

AN INVESTIGATION OF HOW PRESERVICE TEACHERS' ABILITY TO
PROFESSIONALLY NOTICE CHILDREN'S MATHEMATICAL THINKING RELATES TO
THEIR OWN MATHEMATICAL KNOWLEDGE FOR TEACHING

By

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Abstract

The National Council of Teachers of Mathematics and the standards movement promoted reform-based instructional practices as the most effective way to teach mathematics. Thus, teachers are encouraged to teach students mathematics to build conceptual understanding through providing students with opportunities to problem solve, draw conclusions, justify answers, communicate with peers, and make connections to the real world and other content areas. Due to the robust nature of mathematics, it is critical for preservice teachers to develop a strong content and pedagogical-content knowledge of mathematics. This combination of knowledge is referred to as Mathematical Knowledge for Teaching (Ball & Hill, 2009). The ability to professionally notice a child's mathematical thinking is also needed to effectively teach mathematics using reform-based methods. Research has shown that both Mathematical Knowledge for Teaching and professional noticing of mathematical thinking is developed over time. The intent of this study was to determine if preservice teachers' Mathematical Knowledge for Teaching and their ability to professionally notice a child's mathematical thinking developed over the course of a semester, in which they were involved in a mathematics methods course and a field experience in an elementary classroom. The study also examined if there was a relationship between preservice teachers' Mathematical Knowledge for Teaching and their ability to professionally notice mathematics thinking. Data were gathered through child response videos with preservice teachers noticing different components of a child's mathematical thinking and preservice teacher completion of the Learning Mathematics for Teaching instrument, which evaluates Mathematical Knowledge for Teaching. Analysis of Variance and Pearson-Product Moment Correlation were used to analyze the data from those instruments.

The results of the study showed a positive statistical change in the preservice teachers' abilities to make appropriate instructional decisions for a child who was answering mathematics questions, which is one component of professional noticing. There was however no statistical change in the other components of professional noticing or in their Mathematical Knowledge for Teaching scores. These results emphasize the necessity for teacher education to provide more opportunities for preservice teachers to grow in both their content and pedagogical-content knowledge. Expanded opportunities during their teacher education program will help preservice teachers develop Mathematical Knowledge for Teaching and the ability to professionally notice a child's mathematical thinking, which will better prepare them for their time in the classroom.

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Chapter 1

The Research Problem

Introduction

The National Council of Teachers of Mathematics (NCTM) and the standards movement encouraged the creation of reform-based mathematics education. The NCTM (2000) Principles and Standards is an important document that the council developed in order to guide the reform movement (Van De Walle, 2007). The reform movement started in the early 1980s by encouraging the use of problem solving in mathematics classrooms (Van De Walle, 2007). Specifically, the work of Piaget, Dewey, and Vygotsky helped focus research on how children can best learn mathematics, which has helped encourage the reform of mathematics education (Van De Walle, 2007). Reform-based mathematics education, also known as standards-based mathematics education, focuses in part on teaching the five main content areas of mathematics, which are numbers and operations, algebra, geometry, data analysis and probability, and measurement (Van De Walle, 2007). The NCTM has also developed five process standards that accompany the content standards. They are problem solving, reasoning and proof, communication, connections, and representation (Van De Walle, 2007). “These process standards should not be regarded as separate content or strands in the mathematics curriculum. Rather, they direct the methods or processes of doing all mathematics, and therefore, should be seen as integral components of all mathematics learning and teaching” (Van De Walle, 2007, p. 5). Because of the robust mathematics content that children need to learn, it is critical that preservice teachers are thoroughly prepared to help children learn with deep understanding, which includes teachers having a strong content and pedagogical content knowledge (Ball, 2008; Hill et. al., 2005). Not only do teachers need to have strong content and pedagogical knowledge, but they

also need to have a thorough understanding of how to recognize children's mathematical knowledge and thinking (Carpenter, 1989).

It is essential for teachers to develop strong mathematical knowledge for teaching, and that development starts at the preservice level. Mathematical Knowledge for Teaching, or MKT, is the mathematical knowledge that is needed in order to carry out the work of teaching mathematics (Ball, 2008). MKT focuses on the tasks involved in teaching mathematics and the mathematical demands of the tasks involved in teaching (Ball, 2008). MKT will vary from one individual to another. It is anticipated that MKT will also change over time, as a result of experiences working with children, participation in educational classes, and educational research and reading (Borko, 1992). MKT consists of two main, distinguishable domains, with multiple subdomains within each main domain, that represents the mathematical knowledge important for teaching (Ball & Thames, 2010; Ball, 2008). The two main domains are subject matter knowledge and pedagogical-content knowledge (Ball & Thames, 2010; Ball, 2008).

Subject matter knowledge consists of various subdomains, which all revolve around how well the teacher understands the subject content, the teacher's ability to perform mathematical calculations, and effectively teaching the content to the students (Ball & Thames, 2010; Ball, 2008). Pedagogical-content is a teacher's ability to analyze students' answers and to provide probing questions, which will keep students thinking and learning (Ball & Thames, 2010). It involves bridging content knowledge and the practice of teaching (Ball, 2008). A teacher's ability to recognize and use a child's mathematical knowledge during instruction is another critical component of high quality mathematics teaching (James et. al., 2010).

Children have a variety of mathematics strategies, which they use to solve a problem and that makes it important for teachers to have the ability to recognize those strategies and to utilize

them in their teaching. Preservice teachers must have the opportunity to strengthen their pedagogical content knowledge, which includes their ability to notice children's mathematical thinking and reasoning (Borko, 1992). A teacher's attention to a student's thinking and the strategies that they use will evolve through experience and reflecting, but it is an essential component to helping children progress through all of the mathematics content standards, while using the process standards that the NCTM created (Fennema, 1993; Jacobs et al., 2010).

Rationale

This study was conducted in order to gain a better understanding of how preservice teachers' MKT develops through experiences in the college classroom and through work with children in an elementary school. Also through this study teacher educators can gain a more thorough comprehension about how preservice teachers' pedagogical-content knowledge and ability to notice a child's mathematical thinking changes as a result of these experiences. The preservice teachers in this study took part in a mathematics methods course and also worked in elementary schools, with children in mathematics settings. They had the opportunity to analyze the children's mathematical knowledge in an actual classroom setting and then discussed their observations and conclusions under the guidance of an instructor. They also learned a variety of instructional strategies to utilize in teaching students mathematics in an inquiry environment, where students were problem solving, explaining their thinking, and asking questions. Throughout this study, preservice teachers' MKT and their ability to professionally notice a child's mathematical thinking will be analyzed in order to determine if there is a change in their ability, as well as to find out if there is a relationship between their MKT and ability to professionally notice.

Previous research has been conducted in which researchers analyzed MKT with teachers at varying levels of their career. Also, research has examined teachers' ability to professionally notice a child's mathematical thinking. But, there has been little research that has looked at both MKT and professional noticing of preservice teachers together. Preservice teachers have been minimally researched in regards to both MKT and professional noticing. Previous research has also not combined these two areas in mathematics. Therefore, the current study not only focuses on preservice teachers' and their development of MKT and professional noticing, but it also examines if there is a relationship in the development of these two areas.

In order to be an effective teacher, an individual must possess both pedagogical knowledge and content knowledge, or as it is now referred to by researchers, pedagogical-content knowledge. Pedagogical knowledge includes the following: knowing the subject, understanding numerous teaching strategies, (e.g. how to use manipulatives and representations to illustrate a problem, how to ask questions which encourage thinking, or how to use a child's previous knowledge to create a mathematics problem), and knowing how to effectively teach the subject so that all students of varying abilities can learn and understand (Shulman, 1987). "Effective teachers know much more than their subjects, and more than good pedagogy. They know how students tend to understand and misunderstand their subjects; they know how to anticipate and diagnose such misunderstandings; and they know how to deal with them when they arise. Such knowledge differs from knowledge of generic teaching skills because it is content specific" (Darling-Hammond, 2005, p. 205). Deborah Loewenberg Ball and her colleagues expanded these ideas of content knowledge and pedagogical-content knowledge and created the concept of MKT, which is a necessary component to effective mathematics

instruction (Ball, 2008; Hill et al., 2005). Therefore, it is important to discover how MKT develops with preservice teachers since it is critical to effective instruction.

As preservice teachers experience working with children in the schools and learning about children's mathematical knowledge during mathematics methods courses, it is hypothesized that the preservice teachers' pedagogical content knowledge will improve, which will strengthen their overall MKT. Pedagogical-content knowledge is necessary for good mathematics instruction (Shulman, 1987). Pedagogical-content knowledge requires that teachers know the curriculum and the curricular tools available (Ball, 2008; Darling-Hammond, 2005). Teachers need to understand mathematics content, but it is also critical that they know how their students learn best and what struggles they may have, as well as understanding multiple methods for teaching the content to the variety of student needs in their classroom (Ball, 2008; Darling-Hammond, 2005). "It includes an understanding of how novices typically struggle as they attempt to master a domain and an understanding of strategies for helping them learn" (Darling-Hammond, 2005, p. 48). Pedagogical-content knowledge is an essential component for teachers in a reform-based mathematics classroom. Teachers require an overwhelming amount of knowledge to teach mathematics effectively to their students (Ball & Forzani, 2009). In order to properly bring in the process standards that the NCTM suggested, all of the domains within MKT are required. Previous studies have focused on the MKT of teachers who are out in the field working with children regularly. This study focuses on preservice teachers, who are taking methods courses that focus on developing the skills of MKT and professional noticing. This study will focus on how MKT and professional noticing begins to develop, rather than focusing on experienced teachers.

MKT develops over time. It requires working with children, continued education through classroom study and research, reflecting on how children's thinking and their own mathematical thinking changes, and constant effort to increase and improve content knowledge (Ball & Hill, 2009). Preservice teachers need to be engaged in inquiry tasks that are meaningful as they develop their mathematical knowledge (Chapman, 2007). When they are asking questions, participating in discussions, watching and working with students as they problem solve, their own mathematical knowledge will develop. Because MKT consists of both content and pedagogical knowledge, preservice teachers need experiences that will enhance both of these areas. "Preservice elementary teachers are likely to need help to deepen their understanding of concepts from both a mathematical and pedagogical perspective" (Chapman, 2007, p. 342). Through coursework and experiences with children in a classroom setting, preservice teachers' MKT should increase. This study was conducted in order to examine the development of MKT of preservice teachers over the course of a semester, in which the preservice teachers took part in mathematics methods courses where they learned about the mathematics content and how to best teach that content to students, as well as worked in classrooms with children.

Prior to student teaching, preservice teachers at the University of Kansas take a mathematics methods inquiry course. As part of that course, the students learn different methods for teaching the five mathematical content areas and they also have the opportunity to go into the field and work with students in a school setting. Prior to beginning this study, it was anticipated that the preservice teachers' MKT would grow throughout the semester, in part because of the coursework that they were taking and in part because of their experiences working with children in the elementary schools.

Research was completed in 1993 by Fennema and colleagues, where a group of first grade teachers participated in a summer workshop and learned about how students learn addition and subtraction. They created their own instructional plans, using different instructional strategies, and when implementing their instructional plans they learned about students' thought processes when problem solving. As a result of this workshop, "students in experimental classrooms exceeded students in control classes in number fact knowledge, problem solving, reported understanding, and reported confidence in their problem solving abilities" (Fennema, 1993, p. 559). The teachers who took part in this study learned valuable information about children's mathematical thinking and were able to take that information and apply it when teaching their own students (Fennema, 1993). The teachers in this study were experienced teachers and it was evident by the results that these teachers gained a great deal of understanding in how children think mathematically. The current study is examining whether or not a similar development can happen with preservice teachers over the course of a semester while working in a mathematics methods course and in the elementary classroom with children doing mathematics.

Randolph Phillip and colleagues (2007) conducted a similar study with preservice teachers. They examined how the teachers' MKT changed as a result of either being in a university mathematics methods courses and focusing on children's mathematical thinking, or being out in the field in mathematics classrooms working with children. Those students who were out in the field, did not take a mathematics methods course, but had participated in mathematics content courses. Those students, who took the mathematics methods course, did not have any actual opportunities to work with students, but they analyzed them on video and had teacher led discussions focusing on the student's mathematical knowledge. "The results that are

significant indicate that the [preservice teachers] who focused on children's mathematical thinking (in the methods course) developed more sophisticated beliefs about mathematics and mathematics understanding and learning than those who did not focus on children's mathematical thinking" (Philipp, 2007, p. 458). By analyzing children's mathematical thinking under the guidance of an instructor, preservice teachers have the opportunity to reflect on how children think about mathematics differently than how adults think and reason through a mathematics problem (Philipp, 2007).

In this same study, Philipp and his colleagues also noticed that MKT does in fact change for preservice teachers who are out in the field in a classroom setting. These teachers were able to make practical application of what they had learned in mathematics content courses and they saw how that knowledge could be used when working with children (Philipp, 2007). Having the opportunity to work with children provides preservice teachers the opportunity to see how the students worked to make sense of mathematics. One student from the program said, "Working with children is a very valuable experience. It is really easy to say or think what you are going to do in a situation, but sometimes in reality it doesn't work out or you think of something better. Working with children early helps you get comfortable and prepares you for what's to come" (Philipp, 2007, p. 461). Preservice teacher's MKT will develop in some capacity as a result of being in classroom mathematics methods courses and out in the schools working with children (Philipp, 2007). The current study analyzes preservice teachers who participate in both a mathematics methods course and are in the field working with students simultaneously. It includes the development of MKT, but also focuses on the development of professional noticing.

Beginning and preservice teachers must develop a good knowledge, understanding, and ability to attend to children's mathematical thinking in order to provide effective instruction,

which can come from methods courses, professional development, or working with children (Jacobs & Phillip, 2010). Preservice teachers need to realize that the ability to notice how children think is an important instructional tool that all teachers need to develop in order to help children of varying abilities learn (Fennema, 1993; Jacobs et al., 2010). “Although teachers may be able to achieve short-term computational goals without attending to students’ knowledge, they need to understand students’ thinking to facilitate students’ growth in understanding and problem solving” (Carpenter, 1989, p. 502). Therefore, it is critical to analyze how preservice teachers develop the ability to professionally notice a child’s mathematical thinking. Teacher educators want preservice teachers to be able to go into their own classroom and provide effective mathematics instruction so that children can be successful. In order to do this, preservice teachers need to learn how to professionally notice a child’s mathematical thinking. This study will analyze the different components of professional noticing and how this develops with preservice teachers, so that teacher educators can gain a better understanding of this development in order to create university classes that foster this continued development.

Children’s mathematical knowledge is constantly evolving and advancing as they grapple with different mathematical content areas and have different experiences (Fennema, 1993). Having an understanding of this changing knowledge is an important component of a preservice teacher’s MKT. MKT includes pedagogical content knowledge and that includes the ability to understand the students in the classroom, as well as the content, and teaching (Ball & Thames, 2010). Fennema wrote about a first grade teacher and her instructional strategies, “Knowledge about children’s thinking is increasing almost daily, and this teacher and her classroom demonstrate that building instruction on knowledge of children’s thinking is not only possible, but it results in children’s learning mathematics that is clearly in line with the standards proposed

by the National Council of Teachers of Mathematics” (Fennema, 1993, pp. 577-578).

Development of both MKT and professional noticing is an important part of teacher education. This development does not happen immediately. From this study, teacher educators will notice how this development occurs and then be able to make instructional decisions based on encouraging additional growth.

The ability to use a student’s prior knowledge and understanding of mathematics is an important component to being an effective teacher and is included in both MKT and professional noticing. Understanding a student’s prior knowledge is critical to creating valuable learning experiences because students are able to relate their new experiences to information already stored in their long-term memory (Ormrod, 2008). “Learners who have a large body of information already stored in long-term memory have more ideas to which they can relate their new experiences and so can more easily engage in such processes as meaningful learning and elaboration. Learners who lack relevant knowledge must resort to inefficient rote-learning strategies” (Ormrod, 2008, p. 211). Without a good understanding of a student’s prior knowledge, the learning experiences created could be ineffective and will result in more inefficient rote learning strategies rather than meaningful learning and elaboration (Ormrod, 2008). “There was a consensus that as a field we need (a) to develop better ways to characterize partial understandings, (b) to understand how knowledge evolves and is built on the structures one has, and (c) to figure out the kinds of activities that connect with people’s partial understandings and promote conceptual growth in reliable ways” (Schoenfeld, 1994, p. 325) Learning activities will be more beneficial to students, if their teachers have a better understanding of the knowledge and any misconceptions that the students are bringing into the classroom (Ball, 2008). Having an understanding of a student’s prior knowledge, will aid the

teacher in noticing a child's mathematical thinking (Jacobs et al., 2010). This understanding is a component of both MKT and professional noticing, which is analyzed in this study.

A part of pedagogical-content knowledge is a teacher's development of knowledge, dispositions, and practices that support building on children's mathematical thinking (Turner, 2012). A teacher's attention, which includes the depth and detail that they notice, changes with experience. "We found that initially [preservice teachers] focused on their own teaching moves and/or those of an experienced mentor, not attending to what children think or understand, nor to aspects of children's home or community experiences that might be relevant to their mathematics learning" (Turner, 2012, p. 73). Through repeated opportunities to attend to children's mathematical thinking, preservice teacher's attention will progress (Jacobs et al., 2010; Turner, 2012). But, in order for that progress to occur, preservice teachers need to have a general awareness and understanding of a child's mathematical thinking (Jacobs et al., 2010). "It is reasonable to think about teachers becoming concurrently more sophisticated in their thinking as they spend time in the classroom and are supported by opportunities for learning and professional development" (Schneider, 2011, p. 538) Another purpose of this research is to determine if there is a relationship between the development of a preservice teacher's MKT and their ability to notice a child's mathematical thinking and reasoning. It is believed that MKT progresses and the ability to notice a child's thinking improves, but do they improve concurrently? The results of this study will substantially contribute to our understanding of this relationship.

Statement of the Problem

The work of teaching is unnatural (Ball & Forzani, 2010). "To listen to and watch others as closely as is required to probe their ideas carefully and to identify key understandings and

misunderstandings, for example, requires closer attention to others than most individuals routinely accord to colleagues, friends, or even family members” (Ball & Forzani, 2009, p. 499). Teachers have to pay close attention to their student’s thinking and reasoning skills. There are multiple ways that a student will think when solving a problem and there are just as many ways that a teacher can respond (Jacobs et al., 2010). Teachers need to be able to notice a child’s thinking and then instruct them based on the child’s mathematical knowledge, not on how they would personally solve the mathematics problem (Jacobs et al., 2010). This does not always happen. Many teachers are entering the classroom lacking in their own MKT. Within MKT, they could have a low content knowledge or a low pedagogical-content knowledge. In general, with any lack of teacher knowledge, the ability to understand and notice a child’s mathematical thinking is much more difficult. Therefore, instruction is not effective and the children are not learning to conceptually understand and apply mathematics.

Purpose of the Study

This study was designed to examine preservice teachers’ MKT and the extent to which it develops over the course of a semester, in which they will take a mathematics methods course and work with elementary-aged students in a mathematics classroom setting. The ability of the participating preservice teachers to notice a child’s mathematical thinking will also be examined. The purpose of this study was also to determine if there is a relationship between preservice teachers’ MKT and their ability to notice a child’s mathematical reasoning and thinking. It was hypothesized that as preservice teachers take part in mathematics methods courses and have the opportunity to work in the elementary schools, with students in a mathematics setting, their MKT will grow. As their MKT grows, their ability to notice a student’s mathematical thinking and make appropriate instructional decisions based on their thinking will also improve.

Research Questions

1. Does preservice teachers' Mathematical Knowledge for Teaching change during a semester, in which they participate in both a mathematics methods course and work with children in a mathematics setting?
2. Does preservice teachers' noticing of a child's mathematical thinking and reasoning develop over the course of a semester, in which they participate in a mathematics methods course and work with children in a mathematics setting?
3. Is there a relationship between Mathematical Knowledge for Teaching and noticing of children's mathematical reasoning and thinking?

Summary of Chapter 1

In summary, developing a strong MKT, as well as having the ability to notice a child's mathematical thinking and reasoning is a process which begins at the preservice level. "Strong MKT seems to correlate with certain habits of mind, such as careful attention to mathematical detail and well-explicated reasoning, as well as agility with a variety of mathematical productions from textbooks and students" (Ball & Hill, 2009, p. 70). Therefore it is important for preservice teachers to have experience in the schools, working with children. Ball and her colleagues found that mathematics workshops and professional development also aided in developing mathematics knowledge in teachers (Ball & Hill, 2009). Therefore, preservice teachers need to have experience in a mathematics methods classroom, where they learn how to make mathematical representations, explanations, and to communicate about mathematics to their students. This should help teachers improve their MKT, which will in turn improve their ability to work with children in mathematics classrooms.

In order for preservice teachers to better understand children's mathematical knowledge and thinking, they need to have experience analyzing children's thinking, while working with them (Jacobs et al., 2010). Randolph A. Philipp found that giving preservice teachers an opportunity to work directly with children, under the guidance of an instructor who helps the teachers decipher and analyze the situations they experience, is very worthwhile in aiding the development of a better understanding of children's mathematical thinking (Philipp, 2007).

The students, who participated in this research, had the opportunity to be in a mathematics methods classroom, where they worked with an instructor, who helped them learn about children's mathematical thinking, how to understand and analyze that thinking and most importantly, how to utilize it in planning their lessons. The participating students also had the opportunity to go to elementary schools, where they worked with children in an actual school setting.

The purpose of this study is two-fold. First, the researcher hopes to determine if the MKT of preservice teachers develops over the course of a semester in which they learn about children's mathematical thinking with an instructor and work with children in a classroom environment. Secondly, the researcher hopes determine if a relationship exists between a preservice teacher's MKT and their ability to notice a child's mathematical thinking and reasoning. The study intends to find out if a teacher's MKT has an effect on their ability to notice a child's mathematical thinking.

Chapter 2

Review of the Literature

Introduction

Reform mathematics curriculum is based on teaching children how to conceptually understand mathematics by utilizing problem solving and reasoning in instruction (Battista, 1994). Many teachers that are entering the education field need to deepen their understanding of concepts from both a mathematical and pedagogical perspective, which includes their ability to understand and recognize their student's mathematical thinking and reasoning (Chapman, 2007). Because of the robust nature of the mathematics curriculum, it is critical for teachers to have a strong mathematical knowledge for teaching, which consists of subject matter knowledge and pedagogical content knowledge (Hill et al., 2005). Without this, they will have a difficult time teaching their students how to conceptually understand and learn mathematics (Ball & Forzani, 2010). It is very important for teachers to be able to notice their students' mathematical thinking and reasoning skills and then make instructional decisions based on those skills, which will enable children to effectively learn mathematics (Fennema, 1993).

During the undergraduate years, preservice teachers need repeated opportunities to practice the interactive work of teaching and grow in their own Mathematical Knowledge for Teaching (MKT), as well as gain a better understanding of how children mathematically think and reason (Ball & Forzani, 2009). The purpose of this study is to determine if there are relationships between preservice teachers' MKT and their ability to notice and respond to children's mathematical thinking. The second part of this study intends to determine if, as preservice teachers' MKT progresses, there will be similar progression in their ability to recognize mathematical thinking of children.

This chapter will provide a historical context of how and why mathematical knowledge for teaching was developed. It will include an overview of the history, components, and instruction in reform mathematics education and what is needed to make reform mathematics instruction successful in the classroom. How preservice teachers tend to progress in their MKT will be discussed, as well as their ability to notice and effectively respond to a child's mathematical thinking and reasoning.

The Beginnings of Reform Mathematics Education

The National Council of Teachers of Mathematics (NCTM) and the standards movement encouraged the creation of reform-based mathematics education. The NCTM Principles and Standards (2000) have helped to shape a major reform movement in school mathematics (Battista, 1994). Reform-based mathematics education, also known as standards-based mathematics education, is focused on teaching the five main content areas of mathematics, which are numbers and operations, algebra, geometry, data analysis and probability, and measurement (NCTM, 2000). “The vision for mathematics education described in *Principles and Standards for School Mathematics* (2000) is highly ambitious. Achieving it requires solid mathematics curricula, competent and knowledgeable teachers who can integrate instruction with assessment, education policies that enhance and support learning, classrooms with ready access to technology, and a commitment to both equity and excellence” (NCTM, 2000, p. 8). The NCTM also developed five process standards to go along with the content standards. They are problem solving, reasoning and proof, communication, connections, and representation (NCTM, 2000). “These process standards should not be regarded as separate content or strands in the mathematics curriculum. Rather, they direct the methods or processes of doing all mathematics, and therefore, should be seen as integral components of all mathematics learning and

teaching”(Van De Walle, 2007, p. 5). Reform-based mathematics education encourages a conceptual understanding of mathematics and has students do and know mathematics (Battista, 1994; Hiebert, 2000). NCTM (2000) recommends that children solve complex problems and to read, write, and discuss mathematics, rather than memorize computations and procedures (Battista, 1994; Hiebert, 2000).

Constructivism is a theory of knowledge that was developed many years ago and plays a significant role in the development of curriculum, specifically reform-based mathematics curriculum. Constructivist theorists believe that

students will be actively engaged with interesting and relevant problems; they will be able to discuss with each other and with the teacher; they will be active inquirers rather than passive; they will have adequate time to reflect; they will have opportunities to test or evaluate the knowledge that they have constructed; and they will reflect seriously about the constructions produced by other students and the teachers (Phillips, 2004, p. 52).

Knowledge is constantly emerging and the learner is continually looking for meaning and understanding (Joyce, 1996). Reform-based mathematics education was designed to have students problem solve, reason, communicate, and reflect, while supporting their own personal construction of ideas, which is a similar principle to constructivism (Battista, 1999).

Jean Piaget was one of the first constructivist theorists and many others have built upon what he started. Piaget believed that children are active and motivated learners and that they are naturally curious and desire to seek out information and construct cognitive structures (Phillips, 2004). He believed that children have the ability to personally organize what they learn (Phillips, 2004). “Piaget depicted learning as a very constructive process: children create (rather than simply absorb) their knowledge about the world” (Ormrod, 2008, p. 310). Interaction with the environment is also critical for children’s learning because it can be assimilated into knowledge (Harlow, 2006). Piagetian children typically construct knowledge individually as they interact

with the external world and progress through the stages of development at their own rate (Harlow, 2006). Piaget believed that children move through the stages as their brains mature. With maturity, they are able to obtain the thought processes required for the next stage (Ormrod, 2008).

Piaget's four stages of cognitive development are sensorimotor, preoperational, concrete operations and formal operations (Ormrod, 2008). Each of these developmental stages builds upon one another. The stages are qualitatively different, so children at the different stages think differently. Each stage has its own characteristics of thinking. The stages are also invariant. Children cannot skip stages, but rather they must go through the stages in a particular order. Finally, the stages are universal and are seen across all environments and around the world. The sensorimotor stage generally occurs from ages birth until two years old. In this stage, Piaget believed children's understandings were based on their interactions with the world (Ormrod, 2008). The next stage is the preoperational stage, which generally occurs from age two years old through age seven years old. In this stage, children can begin to reason about events, as well as think and talk about things outside of their immediate experience (Ormrod, 2008). Children in the concrete operations stage are able to reason in a logical, adult like way. They begin to understand that their own perspectives are not always shared by others. This stage generally happens between the ages of six and 11 years old (Ormrod, 2008). Finally, in the formal operations stage, which occurs from age 11 years old through adulthood, logical and abstract thought occurs. Advanced reasoning with hypothetical situations happens during this stage (Ormrod, 2008).

Lev Vygotsky also played a role in the development of constructivism. Unlike Piaget, Vygotsky did not focus on the stages of development of a child, but instead he looked more at a

child's potential to learn (Phillips, 2004). Vygotsky believed that learning takes place in social settings, rather than individually. He proposed that children learn from others through interactions, observations, and imitations (Phillips, 2004). Vygotsky distinguished between two ability levels that characterize skills of children. The first is their actual developmental level, which is "the upper limit of tasks that he or she can perform independently, without help from anyone else" (Ormrod, 2008, p. 332). The other ability level is the level of potential development, which is the upper limit of tasks that can be performed with the help of someone more competent (Ormrod, 2008). Vygotsky believed that children should be taught in their zone of proximal development (ZPD), which includes problem solving and learning tasks that they can only accomplish in collaboration with a more competent individual (Ormrod, 2008). Every person has a certain level of competence and what they can achieve with help and guidance from another is their zone of proximal development (Ormrod, 2008). Children develop by attempting tasks that they can only accomplish when working with a more competent individual, which is when they are attempting tasks within their ZPD (Ormrod, 2008). As new concepts are learned, the ZPD will change (Ormrod, 2008).

Darling-Hammond (2005) provided an example about teachers learning to teach students in their zone of proximal development. Students may be able to add numbers and teaching them to subtract numbers would be an appropriate next step to teach, but moving directly onto division may be beyond their current ability. In their zone of proximal development, they would be able to subtract numbers, with the help of someone, but they would be unable to divide numbers. Eventually, they will learn to subtract numbers and by doing so their zone of proximal development will change and, then with the help of someone, they will begin to learn to divide numbers, which will then be their new zone of proximal development.

All of these theorists played a role in the development of the constructivist theory for learning, which in turn helped create reform-based mathematics education. The reform-based mathematics curriculum is based on many of the ideas of these theorists. Battista (1994, p. 464) stated:

Because its instructional goals are cognitive rather than behavioral and because it seeks to mold students' own personal mathematical ideas, teaching that is consistent with the reform movement requires an extensive knowledge of how students learn mathematics. Teaching based on a constructivist view of learning must be guided by knowledge of the conceptual advances that students need to make for various mathematical topics and of the processes by which they make these advances.

In the reform movement, the teacher is considered a facilitator, by guiding children's constructive activities, which is similar to the constructivist view (Battista, 1994). John Dewey, who had some similar views of constructivism, believed that students should be consistently and actively engaged in solving relevant, interesting, and meaningful problems (Phillips, 2004). Students should be communicating with one another as well as with the teacher. "The alert educator, then, will be concerned to select material that is appropriate to the developmental stage of the learner" (Phillips, 2004, p. 49). In reform-based mathematics education, the job of teachers is also to provide students with opportunities to solve complex problems and formulate and test mathematical ideas in order to draw their own conclusions (Battista, 1999). "To develop powerful mathematical thinking in students, instruction must focus on, guide, and support their personal construction of ideas. Such instruction encourages students to invent, test, and refine their own ideas rather than to blindly follow procedures given to them by others" (Battista, 1999, p. 429).

Reform-based mathematics education encourages learning by "doing". Teachers pose complex and interesting problems and students work toward finding solutions, as well as forming

mathematical and logical arguments to support their solutions (Battista, 1999). Students are actively figuring things out, testing ideas, making hypotheses, developing reasons for their strategies, and providing explanations (Van De Walle, 2007). The reform movement was based on a Constructivist view of learning. Constructivists believe that with experience people will learn to make sense of a concept by connecting it to other information or mental organizations, as opposed to just knowing the concept at surface level and not understanding the reasons for it (Fenstermacher, 2004). Proponents of reform-based mathematics education believe in a principle of Piaget, “Actually, in order to know objects, the subject must act upon them, and therefore transform them: he must displace, correct, combine, take part, and reassemble them” (Phillips, 2004, p. 44).

The Reform-based Mathematics Classroom

A reform-based mathematics classroom is different than a traditional classroom. In a traditional classroom focus is spent on computations and procedures (Battista, 1999). Students in a traditional classroom see a mathematics problem as having only a single method for finding the correct answer. Memorization and mechanical application are the main focus of that classroom. “When mathematics is taught as received knowledge rather than as something that (a) should fit together meaningfully, and (b) should be shared, students neither try to use it for sense-making nor develop a means of communicating with it” (Schoenfeld, 1994, p. 57). In a reform-based mathematics classroom, mathematics is taught through problem solving. Students are expected to “do” mathematics and make sense of the different mathematical concepts (Battista, 1999; Hiebert & Stigler, 2004). Battista (1999, p. 426) explained the reform mathematics environment:

In the classroom environment envisioned by NCTM, teachers provide students with numerous opportunities to solve complex and interesting problems; to read, write and discuss mathematics; and to formulate and test the validity of personally constructed

mathematical ideas so that they can draw their own conclusions. Students use demonstrations, drawings, and real-world objects-as well as formal mathematical and logical arguments-to convince themselves and their peers of the validity of their solutions.

In this new reform-based environment, students are actively involved in the learning process.

They develop their own mathematical understanding through reasoning, collaborating, reflecting, and refining their own ideas (Battista, 1999).

This classroom is also collaborative, with students working together to solve problems.

They actively engage with their peers to discover new avenues for solving problems.

“Mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns” (Schoenfeld, 1994, p. 60).

Mathematics should not be taught in isolation. It is important for students to be able to talk about mathematics with their peers and to explain their ideas and thoughts to one another. “Learning to communicate in mathematics fosters interaction and exploration of ideas in the classroom as students learn in an active, verbal environment” (Van De Walle, 2007, p. 5).

The NCTM (2000) developed process standards that are intended to be used in conjunction with the teaching of content standards, which will then direct the processes and methods of doing mathematics (Van De Walle, 2007). While learning the content, students are expected to problem solve, communicate with their classmates and teacher, reason and prove their answers, make connections to other content or the real world, and represent the mathematical problems through models, drawings, or words (NCTM, 2000). These standards are meant to build students’ conceptual mathematics understanding as well as their procedural fluency, which involves knowing why and how they acquired a certain answer through active engagement (Maccini, 2002). It includes making connections to other content areas of

mathematics or other subject areas in order to solve a problem (Van De Walle, 2007). It focuses on conceptual understanding, rather than rule-driven computation (Maccini, 2002).

In the reform-based classroom teachers are seen as facilitators. They pose problems to students, and it is their responsibility to invent methods of solution, carry them out, and evaluate the degree of success (Stanic, 2003). They help create an environment in which students actively engage in mathematical sense-making (Schoenfeld, 1994). “From our perspective, the essential pedagogical task is not to instill the correct ways of doing but rather to guide children’s constructive activities until they eventually find viable techniques. Such guidance must necessarily start from points that are accessible to the children” (Battista, 1994, p. 464). In order to do this properly, teachers must have an understanding of their students’ conceptual structures and methods and be able to monitor what the students are experiencing, thinking and learning (Battista, 1994; Hiebert & Stigler, 2009). Battista (1994) expresses the importance of teachers understanding their students’ current mathematical structures and conceptual understanding, which will be more successful in helping all students conceptually learn mathematics. In general, teachers should teach mathematics by encouraging sense-making through communication and meaningful problem solving, rather than teaching it as an assortment of received knowledge (Schoenfeld, 1994).

A reform-based mathematics classroom actively engages students in mathematics. Students learn to understand mathematics by discovering the reasons for different procedures and calculations making sense of them (Schoenfeld, 1994). They learn to make and test conjectures, to reason and provide rationale for their solutions to problems (Van De Walle, 2007). In order to develop these abilities, students must be given the opportunity to do these things in the classroom (Hiebert, 2000). It is important for teachers to help their students construct mathematical

understanding, which will help them utilize this understanding more often and in new situations (Hiebert, 2000; Hiebert & Stigler, 2009). If they have a conceptual understanding, they will then be able to apply the procedures to other problems. James Hiebert (2000, p. 437) stated:

Those who study learning, including many teachers, are not surprised that understanding does not detract from skill proficiency and may even enhance it. If you understand how and why a procedure works, you will probably remember it better and be able to adjust it to solve a new problem. If you memorize a procedure and do not have a clue about how it works, you have little chance of using it flexibly.

The Teacher in a Reform Mathematics Classroom

In order to teach effectively in a reform-based mathematics classroom, teachers need to have a large range of mathematics knowledge, which includes having a thorough understanding of the content, with the ability to unpack it for the learner (Ball & Forzani, 2010; Ball, 2008). To be considered highly qualified, elementary mathematics teachers must possess a variety of skills and abilities and be able to utilize them effectively in the classroom. First, they need to have a strong knowledge of their subject and be familiar with and able to use a variety of methods to represent that content for learners of all kinds (Ball, 2008; Darling-Hammond, 2005). They also need a strong pedagogical knowledge, or knowledge of how to teach mathematics to all of the students in their class (Ball, 2008; Hill et al., 2005). Highly qualified and effective teachers understand how their students think and are able to answer their questions (Ball, 2008; Jacobs et al., 2010). They know how to evaluate their students' learning abilities and can teach to a variety of those abilities. "Instruction that builds on children's ways of thinking has been linked to rich instructional environments for students" (Jacobs et al., 2010, p. 172). All of these skills are necessary for teachers to effectively teach mathematics.

Highly qualified mathematics teachers need subject matter competence; they need to

know how to perform a wide range of mathematical tasks and know that there are multiple approaches to solving the problem (Ball, 2008; Darling-Hammond, 2005). “With a better grasp of what they are responsible for teaching, post induction teachers are in a good position to identify areas of content they want to strengthen”(Feiman-Nemser, 2001a, p. 1039). Highly qualified teachers not only need to understand mathematics, but they also need to a strong understanding and familiarity with the NCTM Principles and Standards. They should have a good grasp of how the content standards and curriculum fit together and they should know the ultimate goal of their instruction (Darling-Hammond, 2005). The NCTM also introduced the use of process standards in teaching mathematics and it is important for teachers to understand those standards so that they can properly implement them so their students can properly acquire the content knowledge (NCTM, 2000). Overall, “effective teachers need subject matter competence: they need to know how to solve the problems they pose to students and to know that there are multiple approaches to solving many problems” (Darling-Hammond, 2005, p. 205).

Teachers must possess more than subject matter competence. They must also have a strong pedagogical knowledge (Darling-Hammond, 2005). Teaching students the mathematics content requires not only the ability to be able to solve a mathematics problem but also the capacity to make the content accessible to all learners by providing multiple instructional methods so students learn the content and grow in their mathematical understandings (Ball & Forzani, 2009; Hiebert & Stigler, 2009). They need to know how to teach the mathematics content, pose questions, represent problems and delve into their students’ work and questions in order to identify their mistakes (Ball & Hill, 2009). They need to have a variety of instructional strategies for teaching content to their students, so as to engage them in meaningful mathematics work (Hiebert & Stigler, 2009).

Researchers have begun combining pedagogical and content knowledge and calling it pedagogical-content knowledge, which includes knowing the subject, numerous teaching strategies, and specifically how to effectively teach the subject (Shulman, 1987). Darling-Hammond (2005, p. 205) discussed the need for pedagogical-content knowledge:

Effective teachers know much more than their subjects, and more than good pedagogy. They know how students tend to understand and mis-understand their subjects; they know how to anticipate and diagnose such misunderstandings; and they know how to deal with them when they arise. Such knowledge differs from knowledge of generic teaching skills because it is content specific.

Pedagogical-content knowledge includes knowing the kinds of curricular tools that are available for teaching a particular subject (Ball, 2008; Darling-Hammond, 2005). For example, highly qualified mathematics teachers would know how to use geoboards, pattern blocks or a hundreds chart to help teach a particular concept to a group of students.

Deborah Loewenberg Ball and her colleagues extended these ideas of content and pedagogical-content knowledge and created the idea of MKT. Ball (Ball, 2008) discussed the components of MKT. MKT consists of two main categories: subject matter knowledge and pedagogical-content knowledge. As explained by Ball (2008) within subject matter knowledge, there are three main elements. The first element is common content knowledge (CCK), which is mathematical knowledge that is not necessary for teaching, but it is the ability to do the work that is assigned to the students (Ball, 2008). Also, specialized content knowledge (SCK) is included under this umbrella. This includes knowing how to represent a mathematics problem by using drawings, manipulatives, models, or demonstrations and making sense of students' solutions (Ball & Thames, 2010). SCK is mathematical knowledge that is unique to teaching (Ball, 2008). Finally, within subject matter knowledge is horizon knowledge, which is having an awareness of

how mathematical topics are related over the larger mathematics landscape (Ball, 2008). With horizon knowledge a teacher can add new perspectives to the content being taught and make connections with other mathematical content areas (Ball & Thames, 2010).

Ball and colleagues (2010) extended the ideas of Shulman (1997) and discussed the other category of MKT which is pedagogical-content knowledge. The first element of pedagogical-content knowledge is knowledge of the curriculum, which means that teachers know what they need to teach. Pedagogical-content knowledge also includes knowledge of content and students (KCS), which combines what a teacher knows about his or her students and mathematics, as well as knowledge of content and teaching (KCT), which combines knowing about how to teach and mathematics (Ball, 2008). KCS and KCT blend together and are referred to as pedagogical-content knowledge (Ball, 2008). All of these different components are necessary parts of all teachers' MKT. "Strong MKT seems to correlate with certain habits of mind, such as careful attention to mathematical detail and well-explicated reasoning, as well as agility with a variety of mathematical productions from textbooks and students" (Ball & Hill, 2009, p. 70). Teachers with a strong MKT teach students more effectively by providing better mathematical explanations, using better representations, and having a clearer understanding of the structures of mathematics and how they connect to each other (Hill et al., 2005).

Effective mathematics teachers, in reform-based classrooms, also need to have a good understanding of their students' mathematical knowledge and thinking. Included in this is knowledge of the students' development, as well as their previous mathematical knowledge and experiences (Ormrod, 2008). They know how development of children progresses and how it can vary from one child to another, as well as the complexities that children's mathematical thinking involves (Jacobs et al., 2010). Children are different and learn at different rates and

have a variety of thinking strategies, therefore, teachers need to know their students and their abilities, while having a good overall understanding of how children develop and what aids in their development over time (Darling-Hammond, 2005; Jacobs et al., 2010). “Teachers need to understand children’s development and how it influences, and is influenced by, their learning. A foundation of knowledge about child development is essential for planning curriculum; designing, sequencing, and pacing activities; diagnosing student learning needs; organizing the classroom; and teaching social and academic skills” (Darling-Hammond, 2005, p. 88). If a teacher does not know where a child is developmentally, the learning experiences that are used will not be effective because they will either be too easy or too difficult (Ormrod, 2008). Highly qualified teachers are able to accentuate children’s conceptual understanding of mathematics, because they have a better knowledge of their students’ understandings and ways of learning (Ball, 2008; Battista, 1994).

While understanding children’s backgrounds and development are part of gaining a good understanding of their mathematical thinking, teachers also need to learn how and why they respond to different mathematical problems and experiences while they are in-the-moment (Ball, 2008; Jacobs et al., 2010). Research has found that teachers do not make instructional decisions based on their students’ knowledge or misinterpretations of a problem, but rather tend to follow the curriculum and will only make minor changes based on student understanding (Jacobs et al., 2010). “Without listening thoughtfully to your students (assessing) on a daily basis, tomorrow’s lesson can be based only on a guess at their needs” (Van De Walle, 2007, p. 77). If teachers are not focusing on students’ knowledge and understanding, it will be harder for them to progress in problem solving and conceptual mathematics understanding (Carpenter, 1989; Jacobs et al., 2010).

Carpenter and colleagues (Fennema, Peterson, Chiang, & Loef, 1989) did a study in which first grade teachers attended a summer workshop aimed at teaching them about how children develop their addition and subtraction concepts. As part of the workshop, they planned instruction based on what they had learned about children's mathematical thinking. As a result of this workshop, teachers tended to allow students to use more problem solving in their classrooms and gave students the opportunity to use multiple strategies to solve a problem. Carpenter (1989, p. 528) explained the results:

By allowing students to use any strategy they chose, the teacher was able to assess how each student was thinking about the problem rather than requiring the student to imitate one strategy that the teacher specified. This approach was more significant with the belief that instruction should facilitate children's construction of knowledge rather than present information and procedures to children.

In these classrooms, teachers were able to assess the ability of their students to problem solve, which provided them with important information that could guide future instruction, as well as provide students with feedback on their strategies.

Preservice Teacher Preparation

Preservice teachers do not always go into the elementary schools with a strong content and pedagogical knowledge. Preservice preparation is a time to begin forming the skills necessary to develop a good understanding of mathematics so that they can effectively teach it to children (Feiman-Nemser, 2001a). "Subject matter preparation that prospective teachers currently receive is inadequate for teaching toward high subject matter standards, by anyone's definition" (Wilson, 2001, p. 192). Preservice teachers must be given the opportunity in their university course work to strengthen their subject matter knowledge (Borko, 1992). "Content knowledge is immensely important to teaching and its improvement" (Ball, 2008). It is critical to

learn how content knowledge can be used to help preservice teachers learn how to teach effectively (Ball, 2008).

Pedagogical knowledge is also important. Teachers have to know how to teach the content to students (Ball & Forzani, 2010). Ball (2009) discussed the importance for teachers to not only understand how to do the mathematics, but they also need to have an understanding of how to teach the content. “In one of our own studies, we found that summer professional development sites that focused on teachers’ work on mathematical representation, explanation, and communication outperformed similar sites with less focus on those topics” (Ball & Hill, 2009, p. 70). Just knowing how to do mathematics will not make someone a good mathematics teacher (Ball, 2008). “Pedagogical preparation means many things: instructional methods, learning theories, educational measurement and testing, educational psychology, sociology, and history” (Wilson, 2001, p. 193). Having a variety of experiences in all of these areas will help better prepare preservice teachers for when they have their own classroom.

Another area that should be considered when discussing teacher preparation is providing preservice teachers with adequate time in the field, working with students of varying mathematical abilities. Preservice teachers’ need time in the schools working with children and learning to do particular tasks such as creating respectful learning environments, assessing student’s mathematics skills, and adapting curriculum to fit the needs of all learners (Ball & Forzani, 2009). It is important for them to watch children think and reason about mathematics (Jacobs et al., 2010). “From field experience, prospective teachers reported acquiring survival skills, learning about students, and recognizing that their students’ understandings vary, are complex, and differ from the teachers” (Wilson, 2001, p. 196). Field experience will provide hands-on learning experiences for future teachers that classes cannot provide and it will give

them the opportunity to take what they have learned in their mathematics content and methods courses and apply that knowledge when working with children (Wilson, 2001).

While out in the field, it is important for preservice teachers to have worthwhile opportunities to utilize and test the theories that they have learned during their coursework and to apply the practices in a classroom with the students with whom they are working (Feiman-Nemser, 2001a). A preservice teacher needs opportunities to try out new ideas, practice activities learned in classes, use technology, create relationships, and work with children with differing abilities and this can only happen when they are out in the schools.

While in their undergraduate courses, preservice teachers also should have experience creating and using assessments. Assessments are valuable tools for teachers to acquire a better understanding of children's mathematical thinking and reasoning, as well as their progress and difficulties with learning (Black, 1998). However, if they do not know how to create and use formative and summative assessments, these tools are not valuable to them (Black, 1998). Since frequent assessment is a critical part of the daily work of a mathematics teacher, future teachers need to have opportunities to practice using these assessments, so that they will have a level of comfort when working with them in the classroom and so they will know how to effectively use them with their students.

Preservice teachers should have a strong familiarity with the NCTM standards and they need experience creating lessons utilizing these standards. "The five process standards should not be regarded as separate content or strands in the mathematics curriculum. Rather, they direct the methods or processes of doing all mathematics and therefore, should be seen as integral components of all mathematics learning and teaching" (Van De Walle, 2007, p. 5) Universities need to provide ample opportunities for preservice teachers to create lessons incorporating the

standards or children will not learn how to elaborate, justify their answers, or systematically test different approaches to solving a problem (Maccini, 2002). There will always be a variety of abilities in general education mathematics classrooms, so it is critical that preservice teachers understand the mathematics standards at their particular grade level and the teaching expectations that the standards place on them (NCTM, 2000). If they do not have the background or the confidence in how to incorporate the process standards into their mathematics lessons, then it will be more difficult for all learners to gain a conceptual mathematics understanding (Maccini, 2002). Process standards will help keep activities student-centered and then teachers can adapt them to the different ability levels in their classroom, as well as to fit the differing ways that students think and reason about mathematics (Van De Walle, 2007).

The Progression of a Teacher's Mathematical Knowledge for Teaching

Preservice and experienced teachers face a great deal of mathematical demands when teaching (Ball & Thames, 2010). There are a variety of tasks they must learn to do effectively in order to be an effective mathematics teacher, which include having both a mathematical and pedagogical perspective and understanding (Chapman, 2007). Preservice teachers' MKT will progress during the preservice stage, as well as once they have their own classroom and students if they are paying careful attention to mathematical detail in their students and developing their own mathematical knowledge (Ball & Hill, 2009). Generally, there are five levels of skill development: novice, advanced beginner, competent, proficient, and expert. Research says that it takes approximately ten years to become an expert teacher (Schneider, 2011). During the preservice stage, these teachers need assistance in developing their mathematical and pedagogical knowledge by being provided with opportunities to practice their growing conceptions of teaching while using resources to help them (Bryan, 2003). Mathematical and

pedagogical knowledge does not just come naturally to preservice teachers (Ball & Forzani, 2009). Tasks which enable preservice teachers to inquire, collaborate, analyze student work, practice conceptual learning and teaching under supervision, as well as increase their content knowledge will help them develop their MKT (Ball & Forzani, 2009; Ball, 2008).

Schneider (2011) believed in order to develop pedagogical-content knowledge, preservice teachers first need to learn to think about the learners, then to focus on the teaching, and finally to reflect both on the learners and teaching. Teachers require instruction in these areas and they need to be taught how to understand a student's thinking so they can make appropriate instructional decisions. Schneider (2011) conducted a study which analyzed how teachers grow in their understandings of how children think. He noticed that teachers need instruction in learning how to analyze children's understandings. Experience alone will not teach teachers how to determine a child's understandings. "Instruction helped teachers make progress toward understanding how students develop ideas. Those only with experience did not" (Schneider, 2011, p. 547). When students are at universities working to develop and progress in their own MKT, it is important for them to have instructors who possess a strong MKT and have knowledge about what is required to develop it in others (Ball & Hill, 2009). "In order for teachers to have opportunities to learn MKT, those who prepare teachers and provide professional development will themselves need to have adequate support. Better materials, more specific guidance focused on the teaching of MKT, and better design of opportunities to learn from practice are essential" (Ball & Hill, 2009, p. 71).

Not only do preservice teachers need to have proper instruction in order to develop their MKT, but they also should have opportunities to work with children and practice teaching. A practice-focused teacher education program should provide preservice teachers opportunities to

create productive learning environments, to assess students' mathematical abilities, to talk with students about their mathematical school work, and finally, to use the skills that they have developed in their mathematical courses in an engaging environment (Ball & Forzani, 2009). Ball and Forzani (2009) believed it is necessary for preservice teachers to work in the school environment, in order to improve their MKT. "We argue not that practice with the pre-active or cognitive aspects of teaching should be eliminated but that teacher education should offer significantly more-and more deliberate- opportunities for novices to practice their interactive work of instruction" (Ball & Hill, 2009, p. 503). When preservice teachers are in the classroom working with children, it is important for them to have worthwhile experiences, such as using reflective logs, journals, and individual conferences, which will help apply their own knowledge and develop a better understanding of children's mathematical knowledge (Feiman-Nemser, 2001a).

It is safe to say that in order to improve preservice teachers' MKT, they must have worthwhile experiences in the university classroom, as well as in the schools working with children (Philipp, 2007). It is important for teachers to have mathematically rich experiences within their own learning experiences, which will enhance their ability to analyze children's mathematical knowledge (Turner, 2012). In a study that Philipp and colleagues (Philipp, Ambrose, Lamb, Sowder, Schappelle, Thanheiser, & Chauvot, 2007) did he found that preservice teachers "who focused on children's mathematical thinking (those in mathematics methods courses) developed more sophisticated beliefs about mathematics and mathematics understanding and learning than those who did not focus on children's mathematical thinking" (Philipp, 2007, p. 458). Preservice teachers should also have the opportunity to work with children and to use the information they have learned in their methods courses. Philipp also

found, “that the experience of working with children enabled [preservice teachers] not only to observe and investigate children’s thinking in general but also to grapple with understanding and supporting a particular child who was trying to make sense of mathematics” (Philipp, 2007, p. 461). Therefore, the experiences of working in the classroom, guided by an instructor, and working in the classroom with children will help develop a preservice teacher’s MKT (Philipp, 2007). “It is reasonable to think about teachers becoming successively more sophisticated in their thinking as they spend time in the classroom and are supported by opportunities for learning and professional development” (Schneider, 2011, p. 540).

Mathematical Knowledge for Teaching and Student Achievement

MKT consists of two main domains, which are subject matter knowledge and pedagogical-content knowledge (Ball & Thames, 2010). Ball and colleagues developed an instrument designed to measure a teacher’s ability to teach the mathematical content that is required, as well as the specialized knowledge that is required to teach the content (Hill et al., 2005). The instrument, which is called the Learning Mathematics for Teaching instrument (LMT), includes measurement tasks that measure teachers’ ability to provide appropriate explanations and representations for the mathematical problems that students solve (Hill et al., 2005). The instrument was designed to measure the intersection of the knowledge teachers have about content and students (Ball, 2008). In order to test the LMT instrument, developers used it on a group of first and third grade teachers. The results found that “knowledgeable teachers may provide better mathematical explanations, construct better representations, better “hear” students’ methods, and have a clearer understanding of the structures underlying elementary mathematics and how they connect” (Hill et al., 2005, p. 401). Therefore, one might conclude that MKT is an important component in helping students understand and learn mathematics (Hill

et al., 2005). If they do not have strong content knowledge and the ability to teach mathematics in an effective way, students may not develop a strong conceptual understanding of mathematics (Hill et al., 2005).

Teachers need strong pedagogical-content knowledge in order to effectively teach mathematics and content knowledge is a critical component of pedagogical-content knowledge (Ball & Thames, 2010). “Qualitative research on teacher knowledge acknowledges that the mathematical content knowledge required for high-quality instruction is not general mathematical knowledge that is picked up incidentally but profession-specific knowledge that is acquired in University-level training and can be cultivated through systematic reflection on classroom experience” (Baumert, 2010, p. 140). MKT, with specialized mathematical knowledge and skill is necessary to help students learn to conceptually understand mathematics (Ball & Thames, 2010). Teachers with a deep conceptual understanding of mathematics tend to have a larger repertoire of teaching strategies and mathematical representations and explanations (Baumert, 2010). It has also been concluded by researchers that “the efforts of teachers with a limited conceptual understanding fell short of providing students with powerful mathematical explanations” (Baumert, 2010, p. 139). Therefore, teachers with a strong MKT will help students make more gains in mathematics learning (Hill et al., 2005). With a strong MKT, teachers will be able to present material more clearly, in an error-free environment (Hill, 2010).

Progression of the Ability to Notice a Child’s Mathematical Knowledge and Thinking

The ability of teachers to notice children’s mathematical knowledge and thinking improves as a result of working with children and from participating in professional development, particularly professional development which focuses on how children think (Jacobs et al., 2010). A college course can teach preservice teachers how to blend together

children's knowledge with content, but it cannot teach preservice teachers how to make spur of the moment decisions in specific situations (Feiman-Nemser, 2001b). Teachers become more sophisticated in their thinking as they work with students, resulting from working directly with children, analyzing student work, talking with colleagues and instructors about children's thinking, and engaging in activities that promote a better understanding of how children's thinking progresses (Jacobs et al., 2010; Sherin, 2011). Therefore, in order to initially begin to develop expertise in attending to children's strategies, preservice teachers should have experience working with students, as well as learning about different instructional methods and the complexities of children's mathematical thinking under the guidance of an instructor (Jacobs et al., 2010).

Teachers engage in important and complex work, when teaching mathematics. "If instruction is to build on children's thinking, teachers must be able to attend to children's strategies, interpret their understandings, and use these understandings in deciding how to respond" (Jacobs et al., 2010, p. 192). Knowing how children think and paying attention to their students' thinking provides teachers with the ability to better understand what their students understand and exactly what they are learning (Jacobs & Phillip, 2010). But, this is not an easy task and it does not happen immediately (Jacobs et al., 2010). "We found that initially [preservice teachers] focused on their own teaching moves and/or those of an experienced mentor, not attending to what children think or understand, nor to aspects of children's home or community experiences that might be relevant to their mathematics learning" (Turner, 2012). It is important that mentor teachers and college instructors help their students learn to attend to the children's thinking, which can be through watching videos of children doing mathematics following a specific framework for noticing the children and mathematics environment (Jacobs

et al., 2010; Sherin, 2011). Preservice teachers need to have this experience in order to continue to develop and improve their ability to notice a child's thinking (Jacobs et al., 2010).

“Preoccupations of beginning teachers follow a developmental pattern that starts with concerns about self, moves on to concerns about teaching and finally arrives at concerns about pupils” (Feiman-Nemser, 2001b, p. 24). Preservice and beginning teachers may not be able to notice what experienced teachers do, because they do not yet have an elaborate understanding of how to notice, what to notice, and how to respond (Jacobs et al., 2010; Sherin, 2011).

The Importance of Noticing Children's Mathematical Thinking

In order to teach mathematics effectively and facilitate students' growth it is critical to notice a child's mathematical thinking and reasoning and respond to it appropriately (Carpenter, 1989; Jacobs et al., 2010). The focus of the new standards, or reform-based mathematics education encourages teachers to build a conceptual understanding of mathematics with their students (Maccini, 2002). In order for students to develop conceptual understanding, teachers need to understand the problem solving skills of their students and build on them (Carpenter, 1989). Fennema and colleagues (Franke, Carpenter, & Carey, 1993) followed a teacher as she learned how to analyze children's mathematical thinking and then use that knowledge in her instruction. Through this study, they found that the students in this teacher's classroom exceeded other students in the same grade level in meeting and exceeding the NCTM standards (Fennema, 1993). Carpenter (1989) also did a study with first grade teachers and found that “teachers' knowledge of their own students' abilities to solve different addition and subtraction problems was significantly positively correlated with student achievement on both computation and problem solving tests” (Carpenter, 1989, p. 502). Therefore, it is important for teacher educators to take time to teach and prepare preservice teachers in noticing their students' mathematical

thinking (Sherin, 2011). Preservice teachers require guidance and support in learning how to do this, which should occur at the preservice level (Sherin, 2011). Then, when preservice teachers have their own classroom, their ability to notice and appropriately respond to their students' mathematical thinking and reasoning will continue to positively progress (Jacobs et al., 2010).

Summary of Chapter Two

As researchers look back at the history of education, it is easy to see how reform-based mathematics education came into existence. As it is based on constructivism and ideas from John Dewey, it has taken a long time to develop. When the NCTM created the new content and process standards, the intent was to increase students' conceptual understanding of mathematics, which includes reasoning, using logic and mathematical evidence, problem solving and conjecturing (Van De Walle, 2007). Because of these robust standards, it is critical that teachers are prepared to effectively teach their students. It is important that they have a strong MKT, as well as the ability to notice children's mathematical thinking. All teachers do not immediately develop these two skills. Teachers need continued mentorship and guidance in these areas in order to continue to develop these skills (Darling-Hammond, 2005). Therefore, it is crucial that this development starts at the preservice level. Developing a strong MKT and the ability to notice children's mathematical thinking and reasoning will improve student achievement and their ability to gain a conceptual mathematics understanding.

In chapter three, the methodology that was used to discover if there was a significant change in the preservice teachers' ability to professionally notice a child's mathematical thinking, as well as if there was a significant change in their MKT will be discussed. Also, the methodology that was used to determine if there was a relationship between preservice teachers' MKT and their ability to notice children's mathematical thinking and reasoning is presented.

Chapter 3

Research Design

Introduction

This study will use a mixed methods design, which is a “procedure for collecting, analyzing, and linking both quantitative and qualitative data in a single study” (Creswell, 2005, p. 53). The rationale for using mixed methods design is that neither quantitative nor qualitative methods are sufficient, by themselves, to capture all of the data necessary for this study. When used in combination, they complement each other and allow for a more complete analysis. The purpose of this study is to determine if and to what extent preservice teachers’ mathematical knowledge for teaching changes over a semester and also to determine any changes in their ability to professionally notice children’s mathematical thinking and reasoning. The study also is intended to determine if there is a relationship between preservice teachers’ mathematical knowledge for teaching and their professional noticing of children’s mathematical thinking and reasoning. Using both quantitative and qualitative methods will allow for the most complete analysis and will build upon the strengths of both the qualitative and quantitative data (Creswell, 2005).

This study utilized both quantitative and qualitative research methods. Quantitatively, the study tested the statistical significance of preservice teachers’ MKT, over a period of time. It also attempted to determine if there was a correlation between preservice teachers’ MKT score and the scores they received on the child response questions (See Appendix A) which were used to determine their ability to professionally notice children’s mathematical thinking and reasoning. Qualitatively, the researcher interpreted the preservice teachers’ responses on the child response questions (See Appendix A) in order to give them a numerical code. The preservice teachers

were given codes for each of the three child response questions (See Appendix A) based on a scale determined by the researcher.

This study used multiple instruments, which provided numeric data that could be statistically tested. Experimental design is the basis of the research, which tested the impact of a treatment (Creswell, 2005). A within-subjects design approach was used since all participants were part of the same group and their ability was measured over time. Both descriptive and inferential statistics provided useful information about the data that was collected.

The study tested the statistical significance of preservice teachers' MKT, over a period of time. The codings from the video questions, which were created by the raters, were also used to quantitatively test for a statistical significance in the preservice teachers' ability to notice a child's mathematical understanding and reasoning. Quantitative analysis also attempted to determine if there was a correlation between preservice teachers' MKT scores and the scores that they received on the child response questions (See Appendix A). Various types of descriptive and inferential statistics will be used to analyze the data.

Purpose of the Study

This study is designed to determine if preservice teachers' MKT changes over the course of a semester, in which they took a mathematics methods course and worked with elementary-aged students in a mathematics classroom setting. The extent of the preservice teachers' ability to professionally notice a child's mathematical thinking was also examined. The purpose of this study is to determine if there is a relationship between preservice teachers' MKT and the extent to which they professionally notice children's mathematical reasoning and thinking. It was hypothesized that as preservice teachers take part in mathematics methods courses and have the opportunity to work in the elementary schools, with students in a mathematics setting, their MKT

will improve. As their MKT improves, their professional noticing of students' mathematical thinking will develop, therefore they will be able to analyze a child's mathematical strategies and make inferences about their understandings in order to encourage future learning and development. This information will be beneficial to learn to provide teacher education more information about how preservice teachers develop in their own mathematical thinking and their ability to notice children's mathematical thinking. It is useful for instructors to be aware of how these important components of teaching develop in preservice teachers. Courses can then be created and improved to encourage this development.

Research Questions

Through this study, the researcher answered the following research questions.

1. Does preservice teachers' Mathematical Knowledge for Teaching change during a semester, in which they participate in both a mathematics methods course and work with children in a mathematics setting?
2. Does preservice teachers' noticing of children's mathematical thinking and reasoning develop over the course of a semester, in which they participate in a mathematics methods course and work with children in a mathematics setting?
3. Is there a relationship between Mathematical Knowledge for Teaching and noticing of children's mathematical reasoning and thinking?

Participants

The sample for this study consists of 44 preservice teachers, enrolled in Curriculum and Teaching (C&T) 351, entitled Mathematics for the Elementary Classroom, which is an undergraduate course at the University of Kansas. All of the preservice teachers are juniors and

have been admitted into the School of Education at the University of Kansas. The preservice teachers consist of one male and 43 females. The ages of the preservice teachers range from 20 to 26. C&T 351 is the first mathematics methods course the preservice teachers have taken. It is designed to teach them how to effectively teach reform-based mathematics education in an inquiry classroom. The preservice teachers were given a consent form to sign prior to the beginning of the study. The consent form explained that if they agreed their scores on the LMT instrument and child response questions (See Appendix A) would be used in a research study. Their names would be kept anonymous and their scores on the instruments would not affect their class grade. Although both of these activities are mandatory requirements for the class, they had the option to not allow their scores to be used in the research study. They had the option not to participate or to withdraw at anytime and their grade for the class would not be affected (See Appendix D). Table 1 presents a summary of the basic demographics of the participants (See Appendix B).

Table 1

Participant Demographic Information

Student ID	Gender	Age	Number of previous collegiate mathematical classes taken
001	F	21	4
002	F	20	4
003	F	21	3
004	F	22	4
005	F	20	4
006	F	20	4
007	F	21	4
008	F	21	5
009	F	21	4
010	F	22	4
011	F	22	5
012	F	21	4
013	F	21	4
014	F	26	4
015	M	23	5
016	F	20	4
017	F	21	5
018	F	22	3
019	F	20	4
020	F	20	3
021	F	21	3
022	F	21	3
023	F	22	3
024	F	20	4
025	F	21	2
026	F	21	4
027	F	20	3

(table continues)

Student ID	Gender	Age	Number of previous collegiate mathematical classes taken
028	F	21	5
029	F	22	5
030	F	22	4
031	F	21	4
032	F	22	4
033	F	21	5
034	F	22	4
035	F	22	5
036	F	22	4
037	F	21	4
038	F	22	3
039	F	21	5
040	F	20	4
041	F	22	4
042	F	21	4
043	F	20	2
044	F	21	3

Instruments

A variety of instruments were used to gather the data. The preservice teachers watched three different videos over the course of the semester. In order to determine their level of professional noticing, the preservice teachers' responses to questions about the videos were analyzed and coded (See Appendix E). The first video is from a research project that Jacobs and Philipp published. It is a three minute clip of a student named Rex (Jacobs & Phillip, 2010). The video can be found at <http://www.nctm.org/publications/article.aspx?id=27348>.

In the video, Rex is asked a variety of numbers and operations questions and he uses multiple mathematics strategies to figure out the answers. This video was shown on the first day of class. Rex had manipulatives in a bucket to use for counting, although he never used them and no reference was made to them. The first question involved subtraction and asked how many he has left. Rex used his fingers and counted down to find the answer. The second question asked how many days until his birthday. For this question, Rex used his counting up strategy, again with his fingers, but struggled for a moment when he ran out of fingers to count. He then started over and specifically said “that’s 10” when he counted ten fingers and then he continued to count and was able to get the correct answer. The third was a multiplicative type question involving putting items into equal groups. He was asked to separate fifteen tadpoles by putting three into each jar. He was then asked how many jars he needed. Rex immediately responded that he did not know how to answer. The video ended.

The second video is a home video of a first grade girl named Stella. Stella was also asked a variety of numbers and operations questions. She had cotton balls to use for counters, if she wanted to use them, a piece of paper and pencil. The researcher in the video asked her questions, but did not provide any prompting or help. The questions the researcher asked came from research done by Fennema (1993) and colleagues. The researcher adapted questions that Fennema used when analyzing first grade students’ mathematical knowledge. This video of Stella was shown at the midpoint of the semester.

In the video, Stella was given six questions. The first and second questions involved addition, with the second question involving numbers greater than ten. Stella used a pencil and made tally marks to answer the first question and figured out the second question in her head by using base-ten knowledge. The third and fourth questions were subtraction questions. Stella did

not actually do subtraction on the subtraction questions, instead she added in her head. The final two questions were addition questions, with much larger numbers. The first question required Stella to determine the total number of paint cans on a shelf, if originally there were 24 cans and 36 more were added. Stella made 15 tally marks in groups of five and also used the counters. When she explained what she did she did not say anything about the tally marks or cotton balls instead she said she added 10 to 36 and got 46 and then added 10 more to 46 to get the answer of 56. The last question involved larger numbers. She needed to find how many paint cans there were if a shelf initially contained 58 and then 60 more were added. Stella, once again, made many tally marks on her paper and then counted the tally marks. She then used the cotton balls and continued to count up from where she left off with the tally marks. The video ended after that question.

The third video was a home video made of a first grade boy named Carson. The questions for this video were also numbers and operations items, but they were multiplicative, which could include using multiplication or division strategies (Mulligan, 1997). He also had cotton balls and paper and pencil to use if he wanted to. The questions that the parent used were adapted from research that Mulligan and colleagues (Mulligan & Mitchelmore, 1997) used when they studied the strategies that second and third graders use when solving multiplication and division problems. The video of Carson was shown during the last week of the semester.

Carson was asked six questions. The first two questions were equivalent group multiplicative questions. Carson quickly figured out the answers to both of these questions. He used his fingers on one of them, but on the other he answered immediately. The third question was a rate question, in which Carson answered immediately and did not show any specific strategy. The fourth question was an array question. Carson needed to find how many children

there were if there were four lines of children, with three children in each line. He used the cotton balls to figure out this question. On the fifth question, Carson was told that there were eight children and two tables. He needed to figure out how many children were seated at each table. This question was a partition question and he used the cotton balls again to help him answer. On the final question, Carson was told that someone had three books and another child had four times as many books. Carson used the cotton balls to find the answer, but was incorrect. The video ended after that question.

After the preservice teachers watched each video, they were given a response sheet, which asked three open ended questions (See Appendix A). The three questions were:

1. What do you think Rex (or Stella or Carson) was doing to figure out the answers to the problems?
2. What did you learn about Rex's (or Stella's or Carson's) mathematical understandings?
3. What would you do if you were Rex's (or Stella's or Carson's) teacher at the end of the video?

These questions were adapted from a research project conducted by Phillip (2010) and colleagues in which they analyzed the various levels of professional noticing of teachers. The answers the preservice teachers provided were coded on a discrete scale and used to help answer the research questions. The videos of Carson and Stella were videos made by the researcher. Prior to making the videos of the children, the parents signed a consent form stating that they understood the children would be asked a variety of mathematics questions and the videos would be shown to preservice teachers at the University of Kansas (See Appendix C).

A final instrument that was used is the Learning Mathematics for Teaching (LMT) instrument which measures MKT (Hill et. al., 2004). This instrument “probes whether teachers

can solve mathematical problems, evaluate unusual solution methods, use mathematical definitions, and identify adequate mathematical explanations in the content areas” (Hill et. al., 2010, p. 7). The instrument created by Hill (2004) and colleagues contains items that measure teachers’ MKT in five areas. The areas are: elementary number and operation; elementary patterns, functions, and algebra; grades 3-8 geometry; middle school number and operations; middle school pre-algebra and algebra. Each content area contains a content knowledge instrument, which measures the knowledge required for effective mathematical teaching, including knowledge of content and students (Ball, 2008). The LMT includes tasks that measure teachers’ ability to provide appropriate explanations and representations for the mathematical problems that students solve (Hill et al., 2005). For the purpose of this study only the items from elementary number and operations were used.

The elementary number and operations section contains two instruments. One instrument is the content knowledge test and the other is an instrument that measures the knowledge of content and student. The content knowledge test has two forms (Hill et al., 2004). The content knowledge items “probe whether teachers can solve mathematical problems, evaluate unusual solution methods, use mathematical definitions, and identify adequate mathematical explanations” (Hill et al., 2004, p. 7). The number and operations instrument also includes a form that measures teachers’ knowledge of students and content. There are three forms of this instrument (Hill et al., 2004). “These items probe whether teachers recognize common student errors, commonly used strategies, and recognize what makes material difficult or easy for students” (Hill et al., 2004, p. 7). For several reasons, the researcher only used the content knowledge instrument. First, the items in knowledge of content and students section are not meant to be taken by preservice teachers, because they do not have experience teaching. In order

to answer these questions, the preservice teachers would need to have ample experience working with children. Since they will only be in the classroom for a limited time, it was decided that this test would not provide an accurate representation of their MKT. Second, Hill and colleagues (2004) found that the reliability was low on these items. “Generally, reliabilities here are not sufficient to detect moderate effects in mid-sized groups. We recommend construction of new scales and forms from the item pool” (Hill et al., 2004, p. 7). Therefore, the preservice teachers only took the elementary numbers and operations content knowledge instrument, which will evaluate preservice teachers’ ability to solve mathematics problems, but also evaluate student mathematical methods and explanations for mathematics problems. Although the instrument is labeled the content knowledge, it evaluates all of the areas of content knowledge within MKT, which are common content, specialized content, and horizon knowledge.

Form A (2008) of the instrument, which contains twenty-eight items, was taken by the students on the first class day of the semester. They took Form B (2008) of the instrument, which has twenty-nine items during the final week of the semester. The two forms of the LMT are equated in order to be able to be used to analyze growth or difference between the two. Each item took approximately two minutes for the preservice teachers to answer. At the end of Form A, there was a brief demographic questionnaire, which asked for gender, age, and previous collegiate level mathematics classes taken (See Appendix B).

Hill (2004) explained how the items for the instruments were created. They were not aligned with any particular curriculum or professional preparation program. The reliability of the number concepts and operation instruments were determined by using item response theory. “Overall, the reliabilities were adequate (.70 or above) for most content knowledge measures piloted” (Hill et al., 2004, p. 21). The LMT instrument is not meant to make statements about

individual teacher's level of mathematical knowledge, but instead can be used to look at a group as a whole and how it changes over time.

After the preservice teachers completed each instrument, the raw scores were then entered into an Excel file and recorded as correct or incorrect. They were then changed into standard deviation units and imported into the statistical program, Minitab 16, for further statistical analysis, in order to determine if there have been changes in the preservice teachers' MKT during the semester.

Procedure

Prior to the spring semester in which the study was conducted, a pilot study was used with students in C&T 351 during the fall semester of 2012. Students in the pilot class were shown the three child response videos and asked to answer the three questions about the child's mathematical thinking. After the responses were recorded, all of the data was double-coded by two raters in order to help determine the coded discrete scale which will be used as a score for each question. They then met to discuss the criteria for each score for each question. The scores were based on theoretical categories that are derived from a theory developed by Jacobs and Philipp (2010) (Maxwell, 2005). Jacobs and Philipp (2010) used three categories when discussing students' ability to professionally notice a children's mathematical thinking. These categories are robust evidence, limited evidence and lack of evidence (Jacobs et al., 2010). The categories that were used are similar: beginner, novice, emerging, transitional. After the descriptions of the codes were determined, the two raters coded each question individually. They then discussed their individual codes and resolved any discrepancies in order to determine a consensus (See Appendix E).

For the first child response question, the preservice teacher received a score of 0, if there was a lack of evidence in their ability to attend to the child's mathematical strategies (Jacobs et al., 2010). They were considered beginner because their responses incorrect or consisted of a lack of mathematical knowledge. Also, their responses did not provide a description of what the child was doing to figure out the problem. A score of 1 indicated a novice mathematical noticing ability. In this case, their responses consisted mostly of general teaching moves, with a lack of specificity about the children's mathematical strategies (Jacobs, 2010). Their responses may indicate that they have noticed more than one strategy that the child was using to figure out the problem, but their description was very general. A score of 2 would indicate the preservice teacher is at the emergent stage. They are able to identify multiple strategies that the child is using, with a developing description of the strategies. A score of 2 would indicate that the students' responses did not provide evidence of attention to the details of the children's strategies or their understandings of how that particular child mathematically thinks (Jacobs et al., 2010). Finally, a score of 3 would suggest that the preservice teacher is at the transitional stage. In this stage they are able to identify multiple mathematical strategies, with specific descriptions of what the child is doing to figure out the problem. Also, their responses would also include a connection to what the child is doing to figure out the problem and to mathematical concepts.

The second question that the preservice teachers responded to focuses around their ability to notice what a child is doing to figure out mathematics problems and then determine what the child's mathematical understandings are. Preservice teachers received a score of 0 if there was no mathematics connection to the strategy that the child was using. Some of the responses did not provide any interpretation of the child's understanding (Jacobs et al., 2010). Preservice teachers received a score of 1 if there was limited or novice evidence of their understanding and

interpretation of the child's mathematical thinking (Jacobs et al., 2010). They are able to observe a child's strategy, but they make incorrect or immature inferences about their mathematical understandings. The responses are general enough that they could be applied to any problem or child (Jacobs & Phillip, 2010). A score of 2 indicated that the students are able to infer some basic mathematical understandings. They are able to make limited connections to what the child is doing and what that means mathematically. The preservice teacher is able to describe the child's understandings, but in broad, general terms that may be undefined (Jacobs et al., 2010). Finally, a score of 3 indicated that the preservice teacher is at the transitional stage and they are able to infer richer mathematical understanding, with a connection to specific examples of what the child is doing. The preservice teacher is able to interpret the child's understanding by making sense of the details of the strategy and reference how these details reflect what the child did understand (Jacobs et al., 2010).

The third question had the preservice teachers explain what they would do in order to continue to encourage the growth and development of the child's mathematical understandings and abilities. The same four point scale was used, but each code has a description that relates to the third question. A score of 0 indicates that the preservice teacher is at the beginner stage. Their responses for future instruction provide no evidence based on the child's understanding. The preservice teacher makes little or no reference to building upon the child's understandings for future problems (Jacobs et al., 2010). Also, their responses are teacher-centric in nature. They indicate what they, as the teacher will do or show the student based on what the teacher thinks, rather than using the child's current mathematical understandings. To receive a score of 1, the preservice teachers may be using the children's strategies, but only in broad, vague terms. The responses are not specific about the child's strategy, and may include some of their own

mathematical thinking, rather than focusing solely on the child's thinking (Jacobs et al., 2010). Their responses will show some awareness of the child's mathematical thinking, but the awareness will lack depth (Jacobs et al., 2010). The preservice teacher will have a lack of student customization when referring to how they would respond to the student in future situations (Jacobs et al., 2010). Preservice teachers received a score of 2 if they were able to consider the child's mathematical strategy when thinking about future instruction, but they were not looking at future concept development. Although, they did not look at how the mathematical concepts will continue to develop, they were able to clearly consider the child's strategy and understanding, which is a characteristic missing from the score of 1 (Jacobs et al., 2010). Finally, to receive a score of 3, there is robust evidence in their ability to professionally notice the child's mathematical thinking, which means that they understood and responded to the child based on the individual child's mathematical thinking (Jacobs et al., 2010). Preservice teachers in this category show that they are aware of the children's strategies and provide instruction to the children, while building on previous strategies and learning. Their descriptions of responses in this category include "mathematically significant details such as how children counted, used tools or drawings to represent quantities, or decomposed numbers to make them easier to manipulate" (Jacobs et al., 2010, p. 183).

During the pilot study, the researcher also had the opportunity to strengthen the definition of the codes in order to have a very consistent and clear definition of the codes for the study. The following tables provide the definitions created during the pilot study for each of the codes, as well as examples from statements the preservice teachers in the pilot study made when answering the child response questions (See Appendix E).

Table 2

Question 1: Coding, Description, and Example

Code	Description	Preservice Teacher Example
0 (Beginner)	<ul style="list-style-type: none"> ○ A list without a description ○ Incorrect analysis ○ Lack of mathematical knowledge 	<p>“Pictures, manipulatives, counting by 5s, fingers”</p>
1 (Novice)	<ul style="list-style-type: none"> ○ Identified only one strategy ○ Novice description of the strategy 	<p>“Drew the tables and the 5 dots in the middle of each 3 tables. Used cotton balls to sort numbers. Used head math. Use manipulatives.”</p>
2 (Emerging)	<ul style="list-style-type: none"> ○ Identified 2 or more strategies ○ Emerging or developing description that may not be connected to mathematical concepts 	<p>“Carson would draw the picture of the word problem. He would also use the manipulatives given. He would group 2 balls together 5 times. Carson would also count in his head. Carson counted on his hands a few times. He grouped manipulatives together for a lot of problems.” (table continues)</p>
3(Transitional)	<ul style="list-style-type: none"> ○ Multiple strategies identified with specific description of the strategies 	<p>“Pictorially representing the problems and counting. Representing the</p>

Code	Description	Preservice Teacher Example
○	Makes connections to mathematical concepts	problems with manipulatives and counting. Did 5×2 in his head. Counts by 5. Using fingers-adding repetitively for multiplication. Mental math for single digit multiplication and division.”

Table 3

Question 2: Coding, Description, and Example

Code	Description	Preservice Teacher Example
0 (Beginner)	<ul style="list-style-type: none"> No math connection with the strategy 	<p>“I feel he has a very good and advanced understanding of math for his age. The way his brain worked to figure out problems was very impressive.”</p>
1(Novice)	<ul style="list-style-type: none"> Able to see strategies with incorrect or immature inferences An observation that non-educators would be able to make 	<p>“He is a visual learner, who likes to use manipulatives. He is beginning to do multiplication. He used more mental math.”</p>
2(Emerging)	<ul style="list-style-type: none"> Able to infer some basic mathematical understandings Makes some connections of what the child is doing and what that means mathematically 	<p>“He knows how to group numbers well, a precursor to multiplying. “10 more” or “5 more”-likes addition. He understands grouping numbers. He understands “division” problems. He understood 5 was half of ten without mentioning fraction.”</p>

(table continues)

Code	Description	Preservice Teacher Example
3(Transitional)	<ul style="list-style-type: none"> ○ Able to infer richer mathematical understandings and able to connect to specific examples from the video 	<p>“Stella can add 2 one-digit numbers with ease. She can do these in her head with small numbers and tally marks with larger one-digit numbers. Stella also understands 12 is one group of ten and two ones. She uses this understanding to add $10 + 12$ as $10 + 10 + 2$. Stella has not learned how to add 2-digit numbers and is thrown off by these higher numbers.”</p>

Table 4

Question 3: Coding, Description, and Example

Code	Description	Preservice Teacher Example
0 (Beginner)	<ul style="list-style-type: none"> ○ No evidence based on child understanding ○ Incorrect mathematical statement ○ Teacher centric (what the teacher would do or how they think the child should learn) 	<p>“For the last question, I would have him get 15 small objects or pieces of paper to represent the tadpoles, and then have him group them by 3, then have him count the groups.”</p>
1(Novice)	<ul style="list-style-type: none"> ○ Lack of specificity ○ Very general description ○ Statement could be said without seeing the actual video ○ Little description of why the specific strategy is being suggested 	<p>“I would provide him with visuals or cubes to use so that he could see what he was doing and so he could do problems that need more than 10 fingers.”</p>

(table continues)

Code	Description	Preservice Teacher Example
2(Emerging)	<ul style="list-style-type: none"> ○ The answer considers the strategy that the child is using, but the suggestion does not get to mathematical concept development for the child. 	<p>“Rex clearly has started to develop a firm understanding of addition and subtraction. I would use this to start to build his understanding of multiplication and division. By this, I mean that multiplying by 3 is like adding the number together 3 times.”</p>
3(Transitional)	<ul style="list-style-type: none"> ○ Robust description of future teaching ○ The future teaching is individualized for the student based on the student’s understandings and future concept development. 	<p>“I think to help him I would build on the idea of grouping, but also the idea of multiplication and division as being reverse operations. For example with 8 people and 2 tables or 4 people and 2 tables. This problem can be easily represented and he could get a visual that they are one in the same.”</p>

On the first day of the 2013 spring semester the students of C&T 351 were shown the three-minute video of Rex. They were then provided with the response sheet and asked to answer the three questions about Rex and his mathematical thinking (See Appendix A). They were also given the Learning Mathematics for Teaching Numbers and Operations Instrument, Form A (2008). After they completed both tasks, the researcher scored their responses.

The students were then in the university classroom, learning from the instructor, for approximately the first three weeks of the semester. During this time in the classroom, the instructor engaged the students in activities that focus on teaching, by using the NCTM (2000) process standards, problem based planning, the use of assessment in instruction, and teaching through problem solving (Van De Walle, 2007). Then, the students began to learn about how to teach the various NCTM (2000) content standards, specifically numbers and operations. During this time, the students conversed among themselves and with the instructor about the different ways children work through problem situations. Discussion about children's mathematical thinking occurred simultaneously during each day's lesson and activities.

After about three weeks, the students went into the field, where they worked with students in elementary classrooms. Two to three students were placed in each elementary classroom and remained in that classroom for a total of six weeks. The experience was broken up into three, two week segments. While in the classroom, they were required to do three different mathematics reflection activities and to teach two mathematics lessons. In the reflection activities, they analyzed the mathematics curriculum at their school, reflected on the learning environment in their classroom, and determined how the NCTM (2000) process standards are being used in mathematics lessons in their classroom. After teaching their two mathematics lessons, they wrote a reflection in response to various prompts regarding children's mathematical thinking, problem based techniques used, what could be done differently next time, and how the lesson will be followed up in the future.

After their first two weeks in the field, the students returned to the university classroom for additional instruction. They were in the classroom for about two weeks, before returning to the field for their second segment. While in the university classroom, the students watched the

video of Stella. They answered the same three questions about Stella's mathematical thinking and reasoning as were asked about Rex (See Appendix A). The response sheets were scored in the same way that they were scored from the Rex video.

While the students were in the classroom, they continued to learn effective ways to teach the other NCTM (2000) content area standards. They then returned to the field for two more, two-week segments, which were split up with a brief time in the university classroom in between. After their final two-week segment in the field, they were back in the classroom for the rest of the semester. At that time, they watched the third of the three videos and filled out the last response sheet (See Appendix A). They also took the Learning Mathematics for Teaching Numbers and Operations Instrument, Form B (2008). This instrument used the same techniques that were used to score Form A (2008) at the beginning of the semester.

Data Analysis

The data that was collected consisted of student response sheets from the three videos and the raw scores from the two forms of the LMT instrument. The researcher first coded the preservice teachers' responses on the scale from zero to three that were determined during the pilot study (See Appendix E). The codes that were used were created during the pilot study. The preservice teachers' responses were coded based on the codes previously determined.

Preservice teachers received three different scores, one score for each of the three questions. The preservice teachers' answers on the response sheets were coded on a discrete, interval scale, which indicated the extent to which the preservice teachers engaged with the children's mathematical thinking.

The researcher coded the students' responses three times over the course of the semester, once at the beginning, in the middle and finally at the end. The researcher interpreted their responses to determine how they have changed over time. A one-way analysis of variance (ANOVA) was used to test for a statistically significant change in the students' ability to notice a child's mathematical thinking. An ANOVA was used because there are three nominal variables, which are the three videos that the preservice teachers watched, and one dependent variable, which is each question on the child response sheet (Nolan, 2008). Since all of the questions are a little bit different, each question was analyzed from one video to the next. Therefore, a total video score was not used. The researcher looked for a statistically significant change in the students' ability to see what the child is doing to answer the problems, learn about the child's mathematical understandings, and suggest future teaching in order to encourage mathematical understanding and growth. A post hoc test was also used when the null hypothesis was rejected because there was a significant change in the preservice teachers' scores, in order to make specific comparisons between each mean (Salkind, 2011). The Tukey HSD was used since multiple comparisons were made and it makes a determination between the means in terms of standard error (Nolan, 2008).

Finally, the researcher used the coded scaled numbers that were found from the response sheets to determine if there was a relationship between the students professional noticing of children's mathematical thinking and their own mathematical knowledge for teaching. A Pearson-product moment correlation was used to determine the relationships between the score on the LMT 2008B and each score from the third video. The statistical package, MiniTab 16 was used to aid in the statistical analysis.

On the LMT instrument, the researcher found the students raw scores both at the beginning of the semester on Form A and then at the end of the semester on Form B. The raw scores were analyzed and tested for statistical significance by using a paired-samples t-test. A paired-samples t-test was used because two samples were compared over an amount of time, and every participant was in both samples (Nolan, 2008). The researcher then determined if there was statistical significance between the change of the preservice teachers' MKT scores at the beginning of the semester and at the end of the semester. A Pearson product-moment correlation was performed to analyze the relationship between the preservice teachers' MKT score and their coded scores on their professional noticing responses. After the scores were graphed, the researcher interpreted the meaning of the association between scores and looked to find a positive, negative, or no association (Creswell, 2005). After analyzing the association of the scores, it was determined what type of relationship exists between the preservice teachers' MKT score and their professional noticing of children's mathematical thinking.

By using quantitative analysis, the researcher was able to determine if and to what extent a preservice teachers' MKT and professional noticing of children's mathematical thinking and reasoning develops over time. It was also determined if there was a relationship between MKT and noticing of children's mathematical reasoning and thinking.

Summary of Chapter 3

This chapter covers the various procedures, instruments, and analysis methods that were used to determine how preservice teachers' MKT initially develops over the course of a semester. The data analysis will also determine how their ability to professionally notice children's mathematical thinking and reasoning develops over time. Through the use of quantitative research the researcher analyzed the preservice teachers' mathematical knowledge

development. The LMT instrument measured the preservice teachers' MKT while the professional noticing response sheets measured their ability to notice children's thinking. After statistical analysis the extent to which preservice teachers' mathematical knowledge develops and if there is a relationship between their MKT and their ability to professionally notice students' mathematical thinking was analyzed.

In the chapter four, the results of the research will be discussed. An explanation of the information found from the analysis will be provided.

Chapter 4

Analysis of Data

Introduction

The purpose of this study is to determine if and to what extent preservice teachers' mathematical knowledge for teaching changes over a semester and also to identify any changes in their ability to professionally notice children's mathematical thinking and reasoning. The study also is intended to determine if there is a relationship between preservice teachers' mathematical knowledge for teaching and their professional noticing of children's mathematical thinking and reasoning. Quantitative analysis was used to determine if there were specific changes with the preservice teachers. Specifically, multiple paired-samples t-tests were used to compare the means for a within groups design to determine if there was a change in preservice teachers' mathematical knowledge for teaching (Nolan, 2008). An analysis of variance (ANOVA) was used to determine if there was a statistically significant change in the preservice teachers' ability to professionally notice a child's mathematical thinking. Finally, the Pearson-product moment correlation coefficient was also used to determine if there was a relationship between the preservice teachers' mathematical knowledge for teaching and their ability to professionally notice children's mathematical thinking and reasoning (Nolan, 2008).

Research Questions

This study will provide answers to the following research questions, previously introduced in chapter three:

1. Does preservice teachers' Mathematical Knowledge for Teaching change during a semester, in which they participate in both a mathematics methods course and work with children in a mathematics setting?
2. Does preservice teachers' noticing of children's mathematical thinking and reasoning develop over the course of a semester, in which they participate in a mathematics methods course and work with children in a mathematics setting?
3. Is there a relationship between Mathematical Knowledge for Teaching and noticing of children's mathematical reasoning and thinking?

Organization of Data Analysis

The data are presented in three sections, which correspond to the research questions. The first section includes an analysis of the preservice teachers' scores on the Learning Mathematics for Teaching (LMT) instrument, which measures Mathematical Knowledge for Teaching (MKT) (Hill et al., 2004). The instrument created by Hill (2004) and colleagues contains items that measure teachers' MKT in the elementary mathematics standard, number and operations. Two forms of the LMT, 2008A and 2008B, were used to evaluate the preservice teachers who participated in the study. The two forms of the LMT are equated so they can be used to analyze for growth or difference between the two. The results of both LMTs will be reported as will any changes that occurred between 2008A and 2008B.

The second section presents results of the study which provide analysis of the preservice teachers' ability to notice children's mathematical thinking and reasoning. The coding system that was developed to score the preservice teachers' responses to the three questions about children's mathematical thinking and reasoning is described (See Appendix E). The codes that were determined were used to analyze if there was a change in the results of question one on the

video response sheet from video one to two, two to three, and one to three. The same results will be reported for questions two and three from the video response sheet.

The third section provides results of the relationship of preservice teachers' mathematical knowledge for teaching and their ability to notice children's mathematical thinking and reasoning. Correlation results from the Pearson product-moment correlation are reported. These results include information on the relationship or lack of relationship between the preservice teachers' scores on the Learning Mathematics for Teaching 2008B and their coded scores on each question of the third video.

Learning Mathematics for Teaching Results

The participating preservice teachers initially took the LMT instrument, 2008A in January, 2013, in order to measure their MKT at the beginning of the college semester. The instrument consisted of twenty-eight questions. The instrument was scored and each student was given a raw score based on their correct answers. Raw scores were then changed into standard deviation units. All raw scores were initially placed into a histogram and bell curve, which showed the distribution of the data. "A standard assumption for the t-test to be valid when you have small sample sizes is that the outcome variable measures are normally distributed. That is, when graphed as a histogram, the shape approximates a bell curve" (Elliott, 2007). The histogram and bell curve show that it was valid to conduct the t-test since it is very similar to a normal bell curve.

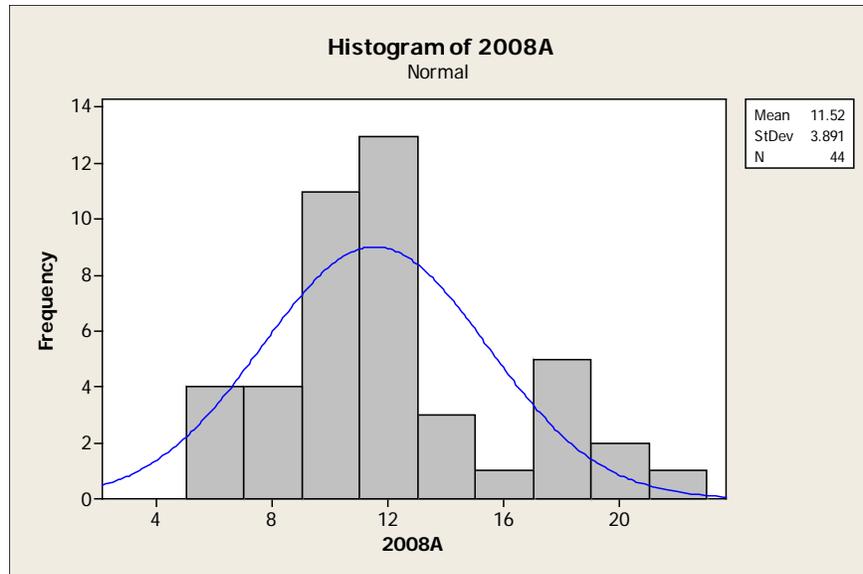


Figure 1. Histogram and Bell Curve of LMT 2008A Results

The participating preservice teachers then took the LMT instrument, 2008B in May, 2013, after participating in mathematical methods coursework and working with elementary children in classrooms. Form B consisted of twenty-nine questions. The instrument was scored and each preservice teacher was given a raw score based on their correct answers. All scores were placed into a histogram and bell curve to analyze the distribution of the data.

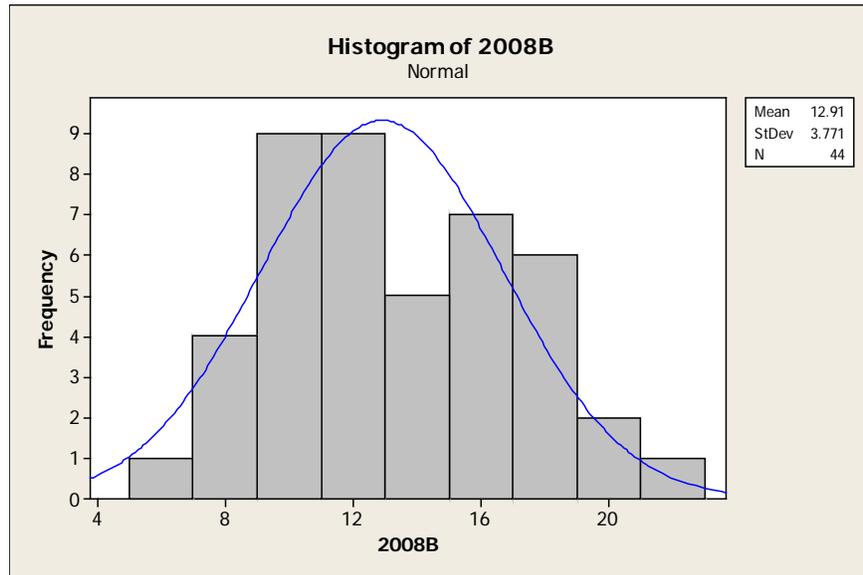


Figure 2. Histogram and Bell Curve of LMT 2008B Results

The histogram and bell curve of these data are once again, very similar to a normal bell curve.

Although, the second set of data are a bit more leptokurtic than the 2008A data, the data are still normally dispersed.

Finally, after the raw scores of both LMT instruments were calculated and turned into standard deviation units, the percentage of change in the scores was determined. Again, these data were placed in a histogram and bell curve to check for normal distribution and relative frequency of the data. Once again, the data were very similar to a normal bell curve.

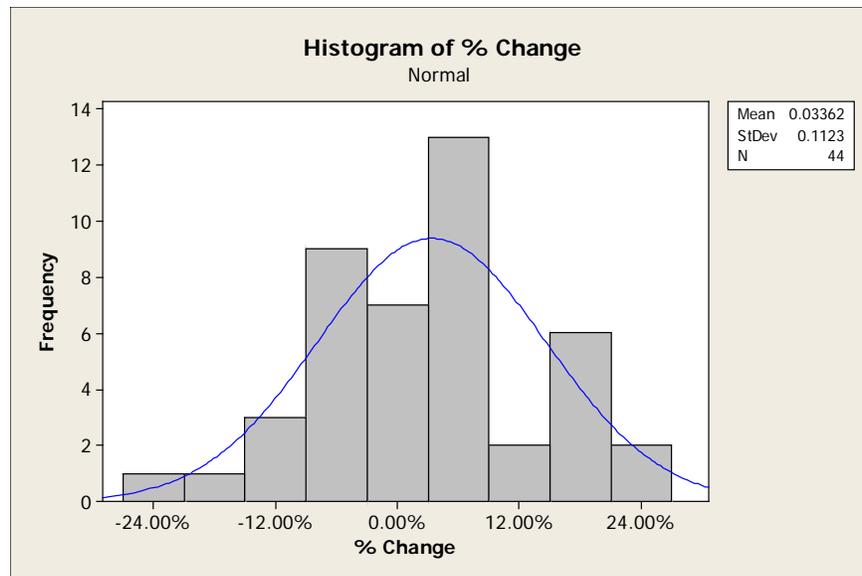


Figure 3. Histogram and Bell Curve of Percent of Change from 2008A and 2008B

The original analysis looked at the percentage of change for each preservice teacher, the entire group, and several subgroups. For the entire group, the average change was 3.36%. The group had an average score on LMT 2008A of 41.15% and on the LMT 2008B an average score of 44.51%. The group was then divided into four subgroups, each consisting of eleven participants. The division of participants was based on their total scores on the two instruments. The groups are identified as highest, high middle, low middle and low. The highest group had an increase of 7.54%, while the high group had an increase of 7.32% and the middle group had an increase of 3.51%. The low group had a decrease of 4.94%. The following graph shows the average percentages of the entire group, as well as the four subgroups for both the LMT 2008A and 2008B.

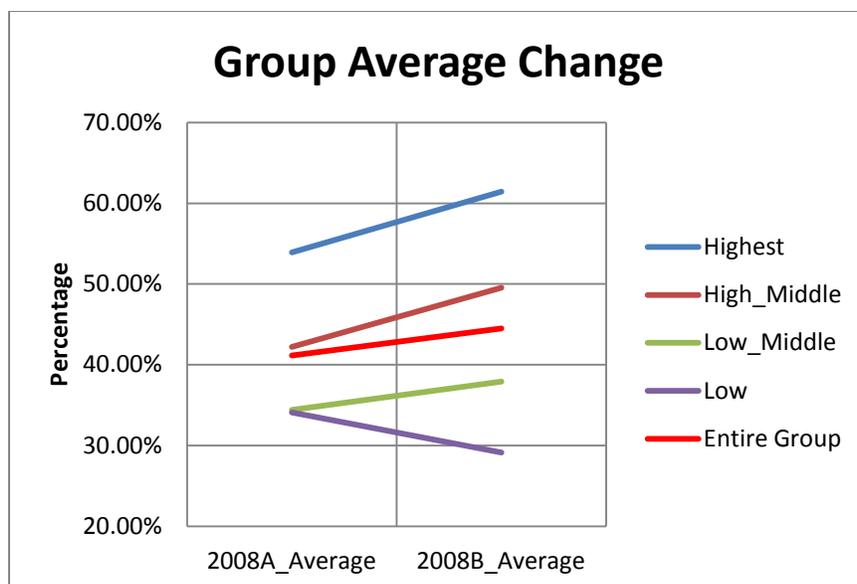


Figure 4. Group Average Percentage Change from 2008A to 2008B

A paired-samples t-test was conducted to compare the preservice teachers' average percentage scores on the LMT 2008A and LMT 2008B conditions. There was not a significant difference in the scores for the LMT 2008A ($M=0.4115$, $SD=0.1390$) and the LMT 2008B ($M=0.4451$, $SD=0.1301$) conditions; $t(43) = -1.98$, $p = .054$. These results suggest that the null hypothesis is true and there is no significant change in the average percentage score on the LMT instrument for preservice teachers over the course of a semester. With $p < .05$, the probability is less than 5% that on any one test of the null hypothesis, the sample averages will differ (Salkind, 2011).

A paired-samples t-test was also conducted on the four separate subgroups of preservice teachers. The first subgroup analyzed consisted of the eleven preservice teachers who had the highest total score on the LMT 2008A and 2008A conditions. There was a significant difference in the scores from the preservice teachers' LMT 2008A ($M=0.5390$, $SD=0.1135$) and their scores on the LMT 2008B ($M=0.6144$, $SD=0.0552$) conditions; $t(10) = -2.64$, $p = .025$. These results

suggest that the null hypothesis is rejected and there is a significant change in the average percentage score on the LMT instrument for preservice teachers with the highest percentage of change over the course of a semester. With $p < .05$, there is a 95% confidence that there will be a significant change with the preservice teachers who have the highest total scores on the LMT instrument over the course of a semester.

The next subgroup analyzed consisted of the eleven preservice teachers who had the second highest total score on the LMT 2008A and 2008B conditions. There was not a significant difference in the scores from the preservice teachers' LMT 2008A ($M=0.4221$, $SD=0.1418$) and their scores on the LMT 2008B ($M=0.4953$, $SD=0.0416$) conditions; $t(10) = -2.07$, $p=.065$.

These results suggest that there is 95% confidence that the null hypothesis is true and there is no significant change in the average percentage score on the LMT instrument for the subgroup of preservice teachers with the second highest total score over the course of a semester.

The third subgroup analyzed was the group identified as middle. There was not a significant difference in the scores from the preservice teachers' LMT 2008A ($M=0.3442$, $SD=0.1320$) and their scores on the LMT 2008B ($M=0.3793$, $SD=0.0267$) conditions; $t(10) = -0.90$, $p=.387$. These results suggest that there is 95% confidence that the null hypothesis is true and there is no significant change in the average percentage score on the LMT instrument for the subgroup of preservice teachers with the third highest or middle total score on the instrument over the course of a semester.

The final subgroup analyzed was the eleven preservice teachers who had the lowest total score on the LMT 2008A and 2008B conditions. There was a significant difference in the scores from the preservice teachers' LMT 2008A ($M=0.3409$, $SD=0.0646$) and their scores on the LMT 2008B ($M=0.2915$, $SD=0.0472$) conditions; $t(10) = 2.72$, $p=.022$. These results suggest two

significant results. First, there is 95% confidence that the null hypothesis is rejected, and the true mean difference lies between 0.3409 and 0.2915. Second, the percentage score means decreased from LMT 2008A to 2008B. Therefore, not only was there a significant change in the scores, but the change was a decrease.

In summary, $p > .05$ indicates that the probability is greater than 5% that when the null hypothesis is tested with the average group percentage score, the high subgroup, and middle subgroup, their scores on the LMT 2008A and 2008B will not differ because of the way that they were taught. For the highest and lowest subgroups, the p value indicates that when the null hypothesis is tested their scores on the LMT 2008A and 2008B will differ over the course of a semester.

Child Response Results

The study also analyzed preservice teacher's ability to notice a child's mathematical thinking and reasoning. In order to do this, the teachers watched a video of a child answering mathematics questions in January, again in March, and finally in May. The children in the videos were of different ages, but were asked similar mathematical, numbers and operations, questions. The preservice teachers were asked the same three questions after watching each video (See Appendix A).

1. What do you think the child was doing to figure out the answers to the problems?
2. What did you learn about the child's mathematical understandings?
3. What would you do if you were the child's teacher at the end of the video to continue to encourage his mathematical understanding and growth?

Before the videos were shown to the preservice teachers in the study, the videos and questions were presented to a group of preservice teachers who took the same mathematics

methods class the prior semester. This pilot study was used to help examine the videos and the child response questions (See Appendix A), as well as to provide consistency in determining the codes for the students' answers. Two separate raters rated the pilot group of preservice teachers' responses based on an interval scale from zero to three. The raters first rated the answers individually and then met together to discuss their scores so as to find a common score for each question, on each video that each student answered. At the end, each preservice teacher in the pilot study had three scores for video one, three for video two, and three for video three. The same process was used when coding the preservice teachers' child response answers in the study.

Each preservice teacher received a score of 0, 1, 2, or 3 for each of the three questions for each video (See Appendix E). Question one asked the preservice teachers what they thought the child in the video was doing to figure out the answers to the problems. As described in chapter three, preservice teachers received a score of 0 if they provided a list only, had an incorrect analysis, or showed a lack of mathematical knowledge. One preservice teacher answered this question by saying, "pictures, manipulatives, counting by fives, fingers". This response received a score of 0 because her response only included a list, without any description. Another preservice teacher wrote, "round to 10, add the remaining and tallies, individual tallies, not grouping". Again, this response received a score of 0 because her response was also a list, as well as a lack of mathematical knowledge. By this response, the preservice teacher did not show mathematical knowledge because she did not connect any of the strategies to mathematic concepts or mathematical development. Preservice teachers received a score of 1 if they identified only one strategy or they had a very novice description of the strategy. One preservice teacher who received a score of 1 wrote, "Rex seemed to be counting on his fingers to find out most of the answers to each question." This preservice teacher did not notice any other strategy

the student in the video was doing and had a very novice description of the strategy. Another preservice teacher wrote, “He used his fingers to count down to six. He counted down out loud while keeping track of how many he’d counted on his fingers.” In order to receive a score of 2 on this question, preservice teachers needed to identify two or more strategies with a developing description of the strategy. A connection to mathematical concepts was not necessary in order to receive a 2. One student wrote, “Used manipulatives to group and then counted them, used increments of five to add and do multiplication problems, repeated addition, drew a picture, and mental math.” She listed multiple strategies but also had a description with an example of what the child on the video was doing. Another student who received a score of 2 wrote, “Most of the time, she relied on direct counting with tallies or cotton balls. Breaking numbers into more manageable addends, such as 10, was done often. There was some mental math for a few problems.” Finally, to receive a score of 3, multiple strategies with specific descriptions needed to be identified. Also, the preservice teacher needed to make a connection to mathematical concepts. One example of a score of 3 is, “Rex was counting down and up on his fingers in order to answer the problems. I do not think that he is very aware of the actual concepts of plus and minus, but he was able to count down from 13 to discover that 7 cookies were left and was able to count from 5 to 19 to discover there were 14 days until his birthday.” Another student wrote, “Making tally’s and then counting all of them up at the end. Used estimation methods for how many more question. Knew $6+6=12$, so she subtracted one. For larger 2-digit numbers, she used the manipulatives to help her solve the problem. Adding two 2-digit numbers-used tally’s and manipulatives.” Both of these students linked the strategies to specific examples, as well as the mathematical concepts.

Question two asked the preservice teachers what they learned about the child's mathematical understandings. Once again they could receive a score of 0, 1, 2, or 3. Preservice teachers who received a score of 0 did not make a mathematics connection to the strategy that they observed or there was not a description of what the child understood mathematically. One student wrote, "He is great at mental math. He is great at grouping." Another student who received a score of 0 answered, "Carson has had a lot of practice using many different ways of thinking to solve a problem. He has a lot of mathematical knowledge introduced to him and especially counting using his head. Carson is ready to learn multiplication because he has a good foundation set in stone." In the video, Carson was not performing any multiplication. He viewed all of the questions as addition or grouping questions. He was not aware that the questions could be answered using multiplication. In order to receive a score of 1 on this question, preservice teachers were able to observe multiple strategies but with incorrect or immature inferences. The responses could be something that a noneducator would say or could be said without seeing the video. "He was able to use his fingers to count up or down. It became more difficult when he needed more than 10 fingers. He could not do the math simply in his head." Another student answered, "Can add in head, cannot add double digit problems yet, had to use manipulatives, knows a lot of strategies for counting faster, and visual learner." Both of these students observed some strategies, but their descriptions were immature. Preservice teachers who could infer basic mathematics understandings and make a connection to what the child is doing in the video and what that means mathematically received a score of 2. An example of a preservice teacher who received a 2 wrote, "Rex's mathematical understanding seemed to be addition and subtraction. He was able to count with his fingers to help him gather answers. It seemed as if he is beginning to learn subtraction and addition of 2-digit numbers." Another example of a score of 2 is, "She

understands addition and subtraction and has a basic understanding of breaking numbers into more even numbers like $10+2+10$ instead of $12+10$. She can do some basic problems in her head.” Both of these preservice teachers inferred some basic mathematics understandings and connected the strategy to mathematical concepts. Finally, to receive a score of 3, preservice teachers needed to infer rich mathematics understandings and were able to connect the mathematics strategy to specific examples. One student who received a score of 3 wrote, “Rex seems to understand addition and subtraction, however I don’t think he would be able to solve problems with larger numbers because he wouldn’t be able to use his fingers to count. It might be helpful if he could draw pictures or learn operations for subtraction and addition. Also, Rex has not yet learned how to multiply or divide. He also has a good sense of number order.”

Another preservice teacher wrote, “She is able to add and subtract but really only numbers under 20. She uses doubles or familiar numbers to add in an easier way, like $10+10+2$. She understands that numbers contain ones and tens because she split them apart. She has no understanding of how to add two large numbers by adding the ones and then tens place.” Both of these preservice teachers connected the strategies to specific mathematics examples and provided richer descriptions which showed a greater knowledge of the child’s mathematics understandings.

The last question that the preservice teachers were asked to answer asked them what they would do at the end of the video to continue to encourage mathematical understanding and growth. A score of 0 was given if there was no evidence on the response based on the child’s understandings or if the response was teacher centric, meaning what “I” would do. One student wrote, “I would help him break up the problem. I would use manipulatives to represent the tadpoles and I’ll have him count out 15 and then I’d have him put them in piles of three. I would then have him count the piles.” This response received a score of 0 because the preservice

teacher is suggesting that she would do most of the work for the child, rather than guiding his work or having him discover the answer on his own. Another response was, “I would encourage Stella to avoid using tallies for adding larger numbers and demonstrate several algorithms for double digit numbers.” Once again, the preservice teacher was suggesting taking away the main strategy that the child in the video used and teaching her a strategy that did not build on any of her prior understandings. A score of 1 was a statement or teaching strategy that could be said without seeing the video or a very general, small description of the strategy was suggested. An example of a response that received a 1 was, “I would continue using the puff balls because he responds well to the visuals. I would also monitor that he is getting enough time. Lastly, I would ask how he is getting his answers so quickly and adapt that method into the questions that take longer.” Another response was, “If she likes using tallies, I would first teach her to group them in five’s so it is easier and faster for her. Then I would encourage her to use the same strategies she used for small numbers, for big numbers.” Both of these responses took into account the child’s strategies but did not provide a thorough description. A score of 2 was given when the child’s strategy was taken into account with a more detailed description of the future teaching. One student wrote, “I would continue to have Carson answer more problems similar to the ones above. He could also further practice counting by different numbers, 2’s, 3’s, 5’s, etc. Maybe gradually take away manipulatives so he has to write out and show his work. Develop why an answer is right.” Another student wrote, “Rex clearly has started to develop a firm understanding of addition and subtraction. I would use this to start to build his understanding of multiplication and division. By this, I mean that multiplying by 3 is like adding the number together 3 times.” Both of these students took into account the child’s strategies when thinking about future teaching and provided a more detailed example of that what they would do to help the child’s

mathematical understandings develop. Lastly, to receive a score of 3, the preservice teacher needed to provide a robust description of future teaching and it is individualized for the student, based on their understandings and future concept development. One preservice teacher received a score of 3 on this question. She wrote, “If I were Carson’s teacher, I would continue to encourage his mathematical understanding by allowing him to continue to use strategies such as manipulatives and drawing when needed. I would also have him use labels with grouping to help him organize and solve problems. In addition, I would have him write down his problems and have him explain his reasoning. I would also have him work on more two step problems since he struggled with those.”

As stated, each preservice teacher received a score of 0, 1, 2, or 3 for each of the three questions for each video. The participants’ scores on each question were averaged. The graph below shows the average trend of each of the three questions.

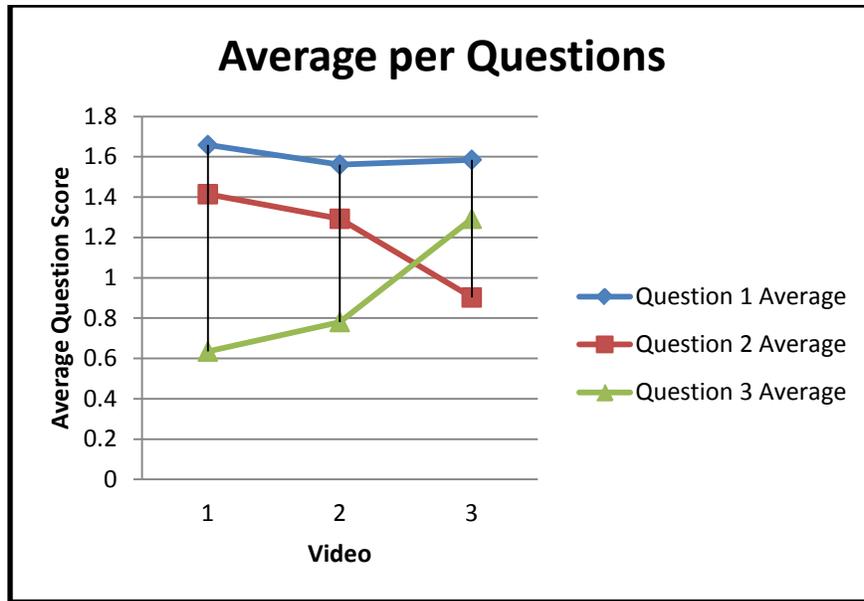


Figure 5. Average Scores per Questions

An analysis of variance (ANOVA) was conducted for each of the three child response questions (See Appendix A) to see if there was a statistically significant change in the preservice teachers' scores on each question from one video to the next. First a one-way between subjects ANOVA was conducted to compare the effect of the university and elementary classroom experiences on the scores that the preservice teachers achieved on question one in video one, video two, and video three conditions. There was not a significant effect of the university and elementary classroom experiences on the scores that the preservice teachers received at the $p < .05$ for the three conditions [$F(2, 120) = 0.19, p=0.828$]. These results suggest that there is 95% confidence that the null hypothesis is true and there is no significant change between the three videos in the preservice teachers' ability to analyze what the child was mathematically doing to answer the questions.

A one-way between subjects ANOVA was also conducted to compare the effect of the university and elementary classroom experiences on the scores that the preservice teachers

achieved on question two in video one, video two, and video three conditions. There was a statistically significant effect of the university and elementary classroom experiences on the scores that the preservice teachers received at the $p < .05$ for the three conditions [$F(2, 120) = 5.18, p=0.007$]. Post hoc comparisons using the Tukey HSD test indicated that the mean score for video one ($M=1.415, SD=0.706$) was significantly different than the scores from video three ($M=0.902, SD=0.768$). Although these results show that the mean score decreased from video one to video three. Post hoc comparisons also indicated that the mean scores for video two ($M=1.293, SD=0.782$) was significantly different than the scores from video three ($M=0.902, SD=0.768$). The mean scores decreased from video two to three also. These results suggest that there is 95% confidence that the null hypothesis is false and there is a significant change in the preservice teachers' ability to analyze the child's mathematical understandings. However, the scores from video one ($M=1.415, SD=0.706$) to video two ($M=1.293, SD=0.782$) did not significantly differ as a result of the experiences the preservice teachers had in the university and elementary classroom.

Finally, a one-way between subjects ANOVA was conducted to compare the effect of the university and elementary classroom experiences on the scores that the preservice teachers achieved on question three in video one, video two, and video three conditions. There was a statistically significant effect of the university and elementary classroom experiences on the scores that the preservice teachers received at the $p < .05$ for the three conditions [$F(2, 120) = 8.78, p=0.000$]. Post hoc comparisons using the Tukey HSD test indicated that the mean score for video one ($M=0.634, SD=0.698$) was significantly different than the scores from video three ($M=1.293, SD=0.750$). Post hoc comparisons also indicated that the mean scores for video two ($M=0.780, SD=0.791$) was significantly different than the scores from video three ($M=1.293,$

SD=0.750). These results suggest that there is 95% confidence that the null hypothesis can be rejected and there is a significant change in the preservice teachers' ability to suggest future mathematical instruction for the child. Therefore, the difference between the groups is due to the treatment, which consisted of both university and elementary classroom experiences. However, the scores from video one (M=0.634, SD=0.698) to video two (M=0.780, SD=0.791) did not significantly differ as a result of the university and elementary classroom experiences.

After reviewing the results of the ANOVA, the effect size was calculated in order to account for the influence of the sample size (Nolan, 2008). "Eta squared is the proportion of variance in the dependent variable that is accounted for by the independent variable" (Nolan, 2008, p. 548). The effect size for all of the conditions was calculated and the results are shown in the table below, along with a summary of the ANOVA results.

Table 5

Summary for Three Questions

Conditions	<i>p</i>-value, $p < .05$	Effect Size (<i>d</i>)
Q1- V1, V2, V3	$p = .828$	$d = 0.003$ (very small)
Q2- V1, V2, V3	$p = .007$	$d = 0.079$ (medium)
Q3- V1, V2, V3	$p = .000$	$d = .127$ (large)

For question one, the effect size is very small, which is appropriate since the null was not rejected. For question two, there is a medium effect size. On question three, the effect size is considered large according to Cohen’s Conventions for Effect Sizes (Nolan, 2008).

Approximately 13% of the variability in the scores from video one, two, and three on question three is due to the university and elementary classroom experiences in which the preservice teachers participated.

The Relationship between MKT and Noticing Mathematical Understandings

Finally, this study also analyzed whether or not there was a relationship between the preservice teachers’ MKT and their ability to notice a child’s mathematical thinking and reasoning. In order to do this a Pearson-product moment correlation was conducted. By finding the Pearson correlation coefficient, a linear relationship between the two variables was quantified (Nolan, 2008).

Initially the raw score data from the LMT scores were changed to percentages correct based on the number of questions in order to plot the data consistent with each other. The LMT had values ranging from six to 21, while the child response codes ranged from zero to three. By

showing the LMT values as percentages, it was easier to see if there was a trend. The graph below shows that all of the lines have an upward slope, which shows some correlation and a positive association. But, because of only having limited data choices for the child response scores, the confidence of the data could be questioned.

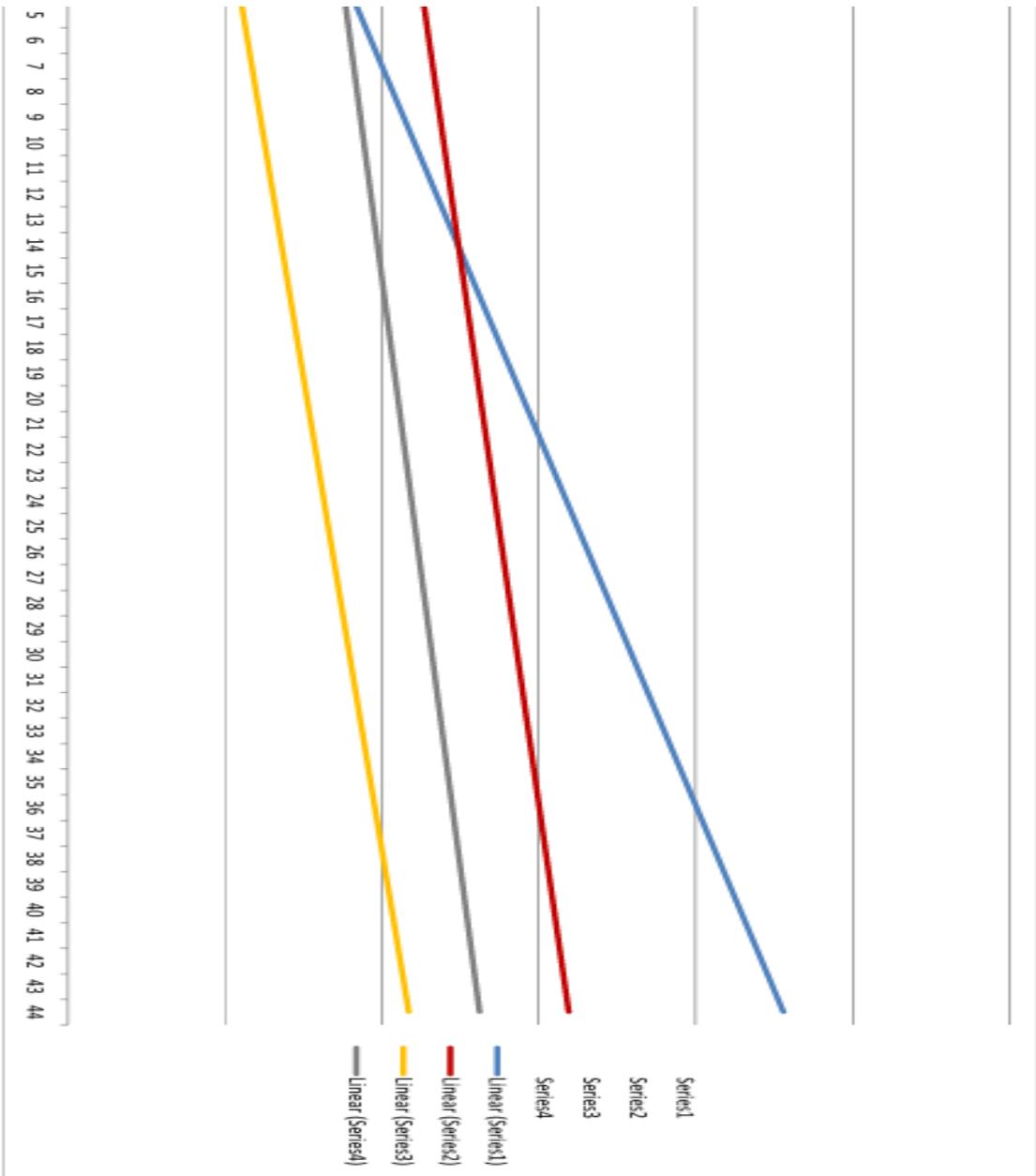


Figure 6. Graph Trend Lines for LMT to V3Q1, V3Q2, V3Q3

Next, a Pearson-product moment correlation was computed to assess the relationship between the preservice teachers' scores on the 2008B LMT instrument and their coded scores on question one of the child response questions (See Appendix A) from the third video (V3Q1). There was not a statistically significant relationship between the two variables, $r = 0.286$, $n = 44$, $p = .060$.

A Pearson Product Moment Correlation was also computed to assess the relationship between the preservice teachers' scores on the 2008B LMT instrument and their coded scores on question two of the child response questions (See Appendix A) from the third video (V3Q2). Once again, there was not a significantly significant relationship between the two variables, $r = 0.292$, $n = 44$, $p = .054$.

Finally, a Pearson Product Moment correlation was computed to assess whether or not there was a correlation between scores on the 2008B LMT instrument and the coded scores on the third question on video three (V3Q3). These results suggested that there was not a statistically significant relationship between the two variables, $r = 0.163$, $n = 44$, $p = .291$.

Although there is a slight positive correlation between the different variables, there is not a statistically significant relationship. It could therefore be interpreted that there is a minimal relationship between the two variables. A preservice teachers' MKT is not significantly correlated with their ability to notice a child's mathematical thinking and reasoning.

Summary

In summary, the preservice teachers' scores on the LMT instrument, as well as the coded scores they received on the child response questions (See Appendix A) were analyzed in order to answer the research questions. The paired-samples t-test showed that there was not a significant

difference in the preservice teachers' MKT from the LMT 2008A to 2008B. An ANOVA analysis determined that there was a change in the preservice teachers' ability to analyze a child's mathematical understandings and suggest future mathematics instruction for a child. Finally, a Pearson-product moment correlation showed no statistically significant correlation between the preservice teachers' LMT scores on the 2008B instrument and their scores on each of the three questions from the third child response video. The implications that the results presented in this chapter provide for future studies, as well as working with preservice teachers in the future will be discussed thoroughly in the final chapter.

Chapter 5

Findings and Conclusions

Introduction

This study examined preservice teachers' ability to professionally notice a child's mathematical thinking, as well as the development of their own Mathematical Knowledge for Teaching (MKT) over the course of a college semester. This chapter begins with a brief summary of the study, followed by an overview of the findings about the development of preservice teachers' ability to professionally notice a child's mathematical thinking and their own MKT, as well as the correlation between the two. The conclusions from the analysis based on the research questions are then discussed. Finally, suggestions for future research based on the findings are provided.

Summary of the Study

The work of teaching is unnatural (Ball & Forzani, 2010). "To listen to and watch others as closely as is required to probe their ideas carefully and to identify key understandings and misunderstandings, for example, requires closer attention to others than most individuals routinely accord to colleagues, friends, or even family members" (Ball & Forzani, 2009, p. 499). Teachers have to pay close attention to their student's thinking and reasoning skills. There are multiple ways that a student will think when solving a problem and there are just as many ways that a teacher can respond (Jacobs et al., 2010). Teachers need to be able to notice a child's thinking and then instruct them based on the child's mathematical knowledge, not on how they would personally solve the mathematics problem, which does not always happen (Jacobs et al., 2010).

Many teachers are entering the classroom lacking in their own mathematical knowledge for teaching, which includes both content and pedagogical knowledge. MKT develops and changes over time for teachers and begins before they enter the preservice level. Individuals' MKT varies from person to person and research has shown that it will develop as a result of working with children in mathematical situations and active participation in education classes that focus on inquiry and educational research, reading, and professional development (Borko, 1992). Because research has shown a relationship between a teacher's mathematical knowledge and their students' mathematics achievement, it is critical for teachers to continue to improve their mathematics content and pedagogical knowledge (Panel, 2008). In general, any lack of teacher knowledge will make it more difficult for a teacher to understand and notice a child's mathematical understanding. As a result, instruction is generally not effective and the children are not learning to conceptually understand and apply mathematics.

The following questions were answered:

1. Does preservice teachers' Mathematical Knowledge for Teaching change during a semester, in which they participate in both a mathematics methods course and work with children in a mathematics setting?
2. Does preservice teachers' noticing of a child's mathematical thinking and reasoning develop over the course of a semester, in which they participate in a mathematics methods course and work with children in a mathematics setting?
3. Is there a relationship between Mathematical Knowledge for Teaching and noticing of children's mathematical reasoning and thinking?

The participants in the study consisted of 44 preservice teachers who were enrolled in Curriculum and Teaching (C&T) 351 at a Midwestern university. C&T 351 is a mathematics

methods course in which the students spend time both with the professor and colleagues in the university classroom and out in the field in elementary school classrooms. A variety of instruments were used to gather the information required to answer the research questions. The Learning Mathematics for Teaching (LMT) instrument was developed to measure a teacher's MKT (Hill et al., 2004). The preservice teachers took this instrument both at the beginning and end of the semester in order to determine if there was a change in their MKT over the course of the semester. They also watched three videos of a child answering mathematics questions. They watched one video at the beginning of the semester, one in the middle and one at the end. Each video showed a child answering multiple numbers and operations questions. The preservice teachers answered three questions that helped show their ability to attend to the child's mathematical strategies, interpret their understandings, and respond on the basis of the child's understandings (Jacobs et al., 2010). Both of these instruments, the LMT and the child response videos and questions, were used to determine if there was a relationship between the preservice teachers' mathematical knowledge for teaching and their ability to professionally notice a child's mathematical thinking and reasoning.

In summary, due to the reform of mathematics curriculum, it is critical for preservice teachers to not only have strong MKT, but also to have ample experience learning how students think mathematically. In a reform-based mathematics classroom, mathematics is taught through problem solving. Students are expected to "do" mathematics and make sense of it. "In the classroom environment envisioned by NCTM, teachers provide students with numerous opportunities to solve complex and interesting problems; to read, write and discuss mathematics; and to formulate and test the validity of personally constructed mathematical ideas so that they can draw their own conclusions. Students use demonstrations, drawings, and real-world objects-

as well as formal mathematical and logical arguments-to convince themselves and their peers of the validity of their solutions” (Battista, 1999, p. 426). This new environment encourages students to take an active role in their learning, reasoning, and reflecting, and as a result, teachers need to have a strong mathematics understanding, as well as the ability to notice how a child is making sense of mathematics. The growth in MKT and professional mathematics noticing does not happen automatically or immediately. It is a process. This study examined how that process develops for teachers during a semester in their collegiate years.

Findings

A paired-samples t-test, was used to determine if there was a change in the preservice teachers’ MKT from the beginning to the end of the university semester. The entire group did not have a significant change in their scores from the LMT 2008A to 2008B. Therefore, it is believed that as a whole, preservice teachers need more time working with students in the elementary classroom, while building their own pedagogical and content mathematics knowledge than just one college semester can provide. However, there was a significant change with two of the subgroups; the highest and lowest. The 44 preservice teachers were split into four subgroups based on their total score on the 2008A and 2008B LMT instruments. The 11 preservice teachers, who had the highest scores, had a significant change in scores with the effect size of 1.12. These findings provided evidence that those preservice teachers who had the highest total score grew in their mathematics pedagogical and content knowledge. Also, the 11 preservice teachers who had the lowest total scores had a significant change in scores with the effect size of 0.87. These findings provide evidence that those preservice teachers had a significant change in their scores from LMT 2008A to 2008B. The overall mean scores from 2008A to 2008B decreased 0.05, which shows that the significant change was a decrease in scores, rather than an

increase. The pairwise comparison between the other two middle subgroups was not significant and therefore did not provide evidence of additional gains or losses in the MKT scores.

An Analysis of Variance (ANOVA) was used to analyze the preservice teacher's scores on the child response questions (See Appendix A). Question one consisted of the preservice teachers' ability to attend to what the child was doing to answer the mathematics questions. The ANOVA showed that there was not a significant change on question one between each video and did not provide evidence of additional gains in the preservice teachers' ability to see what the child was doing to answer the questions.

Question two had preservice teachers interpreting the child's mathematical understandings based on the strategies that they used to answer the questions. An ANOVA was once again used and it showed there was a statistically significant change in the preservice teachers' scores on question two. A post hoc test was used and it found that there was a statistically significant change in scores from video one to video three and video two to video three with effect size of 0.079. These findings provide evidence that over time the preservice teachers' ability to interpret the child's mathematical understandings changed. Specifically, the change occurred from the beginning of the semester (video one) to the end (video three). In contrast, the mean scores decreased over time and therefore, while there were no gains in this area, the change was significant. The post hoc test between video one and video two was not significant and provided no evidence of a change in the preservice teachers' ability to interpret a child's mathematical understandings.

Question three had the preservice teachers decide how to respond to the child based on their mathematical understandings and strategies. An ANOVA analyzed the preservice teachers' scores on question three. The ANOVA and post hoc test showed that there was a statistically

significant change in the preservice teachers' scores from video one to video three and video two to video three with the effect size of 0.127. These findings provide evidence that the preservice teachers grew in their ability to respond to the student based on the child's mathematical understandings over the course of the semester. The ANOVA on question three illustrated that there was not a statistically significant change in the preservice teachers' scores between video one and video two and therefore did not provide evidence of a change in the preservice teachers' ability to respond appropriately to the child from the beginning of the semester to the middle.

Finally a Pearson-product moment correlation was computed to assess the relationship between the preservice teachers' scores on the 2008B LMT instrument and their scores on each question from the third video of the child response videos. The results showed that there was not a statistically significant relationship between 2008B and each of the questions. Therefore, there is not a significant relationship between the students' MKT and their ability to professionally notice a child's mathematical understanding and reasoning while they are at the preservice level.

Conclusions

The preservice teachers' scores on the LMT instrument, which they took at the beginning and end of the semester did not show a statistically significant change in their MKT because of the lack of a change in the overall score of the class it can be deduced that more time or opportunity is required for preservice teachers to develop their MKT, which includes the development of both their pedagogical and content knowledge in mathematics.

Preservice teachers need to have a strong mathematics content understanding and knowledge and it is important for them to have ample opportunity during their undergraduate years to develop their content knowledge. Overall, "effective teachers need subject matter competence: they need to know how to solve the problems they pose to students and to know

that there are multiple approaches to solving many problems” (Darling-Hammond, 2005, p. 205). For the MKT of preservice teachers to develop and expand it is critical that they have more opportunities and experiences to achieve that result. The 2008 National Mathematics Advisory Panel recommended, “Teachers must know in detail the mathematical content they are responsible for teaching and its connections to other important mathematics, both prior to and beyond the level they are assigned to teach” (Panel, 2008, p. 37).

Preservice teachers need to have more enhanced opportunities to expand their knowledge within the five major mathematics content areas, as opposed to the rather superficial exposure most presently receive. Development of preservice teachers’ MKT requires adequate time working with the mathematics content, which can be aided by exposure to and opportunities to work with people in their field of study. Preservice teachers need to have time with the mathematics content and with people who are very familiar and comfortable with all of the areas of mathematics in order to develop their own content knowledge (Stiff, 2000). With a stronger content knowledge their overall MKT will continue to grow and develop.

This study also leads to the conclusions that preservice teachers likely need more opportunities to develop their pedagogical knowledge. In addition to knowing mathematics content, they need to know how to teach the mathematics content that they are expected to teach in the elementary schools. Qualified mathematics teachers know how to pose questions, represent problems and delve into their students’ work and questions in order to identify their mistakes. They must have a variety of instructional strategies for teaching content to their students, so as to engage them in meaningful mathematics work (Hiebert & Stigler, 2009). In the mathematics methods courses taken by preservice teachers, they should be provided with more opportunities to use the variety of curricular tools that are available to teach all of the

mathematics content areas. “Recent studies of learning to teach suggest that immersing teachers in the materials of practice and working on particular concepts using these materials has the potential to be particularly powerful for teachers’ learning” (Darling-Hammond, 2005, p. 401).

Researchers have begun combining pedagogical and content knowledge and calling it pedagogical-content knowledge, which includes knowing the subject, numerous teaching strategies, and specifically how to effectively teach the subject. Preservice teachers need to develop this type of knowledge also. “Effective teachers know much more than their subjects, and more than good pedagogy. They know how students tend to understand and mis-understand their subjects; they know how to anticipate and diagnose such misunderstandings; and they know how to deal with them when they arise. Such knowledge differs from knowledge of generic teaching skills because it is content specific” (Darling-Hammond, 2005, p. 205). Pedagogical-content knowledge includes the ability to interpret the subject matter, find different ways to represent it, and make it accessible to learners (Koehler, 2009). “Pedagogical preparation means many things: instructional methods, learning theories, educational measurement and testing, educational psychology, sociology, and history” (Wilson, 2001, p. 193). Having a variety of experiences in all of these areas will help better prepare preservice teachers for when they have their own classroom. Pedagogical-content knowledge does not occur naturally. Preservice teachers require opportunities to work with students, the mathematics curriculum, and different curricular tools in order to develop their pedagogical-content knowledge.

Deborah Loewenberg Ball extended these ideas of content and pedagogical-content knowledge when she created the idea of MKT. MKT consists of two main categories: subject matter knowledge and pedagogical-content knowledge. Therefore, not only do preservice teachers need to have a strong content knowledge, but they also need pedagogical-content

knowledge in order for their MKT to develop. They need additional time and opportunity in order to develop their pedagogical-content knowledge, which will improve their MKT.

They also need to be surrounded by instructors who are using a variety of tools and strategies to teach mathematics concepts, which will help improve their own pedagogical knowledge. “Learning in the ways they are expected to teach may be the most powerful form of teacher education. Most people tend to teach in ways that mirror how they were taught” (Darling-Hammond, 2005, p. 76). It is important for teacher education programs to examine how they are teaching the methods classes offered to preservice teachers and to ensure that they are replicating how preservice teachers are expected to teach.

Preservice teachers need to be provided with ample opportunities to learn about the development of children’s understandings. Having an immature understanding of this development will make it much more difficult for them to teach children at their instructional level. At each grade level, children have a wide span of skills, abilities and developmental stages. “Understanding developmental pathways and progressions is extremely important for teaching in ways that are optimal for each child” (Darling-Hammond, 2005, p. 92). It is important for preservice teachers to have experience learning how a child physically, socially, emotionally, cognitively, and linguistically develops and gain an understanding that this development does not occur at the same age for each child. Teacher education should provide opportunities for preservice teachers to take coursework on child development, as well as experience in field work that fosters child observation and analysis of their learning in school environments (Darling-Hammond, 2005).

Focusing on some of these areas in teacher education has the potential to improve the preservice teachers’ MKT. But, in order for major growth to take place, preservice teachers’

need more time working with students, engaging in professional development that focuses on mathematics pedagogical-content knowledge, and analyzing with colleagues how the mathematics curriculum changes from grade level to grade level, as well as how students learn and develop in their mathematics thinking. “By engaging in professional discourse with like-minded colleagues grounded in the content and tasks of teaching and learning, teachers can deepen knowledge of subject matter and curriculum, refine their instructional repertoire, hone their inquiry skills, and become critical colleagues” (Feiman-Nemser, 2001a, p. 1044).

Professional noticing consists of attending to a child's strategies, interpreting a child's understandings, and deciding how to respond to the child based on their understandings (Jacobs et al., 2010). This study also observed and analyzed the preservice teachers' ability to notice a child's mathematical thinking and reasoning. The results did not show a statistically significant change in their ability to attend to the child's mathematics strategies. It appears that a semester in the university classroom along with working in an elementary classroom does not provide adequate time or opportunity for the preservice teachers to develop their abilities to attend to what the child was mathematically doing when attempting to determine the answers to the problems presented to them. Teachers' own knowledge influences how they perceive what a child is doing to figure out an answer in the classroom (Sherin, 2011). Based on the study's results, it can be concluded that the preservice teacher's did not have enough pedagogical and content knowledge to grow in their ability to attend to what the child was doing when attempting to solve the mathematics problems. It is also believed that attending to a child's strategies is not a typical activity that adults do, so they need additional teaching and development opportunities in order to grow (Jacobs et al., 2010).

Also, each child in the videos used a different number of mathematics strategies. In the second video, Stella did not use many strategies to answer the mathematics questions. Therefore, the preservice teachers did not notice and describe a lot of different strategies. In many of their answers for video two, question one they focused specifically on one or two strategies that were used, which gave them a lower score on the codings. On the other videos, many strategies were used by the children. It was easier for the preservice teachers to describe more strategies. Therefore, the lack of a significant change in the preservice teachers' ability to attend to a child's mathematical strategies may have occurred because of the different types of questions in the videos and the different number of strategies that the children used.

To foster improvement in preservice teachers' abilities, teacher education courses need to provide more opportunities for preservice teachers to practice attending to a child's mathematics strategies. Preservice teachers should have more time watching children do mathematics and discussing with their colleagues what they see. "One approach involves providing teachers with samples of others' teaching and asking them to describe what they notice" (Sherin, 2011, p. 81). Providing preservice teachers with more opportunities to watch children whether it is in an elementary classroom or on video would help them continue to improve in their ability to attend to their strategies. There should be continued dialogue between the instructor and the students about what they are seeing and recalling about the strategies the child is using.

The second component of professional noticing is interpreting the child's mathematics understanding by what they are doing to solve the problems. The study showed that there was a statistically significant change in the preservice teachers' abilities to interpret the child's understandings. But, the change that took place was a decrease in the overall mean scores. In order to interpret a child's understandings, one must be able to attend to the child's strategies and

have a mathematical understanding on how those strategies coincide with mathematical concepts (Jacobs et al., 2010). Considering that the preservice teachers' mean scores for the attending question on the child response questions (See Appendix A) did not have a statistically significant positive change, they must not have had enough knowledge and experience to attend to the strategies and then interpret what those strategies meant in relation to the child's understandings. Interpreting requires an ability to notice what the child is doing. Without having a strong basis in attending to the strategies, the preservice teacher would have a very difficult time interpreting the mathematics understandings associated with the strategies. Also, a lack of deep understanding in how a child's understanding develops could be another reason why their ability to interpret mathematical understandings decreased. "An effective teacher needs to understand the components of the tasks she assigns and what they require, and she must be able to observe students carefully to evaluate not only what they know but how they learn and perform" (Darling-Hammond, 2005, p. 90). Without a strong understanding of how students learn, develop, and show their understandings, the preservice teachers may have had a difficult time interpreting the child's mathematical understandings. The ability to interpret a child's mathematics understandings requires a teacher to have a strong MKT. MKT consists of having an understanding of the subject, content and students. With the results of this study indicating that the preservice teachers MKT did not increase over the course of the semester, their ability to interpret a child's mathematical understandings may not have been able to increase also due to a lack in their MKT.

Finally, another possible explanation for a decrease in the overall mean scores on the second video response question could be because of the wording of the question. The preservice teachers may not have had a definite understanding of how to respond to the question: "What did

you learn about Rex's (or Stella's or Carson's) mathematical understandings." Jacobs and Philipp (2010) noticed a similar result from their study, in which they asked multiple levels of teachers a similar question after watching a video of a child answering mathematics questions. In their study, they noticed that the teachers would often discuss the child's affect, what they learned more generally about mathematics or their evaluation of the teacher's actions from the video (Jacobs et al., 2010). Similarly, the preservice teachers in this study may not have understood exactly how they were supposed to answer the question in relation to professional noticing.

Once again, instructors need to have explicit discussions about different strategies that children use to answer mathematics problems and how these strategies show their mathematical understandings with preservice teachers during methods courses. Preservice teachers need to have a strong understanding of how children learn and grow in their understandings, so that they can then move on to interpreting those understandings. "Teachers need to understand children's development and how it influences, and is influenced by, their learning. A foundation of knowledge about child development is essential for planning curriculum; designing, sequencing, and pacing activities; diagnosing student learning needs; organizing the classroom; and teaching social and academic skills" (Darling-Hammond, 2005, p. 88). If a teacher does not know the developmental stage of a child then the learning experiences will not be effective. Highly qualified teachers are able to accentuate a child's conceptual understanding of mathematics because they have a better knowledge of the student's understandings and ways of learning (Battista, 1994). In order to notice a child's mathematics strategies and then interpret what their understandings are, preservice teachers must know how children develop and conceptually understand mathematics.

This study's results also showed a statistically significant change in the preservice teachers' ability to respond to the child based on their understandings. The preservice teachers showed improvement in their ability to appropriately decide how to respond to the child. During the methods course taken by the preservice teachers considerable emphasis was placed on teaching in a student-centered way and building conceptual understanding based on what the child already knows. The preservice teachers were taught to teach using the reform-based mathematics concept. In reform-based mathematics classroom teachers pose problems for students, ask questions, and encourage them to elaborate on their own thoughts. Teachers help create activities for students to be able to actively engage in problem solving. "From our perspective, the essential pedagogical task is not to instill the correct ways of doing but rather to guide children's constructive activities until they eventually find viable techniques. Such guidance must necessarily start from points that are accessible to the children" (Battista, 1994, p. 464). In general the focus of the methods class was to help preservice teachers understand that a teacher's job is to guide and help students make sense of mathematics, rather than showing them how to do it. This could prove to be part of the reason why the preservice teachers grew in their ability to respond to the students in a student-centered way.

In order to continue to encourage growth in preservice teachers' abilities to respond appropriately to a child, as well as interpreting a child's mathematics understandings, preservice teachers need to have a strong understanding of how children develop and how their mathematics learning progresses. Also, focused instruction where preservice teachers can learn how the mathematics curriculum and standards change throughout the grade levels would be very beneficial. It is important for preservice teachers to know how the mathematics content changes

and develops throughout the grade levels, so that they know what their students are expected to achieve in the future and what they have already been exposed to.

Teacher education can also use discussion prompts to help analyze the child's mathematics understandings and then make decisions about how to respond. These prompts can be useful to explore the preservice teachers own perspectives, as well as provide opportunity for instruction interpreting the understandings and responding to the students in a way that promotes reform-based mathematics education. Previous studies have shown that many times how teachers decide how to respond to children is based on their own perspectives. Many times teachers offer instruction based on their own understandings and strategies. Teacher education can work to build on these perspectives and focus on attending and using the specific details of the child's strategy when providing instruction (Sherin, 2011).

Overall, all three of these skills work together when professionally noticing a child's mathematics understanding and reasoning. "Deciding how to respond on the basis of children's understandings can occur only if teachers interpret children's understanding, and these interpretations can be made only if teachers attend to the details of children's strategies" (Jacobs et al., 2010). Therefore, teacher education should focus on all three of these skills in an integrated way, while still also focusing on encouraging growth in each independent area.

This study also looked for a relationship between the preservice teachers' MKT and their ability to professionally notice a child's mathematical thinking. A statistically significant relationship between each of the three questions on the child response sheet with the preservice teachers' LMT scores was not discovered. Therefore the values of the preservice teachers' scores on the LMT are not predictive of the values on each question from the child response videos. The lack of a statistically significant relationship could be a result in part because of the limited

variability in the participants and in the instruments used (Salkind, 2011). Also, the LMT may not be sensitive enough to measure the small changes in the preservice teachers' MKT.

Therefore, a strong relationship between the preservice teachers' MKT and professional noticing scores would not be able to occur.

Implications for Teacher Education

Teacher educators can help improve preservice teachers' MKT and their ability to professionally notice a child's mathematical thinking and understandings. Preservice teachers need to be provided with adequate time in the field working with students of varying abilities. Preservice teachers need time in the schools working with children and practicing their skills. "From field experience, prospective teachers reported acquiring survival skills, learning about students, and recognizing that their students' understandings vary, are complex, and differ from the teachers" (Wilson, 2001, p. 196). Field experience will provide hands-on learning experiences for future teachers that classes cannot provide, in which they will be able to learn first-hand the strategies that children use to figure out mathematics problems and how this represents their mathematical understandings.

While out in the field, it is important for teachers to have worthwhile activities to do; so that they can apply what they have learned during their coursework to the students they are working with. "Most use some combination of reflective logs, dialogue journals, weekly cohort-based seminars, and individual conferences to help teacher candidates develop the capacity to learn from the experience and analysis of their own and other's practice" (Feiman-Nemser, 2001a, p. 1024). A preservice teacher needs opportunities to try out new ideas, practice activities learned in classes, use technology, create relationships, and work with children with differing abilities and this can only happen when they are out in the schools.

Teacher education also should provide mathematics content and methods courses that will improve their subject matter and pedagogical-content knowledge. With the variety of abilities in the classroom, having both a strong content and pedagogical knowledge is essential. Preservice teachers need to take classes that will strengthen their content knowledge. It is necessary for them to understand the content that they are going to be teaching. They should have had an ample number of mathematics classes so that they are confident in their own mathematics ability.

Even if preservice teachers have a strong mathematics understanding, it does not mean that they will be good at teaching mathematics and analyzing student understanding. “Teachers not only need to be able to figure out, swiftly, what student thought processes might lead to difficulties, but they must also be able to explain in ways that students can understand. Being able to do this is more than simply knowing the subject” (Ball & Forzani, 2010, p. 223). It is important for them to know how to analyze students’ answers and provide probing questions to keep the students thinking. Preservice teachers do not just gain pedagogical-content knowledge naturally. They need methods courses that will teach them how to teach all five of the different mathematics content areas. When teaching mathematics, in order to be an effective teacher, it is important to not only understand the mathematics content, but teachers also need to have an understanding of how to teach the content. “Pedagogical preparation means many things: instructional methods, learning theories, educational measurement and testing, educational psychology, sociology, and history” (Wilson, 2001, p. 193). Having a variety of experiences in all of these areas during their university coursework will help better prepare preservice teachers.

During university coursework, preservice need to have learning experiences that are similar to how they are expected to teach. “When teachers have opportunities to interact with

their subject matter in ways that they aim for their own students to do, they are more likely to engage in those practices in their classrooms” (Darling-Hammond, 2005, p. 396). In mathematics, it is important for them to have opportunities to learn the content by analyzing, discussing, and thinking about their own as well as their colleague’s mathematical strategies and understandings. It is important for teacher educators to have guided discussions with preservice teachers about the students’ mathematics strategies and understandings. Methods courses should include discussions that help preservice teachers dig deeper into their own perspectives on children’s understandings, but also have targeted opportunities to discuss and analyze the children’s mathematical strategies and how this relates to their understandings (Jacobs et al., 2010).

Teacher education must also include coursework that focuses on how children learn and how their understanding develops. They need multiple courses in learning and development so that they learn how to create and teach developmentally appropriate instructional materials and lessons (Darling-Hammond, 2005). They need to frequently be exposed to the learning theories that help improve preservice teachers’ understandings of how children develop, which is critical to effective teaching. Teacher education not only needs to provide preservice teachers opportunities to gain an understanding of the stages of development, but they also need to learn how to analyze all of the components of the mathematics activities they are using, as well as learn to evaluate what the child knows, and how they learn and perform (Darling-Hammond, 2005). “Understanding where a child is developmentally is one of the most important keys to shaping appropriate learning tasks that are engaging for students-tasks that are both interesting and appropriately challenging” (Darling-Hammond, 2005, p. 89). It is critical for teacher education to continuously provide these opportunities for preservice teachers.

It also may be beneficial for preservice teachers to learn about learning progressions in their teacher education courses. Learning progressions are a “carefully sequenced set of building blocks that students must master en route to a more distant curricular aim. The building blocks consist of sub skills and bodies of enabling knowledge” (Popham, 2007, p. 83). If preservice teachers have a better understanding of learning progressions they will learn to focus on what the student needs to learn, not on what activities they should be doing. In planning lessons, they will first figure out the learning goal, and then create activities that will be done in order to achieve the goal. This understanding will help them focus more on what the child already does mathematically know. Knowledge of learning progressions will also help them make connections between what comes before and after the learning goal. With this additional knowledge, preservice teachers will make instructional decisions about the child based on their understandings and on how the child will best learn and grow.

The scores on the preservice teachers’ LMT scores and child response scores for each of the three questions did not increase much over the course of the semester. A teacher’s MKT and their ability to professionally notice children’s mathematical thinking improves with increased pedagogical-content knowledge, experience, and focused professional development. Thus it can be concluded that preservice teachers need more pedagogical-content knowledge, experience, and a better understanding of how a child’s understanding develops over time in order to improve their MKT and ability to professionally notice children’s mathematical thinking. MKT includes all of the many aspects of teaching, including professional noticing. “It (MKT) is concerned with the tasks involved in teaching and the mathematical demands of these tasks. Because teaching involves showing students how to solve problems, answering students’ questions, and checking students’ work, it demands an understanding of the content of the school

curriculum” (Ball, 2008, p. 396). In order to be effective at professionally noticing students’ thinking these similar skills are needed, as well as a focus on attending to the individual child and mathematically important details, a focus on the child’s understandings, and a knowledge of possible future strategies that correlate with the mathematics curriculum in order to encourage mathematics growth (Jacobs et al., 2010).

Future Research

Through this study, it has become evident that MKT and the ability to professionally notice a child’s mathematical thinking does not improve or change substantially over the short period of time that makes up a university semester. MKT and professional noticing is a complex process and additional time and resources are required for preservice teachers in order to continue to grow in these areas. The researcher focused specifically on preservice teachers, who are undergraduate students completing their final semester of coursework before starting their year-long student teaching experience. Future research could follow these preservice teachers throughout their student teaching experience and even during their first few years of classroom teaching. Similar instruments could be used to determine if time, experience with children, and/or focused professional development are key components to growth in MKT and professional noticing.

Previous research has discussed the importance of professional development centered on helping teachers learn how to professionally notice students thinking. Teachers need development opportunities that focus on how to attend to and interpret what they see, as well as how children think mathematically. This would be an important continuing educational area for teachers as they continue through their career. The amount and type of professional development

could be monitored in order to determine if professional development does help the development of professional noticing.

This research could be extended by including information on the preservice teachers' self-efficacy. It would be interesting to determine if a preservice teacher's self-efficacy has any relationship to their ability to professionally notice a child's thinking, as well as their own MKT. Previous research has analyzed the importance of preservice teachers having a positive outlook for teaching different subjects, therefore, it would be useful to determine how this variable interacts with MKT and professional noticing.

Also, when looking at the preservice teachers' MKT, it was determined that when the students were broken up into subgroups, the different subgroups produced different statistical results. Therefore, future research could include a larger sample size so that the subgroups could have a larger number of preservice teachers which would increase reliability and validity. Then, each subgroup could be analyzed to see how the MKT of the preservice teachers' change over the course of a semester or a longer period of time.

The LMT instrument that was used was the LMT 2008A and 2008B. The developers of the instrument recommended using form A of the instrument and then using the equated form B. The forms were very similar, but contained many different questions. Future research could analyze the preservice teachers' responses on the LMT a little deeper. Only one form of the instrument could be used, once at the beginning of the semester and again at the end, which would allow for more focus on each individual question. Are the preservice teachers' struggling with the same questions? Has there been improvement in one area? Has any specific area or question stayed consistent? Future research in these areas could help teacher educators to hone in on the areas where a majority of the preservice teachers are struggling and to develop

coursework to help them improve in the specific parts of the content area of numbers and operations, as well as the other mathematics content areas.

Finally, the development of a more sensitive measure of preservice teachers MKT could be developed which may provide more specific information on how their MKT develops at this preservice level. The LMT instrument has been used with preservice teachers in the past, but it is generally meant to analyze a teacher's MKT that is regularly in the field working with children. An instrument that analyzed how MKT develops for preservice teachers' who have had minimal experience working with students may provide much more specific information for how MKT develops.

It is believed that this study provides some useful insight into the educational development of preservice teachers'. Although, the results were somewhat inconclusive, it is believed that they did show the need for college-level courses to emphasize building preservice teachers' content and pedagogical knowledge of mathematics by providing opportunities to collaborate with colleagues, analyze children's mathematical strategies and understandings, and develop their own knowledge of mathematics. Hopefully this research will provide more information for future studies so that the education field can continue to learn more about how MKT and professionally noticing develops.

References

- Ball, D. L., & Forzani, F.M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education*, 60(5), 14.
- Ball, D. L., & Forzani, F.M. (2010). What does it take to make a teacher? *Phi Delta Kappan*, 92(2), 5.
- Ball, D. L., & Hill, H. . (2009). The curious-and-crucial case of mathematical knowledge for teaching. *Phi Delta Kappan*, 91(2), 3.
- Ball, D. L., & Thames, M. H. (2010). What math knowledge does teaching require? *Teaching Children Mathematics*, 17(4), 9.
- Ball, D. L., Thames, M. H., & Phelps, G. . (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 18.
- Battista, M. (1994). Teacher beliefs and the reform movement in mathematics education. *Phi Delta Kappan*, 75, 5.
- Battista, M. T. (1999). The mathematical miseducation of America's youth: Ignoring research and scientific study in education. *Phi Delta Kappan*, 80(6), 9.
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klasmann, U., Krauss, S., Neubrand, M., & Tsai, Y. . (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 47.
- Black, P., & William, D. (1998). Inside the black box: Raising standards through classroom assessment. [black article]. *Phi Delta Kappan*, 80(2), 9.

- Borko, H. E., M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education*, 23(3), 28.
- Bryan, L. A. (2003). Nestedness of beliefs: Examining a prospective elementary teacher's belief system about science teaching and learning. *Journal of Research in Science Teaching*, 40(9), 33.
- Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. . (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26(4), 32.
- Chapman, O. (2007). Facilitating preservice teachers' development of mathematics knowledge for teaching arithmetic operations. *Journal for Mathematics Teacher Education*, 10(2), 8.
- Creswell, J. W. (2005). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research*. Upper Saddle River, New Jersey: Pearson Education, Inc.
- Darling-Hammond, L., & Bransford, J. (2005). *Preparing teachers for a changing world*. San Francisco, CA: John Wiley & Sons, Inc.
- Elliott, A. W., W. . (2007). *Statistical analysis quick reference guidebook: With SPSS examples*. Thousand Oaks, CA: Sage Publications, Inc.
- Feiman-Nemser, S. (2001a). From preparation to practice: Designing a continuum to strengthen and sustain teaching. *Teachers College Record*, 103(6), 42.
- Feiman-Nemser, S. (2001b). Helping novices learn to teach: Lessons from an exemplary support teacher. *Journal of Teacher Education*, 52(17), 13.

- Fennema, E., Franke, M. L., Carpenter, T. P., & Carey, D. . (1993). Using children's mathematical knowledge in instruction. *American Educational Research Journal*, 30(3), 28.
- Fenstermacher, G. S., J. F. (2004). *Approaches to teaching*. Columbia University: Teachers College.
- Harlow, S., Cummings, R., & Aberasturi, S. (2006). Karl Popper and Jean Piaget: A rationale for constructivism. *The Educational Forum*, 71(1), 7.
- Hiebert, J. (2000). What can we expect from research? *Teaching Children Mathematics*, 6(7), 2.
- Hiebert, J., & Stigler, J. (2004). A world of difference. *Journal of Staff Development*, 25(4), 5.
- Hiebert, J., & Stigler, J. (2009). Closing the teaching gap. *Phi Delta Kappan*, 91(3), 5.
- Hill, H. C. (2010). The nature and predictors of elementary teachers' mathematical knowledge for teaching. *Journal for Research in Mathematics Education*, 41(5), 32.
- Hill, H. C., Ball, D. L., Phelps, G. . (2010). *Learning mathematics for teaching project*. Paper presented at the Instrument Dissemination Workshop, University of Michigan.
- Hill, H. C., Rowan, B., & Ball D. L. . (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 35.
- Hill, H. C., Schilling, S. G., & Ball, D. L. . (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal*, 105, 19.
- Hoy, W. K. (2010). *Quantitative research in education: A primer*. Thousand Oaks, CA: Sage Publications, Inc.

- Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. . (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 33.
- Jacobs, V. R., Phillip, R. A. . (2010). Supporting children's problem solving. *Teaching Children Mathematics*, 17(2), 8.
- Joyce, B., & Weil, M. . (1996). *Models of teaching*. Needham Heights, MA: Allyn & Bacon.
- Koehler, M. J. M., P. . (2009). What is technological pedagogical content knowledge? *Contemporary Issues in Technology and Teacher Education*, 9(1), 10.
- Maccini, P., & Gagnon, J. C. (2002). Perceptions and applications of NCTM standards by special and general education teachers. [journal]. *Council for Exceptional Children*, 68(3), 19.
- Maxwell, J. A. (2005). *Qualitative research design: An interactive approach* (2nd ed.). Thousand Oaks, CA: Sage Publications, Inc.
- Mulligan, J. T. M., M.C. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education*, 28(3), 21.
- Nolan, S. H., T. E. (2008). *Statistics for the behavioral sciences*. New York, NY: Worth Publishers.
- Ormrod, J. E. (2008). *Human Learning* (Vol. 5). Upper Saddle River, New Jersey: Pearson Education, Inc.
- Panel, N. M. A. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington DC: U.S. Department of Education.
- Philipp, R. A., Ambrose, R., Lamb, L. L., Sowder, J. T., Schappelle, B. P., Sowder, L., Thanheiser, E. & Chauvot, J. . (2007). Effect of early field experiences on the

- mathematical content knowledge and beliefs of prospective elementary school teachers:
An experimental study. *Journal for Research in Mathematics Education*, 38(5), 38.
- Phillips, D. C., & Soltis, J. F. . (2004). *Perspetives on learning*. New York, NY: Teachers
College Press.
- Popham, J. W. (2007). The lowdown on learning progressions. *Educational Leadership*, 64(7), 2.
- Salkind, N. (2011). *Statistics for people who think they hate statistics* (4 ed.). Thousand Oaks,
CA: Sage Publications, Inc. .
- Schneider, R. M. P., K. (2011). Science teacher learning progressions: a review of science
teachers' pedagogical content knowledge development. *Review of Educational Research*,
81(4), 35.
- Schoenfeld, A. H. (1994). *Mathematical Thinking and Problem Solving*. Hillsdale, NJ: Lawrence
Erlbaum Associates, Inc.
- Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (2011). *Mathematics teacher noticing*. New York,
NY: Taylor & Francis.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard
Educational Review*, 57(1), 22.
- Stanic, G. (2003). *A history of school mathematics*. Ann Arbor, MI: National Council of
Teachers of Mathematics.
- Stiff, L. (2000). Preparing to teach young children. *NCTM News Bulletin*.
- Turner, E. E., Drake, C., McDuffie, A. R., Aguirre, J., Bartell, T. G., & Foote, M. Q. (2012).
Promoting equity in mathematics teacher preparation: a framework for advancing teacher

learning of children's multiple mathematics knowledge bases. *Journal for Mathematics Teacher Education*, 15, 15.

Van De Walle, J. A. (2007). *Elementary and middle school mathematics* (6 ed.). Boston, MA: Pearson Education Inc.

Wilson, S. M., Floden, R. E., & Ferrini-Mundy, J. (2001). Teacher preparation research. An insider's view from the outside. *Journal of Teacher Education*, 53(3), 12.

APPENDIX A

CHILD RESPONSE SHEETS

APPENDIX B

DEMOGRAPHIC QUESTIONNAIRE

Demographic Questionnaire

Name: _____

Age: _____ Gender: _____

How many college level mathematics courses have you taken?

Please list the names of the courses you have taken (I.E. PreCalculus, Calculus 1, Linear Algebra, etc.)

APPENDIX C

PARENT CONSENT FORM

INTRODUCTION

The Department of Curriculum and Teaching at the University of Kansas supports the practice of protection for human subjects participating in research. The following information is provided for you to decide whether you wish to participate in the present study. You may refuse to sign this form and not participate in this study. You should be aware that even if you agree to participate, you are free to withdraw at any time. If you do withdraw from this study, it will not affect your relationship with this unit, the services it may provide to you, or the University of Kansas.

PURPOSE OF THE STUDY

The aim of this study is to determine how preservice teachers' mathematical knowledge for teaching changes over the course of a semester, in which they will take a mathematics methods course and work with elementary aged students in a mathematics classroom setting. The extent to how the preservice teachers' ability to professionally notice children's mathematical thinking will also be examined. The other purpose is to determine if there is a relationship between preservice teachers' mathematical knowledge for teaching and the extent to which they professionally notice children's mathematical reasoning and thinking.

PROCEDURES

If I agree to participate in this study, I allow the researchers:

1. To video record my three children Claire Ain, Mason Ain, and Stella Ain while answering various mathematics questions.
2. To show the videos to undergraduate students at the University of Kansas in order to help them learn how to analyze children's mathematical thinking.
3. To analyze and discuss the mathematical processes that the children use to answer the mathematics questions on the videos with undergraduate students at the University of Kansas.

RISKS and BENEFITS

No risks are anticipated for participating in this study.

PARTICIPANT CONFIDENTIALITY

Only the children's first names and grade level will be used in this study. No other information about the children will be provided. **Only the researcher, Dr. Kelli Thomas, the researcher's advisor, and the students in C&T 351** will have view the videos. Videos will be kept for **three years** to ensure thorough analysis and then destroyed.

REFUSAL TO SIGN CONSENT AND AUTHORIZATION

You are not required to sign this Consent and Authorization form and you may refuse to do so. However, if you refuse to sign, you cannot participate in this study.

CANCELLING THIS CONSENT AND AUTHORIZATION

You may withdraw your consent to participate in this study at any time. You also have the right to cancel your permission to use and disclose further information collected about you, in writing, at any time, by sending your written request to: Mari Wheeler Flake, 1122 W. Campus Rd. Joseph R. Pearson Hall, University of Kansas, Lawrence, Kansas 66045.

QUESTIONS ABOUT PARTICIPATION

Questions about procedures should be directed to the researcher listed at the end of this consent form.

PARTICIPANT CERTIFICATION:

I have read this Consent and Authorization form. I have had the opportunity to ask and **to** receive answers to any questions that I had regarding the study. I understand that if I have any additional questions about my rights as a research participant, I may call (785) 864-7429, write to the Human Subjects Committee Lawrence Campus (HSCL), University of Kansas, 2385 Irving Hill Road, Lawrence, Kansas 66045-7568, or email irb@ku.edu.

I agree to allow my children to be part of this study by participating in a short mathematics video. By my signature I affirm that I have received a copy of this Consent and Authorization form

Print Name

Signature

Date

Researcher Contact Information:

Mari Wheeler Flake.

Principal Investigator

Joseph R. Pearson Hall

1122 W. Campus Rd.

University of Kansas

Lawrence, KS 66045

586-945-5429

Ku1@ku.edu

Audio-Visual Permission

If I give permission, I may be tape-recorded for research and educational purposes. “Research and educational purposes” includes sharing the audio tapes during research presentations, in undergraduate and graduate classes, and professional workshops to demonstrate exemplary instructional practices.

Please initial below each purpose for which you give permission:

_____ I give permission to be tape recorded during focus group meetings at KU to discuss what I have noticed about children’s mathematical thinking while out in the field.

_____ I give permission for my audio-recordings, which demonstrate professional noticing of children’s mathematical thinking, to be used for research and educational purposes.

_____ **I am aware that I can still participate in this study without participating in the audio recordings. At any time, I can ask not to be recorded or to have the tape recorder turned off.**

_____ Permission denied **to be recorded**. I may be participate in discussions, but not recorded.

My signature below indicates that I have read and understand the uses of audio-recordings information pertaining to my participation in this study. My questions have been answered to my satisfaction. I have been given a copy of this form.

Print Name

Signature

Date

APPENDIX D

PRESERVICE TEACHER CONSENT FORM

INTRODUCTION

The Department of Curriculum and Teaching at the University of Kansas supports the practice of protection for human subjects participating in research. The following information is provided for you to decide whether you wish to participate in the present study. You may refuse to sign this form and not participate in this study. You should be aware that even if you agree to participate, you are free to withdraw at any time. If you do withdraw from this study, it will not affect your relationship with this unit, the services it may provide to you, or the University of Kansas.

PURPOSE OF THE STUDY

The aim of this study is to determine how preservice teachers' mathematical knowledge for teaching changes over the course of a semester, in which they will take a mathematics methods course and work with elementary aged students in a mathematics classroom setting. The extent to how the preservice teachers' ability to professionally notice children's mathematical thinking will also be examined. The other purpose is to determine if there is a relationship between preservice teachers' mathematical knowledge for teaching and the extent to which they professionally notice children's mathematical reasoning and thinking.

PROCEDURES

If I agree to participate in this study, I allow the researchers:

1. To analyze my scores on the LMT instrument which is designed to measure my MKT twice during the semester, once at the beginning and once at the end. This instrument is expected to take about 30 minutes.
2. To analyze my written reflections about 3 videos of children doing mathematics problems and responding to the videos by answering 3 questions regarding the student's mathematic thinking and reasoning. **The videos will take approximately 5 minutes to watch, while the written responses will also take about 5 to 10 minutes to respond.**
3. To interview **and record me on a video recorder (the iPad)**, in a focus group setting, about what I have noticed about the students' mathematical thinking, in my field experience. The recorded interviews will occur three times over the course of the semester and will take 10 to 15 minutes. **I am aware that if I do not want to be recorded, I can participate in the focus groups interview without being recorded. I can also ask for the recording to be stopped at anytime.**

RISKS and BENEFITS

I understand that this method of data collection is not expected to interfere with my teaching or learning. No risks are anticipated for participating in this study. Participating in this study may help me to think about instructional practices, specifically my own mathematical thinking, as well as children's mathematical thinking. **I understand that it is not mandatory for me to participate in this study. Although, the activities (taking the LMT instrument and analyzing students' mathematical thinking by watching the videos) are mandatory for**

C&T 351, my scores on the instruments and reflections will not affect my grade. I understand that these activities for the study are part of the course requirements, but if I chose not to participate, my scores will not be used in the research study. As part of C&T 351, I will be required to participate in focus group discussions, but I have the option of not being tape recorded. If I chose not to be recorded, it will not affect my grade for the course.

PARTICIPANT CONFIDENTIALITY

My name will not be associated with any publication or presentation with the information collected about me or with the research findings from this study. Instead, the researcher will use a pseudonym. Any identifiable information about me (e.g., MKT instrument scores, response sheet answers) will not be shared unless (a) it is required by law or university policy, or (b) I give written permission. Data will be stored in a locked filing cabinet and a password-protected computer. **Only the researcher and Dr. Kelli Thomas, the researcher's advisor,** will have access to the data. Data will be kept for **three years** to ensure thorough analysis and then destroyed. **Mari Flake will be responsible for transcribing all of the focus group interviews.**

REFUSAL TO SIGN CONSENT AND AUTHORIZATION

You are not required to sign this Consent and Authorization form and you may refuse to do so without affecting your right to any services you are receiving or may receive from the University of Kansas or to participate in any programs or events of the University of Kansas. However, if you refuse to sign, you cannot participate in this study.

CANCELLING THIS CONSENT AND AUTHORIZATION

You may withdraw your consent to participate in this study at any time. You also have the right to cancel your permission to use and disclose further information collected about you, in writing, at any time, by sending your written request to: Mari Wheeler Flake, 1122 W. Campus Rd. Joseph R. Pearson Hall, University of Kansas, Lawrence, Kansas 66045.

QUESTIONS ABOUT PARTICIPATION

Questions about procedures should be directed to the researcher listed at the end of this consent form.

PARTICIPANT CERTIFICATION:

I have read this Consent and Authorization form. I have had the opportunity to ask and to receive answers to any questions that I had regarding the study. I understand that if I have any additional questions about my rights as a research participant, I may call (785) 864-7429, write to the Human Subjects Committee Lawrence Campus (HSCL), University of Kansas, 2385 Irving Hill Road, Lawrence, Kansas 66045-7568, or email irb@ku.edu.

I agree to take part in this study as a research participant. By my signature I affirm that I am at least 18 years old and that I have received a copy of this Consent and Authorization form

Print Name

Signature

Date

Researcher Contact Information:

Mari Wheeler Flake.
Principal Investigator
Joseph R. Pearson Hall
1122 W. Campus Rd.
University of Kansas
Lawrence, KS 66045
586-945-5429
Ku1@ku.edu

Audio-Visual Permission

If I give permission, I may be video-recorded for research and educational purposes. “Research and educational purposes” includes sharing the audio tapes during research presentations, in

undergraduate and graduate classes, and professional workshops to demonstrate exemplary instructional practices.

Please initial below each purpose for which you give permission:

_____ I give permission to be video recorded during focus group meetings at KU to discuss what I have noticed about children's mathematical thinking while out in the field.

_____ I give permission for my video-recordings, which demonstrate professional noticing of children's mathematical thinking, to be used for research and educational purposes.

_____ **I am aware that I can still participate in this study without participating in the video recordings. At any time, I can ask not to be recorded or to have the tape recorder turned off.**

_____ Permission denied **to be recorded**. I may be participate in discussions, but not recorded.

My signature below indicates that I have read and understand the uses of audio-recordings information pertaining to my participation in this study. My questions have been answered to my satisfaction. I have been given a copy of this form.

Print Name

Signature

Date

APPENDIX E

CODEBOOK FOR CHILD RESPONSE QUESTIONS

Coding Descriptions for Child Response Sheets

Question 1: What do you think the child was doing to figure out the answers to the problems?

A score of **0 (Beginner)**:

- Incorrect mathematical strategies stated
- Incorrect use of mathematical strategies and knowledge
- Lack of mathematical knowledge
- List without description of strategy

A score of **1 (Novice)**:

- Identified only one strategy
- A novice description of strategies used
- Lack of attention to details of the strategies used and how the problem was solved

A score of **2 (Emerging)**:

- Identified two or more strategies
- Emerging/developing description of the strategies used
- Description of strategies is not connected to math concepts

A score of **3 (Transitional)**:

- Multiple strategies identified
- Specific description of the strategies used
- Connections are made to mathematical concepts (How the strategies used connect to the math concept of the question being asked)
- Descriptions include mathematically significant details

Question 2: What did you learn about the child's mathematical understandings?

A score of **0 (Beginner)**:

- No mathematical connection to the strategies used
- No description of what the child mathematically understands

A score of **1 (Novice)**:

- Novice description of mathematical strategies with incorrect or immature inferences about the mathematical understandings of the child

- Lack of focus on the particular child
- No evidence of interpretation of the child's understandings
- Commentary on the child but not on the understandings
- An observation noneducators could make

A score of **2 (Emerging)**:

- Descriptions show the ability to infer some basic mathematical understandings
- Limited depth on the child's understandings
- Some broad or undefined descriptions
- Makes some connection of what the child is doing and what that means mathematically

A score of **3 (Transitional)**:

- Inferences have a richer description of mathematical understandings
- Descriptions connect specific examples
- Used details of the strategy to explain what the child understood

Question 3: What would you do if you were the teacher to continue to encourage mathematical understandings and growth?

A score of **0 (Beginner)**:

- No evidence based on child understandings
- Incorrect interpretation of the future mathematical steps needed for the child
- Teacher centric

A score of **1 (Novice)**:

- Very general descriptions for future mathematical teaching
- Descriptions could be said without seeing the actual video of the child
- Little or no reference to building on the child's understandings
- Little description of why the strategy is suggested

A score of **2 (Emerging)**:

- Descriptions consider the child's strategy but does not consider the strategy in relationship to future concept development
- Used the child's understandings in more of a general way

A score of **3 (Transitional)**:

- Robust description of how the child's strategies will be used for future concept development
- Individualized for the student based on their understandings for concept development
- Explicitly considers the child's existing strategies when thinking about the next steps
- Knowledge about the next steps for children's mathematical development

