Measurement of the $B_s^0$ Lifetime in the Flavor-Specific Decay Channel $B_s^0 \to D^- \mu^+ \nu X$

The decays of hadrons containing a b quark are dominated by the weak interaction of the b quark. In first-order calculations, the decay widths of these hadrons are independent of the flavor of the accompanying light quark(s). Higher-order predictions break this symmetry, with the spectator quarks having roles in the time evolution of the B hadron decay [1,2]. The flavor dependence leads to an expected lifetime hierarchy of \( \tau(B_s^+) < \tau(B_{s0}) < \tau(B_s^-) < \tau(B^0) \), which has been observed experimentally [3].

The ratios of the lifetimes of different B hadrons are precisely predicted by heavy quark effective theories and provide a way to experimentally study these higher-order effects, and to test for possible new physics beyond the standard model [4]. Existing measurements are in excellent agreement with predictions [3] for the lifetime ratio \( \tau(B_s^+)/\tau(B^0) \), but until recently the experimental precision has been insufficient to test the corresponding theoretical prediction for \( \tau(B_s^0)/\tau(B^0) \). In particular, predictions using inputs from unquenched lattice QCD calculations give 0.996 < \( \tau(B_s^0)/\tau(B^0) \) < 1 [2]. More precise measurements of both \( B^0_s \) lifetime and the ratio to its lighter counterparts are needed to test and refine the models.

A flavor-specific final state such as \( B_s^0 \rightarrow D^-\mu^+\nu \) is one where the charges of the decay products can be used to know whether the meson was a \( B_s^0 \) or \( \bar{B}_s^0 \) at the time of decay. As a consequence of neutral B meson flavor oscillations, the \( B^0_s \) lifetime as measured in semileptonic decays is actually a combination of the lifetimes of the heavy and light mass eigenstates with an equal mixture of these two states at time \( t = 0 \). If the resulting superposition of two exponential distributions is fitted with a single exponential function, one obtains to second order [5]

\[
\tau_{fs}(B^0_s) = \frac{1}{\Gamma_s} \left[ 1 + \left( \frac{\Delta \Gamma_s}{2 \Gamma_s} \right)^2 \right],
\]

where \( \Gamma_s = (\Gamma_{sL} + \Gamma_{sH})/2 \) is the average decay width of the light and heavy states, and \( \Delta \Gamma_s \) is the difference \( \Gamma_{sL} - \Gamma_{sH} \). This dependence makes the flavor-specific lifetime an important parameter in global fits [6] used to extract \( \Delta \Gamma_s \), and hence, to constrain possible CP violation in the mixing and interference of \( B^0_s \) mesons.

Previous measurements have been performed by the CDF [7], D0 [8], and LHCb [9,10] Collaborations, with additional earlier measurements from LEP [11]. During Run II of the Tevatron collider from 2002–2011, the D0 detector [12] accumulated 10.4 fb\(^{-1}\) of \( pp \) collisions at a center-of-mass energy of 1.96 TeV. We present a precise measurement of the \( B^0_s \) lifetime that uses the flavor-specific decay \( B^0_s \rightarrow D^-\mu^+\nu \), with \( D^- \rightarrow \phi\pi^- \) and \( \phi \rightarrow K^+K^- \) [13], selected from this dataset. It is superseding our previous measurement [8].

A detailed description of the D0 detector can be found elsewhere [12]. The data for this analysis were collected with a single muon trigger. Events are considered for selection if they contain a muon candidate identified...
through signatures both inside and outside the toroid magnet [12]. The muon must be associated with a central track, have transverse momentum ($p_T$) exceeding 2.0 GeV/c, and a total momentum of $p > 3.0$ GeV/c. Candidate $B_0^0 \rightarrow D_\tau^- \mu^+ \times \pi$ decays are reconstructed by first combining two charged particle tracks of opposite charge, which are assigned the charged kaon mass. Both tracks must satisfy $p_T > 1.0$ GeV/c, and the invariant mass of the two-kaon system must be consistent with a $\phi$ meson, 1.008 GeV/c$^2 < M(K^+K^-) < 1.032$ GeV/c$^2$. This $\phi$ candidate is then combined with a third track, assigned the charged pion mass, to form a $D_\tau^- \rightarrow \phi \pi^-$ candidate. The pion candidate must have $p_T > 0.7$ GeV/c, and the invariant mass of the $\phi \pi^-$ system must lie within a window that includes the $D_\tau^- \rightarrow \phi$ meson, 1.73 GeV/c$^2 < M(\phi \pi^-) < 2.18$ GeV/c$^2$. The combinatorial background is reduced by requiring that the three tracks create a common $D_\tau^-$ vertex as described in Ref. [14]. Lastly, each $D_\tau^-$ meson candidate is combined with the muon to reconstruct a $B_0$ candidate. The invariant mass must be within the range $3$ GeV/c$^2 < M(D_\tau^- \mu^+) < 5$ GeV/c$^2$. All four tracks must be associated with the same $p\bar{p}$ interaction vertex (PV), and have hits in the silicon and fiber tracker detectors.

Muons and pion tracks from genuine $B_0^0$ decays must have opposite charges, which defines the right-sign sample. The wrong-sign sample is also retained to help constrain the background model. In the right-sign sample, the reconstructed $D_\tau^-$ meson is required to be displaced from the PV in the same direction as its momentum in order to reduce background.

The flavor-specific $B_0^0$ lifetime, $\tau(B_0^0)$, can be related to the decay kinematics in the transverse plane, $c\tau(B_0^0) = L_{xy} M / p_T(B_0^0)$, where $M$ is the $B_0^0$ mass, taken as the world average [3], and $L_{xy} = \vec{X} \cdot \vec{p}_T / |\vec{p}_T|$ is the transverse decay length, where $\vec{X}$ is the displacement vector from the PV to the secondary vertex in the transverse plane. Since the neutrino is not detected, and the soft hadrons and photons from decays of excited charm states are not explicitly included in the reconstruction, the $p_T$ of the $B_0^0$ meson cannot be fully reconstructed. Instead, we use the combined $p_T$ of the muon and $D_\tau^-$ meson, $p_T(D_\tau^- \mu^+)$. The reconstructed parameter is the pseudorapidity decay length, $\eta = L_{xy} M / p_T(D_\tau^- \mu^+)$. To model the effects of the missing $p_T$ and of the momentum resolution when the $B_0^0$ lifetime is extracted from the PDPL distribution, a correction factor $K$ is introduced, defined by $K = p_T(D_\tau^- \mu^+) / p_T(B_0^0)$. It is extracted from a Monte Carlo (MC) simulation, separately for a number of specific decays comprising both signal and background components.

MC samples are produced using the PYTHIA event generator [15] to model the production and hadronization phase, interfaced with EVTGEN [16] to model the decays of $b$ and $c$ hadrons. The events are passed through a detailed GEANT simulation of the detector [17] and additional algorithms to reproduce the effects of digitization, detector noise, and pileup. All selection cuts described above are applied to the simulated events. To ensure that the simulation fully describes the data, and in particular, to account for the effect of muon triggers, we weight the MC events to reproduce the muon transverse momentum distribution observed in data.

Table I summarizes the semileptonic $B_0^0$ decays that contribute to the $D_\tau^- \mu^+$ signal. Experimentally, these processes differ only in the varying amount of energy lost to missing decay products, which is reflected in the final $K$-factor distribution. Table II shows the list of non-negligible processes from subsequent semileptonic charm decays which also contribute to the signal. These two tables represent the sample composition of the $D_\tau^- \mu^+$ signal.

### Table I

<table>
<thead>
<tr>
<th>Decay channel</th>
<th>Contribution</th>
<th>$^{\pm}$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_\tau^- \mu^+ \nu_\mu$</td>
<td>(27.5 ± 2.4)%</td>
<td></td>
</tr>
<tr>
<td>$D_\tau^- \mu^+ \bar{\nu}<em>\mu (D</em>\tau^- \rightarrow D_\tau^- \gamma / D_\tau^- \bar{\nu}_\mu)$</td>
<td>(66.2 ± 4.4)%</td>
<td></td>
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<tr>
<td>$D_\tau^- \mu^+ \nu_\mu (D_\tau^- \rightarrow D_\tau^- \bar{\nu}<em>\mu / D</em>\tau^- \gamma)$</td>
<td>(0.4 ± 0.3)%</td>
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<tr>
<td>$D_\tau^- \mu^+ \nu_\tau (\tau^- \rightarrow \mu^+ \nu_\tau)$</td>
<td>(5.9 ± 2.7)%</td>
<td></td>
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</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th>Decay channel</th>
<th>Contribution</th>
<th>$^{\pm}$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \rightarrow D_\tau^- DX$</td>
<td>(3.81 ± 0.75)%</td>
<td></td>
</tr>
<tr>
<td>$B^0 \rightarrow D_\tau^- DX$</td>
<td>(4.13 ± 0.70)%</td>
<td></td>
</tr>
<tr>
<td>$B_0^0 \rightarrow D_\tau^- D_\tau^- DX$</td>
<td>(1.11 ± 0.36)%</td>
<td></td>
</tr>
<tr>
<td>$B_0^0 \rightarrow D_\tau^- D_\tau^- (\gamma) X$</td>
<td>(0.92 ± 0.44)%</td>
<td></td>
</tr>
<tr>
<td>$c\bar{c} \rightarrow D_\tau^- \mu^+$</td>
<td>(9.53 ± 1.65)%</td>
<td></td>
</tr>
</tbody>
</table>
for each of the five data samples. The $K$ factors are extracted independently in each sample, with significant shifts observed due to the changing trigger conditions. The $K$-factor distribution peaks at $\approx 0.9$ for the $D_{s}^{+}$ signal and at $\approx 0.8$ for the first four backgrounds listed in Table II. The $K$-factor distribution populates $0.5 < K < 1$ for both the signal and background components.

To determine the number of events in the signal region and define the signal and background samples, we fit a model to the $M(\phi\pi^{-})$ invariant mass distribution as shown in Fig. 1. The $D_{s}^{+}$ and $D^{-}$ mass peaks are each modeled using an independent Gaussian distribution to represent the detector mass resolution, and a second-order polynomial is used to model the combinatorial background. Using the information obtained from these fits, we define the signal sample (SS) as those events in the $M(\phi\pi^{-})$ mass distribution that are within $\pm 2\sigma$ of the fitted mean $D_{s}^{+}$ meson mass, where $\sigma$ is the Gaussian width of the $D_{s}^{+}$ mass peak obtained from the fit. We find a total of 72028 $\pm 727$ $D_{s}^{+}\mu^{+}$ signal events in the full dataset. Yields observed in the different periods are consistent with expectations taking into account changing trigger conditions and detector performance. The background sample (BS) includes those events in the sidebands of the $D_{s}^{+}$ mass distribution given by $-9\sigma$ to $-7\sigma$ and $+7\sigma$ to $+9\sigma$ from the fitted mean mass. Wrong-sign events in the full $M(\phi\pi^{-})$ range are also included in the background sample, yielding more events to constrain the behavior of the combinatorial background.

The extraction of the flavor-specific $B_{s}^{0}$ lifetime is performed using an unbinned maximum likelihood fit to the data, based on the PPDL of each candidate [18]. The effects of finite $L_{xy}$ resolution of the detector and the $K$ factors are included in this fit to relate the underlying decay time of the candidates to the corresponding observed quantity. The signal and background samples defined above are fitted simultaneously, with a single shared set of parameters used to model the combinatorial background shape. To validate the lifetime measurement method, we perform a simultaneous fit of the $B_{s}^{0}$ lifetime using the Cabibbo suppressed decay $B_{s}^{0} \rightarrow D^{-}\mu^{+}\nu X$ seen in Fig. 1 at lower masses. This measurement also enables the ratio $\tau_{s}(B_{s}^{0})/\tau(B_{s}^{0})$ to be measured with high precision, since the dominant systematic uncertainties are highly correlated between the two lifetime measurements. For simplicity, the details of the fitting function are illustrated for the $B_{s}^{0}$ lifetime fit alone. In practice, an additional likelihood product is included to extract the $B_{s}^{0}$ lifetime in an identical manner.

The likelihood function $L$ is defined as

$$L = \prod_{i \in \text{SS}} [f_{D_{s},\mu}^{i}f_{D_{s},\mu}^{i} + (1 - f_{D_{s},\mu})f_{\text{comb}}^{i}] \prod_{j \in \text{BS}} f_{\text{comb}}^{j},$$

where $f_{D_{s},\mu}$ is the fraction of $D_{s}^{+}\mu^{+}$ candidate events in the signal sample, obtained from the fit of the $D_{s}^{+}$ mass distribution, and $f_{\text{comb}}^{i}$ is the candidate (combinatorial background) probability density function (PDF) evaluated for the $i$th event. The probability density $f_{\text{comb}}^{i}$ is given by

$$f_{\text{comb}}^{i} = f_{cc}F_{cc}^{i} + f_{B1}F_{B1}^{i} + f_{B2}F_{B2}^{i} + f_{B3}F_{B3}^{i} + f_{B4}F_{B4}^{i} + (1 - f_{cc} - f_{B1} - f_{B2} - f_{B3} - f_{B4})F_{X}^{i},$$

Each factor $f_{X}$ is the expected fraction of a particular component $X$ in the signal sample, obtained from simulations and listed in Tables I and II. The first term accounts for the prompt $c\bar{c}$ component, and the decays $B_{1}\rightarrow B_{2}$ represent the first four components listed in Table II. The last term of the sum in Eq. (3) represents the signal events $S = (B_{s}^{0} \rightarrow D^{-}\mu^{+}\nu X)$ listed in Table I. The factor $F_{X}$ is the lifetime PDF for the $c\bar{c}$ events, given by a Gaussian distribution with a mean of zero and a free width. Each $B$ decay mode is associated with a separate PDF, $F_{X}$, modeling the PPDL distribution, given by an exponential decay convoluted with a resolution function and with the $K$-factor distribution. All $B$-meson decays are subject to the same PPDL resolution function. A double-Gaussian distribution is used for the resolution function, with widths given by the event-by-event PPDL uncertainty determined from the $B_{s}^{0}$ candidate vertex fit multiplied by two overall scale factors and a ratio between their contributions that are all allowed to vary in the fit.

The combinatorial background PDF, $F_{\text{comb}}^{i}$, is chosen empirically to provide a good fit to the combinatorial background PPDL distribution. It is defined as the sum of the double-Gaussian resolution function and two exponential decay functions for both the positive and negative PPDL regions. The shorter-lived exponential decays are fixed to have the same slope for positive and negative regions, while different slopes are allowed for the longer-lived exponential decays. Figure 2 shows the PPDL
distribution for one of the five data periods for the signal sample, along with the comparison with the fit model. The corresponding $\chi^2$ per degree of freedom for each data-taking period are 1.58, 1.21, 1.29, 1.18, and 1.14.

The corresponding $B^0$ lifetime measurement uses exactly the same procedure for events in the $D^-$ mass peak, including a calculation of dedicated $K$ factors and background contributions from semileptonic decays.

The lifetime fitting procedure is tested using MC pseudoexperiments, in which the generated $B^0$ lifetime is set to a range of different values, and the full fit is performed on the simulated data. Good agreement is found between the input and extracted lifetimes in all cases. As an additional cross-check, the data are divided into pairs of subsamples, and the fit is performed separately for both samples. The divisions correspond to low and high $p_T(B^0)$, central and forward pseudorapidity $|\eta(B^0)|$, regions, and $B^0$ versus $B^{\pm}$ decays. In all cases, the measured lifetimes are consistent within uncertainties.

To evaluate systematic uncertainties on the measurements of $c\tau(B^0)$, $c\tau(B^0_\text{s})$, and the ratio $\tau_{00}(B^0_\text{s})/\tau(B^0)$, we consider the following possible sources: modeling of the decay length resolution, combinatorial background evaluation, $K$-factor determination, background contribution from charm semileptonic decays, signal fraction, and alignment of the detector. All other sources investigated are found to be negligible. The effect of possible mis-modeling of the decay length resolution is tested by repeating the lifetime fit with alternative resolution models, using a single Gaussian component. A systematic uncertainty is assigned based on the shift in the measured lifetime. We repeat the fit using different combinatorial background samples using only the sideband data or only the wrong-sign sample. The maximum deviation from the central lifetime measurement is assigned as a systematic uncertainty. To determine the effect of uncertainties on the $K$ factors for the signal events, the fractions of the different components are varied within their uncertainties given in Table I. We also recalculate the $K$ factors using different MC decay models [16], leading to a harder $p_T$ distribution of the generated $B$ hadrons. The fraction of each component from semileptonic decays is varied within its uncertainties, and the shift in the measured lifetime is used to assign a systematic uncertainty. The signal fraction parameter, $f_{\mu\nu}$, is fixed for each mass fit performed. We vary this parameter within its statistical and systematic uncertainty, obtained from fit variations to the background and signal model of the mass PDFs, and assign the observed deviation as the uncertainty arising from this source. Finally, to assess the effect of possible detector misalignment, a single MC sample is passed through two different reconstruction algorithms, corresponding to the nominal detector alignment and an alternative model with tracking detector elements shifted spatially within their uncertainties. The observed change in the lifetime is taken as systematic uncertainty due to alignment.

Table III lists the contributions to the systematic uncertainty from all sources considered. The most significant effect comes from the combinatorial background determination. Correlations in the systematic uncertainties for the $B^0_\text{s}$ and $B^0$ meson lifetimes are taken into account when evaluating the effect on the lifetime ratio, where the $K$ factor determination dominates.

The measured flavor-specific lifetime of the $B^0_\text{s}$ meson is $c\tau(B^0_\text{s}) = 443.3 \pm 2.9\,(\text{stat}) \pm 6.3\,(\text{syst}) \,\mu\text{m}$, which is consistent with the current world average of $439.2 \pm 9.3\,\mu\text{m}$ [3,6] and has a smaller total uncertainty of $6.9\,\mu\text{m}$. The uncertainty in this measurement is dominated by systematic effects. The $B^0$ lifetime in the semileptonic decay $B^0 \rightarrow D^- \mu^+ \nu\bar{\nu}$ is measured to be $c\tau(B^0) = 459.8 \pm 5.6\,(\text{stat}) \pm 6.4\,(\text{syst}) \,\mu\text{m}$, consistent with the world average of $c\tau(B^0) = 455.4 \pm 1.5\,\mu\text{m}$ [3]. Using both lifetimes obtained in the current analysis, their ratio is determined to be $\tau_{00}(B^0_\text{s})/\tau(B^0) = 0.964 \pm 0.013\,(\text{stat}) \pm 0.007\,(\text{syst})$. Both results are in reasonable agreement with theoretical predictions from lattice QCD [1,2]; the flavor-specific

![FIG. 2. Top: PPDL distribution for $D^-\mu^+$ candidates in the signal sample for one of the five data periods. The projections of the lifetime fitting model, the background function, and the signal function are superimposed. Bottom: fit residuals demonstrating the agreement between the data and the fit model.](image-url)
lifetime has a better precision than the current world average \cite{3,6}, and agrees reasonably well with the slightly more precise recent measurement from the LHCb Collaboration \cite{10}.

In summary, we measure the $B_s^0$ lifetime in the inclusive semileptonic channel $B_s^0 \to D_s^- \mu^+ \nu X$ and obtain one of the most precise determinations of the flavor-specific $B_s^0$ lifetime. Combining this result and that of Ref. \cite{10} with global fits of lifetime measurements in $B^0 \to J/\psi K^+ K^-$ decays \cite{6} gives the most precise determination of the fundamental parameters $\Delta \Gamma_i$ and $\Gamma_i$ which are important for constraining $CP$ violation in the $B^0$ system. Our precise measurement of the ratio of $B_s^0$ and $B^0$ lifetimes can be used to test and refine theoretical QCD predictions and offers a sensitive test of new physics \cite{4}.

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