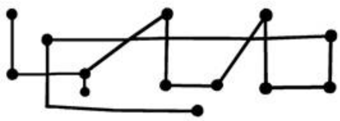
*Kid A*lgebra: Radiohead's Euclidean and Maximally Even Rhythms



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The British rock group Radiohead has carved out a unique place in the post-millennial rock milieu by tempering their highly experimental idiolect with structures more commonly heard in Top Forty rock styles.[1] In what I describe as a Goldilocks principle, much of their music after *OK Computer* (1997) inhabits a space between banal convention and sheer experimentation—a dichotomy which I have elsewhere dubbed the 'Spears–Stockhausen Continuum.'[2] In the timbral domain, the band often introduces sounds rather foreign to rock music such as the ondes Martenot and highly processed lead vocals within textures otherwise dominated by guitar, bass, and drums (e.g., 'The National Anthem,' 2000), and song forms that begin with paradigmatic verse–chorus structures often end with new material instead of a recapitulated chorus (e.g., 'All I Need,' 2007). In this article I will demonstrate a particular rhythmic manifestation of this Goldilocks principle known as *Euclidean rhythms*. Euclidean rhythms inhabit a space between two rhythmic extremes, namely binary metrical structures with regular beat divisions and irregular, unpredictable groupings at multiple levels of structure.

After establishing a mathematical model for understanding these rhythms, I will identify and analyze several examples from Radiohead's post-millennial catalog. Throughout the article, additional consideration will be devoted to further ramifications for the formalization of rhythm in this way, as well as how hearing rhythm in this way may be linked to interpreting the lyrical content of Radiohead's music. After doing so, I will suggest a prescriptive model for hearing these rhythms, and will then conclude with some remarks on how Radiohead's rhythmic practices may relate to larger concerns such as style and genre.

Euclidean Rhythms: Some Mathematical Preliminaries

A familiar example from Radiohead's recent song 'Codex' (2011, see Example 1) will serve to illustrate the mathematics involved in the rest of this article. Stylistically competent rock listeners will entrain to a sixteen-semiquaver subdivision in each measure. That is to say, the lowest common denominator that encompasses all rhythmic onsets in the voice, piano, and quiet kick drum involves sixteen evenly spaced time points per notated measure.[3] Observing the underlying piano rhythm in this section (notated in full in measures 1, 3, and 5–7 of example 1—the remaining measures are rhythmic subsets of this same riff), notice that there are five onsets per measure, meaning that there are five onsets played for every sixteen possible time points. But, these are not just *any* five onsets within that space of sixteen. In fact, the particular spacing of those five onsets represents a maximally even spacing of five onsets within a grid of sixteen evenly spaced time points. That is to say, although one cannot divide sixteen by five evenly, the rhythm we hear in the piano is the *closest-to-even* distribution possible of sixteen units into five sets (4+3+3+3+3). There exists only one such possible distribution given k onsets over n evenly spaced time points, and this is the rhythm that I will henceforth refer to as the Euclidean distribution.[4]

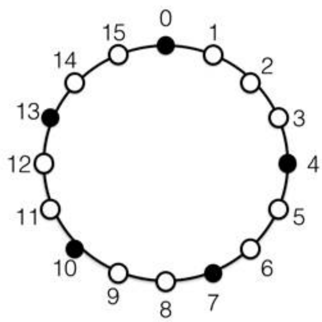
Here it will be useful to establish a notation system that will be used consistently throughout the rest of the article. For any abstract Euclidean distribution, the unordered set will be notated within curly

EXAMPLE 1: EUCLIDEAN RHYTHM IN 'CODEX' (2011, 1:04)

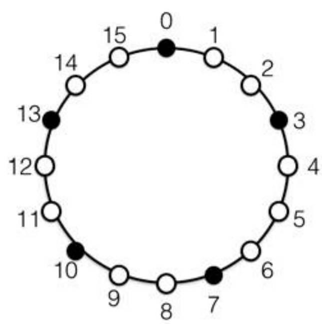
braces with the smallest inter-onset intervals packed to the left. Specific ordered presentations of that rhythm will be enclosed within angle brackets, placing the inter-onset intervals in order beginning on the perceptual downbeat. 'Codex' could then be related abstractly to other Euclidean distributions of five in sixteen by notating the unordered multi-set of inter-onset intervals {3,3,3,3,4} (notated in order from lowest to highest values), and, of the five possible rotations of that set, the specific ordering heard here is <4,3,3,3,3>. At other times in this article, I will find it necessary to notate numbered onsets in a string of n evenly spaced time points (numbered 0 through n–1), so that the attack points heard in the 'Codex' groove could be identified by their placement within a measure of consecutively ordered semiquaver subdivisions: [0,4,7,10,13]. The use of curly braces, angle brackets, and square brackets will be consistent for these purposes, unless noted otherwise.

Recent work in cyclically repeating rhythmic phenomena has made extensive use of a notational system known as the 'rhythm necklace.'[5] The beauty of the necklace is that for every k number of onsets in a given rhythm one can rotate the entire constellation k different ways that are perceptually distinct—inasmuch as they shift the perceived downbeat—yet are mathematically and structurally equivalent. Two such necklaces are given in Example 2.[6] Both of the necklaces, which represent the 'Codex' rhythm shown in Example 1, contain the same pattern of five onsets over the course of sixteen evenly spaced time points. The top necklace represents the rhythm beginning on the notated downbeat of Example 1. Spatially, one could explain the bottom as merely rotated clockwise by three units, which is to say that the buttom necklace is, algebraically, T3 of the top.[7] Since $k = 5$, there are five possible ordered rotations that begin with an onset on zero. Two are notated in Example 2; the other three are:

<0,3,6,10,13> <0,3,6,9,13> <0,3,6,9,12>.



ROTATION NOTATED IN EXAMPLE 1 <0,4,7,10,13>



ROTATION CLOCKWISE BY THREE UNITS (T3) <0,3,7,10,13>

EXAMPLE 2: TWO ROTATIONALLY EQUIVALENT RHYTHM NECKLACES FROM 'CODEX'

While spatially oriented diagrams can be helpful models for representing structure, they can also be clumsy metaphors for representing what is at its heart, a temporal phenomenon. We can represent rhythms such as these much more elegantly based on their inherent mathematical properties.[8] The Euclidean algorithm, represented mathematically as $E(k,n)$, concerns dividing n elements into k sets. When one divides elements among sets, there are four types of results. These results are given in Example 3.

The first, and most trivial, is when k is a factor of n, which is to say that k divides n evenly. The second is when k is not a factor of n, yet k and n are not co-prime (in other words, there exists some integer smaller than k or n, other than one, which divides both evenly—in this case that number is two). In the third case, k is once again not a factor of n, but k and n are co-prime, meaning that no smaller integer other than one can divide both evenly. Additionally, and crucially from a mathematical standpoint, even though k and n are co-prime, k is not a factor of $n+/-1$.

The fourth type, which the 'Codex' example illustrates, is where k is once again not a factor of n, k and n are again co-prime, and k is indeed a factor of $n+/-1$. This $n+/-1$ consideration becomes important when we compare type four to type three distributions, a factor which becomes crucial to analyzing the bass riff from 'The National Anthem.' This fourth case is fascinating from a rhythmic perspective because, with a single duration only one unit longer than the others, perceptually, they seem to divide the underlying pulse stream into larger onsets *almost* evenly, but not quite, in fact, the closest to even possible, and there exists a single unique way, given k

Type	Description	Example
1	k s a factor of n (k divides n evenly)	$E(4,8) = \{2,2,2,2\}$
2	k is NOT a factor of n; k and n are NOT co-prime	$E(4,10) = \{2,2,3,3\}$
3	k is NOT a factor of n; k and n ARE co-prime; k is NOT a factor of $[n+/-1]$	$E(7,16) = \{2,2,2,2,2,3,3\}$
4	k is NOT a factor of n; k and n ARE co-prime; k IS a factor of $[n+/-1]$	$E(5,16) = \{3,3,3,3,4\}$

EXAMPLE 3: FOUR TYPES OF EUCLIDEAN DISTRIBUTIONS

onsets and *n* evenly spaced time points, how this distribution must be proportioned. In all four cases mentioned above, we will call this unique distribution the Euclidean distribution, and I will reserve comment on how this phrase relates to the more commonly used term 'maximally even' until after analyzing some examples of Euclidean rhythms and their interpretations in Radiohead's music.

Euclidean Rhythms in Radiohead's Post-Millennial Corpus

Example 4 shows a reduction of the well-known groove from 'Pyramid Song' (2001).[9] Many have pointed out at least two peculiar rhythmic properties of this groove. First, and perhaps most obviously, the piano's recurring rhythmic pattern is a palindrome—it sounds the same played backwards as played forwards. Secondly, the *metric* subdivision that many listeners hear <3,3,4,3,3> is only possible relative to an underlying metrical framework provided by the drums, which nonetheless do not appear until the second verse, rendering the song's piano-and-voice-driven first half a much trickier affair from a perceptual standpoint, one which many have simply been content to analyze retrospectively: after the second verse, we have learned to hear that piano rhythm in common time, and thus we hear the piano/vocal opening *mutatis mutandis.*

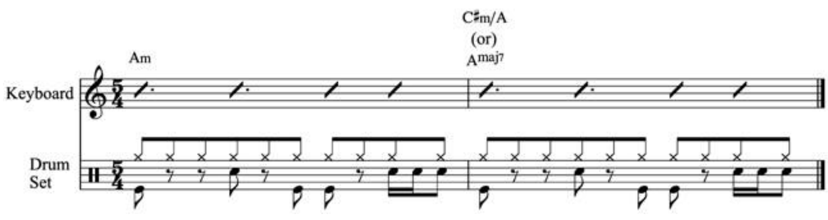
The Euclidean algorithm can be used to explain both of these facets simultaneously. If we count the underlying time points in the drum pattern that enters in verse two, there are sixteen, and if we count the number of piano onsets that happen within those sixteen pulses, we come up with the Euclidean distribution E(5,16), specifically its

EXAMPLE 4: SECOND VERSE OF 'PYRAMID SONG' (2001, 2:07)

<3,3,4,3,3> ordered rotation, which is of course a palindrome because the largest beat is sandwiched exactly in the middle.[10] As for the opening, if we agree that those sixteen time points which grant us the <3,3,4,3,3> rotation are only possible because of the drum pattern, then of course we should not hear the opening in common time by default, because there are no underlying *n* time points present by which group *k* onsets. Some might be tempted to categorize the opening as a 'non-isochronous meter,' but without the underlying pulses present, meter, at least in the hierarchically defined sense we are used to (that is, *k* onsets composed of *n* time points, where *n* divided by *k* equals some number between two and four), may be something of an analytical fiction. While we might hear the ordered <3,3,4,3,3> distribution in verse two, a more faithful representation of the opening music might be <.1875, .1875, .25, .1875, .1875>, an awkward method for describing rhythm to be sure, but one which, when these proportions are imagined spatially, gives an altogether different meaning to the imagery of pyramids in the song's title.

With the mathematical conceptions of Euclidean rhythms now in place, we may progress one step further with these analyzed examples and begin the search for meaning. Throughout this process a listener may search a number of cognitive vaults, but one of the first places many Radiohead fans might visit is their knowledge of the lyrical text, attempting to make sense of the music and the lyrics concomitantly.

A particularly interesting case for such an interpretation occurs in 'Morning Bell' (album version from *Kid A*, 2000). See Example 5. Turning to the Euclidean algorithm, we can see that the evenly spaced ten hi-hat pulses are divided more or less evenly by the four onsets in the keyboard. This Euclidean distribution in the keyboard also helps to group the more complicated pattern of onsets in the drums—we can now understand the singleton on the sixth ordered semiquaver as an anacrusis. Before turning to an exact interpretation of the lyrics, this



EXAMPLE 5: INTRODUCTION OF 'MORNING BELL' (2000, 0:08)

example provides the occasion to discuss the other key phrase from this article's title, 'maximally even.' Though these terms are typically conflated in scholarly discourse,[11] I will argue that 'Morning Bell' provides evidence for a fruitful separation of these two very distinct rhythmic experiences. In fact, my interpretation of the lyrics relies heavily on the difference between Euclidean and maximally even rhythms.

The four types of Euclidean distributions provided in Example 3 should suffice to demonstrate that all maximally even rhythms—from the most obvious case of eight quavers divided into four crotchets all the way to the E(5,16) bossa nova rhythm—can be mathematically formalized as Euclidean distributions of k onsets over n time points. However, the inverse is not true—from a rhythmic and temporal perspective, not all Euclidean necklaces are maximally even, and this distinction only arises for values of k and n where k is not a factor of either n or $n+/-1$.

In type four Euclidean rhythms, such as E(5,16), there will be exactly one instance of the duration which is longer or shorter than the rest, and thus the necklace will be maximally even despite any of its k rotations. In Euclidean distributions where k is neither a factor of n nor $n+/-1$ (as in type two and type three), there will be more than one instance of these longer or shorter durations (here, for example, we see not one, but two sets of three). As such, I suggest that, even though it is mathematically Euclidean, from a temporal perspective, the 'Morning Bell' groove fails to be maximally even because the ordered rotation <3,3,2,2> heard here 'crams' the two longer durations unnecessarily together. The maximally even property should be reserved for rhythms which space onsets in time as far apart as possible. Any unordered Euclidean distribution of E(4,10) is exactly two twos and two threes {2,2,3,3}, but the maximally even spacing must place the threes as far apart as possible, ordered either as <2,3,2,3> or <3,2,3,2>.

The song's repeated lyrics 'cut the kids in half' are, I believe, instructive in interpreting this non-maximally-even Euclidean rhythm in 'Morning Bell.' Central to my interpretation of the rhythmic/lyrical assemblage is a passage from I Kings which reads 'cut the living child in two, and give half to one woman and half to the other.' Imagine a rather large family that contains ten children. The parents are going away on vacation, and four volunteers are each clamoring to babysit as many of the ten children as possible. Since the parents cannot rightly sever the bodies of two of these ten children in order to distribute 2.5 children to every babysitter, they must settle for the Euclidean distribution of whole children. Thus, two babysitters receive two children each, and two lucky babysitters each receive three—the exact rhythmic figure, E(4,10) we hear throughout 'Morning Bell.'

Turning now to the signature bass riff from 'The National Anthem' (2000), I hope to show that two different ways of hearing the Euclidean rhythm will demonstrate the applicability of the mathematical model to musical experience. After making this point, I will again turn toward a possible interpretation of meaning in this passage that relies not on lyrical content, but on biographical details. See Example 6 for a transcription of this bass riff, along with the ondes Martenot that sounds above it just before the second verse.

The Euclidean algorithm proves itself representative of musical experience here, inasmuch as, whether or not one hears the lightly articulated anacruses (notated in smaller noteheads) as bonafide onsets—that is, whether one hears seven or nine numbered onsets—both E(7,16) and E(9,16) work out to be maximally even, type three Euclidean rhythms.[12] Unlike 'Morning Bell,' both of these are maximally even in their temporal presentation because they space the longer durations as evenly as possible.[13] Assuming only seven onsets, the Euclidean distribution {2,2,2,2,2,3,3} is heard here with the two longer durations as far apart as possible <2,2,3,2,2,2,3>. Assuming nine onsets, the Euclidean distribution {1,1,2,2,2,2,2,2,2} is heard here with the two shorter durations as far apart as possible <2,2,2,1,2,2,2,2,1> which, via the group-theoretical principle of complementation, also ensures that the remaining seven longer beats are also spread as widely as possible.[14] If one agrees that these two rhythms are perceptually very similar, then, in order for the math to represent musical experience, they should both either be maximally even, type three Euclidean rhythms, or they should both not. The undeniable similarity between the nine-onset and seven-onset rhythms can also be expressed using Pressing's fusion/fission relationship, whereby the shorter singleton in the local inter-onset durations may be 'fused' with an adjacent two-pulse duration to form the longer three-pulse duration of the latter (and vice versa).[15]

Pausing for a moment to explain a further ramification of my algebraic formalization will set the stage for a biographical interpretation of 'The National Anthem.' Another key component of Pressing's early work in the relationship between pitch and rhythmic sets was his idea that the most common instances of each were the result of *prime generators*. Any interval *k* which is co-prime to the modulus *n* will generate every possible time point in that modulus before repeating. His most wide-reaching example of prime generators relates both the diatonic scale E(7,12) = {1,1,2,2,2,2,2} and its mod_{12} complement, the pentatonic scale E(5,12) = {2,2,2,3,3}, to the maximally even African bell pattern <2,2,1,2,2,2,1> and its complement

<2,3,2,2,3>. The domains of pitch and rhythm are isomorphic if we conceive of a twelve-tone equal tempered scale and a cycle of twelve evenly spaced time points in the same manner. Notice that, just like 'The National Anthem,' a five-beat rhythm derived from the set <2,3,2,2,3> could be equivalent to the seven-beat version if we consider the fusion/fission relationship, which could yield <2,2,1,2,2,2,1>. Pressing argues that the cognitive basis of rhythms such as these could be related to maximal individuation (Pressing 1983, 47). Like finding tonic in the diatonic scale,[16] a listener can easily discern where a rhythmic cycle begins so long as each beat is maximally individuated. Thus, maximal individuation makes finding the downbeat in a Euclidean bell pattern, and the tonic in a diatonic scale, possible (both are E(7,12) = <2,2,1,2,2,2,1>), unlike the impossibly underdetermined 'tonic' of a whole-tone scale or 'downbeat' of an undifferentiated set of crotchets in sextuple simple time (both of which are E(2,12) = <2,2,2,2,2,2>).

Notice that both of these rhythms use a twelve-unit modulus, while most of those heard in Radiohead's music—as well as post-millennial rock music in general—use sixteen. Indeed, various rotations of the E(5,16) Euclidean rhythm can be heard throughout modern rock music. One might then take issue with Pressing's explanation for the ubiquity of E(5,16) as only explained via its 'analogue transformation' of E(5,12), whereby each of the five durations in the twelve-cycle is elongated to yield each of the five durations in a sixteen-cycle, a system that Pressing himself admits is 'too imprecise for many purposes' (Pressing 1983, 43). I argue that, because prime generators also explain the asymmetrical Euclidean distributions possible for the sixteen-beat cycles Radiohead uses frequently, we should consider E(5,16) a 'standard pattern' in its own right for the analysis of Radiohead's music.[17]

Now to relate the mathematics of the mod-sixteen cycle in 'The National Anthem' (whether heard in seven or nine onsets) to an interpretation of meaning. Earlier, in my interpretation of 'Morning Bell,' I explained a rhythmic anomaly with reference to the song's lyrical content. Rather than search for meaning in lyrical content, critics often choose to link Radiohead's expressive gestures with biographical details of the band. And though there are obvious logical pitfalls to this approach, logic does not necessarily have the final say in our personal searches for meaning. Furthermore, knowing something about the band members puts me (as well as most fans) in a situation where, no matter how much I try not to think about that aspect of the band, it is too late; that information has always-already influenced my hearing of the music. Even knowing that 'The National Anthem' is a song by Radiohead has erased all hope of completely avoiding the intentionalist fallacy.

The underlying rhythmic counterpoint in Example 6 is largely the product of a bass riff recorded by Radiohead lead vocalist Thom Yorke and an ondes Martenot melody performed by lead guitarist Jonny Greenwood. Greenwood is undoubtedly the member of the band with the most formal composition experience. He composed the scores to both *There Will Be Blood* and *The Master*, shared a premiere with Penderecki in 2011, and most relevant to our purposes here, is also something of a moonlighting Messiaen enthusiast.[18]

When I hear the ondes Martenot part enter, my mind first registers something like <this is Greenwood's favorite instrument>, then I go to <Greenwood loves the ondes Martenot because of Messiaen's use of it in the *Turangalîla* Symphony>, and then my mind becomes stuck on Messiaen for a while. Once I find myself pondering Messaien, I hear the pitches in the ondes Martenot part as one of his modes of limited transposition—in this case, a subset [679t] of the octatonic collection. Returning to Pressing's notion that pitch structures can be related to rhythmic structures, we may observe that the highly symmetrical octatonic collection is also a maximally even Euclidean distribution of eight pitch classes in twelve-tone equal temperament: E(8,12) =<1,2,1, 2,1,2,1,2>. And this correlation is not a simple coincidence. In fact, Greenwood's use of the octatonic scale is a clever solution to the problem presented by Yorke's mode-mixed bass riff. In the first measure

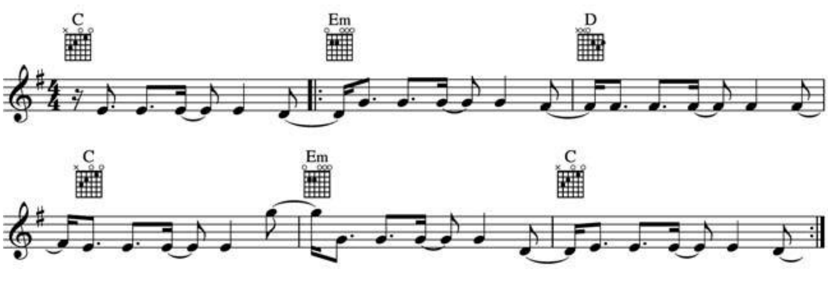
EXAMPLE 6: BASS AND ONDES MARTENOT IN 'THE NATIONAL ANTHEM' (2000, 1:57)

we hear Greenwood using the dyad A/F♯ to reinforce the D major half of the riff, then transposing that dyad up a semitone to complement the D minor portion.

Admittedly, this mode of meaning acquisition, based both on intertextuality and extra-musical interpretation, may seem idiosyncratic to some readers. Creating a mental map of the foregoing interpretation may yield something like the following four steps: recognize the bass rhythm as E(7,16); recognize the ondes Martenot pitches as an octatonic subset; simultaneously recognize the link between steps one and two, as well as the link between step two and Messiaen's modes of limited transposition; recognize the autobiographical connections between Messiaen and Greenwood.

Two justifications may be provided for this process. First, the process is far from immediate, and is a result of listening to and thinking about the music for an extended period of time. In other words, it is not so much how one hears the first time, but how one can *learn* to hear over time. Secondly, the process of learning to hear this way is the product of kinesthetic engagement with the piece. I *feel* the maximally even rhythm in my picking hand, and I feel the muscular movements needed to transpose vocally that minor third in the ondes Martenot up a semitone in the third and fourth measures. Far from a mathematical abstraction, the meaning is intimate, interactive, and the direct result of engaged musical experience.

One final example from Radiohead's post-millennial corpus will serve to highlight yet another aspect of this mathematical formalization, and will ultimately present a question that can only be answered with a prescriptive model for hearing these rhythms. Example 7 shows a reduction of the signature riff from 'Reckoner' (2007, 0:09).[19] Like 'Pyramid Song' and 'Codex,' this riff also relies on the



EXAMPLE 7: INTRODUCTION OF 'RECKONER' (2007, 0:09)

maximally even type four Euclidean E(5,16) rhythm. In 'Reckoner,' the five onsets can be heard as a surface-level manifestation of the five-bar hypermeter present throughout the track. Whereas naming the ordered rotation of that rhythm necklace in 'Pyramid Song' <3,3,4,3,3> and 'Codex' <4,3,3,3,3> was straightforward, the non-articulated downbeat in 'Reckoner' makes this process problematic. Relative to the strong backbeat heard in the drums throughout the track, which, along with the chord changes, make for an incontrovertible downbeat to each measure, the numbered onsets in that sixteen-pulse space form the set [1,4,7,10,14]. Of the five possible rotations for the E(5,16) necklace, this is closest to the relatively rare rotation <3,3,3,4,3>, though displaced late by one semiquaver.

Data from studies by both Toussaint and Pressing suggest that the vast majority of Euclidean and maximally even rhythms either begin or end with the longest duration.[20] This makes two rotations of the E(5,16) rhythm most common: <4,3,3,3,3> and <3,3,3,3,4>, the first of which is heard in 'Codex.' The latter is ubiquitous in myriad styles of rock music because it 'delays' the resolution of a hemiola until the end of the cycle in a manner similar to the even more ubiquitous E(3,8) = <3,3,2> heard in countless rock songs such as Coldplay's 'Clocks' (2002).[21]

The question then remains: how do we explain this extraordinarily rare rotation in 'Reckoner,' and does this facet of the rhythm comment on our mode of hearing Euclidean rhythms generally? The answer to the first part of this question might be stated in its premises. Since both our notions of 'downbeat' and 'beginning' rely on cues from rhythmic strata other than the guitar part in question (namely an assumed downbeat derived from the backbeat in the drum set and, secondarily, the changes in harmony that coincide with said downbeat), the listener perceiving this downbeat will likely sense the guitar onsets as a syncopation against an otherwise unaffected common-time pulse stream. As such, there is no 'beginning' to the E(5,16) rhythm, and the ordered rotation is irrelevant. In order to answer the larger question at stake here, we must make an active choice as listeners either to hear Euclidean rhythms as syncopations against a regular meter, or, alternatively, as carrying a metrical component in their own right.

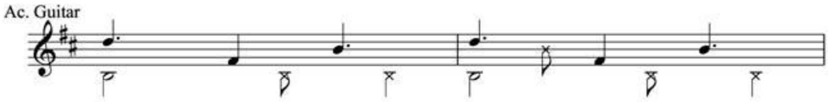
Hearing Euclidean Rhythms: A Prescriptive Turn

What may be concluded from the examples presented in this article is that, given any Euclidean rhythm E(k,n), when $k < .5n$ we are more likely to interpret that rhythm as having some sort of metrical component than when $k > .5n$. Of course, it all depends on the specific musical context, but, all other things equal, this may serve as a guideline for when to hear maximally even and/or Euclidean rhythms either as metrical or as syncopations.[22]

This guideline involving half the value of n is not merely arbitrary, but rooted in a fundamental property of human rhythmic perception that has been experimentally validated by many researchers: humans, as a rule, entrain to beats that are equal to groups of either two or three evenly spaced time points that act as subdivisions of said beat (London 2012, 128). For cases where k is equal to exactly $.5n$, each beat would comprise exactly two subdivisions, which is uninteresting for our purposes here. However, in rhythms such as the E(4,10) in 'Morning Bell' and the E(7,16) in 'The National Anthem,' where k is slightly less than $.5n$, some of the beats will comprise two subdivisions and some will comprise three. When k exceeds $.5n$—for instance in the alternative hearing of 'The National Anthem,' E(9,16)—we are left with beats which are equal to the single subdivided unit. Since this singleton division is not commensurate with the experimental evidence that humans entrain metrically to beats comprising either two or three units, it stands to reason that these necklaces may be more aptly described as rhythmic entities against a standard metrical grid, rather than metrical entities in their own right. Two Radiohead examples involving an eight-unit cycle will help to bolster this point, and will also reveal the role complementation plays in this theory.

Any two rhythms a and b are complementary if the articulated onsets of a form the non-articulated 'rests' of b, and vice versa. Examples 8*a* and *b* compare two co-prime Euclidean rhythms heard on *In Rainbows* (2007). The recurring guitar rhythm heard throughout 'Faust Arp' is the <3,2,3> rotation of E(3,8) (Example 8A), while the piano ostinato that ushers in the climax of 'All I Need' is the E(5,8) complement of that same rhythm <1,2,1,2,2> transposed forward one quaver [1,2,4,5,7] (Example 8B). Especially since there is no underlying percussive layer present in 'Faust Arp,' the guitar rhythm <3,2,3> is likely the metrical layer to which listeners will entrain, while the backbeat present throughout 'All I need' ensures that the piano rhythm <1,2,1,2,2> will be interpreted as a maximally even E(5,8) syncopation against the even crotchet pulse in the drums.[23]

What should be readily perceptible from this side-by-side comparison is that, as long as k does not divide n evenly, the k values which are symmetrical about $.5n$ will be complementary to the modulus. E(3,8) and E(5,8) are complementary because the five unarticulated



EXAMPLE 8A: GUITAR IN 'FAUST ARP' (2007, 0:02)

EXAMPLE 8B: PIANO IN 'ALL I NEED' (2007, 2:45)

time points in the former equal the onsets in the latter. This observation also bears heavily on our two possibilities for hearing the 'The National Anthem' groove. E(7,16) = <2,2,3,2,2,2,3> is the closest possible to $.5n$ where $k < .5n$, and E(9,16) is the closest possible to $.5n$ where $k > .5n$. The two rhythms are complementary since seven and nine are mod-sixteen complements. At least before the drums enter with their backbeat, the seven-onset interpretation could be felt as metrical, while the nine-onset interpretation would probably rely on an imagined steady pulse since it contains two singletons.

Given the rather strict metrical preferences for two- and three-unit durations, what is the role of the E(5,16) rhythm which, despite containing a four-unit duration, is so ubiquitous in Radiohead's music? Is it a rhythmic syncopation over a steady metric pulse, or can it be metrical in its own right? The answer is four-fold. First, Justin London's (2012, 128) work shows that, within a given tempo range, four-unit durations are also possible, which allows for the E(5,16) rhythm within tempo constraints germane to the rock music in which it commonly occurs. Second, for tempo values too slow for this four-unit duration to exist, we hear, at least subconsciously, the would-be four-unit duration as divided into two, two-unit durations, which yields the non-maximally-even Euclidean rhythm E(6,16)—yet another example of Pressing's fission relationship. Third, Toussaint (2010, 5) highlights two inherent mathematical properties of E(5,16) that it shares with a host of patterns heard in various music cultures—the 'rhythmic oddity' and 'deep' properties—and suggests that there is something mathematically transcendent about these properties which yields a 'good' rhythm. Fourth, and most far-reaching, certain rotations of E(5,16), especially those with the longer duration at either the beginning or end (<4,3,3,3,3> and <3,3,3,3,4>), are so ubiquitous in modern rock music that they may form a category all their own. Neither purely rhythmic nor purely metrical, they may exist on their own terms somewhere between the two as a metric-rhythmic hybrid similar to clave in Latin-influenced genres.

In summary, there are plenty of reasons not to hear Euclidean and/or maximally even rhythms as metrical. After all, phenomena such as hemiola and grouping dissonances actually reward hearing surface-level onset durations within the context of steady-pulse meters. What I might advocate, based on some of the examples I have presented in this article, is that, when k exceeds $.5n$, we should consider the resulting rhythmic structure properly rhythmic, rather than metric. After all, this '$> .5n$' clause would keep us, on average, from entraining to singletons as metrical units. However, given the mathematical and tempo constraints I have suggested throughout this article,[24] I hope to have shown that entraining to Euclidean patterns as metrical, without the regular pulse stream acting as an underlying grid—especially when $k < .5n$—can be a viable mode of hearing these structures. The choice between these modes of hearing may represent the choice between 'immanent' and 'imaginative' modes of hearing. As Martin Scherzinger notes:

> By 'immanent' I mean an account in which everything that is analytically relevant persists within the *system* under investigation [italics mine]. . . . The second orientation for listening, the 'opening possibility' sort, is one that widens the horizon of musical meaning by marking various moments of musical undecidability. . . . It would resist meanings whose unity is determined by the totalizing tendencies that structure the multiplicity of the text. (Scherzinger 2004, 272–73)

The 'system' that Scherzinger refers to could be diatonic tonality, steady-pulse metrical preferences, or, more generally, any *a priori* framework inherited from other musical cultures (e.g., common practice music). The more imaginative strategy for interpreting music ('the opening possibility sort') involves resisting the meanings that are merely direct correlates of these systems. To be sure, it does not mean always contradicting them. Our choice whether to interpret Euclidean and maximally even rhythms as metrical, rhythmic, or perhaps some sort of paradigmatic metric–rhythmic hybrid, may ultimately come down to a choice between, on the one hand, deferring our modes of hearing to underlying systems, and, on the other hand, allowing ourselves to interact with this music in imaginative and personal ways that lead to new modes of understanding. These modes of understanding have direct bearing on an individual's search for musical meaning and, just as importantly, on the ways we allow our bodies to interact kinesthetically with and interpret music.

Conclusion: The Goldilocks Principle in Radiohead's Rhythmic Practices

Radiohead's music regularly confronts the listener with structural elements—in the domains of pitch/harmony, timbre, form, and rhythm—which exist in a space between pure Top Forty convention and sheer experimentation. Within the domain of rhythm, the Euclidean and maximally even rhythms examined throughout this article are only one manifestation of this Goldilocks principle. Some characteristic rhythmic gestures heard throughout Radiohead's post-millennial catalogue include: odd cardinalities (e.g., '2+2=5' [2003] and '15 step' [2007]); changing meter (e.g., 'Sail to the Moon' [2003]); grouping dissonance (e.g., 'Weird Fishes' [2007] and 'Lotus Flower' [2011]); and polytempo (e.g., 'The Gloaming' [2003] and 'Bloom' [2011]). Crucially, none of these rhythmic gestures represents a sheer break from what a listener expects from rhythm in rock music. Rather, they seem to excite our listening systems by presenting novel methods of organizing internal beats within larger, familiar frameworks that loop and repeat.

Radiohead uses Euclidean and maximally even rhythms that range from almost completely banal rotations of E(5,16) in 'Codex,' to the jarring E(3,7) opening groove of '2+2=5,' to the non-isochronous yet smooth E(4,10) recurring grooves in both 'Morning Bell' and '15 Step.' The more experimental rhythms that Radiohead uses involve odd-cardinality meters, and most of these odd-cardinality meters involve maximally even distributions, since they are usually either E(2,5) (as in 5/8), or E(3,7) (as in 7/8), both of which are type four Euclidean distributions, which are necessarily maximally even in any rotation.

When we hear these expressive rhythms in Radiohead's music, how does it change our opinion of the band's position within post-millennial experimental rock music? My hypothesis is that Radiohead's compositional idiolect has been so commercially successful precisely because it inhabits this zone between, on one extreme, music that is so transparently native to an established style, and, on the other extreme, music that is so radically unique as to defy stylistic conventions altogether. Radiohead's position on this continuum puts them in a rather unique position within the post-millennial milieu,[25] while perhaps bringing up historical associations with late-sixties Pet Sounds-, Sgt. Pepper's-, and White Album-era compositional practice.

My continued work on Radiohead and post-millennial rock music seeks to explain how other musical domains that exhibit this Goldilocks principle—namely harmony, form, and timbre—by both conforming to certain stylistically dependent expectations while at the same time bending and defying others, might open a richer space for interpretation than music on the extremities of these domains. Since the extremities of these domains either conform to all or none of our previous experiences, music which presents the most sustained engagement with expectation–realization chains may also offer the most sustained opportunity for the kinds of interpretation I have suggested in this article. The maximally even and Euclidean rhythms presented throughout this article can thus be heard as a microcosm of this Goldilocks principle, since they at once present us with an expected metrical framework (or at least the choice to hear one), yet realize individual onsets within that framework in sometimes unexpected ways.

NOTES

1. My dissertation on form in experimental rock music argues that understanding this dichotomy between experimental and conventional styles in post-millennial rock music is crucial to understanding genre in the new millennium (see Osborn 2010a, Chapter One).

2. A previous version of this paper was presented at the Society for Music Theory national meeting, November 2012, New Orleans. I am grateful to Julian Hook and Richard Cohn for their perceptive advice vis-à-vis the mathematical intricacies of this work, to Guy Capuzzo for suggestions regarding an early draft of this article, and to the many conference participants who were willing to share the individual ways they hear and feel these rhythms.

3. Characterizing meter in terms of lowest common denominators can act as a tool for performers and listeners alike to negotiate changing meters by entraining to a pulse level I have elsewhere called the 'pivot pulse' (2010b, 45).

4. Toussaint (2005, 4) introduces this term for a rhythm which is the product of the Euclidean algorithm—an ancient, and purely mathematical conception—which he also relates to other applications including the calculation of leap years in calendar design, drawing digital straight lines, and others non-musical applications. I will use the terms 'Euclidean distribution' and 'Euclidean rhythm' somewhat interchangeably throughout this article.

5. The rhythm necklace features prominently in works by Taylor (2009), London (2012), Demaine et. al (2009), and Toussaint (2005; 2010). This spatial conception of rhythmic sets is isomorphic to a set of elements from $\mathbb{Z}_n$ and therefore a pc-set from mod_n; the rotations of the necklace are transpositions of this pc-set mod_n.

6. Since there are five onsets, and the pattern is not transpositionally symmetrical, there are five possible rotations for the given set, though only two rotations are pictured here.

7. In terms of group theory, one could say the same thing by noting that the number 3 acts as an operator on the first, ordered mod_{12} set $(0,4,7,10,13)_{16}$ to yield the second, ordered mod_{12} set $(0,3,7,10,13)_{16}$. Here I am using parentheses not to denote inter-onset intervals, but to represent a cyclic ordered notation. See

Cohn (1992, 156–62) for more on how group theoretical and set-theoretical operations can be used to analyze recurring rhythmic patterns.

8. Though it is true that the domains of mathematics and temporally based rhythm are also not isomorphic, mathematical models, when devoid from spatial representations of said mathematics, do less to concretize the fluid and cyclical modes of experiencing rhythm.

9. Readers may notice in Example 3 the designation of "swing 8th feel" and infer a 24-pulse pattern, whereas my analysis is based on a "straight" eighth, 16-pulse pattern. Most microtiming analysis suggests that the underlying triplet division of "swung" eighths is only slightly closer to the actual heard rhythm, which lies ever between possible notations. As such, I am treating the notated 16-pulse as the *structural* basis for the expressive performed rhythm, which, qua the above, would be un-notatable by traditional means. The same could be said for virtually all transcriptions of performed music.

10. The rhythmic counterpoint between the piano and drums is altogether much more complex. The drums' accent pattern within these 16 divisions is <3,2,1,3,2,1,1,3>. Grouping the singletons in that accent pattern as anacruses yields the more properly metric division of <3,3,3,3,4>. Thus, we really have two different rotational/transpositional instances of E(5,16) occurring over the same period.

11. London (2012, 127) does refer to 'Euclidean Rhythms' in citing Demaine et. al (2009), but goes no further in discussing the distinction other than to say '[a] Euclidean rhythm is a canonical form of a maximally even pattern of *k* onsets in a cycle of *n* isochronously spaced time points.'

12. Both are also examples of what Toussaint calls a 'toggle rhythm.' If one considers alternating hands at the lowest level of evenly spaced time points, only accenting the proper onsets that compose the rhythm, a toggle rhythm is one in which the first set of onsets is played exclusively on one hand, then the second set of accents would fall exclusively onto the opposite hand. If felt as E(7,16) then 'The National Anthem' is a *smooth toggle rhythm*, meaning that it has no adjacent opposite-hand onsets. If felt as E(9,16) then 'The National Anthem' is a *sharp toggle rhythm*, meaning that there are no intervening unaccented time points at the moment the hands switch (see Toussaint 2010, 10).

13. One difference between the 'Morning Bell' E(4,10) rhythm and the 'Codex' E(5,16) has to do with the fact that 5 and 16 are co-prime and 4 and 10 are not. However, when we think about the 'The National Anthem' E(7,16), it exists somewhere between these two poles. For although 7 and 16 are co-prime (like type four), because 7 is not a factor of 16 +/–1, the Euclidean distribution makes for two instances of its longest duration (as in type two). However, unlike 'Morning Bell,' the specific temporal ordering of these onsets in 'The National Anthem' renders this groove not only Euclidean, but also maximally even, since its two longest durations are indeed as far apart as possible. But, another temporal ordering of the same E(7,16) set, for example <2,2,3,3,2,2,2>, would not be maximally even because the two threes are not spaced as evenly as possible. Even though type two Euclidean distributions are not co-prime and type three are, both have the possibility of being either maximally even or not.
14. The *Kendang* pattern than Michael Tenzer describes in Balinese music uses the same {1,1,2,2,2,2,2,2,2} multiset, though I hesitate to make any claims about distinct orderings of this pattern, because doing so relies on rather Western conceptions of downbeat that may or may not be foreign to *Kendang* musical practice.
15. This fission/fusion relationship features as one of Pressing's five ways in which two mathematically distinct rhythms may be related to one another with varying degrees of similarity (see Pressing 1983, 52).
16. Of course a great many factors besides the mathematical structure of the diatonic scale—for example register, duration, volume, and other purely musical details—contribute to tonic-finding in tonal music.
17. Whether divided into five, seven, or nine onsets, each of those intervals, when applied mod_{16}, will generate every unit in the set before repeating any. For example, completing a mod_{16} cycle using the prime generator 5 will yield each of the sixteen time points in the cycle before ultimately repeating the first: <0,5,10,15,4,9,14,3,8,13,2,7,12,1,6,11,0>.
18. Alex Ross sheds light on the connection between Jonny Greenwood and Messiaen in a 2001 interview: 'I heard the *Turangalîla* Symphony when I was fifteen and I became round-the-bend obsessed with it' (see Ross 2001).

19. Various bends, slides, and grace notes embellish this reduction throughout the track, though none of these distorts the notated rhythmic structure in any way.

20. This is my interpretation of Toussaint's data tables (2005, 6–13), though Pressing (1983, 49) makes this point explicitly.

21. Both <3,3,3,3,4> and <3,3,2> rely on the timeless musical principle of delayed satisfaction, the former delaying the resolution of a hemiola one measure longer than the latter.

22. Perhaps the most notable non-mathematical variable to this proposition is the presence or absence of a backbeat or some other recognizable rhythmic–metric paradigm (such as clave or waltz). The presence of one of these frameworks may render the hearing of a Euclidean rhythm closer to a syncopation, while the absence of any recognizable metric framework may encourage a hearing of the Euclidean rhythm as metric itself.

23. Using Toussaint's 'alternating hands method,' E(5,8) can actually generate E(5,16) by doubling the length of the period from eight to sixteen. Playing E(5,8) twice consecutively on alternating hands will yield E(5,16) on both hands simultaneously over the course of the period (see Toussaint 2010, 9).

24. Of course, I have not accounted for every possible variable that may affect this .5*n* clause. Perhaps the greatest variable may be cultural practice. For example, I am not claiming that my theory works for analyzing *tala* or *korvai* timelines. Timbre and/or pitch parameters may also affect, or even override this mathematical clause. Our perception of accent, rhythm, and meter might be changed drastically if, for example, the kick drum and snare drum parts in many of these examples were played on the opposite drum.

25. Although, comparisons with other experimental yet commercially successful artists in the twenty-first century, such as Sigur Rós and Björk, are certainly valid.

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