Test of QED to fourth order by study of four-lepton final states in e^+e^- interactions at 29 GeV with the HRS detector

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Data taken with the High Resolution Spectrometer detector at the SLAC storage ring PEP were used to test QED to fourth order in the coupling constant α. The experiment studied four-lepton final states produced at high Q^2 in e^+e^- interactions at $\sqrt{s}=29$ GeV. All four final-state particles were detected at large angles with respect to the beam axis. We observed 17 $e^+e^-e^+e^-$, 24 $e^+e^-\mu^+\mu^-$, and 1 $\mu^+\mu^-\mu^+\mu^-$ events with pair masses greater than 1 GeV/c^2 for an integrated luminosity of 291 pb^{-1}. The complete α^4 QED calculation agrees reasonably well with the data.

I. INTRODUCTION

We have tested QED to fourth order in the coupling constant α using data taken with the High Resolution Spectrometer (HRS) at the SLAC storage ring PEP. The following processes were measured and compared with theory:

$$e^+e^- \to e^+e^-e^+e^- ,$$

$$e^+e^- \to e^+e^-\mu^+\mu^- ,$$

$$e^+e^- \to \mu^+\mu^-\mu^+\mu^- .$$

In the region of large pair masses and large scattering angles, the cross section for these reactions is about 0.1 pb, a factor of 1000 below the lowest-order annihilation cross section. Despite the smallness of the cross section, these four-lepton processes are easily identified since backgrounds from other processes in the corresponding kinematical region are not significant.

We used in this analysis a Monte Carlo program written by Berends, Daverveldt, and Kleiss[1] to compare the data with theory. All Feynman diagrams and all possible photon and Z^0 exchanges contributing to fourth order were taken into account, even though, the Z^0-exchange contributions are negligible at a center-of-mass (c.m.) energy of 29 GeV. Figures 1(a)–1(d) show examples from the four groups[1] of Feynman diagrams resulting in four-lepton final states. The "bremsstrahlung," the "annihilation," and "conversion" groups [Figs. 1(a)–1(c)] contribute significantly, while the "multiperipheral" group [Fig. 1(d)] contributes the least since all four leptons were required to be at large angles with respect to the beam axis.

The data were taken with the HRS at the PEP e^+e^- storage ring at a c.m. energy $\sqrt{s}=29$ GeV and corresponded to an integrated luminosity of 291 ± 7 pb^{-1} as measured with wide-angle Bhabha-scattering events. The HRS shown in Fig. 2 is well suited for identifying final states with four charged leptons. Its main characteristics have been described previously;[2] here we only present the features essential to this analysis. In the central region, the vertex and the inner and outer drift chambers (DC) provided charged-particle tracking over 90% of the solid angle with an excellent momentum resolution. A barrel ($|\cos\theta|\le0.6$) and end-cap ($0.7\le|\cos\theta|\le0.9$) shower counters were used for lepton identification. The drift chamber and shower counters were immersed in a 1.62-T solenoidal magnetic field. The HRS had no external muon-detection system. However, muons could be distinguished from electrons using the shower-counter information.

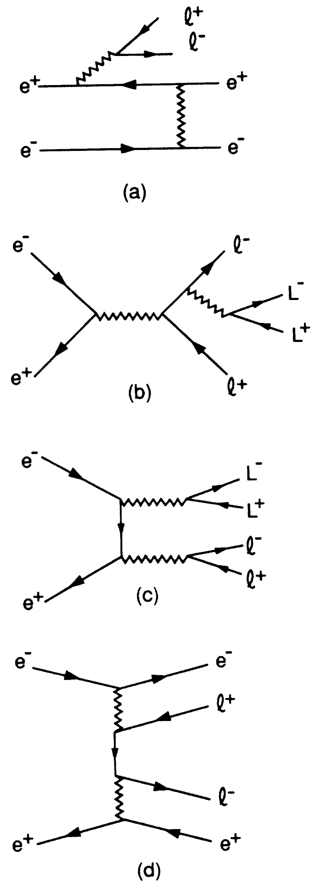

FIG. 1. Diagrams contributing to four-lepton final states; an example from the (a) bremsstrahlung; (b) annihilation; (c) conversion; and (d) multiperipheral groups.

II. EVENT SELECTION

The analysis presented here tests QED to fourth order at large values of Q^2, the four-momentum squared of photons exchanged in the diagrams of Fig. 1. Two characteristic features of the four-lepton final states examined in this paper are (1) the events are composed of four charged leptons at large angles with respect to the e^+e^- beam direction, and (2) the energy of an event, as approximated by the total scalar momentum, $\sum_{i=1,4} c|P_i|$, peaks at 29 GeV.

The requirement that each event must have only four charged tracks with zero net charge and an energy above 20 GeV keeps most of the signal while rejecting a large fraction of background events as described below. Additional selection criteria were designed to reject specific types of background.

Each event was required to satisfy the following criteria.

(1) It must contain four *good* charged tracks, with zero net total charge, in the region $|\cos\theta| \leq 0.9$. A *good* track had a momentum of at least 0.6 GeV/c, passed close to the interaction point (IP) (its radial distance from the IP was required to be $r \leq 0.05$ m, while its distance along the beam direction was $z \leq 0.1$ m), and traversed at least 6 DC layers depending on the angle θ of the track with the beam direction. Specifically, we asked that all tracks satisfy the following conditions: for $0.866 < |\cos\theta| \leq 0.9$, $N_{\rm DC} \geq 6$; for $0.766 < |\cos\theta| \leq 0.866$, $N_{\rm DC} \geq 7$; for $|\cos\theta| \leq 0.766$, $N_{\rm DC} \geq 9$; where $N_{\rm DC}$ is the number of drift-chamber layers (maximum = 17), contributing to the track signals.

(2) The energy of an event, as approximated by the scalar sum of the charged-particle momenta, $\sum_{i=1,4} c|P_i|$, must be between 20 and 40 GeV. This is an appropriate cut for reducing a large part of the background as discussed below. It also allows for radiation of photons from the initial- or final-state particles. Figure 3 shows the energy distribution of the events passing the previous requirements.

(3) All oppositely charged particle pairs must have an invariant mass of at least 1 GeV/c^2. This cut helps reduce backgrounds from hadronic-resonance production and single-photon conversion.

(4) There must be at least three identified leptons or two identified leptons of the same charge sign in each event. All identified leptons were required to have a momentum of at least 1 GeV/c in order to minimize the misidentification of hadrons as leptons. An electron was

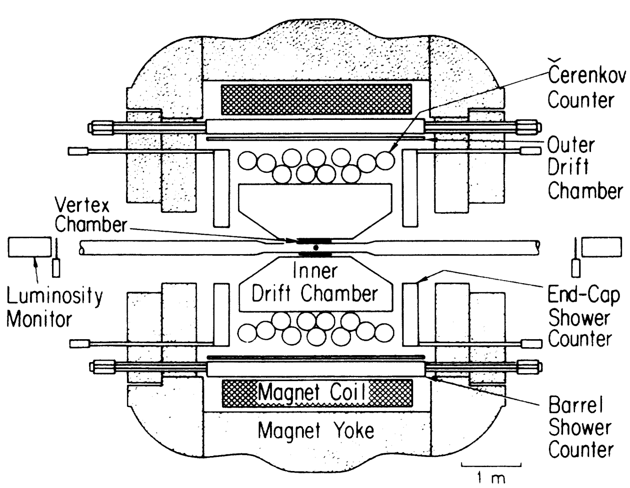

FIG. 2. The HRS detector.

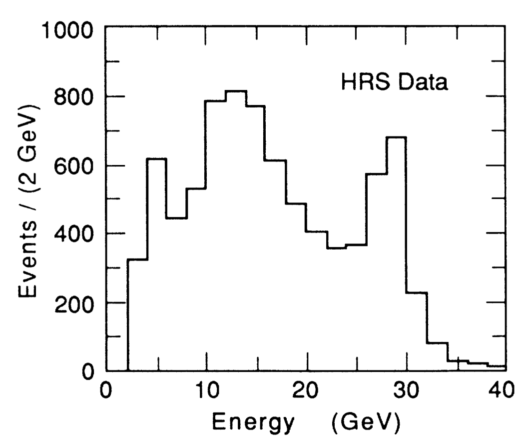

FIG. 3. Energy distribution of HRS events with four good tracks and zero total charge.

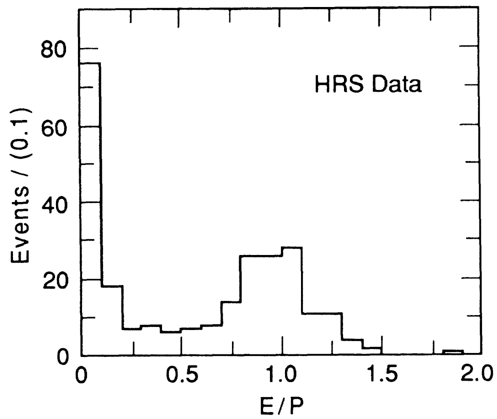

FIG. 4. E/P distribution of all tracks in four-lepton candidate events after cut (3).

required to satisfy $0.55 < E/P < 1.45$ where E is the energy measured with the shower counters and P the momentum as determined with the drift chamber. A track was identified as a muon if it entered the active region of the shower counters and deposited less than 0.5 GeV. A typical value of the deposited energy in the HRS shower counters by a minimum-ionizing particle is 0.2 GeV (Ref. 3). Figure 4 shows the E/P distribution of tracks for all events which satisfied criteria (1)–(3) described above. The energy deposited in the shower counters by tracks with $E/P \leq 0.55$ is presented in the range 0–2 GeV in Fig. 5.

When only three leptons were identified in an event, the identity of the remaining, unidentified track was assigned according to lepton-flavor conservation. Similarly, in events with two identified leptons of the same sign, the identities of the other two tracks were also determined by the assumption of lepton flavor conservation. Failure to identify tracks and events resulted primarily from the finite shower-counter coverage. Twenty-seven percent of the tracks were outside the fiducial volume of these counters. Another 4% of tracks had momenta below 1 GeV/c.

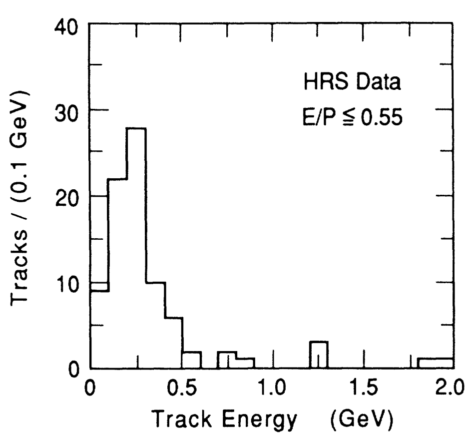

FIG. 5. Energy distribution of individual tracks with $E/P \leq 0.55$ in four-lepton candidate events.

73 events satisfied requirements (1)–(3). Of these, 43 events were uniquely identified: 17 as $e^+e^-e^+e^-$, 24 as $e^+e^-\mu^+\mu^-$, and 2 as $\mu^+\mu^-\mu^+\mu^-$. Of the two $\mu^+\mu^-\mu^+\mu^-$ events, one was rejected upon further examination. The rejected event had the typical characteristics of an annihilation process into hadrons accompanied by energetic initial-state radiation.

III. BACKGROUND ESTIMATION

We have studied possible background reactions in the region of large Q^2 and large scattering angles and have found them to be small.[4] Reactions which can *a priori* contribute as background to the processes studied in this paper are the following:

$$e^+e^- \to e^+e^-\tau^+\tau^- ,$$

$$e^+e^- \to e^+e^-h^+h^- ,$$

$$e^+e^- \to \tau^+\tau^- ,$$

$$e^+e^- \to q\bar{q} \to \text{hadrons} .$$

A potentially important source of background is $e^+e^- \to e^+e^-\tau^+\tau^-$. Both τ's may decay into a charged lepton (an electron or a muon) plus neutrinos. The Monte Carlo calculation of Berends *et al.*[1] was used to simulate events from this reaction. The generated events corresponded to an integrated luminosity of 2530 pb^{-1}. We found no events satisfying all the selection criteria. When scaled to the integrated luminosity of the HRS data (291 pb^{-1}) this gives less than 0.3 background events at 90% confidence level (C.L.) for each of the three processes $e^+e^-e^+e^-$, $e^+e^-\mu^+\mu^-$, and $\mu^+\mu^-\mu^+\mu^-$.

Using the same Monte Carlo programs we generated $e^+e^- \to e^+e^-q\bar{q}$ events in order to determine the background from $e^+e^- \to e^+e^-h^+h^-$, where h is a hadron. The Lund Monte Carlo program[5] was used to fragment the quarks into hadrons. The generated events corresponded to an integrated luminosity of 933 pb^{-1}. We again found no events satisfying all the selection criteria. When scaled to the integrated luminosity of the HRS data this gives less than 0.7 background events at 90% C.L. for each of the three processes $e^+e^-e^+e^-$, $e^+e^-\mu^+\mu^-$, and $\mu^+\mu^-\mu^+\mu^-$.

The process $e^+e^- \to \tau^+\tau^-$ is a possible background when one τ decays into an electron or muon plus neutrinos and the other τ decays into charged and neutral pions plus a neutrino, and the pions simulate electrons or muons. The generated events corresponded to an integrated luminosity of 294 pb^{-1} and we found one event satisfying all the selection criteria. When scaled to the integrated luminosity of the HRS data this gives an upper limit of 2.3 background events at 90% C.L. for $e^+e^-e^+e^-$ and $\mu^+\mu^-\mu^+\mu^-$ processes and 1.0 ± 1.0 background event for $e^+e^-\mu^+\mu^-$.

Events from $e^+e^- \to q\bar{q} \to \text{hadrons}$ may contribute through the decay of a hadron to an electron or muon, or through the misidentification of a hadron as an electron

TABLE I. Number of observed and expected events for $e^+e^-e^+e^-$, $e^+e^-\mu^+\mu^-$, and $\mu^+\mu^-\mu^+\mu^-$ and for unidentified four-prong final states. Upper limits for expected backgrounds are at the 90% C.L.

Data	$e^+e^-e^+e^-$ 17	$e^+e^-\mu^+\mu^-$ 24	$\mu^+\mu^-\mu^+\mu^-$ 1	Events failing lepton ID requirements 31
Expected signal				
$e^+e^-\to e^+e^-e^+e^-$	23.5			10.4
	$\pm$2.8			$\pm$1.6
$e^+e^-\to e^+e^-\mu^+\mu^-$		31.5		8.9
		$\pm$3.5		$\pm$1.5
$e^+e^-\to \mu^+\mu^-\mu^+\mu^-$			2.9	0.14
			$\pm$0.7	$\pm$0.14
Expected background				
$e^+e^-\to e^+e^-\tau^+\tau^-$	$<$0.2	$<$0.2	$<$0.2	$<$0.2
$e^+e^-\to e^+e^-h^+h^-$	$<$0.7	$<$0.7	$<$0.7	0.3
				$\pm$0.3
$e^+e^-\to \tau^+\tau^-$	$<$2.3	1.0	$<$2.3	1.0
		$\pm$1.0		$\pm$1.0
$e^+e^-\to q\bar{q}$	$<$3.6	$<$3.6	$<$3.6	$<$3.6
Total	23.5	32.5	2.9	20.7
expected events	$\pm$2.8	$\pm$3.6	$\pm$0.7	$\pm$2.4
	$\pm$0.5	$\pm$0.7	$\pm$0.06	$\pm$0.4

or muon. The Lund Monte Carlo program[5] was used to generate events corresponding to an integrated luminosity of 185 pb^{-1}. We found no events satisfying all selection criteria. When scaled to the integrated luminosity of the HRS data this gives at most 3.6 background events at 90% C.L. for each of the three processes $e^+e^-e^+e^-$, $e^+e^-\mu^+\mu^-$, and $\mu^+\mu^-\mu^+\mu^-$.

The backgrounds are summarized in Table I.

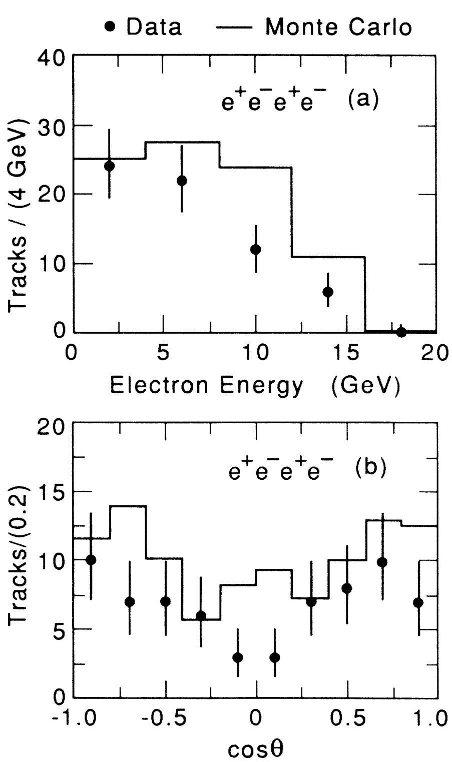

FIG. 6. Distribution of electrons ($e^{\pm}$) in $e^+e^-e^+e^-$ events: (a) electron energy; (b) cos(θ). The histogram is the QED prediction to order α^4.

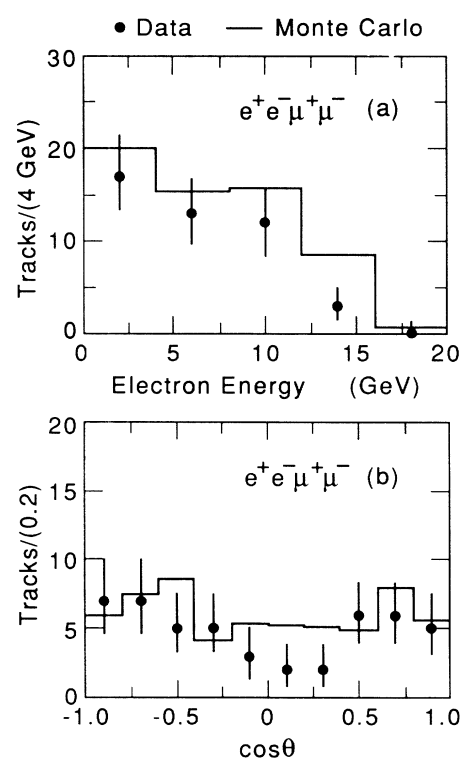

FIG. 7. Distribution of electrons ($e^{\pm}$) in $e^+e^-\mu^+\mu^-$ events: (a) electron energy; (b) cos(θ). The histogram is the QED prediction to order α^4.

IV. COMPARISON WITH QED

The data from this experiment were compared to the theoretical expectations obtained from a Monte Carlo event generator[1] especially designed to describe four-lepton processes where all four leptons are emitted at large angles. All Feynman diagrams contributing to fourth order were taken into account. It included all possible virtual photon and Z^0 exchanges. Four-lepton final states were generated over a kinematic region that extended well beyond the final acceptance criteria. These initial kinematical requirements were as follows.

(a) The scattering angle of the final-state leptons was required to be within the angular interval $20° \le \theta \le 160°$ with respect to the beam.

(b) All leptons were required to have a momentum of at least 0.1 GeV/c.

(c) All oppositely charged lepton pair combinations were required to have an invariant mass of at least 0.5 GeV/c^2.

The generated events for each of the processes $e^+e^-e^+e^-$, $e^+e^-\mu^+\mu^-$, and $\mu^+\mu^-\mu^+\mu^-$, corresponded to the integrated luminosities of 1598 pb^{-1}, 1590 pb^{-1}, and 2059 pb^{-1}, respectively. The generated events were then passed through a full detector simulation and the simulated events were analyzed with the same code and selection criteria applied to the real events.

The number of expected events from the Monte Carlo simulation was 23.5±2.8 $e^+e^-e^+e^-$, 31.5±3.5 $e^+e^-\mu^+\mu^-$, and 2.9±0.7 $\mu^+\mu^-\mu^+\mu^-$, while 17, 24, and 1 events were observed, respectively. The errors attached to the predicted values are statistical only. The number of observed events in each channel is less than expected but the deviations are not statistically significant.

We have also compared the number of observed and expected events before particle identification. This comparison represents a clean test of QED since the contribution from background processes is small (about 4%). While it does not distinguish between different four-lepton final states, the comparison is statistically more significant because of the larger event sample and is also free of systematic errors in particle identification that may result from imperfections in the Monte Carlo simulations of the shower-counter response. The total number of Monte Carlo signal and background events, taking into account those events which failed the lepton identification criteria, is $N_{MC} = 79.6 = 23.5 + 32.5 + 2.9 + 20.7$ (from Table I). The corresponding total number of data events is $N_D = 73 = 17 + 24 + 1 + 31$. The total data and Monte Carlo results differ by less than 1 standard deviation.

In the observed identified sample there was one $e^+e^-\mu^+\mu^-$ event which radiated a photon of 4.6 GeV of energy that made an angle of 31° with the nearest track. This event cannot be compared separately with theory, since there is no complete calculation[6] for the radiative corrections to order α^4.

The observed cross section can be affected by the emission of a real or virtual photon from the electron or positron before they annihilate (initial-state radiation). This

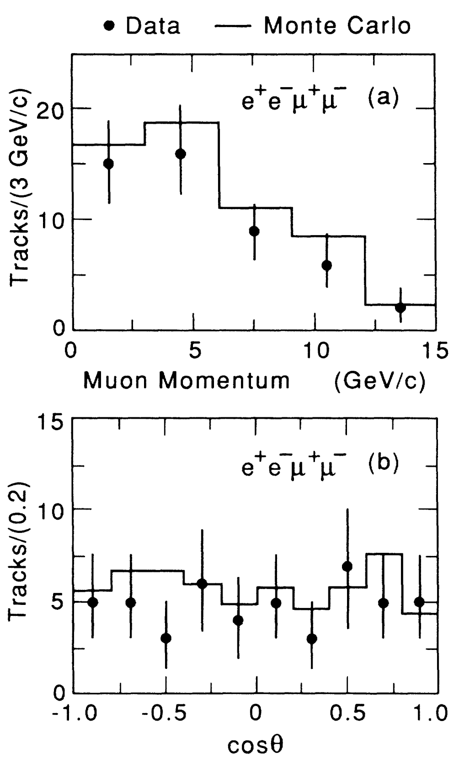

FIG. 8. Distribution of muons ($\mu^\pm$) in $e^+e^-\mu^+\mu^-$ events: (a) muon momentum; (b) cos(θ). The histogram is the QED prediction to order α^4.

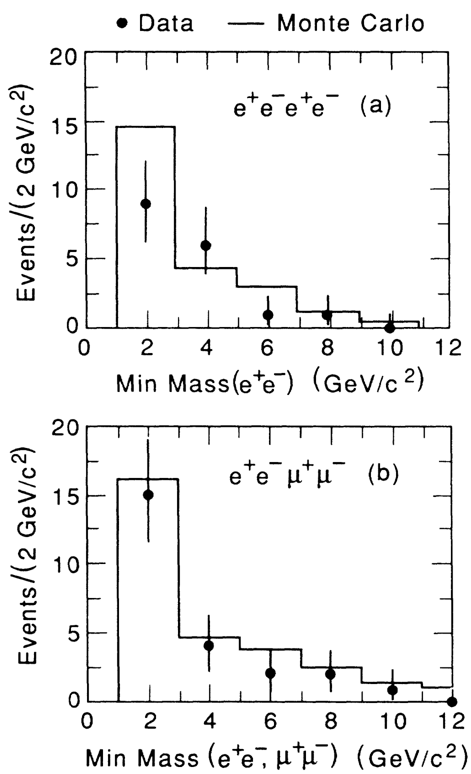

FIG. 9. Lowest-invariant-mass distribution in (a) $e^+e^-e^+e^-$ events; (b) $e^+e^-\mu^+\mu^-$ events. The histogram is the QED prediction to order α^4.

effect was not included in the Monte Carlo calculation of Berends *et al.* We estimated the correction to the four-lepton cross section for initial-state radiation. In our estimate we made use of the factorization of the infrared contributions and the strong peaking of the photon cross section in directions parallel to the motion of the charged particles. We used the probability function of Kuraev and Fadin[7] to describe the emission of real photons. We generated $e^+e^-e^+e^-$ events, which we simulated and passed through the same analysis programs as the real data, at various center-of-mass energies below 29 GeV. All appropriate kinematical transformations between the center-of-mass and the laboratory frames were taken into account. We found[4] that the ratio of the cross section of $e^+e^-\to e^+e^-e^+e^-$ corrected for initial-state radiation over the lowest order (α^4) $e^+e^-\to e^+e^-e^+e^-$ cross section is 0.98 ± 0.08, where the error is due to limited Monte Carlo statistics.

Table I summarizes the results for the predicted number of signal and background events which are expected to pass all cuts for an integrated luminosity of 291 pb^{-1}. The number of data events is presented without background subtraction. The number of expected data and background events has been corrected for initial-state radiation.

The errors attached to the expected values (data and background) are statistical, where the error on the correction for initial-state radiation has also been included. The errors in the total expected events are statistical (first error) and systematic (second error). The systematic error is due to the uncertainty in the integrated luminosity of the data (2%). The 90%-C.L. upper limits are given in background processes where zero events passed the identification cuts.

Various distributions are shown for the final samples and compared with order-α^4 QED calculations normalized to the experimental integrated luminosity of 291 pb^{-1}. These distributions include all four tracks of an identified event, even when only three tracks, or two tracks of the same charge sign have been individually identified. In such cases the assumption of lepton-number conservation determines the identity of the remaining tracks in the event. Figures 6 and 7 show the energy and angular distribution of electrons $(e^\pm)$ in $e^+e^-e^+e^-$ and $e^+e^-\mu^+\mu^-$ events, whereas Fig. 8 shows the momentum and angular distribution of muons $(\mu^\pm)$ in $e^+e^-\mu^+\mu^-$ events. Figure 9 shows the minimum mass distribution of two leptons. We took the lowest invariant mass of all four possible combinations (e^+e^-) in

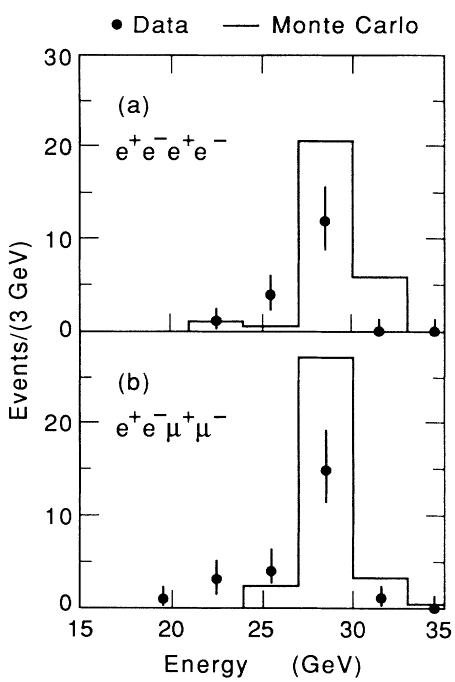

FIG. 10. Energy distribution in (a) $e^+e^-e^+e^-$ events; (b) $e^+e^-\mu^+\mu^-$ events. The histogram is the QED prediction to order α^4.

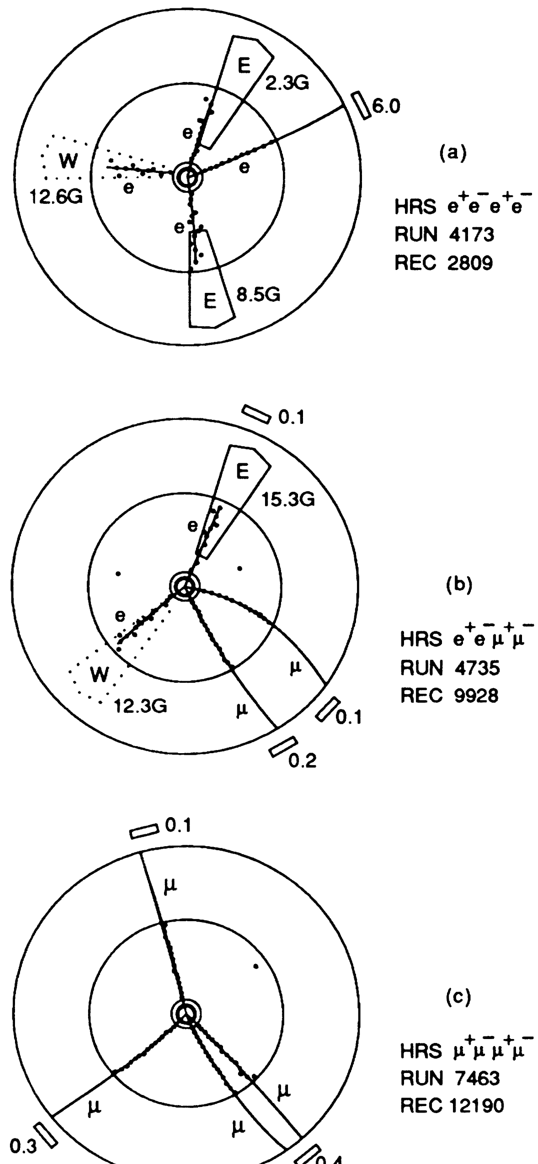

FIG. 11. Characteristic event pictures of $e^+e^-e^+e^-$, $e^+e^-\mu^+\mu^-$, and $\mu^+\mu^-\mu^+\mu^-$ final states.

$e^+e^-e^+e^-$ events, and the lower invariant mass of the e^+e^- or $\mu^+\mu^-$ combinations in $e^+e^-\mu^+\mu^-$ events. Figure 10 shows the energy (total scalar momentum) distribution of $e^+e^-e^+e^-$ and $e^+e^-\mu^+\mu^-$ events. The data show a tail of lower-energy events consistent with initial-state radiation. This effect is not included in the order-α^4 calculation of Berends *et al.* which was used for the comparison of the data with theory. The various distributions show reasonably good overall agreement between the data and the QED predictions. Finally, Fig. 11 shows characteristic event pictures for each of the three processes $e^+e^-e^+e^-$, $e^+e^-\mu^+\mu^-$, and $\mu^+\mu^-\mu^+\mu^-$.

V. CONCLUSIONS

Production of four-lepton final states in e^+e^- interactions was studied using the HRS detector at PEP at a c.m. energy of 29 GeV. The leptons were required to be at angles greater than 26° with respect to the beam axis and all oppositely charged-lepton pairs in a given event were required to have a mass greater than 1 GeV/c^2. The QED Monte Carlo calculation of Berends *et al.* is in reasonably good agreement with the data for $e^+e^-\to e^+e^-e^+e^-$, $e^+e^-\to e^+e^-\mu^+\mu^-$, and $e^+e^-\to\mu^+\mu^-\mu^+\mu^-$ processes. We observe a combined total of 42 such identified events compared to an expected mean number of 58.9. Taking into account statistical fluctuations and systematic errors, these numbers agree within 2 standard deviations. The excellent intrinsic momentum resolution of the HRS ($\delta p/p=0.002p$, with p in GeV/c) allows the identification of four-lepton events by kinematics alone without introducing significant backgrounds. Elimination of electron and muon identification requirements reduces inefficiencies and systematic errors associated with the identification process and yields a total of 73 observed events compared to the expected mean number of 79.6. Agreement in this case is within 1 standard deviation. The various kinematical distributions show good overall agreement.

Taking into account the acceptance of the HRS detector, the selection criteria applied to the data, the total integrated luminosity of the examined data sample, we find agreement between our results and those from other experiments.[8] We, too, do not observe any significant deviation from the QED calculations to α^4.

ACKNOWLEDGMENTS

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