## Direct Limits on the $\boldsymbol{B}_{s}^{\mathbf{0}}$ Oscillation Frequency

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We report results of a study of the $B_{s}^{0}$ oscillation frequency using a large sample of $B_{s}^{0}$ semileptonic decays corresponding to approximately $1 \mathrm{fb}^{-1}$ of integrated luminosity collected by the D 0 experiment at


#### Abstract

the Fermilab Tevatron Collider in 2002-2006. The amplitude method gives a lower limit on the $B_{s}^{0}$ oscillation frequency at $14.8 \mathrm{ps}^{-1}$ at the $95 \%$ C.L. At $\Delta m_{s}=19 \mathrm{ps}^{-1}$, the amplitude deviates from the hypothesis $\mathcal{A}=0(1)$ by 2.5 (1.6) standard deviations, corresponding to a two-sided C.L. of $1 \%$ ( $10 \%$ ). A likelihood scan over the oscillation frequency, $\Delta m_{s}$, gives a most probable value of $19 \mathrm{ps}^{-1}$ and a range of $17<\Delta m_{s}<21 \mathrm{ps}^{-1}$ at the $90 \%$ C.L., assuming Gaussian uncertainties. This is the first direct two-sided bound measured by a single experiment. If $\Delta m_{s}$ lies above $22 \mathrm{ps}^{-1}$, then the probability that it would produce a likelihood minimum similar to the one observed in the interval $16-22 \mathrm{ps}^{-1}$ is $(5.0 \pm 0.3) \%$.


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Measurements of flavor oscillations in the $B_{d}^{0}$ and $B_{s}^{0}$ systems provide important constraints on the Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle and the source of $C P$ violation in the standard model (SM) [1]. The phenomenon of $B_{d}^{0}$ oscillations is well established [2], with a precisely measured oscillation frequency $\Delta m_{d}$. In the SM , this parameter is proportional to the combination $\left|V_{\mathrm{tb}}^{*} V_{\mathrm{td}}\right|^{2}$ of CKM matrix elements. Since the matrix element $V_{\mathrm{ts}}$ is larger than $V_{\mathrm{td}}$, the expected frequency $\Delta m_{s}$ is higher. As a result, $B_{s}^{0}$ oscillations have not been observed by any previous experiment and the current $95 \%$ C.L. lower limit on $\Delta m_{s}$ is $16.6 \mathrm{ps}^{-1}$ [2]. A measurement of $\Delta m_{s}$ would yield the ratio $\left|V_{\mathrm{ts}} / V_{\mathrm{td}}\right|$, which has a smaller uncertainty than $\left|V_{\mathrm{td}}\right|$ alone due to the cancellation of certain theory uncertainties. If the SM is correct, and if current limits on $B_{s}^{0}$ oscillations are not included, then global fits to the unitarity triangle favor $\Delta m_{s}=$ $20.9_{-4.2}^{+4.5} \mathrm{ps}^{-1}$ [3] or $\Delta m_{s}=21.2 \pm 3.2 \mathrm{ps}^{-1}$ [4].

In this Letter, we present a study of $B_{s}^{0}-\bar{B}_{s}^{0}$ oscillations carried out using semileptonic $B_{s}^{0} \rightarrow \mu^{+} D_{s}^{-} X$ decays [5] collected by the D 0 experiment at Fermilab in $p \bar{p}$ collisions at $\sqrt{s}=1.96 \mathrm{TeV}$. In the $B_{s}^{0}-\bar{B}_{s}^{0}$ system there are two mass eigenstates, the heavier (lighter) one having mass $M_{H}$ $\left(M_{L}\right)$ and decay width $\Gamma_{H}\left(\Gamma_{L}\right)$. Denoting $\Delta m_{s}=M_{H}-$ $M_{L}, \quad \Delta \Gamma_{s}=\Gamma_{L}-\Gamma_{H}, \quad \Gamma_{s}=\left(\Gamma_{L}+\Gamma_{H}\right) / 2$, the timedependent probability $P$ that an initial $B_{s}^{0}$ decays at time $t$ as $B_{s}^{0} \rightarrow \mu^{+} X\left(P^{\text {nos }}\right)$ or $\bar{B}_{s}^{0} \rightarrow \mu^{-} X\left(P^{\text {osc }}\right)$ is given by $P^{\mathrm{nos} / \text { osc }}=e^{-\Gamma_{s} t}\left(1 \pm \cos \Delta m_{s} t\right) / 2$, assuming that $\Delta \Gamma_{s} / \Gamma_{s}$ is small and neglecting $C P$ violation. Flavor tagging a $b$ $(\bar{b})$ on the opposite side from the signal meson establishes the signal meson as a $B_{s}^{0}\left(\bar{B}_{s}^{0}\right)$ at time $t=0$.

The D0 detector is described in detail elsewhere [6]. Charged particles are reconstructed using the central tracking system which consists of a silicon microstrip tracker (SMT) and a central fiber tracker (CFT), both located within a 2 T superconducting solenoidal magnet. Electrons are identified by the preshower and liquid-argon and uranium calorimeter. Muons are identified by the muon system which consists of a layer of tracking detectors and scintillation trigger counters in front of 1.8 T iron toroids, followed by two similar layers after the toroids [7].

No explicit trigger requirement was made, although most of the sample was collected with single muon triggers. The decay chain $B_{s}^{0} \rightarrow \mu^{+} D_{s}^{-} X, D_{s}^{-} \rightarrow \phi \pi^{-}, \phi \rightarrow$ $K^{+} K^{-}$was then reconstructed. The charged tracks were required to have signals in both the CFT and SMT. Muons were required to have transverse momentum $p_{T}\left(\mu^{+}\right)>$

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$2 \mathrm{GeV} / c$ and momentum $p\left(\mu^{+}\right)>3 \mathrm{GeV} / c$, and to have measurements in at least two layers of the muon system. All charged tracks in the event were clustered into jets [8], and the $D_{s}^{-}$candidate was reconstructed from three tracks found in the same jet as the reconstructed muon. Oppositely charged particles with $p_{T}>0.7 \mathrm{GeV} / c$ were assigned the kaon mass and were required to have an invariant mass $1.004<M\left(K^{+} K^{-}\right)<1.034 \mathrm{GeV} / c^{2}$, consistent with that of a $\phi$ meson. The third track was required to have $p_{T}>0.5 \mathrm{GeV} / c$ and charge opposite to that of the muon charge and was assigned the pion mass. The three tracks were required to form a common $D_{s}^{-}$vertex using the algorithm described in Ref. [9]. To reduce combinatorial background, the $D_{s}^{-}$vertex was required to have a positive displacement in the transverse plane, relative to the $p \bar{p}$ collision point [or primary vertex (PV)], with at least $4 \sigma$ significance. The cosine of the angle between the $D_{s}^{-}$momentum and the direction from the PV to the $D_{s}^{-}$ vertex was required to be greater than 0.9. The trajectories of the muon and $D_{s}^{-}$candidates were required to originate from a common $B_{s}^{0}$ vertex, and the $\mu^{+} D_{s}^{-}$system was required to have an invariant mass between 2.6 and $5.4 \mathrm{GeV} / c^{2}$.

To further improve $B_{s}^{0}$ signal selection, a likelihood ratio method [10] was utilized. Using $M\left(K^{+} K^{-} \pi\right)$ sideband ( $B$ ) and sideband-subtracted signal $(S)$ distributions in the data, probability density functions (PDFs) were found for a number of discriminating variables: the helicity angle between the $D_{s}^{-}$and $K^{ \pm}$momenta in the $\phi$ center-of-mass frame, the isolation of the $\mu^{+} D_{s}^{-}$system, the $\chi^{2}$ of the $D_{s}^{-}$ vertex, the invariant masses $M\left(\mu^{+} D_{s}^{-}\right)$and $M\left(K^{+} K^{-}\right)$, and $p_{T}\left(K^{+} K^{-}\right)$. The final requirement on the combined selection likelihood ratio variable, $y_{\text {sel }}$, was chosen to maximize the predicted ratio $S / \sqrt{S+B}$. The total number of $D_{s}^{-}$candidates after these requirements was $N_{\text {tot }}=$ $26710 \pm 556$ (stat), as shown in Fig. 1(a).

The performance of the opposite-side flavor tagger (OST) [11] is characterized by the efficiency $\epsilon=$ $N_{\text {tag }} / N_{\text {tot }}$, where $N_{\text {tag }}$ is the number of tagged $B_{s}^{0}$ mesons; the purity $\eta_{s}$, defined as $\eta_{s}=N_{\text {cor }} / N_{\text {tag }}$, where $N_{\text {cor }}$ is the number of $B_{s}^{0}$ mesons with correct flavor identification; and the dilution, related to purity as $\mathcal{D} \equiv 2 \eta_{s}-1$. Again, a likelihood ratio method was used. In the construction of the flavor discriminating variables $x_{1}, \ldots, x_{n}$ for each event, an object, either a lepton $\ell$ (electron or muon) or a reconstructed secondary vertex (SV), was defined to be on the opposite side from the $B_{s}^{0}$ meson if it satisfied


FIG. 1 (color online). $\left(K^{+} K^{-}\right) \pi^{-}$invariant mass distribution (a) for the untagged $B_{s}^{0}$ sample, and (b) for candidates that have been flavor tagged. The left and right peaks correspond to $\mu^{+} D^{-}$ and $\mu^{+} D_{s}^{-}$candidates, respectively. The curve is a result of fitting a signal plus background model to the data.
$\cos \varphi\left(\vec{p}_{\text {eorSv }}, \vec{p}_{B}\right)<0.8$, where $\vec{p}_{B}$ is the reconstructed three-momentum of the $B_{s}^{0}$ meson, and $\varphi$ is the azimuthal angle about the beam axis. A lepton jet charge was formed as $Q_{J}^{\ell}=\sum_{i} q^{i} p_{T}^{i} / \sum_{i} p_{T}^{i}$, where all charged particles are summed, including the lepton, inside a cone of $\Delta R=$ $\sqrt{(\Delta \varphi)^{2}+(\bar{\Delta})^{2}}<0.5$ centered on the lepton. The SV charge was defined as $Q_{\mathrm{SV}}=\sum_{i}\left(q^{i} p_{L}^{i}\right)^{0.6} / \sum_{i}\left(p_{L}^{i}\right)^{0.6}$, where all charged particles associated with the SV are summed, and $p_{L}^{i}$ is the longitudinal momentum of track $i$ with respect to the direction of the SV momentum. Finally, event charge is defined as $Q_{\mathrm{EV}}=\sum_{i} q^{i} p_{T}^{i} / \sum_{i} p_{T}^{i}$, where the sum is over all tracks with $p_{T}>0.5 \mathrm{GeV} / c$ outside a cone of $\Delta R>1.5$ centered on the $B_{s}^{0}$ direction. The PDF of each discriminating variable was found for $b$ and $\bar{b}$ quarks using a large data sample of $B^{+} \rightarrow \mu^{+} \nu \bar{D}^{0}$ events where the initial state is known from the charge of the decay muon.
For an initial $b(\bar{b})$ quark, the PDF for a given variable $x_{i}$ is denoted $f_{i}^{b}\left(x_{i}\right)\left[f_{i}^{\bar{b}}\left(x_{i}\right)\right]$, and the combined tagging variable is defined as $d_{\text {tag }}=(1-z) /(1+z)$, where $z=$ $\prod_{i=1}^{n}\left(f_{i}^{\bar{b}}\left(x_{i}\right) / f_{i}^{b}\left(x_{i}\right)\right)$. The variable $d_{\mathrm{tag}}$ varies between -1 and 1 . An event with $d_{\mathrm{tag}}>0(<0)$ is tagged as a $b$ $(\bar{b})$ quark.

The OST purity was determined from large samples of $B^{+} \rightarrow \mu^{+} \bar{D}^{0} X \quad$ (nonoscillating) and $\quad B_{d}^{0} \rightarrow \mu^{+} D^{*-} X$ (slowly oscillating) semileptonic candidates. An average value of $\epsilon \mathcal{D}^{2}=\left[2.48 \pm 0.21(\text { stat })_{-0.06}^{+0.08}(\mathrm{syst})\right] \%$ was obtained [11]. The estimated event-by-event dilution as a function of $\left|d_{\text {tag }}\right|$ was determined by measuring $\mathcal{D}$ in bins of $\left|d_{\text {tag }}\right|$ and parametrizing with a third-order polynomial for $\left|d_{\text {tag }}\right|<0.6$. For $\left|d_{\text {tag }}\right|>0.6, \mathcal{D}$ is fixed to 0.6 .

The OST was applied to the $B_{s}^{0} \rightarrow \mu^{+} D_{s}^{-} X$ data sample, yielding $N_{\text {tag }}=5601 \pm 102$ (stat) candidates having an identified initial state flavor, as shown in Fig. 1(b). The tagging efficiency was ( $20.9 \pm 0.7$ ) \%.

After flavor tagging, the proper decay time of candidates is needed; however, the undetected neutrino and other missing particles in the semileptonic $B_{s}^{0}$ decay prevent a
precise determination of the meson's momentum and Lorentz boost. This represents an important contribution to the smearing of the proper decay length in semileptonic decays, in addition to the resolution effects. A correction factor $K$ was estimated from a Monte Carlo (MC) simulation by finding the distribution of $K=p_{T}\left(\mu^{+} D_{s}^{-}\right) / p_{T}(B)$ for a given decay channel in bins of $M\left(\mu^{+} D_{s}^{-}\right)$. The proper decay length of each $B_{s}^{0}$ meson is then $\operatorname{ct}\left(B_{s}^{0}\right)=l_{M} K$, where $l_{M}=M\left(B_{s}^{0}\right) \cdot\left(\vec{L}_{T} \cdot \vec{p}_{T}\left(\mu^{+} D_{s}^{-}\right)\right) /\left(p_{T}\left(\mu^{+} D_{s}^{-}\right)\right)^{2}$ is the measured visible proper decay length (VPDL), $\vec{L}_{T}$ is the vector from the PV to the $B_{s}^{0}$ decay vertex in the transverse plane and $M\left(B_{s}^{0}\right)=5.3696 \mathrm{GeV} / c^{2}[1]$.

All flavor-tagged events with $1.72<M\left(K^{+} K^{-} \pi^{-}\right)<$ $2.22 \mathrm{GeV} / c^{2}$ were used in an unbinned fitting procedure. The likelihood, $\mathcal{L}$, for an event to arise from a specific source in the sample depends event-by-event on $l_{M}$, its uncertainty $\sigma_{l_{M}}$, the invariant mass of the candidate $M\left(K^{+} K^{-} \pi^{-}\right)$, the predicted dilution $\mathcal{D}\left(d_{\mathrm{tag}}\right)$, and the selection variable $y_{\text {sel }}$. The PDFs for $\sigma_{l_{M}}, M\left(K^{+} K^{-} \pi^{-}\right)$, $\mathcal{D}\left(d_{\text {tag }}\right)$, and $y_{\text {sel }}$ were determined from data. Four sources were considered: the signal $\mu^{+} D_{s}^{-}\left(\rightarrow \phi \pi^{-}\right)$; the accompanying peak due to $\mu^{+} D^{-}\left(\rightarrow \phi \pi^{-}\right)$; a small (less than $1 \%$ ) reflection due to $\mu^{+} D^{-}\left(\rightarrow K^{+} \pi^{-} \pi^{-}\right)$, where the kaon mass is misassigned to one of the pions; and combinatorial background. The total fractions of the first two categories were determined from the mass fit of Fig. 1(b).

The $\mu^{+} D_{s}^{-}$signal sample is composed mostly of $B_{s}^{0}$ mesons with some contributions from $B_{d}^{0}$ and $B^{+}$mesons. Contributions of $b$ baryons to the sample were estimated to be small and were neglected. The data were divided into subsamples with and without oscillation as determined by the OST. The distribution of the VPDL $l$ for nonoscillated and oscillated cases was modeled appropriately for each type of $B$ meson, e.g., for $B_{s}^{0}$ :

$$
\begin{align*}
p_{s}^{\mathrm{nos} / \mathrm{osc}}\left(l, K, d_{\mathrm{tag}}\right)= & \frac{K}{c \tau_{B_{s}^{0}}} \exp \left(-\frac{K l}{c \tau_{B_{s}^{0}}}\right) \\
& \times\left[1 \pm \mathcal{D}\left(d_{\mathrm{tag}}\right) \cos \left(\Delta m_{s} K l / c\right)\right] / 2 \tag{1}
\end{align*}
$$

The world averages [1] of $\tau_{B_{d}^{0}}, \tau_{B^{+}}$, and $\Delta m_{d}$ were used as inputs to the fit. The lifetime, $\tau_{B_{s}^{0}}$, was allowed to float in the fit. In the amplitude and likelihood scans described below, $\tau_{B_{s}^{0}}$ was fixed to this fitted value, which agrees with expectations.

The total VPDL PDF for the $\mu^{+} D_{s}^{-}$signal is then the sum over all decay channels, including branching fractions, that yield the $D_{s}^{-}$mass peak. The $B_{s}^{0} \rightarrow \mu^{+} D_{s}^{-} X$ signal modes (including $D_{s}^{*-}, D_{s 0}^{*-}$, and $D_{s 1}^{\prime-}$; and $\mu^{+}$ originating from $\tau^{+}$decay) comprise ( $85.6 \pm 3.3$ ) \% of our sample, as determined from a MC simulation which included the PYTHIA generator v6.2 [12] interfaced with the EVTGEN decay package [13], followed by full GEANT v3.15 [14] modeling of the detector response and event reconstruction. Other backgrounds considered were decays via
$B_{s}^{0} \rightarrow D_{(s)}^{+} D_{s}^{-} X$ and $\bar{B}_{d}^{0}, B^{-} \rightarrow D D_{s}^{-}$, followed by $D_{(s)}^{+} \rightarrow$ $\mu^{+} X$, with a real $D_{s}^{-}$reconstructed in the peak and an associated real $\mu^{+}$. Another background taken into account occurs when the $D_{s}^{-}$meson originates from one $b$ or $c$ quark and the muon arises from another quark. This background peaks around the PV (peaking backgrounds). The uncertainty in each channel covers possible trigger efficiency biases. Translation from the true VPDL, $l$, to the measured $l_{M}$ for a given channel, is achieved by a convolution of the VPDL detector resolution, of $K$ factors over each normalized distribution, and by including the reconstruction efficiency as a function of VPDL. The lifetimedependent efficiency was found for each channel using MC simulations and, as a cross check, the efficiency was also determined from the data by fixing $\tau_{B_{s}^{0}}$ and fitting for the functional form of the efficiency. The shape of the VPDL distribution for peaking backgrounds was found from MC simulation, and the fraction from this source was allowed to float in the fit.

The VPDL uncertainty was determined from the vertex fit using track parameters and their uncertainties. To account for possible mismodeling of these uncertainties, resolution scale factors were introduced as determined by examining the pull distribution of the vertex positions of a sample of $J / \psi \rightarrow \mu^{+} \mu^{-}$decays. Using these scale factors, the convolving function for the VPDL resolution was the sum of two Gaussian functions with widths (fractions) of $0.998 \sigma_{l_{M}}(72 \%)$ and $1.775 \sigma_{l_{M}}(28 \%)$. A cross check was performed using a MC simulation with tracking errors tuned according to the procedure described in Ref. [15]. The $7 \%$ variation of scale factors found in this cross check was used to estimate systematic uncertainties due to decay length resolution.

Several contributions to the combinatorial backgrounds that have different VPDL distributions were considered. True prompt background was modeled with a Gaussian function with a separate scale factor on the width; background due to fake vertices around the PV was modeled with another Gaussian function; and long-lived background was modeled with an exponential function convoluted with the resolution, including a component oscillating with a frequency of $\Delta m_{d}$. The unbinned fit of the total tagged sample was used to determine the various fractions of signal and backgrounds and the background VPDL parametrizations.

Figure 2 shows the value of $-\Delta \log \mathcal{L}$ as a function of $\Delta m_{s}$, indicating a favored value of $19 \mathrm{ps}^{-1}$, while variation of $-\log \mathcal{L}$ from the minimum indicates an oscillation frequency of $17<\Delta m_{s}<21 \mathrm{ps}^{-1}$ at the $90 \%$ C.L. The uncertainties are approximately Gaussian inside this interval. The plateau of the likelihood curve shows the region where we do not have sufficient resolution to measure an oscillation, and if the true value of $\Delta m_{s}>22 \mathrm{ps}^{-1}$, our measured confidence interval does not make any statement about the frequency. Using 100 parametrized MC samples with similar statistics, VPDL resolution, overall tagging


FIG. 2 (color online). Value of $-\Delta \log \mathcal{L}$ as a function of $\Delta m_{s}$. Star symbols do not include systematic uncertainties, and the shaded band represents the envelope of all $\log \mathcal{L}$ scan curves due to different systematic uncertainties.
performance, and sample composition of the data sample, it was determined that for a true value of $\Delta m_{s}=19 \mathrm{ps}^{-1}$, the probability was $15 \%$ for measuring a value in the range $16<\Delta m_{s}<22 \mathrm{ps}^{-1}$ with a $-\Delta \log \mathcal{L}$ lower by at least 1.9 than the corresponding value at $\Delta m_{s}=25 \mathrm{ps}^{-1}$.

The amplitude method [16] was also used. Equation (1) was modified to include the oscillation amplitude $\mathcal{A}$ as an additional coefficient on the $\cos \left(\Delta m_{s} K l / c\right)$ term. The unbinned fit was repeated for fixed input values of $\Delta m_{s}$ and the fitted value of $\mathcal{A}$ and its uncertainty $\sigma_{\mathcal{A}}$ found for each step, as shown in Fig. 3. At $\Delta m_{s}=19 \mathrm{ps}^{-1}$ the measured data point deviates from the hypothesis $\mathcal{A}=0(\mathcal{A}=1)$ by 2.5 (1.6) standard deviations, corresponding to a twosided C.L. of $1 \%$ ( $10 \%$ ), and is in agreement with the likelihood results. In the presence of a signal, however, it is more difficult to define a confidence interval using the amplitude than by examining the $-\Delta \log \mathcal{L}$ curve. Since, on average, these two methods give the same results, we chose to quantify our $\Delta m_{s}$ interval using the likelihood curve.


FIG. 3 (color online). $\quad B_{s}^{0}$ oscillation amplitude as a function of oscillation frequency, $\Delta m_{s}$. The solid line shows the $\mathcal{A}=1$ axis for reference. The dashed line shows the expected limit including both statistical and systematic uncertainties.

Systematic uncertainties were addressed by varying inputs, cut requirements, branching ratios, and PDF modeling. The branching ratios were varied within known uncertainties [1] and large variations were taken for those not yet measured. The $K$-factor distributions were varied within uncertainties, using measured (or smoothed) instead of generated momenta in the MC simulation. The fractions of peaking and combinatorial backgrounds were varied within uncertainties. Uncertainties in the reflection contribution were considered. The functional form to determine the dilution $\mathcal{D}\left(d_{\text {tag }}\right)$ was varied. The lifetime $\tau_{B_{s}^{0}}$ was fixed to its world average value, and $\Delta \Gamma_{s}$ was allowed to be nonzero. The scale factors on the signal and background resolutions were varied within uncertainties, and typically generated the largest systematic uncertainty in the region of interest. A separate scan of $-\Delta \log \mathcal{L}$ was taken for each variation, and the envelope of all such curves is indicated as the band in Fig. 2. The same systematic uncertainties were considered for the amplitude method using the procedure of Ref. [16], and, when added in quadrature with the statistical uncertainties, represent a small effect, as shown in Fig. 3. Taking these systematic uncertainties into account, we obtain from the amplitude method an expected limit of $14.1 \mathrm{ps}^{-1}$ and an observed lower limit of $\Delta m_{s}>$ $14.8 \mathrm{ps}^{-1}$ at the $95 \%$ C.L., consistent with the likelihood scan.

The probability that $B_{s}^{0}-\bar{B}_{s}^{0}$ oscillations with the true value of $\Delta m_{s}>22 \mathrm{ps}^{-1}$ would give a $-\Delta \log \mathcal{L}$ minimum in the range $16<\Delta m_{s}<22 \mathrm{ps}^{-1}$ with a depth of more than 1.7 with respect to the $-\Delta \log \mathcal{L}$ value at $\Delta m_{s}=$ $25 \mathrm{ps}^{-1}$, corresponding to our observation including systematic uncertainties, was found to be $(5.0 \pm 0.3) \%$. This range of $\Delta m_{s}$ was chosen to encompass the world average lower limit and the edge of our sensitive region. To determine this probability, an ensemble test using the data sample was performed by randomly assigning a flavor to each candidate while retaining all its other information, effectively simulating a $B_{s}^{0}$ oscillation with an infinite frequency. Similar probabilities were found using ensembles of parametrized MC events.

In summary, a study of $B_{s}^{0}-\bar{B}_{s}^{0}$ oscillations was performed using $B_{s}^{0} \rightarrow \mu^{+} D_{s}^{-} X$ decays, where $D_{s}^{-} \rightarrow \phi \pi^{-}$ and $\phi \rightarrow K^{+} K^{-}$, an opposite-side flavor tagging algorithm, and an unbinned likelihood fit. The amplitude method gives an expected limit of $14.1 \mathrm{ps}^{-1}$ and an observed lower limit of $\Delta m_{s}>14.8 \mathrm{ps}^{-1}$ at the $95 \%$ C.L. At $\Delta m_{s}=19 \mathrm{ps}^{-1}$, the amplitude method yields a result that deviates from the hypothesis $\mathcal{A}=0(\mathcal{A}=1)$ by 2.5 (1.6) standard deviations, corresponding to a two-sided C.L. of $1 \%(10 \%)$. The likelihood curve is well behaved near a preferred value of $19 \mathrm{ps}^{-1}$ with a $90 \%$ C.L. interval of $17<\Delta m_{s}<21 \mathrm{ps}^{-1}$, assuming Gaussian uncertainties. The lower edge of the confidence level interval is near the world average $95 \%$ C.L. lower limit $\Delta m_{s}>16.6 \mathrm{ps}^{-1}$ [2]. Ensemble tests indicate that if $\Delta m_{s}$ lies above the
sensitive region, i.e., above approximately $22 \mathrm{ps}^{-1}$, there is a $(5.0 \pm 0.3) \%$ probability that it would produce a likelihood minimum similar to the one observed in the interval $16<\Delta m_{s}<22 \mathrm{ps}^{-1}$. This is the first report of a direct two-sided bound measured by a single experiment on the $B_{s}^{0}$ oscillation frequency.

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