DYNAMIC CHARACTERISTICS
OF AN
OSCILLATING ELECTRON TUBE

by

LEWIS M. HULL.

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Approved M. E. Rice

Department of Physics.

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CONTENTS.

DYNAMIC CHARACTERISTICS OF AN OSCILLATING ELECTRON TUBE.

Within the last three years the three element vacuum tube has advanced in importance from its position as a more or less uncertain detecting device, whose behavior was shrouded in mystery and apparently governed by no consistent laws or principles, to a most unique position in the radio field; its immense variety of possible applications in both in laboratory measurements and in radio field work make its developement one of the most important developements of present day radio research. The vacuum tube is undoubtedly the only logical radio transmitter of the future for all cases where excessive power is not required, and while its experimental developement has already outstripped all mathematical analyses of its behavior, it appears that its usefulness wherever high or low frequency alternating currents are involved is limited as yet only by the ingenuity of its investigators.

The first detailed and plausible analysis of operating conditions in a vacuum tube was given by Armstrong(1). In his analysis, the pioneer work in this direction, the idea of the"regenerative" action of the audion was first advanced, forecasting the use of the electron tube as a generator of spontaneous oscillations. Immediately there followed the developement of the "pliotron" by the engineers of the General Electric Company. Dr. Langmuir pointed out the advantages of replacing the original uncertain and erratic "audion", in which the phenomena of thermionic emission were complicated by the presence in the tube of streams of ionized gas molecules, by a tube utilizing a pure electron discharge.[2] As soon as the well-known laws regarding thermionic emission from a heated filament[3] became applicable the problem of developement of the vacuum tube was greatly simplified. At present the pliotron, a three element pure electron discharge tube, is the standard type as a generator of spontaneous oscillations.

The first mathematical discussions of dynamic conditions in an oscillating vacuum tube were furnished by Latour,[4] and by Bethenod.[5] Latour considered only the most general cases of audion operation; a feature of his analysis is his employment of the so-called "static constants" of the tube for the predetermination of its amplifying power. Bethonod treats in detail a simple regenerative circuit but he makes his mathematical expressions so comprehensive as to be impossible of complete solution, defining only certain conditions under which the tube can be made to oscillate.

A comprehensive treatment of the three element tube has been given by Vallauri.[6] Vallauri's work, both analytical and experimental, has provided a worthy basis for later researches. His treatment is based wholly upon static characteristics, obtained with the audion in a stationary condition; he relies upon a plane characteristic surface to indicate the constants of the tube at all amplitudes of oscillation, assuming the electron current to be a function, of purely linear form, of both grid and plate potentials over an extended range.

The results of the present research are advanced not with the expectation of adding anything distinctly original to the rather complete store of information on the electron tube that has accumulated in the last two years, but with the hope of presenting the behavior of the fascinating device from a slightly new viewpoint, emphasizing with experimental data on dynamic relations in the tube facts which are discussed rather uncertainly in all treatises now available. The developement of the electron tube has proceeded with such leaps and bounds owing to the tremendous variety of its applications that already the essential features of its operation are common property in the radio world.

The three element electron tube is composed of a heated filament, a cold anode called the "plate", and a "grid" member interposed between filament and plate, all sealed in a tube which is exhausted to such a degree that the effect of gas molecules upon the electron stream is negligible. The filament is heated to incandescance by a local battery, and the electrons which it emits are drawn off to the plate, which is maintained at a large positive potential with respect to the filament. Variations in the potential of the plate and of the intermediate grid have similar but not equal effects upon the electron current. An increase in the potential of the grid with respect to the filament produces a relatively greater increase in the plate current than an equal increase in the potential of the plate. Upon this fact hinges the whole behavior of the tube as an amplifying device, which makes possible its use as a generator. The fundamental "constants" of such a device are: first, the admittance of the tube with respect to the plate potential, or the partial derivative of the plate current with respect to plate potential; second, its admittance with respect to the grid potential, or the partial derivative of plate current with respect to grid potential.

Static Characteristics.

Experimental work was performed upon a Western Electric Pliotron, Type "E". The validity of all general deductions based thereupon may be assumed, on account of the well-known uniformity of behavior of all pure electron tubes. The following symbols will be employed:

I --- electron current to plate.

V --- potential of plate with respect to filament.

v --- potential of grid with respect to filament.

i --- electron current to grid.

a_1--- dI/dV.

a_2--- dI/dv.

The effect of plate potential upon plate current, of plate potential upon grid current, of grid potential upon plate current, and of grid potential upon grid current, all for slow variations in the potentials involved, are shown in Figs. 1a, 2a, 3, and 4, respectively. Both sets of curves having V as the independent variable are plotted for different constant values of v, and those involving a variable v are plotted for different constant values of V. Such curves are the so-called "static Characteristics" and were obtained by varying the potentials step by step, and waiting for the electron current to assume a steady value. Fig. 1 shows the type of circuit employed in such work.

In these characteristics a positive current signifies the flow of electrons to the cold electrode in question. The potentials are given in volts, with respect to the filament. It is evident that the currents into or out of the grid are insignificant compared with the plate current; this is characteristic of pure electron tubes. On Fig. 4 the grid current is plotted to the same acale as the plate current; in order to emphasize this fact. At high grid potentials the grid-filament electron circuit acts as a negative resistance, thereby strengthening the tube's amplifying power, but the effect is practically insignificant owing to the small magnitude of the grid current. a_1 is evidently constant over a large range of plate potentials, while a_2, always very much larger, is constant only for a relatively small range of grid potentials. The curves are somewhat uneven, owing to unavoidable variations in the temperature

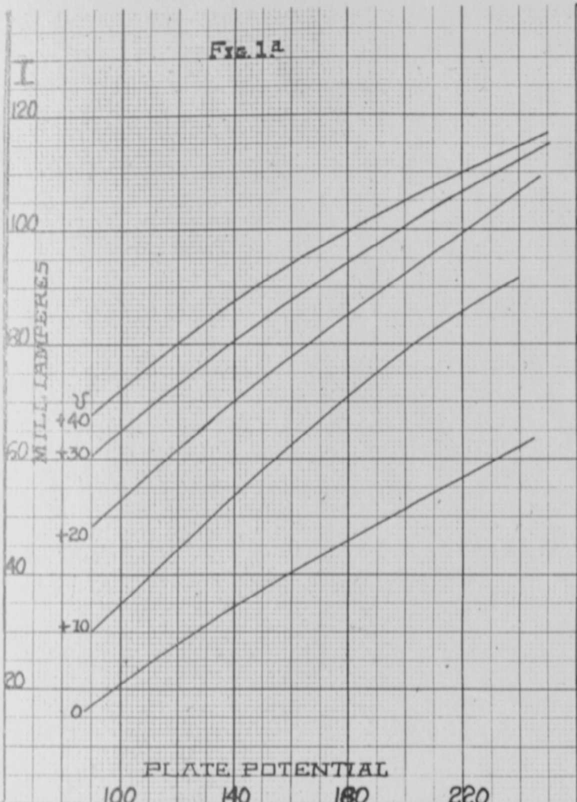


FIG. 1a

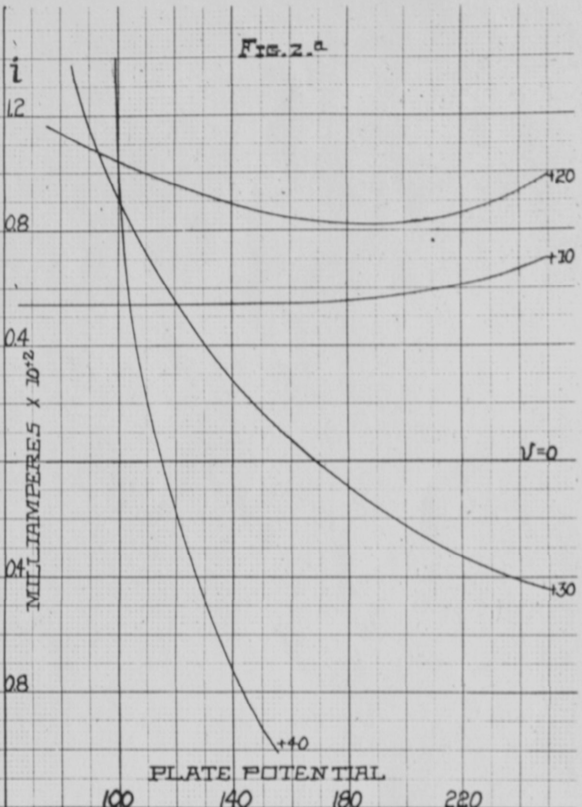


FIG. 2a

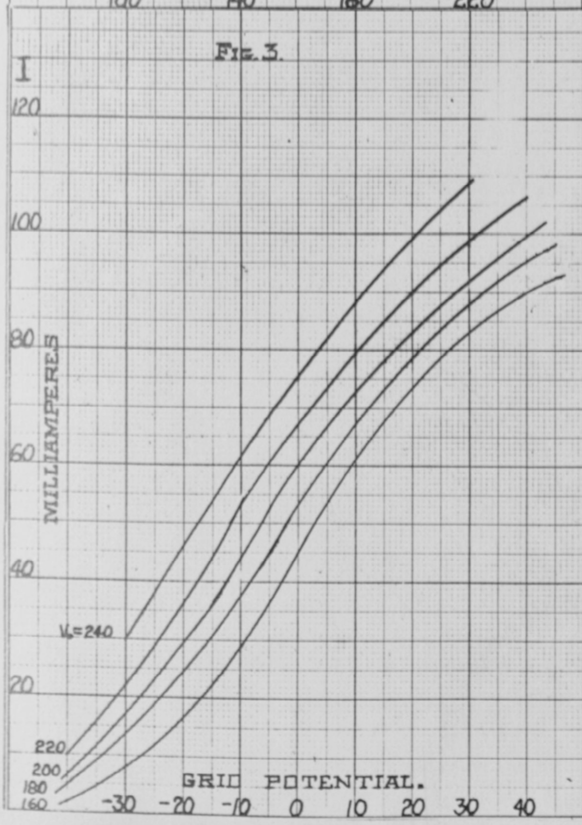


FIG. 3.

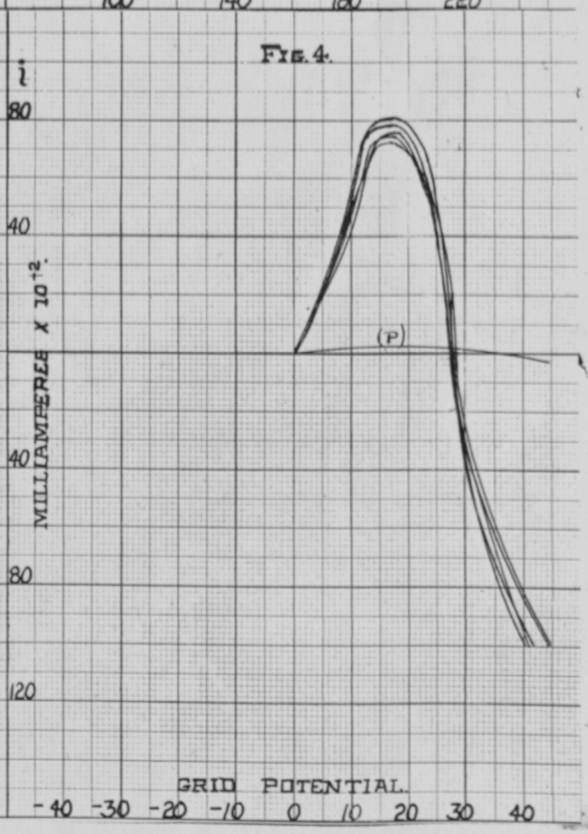


FIG. 4.

of the filament, brought about by slight changes in its resistance at high temperatures.

It is apparent from the figures that I as a function of v and V cannot be adequately represented by a plane surface. Dr. H. J. Van der Bijl suggests the use of the empirical function

$$I = \alpha(V + \mu v^2)$$

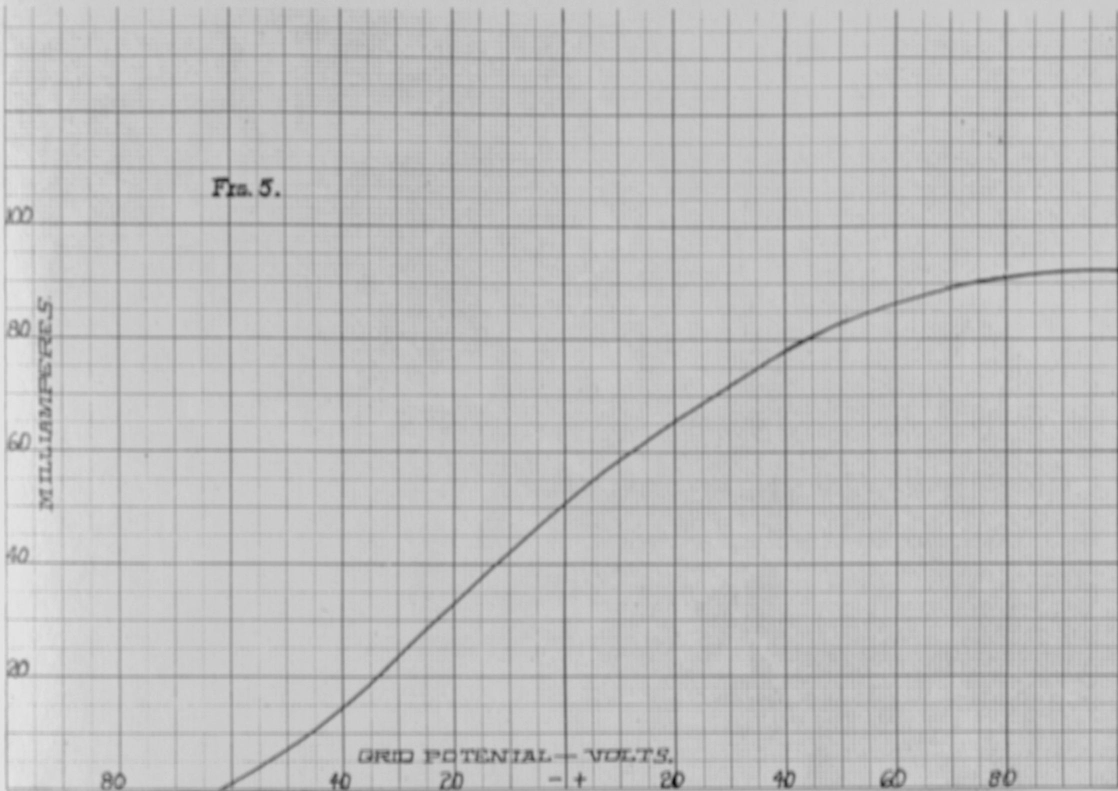
where α is a constant and μ is the "amplification" constant" or the ratio of an increment in plate potential to the increment in grid potential which will produce the same change in plate current.

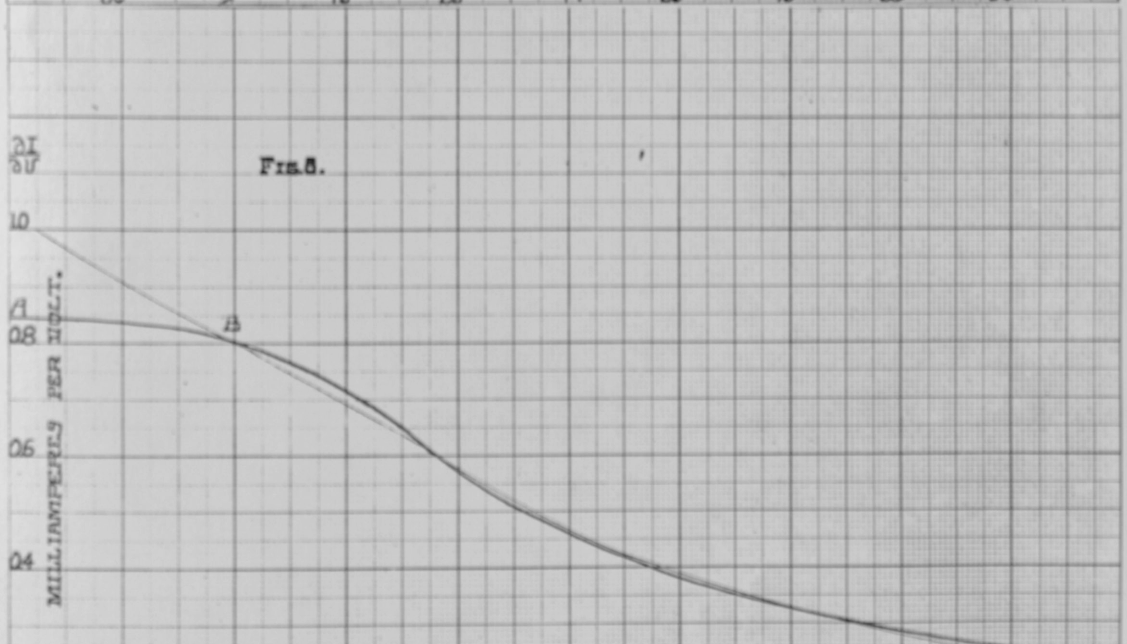
The variation in a_2 is important, and Fig. 5 shows the I and v characteristic over an extended range, compared with Fig. 3. Fig. 6 shows the average value of a_2 throughout a cycle of v as a function of the amplitude of that cycle. It is derived directly from Fig. 5. a_2 as a function of v_0, the grid potential amplitude of equal changes on opposite sides of v=0, is expressed quite approximately above 40 volts by an expression of the form:

$$v_0 = a + b \tan (g - ha_2) \qquad (1)$$

a, b, g, and h, being constants.

The characteristic was obtained over this extended range by rapidly reversing v upwards of a hundred times after each change in its numerical value, as a somewhat gross approximation of an oscillating grid potential. Above an amplitude of 80 volts, under such conditions, the plate current was found to remain constant until v had reached a value of + 200 volts, when I began to fall off with increasing v.


MILLIAMPERES
100
80
60
40
20
GRID POTENTIAL — VOLTS.
80
40
20
— +
20
40
60
80
FIG. 5.


$\frac{\partial I}{\partial V}$
MILLIAMPERES PER VOLT.
1.0
A
B
0.8
0.6
0.4
0.2
GRID POTENTIAL AMPLITUDE — VOLTS.
20
40
60
80
100
120
140
160
180
FIG. 6.

A study of the effect of different frequencies upon these characteristics is the next logical step, for in this form they cannot be applied to an oscillating tube without far reaching assumptions. To accomplish this the cathode ray tube of F. Braun was employed. The deflection of a cathode ray stream by oscillating currents and potentials is a familiar method of analysis in radio work,[7] and needs no extensive description here. It is believed, however, that this device has never beem used before in connection with the pliotron oscillator.

The only Braun tube available for this work was not provided with potential deflection plates inside the tube; the tube was also equipped with only one focussing screen. The latter deficiency was partially remedied by the use of a focussing coil placed around the tube close to the cathode, as shown in Plate IIa. Deflections of the ray by potential changes were obtained by the use of exterior deflecting plates, placed closely against the glass and insulated therefrom to prevent their acting as an outlet for charges collected of the tube; thin copper plates, 2X5 cm., were used. The current deflecting coils, made rectangular in shape, 7.5 cm. by 5 cm., and spaced with planes accurately parallel 4 cm. apart, were wound with 48X38 silk covered Litzendraht.

The Braun tube was excited by a two-plate Wimshurst machine, motor driven. The characteristics could not be photographed from the back of the fluorescent screen, and since the camera could not be placed parallel to the tube in front of the screen, reproduction of the characteristics by hand was resorted to. The sketches were made upon cross section paper which was an exact reproduction of the divisiohs upon the fluorescent screen. Thus they could be scaled to correspond exactly to the deflections; the drawing was done by a person skilled in scientific drawing who was entirely ignorant of the nature of the curves he was reproducing. The original pencil drawings were then mounted and reproduced photographically.

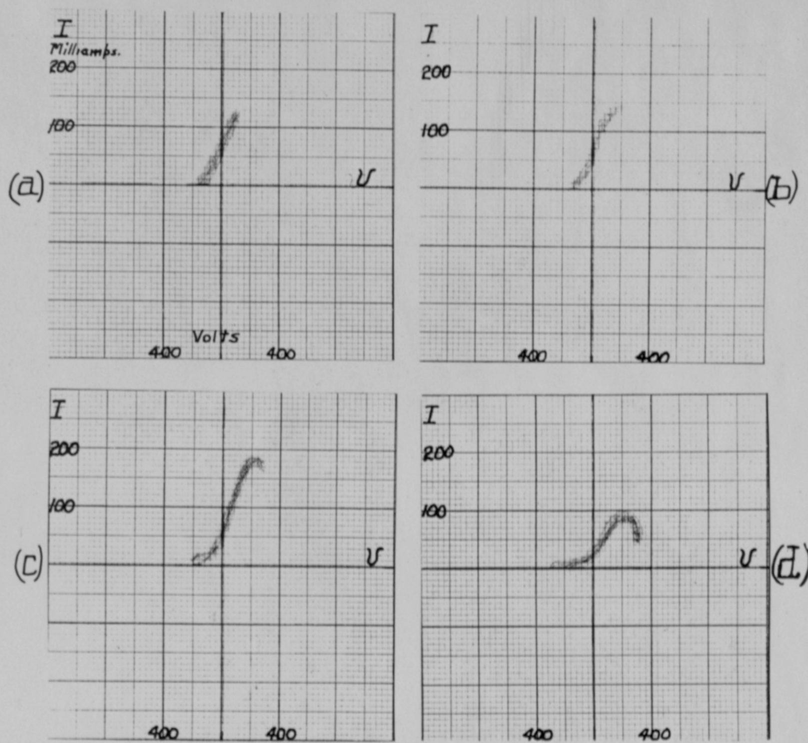
Low Frequency Characteristics.

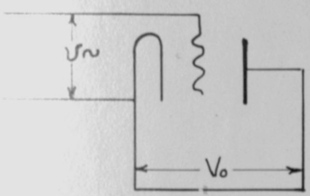
Plate I shows the characteristics obtained by impressing a sinusoidal volatge wave of periodicity 377 upon the plate and upon the grid. Curves (a) to (d) show the effect of increasing the amplitude of the cycle of grid potentuals at a constant plate potential, V_o = 230 volts. They are similar in shape to the corresponding static characteristicst The negative value of a_2 at the extremity of the curve for large amplitudes of grid voltage is shown by the drooping form of the curve. From curve (e) it is evident that the linear relation between V and I is not confined to the static condition. A certain positive plate potential is necessary to start the electron current, the current on the negative side of the V cycle being zero. (f) and (g) were obtained by coupling the grid so that its potential would oppose that of the plate in its effect on the electron current, and applying the alternating voltage to the plate. Here we have the type of coupling essential if the tube is to act as a spon-

PLATE I.

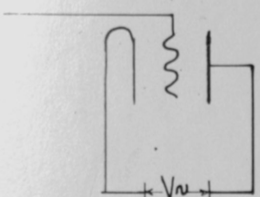
LOW FREQUENCY CHARACTERISTICS.

$\beta = 377.$

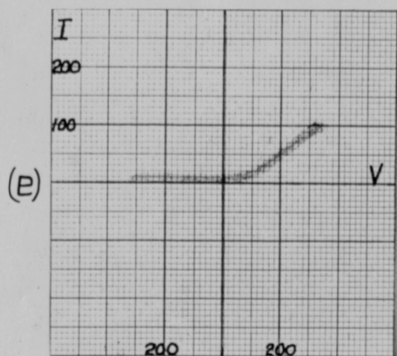


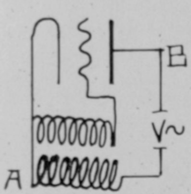


(a) to (d)

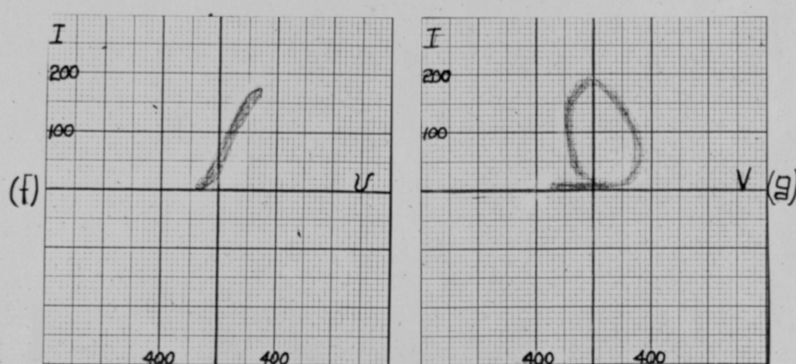


(e) Grid free.





(f)-(g) Grid coupled.



taneous oscillator. The plate current is practically in phase with the grid voltage, while with the whole arrangement the dominant effect throughout the cycle is that of a negative resistance with respect to potentials applied between A and B; it is between these points, of course, that V is measured. While the apparent dI/dV assumes all possible values throughout a cycle owing to the phase displacements of I and V, yet the dynamic or combination value of a_1 throughout the whole cycle is more often negative than positive as shown by the tilting of the ellipsoid to the left. Since any device having a "falling characteristic" will produce spontaneous oscillations when shunted by a resonance circuit (Fleming) curve (g) suggests that if the electron current were to be maintained by means not affecting, or affected by, any oscillations which might occur, a condenser shunted from A to B should complete a circuit capable of generating such oscillations. This circuit is, in fact, of the type used extensively in practice.

Operation of a Simple Oscillating Circuit.

In accordance with the foregoing discussion, consider the circuit shown in Fig. 2. Obviously no mathematical discussion based upon the characteristics of the tube as described above can exactly cover the behavior of such a device, but the actions of the tube can be adequately treated by a familiar form of analysis, unique only in this connection.

Referring to Fig. 3, let e be the instantaneous po-

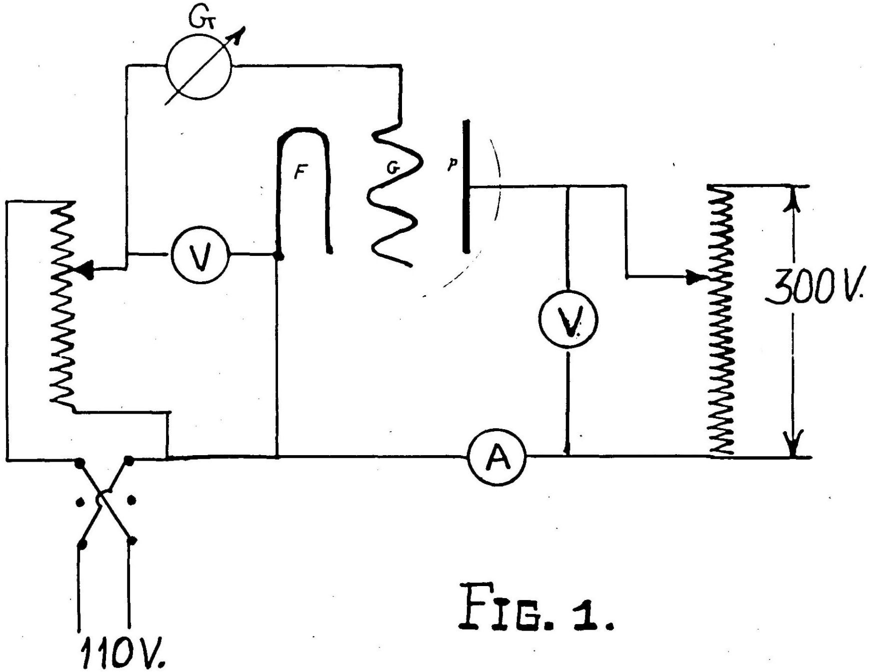


FIG. 1.

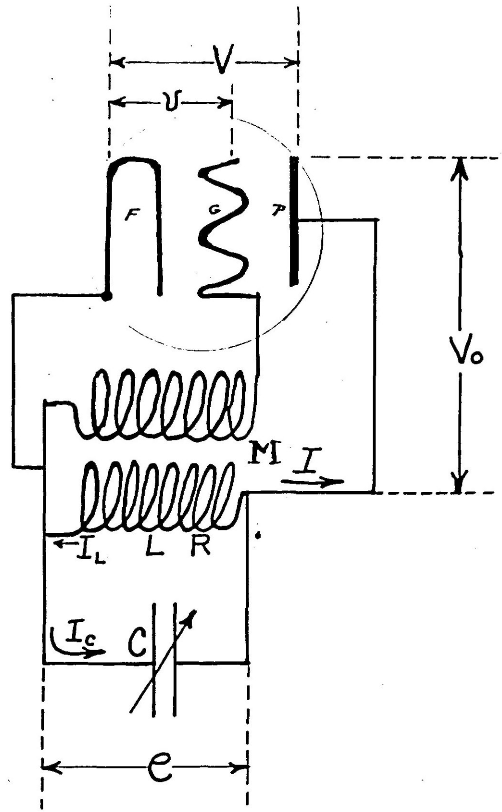


FIG. 2.

tential across the terminals of the condenser, C; V_0 is the constant plate potential, supplied by a storage battery; I_L is the instantaneous value of the current through the inductance, L, I_C the current through the condenser, while I is the total current flowing from the plate. The plate and grid inductances are coupled together with a mutual inductance M.

First confine the attention strictly to conditions at the beginning of any disturbance in the circuit, such as the sudden introduction of the plate potential V_0. In practice the battery supplying V_0 is shunted by a capacity of such magnitude that its high frequency reactance is insignificant compared with that of L or C, and the circuit represented by Fig. 2 is realized very closely.

Then while the amplitude of the disturbance is small, v not exceeding 40 volts, the plate current is given by the expression

$$I = a_1 V + a_2 v.$$

V, now and hereafter, is the potential of the plate with respect to the filament in excess of that necessary to start the electron current. Then for perhaps the first microsecond the following relations hold true:

$$V + e = V_0.$$
$$RI_L + L\, dI_L/dt = e$$
$$I + I_L = -C\, de/dt$$
$$v = M\, dI_L/dt$$

The assumption is here made that the grid current is insignificant in comparison with I. The currents considered above are taken as positive in the direction of the arrows (Fig. 2). Eliminating e and combining:

$$d^2I_L/dt^2+\left[\frac{a_2M+a_1L+RC}{LC}\right]dI_L/dt+\left[\frac{1+a_1R}{LC}\right]I_L=\frac{a_1V_0}{LC}. \qquad (2)$$

In this equation a_1R expressed in practical units is insignificant compared with 1 and may be neglected.

It should be noticed that this equation holds not only for such limited amplitudes of oscillation as occur when v_0 is within the range A to B and a_2 is constant (Fig.6), but for any subsequent period at which a_2 has become constant.

The solution is of the usual form:

$$I_L=A\epsilon^{m_1t}+B\epsilon^{m_2t}+a_1V_0, \text{ where:}$$

$$m_1=-\frac{a_2M+a_1L+RC}{2LC}+\sqrt{\left(\frac{a_2M+a_1L+RC}{4LC^2}\right)^2-\frac{1}{LC}}$$

$$m_2=-\frac{a_2M+a_1L+RC}{2LC}-\sqrt{\left(\frac{a_2M+a_1L+RC}{4LC^2}\right)^2-\frac{1}{LC}}$$

In order that I_L be periodic: $(a_2M+a_1L+RC)^2<4LC$

Let the constants of the circuit be chosen to fulfill this condition.

Call: $m_1=-\alpha+j\beta$

$m_2=-\alpha-j\beta,$

Where: $\alpha=\dfrac{a_2M+a_1L+RC}{2LC}$

$\beta=\sqrt{1/LC-\alpha^2}$

Calling $a_1V_0=I'$ the solution becomes:

$$I_L=\epsilon^{-\alpha t}[(A+B)\cos\beta t+j(A-B)\sin\beta t]+I'. \qquad (3)$$

A and B being fixed by the initial conditions:

(1) $\frac{dI_L}{dt}=0$ when $t=0$, (2) $I_L=0$ when $t=0$, the solution becomes:

$$I_L=I'-I'\epsilon^{-\alpha t}\cos\beta t. \qquad (4)$$

By substituting this in the differential equations expressions can readily be gotten for I_c, V, and I, applicable only for very small amplitudes.

In order that the oscillations be not quickly damped out, α must be either zero or negative, which requires that M be negative. This is a familiar experimental condition; it is found that unless the coils are coupled so that the grid potential opposes V in its effect upon the electron current the tube will not oscillate, under any circumstances. With the capacity and inductance of the magnitudes usual in circuits of this type the fulfillment of this condition makes α negative for an extended range of values of M. While the oscillations are building up under this negative damping factor the effect upon a_1 is insignificant, but, as shown by Fig. 6 the average value of a_2 for the whole cycle decreases rapidly. This, in turn, decreases α numerically, and the oscillations eventually reach an amplitude at which α is zero. Thenceforth the oscillations are sustained steadily at this amplitude.

If we use the empirical expression(1) for a_2 as a function of v_0, a value for v_0 at which α becomes zero is immediately forthcoming. Undoubtedly the investigation of a single tube furnishes no valid reason for assuming that such an empirical expression is generally applicable, but in this case the constancy of behavior of all pure electron tubes may be trusted to give it some general significance.

For this particular type of tube, all constants of the circuit being measured in practical units, the constants in equation (1) have the following values: a = 40, b = 91.7, g = 80, h = 100. Since the damping factor becomes zero when $a_2M = RC + a_1L$, the amplitude of grid potential for the steady oscillation condtiion is then given by:

$$V_0 = 40 + 91.7 \tan\left[80 - \frac{100(a_1L + RC)}{M}\right]. \quad (5)$$

After the oscillations have attained an amplitude making a_2 perceptibly a function thereof, the differential equation (2) is no longer adequate and its solution (3) does not describe the behavior of I_L. As soon as the stable condition has been reached, however, α is again constant and equal to zero, and a_2 no longer changes with time from its average value through the cycle of v. Thus with a new set of condition for fixing the constants of integration the solution (3) is still valid.

Accordingly if t be reckoned from the instant when a_2 becomes equal to $\frac{a_1L+RC}{M}$ and $\alpha = 0$, these conditions obtain: $dI_L/dt = 0$, when $t=0$, and also $I_L = 0$, and are applicable to equation (3).

A and B being determined from these conditions,:-

$$I_L = I' - {}_0I_L \cos(\beta t + \theta), \quad (6)$$

whence:
$$I_c = -{}_0I_L \cos(\beta t + \theta). \quad (7)$$

$$I = -{}_0I_L RC\beta \cos(\beta t + \theta) - I'. \quad (8)$$

$$V = {}_0I_L \sqrt{R^2 + L\beta^2} \cos(\beta t + \theta + \tan^{-1} L\beta/R) + V_0 \quad (9)$$

$$v = -v_0 \sin(\beta t + \theta). \quad (10)$$

Where: ${}_0I_L = I'\sqrt{1 + (v_0/M\beta I')^2}$ and $\tan\theta = v_0/M\beta I'$.

The following assumptions have been made:

RI' negligible compared with V_0.

$RC\beta$ " " " 1.

In practice, when the tube is put in oscillation with this arrangement of inductances a number of harmonic frequencies is present. These frequencies were first detected by their distortion of the dynamic characteristics, particularly those involving I_C. When singled out, and reinforced by a closely coupled wavemeter, as many as six such frequencies could be detected, all multiples of the main frequency $1/2\pi\sqrt{LC}$, and each causing a small loop in such curves as $I_L - dI_L/dt$. It was noted that these harmonics could be practically eliminated by inserting a relatively large self-inductance either in the filament connection or in the plate connection. Accordingly an inductance (L',R') was inserted in the plate lead and was used at times for a current deflection coil. This changes the coefficients but not the form of the equations for oscillation.

If we assume LL'C insignificant compared with a_1L; RLC and RL'C insignificant compared with LC/a_1; and RR'C insignificant compared with L, the higher derivatives of I_L vanish, and the differential equation as modified to take account of L' becomes:

$$d^2I_L/dt^2 + \left[\frac{a_2M + RC + a_1(L-L')}{LC}\right] dI_L/dt + \frac{I_L}{LC} = \frac{a_1V_0}{LC} \qquad (11)$$

Owing to the extremely small oscillating component of I the effect of L' and R' upon V and its related quantities is slight. Consequently the only important result of the insertion of L' into the circuit is that the condition for stability becomes: $a_2M = RC + a_1(L-L')$, and the grid potential amplitude at the steady oscillating state is given by:

$$V_0 = 40 + 91.7 \tan\left[\frac{80 - 100[RC + a_1(L-L')]}{M}\right] \qquad (12)$$

With this <u>increased</u> value for v_0 (due to the fact that L' is greater that L) the equations for oscillation are identical with equations (6) to (10) in all other respects.

From these expressions it is apparent that the oscillating component of I is quite small and is practically ninety degrees out of phase with V, since $L\beta/R$ is always large. V and I are important quantities in determining the conditions for oscillation, since it is upon the small plate current that the useful and relatively large current in the condenser circuit is built up.

The dynamic value of the derivative dI/dV assumes all possible values throughout a cycle and since I and V are, according to this treatment, pure sinusoids, any inclination of dI/dV to a negative value throughout a cycle must come from causes not taken into account by equation (2). While the ideal I and V characteristic would in general be ellipsoidal, and while even with this type of characteristic oscillations are mathematically possible, it is found that whatever distortion occurs in this characteristic is such as to favor the production of oscillations.

The periodicity of the oscillations is $1/\sqrt{LC}$. This periodicity was determined experimentally with a standard wavemeter over a somewhat limited range of capacities, all quantitative work being limited by the fact that only a few capacities were available on which the calibration was entirely trustworthy. L was measured by the resonance method; consequently the effective parallel capacity between adjacent turns makes the apparent value of L, which was used in the following computations, a trifle too large.

(computed)	(measured).
296×10^4	298×10^4
283	296
267	273
260	262
251	251
242	242
235	236
226	228

Equations (6), (10), and (12) indicate partially the conditions for the starting of oscillations. Apparently as soon as C is reduced to $\frac{a_1(L-L')-a_2M}{R}$ or below, the damping factor becomes negative, the oscillations begin, and become stable at amplitudes given by equation (12) and the relations involving ${}_oI_L$ and the other amplitudes derived therefrom. The currents in the LC loop increase almost proportionately with $\sqrt{C}$; this causes sufficient decrease in the amplitude of the grid potential due to decreasing periodicity to give the decreased v_o at which stability obtains, according to (12).

It is found in practice that the value of C at which the oscillations actually start is not given absolutely by the condition $RC < a_2M - a_1(L-L')$. It appears that oscillations maintained at any low amplitude of v are relatively unstable and that in order for them to begin spontaneously, grid potentials are required higher than than those at which the oscillations can be ultimately maintained. As might be expected, there is no fixed value of C and amplitude of v at which the oscillations will normally start. The tube is very erratic in this particular and the starting conditions do not lend themselves to rigorously complete mathematical analysis.

In one particular the circuit is distinctly different in the oscillating condition from its characteristic static condition; that is, v and I are thrown ninety degrees out of phase by the oscillations. Thus the dynamic a_2, like a_1, would apparently assume all possible values during a cycle, from $-\infty$ to $+\infty$. It is a debatable question at present as to how far this complication affects the applicability of the static characteristic to the tube while in the oscillating condition. Of course the equations based upon the static characteristic lead directly to this constant phase difference between v and I.

The power expended in the tube is negative, making the tube a generator. The power dissipated in the closed LC loop is confined to ${}_oI_L^2$ R (assuming the condenser to have no "phase difference"). It is probable that the tube will supply the power dissipated only within a limited range of frequencies, since at at definite high values of C the oscillations "break" in a maner not explained by the foregoing discussion. It appears that this failure to oscillate is a question of the power dissipated as well as of the damping.

The positive and negative signs of the currents and emfs. give their direction with respect to the arrows on Fig. 2. The condenser and inductance currents are practically of the same amplitude. The grid and plate potentials always oppose each other. Apparently all oscillations would be sinusoidal were it not for the fact that a_2 departs greatly in different parts of a cycle from its average value throughout that cycle.

High Frequency Characteristics.

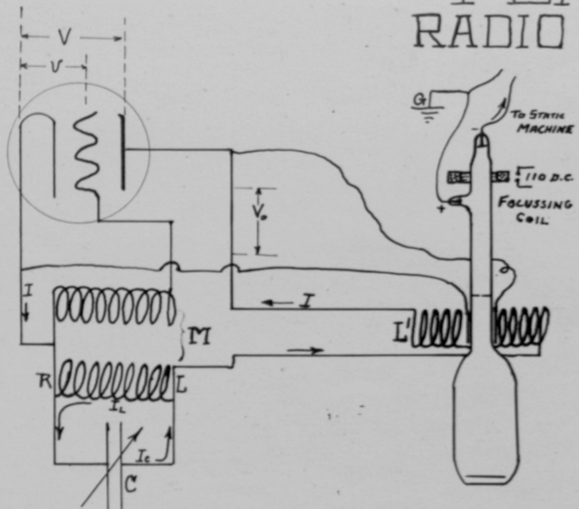
Details of Apparatus.

The use of the Braun tube in connection with an oscillating pliotron of low power presents difficulties not present in the investigation of arc circuits., since the conditions of stability in the former circuit are likely to be profoundly affected by the insertion of "current deflection coils". Two such current deflection coils were placed permanently in the circuit, although only one could be used at a time. The coil L', included in the foregoing mathematical discussion, was composed of 60 turns, and served the two-fold purpose of choking out undesirable harmonics and of providing the necessary magnetic field for appreciable deflections of the cathode stream when the small oscillating plate current passed through it. The I_L and I_C deflections were obtained with a coil of 10 turns whose ~~reac~~tance was calculated to be small, and assumed to be negligible in comparison with Lp and $1/cp$. The only observable effect of the coil upon the oscillations was that it made them harder to start. When inserted in the condenser branch it reduced the amplitude of the oscillations. When placed in series woth L it naturally increased the wavelength of the oscillations quite appreciably.

No pretense is made at accurate quantitative work

PLATE II

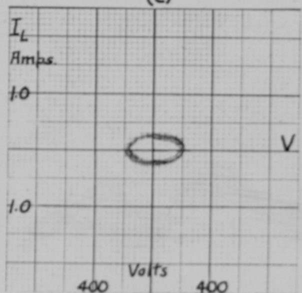
RADIO FREQUENCY CHARACTERISTICS.



(a)

(b)

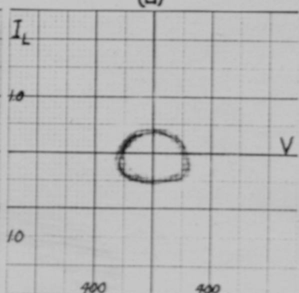
(c)



$C = 200\,\mu\mu f.$

$\beta = 797 \times 10^4$

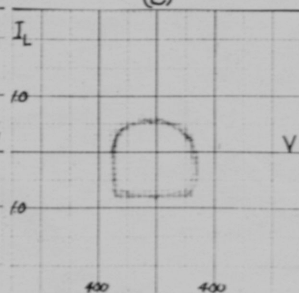
(d)



$C = 600\,\mu\mu f.$

$\beta = 490$

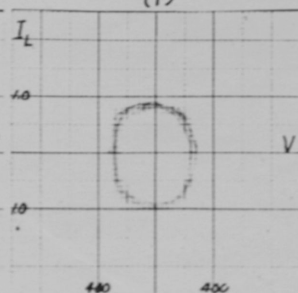
(e)



$C = 1000\,\mu\mu f.$

$\beta = 379.$

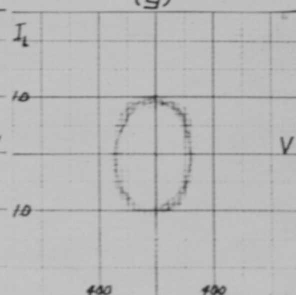
(f)



$C = 2000\,\mu\mu f.$

$\beta = 267.$

(g)



$C = 3000\,\mu\mu f.$

$\beta = 218.$

in the high frequency characteristics here reproduced, although the fluorescent screen was calibrated as accurately as possible for the coils and the potential deflection plates, using low frequency alternating currents and voltages, so that the numerical scales attached to the characteristics at least give some idea of the relative magnitudes involved.

Discussion of Characteristics.

Plate II shows the relation of inductance current to plate plate potential for five different frequencies. In this series, as in all subsequent ones, the coils were so connected that positive currents correspond to the directions of the arrows in Plate II(a). Potential deflections to the right of the zero line indicate a positive potential on the electrode considered. In all these plates, characteristics taken at the same frequency are placed in the same vertical line.

V and I_L are ninety degrees out of phase, in accordancr with equations (6) and (9). In these, as in other characteristics the distortion due to changing values of a_2 throughout a cycle is particularly noticable at the lower frequencies and smaller amplitudes of grid potential. At the higher v amplitudes the change in a_2 while passing over the bend in the characteristic plays a relatively smaller part in its cycle of values and the out-of-phase characteristics are very nearly ellipsoidal. At the lower frequencies a_2 is changing rapidly throughout practically the whole cycle.

Plate III shows first the dynamic relations of plate potential and condenser current, the latter being a measure

PLATE III.

CONDENSER CURRENT.

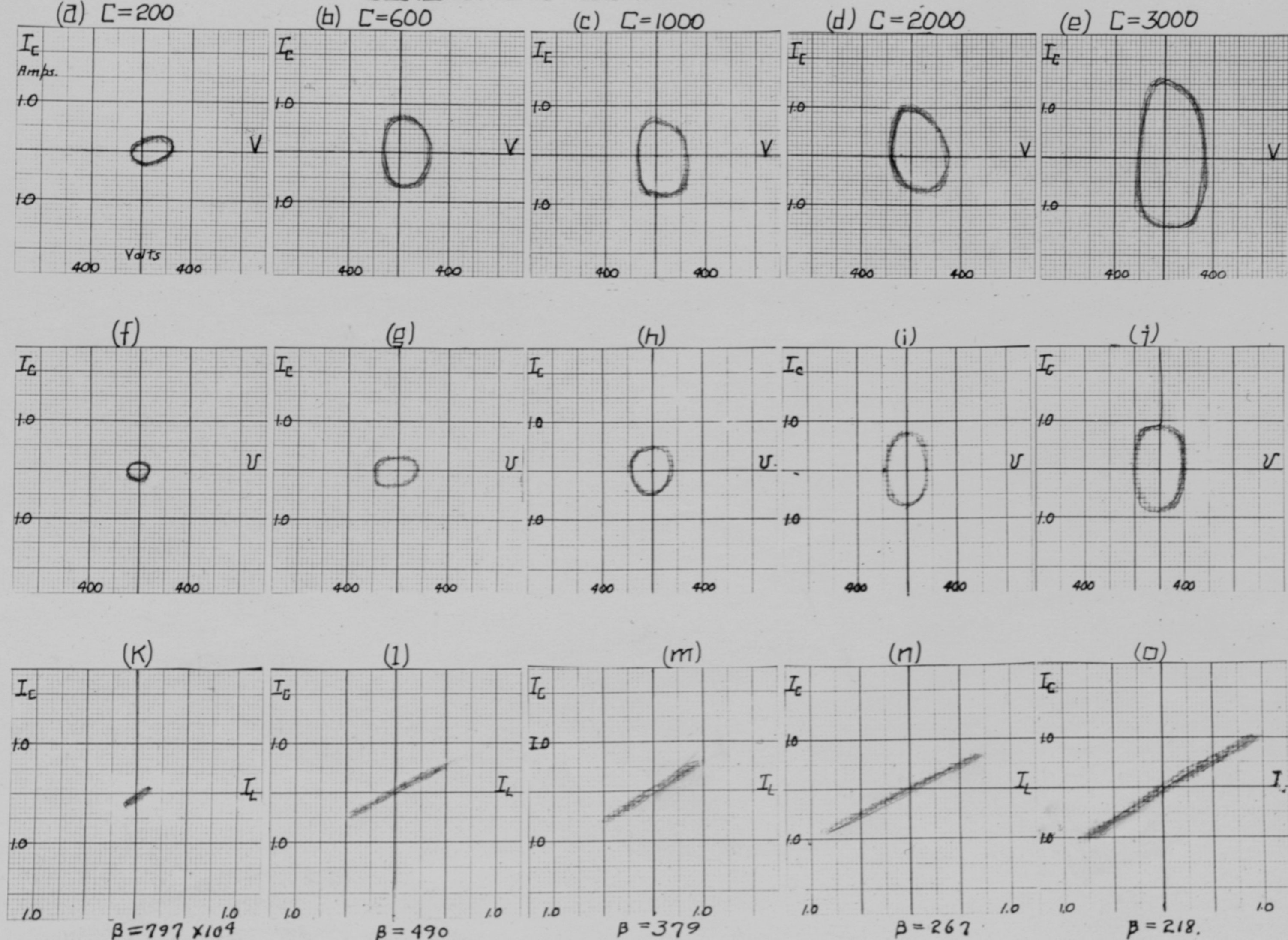
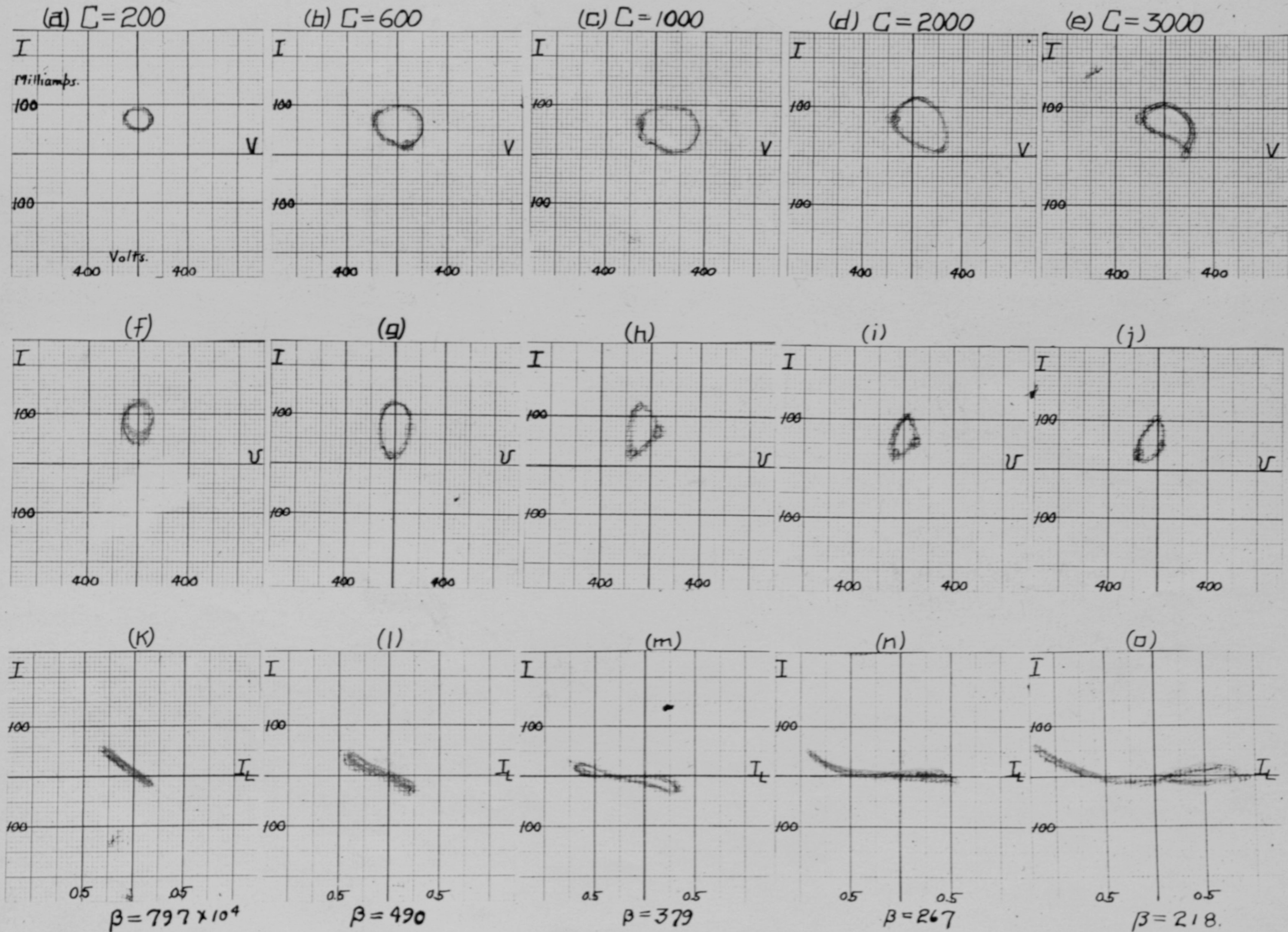


PLATE IV.

ELECTRON CURRENT.



of the time derivative of V and having of course a phase of ninety degrees with respect to it. The waves are distorted from pure sinusoids at the lower frequencies, as explained in connection with the I_c and V curves. The effect of V_o is apparent in the displacement of the loops to the right of the origin. Equations (7) and (9) seem to hold rigorously for the smaller amplitudes. This distortion is apparently due solely to the variation in a_2, also, since in the next series, taken at the same frequencies, the characteristics are also undistorted when the frequency is high. I_c and I_L are shown to be exactly opposite in phase at any particular instant.

Plate IV(a)-(e) shows the persisting tendency of the tube to behave as a negative resistance, dI/dV being negative for the greater part of a cycle (cf. Plate Ig). A distortion peculiar to the oscillating electron current, I, is apparent in these, as well as in subsequent diagrams. The small loops in the curves are not harmonic frequencies, or their effects would be apparent in the other oscillating quantities. Evidently equation (8) does not adequately represent the electron current at higher amplitudes. The increasing distortion of the electron current wave is wotthy of fyrther study, since it is shown in all characteristics involving I. The fact that dI/dV does not change uniformly throughout a cycle should prove a significant feature in connection with the conditions for starting oscillations.

In the next series similar distortions are shown while there is no predominate negative slope. The negative displacement of v in Figs. (h) and (j) is due probably to errors in reproducing the characteristic, since no fixed potential was ever impressed upon the grid. The 180° phase relation between I_L and I is brought out by the next series. The effect of I' upon both I and I_L is apparent in the constant dispalcement down and to the left, while the loops in the curves are due wholly to the distorted I wave.

Summary.

Static characteristics, showing the relation of electron current to grid potential and to plate potential were obtained for a pure electron discharge, "pliotron" tube. The grid voltage - plate current characteristic was extended over a large range of values by applying to the grid the equivalent of an alternating potential of low frequency. An empirical expression was derived for the average value of dI/dv throughout a cycle as a function of the amplitude of that cycle. A Braun tube was employed to plot dynamic characteristics at low frequencies, obtained by applyibg alternating voltages of periodicity 377 radians per second to the plate and to the grid. These were found to be similar in shape to the static characteristics. The net effect of both grid and plate potentials when the grid was couple negatively to the plate circuit was found to be the behavior of the tube as a negative resistance to potentials applied between plate and filament.

A simple oscillating circuit was discussed in detail mathematically. The differential equation for the inductance current in this circuit was solved, its solution being based

on the assumption of constant dI/dv. Two separate cases of constant a_2 were emphasized, first for such low amplitudes that the I and v characteristic is linear, and second, for the particular amplitude at which the damping factor becomes zero, and the oscillations are stable. It was shown that the damping factor is negative when the oscillations begin, that it decreases with increasing amplitude, and that it finally becomes zero when $a_2M = a_1L + RC$. By expressing the initial conditions corresponding to the first case mentioned above a complete solution for I_L for low amplitudes was obtained. With a new set of conditions, based on the damping factor equal to zero at an amplitude of grid potential (furnished by the empirical expression) which makes $a_2M = RC + a_1L$, an expression for I_L was deduced which holds true only after the steady state is reached. From this were derived expressions for all the other variables of the circuit, I_c, V, v, and I. These quantities were found to be sinusoidal functions of time. V is practically ninety degrees out of phase with I_c, I_L, and I; I_c and I_L are directly opposite in phase, while V and v are 180° our of phase.

These relations were all verified by taking Braun tube characteristics at radio frequencies, the tube generating spontaneous oscillations. The waves were found to be distorted by the departure of a_2 during a cycle from its average value for the whole cycle.

While the circuit investigated at this time was an extremely simple one, the results obtained indicate that the Braun tube analysis will prove very profitable when applied to the more complex circuits. The work is being continued with a type of harmonic circuit which cannot be readily analyzed mathematically.

I am greatly indebted to Dr. F. A. Kolster of the U. S. Bureau of Standards for having provided the pliotron tube with which these experiments were made. I am also grateful to Professor M. E. Rice of the Department of Physics for his kindly and valuable suggestions and criticisms upon the methods employed in the work.

Blake Physical Laboratory,

June 1, 1918.

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