# Circuit Elements at Optical Frequencies: Nanoinductors, Nanocapacitors, and Nanoresistors

Nader Engheta,[1,*] Alessandro Salandrino,[1,2] and Andrea Alù[1,2]

[1]*Department of Electrical and Systems Engineering, University of Pennsylvania, Philadelphia, Pennsylvania, USA*

[2]*Department of Applied Electronics, University of Roma Tre, Rome, Italy*

(Received 17 November 2004; published 23 August 2005)

We present the concept of circuit nanoelements in the optical domain using plasmonic and nonplasmonic nanoparticles. Three basic circuit elements, i.e., nanoinductors, nanocapacitors, and nanoresistors, are discussed in terms of small nanostructures with different material properties. Coupled nanoscale circuits and parallel and series combinations are also envisioned, which may provide road maps for the synthesis of more complex circuits in the IR and visible bands. Ideas for the optical implementation of right-handed and left-handed nanotransmission lines are also forecasted.

DOI: 10.1103/PhysRevLett.95.095504 PACS numbers: 61.46.+w, 07.50.Ek, 78.20.−e, 78.67.Bf

Optical wave interaction with metallic and nonmetallic nanoparticles is currently one of the important problems in nanotechnology and nanophotonics. It is well known that in certain noble metals such as Ag, Au, the plasma frequency is in the visible or ultraviolet (UV) regimes, and thus these metals behave as plasmonic materials in the optical frequencies, i.e., their permittivity has a negative real part [1]. As a result, the interaction of optical signals with plasmonic nanoparticles involves surface plasmon resonances [1,2]. Since these particles may be much smaller than the optical wavelength, the following question may naturally arise: ''can such metallic (and nonmetallic) nanoparticles be treated as circuit ''lumped nanoelements,'' such as nanoinductors, nanocapacitors, and nanoresistors in the optical regimes?'' The conventional circuits in the lower frequency domains (such as in the rf and lower frequency range) indeed involve elements that are much smaller than the wavelength of operation, and the circuit theory may be regarded as the ''approximation'' to the Maxwell equations in the limit of such small sizes. Similarly, in the following we explore quantitatively how these circuit concepts and elements may be extended to the optical frequencies when dealing with nanoparticles. It should be pointed out that a mere scaling of the circuit component concepts conventionally used at lower frequencies may not work at frequencies beyond the far infrared, since conducting materials (e.g., metals) behave differently at these higher frequencies. Here, however, we present a concept to synthesize circuit elements, and more complex circuits, in the optical regime. Such optical nanoelements may find future applications in areas such as biological circuits, nano-optics, optical information storage, biophotonics, and molecular signaling.

To begin, we consider a nanosphere of radius $R$ made of a homogeneous material with dielectric function $\varepsilon(\omega)$, which is in general a complex quantity. The sphere is assumed to be much smaller than the wavelength of operation in vacuum and in the material, i.e., $R \ll \lambda_0$ and $R \ll \lambda_0/\sqrt{|\mathrm{Re}\,(\varepsilon)|/\varepsilon_0}$. Consider an incident electromagnetic wave $\mathbf{E}_0$ illuminating this sphere under a monochromatic excitation $e^{-i\omega t}$. Because of the small size of the particle (with respect to wavelength), the scattered electromagnetic (EM) fields in the vicinity of the sphere and the total fields inside it may be obtained with very good approximation using the well-known time-harmonic, quasistatic approach. This leads to the following approximate expressions for the fields inside and outside the sphere (e.g., [3]):

$$\mathbf{E}_{\mathrm{int}} = 3\varepsilon_0\mathbf{E}_0/(\varepsilon + 2\varepsilon_0), \tag{1}$$

$$\mathbf{E}_{\mathrm{ext}} = \mathbf{E}_0 + \mathbf{E}_{\mathrm{dip}} = \mathbf{E}_0 + [3\mathbf{u}(\mathbf{p}\cdot\mathbf{u}) - \mathbf{p}]/(4\pi\varepsilon_0 r^3), \tag{2}$$

with $\mathbf{p} = 4\pi\varepsilon_0 R^3(\varepsilon - \varepsilon_0)\mathbf{E}_0/(\varepsilon + 2\varepsilon_0)$, $\mathbf{u} = \mathbf{r}/r$, $\varepsilon_0$ being the permittivity of the outside region, $\mathbf{r}$ being the position vector from the sphere's center to the observation point, and $r = |\mathbf{r}|$. At every point on the surface of the sphere the normal component of the displacement current $-i\omega D_n$ is continuous, implying that:

$$-i\omega(\varepsilon - \varepsilon_0)\mathbf{E}_0\cdot\hat{\mathbf{n}} = -i\omega\varepsilon_0\mathbf{E}_{\mathrm{dip}}\cdot\hat{\mathbf{n}} + i\omega\varepsilon\mathbf{E}_{\mathrm{res}}\cdot\hat{\mathbf{n}}, \tag{3}$$

where $\hat{\mathbf{n}}$ is the local outward unit vector normal to the surface of the sphere. In this equation, $\mathbf{E}_{\mathrm{res}} \equiv \mathbf{E}_{\mathrm{int}} - \mathbf{E}_0$ represents the residual field internal to the nanosphere when the incident field is subtracted from the total internal field.

If $\mathbf{E}_0$ is oriented as shown by the black arrows in Fig. 1, when Eq. (3) is integrated over the upper hemispherical surface we get the ''total'' displacement current for each relevant term in Eq. (3):

$$\underbrace{-i\omega(\varepsilon - \varepsilon_0)\pi R^2|\mathbf{E}_0|}_{I_{\mathrm{imp}}} = \underbrace{-i\omega\varepsilon\pi R^2(\varepsilon - \varepsilon_0)|\mathbf{E}_0|/(\varepsilon + 2\varepsilon_0)}_{I_{\mathrm{sph}}} \underbrace{-i\omega\varepsilon_0 2\pi R^2(\varepsilon - \varepsilon_0)|\mathbf{E}_0|/(\varepsilon + 2\varepsilon_0)}_{I_{\mathrm{fringe}}}. \tag{4}$$

 

We have named the three terms in Eq. (4) according to their function: the ''impressed displacement current source,'' $I_{\text{imp}}$, the ''displacement current circulating in the nanosphere,'' $I_{\text{sph}}$, and the ''displacement current of the fringe (dipolar) field,'' $I_{\text{fringe}}$, respectively. All of them are related to the polarization charges on the surface of the nanosphere, induced by the excitation. The above relationship among the various segments of the displacement current can be interpreted as the branch currents at a node in a parallel circuit, as shown in Fig. 1. Indeed, such currents as defined above obey the Kirchhoff current law, represented by Eq. (4). The Kirchhoff voltage law is also satisfied, since $\nabla \times \boldsymbol{E}$ is locally near zero in this quasistatic approximation.

The equivalent impedance for the ''nanosphere'' and the ''fringe'' branches of the circuit shown in Fig. 1 are calculated as the ratio between the ''average'' potential difference (due to $\mathbf{E}_{\text{res}}$) between the upper and lower hemispherical surfaces of the sphere

$$\langle V \rangle_{\text{sph}} = \langle V \rangle_{\text{fringe}} = R(\varepsilon - \varepsilon_0)|\mathbf{E}_0|/(\varepsilon + 2\varepsilon_0) \quad (5)$$

and the effective currents evaluated in Eq. (4). Thus, we get:

$$Z_{\text{sph}} = (-i\omega\varepsilon\pi R)^{-1}, \quad Z_{\text{fringe}} = (-i\omega 2\pi R\varepsilon_0)^{-1}. \quad (6)$$

From Eq. (6) we can clearly see that the two parallel elements in the circuits shown in Fig. 1 may behave differently according to the sign of the nanosphere's permittivity. Let us consider the following two cases:

*Nonmetallic sphere as a nanocapacitor.*—In this case, the real part of $\varepsilon$ is a positive quantity, and thus $Z_{\text{sph}}$ in Eq. (6) is capacitive along with the resistive part related to the imaginary part of permittivity. The impedance of the outside fringe is always capacitive, since we assume that the permittivity of the outside region is positive. Thus the equivalent nanocircuit for a nonplasmonic nanosphere can be shown as in the bottom left part of Fig. 1. Here the equivalent circuit elements can be expressed in terms of parameters of the nanospheres as follows:

$$C_{\text{sph}} = \pi R\,\text{Re}[\varepsilon], \qquad G_{\text{sph}} = \pi\omega R\,\text{Im}[\varepsilon], \qquad C_{\text{fringe}} = 2\pi\omega R\varepsilon_0. \quad (7)$$

Since there are two capacitive elements, there is no resonance present in this case—a fact that is consistent with the absence of resonance for optical wave interaction with the small nonplasmonic nanosphere.

*Metallic sphere as a nanoinductor.*—In this case, we assume the sphere to be made of a plasmonic material, such as noble metals in the visible or IR band (e.g., Ag, Au), and as a result the real part of the permittivity may attain a negative value in these frequency bands. Therefore, the equivalent impedance of the nanosphere [Eq. (6)] can be ''negatively capacitive'' at any given frequency for which $\text{Re}[\varepsilon] < 0$. This can be interpreted as a positive *effective* ''inductance,'' as discussed in [4–6]. Therefore, the equivalent circuit for the case of optical wave interaction with a plasmonic nanosphere may be presented as in the bottom right of Fig. 1. Here the equivalent circuit element for the sphere becomes:

$$L_{\text{sph}} = (-\omega^2\pi R\,\text{Re}[\varepsilon])^{-1}. \quad (8)$$

In this case, since there is an inductor in parallel to the fringe capacitor, the circuit may exhibit resonance, which corresponds to the plasmonic resonance for the optical wave interaction with the metallic nanoparticles, as mentioned in [4]. It may be verified that the resonance condition for the circuit $L_{\text{sph}}C_{\text{sph}} = \omega^{-2}$ requires the well-known condition of plasmonic resonance for a nanosphere $\text{Re}[\varepsilon] = -2\varepsilon_0$ [1].

It follows from the above discussion that a small nanosphere excited by an optical signal may effectively behave as a ''nanocapacitor'' or a ''nanoinductor'' at the optical frequency if the sphere is made of nonplasmonic or plasmonic materials, respectively. The imaginary part of the material permittivity may provide an equivalent nanoresistor. It is interesting to note that at lower frequencies the conventional design for an inductor requires the form of ''wound wires,'' whereas here the plasmonic characteristics of natural noble metals may provide us with an effective intrinsic inductance, whose value can be designed by properly selecting the size, shape, and material contents of the nanostructure.

These concepts indeed provide new possibilities for miniaturization of electrical circuits operating at the optical frequencies. The conventional circuits in the rf and lower frequencies, relying on the conduction current circulating in metallic wires along the lumped elements, cannot be straightforwardly scaled down to the infrared and optical frequencies, at which conducting metallic materials behave quite differently. However, introducing plasmonic and nonplasmonic nanoparticles as basic elements

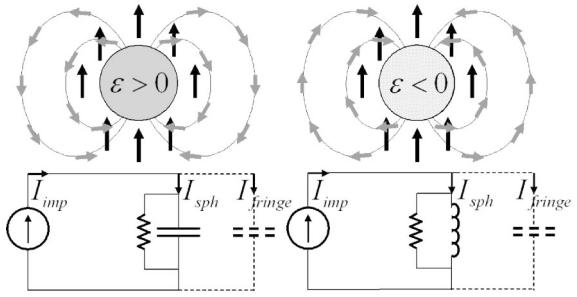


FIG. 1. A basic nanoscale circuit in the optical regime. Left: a nonplasmonic sphere with $\varepsilon > 0$, which provides a nanocapacitor and a nanoresistor; right: a plasmonic sphere with $\varepsilon < 0$, which gives a nanoinductor and a nanoresistor. Solid black arrows show the incident electric field, and the thinner field lines together with the gray arrows represent the fringe dipolar electric field from the nanosphere.

of optical nanocircuits, in which effectively the "*displacement*" current can similarly "circulate," may provide analogous functionalities at the optical frequencies. One may essentially have the three basic circuit elements, i.e., nanoinductor, nanocapacitor, and nanoresistor, operating in the optical frequency, which form the building blocks for the design of more complex circuits at these wavelengths. To get an idea about the values of these nanoelements, let us assume a nanosphere with $R = 30$ nm made of silver. At the wavelength $\lambda_0 = 633$ nm, the permittivity of silver is known to be $\varepsilon_{\mathrm{Ag}} = (-19 + i0.53)\varepsilon_0$ [7]. From Eq. (8), we can then find $L_{\mathrm{sph}} \simeq 7.12$ femtoH, $G_{\mathrm{sph}} = 1.32$ mS, and $C_{\mathrm{fringe}} \simeq 1.67$ attoF. If the particle is made of $Au_2S$ with permittivity $\varepsilon_{\mathrm{Au_2S}} = 5.44\varepsilon_0$ at $\lambda_0 = 633$ nm, the capacitance of the nanosphere will be $C_{\mathrm{sph}}$ 4.53 attoF.

We point out that for a given wavelength and a specific material the values of these nanoelements depend directly on the radius of the nanosphere. However, if one wants to have more flexibility in their design, one can use nanoparticles with different geometries. (Ellipsoids, for instance, would have 3 degrees of freedom, corresponding to the three axes.)

Expanding this concept to configurations with more than one nanoparticle, e.g., the case of two nanospheres with radii $R_1$ and $R_2$, permittivities $\varepsilon_1$ and $\varepsilon_2$, and with a certain distance $d$ apart, an electromagnetic analysis of the field distribution shows that in the "quasistatic" limit considered here these configurations may be effectively treated as "coupled" nanocircuits, each representing one of the nanoparticles. In addition to the elements depicted in Fig. 1 for the isolated nanoparticle, here each circuit should also include a "dependent current source" representing the influence of the field of other particle(s) on this sphere. In other words, the interaction among the particles here may be exhibited using "dependent sources." The value of each dependent current source may be explicitly derived in terms of the induced potential difference across the other nanosphere, in analogy with the previous formulas.

In order to form parallel or series circuit elements with these nanoparticles, one would need to juxtapose two (or more) of them very closely with specific orientations with respect to the illuminating electric field. Figure 2 (top) shows the geometry of a structure that consists of two tightly paired semicylinders of differing permittivities. (Here, for the sake of mathematical simplicity, lossless *cylinders* are considered.) The potential distribution around this fused structure, when illuminated with an electric field, provides useful information about its behavior as combined circuit elements. The middle panels present the potential distribution and equipotential surfaces for the two cases of electric field being parallel (left column) and perpendicular (right column) to the plane interface between the two halves with permittivities $\varepsilon$ and $-\varepsilon$, calculated under the "quasistatic" approximation. We note that the equipotential surfaces near the fused cylinder in the left column of Fig. 3 become perpendicular to its outer surface, implying that the normal component of the total electric field is zero at this surface. However, there is indeed a certain potential difference between the top and bottom parts of the cylinder's surface. As seen from the outside, this fused structure might be regarded as a *parallel* resonant *L-C* circuit (which in fact has an infinite impedance at its resonance, and hence zero net displacement current flowing into it), in parallel with the fringe capacitor, as depicted in the equivalent circuit in the bottom left column of Fig. 2. In an analogous way, the fused semicylinders in the right column of this figure, having the

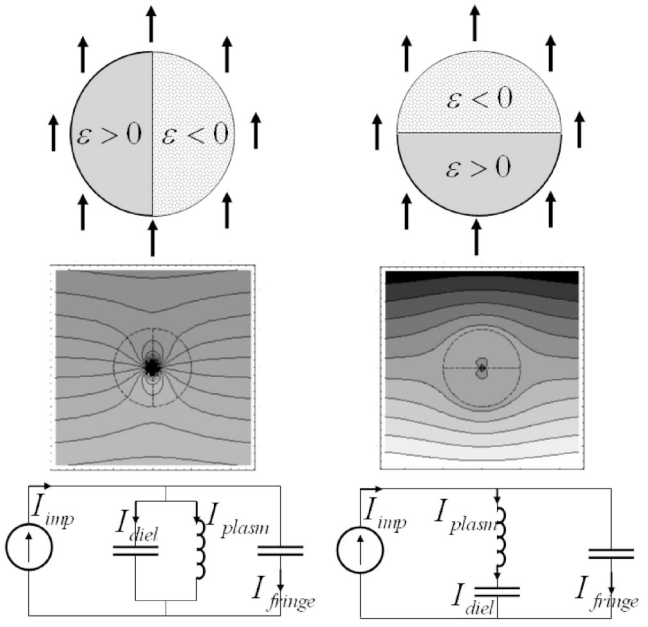


FIG. 2. Parallel and series nanoelements. Top: two fused lossless semicylinders with positive and negative permittivities, illuminated by an optical field; middle: quasistatic potential distributions around and within the structure (solid lines show equipotential surfaces); bottom: equivalent circuits showing parallel and series elements representing the fused structure as seen from the outside.

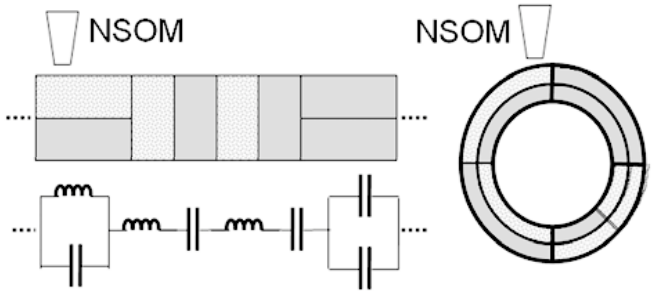


FIG. 3. Nanocircuit synthesis. Top left: conceptual nanoscale circuit formed by rectangular blocks of plasmonic and nonplasmonic segments; bottom left: its equivalent circuit; right: a closed "nanocircuit loop."

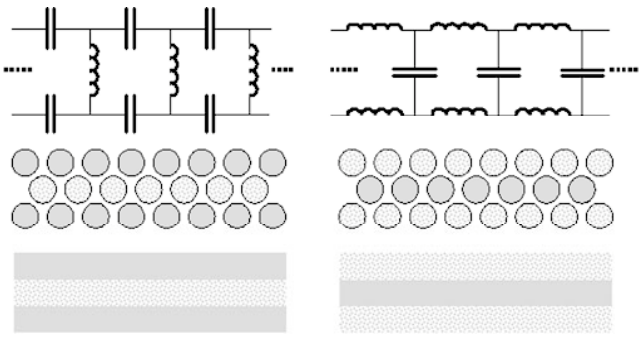

FIG. 4. An idea for optical implementation of right-handed and left-handed nanotransmission lines. Top: conventional RH and LH lines using distributed (or lumped) inductor and capacitor elements; middle: plasmonic and nonplasmonic nanostructures may play the role of nanoinductors and nanocapacitors; bottom: as the nanostructures gets closer, in the limit, plasmonic and nonplasmonic layers may be envisioned to constitute layered transmission lines with forward and backward operation.

external electric field perpendicular to the boundary interface between the two halves, may be regarded as a *series* resonant *L*-*C* circuit, as observed from the outside. This equivalent circuit is reported in the bottom right column of Fig. 2. In fact, as seen in the middle right panel, the equipotential surfaces in this case become parallel with the fused cylinder's surface, implying that the potential difference at the surface of this structure is effectively zero, whereas the displacement current flows in and out of it. The resonant behavior of these examples is present due to the particular choice of oppositely signed (but equal magnitude) permittivities for the two halves. However, different pairs would behave as nonresonant series or parallel elements, depending upon their pairing and orientation with the external excitation. Moreover, other geometries for nanostructures similarly paired may lead to analogous parallel and series configurations. For instance, Fig. 3 (top left) shows a nanostructure conceptually formed by rectangular blocks of plasmonic and nonplasmonic materials. We speculate that when this structure is excited by a local electric field [e.g., by a near-field scanning optical microscope (NSOM)] the plasmonic and nonplasmonic ''blocks'' might act as nanoinductors and nanocapacitors (along with some nanoresistor), respectively, and the structure might thus operate as the more complex circuit depicted in the bottom left panel of Fig. 3. Such nanoscale circuits might indeed behave as plasmonic nanobarcodes and plasmonic data storage systems. Such a circuit may also be configured as a ''closed'' loop, as shown in the right part of Fig. 3. When this optical closed circuit is excited by an NSOM at one point, we speculate that the displacement current along this loop may behave as the current in a circuit formed by equivalent inductors and capacitors. Since the nanotechnology and fabrication techniques for making nanostructures with different metallic and oxide segments are actively being investigated by various groups (see, e.g., [8]), construction of our proposed nanoelements and circuits is within the realm of possibility. Such optical circuits can also be interfaced with biological elements, such as molecules, when these elements can substitute one of the plasmonic or nonplasmonic elements in the circuit. We are currently exploring some of these concepts.

Finally, it is interesting to point out that by properly arranging these nanoscale circuit elements one may form optical nanotransmission lines. If the arrangement involves series nanoinductors and shunt nanocapacitors, for the dominant even mode this will provide conventional [also known as right-handed (RH)] transmission lines in the optical frequency. However, if the shunt nanoinductors and series nanocapacitors are used, we may synthesize negative-index [or left-handed (LH)] transmission lines in the optical domain, similarly to what was recently proposed in the microwaves [5,9]. This may lead to interesting subwavelength focusing effects in the optical frequencies, together with a road map for development of negative-refractive-index (or left-handed) materials in the IR and visible regimes. Such planar RH and LH structures are somehow consistent with the planar geometries suggested in [10], albeit arrived at with a different approach and with a different operation. Figure 4 shows a sketch of such concepts.

A. A. has been partially supported by the SUMMA Foundation. The authors thank Dr. Natalia Bliznyuk for useful discussion during the early stage of this work.

*To whom correspondence should be addressed.
Email address: engheta@ee.upenn.edu

[1] C. F. Bohren, and D. R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley, New York, 1983).
[2] B. Lamprecht *et al.*, Appl. Phys. Lett. **79**, 51 (2001).
[3] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1999).
[4] A. K. Sarychev, D. A. Genov, A. Wei, and V. M. Shalaev, ''Periodical Arrays of Optical Nanoantennas,'' in *Proceedings of SPIE*, Complex Mediums IV: Beyond Linear Isotropic Dielectrics Vol. 5218, edited by Martin W. McCall and Graeme Dewar (SPIE, Bellingham, WA, 2003), pp. 81–92.
[5] G. V. Eleftheriades, A. K. Iyer, and P. C. Kremer, IEEE Trans. Microwave Theory Tech. **50**, 2702 (2002).
[6] N. Engheta, N. Bliznyuk, and A. Alù, ''Interaction between Plasmonic and Non-Plasmonic Nanospheres and Their Equivalent Nanocircuit Elements,'' *Dig. USNC-URSI National Radio Science Meeting, Monterey, CA* (IEEE, Piscataway, NJ, 2004), p. 276.
[7] P. B. Johnson and R. W. Christy, Phys. Rev. B **6**, 4370 (1972).
[8] S. R. Nicewarner-Pena *et al.*, Science **294**, 137 (2001).
[9] L. Liu *et al.*, J. Appl. Phys. **92**, 5560 (2002).
[10] G. Shvets, Phys. Rev. B **67**, 035109 (2003).