Hydraulic tests in highly permeable aquifers

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[1] A semianalytical solution is presented for a mathematical model describing the flow of groundwater in response to a slug or pumping test in a highly permeable, confined aquifer. This solution, which is appropriate for wells of any degree of penetration and incorporates inertial mechanisms at both the test and observation wells, can be used to gain new insights into hydraulic tests in highly permeable settings. The oscillatory character of slug- and pumping-induced responses will vary considerably across a site, even in an essentially homogeneous formation, when wells of different radii, depths, and screen lengths are used. Thus variations in the oscillatory character of responses do not necessarily indicate variations in hydraulic conductivity (K). Existing models for slug tests in partially penetrating wells in high-K aquifers neglect the storage properties of the media. That assumption, however, appears reasonable for a wide range of common conditions. Unlike in less permeable formations, drawdown at an observation well in a high-K aquifer will be affected by head losses in the pumping well. Those losses, which affect the form of the pumping-induced oscillations, can be difficult to characterize. Thus analyses of observation-well drawdown should utilize data from the period after the oscillations have dissipated whenever possible. Although inertial mechanisms can have a large impact on early-time drawdown, that impact decreases rapidly with duration of pumping and distance to the observation well. Conventional methods that do not consider inertial mechanisms should therefore be viable options for the analysis of drawdown data at moderate to large times. *INDEX TERMS:* 1829 Hydrology: Groundwater hydrology; 1894 Hydrology: Instruments and techniques; 5114 Physical Properties of Rocks: Permeability and porosity; *KEYWORDS:* highly permeable aquifers, pumping tests, slug tests

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1. Introduction

[2] Head responses to a pressure perturbation at a well in a highly permeable aquifer can be affected by mechanisms that are of little significance in less permeable media. The inertia-induced oscillations observed during hydraulic tests in aquifers of high hydraulic conductivity (K) are the most common manifestation of such mechanisms. These oscillations last for the entire duration of a slug test (Figure 1) but are normally limited to the early portions of a pumping test or the subsequent recovery (Figure 2a). However, in extremely conductive media, pumping-induced oscillations may extend through the period during which virtually all of the head change occurs (Figure 2b). Although analytical solutions that incorporate such mechanisms have been proposed, these solutions are restricted to a subset of the conditions commonly encountered in the field. The purpose of this paper is to present a general solution that removes many of the limitations imposed by these earlier approaches, so that new insights into hydraulic tests in highly permeable aquifers can be obtained.

[3] *Bredehoeft et al.* [1966] were among the first to discuss the role of water-column inertia in hydraulic tests. They described the impact of inertia-induced oscillations on both pumping and slug tests and demonstrated that the magnitude of these oscillations is determined primarily by the transmissivity of the test interval and the length of the water column in the well. A number of models for slug tests in highly permeable systems have been developed on the basis of that work [e.g., *Van der Kamp*, 1976; *Krauss*, 1977; *Kipp*, 1985; *Kabala et al.*, 1985; *Springer and Gelhar*, 1991; *McElwee and Zenner*, 1998; *Zurbuchen et al.*, 2002]. These models differ in their representations of slug-induced flow in the well and adjacent portions of the aquifer. Theoretically rigorous models have apparently only been developed for slug tests in wells that are screened across the entire aquifer thickness (fully penetrating wells [e.g., *Kipp*, 1985]). Most slug tests, however, are carried out in wells that are screened across a limited portion of the aquifer (partially penetrating wells). Although approximate models are used for the analysis of slug tests performed in this configuration [e.g., *Springer and Gelhar*, 1991], the viability of these approximate representations has not been assessed.

[4] Relatively few models consider the impact of water-column inertia on pumping-induced drawdown or recovery data. *Shapiro* [1989] and *Shapiro and Oki* [2000] proposed models for drawdown in an observation well and pumping well, respectively, to exploit the information available from pumping-induced oscillations in water levels. In both cases, the wells are assumed to be screened across the entire

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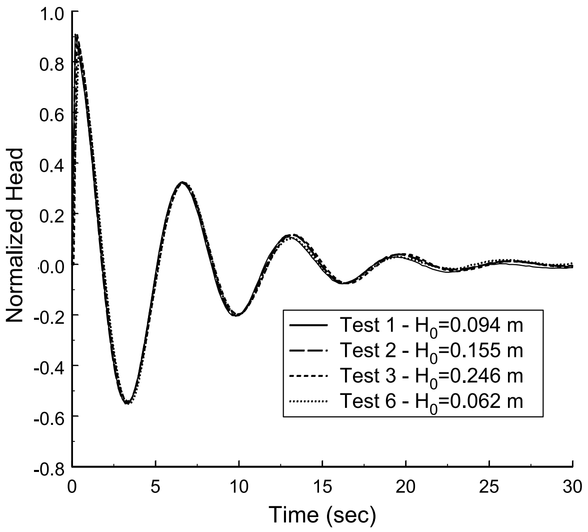

Figure 1. Normalized head ($H(t)/H_0$, where $H(t)$ is deviation from static and H_0 is magnitude of initial displacement) versus time plots for a series of slug tests performed on 5 April 2000 in a temporary well (DP43C) screened in the sand and gravel aquifer underlying the Geohydrologic Experimental and Monitoring Site (GEMS) near Lawrence, Kansas [after *Butler et al.*, 2002].

aquifer thickness. These authors only considered inertial effects related to the water column at a single well, the observation well [*Shapiro*, 1989] or the pumping well [*Shapiro and Oki*, 2000], as a result of the test configurations used in their studies. Apparently, no existing model considers inertial mechanisms operating in both the observation and pumping wells, a potentially significant limitation for many practical applications.

[5] The preceding literature review indicates that existing solutions for hydraulic tests in highly permeable aquifers do not consider a number of the conditions commonly faced in practice. The major objective of this paper therefore is to present a new general solution that incorporates these previously neglected conditions. The paper will begin with a presentation of the governing equations and auxiliary conditions and an overview of the integral-transform approach utilized to obtain the solution. The solution will then be used as the basis for a theoretical assessment of hydraulic tests in highly permeable formations. Insights of practical importance drawn from this assessment will be demonstrated with field examples. The paper will conclude with an overview of the major results and a discussion of the implications of most significance for hydraulic tests in highly permeable systems.

2. Problem Statement

[6] The problem of interest here is that of the head response produced by a slug or pumping test in a confined aquifer of infinite areal extent and constant thickness (Figure 3). Responses at both the test and observation wells are considered, and the wells may be screened/open across any portion of the aquifer. Flow properties are assumed uniform, but the vertical (K_z) and radial (K_r) components of hydraulic conductivity may differ. The inertia of the water column in the well is considered, but inertial effects in the aquifer are assumed negligible [*Bredehoeft et al.*, 1966].

[7] The model developed here borrows elements from previous work. Inertial mechanisms at the test well are incorporated following *Kipp* [1985], while inertial mechanisms at the observation well are incorporated using the approach of *Shapiro* [1989]. The partially penetrating well representations of *Dougherty and Babu* [1984] and *Hyder et al.* [1994] are used at both the test and observation wells, and frictional losses in the wellbore are incorporated following *Van der Kamp* [1976] and *Ross* [1985]. In all cases, the equations are written in a general form that is applicable for both slug and pumping tests. Although the emphasis here will be on issues of importance for highly permeable aquifers, the model is not restricted to those conditions.

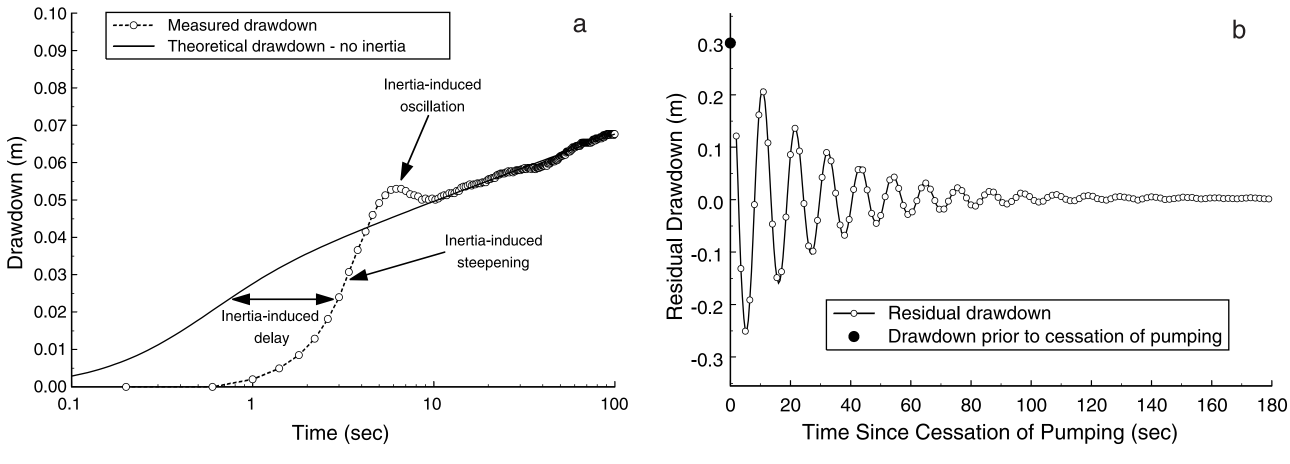

Figure 2. (a) Drawdown at well 7-1 versus logarithm of time plot for 13 August 1999 pumping test in well Gems4N (well 7-1 is 2.45 m from Gems4N; both wells screened in same aquifer as in Figure 1; theoretical drawdown generated using solution of *Moench* [1985] assuming no vertical leakage and no inertia; data from *Butler et al.* [2002]). (b) Residual drawdown at Thompson Corner Exploratory Well 1 (Oahu, Hawaii) versus time since the cessation of pumpage plot for 5–7 August 1993 pumping test (residual drawdown measured at the pumping well, every third data point plotted [*Presley and Oki*, 1996; *Shapiro and Oki*, 2000]).

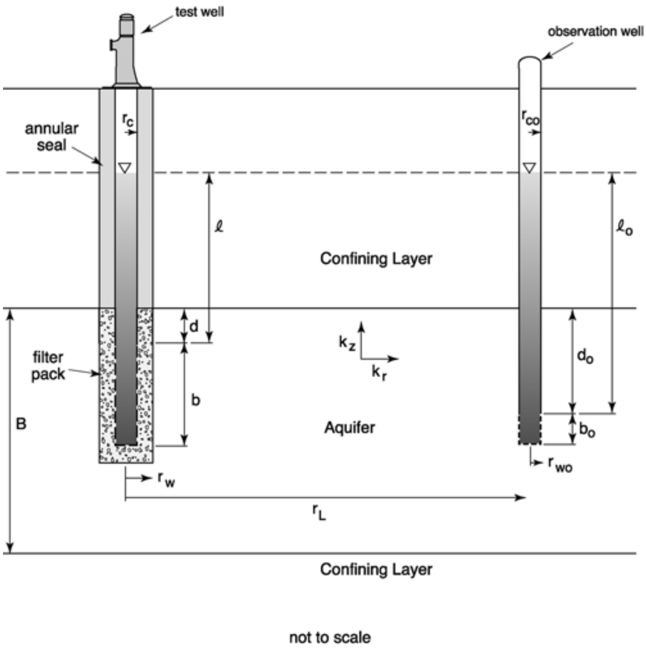

Figure 3. Cross-sectional view of two wells in a hypothetical confined aquifer (notation defined in text).

[8] For the test well and adjacent aquifer of Figure 3, governing equations and auxiliary conditions can be defined as follows:

Aquifer flow

$$\frac{\partial^2 h(r,z,t)}{\partial r^2} + \frac{1}{r}\frac{\partial h(r,z,t)}{\partial r} + \frac{K_z}{K_r}\frac{\partial^2 h(r,z,t)}{\partial z^2} = \frac{S_s}{K_r}\frac{\partial h(r,z,t)}{\partial t} \quad (1)$$

$$h(r,z,t=0) = 0 \quad (2)$$

$$h(r=r_w,z,t) = h_s(z,t),\ d < z < d+b \quad (3)$$

$$h(r=\infty,z,t) = 0 \quad (4)$$

$$\frac{\partial h(r,z=0,t)}{\partial z} = 0 \quad (5)$$

$$\frac{\partial h(r,z=B,t)}{\partial z} = 0 \quad (6)$$

Mass balance in test well

$$\left(\pi r_c^2 \frac{dH(t)}{dt} + Q\right)(H_v(z-d) - H_v(z-d-b)) = 2\pi r_w b K_r \frac{\partial h(r_w,z,t)}{\partial r} \quad (7)$$

Momentum balance in test well

$$\frac{d^2H(t)}{dt^2} + \frac{8\upsilon L}{r_c^2 L_e}\frac{dH(t)}{dt} + \frac{g}{L_e}H(t) = \frac{g}{L_e b}\int_d^{d+b} h_s(z,t)dz \quad (8)$$

Initial conditions in test well

$$H(t=0) = H_0 \quad (9)$$

$$\frac{dH(t=0)}{dt} = H_0' - \frac{Q}{\pi r_c^2} \quad (10)$$

where

- B aquifer thickness, [L];
- b screen length of test well, [L];
- d distance from top of aquifer to top of screen at test well, [L];
- g gravitational acceleration, [L/T^2];
- $H(t)$ deviation of water level in test well from static conditions, [L];
- H_0 initial deviation of water level from static conditions (= 0 for pumping test), [L];
- $H_0{}'$ initial velocity of water level as a result of slug-test initiation, [L/T];
- $H_v(z\text{-}d)$ Heaviside function (= 0 for z-d < 0, = 1 for z-d > 0);
- $h(r, z, t)$ deviation of hydraulic head in aquifer from static conditions, [L];
- $h_s(z, t)$ deviation of hydraulic head within screen of test well from static conditions, [L];
- L $l + (r_c^4/r_w^4)(b/2)$ [*Butler*, 2002], [L];
- L_e effective length of water column in well [*Kipp*, 1985; *Zurbuchen et al.*, 2002], [L];
- l length of water column above top of screen, [L];
- Q pumping rate (= 0 for slug test), [L^3/T];
- r radial distance from center of test well, [L];
- r_c casing radius for test well, [L];
- r_w screen radius for test well, [L];
- S_s specific storage of aquifer, [L^{-1}];
- t time since test initiation, [T];
- ν kinematic viscosity of water, [L^2/T];
- z vertical direction (z = 0 at aquifer top and increases downward).

Note that a constant rate of pumping is assumed for this development. A variable rate of pumping could be readily incorporated using standard convolution approaches [*Streltsova*, 1988].

[9] *Zhan and Butler* [2003] provide the details of the solution derivation. In summary, the approach uses a Laplace transform in time and a finite Fourier transform in the z direction to obtain transform-space analogues of equations (1), (3)–(8). A general solution for the aquifer-flow equation in transform space can be obtained in terms of modified Bessel functions and two constants. Evaluation of the constants and straightforward substitutions yield the following dimensionless Laplace-space functions for the water level in the test well, the head within the screen, and the head in the aquifer:

$$\overline{\Phi}(p) = \frac{\alpha^2\beta\Phi_0\, p^2 + \alpha F_l \Phi_0 p - \alpha\beta q\, p + (\alpha\Phi_0\, p - q)\Omega_w(p) + \alpha^2\beta\Phi_0'\, p}{p(1 + \alpha F_l p + \alpha^2\beta p^2 + \Omega_w(p)\alpha\, p)} \quad (11)$$

$$\overline{\phi_s}(\eta,p) = \frac{\left(\alpha\Phi_0\, p - \alpha^3\beta\Phi_0'\, p^2 - q - \alpha F_l q p\right)\Omega_a(\xi=1,\eta,p)}{p(1 + \alpha F_l p + \alpha^2\beta p^2 + \Omega_w(p)\alpha\, p)}, \quad \zeta < \eta < \zeta + 1 \quad (12)$$

$$\overline{\phi}(\xi,\eta,p) = \frac{(\alpha\Phi_0\, p - \alpha^3\beta\, \Phi_0'\, p^2 - q - \alpha F_l q\, p)\Omega_a(\xi,\eta,p)}{p(1 + \alpha F_l p + \alpha^2\beta p^2 + \Omega_w(p)\alpha\, p)},\ \xi \geq 1 \quad (13)$$

where

$\overline{\Phi}$, $\overline{\phi}_s$, $\overline{\phi}$ = Laplace transform of Φ, ϕ_s, and ϕ, respectively (e.g., $\overline{\phi} = \int_0^\infty \Phi e^{-p\tau} d\tau$);

$$\Phi = \frac{H(t)}{Q_0}; \phi_s = \frac{h_s(z,t)}{Q_0}; \phi = \frac{h(r,z,t)}{Q_0}; \Phi_0 = \frac{H_0}{Q_0}; \Phi_0' = \frac{H_0' r_w^2 S_s}{Q_0 K_r};$$

$$\xi = \frac{r}{r_w}; \eta = \frac{z}{b}; \mathrm{B} = \frac{B}{b}; \zeta = \frac{d}{b}; \tau = \frac{tK_r}{r_w^2 S_s};$$

$$F_l = \frac{16b\upsilon LK_r}{gr_c^4}; \psi = \sqrt{\frac{r_w^2 K_z}{b^2 K_r}};$$

$$\alpha = \frac{r_c^2}{2r_w^2 bS_s}; \beta = \frac{4L_e b^2 K_r^2}{gr_c^4};$$

$$q = \frac{Q}{2\pi K_r b Q_0} = 1 \text{ for pumping tests}, = 0 \text{ for slug tests};$$

$$\Omega_{\mathrm{w}}(p) = \int_\zeta^{\zeta+1} \Omega_{\mathrm{a}}(\xi = 1, \eta, p) d\eta;$$

$$\begin{aligned}
\Omega_a(\xi,\eta,p) &= F_c^{-1}\left(\frac{F_c(H_v(\eta-\zeta) - H_v(\eta-\zeta-1))K_0\left(\sqrt{\psi^2\omega^2+p}\,\xi\right)}{K_1\left(\sqrt{\psi^2\omega^2+p}\right)\sqrt{\psi^2\omega^2+p}}\right) \\
&= \frac{1}{\mathrm{B}}\frac{K_0\left(\sqrt{p}\xi\right)}{K_1\left(\sqrt{p}\right)\sqrt{p}} \\
&\quad + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi}{2\mathrm{B}}\right)\cos\left(\frac{n\pi}{2\mathrm{B}} + \frac{n\pi}{\mathrm{B}}\zeta\right)K_0\left(\sqrt{\psi^2\left(\frac{n\pi}{\mathrm{B}}\right)^2 + p}\,\xi\right)}{nK_1\left(\sqrt{\psi^2\left(\frac{n\pi}{\mathrm{B}}\right)^2 + p}\right)\sqrt{\psi^2\left(\frac{n\pi}{\mathrm{B}}\right)^2 + p}} \\
&\quad \cdot \cos\left(\frac{n\pi}{\mathrm{B}}\eta\right)
\end{aligned}$$

$$\begin{aligned}
F_c(H_v(\eta-\zeta) - H_v(\eta-\zeta-1)) &= \int_0^{\mathrm{B}} (H_v(\eta-\zeta) - H_v(\eta-\zeta-1))\cos\left(\frac{n\pi\eta}{\mathrm{B}}\right)d\eta \\
&= \int_\zeta^{\zeta+1}\cos\left(\frac{n\pi\eta}{\mathrm{B}}\right)d\eta = \frac{2\mathrm{B}}{n\pi}\sin\left(\frac{n\pi}{2\mathrm{B}}\right)\cos\left(\frac{n\pi}{2\mathrm{B}} + \frac{n\pi}{\mathrm{B}}\zeta\right) \\
&= \frac{2}{\omega}\sin\left(\frac{\omega}{2}\right)\cos\left(\frac{\omega}{2} + \omega\zeta\right), \omega = \frac{n\pi}{\mathrm{B}};
\end{aligned}$$

K_0, K_1 modified Bessel function of the second kind of order zero and one, respectively;

ω, p Fourier and Laplace transform variables, respectively;

F_c, F_c^{-1} finite Fourier cosine transform and its inverse, respectively;

Q_0 normalizing parameter = $Q/2\pi K_r b$ for pumping tests, = H_0 for slug tests.

The above notation is similar to that of *Kipp* [1985] except for the β term, which has been redefined to remove the dependence on specific storage.

[10] Equation (11) reduces to the solution of *Kipp* [1985] for a slug test in a fully penetrating well when B is set to one and q and F_l are set to zero, and to the solution of *Hyder et al.* [1994] for a slug test in a partially penetrating well when q, F_l, Φ_0', and β are set to zero. Similarly, equation (13) reduces to the solution of *Shapiro and Oki* [2000] for a pumping test in a fully penetrating well with negligible turbulent losses, when B is set to one, Φ_0 and F_l are set to zero, and leakage is neglected. In addition, substitution of equations (11)–(13) into the Laplace-space forms of equations (1), (3)–(8) will demonstrate that the solutions honor the governing equations and auxiliary conditions.

[11] The preceding development ignored inertial effects produced by the water column in the observation well. However, *Shapiro* [1989] demonstrated that inertial mechanisms at the observation well can be of practical importance. Thus a general model should incorporate inertial mechanisms at both the test and observation wells. Governing equations and auxiliary conditions can be defined for flow in the vicinity of the observation well using a coordinate system that is centered at the observation well (Figure 4) and the principle of superposition [*Tongpenyai and Raghavan*, 1981]:

Aquifer flow

$$\frac{\partial^2 h_o(r_o,z,t)}{\partial r_o^2} + \frac{1}{r_o}\frac{\partial h_o(r_o,z,t)}{\partial r_o} + \frac{K_z}{K_r}\frac{\partial^2 h_o(r_o,z,t)}{\partial z^2} = \frac{S_s}{K_r}\frac{\partial h_o(r_o,z,t)}{\partial t} \tag{14}$$

$$h_o(r_o,z,t=0) = 0 \tag{15}$$

$$h_o(r_{wo},z,t) = h_{so}(z,t),\ d_o < z < d_o + b_o \tag{16}$$

$$h_o(r_o = \infty, z, t) = 0 \tag{17}$$

$$\frac{\partial h_o(r_o, z=0, t)}{\partial z} = 0 \tag{18}$$

$$\frac{\partial h_o(r_o, z=B, t)}{\partial z} = 0 \tag{19}$$

Mass balance in observation well

$$\left(\pi r_{co}^2 \frac{dW_o(t)}{dt}\right)(H_v(z-d_o) - H_v(z-d_o-b_o)) = 2\pi r_{wo} b_o K_r \frac{\partial h_o(r_{wo},z,t)}{\partial r_o} \tag{20}$$

Momentum balance in observation well

$$\frac{d^2 W_o(t)}{dt^2} + \frac{8\upsilon L_o}{r_{co}^2 L_{eo}}\frac{dW_o(t)}{dt} + \frac{g}{L_{eo}}W_o(t) = \frac{g}{L_{eo} b_o}\int_{d_o}^{d_o+b_o}(h(r_L,z,t) + h_{so}(z,t))dz \tag{21}$$

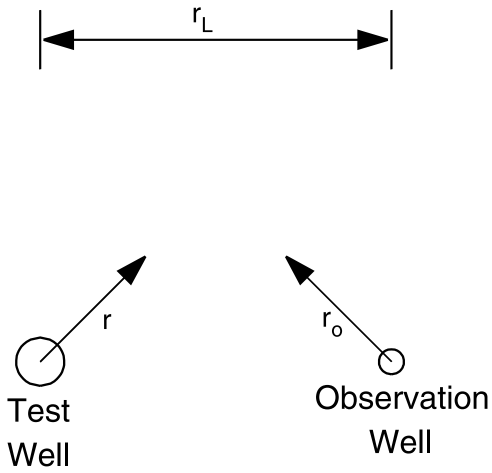

Figure 4. Areal view of configuration used in this work. Test and observation wells are the origins of the r and r_o coordinate systems, respectively. Figure not to scale.

Initial conditions in observation well

$$W_o(t=0) = 0 \tag{22}$$

$$\frac{dW_o(t=0)}{dt} = 0 \tag{23}$$

where

- b_o screen length of observation well, [L];
- d_o distance from top of aquifer to top of screen at observation well, [L];
- $h_o(r, z, t)$ deviation of aquifer head from static conditions due to a perturbation at observation well, [L];
- $h_{so}(z, t)$ deviation from static head within screen of observation well due to a perturbation at observation well, [L];
- L_o $l_o + \frac{r_{co}^4}{r_{wo}^4}\left(\frac{b_o}{2}\right)$ [*Butler*, 2002], [L];
- L_{eo} effective length of water column in observation well [*Kipp*, 1985; *Zurbuchen et al.*, 2002], [L];
- l_o length of water column in observation well above top of screen, [L];
- r_{co} casing radius for observation well, [L];
- r_L distance from test well to observation well, [L];
- r_o radial distance from center of observation well, [L];
- r_{wo} screen radius for observation well, [L];
- $W_o(t)$ deviation of water level in observation well from static conditions, [L].

Note that in equation (21) the head change in the aquifer produced by the stress at the test well is a component of the forcing function at the observation well. This head change is assumed to be constant along the circumference of the observation well. *Shapiro* [1989] points out that this assumption should be reasonable when r_{wo} is small relative to r_L, as in the vast majority of field applications. The perturbation at the observation well is also assumed to have no influence on the head at the test well, again a reasonable assumption when r_{wo} is small relative to r_L.

[12] As shown by *Zhan and Butler* [2003], the Laplace-space functions for the water level in the observation well, the head within the screen of the observation well, and the head in the aquifer can be written in dimensionless form as

$$\overline{\Phi_o}(p) = \frac{\Omega_{aL}(p)}{\alpha_o^2\beta_o R_w^2 p^2 + \alpha_o F_{lo} R_w p + 1 + \Omega_{ow}(p)\alpha_o R_w p} \tag{24}$$

$$\overline{\phi_{so}}(\eta_o, p) = -\frac{\alpha_o R_w p \Omega_{aL}(p)\Omega_o(1, \eta_o, p)}{\alpha_o^2\beta_o R_w^2 p^2 + \alpha_o F_{lo} R_w p + 1 + \Omega_{ow}(p)\alpha_o R_w p} \tag{25}$$

$$\overline{\phi_o}(\xi_o, \eta_o, p) = -\frac{\alpha_o R_w p \Omega_{aL}(p)\Omega_o(\xi_o, \eta_o, p)}{\alpha_o^2\beta_o R_w^2 p^2 + \alpha_o F_{lo} R_w p + 1 + \Omega_{ow}(p)\alpha_o R_w p} \tag{26}$$

where

$\overline{\phi_o}$, $\overline{\phi_{so}}$, $\overline{\phi_o}$ = Laplace transform of Φ_o, ϕ_{so}, and ϕ_o, respectively;

$$\Phi_o = \frac{W_o(t)}{Q_0}; \phi_{so} = \frac{h_{so}(z,t)}{Q_0}; \phi_o = \frac{h_o(r_o,z,t)}{Q_0}; \xi_o = \frac{r_o}{r_{wo}}; \mathrm{B}_o = \frac{B}{b_o};$$
$$\zeta_o = \frac{d_o}{b_o}; \eta_o = \frac{z}{b_o}$$

$$R_w = \frac{r_{wo}^2}{r_w^2}; F_{lo} = \frac{16 b_o \upsilon L_o K_r}{g r_{co}^4}; \alpha_o = \frac{r_{co}^2}{2 r_{wo}^2 b_o S_s}; \beta_o = \frac{4 L_{eo} b_o^2 K_r^2}{g r_{co}^4};$$
$$\psi_o = \sqrt{\frac{r_{wo}^2 K_z}{b_o^2 K_r}};$$

$$\Omega_o(\xi_o, \eta_o, p) =$$
$$F_c^{-1}\left(\frac{F_c(H_v(\eta_o - \zeta_o) - H_v(\eta_o - \zeta_o - 1))K_0\left(\sqrt{\psi_o^2\omega_o^2 + R_w p}\,\xi_o\right)}{K_1\left(\sqrt{\psi_o^2\omega_o^2 + R_w p}\right)\sqrt{\psi_o^2\omega_o^2 + R_w p}}\right.$$

$$= \frac{1}{\mathrm{B}_o}\frac{K_0(\sqrt{R_w p}\,\xi_o)}{K_1(\sqrt{R_w p})\sqrt{R_w p}}$$
$$+ \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left(\frac{n\pi}{2\mathrm{B}_o}\right)\cos\left(\frac{n\pi}{2\mathrm{B}_o} + \frac{n\pi}{\mathrm{B}_o}\zeta_o\right)K_0\left(\sqrt{\psi_o^2\left(\frac{n\pi}{\mathrm{B}_o}\right)^2 + R_w p}\,\xi_o\right)}{nK_1\left(\sqrt{\psi_o^2\left(\frac{n\pi}{\mathrm{B}_o}\right)^2 + R_w p}\right)\sqrt{\psi_o^2\left(\frac{n\pi}{\mathrm{B}_o}\right)^2 + R_w p}}$$
$$\cdot \cos\left(\frac{n\pi}{\mathrm{B}_o}\eta_o\right)$$

$$\Omega_{aL}(p) = \int_{\zeta_o}^{\zeta_o+1} \overline{\phi}(\xi = \xi_L, \eta_o\gamma_b, p)\, d\eta_o;$$

$$\Omega_{ow}(p) = \int_{\zeta_o}^{\zeta_o+1} \Omega_o(\xi_o = 1, \eta_o, p)\, d\eta_o;$$

$$\xi_L = \frac{r_L}{r_w}; \gamma_b = \frac{b_o}{b}.$$

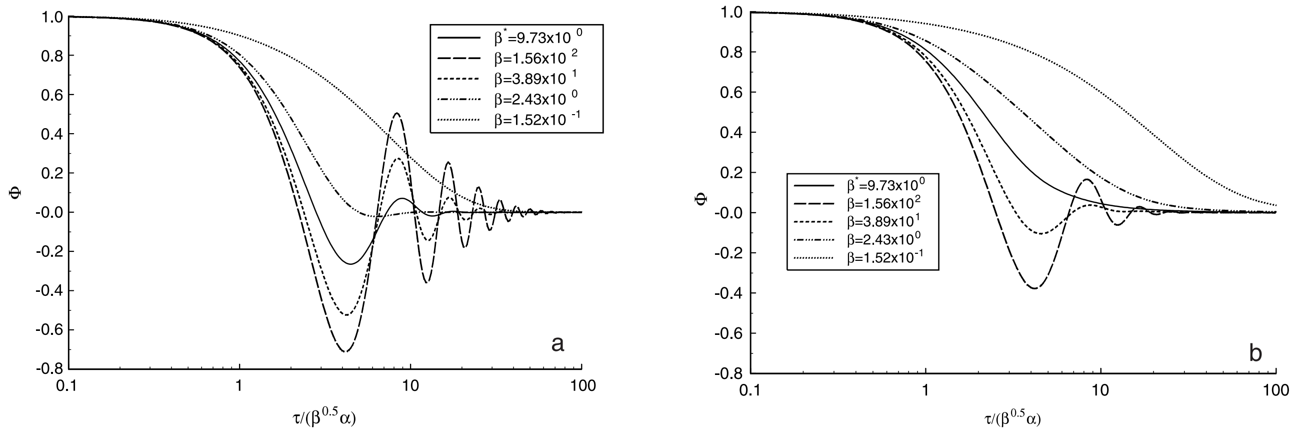

Figure 5. (a) Normalized head ($H(t)/H_0$) versus modified dimensionless time ($t\sqrt{g/L_e}$) plots as a function of $\beta(4L_e b^2 K_r^2/g r_c^4)$ for a partially penetrating test well (ψ = 0.077, B = 48.4, ζ = 36.8). (b) Normalized head versus modified dimensionless time plots as a function of β for a fully penetrating test well (asterisk designates base case of Table 1; $\alpha = 2.84 \times 10^4$, $F_l \approx 0.00$; fully penetrating well plots use same screen length as partially penetrating plots to facilitate comparisons between the two).

Equation (24) reduces to the solution of *Shapiro* [1989] for a pumping test in a fully penetrating well when Φ_0, Φ_0', F_l, F_{lo}, and β are set to zero. In addition, substitution of equations (24)–(26) into the transform-space analogues of equations (14), (16)–(21) will demonstrate that the proposed solutions honor the governing equations and auxiliary conditions.

[13] For expressions of the complexity of equations (11)–(13) and (24)–(26), the analytical back transformation from Laplace space is tedious and only readily performed under quite limited conditions. In the general case, the transformation is best performed numerically. The numerical inversion was performed here using the approach of *D'Amore et al.* [1999a, 1999b], which is based on a Fourier series representation. Further details concerning this approach are given by *Zhan and Butler* [2003].

[14] The expressions presented in equations (11)–(13) and (24)–(26) reduce to existing solutions and honor the original governing equations and auxiliary conditions, so they are assumed to be reasonable representations of head responses to a pressure perturbation in a high-K aquifer. Thus, for the remainder of this paper, these expressions will be used to assess the impact of a variety of factors on hydraulic tests in highly permeable systems.

3. Slug Tests

3.1. Theoretical Assessment

[15] The character of slug-induced responses in high-K aquifers is largely controlled by the inertial parameter, β. As shown in Figure 5a, variation in β over 3 orders of magnitude produces the range of responses observed in highly permeable systems. The dimensionless time plotted in Figures 5a and 5b is $\tau/(\beta^{0.5}\alpha)$, as is the convention for slug tests in high-K aquifers [e.g., *Kipp*, 1985], and the parameters (see Table 1) in this and the following figures are based on examples described by *Butler et al.* [2002]. Given the dependence on β illustrated in Figure 5a, the character of slug-induced responses might be expected to vary considerably across a site. If test wells are of different radii, depths, and screen lengths, a significant degree of variation could occur even in an essentially homogeneous system. The character of responses also varies as a function of the proportion of slug-induced vertical flow. Figure 5b depicts responses for the same conditions as Figure 5a except that vertical flow has been completely suppressed (fully penetrating well). The differences between these two figures clearly illustrate the importance of the vertical component of flow and are consistent with the differences seen in less permeable settings [e.g., *Hyder et al.*, 1994, Figure 2].

[16] Most slug tests are performed in wells that are screened across a limited portion of an aquifer. As shown by the differences between Figures 5a and 5b, vertical flow is an important consideration in that situation. Previously, in the absence of a rigorous solution, approximate methods were developed for the analysis of slug tests performed in partially penetrating wells. For example, *Springer and Gelhar* [1991] extended the method of *Bouwer and Rice* [1976] to highly permeable unconfined aquifers, while analogous approaches have been used to extend the *Hvorslev* [1951] method to high-K confined aquifers [*Butler*, 1998]. All of these approximate approaches, however, are based on the assumption that the contribution

Table 1. Base Parameter Set for Figures[a]

Parameter	Value
$K_r = K_z$	0.00194 m/s
S_s	1.0×10^{-4} m^{-1}
b	0.22 m
B	10.64 m
d	8.10 m
r_w	0.017 m
r_c	0.019 m
L	13.29 m
L_e	16.99 m
ν	1.20×10^{-6} m^2/s

[a]Notation defined in text.

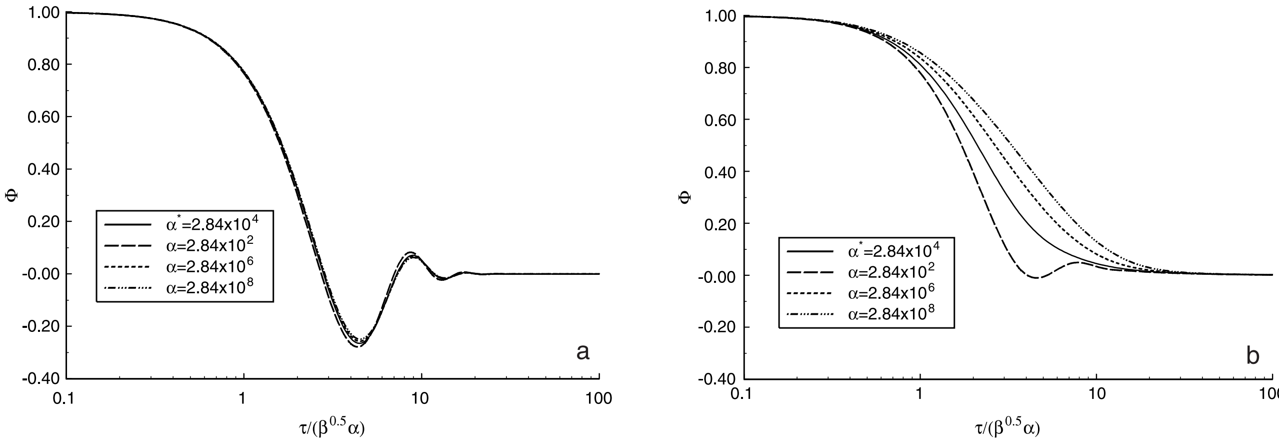

Figure 6. (a) Normalized head versus modified dimensionless time plots as a function of α ($r_c^2/2r_w^2bS_s$) for a partially penetrating test well (ψ = 0.077, B = 48.4, ζ = 36.8). (b) Normalized head versus modified dimensionless time plots as a function of α for a fully penetrating test well (asterisk designates base case of Table 1; β = 9.73, $F_l \approx 0.00$; fully penetrating well plots use same screen length as partially penetrating plots to facilitate comparisons between the two).

of specific storage can be ignored (quasi steady state assumption). The viability of that assumption can be readily assessed by varying the dimensionless storage parameter α in equation (11), while other parameters are held constant. As shown in Figure 6a, variations in α have a negligible impact on slug-induced responses when ψ is greater than 0.08 (screen length <13 r_w in an isotropic aquifer). Additional work has shown that variations in α have a very small impact when ψ is greater than 0.008 (screen length <130 r_w in an isotropic aquifer) for the α range expected in highly permeable aquifers. These results indicate that the quasi steady state assumption is reasonable and that the high-K extensions of the Bouwer-Rice and Hvorslev methods are appropriate approaches for the analysis of slug tests in partially penetrating wells. Note that these results were obtained for a well screened at a relatively large distance from the upper or lower boundary of the aquifer. Additional work has shown that as long as the screen length is small relative to the aquifer thickness (B/b $\gg$ 1), boundary effects will not have a significant impact except when the screen is immediately adjacent to a boundary, a finding that is consistent with the results reported for less permeable systems [*Hyder et al.*, 1994, Figure 5]. However, when the length of the well screen and the aquifer thickness are similar (B/b $\approx$ 1), the upper and lower boundaries of the aquifer will restrict the slug-induced flow to the horizontal plane. In that case, variations in α will have a large impact on test responses (Figure 6b) and quasi steady state approaches may no longer be appropriate.

[17] Frictional losses within the well casing are commonly assumed negligible during a slug test. This assumption, however, may not be appropriate for all tests in highly permeable systems. A number of authors [e.g., *Butler et al.*, 1996; *Butler*, 1998; *McElwee and Zenner*, 1998; *Zurbuchen et al.*, 2002] have described tests in which nonlinear (turbulent) frictional losses in the well affect slug-induced responses. *McElwee and Zenner* [1998] developed an approximate model that incorporated these nonlinear losses, while *Butler* [1998] and *Zurbuchen et al.* [2002] recommended that slug tests be initiated with small initial displacements (H_0) to minimize their impact. Recently, *Butler* [2002] demonstrated that linear (laminar) frictional losses within the casing can be significant in wells of small diameter. This and additional work has shown that the impact of linear well losses cannot be diminished by initiating tests with small H_0, as in the case of nonlinear losses. Thus the model developed here includes linear frictional losses in the casing. The effect of these losses can be assessed by varying the linear loss term F_l over a range of conditions that would be expected in high-K aquifers. As shown in Figure 7, the linear loss term has to significantly exceed 0.3 to have a sizable impact on the K estimate. For the conditions of Figure 7, a F_l value larger than 0.3 translates into a casing radius of less than 0.019 m. *Butler* [2002] has shown that errors introduced by neglect of linear well losses can lead to an underestimation in K that exceeds a factor of 2 for the parameters of Table 1 when small-diameter (r_c = 0.0074 m) wells are used. However, at

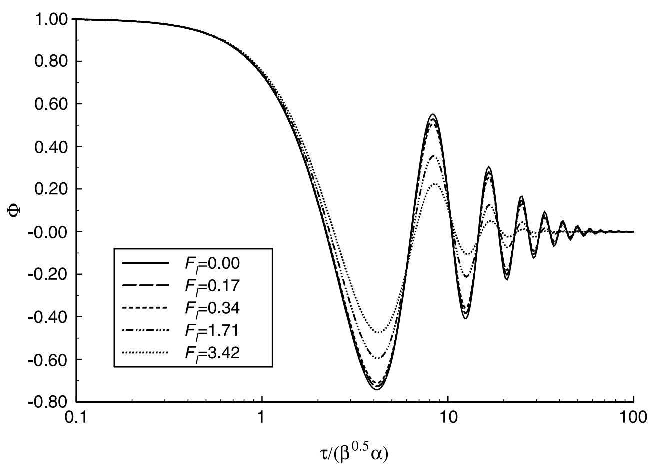

Figure 7. Normalized head versus modified dimensionless time plots as a function of F_l ($16b\upsilon\, LK_r/gr_c^4$) for the largest β value of Figure 5a ($\beta = 1.56 \times 10^2$, $\alpha = 2.84 \times 10^4$, ψ = 0.077, B = 48.4, ζ = 36.8).

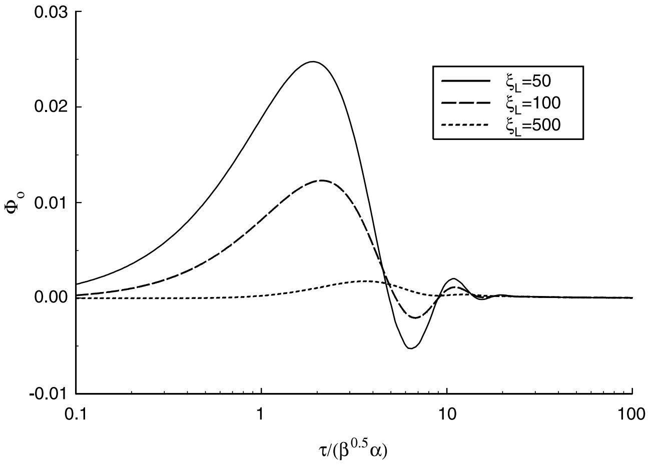

Figure 8. Normalized head at the observation well ($W_o(t)/H_0$) versus modified dimensionless time plots as a function of ξ_L (r_L/r_w) for the parameters of Table 1 (β = 9.73, $\alpha = 2.84 \times 10^4$, ψ = 0.077, B = 48.4, ζ = 36.8, $F_l \approx 0.00$; observation well screened over the same interval as test well, effects of well-bore storage and water-column inertia are assumed negligible for the observation well).

much smaller β values, the linear loss term has no effect on slug-induced responses.

[18] Slug tests are normally performed using the test well as both the site of the perturbation and the site at which measurements are taken. In certain applications, however, there are advantages to performing slug tests with observation wells [*Karasaki et al.*, 1988; *Butler*, 1998]. Figure 8 demonstrates that the oscillatory character of head responses is maintained at observation wells, but the signal is damped quickly with distance. This damping with distance would encourage use of a large H_0 to improve the signal-to-noise ratio, which could produce nonlinear losses in the test well [e.g., *McElwee and Zenner*, 1998] and greatly complicate the analysis.

3.2. Field Example

[19] The use of equation (11) for the analysis of slug-test data can be demonstrated with an example described by *Butler* [2002]. Figure 1 is a plot of normalized response data versus time for a series of slug tests performed in a small-diameter well in a confined sand and gravel aquifer. The coincidence of the normalized plots indicates that nonlinear head losses can be ignored, consistent with the model developed here. Figure 9 is a plot of the normalized head data from test 3 and the responses simulated using equation (11) for a range of values for specific storage. The plot also includes the responses predicted by the approximate high-K extension of the Hvorslev method [*Butler et al.*, 2003]. As shown earlier, slug-induced responses in partially penetrating wells are relatively insensitive to the value of the storage parameter, so a quasi steady state approach would be expected to produce a reasonable K estimate. That expectation was realized in this example as the quasi steady state K estimate is, for all practical purposes, indistinguishable from that obtained using equation (11). Note that equation (11) represents the movement of the water surface in the well, while field data are collected using a pressure transducer within the water column. As shown by the difference between equations (11) and (12), the traditional hydrostatic relationship between water level and transducer-measured head is not appropriate in wells in high-K systems [*McElwee*, 2001; *Zurbuchen et al.*, 2002]. Thus use of equation (11) for analysis of transducer-measured head can introduce error into K estimates unless the simulated water levels are adjusted for the depth of the transducer below static and for the acceleration of the water column [*Zurbuchen et al.*, 2002]. However, as shown by *Butler et al.* [2003], this error is negligible when the maximum normalized head is greater than 0.90, as in the case presented here. *Butler et al.* [2003] recommend keeping the pressure transducer within 0.5 m of the static water level to minimize the need for this adjustment. If this is not possible, the method of *Zurbuchen et al.* [2002] can be used with equation (11) to correct for transducer position, or, if the transducer is situated within the screen, equation (12) can be utilized without any corrections to analyze the test data.

4. Pumping Tests

4.1. Theoretical Assessment

[20] The character of pumping-induced responses at early times in highly permeable aquifers is primarily a function of the inertial parameter. Figures 10a and 10b display the range of responses as a function of β for drawdown in a partially and fully penetrating pumping well, respectively. As β decreases, drawdown converges on that predicted by existing analytical solutions that neglect inertial mechanisms. For example, drawdown at the fully penetrating pumping well in Figure 10b converges on that given by the model of *Papadopulos and Cooper* [1967] for drawdown in a finite-radius pumping well. In all cases, the responses converge with time on the drawdown that would be predicted when inertial effects are neglected. However, the time of that convergence may be relatively large at the upper end of the β range.

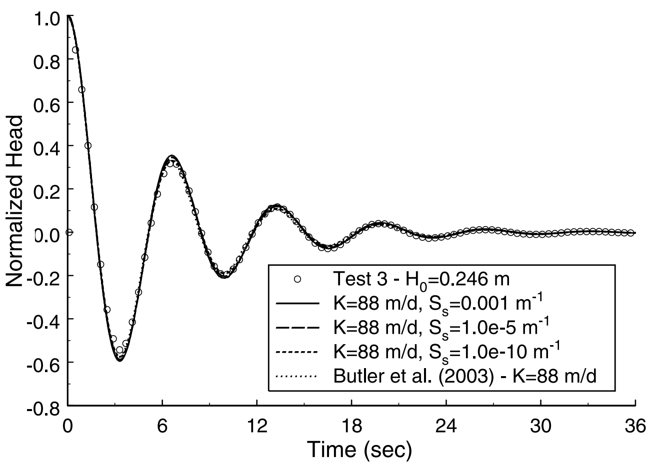

Figure 9. Normalized head versus time plot for test 3 of Figure 1 (every second data point plotted) with theoretical response curves generated using equation (11) and the high-K extension of the Hvorslev model [*Butler et al.*, 2003].

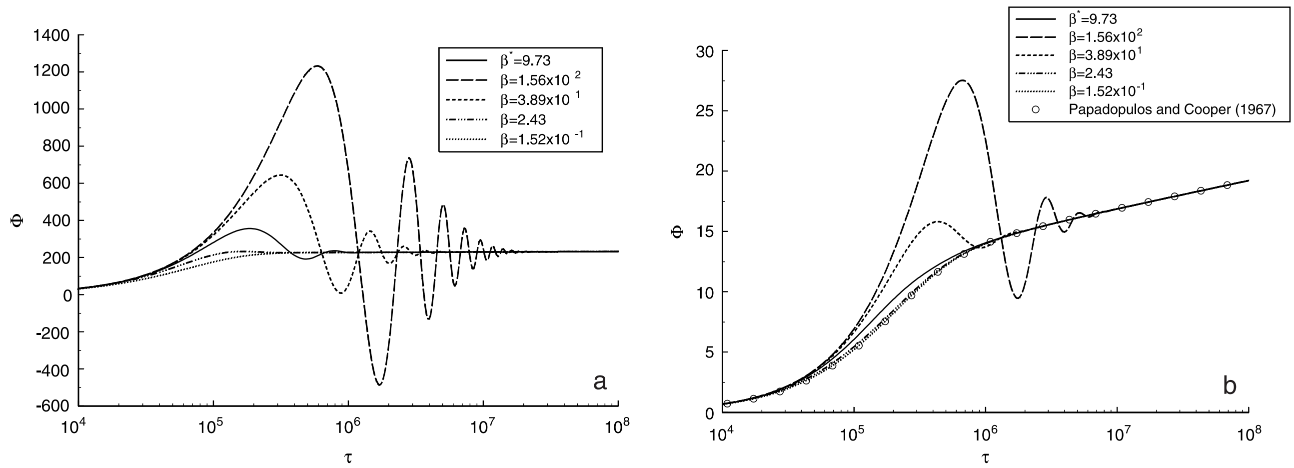

Figure 10. (a) Dimensionless drawdown ($2\pi K_r bH(t)/Q$) versus dimensionless time ($tK_r/r_w^2 S_s$) plots as a function of β for a partially penetrating test well (ψ = 0.077, B = 48.4, ζ = 36.8). (b) Dimensionless drawdown versus dimensionless time plots as a function of β for a fully penetrating test well (asterisk designates base case of Table 1; $\alpha = 2.84 \times 10^4$, $F_l \approx 0.00$; fully penetrating well plots use same screen length as partially penetrating plots to facilitate comparisons between the two).

[21] Pumping tests are commonly performed with observation wells, so the impact of inertial mechanisms on observation-well drawdown is an issue of practical significance. Figure 11 displays the dependence of observation-well drawdown on distance from a fully penetrating pumping well for the largest β value from Figure 10b. Clearly, as the distance from the pumping well increases, the impact of inertial mechanisms diminishes. Regardless of the β value, the effect of inertial mechanisms diminishes with time and distance, and observation-well drawdown converges on that predicted by models that neglect inertia [e.g., *Theis*, 1935; *Hantush*, 1964; *Papadopulos and Cooper*, 1967]. The parallel semilog straight lines displayed on Figure 11 demonstrate that even in the case of very pronounced oscillations in early-time drawdown, the *Cooper and Jacob* [1946] semilog method will produce reasonable parameter estimates at times appropriate for practical applications. Thus the Thiem equation [*Kruseman and de Ridder*, 1990] and methods based on steady-shape approaches [e.g., *Bohling et al.*, 2002] will also be applicable in high-K systems.

[22] The preceding figures were generated assuming that inertial mechanisms are acting only at the test well. Although this condition can occur when the observation interval is isolated with packers, the more general case is that of inertial mechanisms operating at both wells. Figure 12 displays observation-well drawdown as a function of the

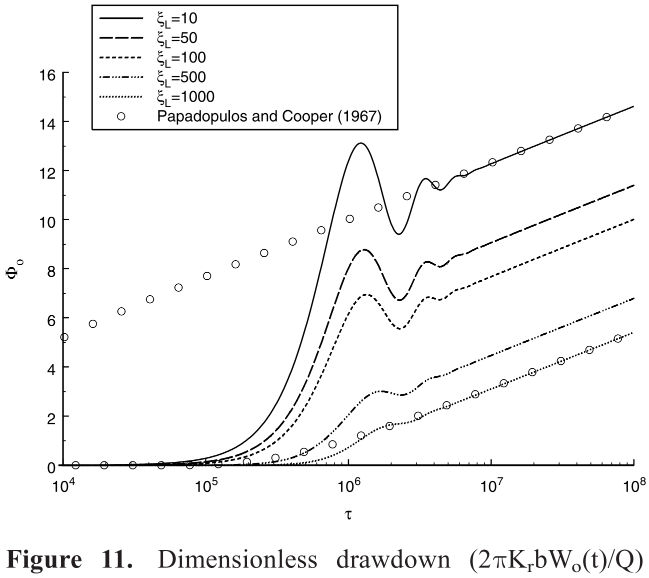

Figure 11. Dimensionless drawdown ($2\pi K_r bW_o(t)/Q$) versus time plots as a function of distance to the observation well ξ_L (r_L/r_w) for the largest β value of Figure 10b ($\beta = 1.56 \times 10^2$, $\alpha = 2.84 \times 10^4$, $F_l \approx 0.00$; observation well screened over the same interval as test well; effects of well-bore storage and water-column inertia are assumed negligible for the observation well).

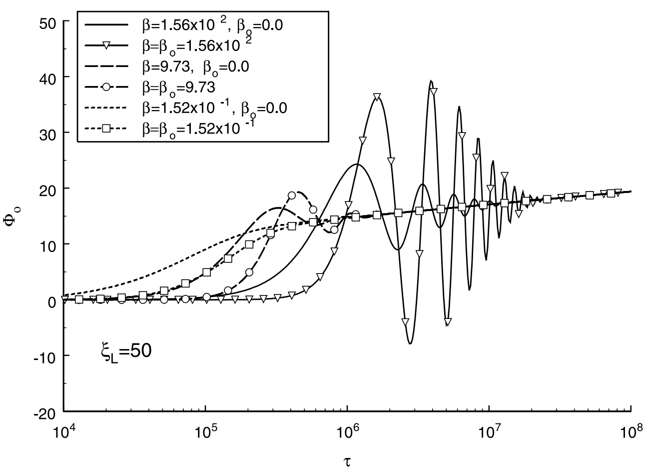

Figure 12. Dimensionless drawdown versus time plots as a function of β and β_o ($4L_{eo}b_o^2K_r^2/gr_{co}^4$) for an observation well at a distance of $\xi_L = 50$ from a partially penetrating test well (test well parameters same as in Figure 10a (ψ = 0.077, B = 48.4, ζ = 36.8, $\alpha = 2.84 \times 10^4$, $F_l \approx 0.00$)); observation well has the same construction parameters as the test well).

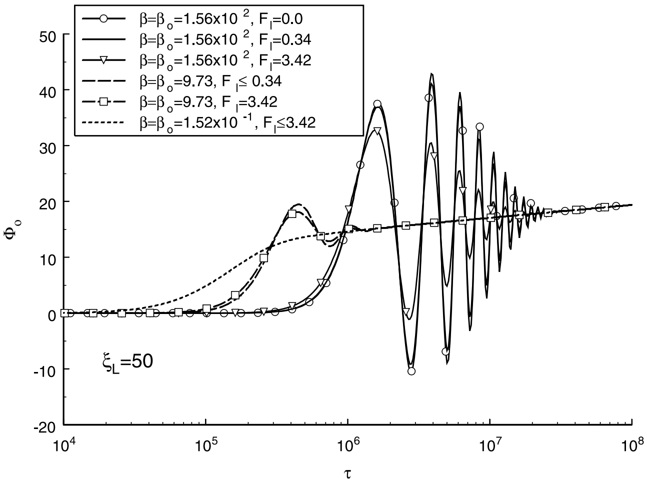

Figure 13. Dimensionless drawdown versus time plots as a function of inertial (β and β_o) and friction (F_l) parameters for an observation well at a distance of ξ_L = 50 from a partially penetrating test well (test well parameters same as in Figure 10a (ψ = 0.077, B = 48.4, ζ = 36.8, α = 2.84 × 10^4); observation well has the same construction parameters as the test well except that friction is ignored (F_{lo} = 0)).

inertial parameters at the test (β) and observation (β_o) wells. Clearly, inertial mechanisms at the observation well can be an important factor at small to moderate dimensionless times in highly permeable aquifers. Failure to include the inertial mechanisms operating at the observation well (assumption of β_o = 0) can therefore lead to sizable errors in parameter estimates in high-K aquifers. The significance of inertial mechanisms at the observation well is a function of the distance from the test well and the time interval of interest, similar to the relationship for inertial mechanisms at the pumping well shown in Figure 11.

[23] Figure 12 displays the effect of inertial mechanisms in the absence of frictional losses at either well. As discussed earlier, linear frictional losses can have a sizable impact on head data at the test well. Figure 13 shows that these losses can also impact drawdown at observation wells. As with inertial mechanisms, the effect of frictional losses in the test well depends upon the distance to the observation well and the time interval of interest. Although only linear losses were considered here, observation-well drawdown will be affected by all types of pumping-well losses because of the impact of those losses on the form of the pumping-induced oscillations. This is in marked contrast to the situation in less permeable formations. In that case, head losses in the pumping well will have no effect on drawdown at an observation well [e.g., *Butler*, 1988]. The dependence of observation-well drawdown on head losses in the pumping well greatly complicates the analysis because of the difficulty of accurately representing those losses. Thus analyses of observation-well drawdown should utilize data from the period after inertial mechanisms have dissipated whenever possible.

4.2. Field Example

[24] The use of equation (24) for the analysis of pumping-test data can be demonstrated with an example described by *Butler et al.* [2002]. Figure 2a is a plot of observation-well drawdown versus the logarithm of time for a pumping test in the same confined sand and gravel aquifer as in the slug-test example. As shown in the figure, inertial mechanisms have a significant impact on the early-time drawdown data but can be neglected at larger times, consistent with the discussion of the preceding section. Figure 14 is a plot of that early-time drawdown data with theoretical responses generated using equation (24) and the model of *Moench* [1985]. The solid line depicts the responses generated using equation (24) with the nominal well parameters, assuming the well-formation configuration of Figure 3 and the K and S_s values determined from previous pumping tests. The pumping test of this example was performed in a packer-isolated interval of a longer screened well with the pump located a short distance above the interval, a configuration that undoubtedly results in a β value that is much less than that obtained with the nominal well parameters. The dotted line depicts the responses generated using equation (24) assuming no inertial effects at the pumping well (β = 0), demonstrating that a lower β value is more appropriate for this example. No further attempt was made to modify the parameters to fit the test data because of the uncertainty regarding nonlinear well losses in the test well and how to incorporate those losses into the analysis. Although the agreement between the test data and the theoretical responses generated using equation (24) is reasonably good, this example demonstrates the difficulty of analyzing oscillatory responses from observation wells because of the dependence of observation-well drawdown on well losses in the test well. Note that the drawdown data for this example were obtained using a pressure transducer located 1.5 m below the static water level; thus the dynamic-pressure mechanisms discussed by *Zurbuchen et al.* [2002] could be responsible for a portion of the difference between the theoretical responses and the test data. In this case, however, the correction of *Zurbuchen et al.* [2002] was not of practical importance. Although the impact of inertial mechanisms lessens with distance from the pumping

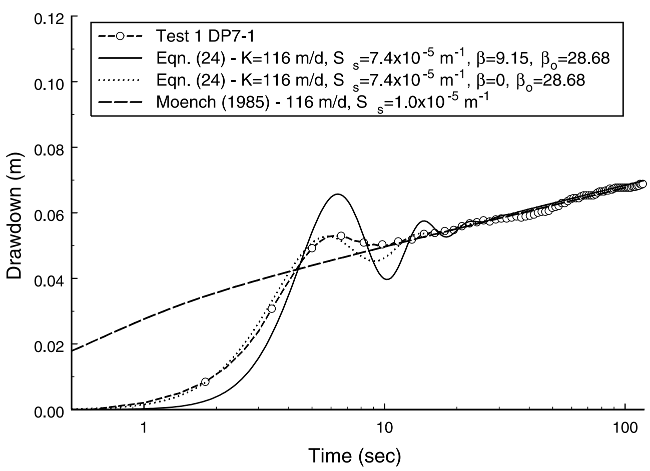

Figure 14. Drawdown versus logarithm of time plot for pumping test of Figure 2a with theoretical response curves generated using equation (24) and the *Moench* [1985] solution (parameters for Moench solution determined from fit to test data between 10 and 100 s; S_s estimates differ due to small differences in fits between 10 and 100 s).

well, *Butler et al.* [1999] observed inertial effects at observation wells more than 10 m from a fully penetrating pumping well in this same aquifer.

5. Summary and Conclusions

[25] Hydraulic tests in highly permeable aquifers are affected by mechanisms not considered in the conventional models of the well hydraulics literature. A new semianalytical solution has been developed for the analysis of hydraulic tests performed in highly permeable systems. This solution, which builds upon the earlier work of *Kipp* [1985] and *Shapiro* [1989], is appropriate for wells of any degree of penetration and includes inertial effects at both the test and observations wells. The solution has been used here to derive new insights into hydraulic tests performed in formations of high hydraulic conductivity.

[26] The character of slug- and pumping-induced responses can vary considerably across a site, even in an essentially homogeneous formation, when wells of different radii, depths, and screen lengths are used. Thus variations in the oscillatory character of responses do not necessarily indicate spatial variations in the hydraulic properties of the formation.

[27] Slug tests in partially penetrating wells in highly permeable aquifers are often analyzed using approximate models based on a quasi steady state representation of the slug-induced flow. The quasi steady state representation should be reasonable for most partially penetrating wells in high-K formations (screen length < 130 r_w in an isotropic aquifer). However, that representation is questionable in wells that are screened across the entire thickness of the aquifer.

[28] Inertial mechanisms can have a dramatic impact on early-time drawdown during a pumping test, but in most cases, that impact decreases rapidly with time. Thus conventional methods that do not consider these mechanisms [e.g., *Cooper and Jacob*, 1946] should be viable options for the analysis of moderate- to large-time drawdown. The impact of inertial mechanisms also diminishes in space, so conventional methods will often be appropriate for observation wells at a considerable distance from the pumping well, regardless of the time at which the drawdown measurements were obtained. Methods based on the attainment of an approximately constant hydraulic gradient (steady-shape conditions), such as the Thiem analysis [*Kruseman and de Ridder*, 1990], will also be appropriate once the impact of inertial mechanisms has dissipated.

[29] Previous work has been limited to consideration of inertial mechanisms operating at a single well, largely as a result of the test configurations that motivated that work. In many cases, however, inertial mechanisms at both the test and observation wells must be considered in the analysis of hydraulic tests in highly permeable aquifers. Consideration of inertial effects only at a single well can introduce sizable error into K estimates.

[30] Linear frictional losses within the well casing are commonly assumed negligible during slug and pumping tests. That assumption, however, may not be appropriate for all tests in highly permeable aquifers. Linear well losses should be considered when tests are performed in small diameter wells, as those losses can significantly influence the character of the response data. Previous work has shown that neglect of those losses can result in errors in K estimates obtained from slug tests that exceed a factor of 2 [*Butler*, 2002].

[31] A frequently cited advantage of observation wells is that head changes at those wells are not dependent on well losses in the test well. However, in high-K settings, drawdown at observation wells will be affected by head losses in the test well. Given the difficulty of accurately representing well losses in the test well, analyses should use observation-well drawdown from the period after inertial mechanisms have dissipated whenever possible.

[32] The solution developed here does not consider all possible conditions that may arise in hydraulic tests in highly permeable aquifers. In particular, the nonlinear losses discussed by *McElwee and Zenner* [1998] and the dynamic-pressure correction of *Zurbuchen et al.* [2002] have not been incorporated. For slug tests, the impact of these additional complexities can be made negligible for most well-formation configurations by utilizing the field practices recommended by *Butler* [1998], *Zurbuchen et al.* [2002], and *Butler et al.* [2003]. For pumping tests, however, the nonlinear losses are not as easily neglected because of the dependence of pumping-induced oscillations on those losses. Thus, as discussed above, drawdown data from the period after the pumping-induced oscillations have dissipated should be used in analyses if possible.

[33] Finally, the results of this work must be considered in light of the major assumptions used in the definition of the mathematical model. Two assumptions in particular are worthy of additional comments. First, in equations (7) and (20), we adopted the commonly used assumption of a uniform radial hydraulic gradient along the screened interval as a mathematical convenience. Previous work [e.g., *Cassiani and Kabala*, 1998] has shown that the use of this mathematical convenience introduces a negligible degree of error to virtually all practical applications, so it should not have any impact on the results presented here. Second, we assumed that the observation well was sufficiently far from the test well so that the head could be assumed constant along the circumference of the observation well and the impact of the perturbation at the observation well on the head at the test well could be ignored. Although this simplified representation is appropriate when the distance between wells is large relative to their radii, further work is needed to assess conditions when the separation distance is small.

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