Measurement of the Topological Branching Fractions of the \( \tau \) Lepton


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We report new and precise measurements of the decay branching fractions of the \( \tau \) lepton to one and three charged particles. The data, corresponding to an integrated luminosity of 176 pb\(^{-1}\), were collected by the high resolution spectrometer HRS at the \( e^+e^- \) storage ring PEP at SLAC operated at \( \sqrt{s} = 29 \) GeV. The fractions of \( \tau \) decays into one and three charged particles are 0.869 ± 0.002 ± 0.003 and 0.130 ± 0.002 ± 0.003, respectively.

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We report new measurements of the topological decay branching fractions of the \( \tau \) lepton to one and three charged particles, denoted by \( B_1 \) and \( B_3 \), respectively. The study of \( \tau \) decay is of particular interest since the high lepton mass allows decays to a variety of final hadronic states and the \( V - A \) coupling of the hadronic weak current limits the quantum numbers to a restricted class of resonances such as \( \rho, A_1, \rho', \) etc. Although our overall understanding of the \( \tau \) decay is satisfactory,\(^1\) many decay modes are poorly known and in particular the sum of the individually measured one-prong modes falls below the measurements of \( B_1 \), a difference highlighted by the results reported in this paper.

The \( e^+e^- \) storage ring PEP at the Stanford Linear Accelerator Center provides an excellent laboratory for these studies since high-statistics samples of 14.5-GeV \( \tau \)'s are available. At such energies the \( \tau^+\tau^- \) final state can be identified with little background. The data sample used in this analysis, which corresponds to an integrated luminosity of 176 pb\(^{-1}\), was collected by the high resolution spectrometer (HRS).

The HRS detector has been described in detail elsewhere.\(^2\) Features of the detector relevant for this analysis include charged-particle tracking over 90% of the solid angle in a solenoidal magnetic field of 16.2 kG, and detection of electromagnetic showers by a series of lead-scintillator calorimeters. To ensure a uniformly high detection efficiency we only use events with \( |\cos \theta| < 0.55 \), where \( \theta \) is the angle of the thrust axis with respect to the beam direction. Within this fiducial region, the momentum resolution for high-momentum tracks is \( \sigma_p/p = 2 \times 10^{-3} p \) (\( p \) in GeV/c).

The resolution for low-momentum tracks is dominated by multiple scattering, but the momentum error on such tracks is less than 1%. The high magnetic field and large tracking radius minimize track overlap and thus ensure very high track reconstruction efficiency. The 40-module barrel-shower counter system has an energy resolution of \( \sigma_E/E = 16\%\sqrt{E} \) (\( E \) in GeV) and a time-of-flight resolution of \( \sigma_t = 360 \) ps. The positions of the electromagnetic showers are measured by means of a set of proportional wire chambers to an accuracy of \( \pm 3 \) cm. There was very little material between the \( e^+e^- \) annihilation point and the first layer of the main tracking system.\(^3\)

The recorded events were required to satisfy one of the three neutral and/or charged triggers: (a) neutral energy \( E > 4.8 \) GeV; (b) \( E > 2.4 \) GeV and at least one charged track; (c) two or more charged tracks, at least one of which is in time with the beam crossing. Most of the \( \tau \) events satisfy both (b) and (c).

The \( \tau \)-pair production at \( \sqrt{s} = 29 \) GeV yields a final state with a clear back-to-back topology. The analysis is restricted to cases in which the \( \tau \) decays to one or three charged particles. We only used events that had zero net charge and contained two, four, or six tracks, with one or three tracks in each hemisphere. The characteristics of the HRS lead to very few events in the 1-2, 1-4, or 2-3 topologies.\(^4\) These charge-nonconserving events have a large background contamination and were only used to estimate the systematic error in the tracking simulation. All the charge-conserving events were required to satisfy the following selection criteria: The distance of closest approach of a track to the interaction point was less than...
1 cm radially and less than 9 cm along the beam direction. The scalar sum of the charged momenta was between 7.25 and 23.2 GeV/c. The background from Bhabha scattering was reduced by requiring that the total momentum of any jet be less than 13 GeV/c. The contamination from two-photon interactions was suppressed by requiring that the acollinearity angle between the two jets be less than 45°.

Additional selection criteria were applied separately to the two-prong, and to the four- and six-prong events. For two-prong events, the momentum of each track was required to be greater than 1 GeV/c. Cosmic rays were rejected by requiring that the time of flight of each track be within 3 ns of the expected time. To reject backgrounds from $e^+e^- \rightarrow e^+e^-$ and $e^+e^- \rightarrow e^+e^-$ processes, no event was allowed to contain more than one electron. An electron was defined as a particle that deposited an energy in the shower counters larger than 3 GeV or greater than one-half of its momentum. The backgrounds were further suppressed by requiring that the shower energy of the event be less than 14.5 GeV. For events with neither track identified as an electron, the background from the $e^+e^--e^+e^-\mu^+\mu^-$ process was minimized by requiring that the momentum of at least one track be greater than 5 GeV/c. The background from the $e^+e^-\mu^+\mu^-$ process was minimized by requiring that the momentum of each track be less than 11 GeV/c. Noninstrumented regions of the detector were avoided by requiring that each track be at least 2 cm from the edge of the shower-counter modules. This ensured a uniform detection efficiency and so facilitated the Monte Carlo simulation.

A source of background in the four-prong events is radiative Bhabha scattering with the photon converting in the beam pipe. These events were rejected by requiring that no more than one track deposit a shower energy greater than 90% of its momentum. The background from low-multiplicity hadronic events was reduced by requiring that, for four-prong events, the invariant mass of the three charged particles be less than 1.6 GeV/c², and that, for six-prong events, the invariant mass of the three charged particles in each jet plus any associated photons be less than 2.0 GeV/c².

The resulting numbers of events are 2235 two-prong, 1760 four-prong, and 103 six-prong events. The detection efficiencies for the three different topological final states have been calculated by use of a Monte Carlo simulation. The simulation produces $\tau$-pair events according to the standard electroweak theory, including the $\alpha^3$ QED radiative corrections. We used the $\tau$ decay branching fractions and the assigned errors listed in Table I. The table is compiled from the results given in Refs. 6–9. The $\pi^\pm 2\pi^0$ and $\pi^\pm \pi^+\pi^-$ decay modes are assumed to come from $A_1$ decay. For the $\pi^\pm 3\pi^0$ branching fraction, we used the theoretical prediction of Gilman and Rhie.1 The $3\pi^\pm \pi^0$ rate has been increased to normalize the sum of all individual decay modes to 100%. The measurements of $B_1$ and $B_2$ are insensitive to this adjustment. The four-pion final states are assumed to be dominated by phase space.

The backgrounds were estimated both from data and from Monte Carlo calculations. The Monte Carlo predictions were crosschecked with the data. By use of the calculation of Berends and Kleiss,10 Bhabha scattering was found to contribute less than 0.1% to the two-prong topology. A similar calculation5 was also used to estimate the background from the $e^+e^-\mu^+\mu^-$ reaction. The backgrounds from the two-photon processes were determined by use of the calculations of Smith, Vermaseren, and Grammer.11 The calculated rates agreed with the observed number of events with large acollinearity angle. The hadronic contamination in the four- and six-prong events was measured by use of six-prong events with a three-prong jet mass above the $\tau$ mass and the assumption that the two jets in the events fragment independently. Beam-gas and cosmic-ray backgrounds were found to be negligible. The total background in the sample is (7.0 ± 0.5)%.

The trigger efficiency for two-prong events was (99.93 ± 0.01)% measured by use of Bhabha events since they always satisfy both the charged trigger and the neutral trigger. The efficiencies for the higher-

<table>
<thead>
<tr>
<th>Topology</th>
<th>Final state</th>
<th>Branching fraction (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One prong</td>
<td>$\pi^\pm \pi^\mp\pi^\mp$</td>
<td>7.7 ± 1.0</td>
</tr>
<tr>
<td></td>
<td>$\pi^\pm \pi^\mp\pi^\mp\pi^0$</td>
<td>13.5 ± 3.0</td>
</tr>
<tr>
<td></td>
<td>$\pi^\pm \pi^\mp\pi^\mp\pi^0$</td>
<td>0.05 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>$\pi^\pm \pi^\mp\pi^\mp\pi^0$</td>
<td>0.05 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>$\pi^\pm \pi^\mp\pi^\mp\pi^0$</td>
<td>Total 21.3 ± 3.2</td>
</tr>
<tr>
<td>Three prong</td>
<td>$\pi^\pm 2\pi^\mp 2\pi^\mp\pi^0$</td>
<td>0.05 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>$\pi^\pm 2\pi^\mp 2\pi^\mp\pi^0$</td>
<td>0.05 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>$\pi^\pm 2\pi^\mp 2\pi^\mp\pi^0$</td>
<td>Total 0.10 ± 0.04</td>
</tr>
</tbody>
</table>
multiquark 1-3 and 3-3 topologies are higher. The overall detection efficiencies, including geometrical and kinematical cuts, as well as the background summary are given in Table II.

To determine the branching ratios we also use the Monte Carlo simulation to measure the topological migration. If $N_{11}$, $N_{13}$, and $N_{33}$ are the measured number of events in the two-, four-, and six-prong topologies, and $\varepsilon$ is the migration matrix between different topologies, then

$$
\begin{pmatrix}
N_{11} \\
N_{13} \\
N_{33}
\end{pmatrix} = \begin{pmatrix}
N_0^{11} & N_0^{13} & N_0^{33} \\
N_1^{11} & N_1^{13} & N_1^{33} \\
N_3^{11} & N_3^{13} & N_3^{33}
\end{pmatrix} \begin{pmatrix}
\varepsilon_{11} & \varepsilon_{13} & \varepsilon_{33} \\
\varepsilon_{11} & \varepsilon_{13} & \varepsilon_{33} \\
\varepsilon_{11} & \varepsilon_{13} & \varepsilon_{33}
\end{pmatrix} \begin{pmatrix}
N_0^{11} \\
N_0^{13} \\
N_0^{33}
\end{pmatrix},
$$

where $N_0^{ij}$ are the number of events originally produced in the $i-j$ topology and $N_0^{00}$ are the number of background events in that topology. In the $3 \times 3$ matrix, $\varepsilon_{ij}^{00}$, for example, is the probability that a 1-3 event migrated to the 3-3 topology and was detected. For an ideal detector the migration matrix would be diagonal. The off-diagonal elements of our migration matrix are small.12

If $N_0^{ij}$ is the total number of produced $\tau$ pairs then

$$
N_0^{ij} = N_0 B_i B_j, \quad N_0^{00} = 2 N_0 B_i B_3, \quad \text{and} \quad N_0^{03} = N_0 B_3^2. 
$$

Ignoring any $\tau$ decay to seven charged particles, we have the additional constraint $B_1 + B_3 + B_5 = 1$. Our measurement9 of $B_5 = 0.001$ gives $B_1 + B_3 = 0.999$. $B_3$ is measured by assuming Poisson statistics and performing a maximum-likelihood fit to minimize the differences between the background-subtracted measurements and the predicted values of $N_0^{00}$, $N_0^{03}$, and $N_0^{33}$. The result13 is $B_3 = 0.130 \pm 0.002$. The decay $K^0 \rightarrow \pi^+ \pi^-$ is included in this $B_3$ measurement.

The measurement relies on the excellent charged-particle tracking system for event selection, with minimum reliance on the shower-counter system. Since $B_3$ is measured by comparing the number of two-, four-, and six-prong events, many systematic errors cancel, allowing the value of $B_0$ to be determined with high precision. In particular the result does not depend on our knowing the absolute value of the integrated luminosity for the experiment.

To study the systematic error that results from the uncertainties in the $\tau$ branching fractions, the analysis has been repeated with variation of the branching fractions within the errors given in Table I. As part of this investigation, we scaled all the one-prong branching fractions to give a total one-prong branching fraction of 0.87. We assume that the only source of $K^0$ mesons comes from the $K^+ \rightarrow K^- \pi^0$ decay. Other modes such as $\tau \rightarrow K^+ K^- \nu$ and $\tau \rightarrow K^- Q\nu$ would contribute at the fraction of a percent level14 and could increase $B_3$ by 0.0006 \pm 0.0005. From these calculations a systematic error of 0.002 was estimated. A systematic error due to the uncertainty in the tracking-inefficiency simulation was estimated by comparing the observed number of 1-2 events with the Monte Carlo prediction. Although such events are at the few percent level and are well simulated,4 a systematic error of 0.001 was assigned to $B_3$. Uncertainty in the simulation of the photon conversion was investigated by comparing the observed number of four-prong Bhabha events, in which the photon converted in the beam pipe, with the Monte Carlo calculation.10 There were 151 \pm 12.3 Bhabha events13 with external photon conversion in the data, compared with the 123 \pm 12.7 events predicted by the Monte Carlo calculation. From this comparison, a systematic error of 0.001 was assigned to $B_3$. The uncertainties coming from the background subtraction and from the statistical error in the migration matrix calculation each contribute a systematic error of 0.001 to $B_3$. Because all the systematic errors are independent, the total systematic error of 0.003 is obtained by adding the above errors in quadrature.

The analysis has been repeated with use of data in a larger solid angle, $|\cos \theta| < 0.85$. The result for $B_3$, although with larger systematic uncertainties, is in good agreement with the value quoted above.

The final results are $B_1 = 0.869 \pm 0.002 \pm 0.003$ and $B_3 = 0.130 \pm 0.002 \pm 0.003$. As seen in Table III, our precise value for $B_3$ is somewhat smaller than, but in agreement with, earlier published measurements. We are in excellent agreement with the recent but less accurate result from Fernandez et al.4

Our measurement of $B_1$ is considerably greater than the sum of the exclusive channels listed in Table I. One possible explanation is that some of the

<table>
<thead>
<tr>
<th>Event topology</th>
<th>Number of events</th>
<th>Defection efficiency (%)</th>
<th>$e^+ e^-$</th>
<th>$\mu^+ \mu^-$</th>
<th>$e^+ e^- \mu^+ \mu^-$</th>
<th>$e^+ e^- \pi^+ \pi^-$</th>
<th>$e^+ e^- q\bar{q}$</th>
<th>$q\bar{q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two prong</td>
<td>2235</td>
<td>10.7 \pm 0.11</td>
<td>&lt; 0.1</td>
<td>1.8 \pm 0.1</td>
<td>5.5 \pm 0.3</td>
<td>0.3 \pm 0.1</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>Four prong</td>
<td>1760</td>
<td>28.0 \pm 0.2</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>5.2 \pm 1.0</td>
</tr>
<tr>
<td>Six prong</td>
<td>103</td>
<td>20.8 \pm 0.6</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>0.7 \pm 0.5</td>
<td>...</td>
<td>9.4 \pm 3.0</td>
</tr>
</tbody>
</table>
TABLE III. Measured values for $B_3$.

<table>
<thead>
<tr>
<th>Detector</th>
<th>Reference</th>
<th>$B_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRS</td>
<td>Present</td>
<td>0.130 ± 0.002 ± 0.003</td>
</tr>
<tr>
<td>MAC</td>
<td>8</td>
<td>0.133 ± 0.003 ± 0.006</td>
</tr>
<tr>
<td>Mark II</td>
<td>16</td>
<td>0.14 ± 0.02 ± 0.01</td>
</tr>
<tr>
<td>TPC</td>
<td>17</td>
<td>0.148 ± 0.009 ± 0.015</td>
</tr>
<tr>
<td>CELLO</td>
<td>7</td>
<td>0.147 ± 0.015 ± 0.013</td>
</tr>
<tr>
<td>TASSO</td>
<td>18</td>
<td>0.153 ± 0.011 ± 0.019</td>
</tr>
</tbody>
</table>

branching-fraction measurements are in error. Other possibilities include conventional decay modes not yet observed such as $\pi^\pm \eta \pi^0 \nu$, or decays involving new particles. More accurate measurements of the individual decay modes are needed to investigate further such possibilities.

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3Three different arrangements were used with the thickness of 0.010$X_0$ for an integrated luminosity of 20 pb$^{-1}$, 0.013$X_0$ for the next 86 pb$^{-1}$, and 0.015$X_0$ for the final 70 pb$^{-1}$ of data.
4The percentages of 1-1 and 1-3 events that migrated into 1-2 topology are as follows:

<table>
<thead>
<tr>
<th>Migration</th>
<th>Data (%)</th>
<th>Monte Carlo (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1 → 1-2</td>
<td>0.5 ± 0.3</td>
<td>0.2 ± 0.1</td>
</tr>
<tr>
<td>1-3 → 1-2</td>
<td>3.3 ± 0.9</td>
<td>2.0 ± 0.3</td>
</tr>
</tbody>
</table>