# A COMPARISON OF DYNAMIC AND STATIC METHODS OF DETERMINING AMPLIFICATION CONSTANTS AND INTERNAL RESISTANCE OF THREE-ELECTRODE VACUUM TUBES.

by

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Approved by:

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Department of Physics

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Symbols used.

$i_p$ = plate current.

$i$ = a. c. component of plate current.

$v_p$ = plate voltage.

$v_g$ = grid voltage.

$k$ = $a/b$ = amplification constant.

$a$ = slope of plate-current grid-voltage characteristic.

$b$ = slope of plate-current plate-voltage characteristic.

$R_i$ = $1/b$ = internal a. c. resistance of tube.

$e_g$ = a. c. impressed e. m. f. on the grid.

# A COMPARISON OF DYNAMIC AND STATIC METHODS OF DETERMINING THE AMPLIFICATION CONSTANTS AND INTERNAL RESISTANCES OF THREE-ELECTRODE VACUUM TUBES.

## Introduction

In three-electrode vacuum tubes we are concerned with two circuits, that between the grid and filament, or input circuit, and that between the plate and filament, or output circuit. In studying the amplifying properties of a tube the current of the grid-filament circuit, or input current is of little importance and will not be considered here.

The plate current at a given instant is a function of the grid voltage and of the plate voltage. This function may be represented by a characteristic surface for which Langmuir[1] has given the equation:-

$$i_p = A(v_p + kv_g)^{\frac{3}{2}}$$

In this equation $i_p$ is the plate current, $v_p$ is the plate voltage, $v_g$ is the grid voltage, and k is a constant for a given tube construction.

In investigating the functioning of tubes experimentally it has been the custom to determine the static characteristics. These are the curves made by the intersection of the plane surfaces $v_p$ equal a constant, or

[1] I. Langmuir, "PROC. INST. RADIO ENGRS., 3, 1915

$v_g$ equal a constant, and the characteristic surface. These are somewhat unsatisfactory as they do not show directly the values needed in studying amplifier tubes. Blondel[1] has shown a graphical method of deriving dynamic characteristics from these static characteristics. These are direct reading but rather slowly arrived at. Miller[2], of the Bureau of Standards, has shown a direct method of deriving dynamic characteristics of amplifier tubes which is both simple and rapid. It will be the purpose of this paper to show some of the advantages of the dynamic method of Miller at audio frequencies, and to some extent a modification of this method, over the static method, which is both slow and inaccurate. In many cases the operation of a tube takes place about a point on the characteristic surface which is nearly plane. The equation of the characteristic surface about an operating point ($i_p$, $v_g$, $v_p$) may be written as given by Vallauri[3] in the form:-

$$i_p = av_g + bv_p + c$$

where a is the slope of the plate-current grid-voltage curve and b is the slope of the plate-current plate-voltage curve. These quantities have the dimensions of

1. M. Andre Blondel, COMPTUS RENDUS. 16. Dec. 1919.
2. John M. Miller, PROC. INST. RADIO ENGRS., 3, 1918.
3. G. Vallauri, "L'Elettrotecnica," IV, 3, 1917.

conductances. Vallauri has shown that these quantities are fundamental in showing the behavior of a three-electrode vacuum tube used as an amplifier or as an oscillator.

The quantities a/b and 1/b will be determined experimentally. The quantity a/b has been called by H. J. Van der Bijl[1], the amplification constant and is identical with k in Langmuir's equation. 1/b is the internal a. c. resistance of the tube and will be designated by R .

Assuming that the plate circuit is closed through an impedance z = (x + jy) Miller derives the following equation:-

$$ke_g = iR_i + i(x + jy)$$

where k is the amplification constant, $e_g$ is the a. c. impressed e. m. f. on the grid, i is the a. c. component of the plate current, and $R_i$ is the internal resistance of the tube in the plate-filament circuit.

## Method

The method used in determining the amplification constant at audio frequencies was a modification of Miller's method. The method used in determining Internal Resistance was also a modification of Miller's method. The circuit is that shown in figure 1, in which cd is a slide wire supplied with a small audio frequency current

[1] Unpublished Paper of September 17, 1917.

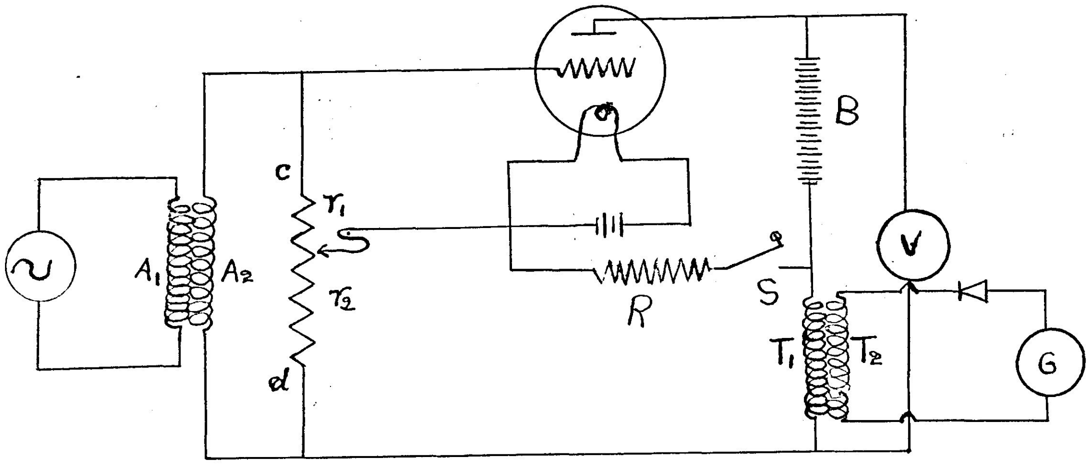



Figure 1.

from a buzzer source, through a transformer $A_1$ $A_2$. R is a variable resistance of about 6000 ohms formed by a graphite mark on a marble slab, which can be connected into the circuit by means of the switch S.

First the amplification constant, k, is determined with the switch S open by adjusting the slider until there is no deflection on the galvanometer when $k = \frac{r_2}{r_1}$.

This statement may be proven as follows:-

When an e. m. f., $e_g$ which is proportional to $r_1$ is introduced between the grid and the filament it has the effect of impressing an e. m. f. $ke_g$ in the plate circuit. The e. m. f. across $r_2$ which is $\frac{r_2}{r_1}e_g$ is 180 degrees out of phase with the e. m. f. $ke_g$ and will balance it when $\frac{r_2}{r_1}e_g = ke_g$. Therefore:-

$$k = \frac{r_2}{r_1}$$

Also

$$R_i = (\frac{r_1}{r_2}k - 1)R.$$

This may be proven as follows.

An e. m. f. $e_g$ proportional to $r_1$ is impressed between the grid and the filament. This will produce an alternating current $\frac{Ke_g}{R_i+R}$ in the plate circuit. The e. m. f. across the resistance R is $\frac{Ke_g R}{R_i+R}$. This is balanced by the e. m. f. $\frac{r_2}{r_1}e_g$ across $r_2$ when $\frac{r_2}{r_1}e_g = \frac{Ke_g R}{R_i+R}$ or

$$R_i = (\frac{r_1}{r_2}k - 1)R$$

In the actual set-up the resistance cd was a straight piece of No. 36 German silver wire one meter long. The resistance R was a graphite mark on a marble slab.

In this work an attempt was made to make these same determinations at radio frequencies using an oscillating tube as a source of e. m. f. In this attempt the chief difficulty was in getting the e. m. f.'s proportional to $r_1$ and $r_2$ 180 degrees out of phase, or if this was done knowing when they were 180 degrees out of phase. It is probable that if a non-inductive, sensitive, high-frequency, current measuring device be inserted in the circuit in place of the coil $T_1$ that this determination could be made directly. With this set-up there would be a direct current passing thru the instrument all the time but with an instrument giving a deflection proportional to the square of the current this would make it more sensitive to the changes due to the a. c. voltage applied.

The static characteristics were made in the usual way. To obtain the grid-voltage plate-current curves the plate voltage was held constant and the grid voltage varied through small steps, usually from -1 volt to +1 volt with the Western Electric type 203B tube as it was designed to operate on zero grid voltage with about 40 volts on the plate. To obtain the plate-voltage plate-current curves the grid voltage was held constant and the plate voltage varied.

## Results

With the dynamic method described above it was possible to read to within one per cent of the correct value. With the static method using a 150 milli-ampere meter to measure the plate current, and a Weston model 203 voltmeter it was possible to read to within about five per cent of the correct value, that is the value which would give the correct slope to the static curve at a given point.

It is possible to obtain these direct reading, dynamic curves from the static curves. In the case of the internal resistance $R_i$ , its values at various points are the reciprocals of the slopes of the plate-current plate-voltage curves. These values may be computed and plotted, giving the direct reading dynamic curve of internal resistance. This process is somewhat slow and subject to all the inaccuracies of the static curves.

In the case of the amplification constant it is necessary to divide the slope (a) of the grid-voltage plate-current characteristic by the slope (b) of the plate-voltage plate-current characteristic to obtain the values of the amplification constant which may be plotted thus giving the direct reading curve for the amplification constant. The accuracy of this curve is subject to the errors of both the plate-voltage plate-current characteristics and the grid-voltage plate-current characteristics. These errors may be additive or they may neutralize one another.

Curves showing these results are given below. It will be observed that there are some cases in which the results of the two cases do not agree. In such cases the dynamics obtained by Miller's method are to be regarded as the more accurate due to the greater accuracy of the readings taken by this method.

The amplification constant, except at low voltages, is very nearly constant and is nearly the same for the two tubes of the same type shown below, although it may have a very different value for a tube of another type.

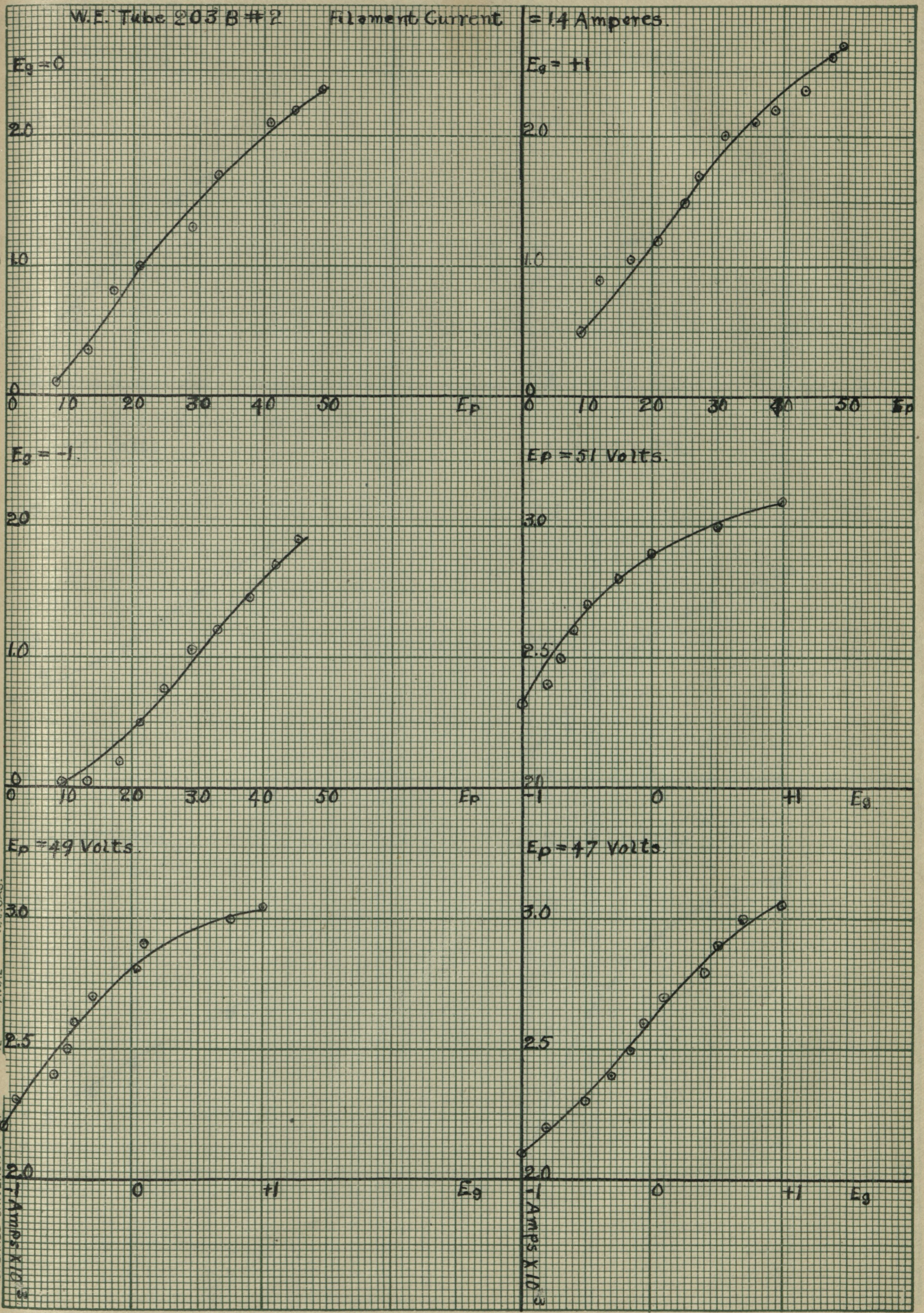
The internal resistance is rather high at low plate voltages but gradually falls as the voltage rises to about 25 volts and then rises as the plate voltage rises. This last effect is due to the space charge effect in the tube.

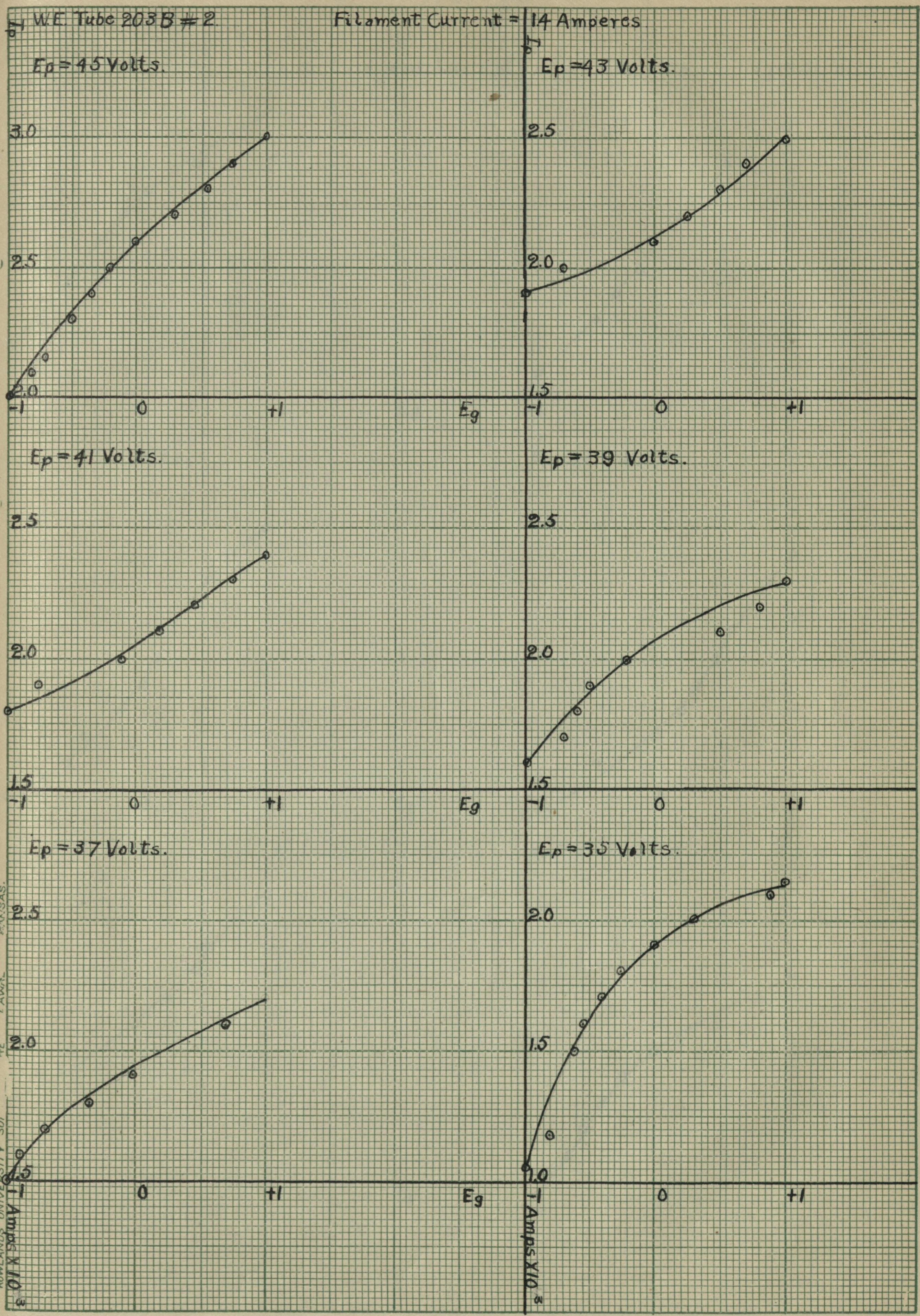
So far as possible these results were all taken as near to the working conditions of the tube as possible. The voltage from the buzzer source was no greater than the tubes might be called upon in practice to amplify. The filament current was kept at about the average for the type of tube used.

In conclusion I wish to express my appreciation to the Physics Department of the University of Kansas for making available to me liberal supplies of apparatus with which to carry on experimental work.

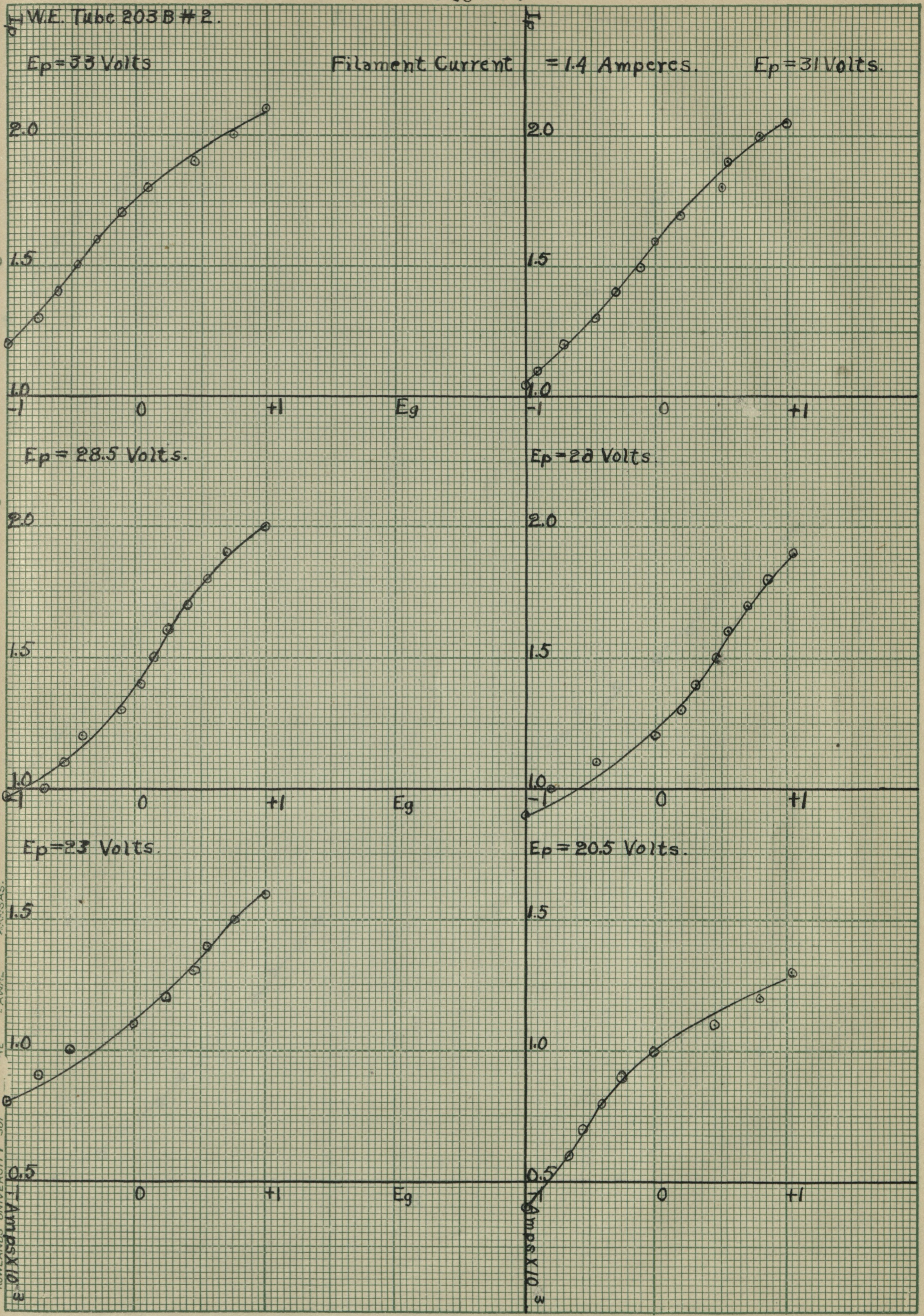
University of Kansas,

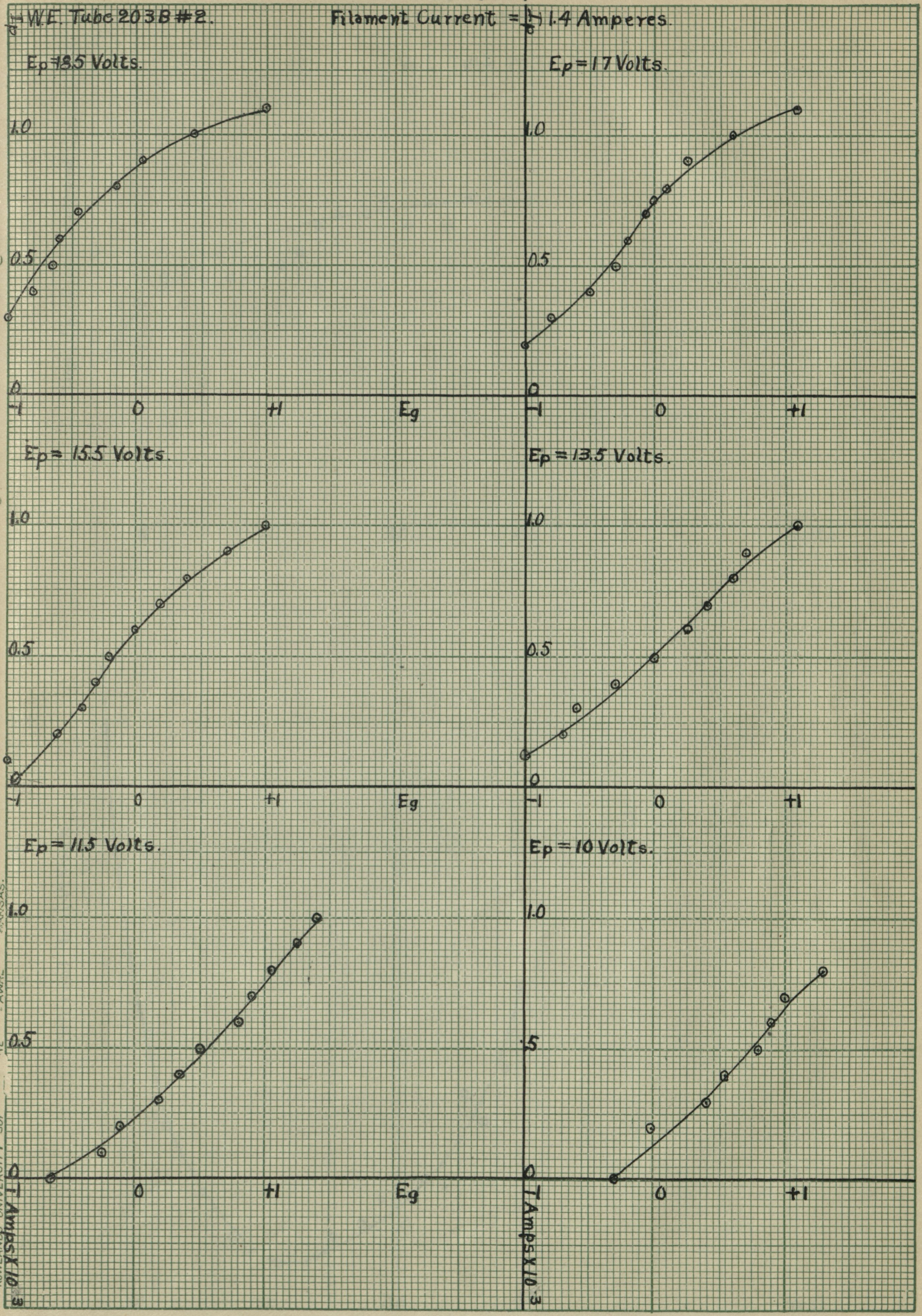
December 31, 1920.

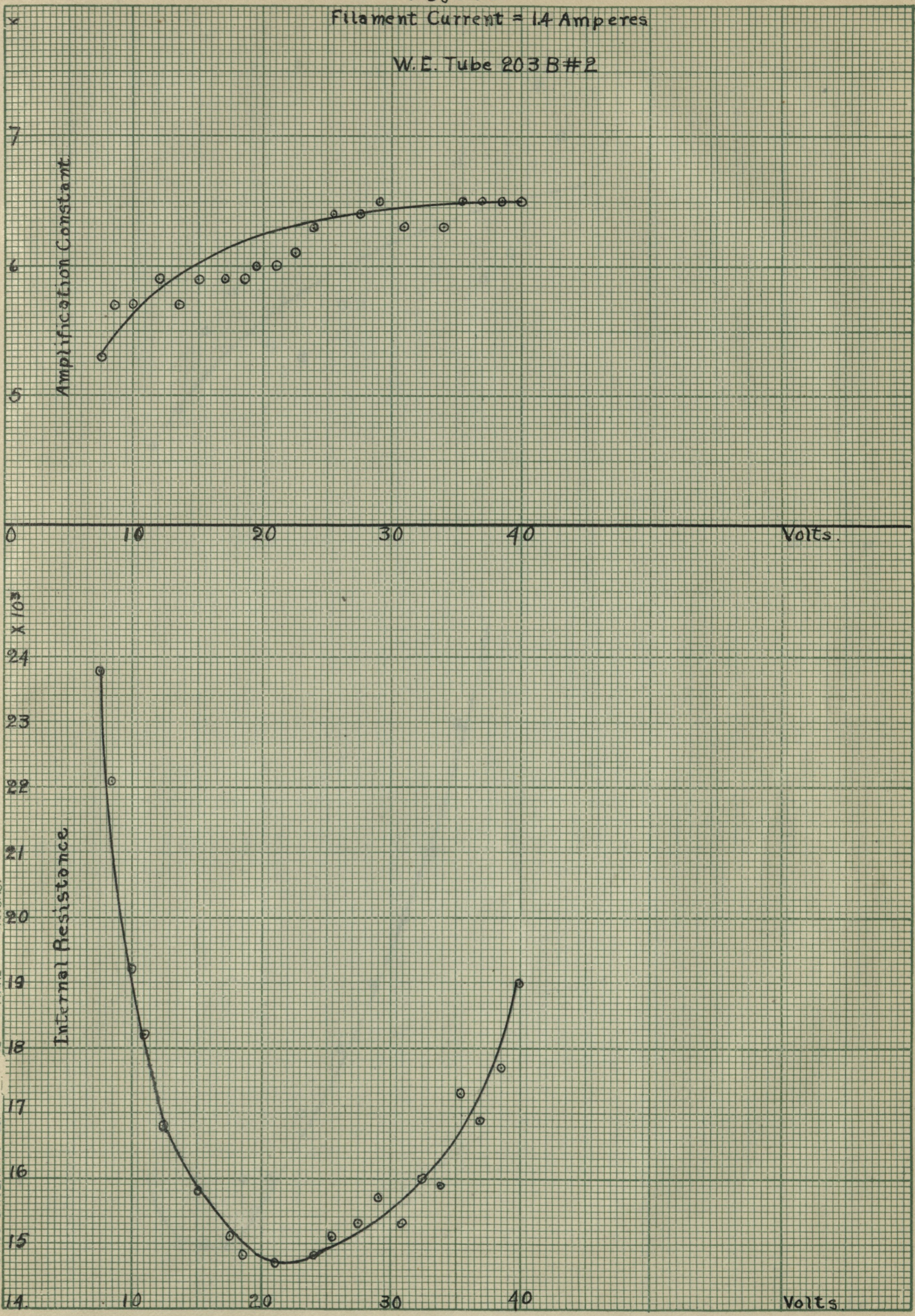

W.E. Tube 203 B #2 Filament Current = 1.4 Amperes.
Eg = 0
Eg = +1
Eg = -1.
Ep = 51 Volts.
Ep = 49 Volts.
Ep = 47 Volts.
Ep
Eg
I Amps X 10^-3

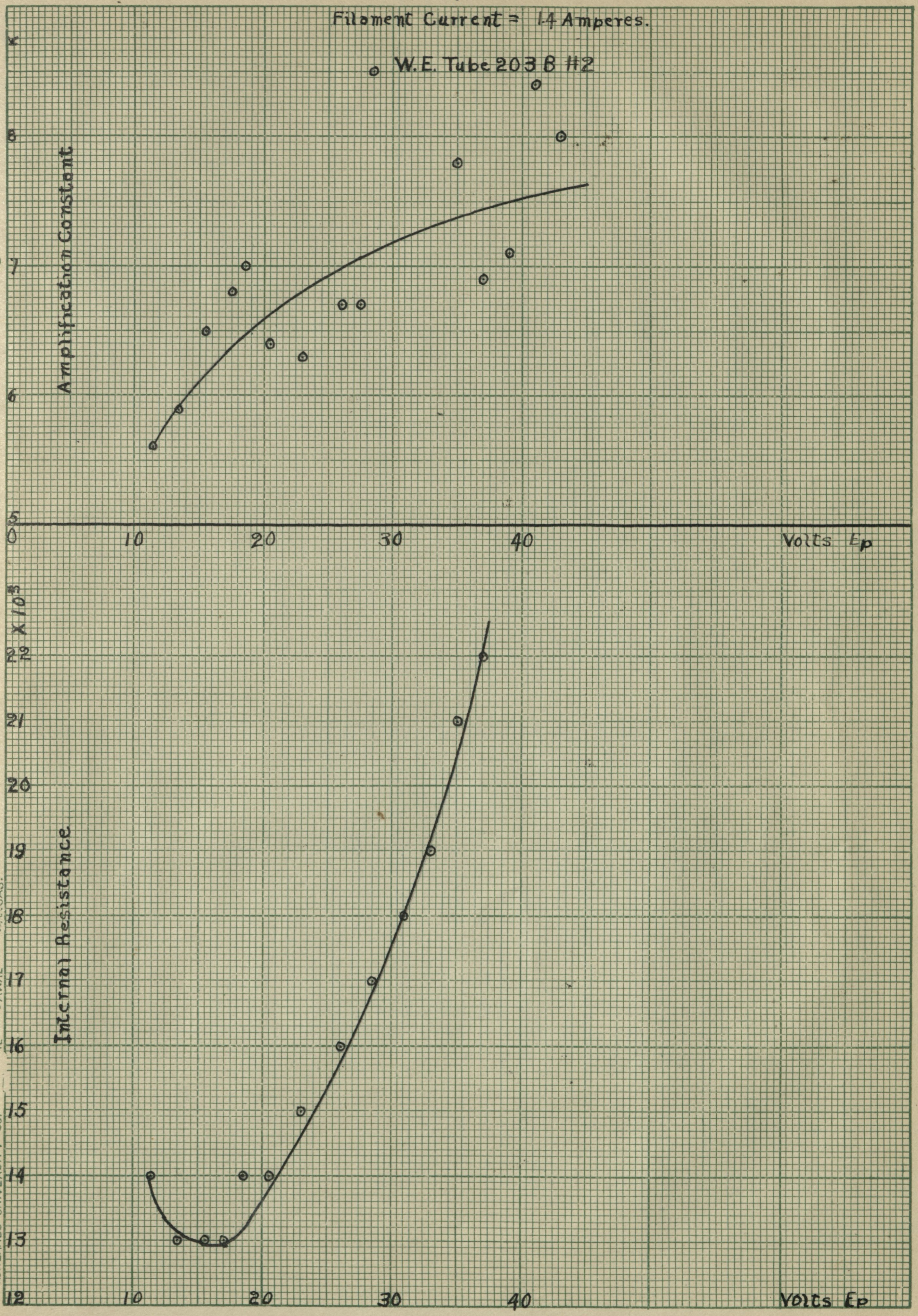


W.E. Tube 203B # 2.
Filament Current = 14 Amperes
Ep = 45 Volts.
Ep = 43 Volts.
Ep = 41 Volts.
Ep = 39 Volts.
Ep = 37 Volts.
Ep = 35 Volts.
Ip
Eg
Amps X 10-3


W.E. Tube 203 B # 2.
Ip
Ep = 33 Volts
Filament Current = 1.4 Amperes.
Ep = 31 Volts.
2.0
1.5
1.0
-1
0
+1
Eg
Ep = 28.5 Volts.
Ep = 28 Volts.
Ep = 23 Volts.
Ep = 20.5 Volts.
0.5
Amps x 10⁻³


W.E. Tube 203B #2.
Filament Current = 1.4 Amperes.
Ep = 18.5 Volts.
Ep = 17 Volts.
Ep = 15.5 Volts.
Ep = 13.5 Volts.
Ep = 11.5 Volts.
Ep = 10 Volts.
Eg
I Amps X 10-3
1.0
0.5
0
-1
+1


Filament Current = 1.4 Amperes
W. E. Tube 203 B #2
Amplification Constant
7
6
5
0
10
20
30
40
Volts.
Internal Resistance
X 10³
24
23
22
21
20
19
18
17
16
15
14
10
20
30
40
Volts.


Filament Current = 1.4 Amperes.
W.E. Tube 203 B #2
Amplification Constant
Volts $E_p$
Internal Resistance
$\times 10^3$
Volts $E_p$
ROWLANDS UNIVERSITY SUPPLY STORE, LAWRENCE, KANSAS.