Bayesian Estimation of a Continuous-Time Model for Discretely-Observed Panel Data

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Abstract

Continuous-time models are used in many areas of science. However, in psychology and related fields, continuous-time models are often difficult to apply because only a small number of repeated observations are typically available. One promising model that has been suggested for such data is the Exact Discrete Model (EDM)—a set of mathematical relations that connect the discrete-time autoregressive cross-lagged (ARCL) panel model to an underlying continuous-time model. To date, several frequentist approaches have been developed for estimating the underlying continuous-time model parameters via the EDM. On the contrary, Bayesian approaches have not yet been explored. Therefore, the purpose of this project was to outline a Bayesian implementation of the EDM with non-informative priors and compare its performance to two frequentist approaches—EDM-SEM (Oud & Jansen, 2000) and Oversampling (Singer, 2012)—under proper model specification and variable experimental conditions. Data were generated under different combinations of sample size, number of time points, and population parameter values for a bivariate panel model. In addition, starting values for the frequentist methods were set to data generating values or randomly perturbed. Results from the three estimation approaches were equivalent at moderate and large sample sizes. The Bayesian implementation resulted in fewer non-converged and improper solutions compared to the frequentist approaches in nearly all experimental conditions. Parameter estimates were slightly less biased and less variable under frequentist estimation at small sample sizes. The Bayesian approach and Oversampling generally provided equivalent or better interval coverage compared to the EDM-SEM procedure across all conditions. Finally, model fit statistics calcu-

lated under the Bayesian approach via posterior predictive modeling checking were less sensitive to sample size than those calculated for the frequentist methods; however, proposed cutoff values did not correspond to Type I error rates. To summarize, preliminary support for a non-informative Bayesian implementation of the EDM was found. In addition, Oversampling appears to be a promising method for frequentist estimation of the EDM. Alternative prior specifications, modeling extensions, and the performance of these approaches under less ideal analytic conditions are important areas for further study.

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Chapter 1

Introduction

Social, behavioral, and biological phenomena change continuously over time. An individual's level of wakefulness, for example, will continually rise and fall over the course of a single day. Such change occurs in infinitely small increments as time elapses in infinitely small intervals. Put another way, the process will exhibit smooth change as the scale of time it is observed on becomes very small; therefore, the process is said to evolve in *continuous-time*. It is difficult to find exception to the idea of continuous-time processes[1] despite the fact that researchers can only hope to measure such processes approximately through frequent observation. Some phenomena are amenable to high-frequency measurement (e.g., heart-rate variability, skin conductance, vocal frequency) and thus a reasonable approximation to continuous time can be made, but many others require expensive or fatiguing assessments that are impractical to administered often (e.g., census surveys, diagnostic interviews, laboratory assays). As a result, researchers can often only measure "snapshots" of truly continuous processes such that observations appear to jump instantaneously from one level of a scale to the next (Brewer, Barenco, Callard, Hubank, & Stark, 2008). Such processes, though truly continuous, are said to be observed in *discrete-time*.

The concepts of continuous and discrete time also extend to the statistical treatment of data. One discrete-time model that has received considerable attention in psychology and related fields

[1] A notable exception occurs in mathematical finance. The pricing of some financial instruments exhibit behavior akin to so-called "jump processes" in which infinitesimally small changes are interspersed with large discrete movements over time (Cont & Tankov, 2004).

is the autoregressive and cross-lagged (ARCL) panel model (Little, Preacher, Selig, & Card, 2007; Voelkle, Oud, Davidov, & Schmidt, 2012). The ARCL panel model is used to model the stability of and dynamic relations between variables over time. It is similar to the vector autoregressive (VAR) model used widely in economics and mathematical finance (Shumway & Stoffer, 2011) with one exception: In the VAR model, multivariate, single-unit data are modeled (i.e., *time series* data) whereas in the ARCL panel model, multivariate, multiple-unit data are modeled (i.e., *panel* data). Moreover, data streams observed in economics and finance are typically larger such that more observations are recorded over time, regardless of the length of the study.

Many authors have noted disadvantages of the ARCL panel model as a result of being a discrete-time model. Notably, estimates of the ARCL panel model parameters vary as a function of the amount of time that elapses between observations (Gollob & Reichardt, 1991; Voelkle et al., 2012). As a result, applied researchers are required to choose the most theoretically relevant sampling interval as model estimates can only be generalized to the specific interval chosen. This is not the case for continuous-time models. In continuous-time models, parameters are invariant to the length of time between assessments. Fortunately, econometrician Albert Bergstrom long ago published a set of mathematical relations that connect the parameters of the discrete-time ARCL panel model to a continuous-time model—the so-called *exact discrete model* (EDM; Bergstrom, 1988)[2]. By using the EDM relations, researchers can estimate the parameters of the underlying continuous-time model from discretely observed data. The EDM was originally developed for use with time series data but was eventually adapted for panel data (Oud & Jansen, 2000; Singer, 1991, 1993). Although not yet widely applied, it has recently been used to model the dynamics of family relationships and adolescent drinking behavior (Delsing, Oud, & De Bruyn, 2005), authoritarianism and anomia (Voelkle et al., 2012), and internalizing and externalizing problem behavior in children (Oud & Delsing, 2010).

Several procedures have been developed for estimating the parameters of the EDM with panel

[2]Although the EDM is technically a set of mathematical relations that connect the discrete-time ARCL panel model to an underlying continuous-time model, throughout this document, "EDM" is used interchangeably in reference to the underlying continuous-time model, e.g., "Estimation of the EDM requires...". In addition, the terms "ARCL model" and "ARCL panel model" are used interchangeably.

data. These procedures generally fall into one of two categories: state-space filtering/smoothing algorithms (Singer, 1993, 1995, 1998) and structural equation modeling (SEM) approaches (Oud & Jansen, 2000; Singer, 2012; Voelkle & Oud, 2013). An important similarity between these methods is that all originate from the frequentist paradigm. In contrast, Bayesian estimation procedures for the EDM have not yet been proposed or studied. Ignoring for a moment the philosophical divide between the two paradigms, it has been shown that Bayesian procedures may outperform frequentist procedures in certain modeling contexts (Lee, 2007; Muthén & Asparouhov, 2012; X.-Y. Song & Lee, 2012; Wang & McArdle, 2008). Therefore, the purpose of the proposed project is to introduce a Bayesian implementation of the EDM—estimable in the JAGS program (Plummer, 2003)—for use with panel data. Additionally, this Bayesian implementation will be evaluated and compared to two existing frequentist procedures.

This document will proceed as follows. First, the discrete-time ARCL panel model is introduced and problems associated with it highlighted. Next, the EDM is introduced. In particular, the method by which Bergstrom (1966) linked the discrete-time ARCL model to a continuous-time model is explained. Accompanying this exposition is an overview of two frequentist procedures that have been developed to estimate the EDM with panel data. Next, the Bayesian paradigm is briefly introduced. The Bayesian implementation of the EDM is then described and contrasted with Bayesian implementations of similar models. Finally, a Monte Carlo investigation is reported in which the performance of the Bayesian approach is compared to the two frequentist alternatives described.

1.1 Problems with the Discrete-Time ARCL Panel Model

The discrete-time ARCL panel model is a multivariate autoregressive model that defines a set of within- and between-variable relations over time; it is typically estimated within the SEM framework. As an SEM model, researchers have considerable flexibility over its specification. That said, researchers typically estimate ARCL models of order 1—i.e., AR(1) processes—which means that

any observation in a data series depends only on data points that immediately preceded it in time. For each unit in a sample, the ARCL model of order 1 is specified in matrix form as follows:

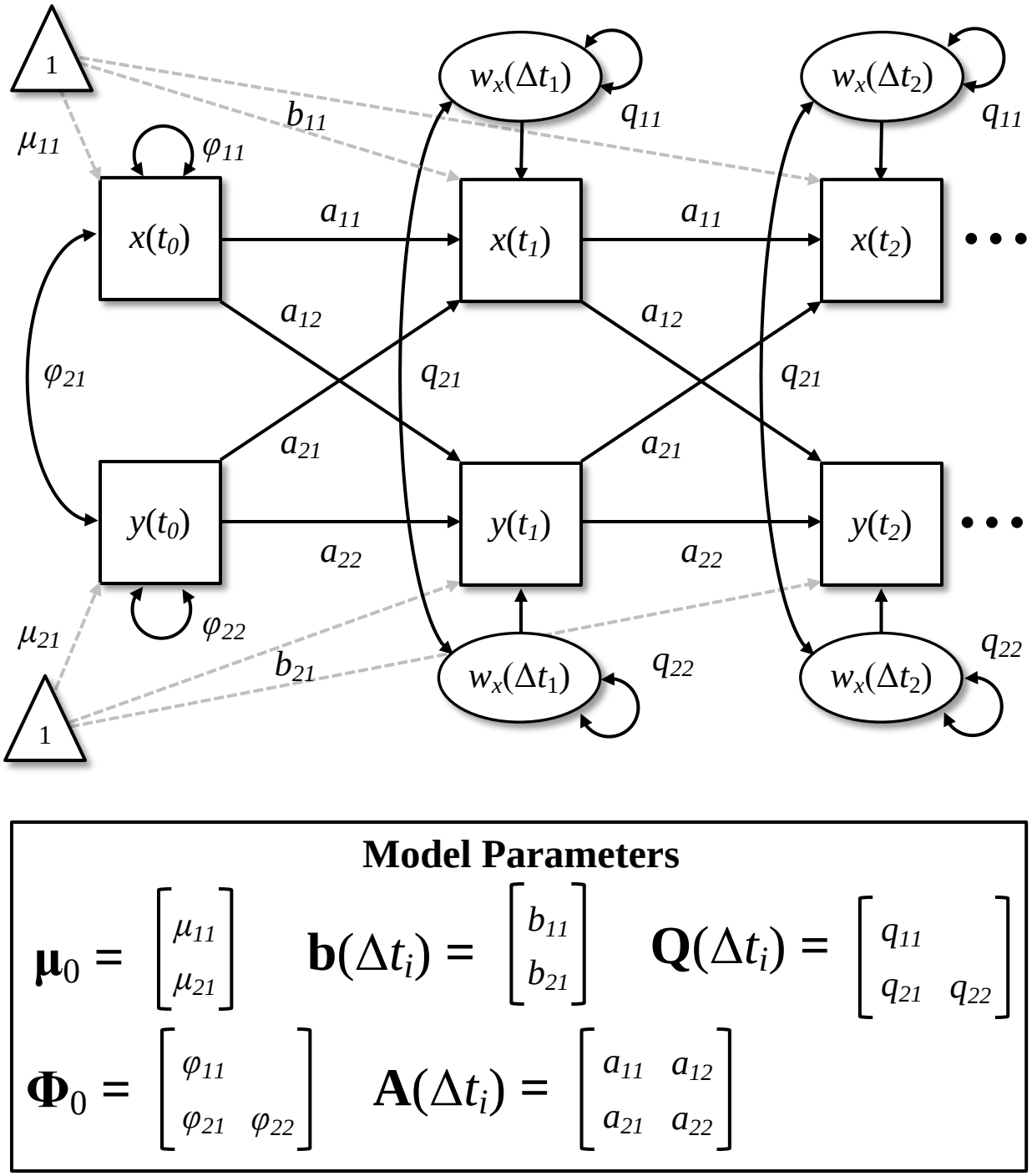
$$\mathbf{x}(t_i) = \mathbf{A}(\Delta t_i)\mathbf{x}(t_i - \Delta t_i) + \mathbf{b}(\Delta t_i) + \mathbf{w}(\Delta t_i). \tag{1.1}$$

The model in Equation 1.1 defines a unit's[3] score vector on a set of measured outcomes as a function of: (a) scores measured one time point prior, (b) a linear trend effect, and (c) unmeasured influences. Specifically, the model for V variables and all units in the sample consists of a $V \times 1$ score vector $\mathbf{x}(t_i)$, a $V \times 1$ vector of lagged scores $\mathbf{x}(t_i - \Delta t_i)$, a $V \times V$ autoregressive/cross-lagged coefficient matrix $\mathbf{A}(\Delta t_i)$, a $V \times 1$ intercept coefficient vector $\mathbf{b}(\Delta t_i)$, and a $V \times 1$ vector of residual terms $\mathbf{w}(\Delta t_i)$. The residual terms are assumed to be multivariate normally distributed with a $V \times 1$ zero mean vector and $V \times V$ covariance matrix $\mathbf{Q}(\Delta t_i)$. At the first measurement occasion, it is standard practice to estimate the mean vector ($\mu_0; V \times 1$) and covariance matrix ($\Phi_0; V \times V$) of the outcome variables. A path diagram of a bivariate model is shown in Figure 1.1.

The notation used to index time in Equation 1.1 is important and requires further examination. The specific points in time when observations were recorded are denoted t_i. Index i reflects the ordering of the observations such that for T time points, $i = 0, 1, 2, ..., T-1$. Note that the first observation occurs at time t_0. Note also that the i index implies observations are made in discrete-time. The intervals of time that elapse between measurements are denoted Δt_i. For these intervals, i begins at 1 and is defined up to $T-1$. For example, if the third occasion of measurement occurred in 2011, it would be denoted as $t_2 = 2011$. Assuming yearly measurements, the time of observation immediately preceding t_2 is $t_1 = 2010$ and the interval between these observations is denoted $\Delta t_2 =$ 1 year.

The notation used here is unconventional but also illuminating. It was used by Voelkle et al. (2012) to highlight two important assumptions made in the ARCL panel model. First, note that the parameter matrices $\mathbf{A}(\Delta t_i)$, $\mathbf{b}(\Delta t_i)$, and $\mathbf{Q}(\Delta t_i)$ are depicted as functions of the time intervals

[3]For notational convenience, an index denoting any arbitrary unit in the sample is omitted from Equation 1.1 and those that follow.



Model Parameters

$$\boldsymbol{\mu}_0 = \begin{bmatrix} \mu_{11} \\ \mu_{21} \end{bmatrix} \quad \mathbf{b}(\Delta t_i) = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} \quad \mathbf{Q}(\Delta t_i) = \begin{bmatrix} q_{11} & \\ q_{21} & q_{22} \end{bmatrix}$$

$$\boldsymbol{\Phi}_0 = \begin{bmatrix} \varphi_{11} & \\ \varphi_{21} & \varphi_{22} \end{bmatrix} \quad \mathbf{A}(\Delta t_i) = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Figure 1.1: Example path diagram of bivariate ARCL panel model

between measurements. This notation makes explicit the fact that estimates of model parameters in an ARCL model depend on the length of time that elapses between measurements. Second, note that the measurement interval is denoted $t_i - \Delta t_i$. Equidistant measurement intervals are typically assumed in the ARCL panel model although, as this notation suggests, violations can occur in practice. In SEM, the equidistant interval assumption can be relaxed—for instance, separate estimates for a_{11} could be obtained in Figure 1.1—although resulting estimates will be difficult to compare within a study. Moreover, the ARCL panel model cannot accommodate *individually-varying* time intervals—those that differ within and between individual units in a study (Voelkle & Oud, 2013).

To better understand how these assumptions become problematic, imagine a scenario in which biweekly measurements are obtained for a set of outcomes in a panel study. The ARCL model is fit to the data and estimates computed. In Figure 1.2, an *autoregression function* is depicted. The figure shows values for an autoregressive coefficient in the model—a measure of construct stability, represented as a_{11} and a_{22} in Figure 1.1—as a function of the measurement interval. Note that for a measurement interval of 0, the autoregressive coefficient is 1 as time has not yet elapsed and thus change has not yet occurred. As the measurement interval increases, however, the value of the autoregressive coefficient decreases. The estimated coefficient using the biweekly assessments is approximately .40. Figure 1.2 highlights the contrast between discrete- and continuous-time. In discrete-time, results can only be generalized to the measurement interval used in the study. In continuous-time, a richer portrait emerges that depicts the evolution of model estimates as time unfolds.

Oud and Delsing (2010) and Voelkle et al. (2012) discuss various contradictions that can arise in discrete-time models. For instance, the magnitude, statistical significance, and even direction (i.e., positive versus negative effect) of discrete-time cross-lagged coefficients depend on the length of the measurement intervals. Therefore, comparing discrete-time results from studies using different measurement intervals can often lead to conflicting conclusions; as Voelkle et al. (2012) note: "it is easy to imagine the fierce debate in the scientific community on the true nature of the relationship between two (or more) constructs" (Voelkle et al., 2012, p. 179).

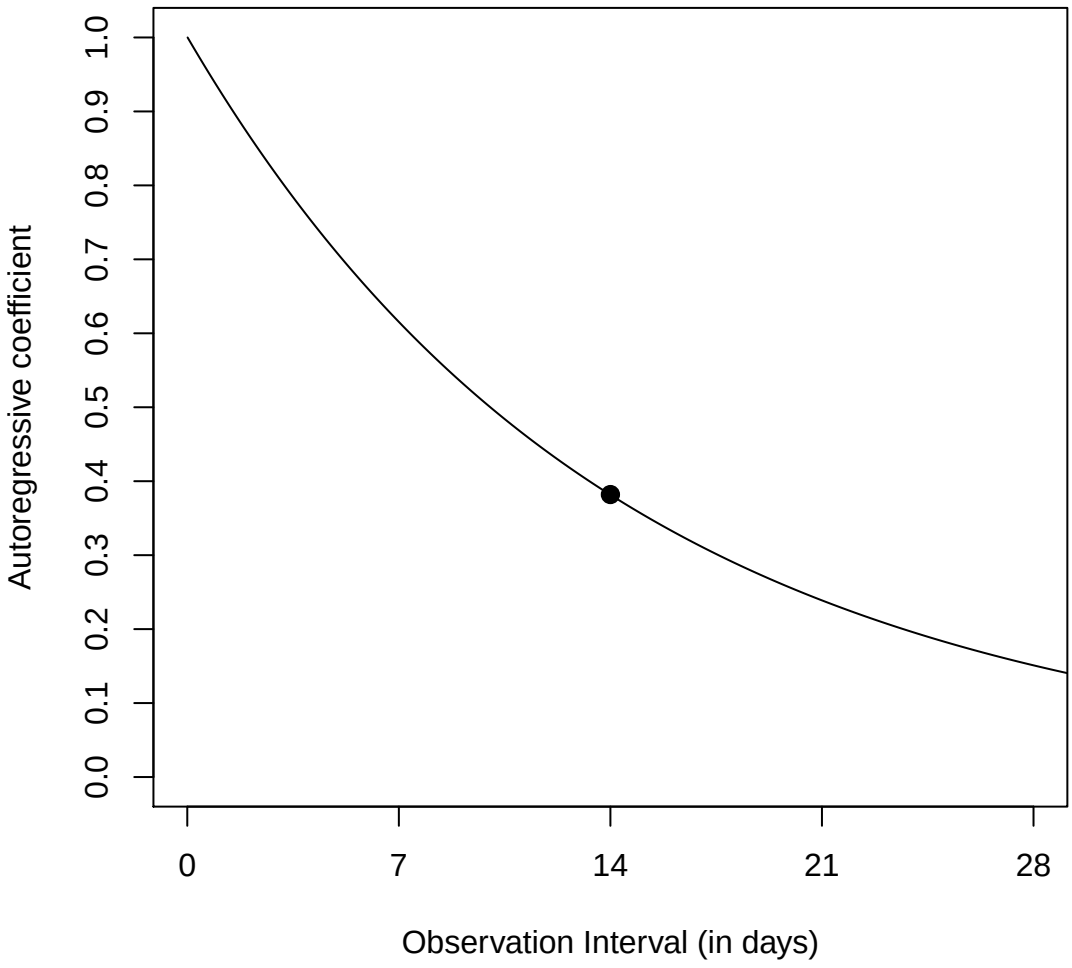


Figure 1.2: Autoregression function

1.2 The Exact Discrete Model

To avoid inferential fallacies like those discussed above, one can instead estimate a continuous-time model. Continuous-time models have been in existence since the formulation of differential and integral calculus in the 17th century. Some important developments occurred later in the early and middle parts of the 20th century that allowed for stochastic continuous-time models—those in which deterministic and stochastic elements are included. Regarding the discrete-time ARCL panel model and its continuous-time counterpart, two other events are worth noting. First, Bergstrom (1966) published the EDM—the collection of mathematical derivations required to connect the discrete-time ARCL model to a continuous-time model. Second, Singer (1991, 1993, 1995, 1998) and Oud and Jansen (2000) adapted the EDM for use with panel data; prior to this work, the EDM was only applicable to multivariate time series data.

The EDM connects the discrete-time ARCL model to the following continuous-time model:

$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = \mathbf{A}\mathbf{x}(t) + \mathbf{b} + \mathbf{G}\frac{\mathrm{d}\mathbf{W}(t)}{\mathrm{d}t}. \tag{1.2}$$

Equation 1.2 is a stochastic differential equation (Øksendal, 2003) and will hereafter be referred to as the *first-order model*. Equation 1.2 defines the rate of change of the variables in $\mathbf{x}$ at time t to be a linear combination of the levels of the variables at time t, a set of (growth) constants, and the rate of change in a stochastic error process. In this model, $\mathbf{x}(t)$ is a $V \times 1$ data vector of V outcome variables, $\mathbf{A}$ is a $V \times V$ matrix of continuous-time regression coefficients, $\mathbf{b}$ is a $V \times 1$ vector of continuous-time intercept coefficients, and $\mathbf{G}$ is a $V \times V$ matrix that scales the stochastic error term $\frac{\mathrm{d}\mathbf{W}(t)}{\mathrm{d}t}$, where $\mathbf{W}(t)$ denotes the Weiner process[4]. Note that the time variable t without the i index is

[4]The *Wiener process*—named after mathematician Norbert Wiener (1894-1964)—is a stochastic process that is used to model unobserved effects in continuous-time models. Loosely-speaking, one can think of it as the continuous-time analogue of a residual term in linear regression. Indeed, the Wiener process is based on a Gaussian formulation just as residual terms are in regression. In particular, it characterizes the distribution of sample paths that exhibit random walk behavior—the accumulation of normally-distributed errors across time. Three important properties of the Wiener process are: (1) The process starts at zero, $\mathbf{W}(t_0) = 0$; (2) the process is continuous everywhere (almost surely); and (3) increments in the process are normally-distributed with mean equal to zero and variance equal to length of time elapsed in the increment, $\mathbf{W}(t) - \mathbf{W}(s) \sim N(\mu = 0, \sigma^2 = t - s)$ for $0 < s < t$.

Technically-speaking, the differential notation shown in Equation 1.2 is incorrect. The Wiener process is differen-

indicative of an arbitrarily chosen point in continuous time as opposed to a specific observed point in discrete time. The **A** matrix is often called the *drift* matrix and contains *auto-* and *cross-effects*. These are analogous to the autoregressive and cross-lagged coefficients in Equation 1.1. However, the parameters in the **A** matrix do not depend on the spacing between observations as do those in the $\mathbf{A}(\Delta t_i)$ matrix. Similarly, the intercept coefficients in **b** are continuous-time analogues of the intercept coefficients in the discrete-time ARCL model, also independent of the measurement interval.

The EDM connects the discrete- and continuous-time models through a set of exact non-linear relations between the model parameters. For instance, the relation between estimates in $\mathbf{A}(\Delta t_i)$ in Equation 1.1 and **A** in Equation 1.2 is:

$$\mathbf{A}(\Delta t_i) = e^{\mathbf{A}\Delta t_i}. \tag{1.3}$$

The term on the right-hand side of Equation 1.3 is the exponential[5] of the **A** matrix, which has been post-multiplied by the ith observation interval Δt_i. At this point, readers might wonder if the analyst could just estimate a discrete-time ARCL panel model to obtain $\mathbf{A}(\Delta t_i)$ and solve for **A** in Equation 1.3. Although intuitively appealing, this so-called *indirect approach* has been criticized as the relationship between $\mathbf{A}(\Delta t_i)$ and **A** is not included during parameter estimation (Hamerle, Nagl, & Singer, 1991). The indirect approach will provide biased results when, for instance, observation intervals are not equal (Voelkle et al., 2012). Therefore, directly including the non-linear constraint in Equation 1.3 during estimation of the model parameters is integral to applications of the EDM. In the follow sections, the complete set of relations between the parameters of a discrete-time ARCL model and the first-order model are discussed. More detailed overviews can be found in Voelkle et al. (2012) and Oud and Delsing (2010).

tiable nowhere almost surely. This notation is used to remain consistent with some of the citations referenced in this article. However, Equation 1.2 is also sometimes written as $\mathrm{d}\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t)\mathrm{d}t + \mathbf{G}\mathrm{d}\mathbf{W}(t)$ for accuracy. Equations in this format are called *difference equations*.

[5]The exponential of a matrix—unless it is a diagonal matrix—is not equivalent to a matrix in which each element is exponentiated. Rather, the exponential of a matrix can be defined by the convergent power series $e^{\mathbf{A}} = \sum_{k=0}^{\infty} \frac{1}{k!}\mathbf{A}^k$. See Moler and Van Loan (2003) for a review of approaches used to calculate matrix exponentials.

1.2.1 Step 1: Solve the Continuous-Time First-Order Model

The first step in connecting the discrete- and continuous-time models is to solve the stochastic differential equation shown in Equation 1.2. This is done by integrating both sides of the equation, which leads to the following solution (Oud & Jansen, 2000):

$$\mathbf{x}(t) = e^{\mathbf{A}(t-t_0)}\mathbf{x}(t_0) + \mathbf{A}^{-1}\left[e^{\mathbf{A}(t-t_0)} - \mathbf{I}\right]\mathbf{b} + \int_{t_0}^{t} e^{\mathbf{A}(t-s)}\mathbf{G}\mathrm{d}\mathbf{W}(s). \tag{1.4}$$

The variable s is used in place of t here because t defines the upper limit of the integral on the right. There are a few features of Equation 1.4 worth noting. First, the left-hand sides of Equations 1.1 and 1.4 are equal with the exception of the time input variable. Therefore, equating the time variables would allow one to set the discrete-time ARCL model and the solution to the first-order model equal to one another. Second, the exponential of the drift matrix is found in all three right-hand side terms. As a consequence, the auto- and cross-effects in $\mathbf{A}$ play a critical role in all components of the model. Third, because the eigenvalues of $\mathbf{A}$ are negative for a stable model (Oud & Delsing, 2010), as $t - t_0$ approaches infinity, the expected value of Equation 1.4 approaches $-\mathbf{A}^{-1}\mathbf{b}$. In other words, the variables in $\mathbf{x}(t)$ converge to stable equilibrium positions defined by $-\mathbf{A}^{-1}\mathbf{b}$ as the amount of time that the equation is integrated over becomes infinitely large. Finally, the last term in the equation represents the integration of the stochastic error process from time t_0 to time t. Formally, it is a multivariate Itō integral with the following expected value and covariance structure:

$$\begin{gathered} E\left[\int_{t_0}^{t} e^{\mathbf{A}(t-s)}\mathbf{G}\mathrm{d}\mathbf{W}(s)\right] = \mathbf{0} \\ \text{and} \\ COV\left[\int_{t_0}^{t} e^{\mathbf{A}(t-s)}\mathbf{G}\mathrm{d}\mathbf{W}(s)\right] = irow\left\{\mathbf{A}_{\#}^{-1}\left[e^{\mathbf{A}_{\#}(t-t_0)} - \mathbf{I}\right] row\mathbf{Q}\right\}. \end{gathered} \tag{1.5}$$

In Equation 1.5, $\mathbf{Q} = \mathbf{GG'}$, $\mathbf{A}_{\#} = \mathbf{A} \otimes \mathbf{I} + \mathbf{I} \otimes \mathbf{A}$, $\mathbf{I}$ is an identity matrix of appropriate dimension, the *row* operator places the elements of a square matrix row-wise into a stacked column vector, the

irow operator represents the reverse procedure, and $\otimes$ is the Kronecker product. The first part of Equation 1.5 defines the expected values of the continuous-time error processes as equal to zero over any arbitrary time interval. The second part of Equation 1.5 defines the constraint needed to connect the error covariance matrix $\mathbf{Q}(\Delta t_i)$ from the discrete-time ARCL model to a $V \times V$ continuous-time error covariance matrix $\mathbf{Q}$—also known as the *diffusion* matrix—similar to how $\mathbf{A}(\Delta t_i)$ and $\mathbf{A}$ are linked in Equation 1.3. As before, $\mathbf{W}()$ represents the Wiener process. The matrix $\mathbf{G}$ is the Cholesky factor (i.e., square root) of $\mathbf{Q}$ and, as stated previously, scales the stochastic error process.

1.2.2 Step 2: Replace the Interval of Integration with the Observed Measurement Intervals

The next step in connecting the discrete- and continuous-time models is to equate the time variables shown in Equations 1.1 and 1.4. When the first-order model is integrated, one is essentially adding up all of the infinitesimally small deterministic and stochastic effects on the variables in $\mathbf{x}(t)$ over the time elapsed since the initial measurement, $t - t_0$. With discretely-observed data values, however, one is interested not in adding up effects occurring over arbitrary time intervals but rather in adding up effects that occur between measurements—that is, during the interval Δt_i. Therefore, one must replace the arbitrary time interval $t - t_0$ in Equation 1.4 by the observation intervals Δt_i. Specifically, for a set of repeated discrete observations, let time point t be replaced by observed time point t_i and let initial time point t_0 be replaced by the preceding time point $t_i - \Delta t_i$. By substitution, time interval $t - t_0$ is replaced by Δt_i. As a result, the left-hand sides of Equations 1.1 and 1.4 are equal, allowing the discrete-time ARCL model to be set equal to the integral solution of the first-order model.

1.2.3 Step 3: Constrain Discrete-Time ARCL Panel Model Parameter Matrices

The final step in connecting the models is to place constraints on the parameter matrices of the discrete-time ARCL panel model. To summarize, the EDM constraints are:

$$\begin{aligned}
\mathbf{A}(\Delta t_i) &= e^{\mathbf{A}\Delta t_i} \\
\mathbf{b}(\Delta t_i) &= \mathbf{A}^{-1}\left[e^{\mathbf{A}\Delta t_i} - \mathbf{I}\right]\mathbf{b} \\
\mathbf{Q}(\Delta t_i) &= irow\left\{\mathbf{A}_{\#}^{-1}\left[e^{\mathbf{A}_{\#}\Delta t_i} - \mathbf{I}\right] row\mathbf{Q}\right\}.
\end{aligned} \tag{1.6}$$

When these constraints are applied, the model can be fit to discretely observed data. Combining Equations 1.4 and 1.5, the full model that is estimated is:

$$\begin{aligned}
&\mathbf{x}(t_i) = e^{\mathbf{A}\Delta t_i}\mathbf{x}(t_i - \Delta t_i) + \mathbf{A}^{-1}\left[e^{\mathbf{A}\Delta t_i} - \mathbf{I}\right]\mathbf{b} + \int_{t-\Delta t_i}^{t_i} e^{\mathbf{A}(t-s)}\mathbf{G}\mathrm{d}\mathbf{W}(s), \\
&COV\left[\int_{t-\Delta t_i}^{t_i} e^{\mathbf{A}(t-s)}\mathbf{G}\mathrm{d}\mathbf{W}(s)\right] = irow\left\{\mathbf{A}_{\#}^{-1}\left[e^{\mathbf{A}_{\#}\Delta t_i} - \mathbf{I}\right] row\mathbf{Q}\right\}.
\end{aligned} \tag{1.7}$$

where $\mathbf{A}_{\#}$ is defined as before and the mean vector (μ_0) and covariance matrix (Φ_0) for the initial observations are estimated. As a result, the parameters of the continuous-time first-order model are estimated based on discretely-observed panel data.

The continuous-time parameters estimated via the EDM are not completely congruent to the discrete-time parameters. That is, the theoretical ranges and interpretations may differ between the discrete- and continuous-time estimates. The auto-effects found along the diagonal of the $\mathbf{A}$ matrix range from $-\infty$ to 0. When $\mathbf{A}$ is a diagonal matrix, $e^{\mathbf{A}}$ is equivalent to a matrix in which the diagonal elements are exponentiated; in this case, it is easy to see that highly negative auto-effects correspond to discrete-time autoregressive coefficients near zero. Conversely, negative auto-effects near zero correspond to large discrete-time autoregressive coefficients near 1. Cross-effects contained in the off-diagonal of $\mathbf{A}$ range from $-\infty$ to ∞, similar to the discrete-time coefficients; however, is generally difficult to determine the size and even direction of the discrete-time estimates by simply

looking at the cross-effects (Oud & Delsing, 2010). Likewise, the continuous-time intercepts in **b** range from $-\infty$ to ∞ and the continuous-time error variances/covariances range from 0 to ∞, just as their discrete-time counterparts do. However, the interpretation of these parameters are different in continuous-time: **b** is a determinant of the long-term means that a stable system converges to and the elements in **Q** scale the stochastic error process that accounts for unmeasured effects on $\mathbf{x}(t)$ not included in the model. Finally, for the parameters in μ_0 and Φ_0, the values and interpretations are equivalent between the discrete- and continuous-time models.

1.3 Frequentist Estimation of the EDM

Frequentist estimation of the EDM with panel data has been developed within the state-space (Singer, 1993, 1995, 1998) and SEM (Oud & Jansen, 2000) statistical frameworks. The former approach was developed using Kalman filtering/smoothing algorithms widely used in engineering (Singer, 1993). Singer originally implemented his method as a stand-alone SAS IML program called Linear Stochastic Differential Equations (LSDE; Singer, 1991) and later updated it as a Mathematica program (Oud & Singer, 2008; Oud & Folmer, 2011). The SEM approach was first implemented by Oud and Jansen (2000) in the Mx program; as it stands, Mx (and its open-source counterpart OpenMx) is the only SEM software package that allows for direct specification of the non-linear matrix constraints shown earlier. Although these two approaches emerged from different modeling paradigms, the differences in model results obtained from each are often moot (Oud, 2004, 2007; Oud & Singer, 2008). Indeed, much debate has occurred over the equivalence between the state-space and SEM paradigms (Chow, Ho, & Hamaker, 2010). In one simulation study, the state-space and SEM approaches provided similar and superior results when compared to two linear approximation methods (Oud, 2007). Some small differences are present, however. For instance, if units are measured at a large number of time points, then the dimensions of SEM model matrices will increase rapidly, making estimation difficult. The state-space approach, on the other hand, is suited to handling longer data series (Oud & Singer, 2008). The SEM approach is

favorable, however, if measurement errors are to be correlated over time, which cannot be done in the state-space framework without violating fundamental assumptions of the estimation process. Nevertheless, in many cases the procedures are exchangeable and a choice between them may be based more on researchers' familiarity with a specific paradigm.

In addition to these nearly equivalent frequentist methods, Singer (2012) also recently proposed a method called *Oversampling* that can be used to estimate the EDM without imposing direct non-linear constraints on the parameter matrices as shown earlier. This method implements a linear approximation to the non-linear constraints and therefore can be implemented more widely in SEM and other general linear modeling programs. Although other linear approximations to the non-linear EDM constraints have been proposed—notably, the so-called *approximate discrete model* (ADM; Oud, 2007; Oud & Delsing, 2010)—the Oversampling approach presents a particularly appealing framework for flexible estimation of the EDM. For instance, individually-varying intervals of measurement can be accommodated and, in one study, it was shown that the identification of certain EDM specifications might be improved (Voelkle & Oud, 2013).

In order to inform the reader exactly how some of these procedures work, the EDM-SEM and Oversampling approaches are briefly reviewed next. These expositions are also necessary as the EDM-SEM and Oversampling methods were investigated as part of the current study. The EDM-SEM approach was chosen for study because to date it is one the most discussed methods for estimating the EDM in this area. The Oversampling approach, on the other hand, has not yet been widely used or studied but, as discussed in greater detail below, may be particularly helpful for estimating the EDM and other continuous-time models in standard SEM and other general modeling software packages. Although the state-space approach developed alongside—in fact, prior to—the EDM-SEM method, it was not included in the present study for the following reasons: (a) it has been shown to provide identical results to the EDM-SEM procedure under many analytic scenarios; (a) state-space modeling in general has not yet permeated social science research at a large scale[6]; and (c) it typically requires considerable technical expertise to program and estimate

[6]This is not to suggest that the state-space modeling framework is without merit nor is it implied that it is not important to the future of social science research. On the contrary, this framework may be particularly advantageous

complex models such as the EDM.

1.3.1 EDM-SEM

Oud and Jansen (2000) showed how the EDM could be estimated as an SEM model by directly specifying the non-linear EDM constraints in SEM software. In discussing this procedure, it is helpful to briefly review the basic SEM formulation. Models in SEM are comprised of two equations. First, a measurement equation is specified such that a set of observed indicators are regressed onto a set of unobserved latent variables:

$$\mathbf{y} = \nu + \Lambda\eta + \varepsilon. \tag{1.8}$$

In Equation 1.8, V observed indicators are contained in the $V \times 1$ column vector $\mathbf{y}$. The variables in $\mathbf{y}$ are linearly related to a set of m latent variables ($m \leq V$) contained in the $m \times 1$ column vector η. Parameters of the measurement equation are contain in the $V \times 1$ intercept vector ν, the $V \times m$ factor loading matrix Λ, and the $V \times V$ covariance matrix Θ for a set of error terms contained in the $V \times 1$ column vector ε, where $E[\varepsilon] = \mathbf{0}$. In SEM, interest often lies in relations between the latent variables contained in η, which is specified in a structural equation:

$$\eta = \alpha + \mathbf{B}\eta + \zeta. \tag{1.9}$$

Intercepts are contained in the $m \times 1$ matrix α, regression coefficients are contained in the $m \times m$ matrix $\mathbf{B}$, and error variances and covariances are contained in the symmetric $m \times m$ matrix Ψ for residual terms contained in the $m \times 1$ column vector ζ ($E[\zeta] = \mathbf{0}$). For simplicity of presentation, exogenous variables are not included in Equations 1.8 or 1.9. Furthermore, let $\mathbf{y} = \eta$ such that all variables in the model are observed; thus, Equation 1.9 is of sole interest. As shown in Voelkle et al. (2012), the latent intercept vector α can be dropped and merged with the fixed effect portion of

for tackling the array of complex models that are increasingly being fit to data by social scientists. For a social science perspective on state-space modeling, see Chow et al. (2010) and H. Song and Ferrer (2009).

the model ($\mathbf{B}\eta$), shown below.

Observed variables in η are measured at T occasions. Let $\eta = \{\mathbf{x}(t_0), \mathbf{x}(t_1), ..., \mathbf{x}(t_i), ..., \mathbf{x}(t_{i=T-1}), \mathbf{1}\}$ be a stacked column vector of dimension $(T+1)m \times 1$ and $\zeta = \{\mathbf{x}(t_0) - \mu_0, \mathbf{w}(\Delta_{t_1}), \mathbf{w}(\Delta_{t_2}), ..., \mathbf{w}(\Delta_{t_{i=T}}), \mathbf{1}\}$ a stack column vector of similar dimension, where $\mathbf{1}$ is an $m \times 1$ vector of 1's. To implement the EDM constraints, one specifies the $\mathbf{B}$ and Ψ matrices as follows:

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mu_0 \\ \mathbf{e}^{\mathbf{A}\Delta_{t_1}} & \mathbf{0} & & \mathbf{0} & \mathbf{0} & \mathbf{A}^{-1}\left[e^{\mathbf{A}\Delta t_1} - \mathbf{I}\right]\mathbf{b} \\ \mathbf{0} & \mathbf{e}^{\mathbf{A}\Delta t_2} & & \mathbf{0} & \mathbf{0} & \mathbf{A}^{-1}\left[e^{\mathbf{A}\Delta t_2} - \mathbf{I}\right]\mathbf{b} \\ \vdots & & \ddots & & & \vdots \\ \mathbf{0} & \mathbf{0} & & \mathbf{e}^{\mathbf{A}\Delta_T} & \mathbf{0} & \mathbf{A}^{-1}\left[e^{\mathbf{A}\Delta t_T} - \mathbf{I}\right]\mathbf{b} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \Psi = \begin{bmatrix} \Phi_{t_0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}(\Delta_{t_1}) & \mathbf{0} & & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Q}(\Delta_{t_2}) & & \mathbf{0} & \mathbf{0} \\ \vdots & & & \ddots & & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & & \mathbf{Q}(\Delta_T) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & 1 \end{bmatrix} \tag{1.10}$$

The matrix Φ_{t_0} is the covariance matrix of the variables in η at initial time point t_0 and the covariance matrices $\mathbf{Q}(\Delta t_i)$ are defined by the constraint shown in Equation 1.5 for time intervals Δt_i.

Once the model is estimated, researchers can conduct hypothesis tests and evaluate the fit of the model as is done in common SEM analyses. Moreover, the continuous-time parameter estimates can be used to calculate discrete-time estimates for time points not observed in the study (e.g., Figure 1.2). As a result, researchers measuring the same outcomes but using different observation intervals can make meaningful comparisons between studies. The model shown in Equation 1.2 can also be extended in various ways. For instance, exogenous inputs that influence the variables in $\mathbf{x}(t)$ can be added. This specification is useful for specifying group-specific trajectories (e.g., males vs. females) or including continuous covariates. A measurement model (Equation 1.8) with appropriate invariance constraints over time can also be included. Finally, one can specify random effects, which Delsing and Oud (2008) refer to as "trait" effects.

The advantage of the EDM-SEM procedure is exactitude; discrete-time parameters in the

ARCL are linked in an exact way to the continuous-time parameters of the underlying stochastic differential equation model. This means that one can use the EDM-SEM approach to estimate continuous-time parameters despite only having access to a few discrete observations over time. The EDM-SEM procedure suffers, however, from a lack of widespread software implementation. Moreover, it has been suggested that even the software that can be used to estimate the model is difficult to use (Steele & Ferrer, 2011) and requires good starting values to reach convergence. A second disadvantage is the limited range of models that can be estimated in the EDM-SEM procedure; despite existing extensions of the model in Equation 1.2 as just discussed, the EDM-SEM procedure is specific to a set of first-order linear differential equations. Models that cannot be re-parameterized to fit within these equations cannot be estimated.

1.3.2 Oversampling

The set of constraints defined by the EDM allows one to integrate over discrete-time measurement intervals while taking stochastic process error, via integration of the stochastic error term, into account; therefore, one is able to connect discrete-time observations to an underlying continuous-time process. In introductory Calculus courses, students are taught the simplest formulation of integration—the Riemann sum. A univariate Riemann sum involves approximating the area under a continuous function by adding together a series of rectangles (or another shape, if desired) that extend from the horizontal axis to the height of the curve. The width of the rectangles are related to the accuracy of the approximation—as the rectangles become slimmer, the error of approximation is reduced.

Singer (2012) outlined a framework called Oversampling that operates in a manner similar to the Riemann sum. In particular, Oversampling involves integrating over discrete-time measurement intervals in discrete-steps—albeit in steps much smaller than observed measurement intervals. Singer (2012) implemented this idea in the SEM and state-space modeling frameworks; here, focus is on the SEM implementation. Consider the following equation (Voelkle et al., 2012, Equa-

tion 3):

$$\frac{\Delta \mathbf{x}(t_i)}{\Delta t_i} = \mathbf{A}_* \mathbf{x}(t_i - \Delta t_i). \tag{1.11}$$

In Equation 1.11, $\Delta \mathbf{x}(t_i)$ represents change in outcome variables over a single observation interval Δt_i and $\mathbf{A}_*$ is an approximation of $\mathbf{A}$. This equation is a discrete-time approximation of the first-order differential equation model[7]. As the time intervals shrink—approaching zero in the limit—the approximation improves.

The relation between approximate drift matrix $\mathbf{A}_*$ and the discrete-time autoregressive parameter matrix $\mathbf{A}(\Delta t_i)$ is:

$$\mathbf{A}(\Delta t_i) = \mathbf{A}_* \Delta t_i + \mathbf{I}. \tag{1.12}$$

When the time interval Δt_i is large, the approximation to $\mathbf{A}$ can be crude. However, as Δt_i becomes arbitrarily small, $\mathbf{A}_*$ will become equivalent to $\mathbf{A}$ within some minuscule limit of approximation error. The observation intervals in most social science studies will not be small enough for $\mathbf{A}_*$ to be a good approximation of $\mathbf{A}$. Therefore, the crux of the Oversampling approach is to make Δt_i smaller than the observed measurement intervals. To implement the procedure, one divides the observed intervals Δt_i into D smaller increments,

$$\delta_{i,d} = \frac{\Delta t_i}{D}, \tag{1.13}$$

such that $\mathbf{A}_*$ provides a reasonable approximation to $\mathbf{A}$ if the variables in $\mathbf{x}(t_i)$ were actually observed at the sampling rate implied by $\delta_{i,d}$. The name Oversampling is in reference to values unobserved between the true measurement intervals in a study—so-called *oversamples*—that are included as part of this discretization scheme. Implementing the approach in SEM requires one to create additional latent variables and place constraints on relevant pathways, similar to latent

[7]For simplicity, counterparts to the continuous-time intercept and stochastic error term are not shown. When included, Equation 1.11 is simply an alternate notation of the discrete-time ARCL model.

difference score models (Voelkle & Oud, in press). If the number of oversamples is sufficient, the following relationship holds:

$$\mathbf{A}(\Delta t_i) = e^{\mathbf{A}\Delta t_i} \approx \lim_{d\to\infty} \prod_{d=0}^{D-1} \left[\mathbf{A}_* \delta_{i,d} + \mathbf{I}\right]. \tag{1.14}$$

One disadvantage of introducing additional latent variables is an increase in computational time. Ultimately, Singer (2012) recommends using state-space routines to estimate the EDM with Oversampling. However, Voelkle and Oud (2013) circumvented the addition of latent variables by calculating $\mathbf{A}_*$ using matrix algebra expressions in a stand-alone OpenMx program. These researchers also used a slightly different expression for the product term in Equation 1.14 that provides a more computationally efficient approximation[8]

The Oversampling approach linearizes the relationship between $\mathbf{A}(\Delta t_i)$ and $\mathbf{A}$. Consequently, SEM software packages that allow for linear constraints can be used. Moreover, individually-varying time intervals between observations can be accommodated. The EDM-SEM is difficult to apply when individuals are sampled at different times over the span of study (Voelkle & Oud, 2013). Such data, however, are easily handled with Oversampling as D can vary between individuals.

The primary limitation of Oversampling is an increase in computation time that may accompany large values of D. This limitation does not apply to the program provided by Voelkle and Oud (2013), however, as oversamples are not truly added as additional latent variables into the SEM matrices. Additionally, Oversampling was only recently introduced and therefore little is known about the number of oversamples needed for a sufficient approximation. Singer (2012) recommended at least 20 oversamples, whereas Voelkle and Oud (2013) recommended 30 oversamples.

Both the EDM-SEM and Oversampling approaches originate from the frequentist paradigm. As a result, one can imagine circumstances under which neither provides answers that are satisfactory. For example, both methods are based on maximum likelihood (ML) estimation. It is well-known that one must rely on asymptotic arguments to achieve the desirable properties of ML

[8]The formula used in (Voelkle & Oud, 2013) is analogous to a trapezoidal approximation of a Riemann sum

estimators (e.g., consistency, efficiency; Hogg, McKean, & Craig, 2012). Therefore, these procedures may not be optimal when sample sizes are small. Indeed, previous simulation studies of EDM estimation via state-space and SEM approaches used sample sizes of $N = 200$ with 11 time points (Voelkle & Oud, 2013) and $N = 700$ with 4 time points (Oud, 2007; Oud & Singer, 2008). At best, then, it is unclear how accurate results are for sample sizes obtained in many social science studies (e.g., < 150). Additionally, because the EDM does not have a closed-form solution, model parameters are estimated via iterative search algorithms. All such algorithms require a set of starting values for the model parameters to initiate the search. Due to the highly non-linear constraints involved, it has been noted that obtaining good starting values for EDM estimation is difficult (Steele & Ferrer, 2011). For these reasons and others discussed in Chapter 2, it may be advantageous to consider alternative estimation schemes for the EDM. In the current study, focus is placed on Bayesian estimation which is increasing in popularity among psychological and other scientists who deal with complex statistical models.

Chapter 2

Bayesian Estimation of the Exact Discrete Model

2.1 Overview of Bayesian Statistics

The goal in a frequentist analysis is to estimate one or more parameters of a statistical model that characterizes a process under study. Estimates will be based on a finite sample of observations and inferences are made with regard to how close the sample-based estimates approach "true" population values. In general, analysts employing a Bayesian perspective aim to satisfy this goal but differ with regard to how they define a population parameter. In the frequentist paradigm, parameters are assumed to be fixed quantities; in the Bayesian paradigm, parameters are assumed to be random variables. The ultimate goal in a Bayesian analysis, then, is to characterize the distribution of the population parameters as opposed to estimating fixed values. In addition, frequentist inferences are predicated on the notion of long-run probability—an argument which is, in some cases, illogical for a given problem (Jackman, 2009)—and are made in isolation. On the contrary, Bayesian inferences center around updating prior evidence for the process under study given new information.

Bayesian analysis is based on a theorem put forth by mathematician Thomas Bayes (1701-

1761) regarding the conditional probability of event occurrence. The theorem states that for two events—say, A and B—the conditional probability of one event (A) given a fixed value of the other event (B) can be calculated as follows:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{2.1}$$

In Bayesian statistics, interest lies in the conditional *distribution* of A given B, not a specific value of $P(A|B)$. In particular, one is interested in characterizing distributions for the random population parameters given a set of fixed values—that is, the sample data. Let $\mathbf{Y}$ represent a set of observed data, and let θ be equal to a set of q unknown population parameters, $\{\theta_1, \theta_2, ..., \theta_q\}$. Bayes theorem states that the conditional distribution of the model parameters can be calculated as follows:

$$P(\theta|\mathbf{Y}) = \frac{P(\mathbf{Y}|\theta)P(\theta)}{P(\mathbf{Y})}. \tag{2.2}$$

In this equation, $P(\theta|\mathbf{Y})$ is called the *posterior distribution* of θ given $\mathbf{Y}$—that is, the multivariate conditional distribution of the q model parameters given the sample data. The numerator on the right-hand side of Equation 2.2 contains two quantities. The first term $P(\mathbf{Y}|\theta)$ represents the *likelihood* of the sample data in $\mathbf{Y}$ given values of the parameters in θ. The likelihood is of primary importance in frequentist analysis as observed data are considered to be random quantities and model parameters fixed. The likelihood is multiplied by a term that represents *a priori* knowledge one has concerning the distribution of θ; this is called the *prior distribution* of θ. As opposed to the frequentist framework, one can incorporate prior information for a set of model parameters—obtained, for instance, from previous research or theoretical considerations—into the estimation process. The term in the denominator of Equation 2.2 is the distribution of the data in $\mathbf{Y}$; as the data are considered fixed quantities, the denominator serves as a normalizing constant such that

the posterior distribution properly integrates to 1. Therefore, Equation 2.2 is often rewritten as:

$$P(\theta|\mathbf{Y}) \propto P(\mathbf{Y}|\theta)P(\theta). \tag{2.3}$$

Equation 2.3 is read as "the posterior distribution of θ given $\mathbf{Y}$ is proportional to the likelihood $P(\mathbf{Y}|\theta)$ times the prior $P(\theta)$".

The frequentist paradigm has dominated applied statistics throughout much of the 20th century. Frequentist procedures are more or less self-contained and easy to implement without substantial computation. Until recently, users of Bayesian statistics were required to derive and summarize posterior distributions analytically—a difficult if not impossible task in most cases (Lynch, 2007). However, in the 1980's, Bayesian statistics re-emerged as the burden of calculating posterior distributions was shifted from the researcher to the computer. Specifically, Monte Carlo sampling procedures were developed such that researchers need only obtain simulated samples from a posterior distribution in order to reconstruct it. Once the posterior distribution is realized, researchers can use a variety of summary statistics (e.g., mean, mode, intervals) to convey their updated beliefs about model parameters. A primary advantage of this simulation approach is that the form of a posterior distribution need not be known to obtain simulated samples from it. As a result, models previously deemed intractable can be fit to data.

2.1.1 Bayesian Estimation of the EDM

The posterior distribution of the EDM model parameters can be constructed via Bayes Theorem. Let $\theta = \{\Phi_0, \mu_0, \mathbf{A}, \mathbf{b}, \mathbf{Q}\}$ be a vector containing the model parameters and $\mathbf{y}_{j,t_i}$ contain a vector of observed scores for individual j at time t_i. The likelihood of the data, $f(\mathbf{y}_{j,t_i}|\theta)$ is derived as:

$$f(\mathbf{y}_{j,t_i}|\theta) \propto \prod_{j=1}^{N} |\Phi_0|^{1/2} e^{(\mathbf{y}_{j,t_0}-\mu_0)\Phi_0^{-1}(\mathbf{y}_{j,t_0}-\mu_0)} \prod_{j=1}^{N}\prod_{t_i=1}^{T-1} |\Sigma|^{1/2} e^{(\mathbf{y}_{j,t_i}-\nu)\Sigma^{-1}(\mathbf{y}_{j,t_i}-\nu)} \tag{2.4}$$

where $\nu = e^{\mathbf{A}\Delta t_i}\mathbf{x}(t_i - \Delta t_i) + \mathbf{A}^{-1}\left[e^{\mathbf{A}\Delta t_i} - \mathbf{I}\right]\mathbf{b}$ and $\Sigma = irow\left\{\mathbf{A}_{\#}^{-1}\left[e^{\mathbf{A}_{\#}\Delta t_i} - \mathbf{I}\right] row\mathbf{Q}\right\}$ for $i \geq 1$. The likelihood in Equation 2.4 is based on the assumption that the data are distributed as multivariate normal. Also, all normalizing constants have been removed. Note that the likelihood is separated into two components: (a) a contribution from the initial occasion of measurement (t_0), and (b) a contribution from the other occasions of measurement.

To obtain the poster distribution, the likelihood is multiplied by the prior distribution of the model parameters, $P(\theta)$. The specification of prior distributions is an area of ongoing debate within Bayesian statistics (Gelman et al., 2013). Critics of the Bayesian perspective charge that choosing priors introduces subjectivity into the modeling process. With large sample sizes, the prior is completely dominated by the likelihood and the choice of prior is largely moot; however, at smaller sample sizes the choice of prior can affect the shape of the posterior distribution.

To ensure comparability between Bayesian estimation of the EDM and the frequentist approaches previously described, non-informative priors are used for the implementation described herein. Following Vandekerckhove, Tuerlinckx, and Lee (2011), priors are chosen to give equal probability weight over the theoretical range of model parameters. The prior for initial mean vector μ_0 is specified as multivariate normal with a zero mean vector and diffuse precision matrix $({}^{1}/_{1000} \times \mathbf{I})$[1]. This prior is also used for the continuous-time intercepts in $\mathbf{b}$. For the initial covariance matrix Φ_0, a matrix decomposition specification first suggested by Barnard, McCulloch, and Meng (2000) is used. Following Gelman and Hill (2006), a uniform prior distribution ranging from -1 to 1 is specified for the initial time point correlation between constructs—denoted $\rho_{\phi_0,21}$; two uniform distributions, each ranging from 0 to 100, are placed on the two initial standard deviations, denoted $\sigma_{\phi_0,11}$ and $\sigma_{\phi_0,22}$; similar uniform priors are used to construct the continuous-time error correlation ($\rho_{q,21}$) and standard deviation ($\sigma_{q,11}$, $\sigma_{q,22}$) parameters. These parameters are then used to construct the relevant covariance matrices during model estimation. This specification is sometimes referred to as a *scaled inverse-Wishart* prior. Although the inverse-Wishart distribution has historically been used as a non-informative prior for covariance matrices, some have recently

[1]It is common in Bayesian statistics to parameterize distributions in terms of precision parameters as opposed to variance parameters. This is primarily done for computational reasons. See Gelman et al. (2013) for more information.

pointed out that it is not entirely uninformative as there is a dependence between the covariance and variance parameters (Barnard et al., 2000; Gelman, 2006). Finally, elements of drift matrix **A** are each given independent univariate priors. In particular, the auto-effects are each assumed to be uniformly distribution between -7 and 0 and the cross-effects are each assumed to be normally distributed with zero means and precision parameters $\tau = {}^{1}/_{1000}$. To reiterate, the priors for the EDM parameters in the proposed implementation are:

$$\mu_0, \mathbf{b} \sim MVN(\mathbf{0}, \frac{1}{1000} \times \mathbf{I})$$

$$\rho_{\phi_0,21}, \rho_{q,21} \sim U(-1,1)$$

$$\sigma_{\phi_0,11}, \sigma_{\phi_0,22}, \sim U(0,100)$$

$$\sigma_{q,11}, \sigma_{q,22}, \sim U(0,100)$$

$$\mathbf{A}_{\textit{auto-effects}} \sim U(-7,0)$$

$$\mathbf{A}_{\textit{cross-effects}} \sim N(0, \frac{1}{1000})$$

Note that the prior distributions for the different EDM parameter matrices are assumed to be independent.

The resulting multivariate posterior distribution does not have a known analytic form. To sample from this distribution, a *slice sampling* algorithm (Neal, 2003) as implemented in the JAGS program (Plummer, 2003) will be used. Slice sampling is an MCMC procedure that can be used to sample from non-standard and unnormalized univariate and multivariate densities. The basic idea behind slice sampling is to uniformly sample from horizontal "slices" of the target distribution which have also been uniformly sampled. Slice sampling is becoming increasingly popular in applied Bayesian statistics because it is either easier to implement or more efficient than other popular MCMC algorithms (e.g., Metropolis-Hastings) (Neal, 2003).

To the author's knowledge, Bayesian estimation of the EDM with panel data has not yet been explored. Some related methods have been proposed, however, and are worth noting. In economics and mathematical finance, Bayesian approaches have been suggested for estimating the parameters

of stochastic differential equations (Sørensen, 2004). The underlying commonality between these methods is the use of *data augmentation* (Tanner & Wong, 1987) to sample unobserved scores between observed data points in a discretization scheme similar to Oversampling, although values are not actually realized in the latter. These approaches are also similar to various numerical methods typically used to approximate differential equations (e.g., Runge Kutta methods). The Bayesian implementation examined in this study does not rely on such explicit extrapolation; moreover, these methods were primarily developed for long time series observed for a single unit of analysis (e.g., the price of a financial product) and have not yet been applied to panel data.

More closely related to the implementation studied here is the work of Oravecz, Tuerlinckx, and Vandekerckhove (2009) and Oravecz, Tuerlinckx, and Vandekerckhove (2011). These researchers developed a Gibbs/Metropolis-Hastings sampling algorithm for the Ornstein-Uhlenbeck (OU) process model. The OU model is a stochastic differential equation very similar to the first-order model presented earlier. In fact, these models are equivalent if the expected value of every variable in $\mathbf{x}(t)$ is zero at all times t. Otherwise, the models are different in form and function. Regarding the latter, the OU model is *mean-reverting* in the sense that it describes processes with strong centralizing forces—that is, the process is continually pulled toward its equilibrium state, and the strength of the pull is dependent on the distance between the current state of the process and its equilibrium value. This feature is not present in the first-order model described earlier. Additionally, these authors did not allow the drift matrix to be asymmetric due to their specific modeling goals.

Although Bayesian estimation routines have not yet been studied for the EDM, they may be advantageous for several reasons. First, studies have shown that Bayesian procedures outperform frequentist procedures in some modeling contexts when small samples are used (Lee & Song, 2004; Wang & McArdle, 2008). Indeed, research regarding the finite sample performance of ML and least squares estimators for simple SEM models has shown that parameter estimate recovery and model selection accuracy are unsatisfactory in small samples (Tanaka, 1987). A recent study also found that variance estimates are positively biased in small samples for state-space time series models estimated with the Kalman filter (H. Song & Ferrer, 2009). Therefore, Bayesian sampling

algorithms may be advantageous for recovering EDM parameters and interval estimates, although this is an empirical question to be investigated. Second, MCMC procedures may be less sensitive to starting values when complex models are estimated (Wang & McArdle, 2008). As a result, convergence issues may be encountered less frequently and local solutions avoided—this is particularly important for estimation of the EDM given the complex non-linear constraints involved and the necessity of good starting values (Steele & Ferrer, 2011). Third, Bayesian interval estimates can be calculated around EDM model parameters that have more direct interpretations (i.e., with what confidence does one believe the parameter is in a specified range?) compared to frequentist intervals (i.e., with repeated sampling, what percentage of intervals contain the population parameter?). Fourth, researchers are able to incorporate prior information into the estimation procedure, which supports cumulative science and meta-analytic thinking (Jackman, 2009). Finally, a feasible Bayesian approach for estimation of the EDM with panel data would provide a foundation for many useful extensions such as random-effects and non-parametric modeling (Chow, Tang, Yuan, Song, & Zhu, 2011; X.-Y. Song & Lee, 2012) that may more readily implemented in the Bayesian framework.

Chapter 3

Methods for Monte Carlo Investigation

The purpose of the present study was to evaluate a Bayesian implementation of the EDM and compare its performance to two frequentist methods. To meet this aim, a large Monte Carlo simulation experiment was conducted that compared Bayesian estimation of the EDM to the EDM-SEM (Oud & Jansen, 2000) and Oversampling (Singer, 2012) approaches under various combinations of sample size, starting values, number of time points, and parameter value configurations. The design and execution of these studies was intended to closely follow the recommendations of Boomsma (2013), Paxton, Curran, Bollen, Kirby, and Chen (2001) and Skrondal (2000) for Monte Carlo research in the social sciences.

3.1 Experimental Design

A summary of the experimental design can be found in Table 3.1. In order to maximize external validity, a small literature review was conducted to inform the simulation conditions. Details of the review can be found in Appendix A. Data was generated from a population model of the form shown in Equation 1.7. The model contains two variables measured repeatedly with a constant measurement lag of one time unit between sampling points. For simplicity, a latent variable measurement model was not included. This model was considered reflective of analyses undertaken in applied research as well as those used for methodological exploration. Specifically, many of

the articles reviewed considered bivariate ARCL models; often, a single outcome was of interest and separate bivariate models were estimated if more than two outcome variables were available. Moreover, most methodological studies of the EDM as applied to panel data have considered two-variable systems, whether as simulated/illustrative examples (e.g., Oud, 2002; Oud & Delsing, 2010; Voelkle et al., 2012) or real applications (e.g., Delsing et al., 2005). The experimental factors manipulated for this study are described in the following sections.

Table 3.1: Monte Carlo Investigation: Design

	Analytic Method		
Experimental Factors	EDM-SEM	Oversampling	Bayesian Estimation
Sample Size	25, 50, 75, 100, 250, 1000	25, 50, 75, 100, 250, 1000	25, 50, 75, 100, 250, 1000
Time Points	2, 4, 8	2, 4, 8	2, 4, 8
Parameter Configurations	Parameter Sets 1-6 (Table 3.2)	Parameter Sets 1-6 (Table 3.2)	Parameter Sets 1-6 (Table 3.2)
Starting Values	Population Values; Perturbed Values ($\pm$.15)	Population Values; Perturbed Values ($\pm$.15)	Perturbed Values ($\pm$.15)
Observation Lag	1	1	1
Number of MCMC chains	N/A	N/A	2
Length of MCMC chains	N/A	N/A	3000

3.1.1 Sample Size

The quality of EDM model results obtained using small sample sizes has not been empirically examined for the EDM-SEM or Oversampling approaches in addition to the Bayesian implemen-

tation proposed herein. Sample size was thus varied as an experimental factor. The samples sizes were chosen to represent a broad range of values likely to be encountered in practice. Indeed, sample sizes found in the literature review ranged from 65 to 4,724 with a mean of 847.30. One study contained two of the largest sample sizes—4,724 and 4,340. The next largest sample size was 1,237 and when the two largest values were removed from consideration, the mean was 526.9. Most of the sample sizes, however, were in a moderate range of 100–500 or a large range of 900–1200. Therefore, sample sizes were chosen to represent a range of small sample sizes (25, 50, 75)—typical, perhaps, of many unpublished studies—as well as moderate (100, 250) and large (1000) sample sizes.

3.1.2 Number of Time Points

In the studies reviewed, the number of time points observed ranged between 2 and 10; the mean number of time points was 3.73. As a result, three levels of the time point factor (2, 4, and 8 time points) were specified for the population model. To date, simulation studies on the various EDM estimation approaches (e.g., Oud, 2007; Oud & Singer, 2008; Voelkle & Oud, 2013) have not considered the effect of the number of time points measured on model results[1]. More measurements provide additional information and it is unclear how reduced information in terms of smaller sample sizes and fewer time points affects model estimation. Conceivably, less longitudinal information could exacerbate the effects of using a small sample size. To what extent this is true for the three estimation methods is an empirical question examined here.

3.1.3 Parameter Configurations

Fourteen parameters were specified for the population model: (a) two initial means ($\mu_{0,11}$, $\mu_{0,21}$), (b) two initial variances ($\phi_{0,11}$, $\phi_{0,22}$), (c) one initial covariance ($\phi_{0,21}$), (d) two auto-effects (a_{11}, a_{22}), (e) two cross-effects (a_{21}, a_{12}), (f) two continuous-time intercepts (b_{11}, b_{21}), (g) two continuous-

[1] Although the *number* of time points has not been examined in simulation studies, the *spacing* between time points has (Voelkle & Oud, 2013). In general, the Voelkle and Oud (2013) study provided support for the use of the Oversampling approach when measurement intervals are not equal within and between individuals.

time error variances (q_{11}, q_{22}), and (h) one continuous-time error covariance (q_{21}). See Figure 1.1 in Chapter 1 for a path diagram of this model. Six configurations of population parameter values were specified for data generation (Table 3.2). In particular, three specifications of drift matrix **A** were combined with two specifications of continuous-time error covariance matrix **Q** and one specification for all other parameter matrices. Values were based on discrete-time estimates that were found in the literature review and converted to continuous-time. In general, results of the review suggested that in discrete-time: (a) autoregressive effects for discrete-time ARCL models are moderate to large in value ($\approx 0.40 - 0.90$); (b) the presence of negative versus positive cross-lagged effects depended on the variables under consideration and were not particularly strong ($\approx -0.20 - 0.20$); (c) initial mean vectors and covariance matrices were variable across studies as the measures employed spanned a wide array of content areas; and (d) little information was available regarding discrete-time intercepts and discrete-time error variances and covariances. More information regarding the choice of parameter values can be found in Appendix A. In Table 3.2, the six continuous-time parameter configurations are shown along with descriptors of the processes implied by each set of values.

3.1.4 Starting Values

It has been noted that estimation of the EDM-SEM requires "good" starting values for convergence of model parameters at the global maximum of the fit function (Steele & Ferrer, 2011). The criterion for a good starting value depends on the modeling context although values that are closer to the true maximum likelihood estimates are more desirable. To determine whether starting values affected model results, two starting value specifications were examined. In the first specification, starting values were specified that matched the parameters used for data generation; in the second specification, values were perturbed from data generating values by obtaining draws from a uniform distribution centered on a given population value and extended .15 units in the positive or negative direction (e.g., for $a_{11} = -0.18$, starting values $\sim U(-0.33, -0.03)$. For some parameter values (e.g., $q_{21} = 0.09$), the uniform distribution was truncated at relevant boundaries

Table 3.2: Parameter Value Sets

Set	μ_0	Φ_0	**A**	**b**	**Q**	Characteristics
1	0.00 0.00	2.00 0.60 0.60 2.00	-0.18 0.04 0.06 -0.29	0.10 0.10	0.30 0.09 0.09 0.30	High autocorrelation Low positive coupling Low stochastic error
2	0.00 0.00	2.00 0.60 0.60 2.00	-0.18 0.04 0.06 -0.29	0.10 0.10	1.00 0.30 0.30 1.00	High autocorrelation Low positive coupling High stochastic error
3	0.00 0.00	2.00 0.60 0.60 2.00	-0.76 0.25 0.17 -0.43	0.10 0.10	0.30 0.09 0.09 0.30	Moderate autocorrelation Moderate positive coupling Low stochastic error
4	0.00 0.00	2.00 0.60 0.60 2.00	-0.76 0.25 0.17 -0.43	0.10 0.10	1.00 0.30 0.30 1.00	Moderate autocorrelation Moderate positive coupling High stochastic error
5	0.00 0.00	2.00 0.60 0.60 2.00	-0.76 -0.25 -0.17 -0.43	0.10 0.10	0.30 0.09 0.09 0.30	Moderate autocorrelation Moderate negative coupling Low stochastic error
6	0.00 0.00	2.00 0.60 0.60 2.00	-0.76 -0.25 -0.17 -0.43	0.10 0.10	1.00 0.30 0.30 1.00	Moderate autocorrelation Moderate negative coupling High stochastic error

(e.g., 0). This approach was chosen to obtain starting values that deviated from the data generating values but would still be within limits of what could be considered reasonable starting values. Convergence in the Bayesian paradigm is assessed using multiple starting values—this process is described in more detail below—and thus the condition in which starting values were set equal to data generating values was specified only for the frequentist approaches.

3.2 Data Generation, Software, Execution

Data were generated using the R statistical computing language (version 3.0.2; R Development Core Team, 2011) and the `MASS` add-on package (version 7.3-33; Venables & Ripley, 2002). Expected covariance matrices and mean vectors based on the model in Equation 1.7 were calculated

for the different combinations of numbers of time points and parameter value sets. These matrices were then used as input to a multivariate normal distribution sampling function included in the MASS package (`mvrnorm`). The analysis model estimated with these data exactly matched the data generating model. The analysis model was estimated in `OpenMx` (version 1.4; Boker et al., 2011) for the frequentist approaches; in particular, two stand-alone `OpenMx` scripts written in the R language by Voelkle et al. (2012) and Voelkle and Oud (2013) were used to implement the EDM-SEM and Oversampling methods, respectively. Following the recommendation of Voelkle and Oud (2013), 30 oversamples were specified between each time point for Oversampling analyses.

The resurgence of Bayesian methods in statistics has resulted in the wide availability of software packages and computing languages capable of estimating models in the Bayesian framework. Indeed, some general statistical programs (e.g., SAS) and dedicated SEM programs (e.g., Mplus, AMOS) used in social science research have added specific Bayesian estimation routines. From a general Bayesian modeling perspective, many options are available. Of these, three programs are notable with regard to their flexibility and capability: WinBUGS/OpenBUGS (Lunn, Spiegelhalter, Thomas, & Best, 2009), JAGS (Plummer, 2003), and STAN (Stan Development Team, 2014). The first two programs in this list utilize variants of the BUGS language (**B**ayesian **I**nference **U**sing **G**ibbs **S**ampling) that was developed throughout the 1980's and early 1990's. In the current study, Bayesian analyses were conducted in the JAGS program via the `rjags` add-on package (Plummer, 2013) for R. The BUGS model syntax and `rjags` code used in the present study can be found in Appendix C

As with any choice of software, there are advantages and disadvantages to using JAGS for EDM model estimation. The primary advantage of JAGS is that it uses a variant of the BUGS language. At the present time, social science researchers that use Bayesian statistics are likely to be familiar with the BUGS language; therefore, specification of the EDM may appear less foreign to these individuals. Indeed, increased use of continuous-time models in general and the EDM in particular may largely depend on the availability of user-friendly and flexible software options. A second advantage of the JAGS program is its capability to calculate the exponential of a matrix via

an add-on module (the `msm` module). To the author's knowledge, neither WinBUGS/OpenBUGS nor STAN contain any built-in or add-on functionality for the calculation of a matrix exponential—a calculation that is necessary to estimate the EDM. Although theoretically a matrix exponential could be calculated in these programs (e.g., via brute-force Taylor series expansion), it may come at the cost of increased computation time or, at the very least, cumbersome programming. One final advantage of the JAGS program for estimating the EDM is its use of efficient slice sampling algorithms.

One of the primary disadvantages of using the JAGS program is in calculating the quantity $\mathbf{A}_{\#}^{-1}$. The matrix inversion function included in JAGS will not compute the inverse of a non-symmetric matrix. As a result, an analytic blockwise inversion method was used (cf. Bernstein, 2005). Although this method is exact, it is not flexible in the sense of accommodating larger numbers of system-level variables without additional programming efforts. Another disadvantage is the lack of support for the scaled-inverse Wishart distribution. In the present study this distribution was specified as a prior for variance/covariance matrices in the model by using a univariate approach (Gelman & Hill, 2006). Model specification/programming would be facilitated if this distribution were allowed.

Data management and execution of the simulation was also handled in the R computing environment. To ensure that results were reproducible, multiple parallel streams of pseudo-random numbers were drawn via the *L'ecuyer* method (L'ecuyer, Simard, Chen, & Kelton, 2002) in R[2]. Computations were completed in a Linux cluster computing environment (Dell PowerEdge 2950 and 1950) located at the author's academic institution. This environment contains 49 compute nodes, each containing eight 2.66 GHz processors; the total amount of random access memory (RAM) per processor is 15.70 GB. The pseudo-random number generation procedure allowed for parallelization of the simulation; however, each individual replication was analyzed using only a single processor, and thus computation time estimates reported in Chapter 3 are based on these processor specifications.

[2]R functions used to implement this method were written by Dr. Paul Johnson, University of Kansas

The simulation contained a total of 216 experimental cells (6 sample size levels $\times$ 3 time point levels $\times$ 6 parameter value sets $\times$ 2 starting value specifications). A total of 1000 replications were analyzed within each cell; therefore, the simulation contained a total of 216,000 replications. For each replication, the three analytic methods under investigation were estimated using the same generated dataset. For the Bayesian approach, 2 *chains*—separate instantiations of the sampling algorithm—were requested. The use of multiple sampling chains allow one to assess convergence of the parameter estimates—that is, convergence of sample draws to the posterior distribution (Jackman, 2009; Gelman et al., 2013). Each chain contained 3000 draws; following the recommendations of Gelman et al. (2013), the first half of each chain was discarded prior to analysis—so-called *burn-in* iterations. This is also the default behavior in the JAGS program when one attempts to summarize the posterior distribution using built-in functions. As a result, parameter and interval estimates produced by the JAGS program were based on 3000 samples (chain 1 + chain 2 = 1500 + 1500) from the posterior distribution.

3.3 Analysis of Results

The outcomes of interest in the present investigation are summarized in Table 3.3. Boomsma (2013) notes that a priori determination of acceptable versus unacceptable outcomes is a critical component of well-designed Monte Carlo studies. Therefore, descriptions of the outcome quantities and guidelines regarding acceptable levels for a select number of these (denoted "Performance Criterion" in Table 3.3) are provided in the following sections. Measures of bias, estimate variability, coverage, and χ^2 Type I error rates are based on the number of replications that successfully converged and resulted in admissible as well as sensible parameter estimates (denoted "R_A"). The issue of whether to include solutions that do not converge or can be considered improper in analyses of Monte Carlo simulations remains unresolved (Boomsma, 2013). In choosing to conduct analyses using only converged and proper solutions, conditions in which the number of analyzed replications was low are noted in the text.

Table 3.3: Monte Carlo Investigation: Outcomes

Outcome	Formula	Performance Criterion
Rate of Convergence	R_C/R	$> 90\%$
Rate of Improper Solutions	R_I/R_C	$< 5\%$
Bias	$\overline{\hat{\theta}_j} - \theta_j$	
Relative Bias	$(\overline{\hat{\theta}_j} - \theta_j)/\theta_j$	$< 5\%$
Root Mean Square Error (RMSE)	$\sqrt{\left[\sum_{r=1}^{R_A} (\hat{\theta}_j - \theta_j)^2\right]/R_A}$	
Coverage	$R_{LB<\theta_j<UB}/R_A$	$> 90\%$
χ^2 Type I Error Rate	$R_{p(\chi^2)<.05}/R_A$	

Note. r = replication. R = total number of replications. R_C = number of converged replications. R_I = number of improper/implausible solution replications. R_A = number of coverged and proper replications used in analysis. $R_{LB<\hat{\theta}_j<UB}$ = number of replications in which interval estimates contained θ_j. $R_{p(\chi^2)<.05}$ = number of replications in which χ^2 test statistic was significant. θ_j = population parameter j. $\hat{\theta}_j$ = parameter j estimate. $\overline{\hat{\theta}_j}$ = mean estimate for parameter j over R.

3.3.1 Convergence Rate

In frequentist statistics, convergence refers to a certain amount of confidence one has that an optimal set of parameter estimates have been found. For all three procedures, convergence was calculated by dividing the number of converged replications (R_C) by the total number of replications (R) per condition. Convergence for the EDM-SEM and Oversampling procedures was determined by inspecting error statuses returned by the `OpenMx` program. Solutions returning error codes of 0 or 1 ("Mx status GREEN") were defined as indicative of model convergence. All other error codes returned by the program were considered indicative of a solution that did not converge; Reasons for non-convergence were varied and discussed in detail in the following chapter.

In Bayesian statistics, when sampling algorithms are used to empirically approximate the posterior distribution of the model parameters via simulation, convergence refers to the condition in which values sampled converge in distribution to the posterior distribution. This type of convergence is typically assessed by running multiple chains using different starting values and assessing whether the chains have "mixed" (Jackman, 2009). This assessment can be done graphically by plotting the sample values against the iteration number—chains that have mixed will be indistinguishable from one another—or by calculating a statistical index. A popular index of convergence in Bayesian statistics is the Potential Scale Reduction Factor (PSRF; Gelman & Rubin, 1992). The PSRF is a measure of the ratio of the between- and within-chain variances. When the PSRF approaches 1 (i.e., the between- and within-chain variances are approximately equal), mixing of the chains is said to occur. PSRF values under 1.1 are generally considered acceptable (Gelman et al., 2013). In the present study, a multivariate generalization of the PSRF (Brooks & Gelman, 1998) was calculated; if the multivariate PSRF was below 1.1, then a set of Bayesian estimates were considered to have converged. To the author's knowledge, guidelines for acceptable rates of convergence have not yet been suggested; in the present study, convergence rates above 90% were considered acceptable.

3.3.2 Improper Solution Rate

Even if a solution converges, the results may not be trustworthy. For example, in SEM, a solution may sometimes result in negative variance estimates or correlation estimates greater than one. Such values—referred to as *Heywood cases* (Bollen, 1989)—exceed theoretical boundaries and are considered inadmissible. In the present study, Heywood cases occurred when: (a) an initial variance or continuous-time error variance was negative; (b) an auto-effect was positive. Solutions containing Heywood cases were considered improper.

Many solutions may not contain Heywood cases but could still be considered improper if the parameter estimates deviate substantially from what would be considered reasonable values. For instance, in SEM, sometimes results "explode" and variance estimates become significantly larger

than one would reasonably expect. Such solutions can influence simulation results considerably. Therefore, outlier analyses were performed prior to analyses of the simulation results. Outliers were identified on a univariate basis for each parameter and within each experimental condition via Tukey's (Tukey, 1977) boxplot method—that is, a parameter estimate was considered an outlier if it deviated more than 3 interquartile ranges from the mean of the parameter estimates within that condition. Contrary to other univariate outlier detection methods, Tukey's method is appropriate for both normal and non-normal distributions. Solutions containing any univariate outliers were also considered improper and removed from further analysis. To calculate the rate of improper solutions, the number of replications flagged as improper (R_I) were divided by the total number of converged replications within each experimental cell. Again, a universal criterion for acceptable rates was not available. Therefore, improper solution rates less than 5% were considered acceptable as a preliminary guiding principle.

3.3.3 Bias / Relative Bias

Estimates of bias and relative bias were calculated in analyzing the simulation results. These measures quantify the difference between a population parameter value (θ_j) used to generate the data and the mean of the parameter estimates obtained over all (converged and proper) replications ($\overline{\hat{\theta}_j}$) within an experimental condition. Bias is based on the metric of the parameter under consideration and thus cannot be compared to other parameters in a model. Therefore, relative bias—defined as bias divided by the true population parameter, $(\overline{\hat{\theta}_j} - \theta_j)/\theta_j$—is often calculated. In the present study, bias was calculated for the initial mean and continuous-time intercept parameters and relative bias was calculated for all other model parameters. This was due to the fact that the initial mean population values were zero in all conditions, and thus relative bias could not be calculated. Results for the continuous-time intercepts are presented in conjunction with the initial means in the next chapter and were also very small in value in all study conditions; for these reasons, bias was also calculated for the continuous-time intercept parameters. Estimates of relative bias were calculated for all other EDM parameters and thus comparable across different parameters.

Using these measures, the accuracy of the parameter estimates produced by each estimation approach was examined. No guidance has been offered for making determinations of acceptable versus unacceptable bias. One must consider the scale of the parameter value in making such judgments, a strategy which is adopted in the current study. Some authors (Muthén & Muthén, 2002; Boomsma, 2013) suggest that values of relative bias less than 5% are reflective of tolerable levels of bias; this criterion was also used in the present study to guide interpretation of the results.

3.3.4 Root Mean Square Error

Statistical estimators are desired that not only produce accurate estimates, but efficient (i.e., less variable) estimates as well. For each EDM model parameter within every experimental condition, an estimate of the Root Mean Square Error (RMSE) was calculated. RMSE quantifies total parameter estimate variability, which can be further partitioned into sampling error and bias (Oud, 2007). The formula for RMSE is shown in Table 3.3. Essentially, RMSE is the square root of the individual deviations of estimates around a population value that have been squared and averaged within a given condition. RMSE values depend in part on the scale of a given parameter; therefore, universal cutoffs of acceptable versus unacceptable levels of RMSE values do not exist. Instead, RMSE values are useful for comparisons across experimental conditions or estimation procedures—such comparisons were made in the present study.

3.3.5 Coverage

Coverage is defined as the proportion of $(1-\alpha)\times 100\%$ intervals that contain the true population parameter value (denoted "$R_{LB<\theta_j<UB}$") over repeated sampling. In simulation experiments, coverage can be calculated for each parameter by dividing the number of intervals which contain the data-generating parameter value by the total number of replications used in the analysis. In this study, an α level of .05 was used. Under frequentist estimation, 95% likelihood-based confidence intervals—the default in `OpenMx`—were calculated. When Bayesian estimation was used, 95% credible intervals were obtained. For a single parameter, credible intervals are formed by

excluding scores below the $(\alpha/2) \times 100\%$ and above the $[1-(\alpha/2)] \times 100\%$ quantiles of sampled posterior distribution values (Jackman, 2009). Coverage is considered optimal when it is close to the nominal interval rate. General guidelines regarding acceptable levels of coverage have not been proposed; in the present study, coverage rates greater than 90% (i.e., $|\text{coverage} - 95\%| < 5\%$) were considered acceptable.

3.3.6 Model fit

Recent efforts have focused on bridging model fit traditions developed within the frequentist and Bayesian frameworks (Levy, 2011). In this study, comparisons in model fit were made between the three estimation approaches[3]. The data generating and analysis model described above is over-identified when 4 or 8 time points are available (30 and 138 degrees of freedom, respectively). As such, the χ^2 test statistic commonly reported in SEM analyses was calculated for each replication under EDM-SEM and Oversampling estimation for conditions containing 4 or 8 time points. The χ^2 test statistic is typically used to evaluate a null hypothesis of *exact fit*—the perfect replication of a sample covariance matrix and mean vector via a model-implied covariance matrix and mean vector. The outcome of interest was the χ^2 Type I error rate which was calculated by dividing the number of replications in which the χ^2 test statistic was significant according to a specified probability threshold ($R_{p(\chi^2)<.05}$) by the number of replications used in the analysis. Two thresholds were examined in this study: .05 and .01.

A similar quantity can be calculated under Bayesian estimation via posterior predictive model-checking (Gelman et al., 2013). An excellent review of this technique as it pertains to frequentist and Bayesian model evaluation is provided by Levy (2011). In the present study, a *posterior predictive p-value* (ppp-value) was calculated that was compared to the p-values generated under frequentist estimation. This ppp-value was constructed by first calculating a set of *realized* χ^2

[3] In the current study, focus is placed on evaluating global model fit—that is, the extent to which a model fits the data according to a universally-applied criterion. In both the frequentist and Bayesian frameworks, a model comparison perspective can also be employed. Debate over which framework should be preferred is beyond the scope of the present study; readers are referred to West, Taylor, and Wu (2012) for more information.

values for each replication. These realized values were calculated by comparing the sample (i.e., simulated data) covariance matrix and mean vector to a model-implied covariance matrix and mean vector generated from each retained iteration of parameter estimates in the MCMC chains. Next, a set of *posterior predictive* χ^2 values were computed. These were calculated in a manner similar to the realized χ^2 values except a posterior predicted data set—generated for each retained MCMC iteration under the analysis model and using the parameter estimates drawn at that iteration—was used in place of the simulated dataset in calculating the χ^2 value. The ppp-value is equal to the proportion of posterior predictive χ^2 values greater than the realized χ^2 values compared at each iteration. A ppp-value of .50 supports the notion that the observed data are indistinguishable from data predicted under the analysis model (good fit) and values close to 0 or 1 are indicative of a model that does not generate data similar to the observed data (poor fit). The resulting ppp-value is not a p-value in the frequentist sense, but utilizing similar cut-off values—specifically, .05—has been proposed (Muthén & Asparouhov, 2012). In this study, the same thresholds examined for the frequentist approaches were also used in evaluating the "Type I error rate" of the Bayesian approach.

3.4 Expectations

The following hypotheses are made with regard to the outcomes of the simulation:

1. *Hypothesis 1*: Convergence rates will decrease and improper solution rates will increase as sample size decreases, fewer numbers of time points are generated, and starting values are perturbed. The Bayesian approach will have the highest rates of convergence and lowest rates of improper solutions across all simulation conditions.

2. *Hypothesis 2*: Across all estimation methods, (relative) bias will increase and coverage rates will deviate more substantially from the nominal 95% rate as sample size and the number of time points decreases. The Bayesian approach will produce more accurate and less variable estimates as well as better coverage rates when sample sizes are small and there are fewer

time points.

3. *Hypothesis 3*: When sample sizes are large and more time points are available, results will be equivalent between the three approaches with regard to bias, parameter estimate variability, and coverage. Regardless of condition, the Oversampling approach will outperform the EDM-SEM approach.

4. *Hypothesis 4*: All three estimation methods will produce nominal Type I error rates at specified levels when sample sizes are large. Rates will become positively biased as sample size decreases.

Chapter 4

Results of Monte Carlo Investigation

Results of the Monte Carlo investigation are presented in this chapter. To reiterate, the general goal of the simulation was to compare Bayesian estimation of the EDM using a non-informative prior distribution over the model parameters to two frequentist methods—EDM-SEM and Over-sampling. Results are presented in the following order. First, rates of convergence and improper solutions are presented. This information is initially provided for the three methods collectively and then the Bayesian and frequentist methods are considered separately in greater detail. Results pertaining to computation time are also presented. Second, results on model estimates are provided. Measures of bias, estimate variability, and coverage are included and reflect the degree to which each estimation approach accurately captured the EDM parameter sampling distributions. Finally, results regarding model fit are compared across the three estimation approaches. Select tables and figures are presented throughout; readers are referred to Appendix B for simulation results not presented in the text.

4.1 Model Convergence

4.1.1 Convergence Rates

Criteria used to determine model convergence for the Bayesian and frequentist approaches were discussed in Chapter 3; in short, a multivariate PSRF value less than 1.1 was indicative of converged Bayesian model estimates and error status codes of 0 or 1 returned by the `OpenMx` program were indicative of converged frequentist estimates. In Table 4.1, rates of convergence and improper solutions for the three estimation methods are presented. These results correspond to the first parameter set which included auto- and cross-effects close to zero (i.e., high stability, low coupling) and lower levels of stochastic error. For the EDM-SEM and Oversampling approaches, convergence information is presented separately for the two different types of starting values—those equal to the data generating parameter values and those perturbed uniformly from data generating values. Additionally, rows are organized according to different values for the sample size and time point conditions.

In general, rates of convergence for the EDM-SEM procedure were lowest among the three estimation approaches in nearly all of the experimental conditions. As shown in Table 4.1, the lowest rate of convergence under the first parameter value configuration (.54) was observed for samples that included 25 units, two time points, and analyzed using the EDM-SEM approach. Rates observed in other conditions for EDM-SEM estimation varied primarily between 70% and 95%. Conversely, convergence was close to 100% for the Bayesian and Oversampling approaches in nearly all experimental conditions. Exceptions to this observation occurred in the smallest sample size condition for Bayesian estimation—for instance, in parameter set 4 (Table B.2), only 29% of replications converged when only two time points were generated. Low rates of convergence also occurred in parameter sets 2 and 6 under this combination of sample size and number of time points for Bayesian estimation.

In Table 4.1, one can see that convergence rates for the frequentist approaches were affected by the starting values. Convergence rates for conditions in which starting values were perturbed from

Table 4.1: Convergence and Improper Solution Rates, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
T	N	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.69 (0.01)	0.54 (0.01)	1.00 (0.01)	0.99 (0.03)	0.96 (0.05)
	50	0.78 (0.00)	0.62 (0.01)	1.00 (0.00)	1.00 (0.02)	1.00 (0.01)
	75	0.83 (0.00)	0.68 (0.00)	1.00 (0.00)	0.99 (0.02)	1.00 (0.01)
	100	0.88 (0.00)	0.70 (0.00)	1.00 (0.00)	0.99 (0.02)	1.00 (0.00)
	250	0.96 (0.00)	0.79 (0.01)	1.00 (0.00)	1.00 (0.02)	1.00 (0.00)
	1000	1.00 (0.00)	0.82 (0.00)	1.00 (0.00)	0.99 (0.02)	1.00 (0.00)
4	25	0.75 (0.01)	0.59 (0.01)	1.00 (0.00)	0.99 (0.02)	1.00 (0.01)
	50	0.89 (0.00)	0.70 (0.00)	1.00 (0.00)	0.99 (0.02)	1.00 (0.00)
	75	0.92 (0.00)	0.74 (0.00)	1.00 (0.00)	1.00 (0.02)	1.00 (0.01)
	100	0.93 (0.00)	0.77 (0.00)	1.00 (0.00)	0.98 (0.02)	1.00 (0.00)
	250	0.99 (0.00)	0.81 (0.00)	1.00 (0.00)	0.99 (0.03)	1.00 (0.01)
	1000	1.00 (0.00)	0.81 (0.00)	1.00 (0.00)	0.99 (0.02)	1.00 (0.00)
8	25	0.82 (0.01)	0.63 (0.00)	1.00 (0.01)	0.99 (0.03)	1.00 (0.01)
	50	0.92 (0.01)	0.73 (0.00)	1.00 (0.01)	0.99 (0.03)	1.00 (0.01)
	75	0.97 (0.00)	0.76 (0.00)	1.00 (0.00)	0.99 (0.03)	1.00 (0.00)
	100	0.99 (0.00)	0.78 (0.00)	1.00 (0.00)	0.99 (0.02)	1.00 (0.00)
	250	1.00 (0.00)	0.81 (0.00)	1.00 (0.00)	0.99 (0.03)	1.00 (0.00)
	1000	1.00 (0.00)	0.83 (0.00)	1.00 (0.00)	0.99 (0.03)	1.00 (0.00)

Note. Convergence rates are shown outside the parentheses. Improper solution rates are shown inside parentheses. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$; $\Phi_0 = \begin{bmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{bmatrix}$; $\mathbf{A} = \begin{bmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{bmatrix}$; $\mathbf{b} = \begin{bmatrix} 0.10 \\ 0.10 \end{bmatrix}$; $\mathbf{Q} = \begin{bmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{bmatrix}$.

population values were uniformly lower across all levels of sample size, number of time points, and parameter value configurations. For parameter set 1, the highest rate of convergence achieved under EDM-SEM estimation and perturbed starting values was 83%. This occurred when samples that included the largest number of units (1000) and time points (8) were analyzed. Additionally, a small number of replications under Oversampling estimation did not converge when perturbed starting values were used. This appeared to occur only when stochastic error was lower (parameter sets 1, 3, and 5).

On the whole, convergence rates increased as sample size and the number of time points in-

Table 4.2: Convergence and Improper Solution Rates, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
T	N	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.90 (0.01)	0.83 (0.01)	1.00 (0.01)	1.00 (0.02)	0.84 (0.07)
	50	0.94 (0.01)	0.89 (0.00)	1.00 (0.00)	1.00 (0.02)	1.00 (0.01)
	75	0.96 (0.00)	0.91 (0.00)	1.00 (0.00)	1.00 (0.02)	1.00 (0.00)
	100	0.97 (0.00)	0.90 (0.00)	1.00 (0.00)	1.00 (0.01)	1.00 (0.00)
	250	0.99 (0.00)	0.93 (0.00)	1.00 (0.00)	1.00 (0.02)	1.00 (0.00)
	1000	1.00 (0.00)	0.96 (0.00)	1.00 (0.00)	1.00 (0.02)	1.00 (0.00)
4	25	0.92 (0.01)	0.84 (0.01)	1.00 (0.01)	0.99 (0.04)	0.99 (0.03)
	50	0.95 (0.00)	0.90 (0.00)	1.00 (0.01)	0.99 (0.04)	1.00 (0.01)
	75	0.97 (0.00)	0.92 (0.00)	1.00 (0.00)	1.00 (0.03)	1.00 (0.00)
	100	0.98 (0.00)	0.92 (0.00)	1.00 (0.00)	1.00 (0.04)	1.00 (0.01)
	250	0.99 (0.00)	0.92 (0.00)	1.00 (0.00)	0.99 (0.04)	1.00 (0.00)
	1000	1.00 (0.00)	0.95 (0.00)	1.00 (0.00)	1.00 (0.04)	1.00 (0.00)
8	25	0.93 (0.01)	0.86 (0.02)	1.00 (0.01)	0.99 (0.04)	1.00 (0.03)
	50	0.97 (0.00)	0.90 (0.00)	1.00 (0.00)	0.99 (0.04)	1.00 (0.00)
	75	0.98 (0.00)	0.90 (0.00)	1.00 (0.00)	0.99 (0.04)	1.00 (0.00)
	100	0.98 (0.00)	0.93 (0.00)	1.00 (0.00)	0.99 (0.03)	1.00 (0.00)
	250	0.99 (0.00)	0.90 (0.00)	1.00 (0.00)	0.99 (0.04)	1.00 (0.00)
	1000	1.00 (0.00)	0.92 (0.00)	1.00 (0.00)	0.99 (0.04)	1.00 (0.00)

Note. Convergence rates are shown outside the parentheses. Improper solution rates are shown inside parentheses. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

creased. Regarding the effect of sample size, nearly all combinations of estimation method, number of time points, parameter value configuration, and starting value type resulted in convergence rates close or equal to 100% at larger sample size sizes (250, 1000). Exceptions occurred under EDM-SEM estimation and perturbed starting values—for instance, convergence rates never exceeded 90% at any sample size for parameter sets 1, 2 (i.e., high stability, low coupling), and 5 (i.e., moderate stability, moderate negative coupling, low stochastic error). Furthermore, the effect of sample size was monotonically increasing except in a limited number of conditions—for example, under parameter set 3 (Table 4.2). Finally, the effect of the number of time points on rates

of convergence was monotonically increasing except for EDM-SEM and Oversampling estimation under perturbed starting values.

The different combinations of model parameter values also appeared to affect convergence rates. In addition to effects already reported, it can be seen that rates of convergence were lower for parameter sets 1 and 2 (low coupling) compared to other parameter configurations (moderate coupling). This is particularly apparent under EDM-SEM estimation and perturbed starting values. The effect of the amount of stochastic error was less clear. For large sample sizes, convergence rates appeared to be larger when stochastic error was high, but such differences were less consistent at lower sample sizes. Finally, there did not appear to be many differences in convergence rates between parameter value sets with positive (3, 4) versus negative coupling (5, 6).

4.1.2 Improper Solution Rates

Rates of improper solutions, defined in Chapter 3 as solutions in which parameter estimates were inadmissible (e.g., negative variance, positive auto-effect) or implausible were generally low. Only in one experimental cell did the improper solution rate exceed 10%; across all conditions, most were less than 5%. There were no salient or consistent effects of sample size or the number of time points. For many conditions, the rate of improper solutions decreased as sample size increased, as can be seen for parameter set 1 in Table 4.1 and the other parameter sets in Appendix B. However, this pattern often did not hold under Oversampling estimation and perturbed starting values.

The most noticeable pattern across the experimental conditions occurred under Oversampling estimation and perturbed starting values: Rates were noticeably higher when levels of stochastic error were lower (i.e., parameter sets 1, 3, 5 versus 2, 4, 6). In addition, improper solutions occurred more often—in many cases, close to a rate of 10%—under Bayesian estimation when sample sizes were small (25, 50). This effect was most noticeable for parameter set 6 (Table B.4) in which the improper solution rate reached 24% for two time points and 25 units.

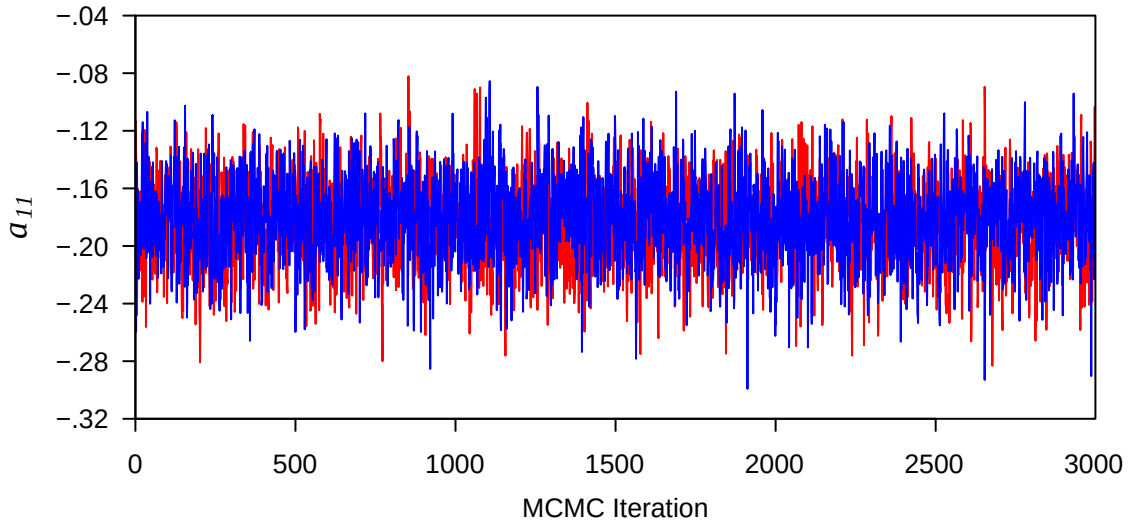


Figure 4.1: Time series plot for replication 1, $N = 100$, $T = 4$, perturbed starting values, parameter set 1

4.1.3 Convergence of the Bayesian Approach

When assessing the convergence of Bayesian estimates, it is helpful to look at time series plots of the simulated parameter draws (Jackman, 2009). In Figure 4.1, a time series plot is shown for the a_{11} parameter estimates from a single replication. The mean of the posterior distribution described by these simulated parameter draws after discarding the burn-in iterations was -.183. This was the first replication in the condition consisting of samples of size 100, four time points, perturbed starting values, and the first parameter set configuration. As shown in the figure, the two chains—one blue, one red—are nearly indistinguishable. The parameter in this plot exhibits strong mixing; both chains appear to traverse the parameter space quickly and the mass of the distribution is clearly centered over the mean of the posterior distribution.

It is also helpful to examine the autocorrelation properties of the time series when assessing convergence (Jackman, 2009). In Figure 4.2, an autocorrelation function is shown for the a_{11} parameter just discussed. Distances between successive iterations of the MCMC chains are plotted on the x-axis and the value of the correlation between values separated at these distances is plotted on the y-axis—not unlike the autoregression function shown in Figure 1.2. The chains containing

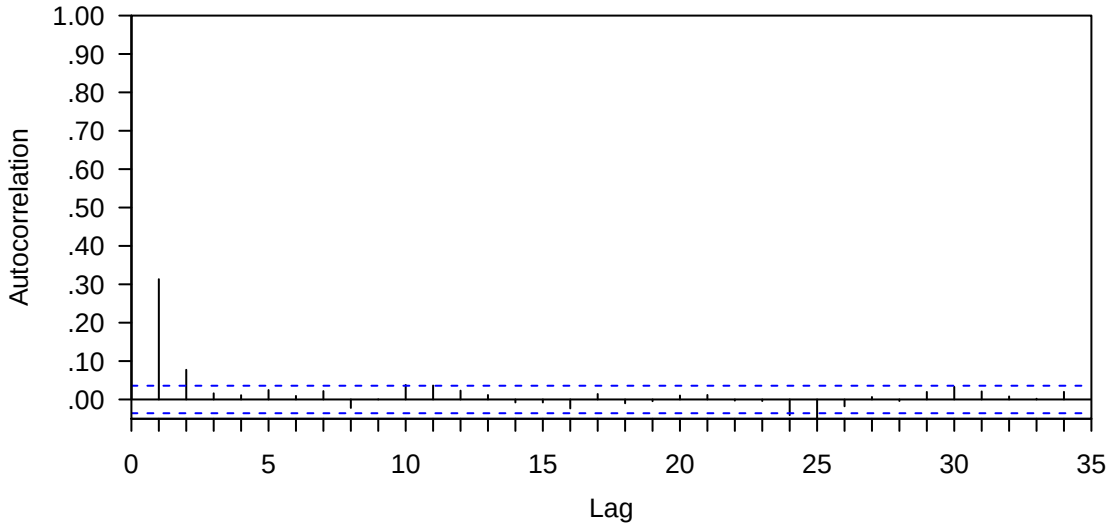


Figure 4.2: Autocorrelation function for replication 1, $N = 100$, $T = 4$, perturbed starting values, parameter set 1

draws of a_{11} in this particular replication do not exhibit patterns of persisting autocorrelation; indeed, most draws are independent of one another after two successive iterations of the MCMC chain.

In the previous section, it was shown that the Bayesian approach demonstrated 100% convergence in nearly all experimental conditions. However, in a small number of extreme data conditions—in particular, conditions containing sample sizes of 25, two time points, and high levels of stochastic error—convergence of the Bayesian approach was poor. In Figure 4.3, a time series plot for the a_{11} parameter in a replication flagged as non-convergent (multivariate PSRF = 1.335) is displayed. This was the 705th replication of the condition that included samples of size 50 and four time points, generated under parameter set 6. Although these chain appear to mix well during the first half of the sampling algorithm, the chains eventually diverge. The mean of these draws for a_{11} was 1.444. When multiplied by the measurement lag of 1 and exponentiated, the drift matrix from this replication produced a discrete-time estimate of .31—a low but reasonable estimate of a discrete-time autoregressive coefficient.

The autocorrelation function for this replication is shown in Figure 4.4. It is seen in this figure that the poor mixing of the MCMC chains also corresponded to some autocorrelation of the

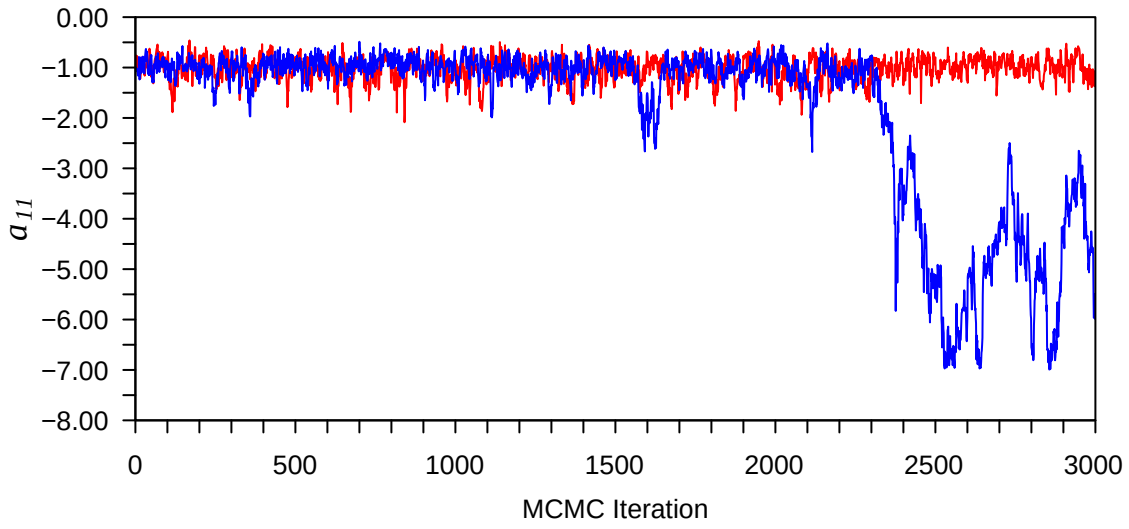


Figure 4.3: Time series plot for replication 703, $N = 50$, $T = 4$, perturbed starting values, parameter set 6

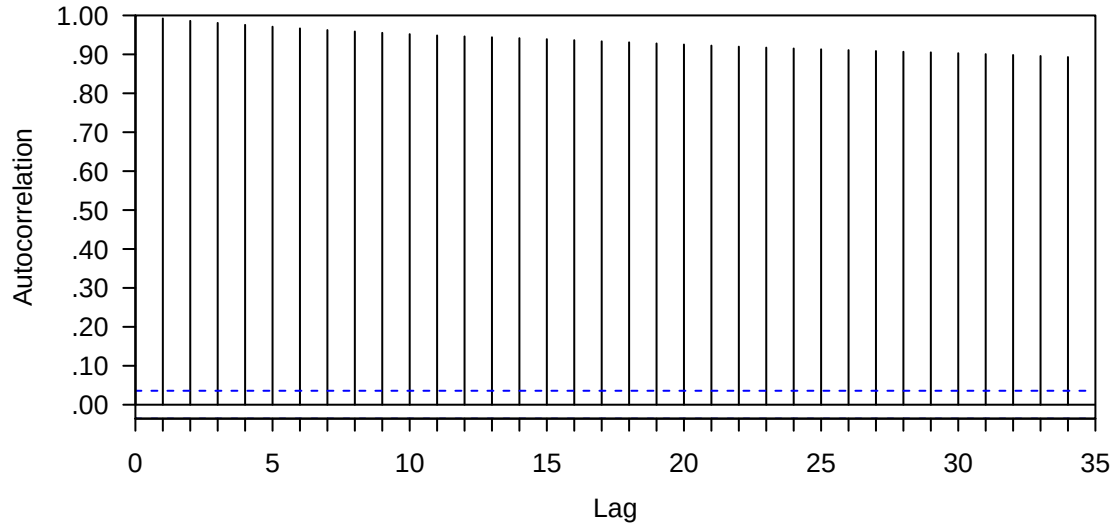


Figure 4.4: Autocorrelation function for replication 705, $N = 50$, $T = 4$, perturbed starting values, parameter set 6

simulated parameters. It appears that draws separated by as many as 35 iterations demonstrated non-negligible autocorrelation. Although high autocorrelation is not always indicative of convergence issues—in fact, it is quite common for posterior distributions to converge in highly autocorrelated series, in which case the MCMC chains are said to evidence *slow mixing*—it is clear in this example that the dependence of the simulated draws on prior iterations may explain why the divergence occurring near iteration 2300 persisted. Therefore, even in cases in which the continuous- and discrete-time estimates appear reasonable, researchers are advised to make use of convergence diagnostic tools such as the PSRF and multivariate PSRF statistics as well as time series and autocorrelation plots.

4.1.4 Convergence of Frequentist Approaches

Although reasons for non-convergence in frequentist estimation may be subtle and complex, the criteria by which convergence is assessed is rather straightforward. When the value of the maximum likelihood or other discrepancy function does not change during successive iterations—that is, within the numerical limits of a convergence criterion—the parameter search algorithm terminates and results are returned to the user. In `OpenMx`, an error code status value of 0 indicates that the program successfully converged in such a manner. An error code status of 1 is labeled as "Mx status GREEN" by the `OpenMx` program and implies that convergence was successful, but arrival at the optimal set of parameter estimates occurred must faster than the search algorithm expected. In general, these solutions are considered acceptable and were included in analyses for the current study. Approximately 14% of all EDM-SEM replications returned an error code status of 1; likewise, roughly 17% of all Oversampling replications returned an error code status of 1.

With regard to non-converged solutions, approximately 12% of all replications failed to converge when the EDM-SEM method was used. Of these, 95% failed due to an interruption in the program calculations; more precisely, the program encountered problems inverting matrices such as the expected covariance matrix or in calculating the exponential of the drift matrix. The other 5% of non-converged solutions failed because the maximum number of iterations in the search

algorithm was reached (error status code 4; less than 0.10% of replications) or no further improvements in the estimates could be found and the optimality criterion was not satisfied (error status code 6; 4.8%). For the Oversampling approach, only 0.14% of replications did not converge. None of these replications were computational failures; 94% of the non-converged solutions returned error status code 6 and the other 6% returned error status code 4.

4.1.5 Computation Time

In this section, results regarding the computational burden of each procedure are presented. The amount of time required for each replication to complete estimation was calculated (in minutes of CPU time) and averaged over the experimental cells; results for parameter set 1 are shown in Table 4.3. As expected, computation time increased monotonically as sample size and the number of time points increased. Computation time was less than 10 minutes across all conditions except under Bayesian estimation and the largest sample size of 1000 as well as a sample size of 250 when eight time points were generated. For the frequentist approaches, computation time was less than five minutes in almost every condition; when a sample size of 1000 was combined with 8 time points, the average computation time exceed 5 minutes in some of the cells. It is important to note that most of the computation time required for the EDM-SEM and Oversampling approaches is likely due to computation of the likelihood-based confidence intervals available in `OpenMx` (Neale & Miller, 1997).

Regarding the different combinations of parameter values, more computation time was required in conjunction with lower amounts of stochastic error; this effect was most noticeable at larger sample sizes. Small differences were also observed such that processes generated with high autocorrelation and low coupling (parameter sets 1 and 2) took longer to compute. Otherwise, differences between parameter value sets were largely negligible. The most noticeable differences in computation time occurred when comparing the Bayesian and frequentist approaches at large sample sizes. For 8 time points and 1000 observed units, the Bayesian approach required more than one hour on average to compute. This is in stark contrast to the frequentist approaches which

Table 4.3: Average CPU Time (in Minutes), Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
T	N	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.03	0.03	0.03	0.03	0.21
	50	0.03	0.03	0.03	0.03	0.42
	75	0.04	0.04	0.04	0.04	0.63
	100	0.06	0.06	0.05	0.05	0.85
	250	0.17	0.16	0.15	0.15	2.30
	1000	0.69	0.66	0.65	0.63	10.47
4	25	0.05	0.05	0.07	0.07	0.52
	50	0.06	0.06	0.09	0.09	1.08
	75	0.09	0.09	0.12	0.12	1.64
	100	0.13	0.12	0.15	0.15	2.28
	250	0.29	0.28	0.31	0.30	6.21
	1000	1.19	1.21	1.20	1.21	39.47
8	25	0.13	0.14	0.21	0.21	1.18
	50	0.21	0.21	0.30	0.30	2.47
	75	0.28	0.28	0.38	0.38	3.85
	100	0.38	0.36	0.47	0.48	5.37
	250	0.69	0.66	0.78	0.79	14.91
	1000	4.02	4.11	3.98	4.11	108.98

Note. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

required approximately five minutes to compute across all parameter set and starting value conditions at eight time points and sample sizes of 1000. Finally, there did not appear to be any consistent pattern of differences in computation time for the frequentist approaches with regard to the two starting value manipulations.

4.2 Model Estimates

The ability to accurately estimate model parameters and describe each parameter's sampling distribution is an important step in any inferential data analysis. In this section, results pertaining to the

accuracy and variability of parameter estimates obtained with each estimation approach are presented. Also, results regarding the quality of interval estimates are reported. The results for these three outcomes are presented in tables organized by parameter groupings in addition to parameter value sets. The groupings were chosen with regard to the type of parameter being estimated; in particular, results for the following pairs of parameters are presented and discussed in tandem: (a) initial means ($\mu_{0,11}, \mu_{0,21}$) and continuous-time intercepts (b_{11}, b_{21}), (b) auto- (a_{11}, a_{22}) and cross-effects (a_{21}, a_{12}), (c) initial variances ($\phi_{0,11}, \phi_{0,22}$) and continuous-time error variances (q_{11}, q_{22}), and (d) the initial covariance ($\phi_{0,21}$) and continuous-time error covariance (q_{21}). The subscripts for each parameter label reflect the positions of each parameter in the appropriate parameter matrix as shown in Figure 1.1, Chapter 1.

4.2.1 Parameter Bias

Recall from Chapter 3 that bias is defined as the discrepancy between the mean of estimates for parameter j ($\overline{\hat{\theta}_j}$) across all replications in a specific condition and the true population parameter value (θ_j) used to generate data—that is, $\overline{\hat{\theta}_j} - \theta_j$. Positive values indicate positive bias (i.e., the estimator tends to overestimate the true population value) and negative values represent negative bias (i.e., the estimator tends to underestimate the true population value). Additionally, relative bias measures the amount of bias relative to the value of the population parameter, i.e., $(\overline{\hat{\theta}_j} - \theta_j)/\theta_j$. As stated in Chapter 3, bias was used to evaluate estimates of the initial means and continuous-time intercepts, and relative bias was used to evaluate the remaining parameters.

Bias was trivial for estimates of the initial means and continuous-time intercepts across all estimation methods and conditions. Results for all six parameter sets are nearly identical and can be found in Appendix B. One exception to this pattern was observed when two time points were generated for samples of size 25. In particular, bias under Bayesian estimation for the first continuous-time intercept was 0.08 under parameter set 2 (Table B.11) and 0.25 under parameter set 6 (Table B.15). A contributing factor to these values could be the low convergence rates observed in these cells.

Table 4.4: Relative Bias for a_{11} and a_{22}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.31, 0.07	0.20, 0.10	0.33, 0.09	0.25, 0.08	4.94, 4.48
	50	0.03, 0.04	0.07, 0.03	0.10, 0.01	0.09, 0.01	0.42, 0.22
	75	0.04, 0.06	0.01, 0.05	0.08, 0.02	0.08, 0.01	0.29, 0.15
	100	-0.02, 0.04	-0.02, 0.05	0.03, 0.01	0.02, 0.02	0.17, 0.11
	250	0.01, 0.01	0.01, 0.02	0.02, 0.00	0.03, 0.01	0.08, 0.04
	1000	0.01, -0.00	0.00, 0.00	0.00, 0.00	0.01, -0.00	0.02, 0.01
4	25	0.15, 0.14	0.08, 0.13	0.17, 0.11	0.12, 0.10	0.32, 0.26
	50	0.04, 0.06	0.04, 0.08	0.06, 0.05	0.07, 0.05	0.16, 0.11
	75	0.01, 0.05	0.02, 0.04	0.02, 0.03	0.04, 0.03	0.09, 0.07
	100	0.03, 0.03	-0.00, 0.04	0.03, 0.02	0.02, 0.03	0.06, 0.06
	250	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.02, 0.01	0.03, 0.02
	1000	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.01, 0.00
8	25	0.07, 0.09	0.06, 0.11	0.09, 0.07	0.09, 0.08	0.16, 0.14
	50	0.04, 0.04	0.04, 0.06	0.04, 0.04	0.06, 0.04	0.09, 0.07
	75	0.02, 0.02	0.03, 0.03	0.02, 0.02	0.04, 0.02	0.06, 0.04
	100	0.02, 0.02	0.01, 0.02	0.02, 0.02	0.02, 0.02	0.03, 0.03
	250	0.01, 0.00	0.01, 0.01	0.01, 0.00	0.01, 0.01	0.01, 0.01
	1000	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Relative bias for the auto- and cross-effects in drift matrix **A** was also low in most conditions. In Table 4.4, relative bias in the auto-effects for parameter set 2 (high stability, no coupling, high stochastic error) is shown. This parameter set contained the largest levels of relative bias observed for the auto-effects. On the whole, relative bias decreased as sample size and the number of time points increased. Within this parameter set, non-negligible levels of relative bias in the auto-effects occurred at the lowest two sample sizes across nearly all combinations of time points and estimation methods; however, in some parameter sets (e.g., parameter set 5, Table B.19) acceptable levels of relative bias were observed for the two frequentist approaches at these smaller sample sizes. In addition, as shown in Table 4.4, relative bias was generally larger under the Bayesian

Table 4.5: Relative Bias for a_{21} and a_{12}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.10, 1.19	0.14, 0.76	-0.18, 0.41	-0.23, 0.34	9.87, 14.92
	50	0.20, 0.53	0.21, 0.50	-0.08, 0.11	-0.04, 0.20	0.04, 0.44
	75	0.27, 0.33	0.31, 0.50	0.00, 0.01	-0.03, 0.25	0.03, 0.38
	100	0.17, 0.21	0.16, 0.18	-0.07, -0.05	-0.02, -0.05	0.03, 0.06
	250	0.07, 0.15	0.09, 0.04	-0.02, 0.03	-0.02, -0.05	-0.00, -0.02
	1000	0.01, 0.05	0.02, 0.01	0.01, 0.03	-0.01, 0.01	-0.01, 0.02
4	25	0.23, 0.45	0.17, 0.33	-0.05, 0.18	0.01, -0.02	0.17, 0.19
	50	0.18, 0.31	0.20, 0.21	0.02, -0.02	0.03, 0.06	0.10, 0.16
	75	0.12, 0.09	0.17, 0.18	0.01, -0.08	0.06, 0.04	0.10, 0.09
	100	0.09, 0.06	0.13, 0.04	0.03, -0.05	0.03, -0.06	0.06, -0.03
	250	0.03, 0.00	0.02, -0.01	0.01, -0.02	-0.01, -0.04	0.01, -0.02
	1000	0.01, 0.01	-0.01, 0.02	0.01, 0.01	-0.01, 0.02	-0.01, 0.02
8	25	0.12, 0.24	0.23, 0.28	-0.00, 0.00	0.07, 0.09	0.15, 0.18
	50	0.07, 0.09	0.13, 0.15	0.03, -0.03	0.03, 0.13	0.07, 0.16
	75	0.04, 0.08	0.03, 0.10	0.01, -0.01	-0.01, 0.07	0.01, 0.09
	100	0.00, 0.03	0.02, 0.00	0.00, -0.02	-0.01, -0.03	0.01, -0.01
	250	0.01, 0.03	0.01, 0.00	0.01, 0.03	-0.00, 0.01	0.01, 0.01
	1000	-0.00, -0.00	0.01, -0.00	-0.00, -0.00	0.00, 0.00	0.01, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

approach compared to the frequentist approach. These differences diminished as sample size and the number of time points increased. For the frequentist approaches, there did not appear to be any effect of the starting values (population versus perturbed) on the relative bias of the auto-effect estimates. Finally, comparisons across the six parameter sets suggests that relative bias in the auto-effects was larger for conditions with higher levels of stochastic error (parameter sets 2, 4, and 6) compared to conditions with lower levels of stochastic error (parameter sets 1, 3, and 5).

Similar patterns were observed for the cross-effects, although one key difference emerged: Under EDM-SEM estimation, relative bias in the cross-effects was much higher compared to the auto-effects. Indeed, in Table 4.5, one can see that relative bias in the cross-effects was larger for

EDM-SEM estimation compared to Bayesian estimation in many of the conditions. Moreover, non-trivial levels of relative bias were observed at moderate sample sizes for EDM-SEM estimation, particularly in the presence of only 2 or 4 time points. Therefore, levels of relative bias were largely comparable between the Bayesian and EDM-SEM approaches. Conversely, lower levels of relative bias were observed in the cross-effects under Oversampling estimation in most conditions, although some instances of non-negligible bias did occur.

Altogether, it appears that at lower sample sizes (< 100 units), non-negligible amounts of relative bias emerged under all three estimation approaches. Relative bias in the auto-effects was higher under Bayesian estimation compared to the two frequentist approaches, and relative bias in the cross-effects was higher for the Bayesian and EDM-SEM approaches compared to the Oversampling approach. As sample sizes and the number of time points increased, however, relative bias became largely ignorable across the three estimation approaches.

Relative bias estimates for the initial variances in parameter set 1 are shown in Table 4.6. The pattern of results observed here are consistent with results from the other parameter sets as reported in Appendix B and similar to those discussed for the auto- and cross-effects. Relative bias levels for the initial variances were negative but low under frequentist estimation across all conditions. The Bayesian approach resulted in positive relative bias estimates greater than 5% at sample sizes of 25 and 50 as well as in some conditions with samples of size 75. For all three estimation methods, relative bias was mitigated as sample size increased but was unaffected by increases in the number of time points. Moreover, the different starting value conditions did not appear to affect either frequentist estimation method.

Relative bias estimates were larger for the continuous-time error variances as opposed to the initial variances across all three estimation methods and experimental factors. Relative bias estimates for continuous-time error variances estimated in parameter set 1 are shown in Table 4.7. Results suggest that relative bias was non-negligible for the three estimation methods at sample sizes of 25 and 50 when two time points were available. Again, relative bias was higher under Bayesian estimation compared to the frequentist approaches at small sample sizes. Trivial levels of

Table 4.6: Relative Bias for $\phi_{0,11}$ and $\phi_{0,22}$, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	-0.03, -0.05	-0.00, -0.04	-0.04, -0.05	-0.01, -0.05	0.22, 0.18
	50	-0.01, -0.02	-0.03, -0.01	-0.01, -0.03	-0.04, -0.01	0.06, 0.09
	75	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.06, 0.06
	100	-0.02, -0.01	-0.00, -0.00	-0.02, -0.01	-0.00, -0.01	0.05, 0.04
	250	-0.00, -0.00	-0.01, -0.01	-0.00, -0.00	-0.01, -0.01	0.01, 0.01
	1000	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.01
4	25	-0.04, -0.04	-0.03, -0.04	-0.04, -0.05	-0.06, -0.06	0.16, 0.16
	50	-0.01, -0.02	-0.01, -0.01	-0.01, -0.02	-0.01, -0.02	0.09, 0.08
	75	-0.01, -0.01	-0.01, -0.02	-0.01, -0.01	-0.01, -0.02	0.05, 0.05
	100	-0.01, -0.00	-0.01, 0.00	-0.01, -0.00	-0.02, -0.00	0.04, 0.05
	250	-0.00, -0.01	-0.00, -0.01	-0.00, -0.01	-0.00, -0.01	0.02, 0.01
	1000	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	0.00, 0.00
8	25	-0.03, -0.04	-0.03, -0.03	-0.04, -0.05	-0.04, -0.04	0.18, 0.19
	50	-0.02, -0.02	-0.02, -0.02	-0.02, -0.01	-0.03, -0.02	0.08, 0.08
	75	-0.02, -0.01	-0.02, -0.01	-0.02, -0.02	-0.01, -0.01	0.05, 0.05
	100	-0.02, -0.01	-0.02, -0.01	-0.02, -0.01	-0.01, -0.01	0.04, 0.04
	250	-0.00, -0.00	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	0.02, 0.01
	1000	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.01, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

relative bias were observed for conditions with larger sample sizes and more available time points. For Bayesian estimation, relative bias was higher when the amount of stochastic error was large, although differences were only noticeable at small and moderate sample sizes. Differences due to parameter set and starting value configurations were minor for the two frequentist approaches.

Relative bias estimates for the initial covariance parameter and the continuous-time error covariance parameter were comparable across estimation methods and generally low. In Table 4.8, results for parameter set 1 are presented. In general, relative bias estimates exceeded 5% only when the sample size was 25, and not in all conditions. Additionally, most conditions in which a frequentist method was used resulted in negative relative bias values whereas positive bias values

Table 4.7: Relative Bias for q_{11} and q_{22}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	-0.12, -0.09	-0.13, -0.11	-0.12, -0.10	-0.12, -0.11	0.28, 0.28
	50	-0.06, -0.06	-0.07, -0.06	-0.05, -0.06	-0.06, -0.06	0.11, 0.11
	75	-0.03, -0.04	-0.04, -0.03	-0.03, -0.05	-0.04, -0.03	0.07, 0.07
	100	-0.04, -0.03	-0.04, -0.03	-0.03, -0.03	-0.03, -0.03	0.04, 0.05
	250	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.02, 0.02
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.01
4	25	-0.03, -0.03	-0.04, -0.03	-0.03, -0.03	-0.03, -0.03	0.08, 0.08
	50	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.02	0.05, 0.04
	75	-0.02, -0.01	-0.01, -0.01	-0.02, -0.01	-0.01, -0.01	0.03, 0.03
	100	-0.00, -0.01	-0.01, -0.01	-0.00, -0.01	-0.01, -0.01	0.02, 0.02
	250	-0.01, -0.00	-0.00, 0.00	-0.01, -0.00	-0.00, -0.00	0.01, 0.01
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
8	25	-0.01, -0.00	-0.02, -0.01	-0.00, -0.01	-0.02, -0.01	0.03, 0.04
	50	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	-0.00, -0.00	0.02, 0.02
	75	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.01, 0.01
	100	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	0.01, 0.01
	250	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
	1000	0.00, -0.00	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

were observed under Bayesian estimation. These results were consistent across all parameter sets as reported in Appendix B

To summarize the preceding section on parameter estimate bias, the following observations are made. First, bias in the EDM parameters decreased as the sample size increased for all estimation methods and across all other experimental factors. The number of time points was also related to the amount of bias in many conditions such that bias diminished as the number of time points increased. Second, at moderate to large sample sizes (e.g., 100 or more sampled units), the three estimation approaches produced comparable estimates of bias, which were within acceptable ranges. At small sample sizes, however, the Bayesian approach resulted in unacceptable

Table 4.8: Relative Bias for $\phi_{0,21}$ and q_{21}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	-0.09, -0.13	-0.03, -0.19	-0.09, -0.10	0.02, -0.15	0.16, 0.08
	50	-0.01, -0.06	-0.01, -0.11	-0.02, -0.03	-0.02, -0.06	0.04, 0.05
	75	0.00, -0.05	0.01, -0.04	0.01, -0.04	-0.01, -0.02	0.03, 0.05
	100	-0.02, -0.04	0.01, -0.06	-0.02, -0.03	-0.00, -0.04	0.03, 0.02
	250	0.01, -0.02	-0.01, -0.02	0.01, -0.02	-0.01, -0.01	0.00, 0.01
	1000	0.00, -0.01	0.00, -0.00	0.00, -0.01	0.00, -0.00	0.01, 0.00
4	25	-0.07, -0.04	-0.05, -0.05	-0.05, -0.00	-0.11, -0.02	0.01, 0.06
	50	0.01, -0.00	0.00, -0.03	0.01, 0.00	-0.01, -0.02	0.06, 0.02
	75	-0.02, -0.02	-0.00, -0.01	-0.01, -0.02	0.01, -0.00	0.05, 0.02
	100	-0.01, -0.02	-0.00, -0.01	-0.01, -0.01	-0.01, -0.00	0.02, 0.01
	250	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.00, -0.01	0.01, 0.00
	1000	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, -0.00	0.01, 0.00
8	25	-0.03, -0.01	-0.04, -0.05	-0.03, -0.01	-0.04, -0.03	0.08, 0.00
	50	-0.02, -0.01	-0.04, 0.00	-0.02, -0.00	-0.04, 0.01	0.02, 0.02
	75	-0.03, -0.01	-0.04, -0.00	-0.02, -0.00	-0.03, 0.00	0.01, 0.01
	100	-0.01, -0.00	-0.02, -0.01	-0.01, -0.00	-0.02, -0.00	0.01, 0.01
	250	-0.00, -0.00	0.01, 0.00	-0.00, -0.00	0.00, 0.00	0.02, 0.01
	1000	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.01, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

levels of relative bias for some EDM parameters—specifically auto-effects, cross-effects, initial variances, and continuous-time error variances—and these levels were larger compared to the two frequentist estimation approaches. A likely explanation for the large values of bias and relative bias observed for Bayesian estimation in cells with sample sizes of 25 and two time points was the low rate of convergence in these cells. In some cases, the EDM-SEM and Oversampling estimation approaches also produced unacceptable levels of bias, most notably for cross-effect and continuous-time error variance parameters. Low levels of bias in initial mean and continuous-time intercept parameters were observed across all three estimation methods and combinations of sample size, numbers of time points, starting values, and parameter value sets. Finally, the parameter

value configurations appeared to influence parameter estimation in some conditions. In particular, higher levels of relative bias were observed in the auto-effects, cross-effects, and continuous-time error variances when levels of stochastic error were higher. These levels decreased as sample size and the number of time points increased.

4.2.2 Root Mean Square Error

Results in the preceding section pertained to the accuracy of the continuous-time model parameter estimates. In this section, results regarding the variability of the continuous-time model parameter estimates are reported. In particular, RMSE values are presented and discussed; as described in Chapter 3, the RMSE is a measure of sampling variability that also accounts for bias in a particular parameter estimate. Recall the formula for RMSE is $\sqrt{\sum_{r=1}^{R_A}(\hat{\theta}_j - \theta_j)^2/R_A}$ where r represents replications (R_A total used in the analysis) and j refers to a specific parameter. Estimators that produce smaller values of RMSE are more desirable.

Prior to discussing specific results, it should be noted that for all continuous-time model parameters examined in this study and across all experimental manipulations, use of population versus perturbed starting values did not appear to affect RMSE values for the two frequentist estimation procedures. RMSE values for the initial mean and continuous-time intercept parameters are reported in Appendix B. Overall, RMSE values were comparable across the three estimation methods. RMSE values were higher for the initial mean parameters as opposed to the continuous-time intercepts. Furthermore, RMSE values decreased as the number of time points increased at small and moderate sample sizes for the continuous-time intercept parameters but not for the initial mean parameters. Finally, the parameter value configurations did not affect RMSE values for the initial mean parameters but did affect RMSE values for the continuous-time intercept values; specifically, higher RMSE values were observed when levels of the stochastic error process were higher.

RMSE results for the auto- and cross-effects are reported in Tables 4.9 and 4.10 for parameter set 2. Results were similar between these two parameter types. As expected, RMSE estimates declined as sample size and the number of time points increased. The only cell in which the three

Table 4.9: RMSE for a_{11} and a_{22}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.17, 0.18	0.15, 0.17	0.17, 0.18	0.16, 0.17	1.66, 2.22
	50	0.11, 0.12	0.11, 0.13	0.11, 0.12	0.11, 0.13	0.14, 0.15
	75	0.10, 0.10	0.09, 0.10	0.10, 0.10	0.09, 0.10	0.11, 0.12
	100	0.08, 0.09	0.08, 0.09	0.08, 0.09	0.08, 0.09	0.09, 0.10
	250	0.05, 0.06	0.05, 0.06	0.05, 0.06	0.05, 0.06	0.06, 0.06
	1000	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
4	25	0.10, 0.12	0.10, 0.12	0.10, 0.12	0.10, 0.12	0.12, 0.15
	50	0.06, 0.07	0.07, 0.08	0.07, 0.08	0.07, 0.08	0.08, 0.09
	75	0.05, 0.06	0.05, 0.06	0.05, 0.06	0.06, 0.06	0.06, 0.07
	100	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.06
	250	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	1000	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.01, 0.02
8	25	0.06, 0.08	0.06, 0.08	0.06, 0.08	0.07, 0.08	0.07, 0.09
	50	0.04, 0.05	0.04, 0.05	0.04, 0.06	0.05, 0.05	0.05, 0.06
	75	0.03, 0.04	0.03, 0.04	0.03, 0.04	0.04, 0.04	0.04, 0.04
	100	0.03, 0.03	0.03, 0.04	0.03, 0.03	0.03, 0.04	0.03, 0.04
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

estimation methods exhibited dissimilar results is shown in the top row in the rightmost column—for two time points and 25 units per sample. In this cell, RMSE values for the Bayesian approach were grossly inflated, likely a result of the low convergence rates observed. Concerning parameter value configurations, parameter sets containing higher levels of stochastic process error resulted in higher RMSE values for all three estimation approaches.

Results for the initial variance parameters are provided in Appendix B; Generally, RMSE values for the initial variance parameters were consistent across the estimation methods and parameter value sets. Similar to results for the initial mean parameters, the number of time points generated did not affect RMSE estimates for these parameters. The only difference between the estimation

Table 4.10: RMSE for a_{21} and a_{12}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.16, 0.18	0.15, 0.18	0.17, 0.19	0.17, 0.19	1.32, 1.44
	50	0.11, 0.12	0.11, 0.12	0.12, 0.12	0.12, 0.12	0.13, 0.13
	75	0.09, 0.09	0.09, 0.10	0.10, 0.10	0.10, 0.10	0.10, 0.11
	100	0.08, 0.08	0.07, 0.08	0.09, 0.09	0.09, 0.09	0.09, 0.09
	250	0.05, 0.05	0.05, 0.05	0.05, 0.06	0.05, 0.06	0.05, 0.06
	1000	0.02, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
4	25	0.09, 0.11	0.09, 0.11	0.10, 0.12	0.10, 0.12	0.11, 0.12
	50	0.06, 0.07	0.06, 0.07	0.07, 0.08	0.07, 0.08	0.07, 0.08
	75	0.05, 0.06	0.05, 0.06	0.06, 0.06	0.05, 0.06	0.05, 0.06
	100	0.04, 0.05	0.04, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05
	250	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	1000	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.01, 0.02
8	25	0.05, 0.07	0.06, 0.07	0.06, 0.07	0.06, 0.07	0.06, 0.08
	50	0.04, 0.05	0.04, 0.05	0.04, 0.05	0.04, 0.05	0.04, 0.05
	75	0.03, 0.04	0.03, 0.04	0.03, 0.04	0.03, 0.04	0.03, 0.04
	100	0.03, 0.03	0.03, 0.03	0.03, 0.04	0.03, 0.04	0.03, 0.04
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

methods occurred at the lowest sample size—Bayesian estimates of the initial variances had larger RMSE values compared to frequentist estimates when samples contained 25 units; again, this is a likely consequence of the low convergence rate in this cell. RMSE estimates for the continuous-time error variances, on the other hand, behaved more similarly to the auto- and cross-effect parameters. In Table 4.11, results are shown for the continuous-time error variances estimated for parameter set 6. At smaller sample sizes and fewer time points, the Bayesian estimation approach contained more estimation error than the two frequentist methods, which produced more or less equivalent results. Finally, in accordance with results presented for the auto- and cross-effects, RMSE estimates became larger for parameter sets that contained higher levels of stochastic error.

Table 4.11: RMSE for q_{11} and q_{22}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.32, 0.32	0.33, 0.34	0.33, 0.32	0.33, 0.34	6.17, 1.94
	50	0.24, 0.22	0.24, 0.23	0.24, 0.23	0.24, 0.23	0.58, 0.34
	75	0.19, 0.19	0.19, 0.19	0.19, 0.19	0.19, 0.19	0.27, 0.26
	100	0.17, 0.17	0.16, 0.15	0.17, 0.16	0.17, 0.15	0.22, 0.20
	250	0.10, 0.10	0.10, 0.11	0.10, 0.10	0.10, 0.11	0.11, 0.11
	1000	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05
4	25	0.21, 0.20	0.21, 0.20	0.21, 0.20	0.21, 0.19	0.59, 0.30
	50	0.15, 0.14	0.15, 0.13	0.15, 0.14	0.15, 0.14	0.21, 0.16
	75	0.12, 0.11	0.12, 0.12	0.12, 0.11	0.12, 0.12	0.14, 0.13
	100	0.10, 0.10	0.11, 0.10	0.10, 0.10	0.11, 0.10	0.12, 0.11
	250	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06
	1000	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
8	25	0.15, 0.13	0.14, 0.13	0.15, 0.13	0.15, 0.13	0.20, 0.16
	50	0.10, 0.09	0.10, 0.10	0.10, 0.10	0.10, 0.10	0.12, 0.11
	75	0.08, 0.07	0.08, 0.07	0.08, 0.08	0.08, 0.07	0.09, 0.08
	100	0.07, 0.07	0.07, 0.06	0.07, 0.07	0.07, 0.06	0.08, 0.07
	250	0.05, 0.04	0.04, 0.04	0.05, 0.04	0.04, 0.04	0.05, 0.04
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Patterns of estimate error discussed throughout this section also held for the initial covariance and continuous-time error covariance parameter. Specifically, RMSE estimates for the initial covariance parameter were unaffected by the number of time points and parameter value configurations. In contrast, RMSE values for the continuous-time error covariance diminished as sample sizes and the number of time points increased and grew when additional stochastic error was added to the system. Again, Bayesian RMSE estimates were more variable than the frequentist RMSE estimates at the smallest sample size.

To review, the three EDM estimation approaches were comparable in terms of parameter estimate error levels in nearly all conditions; the most notable exception occurred in the smallest

sample size condition of 25 units. In conditions utilizing this sample size, it was observed that the Bayesian approach produced greater levels of estimate error. However, when sample sizes increased, RMSE estimates diminished for all model parameters. Moreover, parameters related to the longitudinal dynamics of the system (i.e., auto-effects, cross-effects, continuous-time intercepts, continuous-time variances and covariance) were estimated with less error as the number of time points increased and the amount of process-level stochastic error decreased. Overall, these findings were in agreement with those reported previously for parameter estimate bias.

4.2.3 Interval Coverage

The final outcome of interest with regard to the model estimates was coverage. Recall from Chapter 3 that coverage measures the quality of interval estimates from a frequentist perspective. Specifically, coverage addresses the question: "if one were to repeatedly obtain samples of a given size from a defined population, does the collection of intervals estimated from these samples capture the population parameter at nominal rates?". Recall that 95% profile likelihood confidence intervals (Neale & Miller, 1997) were calculated for the two frequentist approaches whereas 95% credible intervals (Jackman, 2009) were calculated for the Bayesian approach.

In general, coverage rates were close to the nominal 95% rate for all three estimation methods under all experimental conditions. Additionally, it appears that in most cases the two frequentist estimation methods produced coverage rates that were below or at the nominal 95% rate whereas the Bayesian approach produced coverage rates that were above or at the nominal 95% rate. It should be noted, however, that in most cases these deviations were trivial. Results concerning the initial mean and continuous-time intercept parameters are provided in Appendix B. Coverage rates were within the limits defined as acceptable (90%–100%) for all three estimation methods and across all simulation conditions. At lower sample sizes, coverage rates were slightly underestimated by the frequentist approaches and slightly overestimated by the Bayesian approach.

Coverage results for the auto- and cross-effects in parameter set 2 are shown in Tables 4.12 and 4.13. Regarding the auto-effects, coverage rates were below acceptable levels in some of the

Table 4.12: Coverage for a_{11} and a_{22}, Parameter Set 2

		EDM-SEM				Oversampling				Bayesian	
		Population Starts		Perturbed Starts		Population Starts		Perturbed Starts		Perturbed Starts	
T	N	a_{11},	a_{22}	a_{11},	a_{22}	a_{11},	a_{22}	a_{11},	a_{22}	a_{11},	a_{22}
2	25	0.87,	0.87	0.89,	0.87	0.96,	0.97	0.97,	0.97	0.88,	0.79
	50	0.89,	0.89	0.87,	0.86	0.97,	0.97	0.97,	0.95	0.95,	0.95
	75	0.88,	0.88	0.88,	0.85	0.96,	0.95	0.96,	0.94	0.95,	0.94
	100	0.89,	0.87	0.89,	0.86	0.96,	0.95	0.96,	0.95	0.96,	0.94
	250	0.92,	0.91	0.91,	0.91	0.95,	0.95	0.95,	0.95	0.94,	0.94
	1000	0.95,	0.95	0.95,	0.95	0.95,	0.95	0.95,	0.96	0.95,	0.96
4	25	0.88,	0.84	0.87,	0.85	0.96,	0.94	0.95,	0.94	0.94,	0.94
	50	0.92,	0.90	0.89,	0.87	0.95,	0.95	0.95,	0.94	0.94,	0.93
	75	0.91,	0.89	0.90,	0.91	0.95,	0.95	0.94,	0.95	0.94,	0.95
	100	0.93,	0.92	0.94,	0.91	0.96,	0.95	0.95,	0.95	0.94,	0.94
	250	0.95,	0.94	0.96,	0.94	0.96,	0.95	0.96,	0.95	0.96,	0.95
	1000	0.95,	0.95	0.95,	0.96	0.95,	0.96	0.95,	0.95	0.95,	0.96
8	25	0.89,	0.88	0.89,	0.89	0.95,	0.94	0.94,	0.95	0.93,	0.93
	50	0.94,	0.89	0.93,	0.89	0.97,	0.93	0.93,	0.94	0.93,	0.93
	75	0.93,	0.91	0.94,	0.93	0.94,	0.93	0.95,	0.95	0.94,	0.94
	100	0.95,	0.95	0.94,	0.94	0.95,	0.96	0.95,	0.95	0.95,	0.94
	250	0.95,	0.96	0.95,	0.95	0.95,	0.96	0.95,	0.95	0.95,	0.95
	1000	0.94,	0.94	0.95,	0.95	0.94,	0.93	0.94,	0.94	0.94,	0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

small sample size conditions. When the data contained only 2 or 4 time points, the EDM-SEM method provided coverage rates below the nominal 95% level for small and sometimes moderate sample sizes and under both starting value manipulations. The Bayesian estimation approach produced rates close to the nominal 95% level except in the lowest sample size and time point condition. Coverage rates for the Oversampling approach were close to the nominal 95% rate in all study conditions.

Results for the two cross-effect parameters (Table 4.13) were similar to those reported for the auto-effects. Coverage rates were slightly closer to the nominal 95% rate in moderate sample sizes for the EDM-SEM approach. Once again, the Bayesian interval estimates did not sufficiently cover

Table 4.13: Coverage for a_{21} and a_{12}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.85, 0.85	0.83, 0.87	0.95, 0.92	0.94, 0.94	0.87, 0.89
	50	0.87, 0.89	0.89, 0.90	0.95, 0.96	0.95, 0.95	0.97, 0.95
	75	0.91, 0.93	0.91, 0.93	0.95, 0.95	0.94, 0.95	0.95, 0.96
	100	0.92, 0.94	0.91, 0.92	0.94, 0.95	0.94, 0.95	0.94, 0.95
	250	0.95, 0.95	0.93, 0.96	0.96, 0.94	0.95, 0.96	0.96, 0.95
	1000	0.96, 0.96	0.95, 0.96	0.96, 0.94	0.94, 0.95	0.94, 0.94
4	25	0.88, 0.92	0.86, 0.89	0.93, 0.94	0.95, 0.94	0.96, 0.95
	50	0.93, 0.96	0.92, 0.94	0.94, 0.94	0.95, 0.95	0.95, 0.95
	75	0.93, 0.96	0.94, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.95
	100	0.96, 0.96	0.95, 0.95	0.97, 0.95	0.95, 0.94	0.95, 0.94
	250	0.97, 0.97	0.96, 0.97	0.96, 0.97	0.96, 0.96	0.95, 0.96
	1000	0.95, 0.95	0.96, 0.94	0.95, 0.95	0.96, 0.93	0.95, 0.94
8	25	0.95, 0.95	0.93, 0.95	0.96, 0.96	0.94, 0.95	0.94, 0.95
	50	0.94, 0.96	0.96, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.94
	75	0.96, 0.95	0.96, 0.95	0.95, 0.94	0.95, 0.94	0.95, 0.94
	100	0.96, 0.96	0.95, 0.95	0.96, 0.94	0.95, 0.93	0.95, 0.94
	250	0.96, 0.95	0.96, 0.95	0.96, 0.95	0.96, 0.95	0.96, 0.95
	1000	0.94, 0.95	0.95, 0.96	0.94, 0.94	0.94, 0.95	0.95, 0.96

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

the population parameter values for the lowest sample size and time point condition but were otherwise close to the nominal rate. Additionally, the Oversampling approach resulted in acceptable coverage rates for the cross-effects under all conditions. Regarding the different parameter set conditions, coverage rates for both the auto- and cross-effects were lower in the high stochastic error conditions when sample sizes were small.

Acceptable coverage rates were demonstrated in nearly all conditions for the initial variance parameters; results are shown in Appendix B. The sole exception occurred in combinations of the lowest sample size and two time points where coverage rates for the Bayesian approach were slightly outside of acceptable ranges, likely due to the small number of replications analyzed. In

Table 4.14: Coverage for q_{11} and q_{22}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.91, 0.89	0.93, 0.90	0.92, 0.91	0.92, 0.90	0.96, 0.94
	50	0.93, 0.93	0.93, 0.93	0.93, 0.94	0.93, 0.93	0.96, 0.93
	75	0.93, 0.95	0.95, 0.94	0.93, 0.95	0.94, 0.94	0.95, 0.95
	100	0.94, 0.94	0.93, 0.93	0.94, 0.95	0.94, 0.94	0.95, 0.94
	250	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.96, 0.95	0.95, 0.95
	1000	0.96, 0.95	0.93, 0.96	0.96, 0.95	0.93, 0.96	0.94, 0.96
4	25	0.95, 0.94	0.94, 0.93	0.96, 0.94	0.95, 0.94	0.94, 0.94
	50	0.96, 0.94	0.93, 0.95	0.96, 0.94	0.94, 0.94	0.94, 0.94
	75	0.94, 0.95	0.94, 0.94	0.94, 0.95	0.94, 0.95	0.93, 0.95
	100	0.95, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95
	250	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.95
	1000	0.95, 0.94	0.96, 0.95	0.95, 0.94	0.96, 0.95	0.96, 0.95
8	25	0.95, 0.95	0.94, 0.95	0.96, 0.95	0.94, 0.96	0.95, 0.95
	50	0.95, 0.94	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95
	75	0.95, 0.95	0.95, 0.96	0.94, 0.95	0.95, 0.96	0.95, 0.95
	100	0.95, 0.95	0.93, 0.94	0.95, 0.95	0.94, 0.95	0.93, 0.94
	250	0.95, 0.96	0.95, 0.94	0.95, 0.96	0.96, 0.95	0.95, 0.95
	1000	0.94, 0.96	0.95, 0.93	0.94, 0.96	0.95, 0.94	0.94, 0.93

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table 4.14, coverage results for the two continuous-time error variances under parameter set 1 are reported. Similar to the initial variances, rates were acceptable in all conditions except in combinations of 25 sampled units and two time points. For parameter sets in which stochastic error was low, the EDM-SEM and Oversampling procedures produced rates that covered the true data generating value less than the nominal value, many of which lay on the border of the adopted range of acceptability. Conversely, in high stochastic error conditions, the Bayesian approach produced rates below the nominal 95% rate whereas the frequentist approaches resulted in better—although, in some instances, still unacceptable—coverage. Similar patterns held for the continuous-time covariance parameter. Coverage rates for the initial covariance parameter—shown in the same

tables in Appendix B—were also in acceptable ranges under all experimental conditions.

To review the preceding section, coverage rates were close to the nominal 95% rate in most experimental conditions. As observed for parameter bias and variability, exceptions occurred in the smallest sample size conditions. Of note, the EDM-SEM procedure resulted in interval estimates for the auto- and cross-effect parameters that were below acceptable coverage limits. Oversampling and the Bayesian approach provided better coverage in these conditions except under combinations of the lowest sample size and time point levels; in these instances, the Bayesian approach produced unacceptable coverage rates—likely the result of the small number of converged and proper solutions retained for analysis—for some EDM parameters and parameter value sets.

4.3 Model Diagnostics

The final results to be presented are Type I error rates for the χ^2 test statistic. Recall that a p-value for the χ^2 test statistic was calculated for each replication under EDM-SEM and Oversampling estimation and a similar quantity, the ppp-value, was calculated for the Bayesian implementation of the EDM. The primary difference between these two quantities is that the χ^2 p-values are based on the theoretical sampling distribution of the fit statistic and calculated treating model parameters as fixed whereas the ppp-value is an empirically-derived value more akin to a fit index than a test statistic (Levy, 2011) that takes into account sampling variability in the parameter estimates. As noted in Levy (2011) and Muthén and Asparouhov (2012), little guidance has been provided for the ppp-value; therefore, results presented here are intended to provide preliminary guidance for evaluating the fit of the EDM in practice as well as to make comparisons between the frequentist and Bayesian approaches with regard to the sensitivity of model fit measures to design factors included in the simulation.

Table 4.15 shows the Type I error rates for the two different threshold levels (.05, .01) under parameter set 4. Note that results are not presented for the conditions in which two time points were generated as solutions in these categories were saturated (i.e., zero degrees of freedom) and there-

fore could not be evaluated in terms of model-data fit. In general, results between the EDM-SEM and Oversampling approaches were closely aligned. Large differences were observed, however, between the two frequentist approaches and the Bayesian approach. Specifically, Type I error rates were generally higher under Bayesian estimation than the two frequentist approaches, although several exceptions occurred. For EDM-SEM and Oversampling, Type I error rates were close to nominal levels only at the largest sample sizes of 250 and 1000. Furthermore, the error rates became grossly inflated as sample size decreased and the number of time points increased. Rates were generally higher when random starting values were used although differences were trivially and not entirely consistent. Finally, error rates for the Oversampling procedure were generally lower than those observed for the EDM-SEM procedure.

In contrast, Bayesian estimated ppp-values exhibited error rates that were not close to proposed nominal levels. In general, error rates decreased as the number of time points increased; additionally, sample size only appeared to have a small effect on the error rates. Findings regarding sample size were also inconsistent—for instance, under the probability threshold of .01, error rates decreased as sample size increased when four time points were generated but an opposite pattern emerged when eight time points were generated. Also, error rates for Bayesian estimation were not as inflated at lower sample sizes as those obtained under frequentist estimation when eight time points were generated.

Table 4.15: Model Fit, Parameter Set 4

			EDM-SEM		Oversampling		Bayesian
Pr.	T	N	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
.05	4	25	0.233	0.254	0.213	0.226	0.567
		50	0.109	0.139	0.096	0.124	0.595
		75	0.101	0.109	0.075	0.094	0.594
		100	0.097	0.112	0.075	0.096	0.593
		250	0.085	0.080	0.066	0.061	0.586
		1000	0.057	0.067	0.044	0.054	0.572
	8	25	0.886	0.907	0.889	0.897	0.256
		50	0.359	0.375	0.345	0.363	0.319
		75	0.204	0.208	0.190	0.201	0.360
		100	0.182	0.152	0.171	0.143	0.330
		250	0.087	0.091	0.075	0.074	0.312
		1000	0.067	0.068	0.049	0.058	0.327
.01	4	25	0.112	0.105	0.088	0.084	0.269
		50	0.036	0.051	0.028	0.039	0.249
		75	0.028	0.026	0.013	0.012	0.272
		100	0.044	0.036	0.022	0.024	0.246
		250	0.029	0.030	0.014	0.012	0.240
		1000	0.019	0.022	0.008	0.010	0.227
	8	25	0.752	0.766	0.748	0.749	0.078
		50	0.162	0.174	0.150	0.153	0.092
		75	0.081	0.087	0.069	0.071	0.109
		100	0.066	0.050	0.054	0.038	0.129
		250	0.018	0.038	0.009	0.020	0.110
		1000	0.025	0.019	0.007	0.010	0.117

Note. Pr. = probability cutoff value. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Chapter 5

Discussion

5.1 General Discussion

The present study focused on two areas of methodology that have recently garnered the attention of quantitative specialists in the social sciences: Continuous-time modeling and Bayesian statistics. Increased awareness of the former parallels a movement underway in psychological science and related fields focused on the dynamic nature of individuals and groups (Boker, 2007, 2012). Through this new scientific lens, biological, psychological, social, and behavioral constructs are understood as intricately connected systems that evolve continuously over time. However, despite the influx of "big data" and innovative research designs allowing for intensive repeated measurement, many longitudinal studies in the psychological and social sciences continue to collect infrequent repeated observations on large numbers of individuals over time. Therefore, continuous-time models such as the EDM that can be used with panel data present a unique opportunity for researchers studying change.

The second topic explored herein—Bayesian estimation—has also surged in popularity and will continue to do so as researchers in the social sciences utilize increasingly complex statistical models. From a general modeling and philosophical perspective, it can be argued that Bayesian methods encompass a powerful framework for drawing inferences from data (Lynch, 2007; Jack-

man, 2009). However, it may be the practical advantages of Bayesian statistics that appeal most to social scientists, such as the ability to estimate otherwise intractable models as well as the opportunity to incorporate prior evidence into an analysis and directly update existing knowledge through observation. The present study was designed to explore the fusion of these two paradigms in relation to a specific model for panel data—the EDM. In this section, results pertaining to the specific study hypotheses are reviewed and discussed. Overall, results were mixed as they pertain to the hypotheses put forth in Chapter 3.

Hypothesis 1. This hypothesis was largely supported; the Bayesian implementation of the EDM produced higher convergence rates compared to the EDM-SEM approach and comparable convergence rates to the Oversampling approach. Additionally, the Bayesian implementation resulted in fewer improper solutions compared to the Oversampling approach in some conditions. The sole exception occurred for combinations of the lowest sample size and number of time points in which Bayesian convergence rates were low and improper solution rates high compared to the frequentist methods. Finally, across all three estimation methods, it appears rates of non-convergence and improper solutions increased as sample size decreased, the number of time points decreased, and starting values were perturbed from data generating values.

Hypothesis 2. The second hypothesis received little support. Regarding parameter bias, the results suggest that in small sample sizes the Bayesian approach may exhibit positive bias for some EDM model parameters. In particular, estimates of the auto-effects, cross-effects, initial variances, and continuous-time error variances exhibited levels of relative bias in the small sample size conditions that were unacceptable and higher than those observed for the frequentist methods. An exception to this finding was found in results for the cross-effects—in small sample size conditions, the EDM-SEM procedure fared worse than Oversampling and Bayesian estimation. Other model parameters were estimated with more comparable levels of bias across the three analytic methods at low sample sizes, including trivial amounts of bias even at the smallest sample size for the initial mean and continuous-time intercept parameters. Similar patterns were observed with regard to parameter estimate variation, as measured by the RMSE. As the RMSE is a measure of

parameter estimate bias in addition to variability, it is not surprising that the findings were consistent with those reported for parameter bias. Finally, coverage was comparable across the three methods except for estimates of the drift matrix parameters—the EDM-SEM procedure produced intervals that did not sufficiently cover the data generating values at small sample sizes.

In consideration of these findings, there are many plausible explanations for the less optimal performance of the Bayesian implementation at small sample sizes, of which two are discussed here. First, in order to include the large number of experimental factors, levels, and numbers of replications in the simulation study, the length of the MCMC chains were fixed to 3000. This number was chosen based largely on pilot testing in which a balance was sought between estimate precision and computational burden. It is not uncommon to observed much longer chains in the applied literature, sometimes reaching 10,000 or even 100,000 for complex models. Thus, it is possible that longer chain runs would allow for more comparable estimates of bias and parameter variability in smaller sample sizes. This is an important question that deserves further study. A second plausible reason for the small sample size discrepancies is related to the specification of the prior distributions for the model parameters. The priors were intentionally non-informative. In other studies that have found superior performance of Bayesian estimation at low sample sizes (e.g., Lee & Song, 2004), informative priors were specified. The choice of prior distributions for the EDM model parameters is discussed in greater detail below.

Hypothesis 3. Evidence from the present study generally supported the third hypothesis. In moderate and large sample sizes (e.g., ≥ 100), the approaches produced comparable results in terms of parameter bias, parameter variability, and interval coverage. Therefore, if applied researchers secure sufficiently large sample sizes and other modeling assumptions are met, the three approaches studied here would be expected to produce equivalent results. Researchers familiar with Bayesian methods, including model estimation via the BUGS language, may therefore find these results encouraging and conducive to adoption of the model in practice.

One circumstance in which the Bayesian approach may offer an advantage over the frequentist approaches is if the model does not convergence or results in an improper solution. As shown

in this study, even when starting values are close to population targets, the EDM-SEM approach may result convergence failures; although the results reported here suggest that the Oversampling approach is less susceptible to non-convergence, it may nevertheless produce improper or implausible solutions in some cases. Furthermore, if the starting values used are not as optimal as those studied here, one would expect to encounter greater difficulties in arriving at a set of converged and proper parameter estimates.

Hypothesis 4. The final hypothesis received support under frequentist estimation but not Bayesian estimation. The analysis model was specified to match the data generating model exactly and therefore Type I error rates should have been close to nominal levels if influential factors were absent. Rates were indeed close to nominal levels for the EDM-SEM and Oversampling approaches at large sample sizes but became inflated as sample size and the number of time points increased. This finding replicates previously reported results for the χ^2 statistic in the SEM literature (Muthén & Asparouhov, 2012; Curran, West, & Finch, 1996).

Model fit results for Bayesian estimation, on the other hand, did not support the fourth hypothesis. Error rates were higher as compared to the frequentist approaches when sample sizes were large and lower when sample sizes were small. Overall, these findings underscore two important points regarding model fit evaluation for the EDM specifically and SEM models generally under Bayesian estimation. First, although rules of thumb provide convenient guidelines for evaluating model fit, one must exercise caution in applying cutoff values for ppp-values. Many (Levy, 2011; Muthén & Asparouhov, 2012) have noted that ppp-values require further attention in Monte Carlo simulation research to understand their sampling behavior under conditions encountered in practice. Moreover, some note that acceptable limits for ppp-values will depend on the context of a given area and researchers should execute sound judgment (and provide sufficient rationale) for modeling decisions based on their use (Gelman et al., 2013). Second, the ppp-values largely demonstrated resistance to changes in sample size as opposed to the χ^2 statistic. Therefore, at small sample sizes, the ppp-value may be better able to differentiate between well- and poor-fitting models, at least for the EDM model studied here. The similarities and differences between model

fit evaluation in the frequentist and Bayesian paradigms is a fruitful area for further inquiry.

5.2 Prior Distribution Specification

A vital step in conducting any Bayesian analysis is constructing a probability model for unobserved quantities that one wishes to make inferences about—in the present study, this means specification of a prior distribution over the EDM model parameters. The goal of the this project was to compare Bayesian estimation of the EDM to two frequentist approaches; as a result, a non-informative prior specification was used. In this section, alternative specifications of the prior distribution are considered. Although Bayes formula as applied to multi-parameter statistical models typically represents the prior distribution as a multivariate density (e.g., Equation 2.2), in practice it is often broken down into more manageable components as required by commonly-used sampling algorithms (e.g., slice sampling, Gibbs sampling). Therefore, the following discussion will consider alternative prior specifications for the EDM model parameters separately or in small combinations, where appropriate.

Parameters in the initial mean vector μ_0 as well as the continuous-time intercept vector $\mathbf{b}$ were provided multivariate-normal distributions parameterized with mean vectors centered over population values and diagonal precision matrices with small elements along the diagonal. Essentially, these specifications corresponded to univariate normal distribution priors with diffuse variances over each parameter. One could provide more information in the prior specification if sufficient prior knowledge of the parameter distribution exists. Gelman et al. (2013) defines a *weakly-informative prior* as one in which less information is included than is actually available; the goal in using a weakly informative prior is to provide regularization and stabilization of the posterior distribution. This can be achieved by including reasonable bounds and enough information for a given parameter such that the prior distribution is not completely uniform. For the intercept and continuous-time intercept parameters, weakly-informative or informative distributions could be specified by decreasing the variances (increasing the precisions) in the normal distributions shown

here.

Optimal specification of non-informative, weakly-informative, and informative prior distributions for the auto- and cross-effects deserves further study. In the current study, the non-informative prior distributions appeared to work well in reproducing frequentist results at moderate and large sample sizes. Moreover, the "non-informativeness" of the specification used here was somewhat conservative, and thus one could include more information as appropriate, even if one only wishes to specify a weakly informative prior distribution. For instance, a range of -7 to 0 may be rather large for the auto-effects in practice; to include additional information, one could translate results from a discrete-time analysis to get a better idea of what ranges may be reasonable for the auto-effects. Options for informative distributions might include the negative log-gamma distribution (Allella, Chiodo, Lauria, & Pagano, 2001)—a negatively skewed distribution bounded above at 0—or the non-parametric Dirichlet process prior as expounded upon by Chow et al. (2011). A symmetric prior over the cross-effects—in the present case, the normal distribution—also appeared to work well under most conditions in terms of producing low bias and comparable error levels to frequentist estimates. Indeed, estimates of the cross-effects were less biased under Bayesian estimation as compared with the EDM-SEM procedure at low sample sizes. Reducing variability in the prior would provide more information and result in a weakly-informative or informative prior assuming the analyst has sufficient information to do so.

Prior specification of the initial and continuous-time error variances and covariances also deserves further study. In fact, optimal prior specification for a covariance matrix as used in any Bayesian analysis has not been resolved. The inverse-Wishart distribution is still widely used but recently has been noted to result in a dependence between the covariance and variance parameters (Gelman, 2006; Gelman et al., 2013; Barnard et al., 2000). The scaled-inverse Wishart matrix approached used here provided acceptable results in terms of bias, variability, and coverage. For more informative prior distributions over the variance and covariance parameters via the scaled-inverse Wishart, one could use alternatives to the uniform distribution use herein—for instance, a symmetric but non-uniform distribution could be specified for the correlation parameter. For the

standard deviations, Barnard et al. (2000) suggests using a log-normal prior distribution whereas Gelman (2006) proposes a family of half-t distributions. In summary, given increased applications of multi-parameter models to social science data that include covariance matrices (e.g., hierarchical models, SEM models), more research is needed regarding appropriate prior specifications.

5.3 Limitations

All Monte Carlo investigations are limited by scope—it is not possible theoretically or practically to consider every data structure, model specification, or estimation algorithm collectively in an experiment. In the present study, some factors were omitted that may have important implications for estimating the EDM parameters in the frequentist or Bayesian frameworks. Notably, the number of iterations per chain and the specification of the prior distribution were discussed above as areas in need of further exploration for the Bayesian approach. In addition, the bivariate model studied here contained no measurement structure, exogenous predictors, or random effects as described in Oud and Delsing (2010); extensions to trivariate or larger multivariate systems also were not considered. These factors could not be considered given the already sizable scope of the present study—particularly with regard to the computational demands of the Bayesian approach—but deserve attention in future investigations.

A second limitation—in part related to the first—was the absence of various assumption violations in the simulation design. Although important findings are reported in the present study with regard to the performance of each estimation method under varying sample sizes, numbers of repeated observations, parameter value configurations, and starting value variability, it is unclear how these findings may change under less than ideal data conditions (e.g., multivariate normality) and when other important prerequisites are not met (e.g. correct model specification). For instance, omission of the continuous-time intercepts when data contain linear or other trends are present could have important implications for parameter recovery, interval coverage, etc.

Finally, the definition of improper solutions affected model results. Specifically, when solutions

with proper but implausible estimates were included in the analysis, results for the Oversampling approach were less favorable. In some conditions, bias, RMSE, and coverage fell outside acceptable ranges. Results for the EDM-SEM and Bayesian approaches were less affected by inclusion of implausible solutions as rates were lower. The exclusion criterion for solutions with estimate outliers was adopted such that simulation analyses were based on reasonable parameter ranges. To the author's knowledge, the specific issue of including solutions with implausible estimates has not been thoroughly discussed. Ultimately, this issue relates to the larger problem of whether one should include improper solutions in simulation analyses (Boomsma, 2013); both issues deserve increased attention from methodological researchers.

5.4 Future Directions

In addition to the suggestions above pertaining to omitted simulation factors, many other directions for research on Bayesian estimation of the EDM appear promising. Accommodation of individually-varying time intervals—those that vary between units within measurement waves and those that vary across measurement waves—has been a topic of recent interest among methodologists (Voelkle & Oud, 2013; Sterba, in press; Aydin, Leite, & Algina, 2014). The JAGS code shown in Appendix C could be modified to accommodate individually-varying time intervals by switching from wide to long format and including variables for time of measurement where appropriate as shown in Equation 1.7[1]. It would be interesting then to compare these approaches when measurement intervals vary within and between units over time.

The inclusion of random effects for EDM model parameters is also an area deserving further attention. Oravecz et al. (2009) and Oravecz et al. (2011) allow for random effects in their Bayesian implementation of the OU process model. Delsing and Oud (2008) considered random effects

[1]The notation used in Equation 1.7 requires adjustment for individually-varying time intervals. Specifically, an index, say k, would need to be added to signify individual units such that $\Delta_{t_{i,k}}$ would represent the amount of time elapsed during the ith interval for individual k. Note that longitudinal studies with missing data results in uneven numbers of repeated observations between sample units and thus i would need to be allowed to vary across all k. See Voelkle and Oud (2013) and Oravecz et al. (2011) for more information.

analogous to random intercept and slope terms found in latent growth curve (LGC; Bollen & Curran, 2004) and autoregressive latent trajectory (ALT; Bollen & Curran, 2004) models. However, other EDM parameters (e.g., drift coefficients, continuous-time error variances and covariances) are not permitted to vary across individuals or data subgroups[2]. Bayesian random effects models have received considerable attention as their specification arises naturally within the Bayesian paradigm (Jackman, 2009; Gelman et al., 2013; Lee, 2007). Given the constraints imposed in the EDM-SEM (non-linear) and Oversampling (linear) approaches, random effects modeling may be more feasible within the Bayesian paradigm.

5.5 Conclusion

Longitudinal studies in the social sciences have traditionally been analyzed using discrete-time models. In recent years, continuous-time alternatives have been proposed and implemented in software. The EDM is one such model that has been adapted for use with panel data. To date, this model has been developed for use with frequentist statistical procedures (e.g., SEM, state-space modeling) but has not yet been considered from a Bayesian perspective.

The current study was designed to extend this model to the Bayesian framework and compare its performance to two other frequentist methods. From this study, two primary contributions are noted. First, a Bayesian implementation of the EDM was provided in the JAGS program. Applying the EDM in practice, especially the computational components, has been noted as a difficult enterprise (Steele & Ferrer, 2011). Thus, the JAGS program shown in Appendix C may assist researchers familiar with Bayesian statistics to specify and estimate the EDM as shown here or explore model modifications and extensions. Second, results of the Monte Carlo study suggest that with sufficient data, the Bayesian approach with the non-informative priors proposed here will likely provide equivalent results to frequentist methods. However, at small sample sizes and with fewer time points, Bayesian estimation provides slightly less accurate and more variable

[2]Parameters in these matrices *are* allowed to vary across time, as shown in Oud and Jansen (2000).

results compared to the EDM-SEM and Oversampling approaches, although interval coverage is generally equivalent or better. Also, the Bayesian approach often resulted in better rates of model convergence and fewer instances of improper solutions. In conclusion, extension of the EDM to the Bayesian framework has the potential to expand researchers' options for applying this model to data while incorporating philosophical and computational advantages that some contend the Bayesian framework provides. This study is a small step in such a direction and part of a larger movement to adopt continuous-time models in the social sciences. Processes that give rise to human thought, behavior, and biology are most often continuous and should be modeled as such.

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Appendix A

Literature Review

A small literature review was conducted in order to inform the simulation conditions. The keywords *developmental psychology* and *cross-lagged panel* were searched in the PSYCINFO database. Articles were limited to the previous five years. The search resulted in 26 hits. Of these, two were not empirical studies, one article was in a foreign language and could not be interpreted, and one article was a cross-sectional study; these were excluded from further consideration.

The remaining 22 articles were reports of longitudinal research studies that included a discrete-time ARCL panel model analyses. All of these articles were used to inform the sample sizes and number of time points specified in the Monte Carlo simulation. If a study contained multiple samples, they were treated as separate studies. Furthermore, the number of time points from each study was obtained from descriptions of the ARCL panel models; some studies contained additional longitudinal analyses (e.g. latent growth curves) for which a different number of time points were used. Table A.1 contains the values of these variables for each study reviewed.

Unfortunately, reporting was variable and many articles did not contain enough information to inform the parameter value set conditions. To remain as inclusive as possible, parameter matrices were considered separately. This means that parameter values from a study were used to inform the simulation conditions if at least 1 of the 5 model matrices were fully specified. Additional criteria for inclusion were: (a) A bivariate discrete-time ARCL panel model was estimated; and

(b) All estimates of a particular matrix were available—for instance, if only 3 out of 4 estimates were available for **A** (many models only reported significant results), it was not included. Of the 22 longitudinal studies, 4 met the above criteria and were used to inform the conditions. Table A.1 contains the candidate parameter matrices from the studies meeting these criteria. Note that some of the studies contained multiple candidates matrices; these were treated as independent contributions. Furthermore, many studies did not impose stationarity and thus multiple estimates were available for the same parameter across time. In such cases, the average of the estimates for a single parameter was used.

In choosing conditions for the simulation, focus was placed on the **A** and **Q** matrices. Patterns for discrete-time matrix $\mathbf{A}(\Delta t_i)$ shown in Table A.2 suggest that auto-regressive coefficients are typically moderate to large in value and cross-lagged coefficients are typically small in value and may be positive or negative in direction. As such, three discrete-time $\mathbf{A}(\Delta t_i)$ matrices were specified and converted to continuous-time **A** matrices to inform the simulation conditions (Table 3.2). The first **A** matrix (parameter sets 1 and 2) represents a discrete-time process with highly-stable variables and small bi-directional effects. The second two **A** matrices represent bivariate processes with less stable constructs and larger bi-directional effects. The difference between the second (parameter sets 3 and 4) and third (parameter sets 5 and 6) **A** matrices are in the signs of the cross-lagged coefficients, which are positive in the second matrix and negative in the third matrix. These specifications were considered representative of the reviewed studies and are all instantiations of stable systems (i.e., contain negative and real-valued eigenvalues; cf. Voelkle & Oud, 2013).

As Table A.2 shows, there were no examples of discrete-time error covariance matrices available. Consequently, values for the two different **Q** specifications were chosen based on previous EDM illustrations (e.g., Voelkle et al., 2012) and the desire to create low ($q_{11}, q_{22} = 0.30$) and high ($q_{11}, q_{22} = 1.00$) stochastic error conditions. Initial mean vectors and covariance matrices were quite variable among the reviewed articles. In order to keep the number of conditions manageable, only one specification was used for μ_0 and Φ_0. Finally, as no examples were provided for discrete-time intercepts, the continuous-time intercepts were specified for a model with slight linear growth

over time as has been demonstrated in previous analyses (Voelkle et al., 2012).

Table A.1: Literature Review: Sample size, Number of Time Points

#	Article	Sample Size	Time Points
1	Boivin et al. (2010)	1035	4
2	Chen et al. (2012)	1162	4
3	Defoe et al. (2013)	204	4
4	Dhont et al. (2012)	65, 172	2
5	Eisenhower et al. (2013)	245	7
6	Feit et al. (2009)	506	2
7	Fuller-Tyszkiewicz et al. (2012)	4724, 4340	3
8	Gradinger et al. (2012)	447	2
9	Hale et al. (2013)	923	3
10	Keijsers et al. (2010)	309	4
11	Keijsers et al. (2010)	503	10
12	Kelly et al. (2013)	176	3
13	Krahé (2010)	1237	2
14	Leung et al. (2012)	267	2
15	Levy (2009)	288	3
16	McCarty et al. (2012)	521	4
17	Neece et al. (2010)	104	4
18	Neece et al. (2012)	237	7
19	Sneed et al. (2011)	182	4
20	Stevens (2011)	389	2
21	Van Zalk et al. (2011)	916	3
22	Verboom (2014)	1132, 1098	3

Table A.2: Literature Review: Parameter value configurations

Article	μ_0	Φ_0	$\mathbf{A}(\Delta t_i)$	$\mathbf{b}(\Delta t_i)$	$\mathbf{Q}(\Delta t_i)$
Boivin et al. (2010)	Not reported	Could not calculate	.74 .08 .08 .65 .69 .14 .14 .61	Not reported	Could not calculate
Dhont et al. (2012)	2.75 4.45	1.61 -.47 -.47 1.44	.82 -.10 -.19 .79 .88 -.03 -.22 .80 .88 .05 -.19 .75	Not reported	Could not calculate
Feit et al. (2009)	5.27 44.25 5.27 9.19	5.25 -1.95 -1.95 116.64 5.24 -2.06 -2.06 55.95	.50 -.16 -.02 .79 .54 -.11 -.08 .68	Not reported	Could not calculate
Fuller-Tyszkiewicz et al. (2012)	Could not calculate	Could not calculate	.73 .03 .04 .46 .85 .03 .06 .46	Not reported	Could not calculate

Note. μ_0 = Initial mean vector; Φ_0 = Initial covariance matrix; $\mathbf{A}(\Delta t_i)$ = Discrete-time autoregressive/cross-lagged coefficient matrix; $\mathbf{b}(\Delta t_i)$ = Discrete-time intercept vector; $\mathbf{Q}(\Delta t_i)$ = Discrete-time error covariance matrix.

Appendix B

Expanded Results for Monte Carlo Investigation

In this appendix, expanded results for the Monte Carlo simulation are presented. Readers may refer to the Table of Contents for an index of the tables presented here.

Table B.1: Expanded Results, Convergence and Improper Solution Rates, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
T	*N*	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.60 (0.01)	0.57 (0.02)	1.00 (0.01)	1.00 (0.01)	0.32 (0.08)
	50	0.66 (0.00)	0.64 (0.00)	1.00 (0.00)	1.00 (0.00)	0.97 (0.03)
	75	0.69 (0.00)	0.64 (0.01)	1.00 (0.00)	1.00 (0.00)	1.00 (0.01)
	100	0.72 (0.00)	0.65 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.01)
	250	0.86 (0.00)	0.77 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	1000	0.98 (0.00)	0.86 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
4	25	0.70 (0.02)	0.60 (0.01)	1.00 (0.01)	1.00 (0.01)	0.99 (0.04)
	50	0.76 (0.00)	0.69 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.01)
	75	0.81 (0.00)	0.74 (0.00)	1.00 (0.00)	1.00 (0.01)	1.00 (0.01)
	100	0.88 (0.00)	0.76 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.01)
	250	0.96 (0.00)	0.86 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	1000	1.00 (0.00)	0.90 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
8	25	0.77 (0.01)	0.69 (0.01)	1.00 (0.01)	1.00 (0.00)	1.00 (0.01)
	50	0.88 (0.00)	0.79 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	75	0.91 (0.00)	0.81 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	100	0.95 (0.00)	0.85 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	250	0.99 (0.00)	0.88 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	1000	1.00 (0.00)	0.90 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)

Note. Convergence rates are shown outside the parentheses. Improper solution rates are shown inside parentheses. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$; $\Phi_0 = \begin{bmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{bmatrix}$; $\mathbf{A} = \begin{bmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{bmatrix}$; $\mathbf{b} = \begin{bmatrix} 0.10 \\ 0.10 \end{bmatrix}$; $\mathbf{Q} = \begin{bmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{bmatrix}$.

Table B.2: Expanded Results, Convergence and Improper Solution Rates, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
T	*N*	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.72 (0.02)	0.71 (0.02)	1.00 (0.01)	1.00 (0.02)	0.29 (0.09)
	50	0.85 (0.01)	0.81 (0.01)	1.00 (0.01)	1.00 (0.01)	0.81 (0.09)
	75	0.90 (0.00)	0.87 (0.01)	1.00 (0.00)	1.00 (0.00)	0.99 (0.02)
	100	0.92 (0.00)	0.90 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.01)
	250	0.96 (0.00)	0.93 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	1000	0.99 (0.00)	0.96 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
4	25	0.82 (0.01)	0.81 (0.02)	1.00 (0.02)	1.00 (0.01)	0.76 (0.09)
	50	0.91 (0.01)	0.87 (0.00)	1.00 (0.00)	1.00 (0.00)	0.99 (0.01)
	75	0.94 (0.00)	0.90 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.01)
	100	0.95 (0.00)	0.91 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	250	0.97 (0.00)	0.94 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	1000	1.00 (0.00)	0.98 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
8	25	0.93 (0.01)	0.85 (0.02)	1.00 (0.01)	1.00 (0.01)	0.99 (0.03)
	50	0.95 (0.00)	0.93 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.01)
	75	0.96 (0.00)	0.93 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	100	0.98 (0.00)	0.95 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	250	0.98 (0.00)	0.96 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	1000	0.99 (0.00)	0.96 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)

Note. Convergence rates are shown outside the parentheses. Improper solution rates are shown inside parentheses. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$; $\Phi_0 = \begin{bmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{bmatrix}$; $\mathbf{A} = \begin{bmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{bmatrix}$; $\mathbf{b} = \begin{bmatrix} 0.10 \\ 0.10 \end{bmatrix}$; $\mathbf{Q} = \begin{bmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{bmatrix}$.

Table B.3: Expanded Results, Convergence and Improper Solution Rates, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
T	*N*	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.89 (0.01)	0.76 (0.01)	1.00 (0.01)	1.00 (0.02)	0.88 (0.09)
	50	0.95 (0.00)	0.80 (0.00)	1.00 (0.00)	1.00 (0.01)	1.00 (0.01)
	75	0.97 (0.00)	0.82 (0.00)	1.00 (0.00)	1.00 (0.01)	1.00 (0.01)
	100	0.98 (0.00)	0.84 (0.00)	1.00 (0.00)	1.00 (0.01)	1.00 (0.00)
	250	1.00 (0.00)	0.86 (0.00)	1.00 (0.00)	1.00 (0.01)	1.00 (0.00)
	1000	1.00 (0.00)	0.87 (0.00)	1.00 (0.00)	1.00 (0.01)	1.00 (0.00)
4	25	0.91 (0.01)	0.82 (0.01)	1.00 (0.01)	0.99 (0.04)	1.00 (0.03)
	50	0.98 (0.00)	0.86 (0.01)	1.00 (0.00)	1.00 (0.03)	1.00 (0.01)
	75	0.98 (0.00)	0.87 (0.01)	1.00 (0.00)	1.00 (0.03)	1.00 (0.00)
	100	0.98 (0.00)	0.87 (0.00)	1.00 (0.00)	1.00 (0.03)	1.00 (0.00)
	250	0.99 (0.00)	0.88 (0.01)	1.00 (0.00)	1.00 (0.03)	1.00 (0.00)
	1000	1.00 (0.00)	0.90 (0.00)	1.00 (0.00)	1.00 (0.03)	1.00 (0.00)
8	25	0.94 (0.01)	0.84 (0.02)	1.00 (0.01)	1.00 (0.05)	1.00 (0.02)
	50	0.98 (0.00)	0.84 (0.01)	1.00 (0.00)	0.99 (0.04)	1.00 (0.01)
	75	0.98 (0.00)	0.88 (0.01)	1.00 (0.00)	0.99 (0.04)	1.00 (0.00)
	100	0.99 (0.00)	0.89 (0.01)	1.00 (0.00)	1.00 (0.05)	1.00 (0.00)
	250	1.00 (0.00)	0.88 (0.00)	1.00 (0.00)	1.00 (0.05)	1.00 (0.00)
	1000	1.00 (0.00)	0.89 (0.00)	1.00 (0.00)	1.00 (0.03)	1.00 (0.00)

Note. Convergence rates are shown outside the parentheses. Improper solution rates are shown inside parentheses. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$; $\Phi_0 = \begin{bmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{bmatrix}$; $\mathbf{A} = \begin{bmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{bmatrix}$; $\mathbf{b} = \begin{bmatrix} 0.10 \\ 0.10 \end{bmatrix}$; $\mathbf{Q} = \begin{bmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{bmatrix}$.

Table B.4: Expanded Results, Convergence and Improper Solution Rates, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
T	*N*	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.71 (0.02)	0.68 (0.01)	1.00 (0.02)	1.00 (0.02)	0.25 (0.24)
	50	0.83 (0.01)	0.76 (0.01)	1.00 (0.01)	1.00 (0.01)	0.82 (0.09)
	75	0.86 (0.00)	0.86 (0.00)	1.00 (0.01)	1.00 (0.00)	0.99 (0.03)
	100	0.92 (0.00)	0.86 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.01)
	250	0.97 (0.00)	0.95 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	1000	0.99 (0.00)	0.96 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
4	25	0.83 (0.03)	0.79 (0.02)	1.00 (0.02)	1.00 (0.01)	0.82 (0.10)
	50	0.91 (0.01)	0.89 (0.01)	1.00 (0.01)	1.00 (0.00)	0.99 (0.02)
	75	0.95 (0.00)	0.93 (0.00)	1.00 (0.00)	1.00 (0.01)	1.00 (0.01)
	100	0.96 (0.01)	0.95 (0.00)	1.00 (0.01)	1.00 (0.00)	1.00 (0.01)
	250	0.99 (0.00)	0.97 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	1000	0.99 (0.00)	0.99 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
8	25	0.91 (0.01)	0.89 (0.01)	1.00 (0.00)	1.00 (0.01)	0.99 (0.02)
	50	0.96 (0.00)	0.96 (0.01)	1.00 (0.00)	1.00 (0.01)	1.00 (0.01)
	75	0.98 (0.00)	0.97 (0.01)	1.00 (0.00)	1.00 (0.00)	1.00 (0.01)
	100	0.98 (0.00)	0.97 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	250	0.99 (0.00)	0.99 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)
	1000	0.99 (0.00)	0.98 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)

Note. Convergence rates are shown outside the parentheses. Improper solution rates are shown inside parentheses. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.5: Expanded Results, Average CPU Time (in Minutes), Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
T	*N*	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.03	0.03	0.03	0.02	0.21
	50	0.03	0.03	0.03	0.03	0.40
	75	0.04	0.04	0.04	0.04	0.60
	100	0.06	0.06	0.06	0.06	0.81
	250	0.15	0.15	0.15	0.15	2.13
	1000	0.71	0.69	0.69	0.69	9.92
4	25	0.05	0.05	0.07	0.07	0.50
	50	0.07	0.06	0.09	0.09	1.00
	75	0.10	0.10	0.13	0.13	1.55
	100	0.13	0.13	0.17	0.17	2.10
	250	0.28	0.28	0.32	0.33	5.76
	1000	1.83	1.85	1.88	1.85	37.29
8	25	0.13	0.12	0.21	0.20	1.11
	50	0.23	0.22	0.34	0.34	2.33
	75	0.31	0.30	0.44	0.44	3.63
	100	0.38	0.36	0.50	0.50	4.99
	250	0.72	0.71	0.84	0.87	14.17
	1000	4.89	4.98	5.21	5.22	103.31

Note. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.6: Expanded Results, Average CPU Time (in Minutes), Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
T	*N*	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.03	0.03	0.03	0.03	0.22
	50	0.03	0.03	0.03	0.03	0.42
	75	0.05	0.04	0.05	0.04	0.65
	100	0.06	0.05	0.05	0.05	0.86
	250	0.16	0.15	0.14	0.14	2.29
	1000	0.64	0.63	0.61	0.61	10.69
4	25	0.05	0.05	0.07	0.07	0.53
	50	0.07	0.06	0.09	0.08	1.08
	75	0.10	0.08	0.12	0.10	1.67
	100	0.12	0.12	0.14	0.14	2.25
	250	0.27	0.27	0.30	0.30	6.18
	1000	1.10	1.15	1.13	1.15	38.71
8	25	0.13	0.13	0.20	0.20	1.15
	50	0.21	0.18	0.30	0.27	2.47
	75	0.29	0.26	0.40	0.36	3.85
	100	0.33	0.33	0.44	0.44	5.25
	250	0.64	0.63	0.76	0.77	14.52
	1000	3.40	3.49	3.57	3.66	104.93

Note. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.7: Expanded Results, Average CPU Time (in Minutes), Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
T	*N*	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.03	0.03	0.03	0.03	0.21
	50	0.03	0.03	0.03	0.03	0.41
	75	0.04	0.04	0.04	0.04	0.61
	100	0.06	0.06	0.06	0.05	0.82
	250	0.19	0.15	0.18	0.15	2.17
	1000	0.71	0.70	0.71	0.71	9.85
4	25	0.04	0.04	0.06	0.06	0.49
	50	0.06	0.06	0.08	0.08	1.02
	75	0.10	0.10	0.13	0.13	1.56
	100	0.12	0.12	0.16	0.16	2.11
	250	0.27	0.27	0.32	0.32	5.73
	1000	1.54	1.61	1.66	1.70	37.30
8	25	0.12	0.12	0.19	0.19	1.10
	50	0.20	0.20	0.31	0.31	2.33
	75	0.35	0.29	0.51	0.43	3.65
	100	0.39	0.35	0.53	0.49	4.95
	250	0.68	0.66	0.82	0.83	13.78
	1000	4.53	4.68	5.14	5.19	101.88

Note. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.8: Expanded Results, Average CPU Time (in Minutes), Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
T	*N*	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.03	0.03	0.03	0.03	0.22
	50	0.04	0.03	0.04	0.03	0.42
	75	0.04	0.04	0.04	0.04	0.64
	100	0.05	0.05	0.05	0.05	0.86
	250	0.16	0.15	0.14	0.14	2.29
	1000	0.73	0.63	0.69	0.61	10.54
4	25	0.05	0.05	0.07	0.07	0.52
	50	0.06	0.06	0.08	0.08	1.06
	75	0.08	0.08	0.10	0.10	1.63
	100	0.11	0.11	0.14	0.14	2.24
	250	0.27	0.27	0.29	0.30	6.06
	1000	1.11	1.13	1.13	1.15	38.12
8	25	0.13	0.13	0.20	0.20	1.16
	50	0.19	0.18	0.28	0.26	2.42
	75	0.26	0.26	0.35	0.35	3.73
	100	0.33	0.33	0.43	0.43	5.19
	250	0.64	0.62	0.75	0.76	14.22
	1000	3.46	3.50	3.56	3.71	103.25

Note. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.9: Expanded Results, Average CPU Time (in Minutes), Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
T	*N*	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
2	25	0.03	0.03	0.03	0.03	0.21
	50	0.04	0.03	0.03	0.03	0.40
	75	0.04	0.04	0.04	0.04	0.61
	100	0.06	0.06	0.06	0.05	0.81
	250	0.17	0.15	0.16	0.15	2.16
	1000	0.72	0.68	0.69	0.68	9.84
4	25	0.04	0.04	0.06	0.06	0.49
	50	0.07	0.06	0.09	0.08	0.99
	75	0.10	0.10	0.14	0.12	1.54
	100	0.12	0.12	0.16	0.16	2.10
	250	0.27	0.27	0.32	0.32	5.66
	1000	1.59	1.65	1.68	1.71	37.01
8	25	0.12	0.12	0.19	0.18	1.10
	50	0.21	0.20	0.32	0.30	2.30
	75	0.30	0.30	0.43	0.43	3.56
	100	0.36	0.35	0.48	0.48	4.92
	250	0.68	0.67	0.82	0.83	13.47
	1000	4.61	4.71	5.27	5.15	100.01

Note. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}$; $\Phi_0 = \begin{bmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{bmatrix}$; $\mathbf{A} = \begin{bmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{bmatrix}$; $\mathbf{b} = \begin{bmatrix} 0.10 \\ 0.10 \end{bmatrix}$; $\mathbf{Q} = \begin{bmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{bmatrix}$.

Table B.10: Expanded Results, Bias for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.01, 0.00	-0.00, -0.01	0.01, 0.00	-0.00, -0.01	-0.00, -0.01
	50	-0.00, 0.00	-0.00, 0.01	0.00, 0.00	0.00, 0.00	0.00, 0.00
	75	0.00, 0.00	-0.01, 0.01	0.00, 0.00	-0.00, -0.00	-0.00, 0.00
	100	0.00, -0.00	-0.01, -0.00	-0.00, 0.00	-0.01, -0.00	-0.01, -0.00
	250	-0.00, -0.00	-0.01, -0.01	-0.00, -0.00	-0.01, -0.00	-0.01, -0.00
	1000	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, -0.00
4	25	0.02, 0.01	-0.01, -0.00	0.01, 0.01	-0.00, 0.01	-0.01, 0.01
	50	-0.01, 0.00	-0.00, -0.01	-0.00, 0.00	-0.00, -0.01	-0.00, -0.01
	75	0.00, -0.00	0.00, -0.01	0.00, -0.00	0.01, -0.01	0.01, -0.01
	100	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
	250	-0.00, 0.00	0.01, 0.00	-0.00, -0.00	0.01, 0.00	0.01, 0.00
	1000	0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, -0.00
8	25	-0.00, 0.02	-0.01, 0.00	-0.00, 0.01	-0.01, 0.00	-0.01, 0.00
	50	0.00, 0.01	0.00, 0.00	-0.00, 0.01	0.00, 0.00	0.00, -0.00
	75	-0.01, -0.00	0.00, 0.00	-0.01, -0.00	0.00, 0.01	0.00, 0.01
	100	0.00, -0.00	0.01, 0.01	-0.00, -0.00	0.01, 0.01	0.01, 0.01
	250	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	-0.00, -0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	50	0.00, -0.00	-0.00, -0.01	-0.00, -0.01	0.00, -0.01	0.00, -0.01
	75	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	100	0.00, 0.00	0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00
	250	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	-0.00, -0.00
	1000	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, -0.00	0.00, -0.00
4	25	-0.00, -0.00	0.00, 0.01	-0.00, 0.00	0.00, 0.01	0.00, 0.01
	50	-0.00, -0.00	-0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00
	75	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, -0.00	0.00, 0.00
	100	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	-0.00, 0.00
	250	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, 0.00	-0.00, 0.00
	1000	0.00, 0.00	0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, -0.00
8	25	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	50	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
	75	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	100	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	250	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, -0.00
	1000	0.00, 0.00	0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set #1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.11: Expanded Results, Bias for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	-0.02, 0.00	-0.01, -0.01	-0.02, -0.01	-0.01, -0.01	0.03, 0.02
	50	-0.01, -0.01	0.01, 0.00	-0.01, -0.00	-0.00, -0.01	-0.00, -0.01
	75	0.00, 0.00	-0.00, 0.00	-0.00, 0.00	-0.01, -0.00	-0.01, -0.00
	100	0.01, 0.00	-0.01, -0.00	0.00, 0.01	-0.01, 0.00	-0.01, -0.00
	250	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, -0.00	0.00, -0.00
	1000	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, -0.00
4	25	0.02, 0.01	-0.00, -0.01	0.01, 0.01	0.01, 0.00	0.01, 0.01
	50	-0.01, 0.00	-0.01, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, 0.00
	75	0.00, 0.01	0.00, 0.01	0.00, 0.00	0.01, 0.00	0.01, 0.00
	100	-0.00, 0.00	-0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
	250	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, -0.00
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, 0.00	-0.00, 0.00
8	25	-0.01, 0.02	-0.01, 0.01	-0.01, 0.01	-0.01, 0.01	-0.01, 0.01
	50	-0.00, -0.00	-0.01, -0.01	-0.00, -0.00	-0.00, -0.01	-0.00, -0.01
	75	-0.01, -0.01	0.01, -0.00	-0.01, -0.01	0.01, 0.00	0.01, 0.00
	100	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, -0.00
	250	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, 0.00
	1000	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	-0.00, 0.00
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	-0.00, 0.01	0.00, -0.01	-0.01, 0.00	0.00, 0.00	0.08, 0.02
	50	-0.00, -0.01	-0.00, 0.00	0.00, -0.00	-0.00, -0.00	0.00, -0.00
	75	-0.00, 0.00	-0.00, 0.01	0.00, 0.00	-0.00, 0.01	-0.00, 0.01
	100	0.01, 0.00	0.00, -0.00	0.01, -0.00	0.00, -0.00	0.00, 0.00
	250	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
	1000	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, -0.00
4	25	0.00, 0.00	-0.01, 0.01	0.00, 0.01	-0.00, 0.00	0.00, 0.01
	50	-0.00, 0.00	0.01, 0.00	-0.00, 0.00	0.01, 0.00	0.01, 0.00
	75	0.00, 0.00	0.01, 0.01	0.00, 0.00	0.01, 0.01	0.01, 0.01
	100	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00
	250	0.00, 0.00	0.00, 0.00	0.00, 0.00	-0.00, 0.00	-0.00, 0.00
	1000	0.00, -0.00	0.00, -0.00	0.00, -0.00	0.00, -0.00	0.00, -0.00
8	25	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.00	0.01, 0.01
	50	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, -0.00
	75	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00
	100	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	250	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00
	1000	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, 0.00	-0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set #2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.12: Expanded Results, Bias for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	-0.00, -0.00	-0.02, -0.02	-0.00, -0.00	-0.02, -0.02	-0.02, -0.03
	50	-0.00, 0.00	0.00, 0.00	-0.00, -0.00	-0.00, -0.00	-0.00, 0.00
	75	-0.00, -0.00	0.00, -0.00	-0.00, 0.00	0.00, -0.00	0.00, -0.00
	100	0.00, 0.01	0.00, -0.00	0.00, 0.01	0.00, 0.00	0.00, -0.00
	250	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
	1000	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, -0.00
4	25	0.01, -0.00	0.02, 0.01	0.00, -0.01	0.02, 0.02	0.02, 0.02
	50	-0.00, 0.00	0.01, 0.00	-0.00, 0.00	0.01, -0.00	0.01, -0.00
	75	0.00, -0.00	0.01, -0.00	0.00, -0.00	0.01, -0.00	0.01, -0.00
	100	0.00, -0.00	-0.01, 0.00	0.00, 0.00	-0.01, -0.00	-0.01, -0.00
	250	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, -0.00
	1000	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
8	25	-0.01, -0.02	-0.02, -0.01	-0.01, -0.02	-0.01, -0.01	-0.02, -0.00
	50	-0.00, -0.01	-0.01, -0.01	-0.00, -0.01	-0.00, -0.01	-0.00, -0.01
	75	-0.00, -0.01	-0.01, 0.00	-0.00, -0.01	-0.01, 0.00	-0.01, 0.00
	100	-0.01, -0.00	-0.00, 0.01	-0.00, -0.00	-0.00, 0.00	-0.00, 0.00
	250	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	-0.00, -0.00	0.00, -0.00
	1000	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, -0.00
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.00, -0.00	-0.00, -0.00	0.00, 0.00	-0.01, -0.00	-0.00, -0.00
	50	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, 0.00
	75	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, -0.00
	100	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	250	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	1000	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, -0.00
4	25	-0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.01, 0.01
	50	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00
	75	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00
	100	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, 0.00	0.00, 0.00
	250	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, 0.00
	1000	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, 0.00
8	25	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.00
	50	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	75	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00
	100	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	250	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	1000	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, -0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set #3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.13: Expanded Results, Bias for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	-0.02, -0.00	0.01, -0.00	-0.01, -0.01	-0.00, -0.01	-0.00, -0.01
	50	-0.01, -0.00	-0.01, -0.00	-0.01, -0.00	-0.01, -0.00	-0.01, -0.00
	75	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.01	0.00, 0.01
	100	0.00, -0.01	-0.00, 0.01	0.00, -0.00	-0.00, 0.00	-0.00, 0.00
	250	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, -0.00	0.00, -0.00
	1000	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
4	25	-0.01, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, 0.00	-0.01, -0.01
	50	0.01, 0.01	0.00, -0.00	0.01, 0.01	-0.00, -0.00	-0.00, -0.00
	75	-0.01, 0.00	0.01, 0.01	-0.00, 0.00	0.01, 0.01	0.01, 0.01
	100	-0.00, -0.00	-0.01, 0.01	-0.00, -0.00	-0.01, 0.00	-0.01, 0.00
	250	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, 0.00
	1000	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	-0.00, 0.00
8	25	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.00, 0.01	0.00, 0.01
	50	-0.00, 0.01	0.00, -0.02	-0.00, 0.01	0.00, -0.01	0.00, -0.01
	75	0.00, 0.01	-0.00, -0.00	0.00, 0.01	-0.00, -0.00	-0.00, -0.00
	100	0.00, 0.00	0.01, 0.00	0.00, 0.00	0.01, 0.00	0.01, 0.00
	250	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
	1000	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	-0.01, -0.01	-0.01, -0.00	-0.02, -0.01	-0.01, 0.00	0.03, -0.00
	50	0.00, -0.00	-0.01, -0.00	-0.00, 0.00	-0.01, -0.00	-0.00, -0.00
	75	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, 0.00
	100	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, -0.00	0.00, -0.00
	250	-0.00, 0.00	0.01, 0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
	1000	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	-0.00, 0.00
4	25	0.00, 0.01	0.00, 0.00	0.00, 0.01	0.01, 0.00	0.02, 0.01
	50	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
	75	0.00, 0.00	0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00
	100	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.00
	250	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
	1000	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, 0.00
8	25	0.00, 0.00	0.00, 0.01	0.00, 0.00	0.00, 0.01	0.01, 0.01
	50	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	75	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	0.00, -0.00
	100	0.00, -0.00	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, 0.00
	250	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, -0.00
	1000	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set #4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.14: Expanded Results, Bias for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	-0.01, -0.00	0.00, -0.02	-0.01, -0.00	0.00, -0.00	-0.01, 0.00
	50	-0.01, 0.00	0.00, -0.01	-0.01, 0.00	0.00, -0.00	0.00, -0.00
	75	0.01, 0.01	-0.00, 0.00	0.01, 0.01	0.00, 0.00	-0.00, 0.00
	100	-0.00, -0.01	0.00, -0.00	-0.00, -0.00	0.00, -0.00	0.00, -0.00
	250	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00
	1000	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00
4	25	-0.02, 0.00	-0.00, -0.02	-0.02, 0.00	0.00, -0.01	-0.00, -0.01
	50	-0.01, -0.01	-0.00, -0.01	-0.01, -0.01	0.00, -0.01	0.00, -0.01
	75	-0.00, 0.01	0.00, 0.01	-0.00, 0.01	0.00, 0.01	0.00, 0.01
	100	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	-0.00, -0.00
	250	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00
8	25	-0.01, 0.01	0.01, 0.01	-0.00, 0.01	0.02, 0.01	0.02, 0.01
	50	0.01, -0.00	-0.00, -0.00	0.01, -0.00	-0.00, -0.00	-0.00, -0.00
	75	-0.00, -0.00	0.00, -0.00	-0.01, -0.00	0.00, -0.00	0.00, 0.00
	100	0.00, 0.00	-0.01, -0.00	0.00, 0.00	-0.01, -0.00	-0.01, -0.00
	250	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, -0.00
	1000	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, -0.00
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	-0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	-0.00, -0.00
	50	0.00, -0.00	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	-0.00, 0.00
	75	0.00, 0.00	0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00
	100	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, -0.00
	250	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, -0.00
	1000	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	-0.00, -0.00
4	25	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	0.00, 0.00
	50	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, -0.00
	75	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00
	100	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, -0.00
	250	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, -0.00
	1000	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, -0.00
8	25	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.01
	50	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	75	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00
	100	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
	250	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00
	1000	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set #5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.15: Expanded Results, Bias for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.01, 0.01	0.01, -0.00	0.01, 0.01	-0.00, -0.01	0.00, -0.01
	50	-0.00, 0.01	0.01, 0.00	-0.00, 0.02	0.00, 0.00	0.00, 0.00
	75	-0.00, -0.00	0.00, 0.00	-0.01, 0.00	0.00, -0.00	0.00, -0.00
	100	0.00, -0.00	-0.00, -0.00	0.00, 0.00	-0.01, -0.00	-0.01, -0.00
	250	-0.00, 0.00	0.01, 0.00	-0.00, 0.00	0.01, 0.00	0.01, 0.00
	1000	0.00, -0.00	0.00, -0.00	0.00, -0.00	0.00, -0.01	0.00, -0.01
4	25	0.01, -0.00	0.01, 0.01	0.01, 0.01	0.01, 0.02	0.01, 0.01
	50	-0.01, -0.00	0.01, 0.00	-0.01, -0.00	0.01, 0.00	0.01, 0.00
	75	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, 0.00
	100	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	-0.00, -0.00	-0.00, -0.00
	250	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, -0.00
	1000	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, -0.00
8	25	-0.02, 0.01	0.01, 0.01	-0.01, 0.01	0.01, 0.01	0.01, 0.01
	50	0.00, -0.01	0.00, -0.01	-0.00, -0.01	-0.00, -0.01	-0.00, -0.01
	75	0.00, -0.00	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	-0.00, 0.00
	100	0.00, 0.00	0.01, 0.01	0.00, 0.00	0.01, 0.01	0.01, 0.01
	250	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00
	1000	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, -0.00	0.00, -0.00
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.01, 0.00	-0.00, 0.01	0.00, 0.00	0.00, 0.01	0.25, 0.01
	50	-0.00, 0.00	0.01, 0.00	-0.00, 0.00	0.01, 0.00	0.02, -0.00
	75	-0.00, 0.00	0.00, -0.01	-0.00, 0.00	0.00, -0.01	0.00, -0.01
	100	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, 0.00
	250	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00
	1000	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
4	25	0.01, 0.00	0.01, 0.01	0.01, 0.00	0.00, 0.01	0.01, 0.01
	50	0.00, 0.00	0.01, 0.00	0.00, 0.00	0.01, 0.00	0.01, 0.00
	75	-0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, 0.00
	100	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
	250	0.00, -0.00	0.00, -0.00	0.00, -0.00	0.00, -0.00	0.00, -0.00
	1000	0.00, 0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	-0.00, -0.00
8	25	0.00, 0.01	0.01, 0.01	0.00, 0.01	0.01, 0.01	0.02, 0.01
	50	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, -0.00	0.01, 0.00
	75	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.00
	100	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00
	250	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, -0.00	0.00, -0.00
	1000	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set #6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.16: Expanded Results, Relative Bias for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.03, 0.07	-0.00, 0.05	0.05, 0.05	0.05, 0.01	0.28, 0.16
	50	-0.02, 0.03	-0.01, 0.02	0.00, 0.01	0.01, -0.00	0.11, 0.06
	75	0.00, 0.01	-0.03, 0.01	0.02, 0.00	-0.00, -0.01	0.05, 0.03
	100	-0.01, 0.02	-0.01, 0.02	-0.01, 0.01	0.01, 0.00	0.04, 0.03
	250	0.01, 0.00	-0.01, -0.00	0.01, 0.00	-0.00, -0.00	0.01, 0.01
	1000	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00	0.00, 0.00
4	25	0.03, 0.06	0.01, 0.07	0.04, 0.05	0.06, 0.03	0.13, 0.09
	50	0.02, 0.03	0.01, 0.04	0.02, 0.02	0.03, 0.03	0.06, 0.05
	75	0.01, 0.02	0.01, 0.02	0.01, 0.01	0.01, 0.01	0.03, 0.03
	100	0.01, 0.00	0.01, 0.01	0.01, 0.00	0.02, 0.01	0.03, 0.02
	250	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.01
	1000	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
8	25	0.05, 0.06	0.05, 0.06	0.07, 0.04	0.08, 0.04	0.11, 0.07
	50	0.02, 0.02	0.03, 0.03	0.03, 0.02	0.03, 0.02	0.05, 0.03
	75	0.02, 0.01	0.01, 0.01	0.02, 0.01	0.02, 0.01	0.03, 0.02
	100	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.02, 0.01	0.02, 0.02
	250	0.00, 0.00	-0.01, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.01
	1000	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	-0.00, 0.00
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.17, 0.39	0.30, 0.38	-0.05, -0.02	0.04, 0.09	0.16, 0.33
	50	0.19, 0.15	0.22, 0.19	0.03, -0.06	-0.00, 0.08	0.04, 0.16
	75	0.07, 0.16	0.15, 0.06	-0.01, 0.01	-0.00, -0.01	0.02, 0.03
	100	0.09, 0.09	0.07, 0.01	0.03, 0.02	-0.02, -0.04	0.01, -0.02
	250	0.01, 0.04	0.05, -0.02	-0.01, 0.02	0.02, -0.01	0.02, 0.00
	1000	0.00, -0.00	0.01, -0.00	0.00, -0.00	0.01, -0.01	0.01, -0.01
4	25	0.17, 0.17	0.15, 0.17	0.03, -0.08	-0.05, -0.00	0.02, 0.04
	50	0.06, 0.11	0.08, 0.05	0.00, 0.03	0.00, -0.00	0.03, 0.03
	75	0.03, 0.07	0.07, 0.05	-0.00, 0.00	0.03, 0.03	0.05, 0.04
	100	0.02, 0.08	0.00, -0.01	0.01, 0.02	-0.03, -0.00	-0.02, 0.02
	250	0.01, 0.02	-0.00, -0.01	0.01, 0.01	0.00, -0.01	0.00, -0.01
	1000	0.00, -0.01	0.00, -0.01	0.00, -0.01	-0.00, -0.01	-0.00, -0.01
8	25	0.13, 0.14	0.09, 0.18	0.04, 0.02	-0.00, 0.15	0.04, 0.19
	50	0.01, 0.06	0.03, 0.04	-0.02, 0.01	-0.01, 0.04	0.00, 0.07
	75	-0.01, 0.05	0.03, 0.02	-0.02, 0.03	0.02, 0.01	0.03, 0.02
	100	0.00, 0.04	0.01, -0.01	-0.00, 0.03	0.00, -0.02	0.01, -0.01
	250	-0.00, 0.02	0.01, -0.05	-0.00, 0.02	0.00, -0.04	0.01, -0.03
	1000	0.00, 0.01	0.00, -0.01	0.00, 0.01	0.00, -0.01	0.01, -0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.17: Expanded Results, Relative Bias for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.00, 0.01	-0.01, 0.03	0.01, 0.00	0.01, 0.01	0.11, 0.12
	50	-0.00, 0.02	-0.01, 0.02	0.01, 0.01	-0.00, 0.01	0.04, 0.06
	75	-0.00, -0.00	-0.00, 0.01	0.00, -0.00	0.00, 0.00	0.03, 0.03
	100	-0.00, 0.01	0.00, 0.01	0.00, 0.01	0.01, 0.00	0.02, 0.03
	250	0.00, -0.00	-0.00, 0.00	0.00, -0.00	0.00, 0.00	0.01, 0.01
	1000	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
4	25	0.01, 0.03	0.00, 0.04	0.02, 0.03	0.02, 0.03	0.08, 0.09
	50	0.00, 0.02	0.01, 0.03	0.01, 0.01	0.01, 0.02	0.04, 0.05
	75	0.01, 0.01	0.00, 0.01	0.01, 0.01	0.01, 0.00	0.02, 0.02
	100	0.00, 0.01	0.00, 0.01	0.01, 0.01	0.01, 0.00	0.02, 0.02
	250	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.01
	1000	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, 0.00
8	25	0.01, 0.04	0.01, 0.04	0.02, 0.03	0.02, 0.03	0.07, 0.07
	50	0.01, 0.02	0.00, 0.02	0.01, 0.02	0.01, 0.02	0.03, 0.03
	75	0.00, 0.01	0.01, 0.01	0.00, 0.01	0.01, 0.00	0.02, 0.01
	100	0.00, 0.01	0.00, 0.01	0.00, 0.01	0.01, 0.01	0.02, 0.02
	250	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.00
	1000	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, -0.00
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.09, 0.02	0.11, 0.03	0.01, 0.01	-0.00, 0.00	0.07, 0.10
	50	0.08, -0.00	0.07, -0.01	0.04, 0.00	0.02, -0.01	0.07, 0.03
	75	0.02, 0.01	0.05, -0.01	-0.00, 0.02	0.01, -0.01	0.04, 0.02
	100	0.03, -0.01	0.04, 0.00	0.01, -0.01	0.01, 0.00	0.03, 0.02
	250	-0.00, 0.00	0.01, -0.00	-0.01, 0.00	0.00, 0.00	0.01, 0.01
	1000	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00
4	25	0.07, 0.01	0.12, 0.02	0.01, 0.02	0.03, 0.02	0.12, 0.10
	50	0.05, 0.00	0.06, 0.01	0.02, 0.01	0.03, 0.01	0.07, 0.05
	75	0.02, 0.01	0.02, 0.00	0.00, 0.01	-0.00, 0.00	0.02, 0.03
	100	0.02, -0.00	0.02, 0.01	0.01, -0.00	0.01, 0.01	0.02, 0.03
	250	-0.01, 0.00	-0.00, -0.00	-0.01, 0.00	-0.00, -0.00	0.00, 0.00
	1000	-0.00, 0.00	0.01, 0.00	-0.00, 0.00	0.01, 0.00	0.01, 0.00
8	25	0.06, -0.02	0.07, 0.01	0.01, -0.01	-0.00, 0.02	0.06, 0.08
	50	0.03, -0.01	0.05, 0.00	0.01, 0.00	0.02, 0.00	0.05, 0.03
	75	0.01, -0.01	-0.00, 0.01	0.01, -0.00	-0.02, 0.01	-0.00, 0.03
	100	0.01, -0.00	0.02, 0.00	-0.00, -0.00	0.01, 0.01	0.02, 0.02
	250	0.01, 0.00	-0.00, -0.00	0.01, 0.00	-0.00, 0.00	-0.00, 0.01
	1000	0.00, 0.00	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.18: Expanded Results, Relative Bias for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.02, 0.07	0.02, 0.04	0.03, 0.07	0.03, 0.04	3.94, 4.12
	50	-0.01, 0.02	0.00, 0.03	0.00, 0.02	0.02, 0.02	0.24, 0.17
	75	-0.01, 0.02	-0.01, 0.00	0.00, 0.01	0.01, -0.00	0.09, 0.11
	100	-0.00, 0.03	0.01, 0.03	0.01, 0.02	0.01, 0.03	0.08, 0.11
	250	0.00, 0.01	-0.00, 0.01	0.01, 0.00	0.00, 0.00	0.02, 0.03
	1000	0.00, 0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.01, 0.01
4	25	0.03, 0.08	0.04, 0.07	0.05, 0.07	0.06, 0.06	0.37, 0.20
	50	0.01, 0.05	0.01, 0.03	0.03, 0.04	0.03, 0.02	0.10, 0.09
	75	0.01, 0.04	0.00, 0.03	0.02, 0.04	0.01, 0.03	0.06, 0.07
	100	0.01, 0.02	0.01, 0.03	0.01, 0.01	0.01, 0.02	0.05, 0.06
	250	0.01, 0.00	0.00, 0.01	0.01, 0.00	0.01, 0.00	0.02, 0.02
	1000	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00
8	25	0.03, 0.06	0.03, 0.06	0.04, 0.05	0.04, 0.05	0.13, 0.11
	50	0.01, 0.04	0.01, 0.04	0.02, 0.03	0.01, 0.03	0.05, 0.07
	75	0.01, 0.02	0.01, 0.03	0.01, 0.01	0.02, 0.02	0.04, 0.04
	100	0.01, 0.01	0.00, 0.02	0.01, 0.01	0.01, 0.02	0.02, 0.03
	250	0.00, 0.00	0.00, 0.01	0.00, 0.00	0.00, 0.00	0.01, 0.01
	1000	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.30, 0.16	0.40, 0.16	0.06, 0.03	0.13, 0.05	10.78, 7.72
	50	0.15, 0.05	0.18, 0.05	0.05, -0.00	0.02, 0.03	0.10, 0.20
	75	0.07, 0.03	0.07, 0.02	-0.01, 0.02	-0.02, 0.02	0.06, 0.09
	100	0.05, 0.03	0.08, -0.01	-0.01, 0.03	0.01, -0.00	0.08, 0.06
	250	0.03, 0.01	0.02, -0.00	0.01, 0.02	-0.00, 0.00	0.02, 0.02
	1000	0.00, -0.00	0.01, 0.00	-0.00, -0.00	0.01, 0.00	0.01, 0.01
4	25	0.21, 0.04	0.14, 0.06	0.03, 0.01	-0.02, 0.06	0.13, 0.38
	50	0.07, 0.02	0.05, 0.00	-0.02, 0.03	-0.03, 0.00	0.05, 0.09
	75	0.07, -0.00	0.08, -0.01	0.04, 0.01	0.02, -0.01	0.08, 0.05
	100	0.01, -0.00	0.07, 0.00	-0.02, -0.00	0.03, 0.01	0.07, 0.05
	250	0.02, 0.01	0.00, 0.01	0.00, 0.01	-0.01, 0.01	0.01, 0.02
	1000	0.00, -0.00	0.00, -0.00	0.00, -0.00	0.00, 0.00	0.01, 0.00
8	25	0.13, 0.02	0.10, 0.01	0.06, 0.03	0.00, 0.01	0.11, 0.12
	50	0.08, 0.01	0.05, 0.00	0.04, 0.01	0.03, 0.01	0.08, 0.06
	75	0.02, 0.01	0.04, 0.01	-0.01, 0.01	0.01, 0.01	0.04, 0.04
	100	0.01, 0.01	0.04, -0.01	-0.00, 0.01	0.03, -0.01	0.06, 0.02
	250	0.01, 0.00	0.01, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.01
	1000	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, -0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.19: Expanded Results, Relative Bias for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	-0.00, 0.00	0.01, 0.01	-0.00, 0.01	0.02, 0.02	0.13, 0.13
	50	0.01, 0.00	0.01, 0.00	0.01, 0.00	0.01, 0.00	0.05, 0.05
	75	-0.01, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.03, 0.03
	100	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.00	0.02, 0.02
	250	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, -0.00	0.01, 0.01
	1000	0.00, -0.00	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, 0.00
4	25	0.02, 0.03	0.02, 0.03	0.03, 0.03	0.02, 0.03	0.08, 0.08
	50	0.01, 0.01	0.01, 0.02	0.01, 0.01	0.01, 0.01	0.04, 0.04
	75	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.03, 0.02
	100	0.00, 0.01	0.00, 0.01	0.01, 0.01	0.00, 0.01	0.01, 0.02
	250	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.01
	1000	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00
8	25	0.01, 0.04	0.02, 0.03	0.02, 0.04	0.02, 0.03	0.06, 0.06
	50	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.03, 0.03
	75	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.02, 0.02
	100	0.00, 0.00	0.00, 0.01	0.00, 0.00	0.00, 0.01	0.01, 0.02
	250	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.00
	1000	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	-0.00, 0.00
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.07, 0.02	0.16, 0.02	-0.01, 0.00	-0.02, 0.01	0.04, 0.02
	50	0.03, 0.00	0.10, -0.00	0.00, -0.01	0.02, -0.01	0.05, -0.00
	75	0.01, -0.00	0.03, 0.01	-0.00, -0.00	-0.02, 0.00	-0.00, 0.01
	100	-0.00, 0.00	0.04, 0.00	-0.01, -0.00	0.01, 0.00	0.02, 0.01
	250	-0.00, -0.01	0.01, 0.00	-0.00, -0.01	-0.00, -0.00	0.00, -0.00
	1000	0.01, -0.00	0.01, -0.00	0.01, -0.00	0.00, -0.00	0.00, -0.00
4	25	0.03, 0.00	0.10, 0.03	-0.04, -0.00	0.01, 0.02	0.08, 0.05
	50	-0.00, 0.01	0.06, 0.01	-0.01, 0.01	0.02, 0.00	0.05, 0.02
	75	-0.00, 0.00	0.05, -0.01	-0.01, 0.00	0.02, -0.01	0.04, -0.00
	100	-0.00, 0.01	-0.01, 0.01	-0.01, 0.01	-0.02, 0.01	-0.00, 0.01
	250	0.00, -0.00	0.00, 0.01	-0.00, -0.00	-0.00, 0.00	0.00, 0.01
	1000	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	-0.00, 0.00
8	25	0.01, 0.02	0.05, 0.02	-0.02, 0.02	-0.02, 0.02	0.05, 0.05
	50	0.00, 0.01	0.01, 0.01	-0.01, 0.01	-0.00, 0.01	0.02, 0.02
	75	0.01, 0.01	0.00, 0.00	0.00, 0.01	-0.01, 0.00	0.01, 0.01
	100	-0.00, -0.00	0.03, 0.00	-0.01, -0.00	0.02, 0.00	0.03, 0.01
	250	-0.01, -0.00	0.01, -0.00	-0.01, -0.00	0.01, -0.00	0.01, 0.00
	1000	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	-0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix}0.00\\0.00\end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix}2.00 & 0.60\\0.60 & 2.00\end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix}-0.76 & -0.25\\-0.17 & -0.43\end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix}0.10\\0.10\end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix}0.30 & 0.09\\0.09 & 0.30\end{smallmatrix}\right]$.

Table B.20: Expanded Results, Relative Bias for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.04, 0.00	0.05, 0.06	0.02, 0.04	0.03, 0.08	3.27, 1.08
	50	0.01, 0.00	0.01, 0.00	0.01, 0.02	0.01, 0.01	0.17, 0.16
	75	0.01, 0.02	0.00, 0.01	0.01, 0.01	0.01, 0.02	0.10, 0.12
	100	0.00, 0.02	0.00, 0.01	0.01, 0.02	0.01, 0.01	0.07, 0.09
	250	0.00, 0.01	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.03, 0.03
	1000	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	0.00, -0.00	0.01, 0.00
4	25	0.05, 0.06	0.05, 0.06	0.05, 0.06	0.05, 0.07	0.25, 0.19
	50	0.02, 0.05	0.03, 0.04	0.02, 0.04	0.04, 0.04	0.11, 0.10
	75	0.01, 0.02	0.01, 0.02	0.02, 0.02	0.01, 0.02	0.06, 0.06
	100	0.02, 0.02	0.01, 0.02	0.02, 0.01	0.01, 0.02	0.05, 0.05
	250	0.01, 0.01	0.01, 0.00	0.01, 0.01	0.01, 0.00	0.02, 0.02
	1000	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
8	25	0.04, 0.06	0.03, 0.05	0.04, 0.05	0.03, 0.05	0.12, 0.12
	50	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.01, 0.01	0.05, 0.04
	75	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.04, 0.04
	100	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.03, 0.03
	250	0.01, 0.00	0.00, 0.00	0.01, 0.00	0.00, 0.00	0.01, 0.01
	1000	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.33, 0.01	0.37, 0.09	-0.01, -0.03	-0.08, 0.00	0.53, 2.30
	50	0.21, 0.05	0.24, 0.00	0.03, 0.02	0.00, -0.02	0.03, -0.00
	75	0.09, 0.03	0.14, 0.01	-0.04, 0.02	0.05, -0.01	0.11, 0.02
	100	0.04, 0.03	0.09, 0.01	-0.02, 0.02	-0.01, -0.01	0.04, 0.01
	250	0.00, 0.01	0.02, 0.00	-0.01, 0.01	0.01, 0.00	0.02, 0.01
	1000	0.00, -0.00	0.00, -0.00	0.00, -0.00	0.00, -0.00	0.00, 0.00
4	25	0.19, 0.05	0.22, 0.05	0.03, 0.02	-0.01, 0.03	0.13, 0.10
	50	0.07, 0.04	0.13, 0.04	-0.00, 0.04	0.04, 0.03	0.13, 0.08
	75	0.06, 0.02	0.04, 0.01	0.02, 0.02	-0.01, 0.01	0.05, 0.04
	100	0.00, 0.02	0.03, 0.01	-0.01, 0.02	0.01, 0.01	0.05, 0.03
	250	-0.00, 0.01	-0.00, 0.00	-0.01, 0.01	-0.01, 0.00	0.01, 0.01
	1000	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, -0.00	0.00, 0.00
8	25	0.15, 0.03	0.09, 0.04	0.08, 0.03	0.01, 0.02	0.14, 0.09
	50	0.03, 0.02	0.02, -0.00	0.01, 0.01	0.01, -0.00	0.06, 0.03
	75	0.01, 0.02	0.02, 0.01	0.01, 0.02	0.01, 0.00	0.04, 0.02
	100	0.01, 0.01	-0.01, 0.01	0.01, 0.01	-0.01, 0.01	0.02, 0.02
	250	0.01, 0.00	0.01, 0.00	0.01, 0.00	0.01, 0.00	0.02, 0.01
	1000	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.21: Expanded Results, Relative Bias for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	-0.04, -0.04	-0.04, -0.05	-0.04, -0.03	-0.04, -0.05	0.29, 0.25
	50	-0.01, -0.02	-0.01, -0.01	-0.01, -0.03	-0.01, -0.01	0.09, 0.10
	75	-0.00, -0.02	-0.02, -0.01	-0.01, -0.02	-0.02, -0.01	0.05, 0.05
	100	-0.01, -0.01	-0.02, -0.01	-0.01, -0.01	-0.02, -0.01	0.03, 0.04
	250	-0.00, -0.01	0.00, -0.00	-0.00, -0.01	-0.00, -0.00	0.02, 0.02
	1000	0.00, -0.00	-0.00, 0.00	0.00, -0.00	-0.00, -0.00	0.00, 0.00
4	25	-0.03, -0.05	-0.03, -0.05	-0.04, -0.04	-0.04, -0.05	0.19, 0.16
	50	-0.03, -0.01	-0.01, -0.01	-0.03, -0.01	-0.01, -0.01	0.09, 0.09
	75	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.05, 0.06
	100	-0.00, -0.01	-0.02, -0.02	0.00, -0.01	-0.02, -0.01	0.03, 0.04
	250	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.02, 0.02
	1000	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, -0.00	0.01, 0.00
8	25	-0.04, -0.05	-0.05, -0.04	-0.04, -0.05	-0.04, -0.04	0.18, 0.18
	50	-0.02, -0.02	-0.00, -0.02	-0.02, -0.02	-0.00, -0.02	0.10, 0.09
	75	-0.02, -0.02	-0.01, -0.02	-0.02, -0.01	-0.01, -0.01	0.06, 0.05
	100	-0.01, -0.01	-0.00, -0.01	-0.01, -0.01	-0.00, -0.01	0.05, 0.04
	250	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	0.02, 0.02
	1000	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	0.00, 0.00
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	-0.07, -0.08	-0.08, -0.10	-0.06, -0.08	-0.08, -0.09	2.70, 3.68
	50	-0.06, -0.05	-0.05, -0.06	-0.04, -0.06	-0.05, -0.05	0.18, 0.17
	75	-0.03, -0.02	-0.03, -0.03	-0.03, -0.02	-0.02, -0.02	0.12, 0.12
	100	-0.04, -0.01	-0.03, -0.02	-0.02, -0.02	-0.03, -0.02	0.07, 0.08
	250	-0.01, -0.01	-0.00, -0.00	-0.00, -0.01	0.00, -0.01	0.04, 0.03
	1000	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.01, 0.00
4	25	-0.02, -0.00	-0.01, 0.01	-0.01, -0.00	-0.01, 0.00	0.13, 0.15
	50	-0.02, -0.01	-0.02, 0.00	-0.01, -0.01	-0.01, 0.00	0.05, 0.07
	75	-0.01, 0.00	-0.01, -0.01	-0.00, -0.00	-0.00, -0.00	0.04, 0.04
	100	-0.01, -0.00	-0.01, -0.00	-0.01, -0.00	-0.01, -0.00	0.02, 0.03
	250	-0.01, -0.00	-0.00, -0.00	-0.01, -0.00	-0.00, -0.00	0.01, 0.01
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
8	25	-0.00, 0.00	-0.01, 0.01	0.00, 0.00	-0.00, 0.01	0.05, 0.07
	50	-0.01, -0.00	-0.01, 0.01	-0.00, -0.00	-0.00, 0.01	0.02, 0.03
	75	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.02, 0.02
	100	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.01, 0.01
	250	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
	1000	0.00, -0.00	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.22: Expanded Results, Relative Bias for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	-0.03, -0.05	-0.03, -0.04	-0.04, -0.06	-0.04, -0.04	0.22, 0.21
	50	-0.03, -0.03	-0.01, -0.01	-0.03, -0.03	-0.01, -0.01	0.09, 0.10
	75	-0.02, -0.02	-0.02, -0.01	-0.02, -0.02	-0.02, -0.01	0.05, 0.06
	100	0.00, -0.01	-0.01, -0.01	-0.00, -0.02	-0.01, -0.01	0.04, 0.04
	250	-0.01, -0.01	-0.00, 0.00	-0.01, -0.01	-0.00, 0.00	0.02, 0.02
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
4	25	-0.03, -0.04	-0.03, -0.04	-0.03, -0.05	-0.04, -0.04	0.18, 0.18
	50	-0.02, -0.02	-0.02, -0.03	-0.02, -0.02	-0.02, -0.02	0.08, 0.08
	75	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.06, 0.06
	100	-0.02, -0.01	-0.01, -0.02	-0.02, -0.01	-0.01, -0.02	0.03, 0.03
	250	-0.00, -0.00	-0.01, -0.00	-0.00, -0.00	-0.01, -0.00	0.01, 0.02
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
8	25	-0.02, -0.03	-0.03, -0.03	-0.03, -0.03	-0.03, -0.02	0.18, 0.20
	50	-0.02, -0.03	-0.02, -0.02	-0.02, -0.03	-0.02, -0.02	0.08, 0.08
	75	-0.01, -0.02	-0.01, -0.00	-0.01, -0.01	-0.01, -0.01	0.06, 0.06
	100	0.00, -0.01	-0.01, -0.01	0.00, -0.01	-0.01, -0.01	0.04, 0.04
	250	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.01, 0.01
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	-0.12, -0.14	-0.13, -0.11	-0.11, -0.13	-0.12, -0.11	0.33, 0.28
	50	-0.06, -0.06	-0.06, -0.04	-0.06, -0.05	-0.06, -0.04	0.12, 0.13
	75	-0.05, -0.04	-0.04, -0.04	-0.05, -0.04	-0.03, -0.05	0.08, 0.06
	100	-0.03, -0.02	-0.03, -0.03	-0.03, -0.02	-0.03, -0.04	0.05, 0.04
	250	-0.01, -0.01	-0.02, -0.01	-0.01, -0.01	-0.01, -0.01	0.02, 0.02
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.01, 0.01
4	25	-0.04, -0.04	-0.04, -0.02	-0.03, -0.04	-0.03, -0.02	0.11, 0.10
	50	-0.01, -0.03	-0.02, -0.01	-0.01, -0.02	-0.01, -0.01	0.05, 0.04
	75	-0.01, -0.01	-0.01, -0.01	-0.00, -0.01	-0.01, -0.01	0.03, 0.02
	100	-0.01, -0.01	-0.01, -0.01	-0.00, -0.01	-0.00, -0.01	0.03, 0.02
	250	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	-0.00, -0.00	0.01, 0.01
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
8	25	-0.02, -0.01	-0.01, -0.01	-0.01, -0.01	-0.00, -0.01	0.06, 0.04
	50	-0.00, -0.00	-0.01, -0.01	0.00, -0.00	-0.01, -0.01	0.02, 0.02
	75	-0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.02, 0.01
	100	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, 0.00	0.01, 0.01
	250	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.01, 0.01
	1000	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.23: Expanded Results, Relative Bias for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	-0.03, -0.02	-0.04, -0.01	-0.04, -0.04	-0.05, -0.02	0.18, 0.33
	50	-0.01, -0.02	-0.02, -0.02	-0.01, -0.02	-0.03, -0.02	0.10, 0.10
	75	-0.02, -0.02	-0.01, -0.02	-0.02, -0.02	-0.01, -0.02	0.06, 0.04
	100	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.04, 0.04
	250	-0.00, -0.01	-0.00, -0.01	-0.00, -0.01	-0.00, -0.01	0.01, 0.01
	1000	0.00, -0.00	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, 0.01
4	25	-0.03, -0.03	-0.04, -0.03	-0.04, -0.03	-0.04, -0.03	0.22, 0.20
	50	-0.02, -0.02	-0.01, -0.02	-0.03, -0.02	-0.01, -0.02	0.09, 0.08
	75	-0.02, -0.02	-0.01, -0.02	-0.02, -0.01	-0.01, -0.02	0.06, 0.04
	100	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.04, 0.04
	250	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.02, 0.02
	1000	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, -0.00	0.01, 0.00
8	25	-0.04, -0.03	-0.04, -0.03	-0.04, -0.03	-0.04, -0.04	0.19, 0.18
	50	-0.02, -0.02	-0.02, -0.02	-0.02, -0.01	-0.02, -0.02	0.08, 0.08
	75	-0.01, -0.02	-0.01, -0.01	-0.01, -0.02	-0.01, -0.00	0.06, 0.06
	100	-0.01, -0.01	-0.00, -0.01	-0.01, -0.01	-0.01, -0.01	0.04, 0.04
	250	-0.00, -0.01	-0.01, -0.01	-0.00, -0.01	-0.01, -0.01	0.01, 0.01
	1000	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, 0.00
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	-0.12, -0.09	-0.12, -0.12	-0.10, -0.09	-0.11, -0.11	4.31, 2.65
	50	-0.07, -0.06	-0.06, -0.06	-0.06, -0.05	-0.05, -0.06	0.29, 0.18
	75	-0.03, -0.03	-0.04, -0.04	-0.03, -0.03	-0.03, -0.04	0.13, 0.10
	100	-0.03, -0.02	-0.02, -0.03	-0.02, -0.02	-0.01, -0.03	0.10, 0.08
	250	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.03, 0.03
	1000	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, 0.00	0.01, 0.01
4	25	-0.03, -0.02	-0.03, -0.02	-0.01, -0.02	-0.01, -0.02	0.34, 0.16
	50	-0.01, -0.00	-0.01, -0.02	-0.00, -0.00	0.00, -0.01	0.09, 0.06
	75	-0.00, 0.00	-0.00, -0.01	0.01, 0.00	-0.00, -0.01	0.06, 0.04
	100	-0.01, -0.00	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	0.04, 0.03
	250	0.00, 0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.01, 0.01
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
8	25	0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00	0.10, 0.07
	50	-0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, -0.00	0.04, 0.03
	75	-0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.03, 0.02
	100	0.00, -0.00	-0.00, 0.00	0.00, -0.00	-0.00, -0.00	0.02, 0.01
	250	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.01, 0.01
	1000	-0.00, -0.00	0.00, 0.00	-0.00, -0.00	-0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.24: Expanded Results, Relative Bias for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	-0.04, -0.03	-0.03, -0.04	-0.05, -0.04	-0.04, -0.04	0.23, 0.21
	50	-0.02, -0.01	-0.02, -0.03	-0.02, -0.01	-0.01, -0.03	0.09, 0.07
	75	-0.01, -0.02	-0.02, -0.01	-0.01, -0.02	-0.02, -0.01	0.04, 0.05
	100	-0.00, -0.00	-0.02, -0.01	-0.00, -0.00	-0.01, -0.01	0.03, 0.04
	250	-0.00, -0.01	-0.00, -0.01	-0.00, -0.01	-0.00, -0.00	0.02, 0.02
	1000	0.00, -0.00	-0.00, -0.00	0.00, -0.00	0.00, -0.00	0.01, 0.00
4	25	-0.03, -0.04	-0.03, -0.04	-0.04, -0.04	-0.03, -0.04	0.19, 0.18
	50	-0.02, -0.02	-0.01, -0.02	-0.02, -0.02	-0.01, -0.01	0.09, 0.09
	75	-0.01, -0.01	-0.02, -0.01	-0.02, -0.02	-0.01, -0.02	0.05, 0.05
	100	-0.01, -0.01	-0.01, -0.00	-0.02, -0.01	-0.01, -0.00	0.04, 0.05
	250	-0.00, -0.00	-0.00, -0.01	-0.00, -0.00	-0.01, -0.01	0.01, 0.01
	1000	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, 0.00
8	25	-0.02, -0.02	-0.04, -0.05	-0.02, -0.02	-0.05, -0.05	0.17, 0.17
	50	-0.03, -0.02	-0.01, -0.02	-0.02, -0.02	-0.01, -0.02	0.09, 0.09
	75	-0.02, -0.02	-0.01, -0.01	-0.02, -0.02	-0.01, -0.01	0.06, 0.06
	100	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.00	0.04, 0.05
	250	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	-0.00, -0.00	0.02, 0.02
	1000	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, -0.00	0.01, 0.00
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	-0.11, -0.12	-0.10, -0.11	-0.11, -0.12	-0.10, -0.11	0.36, 0.28
	50	-0.06, -0.05	-0.05, -0.06	-0.06, -0.05	-0.05, -0.06	0.13, 0.11
	75	-0.04, -0.04	-0.04, -0.04	-0.03, -0.03	-0.04, -0.04	0.08, 0.07
	100	-0.03, -0.03	-0.02, -0.04	-0.03, -0.03	-0.02, -0.03	0.06, 0.05
	250	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.02, 0.02
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.01, 0.01
4	25	-0.03, -0.03	-0.04, -0.03	-0.03, -0.03	-0.03, -0.04	0.10, 0.09
	50	-0.01, -0.01	-0.02, -0.02	-0.01, -0.01	-0.02, -0.02	0.05, 0.03
	75	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.03, 0.03
	100	-0.00, -0.01	-0.01, -0.01	-0.00, -0.01	-0.01, -0.01	0.02, 0.02
	250	-0.00, -0.00	-0.00, -0.00	-0.01, -0.00	-0.00, -0.00	0.01, 0.01
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
8	25	-0.01, -0.00	-0.00, -0.00	-0.01, -0.00	-0.00, -0.01	0.06, 0.05
	50	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	0.00, -0.01	0.03, 0.02
	75	-0.00, -0.00	-0.00, -0.00	-0.00, -0.01	-0.00, -0.00	0.02, 0.02
	100	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	-0.00, -0.00	0.01, 0.01
	250	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, -0.00	0.01, 0.00
	1000	-0.00, -0.00	-0.00, 0.00	-0.00, -0.00	0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.25: Expanded Results, Relative Bias for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	-0.03, -0.05	-0.02, -0.05	-0.03, -0.04	-0.03, -0.05	0.24, 0.34
	50	-0.03, -0.03	-0.01, -0.02	-0.03, -0.03	-0.02, -0.02	0.12, 0.10
	75	-0.01, -0.01	-0.01, -0.00	-0.01, -0.01	-0.01, -0.00	0.06, 0.06
	100	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.02	0.04, 0.03
	250	-0.00, -0.00	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	0.02, 0.01
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
4	25	-0.02, -0.03	-0.02, -0.03	-0.02, -0.04	-0.03, -0.03	0.25, 0.21
	50	-0.03, -0.01	-0.02, -0.02	-0.03, -0.01	-0.02, -0.02	0.08, 0.08
	75	-0.01, -0.02	-0.01, -0.02	-0.01, -0.01	-0.01, -0.02	0.06, 0.05
	100	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.04, 0.04
	250	-0.01, -0.01	-0.01, -0.00	-0.01, -0.01	-0.01, -0.00	0.01, 0.02
	1000	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, -0.00	0.01, 0.00
8	25	-0.04, -0.03	-0.03, -0.03	-0.04, -0.03	-0.03, -0.03	0.19, 0.20
	50	-0.02, -0.01	-0.03, -0.03	-0.03, -0.01	-0.03, -0.03	0.07, 0.07
	75	-0.02, -0.01	-0.02, -0.01	-0.02, -0.01	-0.02, -0.01	0.05, 0.06
	100	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.04, 0.04
	250	0.00, -0.00	-0.01, -0.01	0.00, -0.00	-0.01, -0.01	0.01, 0.01
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	0.00, 0.00
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	-0.10, -0.11	-0.08, -0.09	-0.10, -0.11	-0.10, -0.09	4.80, 1.03
	50	-0.04, -0.06	-0.05, -0.05	-0.04, -0.06	-0.05, -0.06	0.25, 0.16
	75	-0.03, -0.03	-0.03, -0.02	-0.04, -0.04	-0.03, -0.02	0.14, 0.13
	100	-0.02, -0.02	-0.02, -0.02	-0.01, -0.02	-0.01, -0.02	0.10, 0.09
	250	-0.01, -0.01	-0.02, -0.01	-0.01, -0.01	-0.02, -0.01	0.03, 0.03
	1000	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	-0.00, -0.00	0.01, 0.00
4	25	-0.01, -0.00	-0.01, -0.01	-0.01, -0.01	-0.01, -0.01	0.27, 0.16
	50	-0.00, 0.00	0.00, -0.01	0.00, 0.00	0.01, -0.01	0.10, 0.07
	75	-0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.06, 0.05
	100	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.05, 0.04
	250	0.00, -0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.02, 0.01
	1000	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00
8	25	0.01, -0.00	0.01, 0.00	0.01, -0.01	0.01, -0.00	0.11, 0.07
	50	-0.00, 0.01	0.00, -0.00	-0.00, 0.00	0.00, -0.00	0.05, 0.03
	75	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00	-0.00, -0.00	0.03, 0.02
	100	0.00, 0.00	-0.00, 0.00	0.00, 0.00	0.00, 0.00	0.02, 0.02
	250	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.00, 0.00	0.01, 0.01
	1000	0.00, 0.00	-0.00, 0.00	0.00, 0.00	-0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix}0.00\\0.00\end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix}2.00 & 0.60\\0.60 & 2.00\end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix}-0.76 & -0.25\\-0.17 & -0.43\end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix}0.10\\0.10\end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix}1.00 & 0.30\\0.30 & 1.00\end{smallmatrix}\right]$.

Table B.26: Expanded Results, Relative Bias for $\phi_{0,21}$ and q_{21}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	-0.07, -0.15	-0.07, -0.14	-0.07, -0.05	-0.06, -0.07	0.21, -2.36
	50	-0.03, -0.09	-0.02, -0.07	-0.03, -0.05	-0.01, -0.03	0.06, 0.13
	75	0.01, -0.09	-0.03, -0.07	-0.01, -0.04	-0.04, -0.02	0.00, 0.08
	100	-0.01, -0.04	-0.01, -0.04	-0.01, -0.01	-0.02, -0.01	0.01, 0.06
	250	-0.01, -0.02	0.00, -0.01	-0.01, -0.00	-0.00, 0.00	0.01, 0.03
	1000	-0.01, -0.00	-0.01, 0.01	-0.01, -0.00	-0.01, 0.01	-0.00, 0.01
4	25	-0.04, -0.06	-0.01, 0.01	-0.02, -0.02	-0.02, 0.03	0.10, 0.12
	50	-0.04, -0.04	0.02, -0.05	-0.04, -0.01	0.01, -0.02	0.07, 0.02
	75	-0.03, -0.01	0.02, -0.04	-0.03, 0.00	0.01, -0.02	0.05, 0.00
	100	-0.01, -0.03	-0.03, -0.02	-0.01, -0.01	-0.02, -0.01	0.01, 0.02
	250	-0.01, -0.01	-0.00, 0.00	-0.01, -0.00	-0.00, 0.01	0.01, 0.01
	1000	0.00, -0.00	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	-0.00, 0.00
8	25	-0.08, -0.02	-0.03, -0.04	-0.06, 0.01	-0.02, -0.01	0.12, 0.02
	50	-0.01, -0.02	-0.02, -0.02	-0.01, -0.01	-0.02, -0.01	0.04, 0.01
	75	-0.01, -0.01	-0.04, -0.01	-0.01, 0.00	-0.02, -0.00	0.02, 0.01
	100	-0.01, 0.01	0.00, 0.00	-0.01, 0.01	0.00, 0.00	0.03, 0.01
	250	0.01, -0.00	-0.00, -0.00	0.01, -0.00	-0.00, -0.00	0.01, 0.00
	1000	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, -0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.27: Expanded Results, Relative Bias for $\phi_{0,21}$ and q_{21}, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	-0.08, -0.16	-0.05, -0.15	-0.09, -0.12	-0.06, -0.11	0.04, 0.12
	50	-0.03, -0.04	-0.03, -0.10	-0.02, -0.04	-0.03, -0.09	0.03, 0.02
	75	-0.03, -0.07	-0.01, -0.05	-0.03, -0.07	-0.00, -0.04	0.04, 0.03
	100	-0.01, -0.01	-0.01, -0.05	-0.01, -0.00	-0.02, -0.05	0.01, -0.00
	250	-0.00, -0.00	0.01, -0.02	0.00, -0.00	0.00, -0.02	0.02, 0.00
	1000	-0.00, -0.01	-0.00, 0.00	-0.00, -0.01	-0.00, 0.00	-0.00, 0.01
4	25	-0.04, -0.05	-0.01, -0.06	-0.04, -0.04	-0.03, -0.04	0.09, 0.02
	50	-0.00, -0.03	-0.02, -0.02	0.00, -0.02	-0.01, -0.02	0.05, 0.01
	75	-0.01, -0.00	-0.00, -0.02	-0.01, -0.00	-0.01, -0.02	0.04, 0.00
	100	-0.01, -0.01	-0.02, -0.01	-0.01, -0.01	-0.03, -0.01	0.00, 0.01
	250	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	-0.01, -0.00	0.01, 0.00
	1000	-0.00, -0.00	-0.00, -0.00	-0.00, -0.00	-0.00, 0.00	-0.00, 0.00
8	25	-0.04, -0.03	-0.00, -0.02	-0.04, -0.02	-0.01, -0.01	0.11, 0.01
	50	-0.05, -0.00	-0.02, -0.02	-0.04, -0.00	-0.03, -0.01	0.03, -0.00
	75	-0.01, 0.01	-0.03, -0.00	-0.01, 0.01	-0.03, -0.00	0.01, 0.01
	100	0.01, -0.01	-0.01, -0.00	0.01, -0.01	-0.01, -0.00	0.02, 0.01
	250	-0.00, -0.00	-0.01, 0.01	-0.00, -0.00	-0.01, 0.01	0.00, 0.01
	1000	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00	-0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.28: Expanded Results, Relative Bias for $\phi_{0,21}$ and q_{21}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	-0.03, -0.18	-0.00, -0.21	-0.02, -0.10	-0.01, -0.13	0.34, -5.24
	50	0.01, -0.12	-0.03, -0.10	0.02, -0.07	-0.03, -0.06	-0.00, 0.12
	75	-0.02, -0.06	-0.03, -0.07	-0.02, -0.04	-0.03, -0.05	0.01, 0.04
	100	0.02, -0.05	-0.02, -0.05	0.02, -0.03	-0.02, -0.04	0.01, 0.02
	250	0.00, -0.02	-0.01, -0.02	0.01, -0.02	-0.01, -0.02	-0.00, 0.00
	1000	0.01, -0.00	0.00, -0.00	0.01, -0.00	0.00, -0.00	0.01, 0.01
4	25	-0.03, -0.10	-0.03, -0.05	-0.04, -0.04	-0.02, -0.01	0.08, 0.04
	50	-0.01, -0.00	-0.00, -0.03	-0.01, 0.01	-0.00, -0.01	0.05, 0.02
	75	-0.03, -0.01	-0.02, -0.01	-0.03, -0.00	-0.02, -0.00	0.02, 0.01
	100	-0.02, -0.00	-0.02, -0.03	-0.02, 0.00	-0.02, -0.02	0.01, -0.01
	250	-0.00, -0.01	-0.01, -0.00	-0.00, -0.00	-0.01, 0.00	0.00, 0.01
	1000	-0.01, 0.00	0.00, -0.00	-0.01, 0.00	0.00, 0.00	0.00, 0.00
8	25	-0.02, -0.02	-0.05, -0.02	-0.02, -0.01	-0.04, 0.00	0.09, 0.02
	50	-0.01, -0.01	-0.02, -0.01	-0.01, -0.01	-0.02, -0.00	0.05, 0.00
	75	-0.01, -0.00	-0.02, -0.02	-0.01, 0.00	-0.01, -0.01	0.03, -0.01
	100	-0.01, -0.00	-0.01, -0.01	-0.02, -0.00	-0.01, -0.00	0.02, -0.00
	250	0.00, -0.00	-0.02, -0.00	0.00, 0.00	-0.02, -0.00	-0.01, -0.00
	1000	0.00, -0.00	-0.00, 0.00	0.00, -0.00	-0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.29: Expanded Results, Relative Bias for $\phi_{0,21}$ and q_{21}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	-0.09, -0.06	-0.04, -0.04	-0.08, -0.06	-0.01, -0.09	0.17, 0.22
	50	-0.02, -0.05	-0.04, -0.01	-0.02, -0.06	-0.02, -0.05	0.04, 0.09
	75	-0.01, -0.03	-0.04, -0.03	-0.01, -0.03	-0.03, -0.05	0.00, 0.04
	100	-0.00, -0.02	-0.02, -0.02	0.00, -0.03	-0.01, -0.02	0.02, 0.05
	250	-0.01, -0.03	-0.01, -0.02	-0.01, -0.03	-0.01, -0.02	0.01, 0.01
	1000	-0.00, -0.00	0.00, 0.01	-0.00, -0.00	0.00, 0.00	0.01, 0.01
4	25	-0.03, -0.03	-0.03, -0.00	-0.03, -0.05	-0.03, -0.03	0.10, 0.09
	50	-0.03, -0.03	-0.03, -0.02	-0.04, -0.03	-0.01, -0.03	0.05, 0.03
	75	-0.02, -0.01	-0.03, 0.00	-0.02, -0.01	-0.02, -0.00	0.01, 0.03
	100	-0.02, -0.02	-0.01, -0.01	-0.02, -0.02	-0.01, -0.01	0.02, 0.02
	250	0.00, -0.01	-0.01, -0.00	0.00, -0.01	-0.00, -0.00	0.01, 0.01
	1000	0.01, -0.00	-0.01, -0.00	0.01, -0.00	-0.01, -0.00	-0.00, 0.00
8	25	0.01, 0.01	-0.06, 0.01	0.01, 0.01	-0.06, -0.01	0.07, 0.06
	50	-0.01, -0.00	-0.02, 0.00	-0.01, -0.01	-0.01, -0.00	0.06, 0.03
	75	-0.00, -0.00	0.00, -0.01	-0.00, -0.00	0.00, -0.01	0.05, 0.01
	100	-0.02, -0.00	-0.02, 0.00	-0.02, -0.00	-0.02, 0.00	0.01, 0.02
	250	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	0.00, -0.00	0.01, 0.00
	1000	0.00, -0.00	0.00, 0.00	0.00, -0.00	0.00, 0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.30: Expanded Results, Relative Bias for $\phi_{0,21}$ and q_{21}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	-0.02, -0.05	-0.04, -0.03	-0.03, -0.14	-0.01, -0.11	0.24, 2.25
	50	-0.03, 0.01	-0.04, -0.02	-0.03, -0.05	-0.02, -0.08	0.10, 0.09
	75	-0.03, -0.00	0.00, -0.01	-0.02, -0.04	0.00, -0.03	0.05, 0.11
	100	0.01, 0.00	0.00, -0.01	0.01, -0.01	-0.01, -0.03	0.02, 0.07
	250	-0.01, -0.01	-0.01, -0.00	-0.01, -0.01	-0.01, -0.01	0.00, 0.03
	1000	-0.00, -0.01	-0.00, -0.00	-0.00, -0.01	-0.00, -0.00	0.00, 0.01
4	25	0.02, 0.02	-0.06, 0.06	0.02, -0.01	-0.05, 0.00	0.16, 0.24
	50	-0.04, 0.02	-0.00, 0.03	-0.05, -0.00	-0.00, 0.01	0.06, 0.11
	75	-0.02, 0.02	-0.01, 0.02	-0.02, 0.01	-0.02, 0.01	0.02, 0.08
	100	-0.02, 0.01	-0.01, 0.02	-0.02, 0.00	-0.01, 0.01	0.02, 0.06
	250	-0.01, 0.01	0.00, -0.00	-0.01, 0.00	0.00, -0.00	0.02, 0.02
	1000	-0.00, -0.00	-0.01, 0.00	-0.00, -0.00	-0.01, 0.00	-0.00, 0.01
8	25	-0.02, 0.03	-0.01, 0.02	-0.01, 0.01	-0.00, 0.00	0.13, 0.12
	50	0.00, 0.01	-0.04, 0.01	0.00, 0.01	-0.03, 0.00	0.03, 0.05
	75	-0.00, -0.00	-0.02, -0.00	-0.00, -0.00	-0.02, -0.00	0.02, 0.03
	100	-0.01, 0.01	-0.01, 0.01	-0.01, 0.01	-0.01, 0.00	0.02, 0.03
	250	0.00, 0.00	-0.01, 0.00	0.00, 0.00	-0.01, 0.00	0.00, 0.01
	1000	0.00, 0.00	-0.00, -0.00	0.00, -0.00	-0.00, -0.00	0.00, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.31: Expanded Results, RMSE for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.27, 0.28	0.30, 0.29	0.27, 0.28	0.29, 0.28	0.29, 0.29
	50	0.20, 0.19	0.19, 0.20	0.20, 0.19	0.19, 0.20	0.19, 0.20
	75	0.17, 0.17	0.16, 0.16	0.17, 0.17	0.16, 0.16	0.16, 0.16
	100	0.15, 0.14	0.14, 0.14	0.15, 0.15	0.14, 0.14	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.04, 0.05	0.04, 0.04	0.04, 0.05	0.04, 0.04	0.04, 0.04
4	25	0.30, 0.29	0.28, 0.28	0.30, 0.29	0.27, 0.28	0.27, 0.28
	50	0.19, 0.19	0.20, 0.19	0.20, 0.19	0.20, 0.20	0.20, 0.20
	75	0.16, 0.16	0.16, 0.16	0.16, 0.16	0.16, 0.16	0.16, 0.16
	100	0.14, 0.13	0.14, 0.14	0.14, 0.13	0.14, 0.14	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.05, 0.05	0.05, 0.04	0.05, 0.05	0.04, 0.04	0.04, 0.04
8	25	0.28, 0.28	0.29, 0.27	0.28, 0.28	0.29, 0.27	0.29, 0.27
	50	0.20, 0.19	0.19, 0.20	0.20, 0.19	0.19, 0.20	0.19, 0.20
	75	0.16, 0.16	0.16, 0.17	0.16, 0.16	0.16, 0.17	0.16, 0.17
	100	0.14, 0.15	0.15, 0.14	0.14, 0.15	0.15, 0.14	0.15, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.05, 0.04	0.05, 0.05	0.05, 0.04	0.04, 0.05	0.05, 0.05
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.11, 0.11	0.11, 0.12	0.12, 0.12	0.11, 0.12	0.12, 0.12
	50	0.08, 0.08	0.07, 0.08	0.08, 0.08	0.07, 0.08	0.07, 0.08
	75	0.06, 0.06	0.07, 0.07	0.06, 0.06	0.07, 0.06	0.07, 0.07
	100	0.06, 0.05	0.06, 0.06	0.06, 0.05	0.05, 0.06	0.06, 0.06
	250	0.03, 0.03	0.03, 0.04	0.03, 0.03	0.03, 0.04	0.03, 0.04
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
4	25	0.07, 0.07	0.06, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07
	50	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05
	75	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	100	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
8	25	0.04, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05
	50	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	75	0.02, 0.03	0.03, 0.03	0.02, 0.03	0.03, 0.03	0.03, 0.03
	100	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	250	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.32: Expanded Results, RMSE for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.27, 0.28	0.28, 0.28	0.27, 0.28	0.28, 0.28	0.29, 0.29
	50	0.20, 0.20	0.19, 0.19	0.19, 0.20	0.20, 0.19	0.20, 0.20
	75	0.16, 0.17	0.17, 0.16	0.16, 0.17	0.17, 0.16	0.17, 0.16
	100	0.14, 0.14	0.14, 0.14	0.13, 0.14	0.14, 0.15	0.14, 0.15
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.04, 0.04	0.05, 0.04	0.04, 0.04	0.05, 0.04	0.05, 0.04
4	25	0.28, 0.29	0.27, 0.28	0.28, 0.29	0.28, 0.28	0.28, 0.28
	50	0.19, 0.20	0.19, 0.20	0.20, 0.20	0.20, 0.20	0.20, 0.20
	75	0.16, 0.17	0.17, 0.16	0.16, 0.17	0.17, 0.17	0.17, 0.17
	100	0.14, 0.14	0.14, 0.15	0.14, 0.14	0.14, 0.14	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.05, 0.04	0.04, 0.04	0.05, 0.04	0.04, 0.04	0.04, 0.04
8	25	0.28, 0.28	0.27, 0.29	0.28, 0.28	0.28, 0.29	0.28, 0.29
	50	0.20, 0.20	0.20, 0.20	0.20, 0.20	0.20, 0.20	0.20, 0.20
	75	0.16, 0.16	0.16, 0.16	0.16, 0.16	0.16, 0.16	0.16, 0.16
	100	0.14, 0.15	0.14, 0.14	0.15, 0.14	0.14, 0.14	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.05, 0.04	0.04, 0.04	0.05, 0.04	0.04, 0.04	0.04, 0.04
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.23, 0.21	0.21, 0.21	0.22, 0.21	0.21, 0.21	0.63, 0.68
	50	0.14, 0.14	0.14, 0.15	0.14, 0.15	0.15, 0.15	0.15, 0.15
	75	0.11, 0.12	0.11, 0.12	0.12, 0.12	0.12, 0.12	0.12, 0.12
	100	0.10, 0.10	0.10, 0.10	0.10, 0.10	0.10, 0.10	0.11, 0.10
	250	0.06, 0.07	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.07, 0.06
	1000	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
4	25	0.13, 0.13	0.12, 0.12	0.13, 0.13	0.12, 0.13	0.13, 0.13
	50	0.08, 0.09	0.08, 0.09	0.08, 0.09	0.09, 0.09	0.09, 0.09
	75	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07
	100	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06
	250	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
8	25	0.09, 0.08	0.08, 0.08	0.09, 0.08	0.08, 0.08	0.08, 0.09
	50	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06
	75	0.04, 0.05	0.04, 0.05	0.04, 0.05	0.04, 0.05	0.04, 0.05
	100	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.33: Expanded Results, RMSE for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.28, 0.29	0.29, 0.29	0.28, 0.28	0.29, 0.29	0.28, 0.28
	50	0.20, 0.20	0.20, 0.19	0.20, 0.20	0.20, 0.19	0.20, 0.19
	75	0.16, 0.17	0.16, 0.17	0.16, 0.17	0.16, 0.17	0.16, 0.17
	100	0.14, 0.14	0.15, 0.14	0.14, 0.14	0.15, 0.14	0.15, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.04, 0.04	0.05, 0.04	0.04, 0.04	0.05, 0.05	0.05, 0.05
4	25	0.27, 0.29	0.28, 0.28	0.27, 0.29	0.28, 0.28	0.28, 0.28
	50	0.20, 0.20	0.20, 0.20	0.20, 0.20	0.20, 0.20	0.20, 0.20
	75	0.16, 0.17	0.16, 0.16	0.16, 0.17	0.17, 0.16	0.17, 0.16
	100	0.15, 0.14	0.14, 0.14	0.15, 0.14	0.14, 0.14	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.05	0.04, 0.05
8	25	0.27, 0.28	0.28, 0.28	0.28, 0.28	0.28, 0.28	0.28, 0.28
	50	0.20, 0.19	0.20, 0.21	0.20, 0.20	0.20, 0.21	0.20, 0.21
	75	0.17, 0.16	0.17, 0.16	0.17, 0.16	0.17, 0.16	0.17, 0.16
	100	0.14, 0.14	0.14, 0.14	0.14, 0.14	0.14, 0.14	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.04, 0.04	0.04, 0.05	0.04, 0.04	0.04, 0.05	0.04, 0.05
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.12, 0.12	0.11, 0.12	0.12, 0.12	0.11, 0.12	0.11, 0.12
	50	0.08, 0.08	0.08, 0.08	0.08, 0.08	0.08, 0.08	0.08, 0.08
	75	0.07, 0.07	0.06, 0.07	0.07, 0.07	0.06, 0.07	0.06, 0.07
	100	0.06, 0.05	0.06, 0.06	0.06, 0.05	0.06, 0.06	0.06, 0.06
	250	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
4	25	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07
	50	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05
	75	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	100	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
8	25	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.05
	50	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	75	0.03, 0.02	0.03, 0.03	0.03, 0.02	0.03, 0.03	0.03, 0.03
	100	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	250	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.34: Expanded Results, RMSE for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.28, 0.29	0.28, 0.29	0.28, 0.29	0.29, 0.29	0.29, 0.30
	50	0.20, 0.19	0.20, 0.21	0.20, 0.19	0.19, 0.21	0.20, 0.21
	75	0.16, 0.17	0.16, 0.16	0.17, 0.17	0.16, 0.16	0.16, 0.16
	100	0.14, 0.14	0.14, 0.14	0.14, 0.14	0.14, 0.14	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.05, 0.05	0.04, 0.04	0.05, 0.05	0.04, 0.04	0.04, 0.04
4	25	0.28, 0.29	0.29, 0.28	0.28, 0.29	0.29, 0.28	0.30, 0.28
	50	0.20, 0.20	0.20, 0.20	0.20, 0.20	0.20, 0.20	0.20, 0.20
	75	0.16, 0.17	0.16, 0.17	0.17, 0.17	0.16, 0.17	0.16, 0.17
	100	0.14, 0.14	0.15, 0.15	0.14, 0.14	0.14, 0.15	0.14, 0.15
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.05, 0.04	0.04, 0.04	0.05, 0.04	0.04, 0.04	0.04, 0.04
8	25	0.28, 0.28	0.29, 0.29	0.28, 0.29	0.29, 0.29	0.29, 0.29
	50	0.19, 0.20	0.20, 0.19	0.20, 0.20	0.19, 0.19	0.19, 0.19
	75	0.17, 0.16	0.17, 0.17	0.17, 0.16	0.17, 0.17	0.17, 0.17
	100	0.15, 0.14	0.14, 0.14	0.15, 0.14	0.14, 0.14	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.05, 0.05	0.04, 0.05	0.05, 0.05	0.04, 0.04	0.04, 0.04
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.22, 0.21	0.22, 0.22	0.22, 0.22	0.22, 0.22	0.92, 0.76
	50	0.15, 0.14	0.15, 0.14	0.15, 0.14	0.15, 0.15	0.17, 0.15
	75	0.12, 0.12	0.12, 0.11	0.12, 0.12	0.12, 0.12	0.12, 0.12
	100	0.11, 0.11	0.10, 0.10	0.11, 0.11	0.11, 0.11	0.11, 0.11
	250	0.07, 0.07	0.07, 0.06	0.07, 0.07	0.07, 0.06	0.07, 0.07
	1000	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
4	25	0.13, 0.12	0.12, 0.13	0.13, 0.13	0.13, 0.13	0.16, 0.14
	50	0.08, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	75	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07
	100	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06
	250	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
8	25	0.08, 0.08	0.08, 0.08	0.08, 0.09	0.09, 0.08	0.09, 0.09
	50	0.06, 0.06	0.05, 0.06	0.06, 0.06	0.05, 0.06	0.06, 0.06
	75	0.05, 0.04	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05
	100	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.35: Expanded Results, RMSE for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.28, 0.28	0.29, 0.29	0.28, 0.28	0.29, 0.29	0.29, 0.29
	50	0.20, 0.19	0.20, 0.20	0.20, 0.19	0.20, 0.20	0.20, 0.20
	75	0.16, 0.16	0.17, 0.16	0.16, 0.16	0.17, 0.16	0.17, 0.16
	100	0.14, 0.14	0.14, 0.14	0.14, 0.14	0.14, 0.14	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05
4	25	0.28, 0.28	0.28, 0.27	0.28, 0.28	0.28, 0.27	0.28, 0.27
	50	0.20, 0.21	0.20, 0.20	0.20, 0.21	0.20, 0.21	0.20, 0.21
	75	0.17, 0.16	0.17, 0.16	0.17, 0.16	0.17, 0.16	0.16, 0.16
	100	0.14, 0.14	0.15, 0.14	0.14, 0.14	0.15, 0.14	0.15, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
8	25	0.29, 0.27	0.28, 0.28	0.29, 0.28	0.29, 0.28	0.28, 0.28
	50	0.20, 0.21	0.20, 0.20	0.20, 0.21	0.20, 0.20	0.20, 0.20
	75	0.16, 0.17	0.16, 0.16	0.16, 0.17	0.16, 0.16	0.16, 0.16
	100	0.14, 0.14	0.14, 0.15	0.13, 0.14	0.14, 0.15	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.04, 0.04	0.05, 0.04	0.04, 0.04	0.05, 0.04	0.05, 0.04
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.12, 0.11	0.12, 0.11	0.12, 0.11	0.12, 0.11	0.13, 0.12
	50	0.08, 0.08	0.08, 0.08	0.08, 0.08	0.08, 0.08	0.08, 0.08
	75	0.07, 0.07	0.07, 0.06	0.07, 0.07	0.07, 0.07	0.07, 0.07
	100	0.05, 0.06	0.06, 0.06	0.05, 0.06	0.06, 0.05	0.06, 0.06
	250	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
4	25	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07
	50	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05
	75	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	100	0.03, 0.03	0.04, 0.03	0.03, 0.03	0.04, 0.03	0.04, 0.04
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
8	25	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05
	50	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	75	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	100	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	250	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.36: Expanded Results, RMSE for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.28, 0.28	0.29, 0.28	0.29, 0.28	0.28, 0.28	0.29, 0.30
	50	0.19, 0.19	0.20, 0.20	0.19, 0.19	0.20, 0.21	0.20, 0.20
	75	0.16, 0.16	0.16, 0.17	0.16, 0.16	0.16, 0.16	0.16, 0.16
	100	0.15, 0.14	0.14, 0.13	0.15, 0.14	0.14, 0.13	0.14, 0.13
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.04, 0.04	0.05, 0.05	0.04, 0.04	0.05, 0.05	0.05, 0.05
4	25	0.29, 0.28	0.28, 0.27	0.29, 0.28	0.27, 0.27	0.27, 0.27
	50	0.20, 0.19	0.20, 0.20	0.20, 0.19	0.20, 0.20	0.20, 0.20
	75	0.16, 0.16	0.16, 0.16	0.16, 0.16	0.16, 0.16	0.16, 0.16
	100	0.14, 0.14	0.14, 0.14	0.14, 0.14	0.14, 0.14	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.05, 0.04	0.04, 0.05	0.05, 0.04	0.04, 0.05	0.04, 0.05
8	25	0.29, 0.28	0.30, 0.28	0.29, 0.28	0.30, 0.28	0.30, 0.27
	50	0.20, 0.20	0.20, 0.20	0.20, 0.20	0.20, 0.20	0.20, 0.20
	75	0.17, 0.17	0.16, 0.16	0.17, 0.17	0.16, 0.16	0.16, 0.16
	100	0.14, 0.14	0.14, 0.15	0.13, 0.14	0.14, 0.14	0.14, 0.14
	250	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
	1000	0.04, 0.04	0.05, 0.05	0.04, 0.04	0.05, 0.05	0.05, 0.05
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.22, 0.22	0.22, 0.21	0.22, 0.22	0.22, 0.21	0.88, 0.50
	50	0.15, 0.15	0.15, 0.14	0.15, 0.15	0.16, 0.14	0.18, 0.15
	75	0.12, 0.12	0.12, 0.12	0.12, 0.12	0.12, 0.12	0.13, 0.12
	100	0.11, 0.10	0.11, 0.10	0.11, 0.10	0.11, 0.10	0.11, 0.10
	250	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06
	1000	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
4	25	0.13, 0.13	0.13, 0.13	0.14, 0.13	0.13, 0.13	0.15, 0.14
	50	0.09, 0.09	0.08, 0.08	0.09, 0.09	0.08, 0.08	0.09, 0.09
	75	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07
	100	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.07, 0.06
	250	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
8	25	0.08, 0.08	0.09, 0.08	0.08, 0.08	0.09, 0.08	0.09, 0.09
	50	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06
	75	0.05, 0.04	0.05, 0.05	0.05, 0.04	0.05, 0.05	0.05, 0.05
	100	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	250	0.03, 0.03	0.03, 0.03	0.03, 0.02	0.03, 0.03	0.03, 0.03
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.37: Expanded Results, RMSE for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.09, 0.10	0.09, 0.10	0.09, 0.11	0.09, 0.11	0.10, 0.12
	50	0.06, 0.07	0.06, 0.07	0.06, 0.07	0.06, 0.07	0.07, 0.07
	75	0.05, 0.06	0.05, 0.05	0.05, 0.06	0.05, 0.06	0.05, 0.06
	100	0.04, 0.05	0.04, 0.05	0.04, 0.05	0.05, 0.05	0.05, 0.05
	250	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
4	25	0.06, 0.07	0.06, 0.07	0.06, 0.07	0.06, 0.07	0.07, 0.08
	50	0.04, 0.05	0.04, 0.05	0.04, 0.05	0.04, 0.05	0.04, 0.05
	75	0.03, 0.04	0.03, 0.04	0.03, 0.04	0.03, 0.04	0.04, 0.04
	100	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
8	25	0.04, 0.06	0.04, 0.06	0.05, 0.06	0.05, 0.06	0.05, 0.06
	50	0.03, 0.04	0.03, 0.04	0.03, 0.04	0.03, 0.04	0.03, 0.04
	75	0.02, 0.03	0.03, 0.03	0.02, 0.03	0.03, 0.03	0.03, 0.03
	100	0.02, 0.03	0.02, 0.03	0.02, 0.03	0.02, 0.03	0.02, 0.03
	250	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.01, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.09, 0.10	0.09, 0.09	0.10, 0.11	0.10, 0.10	0.10, 0.11
	50	0.06, 0.06	0.06, 0.06	0.07, 0.07	0.07, 0.07	0.07, 0.07
	75	0.05, 0.05	0.05, 0.05	0.05, 0.06	0.05, 0.06	0.05, 0.06
	100	0.04, 0.04	0.04, 0.05	0.05, 0.05	0.04, 0.05	0.05, 0.05
	250	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
4	25	0.06, 0.07	0.06, 0.06	0.06, 0.07	0.06, 0.07	0.06, 0.07
	50	0.04, 0.04	0.04, 0.04	0.04, 0.05	0.04, 0.05	0.04, 0.05
	75	0.03, 0.04	0.03, 0.04	0.03, 0.04	0.03, 0.04	0.03, 0.04
	100	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
8	25	0.04, 0.05	0.04, 0.05	0.05, 0.05	0.05, 0.06	0.05, 0.06
	50	0.03, 0.04	0.03, 0.03	0.03, 0.04	0.03, 0.04	0.03, 0.04
	75	0.02, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	100	0.02, 0.03	0.02, 0.03	0.02, 0.03	0.02, 0.03	0.02, 0.03
	250	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.01, 0.02	0.01, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.38: Expanded Results, RMSE for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.14, 0.12	0.13, 0.12	0.14, 0.12	0.14, 0.12	0.20, 0.14
	50	0.09, 0.08	0.09, 0.08	0.10, 0.08	0.09, 0.08	0.11, 0.09
	75	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.08, 0.06	0.08, 0.07
	100	0.06, 0.05	0.07, 0.05	0.06, 0.05	0.07, 0.05	0.07, 0.06
	250	0.04, 0.03	0.04, 0.04	0.04, 0.03	0.04, 0.04	0.04, 0.04
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
4	25	0.11, 0.09	0.11, 0.09	0.12, 0.09	0.12, 0.09	0.14, 0.10
	50	0.08, 0.06	0.08, 0.06	0.08, 0.06	0.08, 0.06	0.09, 0.07
	75	0.06, 0.05	0.06, 0.05	0.06, 0.05	0.06, 0.05	0.07, 0.05
	100	0.06, 0.04	0.05, 0.04	0.06, 0.04	0.05, 0.04	0.06, 0.04
	250	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03
	1000	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01
8	25	0.10, 0.07	0.10, 0.07	0.10, 0.07	0.10, 0.07	0.12, 0.08
	50	0.07, 0.05	0.07, 0.05	0.07, 0.05	0.07, 0.05	0.08, 0.05
	75	0.06, 0.04	0.06, 0.04	0.06, 0.04	0.06, 0.04	0.06, 0.04
	100	0.05, 0.04	0.05, 0.03	0.05, 0.04	0.05, 0.03	0.05, 0.04
	250	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02
	1000	0.01, 0.01	0.02, 0.01	0.01, 0.01	0.02, 0.01	0.02, 0.01
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.12, 0.12	0.12, 0.11	0.13, 0.12	0.14, 0.12	0.14, 0.14
	50	0.09, 0.08	0.09, 0.08	0.09, 0.08	0.09, 0.08	0.09, 0.09
	75	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.07	0.07, 0.07
	100	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06
	250	0.04, 0.03	0.04, 0.04	0.04, 0.03	0.04, 0.04	0.04, 0.04
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
4	25	0.11, 0.09	0.11, 0.09	0.11, 0.09	0.11, 0.09	0.12, 0.10
	50	0.07, 0.06	0.07, 0.06	0.08, 0.06	0.08, 0.06	0.08, 0.06
	75	0.06, 0.05	0.06, 0.05	0.06, 0.05	0.06, 0.05	0.06, 0.05
	100	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.06, 0.04
	250	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	1000	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01
8	25	0.09, 0.08	0.09, 0.07	0.09, 0.08	0.10, 0.07	0.10, 0.08
	50	0.06, 0.05	0.07, 0.05	0.07, 0.05	0.07, 0.05	0.07, 0.06
	75	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.05
	100	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04
	250	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.39: Expanded Results, RMSE for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.26, 0.21	0.24, 0.21	0.27, 0.22	0.26, 0.21	3.46, 2.48
	50	0.16, 0.14	0.16, 0.14	0.17, 0.14	0.17, 0.14	0.41, 0.19
	75	0.14, 0.11	0.13, 0.11	0.14, 0.12	0.14, 0.11	0.17, 0.13
	100	0.11, 0.10	0.12, 0.10	0.12, 0.10	0.12, 0.10	0.14, 0.12
	250	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.08, 0.06	0.08, 0.07
	1000	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03
4	25	0.18, 0.14	0.18, 0.15	0.20, 0.14	0.20, 0.15	0.74, 0.21
	50	0.12, 0.10	0.13, 0.10	0.13, 0.10	0.14, 0.10	0.17, 0.11
	75	0.10, 0.08	0.10, 0.08	0.11, 0.08	0.11, 0.08	0.12, 0.09
	100	0.09, 0.07	0.09, 0.07	0.09, 0.07	0.09, 0.07	0.10, 0.08
	250	0.05, 0.04	0.05, 0.04	0.06, 0.04	0.06, 0.04	0.06, 0.04
	1000	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02
8	25	0.14, 0.10	0.14, 0.10	0.14, 0.10	0.15, 0.10	0.19, 0.12
	50	0.10, 0.07	0.09, 0.07	0.10, 0.07	0.10, 0.07	0.11, 0.08
	75	0.08, 0.06	0.08, 0.06	0.08, 0.06	0.08, 0.06	0.09, 0.06
	100	0.07, 0.05	0.07, 0.05	0.07, 0.05	0.07, 0.05	0.07, 0.05
	250	0.05, 0.03	0.04, 0.03	0.05, 0.03	0.04, 0.03	0.04, 0.03
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.22, 0.21	0.24, 0.19	0.24, 0.23	0.25, 0.22	3.10, 2.60
	50	0.16, 0.15	0.16, 0.14	0.16, 0.15	0.16, 0.14	0.20, 0.22
	75	0.12, 0.12	0.13, 0.11	0.13, 0.13	0.13, 0.12	0.14, 0.13
	100	0.11, 0.10	0.11, 0.10	0.12, 0.10	0.12, 0.10	0.12, 0.11
	250	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.07
	1000	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03
4	25	0.18, 0.13	0.18, 0.14	0.18, 0.15	0.19, 0.15	0.24, 0.35
	50	0.12, 0.10	0.13, 0.10	0.12, 0.10	0.13, 0.10	0.14, 0.11
	75	0.09, 0.08	0.10, 0.08	0.10, 0.08	0.10, 0.08	0.11, 0.09
	100	0.09, 0.07	0.09, 0.07	0.09, 0.07	0.09, 0.07	0.09, 0.08
	250	0.05, 0.04	0.05, 0.04	0.06, 0.04	0.06, 0.04	0.06, 0.04
	1000	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02
8	25	0.13, 0.10	0.13, 0.10	0.14, 0.10	0.14, 0.11	0.15, 0.12
	50	0.09, 0.08	0.09, 0.07	0.10, 0.08	0.09, 0.07	0.10, 0.08
	75	0.08, 0.06	0.08, 0.06	0.08, 0.06	0.08, 0.06	0.08, 0.06
	100	0.06, 0.05	0.07, 0.05	0.07, 0.05	0.07, 0.05	0.07, 0.05
	250	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02

Note. P = parameter. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.40: Expanded Results, RMSE for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.13, 0.11	0.13, 0.11	0.14, 0.11	0.13, 0.11	0.21, 0.14
	50	0.09, 0.07	0.09, 0.07	0.09, 0.07	0.09, 0.07	0.10, 0.08
	75	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.08, 0.06
	100	0.06, 0.05	0.06, 0.05	0.06, 0.05	0.06, 0.05	0.07, 0.05
	250	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
4	25	0.11, 0.08	0.10, 0.08	0.11, 0.08	0.11, 0.08	0.13, 0.09
	50	0.07, 0.06	0.07, 0.05	0.07, 0.05	0.07, 0.05	0.08, 0.06
	75	0.06, 0.04	0.06, 0.04	0.06, 0.04	0.06, 0.04	0.07, 0.04
	100	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04
	250	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02
	1000	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01
8	25	0.09, 0.07	0.09, 0.06	0.09, 0.07	0.09, 0.06	0.11, 0.07
	50	0.06, 0.05	0.06, 0.04	0.06, 0.05	0.06, 0.04	0.07, 0.04
	75	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04
	100	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.05, 0.03
	250	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.11, 0.11	0.12, 0.11	0.13, 0.11	0.13, 0.12	0.14, 0.13
	50	0.09, 0.07	0.08, 0.08	0.09, 0.08	0.08, 0.08	0.09, 0.08
	75	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06
	100	0.06, 0.05	0.06, 0.05	0.06, 0.05	0.06, 0.05	0.06, 0.05
	250	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
4	25	0.10, 0.09	0.10, 0.08	0.11, 0.09	0.10, 0.08	0.11, 0.09
	50	0.07, 0.05	0.07, 0.06	0.07, 0.05	0.07, 0.06	0.07, 0.06
	75	0.06, 0.05	0.06, 0.04	0.06, 0.05	0.06, 0.04	0.06, 0.04
	100	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04
	250	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02
	1000	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01
8	25	0.09, 0.07	0.09, 0.07	0.09, 0.07	0.09, 0.07	0.09, 0.07
	50	0.06, 0.05	0.06, 0.04	0.06, 0.05	0.06, 0.04	0.06, 0.05
	75	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04
	100	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03
	250	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.41: Expanded Results, RMSE for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.24, 0.18	0.24, 0.20	0.25, 0.19	0.25, 0.20	3.13, 0.88
	50	0.16, 0.13	0.16, 0.13	0.17, 0.13	0.17, 0.13	0.35, 0.17
	75	0.13, 0.11	0.13, 0.11	0.14, 0.11	0.13, 0.11	0.17, 0.13
	100	0.11, 0.09	0.12, 0.09	0.11, 0.09	0.12, 0.09	0.14, 0.11
	250	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.08, 0.06
	1000	0.04, 0.03	0.03, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03
4	25	0.17, 0.12	0.18, 0.13	0.18, 0.13	0.19, 0.13	0.45, 0.17
	50	0.12, 0.09	0.12, 0.09	0.12, 0.09	0.13, 0.09	0.16, 0.10
	75	0.09, 0.07	0.09, 0.07	0.10, 0.07	0.10, 0.07	0.11, 0.08
	100	0.09, 0.06	0.08, 0.06	0.09, 0.06	0.08, 0.06	0.09, 0.06
	250	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.06, 0.04
	1000	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02	0.03, 0.02
8	25	0.14, 0.09	0.13, 0.09	0.14, 0.09	0.13, 0.09	0.17, 0.10
	50	0.09, 0.06	0.08, 0.06	0.09, 0.06	0.09, 0.06	0.10, 0.06
	75	0.07, 0.05	0.07, 0.05	0.07, 0.05	0.07, 0.05	0.08, 0.05
	100	0.06, 0.04	0.06, 0.04	0.06, 0.04	0.06, 0.04	0.07, 0.05
	250	0.04, 0.02	0.04, 0.03	0.04, 0.02	0.04, 0.03	0.04, 0.03
	1000	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.20, 0.20	0.20, 0.20	0.23, 0.21	0.23, 0.21	3.25, 2.73
	50	0.14, 0.14	0.14, 0.14	0.16, 0.14	0.16, 0.14	0.18, 0.15
	75	0.11, 0.11	0.11, 0.11	0.13, 0.11	0.12, 0.11	0.13, 0.12
	100	0.10, 0.09	0.10, 0.09	0.11, 0.09	0.11, 0.10	0.11, 0.10
	250	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06
	1000	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
4	25	0.15, 0.12	0.15, 0.13	0.17, 0.13	0.17, 0.13	0.21, 0.16
	50	0.11, 0.09	0.11, 0.09	0.12, 0.09	0.12, 0.09	0.13, 0.10
	75	0.09, 0.07	0.09, 0.07	0.10, 0.07	0.10, 0.07	0.10, 0.07
	100	0.08, 0.06	0.08, 0.06	0.08, 0.06	0.08, 0.06	0.08, 0.07
	250	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04	0.05, 0.04
	1000	0.02, 0.02	0.03, 0.02	0.02, 0.02	0.03, 0.02	0.03, 0.02
8	25	0.12, 0.09	0.12, 0.09	0.13, 0.09	0.12, 0.09	0.13, 0.10
	50	0.08, 0.06	0.08, 0.06	0.08, 0.06	0.09, 0.06	0.09, 0.06
	75	0.07, 0.05	0.07, 0.05	0.07, 0.05	0.07, 0.05	0.07, 0.05
	100	0.06, 0.04	0.06, 0.04	0.06, 0.04	0.06, 0.04	0.06, 0.04
	250	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03	0.04, 0.03
	1000	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01	0.02, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.42: Expanded Results, RMSE for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.55, 0.56	0.54, 0.58	0.56, 0.55	0.55, 0.59	0.80, 0.78
	50	0.41, 0.38	0.39, 0.39	0.40, 0.39	0.39, 0.40	0.43, 0.47
	75	0.31, 0.32	0.32, 0.32	0.31, 0.32	0.32, 0.32	0.36, 0.36
	100	0.28, 0.28	0.27, 0.28	0.28, 0.28	0.27, 0.28	0.30, 0.31
	250	0.17, 0.18	0.18, 0.18	0.17, 0.18	0.18, 0.18	0.18, 0.18
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
4	25	0.54, 0.54	0.57, 0.52	0.55, 0.54	0.55, 0.53	0.74, 0.71
	50	0.39, 0.40	0.41, 0.41	0.39, 0.40	0.42, 0.41	0.48, 0.47
	75	0.32, 0.33	0.31, 0.33	0.32, 0.33	0.30, 0.33	0.34, 0.37
	100	0.28, 0.28	0.28, 0.29	0.29, 0.28	0.28, 0.30	0.30, 0.32
	250	0.17, 0.18	0.17, 0.18	0.17, 0.18	0.18, 0.18	0.18, 0.18
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
8	25	0.54, 0.54	0.55, 0.59	0.55, 0.55	0.54, 0.58	0.74, 0.79
	50	0.38, 0.40	0.39, 0.40	0.39, 0.40	0.39, 0.40	0.45, 0.46
	75	0.32, 0.31	0.31, 0.34	0.32, 0.31	0.32, 0.34	0.35, 0.37
	100	0.27, 0.28	0.28, 0.29	0.27, 0.28	0.29, 0.29	0.31, 0.32
	250	0.18, 0.18	0.17, 0.18	0.18, 0.18	0.17, 0.18	0.18, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.09, 0.09	0.08, 0.09	0.09, 0.09	0.09, 0.09	0.15, 0.15
	50	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.08, 0.08
	75	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.06, 0.06
	100	0.05, 0.05	0.04, 0.05	0.04, 0.04	0.04, 0.05	0.05, 0.05
	250	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
4	25	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.06, 0.06
	50	0.03, 0.04	0.04, 0.04	0.03, 0.04	0.04, 0.04	0.04, 0.04
	75	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	100	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
8	25	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.04, 0.04
	50	0.02, 0.03	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.03, 0.03
	75	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	100	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	250	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.43: Expanded Results, RMSE for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.57, 0.53	0.54, 0.55	0.57, 0.55	0.54, 0.55	0.89, 0.84
	50	0.40, 0.41	0.40, 0.38	0.40, 0.40	0.40, 0.38	0.47, 0.46
	75	0.33, 0.32	0.32, 0.30	0.32, 0.33	0.32, 0.32	0.35, 0.36
	100	0.28, 0.27	0.27, 0.28	0.28, 0.28	0.28, 0.28	0.30, 0.30
	250	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.19, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
4	25	0.56, 0.54	0.55, 0.56	0.56, 0.56	0.54, 0.55	0.75, 0.73
	50	0.39, 0.39	0.38, 0.41	0.40, 0.40	0.39, 0.41	0.46, 0.48
	75	0.33, 0.33	0.32, 0.33	0.33, 0.33	0.32, 0.33	0.36, 0.37
	100	0.27, 0.28	0.28, 0.28	0.27, 0.28	0.29, 0.28	0.31, 0.30
	250	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.19, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
8	25	0.55, 0.53	0.53, 0.55	0.56, 0.54	0.54, 0.55	0.76, 0.75
	50	0.40, 0.40	0.39, 0.41	0.40, 0.39	0.39, 0.41	0.47, 0.48
	75	0.32, 0.33	0.33, 0.31	0.33, 0.32	0.33, 0.31	0.37, 0.34
	100	0.28, 0.29	0.28, 0.29	0.28, 0.29	0.28, 0.28	0.31, 0.31
	250	0.19, 0.18	0.17, 0.19	0.19, 0.18	0.17, 0.19	0.18, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.30, 0.32	0.32, 0.31	0.31, 0.33	0.32, 0.32	5.16, 6.44
	50	0.22, 0.23	0.22, 0.23	0.22, 0.22	0.22, 0.23	0.33, 0.34
	75	0.18, 0.18	0.18, 0.19	0.18, 0.18	0.18, 0.19	0.25, 0.25
	100	0.16, 0.16	0.16, 0.16	0.16, 0.16	0.16, 0.16	0.19, 0.20
	250	0.10, 0.10	0.10, 0.10	0.10, 0.10	0.10, 0.10	0.11, 0.11
	1000	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05
4	25	0.18, 0.18	0.17, 0.19	0.18, 0.19	0.18, 0.20	0.24, 0.28
	50	0.13, 0.13	0.13, 0.13	0.13, 0.13	0.13, 0.13	0.15, 0.16
	75	0.10, 0.11	0.11, 0.10	0.10, 0.11	0.11, 0.10	0.12, 0.12
	100	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.10, 0.10
	250	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06
	1000	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
8	25	0.12, 0.12	0.12, 0.13	0.12, 0.12	0.12, 0.13	0.13, 0.15
	50	0.08, 0.09	0.08, 0.09	0.08, 0.09	0.08, 0.09	0.09, 0.10
	75	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.07	0.07, 0.08
	100	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06
	250	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.44: Expanded Results, RMSE for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.56, 0.56	0.56, 0.53	0.57, 0.57	0.56, 0.54	0.80, 0.76
	50	0.40, 0.40	0.39, 0.41	0.40, 0.40	0.39, 0.41	0.46, 0.48
	75	0.33, 0.31	0.32, 0.33	0.32, 0.31	0.32, 0.33	0.36, 0.36
	100	0.28, 0.29	0.28, 0.29	0.28, 0.29	0.28, 0.29	0.30, 0.32
	250	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
4	25	0.53, 0.53	0.56, 0.56	0.53, 0.54	0.56, 0.56	0.76, 0.75
	50	0.39, 0.41	0.39, 0.40	0.39, 0.41	0.40, 0.40	0.46, 0.47
	75	0.33, 0.32	0.31, 0.33	0.33, 0.31	0.31, 0.33	0.35, 0.37
	100	0.29, 0.29	0.26, 0.28	0.29, 0.29	0.27, 0.28	0.28, 0.30
	250	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
8	25	0.55, 0.56	0.56, 0.56	0.55, 0.56	0.56, 0.56	0.75, 0.78
	50	0.39, 0.38	0.40, 0.41	0.40, 0.38	0.39, 0.41	0.47, 0.46
	75	0.32, 0.33	0.33, 0.33	0.33, 0.33	0.33, 0.33	0.37, 0.37
	100	0.28, 0.29	0.29, 0.28	0.28, 0.29	0.28, 0.28	0.30, 0.30
	250	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.18
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.19, 0.15
	50	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.08, 0.08
	75	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.06, 0.06
	100	0.04, 0.04	0.05, 0.04	0.04, 0.05	0.04, 0.04	0.05, 0.05
	250	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
4	25	0.06, 0.05	0.05, 0.05	0.06, 0.05	0.05, 0.05	0.07, 0.07
	50	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	75	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	100	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
8	25	0.04, 0.04	0.04, 0.03	0.04, 0.04	0.04, 0.03	0.04, 0.04
	50	0.03, 0.02	0.02, 0.02	0.03, 0.02	0.02, 0.02	0.03, 0.03
	75	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	100	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	250	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.45: Expanded Results, RMSE for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.53, 0.53	0.58, 0.55	0.54, 0.53	0.57, 0.55	0.74, 0.96
	50	0.38, 0.40	0.39, 0.39	0.38, 0.41	0.39, 0.39	0.45, 0.46
	75	0.33, 0.32	0.32, 0.33	0.33, 0.32	0.32, 0.33	0.36, 0.36
	100	0.27, 0.28	0.27, 0.27	0.27, 0.28	0.27, 0.27	0.29, 0.30
	250	0.17, 0.18	0.18, 0.18	0.17, 0.18	0.18, 0.17	0.19, 0.18
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
4	25	0.55, 0.55	0.55, 0.55	0.56, 0.55	0.56, 0.56	0.81, 0.77
	50	0.38, 0.42	0.41, 0.39	0.38, 0.42	0.41, 0.39	0.49, 0.46
	75	0.33, 0.33	0.32, 0.32	0.33, 0.33	0.33, 0.32	0.36, 0.35
	100	0.28, 0.29	0.27, 0.28	0.28, 0.29	0.27, 0.28	0.30, 0.30
	250	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.19, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
8	25	0.54, 0.54	0.54, 0.58	0.54, 0.55	0.54, 0.58	0.75, 0.78
	50	0.40, 0.41	0.39, 0.40	0.40, 0.40	0.40, 0.40	0.46, 0.46
	75	0.32, 0.32	0.33, 0.33	0.32, 0.32	0.33, 0.33	0.37, 0.37
	100	0.28, 0.29	0.29, 0.27	0.28, 0.29	0.29, 0.27	0.31, 0.29
	250	0.18, 0.18	0.18, 0.17	0.18, 0.18	0.17, 0.18	0.18, 0.18
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.33, 0.33	0.31, 0.33	0.34, 0.34	0.32, 0.33	5.30, 4.06
	50	0.23, 0.23	0.24, 0.23	0.23, 0.23	0.24, 0.23	0.63, 0.36
	75	0.19, 0.19	0.19, 0.18	0.19, 0.19	0.19, 0.18	0.26, 0.24
	100	0.16, 0.16	0.17, 0.16	0.16, 0.16	0.17, 0.16	0.22, 0.20
	250	0.10, 0.10	0.10, 0.10	0.11, 0.10	0.10, 0.10	0.11, 0.11
	1000	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05
4	25	0.20, 0.19	0.20, 0.19	0.21, 0.19	0.21, 0.19	0.79, 0.30
	50	0.14, 0.13	0.14, 0.14	0.14, 0.13	0.14, 0.14	0.19, 0.16
	75	0.12, 0.11	0.12, 0.11	0.13, 0.11	0.12, 0.11	0.14, 0.12
	100	0.10, 0.10	0.10, 0.09	0.10, 0.09	0.10, 0.09	0.11, 0.10
	250	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06	0.06, 0.06
	1000	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
8	25	0.14, 0.12	0.13, 0.13	0.14, 0.13	0.14, 0.13	0.19, 0.16
	50	0.09, 0.09	0.10, 0.09	0.10, 0.09	0.10, 0.09	0.12, 0.10
	75	0.08, 0.07	0.08, 0.08	0.08, 0.07	0.08, 0.07	0.09, 0.08
	100	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.06	0.07, 0.07
	250	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	1000	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02

Note. P = parameter. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.46: Expanded Results, RMSE for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.54, 0.55	0.56, 0.53	0.53, 0.55	0.56, 0.53	0.79, 0.75
	50	0.41, 0.39	0.41, 0.40	0.41, 0.39	0.40, 0.39	0.48, 0.45
	75	0.32, 0.32	0.33, 0.32	0.32, 0.32	0.33, 0.33	0.35, 0.36
	100	0.28, 0.29	0.28, 0.28	0.28, 0.29	0.28, 0.27	0.30, 0.30
	250	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.19, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
4	25	0.56, 0.56	0.54, 0.58	0.56, 0.56	0.55, 0.57	0.77, 0.78
	50	0.40, 0.39	0.39, 0.40	0.40, 0.39	0.40, 0.40	0.47, 0.47
	75	0.32, 0.32	0.32, 0.33	0.32, 0.32	0.32, 0.33	0.36, 0.36
	100	0.28, 0.28	0.28, 0.28	0.28, 0.28	0.28, 0.27	0.30, 0.30
	250	0.17, 0.18	0.18, 0.18	0.17, 0.18	0.17, 0.18	0.18, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
8	25	0.54, 0.56	0.54, 0.57	0.55, 0.56	0.55, 0.57	0.73, 0.75
	50	0.40, 0.39	0.39, 0.40	0.40, 0.39	0.39, 0.41	0.47, 0.48
	75	0.31, 0.33	0.31, 0.32	0.31, 0.33	0.31, 0.32	0.35, 0.36
	100	0.29, 0.29	0.28, 0.29	0.29, 0.29	0.28, 0.30	0.30, 0.32
	250	0.18, 0.17	0.18, 0.18	0.18, 0.17	0.18, 0.18	0.19, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.09, 0.09	0.10, 0.09	0.09, 0.09	0.09, 0.09	0.20, 0.16
	50	0.06, 0.06	0.06, 0.07	0.06, 0.06	0.06, 0.06	0.08, 0.08
	75	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.05, 0.05	0.06, 0.06
	100	0.05, 0.04	0.05, 0.05	0.05, 0.04	0.05, 0.05	0.05, 0.05
	250	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
4	25	0.05, 0.05	0.05, 0.06	0.05, 0.05	0.05, 0.06	0.07, 0.07
	50	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04
	75	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	100	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03	0.03, 0.03
	250	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
8	25	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.04, 0.04	0.05, 0.04
	50	0.03, 0.03	0.03, 0.02	0.03, 0.03	0.03, 0.02	0.03, 0.03
	75	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	100	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02	0.02, 0.02
	250	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01
	1000	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01	0.01, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.47: RMSE for $\phi_{0,11}$ and $\phi_{0,22}$, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.54, 0.53	0.54, 0.54	0.55, 0.54	0.54, 0.55	0.83, 0.97
	50	0.38, 0.40	0.39, 0.39	0.38, 0.39	0.39, 0.39	0.48, 0.46
	75	0.32, 0.32	0.32, 0.33	0.32, 0.32	0.32, 0.33	0.36, 0.37
	100	0.29, 0.28	0.28, 0.28	0.29, 0.28	0.28, 0.29	0.30, 0.31
	250	0.18, 0.18	0.18, 0.17	0.18, 0.18	0.18, 0.17	0.18, 0.18
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
4	25	0.55, 0.54	0.56, 0.57	0.56, 0.55	0.55, 0.56	0.83, 0.79
	50	0.39, 0.40	0.40, 0.40	0.39, 0.40	0.40, 0.40	0.46, 0.47
	75	0.33, 0.33	0.32, 0.32	0.33, 0.33	0.32, 0.32	0.36, 0.35
	100	0.28, 0.28	0.29, 0.27	0.28, 0.28	0.29, 0.27	0.31, 0.30
	250	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.18	0.18, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09
8	25	0.54, 0.55	0.57, 0.56	0.54, 0.55	0.57, 0.56	0.79, 0.79
	50	0.41, 0.41	0.38, 0.40	0.41, 0.41	0.38, 0.40	0.44, 0.46
	75	0.33, 0.31	0.33, 0.33	0.33, 0.31	0.33, 0.33	0.36, 0.37
	100	0.29, 0.27	0.29, 0.28	0.29, 0.27	0.29, 0.28	0.31, 0.30
	250	0.18, 0.17	0.18, 0.18	0.18, 0.17	0.18, 0.18	0.19, 0.19
	1000	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09	0.09, 0.09

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.48: Expanded Results, RMSE for $\phi_{0,21}$ and q_{21}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	0.41, 0.06	0.40, 0.06	0.42, 0.06	0.39, 0.06	0.45, 0.09
	50	0.29, 0.04	0.28, 0.05	0.29, 0.05	0.29, 0.05	0.30, 0.05
	75	0.23, 0.04	0.23, 0.04	0.23, 0.04	0.23, 0.04	0.24, 0.04
	100	0.21, 0.03	0.20, 0.03	0.20, 0.03	0.20, 0.03	0.21, 0.03
	250	0.13, 0.02	0.13, 0.02	0.13, 0.02	0.13, 0.02	0.13, 0.02
	1000	0.07, 0.01	0.07, 0.01	0.06, 0.01	0.07, 0.01	0.07, 0.01
4	25	0.37, 0.04	0.39, 0.04	0.39, 0.04	0.40, 0.04	0.45, 0.04
	50	0.29, 0.03	0.30, 0.03	0.29, 0.03	0.29, 0.03	0.31, 0.03
	75	0.23, 0.02	0.23, 0.02	0.23, 0.02	0.23, 0.02	0.24, 0.02
	100	0.21, 0.02	0.21, 0.02	0.21, 0.02	0.21, 0.02	0.21, 0.02
	250	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.13, 0.01
	1000	0.07, 0.01	0.07, 0.01	0.07, 0.01	0.07, 0.01	0.07, 0.01
8	25	0.41, 0.02	0.40, 0.03	0.41, 0.03	0.40, 0.03	0.45, 0.03
	50	0.29, 0.02	0.28, 0.02	0.29, 0.02	0.29, 0.02	0.30, 0.02
	75	0.23, 0.01	0.24, 0.01	0.23, 0.01	0.23, 0.01	0.24, 0.01
	100	0.21, 0.01	0.20, 0.01	0.21, 0.01	0.21, 0.01	0.21, 0.01
	250	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.14, 0.01
	1000	0.07, 0.00	0.06, 0.00	0.07, 0.00	0.06, 0.00	0.06, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.49: Expanded Results, RMSE for $\phi_{0,21}$ and q_{21}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	0.40, 0.24	0.42, 0.24	0.41, 0.24	0.41, 0.23	0.50, 2.63
	50	0.28, 0.16	0.28, 0.16	0.29, 0.16	0.29, 0.16	0.30, 0.21
	75	0.23, 0.13	0.24, 0.14	0.24, 0.13	0.24, 0.14	0.25, 0.16
	100	0.20, 0.12	0.21, 0.12	0.20, 0.12	0.20, 0.12	0.21, 0.13
	250	0.13, 0.07	0.13, 0.07	0.13, 0.07	0.13, 0.07	0.13, 0.08
	1000	0.07, 0.04	0.06, 0.04	0.07, 0.04	0.07, 0.04	0.07, 0.04
4	25	0.41, 0.14	0.39, 0.15	0.41, 0.14	0.38, 0.15	0.44, 0.17
	50	0.29, 0.10	0.27, 0.10	0.29, 0.10	0.28, 0.10	0.30, 0.10
	75	0.23, 0.08	0.25, 0.08	0.24, 0.08	0.25, 0.08	0.26, 0.08
	100	0.21, 0.07	0.21, 0.07	0.21, 0.07	0.21, 0.07	0.21, 0.07
	250	0.14, 0.04	0.14, 0.04	0.14, 0.04	0.14, 0.04	0.14, 0.04
	1000	0.07, 0.02	0.07, 0.02	0.07, 0.02	0.07, 0.02	0.07, 0.02
8	25	0.42, 0.09	0.40, 0.09	0.42, 0.09	0.40, 0.09	0.46, 0.10
	50	0.30, 0.06	0.31, 0.06	0.30, 0.06	0.31, 0.06	0.32, 0.06
	75	0.25, 0.05	0.24, 0.05	0.25, 0.05	0.24, 0.05	0.25, 0.05
	100	0.21, 0.04	0.21, 0.04	0.21, 0.04	0.21, 0.04	0.22, 0.04
	250	0.13, 0.03	0.13, 0.03	0.13, 0.03	0.13, 0.03	0.13, 0.03
	1000	0.07, 0.01	0.07, 0.01	0.07, 0.01	0.07, 0.01	0.07, 0.01

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.50: Expanded Results, RMSE for $\phi_{0,21}$ and q_{21}, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	0.41, 0.06	0.40, 0.06	0.41, 0.06	0.41, 0.06	0.45, 0.09
	50	0.29, 0.05	0.29, 0.04	0.29, 0.05	0.30, 0.04	0.31, 0.05
	75	0.23, 0.04	0.23, 0.04	0.23, 0.04	0.23, 0.04	0.24, 0.04
	100	0.20, 0.03	0.21, 0.03	0.20, 0.03	0.21, 0.03	0.21, 0.03
	250	0.13, 0.02	0.13, 0.02	0.13, 0.02	0.13, 0.02	0.13, 0.02
	1000	0.07, 0.01	0.07, 0.01	0.07, 0.01	0.07, 0.01	0.07, 0.01
4	25	0.38, 0.04	0.40, 0.04	0.38, 0.04	0.40, 0.04	0.45, 0.04
	50	0.28, 0.03	0.28, 0.03	0.29, 0.03	0.28, 0.03	0.30, 0.03
	75	0.24, 0.02	0.24, 0.02	0.24, 0.02	0.24, 0.02	0.25, 0.02
	100	0.20, 0.02	0.20, 0.02	0.20, 0.02	0.20, 0.02	0.20, 0.02
	250	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.13, 0.01
	1000	0.07, 0.01	0.07, 0.01	0.07, 0.01	0.07, 0.01	0.07, 0.01
8	25	0.42, 0.03	0.42, 0.03	0.42, 0.03	0.42, 0.03	0.47, 0.03
	50	0.29, 0.02	0.30, 0.02	0.29, 0.02	0.30, 0.02	0.32, 0.02
	75	0.24, 0.02	0.24, 0.02	0.24, 0.02	0.24, 0.02	0.25, 0.02
	100	0.21, 0.01	0.21, 0.01	0.21, 0.01	0.20, 0.01	0.21, 0.01
	250	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.13, 0.01
	1000	0.06, 0.00	0.07, 0.00	0.06, 0.00	0.07, 0.00	0.07, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.51: Expanded Results, RMSE for $\phi_{0,21}$ and q_{21}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	0.38, 0.22	0.42, 0.23	0.38, 0.23	0.41, 0.23	0.51, 2.45
	50	0.28, 0.16	0.29, 0.17	0.28, 0.17	0.29, 0.17	0.29, 0.25
	75	0.24, 0.14	0.24, 0.13	0.24, 0.14	0.24, 0.13	0.25, 0.16
	100	0.21, 0.12	0.21, 0.12	0.21, 0.12	0.21, 0.12	0.22, 0.13
	250	0.13, 0.08	0.13, 0.08	0.13, 0.08	0.13, 0.08	0.13, 0.08
	1000	0.07, 0.04	0.07, 0.04	0.07, 0.04	0.07, 0.04	0.07, 0.04
4	25	0.41, 0.15	0.39, 0.15	0.40, 0.15	0.40, 0.15	0.45, 0.24
	50	0.30, 0.10	0.29, 0.10	0.29, 0.10	0.29, 0.10	0.31, 0.11
	75	0.23, 0.09	0.23, 0.08	0.23, 0.09	0.23, 0.08	0.24, 0.09
	100	0.21, 0.07	0.20, 0.07	0.21, 0.07	0.20, 0.07	0.21, 0.08
	250	0.13, 0.04	0.13, 0.04	0.13, 0.04	0.13, 0.04	0.13, 0.04
	1000	0.07, 0.02	0.07, 0.02	0.07, 0.02	0.07, 0.02	0.07, 0.02
8	25	0.41, 0.10	0.40, 0.10	0.41, 0.10	0.41, 0.10	0.47, 0.11
	50	0.30, 0.07	0.30, 0.07	0.30, 0.07	0.30, 0.07	0.32, 0.07
	75	0.23, 0.06	0.24, 0.06	0.23, 0.06	0.24, 0.06	0.25, 0.06
	100	0.21, 0.05	0.20, 0.05	0.21, 0.05	0.20, 0.05	0.21, 0.05
	250	0.13, 0.03	0.13, 0.03	0.13, 0.03	0.13, 0.03	0.13, 0.03
	1000	0.06, 0.02	0.07, 0.02	0.06, 0.02	0.07, 0.02	0.07, 0.02

Note. P = parameter. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.52: Expanded Results, RMSE for $\phi_{0,21}$ and q_{21}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	0.39, 0.06	0.42, 0.07	0.39, 0.06	0.41, 0.07	0.47, 0.10
	50	0.29, 0.04	0.29, 0.05	0.29, 0.04	0.29, 0.04	0.31, 0.05
	75	0.23, 0.04	0.25, 0.04	0.23, 0.04	0.24, 0.04	0.25, 0.04
	100	0.21, 0.03	0.21, 0.03	0.21, 0.03	0.21, 0.03	0.21, 0.04
	250	0.13, 0.02	0.13, 0.02	0.13, 0.02	0.13, 0.02	0.13, 0.02
	1000	0.06, 0.01	0.07, 0.01	0.06, 0.01	0.07, 0.01	0.07, 0.01
4	25	0.41, 0.04	0.40, 0.04	0.41, 0.04	0.40, 0.04	0.46, 0.05
	50	0.29, 0.03	0.29, 0.03	0.29, 0.03	0.30, 0.03	0.31, 0.03
	75	0.24, 0.02	0.24, 0.02	0.24, 0.02	0.24, 0.02	0.25, 0.02
	100	0.21, 0.02	0.21, 0.02	0.21, 0.02	0.21, 0.02	0.21, 0.02
	250	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.13, 0.01
	1000	0.07, 0.01	0.07, 0.01	0.07, 0.01	0.07, 0.01	0.07, 0.01
8	25	0.39, 0.03	0.42, 0.03	0.39, 0.03	0.41, 0.03	0.46, 0.03
	50	0.30, 0.02	0.28, 0.02	0.30, 0.02	0.28, 0.02	0.30, 0.02
	75	0.22, 0.02	0.25, 0.02	0.22, 0.02	0.24, 0.02	0.26, 0.02
	100	0.21, 0.01	0.19, 0.01	0.21, 0.01	0.20, 0.01	0.20, 0.01
	250	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.13, 0.01	0.14, 0.01
	1000	0.06, 0.00	0.07, 0.00	0.06, 0.00	0.07, 0.00	0.07, 0.00

Note. P = parameter. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.53: Expanded Results, RMSE for $\phi_{0,21}$ and q_{21}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	0.40, 0.23	0.42, 0.24	0.41, 0.24	0.42, 0.24	0.47, 1.92
	50	0.29, 0.17	0.29, 0.16	0.29, 0.17	0.30, 0.17	0.32, 0.23
	75	0.24, 0.14	0.24, 0.14	0.24, 0.14	0.24, 0.14	0.25, 0.17
	100	0.21, 0.12	0.20, 0.11	0.21, 0.12	0.20, 0.12	0.21, 0.14
	250	0.13, 0.07	0.13, 0.08	0.13, 0.08	0.13, 0.08	0.13, 0.08
	1000	0.06, 0.04	0.07, 0.04	0.06, 0.04	0.07, 0.04	0.07, 0.04
4	25	0.41, 0.15	0.40, 0.15	0.41, 0.15	0.40, 0.16	0.47, 0.26
	50	0.29, 0.11	0.28, 0.11	0.29, 0.11	0.28, 0.11	0.30, 0.13
	75	0.25, 0.09	0.24, 0.09	0.25, 0.09	0.24, 0.09	0.25, 0.10
	100	0.20, 0.08	0.20, 0.08	0.20, 0.08	0.20, 0.08	0.21, 0.08
	250	0.13, 0.05	0.13, 0.05	0.13, 0.05	0.13, 0.05	0.14, 0.05
	1000	0.07, 0.02	0.07, 0.02	0.07, 0.02	0.07, 0.02	0.07, 0.02
8	25	0.42, 0.11	0.41, 0.11	0.43, 0.11	0.41, 0.11	0.47, 0.13
	50	0.30, 0.07	0.29, 0.08	0.30, 0.07	0.29, 0.08	0.30, 0.08
	75	0.23, 0.06	0.24, 0.06	0.23, 0.06	0.24, 0.06	0.25, 0.06
	100	0.21, 0.05	0.21, 0.05	0.21, 0.05	0.21, 0.05	0.22, 0.05
	250	0.13, 0.03	0.13, 0.03	0.13, 0.03	0.13, 0.03	0.13, 0.03
	1000	0.07, 0.02	0.07, 0.02	0.07, 0.02	0.07, 0.02	0.07, 0.02

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.54: Expanded Results, Coverage for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.95, 0.95	0.93, 0.93	0.95, 0.94	0.94, 0.94	0.95, 0.95
	50	0.93, 0.96	0.96, 0.95	0.94, 0.96	0.96, 0.95	0.97, 0.95
	75	0.93, 0.93	0.95, 0.94	0.94, 0.94	0.94, 0.94	0.95, 0.95
	100	0.94, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.96, 0.96
	250	0.95, 0.94	0.95, 0.95	0.96, 0.94	0.95, 0.95	0.95, 0.95
	1000	0.95, 0.94	0.96, 0.95	0.95, 0.94	0.96, 0.96	0.95, 0.95
4	25	0.94, 0.93	0.94, 0.95	0.94, 0.93	0.95, 0.96	0.96, 0.96
	50	0.96, 0.95	0.96, 0.96	0.96, 0.96	0.95, 0.95	0.96, 0.96
	75	0.95, 0.95	0.96, 0.96	0.95, 0.95	0.96, 0.96	0.96, 0.96
	100	0.95, 0.96	0.94, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.95
	250	0.95, 0.96	0.94, 0.95	0.95, 0.96	0.94, 0.96	0.95, 0.96
	1000	0.95, 0.96	0.95, 0.96	0.95, 0.96	0.95, 0.96	0.95, 0.96
8	25	0.95, 0.94	0.93, 0.95	0.95, 0.95	0.93, 0.95	0.95, 0.97
	50	0.95, 0.94	0.95, 0.94	0.95, 0.95	0.95, 0.95	0.96, 0.95
	75	0.94, 0.95	0.96, 0.96	0.95, 0.95	0.96, 0.95	0.96, 0.96
	100	0.94, 0.94	0.94, 0.95	0.94, 0.94	0.94, 0.96	0.94, 0.96
	250	0.96, 0.95	0.96, 0.94	0.96, 0.95	0.96, 0.95	0.96, 0.95
	1000	0.94, 0.95	0.94, 0.94	0.94, 0.94	0.94, 0.94	0.94, 0.95
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.94, 0.94	0.93, 0.91	0.93, 0.93	0.93, 0.91	0.96, 0.96
	50	0.95, 0.93	0.95, 0.93	0.95, 0.93	0.95, 0.94	0.96, 0.96
	75	0.96, 0.94	0.94, 0.93	0.96, 0.95	0.94, 0.94	0.95, 0.95
	100	0.95, 0.96	0.94, 0.93	0.95, 0.96	0.95, 0.94	0.95, 0.94
	250	0.95, 0.95	0.96, 0.94	0.95, 0.95	0.96, 0.95	0.96, 0.95
	1000	0.95, 0.96	0.97, 0.94	0.95, 0.96	0.97, 0.94	0.97, 0.94
4	25	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.96, 0.96
	50	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95
	75	0.96, 0.95	0.93, 0.94	0.95, 0.96	0.94, 0.95	0.94, 0.95
	100	0.95, 0.95	0.96, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.96
	250	0.93, 0.94	0.95, 0.95	0.94, 0.94	0.95, 0.95	0.95, 0.95
	1000	0.95, 0.95	0.96, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.96
8	25	0.94, 0.93	0.94, 0.93	0.94, 0.94	0.94, 0.93	0.94, 0.93
	50	0.95, 0.94	0.95, 0.94	0.95, 0.94	0.95, 0.94	0.95, 0.94
	75	0.96, 0.95	0.95, 0.94	0.96, 0.95	0.95, 0.94	0.95, 0.94
	100	0.95, 0.95	0.96, 0.94	0.95, 0.95	0.96, 0.95	0.96, 0.94
	250	0.95, 0.94	0.95, 0.96	0.95, 0.94	0.95, 0.96	0.95, 0.95
	1000	0.94, 0.94	0.96, 0.94	0.94, 0.94	0.96, 0.94	0.96, 0.94

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.55: Expanded Results, Coverage for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.95, 0.93	0.95, 0.94	0.96, 0.94	0.96, 0.94	0.97, 0.96
	50	0.95, 0.95	0.96, 0.96	0.96, 0.95	0.96, 0.95	0.96, 0.95
	75	0.95, 0.93	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.95
	100	0.94, 0.94	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.94, 0.94
	250	0.95, 0.94	0.95, 0.94	0.96, 0.94	0.95, 0.93	0.95, 0.93
	1000	0.96, 0.96	0.94, 0.95	0.96, 0.96	0.95, 0.95	0.95, 0.95
4	25	0.94, 0.94	0.96, 0.95	0.95, 0.94	0.95, 0.95	0.97, 0.97
	50	0.95, 0.93	0.95, 0.95	0.94, 0.94	0.96, 0.95	0.96, 0.96
	75	0.94, 0.94	0.93, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.96
	100	0.95, 0.95	0.94, 0.94	0.95, 0.95	0.94, 0.94	0.95, 0.94
	250	0.94, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.95
	1000	0.94, 0.95	0.95, 0.96	0.94, 0.95	0.95, 0.96	0.95, 0.96
8	25	0.94, 0.93	0.94, 0.95	0.95, 0.94	0.95, 0.94	0.96, 0.96
	50	0.93, 0.94	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.96
	75	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.96	0.96, 0.96
	100	0.95, 0.93	0.96, 0.94	0.95, 0.94	0.96, 0.94	0.96, 0.95
	250	0.95, 0.95	0.94, 0.97	0.95, 0.95	0.95, 0.96	0.95, 0.97
	1000	0.94, 0.95	0.96, 0.93	0.93, 0.95	0.95, 0.93	0.96, 0.94
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.91, 0.93	0.91, 0.93	0.91, 0.94	0.93, 0.94	0.96, 0.95
	50	0.93, 0.95	0.94, 0.93	0.95, 0.95	0.94, 0.93	0.95, 0.95
	75	0.94, 0.94	0.94, 0.94	0.94, 0.95	0.95, 0.95	0.95, 0.96
	100	0.95, 0.94	0.93, 0.94	0.96, 0.94	0.94, 0.95	0.94, 0.95
	250	0.96, 0.94	0.95, 0.94	0.96, 0.94	0.95, 0.94	0.95, 0.94
	1000	0.94, 0.96	0.96, 0.95	0.94, 0.96	0.96, 0.96	0.96, 0.95
4	25	0.94, 0.94	0.95, 0.94	0.94, 0.94	0.95, 0.95	0.95, 0.95
	50	0.96, 0.92	0.95, 0.94	0.95, 0.93	0.95, 0.95	0.95, 0.95
	75	0.94, 0.93	0.92, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.95
	100	0.95, 0.95	0.93, 0.93	0.95, 0.94	0.93, 0.94	0.94, 0.94
	250	0.96, 0.95	0.94, 0.94	0.96, 0.95	0.94, 0.94	0.94, 0.94
	1000	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.96
8	25	0.93, 0.94	0.95, 0.93	0.93, 0.94	0.95, 0.94	0.95, 0.95
	50	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.96	0.95, 0.96
	75	0.95, 0.93	0.96, 0.94	0.95, 0.94	0.96, 0.94	0.96, 0.94
	100	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.93, 0.96
	250	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.95
	1000	0.93, 0.95	0.94, 0.95	0.94, 0.94	0.93, 0.94	0.94, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.56: Expanded Results, Coverage for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.93, 0.94	0.93, 0.94	0.94, 0.94	0.93, 0.95	0.95, 0.97
	50	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.96, 0.96
	75	0.94, 0.94	0.95, 0.94	0.94, 0.94	0.95, 0.95	0.95, 0.95
	100	0.94, 0.95	0.93, 0.95	0.94, 0.95	0.94, 0.96	0.94, 0.96
	250	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.94, 0.94	0.94, 0.94
	1000	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.95
4	25	0.95, 0.93	0.94, 0.94	0.95, 0.94	0.95, 0.95	0.96, 0.97
	50	0.94, 0.94	0.94, 0.94	0.94, 0.94	0.94, 0.94	0.95, 0.95
	75	0.95, 0.94	0.95, 0.96	0.95, 0.94	0.94, 0.96	0.95, 0.96
	100	0.93, 0.95	0.95, 0.95	0.93, 0.95	0.95, 0.95	0.96, 0.96
	250	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.94	0.95, 0.94
	1000	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.94	0.95, 0.95
8	25	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.94, 0.96	0.95, 0.97
	50	0.95, 0.95	0.94, 0.93	0.95, 0.95	0.94, 0.93	0.95, 0.94
	75	0.94, 0.96	0.94, 0.96	0.94, 0.96	0.94, 0.96	0.94, 0.96
	100	0.94, 0.95	0.96, 0.95	0.94, 0.95	0.96, 0.94	0.96, 0.95
	250	0.96, 0.95	0.95, 0.96	0.96, 0.95	0.95, 0.96	0.95, 0.97
	1000	0.95, 0.94	0.96, 0.94	0.96, 0.94	0.95, 0.94	0.95, 0.94
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.92, 0.93	0.95, 0.92	0.92, 0.93	0.95, 0.92	0.97, 0.95
	50	0.94, 0.95	0.93, 0.94	0.94, 0.95	0.93, 0.95	0.94, 0.95
	75	0.94, 0.94	0.96, 0.94	0.94, 0.94	0.96, 0.94	0.96, 0.94
	100	0.95, 0.95	0.94, 0.94	0.95, 0.95	0.94, 0.94	0.95, 0.95
	250	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.96
	1000	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.95
4	25	0.94, 0.94	0.95, 0.94	0.95, 0.95	0.95, 0.95	0.95, 0.95
	50	0.96, 0.94	0.95, 0.94	0.96, 0.94	0.95, 0.94	0.96, 0.95
	75	0.95, 0.93	0.93, 0.95	0.95, 0.93	0.93, 0.95	0.94, 0.95
	100	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.93, 0.95
	250	0.93, 0.95	0.95, 0.95	0.93, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.96, 0.94	0.95, 0.94	0.96, 0.94	0.96, 0.94	0.95, 0.94
8	25	0.95, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95
	50	0.95, 0.94	0.95, 0.96	0.94, 0.94	0.94, 0.96	0.94, 0.96
	75	0.95, 0.96	0.96, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.95
	100	0.95, 0.96	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.94, 0.95
	250	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.94, 0.95	0.94, 0.95
	1000	0.93, 0.94	0.96, 0.96	0.93, 0.94	0.96, 0.96	0.95, 0.96

Note. P = parameter. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.57: Expanded Results, Coverage for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.95, 0.95	0.93, 0.94	0.95, 0.95	0.93, 0.94	0.96, 0.97
	50	0.94, 0.96	0.95, 0.94	0.95, 0.96	0.95, 0.95	0.97, 0.96
	75	0.95, 0.94	0.95, 0.96	0.95, 0.94	0.95, 0.96	0.96, 0.96
	100	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
	250	0.95, 0.94	0.94, 0.94	0.94, 0.94	0.94, 0.94	0.94, 0.95
	1000	0.95, 0.94	0.96, 0.96	0.95, 0.94	0.96, 0.96	0.95, 0.96
4	25	0.94, 0.94	0.94, 0.94	0.94, 0.93	0.95, 0.94	0.96, 0.95
	50	0.94, 0.94	0.94, 0.95	0.94, 0.94	0.94, 0.95	0.94, 0.95
	75	0.94, 0.93	0.95, 0.94	0.94, 0.94	0.95, 0.94	0.96, 0.95
	100	0.96, 0.95	0.93, 0.94	0.96, 0.95	0.94, 0.94	0.94, 0.94
	250	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.94, 0.94	0.93, 0.95	0.95, 0.95	0.94, 0.96	0.95, 0.96
8	25	0.95, 0.94	0.94, 0.93	0.95, 0.95	0.94, 0.94	0.96, 0.95
	50	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.96	0.96, 0.96
	75	0.94, 0.95	0.94, 0.94	0.94, 0.95	0.94, 0.94	0.95, 0.95
	100	0.94, 0.94	0.96, 0.95	0.94, 0.94	0.96, 0.95	0.96, 0.95
	250	0.94, 0.96	0.94, 0.95	0.94, 0.96	0.94, 0.95	0.94, 0.95
	1000	0.91, 0.92	0.93, 0.93	0.93, 0.94	0.94, 0.95	0.94, 0.95
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.93, 0.94	0.93, 0.92	0.93, 0.93	0.93, 0.93	0.95, 0.95
	50	0.94, 0.95	0.94, 0.96	0.94, 0.96	0.94, 0.96	0.95, 0.96
	75	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.96, 0.96
	100	0.93, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.94	0.95, 0.94
	250	0.95, 0.94	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.94, 0.96
	1000	0.96, 0.95	0.96, 0.95	0.96, 0.95	0.96, 0.95	0.95, 0.95
4	25	0.94, 0.94	0.95, 0.94	0.95, 0.94	0.94, 0.94	0.94, 0.94
	50	0.95, 0.93	0.95, 0.93	0.95, 0.93	0.95, 0.93	0.95, 0.93
	75	0.95, 0.94	0.94, 0.95	0.95, 0.94	0.94, 0.94	0.94, 0.94
	100	0.95, 0.96	0.94, 0.96	0.95, 0.95	0.94, 0.96	0.94, 0.96
	250	0.96, 0.95	0.96, 0.96	0.96, 0.95	0.95, 0.96	0.95, 0.95
	1000	0.94, 0.94	0.94, 0.94	0.95, 0.95	0.95, 0.95	0.95, 0.95
8	25	0.94, 0.94	0.96, 0.94	0.95, 0.94	0.95, 0.94	0.95, 0.94
	50	0.94, 0.94	0.95, 0.94	0.94, 0.94	0.95, 0.95	0.95, 0.95
	75	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.95	0.94, 0.94
	100	0.94, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.94, 0.95
	250	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.95
	1000	0.93, 0.94	0.94, 0.94	0.95, 0.96	0.95, 0.96	0.96, 0.96

Note. P = parameter. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.58: Expanded Results, Coverage for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.95, 0.94	0.94, 0.94	0.95, 0.94	0.93, 0.94	0.96, 0.96
	50	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.95	0.96, 0.95
	75	0.95, 0.95	0.93, 0.96	0.96, 0.95	0.93, 0.96	0.94, 0.96
	100	0.94, 0.94	0.94, 0.96	0.94, 0.94	0.94, 0.96	0.94, 0.96
	250	0.95, 0.96	0.96, 0.95	0.95, 0.96	0.96, 0.95	0.96, 0.95
	1000	0.95, 0.94	0.93, 0.94	0.95, 0.94	0.93, 0.94	0.93, 0.94
4	25	0.94, 0.95	0.96, 0.96	0.94, 0.95	0.96, 0.96	0.97, 0.97
	50	0.95, 0.94	0.95, 0.94	0.95, 0.94	0.95, 0.93	0.96, 0.94
	75	0.94, 0.96	0.93, 0.94	0.94, 0.95	0.94, 0.94	0.94, 0.95
	100	0.94, 0.94	0.95, 0.96	0.94, 0.94	0.94, 0.96	0.95, 0.96
	250	0.94, 0.94	0.95, 0.94	0.94, 0.94	0.95, 0.94	0.95, 0.95
	1000	0.95, 0.96	0.95, 0.96	0.95, 0.96	0.95, 0.96	0.95, 0.95
8	25	0.93, 0.96	0.94, 0.94	0.93, 0.95	0.94, 0.94	0.95, 0.95
	50	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.96
	75	0.96, 0.93	0.95, 0.95	0.96, 0.93	0.94, 0.94	0.95, 0.95
	100	0.96, 0.94	0.94, 0.94	0.96, 0.94	0.94, 0.94	0.94, 0.95
	250	0.96, 0.95	0.94, 0.96	0.96, 0.95	0.94, 0.96	0.95, 0.95
	1000	0.94, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.96	0.94, 0.95
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.94, 0.94	0.92, 0.95	0.94, 0.94	0.92, 0.95	0.96, 0.97
	50	0.95, 0.94	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.97
	75	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.96
	100	0.95, 0.94	0.95, 0.94	0.95, 0.93	0.95, 0.95	0.95, 0.95
	250	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95
	1000	0.95, 0.96	0.94, 0.95	0.95, 0.96	0.94, 0.94	0.94, 0.94
4	25	0.94, 0.96	0.95, 0.94	0.94, 0.96	0.95, 0.94	0.95, 0.94
	50	0.93, 0.95	0.95, 0.95	0.93, 0.95	0.96, 0.95	0.95, 0.95
	75	0.95, 0.92	0.94, 0.96	0.95, 0.92	0.94, 0.96	0.94, 0.96
	100	0.95, 0.95	0.93, 0.93	0.95, 0.95	0.93, 0.93	0.93, 0.93
	250	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.94, 0.95
8	25	0.94, 0.94	0.93, 0.94	0.95, 0.94	0.94, 0.93	0.94, 0.94
	50	0.94, 0.94	0.93, 0.96	0.94, 0.94	0.94, 0.95	0.94, 0.95
	75	0.94, 0.94	0.96, 0.95	0.94, 0.95	0.96, 0.95	0.96, 0.95
	100	0.96, 0.95	0.96, 0.94	0.96, 0.95	0.96, 0.94	0.96, 0.93
	250	0.94, 0.95	0.96, 0.96	0.94, 0.95	0.96, 0.96	0.96, 0.95
	1000	0.95, 0.95	0.94, 0.94	0.95, 0.95	0.94, 0.94	0.94, 0.94

Note. P = parameter. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.59: Expanded Results, Coverage for $\mu_{0,11}$, $\mu_{0,21}$, b_{11}, and b_{21}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$	$\mu_{0,11}$, $\mu_{0,21}$
2	25	0.94, 0.94	0.94, 0.95	0.94, 0.95	0.94, 0.94	0.96, 0.96
	50	0.95, 0.96	0.94, 0.93	0.95, 0.95	0.95, 0.93	0.96, 0.95
	75	0.95, 0.94	0.95, 0.94	0.95, 0.95	0.96, 0.95	0.96, 0.95
	100	0.94, 0.96	0.94, 0.96	0.94, 0.96	0.94, 0.97	0.94, 0.96
	250	0.95, 0.95	0.96, 0.95	0.96, 0.95	0.96, 0.95	0.96, 0.95
	1000	0.95, 0.96	0.94, 0.94	0.96, 0.96	0.94, 0.94	0.94, 0.94
4	25	0.92, 0.95	0.96, 0.95	0.93, 0.95	0.96, 0.96	0.98, 0.96
	50	0.94, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.94	0.96, 0.95
	75	0.94, 0.95	0.95, 0.96	0.94, 0.95	0.95, 0.97	0.95, 0.97
	100	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95
	250	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.95
	1000	0.94, 0.95	0.94, 0.94	0.95, 0.96	0.95, 0.95	0.95, 0.95
8	25	0.94, 0.94	0.92, 0.94	0.94, 0.95	0.93, 0.94	0.95, 0.96
	50	0.95, 0.94	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
	75	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.96
	100	0.97, 0.96	0.96, 0.94	0.97, 0.96	0.95, 0.95	0.96, 0.94
	250	0.94, 0.95	0.94, 0.95	0.93, 0.95	0.94, 0.95	0.95, 0.95
	1000	0.92, 0.93	0.91, 0.91	0.94, 0.95	0.93, 0.94	0.94, 0.94
T	N	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}	b_{11}, b_{21}
2	25	0.94, 0.93	0.94, 0.94	0.93, 0.93	0.93, 0.95	0.95, 0.98
	50	0.93, 0.92	0.92, 0.95	0.93, 0.92	0.92, 0.95	0.93, 0.96
	75	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.95
	100	0.95, 0.95	0.94, 0.94	0.95, 0.94	0.94, 0.94	0.94, 0.95
	250	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.95, 0.94	0.94, 0.96	0.95, 0.94	0.94, 0.96	0.94, 0.96
4	25	0.93, 0.94	0.95, 0.95	0.93, 0.94	0.95, 0.94	0.94, 0.94
	50	0.95, 0.94	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.95
	75	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.95, 0.95
	100	0.95, 0.95	0.93, 0.94	0.95, 0.95	0.93, 0.94	0.93, 0.94
	250	0.96, 0.95	0.94, 0.96	0.96, 0.95	0.94, 0.96	0.94, 0.95
	1000	0.92, 0.95	0.96, 0.94	0.93, 0.96	0.96, 0.94	0.95, 0.94
8	25	0.93, 0.94	0.94, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95
	50	0.95, 0.94	0.95, 0.96	0.95, 0.95	0.95, 0.95	0.95, 0.95
	75	0.94, 0.96	0.96, 0.95	0.94, 0.96	0.95, 0.95	0.95, 0.95
	100	0.96, 0.95	0.94, 0.94	0.95, 0.94	0.94, 0.94	0.95, 0.94
	250	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.93, 0.93	0.95, 0.94	0.95, 0.94	0.96, 0.94	0.96, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.60: Expanded Results, Coverage for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.91, 0.88	0.91, 0.89	0.96, 0.93	0.96, 0.93	0.97, 0.96
	50	0.90, 0.89	0.90, 0.91	0.95, 0.95	0.94, 0.94	0.95, 0.95
	75	0.92, 0.90	0.93, 0.92	0.95, 0.94	0.96, 0.94	0.96, 0.95
	100	0.94, 0.92	0.93, 0.91	0.95, 0.95	0.94, 0.94	0.95, 0.95
	250	0.95, 0.94	0.93, 0.94	0.95, 0.94	0.94, 0.95	0.95, 0.95
	1000	0.97, 0.95	0.95, 0.95	0.97, 0.95	0.95, 0.95	0.96, 0.95
4	25	0.89, 0.89	0.91, 0.92	0.94, 0.94	0.94, 0.95	0.94, 0.95
	50	0.93, 0.92	0.93, 0.91	0.96, 0.94	0.95, 0.94	0.95, 0.94
	75	0.95, 0.94	0.95, 0.94	0.96, 0.96	0.96, 0.95	0.95, 0.95
	100	0.95, 0.95	0.95, 0.95	0.96, 0.96	0.95, 0.94	0.95, 0.94
	250	0.95, 0.94	0.94, 0.94	0.95, 0.94	0.94, 0.93	0.94, 0.94
	1000	0.94, 0.94	0.96, 0.95	0.94, 0.94	0.95, 0.95	0.95, 0.95
8	25	0.95, 0.91	0.94, 0.91	0.95, 0.95	0.94, 0.94	0.94, 0.94
	50	0.94, 0.94	0.96, 0.94	0.95, 0.95	0.96, 0.95	0.96, 0.95
	75	0.96, 0.95	0.96, 0.95	0.96, 0.95	0.95, 0.95	0.95, 0.95
	100	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.94, 0.96	0.94, 0.95
	250	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.96, 0.94	0.95, 0.94
	1000	0.94, 0.95	0.94, 0.94	0.94, 0.94	0.94, 0.95	0.94, 0.94
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.90, 0.90	0.90, 0.91	0.94, 0.92	0.94, 0.93	0.97, 0.96
	50	0.93, 0.93	0.94, 0.93	0.95, 0.94	0.95, 0.93	0.96, 0.94
	75	0.95, 0.96	0.93, 0.96	0.95, 0.95	0.94, 0.95	0.95, 0.96
	100	0.95, 0.97	0.94, 0.94	0.95, 0.96	0.95, 0.94	0.95, 0.94
	250	0.96, 0.96	0.95, 0.95	0.95, 0.96	0.94, 0.95	0.95, 0.95
	1000	0.96, 0.96	0.96, 0.96	0.96, 0.95	0.96, 0.96	0.96, 0.96
4	25	0.94, 0.94	0.92, 0.94	0.95, 0.94	0.94, 0.94	0.95, 0.95
	50	0.95, 0.96	0.96, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.96
	75	0.97, 0.95	0.96, 0.96	0.96, 0.94	0.95, 0.94	0.95, 0.95
	100	0.95, 0.96	0.95, 0.97	0.94, 0.95	0.94, 0.96	0.94, 0.95
	250	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.96, 0.95	0.96, 0.95
	1000	0.95, 0.95	0.94, 0.94	0.95, 0.95	0.95, 0.94	0.94, 0.94
8	25	0.95, 0.97	0.94, 0.96	0.94, 0.95	0.95, 0.95	0.95, 0.95
	50	0.97, 0.96	0.96, 0.98	0.95, 0.95	0.96, 0.96	0.96, 0.96
	75	0.96, 0.96	0.94, 0.96	0.95, 0.95	0.94, 0.95	0.95, 0.95
	100	0.95, 0.95	0.95, 0.96	0.95, 0.94	0.94, 0.95	0.94, 0.95
	250	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.94, 0.96	0.94, 0.96
	1000	0.94, 0.95	0.96, 0.95	0.94, 0.95	0.96, 0.96	0.96, 0.96

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.61: Expanded Results, Coverage for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.92, 0.93	0.94, 0.92	0.94, 0.93	0.94, 0.93	0.95, 0.95
	50	0.95, 0.94	0.94, 0.93	0.95, 0.93	0.94, 0.94	0.95, 0.95
	75	0.95, 0.94	0.95, 0.95	0.95, 0.95	0.96, 0.95	0.95, 0.95
	100	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.94, 0.95	0.94, 0.96
	250	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95	0.94, 0.95
	1000	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
4	25	0.95, 0.93	0.94, 0.94	0.94, 0.94	0.93, 0.94	0.94, 0.94
	50	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.94, 0.95
	75	0.96, 0.94	0.95, 0.96	0.95, 0.94	0.95, 0.96	0.94, 0.96
	100	0.94, 0.95	0.96, 0.95	0.94, 0.95	0.95, 0.94	0.95, 0.94
	250	0.95, 0.95	0.94, 0.96	0.95, 0.94	0.94, 0.96	0.94, 0.95
	1000	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.94, 0.96	0.95, 0.95
8	25	0.95, 0.95	0.96, 0.95	0.94, 0.96	0.95, 0.95	0.94, 0.95
	50	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.94, 0.95
	75	0.96, 0.96	0.94, 0.94	0.95, 0.95	0.94, 0.94	0.93, 0.94
	100	0.94, 0.95	0.95, 0.95	0.93, 0.95	0.95, 0.96	0.95, 0.96
	250	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.96, 0.94	0.96, 0.95	0.97, 0.94	0.96, 0.95	0.96, 0.94
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.95, 0.94	0.94, 0.95	0.94, 0.94	0.93, 0.94	0.97, 0.95
	50	0.94, 0.93	0.95, 0.94	0.94, 0.94	0.95, 0.93	0.96, 0.95
	75	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.94, 0.94	0.95, 0.95
	100	0.95, 0.94	0.95, 0.95	0.94, 0.94	0.95, 0.95	0.95, 0.96
	250	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.94	0.94, 0.94
	1000	0.95, 0.94	0.96, 0.95	0.95, 0.94	0.96, 0.95	0.95, 0.95
4	25	0.94, 0.94	0.95, 0.95	0.94, 0.95	0.95, 0.96	0.95, 0.96
	50	0.96, 0.95	0.95, 0.96	0.95, 0.96	0.95, 0.96	0.95, 0.96
	75	0.96, 0.95	0.96, 0.94	0.96, 0.95	0.96, 0.95	0.96, 0.94
	100	0.95, 0.94	0.94, 0.94	0.95, 0.94	0.94, 0.94	0.94, 0.94
	250	0.96, 0.95	0.96, 0.95	0.96, 0.94	0.96, 0.95	0.96, 0.95
	1000	0.95, 0.96	0.96, 0.95	0.95, 0.96	0.96, 0.95	0.95, 0.95
8	25	0.96, 0.94	0.96, 0.97	0.95, 0.94	0.95, 0.96	0.95, 0.96
	50	0.96, 0.96	0.95, 0.95	0.96, 0.96	0.95, 0.95	0.95, 0.95
	75	0.95, 0.97	0.95, 0.94	0.95, 0.96	0.94, 0.94	0.95, 0.94
	100	0.94, 0.96	0.95, 0.95	0.94, 0.95	0.94, 0.95	0.95, 0.94
	250	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.94
	1000	0.95, 0.94	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.94, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.62: Expanded Results, Coverage for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.90, 0.90	0.91, 0.89	0.94, 0.95	0.95, 0.94	0.48, 0.68
	50	0.92, 0.93	0.94, 0.92	0.95, 0.96	0.95, 0.95	0.95, 0.96
	75	0.93, 0.94	0.95, 0.94	0.94, 0.94	0.96, 0.95	0.95, 0.95
	100	0.94, 0.95	0.94, 0.94	0.94, 0.95	0.95, 0.95	0.94, 0.93
	250	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.94
4	25	0.93, 0.93	0.93, 0.92	0.95, 0.95	0.94, 0.94	0.92, 0.94
	50	0.95, 0.94	0.94, 0.95	0.95, 0.95	0.94, 0.96	0.92, 0.94
	75	0.95, 0.95	0.96, 0.94	0.95, 0.95	0.95, 0.95	0.93, 0.94
	100	0.95, 0.96	0.95, 0.94	0.94, 0.96	0.95, 0.94	0.93, 0.93
	250	0.95, 0.95	0.96, 0.95	0.95, 0.94	0.95, 0.96	0.95, 0.95
	1000	0.95, 0.94	0.94, 0.93	0.95, 0.95	0.95, 0.94	0.95, 0.94
8	25	0.93, 0.94	0.95, 0.93	0.95, 0.95	0.95, 0.95	0.93, 0.94
	50	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.94, 0.94	0.94, 0.93
	75	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.95, 0.94
	100	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.94, 0.94
	250	0.94, 0.95	0.95, 0.96	0.94, 0.95	0.94, 0.96	0.94, 0.96
	1000	0.94, 0.92	0.94, 0.93	0.95, 0.94	0.95, 0.95	0.96, 0.95
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.94, 0.93	0.93, 0.94	0.95, 0.93	0.94, 0.94	0.67, 0.60
	50	0.95, 0.94	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.96, 0.95
	75	0.97, 0.95	0.95, 0.95	0.96, 0.93	0.95, 0.95	0.96, 0.95
	100	0.96, 0.96	0.95, 0.93	0.95, 0.95	0.95, 0.94	0.95, 0.95
	250	0.96, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.94, 0.93	0.95, 0.96	0.94, 0.93	0.94, 0.95	0.94, 0.95
4	25	0.95, 0.97	0.94, 0.94	0.95, 0.96	0.94, 0.95	0.96, 0.96
	50	0.96, 0.95	0.93, 0.94	0.96, 0.95	0.93, 0.94	0.94, 0.94
	75	0.97, 0.94	0.96, 0.95	0.96, 0.94	0.96, 0.95	0.95, 0.95
	100	0.95, 0.94	0.94, 0.94	0.94, 0.94	0.94, 0.94	0.93, 0.94
	250	0.95, 0.96	0.95, 0.95	0.94, 0.96	0.94, 0.95	0.95, 0.95
	1000	0.95, 0.94	0.94, 0.95	0.96, 0.95	0.95, 0.95	0.95, 0.94
8	25	0.95, 0.95	0.95, 0.95	0.94, 0.95	0.94, 0.95	0.94, 0.95
	50	0.96, 0.94	0.95, 0.95	0.95, 0.94	0.96, 0.95	0.95, 0.95
	75	0.95, 0.96	0.94, 0.95	0.94, 0.96	0.94, 0.95	0.93, 0.95
	100	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95
	250	0.95, 0.94	0.95, 0.96	0.95, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.93, 0.94	0.93, 0.93	0.94, 0.95	0.95, 0.94	0.95, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.63: Expanded Results, Coverage for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.94, 0.92	0.95, 0.93	0.93, 0.93	0.94, 0.93	0.96, 0.94
	50	0.93, 0.94	0.95, 0.95	0.93, 0.94	0.94, 0.94	0.95, 0.95
	75	0.95, 0.93	0.94, 0.96	0.96, 0.94	0.94, 0.96	0.94, 0.96
	100	0.95, 0.96	0.94, 0.94	0.95, 0.96	0.95, 0.95	0.95, 0.95
	250	0.95, 0.95	0.94, 0.97	0.95, 0.95	0.94, 0.96	0.94, 0.96
	1000	0.95, 0.95	0.93, 0.94	0.95, 0.95	0.93, 0.94	0.94, 0.94
4	25	0.96, 0.95	0.95, 0.93	0.95, 0.95	0.95, 0.93	0.95, 0.94
	50	0.96, 0.94	0.96, 0.95	0.96, 0.94	0.96, 0.96	0.95, 0.96
	75	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.94, 0.96	0.94, 0.96
	100	0.94, 0.96	0.95, 0.96	0.94, 0.96	0.95, 0.96	0.95, 0.95
	250	0.96, 0.94	0.94, 0.96	0.96, 0.94	0.94, 0.96	0.94, 0.96
	1000	0.96, 0.94	0.94, 0.95	0.96, 0.94	0.94, 0.95	0.94, 0.95
8	25	0.95, 0.95	0.96, 0.95	0.94, 0.94	0.96, 0.95	0.94, 0.95
	50	0.95, 0.94	0.96, 0.95	0.95, 0.94	0.95, 0.95	0.94, 0.95
	75	0.95, 0.94	0.97, 0.95	0.95, 0.94	0.96, 0.95	0.96, 0.94
	100	0.95, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.96	0.95, 0.95
	250	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.94, 0.95	0.94, 0.95
	1000	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.94
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.95, 0.93	0.95, 0.93	0.94, 0.93	0.94, 0.93	0.96, 0.96
	50	0.95, 0.94	0.97, 0.95	0.94, 0.94	0.95, 0.94	0.96, 0.95
	75	0.95, 0.94	0.96, 0.95	0.94, 0.94	0.95, 0.94	0.95, 0.95
	100	0.96, 0.94	0.96, 0.96	0.95, 0.94	0.94, 0.96	0.95, 0.96
	250	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.94
	1000	0.95, 0.94	0.94, 0.94	0.95, 0.94	0.95, 0.94	0.94, 0.94
4	25	0.96, 0.94	0.95, 0.94	0.94, 0.94	0.94, 0.93	0.95, 0.94
	50	0.95, 0.96	0.96, 0.94	0.95, 0.96	0.95, 0.95	0.95, 0.95
	75	0.95, 0.95	0.95, 0.95	0.95, 0.94	0.95, 0.94	0.94, 0.95
	100	0.96, 0.96	0.95, 0.95	0.96, 0.96	0.95, 0.95	0.95, 0.94
	250	0.96, 0.95	0.95, 0.95	0.97, 0.95	0.95, 0.95	0.95, 0.94
	1000	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
8	25	0.96, 0.95	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.95
	50	0.94, 0.95	0.95, 0.96	0.94, 0.95	0.94, 0.96	0.94, 0.95
	75	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.94
	100	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.95
	250	0.94, 0.94	0.95, 0.95	0.94, 0.94	0.95, 0.95	0.95, 0.95
	1000	0.94, 0.95	0.94, 0.96	0.95, 0.95	0.94, 0.96	0.94, 0.97

Note. P = parameter. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.64: Expanded Results, Coverage for a_{11}, a_{22}, a_{21}, and a_{12}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}	a_{11}, a_{22}
2	25	0.93, 0.93	0.91, 0.91	0.94, 0.96	0.94, 0.94	0.59, 0.95
	50	0.94, 0.95	0.94, 0.94	0.94, 0.95	0.93, 0.95	0.95, 0.95
	75	0.95, 0.94	0.95, 0.93	0.94, 0.95	0.95, 0.94	0.95, 0.95
	100	0.96, 0.95	0.94, 0.96	0.96, 0.96	0.94, 0.95	0.93, 0.95
	250	0.95, 0.95	0.94, 0.93	0.95, 0.95	0.94, 0.94	0.94, 0.93
	1000	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.96
4	25	0.95, 0.94	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.96, 0.96
	50	0.94, 0.94	0.95, 0.94	0.94, 0.95	0.94, 0.94	0.92, 0.93
	75	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.94	0.95, 0.94
	100	0.95, 0.94	0.95, 0.96	0.94, 0.94	0.95, 0.96	0.95, 0.94
	250	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.93, 0.95
	1000	0.95, 0.96	0.94, 0.94	0.95, 0.96	0.95, 0.95	0.95, 0.96
8	25	0.94, 0.93	0.95, 0.96	0.94, 0.94	0.96, 0.96	0.93, 0.94
	50	0.94, 0.96	0.96, 0.95	0.94, 0.96	0.96, 0.95	0.95, 0.95
	75	0.96, 0.94	0.96, 0.95	0.96, 0.95	0.95, 0.95	0.94, 0.95
	100	0.94, 0.96	0.94, 0.95	0.94, 0.96	0.94, 0.95	0.95, 0.94
	250	0.95, 0.97	0.95, 0.95	0.95, 0.97	0.95, 0.96	0.95, 0.96
	1000	0.93, 0.93	0.93, 0.93	0.94, 0.94	0.94, 0.93	0.95, 0.94
T	N	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}	a_{21}, a_{12}
2	25	0.95, 0.92	0.96, 0.92	0.94, 0.93	0.94, 0.94	0.85, 0.86
	50	0.96, 0.95	0.96, 0.95	0.94, 0.94	0.95, 0.95	0.96, 0.96
	75	0.97, 0.95	0.97, 0.94	0.95, 0.95	0.96, 0.94	0.96, 0.95
	100	0.97, 0.96	0.97, 0.96	0.95, 0.96	0.96, 0.95	0.96, 0.96
	250	0.94, 0.94	0.95, 0.94	0.94, 0.94	0.95, 0.94	0.95, 0.94
	1000	0.94, 0.96	0.95, 0.95	0.94, 0.96	0.95, 0.95	0.95, 0.95
4	25	0.96, 0.95	0.96, 0.94	0.95, 0.95	0.95, 0.95	0.97, 0.96
	50	0.96, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.94	0.94, 0.94
	75	0.94, 0.96	0.96, 0.95	0.93, 0.96	0.95, 0.95	0.94, 0.95
	100	0.96, 0.95	0.96, 0.95	0.96, 0.96	0.97, 0.94	0.96, 0.94
	250	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.95
	1000	0.95, 0.93	0.94, 0.93	0.96, 0.94	0.94, 0.94	0.94, 0.94
8	25	0.96, 0.95	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.96
	50	0.97, 0.95	0.95, 0.95	0.96, 0.96	0.94, 0.95	0.95, 0.95
	75	0.95, 0.96	0.96, 0.95	0.95, 0.96	0.96, 0.95	0.95, 0.96
	100	0.95, 0.96	0.95, 0.97	0.95, 0.96	0.95, 0.97	0.94, 0.96
	250	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95
	1000	0.93, 0.93	0.93, 0.94	0.94, 0.94	0.95, 0.95	0.95, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.65: Coverage for $\phi_{0,11}$ and $\phi_{0,22}$, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.95, 0.93	0.97, 0.92	0.95, 0.94	0.96, 0.92	0.95, 0.94
	50	0.94, 0.94	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.96, 0.95
	75	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.96, 0.94
	100	0.95, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.95	0.96, 0.95
	250	0.95, 0.95	0.96, 0.95	0.96, 0.95	0.96, 0.95	0.95, 0.95
	1000	0.95, 0.94	0.95, 0.96	0.95, 0.94	0.95, 0.95	0.95, 0.95
4	25	0.94, 0.95	0.93, 0.97	0.94, 0.95	0.95, 0.97	0.95, 0.96
	50	0.95, 0.94	0.93, 0.95	0.96, 0.94	0.94, 0.95	0.94, 0.94
	75	0.95, 0.94	0.95, 0.93	0.95, 0.94	0.96, 0.94	0.96, 0.94
	100	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.94	0.95, 0.94
	250	0.97, 0.95	0.95, 0.94	0.96, 0.95	0.94, 0.95	0.94, 0.95
	1000	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.96	0.95, 0.95
8	25	0.95, 0.95	0.95, 0.92	0.96, 0.95	0.95, 0.93	0.95, 0.94
	50	0.96, 0.95	0.96, 0.95	0.95, 0.95	0.95, 0.95	0.96, 0.96
	75	0.95, 0.95	0.95, 0.93	0.95, 0.95	0.95, 0.94	0.94, 0.94
	100	0.96, 0.96	0.95, 0.95	0.96, 0.96	0.95, 0.95	0.95, 0.94
	250	0.95, 0.94	0.96, 0.94	0.95, 0.94	0.96, 0.94	0.96, 0.93
	1000	0.95, 0.93	0.94, 0.95	0.95, 0.93	0.94, 0.95	0.94, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.66: Expanded Results, Coverage for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.92, 0.96	0.95, 0.93	0.93, 0.95	0.95, 0.94	0.94, 0.95
	50	0.94, 0.93	0.93, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.96
	75	0.95, 0.95	0.94, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.95
	100	0.94, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.94	0.95, 0.95
	250	0.95, 0.94	0.94, 0.95	0.95, 0.95	0.94, 0.94	0.94, 0.94
	1000	0.96, 0.96	0.94, 0.96	0.96, 0.96	0.95, 0.96	0.94, 0.95
4	25	0.93, 0.93	0.94, 0.94	0.94, 0.94	0.95, 0.94	0.95, 0.95
	50	0.95, 0.95	0.96, 0.93	0.95, 0.94	0.96, 0.94	0.95, 0.94
	75	0.93, 0.95	0.95, 0.94	0.94, 0.95	0.96, 0.95	0.94, 0.94
	100	0.96, 0.94	0.95, 0.95	0.96, 0.94	0.94, 0.96	0.94, 0.95
	250	0.94, 0.94	0.96, 0.95	0.94, 0.94	0.95, 0.96	0.95, 0.96
	1000	0.94, 0.96	0.94, 0.95	0.94, 0.96	0.95, 0.95	0.95, 0.95
8	25	0.93, 0.94	0.94, 0.93	0.95, 0.95	0.95, 0.95	0.95, 0.96
	50	0.94, 0.94	0.95, 0.94	0.94, 0.95	0.96, 0.94	0.95, 0.94
	75	0.96, 0.94	0.95, 0.96	0.96, 0.94	0.96, 0.96	0.94, 0.95
	100	0.95, 0.95	0.94, 0.94	0.95, 0.95	0.95, 0.95	0.95, 0.96
	250	0.94, 0.94	0.95, 0.94	0.94, 0.94	0.95, 0.94	0.95, 0.93
	1000	0.94, 0.93	0.95, 0.94	0.94, 0.93	0.94, 0.94	0.95, 0.94
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.93, 0.91	0.92, 0.92	0.94, 0.92	0.94, 0.93	0.90, 0.81
	50	0.92, 0.93	0.93, 0.92	0.94, 0.94	0.95, 0.94	0.95, 0.94
	75	0.94, 0.95	0.95, 0.91	0.95, 0.95	0.95, 0.93	0.94, 0.94
	100	0.92, 0.95	0.93, 0.93	0.94, 0.96	0.94, 0.94	0.94, 0.94
	250	0.95, 0.94	0.95, 0.95	0.94, 0.95	0.96, 0.95	0.95, 0.95
	1000	0.96, 0.96	0.93, 0.96	0.95, 0.95	0.93, 0.96	0.94, 0.96
4	25	0.93, 0.95	0.94, 0.92	0.94, 0.96	0.96, 0.93	0.94, 0.92
	50	0.93, 0.94	0.94, 0.96	0.94, 0.95	0.95, 0.96	0.94, 0.94
	75	0.94, 0.94	0.94, 0.96	0.95, 0.94	0.95, 0.96	0.95, 0.95
	100	0.95, 0.95	0.94, 0.94	0.96, 0.96	0.95, 0.95	0.95, 0.94
	250	0.95, 0.95	0.93, 0.95	0.95, 0.95	0.94, 0.95	0.94, 0.95
	1000	0.94, 0.94	0.95, 0.95	0.94, 0.94	0.95, 0.95	0.95, 0.95
8	25	0.95, 0.95	0.93, 0.94	0.96, 0.96	0.95, 0.95	0.96, 0.93
	50	0.95, 0.93	0.94, 0.95	0.95, 0.94	0.95, 0.95	0.94, 0.94
	75	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.94
	100	0.95, 0.94	0.94, 0.96	0.95, 0.94	0.95, 0.95	0.95, 0.94
	250	0.95, 0.94	0.96, 0.95	0.94, 0.94	0.95, 0.95	0.96, 0.95
	1000	0.93, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.94	0.95, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.67: Expanded Results, Coverage for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 3

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.93, 0.94	0.93, 0.94	0.93, 0.94	0.94, 0.94	0.94, 0.95
	50	0.94, 0.94	0.95, 0.93	0.95, 0.94	0.96, 0.94	0.95, 0.94
	75	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.95
	100	0.95, 0.94	0.96, 0.94	0.95, 0.94	0.96, 0.94	0.95, 0.95
	250	0.96, 0.94	0.95, 0.95	0.96, 0.94	0.95, 0.95	0.95, 0.95
	1000	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.94
4	25	0.96, 0.94	0.95, 0.95	0.96, 0.94	0.95, 0.95	0.94, 0.94
	50	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.94, 0.95	0.96, 0.94
	75	0.95, 0.96	0.95, 0.96	0.94, 0.96	0.96, 0.96	0.95, 0.95
	100	0.95, 0.94	0.97, 0.94	0.95, 0.94	0.96, 0.94	0.97, 0.95
	250	0.95, 0.94	0.96, 0.95	0.95, 0.94	0.96, 0.95	0.96, 0.94
	1000	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95	0.94, 0.94
8	25	0.96, 0.94	0.95, 0.93	0.96, 0.95	0.95, 0.94	0.96, 0.95
	50	0.95, 0.95	0.95, 0.93	0.95, 0.96	0.95, 0.94	0.94, 0.95
	75	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.94, 0.95
	100	0.95, 0.94	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.94, 0.95
	250	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.95, 0.95	0.95, 0.96	0.94, 0.95	0.95, 0.96	0.95, 0.96
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.91, 0.91	0.90, 0.92	0.92, 0.91	0.91, 0.92	0.97, 0.95
	50	0.93, 0.94	0.94, 0.95	0.93, 0.94	0.94, 0.95	0.96, 0.95
	75	0.93, 0.94	0.94, 0.94	0.94, 0.94	0.94, 0.94	0.94, 0.95
	100	0.95, 0.95	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.94, 0.96
	250	0.94, 0.95	0.94, 0.93	0.94, 0.95	0.94, 0.94	0.95, 0.94
	1000	0.95, 0.94	0.95, 0.96	0.95, 0.94	0.96, 0.96	0.95, 0.95
4	25	0.92, 0.94	0.94, 0.96	0.92, 0.94	0.95, 0.96	0.94, 0.95
	50	0.94, 0.95	0.94, 0.94	0.94, 0.95	0.95, 0.94	0.94, 0.94
	75	0.95, 0.96	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.94, 0.94
	100	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.95, 0.95
	250	0.94, 0.94	0.96, 0.94	0.94, 0.94	0.96, 0.94	0.96, 0.95
	1000	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
8	25	0.94, 0.94	0.94, 0.95	0.95, 0.94	0.95, 0.95	0.94, 0.95
	50	0.96, 0.96	0.96, 0.96	0.96, 0.96	0.96, 0.96	0.96, 0.96
	75	0.95, 0.95	0.95, 0.93	0.95, 0.95	0.95, 0.94	0.95, 0.94
	100	0.94, 0.94	0.95, 0.95	0.94, 0.94	0.95, 0.95	0.96, 0.96
	250	0.94, 0.96	0.94, 0.95	0.94, 0.96	0.94, 0.95	0.94, 0.95
	1000	0.96, 0.94	0.94, 0.94	0.96, 0.95	0.94, 0.95	0.94, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 3 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.68: Expanded Results, Coverage for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.95, 0.95	0.93, 0.95	0.95, 0.95	0.94, 0.96	0.96, 0.88
	50	0.96, 0.95	0.93, 0.95	0.96, 0.95	0.94, 0.95	0.95, 0.96
	75	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95
	100	0.95, 0.95	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.95, 0.95
	250	0.96, 0.94	0.95, 0.94	0.96, 0.94	0.95, 0.95	0.94, 0.95
	1000	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
4	25	0.94, 0.94	0.94, 0.94	0.94, 0.95	0.94, 0.95	0.95, 0.95
	50	0.96, 0.92	0.94, 0.95	0.96, 0.93	0.94, 0.95	0.93, 0.95
	75	0.95, 0.94	0.96, 0.94	0.95, 0.94	0.96, 0.94	0.95, 0.94
	100	0.96, 0.95	0.97, 0.95	0.96, 0.95	0.96, 0.95	0.96, 0.95
	250	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.94, 0.95
	1000	0.94, 0.95	0.94, 0.93	0.94, 0.95	0.95, 0.95	0.95, 0.95
8	25	0.95, 0.96	0.94, 0.93	0.95, 0.96	0.95, 0.93	0.95, 0.94
	50	0.94, 0.94	0.96, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95
	75	0.95, 0.96	0.94, 0.94	0.95, 0.96	0.94, 0.95	0.94, 0.94
	100	0.94, 0.94	0.94, 0.96	0.94, 0.95	0.94, 0.96	0.94, 0.96
	250	0.94, 0.95	0.96, 0.95	0.94, 0.95	0.96, 0.95	0.95, 0.95
	1000	0.92, 0.92	0.93, 0.93	0.94, 0.95	0.94, 0.94	0.95, 0.95
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.89, 0.92	0.93, 0.90	0.91, 0.92	0.93, 0.91	0.63, 0.84
	50	0.92, 0.93	0.91, 0.93	0.93, 0.93	0.93, 0.94	0.95, 0.96
	75	0.93, 0.93	0.94, 0.94	0.93, 0.94	0.94, 0.95	0.95, 0.94
	100	0.95, 0.94	0.94, 0.94	0.95, 0.94	0.95, 0.94	0.93, 0.95
	250	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.93, 0.95
	1000	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95
4	25	0.95, 0.95	0.95, 0.93	0.95, 0.96	0.95, 0.94	0.92, 0.94
	50	0.94, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.94, 0.93
	75	0.93, 0.95	0.95, 0.94	0.93, 0.95	0.95, 0.94	0.94, 0.95
	100	0.97, 0.95	0.95, 0.96	0.96, 0.96	0.95, 0.96	0.95, 0.95
	250	0.94, 0.94	0.96, 0.95	0.94, 0.94	0.95, 0.95	0.95, 0.95
	1000	0.97, 0.95	0.95, 0.95	0.97, 0.95	0.96, 0.96	0.95, 0.95
8	25	0.95, 0.95	0.96, 0.93	0.96, 0.95	0.96, 0.93	0.94, 0.92
	50	0.96, 0.94	0.95, 0.94	0.96, 0.95	0.95, 0.94	0.93, 0.94
	75	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.94, 0.94
	100	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.94
	250	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.95
	1000	0.94, 0.91	0.94, 0.94	0.95, 0.94	0.95, 0.95	0.96, 0.96

Note. P = parameter. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.69: Expanded Results, Coverage for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.95, 0.94	0.94, 0.96	0.95, 0.95	0.94, 0.96	0.95, 0.96
	50	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.93, 0.94
	75	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.94, 0.95
	100	0.94, 0.94	0.95, 0.96	0.94, 0.95	0.94, 0.96	0.95, 0.96
	250	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.94, 0.94	0.94, 0.95
	1000	0.96, 0.96	0.97, 0.94	0.96, 0.96	0.96, 0.94	0.95, 0.94
4	25	0.94, 0.95	0.94, 0.93	0.94, 0.95	0.94, 0.94	0.94, 0.94
	50	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.95, 0.95
	75	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.94, 0.95
	100	0.96, 0.95	0.96, 0.95	0.96, 0.95	0.95, 0.95	0.95, 0.95
	250	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.95
	1000	0.96, 0.94	0.96, 0.96	0.96, 0.94	0.96, 0.95	0.96, 0.95
8	25	0.95, 0.94	0.94, 0.94	0.95, 0.94	0.95, 0.94	0.95, 0.94
	50	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.94
	75	0.96, 0.94	0.95, 0.96	0.96, 0.94	0.95, 0.95	0.96, 0.94
	100	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.94	0.95, 0.94
	250	0.94, 0.96	0.95, 0.95	0.94, 0.96	0.95, 0.95	0.95, 0.95
	1000	0.96, 0.95	0.94, 0.94	0.96, 0.95	0.94, 0.95	0.95, 0.95
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.91, 0.89	0.91, 0.92	0.91, 0.90	0.91, 0.92	0.95, 0.95
	50	0.94, 0.93	0.95, 0.94	0.94, 0.93	0.95, 0.94	0.94, 0.95
	75	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.94, 0.95	0.96, 0.97
	100	0.93, 0.94	0.94, 0.93	0.94, 0.93	0.94, 0.94	0.94, 0.95
	250	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.96, 0.95	0.96, 0.96	0.96, 0.95	0.96, 0.95	0.96, 0.95
4	25	0.95, 0.94	0.95, 0.93	0.95, 0.94	0.95, 0.93	0.95, 0.93
	50	0.95, 0.96	0.94, 0.94	0.95, 0.95	0.95, 0.94	0.94, 0.95
	75	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.96
	100	0.95, 0.94	0.96, 0.94	0.95, 0.94	0.95, 0.94	0.95, 0.94
	250	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
8	25	0.96, 0.95	0.96, 0.94	0.96, 0.95	0.96, 0.94	0.95, 0.94
	50	0.95, 0.95	0.94, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.96
	75	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.94
	100	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.96, 0.96
	250	0.95, 0.94	0.95, 0.94	0.95, 0.94	0.95, 0.95	0.95, 0.95
	1000	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.94, 0.94	0.94, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.70: Expanded Results, Coverage for $\phi_{0,11}$, $\phi_{0,22}$, q_{11}, and q_{22}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$	$\phi_{0,11}$, $\phi_{0,22}$
2	25	0.95, 0.95	0.96, 0.94	0.95, 0.95	0.96, 0.95	0.95, 0.90
	50	0.95, 0.94	0.95, 0.95	0.96, 0.94	0.96, 0.95	0.95, 0.95
	75	0.95, 0.94	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.94
	100	0.95, 0.95	0.95, 0.95	0.95, 0.94	0.95, 0.94	0.95, 0.95
	250	0.94, 0.94	0.95, 0.96	0.94, 0.94	0.95, 0.96	0.95, 0.96
	1000	0.94, 0.95	0.96, 0.95	0.94, 0.95	0.96, 0.95	0.96, 0.95
4	25	0.94, 0.95	0.95, 0.93	0.95, 0.95	0.95, 0.94	0.93, 0.94
	50	0.94, 0.95	0.94, 0.94	0.95, 0.95	0.94, 0.95	0.96, 0.95
	75	0.94, 0.95	0.96, 0.95	0.94, 0.95	0.96, 0.95	0.95, 0.95
	100	0.96, 0.96	0.94, 0.96	0.96, 0.95	0.94, 0.96	0.94, 0.95
	250	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.94
	1000	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.95, 0.97	0.95, 0.96
8	25	0.94, 0.94	0.94, 0.95	0.95, 0.95	0.95, 0.95	0.94, 0.93
	50	0.94, 0.93	0.95, 0.94	0.94, 0.93	0.95, 0.94	0.96, 0.94
	75	0.95, 0.96	0.95, 0.94	0.95, 0.96	0.95, 0.94	0.95, 0.95
	100	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.94, 0.95
	250	0.94, 0.97	0.95, 0.94	0.94, 0.97	0.95, 0.94	0.95, 0.93
	1000	0.92, 0.92	0.93, 0.94	0.93, 0.94	0.94, 0.94	0.95, 0.96
T	N	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}	q_{11}, q_{22}
2	25	0.94, 0.93	0.94, 0.93	0.93, 0.93	0.93, 0.93	0.64, 0.96
	50	0.94, 0.94	0.92, 0.92	0.95, 0.94	0.93, 0.94	0.96, 0.96
	75	0.94, 0.94	0.94, 0.94	0.94, 0.94	0.95, 0.94	0.95, 0.94
	100	0.95, 0.94	0.94, 0.96	0.95, 0.94	0.94, 0.95	0.94, 0.95
	250	0.95, 0.93	0.95, 0.94	0.95, 0.93	0.95, 0.94	0.95, 0.94
	1000	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.94, 0.95
4	25	0.95, 0.93	0.93, 0.94	0.94, 0.95	0.93, 0.95	0.94, 0.96
	50	0.95, 0.94	0.94, 0.95	0.96, 0.94	0.94, 0.95	0.93, 0.95
	75	0.95, 0.96	0.94, 0.93	0.95, 0.96	0.94, 0.94	0.94, 0.94
	100	0.95, 0.95	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.93, 0.94
	250	0.94, 0.94	0.94, 0.94	0.94, 0.94	0.94, 0.94	0.94, 0.94
	1000	0.93, 0.94	0.94, 0.95	0.94, 0.94	0.94, 0.95	0.94, 0.95
8	25	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.96, 0.96	0.93, 0.94
	50	0.96, 0.95	0.95, 0.94	0.96, 0.95	0.96, 0.94	0.94, 0.93
	75	0.96, 0.97	0.96, 0.95	0.96, 0.96	0.96, 0.95	0.95, 0.94
	100	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.93, 0.95
	250	0.94, 0.95	0.96, 0.95	0.94, 0.95	0.96, 0.95	0.95, 0.95
	1000	0.92, 0.92	0.92, 0.93	0.94, 0.93	0.93, 0.94	0.94, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.71: Expanded Results, Coverage for $\phi_{0,21}$ and q_{21}, Parameter Set 1

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	0.94, 0.92	0.96, 0.91	0.94, 0.92	0.96, 0.92	0.98, 0.95
	50	0.94, 0.93	0.95, 0.92	0.94, 0.93	0.95, 0.93	0.96, 0.95
	75	0.95, 0.96	0.95, 0.93	0.95, 0.96	0.95, 0.94	0.96, 0.95
	100	0.95, 0.96	0.94, 0.94	0.95, 0.96	0.95, 0.94	0.95, 0.95
	250	0.96, 0.94	0.94, 0.94	0.95, 0.94	0.95, 0.95	0.95, 0.95
	1000	0.96, 0.95	0.95, 0.94	0.96, 0.95	0.95, 0.94	0.94, 0.95
4	25	0.95, 0.94	0.94, 0.95	0.95, 0.95	0.93, 0.95	0.95, 0.96
	50	0.94, 0.95	0.94, 0.96	0.95, 0.96	0.94, 0.96	0.95, 0.95
	75	0.95, 0.94	0.96, 0.93	0.96, 0.94	0.96, 0.94	0.96, 0.94
	100	0.94, 0.94	0.96, 0.94	0.94, 0.94	0.96, 0.94	0.96, 0.94
	250	0.95, 0.96	0.95, 0.96	0.95, 0.95	0.95, 0.97	0.96, 0.96
	1000	0.96, 0.94	0.96, 0.94	0.96, 0.94	0.95, 0.94	0.95, 0.94
8	25	0.94, 0.95	0.93, 0.94	0.94, 0.95	0.94, 0.95	0.97, 0.95
	50	0.95, 0.96	0.95, 0.94	0.95, 0.96	0.95, 0.94	0.96, 0.94
	75	0.95, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.96	0.97, 0.96
	100	0.94, 0.96	0.95, 0.95	0.94, 0.96	0.95, 0.95	0.95, 0.95
	250	0.96, 0.94	0.95, 0.94	0.96, 0.94	0.95, 0.94	0.95, 0.94
	1000	0.96, 0.95	0.97, 0.93	0.95, 0.95	0.96, 0.94	0.96, 0.93

Note. P = parameter. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.72: Expanded Results, Coverage for $\phi_{0,21}$ and q_{21}, Parameter Set 2

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	0.94, 0.91	0.94, 0.91	0.94, 0.93	0.95, 0.93	0.97, 0.93
	50	0.96, 0.93	0.96, 0.95	0.96, 0.94	0.96, 0.95	0.97, 0.96
	75	0.95, 0.94	0.95, 0.93	0.95, 0.95	0.95, 0.94	0.96, 0.96
	100	0.96, 0.92	0.94, 0.93	0.96, 0.94	0.95, 0.94	0.95, 0.95
	250	0.95, 0.94	0.95, 0.95	0.95, 0.95	0.94, 0.95	0.94, 0.95
	1000	0.94, 0.95	0.96, 0.95	0.94, 0.95	0.96, 0.96	0.95, 0.96
4	25	0.94, 0.93	0.94, 0.91	0.94, 0.94	0.95, 0.93	0.96, 0.95
	50	0.96, 0.93	0.96, 0.93	0.96, 0.94	0.96, 0.94	0.96, 0.95
	75	0.95, 0.95	0.95, 0.94	0.95, 0.96	0.95, 0.94	0.95, 0.95
	100	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.96
	250	0.93, 0.95	0.94, 0.96	0.93, 0.95	0.94, 0.96	0.95, 0.96
	1000	0.95, 0.94	0.96, 0.95	0.95, 0.94	0.95, 0.96	0.95, 0.96
8	25	0.91, 0.94	0.94, 0.94	0.93, 0.95	0.95, 0.95	0.96, 0.95
	50	0.93, 0.95	0.93, 0.94	0.94, 0.95	0.93, 0.95	0.94, 0.96
	75	0.93, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.94	0.96, 0.94
	100	0.95, 0.94	0.94, 0.96	0.95, 0.94	0.94, 0.96	0.95, 0.96
	250	0.95, 0.94	0.94, 0.94	0.95, 0.94	0.94, 0.94	0.95, 0.94
	1000	0.94, 0.94	0.94, 0.95	0.94, 0.94	0.93, 0.95	0.94, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 2 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.73: Expanded Results, Coverage for $\phi_{0,21}$ and q_{21}, Parameter Set 4

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	0.95, 0.91	0.92, 0.91	0.96, 0.93	0.94, 0.93	0.95, 0.86
	50	0.96, 0.95	0.94, 0.90	0.95, 0.94	0.95, 0.92	0.97, 0.96
	75	0.94, 0.94	0.95, 0.95	0.94, 0.94	0.95, 0.95	0.96, 0.96
	100	0.95, 0.93	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.95
	250	0.95, 0.94	0.95, 0.94	0.95, 0.94	0.95, 0.94	0.94, 0.95
	1000	0.94, 0.95	0.96, 0.96	0.94, 0.95	0.96, 0.96	0.95, 0.96
4	25	0.95, 0.94	0.94, 0.93	0.95, 0.94	0.94, 0.94	0.97, 0.96
	50	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95
	75	0.96, 0.93	0.95, 0.94	0.96, 0.94	0.94, 0.95	0.95, 0.95
	100	0.95, 0.94	0.94, 0.94	0.94, 0.94	0.94, 0.94	0.95, 0.94
	250	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.95
	1000	0.94, 0.95	0.93, 0.94	0.95, 0.96	0.94, 0.95	0.94, 0.95
8	25	0.93, 0.95	0.94, 0.96	0.94, 0.95	0.94, 0.96	0.96, 0.96
	50	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.96, 0.95
	75	0.95, 0.94	0.95, 0.95	0.95, 0.93	0.95, 0.94	0.96, 0.95
	100	0.94, 0.94	0.96, 0.94	0.94, 0.95	0.95, 0.94	0.96, 0.95
	250	0.96, 0.95	0.95, 0.95	0.96, 0.95	0.95, 0.95	0.95, 0.94
	1000	0.94, 0.94	0.93, 0.93	0.95, 0.95	0.94, 0.94	0.94, 0.94

Note. P = parameter. T = number of time points. N = sample size. Parameter set 4 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.74: Expanded Results, Coverage for $\phi_{0,21}$ and q_{21}, Parameter Set 5

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	0.95, 0.93	0.94, 0.93	0.95, 0.93	0.94, 0.93	0.97, 0.96
	50	0.95, 0.94	0.94, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.95
	75	0.95, 0.95	0.93, 0.95	0.95, 0.95	0.94, 0.94	0.94, 0.96
	100	0.94, 0.94	0.95, 0.93	0.94, 0.94	0.96, 0.93	0.96, 0.94
	250	0.95, 0.95	0.95, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.94
	1000	0.96, 0.95	0.94, 0.95	0.96, 0.95	0.94, 0.94	0.94, 0.95
4	25	0.94, 0.94	0.95, 0.92	0.95, 0.94	0.95, 0.93	0.97, 0.95
	50	0.94, 0.94	0.94, 0.95	0.94, 0.94	0.94, 0.95	0.95, 0.95
	75	0.96, 0.95	0.96, 0.94	0.96, 0.95	0.96, 0.95	0.96, 0.94
	100	0.95, 0.94	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95
	250	0.95, 0.95	0.96, 0.94	0.95, 0.95	0.95, 0.94	0.95, 0.94
	1000	0.94, 0.96	0.95, 0.95	0.94, 0.96	0.94, 0.95	0.95, 0.95
8	25	0.95, 0.93	0.93, 0.96	0.95, 0.93	0.93, 0.95	0.96, 0.96
	50	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.97, 0.96
	75	0.96, 0.94	0.94, 0.93	0.96, 0.94	0.94, 0.93	0.95, 0.94
	100	0.94, 0.94	0.97, 0.94	0.94, 0.94	0.97, 0.94	0.96, 0.93
	250	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.94, 0.95
	1000	0.96, 0.95	0.94, 0.95	0.96, 0.95	0.95, 0.95	0.94, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 5 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.75: Expanded Results, Coverage for $\phi_{0,21}$ and q_{21}, Parameter Set 6

		EDM-SEM		Oversampling		Bayesian
		Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
T	N	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}	$\phi_{0,21}$, q_{21}
2	25	0.93, 0.92	0.93, 0.92	0.94, 0.93	0.94, 0.93	0.98, 0.92
	50	0.95, 0.94	0.94, 0.95	0.95, 0.94	0.94, 0.95	0.95, 0.97
	75	0.94, 0.94	0.95, 0.94	0.95, 0.95	0.95, 0.95	0.96, 0.96
	100	0.94, 0.95	0.97, 0.96	0.94, 0.95	0.96, 0.96	0.97, 0.96
	250	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
	1000	0.96, 0.96	0.95, 0.94	0.96, 0.96	0.96, 0.94	0.96, 0.94
4	25	0.94, 0.94	0.93, 0.94	0.94, 0.95	0.94, 0.95	0.97, 0.96
	50	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.96, 0.95	0.96, 0.95
	75	0.94, 0.95	0.95, 0.96	0.94, 0.95	0.95, 0.95	0.95, 0.95
	100	0.96, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.96, 0.95
	250	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.94
	1000	0.94, 0.95	0.95, 0.95	0.94, 0.95	0.95, 0.95	0.95, 0.95
8	25	0.94, 0.94	0.93, 0.94	0.93, 0.95	0.94, 0.95	0.96, 0.95
	50	0.94, 0.96	0.94, 0.94	0.94, 0.96	0.95, 0.94	0.95, 0.95
	75	0.94, 0.96	0.95, 0.95	0.94, 0.96	0.95, 0.95	0.95, 0.96
	100	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95	0.95, 0.95
	250	0.95, 0.95	0.95, 0.96	0.95, 0.95	0.95, 0.96	0.96, 0.96
	1000	0.92, 0.94	0.92, 0.93	0.94, 0.95	0.93, 0.94	0.95, 0.95

Note. P = parameter. T = number of time points. N = sample size. Parameter set 6 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.76: Expanded Results, Model Fit, Parameter Set 1

			EDM-SEM		Oversampling		Bayesian
Pr.	T	N	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
.05	4	25	0.222	0.236	0.205	0.218	0.562
		50	0.119	0.119	0.098	0.110	0.596
		75	0.098	0.087	0.084	0.085	0.599
		100	0.090	0.088	0.080	0.078	0.598
		250	0.063	0.054	0.062	0.061	0.611
		1000	0.062	0.042	0.061	0.046	0.608
	8	25	0.892	0.884	0.880	0.894	0.281
		50	0.347	0.383	0.329	0.354	0.291
		75	0.192	0.211	0.186	0.204	0.312
		100	0.148	0.160	0.146	0.157	0.334
		250	0.083	0.071	0.082	0.074	0.317
		1000	0.074	0.056	0.073	0.052	0.327
.01	4	25	0.096	0.084	0.082	0.068	0.242
		50	0.045	0.037	0.024	0.022	0.263
		75	0.034	0.018	0.023	0.017	0.264
		100	0.025	0.035	0.016	0.028	0.268
		250	0.010	0.012	0.008	0.011	0.255
		1000	0.012	0.006	0.011	0.004	0.276
	8	25	0.749	0.727	0.731	0.725	0.087
		50	0.171	0.163	0.153	0.141	0.100
		75	0.072	0.074	0.064	0.061	0.106
		100	0.046	0.048	0.044	0.045	0.131
		250	0.022	0.021	0.021	0.020	0.108
		1000	0.011	0.013	0.010	0.012	0.135

Note. Pr. = probability cutoff value. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.77: Expanded Results, Model Fit, Parameter Set 2

			EDM-SEM		Oversampling		Bayesian
Pr.	T	N	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
.05	4	25	0.247	0.214	0.226	0.188	0.548
		50	0.129	0.130	0.106	0.103	0.598
		75	0.107	0.117	0.081	0.096	0.595
		100	0.098	0.090	0.063	0.068	0.617
		250	0.064	0.077	0.056	0.059	0.610
		1000	0.044	0.056	0.042	0.053	0.607
	8	25	0.873	0.894	0.880	0.894	0.263
		50	0.363	0.370	0.354	0.354	0.325
		75	0.215	0.230	0.200	0.218	0.327
		100	0.146	0.164	0.139	0.160	0.331
		250	0.066	0.074	0.066	0.074	0.331
		1000	0.043	0.059	0.041	0.065	0.364
.01	4	25	0.104	0.110	0.081	0.077	0.236
		50	0.058	0.052	0.033	0.028	0.276
		75	0.042	0.046	0.020	0.025	0.274
		100	0.043	0.045	0.015	0.024	0.284
		250	0.019	0.028	0.010	0.013	0.265
		1000	0.009	0.013	0.007	0.012	0.284
	8	25	0.725	0.776	0.717	0.765	0.087
		50	0.180	0.178	0.163	0.157	0.112
		75	0.095	0.083	0.080	0.069	0.098
		100	0.040	0.057	0.033	0.046	0.111
		250	0.013	0.019	0.013	0.017	0.122
		1000	0.006	0.018	0.004	0.015	0.140

Note. Pr. = probability cutoff value. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.18 & 0.04 \\ 0.06 & -0.29 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Table B.78: Expanded Results, Model Fit, Parameter Set 3

			EDM-SEM		Oversampling		Bayesian
Pr.	T	N	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
.05	4	25	0.205	0.219	0.193	0.201	0.554
		50	0.115	0.103	0.109	0.102	0.608
		75	0.104	0.109	0.092	0.107	0.598
		100	0.091	0.081	0.077	0.073	0.600
		250	0.071	0.079	0.063	0.058	0.580
		1000	0.056	0.070	0.052	0.061	0.576
	8	25	0.893	0.893	0.879	0.898	0.239
		50	0.343	0.364	0.337	0.352	0.304
		75	0.183	0.196	0.173	0.183	0.342
		100	0.155	0.162	0.143	0.160	0.323
		250	0.068	0.079	0.064	0.075	0.354
		1000	0.054	0.062	0.054	0.058	0.332
.01	4	25	0.068	0.079	0.051	0.070	0.240
		50	0.034	0.032	0.025	0.028	0.267
		75	0.025	0.030	0.019	0.025	0.284
		100	0.029	0.025	0.018	0.017	0.272
		250	0.018	0.024	0.010	0.009	0.258
		1000	0.015	0.019	0.011	0.014	0.264
	8	25	0.756	0.747	0.742	0.748	0.076
		50	0.129	0.151	0.122	0.148	0.107
		75	0.071	0.062	0.061	0.055	0.130
		100	0.041	0.048	0.033	0.039	0.099
		250	0.019	0.014	0.015	0.007	0.110
		1000	0.013	0.015	0.012	0.013	0.111

Note. Pr. = probability cutoff value. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & 0.25 \\ 0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.79: Expanded Results, Model Fit, Parameter Set 5

			EDM-SEM		Oversampling		Bayesian
Pr.	T	N	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
.05	4	25	0.203	0.226	0.195	0.214	0.564
		50	0.089	0.118	0.083	0.115	0.557
		75	0.087	0.095	0.077	0.091	0.624
		100	0.094	0.087	0.078	0.079	0.599
		250	0.053	0.081	0.046	0.073	0.621
		1000	0.059	0.066	0.056	0.066	0.598
	8	25	0.884	0.886	0.884	0.890	0.268
		50	0.375	0.329	0.366	0.323	0.308
		75	0.208	0.180	0.199	0.180	0.311
		100	0.155	0.155	0.147	0.148	0.298
		250	0.091	0.085	0.088	0.077	0.339
		1000	0.060	0.052	0.057	0.051	0.362
.01	4	25	0.091	0.082	0.075	0.069	0.269
		50	0.024	0.031	0.018	0.027	0.246
		75	0.029	0.023	0.021	0.018	0.278
		100	0.029	0.029	0.018	0.018	0.264
		250	0.012	0.021	0.006	0.011	0.255
		1000	0.018	0.018	0.014	0.016	0.259
	8	25	0.745	0.720	0.744	0.714	0.070
		50	0.174	0.142	0.164	0.136	0.105
		75	0.067	0.059	0.058	0.059	0.113
		100	0.046	0.050	0.040	0.046	0.107
		250	0.023	0.024	0.018	0.014	0.100
		1000	0.016	0.009	0.012	0.006	0.124

Note. Pr. = probability cutoff value. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 0.30 & 0.09 \\ 0.09 & 0.30 \end{smallmatrix}\right]$.

Table B.80: Expanded Results, Model Fit, Parameter Set 6

			EDM-SEM		Oversampling		Bayesian
Pr.	T	N	Population Starts	Perturbed Starts	Population Starts	Perturbed Starts	Perturbed Starts
.05	4	25	0.242	0.223	0.218	0.213	0.586
		50	0.126	0.110	0.115	0.097	0.603
		75	0.118	0.085	0.106	0.079	0.597
		100	0.080	0.088	0.073	0.070	0.602
		250	0.067	0.072	0.064	0.062	0.595
		1000	0.065	0.055	0.052	0.047	0.583
	8	25	0.895	0.884	0.891	0.892	0.266
		50	0.358	0.375	0.345	0.360	0.316
		75	0.215	0.204	0.202	0.189	0.310
		100	0.168	0.168	0.152	0.147	0.337
		250	0.075	0.096	0.067	0.085	0.346
		1000	0.057	0.078	0.051	0.069	0.358
.01	4	25	0.100	0.080	0.074	0.074	0.247
		50	0.041	0.033	0.028	0.016	0.266
		75	0.039	0.031	0.027	0.023	0.265
		100	0.024	0.030	0.017	0.014	0.288
		250	0.015	0.026	0.014	0.016	0.254
		1000	0.019	0.016	0.009	0.009	0.232
	8	25	0.743	0.754	0.736	0.754	0.080
		50	0.170	0.177	0.156	0.159	0.097
		75	0.085	0.079	0.072	0.062	0.104
		100	0.059	0.063	0.045	0.042	0.113
		250	0.030	0.032	0.020	0.020	0.109
		1000	0.018	0.019	0.012	0.010	0.136

Note. Pr. = probability cutoff value. T = number of time points. N = sample size. Parameter set 1 values: $\mu_0 = \left[\begin{smallmatrix} 0.00 \\ 0.00 \end{smallmatrix}\right]$; $\Phi_0 = \left[\begin{smallmatrix} 2.00 & 0.60 \\ 0.60 & 2.00 \end{smallmatrix}\right]$; $\mathbf{A} = \left[\begin{smallmatrix} -0.76 & -0.25 \\ -0.17 & -0.43 \end{smallmatrix}\right]$; $\mathbf{b} = \left[\begin{smallmatrix} 0.10 \\ 0.10 \end{smallmatrix}\right]$; $\mathbf{Q} = \left[\begin{smallmatrix} 1.00 & 0.30 \\ 0.30 & 1.00 \end{smallmatrix}\right]$.

Appendix C

JAGS/rjags Code for Bayesian Approach

In this appendix, code used to implement the bivariate model used in the simulation is presented. `rjags` code is provided first and followed by the model script. The data are assumed to be contained in a data frame called `mydata` with variables `x` and `y` ordered by time (e.g., `x1`, `y1`, `x2`, `y2`, `x3`, `y3`, `x4`, `y4`). Data and other important quantities are read into a list called `jagsdata`, which `rjags` uses in tandem with the model script ("ct_bivariate.bug") to compile the model and sample from the posterior distribution.

C.1 `rjags` Code

```
## N = Number of individuals
## T = Number of time points
## dat = Data frame
## Ismall = Small identity matrix
## Ibig = Big identity matrix
## Zero = Zero vector
## mu0mean = Mean hyperparameters for mu0
## mu0cov = Covariance hyperparameters for mu0
## bmean = Mean hyperparameters for b
## bcov = Covariance hyperparameters for b

## Define data and other quantities
jagsdata <- list()
jagsdata[["N"]] <- nrow(mydata)
jagsdata[["T"]] <- ncol(mydata) / 2
jagsdata[["dat"]] <- data
```

```
jagsdata[["Ismall"]] <- diag(2)
jagsdata[["Ibig"]] <- diag(4)
jagsdata[["Zero"]] <- matrix(0, nr = 2, nc = 2)
jagsdata[["mu0mean"]] <- matrix(c(0, 0), nr = 2, nc = 1)
jagsdata[["mu0cov"]] <- matrix(c(.001, 0, 0, .001), nr = 2, nc = 2)
jagsdata[["bmean"]] <- matrix(c(.1, .1), nr = 2, nc = 1)
jagsdata[["bcov"]] <- matrix(c(.001, 0, 0, .001), nr = 2, nc = 2)

## Compile model
BayesModel <- jags.model("ct_bivariate.bug", data = jagsdata,
                     n.chains = 2, n.adapt = 1500)
## Sample from posterior
BayesOut <- coda.samples(model = BayesModel,
                 variable.names = c("mu0", "Phi0", "A", "b", "rowQ"),
                 n.iter = 3000, thin = 1)

## Extract model results
summary(BayesOut)
```

C.2 JAGS Model Script

```
##-----------------------------------------------------##
##         Bivariate EDM model with panel data         ##
##-----------------------------------------------------##

## Dimension definitions
var mu0[2,1], Phi0[2,2], A[2,2], b[2,1],
    Ainv[2,2], Acof[2,2], Apdinv[2,2], Apdcof[2,2],
    B[2,2], C[2,2], D[2,2],
    DCAB[2,2], DCABcof[2,2], DCABinv[2,2], iApd[2*2,2*2],
    AI[2*2,2*2], IA[2*2,2*2], Apd[2*2,2*2], rowQ[2*2,1],
    rowQdel[2*2,1], Qdel[2,2],
    fixed[N,(2*T)]

## Model
model {
  for (i in 1:N) {
    dat[i,1:2] ~ dmnorm(mu0, inverse(Phi0))
    for (j in 1:(T-1)) {
      dat[i,((2*j)+1):((2*j)+2)] ~
       dmnorm(fixed[i,((2*j)+1):((2*j)+2)], inverse(Qdel))
```

```
    fixed[i,((2*j)+1):((2*j)+2)] <- mexp(A) %*%
    dat[i,(((2*j)+1)-2):(2*j)] + Ainv %*% (mexp(A) - Ismall) %*% b
  }
}

##-----------##
## Inverse A ##
##-----------##

Acof[1,1] <- A[2,2]
Acof[2,1] <- -A[2,1]
Acof[1,2] <- -A[1,2]
Acof[2,2] <- A[1,1]
Ainv <- (1 / (Acof[2,2] * Acof[1,1] - Acof[1,2] * Acof[2,1])) * Acof

##-----------------------------------------##
## Continuous-time Error Covariance Matrix Q ##
##-----------------------------------------##

##--- A %x% I ---#
AI[1:2, 1:2] <- A[1,1] * Ismall
AI[1:2, 3:4] <- A[1,2] * Ismall
AI[3:4, 1:2] <- A[2,1] * Ismall
AI[3:4, 3:4] <- A[2,2] * Ismall

##--- I %x% A ---#
IA[1:2,1:2] <- A
IA[3:4,3:4] <- A

IA[1:2,3:4] <- Zero
IA[3:4,1:2] <- Zero

##--- Apd = A %x% I + I %x% A ---#
Apd <- AI + IA

#--- Apd Inverse ---#
Apdcof[1,1] <- Apd[2,2]
Apdcof[2,1] <- -Apd[2,1]
Apdcof[1,2] <- -Apd[1,2]
Apdcof[2,2] <- Apd[1,1]
Apdinv <- (1 / (Apdcof[2,2] * Apdcof[1,1] -
Apdcof[1,2] * Apdcof[2,1])) * Apdcof

B <- Apd[1:2,3:4]
C <- Apd[3:4,1:2]
```

```
D <- Apd[3:4,3:4]

DCAB <- D - (C %*% Apdinv %*% B)
DCABcof[1,1] <- DCAB[2,2]
DCABcof[2,1] <- -DCAB[2,1]
DCABcof[1,2] <- -DCAB[1,2]
DCABcof[2,2] <- DCAB[1,1]
DCABinv <- (1 / (DCABcof[2,2] * DCABcof[1,1] -
  DCABcof[1,2] * DCABcof[2,1])) * DCABcof

iApd[1:2,1:2] <- Apdinv + Apdinv %*% B %*% DCABinv %*% C %*% Apdinv
iApd[3:4,1:2] <- -DCABinv %*% C %*% Apdinv
iApd[1:2,3:4] <- -Apdinv %*% B %*% DCABinv
iApd[3:4,3:4] <- DCABinv

##--- Qdel ---#
rowQdel <- iApd %*% (mexp(Apd) - Ibig) %*% rowQ
Qdel[1:2,1] <- rowQdel[1:2,1]
Qdel[1:2,2] <- rowQdel[3:4,1]

##--------##
## Priors ##
##--------##

##--- Priors for mu0 ---##
mu0 ~ dmnorm(mu0mean, mu0cov)

##--- Priors for Phi0 ---##
Phi0[1,1] <- pow(sigma.p1, 2)
Phi0[2,2] <- pow(sigma.p2, 2)
sigma.p1 ~ dunif(0, 100)
sigma.p2 ~ dunif(0, 100)

Phi0[2,1] <- sigma.p1 * sigma.p2 * rho.p
Phi0[1,2] <- Phi0[2,1]
rho.p ~ dunif(-1, 1)

##--- Priors for A ---##
A[1,1] ~ dunif(-7, 0)
A[2,1] ~ dnorm(0, .001)
A[1,2] ~ dnorm(0, .001)
A[2,2] ~ dunif(-7, 0)

##--- Priors for b ---##
b ~ dmnorm(bmean, bcov)
```

```
  ##--- Priors for Q ---#
  rowQ[1,1] <- pow(sigma.q1, 2)
  rowQ[2,1] <- sigma.q1 * sigma.q2 * rho.q
  rowQ[3,1] <- rowQ[2,1]
  rowQ[4,1] <- pow(sigma.q2, 2)

  sigma.q1 ~ dunif(0, 100)
  sigma.q2 ~ dunif(0, 100)
  rho.q ~ dunif(-1, 1)
}
```