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# Simple time-variant, band-pass filtering by operator scaling

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ABSTRACT

A convolutional method of time-variant, band-pass filtering presented shows that a change of filter cutoff frequencies with time is achieved by frequency scaling the amplitude spectrum of a reference operator. According to the scaling property of the Fourier transform, this frequency scaling is actually accomplished by a simple time-domain scaling of the reference operator in which the filter operator at a sample point on a seismogram is obtained by compressing the reference operator after multiplication by a constant value. Therefore, the length of filter operator changes as the cutoff frequencies and the pass band change with time; the higher the cutoff frequencies and the broader the passband, the shorter the operator length. The algorithm does not involve any complex-valued arithmetic that may significantly reduce the computational efficiency if a small computer is used. Because the time-variant convolution formula is exact, the change of cutoff frequencies is not limited to slowly varying or monotonic variations used in other algorithms.

The way of changing cutoff frequencies restricts the passband of the filter to a constant value in terms of octaves. However, this restriction can be relaxed significantly in practical usage by a cascaded implementation if the Nyquist frequency is well above the passband of the filter. Computational efficiency of the method is quite comparable to that of the time-invariant, band-pass filtering. Tests of the method on both real and synthetic data sets confirm the effectiveness of the filter.

## INTRODUCTION

In reflection seismology, a noticeable change in the apparent frequency of reflection events is quite often observed within finite time gates of interest. The dominant cause of this frequency variation is the absorption of seismic energy in the form of heat energy (Dobrin and Savit, 1988, 46). High-frequency energy is attenuated at a greater rate than lower-frequency energy, resulting in a progressive lowering of apparent frequency of reflection events with increasing time. In high-resolution shallow reflection data, this character is observed more frequently than in conventional reflection data. The reasons for this seem to be related to the rapid change of elastic parameters with depth in the near surface (Born, 1941). Time-variant deconvolution can be used to compensate for the loss of high-frequency components of reflection energy, but this process can also accentuate noise if the level of reflection energy has been attenuated below that of the noise. This situation of low signal-to-noise ratio (S/N) is common in shallow reflection data. Therefore, time-variant, band-pass filtering, which honors the time-variant nature of signal and attempts to curtail (instead of boost and balance) noise, can be useful in improving S/N.

Many authors have developed various kinds of time-variant, band-pass filters for geophysical data processing. These filters can be grouped into several distinct categories according to the method of implementing the time variance of the filter cutoff frequencies. One straightforward method uses an overlapping sequence of gate-wise time-invariant filters (Lenihan, 1972). An alternative method uses filter cutoff frequencies that change continuously with time (the true time-variant method) (Nikolic, 1975; Pann and Shin, 1976; Stein and Bartley, 1983; Scheuer and Oldenburg, 1988). According to the method of filter implementation, such filters may be grouped into convolutional (Pann and Shin, 1976; Scheuer and Oldenburg, 1988) and recursive (Shanks, 1967; Stein, 1981; Stein and Bartley, 1983; Nikolic, 1975) methods. The convolutional method is based upon the

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well-known notion that filtering in the frequency domain is equivalent to convolution in the time domain. In the recursive method the filter coefficients are computed based upon the recursive difference equation obtained from the system transfer function (usually of the Butterworth type), and these coefficients are used as weights for input and previous output values to perform the prescribed filtering (Oppenheim and Schafer, 1989, 36). Advantages and disadvantages of each group are well summarized in Scheuer and Oldenburg (1988). However, several filter features are worth mentioning here in the context of the characteristics of our method. The standard method of the gate-wise, time-invariant filter is easiest to implement, however, the transition effects between the adjacent gates may become troublesome. A continuous time-variant method can better account for the variation of the apparent frequency on a real seismogram, but the algorithm is more involved and computationally intensive than is the standard time-invariant method. The recursive method, compared to the convolutional method, is easy to implement and computationally efficient. However, the filter response may differ from the one desired if a small number of filter coefficients are used. Furthermore, the phase response of the recursive filter is not zero-phase and requires both forward- and reverse-time implementations to achieve the desired zero-phase response. The convolutional method, in contrast, allows the filter response to be controlled precisely. Most convolutional methods currently used involve complex-domain calculations to allow the cutoff frequencies to change continuously with time, which may make the algorithm more involved and computationally intensive (Pann and Shin, 1976; Scheuer and Oldenburg, 1988). Efforts to reduce the computational costs in these complex-valued convolutional methods involve the sacrifice of some generality. For example, in the method in Pann and Shin (1976) the bandwidth of the filter is restricted to a constant value and the cutoff frequencies change slowly with time. In the method in Scheuer and Oldenburg (1988) the cutoff frequencies change slowly to increase the computational efficiency.

We present a continuous time-variant, band-pass filter based upon the scaling property of the Fourier transform. The change of the cutoff frequencies with time is accommodated by frequency scaling the amplitude spectrum of a reference operator, and in actual implementation this scaling is accomplished in the time domain by compressing the reference operator after multiplication by a constant value. The scaling (the compressing) ratio at a particular sampling point on a seismogram is determined by the ratio of the low (or high)-cutoff frequency specified at a given sample point to the low (or high)-cutoff frequency of the reference operator. The compression is accomplished using a simple linear interpolation scheme and no other higher-order polynomial scheme needs to be used. Because of the compression, the operator length changes from one sample point to another and is always less than or equal to that of the reference operator. Therefore, the method is simple and computationally efficient. Furthermore, the algorithm does not involve any complex-valued arithmetic that may significantly reduce the computational efficiency if a small computer is used, as is usually the case in engineering seismology. Changing the cutoff frequencies through the frequency scaling of the reference operator restricts the passband of the filter operator to a constant value in octaves. However, this restriction can be relaxed in practical usage by a cascaded implementation if the Nyquist frequency is far above the passband of the filter. Since the time-variant convolution formula is exact, the change in cutoff frequency is not limited to slowly varying or to monotonic types developed by previous investigators. Although the method has been developed for high-resolution shallow reflection data, it is equally applicable to conventional exploration seismic data.

## ALGORITHM

A key assumption in the development of the time-variant, band-pass filtering algorithm described here is that the width of the passband of the filter is assumed to remain a constant value in octaves. Therefore, it is assumed that the low- and high-cutoff frequency functions, $f_L(t)$ and $f_H(t)$, are not independent but are related through the bandwidth $\alpha$. A way to relax this restriction in practical usage will be discussed in the next section.

Let $x(t)$ represent the input seismic trace, $y(t)$ the output filtered trace, and $h_t(\tau)$ the time-domain expression of filter operator at time $t$. Next, $h_t(\tau)$ is assumed to be real and zero-phase. Then the output is obtained by convolution of the input $x(t)$ with the filter operator $h_t(\tau)$:

$$y(t) = \int_{-\infty}^{\infty} h_t(\tau)x(t-\tau)\, d\tau. \tag{1}$$

Let $|H_t(f)|$ represent the amplitude spectrum of $h_t(\tau)$. Since $h_t(\tau)$ is real and symmetrical, $H_t(f)$ is also real and symmetrical (Oppenheim and Willsky, 1983, 203). Consider a reference amplitude spectrum $|H_r(f)|$ of idealized rectangular shape with low- and high-cutoff frequencies of $f_{\ell c}$ and $f_{hc}$, respectively. Assume that the amplitude spectrum $|H_t(f)|$ of the filter operator at time $t$, $h_t(\tau)$, is obtained by scaling $|H_r(f)|$ along the frequency axis with a positive scaling ratio $a$, i.e.,

$$|H_t(f)| = |H_r(af)| \qquad (a > 0), \tag{2}$$

then $h_t(\tau)$ can be expressed as

$$h_t(\tau) = \frac{1}{a}\, h_r\left(\frac{\tau}{a}\right). \tag{3}$$

This is the scaling property of the Fourier transform (Oppenheim and Willsky, 1983, 207). It says that the filter operator at time $t$ is obtained by amplitude and time scaling the reference operator $h_r(\tau)$ with a scaling ratio $1/a$. The cutoff frequencies of the filter at time $t$ have now become $f_{\ell c}/a$ and $f_{hc}/a$, but the bandwidth in octaves remains unchanged, i.e., $\log_2 (af_{hc}/af_{\ell c}) = \log_2 (f_{hc}/f_{\ell c})$. The scaling ratio $a$ is a function of time and is determined by the ratio of either the low-cutoff frequency $f_L(t)$ or the high-cutoff frequency $f_H(t)$ at time $t$ to the corresponding cutoff frequency ($f_{\ell c}$ or $f_{hc}$) of the reference operator. That is, if $f_L(t)$ is given, then

$$a(t) = f_{\ell c}/f_L(t), \tag{4}$$

or, if $f_H(t)$ is given, then

$$a(t) = f_{hc}/f_H(t). \tag{5}$$

Substituting equation (4) into (3) and equation (3) into (1) results in the following convolution formula for a time-variant, band-pass filter in which the low-cutoff frequency function $f_L(t)$ and the bandwidth $\alpha$ in octaves of the filter are given as

$$y(t) = \int_{-\infty}^{\infty} \frac{f_L(t)}{f_{\ell c}} h_r\left(\tau \frac{f_L(t)}{f_{\ell c}}\right) x(t - \tau)\, d\tau. \tag{6}$$

The design of the reference filter operator $h_r(\tau)$ in equation (6) is now the critical factor in implementing this basic algorithm.

## DESIGN OF REFERENCE FILTER OPERATOR

The reference filter operator $h_r(\tau)$ is usually obtained as follows: an amplitude spectrum is designed in the frequency domain with specific cutoff frequencies and rolloff characteristics, then it is inverse-Fourier transformed to generate the time-domain expression, and finally the expression is truncated effectively. In the following discussion the bandwidth of the filter operator is given in octaves unless otherwise stated.

If only a bandwidth $\alpha$ is given for the reference operator, there can be infinitely many choices for the cutoff frequencies $f_{\ell c}$ and $f_{hc} = f_{\ell c} 2^{\alpha}$. If the processing were done in analog form, there would be no advantage in choosing one particular reference operator over another. In discrete signal processing, however, the accuracy of the time scaled operator becomes an issue because the scaling involves interpolation along the time axis as indicated by equation (6). The time scaling may be either compression or stretching, depending upon the ratio $a(t)$. Compression occurs if $a(t)$ is smaller than 1, and stretching results if $a(t)$ is greater than 1. Compression is always more desirable than stretching because stretching requires a higher-order interpolation scheme to achieve the same degree of accuracy (Burden and Faires, 1985, 78). Thus, if the bandwidth $\alpha$ and the low-cutoff frequency function $f_L(t)$ are given, the minimum of the low-cutoff frequencies, MIN $\{f_L(t)\}$, and $2^{\alpha}$MIN $\{f_L(t)\}$ may be used for the low- and high-cutoff frequencies of the reference operator, respectively.

The convolutional method of filtering has the inherent property that both performance and computation time are sensitive to the operator length. The effective operator length is usually determined by truncating the time-domain representation of the desired (ideal) frequency-domain amplitude spectrum at a sample point where the discarded filter energy becomes negligible. This length affects the computation time of filtering because each sample on a seismogram is multiplied and summed as many times as the number of coefficients included in the truncated operator. Truncation in the time domain also affects the accuracy of the filtering because it causes the amplitude spectrum of the actual operator to deviate from the desired one. The truncation of all the reference operators used in this paper was done at the sample point where the cumulative energy is 90% of the total energy. Once the effective operator length $N_{r,eff}$ of the reference operator has been determined, the length $N_{eff}(t)$ of the operator at time $t$ is determined by the scaling ratio $a(t)$ as

$$N_{eff}(t) = a(t) N_{r,eff}, \tag{7}$$

and if the nature of the scaling is to be compression,

$$N_{eff}(t) \le N_{r,eff} \qquad (t \ge 0). \tag{8}$$

This reduction of the operator length can be used to save computation time during the convolutional calculation. This topic will be discussed in a later section on "computational efficiency."

For given cutoff frequencies and bandwidth, the effective length of the reference operator $N_{r,eff}$ depends on the rolloff characteristics (the shape) of the amplitude spectrum. For example, two typical types of amplitude spectra used commonly in filter design are rectangular and Hanning windows (Ziemer et al., 1989, 464). The Hanning window amplitude spectrum results in a shorter effective filter length and less deviation of the actual spectral shape from the desired one than does the rectangular window. Therefore, the Hanning window is preferred in the design of the reference operator unless a strict definition of cutoff frequencies is required during the filtering.

## EXAMPLES OF CONSTANT-OCTAVE, BAND-PASS FILTERING

Figure 1a shows a set of sinusoidal signals in which the frequency changes from 50 Hz to 300 Hz in 5 Hz increments. Each trace is 512 ms long with a 0.5-ms sampling interval. These data were used as input data for the time-variant, band-pass filtering with constant-octave bandwidth. The filtering had a one-octave passband of 50 Hz to 100 Hz at 0 ms, 150 Hz to 300 Hz at 256 ms, and back to 50 Hz to 100 Hz at 512 ms as indicated by the solid lines superimposed on the filtered section. Although the time-varying trend of the cutoff frequency can be any type, a linearly varying trend was used for simplicity. The reference operator was designed with low- and high-cutoff frequencies of 50 Hz and 100 Hz, respectively, using the Hanning window amplitude spectrum with a 17-dB amplitude response at the two cutoff frequencies. The two cutoff frequencies are the minima of the low- and high-cutoff frequencies. The compressed operator was obtained from the reference operator by a simple linear interpolation.

Figure 1b shows the output of the filtering. The time-domain expressions of the operator and the amplitude spectra are also shown below the output section at every 25 ms. These output data confirm the effectiveness of the filter.

Two high-resolution shallow reflection field files (with first-arrival muting applied) are shown in Figure 2a. Several reflections are identifiable at shallow depth. The apparent frequency of the reflections changes rapidly from about 120 Hz for reflection $R_1$ to 40 Hz for reflection $R_2$. Shallow reflection seismology deals with the most attenuative part of the earth, and it is not uncommon to observe this much drastic change in the frequency content of reflection signals over a short time period. Constant-octave, time-variant, band-pass filtering was applied to this data set with low- and high-cutoff frequencies varying linearly with time from 80 Hz–240 Hz to 20 Hz–60 Hz over the hyperbolic time gate defined by the onset times of the two reflections $R_1$ and $R_2$. The cutoff frequencies above and below this time gate were set to be time-invariant at 80 Hz–240 Hz and 20 Hz–60 Hz, respectively. The Hanning window amplitude spectrum with a 20 Hz–60 Hz passband and with −17 dB amplitude response at the cutoff frequencies was used to design the reference operator. The filtered output sections reveal better-defined reflections with improved S/N ratio.

## CASCADED IMPLEMENTATION TO RELAX THE RESTRICTION

The restriction of constant bandwidth described in the previous section raises a limitation in the practical usage of this filter concept. However, as long as the Nyquist frequency is far above the effective signal frequencies, the restriction may be relaxed by using a cascaded implementation of the constant-octave filtering algorithm.

If it were possible to design true low- and high-cut filters based upon the method of operator scaling presented, the restriction would be removed by a cascaded implementation with two independent low- and high-cut filtering passes. The design of a high-cut filter would not be a problem because the low-cutoff frequency could be set to 0 Hz regardless of the scaling ratio (Figure 3a). However, the method cannot be used to design the low-cut filter because of folding about the Nyquist frequency or drifting of the high-cutoff frequency to a lower value than the Nyquist frequency. Scaling the

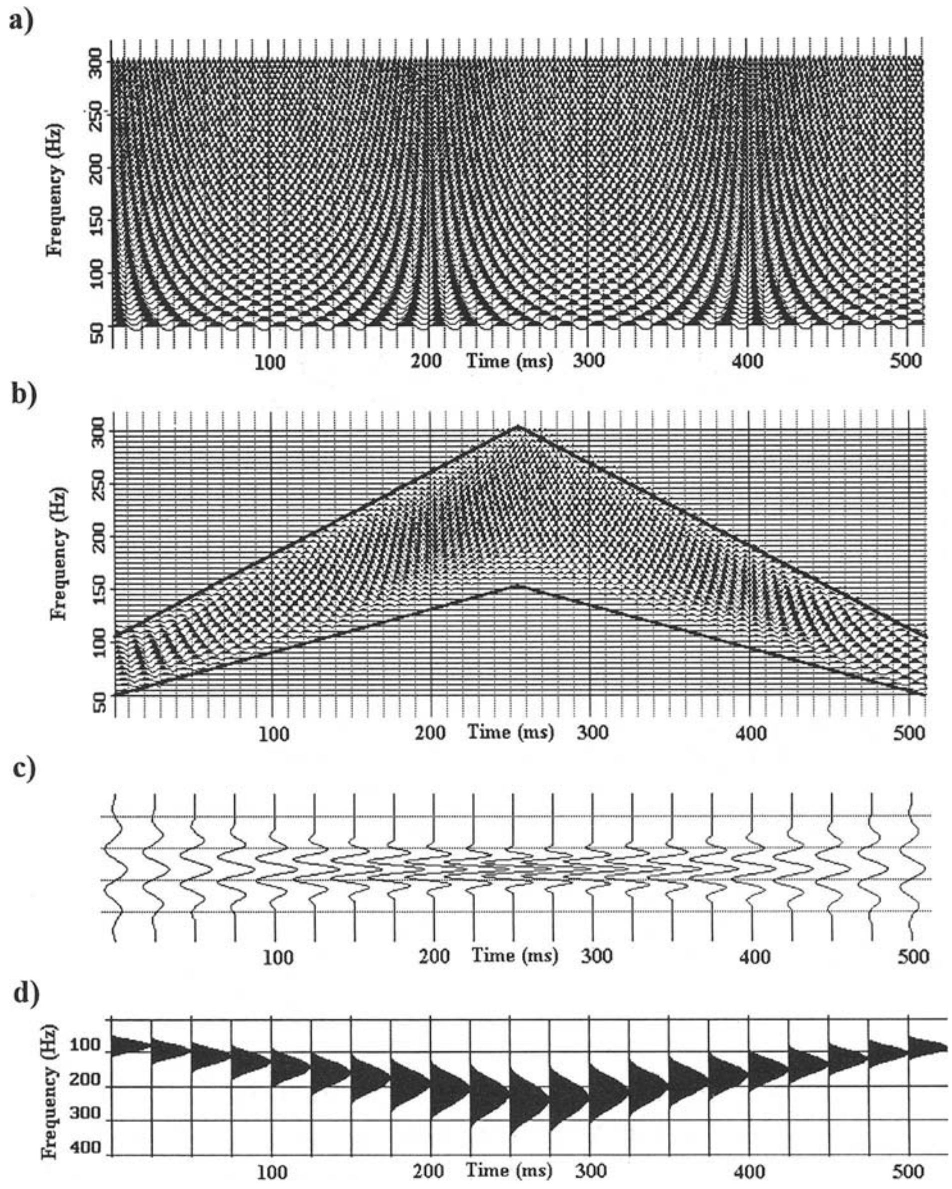


FIG. 1. (a) A set of sinusoidal signals in which the frequency changes from 50 Hz at the bottom trace to 300 Hz at the top trace in 5 Hz increments. A sampling interval of 0.5 ms was used giving a Nyquist frequency of 1000 Hz. (b) The output filtered section obtained by using the method of time-variant, constant (one)-octave, band-pass filtering presented with the cutoff frequencies varying with time as indicated by the solid lines. (c) The time-domain and (d) the frequency-domain expressions of the filter operator displayed at every 25 ms interval. The reference operator was designed using a Hanning-window amplitude spectrum with a passband of 50 Hz–100 Hz.

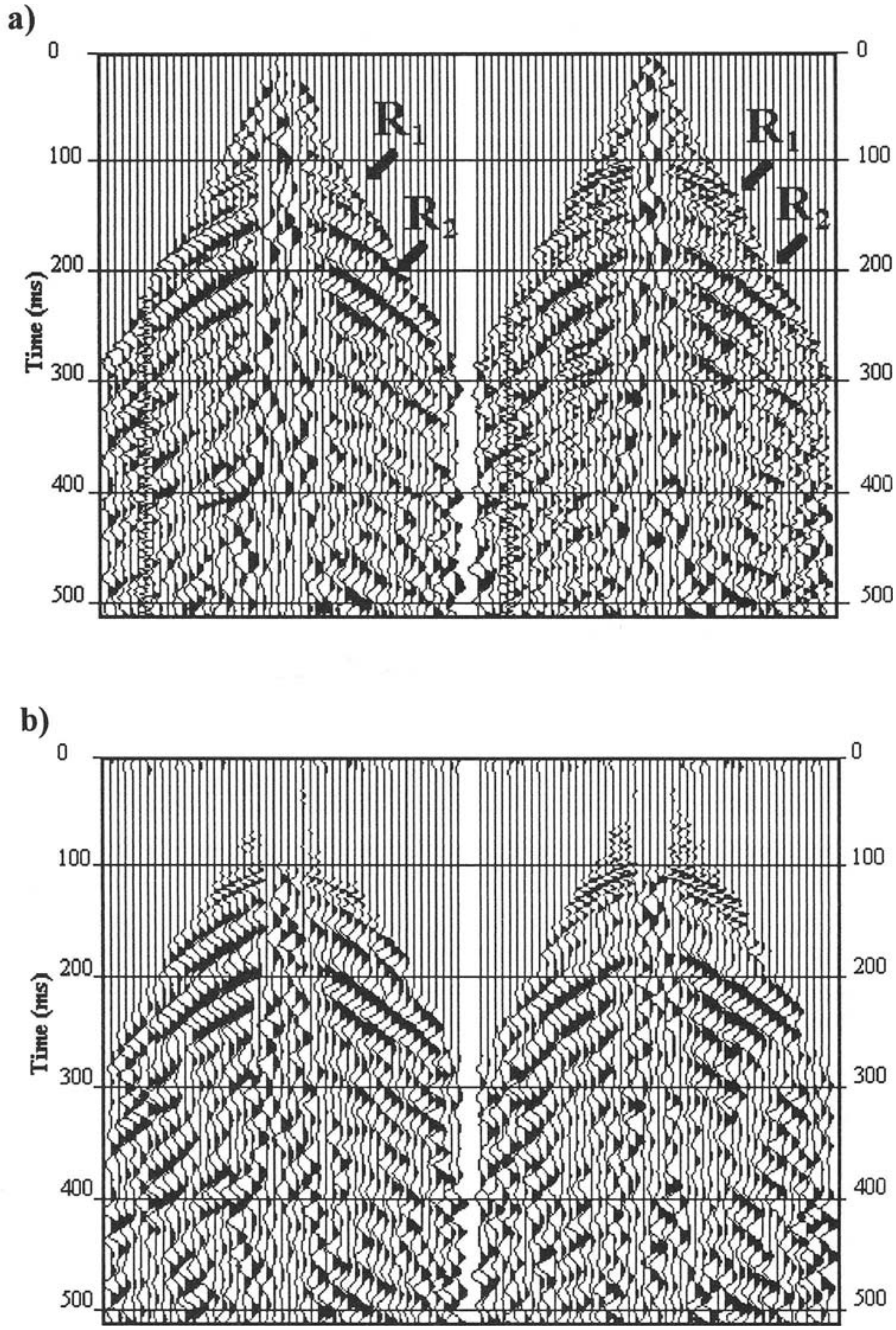


FIG. 2. (a) Two high-resolution shallow reflection field files in which the apparent frequency of reflection changes approximately from 120 Hz at reflection R1 to 40 Hz at reflection R2. (b) Time-variant band-pass filtering of (a) using the method presented. Passband of the filtering varies from 80 Hz–240 Hz to 20 Hz–60 Hz within the hyperbolic time-gate defined by the approximate arrival times of R1 and R2.

low-cut operator will cause "folding back" of the frequencies higher than the Nyquist frequency if the scaling ratio is smaller than unity, and it will cause drifting of the high-cutoff frequency to a lower value than the Nyquist frequency, resulting in a band-pass filter instead of low-cut filter, if the ratio is greater than the unity (Figure 3b).

Arbitrary band-pass filters may be constructed by cascading constant-octave, band-pass filters. Low- and high-cut filtering may be done in a cascaded form using this method if the scaling ratio $a(t)$ and the bandwidth $\alpha$ of the reference operator are properly chosen based upon low- and high-cutoff frequency functions $f_L(t)$ and $f_H(t)$ for two separate filter runs (Figure 4). Assume $f_L(t)$ and $f_H(t)$ are given independently, then for high-cut filtering, the scaling ratio $a(t)$ is determined as a ratio of the high-cutoff frequency $f_{hc}$ of the reference operator to the value given by $f_H(t)$, i.e.,

$$a(t) = f_{hc}/f_H(t). \tag{9}$$

In this implementation, the low-cutoff frequency $f_{\ell c}(t)$ of the operator at time $t$ must be smaller than or equal to the value given by $f_L(t)$, i.e.,

$$f_{\ell c}(t) \le f_L(t). \tag{10}$$

For the high-cut run, a high-cut filter with the passband starting at 0 Hz could be used, but the operator length is longer than that of the band-pass because of the inclusion of excessive low-frequency components in the design of the reference operator. Accordingly, for the low-cut run, $a(t)$ is determined as a ratio of the low-cut frequency $f_{\ell c}$ of the reference operator to the value given by $f_L(t)$, i.e.,

$$a(t) = f_{\ell c}/f_L(t). \tag{11}$$

It must be assured that the high-cutoff frequency $f_{hc}(t)$ of the operator at time $t$ be greater than or equal to the value given by $f_H(t)$, i.e.,

$$f_{hc}(t) \le f_H(t). \tag{12}$$

There is, however, another condition to be met in the low-cut run: the high-cutoff frequency $f_{hc}(t)$ of the operator should not exceed the Nyquist frequency $f_{Nq}$ at any time because of the reasons explained in the previous paragraph, i.e.,

$$f_{hc}(t) \le f_{Nq}(t). \tag{13}$$

Thus, three inequalities, (10), (12), and (13), should be satisfied simultaneously. The remaining issue now is the selection of the bandwidth $\alpha$ for the reference operator. In order to satisfy the two inequalities (10) and (12) simultaneously it is obvious that $\alpha$ should satisfy the following condition:

$$\alpha \ge \text{MAX}\,\{f_H(t)/f_L(t)\}. \tag{14}$$

Therefore, taking the maximum bandwidth set by the two cutoff frequency functions $f_L(t)$ and $f_H(t)$ would be a choice for the bandwidth of the reference operator. Then, the following cutoff frequencies may be selected to design the two reference operators for high- and low-cut runs:

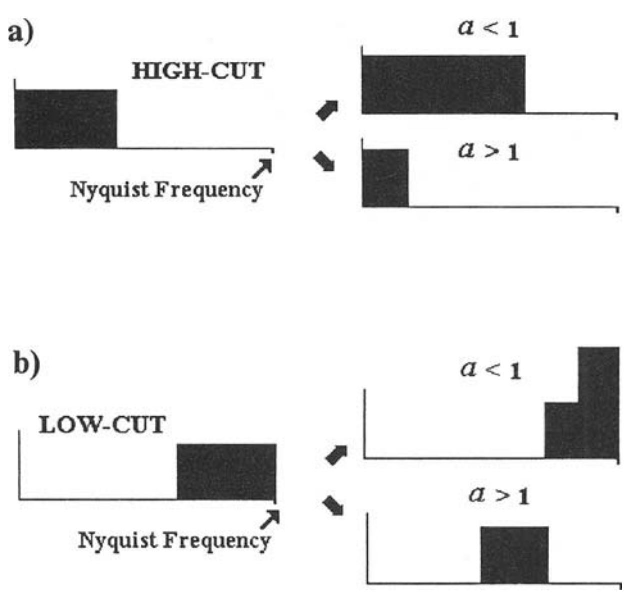


FIG. 3. A schematic illustration showing the feasibility of designing time-variant, high-cut and low-cut filters based on the presented method of frequency scaling a reference amplitude spectrum. (a) Designing the time-variant, high-cut filter is possible because the low-cutoff frequency could be set to 0 Hz regardless of the scaling ratio, but (b) designing the time-variant, low-cut filter is not possible because of either folding about the Nyquist frequency or drifting of the high-cutoff frequency to a lower value than the Nyquist.

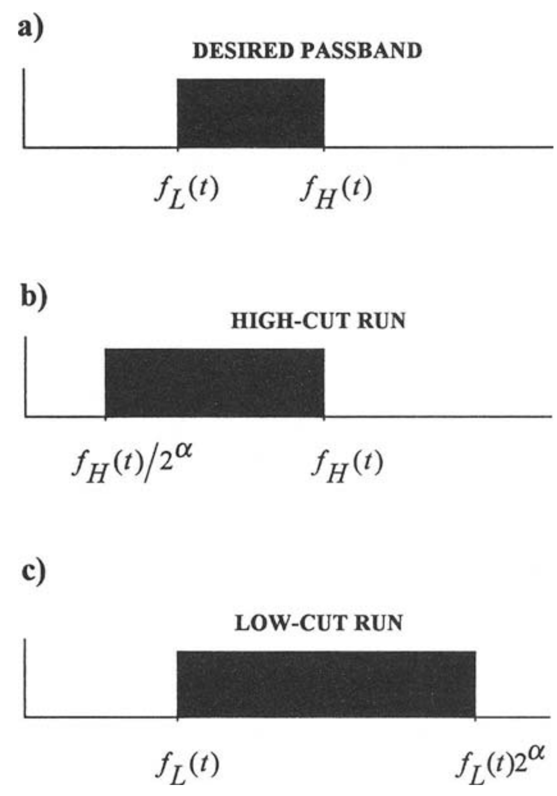


FIG. 4. Band-pass filtering with arbitrary cutoff frequencies can be accomplished by a cascaded run of two constant-octave, band-pass filters with each filter run to set the high-cut and low-cut, respectively.

$$\text{(For High-Cut Run)}\ f_{\ell c} = \text{MIN}\ \{f_H(t)\}/2^{\alpha},$$

$$f_{hc} = \text{MIN}\ \{f_H(t)\}, \qquad (15)$$

and

$$\text{(For Low-Cut Run)}\ f_{\ell c} = \text{MIN}\ \{f_L(t)\},$$

$$f_{hc} = 2^{\alpha}\ \text{MIN}\ \{f_L(t)\},$$

with

$$\alpha = \text{MAX}\ \{f_H(t)/f_L(t)\}. \qquad (16)$$

Inequality (13) can now be expressed in terms of two cutoff frequency functions $f_L(t)$ and $f_H(t)$:

$$\text{MAX}\ \{f_{hc}(t)\} = 2^{\alpha}\ \text{MAX}\ \{f_L(t)\} \leq f_{Nq}. \qquad (17)$$

Therefore, this condition can be checked once the cutoff frequency functions are given. It is this condition that prevents the low- and high-cutoff frequency functions $f_L(t)$ and $f_H(t)$ from being completely independent. However, in most seismic data, the Nyquist frequency is well above the effective band of signal, and the inequality (17) can usually be satisfied.

It has been assumed implicitly that in the cascaded implementation the type of amplitude spectra for the reference operator should be the rectangular window instead of the Hanning window. This is required because the rolloff characteristics in the passband of the filter can remain unchanged only if the rectangular window is used. However, as discussed in an earlier section, the rectangular window amplitude spectrum results in a longer operator than does the

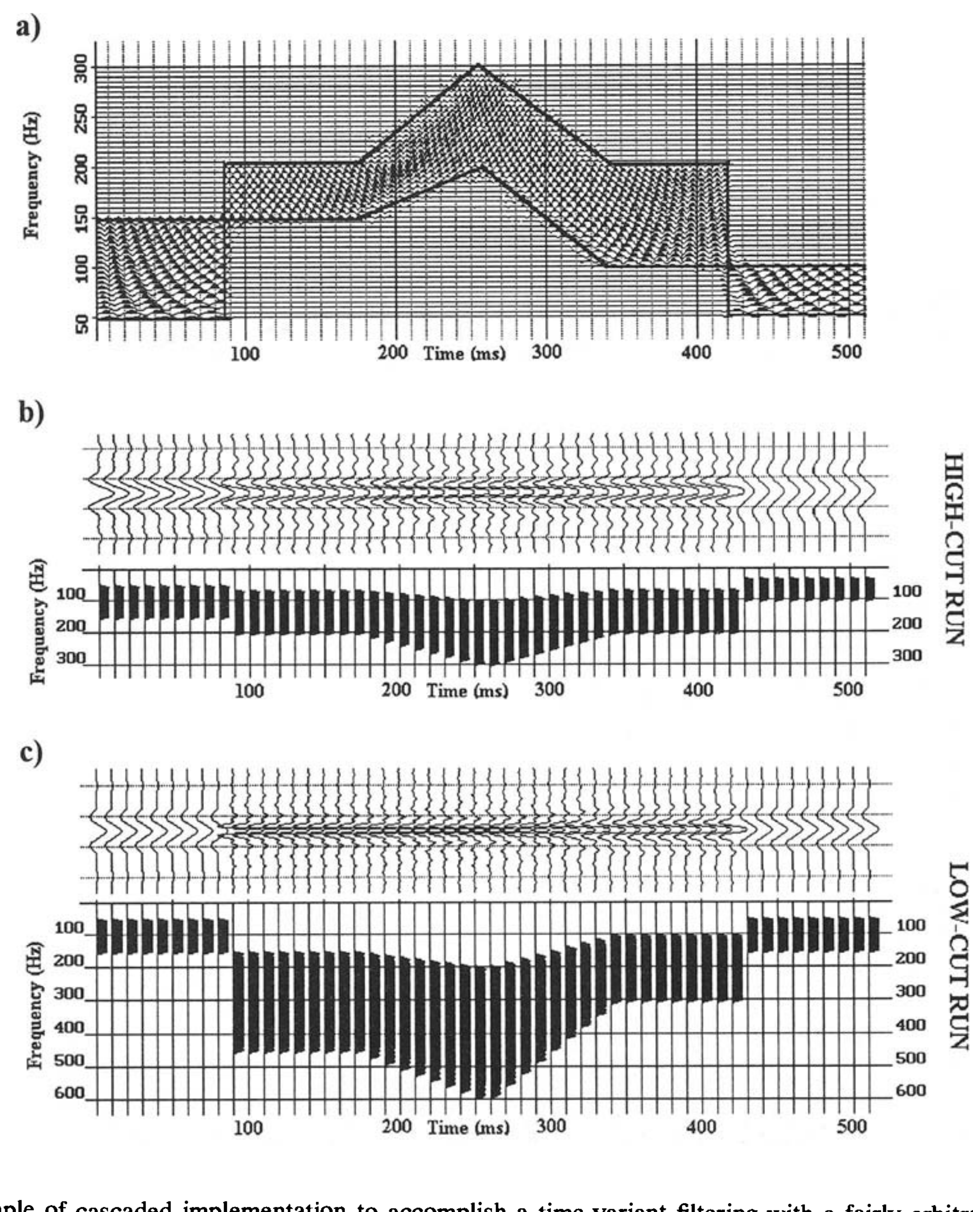


FIG. 5. An example of cascaded implementation to accomplish a time-variant filtering with a fairly arbitrary passband. (a) Filtered output section on which solid lines are superimposed to indicate the passband of the filtering; time- and frequency-domain representations of the operator in (b) the high-cut run, and (c) in the low-cut run. The passband resulting from the cascaded implementation is the overlapping band of these two spectra.

Hanning window amplitude spectrum. Therefore, if the cutoff frequency functions do not deviate significantly from the constant-octave trend and a strict preservation is not a main issue, the Hanning window filter may still be used to increase the computational efficiency.

Figure 5 shows an example of the cascaded implementation. The cutoff frequency functions are indicated by the solid lines. Rectangular amplitude spectra were used during the design of the two reference operators, and a 0.5-ms sampling interval was used to produce a 1000-Hz Nyquist frequency. Low- and high-cutoff frequencies were chosen arbitrarily, and their changing trend included the extreme case of staircase variation. This example illustrates that the cascaded implementation can significantly relax the restriction of the constant-octave bandwidth in practical usage and also that there is no restriction on the changing trend of cutoff frequencies.

## COMPUTATIONAL EFFICIENCY

The computational efficiency of a convolutional method of filtering is directly proportional to the length of the filter operator. Our method of time-variant, band-pass filtering has a filter operator whose length changes with time and is always less than that of the reference operator. Recalculation of the coefficients does not involve any more complicated operation than does a linear interpolation. Because of these two facts, the method is fast, and its computational efficiency is comparable to that of time-invariant, convolutional, band-pass filtering. This is explained in the following using a specific example.

The time-variant passband of our method for a constant-octave case is illustrated in Figure 6a. Cutoff frequencies change linearly with time from 80 Hz–240 Hz to 20 Hz–60 Hz over 512 ms of recording time. The change of operator length with time is illustrated below. The length changes nonlinearly with time and is less than or equal to that of the reference operator (20 Hz–60 Hz). To evaluate the computational efficiency, the operator length for this time-variant filtering is compared to three time-invariant cases. Passbands for the time-invariant cases are those that might be chosen for a seismogram whose effective signal band changes according to the trend shown in Figure 6a. Total cumulative operator lengths for both time-invariant and time-variant cases are shown in Figure 6c and confirm that the length for the time-variant case is about the same as the average length of the three time-invariant cases. Thus, the computation times of both cases should be about the same. However, there is one additional step that takes extra computation time in the case of time-variant filtering, which is the recalculation of the operator because of the compression of the reference operator. A linear interpolation scheme turned out to be accurate enough for this recalculation and no higher-order interpolation scheme needs to be used. The linear interpolation algorithm involves several multiplication and summations for each filter coefficient, therefore, the actual computation time for the time-variant case may be several times longer than that for the time-invariant case.

This ratio, however, can be reduced if an additional checking statement is incorporated into the algorithm to determine if the change of cutoff frequencies is significant enough to warrant recalculation of the coefficients. If the change of cutoff frequencies from one sample point on a seismogram to the next sample is small (say, less than 1 Hz) it may not be necessary to recalculate the coefficients. This will omit the step of linear interpolation and save computation time. From extensive synthetic experiments, we conclude that, regardless of whether we are dealing with high-resolution or conventional exploration seismic data, a recalculation is not necessary over several sample points on a seismogram. Because the cascaded implementation discussed in the previous section simply consists of a serial application of two constant-octave runs, our method in its

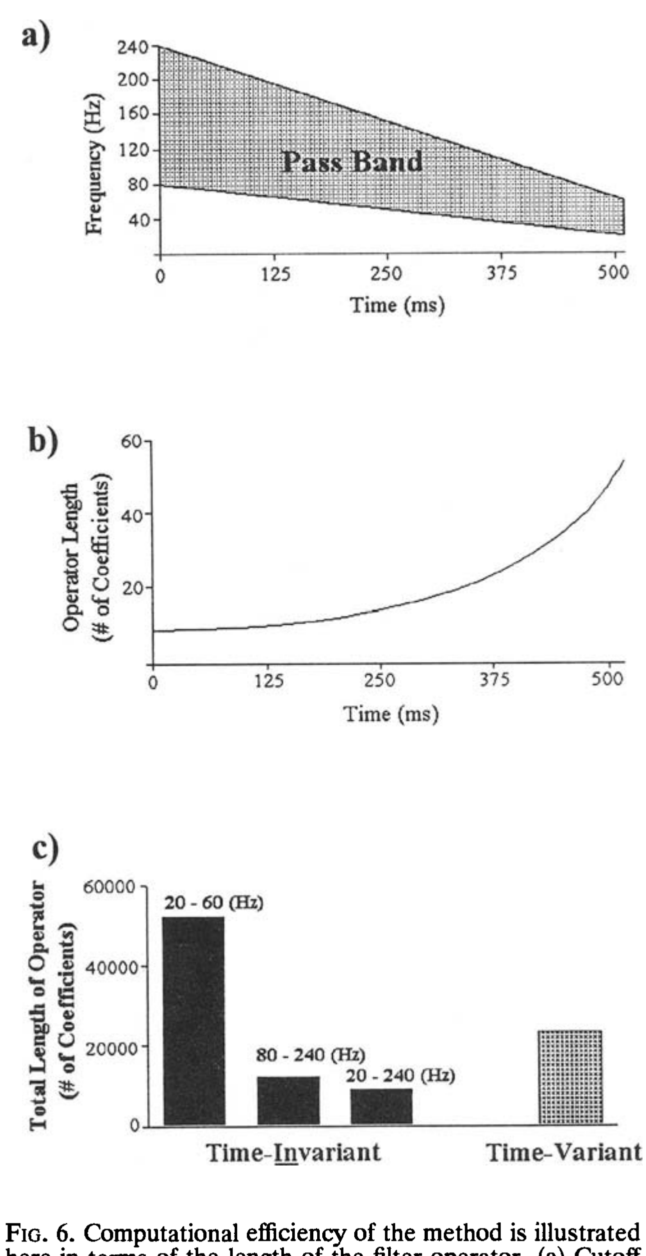


FIG. 6. Computational efficiency of the method is illustrated here in terms of the length of the filter operator. (a) Cutoff frequency functions with the constant-octave bandwidth. (b) Change of the length of the filter operator with time. (c) Comparison of the total cumulative length of the operator for the method presented of a constant-octave, time-variant case with three time-invariant cases whose passbands are indicated above each bar.

most general form will take about twice the computation time as that of the time-invariant, band-pass filtering case.

## CONCLUSIONS

The technique of time and amplitude scaling a reference filter operator can be used to produce an effective time-variant, band-pass filter with constant-octave bandwidth. The scaling makes the filter algorithm simple and computationally efficient. The reference operator should be of as low a frequency as possible to avoid inaccuracies that may occur during the interpolation of the reference operator. The Hanning window amplitude spectrum may be used preferentially to increase the computational efficiency of the filtering. The method can be implemented in a cascaded form to effectively relax the restriction of the constant-octave bandwidth. In this case, the effectiveness of the filter may be limited by the Nyquist frequency and also by the shape of the amplitude spectrum of the reference operator. However, in most practical situations, the limitation will not occur.

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