ARTICLES

Shape studies of quark jets versus gluon jets at $\sqrt{s}=10$ GeV

M. S. Alam,[a] I. J. Kim,[a] B. Nemati,[a] J. J. O'Neill,[a] V. Romero,[a] H. Severini,[a] C. R. Sun,[a] P.-N. Wang,[a] M. M. Zoeller,[a] G. Crawford,[b] R. Fulton,[b] K. K. Gan,[b] H. Kagan,[b] R. Kass,[b] J. Lee,[b] R. Malchow,[b] F. Morrow,[b] M. Sung,[b] C. White,[b] J. Whitmore,[b] P. Wilson,[b] F. Butler,[c] X. Fu,[c] G. Kalbfleisch,[c] M. Lambrecht,[c] P. Skubic,[c] J. Snow,[c] P.-L. Wang,[c] D. Bortoletto,[d] D. N. Brown,[d] J. Dominick,[d] R. L. McIlwain,[d] T. Miao,[d] D. H. Miller,[d] M. Modesitt,[d] S. F. Schaffner,[d] E. I. Shibata,[d] I. P. J. Shipsey,[d] M. Battle,[e] J. Ernst,[e] H. Kroha,[e] S. Roberts,[e] K. Sparks,[e] E. H. Thorndike,[e] C.-H. Wang,[e] M. Artuso,[f] M. Goldberg,[f] T. Haupt,[f] N. Horwitz,[f] R. Kennett,[f] G. C. Moneti,[f] S. Playfer,[f] Y. Rozen,[f] P. Rubin,[f] T. Skwarnicki,[f] S. Stone,[f] M. Thulasidas,[f] W.-M. Yao,[f] G. Zhu,[f] R. Stroynowski,[g] A. V. Barnes,[h] J. Bartelt,[h] S. E. Csorna,[h] Z. Egyed,[h] V. Jain,[h] T. Letson,[h] M. D. Mestayer,[h] P. Sheldon,[h] D. S. Akerib,[i] B. Barish,[i] M. Chadha,[i] D. F. Cowen,[i] G. Eigen,[i] J. S. Miller,[i] J. Urheim,[i] A. J. Weinstein,[i] S. Menary,[j] R. J. Morrison,[j] H. N. Nelson,[j] J. Richman,[j] H. Tajima,[j] D. Schmidt,[j] D. Sperka,[j] M. Witherell,[j] D. Acosta,[k] G. Masek,[k] B. Ong,[k] H. Paar,[k] M. Sivertz,[k] M. Procario,[l] M. Daoudi,[m] W. T. Ford,[m] D. R. Johnson,[m] K. Lingel,[m] M. Lohner,[m] P. Rankin,[m] J. G. Smith,[m] J. Alexander,[n] C. Bebek,[n] K. Berkelman,[n] D. Besson,[n] T. E. Browder,[n] D. G. Cassel,[n] E. C. Cheu,[n] D. M. Coffman,[n] P. S. Drell,[n] R. Ehrlich,[n] M. Garcia-Sciveres,[n] B. Geiser,[n] B. Gittelman,[n] S. W. Gray,[n] D. L. Hartill,[n] B. K. Heltsley,[n] K. Honscheid,[n] C. Jones,[n] J. Kandaswamy,[n] N. Katayama,[n] P. C. Kim,[n] D. L. Kreinick,[n] G. S. Ludwig,[n] J. Masui,[n] J. Mevissen,[n] N. B. Mistry,[n] S. Nandi,[n] C. R. Ng,[n] E. Nordberg,[n] C. O'Grady,[n] J. R. Patterson,[n] D. Peterson,[n] D. Riley,[n] M. Sapper,[n] M. Selen,[n] H. Worden,[n] M. Worris,[n] F. Würthwein,[n] P. Avery,[o] A. Freyberger,[o] J. Rodriguez,[o] R. Stephens,[o] J. Yelton,[o] S. Henderson,[p] K. Kinoshita,[p] T. Liu,[p] M. Saulnier,[p] R. Wilson,[p] H. Yamamoto,[p] A. J. Sadoff,[q] R. Ammar,[r] S. Ball,[r] P. Baringer,[r] D. Coppage,[r] R. Davis,[r] N. Hancock,[r] M. Kelly,[r] N. Kwak,[r] H. Lam,[r] S. Ro,[r] Y. Kubota,[s] M. Lattery,[s] J. K. Nelson,[s] D. Perticone,[s] R. Poling,[s] S. Schrenk,[s] and R. Wang[s]

(CLEO Collaboration)

[a]*State University of New York at Albany, Albany, New York 12222*
[b]*Ohio State University, Columbus, Ohio 43210*
[c]*University of Oklahoma, Norman, Oklahoma 73019*
[d]*Purdue University, West Lafayette, Indiana 47907*
[e]*University of Rochester, Rochester, New York 14627*
[f]*Syracuse University, Syracuse, New York 13244*
[g]*Southern Methodist University, Dallas, Texas 75275*
[h]*Vanderbilt University, Nashville, Tennessee 37235*
[i]*California Institute of Technology, Pasadena, California 91125*
[j]*University of California at Santa Barbara, Santa Barbara, California 93106*
[k]*University of California at San Diego, La Jolla, California 92093*
[l]*Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213*
[m]*University of Colorado, Boulder, Colorado 80309-0390*
[n]*Cornell University, Ithaca, New York 14853*
[o]*University of Florida, Gainesville, Florida 32611*
[p]*Harvard University, Cambridge, Massachusetts 02138*
[q]*Ithaca College, Ithaca, New York 14850*
[r]*University of Kansas, Lawrence, Kansas 66045*
[s]*University of Minnesota, Minneapolis, Minnesota 55455*
(Received 1 June 1992)

Using date from e^+e^- annihilation into hadrons taken on the $\Upsilon(1S)$ and continuum, we are able to compare event properties of three-gluon (ggg) and quark-antiquark ($q\bar{q}$) decays. By tagging radiative decays of the $\Upsilon(3S)$ to the χ_b' states, we can make comparisons of the χ_b' event shapes with continuum

 ©1992 The American Physical Society

and ggg decays of the $\Upsilon(1S)$. Contrary to the notion that two-gluon (gg) events are globally very similar to $q\bar{q}$ jets, we infer from a study of the χ_b' ($J=0$ or 2) states that gg event shapes more closely resemble ggg decays of the $\Upsilon(1S)$, whereas $q\bar{q}g$ decays of the χ_b' ($J=1$) state more closely resemble $q\bar{q}$ continuum events in their decay characteristics.

PACS number(s): 13.87.Fh, 13.25.+m, 13.65.+i

INTRODUCTION

To date, there have been many studies of jet fragmentation [1]. One of the salient issues in studying jet fragmentation is understanding the ways in which hadron production from gluons compares with hadron production from quarks. Previous studies of gluon production in three-jet $q\bar{q}g$ events in e^+e^- colliders at high energy invariably rely on Monte Carlo techniques to identify the gluon jet and thereby determine, e.g., the shape parameters of that jet (usually the lowest-energy jet) and then extrapolate in energy in order to determine differences with a quark jet of equivalent energy.

We can avoid these difficulties at lower energy by examining the shape parameters of the χ_b' triplet, the $J=0$ and $J=2$ states of which are predicted by Barbieri *et al.* [2] to decay into gg, with the $J=1$ state predicted to decay into $q\bar{q}g$. The gg decays therefore give us a source of gluon jets which affords a direct comparison with $q\bar{q}$ jets at the same energy.

At present, among the several experimental observations which indicate differences between gluon vs quark fragmentation are the following.

(1) An enhancement in baryon production on the $\Upsilon(1S)$ [assumed to have an hadronic width nearly saturated by $\Upsilon(1S)\to ggg$] relative to the continuum [3].

(2) Suppression of open charm production on the $\Upsilon(1S)$ [4]. This has a natural connection with the observed baryon enhancement on the $\Upsilon(1S)$ if the baryon production rate in charm events is much smaller than the rate for $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ events [5]. This is also expected from the so-called "string model" where $c\bar{c}$ pairs can only arise in ggg events by quark popping (highly suppressed for higher-mass quark pairs).

(3) Evidence from KEK and CERN e^+e^- colliders TRISTAN and LEP that gluon jets are "fatter" than quark jets [6].

(4) From lower-energy machines operating in the Υ mass region, it has been observed that ggg events are more spherical than $q\bar{q}$ events. However, it has not been demonstrated how much of this effect is due to having three fragmenting partons compared with two fragmenting partons on the continuum and how much is due to some intrinsic property of gluons.

(5) In addition to having a more spherical distribution of energy flow, ggg decays also tend to have a slightly higher charged track multiplicity than $q\bar{q}$ continuum events at 10 GeV. This latter effect is again not inconsistent with having three fragmenting partons and a logarithmic dependence of multiplicity on the center-of-mass energy [7]. However, color considerations alone [8] argue that the multiplicity for jets from gluons relative to quarks should be larger by a factor of $\frac{9}{4}$, reflecting the ratio of the color charge of the gluon to that of the quark. Higher-order calculations reduce this value of $\frac{9}{4}$ by $\sim 10\%$ at $W=34$ GeV and $\sim 15\%$ at $W=10$ GeV [9]. By selecting three-jet events (assumed to be $q\bar{q}g$ events) which have equal opening angles between all three jets, the High Resolution Spectrometer (HRS) Collaboration [10] determined the ratio of gluon-jet multiplicity to quark-jet multiplicity to be $1.29^{+0.21}_{-0.41}\pm 0.20$, much more consistent with unity than 2.

We have made new measurements of the shape distributions of χ_b' decays, $q\bar{q}$ and $\Upsilon(1S)\to ggg$ events using $\Upsilon(3S)$, continuum, and $\Upsilon(1S)$ data collected between October 1989 and June 1991 with the CLEO-II detector at the Cornell Electron Storage Ring (CESR). The data sample of e^+e^- hadronic annihilations includes 193 pb^{-1} of data taken on the continuum at 10.52 GeV [below the $\Upsilon(4S)$ resonance], and 13 pb^{-1} of data taken on the $\Upsilon(1S)$ resonance at 9.46 GeV. We have used $\Upsilon(3S)$ data (121 pb^{-1} of data taken at 10.35 GeV center-of-mass energy) as a source of χ_b' decays produced in the process $\Upsilon(3S)\to\gamma\chi_b'$. We identify such decays by the observations of the transition photon in the CLEO-II electromagnetic calorimeter. Multiplicity and shape distributions of the χ_b' states are studied and compared with similar quantities from three-gluon decays of the $\Upsilon(1S)$ and $q\bar{q}$ events.

The CLEO-II detector is a general purpose solenoidal magnet spectrometer and calorimeter. Elements of the detector and performance characteristics are described elsewhere [11]. Hadronic events are selected by requiring at least five charged tracks (to suppress QED events as well as events from $\gamma\gamma$ collisions that produce low-multiplicity final states) in the central detector and a total neutral energy greater than 15% of the center-of-mass energy. To further suppress beam-related backgrounds (beam gas and beam wall), we remove events which have large momentum imbalance. We also require the fitted event vertex to be within ± 5 cm of the e^+e^- intersection point along the axis defined by the incident beams and ± 2 cm in the transverse plane.

DECAYS OF THE χ_b' STATES USING A PHOTON TAG

Measurements of two-gluon decays of quarkonia with $C=+1$ (e.g., the χ_b or η_b states for bottomonium) allow us to compare more directly quark and gluon hadronization without the complications introduced by having three fragmenting partons in $\Upsilon(1S)\to ggg$ decays compared with two in continuum $q\bar{q}$ events. Unfortunately, since such $C=+1$ states do not couple directly to the virtual photon produced in 10-GeV e^+e^- collisions, they must be produced either in two-photon collisions or as

decay products from one of the $J(PC)=1(--)$ Υ resonances. Although the first mechanism suffers from a small rate at our energies, the second rate can be substantial. For CESR running at the $\Upsilon(3S)$ energy, the cross section for χ_b' production is about 30% of the total $\Upsilon(3S)$ enhancement above the continuum.

For the χ_b' triplet, the $J=2$ and 0 states, having $(PC)=(++)$, can decay into two gluons. Their widths are calculated [2] up to second order in α_s to be

$$\Gamma_{gg}(2P_2)=\frac{8\alpha_s^2}{5m_b^4}|\Psi'(0)|^2(1+\alpha_s/\pi)$$

and

$$\Gamma_{gg}(2P_0)=\frac{6\alpha_s^2}{m_b^4}|\Psi'(0)|^2(1+10.2\alpha_s/\pi)\ .$$

For both P states, the values of $|\Psi'(0)|$ are the same and we expect that the ratio of $J=0$:$J=2$ hadronic widths would be 15:4. Since in $\Upsilon(1S)\to ggg$ most of the energy is carried by the two most energetic gluons in the event [12], Barbieri *et al.* [2] predicted that the event characteristics of $\chi_b'\to gg$ may approach those of $\Upsilon(1S)\to ggg$ in both multiplicity and shape. Experimentally, we parametrize the event shape using the variable R_2 [13], defined as the ratio of the second to the zeroth Fox-Wolfram moments calculated using all the charged tracks and photons with energy in excess of 100 MeV.

As the decay of a spin-1 particle into two real massless vectors (gluons or photons) is prohibited by angular momentum conservation, it is expected that the $J=1$ state will preferentially decay into $q\bar{q}g$, where the gluon is emitted with very low energy [14]. The Gatto-Barbieri prediction for this width is

$$\Gamma_{q\bar{q}g}(2P_1)=\frac{32\alpha_s^3}{9m_b^4\pi}|\Psi'(0)|^2\ln(m_b\langle r\rangle)\ ,$$

where $\langle r\rangle$ is the rms value of the radius of the $b\bar{b}$ state. In the limit where the soft gluon has zero momentum, we expect (modulo possible differences in the relative $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, and $c\bar{c}$ abundances owing to the presence of the gluon) $q\bar{q}g$ decays to be very similar to $q\bar{q}$ events. The predictions for the ratios of the hadronic widths from the formulas above are consistent with measurements from the CUSB-II [15] and CLEO Collaborations [16] of the electromagnetic branching ratios of these states (assuming the total width is the sum of the hadronic and radiative widths).

We can measure explicit properties of χ_b' decays by tagging the photon produced in radiative transitions from the $\Upsilon(3S)$. For the $\Upsilon(3S)$ data sample, there are roughly 70 000 total tags for the $J=2$, 1, and 0 states (30 000, 30 000, and 10 000, respectively). This is the same data set used previously to measure the mass splittings between the three χ_b' states [17]. We improve the significance of our three signals be vetoing photons which pair with other photons of energy greater than 200 MeV to give an invariant mass consistent with a π^0.

MEASUREMENT OF EVENT PARAMETERS

We measure both the observed charged multiplicity and R_2 distributions for the three J states by determining the number of photon tags corresponding to $\Upsilon(3S)\to\gamma\chi_b'(J=2)$, $\Upsilon(3S)\to\gamma\chi_b'(J=1)$, and $\Upsilon(3S)\to\gamma\chi_b'(J=0)$ for a given event charged multiplicity or R_2 value. Thus, for each bin of, e.g., charged multiplicity, we plot the photon spectrum for all events in that bin, then perform a fit to this photon spectrum, and extract the total number of χ_b' events observed. We use the same fit parameters described in our previous publication [17]. In this way we can determine the spectrum of $dN(\chi_b')/dX$, where X is the charged multiplicity of R_2.

Figure 1 shows the observed charged multiplicity for the three χ_b' states using the above technique. Although we observe indications of a slightly larger value of average multiplicity for the $J=0$ and 2 states, we do not observe the enhancement of ~ 2 in gluon-jet multiplicity compared with quark-jet multiplicity, as expected by color arguments [18]. We similarly measure the production cross section for each state of the χ_b' for a given bin in R_2.

Figure 2 illustrates the photon momentum spectra we fit. For this figure we have used events with low R_2 values and compared them to events with high R_2 values. It is clear that the $J=2$ ($J=1$) peak, for example, is much more pronounced in the top (bottom) plot.

We wish to compare the observed χ_b' shape distributions with the shape distributions we observe for $q\bar{q}$ and ggg events. Since we take $\Upsilon(1S)$ data and continuum data at different center-of-mass energies (9.46 and 10.55 GeV, respectively) than the center-of-mass energy for the χ_b' decays, corrections to the observed data are needed so that shape dependences on energy are taken out. We first must correct for the motion of the χ_b' state—since the χ_b' has nonzero momentum, there is some net boost along the χ_b' direction of motion. We do so by calculating the

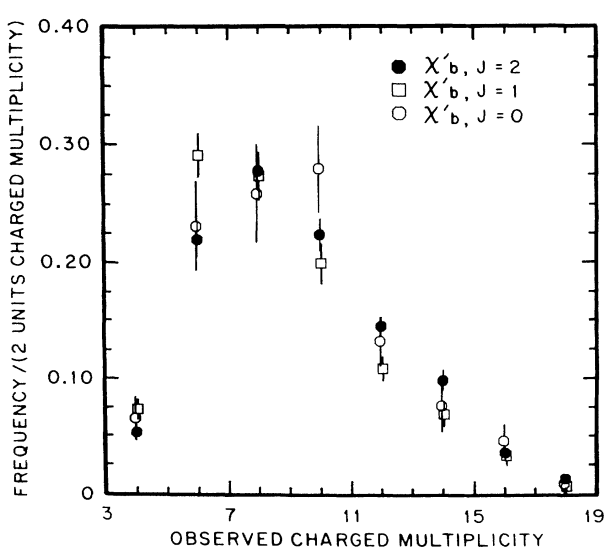

FIG. 1. Charged multiplicity distribution for $\Upsilon(3S)\to\gamma\chi_b'$ events.

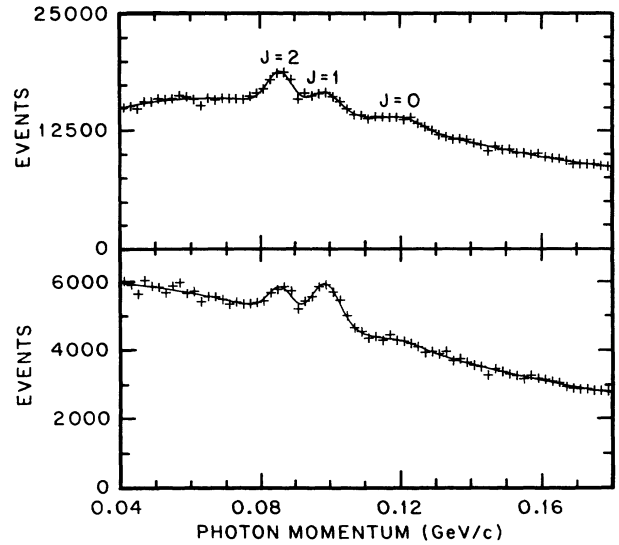

FIG. 2. Inclusive photon momentum spectrum for events with low R_2 ($0.0<R2<0.20$, top) and events with high $R2$ ($0.4<R2<1.0$). The difference in the relative populations of the $J=2$ and 1 lines are evident from the plot.

value of R_2 in the center-of-mass frame recoiling against our photon tag [19]. Next, we must adjust the observed $\Upsilon(1S)\to ggg$ R_2 distribution for the difference in center-of-mass energies from 9.46 to 10.2 GeV. We need similarly to adjust the $q\bar{q}$ distribution from 10.55 to 10.2 GeV. For these last two effects, we use a bin-by-bin shift obtained from Monte Carlo data. The typical magnitude of the shifts in the R_2 values from these two effects are $+0.0038$ and -0.0082, respectively, relative to the uncorrected R_2 values.

Figure 3 shows the R_2 distributions we obtain, where we have included all of the above effects. We see from the figure that the R_2 distributions for the $J=0$ and 2 states are closer in shape to that for the direct decays of the $\Upsilon(1S)\to ggg$, whereas the $J=1$ state is closer in shape to the continuum. This is in contradiction with an assumption used in a previous study [20], where the thrust of continuum-subtracted $\Upsilon(3S)$ events was fit to the sum of a high-thrust component (presumed to originate from decays of the χ_b' states) and a low-thrust $\Upsilon(1S)$-like component [presumed to be a good model of $\Upsilon(3S)\to ggg$ direct decays]. The high-thrust component (i.e., the χ_b' piece) was assumed to have a thrust distribution which was well modeled by continuum data. Our result indicates that the shape distributions given by the $J=0$ and 2 decays are more consistent with $\Upsilon(1S)$ decays than $q\bar{q}$ decays in their event characteristics and that the net χ_b' shape is not far from the mean of the observed $q\bar{q}$ and $\Upsilon(1S)\to ggg$ shape distributions.

We wish to also compare our results with predictions for the shapes of decays of the χ_b' states from the JETSET (version 7.3) event generator [21]. We therefore generate $\Upsilon(3S)\to\gamma\chi_b'$ events, where the χ_b' decays generically. It is necessary to include the effect of cascade decays, e.g., $\chi_b'\to\gamma\Upsilon(2S)$, $\Upsilon(2S)\to ggg$, which will have the effect of making our observed $\chi_b'(J=2$ and 1) distributions more three-gluon like. This effect softens the observed $\chi_b'(J=2$ and 1) R_2 distributions typically by approximately 0.022 ± 0.0005 units [22] of R_2. There is virtually no effect on the $J=0$ state from cascades since the $J=0$ state has an almost negligible branching ratio for these cascade decays. To make this correction, we use branching ratios for these cascades determined by the CUSB [15] and CLEO Collaborations [16] as input to our Monte Carlo event generator. In Fig. 4 we see fair agreement between the LUND model prediction for $\chi_b'(J=2)$ [23] decays and the distributions we obtain from data. In this figure the data distribution is as observed in Fig. 3,

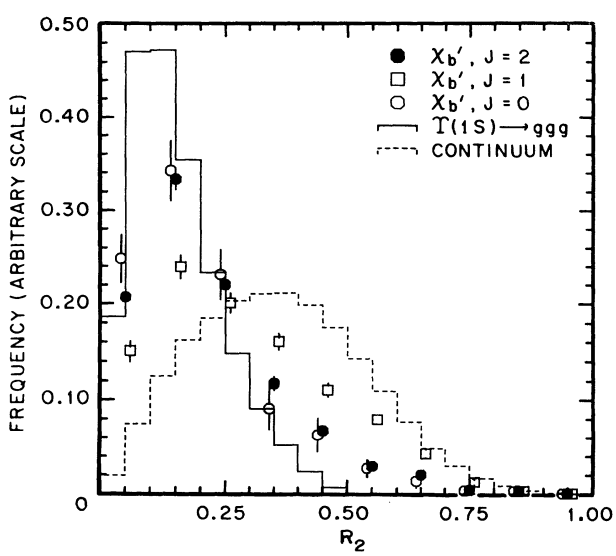

FIG. 3. R_2 distributions for $\Upsilon(1S)\to ggg$, continuum events and the three χ_b' states. The $\Upsilon(1S)$ and continuum distributions have been evolved to the energy of the χ_b' decays.

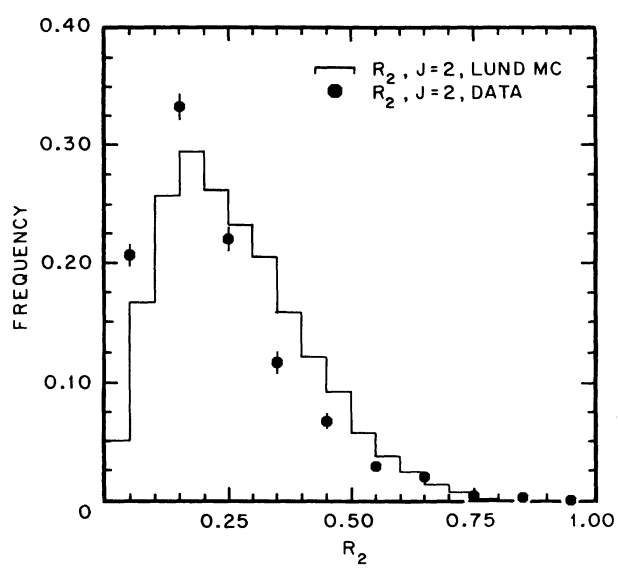

FIG. 4. R_2 distributions for $J=2$ state with LUND predictions overlaid.

TABLE I. Mean event parameters.

Parent state	Observed charged multiplicity	Corrected $\langle R_2 \rangle$
χ_b' direct, $J=2$	$9.339\pm0.090\pm0.045$	$0.220\pm0.003\pm0.016$
χ_b', direct, $J=1$	$8.679\pm0.090\pm0.048$	$0.293\pm0.003\pm0.016$
χ_b', direct, $J=0$	$9.217\pm0.160\pm0.072$	$0.192\pm0.005\pm0.008$

whereas the Monte Carlo distribution includes the previously mentioned cascade effects. Although the JETSET model therefore gives a somewhat harder spectrum, overall it seems to approximate the shape characteristics of two-gluon decays. Qualitatively, the observed difference between the data and Monte Carlo simulation suggests that the two gluons may be fragmenting less independently than is assumed in the LUND Monte Carlo simulation.

Table I gives the mean values of multiplicity and R_2 for the three states, where the "corrected $\langle R_2 \rangle$" column gives the R_2 values including all our corrections. The systematic errors are dominated by uncertainties in the fit parameters appropriate for fitting the photon lines and, for the corrected R_2 values, include errors in the branching ratios for cascade events.

We have qualitatively verified the validity of our technique by measuring the R_2 distribution for $\Upsilon(1S)$ events produced in the decay $\Upsilon(3S)\rightarrow\Upsilon(1S)\pi^+\pi^-$ [for this study we exclude the transition pions and again must boost into the $\Upsilon(1S)$ rest frame in our calculation of R_2] as a function of dipion recoil invariant mass and comparing this distribution with the R_2 distribution we obtain directly from $\Upsilon(1S)$ data. We find that the agreement is satisfactory.

The tag technique can be extended to other applications as well. In principle, it can be used to measure proton, kaon, and pion fractions, as well as inclusive Λ, D^*, K_S^0, lepton, charged-particle, and photon yields. These will be the subject of a forthcoming article.

CONCLUSIONS

We have measured the shape distributions of the χ_b' states. This analysis of global event parameters lends credence to the Barbieri-Gatto model of their decays. We have demonstrated in an unambiguous manner direct differences between gg and $q\bar{q}$ events. Predictions for the shapes of gg and $q\bar{q}$ decays are, in principle, calculable given the Altarelli-Parisi splitting functions [24]. These splitting functions must be folded in with the cross sections for these processes in order to determine the full parton shower profile. However, we do not have the statistical power with this technique to observe three-jet events and verify such predictions for hard QCD radiation. We observe similar multiplicity distributions for the $J=0$ and 2 states as compared with the $J=1$ state, at variance with the expectation based on color charge considerations that the multiplicity of gluon jets should exceed that of quark jets by a factor of ~ 2.

It would be useful to see how these data compare with results from $c\bar{c}$. The photon spectrum for the decay $\psi'\rightarrow\chi_c'\gamma$ has been measured with better signal to noise and better statistics [25] than in this measurement; it would be interesting to see whether the bottomonium results extrapolate to the less perturbative regime of charmonium.

ACKNOWLEDGMENTS

We gratefully acknowledge the effort of the CESR staff in providing us with excellent luminosity and running conditions. J.P.A. and P.S.D. thank the PYI program of the NSF, I.P.J.S. thanks the YI program of the NSF, K.H. thanks the Alexander von Humboldt Stiftung, G.E. thanks the Heisenberg Foundation, K.K.G. and A.J.W. thank the SSC Fellowship program of TNRLC, K.K.G., H.N.N., J.D.R., and H.Y. thank the OJI program of the U.S. DOE, and R.P. and P.R. thank the A.P. Sloan Foundation for support. This work was supported by the National Science Foundation and the U.S. Department of Energy.

[1] For reviews, see P. Mättig, Phys. Rep. **177**, 141 (1989); W. Hoffman, Annu. Rev. Nucl. Part. Sci. **38**, 279 (1988).

[2] R. Barbieri, R. Gatto, R. Kogerler, and Z. Kunszt, Phys. Lett. **57B**, 455 (1975); R. Barbieri, R. Gatto, and E. Remiddi, *ibid.* **61B**, 465 (1976); R. Barbieri, M. Caffo, and E. Remiddi, Nucl. Phys. **B162**, 220 (1980).

[3] S. Behrends *et al.*, Phys. Rev. D **31**, 2161 (1985).

[4] H. Albrecht *et al.*, Z. Phys. C **55**, 25 (1992); CLEO Collaboration, in *Proceedings of the XIVth International Symposium on Lepton and Photon Interactions,* Stanford, California, 1989, edited by A. Riordan (World Scientific, Singapore, 1990).

[5] H. Scheck, Phys. Lett. B **224**, 343 (1989).

[6] AMY Collaboration, Y. K. Kim *et al.*, Phys. Rev. Lett. **63**, 1772 (1989); OPAL Collaboration, G. Alexander *et al.*, Phys. Lett. B **265**, 462 (1991).

[7] As an example, if the average multiplicity is 10 for a $q\bar{q}$ event, then for a ggg event, where the partition of energy among the three gluons is equal, the average multiplicity would be 11.12. As ggg events become more gg like, the average multiplicity in this scheme decreases and approaches 10.

[8] M. B. Einhorn and B. G. Weeks, Nucl. Phys. **B146**, 445 (1978); K. Shizuya and S. H. Tye, Phys. Rev. Lett. **41**, 787 (1978).

[9] J. B. Gaffney and A. H. Mueller, Nucl. Phys. **B250**, 109 (1985).

[10] M. Derrick *et al.*, Phys. Lett. **165B**, 449 (1985).

[11] Y. Kubota *et al.*, Nucl. Instrum. Methods **A320**, 66 (1992).

[12] Roy Cabenda, Ph.D. thesis, Cornell University, 1981.

[13] G. C. Fox and S. Wolfram, Phys. Rev. Lett. **41**, 1581 (1978); Phys. Lett. **82B**, 134 (1979).

[14] This can be phenomenologically understood as a conse-

quence of the running of α_s and the fact that this decay is described as the emission of two gluons, one of which is virtual (for which a longitudinal polarization is allowed) and decays to a $q\bar{q}$ pair. As the lower-energy gluon becomes softer, the coupling at the gluon emission vertex becomes larger, and the rate is increased.

[15] M. Narain *et al.*, Phys. Rev. Lett. **66**, 3113 (1991); U. Heintz, *ibid.* **66**, 1563 (1991).

[16] Helen M. Worden, Ph.D. thesis, Cornell University, 1992.

[17] R. Morrison *et al.*, Phys. Rev. Lett. **67**, 1696 (1991).

[18] By plotting the dependence of our $\Upsilon(3S)\rightarrow\gamma\chi_b'$ candidates on the orientation of the jet axis, we have verified that the orientation of the jet axis in polar angle is the same for the three J states. Not knowing *a priori* the preferred orientation of the jet axis relative to the beam axis in the radiative decays to the $1++$ and the $2++$ states, it cannot be ruled out that the difference in observed multiplicity is an artifact of acceptance—if the jet axis for $1++$ decays is more closely aligned to the beam axis, then we will consequently lose charged tracks down the beam pipe. The fact that there is no dependence on jet axis orientation ensures that the effect we see is not an artifact of acceptance.

[19] More specifically, in our determination of R_2, we boost all the observed particles in the event into the rest frame of the object recoiling against our candidate transition photon in order to obtain the R_2 distribution for a χ_b' particle decaying at rest. The magnitude of this correction is measured to be approximately a shift of -0.0016 relative to the R_2 value before making this correction.

[20] D. Peterson *et al.*, Phys. Lett. **114B**, 277 (1982).

[21] We use JETSET version 7.3 program using LUND fragmentation and take the default parameters and default (parton shower followed by string fragmentation) event generation scheme. In this approach the parton shower initially occurs in the high-Q^2 and therefore perturbative regime. As the Q^2 decreases and the process moves into a nonperturbative regime, QCD confinement is modeled as a stretched color flux tube (i.e., the "string") between two quarks. When the string is stretched past a certain point, the string breaks and hadrons form at the break point; this process continues until the potential energy of the flux tube falls below a cutoff value of 1 GeV.

[22] The error shown here is the error due to our uncertainty in the cascade branching ratios, which we ultimately apply to our overall systematic error.

[23] Since our version of the JETSET event generator (version 7.3) did not produce decays of the variety $\chi_b'(J=1)\rightarrow q\bar{q}g$, we compare our results only with the $J=2$ state. In conjunction with the work that has been done to reproduce global event characteristics and inclusive particle spectra at energies of LEP and the DESY e^+e^- collider PETRA, our data now offer the possibility of extending the model to reproduce other bottomonium decays in addition to the $\Upsilon(1S)$.

[24] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977). These functions define the fractal energy radiatively emitted in the processes $q\rightarrow q'g$, $g\rightarrow g'g$, and $g\rightarrow g'q$.

[25] E. Bloom and C. Peck, Annu. Rev. Nucl. Part. Sci. **33**, 143 (1983).