texarkana	99	65	22

T = temperature exceeded, on the average, only 1 percent of the time during the summer months of June through September

T = temperature during the normal construction season

T = temperature equaled or exceeded, on the average, 99 percent of the time during the winter months of December through February

Temperature in C = $\frac{5}{9}$(Temperature in F-32)

Table 6, continued

Location	Temperature °F T_w	T_m	T_c
California			
Bakersfield	103	65	31
Burbank	97	64	36
Eureka/Arcata	67	52	32
Fresno	101	63	28
Long Beach	87	63	41
Los Angeles	94	62	41
Oakland	85	57	35
Sacramento	100	60	30
San Diego	86	62	42
San Fransisco	83	56	35
Santa Maria	85	57	32
Colorado			
Alamosa	84	60	-17
Colorado Springs	90	61	-1
Denver	92	62	-2
Grand Junction	96	64	8
Pueblo	96	64	-5
Connecticut			
Bridgeport	90	60	4
Hartford	90	61	1
New Haven	88	59	5
Delaware			
Wilmington	93	62	12
Florida			
Daytona Beach	94	70	32
Ft. Myers	94	74	38
Jacksonville	96	68	29
Key West	90	77	55
Lakeland	95	72	35
Miami	92	75	44
Miami Beach	91	75	45
Orlando	96	72	33
Pensacola	92	68	29
Tallahassee	96	68	25
Tampa	92	72	36
West Palm Beach	92	75	40
Georgia			
Athens	96	61	17
Atlanta	95	62	18
Augusta	98	64	20
Columbus	98	65	23

Table 6, continued

Location	Temperature °F T_w	T_m	T_c
Georgia (cont'd.)			
Macon	98	65	23
Rome	97	62	16
Savannah/Travis	96	67	24
Hawaii			
Hilo	85	73	59
Honolulu	87	76	60
Idaho			
Boise	96	61	4
Idaho Falls	91	61	-12
Lewiston	98	60	6
Pocatello	94	60	-8
Illinois			
Chicago	95	60	-3
Moline	94	63	-7
Peoria	94	61	-2
Rockford	92	62	-7
Springfield	95	62	-1
Indiana			
Evansville	96	65	6
Fort Wayne	93	62	0
Indianapolis	93	63	0
South Bend	92	61	-2
Iowa			
Burlington	95	64	-4
Des Moines	95	64	-7
Dubuque	62	63	-11
Sioux City	96	64	-10
Waterloo	91	63	-12
Kansas			
Dodge City	99	64	3
Goodland	99	65	-2
Topeka	99	69	3
Wichita	102	68	5
Kentucky			
Covington	93	63	3
Lexington	94	63	6
Louisville	96	64	8

Table 6, continued

Location	Temperature °F		
	T_w	T_m	T_c
Louisiana			
Baton Rouge	96	68	25
Lake Charles	95	68	29
New Orleans	93	69	32
Shreveport	99	66	22
Maine			
Caribou	85	56	-18
Portland	88	58	-5
Maryland			
Baltimore	94	63	12
Frederick	94	63	7
Massachusetts			
Boston	91	58	6
Pittsfield	86	58	-5
Worcester	89	58	-3
Michigan			
Alpena	87	57	-5
Detroit-Metro	92	58	4
Escanaba	82	55	-7
Flint	89	60	-1
Grand Rapids	91	62	2
Lansing	89	59	2
Marquette	88	55	-8
Muskegon	87	59	4
Sault Ste Marie	83	55	-12
Minnesota			
Duluth	85	55	-19
International Falls	86	57	-29
Minneapolis/St. Paul	92	62	-14
Rochester	90	60	-17
St. Cloud	90	60	-20
Mississippi			
Jackson	98	66	21
Meridian	97	65	20
Vicksburg	97	66	23
Missouri			
Columbia	97	65	2
Kansas City	100	65	4
St. Joseph	97	66	-1
St. Louis	98	65	4
Springfield	97	64	5

Table 6, continued

Location	Temperature °F T_w	T_m	T_c
Montana			
Billings	94	60	-10
Glasgow	96	60	-25
Great Falls	91	58	-20
Havre	91	58	-22
Helena	90	58	-17
Kalispell	88	56	-7
Miles City	97	62	-19
Missoula	92	58	-7
Nebraska			
Grand Island	98	65	-6
Lincoln	100	64	-4
Norfolk	97	64	-11
North Platte	97	64	-6
Omaha	97	64	-5
Scottsbluff	96	62	-8
Nevada			
Elko	94	61	-13
Ely	90	59	-6
Las Vegas	108	66	23
Reno	95	62	2
Winnemucca	97	63	1
New Hampshire			
Concord	91	60	-11
New Jersey			
Atlantic City	91	61	14
Newark	94	62	11
Trenton	92	61	12
New Mexico			
Albuquerque	96	64	14
Raton	92	64	-2
Roswell	101	70	16
New York			
Albany	91	61	-5
Binghampton	91	67	-2
Buffalo	88	59	3
New York	94	59	11
Rochester	91	59	2
Syracuse	90	59	-2

Table 6, continued

Location	Temperature °F T_w	T_m	T_c
North Carolina			
Asheville	91	60	13
Charlotte	96	60	18
Greensboro	94	64	14
Raleigh/Durham	95	62	16
Wilmington	93	63	23
Winston/Salem	94	63	14
North Dakota			
Bismarck	95	60	-24
Devils Lake	93	58	-23
Fargo	92	59	-22
Minot	91		-24
Williston	94	59	-21
Ohio			
Akron/Canton	89	60	1
Cincinnati	94	62	8
Cleveland	91	61	2
Columbus	92	61	2
Dayton	92	61	0
Mansfield	91	61	1
Sandusky	92	60	4
Toledo	92	61	1
Youngstown	89	59	1
Oklahoma			
Oklahoma City	100	64	11
Tulsa	102	65	12
Oregon			
Astoria	79	50	27
Eugene	91	52	22
Medford	98	56	21
Pendleton	97	58	3
Portland	91	52	21
Roseburg	93	54	25
Salem	92	52	21
Pennsylvania			
Allentown	92	61	3
Erie	88	59	7
Harrisburg	92	61	9
Philadelphia	93	63	11
Pittsburgh	90	63	5
Reading	92	61	6

Table 6, continued

Location	Temperature °F T_w	T_m	T_c
Pennsylvania, cont'd.			
Scranton/Wilkes-Barre	89	61	2
Williamsport	91	61	1
Rhode Island			
Providence	89	60	6
South Carolina			
Charleston	95	66	23
Columbia	98	64	20
Florence	96	64	21
Greenville	95	61	19
Spartanburg	95	60	18
South Dakota			
Huron	97	62	-16
Rappid City	96	61	-9
Souix Falls	95	62	-14
Tennessee			
Bristol/Tri City	92	63	11
Chattanooga	97	60	15
Knoxville	95	60	13
Memphis	98	62	17
Nashville	97	62	12
Texas			
Abilene	101	65	17
Amarillo	98	66	8
Austin	101	68	25
Brownsville	94	74	36
Corpus Christi	95	71	32
Dallas	101	66	19
El Paso	100	65	21
Fort Worth	102	66	20
Galveston	91	70	32
Houston	96	68	28
Laredo AFB	103	74	32
Lubbock	99	67	11
Midland	100	66	19
Port Arthur	94	69	29
San Angelo	101	65	20
San Antonio	99	69	25
Victoria	98	71	28
Waco	101	67	21
Wichita Falls	103	66	15

Table 6, continued

Location	Temperature °F T_w	T_m	T_c
Utah			
Salt Lake City	97	63	5
Vermont			
Burlington	88	57	-12
Virginia			
Lynchburg	94	62	15
Norfolk	94	60	20
Richmond	96	64	14
Roanoke	94	63	15
Wahington, D. C.			
National Airport	94	63	16
Washington			
Olympia	85	51	21
Seattle	85	51	20
Spokane	93	58	-2
Walla Walla	98	57	12
Yakima	94	62	6
West Virginia			
Charleston	92	63	9
Huntington	95	63	10
Parkersburg	93	62	8
Wisconsin			
Green Bay	88	59	-12
La Crosse	90	62	-12
Madison	92	61	-9
Milwaukee	90	60	-6
Wyoming			
Casper	92	59	-11
Cheyenne	89	58	-6
Lander	92	58	-16
Sheridan	95	59	-12

Table 7-Results of Temperature Analysis (National Academy of Sciences 1974)

Anal. No.	(1) Col. Size in.	(2) Beam Size in.^{2}	Base Type	Total Length ft.	No. Flrs.	Beam Max. Moment in.-kips	Beam Max. Axial Load kips	Column Max. Moment in.-kips	Column Max. Shear kips
1-1	24	280	F	200	3	70	135	600	69
2-1	24	280	H	200	3	45	77	250	19
3-1	16	280	F	200	3	53	70	170	22
4-1	16	280	H	200	3	25	25	75	6
A-1	24	280	F	200	2	43	182	570	67
A-2	24	1000	F	200	2	70	321	750	95
B-1	24	280	F	400	2	128	280	840	90
M-1	24	280	F	200	3	100	253	740	83
M-2	24	280	F	200	3	50	138	500	60

(1) Column size = one side of a square column.

(2) Beam size = cross-sectional area. Moment of inertia about the vertical axis = 4667 in.^{4} for all cases.

M-1 analysis includes columns at one end of the frame substantially stiffer than the rest of the columns.

M-2 analysis includes hinges placed at the top and bottom of the exterior columns.

F columns fixed at the foundation.

H columns hinged at the foundation.

1 in. = 25.4 mm; 1 ft = 0.305 in.; 1 in.-kip = 113 m-kN; 1 kip = 4.45 kN

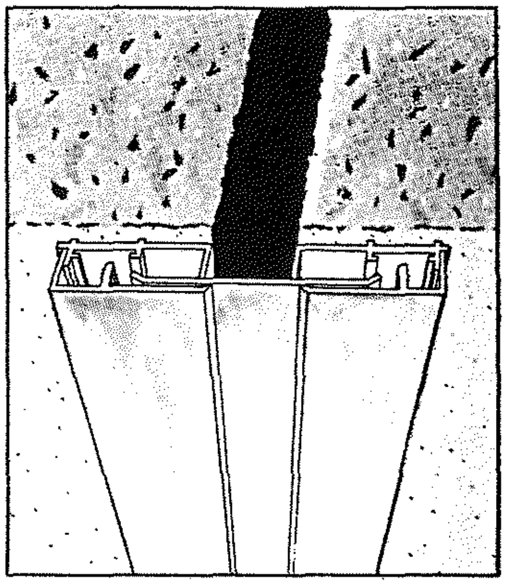


Fig. 1 — Wall expansion joint cover (courtesy Architectural Art Mfg., Inc.)

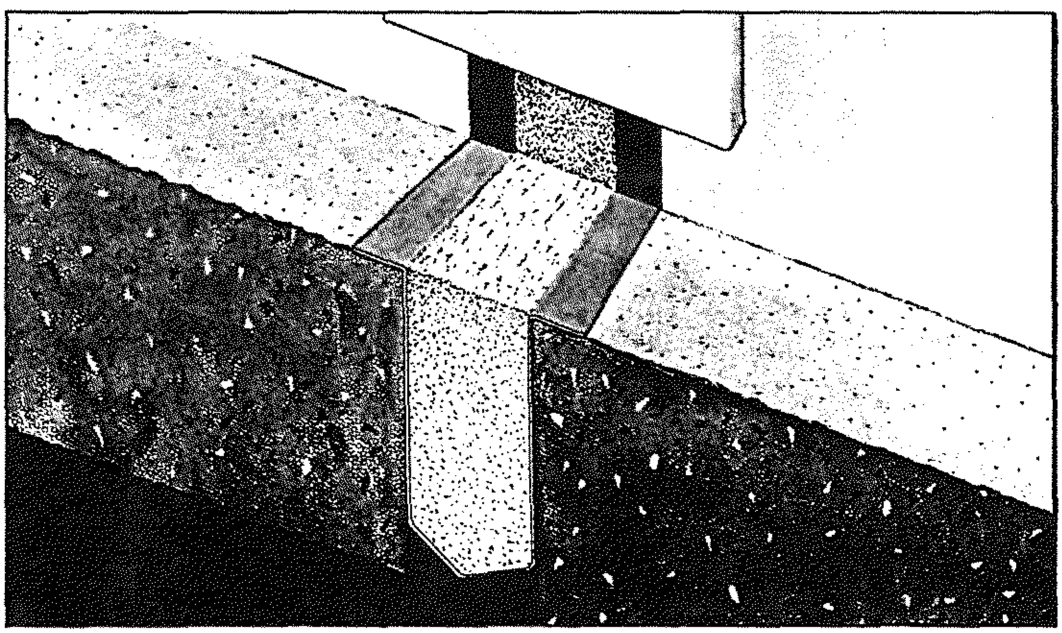


Fig. 2 — Fire rated filled expansion joint (courtesy Architectural Art Mfg., Inc.)

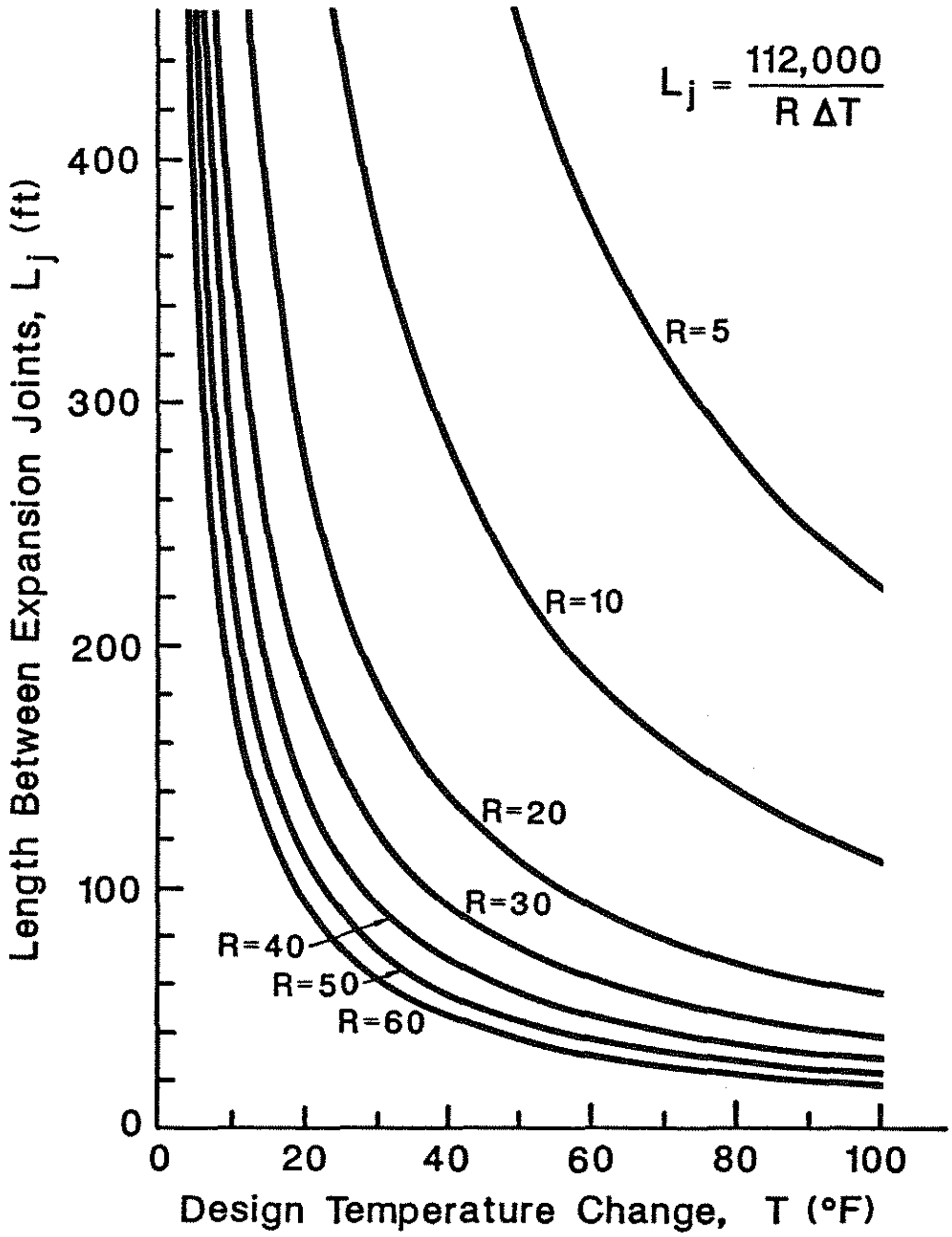


Fig. 3 — Length between expansion joints vs. design temperature change, ΔT (Martin & Acosta 1970)(1 ft = 0.305 m.; 1°F = $\frac{5}{9}$°C)

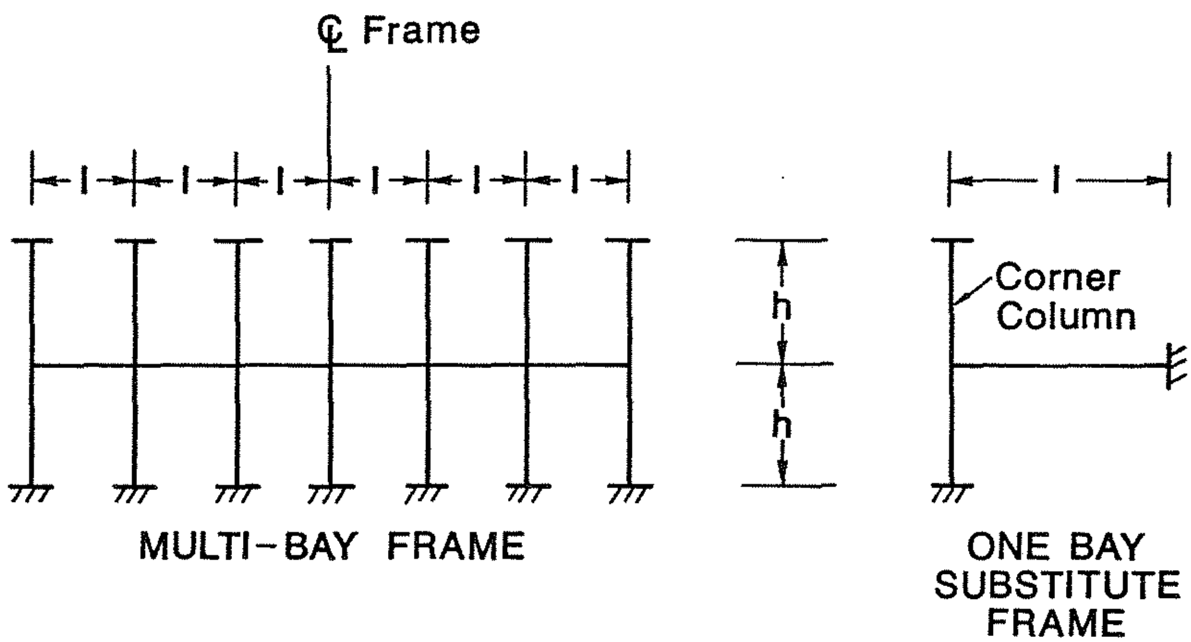
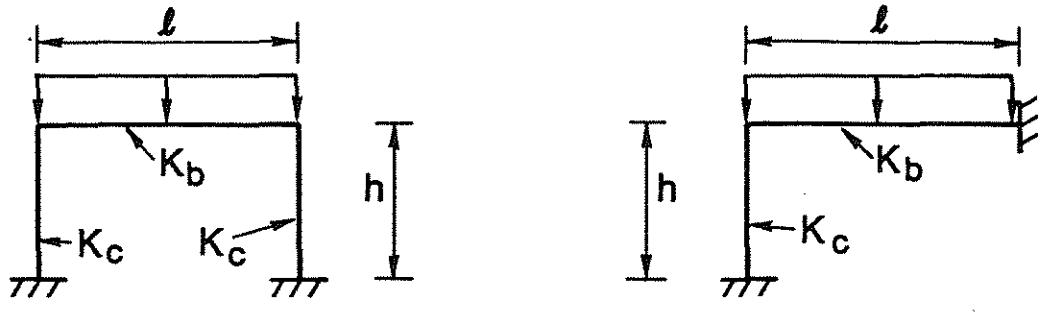


Fig. 4 — Multi-bay frame and one bay substitute frame (after Varyani & Radhaji 1978)

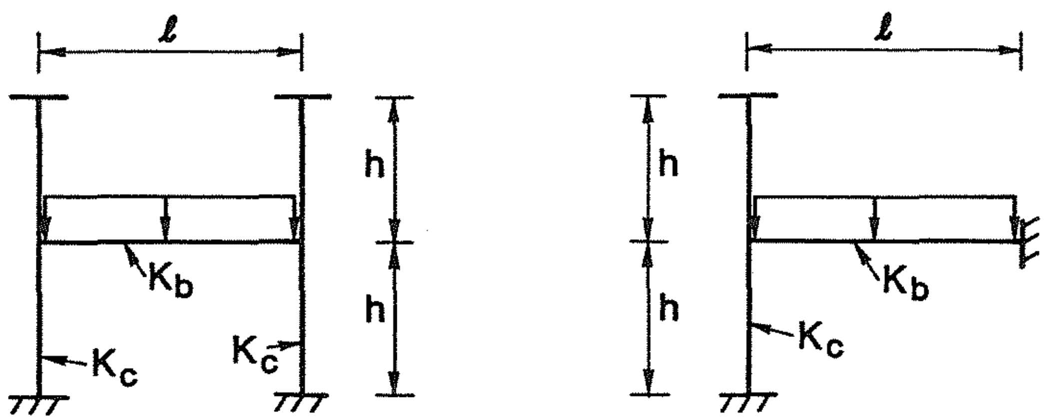


$$M_g = \frac{M_f}{(2+r')}$$

SINGLE-STORY SINGLE BAY

$$M_g = \frac{M_f}{2(1+r')}$$

SINGLE-STORY MULTI-BAY



$$M_g = \frac{M_f}{(4+r')}$$

MULTI-STORY SINGLE BAY

$$M_g = \frac{M_f}{2(2+r')}$$

MULTI-STORY MULTI-BAY

M_f = Beam fixed-end moment

$$r' = r^{-1} = \frac{K_b}{K_c}$$

Fig. 5 — Moments at base of corner columns due to gravity using one bay substitute frames (after Varyani & Radhaji 1978)

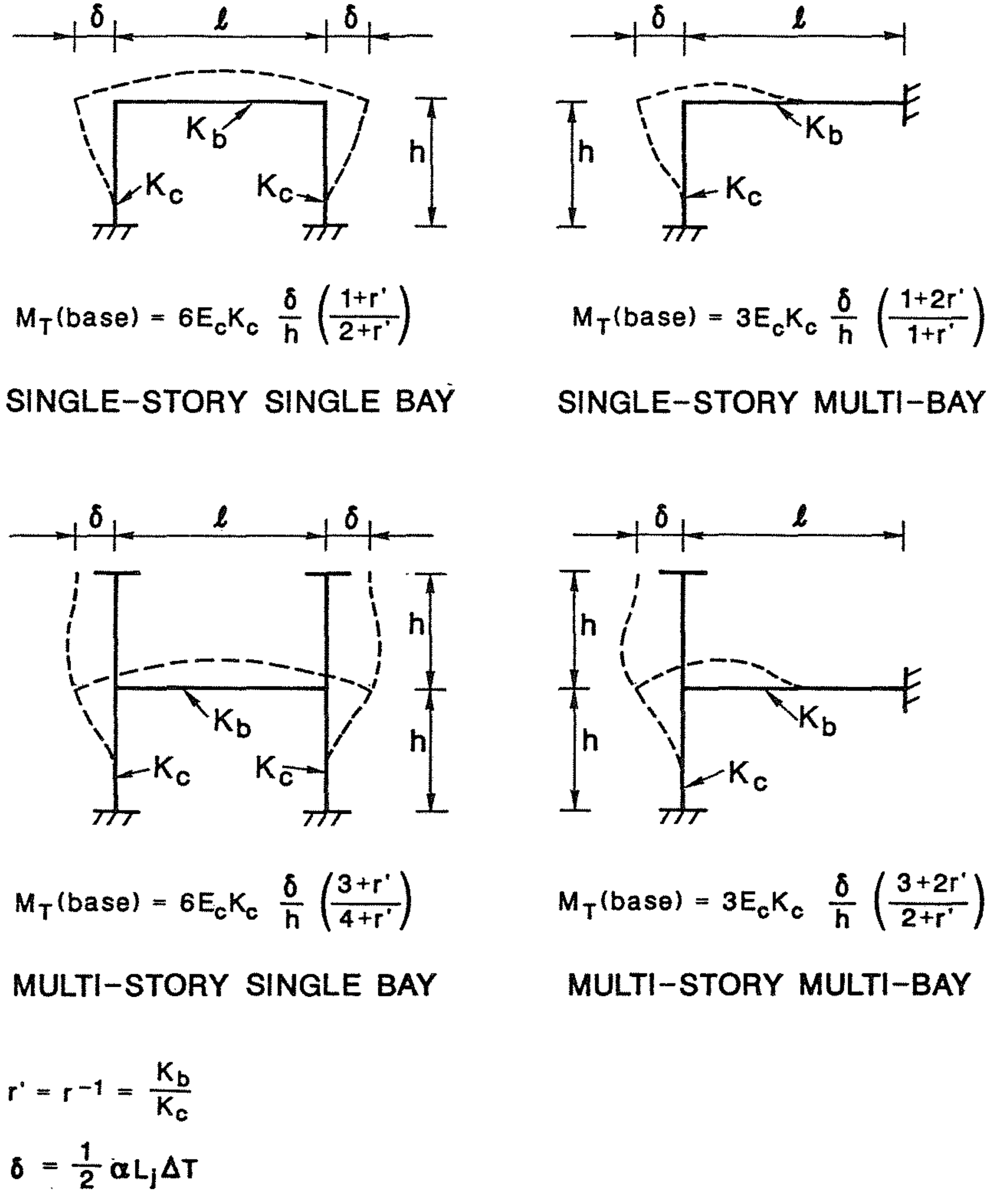


Fig. 6 — Moments at base of corner columns due to temperature change using one bay sub- stitute frame, L_j = total length between expansion joints (after Varyani & Radhaji 1978).

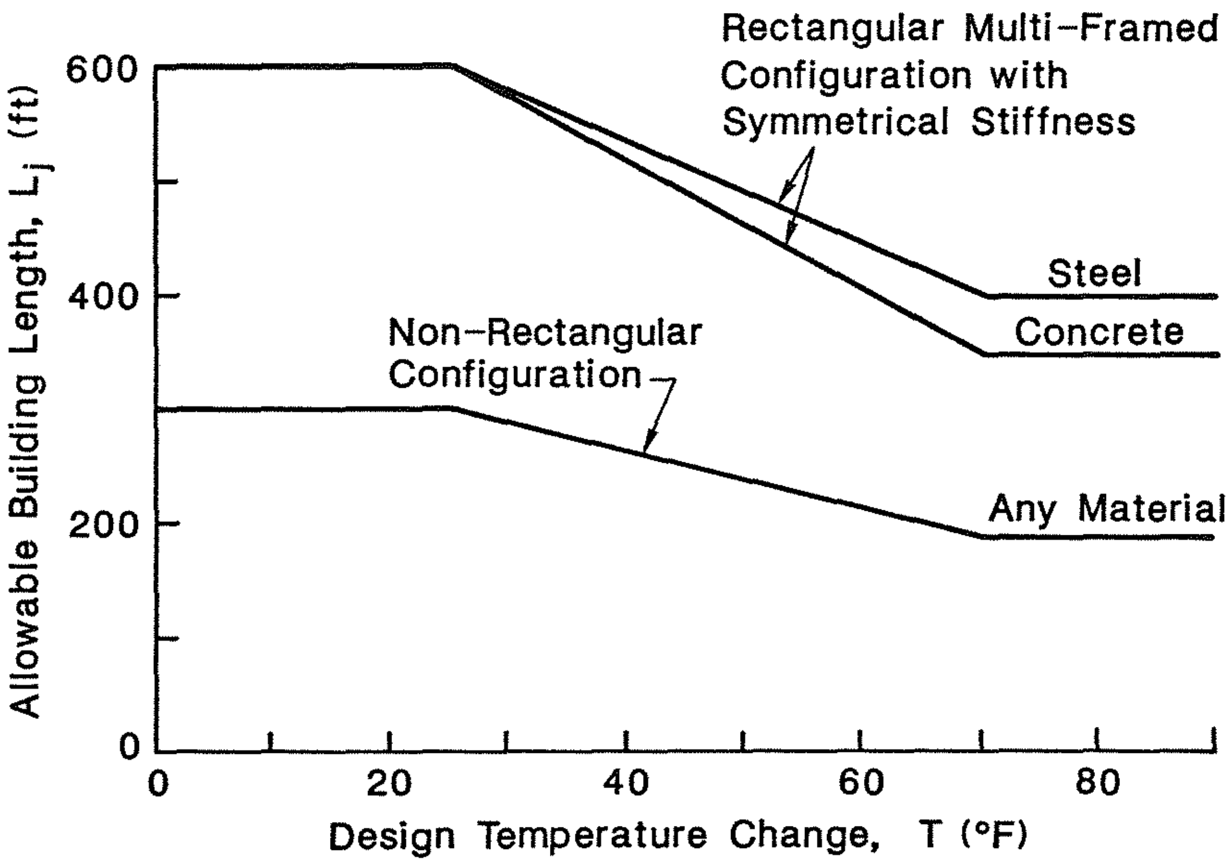


Fig. 7 — Expansion joint criteria of the Federal Construction Council (National Academy of Sciences 1974)(1 ft = 0.305 m; 1°F = $\frac{5}{9}$°C)

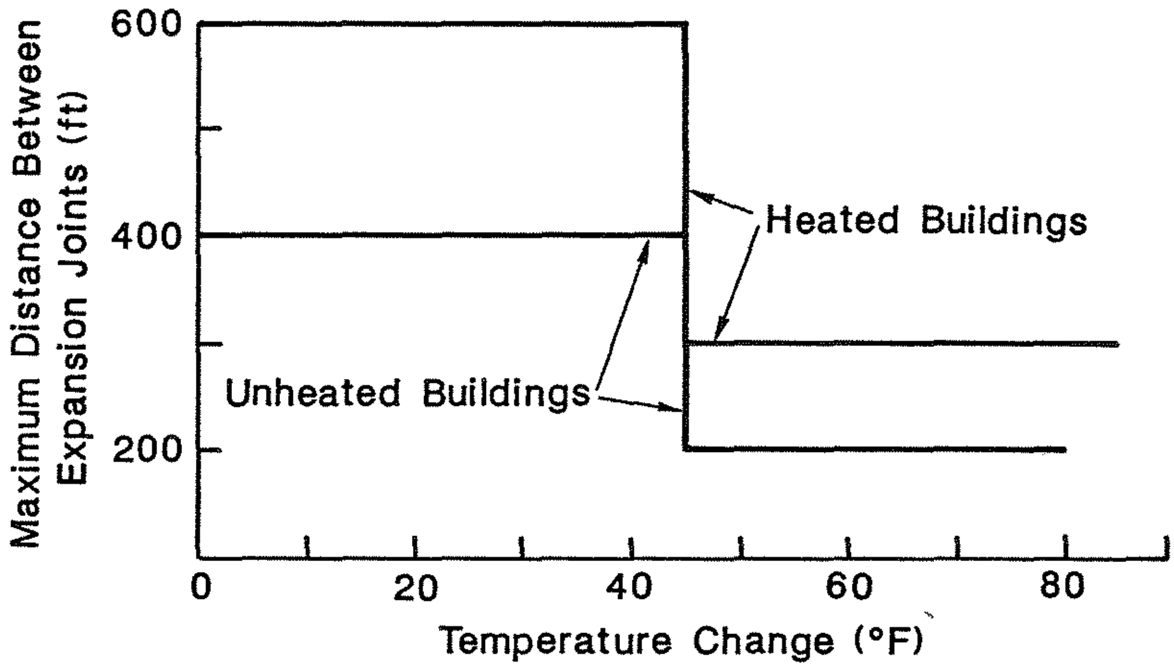


Fig. 8 — Expansion joint criteria of one federal agency (National Academy of Sciences 1974)(1 ft = 0.305 m; 1°F = $\frac{5}{9}$°C)

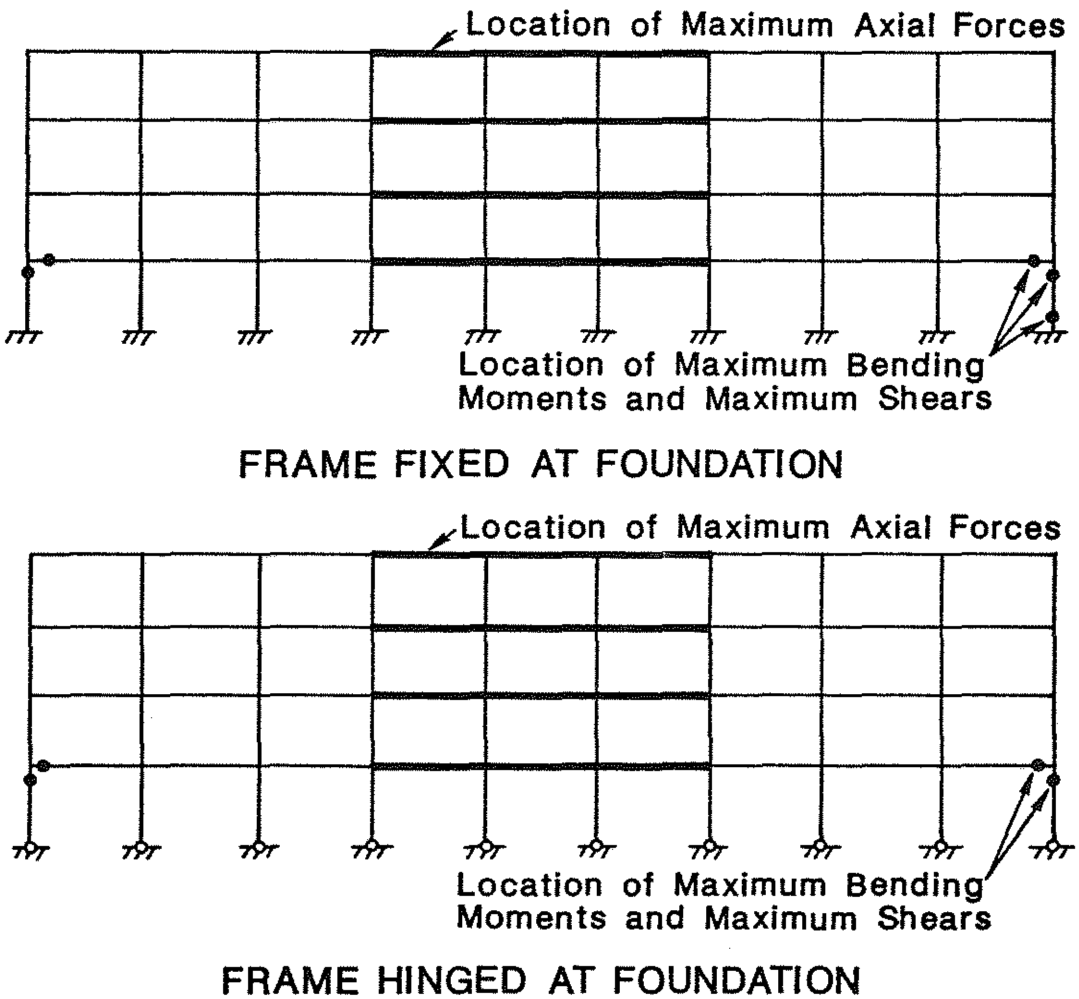


Fig. 9 — Frames subjected to a uniform temperature change (National Academy of Sciences 1974)

APPENDIX A - NOTATION

A	cross-sectional area of the element under consideration;
D	unfactored dead load;
E_c	modulus of elasticity of concrete;
F	force resulting in a restrained member due to a temperature change, ΔT;
f_c'	specified compressive strength of concrete;
f_{tc}	tensile capacity of concrete;
h	column height;
I_b	moment of inertia of the beam;
I_c	moment of inertia of the column;
K_b	beam stiffness factor;
K_c	column stiffness factor;
L	unfactored live load;
l	beam length;
L_j	total length of building between expansion joints;
M_f	fixed-end moments at the beam-column joints due to gravity loading, unfactored;
M_g	column moment due to gravity, unfactored;
M_T	column moment due to temperature, unfactored;
r	ratio of the stiffness factor of the column to the stiffness factor of the beam = K_c/K_b;
r'	inverse of r, $r^{-1} = K_b/K_c$;
T	unfactored thermal load;
T_c	temperature equaled or exceeded, on the average, 99 percent of the time during the winter months of December, January, and February;
T_m	temperature during the normal construction season in the locality of the building;

APPENDIX - NOTATION, cont'd.

T_{max} extreme normal daily maximum temperature;

T_{min} extreme normal daily minimum temperature;

T_w temperature exceeded, on the average, only 1 percent of the time during the summer months of June through September;

U required strength of the element under consideration;

w uniformly distributed load;

α coefficient of thermal expansion for concrete;

ΔT design temperature change; and

δ lateral frame deflection between expansion joints due to temperature change ΔT.

APPENDIX B - EXPANSION JOINT EXAMPLES

Example 1: Single Story-Multi Bay Building

Given:

A one-story, 20 ft high warehouse is to be constructed in St. Louis, Missouri. The building is to be a reinforced concrete frame with plan dimensions of 210 ft (east-west) by 600 ft (north-south). Columns are spaced at 30 ft in each direction.

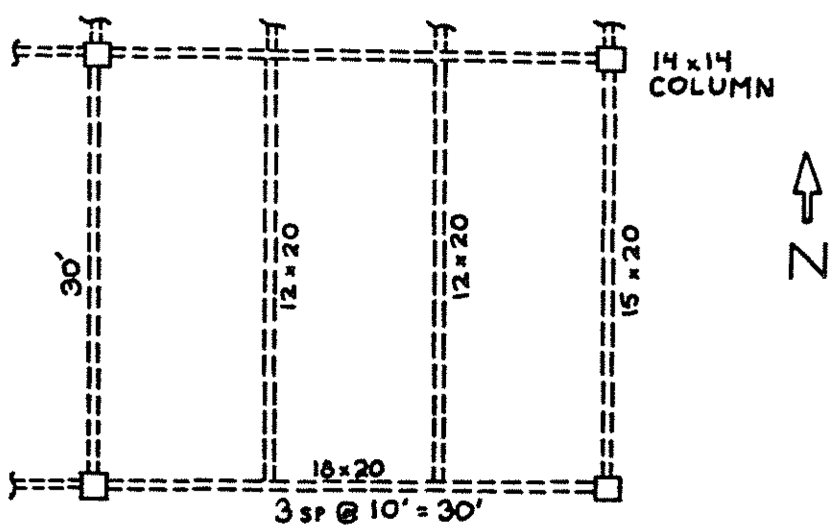
Design Parameters:

Roof dead load is 5 psf (not including slab) and roof live load is 20 psf. Concrete design strength, f'_c = 6 ksi

Assumptions:

Architectural and preliminary design considerations result in the following member sizes: 5 in. roof slab thickness, 12 in. x 20 in. interior beams, 15 in. x 20 in. perimeter beams, 18 in. x 20 in. girders, and 14 in. x 14 in. columns.

Typical Corner Section:



Required:

Using each of the three methods described in the text, determine the maximum length between expansion joints. Base calculations on uncracked gross section properties.

Martin and Acosta (1970)

Step 1: Calculate member properties:

a) Column and beam moments of inertia, I:

$$I_c = \frac{14^4}{12} = 3201 \text{ in.}^4$$

$$\text{N-S: } I_b = \frac{(15)(20)^3}{12} = 10{,}000 \text{ in.}^4$$

$$\text{E-W: } I_b = \frac{(18)(20)^3}{12} = 12{,}000 \text{ in.}^4$$

b) Column and beam stiffness factors, K:

$$K_c = \frac{I_c}{h} = \frac{3201 \text{ in.}^4}{20 \text{ ft x } 12} = 13.34 \text{ in.}^3$$

$$\text{N-S: } K_b = \frac{I_b}{\ell} = \frac{10{,}000 \text{ in.}^4}{30 \text{ ft x } 12} = 27.78 \text{ in.}^3$$

$$\text{E-W: } K_b = \frac{I_b}{\ell} = \frac{12{,}000 \text{ in.}^4}{30 \text{ ft x } 12} = 33.33 \text{ in.}^3$$

c) Ratio of column to beam stiffness factor, r:

$$\text{N-S: } r = \frac{K_c}{K_b} = \frac{13.34 \text{ in.}^3}{27.78 \text{ in.}^3} = 0.480$$

$$\text{E-W: } r = \frac{13.34 \text{ in.}^3}{33.33 \text{ in.}^3} = 0.400$$

Step 2: Calculate design temperature changes, ΔT, using Eq. (4):

$$\Delta T = \frac{2}{3}(T_{max} - T_{min}) + 30°F \tag{4}$$

Substituting from Table 5 for St. Louis, MO,

$$T_{max} = 89.2°F$$

$$T_{min} = 23.25°F$$

$$\Delta T = \frac{2}{3}(89.2°F - 23.5°F) + 30°F$$

$$= 73.8°F$$

Step 3: Calculate value of R, using Eq. (6):

$$R = 144 \frac{I_c}{h^2} \left(\frac{1 + r}{1 + 2r}\right) \tag{6}$$

Substituting h = 20 ft x 12 in., and I_c and r from Step 1,

$$\text{N-S: } R = \frac{(144)(3201 \text{ in.}^4)}{(20 \text{ ft} \times 12)^2} \left(\frac{1 + 0.480}{1 + (2)(0.480)}\right) = 6.043$$

$$\text{E-W: } R = \frac{(144)(3201 \text{ in.}^4)}{(20 \text{ ft} \times 12)^2} \left(\frac{1 + 0.400}{1 + (2)(0.400)}\right) = 6.224$$

Step 4: Calculate expansion joint spacing, L_j, using Eq. (5):

$$L_j = \frac{112{,}000}{R \, \Delta T} \tag{5}$$

Substituting R (Step 3) and ΔT (Step 2),

$$\text{N-S: } L_j = \frac{112{,}000}{(6.043)(73.8)} = 251.1 \text{ ft}$$

$$\text{E-W: } L_j = \frac{112{,}000}{(6.224)(73.8)} = 243.8 \text{ ft}$$

Step 5: Compare expansion joint spacing from Step 4 to the limitation expressed in Eq. (8):

$$L_j \leq \frac{2000h}{\Delta T} \tag{8}$$

$$L_j \leq \frac{2000(20)}{73.8} = 542.0 \text{ ft}$$

The values obtained in Step 4 control.

Varyani and Radhaji (1978)

Step 1: Calculate member properties:

See Step 1 of Martin and Acosta

Step 2: Calculate r':

$$r' = \frac{1}{r}$$

Substituting r from Step 1 of Martin and Acosta

$$\text{N-S: } r' = \frac{1}{0.480} = 2.083$$

$$\text{E-W: } r' = \frac{1}{0.400} = 2.500$$

Step 3: Calculate design temperature change, ΔT, using Eq. (20a) and (20b):

$$\Delta T = \frac{1}{2}[- \frac{2}{3}(T_{max} - T_{min}) - 27°F] \qquad (20a)$$

Substituting values for T_{max} and T_{min} (see Step 2 of Martin and Acosta)

$$\Delta T = \frac{1}{2}[- \frac{2}{3}(89.2 - 23.5) - 27°F]$$

$$= -35.4°F \text{ for contraction}$$

$$\Delta T = \frac{1}{2}[\frac{2}{3}(T_{max} - T_{min}) - 14°F] \qquad (20b)$$

$$= \frac{1}{2}[\frac{2}{3}(89.2 - 23.5) - 14°F)]$$

$$= 29.1°F \text{ for expansion}$$

Note that although the absolute value of ΔT is greater for contraction, the value obtained for expansion will usually control since it adds to the negative gravity moments at exterior beam-column connections.

Step 4: Calculate factored fixed-end moments, M_f:

Design parameters result in the following factored loads.

N-S: W_u(dead) = 0.857 kips/ft

$W_u(\text{live}) = 0.151$ kips/ft

E-W: $W_u(\text{dead}) = 0.525$ kips/ft

$W_u(\text{live}) = 0$

$R_u(\text{dead}) = 18.0$ kips (beam reactions)

$R_u(\text{live}) = 5.1$ kips (beam reactions)

N-S: $M_f(\text{dead}) = \dfrac{(0.857)(30)^2}{12} = 64.3$ ft-kips

$M_f(\text{live}) = \dfrac{(0.151)(30)^2}{12} = 11.3$ ft-kips

E-W: $M_f(\text{dead}) = \dfrac{(0.525)(30)^2}{12} + \dfrac{(18)(10)(20)^2}{30^2} + \dfrac{(18)(10)(20)^2}{30^2}$

$= 159.4$ ft-kips

$M_f(\text{live}) = \dfrac{(5.1)(10)(20)^2}{30^2} + \dfrac{(5.1)(10)(20)^2}{30^2}$

$= 34.0$ ft-kips

Step 5: Calculate factored gravity moments, M_g, at the base and top of the column using Eq. (13) and (27):

Base

$$M_g(\text{base}) = \frac{M_f}{2(1 + r')} \qquad (13)$$

Substituting M_f (Step 4) and r' (Step 2),

N-S: $M_g(\text{base}) = \dfrac{64.3 + 11.3}{2(1 + 2.083)} = 12.26$ ft-kips

E-W: $M_g(\text{base}) = \dfrac{159.4 + 34.0}{2(1 + 2.500)} = 27.63$ ft-kips

Top

$$M_g(\text{top}) = \frac{M_f}{(1 + r')} = 2\ M_g(\text{base}) \qquad (27)$$

N-S: $M_g(\text{top}) = 24.52$ ft-kips

E-W: $M_g(\text{top}) = 55.26$ ft-kips

Step 6: Using Eq. (9) and (10), calculate the unfactored temperature induced moment, M_T, at base and top of the column, assuming column design is governed by the factored gravity moment obtained at the top of the column, M_g(top). This procedure is based on the assumption that the entire column, including the connection at the base, is proportioned based on the maximum gravity moment, which occurs at the top of the column.

$$U = 1.4D + 1.7L \qquad (9)$$

$$U = .75(1.4D + 1.7L + 1.4T) \qquad (10)$$

Substituting $U = 1.4D + 1.7L = M_g$ and $T = M_T$,

$$M_g(\text{top}) = 0.75\ [M_g(\text{base or top}) + 1.4M_T(\text{base or top})]$$

Base

N-S: $24.52 = 0.75(12.26 + 1.4\ M_T)$

$M_T(\text{base}) = 14.60$ ft-kips

E-W: $55.26 = 0.75(27.63 + 1.4\ M_T)$

$M_T(\text{base}) = 32.89$ ft-kips

Top

N-S: $24.52 = 0.75(24.52 + 1.4\ M_T)$

$M_T(\text{top}) = 5.84$ ft-kips

E-W: $55.26 = 0.75(55.26 \text{ ft-kip} + 1.4\ M_T)$

$M_T(\text{top}) = 13.17$ ft-kips

Step 7: Using Eq. (22), calculate expansion joint spacing, L_j, based on the column moments at the base of the column:

$$L_j = \frac{2M_T}{3E_cK_c\alpha\Delta T}\left(\frac{1 + r'}{1 + 2r'}\right) \tag{22}$$

Substituting M_T(base) from Step 6, h = 20 ft x 12, r' (Step 2), $E_c = 57{,}000\sqrt{6000} = 4{,}415{,}201$ psi (ACI 318), K_c (Step 1), $\alpha = 5.5 \times 10^{-6}$, and $\Delta T = 29.1°F$ (Step 3),

$$\text{N-S: } L_j = \frac{(2)(14.60\text{x}12{,}000)(20\text{x}12)\left(\frac{1+2.083}{1+(2)(2.083)}\right)}{(3)(4{,}415{,}201)(13.34)(5.5\text{x}10^{-6})(29.1)}$$

$= 1770$ in. $= 147.5$ ft

$$\text{E-W: } L_j = \frac{(2)(32.89\text{x}12{,}000)(20\text{x}12)\left(\frac{1 + 2.500}{1+(2)(2.500)}\right)}{(3)(4{,}415{,}201)(13.34)(5.5\text{x}10^{-6})(29.1)}$$

$= 3904$ in. $= 325$ ft

Step 8: Using Eq. (28), calculate expansion joint spacing, L_j, based on the column moments at the top of the column:

$$M_T(\text{top}) = 6E_cK_c \frac{\delta}{h} \left(\frac{r'}{1 + r'}\right) \tag{28}$$

Substituting Eq. (7) for δ into Eq. (28) and solving for L_j results in the equation for calculating L_j directly,

$$L_j = \frac{M_T h}{3E_cK_c\alpha\Delta T} \left(\frac{1 + r'}{1 + 2r'}\right)$$

Substituting M_T(top), h, r', E_c, K_c, and ΔT gives,

$$\text{N-S: } L_j = \frac{(5.84\text{x}12{,}000)(20\text{x}12)\left(\frac{1 + 2.083}{2.083}\right)}{(3)(4{,}415{,}201)(13.34)(5.5\text{x}10^{-6})(29.1)}$$

$$= 879.5 \text{ in.} = 73.3 \text{ ft}$$

$$\text{E-W: } L_j = \frac{(13.17\text{x}12{,}000)(20\text{x}12)\left(\frac{1 + 2.500}{2.500}\right)}{(3)(4{,}415{,}201)(13.34)(5.5\text{x}10^{-6})(29.1)}$$

$$= 1878.3 \text{ in.} = 156.5 \text{ ft}$$

Step 9: Summary of expansion joint spacings, L_j:

a) Based on base of column moment calculations (Step 7)

N-S: L_j = 147.5 ft

E-W: L_j = 325.0 ft

b) Based on top of column moment calculations (Step 8)

N-S: L_j = 73.3 ft

E-W: L_j = 156.5 ft

National Academy of Sciences (1974)

Step 1: Calculate design temperature changes, ΔT, using Eq. (33a) and (33b):

$$\Delta T = T_w - T_m \tag{33a}$$

Substituting from Table 6 for St. Louis, MO:

$T_w = 98°F$

$T_m = 65°F$

$\Delta T = 98°F - 65°F$

$= 33°F$

$$\Delta T = T_m - T_c \tag{33b}$$

$T_m = 65°F$

$T_c = 4°F$

$\Delta T = 65°F - 4°F$

$= 61°F$

Therefore, use $\Delta T = 61°F$

Step 2: Using Fig. 5, compute the allowable building length assuming ΔT = 61°F:

From Figure 5, allowable building length is approximately 400 ft

Step 3: Apply modification factors:

Fixed column bases, decrease length by 15%.

$L_j = 400 \text{ ft} - (400)(0.15)$

$= 340 \text{ ft}$

Example 2: Multi-Story, Multi-Bay Building

Given:

A four-story, 56 ft high business complex is to be constructed in St. Louis, Missouri. The building is to be a reinforced concrete frame with plan dimensions of 210 ft (east-west) by 600 ft (north-south). Columns are spaced at 30 ft in each direction. Each story is 14 ft high.

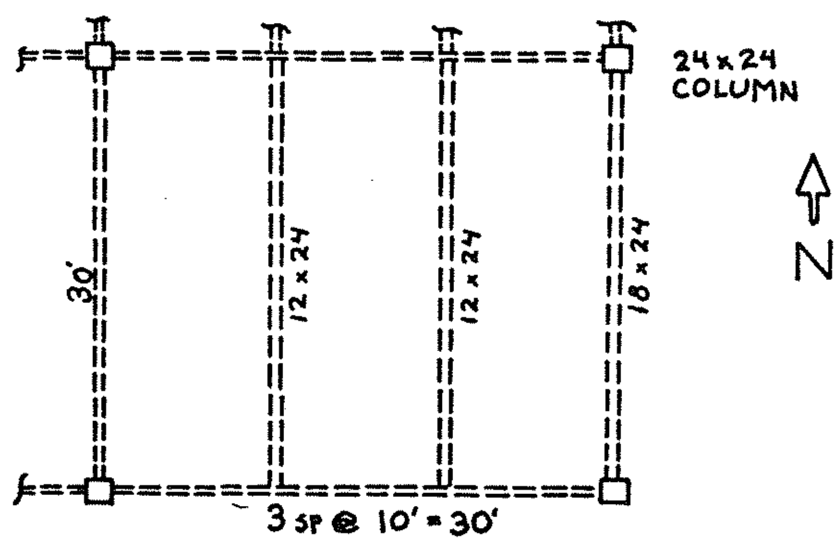
Design parameters:

Roof dead load is 5 psf, roof live load is 20 psf, and floor live load is 125 psf. Concrete design strength, f'_c = 6 ksi.

Assumptions:

Architectural and preliminary design considerations result in the following member sizes: 5 in. slab thickness (roof and floors), 12 in. x 24 in. interior beams, 18 in. x 24 in. perimeter beams, 24 in. x 24 in. girders, and 24 in. x 24 in. columns.

Typical Corner Section:



Required:

Determine the maximum length between joints.

Varyani and Radhaji (1978)

Step 1: Calculate member properties;

a) Column and beam moment of inertia, I,

$$I_c = \frac{24^4}{12} = 27,648 \text{ in.}^4$$

$$\text{N-S: } I_b = \frac{(18)(24)^3}{12} = 20,736 \text{ in.}^4$$

$$\text{E-W: } I_b = \frac{(24)(24)^3}{12} = 27,648 \text{ in.}^4$$

b) Column and beam stiffness factors, K:

$$K_c = \frac{I_c}{h} = \frac{27,648}{14\text{x}12} = 164.57 \text{ in.}^3$$

$$\text{N-S: } K_b = \frac{I_b}{\ell} = \frac{20,736}{30\text{x}12} = 57.6 \text{ in.}^3$$

$$\text{E-W: } K_b = \frac{I_b}{\ell} = \frac{27,648}{30\text{x}12} = 76.8 \text{ in.}^3$$

Step 2: Calculate r':

$$\text{N-S: } r' = \frac{K_b}{K_c} = \frac{57.6}{164.57} = 0.350$$

$$\text{E-W: } r' = \frac{K_b}{K_c} = \frac{76.8}{164.57} = 0.467$$

Step 3: Calculate design temperature change, ΔT, using Eq. (20a) and (20b): See Example 1.

$$\Delta T = 29.1°F$$

Step 4: Calculate fixed-end moments, M_f:

Design parameters result in the following factored loads,

N-S: $W_u(\text{dead}) = 1.013$ kips/ft

$W_u(\text{live}) = 0.929$ kips/ft

E-W: $W_u(\text{dead}) = 0.840$ kips/ft

$W_u(\text{live}) = 0$ kips/ft

$R_u(\text{dead}) = 18.1$ kips

$R_u(\text{live}) = 31.9$ kips

$$\text{N-S: } M_f(\text{dead}) = \frac{(1.013)(30)^2}{12} = 75.98 \text{ ft-kips}$$

$$M_f(\text{live}) = \frac{(0.929)(30)^2}{12} = 69.68 \text{ ft-kips}$$

$$\text{E-W: } M_f(\text{dead}) = \frac{(0.840)(30)^2}{12} + \frac{(18.1)(10)(20)^2}{30^2} + \frac{(18.1)(10)(20)^2}{30^2}$$

$$= 183.75 \text{ ft-kips}$$

$$M_f(\text{live}) = \frac{(31.9)(10)(20)^2}{30^2} + \frac{(31.9)(20)(10)^2}{30^2}$$

$$= 212.5 \text{ ft-kips}$$

Step 5: Calculate factored gravity moments, M_g, at the base and top of the column using Eq. (15) and (31):

Base

$$M_g(\text{base}) = \frac{M_f}{2(2+r')} \qquad (15)$$

Substituting M_f (Step 4) and r' (Step 2),

$$\text{N-S: } M_g(\text{base}) = \frac{75.98 + 69.68}{2(2 + .350)} = 30.99 \text{ ft-kips}$$

$$\text{E-W: } M_g(\text{base}) = \frac{183.75 + 212.5}{2(2 + .467)} = 80.31 \text{ ft-kips}$$

Top

$$M_g(\text{top}) = \frac{M_f}{(2+r')} = 2M_g(\text{base}) \qquad (31)$$

$$\text{N-S: } M_g(\text{top}) = 61.98 \text{ ft-kips}$$

$$\text{E-W: } M_g(\text{top}) = 160.62 \text{ ft-kips}$$

Step 6: Using Eq. (9) and (10), calculate temperature induced moment, M_T, at the base and top of the column assuming column design to be governed by gravity moment obtained at the top of the column, M_g(top). Follow the same procedure as used in Step 6 of Varyani and Radhaji in Example 1.

Base

$$\text{N-S: } 61.98 = 0.75(30.99 + 1.4\ M_T)$$

$$M_T(\text{base}) = 36.89 \text{ ft-kips}$$

$$\text{E-W: } 160.62 = 0.75(80.31 + 1.4\ M_T)$$

$M_T(\text{base}) = 95.61$ ft-kips

Top

N-S: $61.98 \text{ ft-kip} = 0.75(61.98 + 1.4\ M_T)$

$M_T(\text{top}) = 14.76$ ft-kips

E-W: $160.62 \text{ ft-kip} = 0.75(160.62 + 1.4\ M_T)$

$M_T(\text{top}) = 38.24$ ft-kips

Step 7: Using Eq. (24), calculate expansion joint spacing, L_j, based on the column moments at the base of the column.

$$L_j = \frac{2M_T h}{3E_c K_c \alpha \Delta T}\left(\frac{2 + r'}{3 + 2r'}\right) \qquad (24)$$

Substituting M_T(base) from Step 6, h = 14 ft x 12, r' (Step 2), E_c = 4,415,201 psi, K_c (Step 1), $\alpha = 5.5 \times 10^{-6}$, and $\Delta T = 29.1°F$ (Step 3),

$$\text{N-S: } L_j = \frac{(2)(36.89\text{x}12{,}000)(14\text{x}12)\left(\frac{2+0.350}{3+(2)(0.350)}\right)}{(3)(4{,}415{,}201)(164.57)(5.5\text{x}10^{-6})(29.1)}$$

$= 222.4$ in. $= 18.5$ ft

$$\text{E-W: } L_j = \frac{(2)(95.61\text{x}12{,}000)(14\text{x}12)\left(\frac{2+0.467}{3+(2)(0.467)}\right)}{(3)(4{,}415{,}201)(13.34)(5.5\text{x}10^{-6})(29.1)}$$

$= 570.0$ in. $= 47.5$ ft

Step 8: Using Eq. (32), calculate expansion joint spacing, L_j, based on the column moments at the top of the column,

$$M_T = 6E_cK_c \frac{\delta}{h} \left(\frac{1 + r'}{2 + r'}\right) \tag{32}$$

Substituting Eq. (7) for δ into Eq. (32) and solving for L_j results in an equation for calculating L_j directly.

$$L_j = \frac{M_T}{3E_cK_c\alpha\Delta T} \left(\frac{2 + r'}{1 + r'}\right)$$

Substituting M_T(top), h, r', E_c, K_c, and ΔT gives

$$\text{N-S: } L_j = \frac{(14.76\text{x}12,000)(14\text{x}12)\left(\frac{2+0.350}{1+0.350}\right)}{(3)(4,415,201)(164.57)(5.5\text{x}10^{-6})(29.1)}$$

$$= 148 \text{ in.} = 12.4 \text{ ft}$$

$$\text{E-W: } L_j = \frac{(38.24\text{x}12,000)(14\text{x}12)\left(\frac{2+0.467}{1+0.467}\right)}{(3)(4,415,201)(164.57)(5.5\text{x}10^{-6})(29.1)}$$

$$= 371 \text{ in.} = 30.9 \text{ ft}$$

Step 9: Summary of expansion joint spacings, L_j:

a) Based on base of column moment calculations

N-S: L_j = 18.5 ft

E-W: L_j = 47.5 ft

b) Based on top of column moment calculations

N-S: L_j = 12.4 ft

E-W: L_j = 30.9 ft

National Academy of Science (1974)

This method does not distinguish between single and multistory structures. Thus, since the temperature changes and degree of column fixity are the same for this structure as for the single story warehouse in Example 1, the required value of expansion joint spacing is the same.

L_j = 340 ft

Comparison and Discussion

		Single Story	Multi-Story
Martin and Acosta (1970)			
	N-S:	L_j = 251.1 ft	Not Applicable
	E-W:	L_j = 243.8 ft	Not Applicable
Varyani and Radhaji (1978)			
(Based on base of column)	N-S:	L_j = 147.5 ft	L_j = 18.5 ft
	E-W:	L_j = 325.0 ft	L_j = 47.5 ft
(Based on top of column)	N-S:	L_j = 73.3 ft	L_j = 12.4 ft
	E-W:	L_j = 156.5 ft	L_j = 30.9 ft
National Academy of Sciences (1974)			
	N-S:	L_j = 340 ft	L_j = 340 ft
	E-W:	L_j = 340 ft	L_j = 340 ft

As demonstrated by these examples, the resulting expansion joint spacings vary greatly between methods. In some cases, the answers are ridiculously low. All three methods utilize variables that require the designer to make certain assumptions. These assumptions are critical and may be a partial cause of the disparity between the results.

Martin and Acosta's (1970) method requires that the designer to decide whether to use a cracked or uncracked moment of inertia. Using uncracked moments of inertia results in lower values of L_j. As pointed out, Eq. (5) is based on load factors [Eq. (1) and (2)] that are more conservative than those now in use [Eq. (9) and (10)]. Conservative load factors will also decrease the calculated value of L_j. Assumptions concerning which modification factors to use affect the results obtained with the National Academy of Sciences (1974) method. Revising the original assumptions and modifying Eq. (5) to account for the current more liberal load factors will lead to a closer correlation between Martin and Acosta (1970) and the National Academy (1974).

Varyani and Radhaji's (1978) method provides the most unrealistic values of L_j and is clearly sensitive to the design assumptions. Using a cracked, instead of uncracked moment of inertia will increase the calculated values of L_j, but the extremely short joint spacings are primarily the result of the premise that the combined factored gravity and thermal moments [Eq. (10)] should not exceed the factored gravity moments [Eq. (10)]. Typical design practice often results in columns that have a structural capacity that far exceeds the factored moments. If this is the case, a more appropriate way of calculating the "usable" temperature induced moment, M_T, would be to equate the combined gravity and thermal effects to the actual capacity of the column, rather than the factored gravity moment. This is obtained by modifying Eq. (10) as follows:

$$\phi M_n = 0.75(1.4M_D + 1.7M_L + 1.4M_T) \qquad \text{(B1)}$$

Eq. (B1) will yield longer, more realistic joint spacings than Eq. (10). For a fixed building geometry (and L_j), M_T is also fixed. In this case, ϕM_n can be increased to satisfy Eq. (B1).

The preceding examples illustrate the strengths and weaknesses of the proposed methods. The method requiring the least effort (National Academy of Sciences 1974) produces joint spacings that are in line with current practice. The most time-consuming method (Varyani and Radhaji 1978) can produce unreasonably low values of L_j and appears to need additional modifications before an acceptable version is available. The final determination of which method to use rests with the designer and must provide an expansion joint spacing that will limit member forces without adversely affecting structural integrity or serviceability.