Essays in Monetary and Fiscal Policy

By

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Submitted to the Department of Economics and the
Graduate Faculty of the University of Kansas
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

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Date defended: May 1, 2015
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Essays in Monetary and Fiscal Policy

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Date approved: May 1, 2015
Abstract

When it comes to monetary and, especially, fiscal policy, standard focus relates to topics such as changes in policy regimes, general tax levels, fiscal expenditures, and uncertainties therein. These are broad abstractions from reality, which more often deals with policies that are more subtle than these. These seemingly small policy changes possess the potential to have dramatic and lasting impacts on the economy. My dissertation considers some of these smaller policies, their coordination with monetary policy, and their potential impact on the economy in general. Included in these policies are the removal of small currency denominations from circulation, the indexation of the federal income tax code, and the use of differing measures of inflation in policy considerations. My findings suggest that some policies considered to be important may not be, while other policies deemed insignificant can have dramatic consequences.

For example, policy makers have been increasingly concerned with the future of the monetary system’s foundation: currency. While some have focused on larger denominations, for decades a debate has focused on the effects of price-rounding if the smallest denominations are eliminated. In my first chapter, I deviate from the bulk of the literature, which typically considers case-studies with empirical simulations and data manipulation, and evaluate a multiple household, deterministic model with endogenous currency production. My findings suggest that the elimination of the smallest unit of currency has a “nickel-and-dime” effect on the economy, regardless of the rounding policy. This model is constructed and calibrated to emulate a “worst-case scenario”, but it is also robust to the empirical
results in the literature as well as the empirical results in this work.

In the second chapter, I consider a standard DSGE model with a labor income tax code derived from household income levels, finding that subtle alterations in this tax code can cause substantial changes in model dynamics. Specifically, I find that indexing the federal income tax code for inflation in the 1980s had a significant impact on the economy. My results are parallel to those of the recent monetary/fiscal policy coordination literature without considering government debt and to those of the Great Moderation literature without changing monetary policy. This suggests that the reductions in volatility seen in the data were not merely unilateral changes by monetary policy makers, but a combination of single-handed movements on both sides. Thus, even if this combination of policy changes was not sufficient to usher in the period of tranquility seen from the late-1980s through the mid-2000s, this study suggests that they were necessary.

Finally, using a two-sector New Keynesian model with sector-specific levels of price stickiness, I explore the impact of changing the monetary authority’s inflation target from a narrow measure to a broad measure while fiscal policy continues to index its tax codes and transfer payment systems to the narrow measure. I find that a monetary policy adjustment like this can have a large impact on the level and variation of real fiscal debt, but the magnitude and direction of the impact is conditional on the coordinated stance of the policy makers. However, this switch in regimes unconditionally causes most standard deviations to increase. This matches the trends seen in latest time-varying parameter empirical models since 2000, when the Federal Reserve began using forecasts of the personal consumption expenditures (PCE) chain-type price index instead of the consumer price index (CPI).
Acknowledgements

I would like to thank my advisor, Shu Wu, as well as the rest of my committee for their support throughout this process. Their guidance and advise have proven to be invaluable.

I would also especially like to thank my wife, Jennilyn, who has endured nearly a half-decade of the graduate-school experience and is still with me. Her emotional support throughout this process was crucial to my successes thus far and will be in the future.
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Chapter 1

Discontinuing Small Monetary Denominations: Does it Really Matter?

1.1 Introduction

Using a multiple household, deterministic model, I find that the abandonment of small units of currency will have a trivial effect on the economy. This model analyzes the direct, indirect, distributional, and dynamic aspects of such a policy revision, finding that the frivolous nature of these results spans all four dimensions. In the past, studies have been conducted to investigate the overall effect of such an elimination, particularly direction and magnitude of price rounding. Many of these investigations have used menu item simulations with the most common of rounding policies, while others have applied these policies to actual transaction data. The results of these studies have been controversial, citing net rounding-up in some instances and net rounding-down in other instances. This paper diverges from the simulation-based empirical methods and answers a rather simple question: how will the overall economy react to this type of change in Treasury policy? I find that the direction of price rounding does not matter, as even the most exaggerated magnitudes affect the economy minimally. This model is robust to the estimates found in the literature, allowing
me to bridge the gap between their empirical work and this theoretical insight. All in all, the worst-case scenario produces a cost of just over three dollars per person, per year in the United States; with more realistic examples producing even less significant effects.

1.1.1 Background

The future of currency has been increasingly brought into the spotlight in the United States. Some, such as Rogoff (2014), consider the usefulness of large currency denominations and whether or not they are needed. On the other side of the spectrum, there has been growing support in the United States for the elimination of the smallest denominations from the coinage system. Many other countries have already passed policies of this nature, including the United States, which eliminated the half-cent coin in 1857. This would be analogous to eliminating the dime today, and yet I am unaware of any literature suggesting there were serious consequences to this action. More recently, the Canadian government has moved to phase out its one cent coin, citing increasing costs of production and marginal utilization of the medium. Other countries, such as Australia and New Zealand, have also transitioned from their smallest denominated coins rather seamlessly, with New Zealand doing it twice since 1990. Proponents of the move believe that the production cost of the penny is too high for its lack of use, claiming that it costs taxpayers millions of dollars per year. Opponents claim that the elimination of the penny, and the rounding policies that follow, will implicitly establish a severe, regressive “rounding tax” that will drive up prices and cause overall welfare to fall. President Barak Obama even addressed this issue in a Google+ Hangout interview in February, 2013. When asked why the US Treasury has not discontinued the penny, he responded simply by saying “[…] I don’t know,” suggesting that an emotional attachment to the penny has helped preserve it.

One of the most basic concepts of maintaining a stable, working fiat system is that the value of the base materials on which currency is printed must be less than its face value (see Cipolla, 1956; Smith and Sargent, 1997; Sargent and Velde, 1999). In 2012, the US Mint
Annual Report estimated that it cost roughly $0.016 on average for the raw materials to produce a penny, and $0.02 overall. The same report also estimated that the US government generated -$58 million in seigniorage from the penny in 2012 and has generated negative seigniorage from the penny for the past seven years. While this paper focuses mostly on the penny, the trend seen here is not exclusive to this coin. Nickels have the same problem, requiring $0.0829 in raw materials and $0.1009 overall per coin. The fact that these coins cost more to produce than their face value can have some bad consequences. By Gresham’s law, and many of its derivatives, these coins are in danger of being exported or melted down (see Rolnick and Weber, 1986). Figure 1.1 presents the futures prices for both copper and zinc, the raw materials needed to produce a penny. Both prices fell initially in the wake of the financial crisis, but quickly rebounded. The increasing price of copper even prompted a rash of theft across the nation as people tried to take advantage. Reports of stollen copper piping from air conditioning units and construction sites skyrocketed. If the value of these raw materials continued to rise, at what point would a black market emerge for melting the coins down and selling them as their base materials? Of course, conducting such action is illegal in accordance with United States Code, but that didn’t really stop individuals from stealing the copper piping either.¹

¹See Title 18, Part 1, Chapter 17, Section 331 as well as United States Mint Press Release: December 14, 2006.
On the other side of this “coin”, if the penny is eliminated, there will be a significant disturbance in the economy from the ensuing rounding policies. Treasury policies in other nations typically round purchases made with currency to the next-smallest denomination, i.e. with transactions ending in $0.01, $0.02, $0.06, and $0.07 being rounded down and those ending in $0.03, $0.04, $0.08, and $0.09 being rounded up in the United States. Much of the controversy centers around the possibility that the price of the average transaction may be rounded upwards, costing consumers millions of dollars every year. Other studies suggest that, since the nickel’s face value is less than the value of its materials, eliminating the penny would actually cost the government more since demand would shift from the penny to the nickel, though these reports may be subject to bias (see Bosco and Davis, 2012). Also, since any rounding policy would only apply to cash transactions, the elimination of the penny would be a regressive policy, hurting the poor more since they use cash and coin as their primary medium of exchange.

The literature on this topic is fairly thin, boiling down to two simple methods: a simulation-based hypothetical approach, and an empirical-based, data-manipulation approach. The former is used by Lombra (2001, 2007) as well as Chande and Fisher (2003). Lombra bases his simulations on the menu of a convenience store chain, estimating the cost of transactions of three items or less. His results find that the prices of 60-93% of those simulated purchases were rounded up, with 50-83% of them being paid for with cash. This implies an annual cost to consumers of roughly $318-$818 million each year. Chande and Fisher, on the other hand, conduct similar simulations and find that the distribution of the hundredths place approaches uniformity as more items are purchased per transaction. This implies that the rounding effect may be very small in a big picture sense, or even negative. These results, and those of Lombra (2007) include the use of sales tax, whereas Lombra (2001) does not. Distributionally, Lombra estimates that around 9.5% of consumers in the United States round to the $0.01 level. However, this estimate has been criticized as being too high, as it is based on a sample of consumers who are already aware of the rounding policy.

2This report was commissioned by Jarden Zinc Products, which claims to be “North America’s leading plated coin blank producer.” This report may be biased towards not eliminating the penny.
United States do not have any kind of transactions account\(^3\), implying that this policy has the potential to be extremely regressive. The lack of transaction accounts is important because only those purchases made with cash will be rounded, while those paid with other types of money will not be. Thus, poorer households without these transactions accounts will be subjected to this policy change more than richer households.

The second method, used by Whaples (2007), considers actual transactions data from a convenience store chain, rounding those prices in accordance with the proposed policy. Upon review of these estimates, he finds that the “rounding tax” is, on average, slightly negative, finding only one State with a positive result. In all cases, whether positive or negative, these results were not significantly different from zero. Thus, he finds that the net rounding effect of such a policy will be effectively null.

One main cause of this small literature is that most of the nations that have undergone this transition were emerging economies recovering from massive inflation, making the data unreliable and the eliminated denominations nearly worthless. Those nations that consider this policy without a hyperinflation episode are few, and their occurrences are few and far between. For example, policy makers in Australia decided to phase out their one- and two-cent coins in 1992. New Zealand implemented similar policy twice, doing away with their one cent coin in 1990 and their five cent coin in 2006. Even though we have some examples of this policy, in both scenarios global contagion caused increased volatility in both economies.\(^4\) Thus, there are many roadblocks to conducting reliable econometric tests. So, after a simple empirical exercise, this paper takes a theoretical approach to see if, and to what degree, the elimination of the penny will effect the economy. I propose a deterministic, structural framework with two heterogeneous representative households, a producer, a banking sector, and a government sector that uses taxes and worn out currency to produce new currency. Using this simple model, I show the direct, indirect, distributional, and dynamic effects

\(^3\)Credit cards, checking accounts, etc.
\(^4\)“Black Monday” events sparked a US recession in the early 1990s and the global financial crisis beginning in 2007 were felt by both nations either during or immediately after their transition periods.
of eliminating small currency denominations and find that eliminating the penny will have varying effects, but that these effects are too small to matter in the long run. Steady state levels do adjust after the policy change, but the extent of this adjustment relative to the magnitude of the policy change I propose is negligible at best. We examine the effects of a policy change that essentially adds a half cent to the value of every dollar, which is orders of magnitude larger than even the most extreme cases in the literature (see Whaples, 2007; Lombra, 2001). Even with an extremely large change in policy, the effects are very small.

1.2 A Simple Empirical Test: Australia

While the data typically cannot be trusted due to exogenous events, I consider at least one situation. Australia not only eliminated its one-cent coin in 1992, but also phased out its two-cent coin simultaneously. So if there is to be a significant effect, we can expect to see one here. Taking quarterly data from the Australian Bureau of Statistics, I construct a simple VAR model with a dummy variable. This dummy holds value zero when there is no rounding policy, while holding a value of one starting in 1992:1. In this situation, I focus on the coefficient values tied to the dummy. This will show whether there was a significant structural change to the variables in the model or not. The variables included in this model are seasonally adjusted GDP growth, a CPI-measured inflation rate, the log-difference of the stock of currency, and the log-difference of general government final consumption expenditure from 1970:1-2013:3. The AIC suggests a five-lag model, so I simply present the coefficients on the dummy variable along with their 95% confidence intervals. The results are depicted in Table (1.1).

As can be seen, while all coefficients are negative, none of them are significantly different from zero, suggesting that there was no structural change in the Australian economy after the implementation of the rounding policies. Some of the literature (specifically Lombra (2001, 5Australia was the only country which altered its monetary system far enough in the past and recorded all the necessary data at a high enough frequency for a satisfactory test.)
suggests that there could be significant upward pressure on inflation and government spending while hurting the economy. These results suggest that, not only are the effects not significant, but the estimates are in the opposite direction. The theoretical models presented below yield very similar results.

1.3 The Mechanism at Work: A Two-Period Model

Before we can understand how the economy will react as a whole, we need to explore the mechanism involved. Specifically, I am interested in how the households will react to the change, abstracting towards simplifying assumptions such as constant government spending and tax rates. I can then use these results to make inferences about the larger model. For example, if the net effect of the policy change is that prices are rounded down, I find that consumption levels rise and currency holdings fall. While government spending is constant in this model, the increase in consumption and the decrease in the need for new currency implies that there would be downward pressure on overall government expenditures if this was not the case. Thus, the elimination of the penny is welfare-improving.

1.3.1 The Model

Here I present a two-agent, two-period endowment economy with currency production. The first agent is a representative household which receives an endowment each period and has the ability to invest and consume in the first period. In the second period, the household uses all of the savings and investments, along with the second period endowment, to purchase consumption goods. The second agent is the government, which takes in a sales tax from the household and produces the currency used in transactions. After addressing the model, I consider a change in the treasury’s policy that will effect the purchasing power of the currency held by the household.
The Household  The household maximizes the discounted sum of its utility, where its utility function for each period is given by

\[ U_t = \ln c_t - \frac{\chi c_t}{\mu n_t} \]

for period \( t = \{1, 2\} \), where \( c \) denotes real consumption, \( \chi \) and \( \mu \) are parameters representing preferences and the Treasury policy, respectively, and \( n \) represents the stock of currency holdings for the household given by the following law of motion

\[ n_t = n_t^p + (1 - \sigma)n_{t-1}. \]

To simplify the matter, I set \( n_0 = 0 \), that is, it starts with zero currency holdings. The term \( n_t^p \) represents the newly printed currency in period \( t \). The parameter \( \sigma \) represents the rate at which currency wears out each period.\(^6\) The household receives an endowment of \( y_1 \) in the beginning of the first period and has the ability to consume and invest in government bonds \( b \). Combining these characteristics gives me the first-period budget constraint for the household,

\[ \tau c_1 + b + n_1^p = y_1, \]

where \( \tau \) is the gross consumption tax rate levied on the household by the government. We choose the sales tax over a lump sum tax for its distortionary and regressive properties. Choosing this will ensure the maximum overall effects, if such effects exist, giving me the extremes off which to base my analysis.

In the second period, the household once again receives an endowment \( y_2 \) as well as receiving the return on the bonds purchased in the first period and acquiring more currency for purchases in the second period. Thus, the household’s budget constraint in the second period.

\(^6\)For simplicity, I'll refer to it as the depreciation rate of currency in the future, though this may be a slight abuse of the language.
period is as follows:
\[ \tau c_2 + n^p_2 = y_2 + rb, \]
where \( r \) represents the gross interest rate on the government bonds.

**The Government** Here I consider a government which produces units of government spending \( g_t \) as well as currency for the economy \( n^p_t \). In order to finance these expenditures, the government takes in the sales tax revenue from the household, collects the worn out currency in the economy, and sells bonds. It costs the government \( \zeta > 0 \) to produce each unit of new currency. Thus, the government’s real budget constraints for each period are

\[ g_1 + \zeta n^p_1 = b + (\tau - 1)c_1 + \Delta n_1 + \sigma n_0 \]

and

\[ g_2 + rb + \zeta n^p_2 = (\tau - 1)c_2 + \Delta n_2 + \sigma n_1, \]

where \( \Delta n_t \) represents the seignorage income from a change in the amount of currency in the economy.

### 1.3.2 Comparative Statics: Treasury Policy Changes

Here, I analyze the effects of a change in the Treasury policy parameter \( \mu \). Notice that I model this policy parameter so that, if prices are rounded down overall, the parameter will increase, making currency more valuable. While this model is simple and only two periods, the inclusion of currency into the model makes it too complex to solve by hand. Thus, I have to calibrate the parameters and solve for the variables before acquiring numerical results for the comparative statics problem.

**Parameter Calibration** This model contains six parameters which are calibrated to match moments in the data or values commonly used in the literature. First, I normal-
ize total output $y$ to be unity. This is purely for simplification of the model. We then calibrate the exogenous government spending $g$ to be 0.20. This matches the mean quarterly ratio of federal government current expenditures to nominal gross domestic product in the United States between 1947:Q1 and 2013:Q1, which is 0.1971. We also calibrate the net interest rate in the model $r$ to be 1.04, which matches the average one-year Treasury constant maturity rate between January 1987 and May 2013. We calibrate $\beta$, which is the personal discount factor, to 0.995, which matches much of the literature cited in this paper. Our currency depreciation parameter $\sigma$ is calibrated to 0.40 for two reasons. First, it’s large enough to ensure that the government will have to produce new currency in the second period. Second, one-dollar bills in the United States last an average of about 18 months, whereas larger bills and coins last much longer. Since there are considerably more one-dollar bills in circulation than any other paper denominations, this calibration seems to be a reasonable estimate of the average depreciation of all paper currency. The calibration of the tax rate $\tau$ is ten-percent. This sales tax rate is mostly ad hoc and larger than most state and local rates, but since this is the only form of taxes in the model, letting this be a little larger than the data suggests covers the fact that the overall amount of taxes paid is much larger than that suggested by the model. The last two parameters $\chi$ and $\zeta$ are calibrated to ensure that the levels of consumption in each period sum to approximately 0.65. The average quarterly ratio of real personal consumption expenditure to real gross domestic product from 1947:Q1 to 2013:Q1 is 0.6547, meaning that a calibration of this type matches this moment in the data. With this in mind, I calibrate the values to $\chi = 0.10$ and $\zeta = 0.90$.

**Numerical Results for the Model** Using the parameter values outlined above, I can solve for the variables in the model. Using these values, along with the parameter values, I can also numerically solve the comparative statics problem. Since the initial condition is that the Treasury continues to produce the penny, I consider the changes to the variables

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7Of course, the summation includes discounting the second period’s consumption by $\frac{1}{r}$, which is how the total output for the two periods is summed.
to a Treasury policy change when $\mu = 1$, meaning that there is no rounding policy. Table (1.2) presents these results. Notice that, as expected, consumption in both periods will move proportionally with the Treasury policy parameter via an income effect. This means that, if a policy change results in a net round-down situation ($\mu$ increases), then consumption will rise in both periods. The effect on the second period’s currency holdings is inversely related to the policy parameter, which in fairly intuitive since the value of currency increases when prices are rounded down, allowing consumers to purchase more consumption goods with less currency. The effect on the first period’s currency holdings is less certain. This value hovers around zero, moving slightly one way or the other with adjustments to the tax policy or the production cost of currency. This is also fairly intuitive since the consumer, in this model, carries over the total currency holdings (net depreciation) to the second period.

1.4 Deterministic Model with Heterogeneous Households

While a two-period model does give us an idea of how the households will react to this change (holding all else constant), the literature raises questions that require a slightly more complex model, allowing for more endogenous adjustment. To address these questions, I present a deterministic model of the economy with multiple, heterogeneous households and a government sector that produces currency. One of the first questions deals with welfare distribution, so I present one wealthy household and one poor household to the model. The former is a typical Ricardian household that has access to the financial markets, allowing it to consume and save in each period. The latter is a “rule-of-thumb” household as in Campbell and Mankiw (1989). This household is also referred to as a “hand-to-mouth” household in that it has no access to financial markets, meaning that all income is consumed immediately (see Mankiw, 2000; Weber, 2000; Galí et al., 2007, for more examples).

We introduce a banking sector to the economy, which provides an inside money substitute to the currency produced by the government. As in the previous model, the government takes
in taxes through a sales tax and collects worn out currency to produce the new currency. We also introduce a representative firm to analyze any labor-leisure tradeoffs. This firm uses the labor provided by the households and the public goods produced by the government to produce consumer goods.

1.4.1 The Heterogeneous Households

The Ricardian Household The population in this model is divided, consisting of $(1 - \lambda)$ Ricardian households and $\lambda$ “rule of thumb” households, where $\lambda \in [0, 1]$. Following the literature, I’ll denote the Ricardian, or optimizing, household with an “o”, such that a variable $x$ that belongs to this agent is referred to by $x^o$. With a discount factor $\beta$, this household maximizes an intertemporal problem with contemporaneous utility function

$$U^o_t = \ln c^o_t - \eta h^o_t - \frac{1}{\chi} \left( \frac{c^o_t P_t}{M^o_t} \right)^\chi,$$

(1.1)

where $c^o_t$, $h^o_t$, $P_t$, and $M^o_t$ denote real consumption expenditures, hours worked, the price level, and the nominal monetary services aggregate, respectively. The monetary services aggregate is a CES function that aggregates the purchasing abilities of outside money, denoted by nominal currency holdings $N^o_t$, and inside money, denoted by nominal deposits $D_t$, in the following manner

$$M^o_t = \left[ \xi^{\frac{1}{\theta}} (\mu N^o_t)^{\frac{\theta - 1}{\theta}} + (1 - \xi)^{\frac{1}{\theta}} D^o_t \right]^{\frac{\theta}{	heta - 1}},$$

where $\xi \in (0, 1)$ is the distribution parameter as described by Arrow et al. (1961), $\theta > 0$ represents the elasticity of substitution between currency holdings and deposits in providing monetary services, and $\mu$ represents a Treasury policy parameter that embodies the price rounding effect on currency purchases (see Belongia and Ireland, 2012; Ireland, 2012, for more uses of this functional form in this type of scenario.). Thus a net rounding down of prices would be represented by an increase in $\mu$. As a base for further analysis, the pre-policy change value is set to $\mu = 1$. The third part of expression (1.1) is a shopping time
friction, which is a positive function of the consumption good and a negative function of the purchasing medium. This expression is used frequently in the literature.

To encompass the ideas of a money multiplier and creation of inside money, I allow the Ricardian household to save and borrow. In each period, it earns its wage income $W_t h_t^o$, receives its investment income via principle payments on government discount bonds purchased last period $B_{t-1}$ and interest payments on deposits $r^d_{t-1}D_{t-1}$, and takes out loans from the representative bank $L_t$. Here I consider $W_t$ to be the nominal wage rate set by the representative firm, and $r^d_t$ to be the gross nominal interest rate on deposits set by the representative bank. It then takes this income and uses it for consumption, investment in new discount bonds, savings, repayment of debt from last period, and holdings of newly printed currency $N^o_{t}$, where

$$N^o_t = N^o_{t} + (1 - \sigma)N^o_{t-1}.$$  

Currency depreciates at rate $\sigma$, which implies that the government needs to collect this currency and replace it with new currency. Thus, the government needs to print currency in order to keep up with both the change in the amount demanded as well as replace the depreciated currency. With this in mind, the budget constraint of the Ricardian household becomes

$$\tau_t c_t P_t + \frac{B_t}{r_t} + D_t + N^o_{t} + r^d_{t-1}L_{t-1} = W_t h_t^o + B_{t-1} + r^d_{t-1}D_{t-1} + L_t,$$

where $r^d_t$ is the gross interest rate on loans and $\tau_t$ is the gross sales tax rate.

**The Hand-to-Mouth Household** This household has no access to financial markets, implying that it consumes all of its labor income in each period. Thus, it only has to decide how many hours to work each period. As in the literature, I denote the rule-of-thumb household’s variables with an “r” superscript. This household therefore maximizes its
contemporaneous utility function each period, given by

\[ U_r^t = \ln c_r^t - \eta h_r^t - \frac{1}{\chi} \left( \frac{c_r^t P_t}{M_t^r} \right)^\chi, \]

where each of the variables are analogous to those of the Ricardian household. In this case, however, the monetary services aggregate collapses to

\[ M_r^t = \xi \frac{1}{\nu-\tau} \mu N_t^r \]

since this household cannot access financial markets for deposits. Since this household consumes all its labor income, the budget constraint is simply

\[ \tau_t c_r^t P_t = W_t h_r^t \]

and all labor income is converted to currency for consumption purposes, giving me \( \mu N_t^r = W_t h_r^t \).

1.4.2 The Representative Bank

The representative bank’s primary purpose is to provide a substitute for the Ricardian household’s medium of exchange. It’s secondary purpose is to facilitate multiple deposit creation and the money multiplier that follows. Each period, the bank takes in new deposits and payments on matured loans from last period. It also issues new loans, makes interest payments on last period’s deposits, and incurs a linear deposit creation cost \( x_d D_t \), which causes a wedge in the saving/borrowing process. Thus the profits \( \Pi_b^t \) for the representative bank are given as

\[ \Pi_b^t = D_t - L_t + r_{t-1}^d L_{t-1} - r_{t-1}^d D_{t-1} - x_d D_t. \]

Combining this with equation (1.2) gives me an implicit Clower constraint, which seems to be redundant considering I include the shopping time friction, but this allows me to ensure that the only decision this household makes is one of labor hours.
On top of this budget constraint, the bank is also subject to reserve requirements by the monetary authority. In accordance with profit maximization, the bank wants to hold zero excess reserves, holding only those that are required. Thus, I have the equilibrium condition $L_t = (1 - \omega)D_t$ for all $t = 0, 1, \ldots$; where $\omega \in [0, 1]$ represents the required reserves ratio.

### 1.4.3 The Representative Firm

The representative firm produces real output $y_t$ using the aggregate labor hours from the households $h_t$ and the real public goods produced by the government $g_t$. The firm pays out a single wage rate $W_t$, so it does not care where the labor hours come from, maximizing its profits $\Pi^f_t$ by choosing this aggregate, given by $h_t = (1 - \lambda)h^o_t + \lambda h^r_t$. The production function is constant-returns-to-scale Cobb-Douglas, such that $y_t = h^\alpha t g^{1-\alpha}_t$ and $\alpha \in (0, 1)$. The profit function of the representative firm is

$$\Pi^f_t = P_t y_t - W_t h_t.$$ 

### 1.4.4 The Government and Monetary Authority

The government in this model not only produces the public good $G_t$, but also prints new currency $N_t^p$. To do so, the government must collect the sales tax from the households, borrow from the households, and remove the depreciated currency from the economy. With this in mind, the government’s budget constraint becomes

$$(\tau_t - 1)c_t P_t + \frac{B_t}{r_t} + \Delta N_t + \sigma N_{t-1} - G_t = B_t + \zeta N^p_t,$$

where $\Delta N_t$ represents the seignorage from the increase in the aggregate currency levels, $\zeta > 0$ represents the cost of printing new currency, and $c_t$ represents real aggregate consumption. Aggregate consumption and currency levels are given by $c_t = (1 - \lambda)c^o_t + \lambda c^r_t$ and $N_t = c_t$. 

---

9Notice that lower case letters represent the real values of their corresponding upper-case variables.
\[(1 - \lambda)N_t^\rho + \lambda N_t^r,\] respectively. The tax policy used by the government follows the simple rule
\[
\ln \left( \frac{T_t}{\tau} \right) = \phi_r \ln \left( \frac{T_{t-1}}{\tau} \right) + \phi_y \ln \left( \frac{y_t}{y_{t-1}} \right),
\]
where \(\tau\) is the steady state level for the tax rate and \(\phi_r\) and \(\phi_y\) are necessarily positive parameters. The use of a cyclical tax rate ensures that the economy does not diverge in one direction or another, while also being in line with a standard progressive tax code. The autoregressive nature of this fiscal policy rule is considered due to the fact that tax policies generally don’t change often or by very much at any particular time. The monetary authority follows a Taylor (1993) type interest rate rule with smoothing
\[
\ln \left( \frac{r_t}{r} \right) = \rho_r \ln \left( \frac{r_{t-1}}{r} \right) + \rho_\pi \ln \left( \frac{\pi_{t-1}}{\pi} \right),
\]
where \(\rho_r\) and \(\rho_\pi\) are positive parameters and \(r\) and \(\pi\) are steady state values for the bond rate and the inflation rate, respectively.

1.5 Results

Here I present some of the findings in the model. We start with the calibration of the parameter values, which are set using both the literature and historical data. We then provide some graphical representations of the model dynamics after a foreseen, permanent change to the Treasury policy parameter \(\mu\). We consider both rounding up and rounding down scenarios, increasing the value of \(\mu\) from unity to 1.005 and decreasing it to 0.995. Specifically, I consider a situation where every dollar gains/loses a half cent in value. This is much, much larger than any of the simulations in the literature suggest, but I consider this larger value for expositional purposes.\(^{10}\) In addition, a larger value will show us what

\(^{10}\)Lombra estimates that each transaction costs between five and six dollars. Following his method of applying a one cent rounding tax to each transaction implies that Lombra’s analogous value would be \(\mu_L = 0.99828\), or subtracting 0.182 cents from every dollar. Whaples estimates the per transaction benefit to be about 0.025 cents, implying an average increase in value of 0.0045 cents per dollar, or \(\mu_W = 1.0000455\).
happens to the economy if this change results in a massive, permanent shock to the economy. One can think of this value as a best-case/worst-case scenario analysis.

### 1.5.1 Calibration

Whenever possible, I use quarterly data from 1987:1–2006:4 to encompass the beginning of the Great Moderation up to just before the recent financial crisis. Some parameters are set following past literature values. For example, I set $\chi = 5$, $\xi = 0.20$, and $\theta = 0.50$ following Ireland (2012), which uses the same functional forms for the monetary services aggregate and the shopping time friction. Considering Lombra (2001), who suggests that 9.5% of households don’t have access to transaction accounts, I set $\lambda = 0.095$. Other variables are set to fit simple intuition. For example, I calibrate $\eta = 3$ because the typical work day is eight hours, or one third of the day. Setting $\eta$ to this value gives me labor hours close to $h = 1/3$.

We also set $\alpha = 0.80$, assuming that public goods produced by the government do not add much to the production process, which leans heavily on labor hours. Another parameter that can be set fairly easily is the steady state inflation rate. We set this to $\pi = 1.005$, which implies an annual inflation rate of around two percent, the implied inflation target. Considering general data trends found at the Federal Reserve Bank of Richmond, I assume that the typical unit of currency will wear out every five years or so, implying that $\sigma = 0.05$. The same holds true for seigniorage data from the Board of Governors of the Federal Reserve System, implying that $\zeta = 0.30$, a value that ensures that producing currency yields positive seigniorage in general.\footnote{This value for $\zeta$ is probably larger than it should be, but a larger value will again give me a worst-case-scenario if it costs the government large amounts to produce the currency. We are also focusing more on the effects of coin costs and not that of paper currency costs, which are dramatically larger, in a relative sense.}

The rest of the parameters have been estimated using the data. We calibrate $\omega = 0.035$ to match the average ratio of the St. Louis Adjusted Monetary Base less currency in circulation and excess reserves to savings deposits at commercial banks. This is an estimate of the average required reserves ratio during the period 1987:Q1–2006:Q4. For the same time...
period, I estimate the value of $x^d$ by considering a ratio of a deposit rate estimate to the bank prime loan rate. The deposit rate estimate is the over-time average of the mean of the 6-month certificate of deposit secondary market rate and the rate on money market mutual fund accounts. This gives me an arbitrary short-term interest rate that lies somewhere between the highest and lowest available rates on differing types of interest bearing deposits. Doing so implies that $x^d$ should be set to 0.01. The Ricardian household’s discount factor $\beta$ is set by considering the average effective federal funds rate over my calibrated value for inflation. This gives me a value of $\beta = 0.99$. The parameter values in the fiscal policy rule are set with a simple linear regression. Here I regress the values of the federal government current tax receipts-to-personal consumption expenditures ratio against its owned lagged value and the GDP growth rate. Doing so gives me parameter values $\phi_r = 0.9009$ and $\phi_y = 0.0074$. This implies that the tax rate does not react strongly to outside forces, which would coincide with the idea that tax rates are mostly exogenous. The values for the monetary policy rule were calibrated in the same fashion. Regressing the effective federal funds rate against its own lag and a one-period lag of the inflation rate gives me parameter values of $\rho_r = 0.96$ and $\rho_\pi = 0.06$.

1.5.2 Deterministic Shock Simulations

Since this paper considers fiscal policy, any policy change is announced in a particular period, but there is a one year (four period) lag before the policy comes into effect. Figure (1.2) shows the impulse responses for aggregate consumption, aggregate currency holdings, the inflation rate, and government spending after a positive, permanent shock to the Treasury policy parameter, i.e. prices are rounded down and $\mu$ increases to 1.005. Each of the panels in the figure contains two vertical lines. These lines coincide with the announcement and implementation of the policy change, respectively. We chose the aggregate values for each because the individual household impulse response functions are nearly identical in shape, so a broad, macroeconomic view represents a good statistic for these results.
Policy Announcement  As can be seen in the figure, the effects of this relatively large shock are minor. Upon the announcement of the new policy, the households realize that their steady state consumption levels will increase. Therefore, in an effort to smooth their consumption, they deviate from their Euler equations and begin increasing consumption immediately (panel 1). Due to the persistence in the tax level though, this transition is slow and does not reach the new steady state level before implementation of the new policy. In order to reach these higher levels of consumption, they need more currency and deposits, as can be seen by the upwards drift in the second panel. This increase in consumption, currency, and deposits puts upward pressure on inflation (panel 3), but is quickly corrected by monetary policy. As for the government sector of the economy, the balanced budget
assumption and the increased need for currency force the government to divert resources from traditional spending to currency production. This is exacerbated by the fact that output remains relatively unchanged, causing a crowding out effect that forces spending to fall at a faster rate.

**Policy Implementation**  
Upon implementation of the new policy, the currency holdings of the households become more valuable. This income effect causes consumption to spike slightly and currency holdings to fall. This increase in consumption and the fall in seignorage revenues cause government spending to fall temporarily at implementation. In a reversal of the announcement period, the fall in currency holdings causes inflation to fall immediately, but is corrected by monetary policy in the next period. The increase in the value of currency also causes deposits to fall by a substitution effect. After the initial impact of implementation, the households continue to transition to the new steady state level of consumption. This increase causes an increased need for currency, which again combine to push government spending down as in the announcement period.

If I reverse the net effect and follow the rounding direction estimated by Lombra (2001) ($\mu = 0.995$), I get nearly symmetric results, despite the inherent non-linearity of the model, which coincides with his findings. In this scenario, welfare would fall, government spending would increase, and there would be upward pressure on inflation. However, due to monetary policy and an inflation target that is maintained, inflation is contained and the spike is even neutralized, a situation not considered in the literature.

Overall, we can see in Table (1.4) that the long run effects of this permanent shock are very small. This table shows the initial and resulting steady state values for the model, along with the percentage change in each of them. We see that, for either a positive or negative rounding result, most of the variables in the model will settle in close to their initial values. The largest shift in any variable is in the currency holdings, but even this adjustment is by less than a half of one percent in the aggregate. If we were to look at the data, this
adjustment would imply that the currency component of M1 would increase or decrease by around $3.6 million in the first quarter of 2013, or a little over a penny per person. Thus, the long run effects of this policy, even after this large shock, would be insignificant. This matches the empirical results from section 1.2, found in Table (1.1), suggesting that there is virtually no change in GDP growth; while currency growth, inflation, and government spending all fall by minute levels.

1.5.3 Welfare Analysis: Consumption Equivalent Variation

In this section, I conduct a welfare analysis using consumption equivalent variation (CEV). In keeping with Lombra (2001), I consider the first five years after the policy announcement, where the policy is announced at the beginning of the first year and implemented at the beginning of the second year. To evaluate the CEV, I first consider the situation in which the policy doesn’t change, i.e. the penny is not eliminated. We then consider the utility function maximized by each household where policy does change. Adding my measure for welfare gain or loss, I have the following utility functions,

\[
\max_{i \in \{o, r\}} \sum_{t=0}^{20} \beta^t \left[ \ln c_i^t (1 + \phi_w^i) - \eta h_i^t - \frac{1}{\chi} \left( \frac{c_i^t}{m_i^t} \right)^\chi \right],
\]

for \(i = \{o, r\}\), where \(\phi_w^o\) and \(\phi_w^r\) are the equivalent variation measurements for each of the respective households.\(^{12}\) While the poor household is constrained to the point that it can only make contemporaneous decisions, it discounts time in the same fashion as the rich household. Considering the values depicted in Figure (1.2), and comparing these maximized utility values with those considering only the initial steady state values, I find that, in the case of a net-round down situation, both households are better off, though the poor household is considerably more. The results are provided in Table (1.3). They show what percentage of additional consumption is needed each quarter to make the household indifferent between

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\(^{12}\)Negative values for these measurements imply that the household is better off after the policy change, whereas positive values indicate that the household is worse off.
Table 1.1: Dummy Variable Coefficients

<table>
<thead>
<tr>
<th>Point Estimate</th>
<th>Lower Conf. Bound</th>
<th>Upper Conf. Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP Growth</td>
<td>-0.00</td>
<td>-0.5571</td>
</tr>
<tr>
<td>Inflation</td>
<td>-0.22</td>
<td>-0.7818</td>
</tr>
<tr>
<td>Currency Growth</td>
<td>-0.14</td>
<td>-0.3551</td>
</tr>
<tr>
<td>Gov. Spending Growth</td>
<td>-0.11</td>
<td>-0.9240</td>
</tr>
</tbody>
</table>

a A 95% confidence interval is considered here.

Table 1.2: Comparative Statics Results: $\mu$

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>Period 1 Consumption</td>
<td>+</td>
</tr>
<tr>
<td>$c_2$</td>
<td>Period 2 Consumption</td>
<td>+</td>
</tr>
<tr>
<td>$n_1$</td>
<td>Period 1 Currency Holdings</td>
<td>+/-</td>
</tr>
<tr>
<td>$n_2$</td>
<td>Period 2 Currency Holdings</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1.3: Welfare Analysis: Consumption Equivalent Variation

<table>
<thead>
<tr>
<th>Measure</th>
<th>Positive Shock ($\mu = 1.005$)</th>
<th>Negative Shock ($\mu = 0.995$)</th>
<th>Lombra (2001) ($\mu = 0.99828$)</th>
<th>Whaples (2007) ($\mu = 1.0000455$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich CEV, $\phi_w^p$</td>
<td>-0.0441%</td>
<td>0.0444%</td>
<td>0.0152%</td>
<td>-0.0004%</td>
</tr>
<tr>
<td>Poor CEV, $\phi_w^p$</td>
<td>-0.7458%</td>
<td>0.7562%</td>
<td>0.2589%</td>
<td>-0.0069%</td>
</tr>
<tr>
<td>Aggregate CEV, $\phi_w^p$</td>
<td>-0.1108%</td>
<td>0.1120%</td>
<td>0.0384%</td>
<td>-0.0010%</td>
</tr>
</tbody>
</table>
the two situations. The last row of the table considers the aggregated CEV using the same method as with the other terms in the model, \( \phi_w^a = (1 - \lambda)\phi_w^o + \lambda\phi_w^r \). Considering that the rich household has access to financial markets, it is intuitive that its results are less volatile than that of the poor household. Since the poor household is completely constrained and cannot access financial markets, variations in the value of its currency holdings will cause larger equivalency requirements. Thus, we see that the progressive/regressive nature of this "rounding tax" depends on the direction of the rounding. If the net result is that we round down, we find that the welfare gap closes slightly. If we round up, on the other hand, then the gap would widen.

**Comparing to Past Results** We can now take the values from Table (1.3) and see how they compare to the results found in the literature, finding that my model is quite robust. For example, Lombra (2001) claims that these changes will cost consumers no less than $1.5 billion over a five-year period. Using data on personal consumption expenditures (measured in billions of chained 2009 dollars during 1990:Q1–1999:Q4) I find that, if prices are rounded up at this estimated rate, consumers will lose an average of $118 million per quarter, or about $2.36 billion over an average five-year period, which is in his estimated range.\(^{13}\) This averages out to about a $1.60 cost to each consumer per year.\(^{14}\) If I consider the values estimated in Whaples (2007), and conduct the same test as above, I find that the average consumer would benefit by approximately 0.04 cents per year. Thus, my model fits the results in the literature quite well. In any case, the cost or benefit to each consumer in these events is negligible.

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\(^{13}\)$2.36 billion measured in 2009 dollars is approximately equivalent to $1.89 billion measured in 2000 dollars, the publication year of the menu costs in Lombra’s simulations. Thus, applying his results to my model yields nearly the same conservative values he finds.

\(^{14}\)In calculating these values, I multiply the values found in Table (1.3) by the quarterly personal consumption expenditures and take the average. We then take this value and divide it by the average population at the time.
1.6 Concluding Remarks

This paper deviates from the literature through the use of a structural model. In doing so, I find that the effects of eliminating a nation’s smallest unit of currency are insignificant, even when considering shocks that are many times larger than those suggested in the literature. While I consider the larger shock for expositional purposes, the model is robust to shocks which correspond to those past estimates. Future inquiries can build on these results by relaxing some of my base assumptions, such as a balanced government budget, or by considering a more intricate currency production process. The simplicity and robustness of this model, however, makes it a good starting point for future debate on a growing issue. As prices continue to rise gradually, the production and distribution of the penny will become increasingly burdensome for the US government and taxpayers, implying a need for policy change. The governments of Canada, New Zealand, Australia, and others; facing the same pressures; resorted to discontinuation of their smallest denominations. What will the US government decide to do?
Table 1.4: Steady State Analysis Before and After Rounding Policies

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial Value</th>
<th>Round Down $^b$</th>
<th>Round Up $^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Value</td>
<td>Change (%)</td>
</tr>
<tr>
<td><strong>Rich Household</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption, $c_t^o$</td>
<td>0.2876</td>
<td>0.2878</td>
<td>0.0778</td>
</tr>
<tr>
<td>Currency, $n_t^o$</td>
<td>0.0599</td>
<td>0.0597</td>
<td>-0.2490</td>
</tr>
<tr>
<td>Deposits, $d_t$</td>
<td>0.6069</td>
<td>0.6069</td>
<td>0.0004</td>
</tr>
<tr>
<td>Loans, $l_t$</td>
<td>0.5856</td>
<td>0.5856</td>
<td>0.0004</td>
</tr>
<tr>
<td>Hours, $h_t^o$</td>
<td>0.3326</td>
<td>0.3326</td>
<td>0.0004</td>
</tr>
<tr>
<td><strong>Poor Household</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption, $c_t^r$</td>
<td>0.2964</td>
<td>0.2967</td>
<td>0.0749</td>
</tr>
<tr>
<td>Currency, $n_t^r$</td>
<td>0.3323</td>
<td>0.3307</td>
<td>-0.4988</td>
</tr>
<tr>
<td>Hours, $h_t^r$</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.0000</td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spending, $g_t$</td>
<td>0.0382</td>
<td>0.0380</td>
<td>-0.5947</td>
</tr>
<tr>
<td>Curr. Prod., $n_t^n$</td>
<td>0.0043</td>
<td>0.0043</td>
<td>-0.3409</td>
</tr>
<tr>
<td><strong>General Economy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output, $y_t$</td>
<td>0.4146</td>
<td>0.4146</td>
<td>0.0003</td>
</tr>
<tr>
<td>Inflation, $\pi_t$</td>
<td>1.0050</td>
<td>1.0050</td>
<td>0.0000</td>
</tr>
<tr>
<td>Interest Rate, $r_t$</td>
<td>1.0152</td>
<td>1.0152</td>
<td>0.0000</td>
</tr>
<tr>
<td>Wage Rate, $w_t$</td>
<td>0.9970</td>
<td>0.9970</td>
<td>-0.0001</td>
</tr>
</tbody>
</table>

$^a$ Recall that the shock I consider here is orders of magnitude larger than those estimated in the literature. We do this for expositional purposes.

$^b$ Treasury policy parameter $\mu$ set to 1.005.

$^c$ Treasury policy parameter $\mu$ set to 0.995.
Chapter 2

Inflation-Protected Taxes and Monetary Dominance

2.1 Introduction

Beginning with Leeper (1991), the standard monetary/fiscal policy interaction literature has considered lump sum tax rules based on government debt and/or spending. Analysis is typically focused on significant shifts or underlying uncertainties in such policies.\(^1\) But what about subtler fiscal policies such as changes to the basic structure of the tax code itself? In this paper, I implant a labor income tax code derived from household income levels within an otherwise standard New Keynesian model. This tax code is constructed with actual legislation in mind. With it, I explore the impact of indexing the federal income tax code for inflation. Viewing tax policy from a legislative perspective allows me to analyze monetary and fiscal policy coordination while abstracting from fiscal sustainability issues and the empirical estimation problems that follow. Since the Federal Income Tax Code was indexed in 1985, estimating changes to this type of fiscal policy is unnecessary. Even so, my model predicts a similar timeline for differing policy regimes as those who consider

\(^1\)In addition to the example mentioned above, see Sims (1994), Woodford (2001), Davig and Leeper (2011) and Leeper and Zhou (2013).
the fiscal sustainability issues. Specifically, an indexed tax code yields the same “passive” fiscal policy properties as seen in the literature. Only after the indexation of the tax code is active monetary policy allowed to be dominant. Thus, active monetary policy and an indexed income tax code are necessary conditions for stability. This implies that the increased aggressiveness of the Volcker disinflation, nearly five years prior to the indexation of the federal income tax code, was not sufficient in creating the period of relative stability known as the Great Moderation. In fact, unilateral moves by either the monetary or fiscal authorities would not have led to this period of tranquility.

2.1.1 A Legislative View of Policy Coordination

Sargent and Wallace (1981) show that there are situations in which the monetary authority can be very limited in its control of inflation, even if the relationship between the monetary base and the price level remains strong. Also called the fiscal theory of the price level, it suggests that the central bank can be forced to cover the difference between the funds needed for government spending and the public’s demand for government bonds through seignorage. Thus, in this situation, fiscal policy governs inflation dynamics and essentially reverses the standard interest rate channel. This model, along with those of Leeper (1991), Sims (1994), and Woodford (2001), to name a few, focus on tax rate rules that are dependent on the level of real outstanding government debt and government spending. However the federal income tax code has remained relatively steady since the 1990s while debt has fluctuated (see Romer and Romer, 2010). Figure 2.1 presents the legislated federal income tax code as average or effective income tax rates for fixed, evenly spaced real income levels across the the period 1950 to 2011. These data are constructed by applying 200 synthesized real income levels to the Federal Income Tax Code from 1950–2011 to calculate the real tax liability owed at each income level. The effective rates are then calculated by dividing the liability by the income level. This shows that the core of the tax code has remained relatively unaltered for the

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2 Though, as I mentioned, my tax rule is not a function of government debt, so a literal application of the Leeper definition may be a stretch.
last two decades, suggesting that fiscal policy may not be as passive as the literature would suggest it is. For this purpose, I focus not on changing average lump sum taxes based on debt and spending, but more on the tax code as it is structured in legislation.

One specific piece of legislation during this period changed the economic landscape: the Economic Recovery Tax Act of 1981. Not only did it reduce the top marginal federal income tax rate from 70 to 50 percent, but it also established that this particular tax code would be automatically indexed for inflation beginning in 1985. *Indexation* is a policy in which the nominal bounds on the tax code brackets are adjusted annually for inflation.³ Doing so is important even in low inflation economies because, without it, we get a phenomenon known as *bracket creep*, which can be seen in Figure 2.1 especially in the high-inflation period of 1965-1980. This is the process by which a household ascends to higher tax liabilities when its nominal income increases, even if the purchasing power of that income remains constant. This causes the real disposable income of the household to fall over time.

As a more concrete example, consider a household in 1973 making $19,000 annually (roughly $99,600 in 2014 dollars) and filing its taxes. Given that the consumer price index increased by approximately 11% between 1973 and 1974 and assuming that this household received an equivalent cost-of-living adjustment to their salary, the real value of their tax liabilities increased, raising their effective tax rate from about 21.58% in 1973 to 22.42% in 1974. This is shown in more detail in Table 2.1. While this is a tailored exampled used to make a point, this is bracket creep in its simplest form, causing real disposable income to fall over time due to inflationary pressure. It is important to note that this example only considers the change from one year to the next, but the entire period between 1965 and 1980 saw accelerating price levels, meaning that households could lose around one percent of their disposable income every year. Again, refer to the effective tax rates for fixed real income levels in Figure 2.1 to see how quickly tax liabilities can increase in a high inflation period. More distinctly, we can show the changes in the effective tax rates over time for chosen

³There are other methods of indexing a tax code (see Thuronyi, 1996), but is the largest component and thus is the focal point of this paper.
Table 2.1: Changes in Tax Liabilities Due to Inflation: 1973–1974

<table>
<thead>
<tr>
<th>Bracket</th>
</tr>
</thead>
<tbody>
<tr>
<td>14%</td>
</tr>
<tr>
<td>15%</td>
</tr>
<tr>
<td>16%</td>
</tr>
<tr>
<td>17%</td>
</tr>
<tr>
<td>19%</td>
</tr>
<tr>
<td>22%</td>
</tr>
<tr>
<td>25%</td>
</tr>
<tr>
<td>28%</td>
</tr>
<tr>
<td>32%</td>
</tr>
<tr>
<td>36%</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1973</th>
<th>1974</th>
</tr>
</thead>
<tbody>
<tr>
<td>Taxable Income</td>
<td>Liability</td>
</tr>
<tr>
<td>$1,000</td>
<td>$140</td>
</tr>
<tr>
<td>$1,000</td>
<td>$150</td>
</tr>
<tr>
<td>$1,000</td>
<td>$160</td>
</tr>
<tr>
<td>$1,000</td>
<td>$170</td>
</tr>
<tr>
<td>$4,000</td>
<td>$760</td>
</tr>
<tr>
<td>$4,000</td>
<td>$880</td>
</tr>
<tr>
<td>$4,000</td>
<td>$1,000</td>
</tr>
<tr>
<td>$3,000</td>
<td>$840</td>
</tr>
<tr>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>$19,000</td>
<td>$4,100</td>
</tr>
</tbody>
</table>

income levels. Figure 2.2 shows the movements in the effective tax rates for real incomes of $40,000, $70,000, and $100,000 measured in 1982 dollars. The major changes in these time series correspond to legislative alterations to the tax code. These include the initiation and removal of the Vietnam War surtax in 1966 and 1969, respectively, as well as the adjustment for bracket creep in 1976 and the major tax overhauls in 1981 and 1986. But notice that there are persistent changes in these tax rates between the legislative adjustments. Outside of the 1976 adjustment, tax rates rose steadily between 1970 and 1980. Depending on the real income level, we see increases in tax liabilities of anywhere between 0.5 and 2.0 percentage points per year. After 1984, the only tax rate changes are results of legislation, not bracket creep.

Continuing this example at the macro-level, Table 2.2 shows a rough estimate of the additional tax revenue generated by inflation starting with the tax legislation of 1981 and prior to the indexation in 1985. Equivalently, this can be viewed as the loss in real disposable income to inflation. I construct this time series by first calculating the effective personal income tax rates via nominal receipts from income taxes. I then apply them to their corresponding real incomes calculated using the change in average hourly earnings. Since the new tax code and
Table 2.2: Estimates of Additional Tax Revenue Due to Inflation

<table>
<thead>
<tr>
<th>Year</th>
<th>Effective Rate</th>
<th>Additional Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981</td>
<td>11.0%</td>
<td>$ 0.00 billion</td>
</tr>
<tr>
<td>1982</td>
<td>10.7%</td>
<td>$16.27 billion</td>
</tr>
<tr>
<td>1983</td>
<td>9.7%</td>
<td>$26.78 billion</td>
</tr>
<tr>
<td>1984</td>
<td>9.1%</td>
<td>$36.91 billion</td>
</tr>
</tbody>
</table>

brackets were set in 1981, the additional revenue for the first year is null, but since prices rose at a faster pace during this period, these additional tax receipts accumulated quickly, even in the face of falling effective tax rates.\(^4\) While this is a rough estimate, these figures are robust to multiple measures of income and wage inflation.

2.1.2 Volatility Reductions and the Timing of Indexation

At first, theories about the sudden fall in volatility were focused exclusively on monetary policy. Works such as Taylor (1999) and Clarida, Galí and Gertler (2000) suggested that there was a dramatic shift in the way monetary policy was conducted. This break was considered to come from one of two sources: a move from discretionary policy towards interest rate rules or an increased aggressiveness against inflation if rules were already the norm. Other explanations also surfaced, including Blanchard and Simon (2001) and Galí and Gambetti (2009), which suggest a sudden, structural shift in the relationships between variables in the economy. But even with all of the empirical evidence, Stock and Watson (2003) still estimate that 40-60% of the cause remains unknown, prompting a title of “good luck.” By this, it is generally meant that the variance of supply shocks has fallen dramatically since the 1980s.

Contrastingly Athanasios Orphanides and his coauthors dismissed the idea of discretionary policy, citing faulty information as the culprit, finding persistent differences in the real-time estimates of the output gap versus the revised measurements.\(^5\) This is typically

\(^4\)The effective tax rates fell due to the fact that the tax reductions were imposed over multiple years, but the brackets were established in 1981.

\(^5\)A select few of this large literature include Orphanides and van Norden (2002) and Orphanides (2003, 2004).
considered to be an effect of an unanticipated reduction in productivity growth, causing estimates of the output gap to be lower than they truly were. If the Federal Reserve was utilizing the output gap as a primary indicator of economic activity, large measurement errors could easily derail monetary policy, making it seem discretionery. But as Orphanides (2003) notes, there is a sudden reduction in this measurement error in the mid-1980s. For this reason, this literature often refers to this period of high inflation and volatility as the “Great Inflation.”

In either case, the empirical work in this field suggests that the sudden reduction in volatility occurred at about the same time as the indexation of the income tax code. Stock and Watson (2003) narrow the literature’s results, saying that much of the evidence points to a structural break in the first quarter of 1984. Considering the fact that tax laws are annual by nature, if the tax code was first indexed for inflation in 1985, bracket creep would have ended at the beginning of 1984, corresponding the the structural breaks found the in the literature.

2.1.3 The Great Moderation as a Monetary and Fiscal Phenomenon

This paper ties together the monetary/fiscal-interaction and Great Moderation literatures. Using time-varying parameter estimates, my model predicts a similar timeline of breaks in model dynamics as Davig and Leeper (2011) and Bianchi (2012). However, many papers in this line use sophisticated Markov-switching models to estimate regime changes within fiscal policy. My model considers actual legislation, allowing me to pinpoint regime changes in fiscal policy without estimation. This is a more realistic representation of the current income tax code, which has not substantially changed since the early 1990s.

My model predicts that only the combination of active monetary policy and an indexed tax code results in a unique solution, much like the active monetary/passive fiscal policy prescriptions in the literature. Any unilateral policy shift results in either sunspot equilibria

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6Similarly, Keating and Valcarcel (2012) consider this period a “blip” on the metaphorical radar, suggesting that the Great Moderation may not have been the greatest.
or explosive behavior. Extending my model by introducing a tax-labor productivity channel results in a unique solution even in the non-indexed specification, while indexation leads to decreases in variable volatilities and changes in variable correlations similar to the empirical results found by Galí and Gambetti (2009) and others. Additionally, impulse response analysis suggests that bracket creep is a plausible cause of the labor productivity slowdown and that indexation removed much of the noise from the data as described in Orphanides (2003).

The remainder of this paper is organized as follows: Section 2.2 analyzes a standard New Keynesian model with a progressive tax code, Section 2.3 presents the results, including determinacy regions and application of the literature, and Section 2.5 concludes.

2.2 New Keynesian Model with Tax Policy

Here I present a DSGE model with nominal price rigidities. The essence of my model much like those used in Ireland (2004, 2012) and Belongia and Ireland (2012), but with a fiscal agent which enlists an individual income tax code that depends on the households wage income like those of Guo and Lansing (1998) and Chen and Guo (2013).

2.2.1 The Representative Household

In this model the representative household solves

$$\max_{\{c_t, h_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \eta^p_t \ln c_t - \frac{1}{\psi} h_t^\psi \right), \quad (2.1)$$

where $c_t$ and $h_t$ denote real consumption and labor hours, respectively. The parameters $\beta \in (0, 1)$ and $\psi > 0$ represent the subjective discount factor and the elasticity of substitution, respectively. The preference shock $\eta^p_t$ follows an autoregressive process in its natural

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7See Woodford (2011); Galí (2009); and Rotemberg (1982) for details regarding this style of model and nominal price rigidities.
logarithm

\[ \ln \eta^p_t = \rho_p \ln \eta^p_{t-1} + \varepsilon^p_t, \]  

\[(2.2)\]
such that \( \rho_p \in (0, 1) \) and \( \varepsilon^p_t \) is an i.i.d. innovation with zero mean and constant variance \( \sigma^p > 0 \). While solving (2.1), the household has to consider its own budget constraint. In every period, the household earns income via the returns on nominal discount bonds \( B_t \) purchased in the previous period; its disposable labor income, with \( W_t \) and \( \tau_t \) representing the nominal wage and tax rates, respectively; and revenues from dividend payments \( D_t \). We assume the household takes the tax rate \( \tau_t \) as given since it is set by the fiscal authorities.\(^8\) This income is then divided between the purchase of real consumption goods \( c_t \) at price \( P_t \) and nominal bonds at price \( 1/r_t \), where \( r_t \) is the gross nominal interest rate in the economy. All of this yields the following budget constraint:

\[ P_t c_t + B_t/r_t \leq B_{t-1} + (1 - \tau_t)W_th_t + D_t. \]  
\[(2.3)\]

Along with this budget constraint, the first order conditions are

\[ \frac{\eta^p_t}{c_t} = \beta r_t E_t \left[ \frac{\eta^p_{t+1}}{c_{t+1} \pi_{t+1}} \right] \]  
\[(2.4)\]

and

\[ \frac{W_t}{P_t} = \frac{h_t^{\psi-1}c_t}{\eta^p_t(1 - \tau_t)}, \]  
\[(2.5)\]

where

\[ \pi_t = P_t/P_{t-1} \]  
\[(2.6)\]
is the gross inflation rate. Derivation of these optimization conditions can be found Appendix B. As we can see, the Euler equation remains unaltered from the standard New Keynesian models, but our intratemporal condition now depends on the labor income tax rate \( \tau_t \).

\(^8\)The results of Guo and Lansing (1998) show that this specification does not change their determinacy results.
2.2.2 The Final Goods-Producing Firm

Like so many New Keynesian models, I consider a final good-producing firm which simply aggregates the differentiable goods $y_t(i)$ for $i \in [0, 1]$ produced by the continuum of intermediate goods-producing firms for consumption by the households. It does so with a CES production function

$$y_t \leq \left[ \int_0^1 y_t(i) \frac{\eta^s_{t-1}}{\eta^s_t} \, di \right]^\frac{\eta_s^t}{\eta^s_t - 1},$$

where $\eta^s_t$ is an exogenous process governing the elasticity of substitution. I consider innovations to this as price markup shocks, which follow an autoregressive process in its natural logarithm

$$\ln \eta^s_t = (1 - \rho_s) \ln \eta^s_t + \rho_s \ln \eta^s_{t-1} + \varepsilon^s_t,$$ \hspace{1cm} (2.7)

where $\rho_s \in (0, 1)$, $\eta^s > 0$, and $\varepsilon^s_t$ is the i.i.d. innovation with zero mean and constant variance $\sigma^2_s > 0$. The final goods-producing firm maximizes its profits in a perfectly competitive market, yielding its demand for each intermediate good

$$y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\eta_t^s} y_t,$$ \hspace{1cm} (2.8)

for all $i \in [0, 1]$.

2.2.3 The Intermediate Goods-Producing Firms

As was mentioned above, there is a continuum of monopolistically-competitive, intermediate good-producing firms labeled by $i \in [0, 1]$. For simplification, we assume that all of these firms face the same production technology given by

$$y_t(i) \leq z_t h_t(i)$$ \hspace{1cm} (2.9)
for all $i$, where $z_t$ is a labor-augmenting productivity process governed by

$$\ln z_t = (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} + \varepsilon_t^z,$$  \hspace{1cm} (2.10)

such that $z > 0$, $\rho_z \in [0, 1]$, and $\varepsilon_t^z$ is an i.i.d. innovation with zero mean and variance $\sigma_z^2 > 0$.

I also introduce a Rotemberg (1982) cost of price adjustment

$$\Phi_t(i) = \frac{\mu}{2} \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right)^2 c_t$$

for all $i$, which is measured in units of the consumption good. With $\mu \geq 0$ regulating the magnitude, this cost of adjustment constraint makes these firms’ problems dynamic. I also assume that all profits from these firms are remitted to dividends $D_t(i)$, so that the profit functions simplify to

$$D_t(i) = P_t(i)y_t(i) - W_t h_t(i) - P_t \Phi_t(i)$$  \hspace{1cm} (2.11)

for all $i$. With this in mind, each intermediate goods-producing firm’s problem is given by

$$\max_{\{P_t(i), h_t(i), D_t(i)\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \eta^p D_t(i) \frac{c_t}{P_t},$$  \hspace{1cm} (2.12)

subject to constraints (2.8) and (2.11). The first term in (2.12) is the discounted marginal utility value to the household of additional future profits. Each firm’s optimizing conditions are therefore:

$$(1 - \eta_t^p) \left( \frac{P_t(i)}{P_t} \right)^{-\eta_t^p} \frac{y_t}{P_t} + \eta_t^p \left( \frac{P_t(i)}{P_t} \right)^{-\eta_t^p - 1} \frac{y_t W_t}{z_t P_t^2} - \mu \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right) \frac{c_t}{\pi P_{t-1}(i)}$$

$$+ \beta \mu \mathbb{E}_t \left[ \left( \frac{P_{t+1}(i)}{\pi P_t(i)} - 1 \right) \frac{P_{t+1}(i) \eta_{t+1} P_t}{\pi P_t(i)^2} \right] = 0$$  \hspace{1cm} (2.13)

for all $i$, which, when linearized, gives me a New Keynesian Phillips Curve. A derivation of these conditions can be found in Appendix B.
2.2.4 The Fiscal Authority and the Labor Income Tax Code

The fiscal agent in this model is responsible each period for producing real government goods \( g_t \) and settling its debt from last period \( B_{t-1} \). To do so, it sells nominal discount bonds \( B_t \) at price \( 1/r_t \) and collects tax revenue through a labor income tax. Considering this, the government’s budget constraint is

\[
P_t g_t + B_{t-1} \leq \frac{B_t}{r_t} + \tau_t W_t h_t. \tag{2.14}
\]

Similar to the specification used in Chen and Guo (2013), the effective labor income tax rate \( \tau_t \) evolves according to

\[
\tau_t = 1 - \theta \left( \frac{w_t h_t}{w_t h_t P_t^{1/n}} \right)^\phi \tag{2.15}
\]

where \( w_t \equiv w_t/r_t \) is the real wage rate, \( \theta \in [0, 1] \) dictates the steady state tax rate, \( \phi \in (-1, 1) \), and \( 1_n \) is an indicator function that holds a value of one when the tax code is not indexed for inflation and zero when it is indexed for inflation. I assume that, when the tax code is set or adjusted, the price level index as it is related to the new tax code is reset to unity, allowing me to omit a price level in the numerator. With \( \tau_t \) representing the effective rate, I define the marginal tax rate \( \tau^m_t \) to be the change in tax liability relative to the change in labor income. This implies

\[
\tau^m_t \equiv \frac{\partial(\tau_t w_t h_t P_t^{1/n})}{\partial(w_t h_t P_t^{1/n})} = \tau_t + \phi \theta \left( \frac{w_t h_t}{w_t h_t P_t^{1/n}} \right)^\phi. \tag{2.16}
\]

Thus, we can see that if \( \phi > 0 \), the marginal tax rate at any given income level is greater than the effective tax rate, which is the definition of a progressive tax code. If \( \phi < 0 \), the marginal rate is less than the effective rate at every level of income, which defines a regressive tax code. Typical models in this literature assume income tax codes with \( \phi = 0 \), in which case the marginal tax rate and the effective tax rates are equal at all times. This is the standard definition of a flat tax code. From another perspective, this parameter governs
the labor income elasticity of the effective tax rate, given by

$$\frac{\partial \tau_t}{\partial (w_t h_t P_t^{1n})} \frac{w_t h_t P_t^{1n}}{\tau_t} = \phi \frac{1 - \tau_t}{\tau_t},$$

where, combining this with (2.16), we can see that setting \( \phi = 0 \) will make the effective tax rate and the marginal tax rate unresponsive to changes in the income level. When linearized, (2.15) results in a tax policy similar to those in Leeper (1991) and Davig and Leeper (2011), only this is a function of labor income, not government debt. Those models with no labor income tax policy would be equivalent to setting \( \phi = 0 \) and \( \theta = 1 \).

Now let’s consider the theoretical impact of not indexing a labor income tax code. Assuming that the general trend of the price index is upward, as has been historically, then at any time after the price index is reset to one, \( P_t \geq 1 \). If I also assume that the tax code is progressive, as has been the case since its inception in 1913, then by (2.16), the model claims

$$\left( \tau_m^m - \tau_t \right)_{1_{n=1}} \leq \left( \tau_m^m - \tau_t \right)_{1_{n=0}},$$

which implies that the tax code becomes less progressive (becomes “flatter”) as the price level rises. This is intuitive since there is always a top marginal tax rate. Once a household’s income reaches the top marginal rate, bracket creep will cause a larger percentage of said income to fall into that top marginal bracket. The result is that the effective rate approaches the marginal rate from below. While this model does not consider multiple households, the distributional implications of a non-indexed tax code could be significant.

### 2.2.5 The Monetary Authority

The monetary policy rule considered here is standard in the literature:

$$\ln \left( \frac{r_t}{r} \right) = \rho_r \ln \left( \frac{r_{t-1}}{r} \right) + \rho_y \ln \left( \frac{E_t \left[ \pi_{t+1} \right]}{\pi} \right) + \rho_x \ln \left( \frac{x_t}{x} \right) + \varepsilon_t^r, \quad (2.17)$$

37
where $\rho_r$, $\rho_\pi$, and $\rho_x$ are all non-negative; $r$ and $\pi$ represent the steady state value of the interest rate and target inflation rate respectively and $\varepsilon_r^t$ is an i.i.d. innovation to monetary policy with zero mean and variance $\sigma_r > 0$. Also, I assume that the monetary authority targets the output gap $x_t$ as measured using the efficient allocation. Thus, potential output $Q_t$ is given as in Ireland (2004)

$$Q_t = \eta_p^{1/\psi} z_t,$$

(2.18)

which is a measure of output that varies only with the preference shock and productivity. Considering this measure, the output gap $x_t$ is considered to be

$$x_t = \frac{y_t}{Q_t}.$$  

(2.19)

See Appendix B for more details.

### 2.2.6 Symmetric Equilibrium

In equilibrium, I assume that the household and the intermediate goods-producing firms solve their respective optimization problems and that the household does so with a binding budget constraint. Since the production technology remains constant across the continuum of intermediate goods-producing firms, I assume that they make identical decisions. Thus I find that $y_t(i) = y_t$, $D_t(i) = D_t$, $h_t(i) = h_t$, and $P_t(i) = P_t$ for all $i \in (0, 1)$. I also assume that there is no slack in the fiscal authority’s budget constraint and that $B_t = 0$ for all $t$, implying a balanced budget in equilibrium. With these assumptions, my model reduces to equations (2.2)–(2.7), (2.9)–(2.11), (2.13)–(2.15), and (2.17)–(2.19) which can be used to solve for the following 15 variables: $\eta_p^p, \eta_s^p, z_t, P_t, \pi_t, c_t, h_t, W_t, y_t, D_t, r_t, x_t, g_t, \tau_t, Q_t$. As was done already for the tax code with nominal wages, I can express dividends in real terms by dividing by the price level, giving me $d_t \equiv D_t/P_t$. Doing this for both the nominal wage rate and the dividends ensures that the model’s variables, outside of the unit root in the price level itself, will be stationary.
2.2.7 The Linearized Model

Since the model contains a tax code that may or may not be indexed for inflation, it is important to linearize the model and possibly analyze permanent shifts in the variables. Let the "∼" signify each respective variable’s deviation in natural logarithm from its steady state, which we express as the variable without a time subscript. Below is the full list of log-linearized equations.

\[ c\tilde{c}_t = (1 - \tau)wh(\bar{\tilde{w}}_t + \bar{\tilde{h}}_t) - \tau wh\bar{\tilde{r}}_t + dd_t \]

\[ \bar{\tilde{\eta}}_t - \tilde{c}_t = \bar{\tilde{r}}_t + E_t[\bar{\tilde{\eta}}_{t+1} - \tilde{c}_{t+1} - \bar{\tilde{\pi}}_{t+1}] \]

\[ \bar{\tilde{w}}_t = (\psi - 1)\bar{\tilde{h}}_t + \tilde{c}_t - \bar{\tilde{\eta}}_t + \frac{\tau}{1 - \tau} \bar{\tilde{r}}_t \]

\[ \bar{\tilde{\pi}}_t = \bar{\tilde{P}}_t - \bar{\tilde{P}}_{t-1} \]

\[ \bar{\tilde{y}}_t = \bar{\tilde{z}}_t + \bar{\tilde{h}}_t \]

\[ dd_t = y\bar{\tilde{y}}_t - wh(\bar{\tilde{w}}_t + \bar{\tilde{h}}_t) \]

\[ \bar{\tilde{\pi}}_t = \beta E_t[\bar{\tilde{\pi}}_{t+1}] + \frac{\eta^s wy}{\mu z c}(\bar{\tilde{w}}_t - \bar{\tilde{z}}_t) + \frac{\eta^s y}{\mu c} \left( \frac{w}{z} - 1 \right) \bar{\tilde{\eta}}^{s}_t \]

\[ \bar{\tilde{g}}_t = \bar{\tilde{r}}_t + \bar{\tilde{w}}_t + \bar{\tilde{h}}_t \]

\[ \bar{\tilde{\tau}}_t = \frac{1 - \tau}{\tau} \phi(\bar{\tilde{w}}_t + \bar{\tilde{h}}_t + \Pi \bar{\tilde{P}}_t) \]

\[ \bar{r}_t = \rho_r \bar{r}_{t-1} + \rho_\pi E_t[\bar{\tilde{\pi}}_{t+1}] + \rho_x \bar{x}_t + \varepsilon^r_t \]

\[ \bar{Q}_t = \frac{1}{\psi} \bar{\tilde{\eta}}^{p}_t + \bar{\tilde{z}}_t \]

\[ \bar{x}_t = \bar{\tilde{y}}_t - \bar{Q}_t \]

\[ \bar{\tilde{\eta}}^{p}_t = \rho_p \bar{\tilde{\eta}}^{p}_{t-1} + \varepsilon^{p}_t \]

\[ \bar{\tilde{\eta}}^{s}_t = \rho_s \bar{\tilde{\eta}}^{s}_{t-1} + \varepsilon^{s}_t \]

39
\[ \tilde{z}_t = \rho_z \tilde{z}_{t-1} + \tilde{\varepsilon}_t \]

Notice that, when the tax code is not indexed for inflation \((\mathbb{1}_n = 1)\), the price level appears in the tax code, which then makes it appear in the New Keynesian Philips curve. If the monetary authority does not target the price level, then all transitory shocks will have a permanent effect. Another important component of this model is the feedback effect between the tax rate and the wage rate. As the tax rate rises, it puts upward pressure on the wage rate, which then causes the tax rate to rise (holding labor hours constant). Thus, the natural drift in the prices causes explosive behavior through a wage rate channel. Two fiscal policy changes can eliminate the permanent effects of these shocks. The first is by simply indexing the tax code for inflation \((\mathbb{1}_n = 0)\), at which point the price level term falls out of the Philips curve. The second is by considering a flat tax rate \((\phi = 0)\), where the tax rate does not move regardless of whether it is indexed or not. This is the typical assumption in most models that consider taxation and, as can be seen in the equations, will negate the price level term. Considering the legislative history of the United States, a flat tax rate has never been used, though income taxes were indexed for inflation in 1985.\(^9\)

\section*{2.3 Results}

In this section I present the solution process and results of the model outlined in section 2.2. I begin by parameterizing the model. The structural components of the model are calibrated in a simple fashion, while the parameters that guide the exogenous processes are set by matching moments with the literature. I then present the steady state solutions for the model’s variables and analyze the general dynamic properties with determinacy regions. Using these regions, I then apply the data to the model to find the probability of obtaining a determinant solution to my model across time.

\(^9\)It is worth noting that the federal income tax was adopted with the Sixteenth Amendment to the United States Constitution in 1913. Prior to this, Article I, Section 8 required that any taxes be imposed uniformly, making income taxes unpopular.
2.3.1 Parameter Calibration

In order to solve the model above, specific values must be assigned to the 17 parameters in the model: $\beta, \psi, \pi, \mu, \theta, \phi, \eta^s, \rho_p, \rho_s, \rho_z, \rho_r, \rho_\pi, \sigma^p, \sigma^s, \sigma^r, \text{ and } \sigma^z$. A summary table of the parameter values can be found in Table (2.3). Since $r = \frac{1}{\beta}$ in steady state, I calibrate the subjective discount factor $\beta$ to match the fact that the average annual effective federal funds rate was approximately five percent from 1955 to 2013, which sets it at 0.95. I then set the steady state inflation target to $\pi = 1$. While, this is not very realistic, it keeps the model stationary from being explosive in all situations when the tax code is not indexed ($1_n = 1$), providing a basis for analysis. As for the tax code, much of my analysis will involve analyzing the dynamics of the model as $\theta$ and $\phi$ are adjusted. But for those situations requiring fixed values, I calibrate these parameters to be the average of time-varying estimates described in more detail in section 2.3.4.2 below. Doing so implies that these parameters are about $\theta = 0.90$ and $\phi = 0.15$, ensuring that the labor income tax code is progressive. Setting

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective Discount Factor</td>
<td>$\beta$ 0.95</td>
</tr>
<tr>
<td>Elasticity of Substitution</td>
<td>$\psi$ 0.181</td>
</tr>
<tr>
<td>Cost of Price Adjustment</td>
<td>$\mu$ 13.25</td>
</tr>
<tr>
<td>Steady State Inflation Rate</td>
<td>$\pi$ 1.00</td>
</tr>
<tr>
<td>Level of the Tax Code</td>
<td>$\theta$ 0.90</td>
</tr>
<tr>
<td>Progression of the Tax Code</td>
<td>$\phi$ 0.15</td>
</tr>
<tr>
<td>Elasticity of Demand</td>
<td>$\eta^s$ 6.00</td>
</tr>
<tr>
<td>Smoothing Response</td>
<td>$\rho_r$ 0.75</td>
</tr>
<tr>
<td>Inflation Response</td>
<td>$\rho_\pi$ 0.35</td>
</tr>
<tr>
<td>Output-Target Response</td>
<td>$\rho_y$ 0.15</td>
</tr>
<tr>
<td>Preference Shock Persistence</td>
<td>$\rho_p$ 0.50</td>
</tr>
<tr>
<td>Technology Shock Persistence</td>
<td>$\rho_z$ 0.85</td>
</tr>
<tr>
<td>Cost-Push Shock Persistence</td>
<td>$\rho_s$ 0.50</td>
</tr>
<tr>
<td>Preference Shock St. Dev.</td>
<td>$\sigma^p$ 0.01</td>
</tr>
<tr>
<td>Elasticity Shock St. Dev.</td>
<td>$\sigma^s$ 0.01</td>
</tr>
<tr>
<td>Productivity Shock St. Dev.</td>
<td>$\sigma^z$ 0.01</td>
</tr>
<tr>
<td>Monetary Policy Shock St. Dev.</td>
<td>$\sigma^r$ 0.0025</td>
</tr>
</tbody>
</table>
\( \eta^s = 6 \), as in Ireland (2004, 2012), fixes the steady state markup of price over marginal cost at 20%. Similarly, I set the level of price adjustment as is done in King and Watson (1996) and Ireland (2000). These papers suggest that discrepancies between the current price level and the desired price level are eliminated at about a 10% per quarter rate. Given that this is an annual model, I assume a 40% reduction per year, which implies a value of \( \mu = 13.25 \). For the remaining structural parameters, I calibrate the household’s elasticity of labor substitution to be \( \psi = 0.181 \), pinning labor hours at \( h = 0.330 \) and suggesting that households work approximately \( \frac{1}{3} \) of their day.

### 2.3.2 Steady State

Given the calibrated parameters above, the steady state values can be solved analytically. Since I assume that the steady state price level is set to one, the indexed and non-indexed models are identical in steady state. I also normalize output to equal one, just as a simplification. With this, I find that government spending is 8.25% of total output in the model. While this is less than the data-recommended average of 20%, it is calculated using the estimated tax parameters. Another reason to consider this as accurate is that my model only incorporates income taxes, which would make government spending less.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Output</td>
<td>( y_t ) 1.0</td>
</tr>
<tr>
<td>Real Consumption</td>
<td>( c_t ) 0.9175</td>
</tr>
<tr>
<td>Real Government Spending</td>
<td>( g_t ) 0.0825</td>
</tr>
<tr>
<td>Productivity</td>
<td>( z_t ) 3.0303</td>
</tr>
<tr>
<td>Tax Rate</td>
<td>( \tau_t ) 0.10</td>
</tr>
<tr>
<td>Nominal Interest Rate</td>
<td>( r_t ) 1.05</td>
</tr>
<tr>
<td>Real Wage Rate</td>
<td>( w_t ) 2.5</td>
</tr>
<tr>
<td>Labor Hours</td>
<td>( h_t ) 0.330</td>
</tr>
<tr>
<td>Real Dividends</td>
<td>( d_t ) 0.1750</td>
</tr>
</tbody>
</table>
2.3.3 Determinacy Regions

An economic model is said to be determinant if it has a unique solution. The figures below map out not only the determinant areas, but also those parameter spaces which yield an infinite number of solutions and no solution. The first example, shown in Figure 2.6, looks at the interaction between the fiscal and monetary policy parameters governing the progressiveness of the tax code $\phi$ and the reaction to inflation $\rho_{\pi}$, seeing how they work together when the tax code is not indexed for inflation. Due to the permanency of the shocks, only a flat or regressive tax code ($\phi \leq 0$) yields a unique solution to the model. Otherwise the economy will either find itself in a situation of sunspots (infinite solutions) or explosive behavior (no solution). Intuitively, the next question to ask is if the monetary authority can overcome the problems associated with a non-indexed tax code through some combination of inflation and output gap targeting. Figure 2.7 shows that monetary policy makers cannot push the economy to a region of determinacy, at least within the empirically plausible set of parameter values. Though my tax rule does not consider outstanding government debt, these properties are nearly identical to the situation of active fiscal policy in Leeper (1991) and Davig and Leeper (2011), where passive monetary policy creates sunspot equilibria and active policy leads to explosive behavior.

If the tax code is indexed for inflation, such as it was after 1985, the results are drastically different. Figure 2.8 shows, again, the interaction between monetary and fiscal policy, but this time the tax code is indexed. Now there is very little tradeoff between monetary and fiscal policy. At this point, as long as monetary policy is active, the model predicts a determinant, stable solution. With this in mind, how should monetary policy conduct itself to ensure a unique solution? In Figure 2.9 the determinacy regions look fairly similar to those of standard models with slight alterations due to a progressive tax code. For the most part, simply adhering to the Taylor Principle provides the needed unique solution, which again is similar to the dynamic properties explored in the literature when fiscal policy is passive.

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10 See Blanchard and Kahn (1980) for further discussion of causes and implications of each type of result.
Thus, the indexation of the tax code can be viewed as fiscal policy becoming passive, allowing monetary policy to dictate inflation dynamics.

2.3.4 Assessing the Probability of a Determinant Solution

Now that the determinacy regions have been mapped, I consider the values of the policy parameters and the dynamic properties they incur. I start with a simple two-period breakdown, applying estimates generated in the literature. I then extend this so that the parameters are time varying, allowing my model to predict exactly when these dynamic changes occurred.

2.3.4.1 The Two-Period Case

The literature considered here is Clarida et al. (2000), which estimates an interest rate rule similar to that in this model for both the pre- and post-Great Moderation periods. Then,

<table>
<thead>
<tr>
<th>Table 2.5: Solution Probabilities Based on Estimates from the Literature: Clarida et al. (2000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax Code Not Indexed</td>
</tr>
<tr>
<td>Infinite</td>
</tr>
<tr>
<td>Pre-Volcker</td>
</tr>
<tr>
<td>Volcker-Greenspan</td>
</tr>
</tbody>
</table>

taking the estimates given, I draw 50,000 times from normal distributions and apply the resulting parameter values to the model, using the log-linearized version of the model in Dynare for efficiency purposes.\textsuperscript{11} Table 2.5 lays out the resulting probabilities as well as the probabilities associated with related counterfactuals. Once again, I find that the increased probability of a unique solution is not solely a monetary policy phenomenon, but also relies on fiscal policy via indexation. Simply increasing the aggressiveness of monetary policy in the Volcker era would have only moved the economy into a parameter space that results in explosive behavior (active monetary and fiscal policy), but the indexation of the tax code

\textsuperscript{11}Dynare version 4.4.0 with Matlab version R2011a for Mac.
eliminated fiscal policy from having a major impact on the dynamics of the economy, giving monetary policy control and producing a unique solution (active monetary/passive fiscal policy). Monetary policy makers had the right idea, but they needed fiscal policy makers to do their part as well.

2.3.4.2 A Time-Varying Parameter Extension

To see how these policies played out over time, I extend the above analysis to incorporate time-varying parameters for both the fiscal and monetary policy side.

**Estimation Methods** The tax policy used in the model contains two structural parameters \((\phi, \theta)\) which determine the progressiveness and the steady state effective income tax rate in the economy. To get an idea of where these parameters lie, I replicate the results of Chen and Guo (2013) for each year starting in 1950 and ending in 2011. I consider 1,000 nominal incomes spread evenly between $1 and $400,000, which is roughly the cutoff for the bottom 99% of income earners today. Using the nominal income tax brackets from the tax code in each year, I calculate the total tax liability at each income level. Dividing this value by the synthesized taxable income level yields the average income tax rate for each income level. Figure 2.3 shows how these average tax rates progressed for selected years. As can be seen, the progressiveness of the tax code has steadily fallen since World War II.

Using the values gathered for each year, I then use OLS to estimate the natural logarithm of my tax model

\[
\ln(1 - \tau) = \ln \theta + \phi \ln \left( \frac{Y^*}{Y} \right),
\]

where \(Y\) are the 1,000 nominal income values and \(Y^*\) is the average taxable income in each year, calculated by taking the total taxable income divided by the number of individual tax returns filed in that year.\(^{12}\) Doing this for every year gives us time-varying parameter

\(^{12}\)This data can be found in the SOI Tax Stats of the US Internal Revenue Service, specifically Historical Tables 8 and 9.
estimates for the tax code. Figure 2.4 shows these estimates from 1966 to 2011. These estimates show that, while the average tax rate \((1 - \theta)\) didn’t change much over the years, the progressiveness of the tax code did. These estimates capture the substantial high-income tax cuts in 1981 and in 1986, as well as the surtaxes of the late-1960s.

For the monetary policy parameters, I consider a Kalman filter using data on the effective federal funds rate, the inflation rate as calculated by CPI, and the output gap calculated by taking the ratio of GDP to potential GDP. The Kalman filter is a recursive algorithm that estimates an unobserved-state vector. Since interest rates, inflation, and the output gap are known, I can used the data to estimate the unobserved monetary policy parameters \(\beta = [\rho_r \rho_\pi \rho_x]’\) across time. Following the process as explained by Kim and Nelson (1999), I obtain time-varying estimates of these parameters. For this model, I consider the state space model, where the measurement equation is the interest rate rule given by (2.17), where I make a simplifying assumption that expected inflation rate is equal to the current period’s inflation rate. The transition equation, since I am considering policy parameters, is assumed to be a random walk. In matrix notation,

\[
r_t = X_t \beta_t + \varepsilon_t \\
\beta_t = \beta_{t-1} + \nu_t
\]

where \(X_t = [r_{t-1} \pi_t x_t]\) and all variables are natural logarithm form. For the initial estimates, I use the adapted pre-Volcker estimates from Clarida et al. (2000), giving me \(\beta_{0|0} = [0.68 \ 0.2656 \ 0.086]’\). The prediction phase of the filter considers the random walk assumption so that \(\beta_{t|t-1} = \beta_{t-1|t-1}\). Predicting the covariance matrix of the parameters \(V_t\) also becomes simple, such that \(V_{t|t-1} = V_{t-1|t-1} + Q\). From these, I can assess the prediction

\[13\] The years prior to 1966 yielded values of \(\theta\) in excess of one, which is not plausible for the model considered here. Because of this, I simply cut off my estimates where Chen and Guo (2013) did. Somewhat of a concern is the sensitivity of these estimates to the number of sample points within the fixed income bounds.

\[14\] Even though the Federal Reserve currently considers the personal consumption expenditures price index for its inflation targets, it has only done so since 2000. Prior to this it used the consumer price index.
error $\eta_{t-1} = r_t - X_t\beta_{t-1}$ and its conditional variance $f_{t-1} = X_t V_{t-1} X'_t + \sigma^2$. Knowing the prediction error and its variance, I then update my parameter estimates

$$\beta_{t|t} = \beta_{t|t-1} + V_{t|t-1} X'_t f_{t|t-1}^{-1} \eta_{t|t-1}$$

and my conditional variance estimates

$$V_{t|t} = V_{t|t-1} - V_{t|t-1} X'_t f_{t|t-1}^{-1} X_t V_{t|t-1}.$$ 

The term $V_{t|t-1} X'_t f_{t|t-1}^{-1}$ is the weight assigned to new information when updating my estimates, commonly called the Kalman gain. Continuing to do this for all $t$ yields the time-varying parameter estimates. Additionally, we can obtain smoothed parameter estimates by updating our original estimates given all the information. If I denote the final period of the sample as $T$, the smoothed estimates adhere to the recursive algorithm

$$\beta_{t|T} = \beta_{t|t} + V_{t|t} V_{t+1|t}^{-1} (\beta_{t+1|T} - \beta_{t|t})$$

$$V_{t|T} = V_{t|t} + V_{t|t} V_{t+1|t}^{-1} (V_{t+1|T} - V_{t+1|t}) V_{t+1|t}^{-1} V'_{t|t}$$

for all $t$. In this case, I work backwards from $t = T$ to $t = 0$, instead of forwards like I did for the original estimates. Figure 2.5 provides the smoothed results with one standard deviation confidence bands. With these values, I now have a complete rendering of monetary and fiscal policy through the entire sample period.

**Applying the Parameters to the Model**  Figure 2.10 shows the time-varying probabilities across the entire sample period as in Coibion and Gorodnichenko (2011). To be as thorough as possible, for each year I draw 10,000 times from normal distributions for the monetary policy parameters and utilize the time-varying tax code estimates from Figure 2.4,
again plugging them into the log-linearized model and solving via Dynare.\textsuperscript{15} Again, notice that while the probability of sunspot equilibria is very similar in the non-indexed and indexed models, the remaining probabilities go in opposite directions. For the non-indexed model, the increased reaction by monetary policy makers to inflation in the late-1970s results in explosive behavior in the economy.\textsuperscript{16} As for the indexed model, the added aggression yields determinacy in the economy. Thus, the model predicts that monetary policy makers did not achieve stability in 1979. Rather, they had to wait for fiscal policy to catch up and index the income tax code in 1985, which matches the estimated break in volatility in the Great Moderation literature.\textsuperscript{17} Additionally, these results are directly comparable to the timetable presented in Davig and Leeper (2011, Figure 1), which uses a Markov-switching model and a tax policy that targets real outstanding government debt. This suggests that the estimates found in this regime-switching literature may be picking up subtle changes in fiscal policy such as the indexation of the tax code.

### 2.4 Adding a Fiscal Policy Channel to the Model

The model given in section 2.2 is a fairly standard model, but this poses a problem when considering policies that can cause explosive behavior. To get around this and further my analysis of this policy, I introduce a fiscal policy channel through labor productivity. It implies that rising tax rates will cause productivity to decrease, a concept that is in no way new to the empirical literature, but is universally abstracted from in the theoretical literature. Vartia (2008) provides an extensive review of the empirical literature, which typically

\textsuperscript{15}Use of only the fiscal policy estimates is done for two reasons. First, drawing from the fiscal policy parameter distributions dramatically increases the dimensions of this analysis, resulting in an immense amount of computing time. Second, the fact that the estimates of $\theta$ are very close to its upper bound suggests that drawing randomly from an assumed distribution would cause more problems that it would solve.

\textsuperscript{16}In Figure 2.4 the non-smoothed parameter results show a large jump in inflation reaction, while the smoothed results suggests increased persistence. Both results yield similar probabilities.

\textsuperscript{17}Also realize that the tax brackets were first adjusted in 1985, which means the effect of indexation, specifically the elimination of bracket creep, began in 1984, exactly matching the estimated starting period of the Great Moderation in the literature.
considers the implications of various taxes on entrepreneurial activity and research and development. She then finds a negative relationship between top marginal personal income tax rates and total factor productivity, which she claims is also through this entrepreneurial channel. Carroll et al. (2001) also finds that high marginal income tax rates can hurt productivity, especially the growth rates of small firms. Since my model only considers labor income taxes charged to households, this channel could be considered a Laffer channel in which increasing tax rates may cause the tax base to fall through falling productivity, reducing overall tax revenues. To model this, I simply add a tax component to the otherwise exogenous process in equation (2.10)

\[
\ln z_t = (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} - \rho^\tau_z \ln \left( \frac{\tau_t}{\tau} \right) + \varepsilon_t^z, \tag{2.20}
\]

with \( \rho^\tau_z \geq 0 \). This matches the empirical results that increasing tax rates reduces labor productivity. Interestingly, Rogerson and Wallenius (2009) show that micro-level estimates of labor supply elasticity to tax rates are much lower than their macro-level counterparts, suggesting that this channel is stronger in the aggregate sense.\(^\text{18}\) While I do not consider this channel for the baseline analysis in section 2.3, it adds a dynamic to New Keynesian models that most do not contain, making productivity a partly endogenous variable.

One issue modeling this channel in this way is that the parameter \( \rho^\tau_z \) is hard to pin down. The closest empirical analysis that applies to a channel modeled in this way is Bloom et al. (2000), in which they estimate the short-run user cost of capital elasticity to taxes to be -0.1, while the long-run estimate is closer to -1.0. Since this is the closest I can currently get, I calibrate \( \rho^\tau_z = 0.10 \) which means, in a linearized model, that only about ten percent of the deviation in the tax rate from its steady state directly impacts productivity. I choose this value leaning towards the more conservative side. As was mentioned earlier, Rogerson and Wallenius (2009) find that estimates of labor supply elasticity to tax rates are lower at the

\(^{18}\)For a more in depth look at the entrepreneurial channel, see Meltzer and Richard (1981) and Carroll, Holtz-Eakin, Rider and Rosen (2000, 2001).
micro level than at the macro level, so this calibration of $\rho_z^\tau$ should be fair.

The expansion of the model to include this channel is done for three reasons. Considering the strong assumptions on the tax code and the somewhat outside-the-box results that follow, this allows for another plausible angle with which to tackle this debate. Suggesting that there was no way for monetary policy makers to induce determinacy in the economy without an indexed tax code is a strong statement, so coming at the question from a different direction should only strengthen the argument. The second reason behind this arrangement is that, as shown in Section 2.4.1 below, introducing this channel makes the model stationary in both of the active monetary policy scenarios, allowing us to analyze variances, correlations, and impulse responses, which are key for this type of analysis. The third reason is simply because there is a literature that suggests this is an empirically viable channel, whether it is strong or not. Analysis of the model shows that any value of $\rho_z^\tau > 0$ results in stationarity, so this is not a question of how strong the channel is.\textsuperscript{19} The data suggests this channel exists, so it is something worth exploring.

2.4.1 The Impact of this Fiscal Channel on the Model

If the entrepreneurial channel is active ($\rho_z^\tau > 0$) the model becomes completely stationary when the tax code is not indexed for inflation, regardless of monetary policy. This is because the output gap (which is a function of the tax rate) becomes a function of the price level, making monetary policy makers implicitly target the price level. If the tax code is indexed, the price level falls out of the interest rate rule and the monetary policy component of my model is equivalent to those in much of the monetary policy literature. Thus, as it is with fiscal policy, from a monetary policy standpoint, this model is a generalized version of other models. As is presented below, the inclusion of this channel directly impacts the volatility in the model, the correlations between variables, and the variance decompositions.\footnote{\textsuperscript{19} Though larger values of $\rho_z^\tau$ do produce much quicker returns of the variable to steady state.}
2.4.2 Impulse Responses

In this section I present the impulse responses for the expanded model, considering both the indexed and non-indexed situation. For conciseness, I present only the results for negative monetary policy, positive productivity, and positive demand shocks. The impulse responses in Figure 2.11 depict output growth, the output gap, productivity, and the tax rate. The first thing to notice is that, since not indexing the tax code causes the monetary authority to implicitly target the price level along with targeting inflation, the impulse response functions are damped oscillations instead of monotonically converging. The second thing to notice is that, while the dynamics of output growth do not change much from one scenario to the other, the change in the dynamics of the output gap, an input into the interest rate rule, is quite large, especially when it comes to monetary policy shocks.

An important interpretation of a monetary policy shock is a measurement error of one or more of the input arguments. Orphanides (2003) considers a simple interest rate rule like the one below targeting inflation and the output gap and introduces noisy information in both inputs ($\epsilon_\pi^t, \epsilon_x^t$) respectively

\[ r_t - r = \gamma(\pi_t - \pi) + \delta x_t - [(1 - \gamma)\epsilon_\pi^t + \delta \epsilon_x^t], \]

showing that measurement error can derail even theoretically sound monetary policy. When comparing this rule to (2.17), the term in brackets can be defined to be $\epsilon_r^t$, implying that a negative monetary policy shock can be likened to a measurement error that causes monetary policy makers to lower interest rates further than needed. With this intuitive theory, Orphanides (2003, 2004) concludes that the perceived passivity of monetary policy was caused more by mismeasurement of productivity growth than discretionary policy. Looking at the impulse response, I find that a measurement error of this type actually has a larger negative impact on productivity when the tax code is not indexed for inflation. This, in turn, leads to output gap levels that are higher than originally reported, which was the case during the
“Great Inflation,” where ex-post estimates of the output gap were much larger than their original counterparts. If this is not taken into consideration, an initial measurement error can, in theory, lead to further measurement errors, causing interest rates to be low for too long and cause elevated levels of inflation, just as was evident in the 1970s. Additionally, the interest rate response to a technology (productivity) shock in Figure 2.12 shows the difference in monetary policy responses when the tax code is not indexed. In this case, instead of interest rates rising with inflation, they fall with the output gap. Supply shocks were large and occurred often during the 1970s, so if bracket creep was causing measurement error, then the situation is the same as that implied in the equation above.

Extending on this idea, notice that the response of productivity (output gap) always ends up lower (higher) when the tax code is not indexed for inflation. So if the monetary authorities are estimating the state of the economy via the black line, they are under-estimating the output gap. Recall that this is an annual model, which means the under-estimation can actually last for long periods of time. Add in the measurement errors after the initial shocks, and policy makers can easily compound the effects of the initial shock, whether it be a supply- or demand-side shock.\(^\text{20}\) These results imply that inflation-induced bracket creep, while subtle, is a plausible explanation for measurement errors in labor productivity during this time period.

Figure 2.13 presents some additional impulse responses. Here we can see that, after indexation, the response of real disposable income \((1 - \tau_t)w_t h_t\) greatly increases. This is a testament to the impact of bracket creep on households. Having a tax code that allows a household’s tax liabilities to drift upward results in an erosion of its purchasing power. Additionally, the impact on government spending shows how revenues \(\tau_t w_t h_t P_{t}^{1n}\) were im-

\(^{20}\)The evidence suggests that the influence of a non-indexed tax code was not taken into consideration. The only mention of “bracket creep” by the Board of Governors shows up in short conversation in the transcript of the FOMC Meeting for December 21, 1981, after the Economic Recovery Tax Act of 1981 had been passed into law. This suggests that, while they may have been aware of it, they may not have considered it as a major factor in their policy decisions. Thus, when considering these measurement errors, they easily could have thought the economy was following the black line, while the economy was actually following the blue line.
pacted. Since this model assumes a balanced budget, the fiscal authority can only spend what it brings in through tax revenues. Considering the indexed ($\Pi_n = 0$) and non-indexed ($\Pi_n = 1$) models are identical in steady state, its fairly intuitive that the initial impacts on revenues are similar. The bracket creep, however, shows up in the increased persistence of government spending. Although the tax rate is falling back to steady state, it happens more gradually because the price level has to adjust as well. This provides a boost to government revenues for a longer period of time, something that is well known by those familiar with the impact of bracket creep.

It is also worth noting that the decreased impact of non-technology shocks on productivity matches the general empirical results found in Galí and Gambetti (2009) without adjusting the aggressiveness of monetary policy. To an extent the impulse responses for output to non-technology shocks also match after the immediate impact, which are very similar in each scenario. Finally, technology shocks have less of an impact on output growth in the indexed model than they do in the non-indexed version. Thus, it would not necessarily take smaller shocks to induce a reduced volatility, as the “good luck” theorists claim.

2.4.3 Structural Changes in the Theoretical Moments

The important findings regarding the Great Moderation typically consider the theoretical moments of certain variables as well as the correlations between them. Tables 2.6 and 2.7 present the changes in standard deviations and correlations between differenced output, labor hours, and productivity along with whether the directions of these shifts match those found by Galí and Gambetti (2009) and Stiroh (2009). As can be seen, the results presented in this model match those found empirically in the literature outside of the change in labor hours. The difference there can at least partially be attributed to the overly simple modeling of the production process.\textsuperscript{21} Otherwise, we see a decrease in the standard deviation of output growth and productivity growth.

\textsuperscript{21}For example, the labor market matching literature provides a simple approach to modeling a more accurate labor market, but I consider this beyond the scope of this paper.
Table 2.6: Standard Deviations of Selected Variables

<table>
<thead>
<tr>
<th></th>
<th>Non-Indexed</th>
<th>Indexed</th>
<th>Match&lt;sup&gt;a&lt;/sup&gt;</th>
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</thead>
<tbody>
<tr>
<td>$\Delta y_t$</td>
<td>0.0060</td>
<td>0.0053</td>
<td>Yes</td>
</tr>
<tr>
<td>$\Delta h_t$</td>
<td>0.0099</td>
<td>0.0104</td>
<td>No</td>
</tr>
<tr>
<td>$\Delta z_t$</td>
<td>0.0101</td>
<td>0.0097</td>
<td>Yes</td>
</tr>
</tbody>
</table>

<sup>a</sup> This column refers to whether the movements in each variable’s standard deviation matches the movements in Galí and Gambetti (2009).

Similarly, I find that the correlation between output growth and productivity growth, as well as that between labor hours and productivity growth, fall. This also matches the empirical results found in the literature. Again, the correlation between output growth and the growth in labor hours does not match, but a more developed labor market could change that. For the most part, the changes in theoretical moments and variable interactions within this simple, generalized model of monetary-fiscal interaction match what has been found in the data, all without changing the parameters of either the interest rate rule or the tax rule.

Table 2.7: Correlations of Selected Variables

<table>
<thead>
<tr>
<th></th>
<th>Non-Indexed</th>
<th>Indexed</th>
<th>Match&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y_t, \Delta h_t$</td>
<td>0.2663</td>
<td>0.3698</td>
<td>No</td>
</tr>
<tr>
<td>$\Delta h_t, \Delta z_t$</td>
<td>-0.8227</td>
<td>-0.8602</td>
<td>Yes</td>
</tr>
<tr>
<td>$\Delta y_t, \Delta z_t$</td>
<td>0.3283</td>
<td>0.1556</td>
<td>Yes</td>
</tr>
</tbody>
</table>

<sup>a</sup> This column refers to whether the movements in each variable’s standard deviation matches the movements in Galí and Gambetti (2009).

One last result is the fall in the contribution of all supply shocks with the indexation of the tax code. While this does not match that of Galí and Gambetti (2009), it does match the results presented in Arias <i>et al.</i> (2007). This again suggests that the theory that supply shocks are simply smaller now may be flawed.
2.5 Concluding Remarks

While much of the monetary/fiscal policy coordination literature considers lump sum tax policies structured around real outstanding government debt, this paper takes a more legislative approach. Implanting a labor income tax code structured around income levels into an otherwise standard New Keynesian model, I analyze the dynamic impacts of indexing such a tax code for inflation. This type of policy was implemented in the federal income tax code in 1985, eliminating bracket creep in the process. Though this policy has been overlooked in the literature, my model predicts that it had a substantial impact on the dynamics of the economy. Specifically, an indexed tax code yields the same dynamic properties as “passive” fiscal policy in the Leeper (1991) sense. Without indexation, my model predicts that the active monetary policy enacted in the late 1970s would have produced an active/active policy scenario which results in explosive behavior. Only after the indexation of the tax code was monetary policy allowed to be dominant, providing determinacy in the model at the precise moment suggested in the Great Moderation literature. Thus, the reduction in volatility seen in the data was not simply a monetary-policy phenomenon, but a combination of movements from both sides of policy. One change without the other would not have resulted in the period of tranquility seen in the late-1980s through the mid-2000s. This timeline of events matches that found in the Markov-switching model estimates of Davig and Leeper (2011), suggesting that their estimates may have been picking up this subtle fiscal policy change. While some in the literature find the same timeline as my model suggests, the drawback of these regime-switching models is the interpretation of the estimated shifts. My model gets around this by focusing on specific legislation, providing easy interpretation of what actually happened.

Extending the model to include a fiscal policy channel through productivity, I find that the productivity slowdown and associated output gap mismeasurement suggested by Orphanides (2003) may have had its roots in bracket creep. Increasing tax rates caused by bracket creep put downward pressure on labor productivity, causing any shocks to the model
to have substantially larger impacts on productivity and the output gap. Had monetary policy makers not considered the impacts of bracket creep, their estimates of the output gap would be much lower than the true value, which was exactly the case in the mid-1970s. Thus, any initial measurement error is compounded, resulting in seemingly discretionary monetary policy when considering revised variable estimates. Even when holding monetary policy as active, indexation of the tax code results in decreased standard deviations of key variables discussed in the literature. This policy also alters the structural relationships between the variables in the same manner as those explored in the literature. This includes the correlations between variables such as output growth and productivity growth as well as reducing the impact of supply shocks on the economy. Thus, this policy may have also reduced the impact of supply shocks, which would debunk the “good luck” theories of the Great Moderation.

All of these results stem from a fiscal policy that does not consider large changes in government spending or big swings in the tax rates, which are the focus of most of the literature. This policy, on the other hand considers a change in how the tax code is implemented, revealing substantial impacts on the model dynamics. Thus, maybe we should be less worried about the big-ticket policies, and a little more interested in some of these subtler policies. For example, an extension to this paper is to consider the impact of monetary and fiscal policy makers utilizing differing measurements of inflation. The Federal Reserve began targeting inflation based on the personal consumption expenditure price index, while the taxes and transfer payments of the US Government are still indexed using the consumer price index. Do subtleties in monetary and fiscal policy such as indexation or the use of differing inflation measurements have significant impacts on the economy? Maybe these are the questions we should be asking.
Figures

Figure 2.1: Time series of effective tax rates for 24 evenly-spaced, synthesized real income levels between $10,000 and $2 million from 1950-2012 considering only the legislated, federal personal income tax code.

Figure 2.2: Percentage point change in legislated effective labor income tax rates for selected real incomes measured in 1982 dollars.
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Figure 2.4: Tax Code Parameters Estimated with Ordinary Least Squares: 1966–2011
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Figure 2.6: Monetary-Fiscal Interaction Determinacy Regions: Tax Code Not Indexed for Inflation
Figure 2.7: Monetary Policy Determinacy Regions with Fiscal Policy Held Constant: Tax Code Not Indexed for Inflation

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Figure 2.11: Selected Impulse Responses

Figure 2.12: Selected Impulse Responses
Figure 2.13: Selected Impulse Responses
Chapter 3

Broad Inflation and Monetary/Fiscal Coordination

3.1 Introduction

Beginning in the mid-2000s, a small group outside the policy-making circle began to critique the Federal Reserve for what they believed to be an overly-loose stance. The reason behind this dissent began when the federal funds rate reached one percent in June 2003, the lowest level since the late 1950s, and remained there until well into 2004 as shown in Figure (3.1). This opinion reached headline proportions in the midst of the financial crisis, where the source of the housing bubble was assumed to be low borrowing costs driven by excessively easy money. At best, they saw it as a combination of short-term deviations from a Taylor (1993) rule due to high levels of uncertainty and risk exogenous to a model (Alcidi et al., 2011). At worst, they viewed it as a deliberate deviation from the rule which directly caused the housing bubble and subsequent collapse (Taylor, 2009; McDonald and Stokes, 2013).

However, there are two inconsistencies with these conclusions. First, most of these analyses utilize revised data in their policy rule estimates instead of real-time data. Orphanides (2003) shows that the use of revised data can be quite deceiving when discussing the efficacy
Figure 3.1: Target Federal Funds Rate

*Note:* Quarterly data of the target federal funds rate is provided above along with economic recessions (shaded area), defined as the periods between dates designated as peak and trough provided by the NBER.

*Source:* Federal Reserve Bank of St. Louis and the National Bureau of Economic Research

of past decisions. Orphanides and Wieland (2008) correct for this and find that monetary policy was not as easy as originally attested. Mehra and Minton (2007) also make use of real-time data, finding that a model which targets the core consumer price index (CPI) inflation forecasts fits actual policy movements well. Combined, these studies lay to rest the notion that policy was too easy. But the use of core CPI by Mehra and Minton (2007) for the entire sample is where the second caveat lies. What this literature fails to consider is the fact that the inputs to the Federal Reserve’s hypothetical monetary policy rule have changed over time.\(^1\) For example, the measure of inflation used by monetary policy makers alone has been refined multiple times since the mid-1980s. Prior to July 1988, inflation forecasts were presented using the implicit GNP deflator.\(^2\) Humphrey-Hawkins reports then suggest that inflation forecasts were estimated using changes in the headline CPI.\(^3\) Beginning in February, 2000, the Fed began presenting its forecasts via changes in the headline personal consumption expenditures (PCE) chain-type price index, eventually refining it down to core PCE

\(^1\) Orphanides and Wieland (2008) does correct for this by directly including the forecasts from the Humphrey-Hawkins reports, smoothing any changes in the inflation measurement used from one period to another.

\(^2\) Taylor (1993) uses lagged inflation measured in this manner in his original monetary policy rule.

\(^3\) The Humphrey-Hawkins Full Employment Act set forth clearer goals for both fiscal and monetary policy. A requirement of this legislation was that the Board of Governors must send a report (called a Humphrey-Hawkins report) to congress semiannually outlining monetary policy. These reports include both past and forecasted data as motivation for its stance.
in mid-2004. Figure (3.2) presents the differences between these measurements, which can

be considerable at times. The divergence in the headline measures of inflation are generally
seen in the level, though headline PCE is also slightly less volatile. Conversely, the dissimi-
larity between the headline and core measures of inflation are generally seen in the volatility.

Using Greenbook forecasts of different inflation measures, Mehra and Sawhney (2010) find
that monetary policy did as a standard Taylor rule would suggest, meaning that policy be-
tween 2001–2007 did not qualify as “too easy". Additionally, Dokko et al. (2009) consider
both real-time data and the correct measures of inflation, finding that these persistently low
interest rates were not a primary contributing factor to the formation of the housing bubble
and that any aid from it was small. Thus, while many chastised the Fed for its easy-money
stance, use of the correct data suggests that it was acting responsibly.

Looking at the history of Fed considerations, a tangental literature raises the next impor-
tant question: What is the best measure of inflation for monetary policy to target? Typical
analysis of this question only addresses the difference between the use of headline CPI ver-
sus core CPI, which removes effects from changes in food and energy costs (i.e. Mehra and
Minton, 2007).\footnote{There are many versions of core inflation, differing in everything from what goods/services are omitted to how they are omitted.} Mishkin (2008, pg. 7-8) outlines the reasons for targeting core inflation over
headline inflation, saying in reference to the New Neoclassical Synthesis:

Indeed, these models indicate that monetary policy should try to get the economy to operate at the same level that would prevail if all prices were flexible—that is, at the so-called natural rate of output or employment. Stabilizing sticky prices helps the economy get close to the theoretical flexible-price equilibrium because it keeps sticky prices from moving away from their appropriate relative level while flexible prices are adjusting to their own appropriate relative level.

Thus, he claims that the proper measurement for use by monetary policy makers are core inflation statistics. Equivalently, central banks need to realize the difference between transitory, exogenous adjustments to the inflation rate and focus on the underlying trend in inflation. Considering a more general framework, Anand and Prasad (2010) show that this may not hold in all situations. They claim that welfare-maximizing monetary policy targets headline CPI when the economy faces incomplete markets characterized by credit-constrained consumers and financial frictions.\(^5\)

While the literature does consider whether to target headline or core measures of inflation, a few have also debated whether inflation measured by the CPI or the PCE price index is better. From the basic formula to the scope of the basket of goods, Clark (1999) diligently discusses the differences between the two headline measures, suggesting that inflation should be measured by the CPI. On the other hand, Rich and Steindel (2007) compare the core versions of each, finding that neither measure consistently dominates the other. One major difference between the two measures is that the PCE is continually updated as new information surfaces. Croushore (2008) finds that these revisions are actually forecastable, adding another wrinkle to the use of the broader measure. Essentially, this implies that the use of real-time PCE data should be adjusted for the forecasted revisions before being implemented.

\(^5\) Additionally, they find that strict inflation targeting is no longer optimal, finding that welfare-maximizing monetary policy must also react to the output gap. They do find, however, if the only friction in the economy is price-stickiness, strict targeting of core inflation is optimal, which falls directly in line with the Mishkin (2008) analysis.
in monetary policy. Granted, most of the literature finds the Federal Reserve to be forward looking (see Mehra and Minton, 2007; Orphanides and Wieland, 2008, among others), but it is hard to believe that the members of the FOMC do not consider current data at least in relation to past forecasts. So not only is the measurement different, but their applications can also deviate.

While there are many aspects of this literature and all are very thorough in their analyses, nearly all of them lack a fiscal policy authority of any kind. The seminal work of Sargent and Wallace (1981) suggests that there are instances where monetary policy could leave the Taylor principle unsatisfied and yet the economy could still have a stable, unique equilibrium. Showing that, if fiscal policy set taxes as it saw fit, monetary policy makers could undertake a backseat role in simply controlling the level of real outstanding debt. Leeper (1991) further defines the concepts of active and passive monetary and fiscal policy, showing that the dynamics of the model are not always simply a function of the central bank’s decisions. Furthermore, the works of Davig and Leeper (2011) and Bianchi (2012) suggest that the active and passive stances of policy makers often change. Thus, it is important for analysis of these types of monetary policy changes to not abstract away from fiscal policy decisions. How the economy is impacted may not always just depend on the stance of one policy maker.

This paper follows the debate between using broad or narrow inflation measures in monetary policy, but also considers the fact that fiscal policy, such as tax codes and transfer systems, has remained indexed to the narrow measure. I consider a two-sector New Keynesian model with sector-specific price stickiness similar in structure to the works of Anand and Prasad (2010), Du and Yagihashi (2015), and Iacoviello (2005). These sectors represent basic consumption goods as well as healthcare services, with each composed of intermediate and final producers. The inclusion of healthcare services provides a sector of the economy that is not generally considered under CPI, which is my narrow measure of inflation, but is considered under PCE, the broad measure.6 Another motivation for the use of this sector is

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6The scope of CPI focuses on out-of-pocket expenditures by households in urban settings, while PCE expands the scope to include expenses such as healthcare, which is typically provided for by employers and
that healthcare services as a share of personal consumption expenditures in the United States has gradually risen since 1960, yet has been one of the more stable sectors of consumption, as shown in Figures (3.3) and (3.4). As can be seen, the growth of healthcare services in

![Figure 3.3: Healthcare Services as a Share of Personal Consumption Expenditures](image)

![Figure 3.4: Growth Rates of Healthcare Services and Personal Consumption Expenditures](image)

*Note:* These growth rates are measured as annualized month-to-month growth rates from January 1995, to January 2015. Health services are represented by the thicker, black line (which is more stable) while overall personal consumption expenditures are represented by the thinner, blue line. This subsample was chosen due to difficult interpretation caused by the high frequency of the data and the need to show current trends.

*Source:* Bureau of Economic Analysis

the United States economy has been much more stable than overall personal consumption expenditures. This implies that those sectors included in the measurement of CPI must be even more volatile than the overall PCE shown here. Additionally, it is easy to recognize not directly out of the household’s pocket.
that the healthcare sector is growing in importance, as the average household spending has shifted toward this sector more and more over the past decades. Statistically, past studies have found that the prices in the healthcare sector are stickier than those in the general goods markets. For example, Bils and Klenow (2004) find empirical evidence that consumer goods prices adjust about every 7 months, while prices in the healthcare sector clear about every 9.75 months. Because of this, the inclusion of the healthcare sector alone should make the inflation target less volatile, as shown in Figure (3.2). In an sense, PCE-derived inflation is like a more diversified portfolio, yielding slightly lower returns (inflation rates), yet is more stable than a CPI-derived portfolio. Using these sectors allows me to analyze the impacts of the economy when the economy as a whole (especially fiscal policy) focuses on a narrow measure of inflation and monetary policy moves to a broader inflation measure. My findings suggest that switching from a narrow to a broad regime does not impact the model’s determinacy properties (Blanchard and Kahn, 1980) much. However, just because the model may remain stable does not necessarily mean that the dynamics are unaltered overall. Running simulations of the model shows that the choice of monetary policy regime can have a dramatic impact on the means and standard deviations of the variables. These simulations are also dependent on the coordination efforts between the monetary and fiscal authorities (Leeper, 1991). Under the standard active monetary/passive fiscal regime, switching inflation measures can actually put downward pressure on real fiscal debt levels. Conversely, estimation of the policy rules suggests that the model is in a passive monetary/active fiscal regime. In this case switching inflation measures puts substantial upward pressure on real fiscal debt levels.\textsuperscript{7} Interestingly, the switch in inflation measures not only puts extensive upward pressure on fiscal debt, but it also decreases the standard deviation, suggesting that debt is higher yet more stable under these policies. In a general sense, though, this switch causes standard deviations for many key variables to rise, a testament to policy makers reacting to the economy as a whole and being less concerned with sector-specific adjustments.

\textsuperscript{7}The estimation of a passive/active regime is common in the literature during this period of time. See Davig and Leeper (2011) as one of multiple examples.
The rest of the paper is presented as follows: Section 3.2 provides empirical motivation in the form of a time-varying parameter model; Section 3.3 presents the New Keynesian model; Section 3.4 then presents the results of that model, including analysis under different monetary and fiscal policy regimes; Section 3.5 concludes and offers additional thoughts on this topic.

3.2 Motivation: A TVP-FAVAR Model

Empirically analyzing the transmission mechanism of monetary policy has always been a difficult task. The use of vector autoregressions (VAR) has typically been the emphasis, but there are many drawbacks to these models. The most common is simply the lack of observations. In order to completely understand the transmission of monetary policy, many variables need to be included, but the lack of observations makes this impossible. Bernanke et al. (2005) make use of principle component analysis to overcome this, estimating what they call a factor-augmented vector autoregression (FAVAR) model. Doing so allows them to consider many variables while still maintaining parsimony for better estimates. Another issue with the standard VAR approach is that, as mentioned above, there have been many changes to monetary policy over the years. This has prompted many to use time-varying parameter vector autoregression (TVP-VAR) models such as that considered by Primiceri (2005), for example. The logical next step is to attempt to overcome both of these pitfalls simultaneously. Thus, using the methods derived by Del Negro and Otrok (2008), Korobilis (2009) presents a time-varying parameter, factor-augmented vector autoregression (TVP-FAVAR) model in which the information from a large dataset is integrated into a TVP-VAR model.

While the goal of Korobilis (2009) is to consider monetary policy since 1960, this paper considers the changes that have occurred since about the year 2000. Therefore, I utilize his benchmark model and dataset, constructing a TVP-FAVAR with four factors and two lags.
This framework can conveniently be structured in a state-space format, where

\[
\begin{bmatrix}
    x_t \\
    y_t
\end{bmatrix}
= \lambda_t \begin{bmatrix} f_t \\ y_t \end{bmatrix} + \epsilon_t^o
\]

\[
\begin{bmatrix} f_t \\ y_t \end{bmatrix}
= B_t \begin{bmatrix} f_{t-1} \\ y_{t-1} \end{bmatrix} + \epsilon_t^y
\]

are the observation and state equations, respectively. Here, \( x_t \) is an \((n_1 \times 1)\) vector of variables that represent the economy. The variables come from all sectors of the economy and include various measurements of output, payrolls, interest rates, investment, price indices, etc. In a similar fashion, \( y_t \) is an \((n_2 \times 1)\) vector of monetary policy variables, in this case the chosen measurements of inflation, unemployment, and the federal funds rate. The factors are denoted by \( f_t \), which is a \((k \times 1)\) vector with \( k < n_1 \). The matrices \( \lambda_t \) and \( B_t \) represent the time-varying factor loadings and state equation coefficients, respectively. This model is then estimated using Bayesian techniques, which provides the time-varying parameters necessary for this analysis. For a more thorough, technical explanation of the estimation process, see Korobilis (2009), Del Negro and Otrok (2008), and Koop (2003).

This setup is identical to the benchmark model of Korobilis (2009), down to the variables included in \( x_t \). The single adjustment I make to this model is to the vector of monetary policy instruments \( y_t \). Here, I follow with the literature of Mehra and Minton (2007) and Orphanides and Wieland (2008) by substituting out the original inflation data. I replace the GDP Deflator data (which is used for the entire sample period), with the actual inflation measures used in the monetary policy consideration, such as headline CPI, headline PCE, and core PCE.
3.2.1 Monetary Policy Transmission Mechanism

The first important question that needs to be addressed is “how does a change in the inflation target impact the effectiveness of monetary policy?” Figure (3.5) presents some basic impulse responses derived from the TVP-FAVAR model described above. These impulse responses are unit shocks to the interest rate equation and the subsequent reactions of inflation and the unemployment rate. I plot both the 1996Q1 and the 2006Q3 responses to show that the effect of altering interest rates is similar in both situations, even though policy makers are targeting a different inflation measure in each period. With the 80% credible intervals plotted as well, it’s easy to see that these responses are not statistically different. Even the existence of a price puzzle does not change. This is rather intuitive since the two competing measures of inflation are similar in so many ways. Therefore, the effectiveness of monetary policy does not seem to be radically changed when moving from one measure of inflation to
3.2.2 The General Rise in Standard Deviations

While the differences may not be seen in the impulse responses, they can be seen when looking at how the standard deviations of the variables change over time. Figure (3.6) plots the time-varying posterior means of the standard deviations of the variables in the state equation. The first panel looks at the four factors that are derived from the original 115 economics indicators. These factors represent the data in general, and it can be seen that their standard deviations begin to drift upwards starting around 2000, just as the Federal Reserve switched from a headline CPI target to a headline PCE target. What is interesting to note is that the standard deviation from the interest rate equation begins to fall in mid-
2001, just as those of the factors begin to rise. This is fairly intuitive if one notices in Figure 3.2 that the variations of the inflation targets fall as we move from headline CPI to headline PCE to core PCE. Thus, if the measurement used by monetary policy makers is less volatile, so too will be the interest rate they use as an instrument. However, this means that the additional components in the headline measurements are allowed to adjust more freely, causing increases in standard deviations across the factors. Additionally, moving from a narrower measure (CPI) to a broader measure (PCE) makes the inflation target less volatile, since any sector-specific fluctuations are averaged-out by the other sectors. Combining these results suggests that the efficiency of monetary policy may not be altered in a significant manner, but that the variability of the economic indicators could increase by targeting a broader inflation measure.

### 3.3 Two-Sector New Keynesian Model

As was mentioned above, I consider a two-sector New Keynesian model with a representative household and both monetary and fiscal authorities. The sectors include standard consumption goods as well as healthcare services as in Du and Yagihashi (2015). The relationship between these sectors is much like Anand and Prasad (2010), but in this case there different levels of sticky prices in each sector. The fiscal authority indexes its tax code and transfer system to the narrow measure, and all real variables are considered as adjusted by the narrow price index. The monetary authority, however has the option of targeting the standard, narrow measure of inflation or the broader measure of inflation, which also gives weight to price adjustments in the healthcare sector.

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8 Anand and Prasad (2010) considers the choice between monetary policy makers targeting headline or core inflation, which necessitates one sector with sticky prices and one without.
3.3.1 Representative Household

The representative household obtains utility from real consumer goods $c_t$ as well as its health status $x_t$. Consumer goods are valued relative to an internal habit formation system with persistence $\delta$, while the health status is derived from their flow of real healthcare services $h_t$ and normalized leisure $(1 - l_t)$ such that

$$x_t = h_t^{\kappa_h} (1 - l_t)^{\kappa_l},$$

where $l_t$ denotes labor hours and $\kappa_h > 0$ and $\kappa_l > 0$ denote the weights on health services and leisure, respectively. This functional form assumes that not only will doctor’s visits and medicine make the household healthier, but rest and leisure also contribute to health status. Combining these, the one-period utility function for the household is given as

$$U_t = \epsilon_t^\beta \left[ \frac{1}{1 - \sigma_c} (c_t - \delta c_{t-1})^{1 - \sigma_c} + \frac{1}{1 - \sigma_x} x_t^{1 - \sigma_x} \right],$$

where $\sigma_c > 1$ and $\sigma_x > 1$ are the inverse of the intertemporal elasticities of substitution for regular goods consumption and health status, respectively. The term $\epsilon_t^\beta$ is a shock to the discount factor following an autoregressive law of motion in its logarithm

$$\epsilon_t^\beta = (1 - \rho_\beta) \epsilon_{t-1}^\beta + \rho_\beta \epsilon_t^\beta + \eta_t^\beta,$$

with persistence $\rho_\beta \in [0, 1)$ and the zero-mean, serially uncorrelated innovation $\eta_t^\beta$ is normally distributed with standard deviation $\nu_\beta$. Utility in this functional form follows that of Hall and Jones (2007) and Du and Yagihashi (2015), in which instantaneous utility adjusts monotonically with consumption goods and health status.

In each period, the representative household purchases consumption goods at price $P_t^c$ and health services at price $P_t^h$. It also interacts in the financial markets through the purchase of nominal discount bonds $B_t$ at price $1/r_t$, where $r_t$ is the gross nominal interest rate. To
purchase these, the household earns income by supplying labor $l_t$ at nominal wage rate $W_t$, acquiring dividend payments $D_t$ from the intermediate goods-producing firms in each sector, and transfer payments from the fiscal authority. Additionally, there is a tax system which includes taxes on labor. Mathematically, this budget constraint can be expressed as

$$P^c_t c_t + P^h_t h_t + \frac{B_t}{\tau_t} + \tau^*_t = W_t l_t + D_t + B_{t-1} + T_t,$$

where $T_t$ are the nominal transfer payments which are exogenous to the household and $\tau^*_t$ are the total tax liabilities of the household.

The taxes are levied by the fiscal authority, making the total tax liabilities of the household

$$\tau^*_t = \tau_t W_t l_t;$$

where the distortionary tax rate, $\tau_t$, represents the labor income tax. This is possibly time varying and set by the fiscal authority.

Maximizing utility subject to its budget constraints, the representative household’s optimality conditions are shown below. The solution method can be found in the appendix.

$$\epsilon^\beta_t (c_t - \delta c_{t-1} )^{-\sigma_c} - \delta \beta \mathbb{E}_t \left[ \epsilon^\beta_{t+1} (c_{t+1} - \delta c_t )^{-\sigma_c} \right] = \Lambda^1_t$$

$$\epsilon^\beta_t \left[ h_t^{c_h} (1 - l_t)^{c_l} \right]^{-\sigma_x} \frac{\kappa_h x_t}{h_t} = \frac{P^h_t}{P^c_t} \Lambda^1_t$$

$$\epsilon^\beta_t \left[ h_t^{c_h} (1 - l_t)^{c_l} \right]^{-\sigma_x} \frac{\kappa_l x_t}{1 - l_t} = \frac{W_t}{P^c_t} (1 - \tau^*_t) \Lambda^1_t$$

$$\beta \mathbb{E}_t \left[ \Lambda^1_{t+1} \right] = \frac{\Lambda^1_t}{P^c_t \tau_t}$$

Where $\Lambda^1_t$ is the marginal utility of consumption. These equations, in addition to the budget constraint combining (3.1) and (3.2), provide all of the equilibrium conditions necessary for the household sector of the model.
3.3.2 Sectoral Good-Producing Firms

With the inclusion of the healthcare sector, this model will have two sectoral good-producing firms denoted by \( n \in \{c, h\} \), where \( c \) represents the consumption good market and \( h \) represents the healthcare market. Since the focus of this paper is the change in inflation dynamics under a broad policy target, I abstract from the structural differences between the two sectors and allow the stickiness of their price to be the only noteworthy difference. This allows for a simpler analysis, showing that the trends seen in the data can be captured by dissimilarities in the sectors’ price dynamics only.

For each perfectly competitive market, the firm purchases differentiated goods \( y^n_t(j) \) from the continuum of intermediate goods-producing firms in its sector indexed by \( j \in [0, 1] \) at price \( P^n_t(j) \). They then aggregate these goods using a standard constant elasticity of substitution (CES) production function as in Dixit and Stiglitz (1977).

\[
y^n_t = \left( \int_0^1 y^n_t(j) \frac{\epsilon^{s,n}_{t,j-1}}{\epsilon^{s,n}_{t,j-1}} dj \right)^{\frac{\epsilon^{s,n}_{t}}{\epsilon^{s,n}_{t-1}}}
\]

Here, \( \epsilon^{s,n}_t \) represents the elasticity of substitution between intermediate goods in market \( n \) following an AR(1) process

\[
\epsilon^{s,n}_t = (1 - \rho_{s,n})\epsilon^{s,n}_t + \rho_{s,n}\epsilon^{s,n}_{t-1} + \eta^{s,n}_t
\]

where \( \rho_{s,n} \in [0, 1) \) and \( \eta^{s,n}_t \) is a zero-mean, serially uncorrelated innovation which is normally distributed with standard deviation \( \nu_{s,n} \). Considering the profit function

\[
P^n_t y^n_t - \int_0^1 P^n_t(j) y^n_t(j) dj,
\]

optimality for the sectoral good-producing firms is reached when the demand for each inter-
mediate good $j$ is given by
\[
y^n_t(j) = \left( \frac{P^n_t(j)}{P^n_t} \right)^{-\epsilon_s^n} y^n_t. \tag{3.3}
\]

Given the demand for each intermediate good above and the perfectly competitive nature of each market, the aggregate price level for each sector $n$ is given by
\[
P^n_t = \left( \int_0^1 P^n_t(j)^{1-\epsilon^n_t} \, dj \right)^{\frac{1}{1-\epsilon^n_t}}
\]

### 3.3.3 Intermediate Goods-Producing Firms

As was mentioned above, there is a continuum of monopolistically-competitive, intermediate goods-producing firms labeled by $j \in [0, 1]$ in each sector $n \in \{c, h\}$. Each firm combines labor and capital to produce its intermediate good
\[
y^n_t(j) = e^n_p y^n_t(j)
\]

where $e^n_p$ is total factor productivity, which follows an autoregressive process with persistence $\rho_p \in [0, 1]$ and the zero-mean, serially uncorrelated innovation $\eta^n_p$, which is normally distributed with standard deviation $\nu_p$.

\[
e^n_p = (1 - \rho_p)e^p + \rho_p e^{p}_{t-1} + \eta^n_p
\]

With a monopolistically competitive market comes discrepancies in the goods and a deviation from the price-taker scenario. I assume, as in Rotemberg (1982), that there is a quadratic cost of price adjustment in each market, which is measured in terms of the sectoral good.

\[
\Phi^n_t(j) = \frac{\phi^n_p}{2} \left( \frac{P^n_t(j)}{\pi^n P^n_{t-1}(j)} - 1 \right)^2 y^n_t \quad \forall j \in [0, 1] \tag{3.4}
\]

The parameter $\phi^n_p$ governs the magnitude of the cost in market $n$, while $\pi^n$ is the steady state inflation rate in the same market. This price setting friction makes the firms’ problems
dynamic. Thus, they choose a new price $P^n_t(j)$ which maximizes the present value of expected future profits. The stochastic discounting factor is determined by the household’s marginal utility since they reap the benefits of the profits as owners. Their problems then become

$$\max_{P^n_t(j)} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i v^j \Lambda^1_{t+i} \frac{D^n_{t+i}(j)}{P^n_{t+i}} \quad \forall j \in [0, 1].$$

The firms’ nominal profits are

$$D^n_t(j) = P^n_t(j)y^n_t(j) - W^n_t(j) - P^n_t\Phi^n_t(j) \quad \forall j \in [0, 1].$$

Converting this into real profit and considering the sectoral goods-producing firms’ demand (3.3) and the price adjustment cost (3.4), the expanded real profits for each sector are

$$\frac{D^n_t(j)}{P^n_t} = (P^n_t(j))^{1-\epsilon^n_t} y^n_t - (P^n_t(j))^{-\epsilon^n_t} W^n_t y^n_t - \frac{\phi^n_t}{2} (\frac{P^n_t(j)}{P^n_{t-1}(j)} - 1)^2 y^n_t \quad \forall j \in [0, 1].$$

Maximizing these real profits subject to the demand of the final good firm yields the intermediate firms’ optimality condition, which represents the New Keynesian Philips Curve when linearized.

$$\left(1 - \epsilon^n_s(j)\right) \left(\frac{P^n_t(j)}{P^n_{t-1}(j)}\right)^{-\epsilon^n_s(j)} \Lambda^n_t y^n_t + \epsilon^n_s(j) \left(\frac{P^n_t(j)}{P^n_{t-1}(j)}\right)^{-\epsilon^n_s(j)-1} W^n_t \Lambda^n_t y^n_t - \phi^n_t \left(\frac{P^n_t(j)}{P^n_{t-1}(j)} - 1\right)^2 y^n_t = \beta \phi^n_{n+1}$$

for all $j \in [0, 1]$ and each $n \in \{c, h\}$. The derivation of this condition can be found in the appendix.
3.3.4 Fiscal Policy

In this model, I consider real government spending $g_t$ to be exogenous. In addition to this, the fiscal authority also pays off its outstanding debt and issues transfer payments to the households. To pay for these expenses, they levy labor and capital return taxes as well as issue nominal discount bounds. With all of this in mind, the government’s budget constraint becomes

$$P_t g_t + T_t + B_{t-1} = \frac{B_t}{r_t} + \tau_t W_t l_t.$$  

The tax policy and transfer payments are assumed to be set in the legislation ahead of time, making them rule-based. The tax rate, like the Federal income tax system, is dependent on the household’s wages.

$$\tau_t = (1 - \rho_\tau) \bar{\tau} + \rho_\tau \left[ \tau_{t-1} + \gamma_\tau \left( \frac{w_{t-1} l_t}{w_{t-1} l_{t-1}} (\pi^c_{t-1})^{1-\chi} - 1 \right) \right]$$  

Here, $\rho_\tau \in [0, 1]$ determines the stationarity and $\gamma_\tau$ determines the progressiveness of the tax codes, while $\chi > 0$ determines the level of indexation to inflation. Note that there is no restriction on $\gamma_\tau$, allowing me to analyze the policy as a progressive ($\gamma_\tau > 0$), regressive ($\gamma_\tau < 0$), and flat ($\gamma_\tau = 0$) tax code. Indexation also plays an important role here. If we consider the $\bar{\tau}$ to be the intended steady state tax rate set by the fiscal authority, the true steady state tax rate

$$\tau = \bar{\tau} + \frac{\rho_\tau \gamma_\tau}{1 - \rho_\tau} \left[ (\pi^c)^{1-\chi} - 1 \right],$$

actually deviates from the intended tax rate. Both fiscal and monetary policy makers can contribute to the solution of this problem. On the fiscal side, policy makers can completely index the tax code to inflation ($\chi = 1$) or they can simply enact a flat tax ($\gamma_\tau = 0$). Both policies will allow them to hit their targeted tax rate. On the monetary policy side, policy makers can target zero inflation ($\pi^c = 1$). Without at least one of these policies, fiscal policy makers will not be able to hit their intended tax rates. This is a classic case of bracket creep.
in the long run, suggesting that the households will automatically be paying, and expecting, higher tax rates than those set forth in legislation. A rudimentary analysis of the labor wedge response to changes in the level of indexation can be found in the appendix.

On the other side of this coin are the lump-sum real transfers from the fiscal authority to the household, \( t_t = \frac{\tilde{t}_t}{p_t} \). These transfers, like the tax code above, is assumed to be set in legislation and remain, following the rule

\[
t_t = (1 - \rho_T)\bar{t} + \rho_T \left[ t_{t-1} + \gamma_T \left( \frac{y_{t-1}^c}{y_{t-1}^c} (\pi^c)^{\chi^T - 1} - 1 \right) \right].
\]

This rule is constructed in the same manner as (3.5), but is assumed to be dependent on the overall health of the real economy. The parameter \( \rho_T \in [0, 1] \) governs the persistence of the transfer system, while \( \gamma_T \) controls the sensitivity of the transfers to changes in the real aggregate economy. And also like the tax code, \( \gamma_T \) has no restrictions. When \( \gamma_T > 0 \), this policy is considered to be countercyclical, while a negative value constitutes a pro-cyclical policy. Setting this at zero will make the transfers constant. Analyzing the steady state value of these transfers and assuming that \( \bar{t}_t \) is the intended level of real transfers to the household,

\[
t = \bar{t} + \frac{\rho_T \gamma_T}{1 - \rho_T} \left[ (\pi^c)^{\chi^T - 1} - 1 \right],
\]

implies that the steady state level of real transfers will fall below the intended level, assuming \( \chi^T > 0 \) like its counterparts in the tax code. Again, a flat transfer system (\( \gamma_T = 0 \)) or a completely indexed system (\( \chi^T = 1 \)) can alleviate this problem from the fiscal side. Having a zero steady state inflation rate (\( \pi^c = 1 \)) will fix this from the monetary side.

When considering both the tax code and the transfer system, it is easy to see that, if neither the fiscal nor the monetary side step up to control this problem, there will be upward pressure on government revenues while downward pressure on government expenditures, providing some relief for the fiscal debt burden.  

\footnote{While the automatic indexation of the federal income tax code was signed into law in 1981, the policy was legislated to take effect in 1985 in hopes that bracket creep during that time would at least partially}
3.3.5 Monetary Authority: Measurement and Policy

The next goal is to derive the overall price index $P_t$. To do so in a tractable manner, I assume that the monetary authority measures aggregate output $y_t$ using the following CES production function:

$$y_t = \frac{(y_c^t)^\omega (y_h^t)^{1-\omega}}{\omega^\omega(1-\omega)^{1-\omega}} \tag{3.6}$$

This functional form is similar to those used by Aoki (2001) and Erceg and Levin (2006) in which the contribution of regular consumption goods and health care services is constant. A driving force behind the function form used in (3.6) is that it implies a simple aggregate price level

$$P_t = (P_c^t)^\omega (P_h^t)^{1-\omega}. \tag{3.7}$$

This is particularly helpful since the conversion from an aggregate price level to an aggregate inflation rate is also simple. Simply dividing both sides of (3.7) by $P_{t-1}$ yields

$$\pi_t = \frac{(\pi_c^t)^\omega (\pi_h^t)^{1-\omega}}{P_t}$$

This implies a similar functional form to that used in the personal consumption expenditures statistic and the implied chain-type price index. This type of index is broader in scope than the tradition consumer price index in that it includes more of the healthcare sector, such as payments made by employers on behalf of their employees.

In terms of monetary policy, I close the model with a simple Taylor (1993)-type rule to offset the revenue losses from the major reductions in the tax rates. It was thought that the expansionary effects of tax reductions could take multiple years to fully take effect.
which considers rate smoothing as well as both inflation and a measure of output growth.\textsuperscript{10}

\[
r_t = r_{t-1} + (1 - \rho_r) \left[ \rho + \rho_\pi \left( E_t(\pi_{t+1}) - \pi \right) + \rho_y \left( \frac{y_t}{y_{t-1}} - 1 \right) \right] + \eta_t^r
\]

Here, $\eta_t^r$ is a zero-mean, serially uncorrelated shock which is distributed normally with standard deviation $\nu_r$. Additionally, my analysis will consider alterations in the monetary authority’s inputs, be it the aggregate inflation and output measures, or the consumer goods market measures only. In a sense, this represents the Federal Reserve’s choice between using a CPI-based measure, which is narrower in scope, and a PCE Chained Index-based measure, which is broader in scope. The fiscal policy maker indexes (perhaps incompletely) its tax code and transfer payments systems to the narrower measure. Additionally, the value of real variables are measured using the narrower index, reflecting the general public’s preference for CPI when adjusting for inflation.

\subsection{3.3.6 Equilibrium Conditions and Stationarity}

A competitive equilibrium is a set of prices and allocations for which households and firms behave optimally and all markets clear, given values of the exogenous processes and initial values of the endogenous states. That is, households maximize utility subject to it budget constraints and the intermediate goods-producing firms in each market maximize their profits subject to their technology constraints and the demand by each market’s sectoral good-producing firm. Since the firms in each sector produce with the same technology, I assume that they all make the same decisions, allowing me to drop the $j$ indicator. Additionally, the aggregating functions of the monetary authority must hold along with the interest rate and tax/transfer rules.

\textsuperscript{10}The empirical conclusions of Rudebusch (2002, 2006) suggest that monetary policy estimates with high degrees of smoothing may be misspecified. He argues that smoothing is a very short-term (weekly or monthly) phenomenon and that estimates suggesting smoothing at the quarterly or yearly level may be the result of serial correlation of the error terms. While this can be a very important problem in the empirical literature, I consider the inclusion of higher levels of smoothing and the omission of serial correlation in the shock terms to be sufficient for a model such as this one.
In addition to these equations holding, I must also define some equilibrium equations based on the assumptions of the model. First, goods produced in the consumption sector can only be consumed by the household and by government as regular consumption goods. Similarly, goods produced in the health services sector can only be consumed by the household and by the government as health goods. Thus, the following conditions must hold in steady state.

\[ y_t^c = c_t + g_t^c \]
\[ y_t^h = h_t + g_t^h \]

where

\[ g_t = g_t^c + p_t^h g_t^h. \]

Since I assume a symmetric equilibrium, the following conditions hold in the labor market:

\[ l_t = l_t^c + l_t^h, \]
\[ l_t^c = \int_0^1 l_t^c(j) dj, \]
and
\[ l_t^h = \int_0^1 l_t^h(j) dj. \]

With these, we have \( y_t^n = \epsilon_t^n l_t^n \) for both sectors \( n \in \{c, h\} \).

Since the price level \( P_t^n = \pi_t^n P_{t-1}^n \) for both sectors of the economy, the nominal variables inherit a unit root. To ensure their stationarity, we simply adjust the nominal variables for the price level. Universal measures like the market wage rate \( W_t \), the dividend income \( D_t \), and nominal bonds \( B_t \) are indexed to the consumption goods price level since consumption goods make up the general, day-to-day, majority of expenditures relative to health services. Thus, \( w_t = W_t / P_t^c \), \( d_t = D_t / P_t^c \), and \( b_t = B_t / P_t^c \) must hold. There is one additional caveat to the dividend payments. The dividends are profits from each of the production sectors, which
consider different price indexes when maximizing profits. Therefore, real dividend payments from each of the sectors are defined as $d_t^c = D_t^c / P_t^c$ and $d_t^h = D_t^h / P_t^h$. With this in mind, aggregated real dividend payments must be

$$d_t = d_t^c + p_t^h d_t^h,$$

where $p_t^h = P_t^h / P_t^c$ is the relative price level, or the real price of health services, which is also stationary. Considering this gives me

$$p_t^h = \frac{\pi_t^h}{\pi_t^c} p_{t-1}^h$$

which suggests that the steady state path of prices in each sector must grow at the same rate. Combining all of this gives my model 27 equations to solve for 27 stationary variables. These variables include $x_t, h_t, c_t, l_t, p_t^h, b_t, r_t, \tau_t, w_t, d_t^c, d_t^h, \pi_t^c, t_t, \epsilon_t^\beta, \Lambda_t, y_t, y_t^c, y_t^h, \pi_t^h, \pi_t, \pi_t^h, l_t^c, l_t^h, \epsilon_t^p, \epsilon_t^{s,c}, \epsilon_t^{s,h}, g^c$, and $g^h$.

### 3.4 Results

In this section I first calibrate and solve for the steady state values of the model. I then consider the determinacy properties (Blanchard and Kahn, 1980) when looking at varying monetary and fiscal policy stances. Considering arbitrarily assigned monetary and fiscal policy parameters initially, I analyze the model’s properties under what I consider the “standard” policy stances on both sides. I then extend the analysis by estimating the policy parameters and applying them to the model, suggesting that the stances of monetary and fiscal policy makers are important when considering switching from a policy rule that considers a narrow measure of inflation to one which considers a broader measure.
3.4.1 Calibration

This model includes 39 parameters, most of which I will calibrate to match empirical trends. Since this model is smaller than that of Smets and Wouters (2003), using estimation techniques for the parameter values does not yield the same level of intuitiveness. Since tax and transfer policies are adjusted annually, I calibrate to match annual trends. The average effective federal funds rate, which closely matches the rates of one-year bonds, was 4.38 percent between 1990 and 2007. Thus, I fix \( r = 1.04 \). The steady state version of the model implies that the target inflation rates in both sectors and in the aggregate must be equal to ensure stability. Because of this, I calibrate steady state inflation measures to be \( \pi = \pi^c = \pi^h = 1.02 \), reflecting the implied two percent inflation target. This, in turn, fixes the subjective discount factor \( \beta = \pi / r \), which is approximately 0.98. In addition to these, I also fix \( \delta = 0.7 \), which matches the estimates of Sims and Wolff (2013). I use this estimate because they also consider a household with internal habit formation, instead of external habit formation as in the Smets and Wouters (2003) model.

Switching to the health status of the household and its preferences, in general, I calibrate \( \sigma_c = 2.0 \) to match that in Hall and Jones (2007). Additionally, I assume that the elasticity of substitution regarding health status \( \sigma_x \) is identical. Considering that health and utility are quite similar intrinsically, a calibrate \( \kappa_h = 1 \) and fix \( \kappa_l = 3.409 \) to peg total labor hours \( l = 1/3 \). This implies a standard eight hour work day, given total hours normalized to unity.

It is generally assumed that the average markup in the consumer goods sector is about 20%, whereas Dranove et al. (1993) suggest that the price markup in the healthcare sector is about 30% in this context. With this in mind, I calibrate the elasticities of substitution to be \( \epsilon^{s,c} = 6 \) and \( \epsilon^{s,h} = 4.3 \). The first order conditions of the intermediate goods-producing firms in each sector, when linearized around the steady state, yield New Keynesian Philips Curves. Within these curves lies the speed at which each market’s prices completely adjust. King and Watson (1996) suggest that the deviations in the price level in the goods market from its desired value clear at about a ten percent pace per quarter. This translates into
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intertemporal Discount Factor ($\beta$)</td>
<td>0.98</td>
</tr>
<tr>
<td>Habit Persistence ($\delta$)</td>
<td>0.70</td>
</tr>
<tr>
<td>Discount Factor Shock ($\epsilon^\beta$)</td>
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</tr>
<tr>
<td>Elasticity: Consumer Goods Firms ($\epsilon^{s,c}$)</td>
<td>6.00</td>
</tr>
<tr>
<td>Elasticity: Healthcare Firms ($\epsilon^{s,h}$)</td>
<td>4.30</td>
</tr>
<tr>
<td>Total Factor Productivity ($\epsilon^p$)</td>
<td>3.00</td>
</tr>
<tr>
<td>Output Share of Consumer Goods ($\omega$)</td>
<td>0.78</td>
</tr>
<tr>
<td>Indexation: Tax Code ($\chi^r$)</td>
<td>1.00</td>
</tr>
<tr>
<td>Indexation: Transfer System ($\chi^T$)</td>
<td>1.00</td>
</tr>
<tr>
<td>Price Adjustment Cost: Consumer Goods Sector ($\phi^c_p$)</td>
<td>15.65</td>
</tr>
<tr>
<td>Price Adjustment Cost: Healthcare Sector ($\phi^h_p$)</td>
<td>16.98</td>
</tr>
<tr>
<td>Health Status Share: Health Services ($\kappa_h$)</td>
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</tr>
<tr>
<td>Health Status Share: Leisure Hours ($\kappa_l$)</td>
<td>3.00</td>
</tr>
<tr>
<td>Elasticity: Consumer Goods ($\sigma_c$)</td>
<td>2.00</td>
</tr>
<tr>
<td>Elasticity: Health Status ($\sigma_x$)</td>
<td>2.00</td>
</tr>
<tr>
<td>Monetary Policy: Long Run Rate ($r$)</td>
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</tr>
<tr>
<td>Monetary Policy: Inflation Target ($\pi$)</td>
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</tr>
<tr>
<td>Desired Tax Rate ($\bar{\tau}$)</td>
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</tr>
<tr>
<td>Desired Transfer Payments ($\bar{t}$)</td>
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</tr>
<tr>
<td>Consumer Goods: Government ($g^c$)</td>
<td>0.20</td>
</tr>
<tr>
<td>Health Services: Government ($g^h$)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 3.1: Calibrated Parameter Values

about a 46% annual pace in the goods markets. As was mentioned earlier, Bils and Klenow (2004) find empirical evidence that consumer goods prices adjust about every 7 months, while prices in the healthcare sector clear about every 9.75 months. Using these relative rates, I need to calibrate the model such that consumption goods prices adjust at about a 46% pace and healthcare prices adjust at about a 33% pace. Combining these findings and assumptions, I calibrate $\phi^c_p = 15.65$ and $\phi^h_p = 16.98$. On the policy side of the model, I assume that the healthcare component of the monetary authority’s aggregate inflation measure is approximately 22%, implying $\omega = 0.78$ and matching the findings of Blair (2014). As for the spending aspect of the model, I assume in the base model that $g^c = 0.20$ and $g^h = 0.0$, implying that government spending is done purely in the consumer goods markets and is about 20% of output, matching the annual trend.
3.4.2 Steady State

Given the parameter values outlined above, I calculate the steady state values of each of the real variables. For this baseline analysis, I assume that $\chi^\tau = \chi^T = 1$, in which case both the tax code and the transfer system are perfectly indexed to inflation, since this has been the goal since 1985.\textsuperscript{11} The values of these variables in steady state can be found in Table 3.2. Again, an exploration of the impact of various levels of indexation on the labor wedge and these steady state values can be found in the appendix.

3.4.3 Determinacy Regions

Here I present the parameter spaces which yield particular dynamic properties in the model as outlined by Blanchard and Kahn (1980). In this section I will first discuss the tradeoffs that must be considered by monetary and fiscal policy makers. Next, given fiscal policy, I explore what monetary policy can do to maintain a stable economy.

3.4.3.1 Active and Passive Policy Combinations

To get a better sense of what the dynamic properties of the model look like under different policy regimes, I evaluate the determinacy regions of my model below. First, I consider both monetary and fiscal policy. To keep the equations simple, I continue to assume that the tax code and the transfer system are perfectly indexed to inflation ($\chi^\tau = \chi^T = 1$).\textsuperscript{12} Additionally, I assume strict inflation targeting, omitting the output growth rate from monetary policy consideration ($\rho_y = 0$), and a random walk tax code which depends on the growth rate of income ($\rho_T = 1$). This specification of the tax code, I believe, is the most akin to an actual income tax code. Figure (3.7) presents the parameter space in this situation. The $x$-axis depicts the stance of monetary policy $\rho_\pi$ while the $y$-axis represents the stance of fiscal policy

\textsuperscript{11} Automatically indexing the federal income tax code began in 1985, but this policy was signed into law under the Economic Recovery Tax Act of 1981, which also cut top marginal rates from 70\% to 50\%.

\textsuperscript{12} The absence of this assumption requires a new steady state to be calculated for each parameterization of the monetary and fiscal policy rules. Future versions or extensions of this paper will most likely include adjustments to these parameters.
### Table 3.2: Steady State Values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Steady State</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumable Goods: Household ((c_t))</td>
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</tr>
<tr>
<td>Health Services: Household ((h_t))</td>
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</tr>
<tr>
<td>Total Labor Supply ((l_t))</td>
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</tr>
<tr>
<td>Consumption Labor Supply ((l^c_t))</td>
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</tr>
<tr>
<td>Healthcare Labor Supply ((l^h_t))</td>
<td>0.130</td>
</tr>
<tr>
<td>Health Status ((x_t))</td>
<td>0.115</td>
</tr>
<tr>
<td>Aggregate Output ((y_t))</td>
<td>0.937</td>
</tr>
<tr>
<td>Consumer Goods Output ((y^c_t))</td>
<td>0.611</td>
</tr>
<tr>
<td>Healthcare Goods Output ((y^h_t))</td>
<td>0.389</td>
</tr>
<tr>
<td>Inflation: Aggregate ((\pi_t))</td>
<td>1.020</td>
</tr>
<tr>
<td>Inflation: Consumer Goods ((\pi^c_t))</td>
<td>1.020</td>
</tr>
<tr>
<td>Inflation: Healthcare Goods ((\pi^h_t))</td>
<td>1.020</td>
</tr>
<tr>
<td>Gross Nominal Interest Rate ((r_t))</td>
<td>1.040</td>
</tr>
<tr>
<td>Real Price of Health Services ((p^h_t))</td>
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<td>Real Wage Rate ((w_t))</td>
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<tr>
<td>Profit: Consumer Goods Sector ((d^c_t))</td>
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<tr>
<td>Profit: Healthcare Goods Sector ((d^h_t))</td>
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<tr>
<td>Tax Rate ((\tau_t))</td>
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<tr>
<td>Transfers ((t_t))</td>
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</tr>
<tr>
<td>Consumer Goods: Government ((g^c_t))</td>
<td>0.200</td>
</tr>
<tr>
<td>Health Services: Government ((g^h_t))</td>
<td>0.000</td>
</tr>
<tr>
<td>Fiscal Revenues ((\tau_t w_t l_t))</td>
<td>0.200</td>
</tr>
<tr>
<td>Fiscal Debt ((b_t))</td>
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<tr>
<td>Marginal Utility of Consumption ((\Lambda_t))</td>
<td>20.586</td>
</tr>
</tbody>
</table>
in the form of progressivity $\gamma_r$. As can be seen, under these simplifying assumptions, there is a direct parallel to the findings of Leeper (1991) and a clear-cut difference between “active” and “passive” policy on both sides.\(^\text{13}\)

In the upper-right quadrant of the parameter space I find stability under the standard determinacy definition with active monetary policy and passive fiscal policy. In this situation monetary policy makers are controlling the dynamics of the price level, ignoring real fiscal debt. To ensure stability, fiscal policy makers must make the tax code progressive enough to control the path of real outstanding fiscal debt.

As I find in the upper-right quadrant, so too do I find determinacy in the lower-left quadrant of the parameter space. In this instance, however, we have what is considered the \textit{fiscal theory of the price level}, in which the roles of the monetary and fiscal policy makers are reversed. Here fiscal policy is active, meaning that the tax code is not responsive enough to stabilize real debt levels. Thus, monetary policy must then be passive, adjusting interest

\(^{13}\)The parallel mentioned here pertains to the dynamic properties seen in the model when adjusting monetary and fiscal policy. In the Leeper (1991) sense, fiscal policy is a lump sum tax which responds directly to the levels of real outstanding government debt. In this model, I construct fiscal policy as an income tax code, which is dependent on the growth rate of real wages. The results seen here are “parallel” in the sense that fiscal policy must be progressive enough to control the real level of fiscal debt, meaning that the tax rate must increase by enough when debt rises. The only clear difference here is in the structure of the tax rate rule.
rates in a manner that controls real debt levels, but leaves the pricing function to the control of fiscal policy.

If both monetary and fiscal policy makers are passive, then the model falls into the parameter space marred by indeterminacy, or an infinite number of equilibria, shown in the upper-left quadrant. In this case fiscal policy is passive and controlling the level of real government debt, but monetary policy makers are not controlling the price level, leaving its determination to the expectations of the household. In this scenario, the economy can move from one equilibrium to another without any changes to its fundamentals, yielding what are commonly called “self-fulfilling prophesies”. Under this regime, the economy wanders around the various equilibria, producing a highly volatile, yet not explosive situation.

If, however, both policy makers are active, the model has no solution. In this scenario, monetary policy makers are controlling the path of prices, but the tax code is not responsive enough to control the level of real outstanding debt. Thus, this combination of regimes yields explosive behavior in all variables since real debt is allowed to explode. This situation is found in the lower-right quadrant of the parameter space.

3.4.3.2 Monetary Policy Choices

Above I considered both monetary and fiscal policy options and the parameter spaces needed for determinacy in the model. However, as Romer and Romer (2010) show, there have been very few alterations to the core of the tax code since 1993. Thus, it is important to consider the choices of monetary policy makers when fiscal policy remains constant. To begin this analysis, I fix the progressive aspect of fiscal policy at a calibrated level of $\gamma_T = 1.7$. This is the average elasticity of Federal income tax liabilities to changes in income, as measured by the NBER TAXSIM database from 2000-2013. Additionally, I begin the analysis by assuming that the tax code is a random walk ($\rho_T = 1$) as before. Figure 3.8 shows the parameter space in this situation. As can be seen, the response to the growth rate of output does little, especially when considering the parameter space that yields determinacy. It does, however,
increase the size of the parameter space that renders the model explosive. If I increase the stationarity parameter in the tax code, I find that the parameter space that yields explosive behavior (no solution) grows, but the parameter space that results in a unique solution remains steady.

### 3.4.3.3 Altering Monetary Policy Regimes

The primary focus of this paper is to consider what happens to the model when switching monetary policy inputs from a narrower to a broader measure of inflation. From a determinacy-region point of view, the results are nearly identical as those in Figure 3.7, and any differences are too small to be worth a discussion. Even under alternative fiscal policy parameters, these values hold true in both scenarios. While the properties of the parameter space do not change from one monetary policy regime to the other, this does not mean that switching regimes does not have an impact overall.
3.4.4 The Standard Active/Passive Combination

Before getting into the details of estimation and its analysis, I consider an arbitrary parameterization in which monetary policy is active and fiscal policy is passive, placing the model in the upper-right quadrant of Figure (3.7). With this standard policy assumption, I analyze the impulse responses for short-term analysis as well as the simulated moments of the model for long-term analysis. In a later section, I use simple non-linear least squares to get an idea of where these parameters actually fall.

3.4.4.1 Impulse Response Analysis: Monetary Policy

In this section I provide some of the impulse response functions of the model. The first issue that needs to be addressed is whether there is any alterations to the impact of monetary policy when moving from a narrow regime to a broad regime. Recall that the empirical results shown in Figure (3.5) reveal that the impact of monetary policy remains, for the most part, unaltered. Figures (3.9)–(3.11) show a similar result from the structural model. The reactions of the variables are fairly standard for monetary policy shocks. An increase in the monetary policy instrument (interest rate) causes both inflation and output to fall at the aggregate level. It is interesting to note that the impact on output in the consumption goods sector (Figure (3.9), top left panel) is much less relative to that in the healthcare services sector (Figure (3.10), top left panel). The real price of healthcare services doesn't change much and oscillates around the steady state, implying that the impact on each sector is primarily driven by substitutability.

As for how monetary policy translates into the model under each regime, notice that the impulse responses of the two regimes are nearly identical, laying one on top of the other. So while the empirical estimates showed no statistical divergence, the structural model shows almost not difference at all. This is actually a trend among all the model-wide shocks such that the progressiveness of the tax code is held at $\gamma_r = 1.7$ as before, while the aggressiveness of monetary policy towards inflation $\rho_\pi = 1.83$. This calibration of the monetary policy function is arbitrarily set to the value found for the Volcker-Greenspan era in Clarida et al. (2000) plus one to ensure it is well above unity.
Figure 3.9: Consumption-Sector Variables to Monetary Policy Shock

Note: The black, solid lines represent the narrow monetary policy regime, while the blue, dashed lines represent the broad monetary policy regime. These values are calculated using the estimated monetary policy parameters for each respective period.

Figure 3.10: Health Services-Sector Variables to Monetary Policy Shock

Figure 3.11: Aggregate Variables to Monetary Policy Shock

Note: With a steady state of zero, the impulse response reported for fiscal debt is in levels, or deviations from steady state, equivalently. All other impulse responses are in percent deviations from steady state.
as productivity $\eta^p_t$ and demand $\eta^d_t$. However, I do find that shocks to individual sectors yield differing results. For this reason, impulse response functions these broad-based shocks are reserved for the appendix.\footnote{The impacts of these aggregate shocks are not identical, but strikingly close. This is essentially by design since the consumption good and health services sectors are modeled identically outside their parameterization, specifically the calibration of price stickiness. Thus, an aggregate shock will impact both sectors, and consequently the aggregate, in very similar fashions.}

### 3.4.4.2 Impulse Response Analysis: Sector-Specific Shocks

The next impulse response analysis considers a cost-push shock in the consumption goods sector $\eta^{s,c}_t$, which is translated as an exogenous occurrence that decreases the price markup by one percentage point. Figure (3.12) shows the response of the consumption goods-sector variables to this shock. In this sector, such an innovation is seen as a supply shock, with output rising and inflation falling. As the price falls, the household will increase their labor hours in that sector, producing more and causing wages to fall initially. This fall in the wage rate causes an immediate surge in profits for the intermediate goods-producing firms. All of these occurrences are typical of a standard supply shock, but for the health services sector it translates into a demand shock, which is shown in Figure (3.13). Here we see a substitution effect in which the households demand more consumption goods and less health services. Thus, in this sector we see decreases in both output and inflation. The base cause of this is in the real price of health services, which is simply the ratio of the price level for health services to that of consumption goods. The 1.5% decrease in inflation in the consumption sector outweighs the 0.7% fall in health services inflation. This drives the real price of health services up by one percent, causing demand and labor hours to fall. What is interesting, though, is that profits for the intermediate goods-producing firms in this sector rise substantially in the face of falling demand. This is due, for the most part, to a falling wage rate along with the falling labor hours. Typically simple supply and demand dynamics move these two variables in opposite directions, but since the shock originated in the larger sector of the economy, where it materializes as a supply shock, this drove down the wage
rate overall.

When looking at the aggregate economy, this supply shock once again manifests itself as a demand shock as seen in Figure (3.14). This is due to the high level of substitutability between consumption goods and health services. Notice that output in the consumption goods sector only increases by approximately 0.03% at its highest point, while output in the health services sector falls by 0.5%. This translates into a decrease in overall output by about 0.1%. Overall inflation falls by nearly 1.5% at the same time. With falls in the wage rate and overall labor hours, we see fiscal revenues fall by approximately 3% at its lowest point and fiscal debt rise. Real levels of debt see larger increases than one would expect due to the fact that; while interest rates do fall, making debt cheaper; inflation rates fall by a larger percent, pushing the real value of future debt higher. As the wage rate bounces back quickly, even overshooting the original level, revenues are replenished and debt falls back to its steady state level. Something that is also worth noting is the increase in impact from this shock at the aggregate level when monetary policy makers move from the narrow to the broad inflation regime. I find a considerable increase in the variability in the economy from this shock. This is due to the fact that, under the broad measure regime, monetary policy reacts less strongly to adjustments in inflation from a single sector. Since the consumption goods sector is by far the larger of the two sectors, changes in its inflation rate can cause larger swings in variable movements at the aggregate level.

Similarly, I present the impulse response functions for a cost-push shock in the health services sector of the model. This shock represents a reduction in the price markup on health services by about one percentage point. As before, it manifests itself as a typical supply shock in the health services market, as seen in Figure (3.16). Inflation falls and output increases due to a decrease in the real price of health services. Contrary to before, profits for the health services sector fall because of increased labor costs. These increased labor costs come from the increase in labor hours as well as the increase of the real price-adjusted wage rate $\frac{w}{p}$ in that sector. In Figure (3.15), I find that output in the consumption goods sector does
Figure 3.12: Consumption-Sector Variables to Cost-Push Shock in the Consumption Sector

Figure 3.13: Health Services-Sector Variables to Cost-Push Shock in the Consumption Sector

Note: The black, solid lines represent the impulse response functions under what I call the narrow monetary policy regime, which considers expectations of the CPI-like $\pi^c_t$ inflation measure. The blue, dashed lines represent the impulse response functions under what I call the broad monetary policy regime, which considers expectations of the PCE-like $\pi_t$ inflation measure.

Figure 3.14: Aggregate Variables to Cost-Push Shock in the Consumption Sector

Note: With a steady state of zero, the impulse response reported for fiscal debt is in levels, or deviations from steady state, equivalently. All other impulse responses are in percent deviations from steady state.
not respond much to the shock initially. Interestingly, though, the immediate realization of the shock depends on the monetary policy regime. In both instances inflation falls, but if policy makers use the narrow inflation measurement, output increases slightly and the shock seems to come from the supply side. If policy makers use the broad inflation measurement, the shock looks like it comes from the demand side. Since the wage rate falls and labor hours are fairly stable in this sector, profits for the consumption sector actually increase. At the aggregate level (Figure 3.17), this shock manifests itself as a supply shock. This is rather intuitive since its impact on the consumption goods sector is minimal. From a fiscal accounting point of view, decreased wages outweigh increased labor hours, pushing household income and fiscal revenues down and fiscal debt up. The extent to which debt increases again depends on the monetary policy regime. From the perspective of the health services sector, use of the broad inflation measurement dampens this shock, keeping debt levels lower overall. What these impulse responses show is that, while the choice of monetary policy regime may not matter for innovations impacting the overall economy, it is possible that sector-specific shocks can impact the economy in significantly different ways. In generality, it seems that if monetary policy makers consider an inflation measure that excludes all or portions of a specific sector, then shocks to that sector can be dramatically amplified. This is similar to the macro-prudential policy literature which is now considering the inclusion of price movements in the financial markets. The consumer price index does not include the value of financial services that are either free or not paid for by the household directly, meaning that use of such a measurement could cause amplification of shocks originating in that sector.

3.4.4.3 Trends Over Time: Simulated Moments

In this section I expand on the analysis of the previous section and consider simulated moments of the variables in the model. This allows me to consider what might happen in the long term, given that the true economy is not hit by one shock at a time. Specifically, these moments are calculated by randomly drawing the structural shocks from normal distributions
Figure 3.15: Consumption-Sector Variables to Cost-Push Shock in the Health Sector

Figure 3.16: Health Services-Sector Variables to Cost-Push Shock in the Health Sector

Note: Again, the black, solid lines represent the impulse response functions under the narrow monetary policy regime while the blue, dashed lines represent the impulse response functions under the broad monetary policy regime.

Figure 3.17: Aggregate Variables to Cost-Push Shock in the Health Services Sector

Note: Again, with a steady state of zero, the impulse response reported for fiscal debt is in levels, or deviations from steady state, equivalently. All other impulse responses are in percent deviations from steady state.
with their given specifications. The model is then simulated for 20,000 periods, after which the means and standard deviations are calculated. The results are shown in the tables below, along with the percentage change seen when moving from the CPI-based measure to the broader measure of inflation. It needs to be noted that these results come under the assumption that the response parameters in the monetary policy rule remain unchanged from one regime to another and that I assume that the monetary/fiscal mix is active/passive. Thus, all else held constant, what happens if policy makers simply change their input from one measure of inflation to another. The means of the variables are shown in Table 3.3. As can be seen here, overall, the average values of the variables do not change much at all. The major changes are seen in downward pressure on inflation rates and fiscal debt levels. It is interesting to note the fall in inflation rates in the early 2000s, which fell under the broad monetary policy regime at the time. Recall that the inflation variables in the table are expressed in gross terms. Converting these shows that a 0.01% decrease in the gross variables is equivalent to approximately a 0.56% decrease in the net variables. While this is still small, the downward pressure on inflation while maintaining the same values for nearly all of the other variables seems similar to the early 2000s era. Table 3.4 presents the percent change in the standard deviations of the variables when monetary policy moves from one regime to the other. Note that the standard deviations of nearly all variables increase, the source of which can be found in the table through monetary policy. Just as was shown in the impulse responses, shocks to the larger sectors are amplified when monetary policy considers the inflation rates from smaller sectors dually. This is due to the fact that interest rates no longer adjust as much to changes in inflation from that particular sector, as shown in the reduced standard deviation of the interest rate. This also matches the trends seen in Figure 3.6, where the standard deviations of many of the factors increases while the standard deviation of the federal funds rate falls. Recall that these factors represent the general trends of 115 economic indicators, presenting a parsimonious illustration of the economy as a whole.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Inflation Measure</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Narrow</td>
<td>Broad</td>
</tr>
<tr>
<td>Consumable Goods: Household $(c_t)$</td>
<td>0.410</td>
<td>0.410</td>
</tr>
<tr>
<td>Health Services: Household $(h_t)$</td>
<td>0.387</td>
<td>0.387</td>
</tr>
<tr>
<td>Total Labor Supply $(l_t)$</td>
<td>0.334</td>
<td>0.334</td>
</tr>
<tr>
<td>Consumption Labor Supply $(l^c_t)$</td>
<td>0.204</td>
<td>0.204</td>
</tr>
<tr>
<td>Healthcare Labor Supply $(l^h_t)$</td>
<td>0.130</td>
<td>0.130</td>
</tr>
<tr>
<td>Health Status $(x_t)$</td>
<td>0.114</td>
<td>0.114</td>
</tr>
<tr>
<td>Aggregate Output $(y_t)$</td>
<td>0.934</td>
<td>0.934</td>
</tr>
<tr>
<td>Consumer Goods Output $(y^c_t)$</td>
<td>0.609</td>
<td>0.609</td>
</tr>
<tr>
<td>Healthcare Goods Output $(y^h_t)$</td>
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<td>0.388</td>
</tr>
<tr>
<td>Inflation: Aggregate $(\pi_t)$</td>
<td>1.023</td>
<td>1.023</td>
</tr>
<tr>
<td>Inflation: Consumer Goods $(\pi^c_t)$</td>
<td>1.023</td>
<td>1.023</td>
</tr>
<tr>
<td>Inflation: Healthcare Goods $(\pi^h_t)$</td>
<td>1.023</td>
<td>1.023</td>
</tr>
<tr>
<td>Gross Nominal Interest Rate $(r_t)$</td>
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<td>1.043</td>
</tr>
<tr>
<td>Real Price of Health Services $(p^h_t)$</td>
<td>1.084</td>
<td>1.084</td>
</tr>
<tr>
<td>Real Wage Rate $(w_t)$</td>
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<td>2.492</td>
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<tr>
<td>Profit: Consumer Goods Sector $(d^c_t)$</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>Profit: Healthcare Goods Sector $(d^h_t)$</td>
<td>0.089</td>
<td>0.089</td>
</tr>
<tr>
<td>Tax Rate $(\tau_t)$</td>
<td>0.240</td>
<td>0.240</td>
</tr>
<tr>
<td>Transfers $(t_t)$</td>
<td>-0.000</td>
<td>-0.000</td>
</tr>
<tr>
<td>Consumer Goods: Government $(g^c_t)$</td>
<td>0.200</td>
<td>0.200</td>
</tr>
<tr>
<td>Health Services: Government $(g^h_t)$</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Fiscal Revenues $(\tau_t w_t l_t)$</td>
<td>0.200</td>
<td>0.200</td>
</tr>
<tr>
<td>Fiscal Debt $(b_t)$</td>
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<td>-0.012</td>
</tr>
<tr>
<td>Marginal Utility of Consumption $(\Lambda_t)$</td>
<td>20.793</td>
<td>20.793</td>
</tr>
</tbody>
</table>

Table 3.3: Simulated Means

*Note:* These standard deviations are calculated by simulating the model for 20,000 periods using the parameters as estimated in Table 3.5 for each respective model. The percent change is calculated by taking the difference and dividing by the values found in the “Narrow” column.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Inflation Measure</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Narrow</td>
<td>Broad</td>
</tr>
<tr>
<td>Consumable Goods: Household ($c_t$)</td>
<td>0.032</td>
<td>0.032</td>
</tr>
<tr>
<td>Health Services: Household ($h_t$)</td>
<td>0.040</td>
<td>0.041</td>
</tr>
<tr>
<td>Total Labor Supply ($l_t$)</td>
<td>0.024</td>
<td>0.024</td>
</tr>
<tr>
<td>Consumption Labor Supply ($l_c^t$)</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td>Healthcare Labor Supply ($l_h^t$)</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>Health Status ($x_t$)</td>
<td>0.019</td>
<td>0.019</td>
</tr>
<tr>
<td>Aggregate Output ($y_t$)</td>
<td>0.058</td>
<td>0.058</td>
</tr>
<tr>
<td>Consumer Goods Output ($y_c^t$)</td>
<td>0.033</td>
<td>0.034</td>
</tr>
<tr>
<td>Healthcare Goods Output ($y_h^t$)</td>
<td>0.042</td>
<td>0.043</td>
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<tr>
<td>Inflation: Aggregate ($\pi_t$)</td>
<td>0.121</td>
<td>0.123</td>
</tr>
<tr>
<td>Inflation: Consumer Goods ($\pi_c^t$)</td>
<td>0.125</td>
<td>0.127</td>
</tr>
<tr>
<td>Inflation: Healthcare Goods ($\pi_h^t$)</td>
<td>0.109</td>
<td>0.110</td>
</tr>
<tr>
<td>Gross Nominal Interest Rate ($r_t$)</td>
<td>0.119</td>
<td>0.118</td>
</tr>
<tr>
<td>Real Price of Health Services ($p_h^t$)</td>
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<td>0.048</td>
</tr>
<tr>
<td>Real Wage Rate ($w_t$)</td>
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</tr>
<tr>
<td>Profit: Consumer Goods Sector ($d_c^t$)</td>
<td>0.090</td>
<td>0.093</td>
</tr>
<tr>
<td>Profit: Healthcare Goods Sector ($d_h^t$)</td>
<td>0.071</td>
<td>0.073</td>
</tr>
<tr>
<td>Tax Rate ($\tau_t$)</td>
<td>0.202</td>
<td>0.210</td>
</tr>
<tr>
<td>Transfers ($t_t$)</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>Consumer Goods: Government ($g_c^t$)</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>Health Services: Government ($g_h^t$)</td>
<td>0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>Fiscal Revenues ($\tau_t w_t l_t$)</td>
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<td>0.200</td>
</tr>
<tr>
<td>Fiscal Debt ($b_t$)</td>
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<tr>
<td>Marginal Utility of Consumption ($\Lambda_t$)</td>
<td>6.256</td>
<td>6.285</td>
</tr>
</tbody>
</table>

Table 3.4: Simulated Standard Deviations

*Note:* These standard deviations are calculated by simulating the model for 20,000 periods using the parameters as estimated in Table 3.5 for each respective model. The percent change is calculated by taking the difference and dividing by the values found in the “Narrow” column.
3.4.5 Estimating the Policy Functions

As stated above, I now consider the data to estimate the policy functions in the model. This gives me at least an idea of where these parameters may fall. However, it is important to note that these rules are very simple and therefore the true values of these parameters may be subject to measurement and/or model specification error such as that described by Rudebusch (2002, 2006) and others.

3.4.5.1 Data and Results

After considering the parameter space in the above sections, I now estimate the monetary policy rule (3.8) above using nonlinear least squares. The data are quarterly and are expressed in annualized terms during the period 1981:Q1–2008:Q4. For this exercise, I use the Philadelphia Federal Reserve Bank’s one-year expected CPI inflation for the period 1981Q3–1999Q4 and the Greenbook one-year ahead forecast of PCE inflation for the period 2000Q1–2008Q4, just as those utilized by Mehra and Minton (2007) and Mehra and Sawhney (2010). Additionally, I use the effective federal funds rate and output growth rate as provided by the St. Louis Federal Reserve Bank (FRED). Since there is a break in the Federal Reserve’s target variable from CPI to PCE inflation in the first quarter of 2000, I break down the analysis into two groups. For added simplification, and after extensive robustness checks, I continue to assume that the monetary policy rule is strictly targeting inflation, fixing $\rho_y = 0$. As can be seen in the determinacy regions above, the response to the growth rate of output has no impact on the dynamics at least within the vicinity of the parameter’s estimate and the framework of this model. The estimates are provided in Table 3.5. The response to inflation, for the most part, has satisfied the Taylor-principle, but this response dipped slightly after the turn of the century.\footnote{Even thought the parameter estimate for $\rho_\pi$ falls below unity in the second subsample, it is not statistically different from the first subsample estimate, a result found in the empirical estimates of Mehra and Sawhney (2010).}

For the fiscal policy rule, annual data must be used. Thus, I consider the average federal
Table 3.5: Parameter Estimates for Policy Rules

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<thead>
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<th></th>
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</thead>
<tbody>
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<td>(a)</td>
<td>(b)</td>
<td></td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>0.8137</td>
<td>0.9378</td>
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<tr>
<td></td>
<td>(0.0766)</td>
<td>(0.0589)</td>
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<td>$\rho_{\pi}$</td>
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<td>0.9964</td>
</tr>
<tr>
<td></td>
<td>(0.0033)</td>
<td>(0.0231)</td>
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<table>
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<tr>
<td></td>
<td>(0.0136)</td>
</tr>
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<td>$\gamma_\tau$</td>
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<td>(0.0499)</td>
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<td></td>
<td>(0.0396)</td>
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<td>$\gamma_t$</td>
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</tr>
<tr>
<td></td>
<td>(0.0721)</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Equilibrium Probability†</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indeterminate</td>
</tr>
<tr>
<td>Determinate</td>
</tr>
<tr>
<td>No Solution</td>
</tr>
</tbody>
</table>

* Monetary policy parameters are estimated using quarterly data and nonlinear least squares. The values in parentheses are heteroskedasticity-robust standard errors.

* Fiscal policy parameters are estimated using annual data from 1981-2011 since quarterly data is unavailable. Therefore, the changes in the probabilities are caused purely by changes in the monetary policy parameters.

† Probabilities are expressed as a percent and evaluated by drawing 50,000 times from normal distributions with the given moments and applying them to the model. Each result is simply the number of times the parameter selection resulted in the dynamic property divided by the total number of draws.
tax rate as estimated by the NBER and the annual percent change in real personal income from 1960–2011. Additionally, for ease of estimation and to reflect the goal of fiscal policy makers, I assume that the tax code is perfectly indexed for inflation. While this is far from the actual scenario, the lack of data requires me to make these simplifying assumptions.

Along with the empirical estimates of the policy rules, I also include the probability of the model being parametrically located in each of the determinacy regions shown above. As can be seen, the probability of the model being indeterminate is zero, reflecting the push just before this sample period by the Volcker administration to get out of the high volatility environment. Additionally, we see that the probability of either a unique solution or no solution to the model does not change much, a parallel to the lack of substantial change in the determinacy regions mentioned above.

3.4.5.2 Determinacy Regions

Since the determinacy regions presented above were subjected to some fairly strong assumptions with regards to the parameter values, I now present them with the estimated values. Figure (3.18) shows that these regions are quite sensitive to the smoothing parameter $\rho_\tau$.

Figure 3.18: Monetary and Fiscal Policy with Estimated Parameter Values

which was set to unity prior to this exercise. Since the value of $\gamma_\tau$ is much lower than the
original specification, changes to this parameter do not substantially alter the probability of achieving determinacy. The area defined as the passive/active monetary/fiscal regime remains unaltered.

3.4.5.3 Impulse Response Analysis

Here I present the impulse response functions for the two models. This time, however, I use the estimated values of the monetary and fiscal policy parameters. These values place the model under the category of fiscal theory of the price level in which monetary policy is passive and fiscal policy is active. More specifically, the model lies in the lower-left quadrant of the parameter space of Figure (3.18). The first thing to notice is that the initial impact of each shock is dramatically reduced under this regime. This is primarily due to the fact that, both active fiscal policy and passive monetary policy result in a small amount of change to the policy variables. The second thing to notice is that the dynamics of the impulse response functions are much cleaner than what was seen prior. Here, the variables adjust and then return to steady state in a monotonic manner, whereas the responses under the previous regime were subject to oscillations. Next, notice that there is virtually no difference between the two models in most of the variables. Below I present the impulse response functions for the monetary policy and sector-specific supply shocks, just as before. The remaining impulse response functions can be found in the appendix.

Just as with the empirical model and the active/passive scenario above, the efficacy of monetary policy needs to be addressed. To do so in a simple fashion, I analyze the impulse response function to a sudden change in the federal funds rate. Since this model is parameterized under the passive monetary and active fiscal policies, the dynamic impact of an innovation to the interest rate changes. The results are shown in Figures (3.19)–(3.21). As can be seen the variables react in nearly identical fashion regardless of the monetary regime, but notice that inflation actually moves with the change in interest rates (albeit lagged). The way these models are constructed, the monetary policy rule typically determines the
inflation rate. However, since this is a PM/AF scenario, rising interest rates put upward pressure on debt levels, so the fiscal authority raises prices to inflate real debt away. The lack of change in the impulse responses when moving from one monetary policy regime to another matches the trend seen in Figure (3.5). In both situations there is little difference between the reactions of the variables when monetary policy targets narrow inflation and when it targets broad inflation.

Next consider the impact of a cost-push shock in the consumption good sector. Figure (3.22) presents the results in the consumption good sector, in which the shock originated. Here the general movements of the variables are the same as shown above. As a standard supply-side shock, output in this sector rises while the inflation rate falls. Figure (3.23), again, shows that this supply-side innovation manifests itself as a demand shock in the health services sector, with both output and the inflation rate falling. Output in the health services sector falls so much more drastically than in the consumption sector since the real (or relative) price level in the health services sector increases dramatically. At the aggregate level, Figure (3.24) shows again that this supply-side shock from the consumption sector shows up as a demand shock at the aggregate level due to the disproportionate impact on the two sectors. Once again, I find upward pressure on fiscal debt under the broad monetary policy regime relative to its narrow counterpart. This is due to the fact that interest rates, which are very persistent under this specification, do not fall by as much under the broad regime. This stems from the fact that while inflation in the consumer goods sector falls by 0.4%, the shock only pushes inflation rates in the health services sector by 0.17%, resulting in the aggregate inflation rate not falling by as much as that in the consumer good sector. Since the input of the monetary policy rule is not as volatile under the broad regime, interest rates are also not as volatile, but higher interest rates mean a higher cost of borrowing for the fiscal authority, pushing up debt levels.

Next consider an analogous shock to the health services sector. Figure (3.26) presents the impulse response functions for the variables in the health services sector. Notice that,
Figure 3.19: Consumption-Sector Variables to Monetary Policy Shock

Figure 3.20: Health Services-Sector Variables to Monetary Policy Shock

Note: The black, solid lines represent the narrow monetary policy regime, while the blue, dashed lines represent the broad monetary policy regime. These values are calculated using the estimated monetary policy parameters for each respective period.

Figure 3.21: Aggregate Variables to Monetary Policy Shock

Note: With a steady state of zero, the impulse response reported for fiscal debt is in levels, or deviations from steady state, equivalently. All other impulse responses are in percent deviations from steady state.
Figure 3.22: Consumption-Sector to Cost-Push Shock in the Consumption Sector

Note: The black, solid lines represent the narrow monetary policy regime, while the blue, dashed lines represent the broad monetary policy regime. These values are calculated using the estimated monetary policy parameters for each respective period.

Figure 3.23: Health Services-Sector to Cost-Push Shock in the Consumption Sector

Figure 3.24: Aggregate Variables to Cost-Push Shock in the Consumption Sector

Note: With a steady state of zero, the impulse response reported for fiscal debt is in levels, or deviations from steady state, equivalently. All other impulse responses are in percent deviations from steady state.
in this sector, the choice of monetary policy regime does not matter much. This is at least partly due to the small relative size of this sector relative to the other. In fact, the shape of the impulse responses are almost identical to those shown earlier. This shock impacts this market as a standard supply-side innovation should, pushing inflation rates down while increasing output. The main difference between this set of impulse responses and those from the earlier analysis is that the shock has a dramatically diminished impact on the economy. Recall that this shock had different impacts on the consumption goods sector depending on the monetary policy regime in the previous analysis. Under one regime, it manifested itself as a demand shock, while showing up as a supply shock under the other regime. With this new parameterization, a cost-push shock in the health care sector is seen as a demand-side innovation in the consumption good sector due to the large impact on the relative price.

At the aggregate level, Figure (3.27) shows that this supply-side shock does reveals itself as a supply-side innovation at the aggregate level. This time, unlike the previous analysis, the interest rate under the broad regime is higher than under the narrow regime, causing upward pressure on fiscal debt levels. Before, a cost-push shock to this sector actually decreased the pressure on fiscal debt because the shock impacted the health services sector enough to overcome the small weighting, pushing aggregate inflation (and thus, interest rates) lower than inflation in the consumption good sector. Here, the impact on the health services sector does not overcome the small weighting, meaning that aggregate inflation (and thus, interest rates) do not fall as much. This puts more upward pressure on the level of fiscal debt.

3.4.5.4 Simulated Moments of the Model

Just as in the previous analysis, I now turn to the simulated moments in the model under the different regimes using the estimated parameter values. This gives us a longer-term look at the impact of the differing monetary policy regimes. It needs to be noted that the values reported here are sensitive to the number of periods under which the simulation is considered. This is due to the fact that the estimated parameter values are very close to the
Figure 3.25: Consumption-Sector to Cost-Push Shock in the Healthcare Sector

Note: The black, solid lines represent the narrow monetary policy regime, while the blue, dashed lines represent the broad monetary policy regime. These values are calculated using the estimated monetary policy parameters for each respective period.

Figure 3.26: Health Services-Sector to Cost-Push Shock in the Healthcare Sector

Figure 3.27: Aggregate Variables to Cost-Push Shock in the Healthcare Sector

Note: With a steady state of zero, the impulse response reported for fiscal debt is in levels, or deviations from steady state, equivalently. All other impulse responses are in percent deviations from steady state.
determinacy/no solution boundary, meaning that the variables can be very persistent. This has already been seen in the impulse response analysis, where interest rates fell and remained low for a significant time. Therefore, to attain somewhat reliable and stable results, these simulations are carried out for 200,000 periods. Table (3.6) presents the simulated means of the variables under both regimes as well as the percent change from the narrow regime to the broad regime. As can be seen, most of classic variables from the consumption good and health services sectors remain nearly unchanged. The larger changes start with monetary policy, where increased inflation rates (around 150 basis points) causes a need for higher interest rates. These small increases in inflation and in the interest rate cause large (in percent terms) upward pressure on fiscal debt levels. What this says is that, under the estimated policy rules, the economy could see added fiscal pressure simply due to a change in the measure of inflation monetary policy targets.

Not only am I concerned with the simulated means of the variables, but the simulated standard deviations should also be addressed in this case. Table (3.7) provides the results from the simulations. The first thing to notice is that, while the basic variables did not adjust much in terms of their means, the adjustment of the standard deviations is much more pronounced. Output-related variables such as consumption, health services, and their aggregates saw their standard deviations increased by as much as six percent from one regime to the other. The only variables in these sectors that see decreases in their standard deviations are the labor supply variables and the firm profits, which are obviously tied to labor supply. An interesting phenomenon to note is that, while the simulated mean of the fiscal debt level increases dramatically, its standard deviation actually decreases by almost 1.5%. So what we see is a higher, yet more stable, level of government debt when monetary policy moves from the narrow to the broad regime.

Comparing the results of this scenario to the empirical results found above is a little less straightforward. While I do find increases in standard deviations across the board, I also find increases in the standard deviation of the federal funds rate. The former matches
<table>
<thead>
<tr>
<th>Variable</th>
<th>Narrow</th>
<th>Broad</th>
<th>%Δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumable Goods: Household ($c_t$)</td>
<td>0.411</td>
<td>0.411</td>
<td>0.04</td>
</tr>
<tr>
<td>Health Services: Household ($h_t$)</td>
<td>0.389</td>
<td>0.389</td>
<td>0.12</td>
</tr>
<tr>
<td>Total Labor Supply ($l_t$)</td>
<td>0.333</td>
<td>0.334</td>
<td>0.06</td>
</tr>
<tr>
<td>Consumption Labor Supply ($l^c_t$)</td>
<td>0.204</td>
<td>0.204</td>
<td>0.03</td>
</tr>
<tr>
<td>Healthcare Labor Supply ($l^h_t$)</td>
<td>0.130</td>
<td>0.130</td>
<td>0.12</td>
</tr>
<tr>
<td>Health Status ($x_t$)</td>
<td>0.115</td>
<td>0.115</td>
<td>0.02</td>
</tr>
<tr>
<td>Aggregate Output ($y_t$)</td>
<td>0.937</td>
<td>0.937</td>
<td>0.05</td>
</tr>
<tr>
<td>Consumer Goods Output ($y^c_t$)</td>
<td>0.611</td>
<td>0.611</td>
<td>0.03</td>
</tr>
<tr>
<td>Healthcare Goods Output ($y^h_t$)</td>
<td>0.389</td>
<td>0.389</td>
<td>0.12</td>
</tr>
<tr>
<td>Inflation: Aggregate ($\pi_t$)</td>
<td>1.022</td>
<td>1.037</td>
<td>1.49</td>
</tr>
<tr>
<td>Inflation: Consumer Goods ($\pi^c_t$)</td>
<td>1.022</td>
<td>1.037</td>
<td>1.49</td>
</tr>
<tr>
<td>Inflation: Healthcare Goods ($\pi^h_t$)</td>
<td>1.022</td>
<td>1.037</td>
<td>1.49</td>
</tr>
<tr>
<td>Gross Nominal Interest Rate ($r_t$)</td>
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<td>1.058</td>
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<tr>
<td>Real Price of Health Services ($p^h_t$)</td>
<td>1.086</td>
<td>1.085</td>
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</tr>
<tr>
<td>Real Wage Rate ($w_t$)</td>
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<td>2.501</td>
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</tr>
<tr>
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<td>0.101</td>
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</tr>
<tr>
<td>Profit: Healthcare Goods Sector ($d^h_t$)</td>
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<td>0.090</td>
<td>−0.37</td>
</tr>
<tr>
<td>Tax Rate ($\tau_t$)</td>
<td>0.240</td>
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</tr>
<tr>
<td>Transfers ($t_t$)</td>
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<td>−0.000</td>
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</tr>
<tr>
<td>Consumer Goods: Government ($g^c_t$)</td>
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<td>0.200</td>
<td>0.00</td>
</tr>
<tr>
<td>Health Services: Government ($g^h_t$)</td>
<td>−0.000</td>
<td>−0.000</td>
<td>0.00</td>
</tr>
<tr>
<td>Fiscal Revenues ($\tau_t w_t l_t$)</td>
<td>0.200</td>
<td>0.200</td>
<td>0.15</td>
</tr>
<tr>
<td>Fiscal Debt ($b_t$)</td>
<td>0.002</td>
<td>0.018</td>
<td>901.82</td>
</tr>
<tr>
<td>Marginal Utility of Consumption ($\Lambda_t$)</td>
<td>20.611</td>
<td>20.594</td>
<td>−0.08</td>
</tr>
</tbody>
</table>

Table 3.6: Simulated Means under AF/PM Regime

*Note:* These standard deviations are calculated by simulating the model for 200,000 periods using the parameters as estimated in Table 3.5 for each respective model. The percent change is calculated by taking the difference and dividing by the values found in the “Narrow” column.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Narrow</th>
<th>Broad</th>
<th>%∆</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumable Goods: Household ((c_t))</td>
<td>0.033</td>
<td>0.033</td>
<td>2.77</td>
</tr>
<tr>
<td>Health Services: Household ((h_t))</td>
<td>0.030</td>
<td>0.032</td>
<td>6.34</td>
</tr>
<tr>
<td>Total Labor Supply ((l_t))</td>
<td>0.033</td>
<td>0.032</td>
<td>-1.62</td>
</tr>
<tr>
<td>Consumption Labor Supply ((l^c_t))</td>
<td>0.018</td>
<td>0.017</td>
<td>-0.88</td>
</tr>
<tr>
<td>Healthcare Labor Supply ((l^h_t))</td>
<td>0.016</td>
<td>0.016</td>
<td>-2.00</td>
</tr>
<tr>
<td>Health Status ((x_t))</td>
<td>0.019</td>
<td>0.019</td>
<td>0.70</td>
</tr>
<tr>
<td>Aggregate Output ((y_t))</td>
<td>0.054</td>
<td>0.057</td>
<td>4.45</td>
</tr>
<tr>
<td>Consumer Goods Output ((y^c_t))</td>
<td>0.034</td>
<td>0.035</td>
<td>2.52</td>
</tr>
<tr>
<td>Healthcare Goods Output ((y^h_t))</td>
<td>0.033</td>
<td>0.035</td>
<td>5.22</td>
</tr>
<tr>
<td>Inflation: Aggregate ((\pi_t))</td>
<td>0.224</td>
<td>0.232</td>
<td>3.17</td>
</tr>
<tr>
<td>Inflation: Consumer Goods ((\pi^c_t))</td>
<td>0.226</td>
<td>0.233</td>
<td>3.08</td>
</tr>
<tr>
<td>Inflation: Healthcare Goods ((\pi^h_t))</td>
<td>0.219</td>
<td>0.227</td>
<td>3.47</td>
</tr>
<tr>
<td>Gross Nominal Interest Rate ((r_t))</td>
<td>0.209</td>
<td>0.220</td>
<td>5.39</td>
</tr>
<tr>
<td>Real Price of Health Services ((p^h_t))</td>
<td>0.040</td>
<td>0.037</td>
<td>-6.38</td>
</tr>
<tr>
<td>Real Wage Rate ((w_t))</td>
<td>0.335</td>
<td>0.344</td>
<td>2.85</td>
</tr>
<tr>
<td>Profit: Consumer Goods Sector ((d^c_t))</td>
<td>0.094</td>
<td>0.094</td>
<td>-0.64</td>
</tr>
<tr>
<td>Profit: Healthcare Goods Sector ((d^h_t))</td>
<td>0.061</td>
<td>0.060</td>
<td>-1.39</td>
</tr>
<tr>
<td>Tax Rate ((\tau_t))</td>
<td>0.042</td>
<td>0.043</td>
<td>1.13</td>
</tr>
<tr>
<td>Transfers ((t_t))</td>
<td>0.001</td>
<td>0.002</td>
<td>5.20</td>
</tr>
<tr>
<td>Consumer Goods: Government ((g^c_t))</td>
<td>0.010</td>
<td>0.010</td>
<td>0.00</td>
</tr>
<tr>
<td>Health Services: Government ((g^h_t))</td>
<td>0.010</td>
<td>0.010</td>
<td>0.00</td>
</tr>
<tr>
<td>Fiscal Revenues ((\tau_t w_t l_t))</td>
<td>0.074</td>
<td>0.075</td>
<td>1.06</td>
</tr>
<tr>
<td>Fiscal Debt ((b_t))</td>
<td>0.311</td>
<td>0.307</td>
<td>-1.47</td>
</tr>
<tr>
<td>Marginal Utility of Consumption ((\Lambda_t))</td>
<td>4.974</td>
<td>5.110</td>
<td>2.74</td>
</tr>
</tbody>
</table>

Table 3.7: Simulated Standard Deviations under AF/PM Regime

*Note:* These standard deviations are calculated by simulating the model for 200,000 periods using the parameters as estimated in Table 3.5 for each respective model. The percent change is calculated by taking the difference and dividing by the values found in the “Narrow” column.
the empirics, while the latter does not. This can be partially explained by the status of monetary and fiscal policy. The active fiscal/passive monetary policy relationship forces monetary policy to adjust its interest rates in order to maintain the stationarity of real fiscal debt levels. This could lead to larger adjustments in the long run since monetary policy is not being proactive against inflation.

3.5 Concluding Remarks

In this paper I explore the dynamic impact of monetary policy makers moving from a narrower to a broader measure of inflation in their policy rules, while fiscal policy makers maintain the narrower measure for indexation purposes. Using a dynamic stochastic general equilibrium model with two production sectors and nominal rigidities at the intermediate production level, I find that the impact of this regime switch on fiscal debt levels depends greatly on the coordination efforts between monetary and fiscal policy. Under the standard active monetary and passive fiscal policy assumption, this regime shift has very little impact on the levels of basic household variables while putting downward pressure on fiscal debt levels. However, the shift does put upward pressure on standard deviations of most of the model’s variables while reducing that of the policy instrument. This is due to the fact that monetary policy moves to a target that averages out perturbations in more volatile sectors with those of more stable sectors, meaning that the policy instrument will not respond in as dramatic a fashion. So even though the determinacy properties of the model remain mostly unaltered, there is still an impact on the general dynamics of the economy.

Expanding on this by considering the data and estimating the true stance of both policy makers, I find that the model just falls under the passive monetary and active fiscal policy (AF/PM) combination. By “just”, I mean that the parameterization of this model is very close to the active/active combination, which makes the model determinate, but very persistent. When considering a regime shift in this scenario, I find that there are modest increases
in the levels of the classic variables, but there is substantial upward pressure on fiscal debt levels. The interest rate does not adjust as strongly to consumer-sector inflation as it did before, keeping it higher on average. Also because of this, I again find increased standard deviations of most variables not dependent on labor supply. The only additional variable with a lower standard deviation in this scenario is the fiscal debt, suggesting that this regime could cause increased, yet more stable, levels of debt.

What all of these results imply is that moving from a narrower inflation target to a broader target has mixed results on the levels of fiscal debt, but in all scenarios I find increased standard deviations. This matches the results from a TVP-FAVAR model nearly identical to that of Korobilis (2009), showing that standard deviations of economic indicators began to rise in the early 2000s, shortly after the Federal Reserve changed its targeted inflation measure. Specifically, I find that under the AF/PM regime, the model shows increased levels of real fiscal debt and increased variations in the remaining economic indicators, which is similar to the statistical trends seen since the monetary policy adjustment.

Further work on this subject could include other tax rates besides a simple wage income tax, perhaps in an overall tax rule which includes all forms of incomes to better correspond to the generalized data available. The addition of capital and financial markets, as well as introducing matching mechanisms to the labor and government debt markets could make the results richer as well. Such matching mechanisms would allow policy makers to include the unemployment rate to its monetary rule, which is part of the dual mandate and corresponds to the empirical estimates of monetary policy rules. As noted above, Anand and Prasad (2010) show that welfare-maximizing monetary policy is dependent on where the nominal rigidities are located, concluding that targeting core inflation measures is only optimal when the only friction is derived from price stickiness. Simultaneously, adding a matching market to government debt would allow the quantity of debt to be supply-driven, while the price of the debt is equilibrium-driven.
Bibliography


Appendix A

Solving the Models in Chapter 1

A.1 Solution Methods for the Two-Period Model

In this appendix, we solve the problems for each of the sectors of the two-period economic model.

A.1.1 The Household’s Problem

Below is the Lagrangian associated with the household’s problem. The household chooses \( \{c_1, c_2, n_1, n_2\} \) to maximize its intertemporal utility function, letting \( \beta > 0 \) represent the intertemporal discount factor,

\[
L(\cdot) = \ln c_1 - \frac{\gamma c_1}{\mu n_1} + \beta \left[ \ln c_2 - \frac{\gamma c_2}{\mu n_2} \right] + \lambda \left[ (w - (1 + \tau)c_2 - (1 + r)(1 + \tau)c_1 - n_2 - (r + \delta)n_1) \right],
\]

where \( \lambda \) is the Lagrange multiplier associated with the intertemporal budget constraint.
First-Order Conditions

The first order conditions with respect to the choice variables for the household are as follows:

\[
\frac{\partial L}{\partial c_1} = \frac{1}{c_1} - \frac{\gamma}{\mu n_1} - \lambda(1 + r)(1 + \tau) = 0
\]

\[
\frac{\partial L}{\partial c_2} = \frac{\beta}{c_2} - \frac{\beta \gamma}{\mu n_2} - \lambda(1 + \tau) = 0
\]

\[
\frac{\partial L}{\partial n_1} = \frac{\gamma c_1}{\mu n_1^2} - \lambda(r + \delta) = 0
\]

\[
\frac{\partial L}{\partial n_2} = \frac{\beta \gamma c_2}{\mu n_2^2} - \lambda = 0
\]

A.2 Competitive Equilibrium in the Two-Period Model

A.2.1 Intertemporal Budget Constraints

**Household**  We can combine equations (1.1) and (1.1) to form the household’s intertemporal budget constraint,

\[
\tau c_2 + r \tau c_1 + n_2 + (r - 1 + \delta)n_1 = y_2 + ry_1. \tag{A.1}
\]

Since the endowments are exogenous to the household, we can assume that the total endowment is constant, or \(y_2 + ry_1 = y\). As we show below, we make the same type of assumption for government spending.

**Government**  Combining the government’s budget constraints gives us the real intertemporal budget constraint

\[
g_2 + r g_1 = (\tau - 1)c_2 + r(\tau - 1)c_1 + (1 - \zeta)[n_2 + (\sigma + r - 1)n_1]. \tag{A.2}
\]
Here, as mentioned above, we can assume that the total amount of government spending is an exogenous constant such that \( g_2 + rg_1 = g \).

**Equilibrium** Combining equations (A.1) and (A.2) shows us that, across both periods, the goods market clears.

\[
c_2 + rc_1 + g + \zeta [n_2^p + r n_1^p] = y
\]

Combining the individual budget constraints shows that the goods market clears within each period as well. This model’s equilibrium, therefore, is characterized as the values of \( \{ c_1, c_2, n_1, n_2, r, \phi \} \) which satisfy the system of six equations given by budget constraints (A.1) and (A.2) as well as its first order conditions, where \( \phi \) is the Lagrange multiplier associated with the budget constraint.

### A.3 Optimizing Conditions for Deterministic Model

Here we present the optimizing conditions of the agents in the deterministic model. All are solved for using standard Bellman equation methods.

#### A.3.1 Patient Household

\[
\frac{1}{c_t} - c_t^{\chi - 1} \left( \frac{1}{m_t} \right)^\chi = \lambda_t^2(1 + \tau)
\]

\[
n_t = \xi \mu^\theta - 1 m_t \left( \frac{\lambda^1_t}{\lambda^2_t - \beta \lambda^2_{t+1} \frac{1-\delta}{\pi_{t+1}}} \right)^\theta
\]

\[
d_t = (1 - \xi) m_t \left( \frac{\lambda^1_t}{\lambda^2_t - \beta \lambda^2_{t+1} \frac{r^p_t}{\pi_{t+1}}} \right)^\theta
\]

\[
\left( \frac{c_t}{m_t} \right)^\chi = \lambda^1_t m_t
\]

\[
1 = \beta \frac{\lambda^2_{t+1}}{\lambda^2_t} \frac{r_t}{\pi_{t+1}}
\]
A.3.2 Impatient Household

\[
\frac{1}{c_t} - (c'_t)^{\chi - 1} \left( \frac{1}{\xi^{\frac{1}{\mu}} \mu n'_t} \right)^{\chi} = \lambda^3_t (1 + \tau)
\]

\[
n'_t = \left( \frac{c_t}{\xi^{\frac{1}{\mu}} \mu} \right)^{\frac{\chi}{\chi + 1}} \left( \frac{1}{\lambda^3_t - \beta \lambda^3_{t+1} \frac{1 - \delta}{\pi_{t+1}}} \right)^{\frac{1}{\chi + 1}}
\]

\[
1 = \beta \frac{\lambda^3_{t+1}}{\lambda^3_t} \frac{r'_t}{\pi_{t+1}}
\]

A.3.3 Representative Bank

\[
r'_t = \frac{r^d_t}{1 - \omega} + \frac{x^d_t - \omega}{\Lambda_{t+1}(1 - \omega)}
\]

where \( \Lambda_{t+1} \equiv \frac{\beta \lambda^2_{t+1}}{\lambda^2_t} \) is the discount factor derived from the patient household’s Euler equation.
Appendix B

Solving the Model in Chapter 2

This section is devoted to solving the New Keynesian model found in section 2.2. A Bellman method is used because it is generally more tractable and simple than a Lagrangian method.

B.1 The Representative Household’s Problem

Considering equations (2.15), (2.1), and (2.3) above, we can form the Bellman Equation for the representative household.

\[
V_h(B_{t-1}) = \max_{c_t, h_t, B_t} \left\{ \eta^p \ln c_t - \frac{1}{\psi} h_t^\psi + \beta E_t \left[ V_h(B_t) \right] + \Lambda_t \left[ \frac{B_{t-1} + (1 - \tau_t) W_t h_t + D_t - B_t / r_t}{P_t} - c_t \right] \right\},
\]

where \( \Lambda_t \geq 0 \) represents the shadow price of the budget constraint. Solving this problem for consumption, labor hours, and nominal bond holdings yields the following first order conditions:

\[
\frac{\eta^p_t}{c_t} = \Lambda_t, \quad (B.1)
\]

\[
h_t^{\psi-1} = \Lambda_t (1 - \tau_t) \frac{W_t}{P_t}, \quad (B.2)
\]

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\[ \beta \mathbb{E}_t [V'_h(B_t)] = \frac{\Lambda_t}{r_t P_t}. \] (B.3)

The Benveniste-Shienkman condition follows accordingly as

\[ V'_h(B_{t-1}) = \frac{\Lambda_t}{P_t}. \] (B.4)

Combining equations (B.1)–(B.4) yields the optimizing conditions found in (2.4) and (2.5).

### B.2 The Final-Good Firm’s Problem

The profits of the firm are given by

\[
\Pi_f = P_t y_t - \int_0^1 P_t(i) y_t(i) di \\
= P_t \left[ \int_0^1 y_t(i) \frac{\eta_t}{\eta_t - 1} di \right] \frac{\eta_t}{\eta_t - 1} - \int_0^1 P_t(i) y_t(i) di.
\]

In this situation, the final goods-producing firm chooses the amount of each intermediate good \( y_t(i) \) for all \( i \). Since this not a dynamic problem, first order condition is simply

\[
P_t(i) = P_t \left[ \int_0^1 y_t(i) \frac{\eta_t}{\eta_t - 1} di \right] \frac{\eta_t}{\eta_t - 1} y_t(i) \frac{1}{\eta_t} \\
\Rightarrow P_t(i) = P_t y_t \frac{1}{\eta_t} y_t(i) \frac{1}{\eta_t}.
\]

Solving for \( y_t(i) \) provides the demand equation for the intermediate goods by the final goods-producing firm. Using this, the implicit price aggregator is

\[
P_t y_t = \int_0^1 P_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\eta_t}{\eta_t - 1}} y_t di
\]

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B.3 The Intermediate-Good Firm’s Problem

After combining all of the constraints with (2.12), the Bellman equation for each firm \( i \)'s dynamic problem is as follows:

\[
V_f (P_{t-1}(i)) = \max_{\frac{P_t(i)}{P_t}} \left\{ \left( \frac{P_t(i)}{P_t} \right)^{1-\eta_i^p} \frac{y_t \eta^p_t}{c_t} - \left( \frac{P_t(i)}{P_t} \right)^{-\eta_i^p} \frac{y_t W_t \eta^p_t}{Z_t P_t c_t} \right. \\
- \frac{\mu}{2} \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right)^2 \eta_t^p + \beta \mathbb{E}_t [V_f (P_t(i))] \left. \right\}.
\]

Since we combined all the constraints into the problem, there is only one first order condition

\[
(1 - \eta_i^p) \left( \frac{P_t(i)}{P_t} \right)^{-\eta_i^p} \frac{y_t \eta_t^p}{P_t c_t} + \eta_i^p \left( \frac{P_t(i)}{P_t} \right)^{-\eta_i^p-1} \frac{y_t W_t \eta_t^p}{Z_t P_t^2 c_t} \\
- \mu \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right) \frac{\eta_t^p}{\pi P_{t-1}(i)} + \beta \mathbb{E}_t [V'_f (P_t(i))] = 0, \quad (B.5)
\]

for all \( i \in [0,1] \) and one Benveniste-Shienkman condition

\[
V'_f (P_{t-1}(i)) = \mu \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right) \frac{P_t(i)}{\pi P_{t-1}(i)^2} \eta_t^p. \quad (B.6)
\]

for all \( i \in [0,1] \). Combining (B.5) and (B.6) provides the intermediate goods-producing firms’ first order conditions.
B.4 The Efficient Allocation

In order to solve for the output gap, consider a social planner who can overcome the frictions in the economy caused by the nominal price rigidity. Following Ireland (2004), in each period $t$, the social planner instructs $n_t(i)$ units of the representative household’s labor to produce $Q_t(i)$ of the intermediate good, which is then combined into the final good using the same constant returns to scale technology as above. Thus, the social planner maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \eta_t^p \ln Q_t - \frac{1}{\psi} \left( \int_0^1 n_t(i) di \right)^\psi \right]$$

subject to the resource constraint

$$z_t \left( \int_0^1 n_t(i) \frac{\eta_t^s-1}{\eta_t^s} di \right) \frac{\eta_t^s}{\eta_t^s-1} = Q_t.$$  

Solving this problem gives us the efficient allocation
Appendix C

Appendix for Chapter 3

C.1 Solving the Model

Here I present the details of the solution process for the model, particularly those related to the representative household, the intermediate goods firms, the set of equilibrium equations, and the steady state equations.

C.1.1 Representative Household’s Problem

Assuming no issues with time-inconsistency, I can solve the household’s problem using the standard Bellman method. In this case, the Bellman equation is formulated as

\[
\mathcal{V}(c_{t-1}, B_{t-1}) = \max \left\{ \epsilon_t^\beta \left( \frac{b_t}{1 - \sigma_c} (c_t - \delta c_{t-1})^{1-\sigma_c} + \frac{1}{1 - \sigma_x} [h_t^{\kappa x} (1 - l_t)^{\kappa x}]^{1-\sigma_x} \right) \right. \\
\quad + \beta \mathbb{E}_t [\mathcal{V}(c_t, B_t)] \\
\quad + \frac{\Lambda_t^1}{P_t} \left[ W_t l_t + D_t + B_{t-1} - \tau_t w W_t l_t - P_t^c c_t - P_t^h h_t - B_t \frac{B_t}{r_t} + T_t \right] \right\}
\]

where the household chooses \(\{c_t, h_t, B_t\}\). The variable \(\Lambda_t^1\) represents the multiplier on the budget constraint. Using the notation \(\mathcal{V}_j(\cdot)\) to denote the derivative of the value function with respect to the \(j\)th term, the first order conditions of are listed below.

\[
\epsilon_t^\beta (c_t - \delta c_{t-1})^{-\sigma_c} + \beta \mathbb{E}_t \mathcal{V}_1(\cdot) = \Lambda_t^1
\]  

(C.1)
\[ \epsilon_t^{\beta} \left[ h_t^{\kappa} \left( 1 - l_t \right)^{\kappa_l} \right]^{\sigma_x} \frac{\kappa_l x_t}{h_t} = \frac{P_h}{P_c} \Lambda_t^1 \]  
(C.2)

\[ \epsilon_t^{\beta} \left[ h_t^{\kappa} \left( 1 - l_t \right)^{\kappa_l} \right]^{\sigma_x} \frac{\kappa_l x_t}{1 - l_t} = \frac{W_t}{P_c} \left( 1 - \tau_w ^n \right) \Lambda_t^1 \]  
(C.3)

\[ \beta \mathbb{E}_t \mathcal{V}_2 (\cdot) = \frac{\Lambda_t^1}{P^c_t} \]  
(C.4)

Additionally, the Benveniste-Scheinkman conditions are listed below

\[ \mathcal{V}_1 (\cdot) = - \delta \epsilon_t^{\beta} (c_t - \delta c_{t-1})^{-\sigma_c} \]  
(C.5)

\[ \mathcal{V}_2 (\cdot) = \frac{\Lambda_t^1}{P^c_t} \]  
(C.6)

Combining equations (C.1)–(C.4) with equations (C.5)–(C.6) yields the optimality conditions for the household.

### C.1.2 Intermediate Goods-Producing Firms’ Problems

Due to the recursive nature of the intermediate goods-producing firms’ problem and considering \((3.3.3)\) and \((3.3.3)\) above, I can construct a simple Bellman equation for each firm \(j\) in each sector \(n\).

\[ \mathcal{V}^n (P_t^n (j)) = \max_{P_t^n (j)} \left\{ \left( \frac{P_t^n (j)}{P_t^n} \right)^{1 - \epsilon_t^{s,n}} \Lambda_t^1 y_t^n - \left( \frac{P_t^n (j)}{P_t^n} \right)^{-\epsilon_t^{s,n}} \frac{W_t \Lambda_t^1 y_t^n}{P_t^n \epsilon_t^p} \right. 
- \frac{\phi^p}{2} \left( \frac{P_t^n (j)}{\pi^n P_t^n (j)} - 1 \right) \left. \Lambda_t^1 y_t^n + \beta \mathbb{E}_t [\mathcal{V}^n (P_t^n (j))] \right\} \]  
(C.7)

Solving this recursive function yields one first order condition

\[ \left( 1 - \epsilon_t^{s,n} \right) \left( \frac{P_t^n (j)}{P_t^n} \right)^{-\epsilon_t^{s,n}} \frac{\Lambda_t^1 y_t^n}{P_t^n} + \epsilon_t^{s,n} \left( \frac{P_t^n (j)}{P_t^n} \right)^{-\epsilon_t^{s,n}-1} \frac{W_t \Lambda_t^1 y_t^n}{(P_t^n)^2 \epsilon_t^p} 
- \phi^p \left( \frac{P_t^n (j)}{\pi^n P_t^n (j)} - 1 \right) \frac{\Lambda_t^1 y_t^n}{\pi^n P_t^n (j)} + \beta \mathbb{E}_t [\mathcal{V}_1^n (\cdot)] = 0 \]  
(C.8)
and one Benveniste-Scheinkman condition

\[ V^b_t(\cdot) = \phi_p^n \left( \frac{P_t^n(j)}{\pi^n P_{t-1}^n(j)} - 1 \right) P_t^n(j) P_{t-1}^n(j) \Lambda_{t+1}^c y^c_{t+1}. \]  

Combining these two conditions yields the intermediate firms' optimality conditions.

### C.1.3 Equilibrium Equations

Below is the set of equations for the model in the symmetric equilibrium and in real terms.

\[
x_t = h_t^c (1 - l_t)^{\kappa_l}
\]

\[
p_h^t = \frac{\pi_h^t}{\pi_t^h} p_{t-1}^h
\]

\[
e_t^b (c_t - \delta c_{t-1})^{-\sigma_c} - \delta \beta E_t \left[ e_{t+1}^b (c_{t+1} - \delta c_t)^{-\sigma_c} \right] = \Lambda_t^c
\]

\[
e_t^b [h_t^c (1 - l_t)^{\kappa_l}]^{-\sigma_x} \frac{\kappa_t x_t}{h_t} = p_t^h \Lambda_t^c
\]

\[
e_t^b [h_t^c (1 - l_t)^{\kappa_l}]^{-\sigma_x} \frac{\kappa_t x_t}{1 - l_t} = w_t (1 - \tau_t^w) \Lambda_t^c
\]

\[
\beta r_t E_t \left[ \frac{\Lambda_t^{c+1}}{\Lambda_t^c \pi_t^{c+1}} \right] = 1
\]

\[
y_t = \left( \frac{y_t^c}{y_t^h} \right)^{1-\omega} \left( \frac{y_t^c}{\omega (1 - \omega)^{1-\omega}} \right)
\]

\[
\pi_t = \left( \frac{\pi_t^c}{\pi_t^h} \right)^{1-\omega}
\]

\[
y_t^c = e_t^c y_t^c
\]

\[
y_t^h = e_t^h y_t^h
\]

\[
\Lambda_{t}^c y_{t+1}^c \left[ 1 - \epsilon_t^{s,c} \right] + e_t^{s,c} \frac{w_t}{\epsilon_t^p} - \phi_p \left( \frac{\pi_t^c}{\pi_t^c} - 1 \right) \frac{\pi_t^c}{\pi_t^c} + \beta \phi_p E_t \left[ \left( \frac{\pi_t^{c+1}}{\pi_t^c} - 1 \right) \frac{\pi_t^{c+1}}{\pi_t^c} \Lambda_{t+1}^c y_{t+1}^c \right] = 0
\]

\[
\Lambda_{t}^h y_{t+1}^h \left[ 1 - \epsilon_t^{s,h} \right] + e_t^{s,h} \frac{w_t}{p_t^h e_t^p} - \phi_p \left( \frac{\pi_t^h}{\pi_t^h} - 1 \right) \frac{\pi_t^h}{\pi_t^h} + \beta \phi_p E_t \left[ \left( \frac{\pi_t^{h+1}}{\pi_t^h} - 1 \right) \frac{\pi_t^{h+1}}{\pi_t^h} \Lambda_{t+1}^h y_{t+1}^h \right] = 0
\]
\[ \begin{align*}
g_t^c & + p_t^h g_t^h + t_t + \frac{b_t-1}{\pi_t^c} = \frac{b_t}{rt} + \tau_t^w t_t, \\
\tau_t^w & = (1 - \rho_w)\tau_t^w + \rho_w \left[ \tau_t^w t_t - 1 + \gamma_t \left( \frac{w_{t-1}}{w_{t-1}t_t} (\pi_t^c)^1 - 1 \right) \right] \\
t_t & = (1 - \rho_T)\bar{t} + \rho_T \left[ t_t - 1 + \gamma_T \left( \frac{y_{t-1}}{y_t} (\pi_t^c)^1 - 1 \right) \right] \\
\bar{r}_t & = \rho_r \bar{r}_{t-1} + (1 - \rho_r) [\rho_y \bar{y}_{t+1} + \rho_y (\bar{y}_t - \bar{y}_{t-1})] + \eta_t^r \\
l_t & = l_t^c + l_t^h \\
d_t^c & = y_t^c - w_t l_t^c - \frac{\phi_p}{2} \left( \pi_t^c \frac{c_t}{\pi_t} - 1 \right)^2 y_t^c \\
d_t^h & = y_t^h - \frac{w_t l_t^h}{p_t^h} - \frac{\phi_p}{2} \left( \pi_t^h \frac{c_t}{\pi_t} - 1 \right)^2 y_t^h \\
y_t^c & = c_t + g_t^c + \frac{\phi_p}{2} \left( \pi_t^c \frac{c_t}{\pi_t} - 1 \right)^2 y_t^c \\
y_t^h & = h_t + g_t^h + \frac{\phi_p}{2} \left( \pi_t^h \frac{c_t}{\pi_t} - 1 \right)^2 y_t^h \\
\epsilon_t^\beta & = (1 - \rho_{\beta})\epsilon_t^\beta + \rho_{\beta} \epsilon_{t-1}^\beta + \eta_t^\beta \\
\epsilon_t^{s,c} & = (1 - \rho_{s,c})\epsilon_t^{s,c} + \rho_{s,c} \epsilon_{t-1}^{s,c} + \eta_t^{s,c} \\
\epsilon_t^{s,h} & = (1 - \rho_{s,h})\epsilon_t^{s,h} + \rho_{s,h} \epsilon_{t-1}^{s,h} + \eta_t^{s,h} \\
\epsilon_t^p & = (1 - \rho_p)\epsilon_t^p + \rho_p \epsilon_{t-1}^p + \eta_t^p \\
g_t^c & = (1 - \rho_g)\epsilon_t^c + \rho_g \epsilon_{t-1}^c + \eta_t^{g,c} \\
g_t^h & = (1 - \rho_g)\epsilon_t^h + \rho_g \epsilon_{t-1}^h + \eta_t^{g,h} \\
\end{align*} \]

C.1.4 Steady State

In this section I discuss the steady state moments of the model. Below is the list of steady state equations.
\[ x = h^{\kappa h} (1 - l)^{\kappa l} \]

\[ (1 - \delta \beta)(1 - \delta)c^{-\sigma_c} = \Lambda^1 \]

\[ \frac{\kappa_h x^{1 - \sigma_x}}{h} = p^h \Lambda^1 \]

\[ \frac{\kappa_l x^{1 - \sigma_x}}{1 - l} = w(1 - \tau^w) \Lambda^1 \]

\[ y = \frac{(y^c)^\omega (y^h)^{1 - \omega}}{\omega^\omega (1 - \omega)^{1 - \omega}} \]

\[ \pi = (\pi^c)^\omega (\pi^h)^{1 - \omega} \]

\[ y^c = e^p l^c \]

\[ y^h = e^p l^h \]

\[ w = \frac{\epsilon^{s,c} - 1}{\epsilon^{s,c}} \]

\[ \frac{w}{p^h e^p} = \frac{\epsilon^{s,h} - 1}{\epsilon^{s,h}} \]

\[ b \left( \frac{\beta - 1}{\pi^c} \right) = g + t - \tau^w w t \]

\[ \tau^w = \tau^w + \frac{\rho_w \gamma_w}{1 - \rho_w} \left( (\pi^c)^{1 - \chi^w} - 1 \right) \]

\[ t = \tilde{t} + \frac{\rho_T \gamma_T}{1 - \rho_T} \left( (\pi^c)^{1 - \chi^T} - 1 \right) \]

\[ l = l^c + l^h \]

\[ d^c = y^c - w l^c \]

\[ d^h = y^h - \frac{w}{p^h} l^h \]

\[ y^c = c + g^c \]

\[ y^h = h + g^h \]

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C.2 Indexation and the Labor Wedge

Here I analyze the long run impact of indexation in the model by varying the parameters $\chi^\tau$ and $\chi^T$. For this preliminary analysis I fix them to be equal, assuming that the quality of indexation is equivalent on the tax and transfer sides of fiscal policy. Below are graphs which depict consumption, health services, labor hours in both markets, and the impact on effective tax rates, revenues, and fiscal debt levels. As can be seen, indexation seems to have a dramatic impact in the long run. Starting with the first panel of Figure C.1, notice that both consumption and health services increase as the tax code and transfer payments are more perfectly indexed. The responsiveness of these variables to changes in $\chi^\tau$ and $\chi^T$ depends on the calibration of the various preference parameters. Adjusting $\kappa_h = 0.50$ results in a fall in health services demanded and an increase in the consumer goods consumed. Additionally, under this parameterization, the responsiveness of consumer goods increases
to match that of the health services. Likewise, increasing $\kappa_h$ has the opposite impact.

Now considering Figure C.2, notice that as the level of indexation increases, the steady state tax rate falls, eventually settling in to the desired, target tax rate. This shows the long run impact of bracket creep, showing that taxes will be higher in the long run if left exposed to the influences of inflation. Looking at it from a different angle, improving the indexation of the tax code reduces the labor wedge in the model. This is what allows higher levels of labor and, consequently, higher level of consumption in both sectors. From a fiscal revenue point of view, however, indexation puts downward pressure on revenue levels, resulting in higher required long-run surpluses to balance the budget. This was the argument behind the Economic Recovery Tax Act of 1981, which included the automatic indexation policy. Knowing the impact of bracket creep on tax revenues, the bill postponed implementing this automatic index until 1985, hoping to reduce the impact of major tax reductions. As can be seen in the second panel of Figure C.2, going from a non-indexed to a completely indexed
tax code can have a massive impact on sustainable fiscal debt levels.

C.3 Impulse Responses for Model-Wide Shocks

In this section, I provide the impulse responses for shocks that impact the model as a whole. The policy parameters in this model are set to match the estimates in Table 3.5. These are the productivity $\eta^p_t$, demand $\eta^\beta_t$, and monetary policy $\eta^r_t$ shocks.

C.3.1 Productivity Shock

As can be seen in the model, the total factor productivity $\epsilon^p_t$ used by the intermediate-level producers is identical. Thus, a shock to this impacts both sectors simultaneously. The results are shown in Figures (C.3)–(C.5). What is interesting here is that this supply-side shock actually manifests itself as a negative demand shock for the health services sector due to the rise in the real price. What this means is that the inflation in the consumer goods sector falls farther than that in the health services sector, pushing the relative price upward. Once again I find that the interest rate is extremely persistent, yet does not react as much to the changes in the aggregate inflation measure since inflation in the health services sector adjusts by less. Other than those slight differences, the impact of moving from one monetary regime to the other is minimal.

C.3.2 Demand Shock

As I showed with the productivity shock, the demand shock is an all-encompassing shock, causing the model to react in a similar fashion regardless of the monetary regime. The results are shown in Figures (C.6)–(C.8). With this shock, I do find more dissimilarities between the two monetary regimes than from the productivity shock, which is expected since consumption goods and health services are imperfect substitutes. Thus, even though this is a standard demand shock, inflation increases only minimally before it falls, and never
Figure C.3: Consumption-Sector Variables to Productivity Shock

Note: The black, solid lines represent the narrow monetary policy regime, while the blue, dashed lines represent the broad monetary policy regime. These values are calculated using the estimated monetary policy parameters for each respective period.

Figure C.4: Health Services-Sector Variables to Productivity Shock

Figure C.5: Aggregate Variables to Productivity Shock

Note: With a steady state of zero, the impulse response reported for fiscal debt is in levels, or deviations from steady state, equivalently. All other impulse responses are in percent deviations from steady state.
actually increasing in the health services sector, pushing the real price down. There are also higher levels of fiscal debt since interest rates are higher at every point and there is more downward pressure on labor hours in the consumption sector.
Figure C.6: Consumption-Sector Variables to Demand Shock

Figure C.7: Health Services-Sector Variables to Demand Shock

Note: The black, solid lines represent the narrow monetary policy regime, while the blue, dashed lines represent the broad monetary policy regime. These values are calculated using the estimated monetary policy parameters for each respective period.

Figure C.8: Aggregate Variables to Demand Shock

Note: With a steady state of zero, the impulse response reported for fiscal debt is in levels, or deviations from steady state, equivalently. All other impulse responses are in percent deviations from steady state.