TEACHING SHAPE RECOGNITION TO STUDENTS WITH SIGNIFICANT INTELLECTUAL DISABILITIES

By

Kristin Joannou Lyon

Submitted to the graduate degree program in Special Education and the Graduate Faculty of the University of Kansas in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Chairperson Wayne Sailor

Jane Wegner

Jennifer Kurth

Kelli Thomas

Barbara Thompson

H. Rutherford Turnbull III

Date Defended: April 21, 2014

The Dissertation Committee for Kristin Joannou Lyon

certifies that this is the approved version of the following dissertation:

TEACHING SHAPE RECOGNITION TO STUDENTS

WITH SIGNIFICANT INTELLECTUAL DISABILITIES

____

Chairperson Wayne Sailor

Date approved: April 23, 2014

Abstract

Little research on teaching mathematics to students with significant intellectual disabilities exists, and that which does exist is largely in the areas of number and money skills. All students, including students with significant intellectual disabilities, need mathematics skills to lead productive and independent lives. Geometry skills help students make sense of the world around them. Mastery of shape recognition is a beginning geometry skill that is necessary for progressing to more advanced topics in geometry. The purpose of this study was to teach beginning shape recognition skills by combining evidence-based practices in shape recognition instruction with best practices in teaching mathematics to students with significant intellectual disabilities. This study utilized a small sample, interrupted time series (single-case) multiple-probe design across four behaviors that were (a) matching identical shapes, (b) matching shapes that are different sizes, (c) matching shapes that have different orientations, and (d) shape recognition. Although no students reached mastery criteria, all students showed some improvement, and much was learned regarding teaching mathematics to students with significant intellectual disabilities. Limitations, suggestions for future research, and the implications of these findings are discussed.

ACKNOWLEDGEMENTS

As I was interviewing to start my doctorate, a professor explained the program to me as, “best today, better tomorrow.” At that point I did not realize how true that statement would be. My dissertation is the culmination of my doctoral journey. There are so many people to thank who have helped me along the way to reach my best today and who I am sure will push me to be better tomorrow.

I am truly grateful and honored to have the support of my entire committee. My deepest gratitude goes to my advisor, Dr. Wayne Sailor, for your continuous support, guidance, and advice throughout my entire doctoral program. From the first day I met you, you have challenged me to rethink how education should look for all students—thank you. Dr. Jane Wegner, thank you for your advice and support in conjunction with my minor. I have learned so much in your classes and my only regret is that I did not take them before I started teaching. Dr. Jennifer Kurth, thank you for being a late addition to my committee. I have enjoyed getting know you and collaborating with you this past year. Dr. Barbara Thompson, thank you for allowing me to teach and present with you. I have learned a great deal from those experiences and I am certain they will help me become a better professor. Dr. Kelli Thomas, working with you is what guided my research interest of teaching mathematics to students with significant disabilities. Not only have you been a great boss, mentor, and teacher but most of all you have been a great friend. Finally, Dr. Rud Turnbull, while there is so much I could thank you for, it all begins with the fact that you are the reason I am here at the University of Kansas and for that I am grateful.

I am indebted to several others for their support throughout my dissertation study. Thank you Stephen for helping me find participants, conducting inter-rater reliability, and advising me throughout the process. Thank you Rachel, Kelly, and Sheryl for trusting me to work with your students.

The doctoral journey has not been a lonely one, I would like to thank all of my fellow doctoral students who have mentored, supported, and inspired me throughout the last four years. Special thanks to Heather, Marti, and Audra. I would not have finished this dissertation or doctoral degree without your unending support and advice. Thank you for always being a shoulder to lean on.

I am grateful to my family and friends who have always supported me in all my adventures, big or small, this one being no exception. Special thanks to Mom and Dad who have always believed that I could achieve anything I set my mind to, even when I did not always believe it myself.

Finally and with all my heart, thank you Chris. You have seen me through every part of this doctoral journey holding my hand and cheering me on every step of the way. Thank you for your never-ending love and support, I cannot imagine doing this without you. I love you more than anything!

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER I—INTRODUCTION AND REVIEW OF THE LITERATURE

Background

Mathematics skills are used on a daily basis. Whether one wishes to cook a meal, plan a budget, or navigate around town, mastery of mathematics skills is essential to independent living and to attaining a high quality of life. According to the National Council of Teachers of Mathematics (NCTM)(2000a) a high quality mathematics education is critical for all students, however this goal is not currently realized for many. In response to this problem, NCTM developed *Principles and Standards for School Mathematics* (2000a), which describes their view on what constitutes a high quality mathematics education for all students from pre-kindergarten through twelfth grade.

The three main components of *Principles and Standards* include five content area standards, five process standards, and six principles. The five content areas and five process standards describe what all students should learn in mathematics. The content area standards describe the five domains (or topics) of mathematics that all students should master: (a) number and operations, (b) algebra, (c) geometry, (d) measurement, and (e) data analysis and probability (Table 1). The process standards describe the five skills that students must develop in conjunction with the content area standards in order to successfully use mathematics: (a) problem solving, (b) reasoning and proof, (c) communication, (d) connections, and (e) representation (Table 2). The six principles that describe how all students should learn the content area and process standards are (a) equity, (b) curriculum, (c) teaching, (d) learning, (e) assessment, and (f) technology (Table 3). Each of the three components—content area standards, process standards, and principles—is essential to a high quality mathematics education; a lack of any component would necessarily result in a poor mathematics education.

Table 1

Principles and Standards for School Mathematics - Content Area Standards

Standard	Goals
Number and Operations	"Understand numbers, ways of representing numbers, relationships among numbers, and number systems; understand meanings of operations and how they relate to one another; compute fluently and make reasonable estimates" (NCTM, 2000b, p. 7)
Algebra	"Understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; analyze change in various contexts" (NCTM, 2000b, p. 8)
Geometry	"Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships; specify locations and describe spatial relationships using coordinate geometry and other representational systems; apply transformations and use symmetry to analyze mathematical situations; use visualization, spatial reasoning, and geometric modeling to solve problems"(NCTM, 2000b, p. 9)
Measurement	"Understand measurable attributes of objects and the units, systems, and processes of measurement; apply appropriate techniques, tools, and formulas to determine measurements" (NCTM, 2000b, p. 9)
Data Analysis & Probability	"Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them; select and use appropriate statistical methods to analyze data; develop and evaluate inferences and predictions that are based on data; understand and apply basic concepts of probability" (NCTM, 2000b, p. 10)

Throughout the document, *Principles and Standards* uses the word "all." All students need a high quality mathematics education. However, a group of students who often receive a poor quality, if any, mathematics education are students with significant intellectual disabilities. The American Association on Intellectual and Developmental Disabilities defines intellectual disability as, "a disability characterized by significant limitations in both intellectual functioning and in adaptive behavior, which covers many everyday social and practical skills. This disability originates before the age of 18" (American Association on Intellectual and Developmental Disabilities, 2013). NCTM's premise that high quality mathematics education be accessible for all students includes students with intellectual disability. Providing poor quality or limited

Table 2

Principles and Standards for School Mathematics- Process Standards

Standard	Goals
Problem solving	"Build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving" (NCTM, 2000b, p. 10)
Reasoning and proof	"Recognize reasoning and proof as fundamental aspects of mathematics; make and investigate mathematical conjectures; develop and evaluate mathematical arguments and proofs; select and use various types of reasoning and methods of proof" (NCTM, 2000b, p. 11)
Communication	"Organize and consolidate their mathematical thinking though communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others; analyze and evaluate the mathematical thinking and strategies of others; use the language of mathematics to express mathematical ideas precisely" (NCTM, 2000b, p. 12)
Connections	"Recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics" (NCTM, 2000b, p. 13)
Representation	"Create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; use representations to model and interpret physical, social, and mathematical phenomena" (NCTM, 2000b, p. 14)

mathematics education to students with significant intellectual disabilities violates the premise established in the *Principles and Standards* document.

When discussing mathematics education, a major factor to consider for students with significant intellectual disabilities is limited or non-existent exposure to learning opportunities (Browder, Jimenez, Spooner, et al., 2012). Instruction for students with the most significant intellectual disabilities often focuses on functional skills, i.e., skills needed to function on a daily basis such as self-care skills, instead of academic skills such as literacy and mathematics. Emphasis on functional skills persists because this population of students has historically been the victim of low expectations (Wehmeyer, Lattin, & Agran, 2001); often people do not expect these students to learn to read or do math. Because of labels, test results, and levels of support

Table 3

Principles and Standards for School Mathematics- Principles

Standard	Goal
Equity	"Excellence in mathematics education requires equity- high expectations and strong support for all students" (NCTM, 2000b, p. 3)
Curriculum	"A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades" (NCTM, 2000b, p. 3)
Teaching	"Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (NCTM, 2000b, p. 4)
Learning	"Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, 2000b, p. 5)
Assessment	"Assessment should support the learning of important mathematics and furnish useful information to both teachers and students" (NCTM, 2000b, p. 5)
Technology	"Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000b, p. 6)

needed, people often make assumptions about what this population of students can and cannot do. Such assumptions leave some people questioning whether students with significant intellectual disabilities are capable of mastering academic skills.

A perspective that refutes these assumptions of limitations is "the least dangerous assumption" (Donnellan, 1984) that "asserts that in the absence of conclusive data, educational decisions ought to be based on assumptions which, if incorrect, will have the least dangerous effect on the student" (p. 142). We do not know what these students are capable of achieving because we do not frequently set high enough expectations for them. The least dangerous assumption for students with severe intellectual disabilities is to assume that they can learn academic skills, since assuming that students cannot achieve certain skills limits educational opportunities (Jorgensen, 2005). Even if we overestimate capabilities by assuming competence, students could still make progress and increase skill levels. Negative side effects of assuming incompetence may be segregated education, lack of age appropriateness, limited social opportunities, and limited planning for post-secondary education (Jorgensen, 2005), thereby potentially affecting college and/ or career readiness outcomes.

As mentioned before, educators and others have historically set low expectations for students with significant intellectual disabilities historically. Fortunately, this pattern is starting to change. Legislation that reflects a belief that students with significant disabilities can master academic content is evolving. The 1997 amendments of the Individuals with Disabilities Education Act (IDEA; P.L. 105-17) requires that all students' Individualized Education Programs (IEPs) include information about participation and progress in the general curriculum, specifically (a) how the student's disability affects participation and progress in the general curriculum, (b) goals that promote participation and progress in the general curriculum, and (c) supports and modifications that are necessary for participation and progress in the general curriculum. The goal of these amendments is to raise expectations and provide a challenging curriculum for all students with disabilities (Wehmeyer et al., 2001), including students with significant intellectual disabilities.

Teaching mathematics to students with significant intellectual disabilities is a new challenge for many educators (Browder, Trela, et al., 2012). As the field of education moves toward more inclusive education and holds students with significant intellectual disabilities accountable to grade-level standards, the next problem to be addressed is the lack of evidence-based practices for teaching mathematics applicable to this population.

Mathematics Instruction for Students with Significant Intellectual Disabilities

A seminal piece of work in the area of teaching mathematics to students with significant intellectual disabilities was the Browder, Spooner, Ahlgrim-Delzell, Harris, and Wakeman (2008) meta-analysis. Although previous reviews had been published on teaching mathematics to students with disabilities, the meta-analysis addressed a hole in the research in three ways: (a) it was the first review to look a interventions targeted specifically for students with significant cognitive disabilities, while other reviews looked across disability categories or looked a different disability category, i.e. learning disabilities; (b) it looked at the full range of mathematics content as opposed to reviewing a specific topic such as money; and (c) it was the first to evaluate the quality of research using evidence-based research practices.

The purpose of Browder's meta-analysis was to assess what mathematics content area standards and topics have been taught to students with significant cognitive disabilities in research literature, and to evaluate that research to identify best practices for teaching mathematics to this population of students. Sixty-eight studies were included in the meta-analysis, of which 54 studies were single subject design and 14 were group comparative design. Inclusion criteria specified (a) publication in a peer-reviewed journal or a dissertation, (b) English language, (c) publication between 1975 and 2005, (d) intervention that taught a mathematics skill, and (e) experimental or quasi-experimental design for group of single subject studies. The meta-analysis investigated what NCTM content area standards were represented in the research and what topics under each standard were present in the literature. The meta-analysis also explored the available evidence in the literature showing that students with significant cognitive disabilities can learn mathematics and which evidence-based practices had been successful in teaching mathematics to these students.

The vast majority of the studies reviewed were in the area of number (counting, calculation, and number matching) and measurement (almost exclusively money related skills). Of the 68 studies included in the meta-analysis, 37 addressed number and operations skills, 36 addressed measurement skills (money and time), two studies addressed algebra skills, two addressed geometry skills, and two studies addressed data analysis and probability. Clearly more studies are needed in the areas of algebra, geometry and data analysis and probability.

The meta-analysis provides strong evidence that not only can students with significant cognitive disabilities learn mathematics but that systematic instruction with a system of prompting is an effective instruction practice for teaching mathematics to students with significant intellectual disabilities. Time delay and a system of least prompts were shown have strong effects sizes in the meta-analysis. Systematic instruction involves using consistent prompting and feedback to teach to a clearly defined set of responses. Some evidence was present in the literature review that such instruction may be more effective with in vivo settings, using real-life, meaningful applications to teach skills.

For the purposes of the present study, a literature review was conducted to investigate research published on teaching mathematics to students with significant intellectual disabilities. This review examines the relevant literature published since the comprehensive meta-analysis (Browder et al., 2008). Articles were included if they met the same inclusion criteria as the meta-analysis with one exception: dissertations were not included in this literature review. One article was excluded because it did not have an adequate description of the participants resulting in a total of eight studies for the present review (Table 4).

One significant difference in the recent literature is a greater diversity of topics. Three studies addressed an array of topics in the number and operations content area standard. Skibo, Mims, and Spooner (2011) investigated the effect of response cards combined with systematic instruction in teaching number identification to three elementary students with severe intellectual disabilities or multiple disabilities. All students showed significant increases in number identification and maintenance of the skill with later probes. Browder, Jimenez, Spooner et al.

Table 4

Research on Teaching Mathematics to Students with Significant Intellectual Disabilities, 2006-2013

Study	*n*	Age Range	Math Skill(s)	Intervention
Browder, Jimenez, Spooner, et al. (2012)	7	Elementary school	Number identification, counting, composing sets, addition, comparing sets, patterning, linear measurement, and calendar skills	Systematic instruction Story-based instruction Embedded instruction
Browder, Jimenez, and Trela (2012)	4	Middle school	Solving simple one-step equations, drawing line segments, next dollar strategy, data collection	Systematic instruction Story-based instruction Graphic organizers
Browder, Trela, et al. (2012)	11	Middle school/ High school	Finding points on a plane, solving a simple linear equation, interpreting bar graphs, solving purchasing problems	Systematic instruction Story-based instruction Graphic organizers
Collins, Hager, and Galloway (2011)	3	Middle school	Order of operations (for calculating sales tax)	Systematic instruction Pairing core and functional skills
Horn, Schuster, and Collins (2006)	3	Middle school	Telling time	Response Cards
Jimenez, Browder, and Courtade (2008)	3	High school	Solving algebraic equations	Systematic Instruction Concrete representations
Jimenez and Kemmery (2013)	5	Elementary school	Counting with one-to-one correspondence, number identification, rote counting, composing sets, addition with sets, comparing sets, patterning, linear measurement, and calendar skills	Systematic instruction Story-based instruction Embedded instruction
(Skibo et al., 2011)	3	Elementary	Number identification	Response Cards Systematic Instruction

(2012) conducted a pilot study investigating the effect of a story-based curriculum combined with systematic instruction and embedded instruction in the general education curriculum for teaching early numeracy skills to seven elementary students with moderate and severe developmental disabilities. The skills addressed in this study included (a) number identification, (b) counting, (c) composing sets, (d) addition, (e) comparing sets, (f) patterning, (g) linear measurement, and (h) calendar skills. The results showed an increase in skills for all students. Jimenez and Kemmery (2013) replicated the previous study (Browder, Jimenez, Spooner et al., 2012) with five elementary students with moderate intellectual disabilities. Results were similar to those of the earlier study; all students had a significant increase in the early numeracy skills after the intervention.

Three studies targeted skills in the algebra content area standard. Jimenez et al. (2008) used concrete representations with systematic instruction to teach three high school students with moderate developmental disabilities to solve simple linear equations. All three students were able to master the skill. Browder, Jimenez, and Trela (2012) targeted multiple skills across multiple content area standards including solving simple one-step equations (algebra), drawing line segments (geometry), next dollar strategy (measurement), and data collection (data analysis and probability). They used systematic instruction, story-based instruction and graphic organizers to promote learning these skills among four middle school students with moderate intellectual disabilities. Results were mixed with some students showing significant increases in all skills and some students showing little to no improvement in most skills. Browder, Trela et al. (2012) also targeted multiple skills across multiple content area standards including finding points on a plane (geometry), solving a simple linear equation (algebra), interpreting bar graphs (data analysis and probability), and solving purchasing problems (measurement). In a manner similar to that of the previous study, they used a combination of systematic instruction, story-based instruction, and graphic organizers with 11 middle school and high school students with moderate or severe developmental disabilities. This study was different from the other studies reviewed because it used a group design. One group of students received the intervention to address the mathematics skills, and another group of students received the intervention to address a set of four science skills. As predicted, the math group showed a greater degree of improvement in math than the other students, while the science group showed much more improvement in science than the math group. The math group had a 27.9% mean increase in math scores with a standard deviation of 12.3. The highest increase was in geometry skills, followed by algebra, measurement, and data analysis and probability. The mean increase in the different content area standards ranged from 34.7% to 13.1%.

Two studies addressed geometry, both mentioned above. Browder, Jimenez, and Trela (2012) addressed drawing line segments as one of the four target skills in that study. Finding points on a plane was one of four target skills in the Browder, Trela et al. (2012) study. Two studies had skills in the data analysis and probability standard (both studies are described in the previous paragraph). In their studies, Browder, Jimenez, and Trela (2012) taught data collection and Browder, Trela et al. (2012) taught interpreting bar graphs.

Five studies addressed measurement skills, four of which have been reviewed earlier. Browder, Jimenez, Spooner et al. (2012) and Jimenez and Kemmery (2013) taught linear measurement, while Browder, Jimenez, and Trela (2012) taught the next dollar strategy, and Browder, Trela et al. (2012) taught solving purchasing problems. Horn et al. (2006) used response cards to teach telling time to three middle schools students with moderate and severe disabilities. All three students showed significant increases in telling time with the use of response cards.   The studies published since the meta-analysis Browder et al. (2008) show a broader range of topics. Although several studies still addressed number and operations, the topics were more diverse than just counting, number identification, and calculation. Several studies also addressed measurement skills and although the majority was focused on money and time, two studies investigated linear measurement. It was encouraging to observe that many studies addressed geometry, algebra and data analysis, topics that were minimally present in the literature prior to 2005. All but one of the studies used systematic instruction as part of an intervention package. Continued research in a wide array of mathematic skills is still needed.

The literature clearly indicates that more research is needed on teaching mathematics to students with significant intellectual disabilities, particularly in content area standards that are underrepresented such as algebra, geometry, and data analysis and probability. For the present study the researcher chose to address the lack of research in teaching geometry to students with significant intellectual disabilities. All children need a solid understanding of geometry. Geometry enables children to make sense of the spatial experiences that they encounter on a regular basis (Burger, 1985). Geometry develops mathematical reasoning abilities related to visual concepts (Burger, 1985). Not only is geometry critical to mathematics but is also essential to other subjects and the real world because:

> Geometry is the domain that (a) connects mathematics with the real, physical world (critical for numerous fields), (b) studies visual structures and patterns, (c) represents phenomena whose original is not physical or visual (e.g. graphs, networks), and (d) brings coherence to all of these, because they all use the same mathematical language for describing space. (Sarama & Clements, 2009, p. 201)

In short, geometry is essential to understanding the world around us.

Because very little research on teaching geometry to students with significant intellectual disabilities is available in the literature, this study examines a beginning geometry skill: shape recognition. Geometry skills start developing very early as young children make sense of the world around them. The NCTM (2000a) geometry standards for prekindergarten to second grade related to shape recognition state:

> In prekindergarten through grade 2 all children should-
>
> - Recognize, name, build draw, compare, and sort two- and three-dimensional shapes;
> - Describe attributes and parts of two- and three-dimensional shapes;
> - Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes. (p. 44)

Shape Recognition

Instruction in shape recognition needs to move away from rote memorization and toward assisting children to develop meaningful understandings of concepts (Battista, 2002). In order to foster an understanding of the properties of shapes, early geometry instruction must be designed with a deep awareness of how children develop an understanding of the concepts related to shape recognition. Therefore, the purpose of this literature review is to summarize the general mathematics education research on how young children learn shape recognition and the best instructional practices related to shape recognition. Note that the majority of this literature is not specific to children with disabilities, however, does include a summary of the scant research available on teaching shape recognition to students with disabilities.

Children's Development of Shape Recognition

Piaget. The work of Piaget and Inhelder (1967) established the early research on how children learn to identify shapes. Their work had two main principles. The first principle was that learning to identify shapes is not a passive process; instead children learn to understand shapes through active manipulation of the world around them. The second principle was that children learn to distinguish shapes based on topological properties first followed by Euclidean properties. Topological and Euclidean properties are defined by transformations. Topological properties are those properties that remain unchanged with transformations involving bending, stretching, shrinking, pulling and shape distortion (Martin, 1976b). Examples of topological properties include connectedness and openness (versus closedness) of curves. Euclidean properties are properties of shapes that remain unchanged with transformations that preserve the distance between pairs of points (Jahoda, Deregowski, & Sinha, 1974; Martin, 1976b), also known as rigid movements. This property includes reflections, rotations and translations. Examples of Euclidean properties include angle measure and length of sides.

Although the first principle on how children learn through active manipulation of their environments is still supported in the literature, much debate exists on the second principle. Several studies have explored Piaget and Inhelder's theory with mixed results, some in support of their theory (Darke, 1982), others refuting it (Jahoda et al., 1974; Martin, 1976a, 1976b). Several flaws in the research on children's preference for topological versus Euclidean research presented in the literature on shape recognition muddled the results. These flaws included, but were not limited to: (a) terms (topological properties and Euclidian properties) were either vaguely or inaccurately defined; (b) the type of task affected the results; (c) studies differed in the constructs they were assessing, (d) the testing procedures were inconsistent; and (e) children's lack of abilities in other areas (e.g., fine motor skills for drawing shapes) (Geeslin & Shar, 1979). Geeslin and Shar (1979) proposed an explanation for the mixed results in the research on Piaget and Inhelder's theory. They stated that a child's ability to identify a shape is not determined by an understanding of topological properties versus Euclidean properties; instead the level of distortion from one shape to another is the determinant. Their study showed strong support for the distortion theory. In Geeslin and Shar's (1979) study, children at different ages were cognizant of both topological and Euclidean properties. During the last several decades, research has moved away from Piaget and Inhelder's theory.

van Hiele. Presently the preponderance of research on how children learn to recognize shapes is focused on van Hiele's (1986) work. van Hiele proposed hierarchical levels of geometric thought that determined how children learned about shapes. Although van Hiele's levels apply to a variety of geometry concepts beyond shapes, levels reviewed here are described in terms of shape recognition. In the first level, the visual level, children think about shapes based on their appearances. In this level children look at shapes as a whole and do not look at the parts or properties that make up shapes (Hannibal, 1999). Most preschool children are either at or prior to the emergence of this level of geometric thought (Aslan & Arnas, 2007). In the second level, the analytic level, children think about shapes based on their properties such as number of sides and angle measure. Most third or fourth graders have reached this level of geometric thought (Aslan & Arnas, 2007). In the third level, the abstract level, children order properties and form less concrete definitions. In the fourth level, the deductive level, children reason with axioms and theorems. In the fifth level, the rigorous level, children can compare systems with different axioms. Many children do not achieve the fourth and fifth levels of geometric thought. Four assumptions guide van Hiele's levels of geometric thought (Clements, 2003): (a) development through these levels is a discontinuous process, each level is distinct and separate from other levels with observable differences in each level; (b) children go through these levels in a specific order and each subsequent level represents more advanced thought; (c) children must have a full understanding of one level before moving to the next level; and (d) each level has its own discourse and cognitive processes. The bulk of current research on shape recognition substantiates or expands on van Hiele's levels.

Assessment of van Hiele levels. Burger and Shaughnessy (1986) described several tasks used to determine what level of geometric thinking children have attained with respect to shapes. The four tasks are (a) drawing, (b) identifying and defining, (c) sorting, and (d) mystery shapes. During the drawing task, children drew one triangle, and then were asked to draw one that was different from the first. The request for different drawings continued until the activity lost its usefulness, at which time, children had to explain how the triangles were different and how many different triangles could be drawn. The identifying and defining task required children to label a sheet of quadrilaterals as a square, rectangle, parallelogram and rhombus. The children then had to explain their answers. In the sorting activity children were given a variety of triangles and were asked to sort them in some way based on how they were alike. Again children had to explain their reasoning. The mystery shape tasks gave children a series of clues about a shape and they had to determine what the shape was. From these observations, Burger and Shaughnessy (1986) created a list of indicators to determine what level children had achieved (Table 5). The indicators only went up to the deductive level because no students in the study reached the rigorous level. This limit is reasonable since the majority of learners, irrespective of age, do not reach the most advanced stage of geometric thought.

Due to the time-consuming nature of the Burger and Shaughnessy (1986) assessment, Jaime and Gutiérrez (1994) created a new assessment based on key thinking processes demonstrated in the van Hiele levels. The key thinking processes are identification, definition, classification and proof of properties. However, not all thinking processes are exhibited at each

Table 5

Indicators of van Hiele Levels of Thought (Burger & Shaughnessy, 1986)

Level of Geometric Thought	Indicators
Visual level	Imprecise properties Visual prototypes Irrelevant attributes Inconsistent classifications No use of properties
Analytic Level	Compares shapes based on properties Sorts shapes based on a single property Refers to shapes based on properties, not based solely on shape names Creates personal definitions for shapes No understanding of mathematical proof
Abstract Level	Forms complete definitions of shapes Can modify definitions of shapes Understands hierarchical nature of shape classes Sorts shapes on a variety of properties Uses if, then statements Uses informal deduction Confusion around axiom and theorem
Deductive Level	Rephrases unclear questions Frequently uses deductive logic Uses proofs to make decision Understands axiom and theorem

level. In level one, the visual level, children exhibit identification, definition and classification. Level two, the analytic level, involves all four processes. Level three, the abstract level, involves definition, classification and proof of properties. Level four, the deductive level, involves definition and proof of properties.

Gutiérrez, Jaime, and Fortuny (1991) proposed an alternate method of evaluating children's levels of geometric reasoning. This work refuted the assumption that children can be in only a single level at a time. The researchers believed that children's work might demonstrate more than one level of reasoning. Additionally, their research refuted the belief that each level is discrete, and that a child has either achieved or not achieved a given level. Acquisition of a level is a long, on-going process; therefore children should be assessed on degrees of acquisition. The first degree of acquisition is no acquisition, wherein children have no awareness of the given level of geometric thought. The second is low acquisition where children are aware of a level and try to use it but have little success and therefore retreat to the prior level. The third is intermediate acquisition, wherein children use the level more often but still lack mastery and will retreat to the lower levels in certain situations. The fourth is high acquisition, wherein children regularly use the level and only occasionally move to the lower levels when a mistake is made. The fifth and final degree of acquisition is complete acquisition, wherein children have mastered this level of thinking and have used it consistently with no challenges. When applying these degrees to the children in their study, the results supported the beliefs of Gutiérrez et al. that children could be developing more than one level at the same time, although the earlier levels were usually acquired more rapidly than later levels.

Extensions of van Hiele's levels. Beginning in 1986 and continuing to 2012, research has resulted in many modifications to van Hiele's original levels. Burger and Shaughnessy (1986) drew three conclusions about levels of geometric thinking. First, the levels involve developing both an understanding of concepts and reasoning strategies. Second, the levels are not as discrete as van Hiele described. Determining which level a child is in is often a challenge because ambiguity exists between any two levels. Third, children can move between levels depending on the tasks.

In their study of four- to seven-year-old children's abilities to classify shapes, Clements, Swaminathan, Hannibal, and Sarama (1999) proposed some changes to van Hiele's levels. First, a shortcoming of van Hiele's work is that he studied only older children, and so his work does not give an accurate picture of geometric thought in early childhood. Therefore, the first change

Clements et al. (1999) proposed was the addition of an earlier level to describe the abilities of young children. In this level, which they named the prerecognitive level, children have an awareness of shapes but cannot accurately distinguish between shapes. They are just starting to think about shapes and form schemas related to them. The second recommended change was a restructuring of the visual level. In this level children classify shapes by both visual characteristics and also by using some properties without analysis. Clements et al. (1999) recommended that this level be relabeled as the syncretic level, as children use both visual prototypes and a beginning understanding of properties to classify shapes. Aslan and Arnas (2007) conducted a follow-up study that affirmed these recommendations.

Hannibal (1999) developed several insights about factors affecting levels of geometric thought. First, classification was influenced by the stimuli presented. For example, if a variety of triangles were presented with other shapes that were quite different from triangles, such as squares and circles, children were more likely to correctly identify less common triangles (such as scalene triangles as opposed to equilateral triangles) than if the triangles were just presented with similar looking figures, such as pentagons and certain quadrilaterals that young children could describe as pointy. Also, consistency with categorization increases from ages four to six. Younger children were inconsistent with their classifications from one assessment to the next, while older children usually classified in a consistent manner. A second insight was that children tended to make more correct categorizations when asked to justify their reasoning. If children were asked to justify a shape classification that was incorrect, they would often correct themselves. Finally, at ages three to six, children are becoming increasingly aware of essential (such as number of sides) and nonessential (such as orientation) properties of shapes. These three insights supported and further clarified van Hiele's early stages of geometric thought.

More recently, Sinclair and Moss (2012) extended the van Hiele levels of geometric thought to levels of geometric discourse. These levels are centered around the process they dubbed "saming." Saming occurs when children who are learning shapes are capable of giving one name to a number of things (e.g., triangles). Young children struggle with this skill because they rely on visual prototypes when classifying shapes. If a figure varies too greatly from the prototype they will not identify it as the same. These levels of geometric discourse demonstrate that children shift from using visual prototypes to verbal definitions for shape recognition. The first level of geometric discourse is that of elementary discursive objects, wherein children work with concrete shapes. Saming is equivalent to matching. A shape name refers to one concrete object. Objects are the same if they are not transformed too far from the first shape. The second level is that of concrete discursive objects, wherein children are more flexible with the level of transformation that they will allow from one shape to another to consider it the same. A shape name does not refer to one object but to a group of objects. In the third level of geometric discourse, that of abstract objects, children no longer rely on visual transformations for saming of shapes. Instead children use verbal definitions to determine if two shapes are the same. Children do not make a smooth transition from one level to another, instead they can move between levels.

Learning trajectory for shapes. Sarama and Clements (2009) extended the work of van Hiele by developing a learning trajectory that describes the different stages children must go through in learning shapes. This learning trajectory addresses the first two of van Hiele's levels, wherein children are learning to identify shapes first based on visual characteristics and then based on properties (Table 6). The trajectory provides a sequential progression of the stages students go through when learning to recognize shapes. Shape recognition is first based on

Table 6

Shape Recognition Learning Trajectory (Sarama & Clements, 2009)

Stage	Description	Age
Same thing comparer	Determine whether two things in the environment are the same or different	0-2
Shape matcher–identical	Match familiar shapes (circle, square, typical triangle) that are identical	
Shape matcher–sizes	Match shapes with different sizes in the same orientation	
Shape matcher–orientation	Match shapes that are in different orientations	
Shape recognizer–typical	Identify circles, squares, and sometimes typical triangles	3
Shape matcher–more shapes	Learn to match more shapes (such as rectangles),	3-4
Shape recognizer–circles, squares, and triangles+	Recognize circles, squares, and triangles (both typical and less typical)	4
Part comparer	Match one part on each of two shapes and says it is the same.	
Constructor of shapes from parts–looks like	Use manipulatives to create a shape that looks like another shape	
Some attributes comparer	Distinguish differences between shapes not looking at entire shape	
Shape recognizer–all rectangles	Recognize a wide array of rectangles	4-5
Side recognizer stage	Learn that sides are distinct properties of shapes	
Most attributes comparer	Learn to look at the whole shape when comparing but still may miss some spatial relationships	
Corner recognizer stage	Learn that angles are distinct properties of shapes	
Shape recognizer–more shapes	Learn to recognize a wider variety of shapes (typical examples) such as hexagons and trapezoids	5
Shape identifier	Name wide variety of shapes avoiding common mistakes	6
Angle recognizer	Learn about angles in contexts beyond corners	7
Parts of shapes identifier	Identify shapes based on properties.	
Congruence determiner	Identify congruent shapes by looking at all parts	
Congruence superposer	Decide congruency by placing a shape on top of another	
Constructor of shapes from part–exact	Use manipulatives to accurately create a model of a shape	
Angle representer	Represent angles in a variety of contexts	8+
Congruence representer	Describe properties in transformations	
Shape class identifier	Classify shapes based on attributes, not solely properties	
Shape property identifier	Classify shapes based solely on properties	
Shape property class identifier	Sort shapes hierarchically based on properties	
Angle synthesizer	Learn multiple meanings of angles	

visual characteristics and later based on properties of shapes, and is a lengthy process that starts when the child is an infant and continues until the child is at least eight.

Best Practices for Teaching Shape Recognition

van Hiele stated that instruction has a greater effect than development on progression through levels of thought. van Hiele believed that students need a variety of geometric experiences (Clements & Sarama, 2000; Crowley, 1987). Because of this he proposed five phases of learning essential for progress through the levels (van Hiele, 1986). The first phase is inquiry and information, during which the teacher and children have an introductory talk about the content to be studied. In the second phase, directed orientation, the children delve deeper into the content with teacher-created activities. Explication is the third phase, in which children discuss with each other their developing understandings of the content. Phase four is free orientation, when children engage in more difficult, open-ended problems. Finally phase five, integration, involves a review and synthesis of the content. Although little expansion of these phases of learning can be found in the research, many recommendations of best practice for teaching shape recognition are based on the research around levels of geometric thought.

Many educators have underestimated the geometric abilities of young children (Clements et al., 1999); however, young children enter school with a developing understanding of shapes and teachers need to extend the understandings that children already possess (Clements & Sarama, 2000). The recommendations of best practice, which will be discussed further, fall into two main categories: focusing on the properties and creating a geometry-rich environment.

Focus on properties. Too often shape recognition is taught through examples, by showing children pictures of certain shapes until they learn to associate the name with a figure. This approach may work for shapes that do not have many variations, such as a square and a circle, but can lead to problems with shapes that can vary a great deal in form, such as rectangles and triangles (Clements & Sarama, 2000). Recognition by example can lead children to limit a class of shapes; for example the child may believe that only equilateral triangles are triangles and fail to identify an isosceles triangle as a triangle. Teachers need to guide children attend to the necessary properties that make up a class of shapes. When children are classifying shapes teachers should ask them to describe the criteria for classification. Initially children's answers will be visual but if they are encouraged to discuss the properties of each group of shapes (Clements et al., 1999), they will then be able to describe the essential and the nonessential attributes of a shape (Hannibal, 1999). For example, an essential attribute of a triangle is that it has three sides while a nonessential attribute is the orientation (i.e., not all triangles are flat on the bottom and have a pointy top). Number of sides is always an essential attribute for shapes, while size of angles, congruency of sides, aspect ratio and symmetry are sometimes essential attributes depending on the shape. Size and orientation are never essential attributes. When helping children focus on properties, some important considerations include avoiding misconceptions and providing multiple examples.

Avoid common misconceptions. Many misconceptions children have about shapes, such as the mistaken idea that a square is not a rectangle, result from the shape instruction they receive. Such misconceptions can be avoided in multiple ways. One way is for teachers to use accurate geometric language (Brown, 2009; Hannibal, 1999). When talking to children about shapes, teachers should use words like corners, lines and angles. Teachers' instruction can be more accurate by addressing frequent misunderstandings that children have about shapes, e.g. all diamonds are squares, a square is not a rectangle, and two triangles put together make a square (Clements & Sarama, 2000). Hannibal (1999) created guidelines for presenting triangles and rectangles to children. When discussing triangles, teachers should emphasize that the shape is made of the three connected lines; and they should show children how size, orientation, aspect ratio and angle size can vary. The teacher must avoid describing a triangle a having a flat bottom and a pointed top. When discussing rectangles, emphasis should be given to the concept that a rectangle has four connected sides, with opposite sides being equal, and four right angles. Children can be taught to check for right angles using the corner of a piece of paper. Rectangles should not be describe as having two long sides and two short sides. Neither triangles nor rectangles should be compared to three-dimensional shapes. For example: an ice cream *cone* is not a triangle.

Provide multiple examples. Curriculum and classroom materials usually provide a narrow selection of shapes (Clements & Sarama, 2000). Rectangles tend to look like doors and triangles tend to be equilateral, which does not give children a complete understanding of shape classes (Clements et al., 1999). Teachers need to provide examples of a wide variety of shapes in a variety of positions (Clements, Copple, & Hyson, 2010) with different sizes, materials and colors (Burger, 1985; Clements & Sarama, 2000). Teachers should provide examples and non-examples of each shape (Clements & Sarama, 2000).

Geometry-rich environments. Classrooms need to be organized so that children have opportunities to actively engage in meaningful geometry activities, which leads to exploration and development of concepts of shapes. Brown (2009) stated that:

> Early childhood environments should provide opportunities for children to explore materials, engage in activities, and work in collaboration with peers and teachers to construct their own knowledge of the world around them. Teachers can organize the environment to encourage children to explore shapes and their characteristics by

providing pictures of various typical and atypical shapes in different sizes and orientations throughout the classroom. (p. 476)

Aspects of geometry-rich environments include: (a) providing manipulatives, visuals and models; (b) creating engaging activities; and (c) having children actively manipulate shapes with computers.

Manipulatives, visuals, and models. As discussed earlier, children need to have multiple examples of shapes to help understand shape categories and properties. Use of a variety of manipulatives, visuals and models can help provide those examples and increase children's ability to understand shape properties and concepts (Clements & Battista, 1986, 1992).

Engaging activities. A variety of activities that match children's levels of geometric thought should be created to help children develop an understanding of shapes. Clements and Sarama (2000) provided guidelines and sample activities for young children at the prerecognitive level and the visual level. Children at the prerecognitive level should be exploring shapes in the environment such as sorting shapes, identifying shapes around them, and constructing shapes. Children in the visual level should have the opportunity to measure and manipulate shapes to develop property understanding such as determining why a figure does or does not belong in a shape category, folding shapes to explore symmetry, and using a computer to create and manipulate shapes. The goal of such activities should be to help children develop the skills they need to progress towards the next level of geometric thought.

Use of computers. Multiple studies (Clements, 2003; Sinclair & Moss, 2012) have explored using computer software programs, such as *Sketchpad* and *Logo*, to have children manipulate and construct shapes in order to help them develop an understanding of shape properties. Computers expand the opportunities children have to work with shapes (Clements &

Sarama, 2000). Computer software addresses the shortcomings of diagrams and models where children cannot dynamically change the shapes.

Sinclair and Moss (2012) explored the effectiveness of *Sketchpad* as a dynamic geometry environment (DGE) to help children progress from one level of geometric thought to another. *Sketchpad* allows children to transform shapes by dragging a corner of a given figure while still maintaining the properties of that shape. Sinclair and Moss (2012) described three advantages of the *Sketchpad*: (a) the corners are marked with circles making them more noticeable to children; (b) the shapes can take up a large or small amount of space, which is not the case with most curricular materials; and (c) a shape can be dragged to show the different orientations, aspect ratios and angle sizes the shape can take on. A shortcoming of this study is that, although the tool was shown to help children progress from visual to the analytic with the shape (triangles) they were studying, the change in geometric thought was not generalized to other shapes.

Logo is by far the most researched geometry software (Clements, 2003). *Logo* allows children to type in basic commands to direct a turtle in drawing shapes. *Logo* has been shown to help children progress to more advanced van Hiele levels by helping them explore shape properties, motivating them in learning shapes, and helping them to express mathematical ideas (Clements, 2003).

Shape Recognition and Students with Disabilities

Despite the importance of all students developing an understanding of geometry, including students with disabilities, very little research is available for teachers to rely upon. Berla and Butterfield (1977) were interested in teaching shape recognition to elementary students who were blind. Their study focused on students matching shapes of countries instead of geometric figures; however, their findings are relevant to shape recognition. They found that students who were blind had three deficits that affect shape recognition: (a) they did not have a systematic approach to tactually exploring figures, (b) they struggled with tracing lines, and (c) they had not developed cognitive strategies to aid with tactual exploration (Berla & Butterfield, 1977). In their study they taught students to look for three key features in each shape when feeling the shape, which resulted in improved shape matching.

Hitchcock and Noonan (2000) also addressed shape matching but they worked with preschoolers with early childhood learning impairments. They used constant time delay to support students in matching shapes. Constant time delay is when a stimulus is presented (e.g., a shape) and then the teacher waits a set amount of time, such as four seconds, before responding (e.g., prompting or correcting). The study investigated the effectiveness of computer-assisted instruction as compared to teacher-assisted instruction with manipulatives and showed that when using an adapted alternating treatment design computer-assisted instruction was as effective as or more effective than teacher-assisted instruction in improving shape matching in preschool students with disabilities.

In their study with students who had intellectual disabilities aged 11 to 20 years, Mackay, Soraci, Carlin, Dennis, and Strawbridge (2002) showed that aspects of how objects are presented can facilitate shape matching. They used an increased number of identical non-matches to support students in matching objects. Students were shown a shape followed by an array of nine objects, wherein one figure was the match to the stimulus and the other eight figures were identical objects that did not match the stimulus. They believed that increasing the number of identical non-matches would help students find the figure that matched the stimulus. When students were successful in matching the correct shapes, the number of identical non-matches was decreased one at a time until students were successful at matching figures from an array of two: a match and a non-match. Displays were presented on the computer and when a correct match was made stars would come across the screen to indicate the correct response. The results showed this is an effective and efficient method for teaching shape matching to students with intellectual disabilities.

Herberg, McLaughlin, Derby, and Gilbert (2011) investigated the use of direct instruction flashcards in teaching shape recognition to two students with developmental delays. This procedure involved showing one student a flashcard with a shape, waiting several seconds for the student to respond, and then providing feedback. This technique was effective in teaching one student to identify shapes as well as supporting the other student in distinguishing between a triangle and a non-triangle.

Conclusion

Early developmental research on shape recognition was founded on Piaget and Inhelder's (1967) work, which stated that children first develop an understanding of topological properties and then develop an understanding of Euclidean properties in geometry. This proposition has since been refuted and later work on shape recognition has centered around van Hiele's (1986) levels of geometric thought. Current research still relies on his work, although modifications to his levels have been made, such as adding an earlier level to reflect the understanding of young children and renaming and redefining the visual level. Sarama and Clements (2009) extended van Hiele's levels of geometric thought to create a 27 step learning trajectory that students move through sequentially as they learn to recognize shapes.

Many best practices for teaching shape recognition have emerged from the research. These recommendations fall under two categories: focusing on properties and creating a geometry-rich environment. Teachers need to help students develop an understanding of the properties of shapes by avoiding common misconceptions and providing multiple examples of shapes with different materials, orientations, and aspect ratios. Teachers need to provide geometry-rich environments by using manipulatives, visuals, and models; creating engaging activities where children actively explore shapes; and having students use computer software that allows them to manipulate shapes.

The literature contains scant research on teaching shape recognition to students with disabilities. Explicit strategy instruction, constant time delay, computer-assisted instruction, manipulation of object presentation, and direct instruction flashcards have been shown to be effective methods for teaching aspects of shape recognition to some students with disabilities. Several weaknesses are evident in the research literature on teaching shape recognition to students with disabilities in addition to the obviously small number of studies. One weakness is that the current research has only reported investigations of a small number of disability categories and usually no clear description of those disabilities. Terms such as "developmental delay" can be applied to a wide variety of students with very different strengths and support needs. Another weakness of these studies is that they employed very few, if any, of the best practices described above. While a few studies used computer software, they presented static images of the shapes instead of allowing student to manipulate shapes in order to facilitate an understanding of shape properties. None of the studies emphasized properties. The environments in which the studies were conducted cannot be described as geometry-rich. Although these studies may have resulted in only rote matching or identifying shapes, they still promoted an understanding of shape properties needed for more advanced shape activities.

Significant research on how children develop shape recognition skills exists; however, more studies are needed on best practices for teaching shape, especially for teaching shape recognition to students with disabilities. More research is needed specifically addressing a wider variety of disabilities such as students with learning disabilities and students with multiple disabilities. Such studies will need to integrate best practices in teaching students with disabilities with best practices in teaching shape recognition.

Conceptual Model

The present researcher hypothesized that combining best practices in teaching mathematics to students with significant intellectual disabilities with evidence-based practices for teaching shape recognition would be effective for teaching shape recognition to students with significant intellectual disabilities (Figure 1). Best practices in this case include systematic instruction with a system of prompts (either time delay or a system of least prompts). Evidence-based practices for teaching shape recognition include: (a) avoiding common misconceptions; (b) using multiple examples; (c) creating engaging activities; (d) using manipulatives, visuals, and models; and (e) using computers. In order to support inclusion and access to the general education curriculum, instructional methods need to combine best practices in teaching students with significant intellectual disabilities with evidence-based practices in the given subject area. Failure to use best practices in teaching students with significant intellectual disabilities will result in not meeting individual student's needs. Failure to use evidence-based practice in the given subject area will result in segregation and/or lack of access to the general education curriculum. The purpose of this study was to investigate effective teaching practices in an area of mathematics that not only meets the support needs of students with intellectual disabilities, but also supports inclusion and access to the general curriculum.

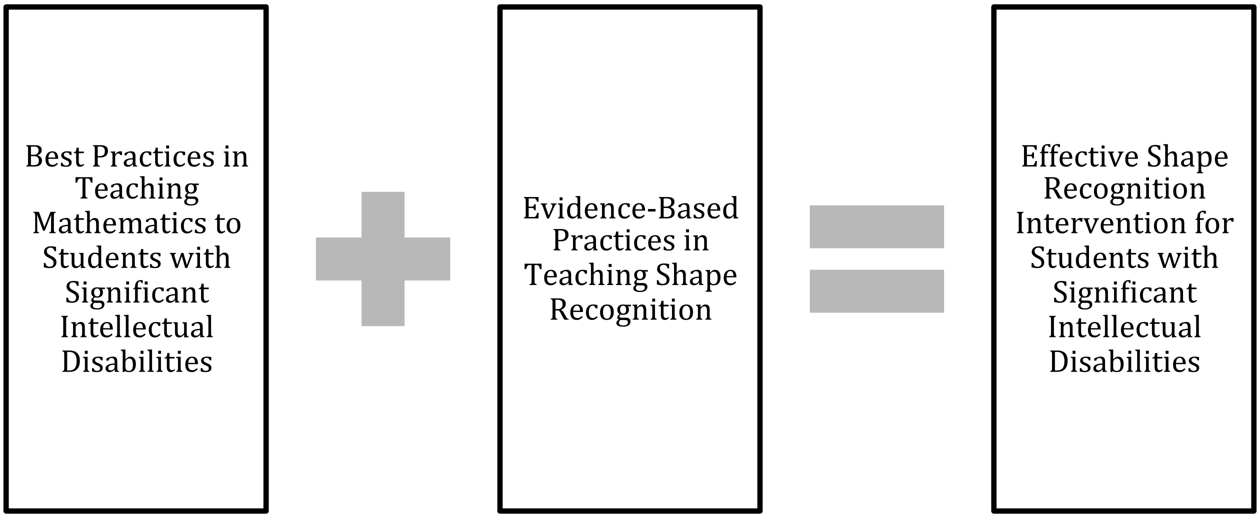


Figure 1. Conceptual model for teaching shape recognition to students with significant intellectual disabilities

Purpose of the Study and Research Question

The purpose of this study was to investigate teaching mathematics to students with significant intellectual disabilities in a domain where currently very little research is available, i.e., geometry. Specifically, this study investigates the beginning geometry skill of shape recognition. To meet the significant support needs of this population while promoting access to the general curriculum and inclusion, the study combines best practices in mathematics instruction for students with significant disabilities with evidence-based practices in teaching shape recognition. The intent of the study is to develop a preliminary line of research that will lead to a further understanding of effective geometry instruction for students with significant intellectual disabilities. The research question is: Does combining evidence-based practices in shape recognition instruction with best practices in teaching mathematics to students with significant intellectual disabilities produce effective outcomes for teaching shape recognition to those students?

CHAPTER II—METHODS

Participants and Setting

This study was conducted at a small, urban elementary school in the Midwest. All sessions took place during regular school hours. The researcher, who is certified to teach both elementary education and special education and has many years of experience working with students with significant disabilities, conducted all interventions. Lessons took place in either a teacher workroom or in a boardroom, were conducted individually with students every school day, and lasted approximately 30 minutes. To improve the internal validity of this study, interventions were in place to control for other instruction that may have affected the results. None of the participants received instruction related to shapes beyond the intervention.

Four students, who all received special education services in a self- contained classroom with a functional life skills instructional program in which they received significant adult support, participated in this study. All of the students were included with their same age peers for homeroom, lunch, recess, and specials (with the exception of adaptive physical education), but they spent the majority of their day in the self-contained classroom. The researcher together with two teachers working in functional life skills classrooms selected the participants. The teachers nominated students based on the researcher-determined eligibility criteria: (a) diagnosis of an intellectual disability with extensive support needs in both academic and functional skills, (b) participation in an alternate assessment based on alternate achievement standards for statewide testing, (c) enrollment in elementary school, (d) adequate vision and hearing required to participate in activities, and (e) lack of shape matching and shape recognition skills. Two other students were nominated but were not included because they did not meet all of the inclusion criteria. One student who was not included had already mastered the shape matching skills and had some shape recognition skills. The second student nominated was not included due to a significant visual impairment and hearing loss. Institutional Review Board (IRB) approval was granted prior to beginning this study. Parental consent and student assent were obtained prior to collecting data (Appendix A). Table 7 provides additional information about the student demographics. A pseudonym was given to each participant to protect confidentiality.

Miguel was a bilingual, nonverbal communicator who used a picture exchange system and gestures to express his wants and needs. He had significant delays in language comprehension but could follow classroom routines and some one-step commands. He could not make a selection between two preferred items. Miguel greeted adults and peers with handshakes or high fives but did not continue interaction beyond the initial greeting. He could play interactively with adults but did not play interactively with peers, requiring structured play experiences. Miguel needed assistance with personal hygiene such as toileting. He could inconsistently recognize his name. He was making slow progress toward matching colors and was unable to match or recognize any numbers. Miguel needed structured, hands-on learning activities to learn new information.

Carter was a nonverbal communicator with limited expressive language who struggled to communicate his wants and needs through the use of a communication book and nonverbal form of communication, i.e. gestures and facial expressions. His strengths were vocabulary comprehension and following one-step directions. He greeted adults and peers with high fives when requested. He had difficulty playing appropriately with peers and sharing toys. Carter was able to feed himself, wash his hands and assist with dressing, but he needed assistance with some personal hygiene such as toileting. Carter could recognize approximately five letters and identify his name from an array of 5-7 names. He could complete simple shape sorters and puzzles

Table 7

Student Demographics

Student	Gender/ Ethnicity	Age/ Grade	Disability	Intellectual Functioning	Adaptive Functioning
Miguel	M/ Hispanic	7/ 2^{nd}	Intellectual Disability	Bayley: Raw Score = 59 Developmental Age = 22 months	Vineland = 40
Carter	M/ Two races	8/ 2^{nd}	Intellectual Disability	Bayley: Raw Score = 65 Developmental Age = 19 months	Vineland = 36
Kyle	M/ Black	8/ 3^{rd}	Intellectual Disability	Leiter: Full Scale IQ = 44	Vineland = 46
Nathan	M/ Black	10/ 5^{th}	Intellectual Disability	Leiter: Full Scale IQ = 44	Vineland = 50

Notes: Bayley = Bayley Scale of Infant and Toddler Development; Leiter = The Leiter International Performance Scale Revised; Vineland = The Vineland Adaptive Behavior Scales, 2^{nd} Edition (Composite Score)

independently and could match letters and colors but was unable to identify shapes and numbers.

Kyle could communicate verbally with one- and two-word utterances but had low intelligibility with unfamiliar communication partners. He was learning to use the Picture Exchange Communication System (PECS) to communicate his wants and needs but predominantly relied on gestures. He had delays in the area of vocabulary and responding to yes/ no questions. He learned new vocabulary after frequent repetitions and practice. Kyle could recognize, greet, and say the name of familiar peers and adults but had difficulty initiating play with peers. Kyle needed assistance with eating with utensils, with washing his hands, and with dressing. He had difficulty with all functional academic and activities and required assistance in completing activities.

Nathan could communicate verbally with moderate intelligibility issues. He could identify objectives or photos from an array of three when presented with a function or feature of the object. He interacted with the adults who worked most frequently with him but had little positive peer interaction and some issues with aggression towards peers. He required one-on-one assistance with activities of daily life. Nathan could recognize five letters, hold a pencil and scribble lines, and count up to 10 with verbal prompting.

Target Skills

The intention of this study was to address four target skills related to shape matching and shape recognition: (a) matching shapes that are identical, (b) matching shapes that are different sizes, (c) matching shapes that have different orientations, and (d) shape recognition. The choice and order of these targets skills was based on the Sarama and Clements (2009) learning trajectory discussed earlier. All four of the target skills were to be applied to familiar shapes: circles, squares, and typical triangles. The term "familiar shapes" comes from the learning trajectory (Sarama & Clements, 2009) to describe these three basic shapes, although they may not be familiar to students who have yet to learn them. Again, the decision to use circles, squares, and typical triangles was based on the learning trajectory discussed earlier (Sarama & Clements, 2009). These four target skills were the foundation for all assessments and activities throughout the study.

Research Design

This study utilized a small sample, interrupted time series (single case) multiple-probe design across behaviors within four participants. The four behaviors were the four target skills of (a) matching shapes that are identical, (b) matching shapes that are different sizes, (c) matching shapes that have different orientations, and (d) shape recognition. This design is frequently used for measuring the effect of an independent variable on a chain sequence of behaviors (Horner & Baer, 1978) such as learning progression of shape recognition. A multiple-probe design was chosen because it can evaluate (a) the initial achievement of each target skill, (b) the level of achievement of each target skill after multiple opportunities to perform the skill before instruction, (c) the level of achievement of each target skill after instruction, and (d) the level of achievement of sequential skills after achievement has been mastered in earlier skills (Horner & Baer, 1978). The three reasons to choose a multiple-probe design over a multiple-baseline design are if (a) a continuous baseline would be impractical, (b) repeated testing may prove reactive, and (c) a strong assumption can be made that the baseline would be stable (Horner & Baer, 1978). All three of these reasons applied to this study. A continuous baseline for this study would not be feasible due to the time required and concerns that continuous assessment might affect student performance. Finally, a strong assumption could be made that the students' baseline performances would be stable because the target skills are progressive; a student must master the first skill before showing improvement in later skills.

The following four-stage procedure was planned for this study, with each stage addressing one target skill. The first stage was matching identical shapes. First for each student a baseline was established, lasting at least three sessions until a stable baseline had been achieved. During all baseline assessments, students received no instruction in shape recognition. Next, the researcher delivered the intervention through individualized instruction, that is, daily lessons conducted around matching identical shapes, and continued the instruction until each student reached a criterion of at least 80% correct matching of identical shapes on three consecutive sessions. Throughout the intervention, random probes were taken on the other three target skills. The next three stages (matching shapes with different sizes, matching shape in different orientations, and shape recognition) followed the same procedures with random probes conducted on the other three target skills, including skills that had already been taught. See Figure 2 for sample data illustrating the design.

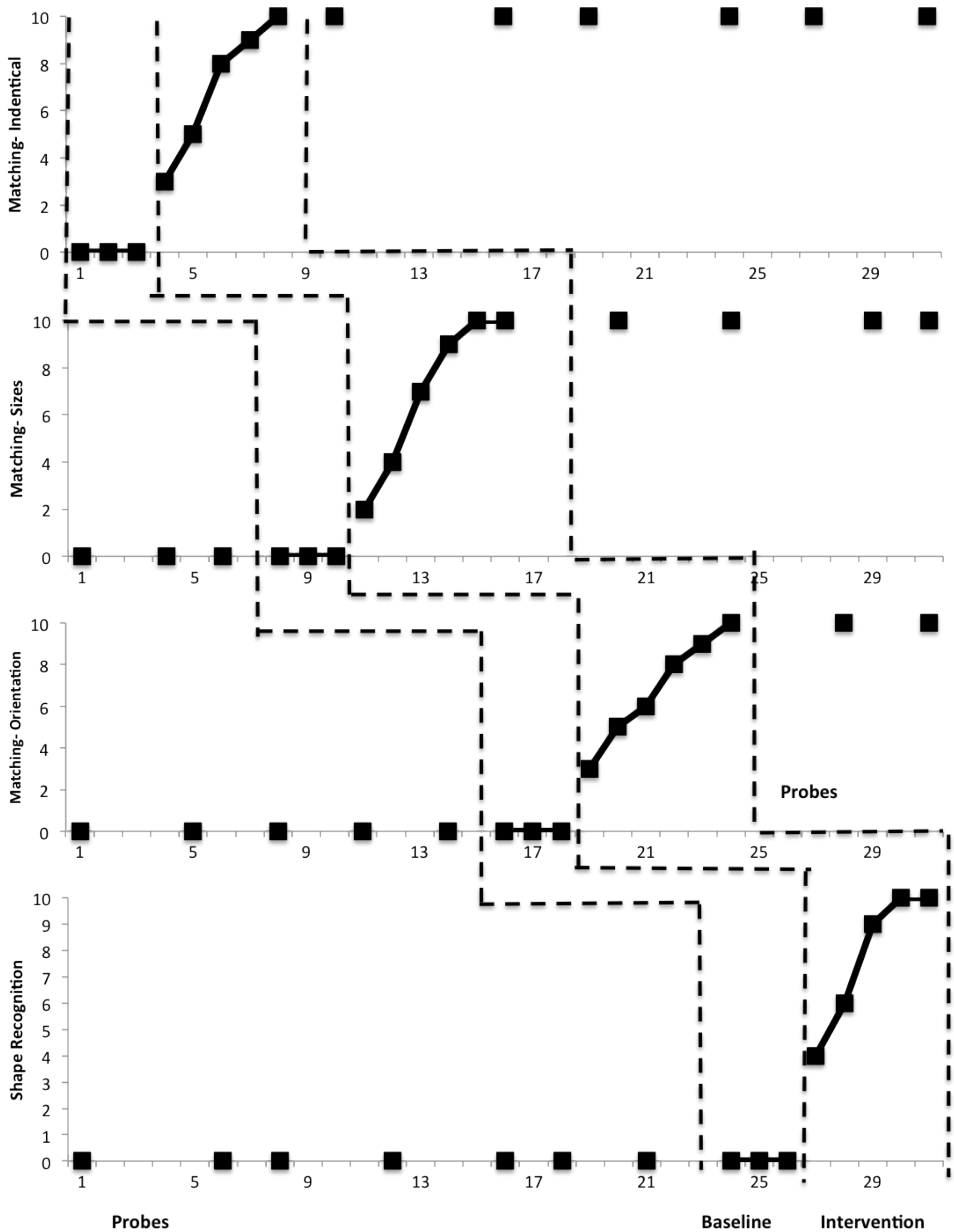


Figure 2

Interrupted time series (single case) multiple-probe design across behaviors sample data

Intervention

A multi-component intervention was used that included (a) evidence-based practices in teaching shape recognition, (b) systematic instruction with a system of least prompts, and (c) communication supports.

Evidence-Based Practices in Teaching Shape Recognition

The Building Blocks curriculum (Clements & Sarama, 2013) was used for planning interventions because it reflects the general mathematics education literature on best practices in teaching shape recognition. As discussed above, operationally defining those best practices includes: (a) avoiding common misconceptions; (b) using multiple examples; (c) creating engaging activities; (d) using manipulatives, visuals, and models; and (e) using computers. The Building Blocks curriculum meets all of the elements of best practice for teaching shape recognition described above; in fact, much of the research used to define the elements of best practice was conducted by the authors of the curriculum, Douglas H. Clements and Julie Sarama. Building Blocks is a research-based pre-kindergarten mathematics curriculum that was developed under a grant from the National Science Foundation. The curriculum is centered on engaging activities using manipulatives, visuals, and models with computer software activities that supplement the curriculum. Furthermore, the curriculum focuses on the properties of shapes while using multiple examples of shapes and avoiding common misconceptions.

Although a pre-kindergarten curriculum, Building Blocks is appropriate for the elementary students in this study because they had not mastered basic shape recognition skills. Modifications were made to make the curriculum appropriate for students with significant intellectual disabilities. The curriculum contains 30 weekly lesson plans, each with a set of big ideas and objectives. For this study, the researcher looked only at lesson plans and objectives that pertained to the target skills of shape matching and shape recognition, which consisted of 14 weekly lessons plans. Weekly lesson plans always included multiple objectives and big ideas, sometimes in the same content area standard, such as shape recognition and shape composition, and sometimes in different content area standards, such as shape matching and counting. Because of the intense support needs and difficulty mastering new content, this population often needs to be taught skills in smaller chunks with more opportunity for practice and repetition (Browder, Jimenez, Spooner, et al., 2012). With this need in mind, the researcher decided that lesson plans would focus on one target skill at a time (either one of the shape matching skills or shape recognition).

To accomplish this goal, lists of all activities in the 14 weekly lesson plans that included shape matching (Table 8) and shape recognition (Table 9) were created. The descriptions in Tables 8 and 9 provide a general overview of each activity as modified for students with significant intellectual disabilities. To reflect best practice a variety of materials, manipulatives, and examples were used. The curriculum does not distinguish between the three levels of shape matching (identical shapes, shapes with different sizes, shapes in different orientations). For this study the same activities would be used for all three target skills, but different manipulatives would be used at the different levels. For example, only identical shapes would be used for the shape matching activity in the first target skill, and shape manipulatives with different sizes would be used in the shape matching activity for the second target skill. One activity was not selected for this study because of the doubtful feasibility of success with the given population of students. That activity was a “feely box” where the teacher secretly puts a shape into a large box with a hole large enough for the student’s hand. The student feels the shape but does not pull it out and then points to the matching shape on the display.

Table 8

Shape Matching Activities Adapted from Building Blocks

Activity	Description
Match Shapes	This activity can be done two ways: (1) A student selects one shape from a bag (or is given a shape if the student is unable to make a selection from the bag). The student is then shown three other shapes and is asked to, "Choose the same shape." (2) A student selects two shapes from a bag and uses picture symbols to indicate if the two shapes are "same" or "different."
Mystery Pictures	On the computer, students are asked to match shapes to an outline. When completed a mystery shape is revealed. This activity can only be done by matching shapes with different orientations and matching shapes in different orientations target skill because none of the pictures have identical shapes.
Shapes Around Us	Students are given a shape and then asked find the same shape in real world examples. This can be done by looking at pictures of real world shapes such as shapes in a house or shapes on a car or by walking around the classroom or school.
Shape Flip Book	A flipbook is created with three panels of shapes. Students are shown two panels at a time and are asked if those are the same shape or different shapes. If the shapes are different the student flips that panel to the next shape. This continues until all three panels are the same. The researcher then reiterates that all three panels are the same shape. They can then move on to another shape and continue the process.
Memory Geometry	This activity is based on the classic board game in which children turn over two cards trying to match shapes. A student selects two cards, flips them over, and then is asked if the shapes are the same are different. If the shapes are the same, the match is set aside. If the shapes are different, the cards are flipped back over. Because many students always select cards in certain positions, the researcher moves around the cards between turns. When all matches are found, the researcher reiterates that they are the same shape.

Lesson plans were created that had the same structure as the weekly lesson plans in the Building Block (Clements & Sarama, 2013) curriculum but only addressed one skill at time. Weekly lesson plans were structured into five daily lesson plans that followed the same order: an introductory activity, work time (two different activities), a reflection (Table 10), and assessment. Activities in the curriculum varied from day to day, but the structure remained the same. For this study, only one work activity was performed each day because of time constraints and to give students more opportunities to practice a given activity. Also, considerable changes

Table 9

Shape Recognition Activities Adapted from Building Blocks

Activity	Description
Match and Name Shapes	A student selects one shape from a bag. The student is then shown three other shapes and is asked to, "Choose the same shape." Once the student has a match, the student is asked to identify the shape.
Shapes Around Us	A shape is introduced and described to the student (such as a circle). The student finds the shape in real world examples. Once there is a match, the student is asked to label the shape.
Is it or Not?	The researcher draws or shows a shape. The student is asked to label it. Then the researcher draws a similar shape (such as an ellipse or an oval if you showed a circle) and the student uses picture symbols to indicate if the two shapes are "same" or "different." This is done with several examples.
Shape Show	The researcher shows and names a specific shape, then describes that shape as she walks her fingers around the perimeter. The student is questioned about the properties of the shape (i.e. how many sides does it have?). The student is asked to identify an object that is the same shape.
Mystery Pictures	On the computer, students are asked to match shapes to an outline. When completed a mystery shape is revealed. With each match, the shape is verbally named.
Shape Flip Book	A flipbook is created with three panels of shapes. Students are shown two panels at a time and are asked if those are the same shape or different shapes. If the shapes are different the student flips that panel to the next shape. This continues until all three panels are the same. The researcher then reiterates that all three panels are the same shape. Students then label the shape. To do this, two or three shape names printed on cards are presented, the names are read aloud, and the student selects the correct label. The researcher then reiterates the shape name, "This is a (shape name)." They can then move on to another shape and continue the process.
Shape Songs	Teacher leads students in singing songs that describe the properties of a shape.
I Spy	Describe an object in the classroom, for example, "I spy something with three sides." Picture symbols are used to show the properties. Students are then given to or three options and have to select the correct shape. For example, "Is it the slice of pizza or is it the book?"
Shape Steps	A variety of large shapes are placed on the floor (either in chalk, tape, or laminated paper). A diagram of a specific shape is shown. Students are asked to step on just that shape. Repeat with other shapes.
Guess my Rule	Students watch as the researcher sorts shapes into two piles. They are then asked guess the rule used to sort by giving them two or three options (i.e. circles vs. squares, four-sided shapes versus other shapes).
Mr. Mix Up	Explain to students that Mr. Mix Up always gets things wrong. Show how Mr. Mix Up labels a shape incorrectly and then have the students correct it. "Mr. Mix Up said this is a square. It is not a square. Is it a circle or a triangle?"

Table 10

Sample Reflection Questions from Building Blocks

Shape Matching	How do you know two shapes are the same? How do you know whether two shapes are exactly the same size and shape? How do you match shapes? How can you find shapes that match each other?
Shape Recognition	How do you know some shapes are (insert shape) and some are not? Show an shape from the Big Book Building Shapes and ask, "Why is this shape not a (insert shape)?" How do you know for sure a shape is a (insert shape)? How is a (insert shape) different from a (insert shape)? Is this a (insert shape)? Why or why not? How are these shapes different? What pictures did you make on the computer? What shapes are in the pictures? What shapes did you find today? What is this? How do you know?

were made to the reflection questions, and picture symbols were used to make them more accessible to students. Two sample lesson plans are provided in Table 11.

Materials. Some materials used in this study were provided with the Building Blocks curriculum, some materials were purchased separately, and some materials were made by the researcher. A large and diverse set of materials and manipulatives came with Building Blocks, only a portion of which were used for this study, specifically (a) the *Building Shapes* big book, (b) a foam shape set, (c) attribute blocks, and (d) pattern blocks. The *Building Shapes* big book featured one at a time the following shapes: lines, rectangles, squares, rhombi, trapezoids, triangles, curves, circles, and ovals. Each shape is described, and the text is accompanied by drawings of various examples of the shape and a photograph of a building that includes the shape. The foam shape set included a variety of types of triangles, quadrilaterals, pentagons, and hexagons in blue or yellow. The attribute blocks consisted of a set of 60 plastic shapes,

Table 11

Sample Lesson Plans

Target Skill	Introductory Activity	Work Time	Reflection
Shape Matching (Identical)	Shapes Around Us (Magnetic Shapes Book)	Shape Matching (Use plastic manipulatives- show me the same shapes)	When matching shapes, what do you look at?
Shape Recognition	I Spy	Shape Flip Book	How do you know this is a triangle?

specifically triangles, rectangles, squares, circles, and hexagons. Half of the manipulatives for each type of shape were large (approximate height 2 ½ inches) and half were small (approximate height of 1 ½ inches). All types of shapes came in three colors (i.e., red, yellow, and blue) and varied in thickness. The pattern blocks were plastic blocks in six colors (i.e., red, orange, yellow, green, blue and beige) and six shapes (i.e., triangles, squares, trapezoids, two types of rhombuses, and hexagons). Each color was associated with one shape and the shapes varied in size, the largest of which was 1 ¾ inches.

Purchased materials included books to use for the Shapes Around Us activity, various sets of shapes flashcards, the *Sing and Read Shapes* CD (Burnett, Noble, & Wood, n.d.), and a shapes memory game. The following books were purchased for the Shape Around Us Activity: *A circle here, a square there* (Diehl, 2007); *The Greedy Triangle* (Burns, 1994); *The Shape of Things* (Dobbs, 1994); *Shapes* (2004); *Shapes, Shapes, Shapes* (Hoban, 1986); and *Shapes!* (2012).

Researcher-made materials included a flipbook, all assessments, a variety of picture symbols, reflections, a variety of shape cards, and shape name cards. The flipbook was made by hole punching the top center of the flashcard and putting them in rows of three in a three-ring binder, arranging them so that one of each type of shape was on each ring of the binder. Picture symbols and reflections were printed from Boardmaker Plus! software (DynaVox Systems, 2011). Assessments, shape cards, and shape name cards were designed in Microsoft Word and PowerPoint.

Systematic Instruction with a System of Least Prompts

Operationally defining best practices in teaching students with significant intellectual disabilities includes providing the support needed to help students succeed in the lesson. For this study such support was in the form of systematic instruction. The first step of systematic instruction involves selecting the specific behaviors that will be targeted. A process that can help identify the specific behaviors to be targeted is task analysis, a detailed step-by-step procedure for completing a task (e.g., matching shapes) and then teaching one specific step at a time. This step was done with every activity in the intervention. The researcher taught the first step and, once that step was mastered, taught the second step. The process kept moving to a subsequent step as students mastered each previous step. A sample task analysis follows:

Match Shapes Task Analysis

Description of Activity: A student is given a shape from the shape set and then is asked to match it to another example of the same shape in the set.

1. Student looks at given shape
2. Student looks at three choices
3. Student selects the shape that is the same as the given shape (by touch, pointing, eye gazes, etc.)

The next step of systematic instruction is use of a systematic method for prompting the behavior. The researcher chose to use a system of least prompts based on the aforementioned meta-analysis (Browder et al., 2008) that showed that a system of least prompts and time delay were effective practices in teaching mathematics to students with significant intellectual disabilities. A system of least prompts was preferred because it offered different intensities of support, and thus would address the researcher-predicted need for more intense levels of support as these students learned an activity or skill.

Systematic instruction with a system of least-prompts was used for each lesson. This instruction involved presenting a task to the student, usually accompanied by a verbal stimulus. For example, for the shape matching activity the verbal stimulus was "Which one is the same shape?" The researcher waited a given amount of time for the student to respond (for this study wait time was five seconds); if a student did not respond after the given wait time, the researcher presented the prompt that provided the lowest level of support. If the student still did not respond the researcher continued prompting, each time increasing the level of support the prompt provided. For this study three prompts were used (from lowest to highest level of support): (a) verbal prompt, (b) model prompt, and (c) physical prompt. A verbal prompt consisted of the researcher telling the student how to proceed. For example, in the shape matching activity the verbal prompt would be, "Touch the shape that is the same shape as the one you chose."   For a model prompt, the researcher showed the student how to accomplish the step while verbalizing, "Watch me, I'm touching the shape that is the same shape as you chose. Now you try." Finally, a physical prompt was hand-over-hand assistance to complete the task. When the student completed the task the researcher gave him praise.

Communication Supports

An important area of support for students with significant intellectual disabilities is in the area of communication. These students may struggle to communicate in many ways including speech, facial expressions, gestures, and print (Downing, 2005). All students in the study relied on augmentative and alternative communication (AAC), to some extent, to facilitate language comprehension and language expression. AAC is a system of communication that can include signing, gestures, symbols, residual speech, and speech generating devices. AAC is critical for all students who cannot communicate effectively through speech. The form of AAC used by all four students in this study was picture symbols. Supporting communication and AAC use needed to be a focus of all activities and assessments throughout this study.

The researcher supported student communication through aided language input (also known as aided language stimulation or aided language modeling). Often, students who use AAC struggle with an asymmetry between language input and language output; children who use AAC usually express themselves (language output) through signs, symbols, and speech-generating devices; but most of the language they receive from others (language input) is spoken language. Aided language input seeks to minimize this problem by having the communication partner (such as the researcher) use a student's AAC system to communicate with the student (Beukelman & Mirenda, 2005). When speaking with a student, the researcher used the pictures symbols to supplement the speech. For example, in the shape memory activity the researcher used picture symbols when telling the student to flip over a card, when asking the student if the shapes were the same or different, and when describing the properties of the shape. By using aided language input, the students received language input in the same manner that they used for output, that is, the AAC device. With aided language input, students did not have to rely solely on the auditory input of speech; they also had the visual input of their AAC system to support comprehension.

Measurement

Assessment

Four measures were used throughout the study: a measure of matching identical shapes, a measure of matching shapes with different sizes, a measure of matching shapes with different orientations, and a measure of shape recognition. The shapes in the measures were the same shapes targeted in the intervention (i.e., circle, square, and typical triangle). In the three shape-matching measures the students were shown one shape and were prompted, "Choose the same shape." They chose from three different shapes and the distractors were shapes that looked similar. For circles the distractors were semicircles, crescents, and ovals. For squares the distractors were rectangles, trapezoids, and parallelograms. For triangles the distractors were pentagons and rhombi. Sample assessments are shown in Appendix C through E. These samples show the shape stimulus and the response choices on one page. For the actual assessment these shapes were presented as individual picture symbols. For the shape recognition measure, students were shown a set of three shapes and were prompted, "Show me the (insert shape)." Distractors similar to the shape matching assessments were used. A sample assessment is shown in Appendix F. The researcher administered all assessments.

Maintenance

Measures of maintenance were built into to the design of the study, consisting of probes taken even after the intervention phase. The only skill that did not have built-in maintenance measures was the final target skill, shape recognition. Intermittent maintenance measures were planned for a few weeks following completion of the study if time permitted.

Generalization

Measures of generalization were planned in two ways. For the first three phases (shape matching), the plan was to assess whether the shape matching skill was generalized to novel shapes (i.e., rectangles and atypical triangles). For the final phase of shape recognition, the plan was to assess whether shape recognition was generalized to novel examples of the shapes such as manipulatives the student had not seen before. Unfortunately, as will be discussed later, no student reached mastery criterion, thus eliminating the possibility of generalization measures.

Implementation Fidelity

A doctoral student took data on implementation fidelity (Table 12) on 27% of intervention sessions. Implementation fidelity was calculated as the number of fidelity criteria fully implemented divided by the total number of fidelity criteria multiplied by 100.

Inter-rater Reliability

A doctoral student conducted inter-rater reliability (IRR) on 27% of all assessments across baseline, intervention, and probes. IRR was calculated as the total number of agreements divided by the total number of disagreements and agreements multiplied by 100.

Table 12

Fidelity Checklist

Implementation Fidelity Checklist					
Completed by:	Score: F–Fully Present, P–Partially Present, N–Not Present, N/A–Not Applicable				
Date Observed:					
Lesson Structure					
1. Daily lesson plan created					
2. All activities come from Building Blocks Curriculum					
3. Introductory Activity					
4. Work Time Activity					
5. Reflection					
6. Assessment					
Systematic Instruction					
7. Task analysis available					
8. 5 seconds wait time before prompting					
9. Prompt from least to most support (verbal, modeling, and physical assistance)					
10. 5 seconds wait time for each successive prompting (if needed)					
11. Prompts provided for individual steps as needed					
Communication Supports					
12. System of communication available					
13. System of communication used by teacher frequently to supplement spoken language					
Best Practices					
14. Avoids common misconceptions					
15. Focuses on the properties					
16. Provides multiple examples					
17. Uses manipulatives, visuals, and/ or models					
18. Activities are engaging					
19. Uses computers					

CHAPTER III—RESULTS

Implementation Fidelity

Implementation fidelity was conducted for 27% of all intervention sessions measuring adherence to the three components of the intervention package, which were (a) systematic instruction with a system of least prompts, (b) evidence-based practices for teaching shape recognition, and (c) communication supports. Implementation fidelity was found to be 100% for all criteria except for one. One evidence-based practice for teaching shape recognition is use of computers; however computers were not used for the first target skill of matching identical shapes because the Building Blocks computer programs included shapes in different orientations or sizes. Thus, the plan was to implement computer activities once the later target skills were addressed; unfortunately students never progressed to those stages.

Inter-rater Reliability

Inter-rater reliability was conducted for 27% of all assessments across baseline, intervention, and probes. Inter-rater reliability was 99.1% with a range from 90% to 100%.

Intervention

The purpose of this study was to determine if combining best practices in teaching mathematics to students with significant intellectual disabilities and evidence-based practices for teaching shape recognition would result in mastery of basic shape recognition skills for students with significant intellectual disabilities. The research used a small sample, interrupted time series (single case) multiple-probe design across behaviors within four participants. Figures 3 through 6 provide the total number of correct responses across each of the four target skills for each student. Mastery criterion for this study was set at three consecutive days at 80% or higher. The plan for the study was that after a student reached mastery in the first target skill (matching identical shapes), baseline and intervention would be conducted for the later skills. After approximately 30 days of data collection, no students reached mastery criterion. At that time, visual analysis and statistical estimation of effect size indicated the intervention had limited effectiveness and the study was discontinued because it did not seem likely that students would reach criterion with further intervention sessions. Therefore, baseline and intervention data will be reported for only matching identical shapes. Random probes for the other three target skills were taken during the matching identical shapes stage and will be discussed later.

Although no students reached mastery criteria of 80% or higher for three consecutive sessions, all students approached this goal. Miguel scored 80% on one session, declined to 70% the next session and then returned back to 80% the following session. However, this achievement was followed by a sharp decrease in his performance with scores ranging from 20% to 40%. Carter scored 80% for two sessions in a row and then his scores decreased. Figure 5 shows a pattern of a steady increase of performance and then a dip in performance and then an increase again. After the aforementioned decrease in skills following two days at 80% correct response, he was absent for 6 school days due to illness after which his scores stayed low (10% to 30% correct). Had attendance been consistent, Carter's scores may have increased in a manner similar to earlier data. Kyle reached 80% on one day but his scores were variable with no consistency or patterns to performance. Nathan was the only student who never reached 80%; however he approached this desired score with two consecutive days at 70% correct matching of identical shapes.

Visual analysis with effect size estimation was used to analyze the data. Two types of visual analysis are used with small sample, interrupted time series designs: level analysis and trend analysis. Level analysis is more appropriate for a behavioral intervention, wherein a student is capable of performing a particular behavior but needs an intervention to support occurrence or frequency of the behavior. For example, consider a token system to encourage homework completion: a student is capable of completing the homework on his own, but the token system increases the likelihood of the behavior (homework completion) occurring. As soon as the token system is implemented a sharp increase in the level of homework completion would be expected. In contrast, this shape recognition intervention was learning-based rather than performance-based. At the beginning of the study the students were not capable of matching shapes; they had to learn how to do so. With learning-based interventions, a sharp increase in learning acquisition as the intervention is implemented is not expected; instead, a typical learning curve is to be anticipated. Therefore trend analysis, as the more appropriate estimate of learning-based outcomes, focuses on differences in the slopes of the lines of data points in each stage.

For each student, baseline and intervention condition data were displayed with a line of best fit calculated in Excel, which uses a least-squares regression formula. Two types of visual trend analysis were conducted using these linear trend lines: within and between conditions. For within conditions trend analyses, the researcher looked solely at the intervention trend lines. Because students' scores should have been improving throughout the intervention, the linear trend lines should have demonstrated a strong positive slope. However, only one student (Carter) showed a positive slope. The intervention trend lines for the other three students were close to flat trend lines (i.e., a slope of zero), which would be expected in baseline but not with an intervention. Although only one student showed a consistent trend, all students had isolated periods of improvement.

To complete between condition trend analyses, the linear trend lines for both the baseline and the intervention conditions were evaluated. The baseline linear trends for each student should be close to a flat line or a slope of zero. None of the baseline trend lines were perfectly flat. There was minor variability in the baseline data, though this was expected due to the nature of the assessment. When measuring the different target skills, students were given three choices, so they had a one-in-three chance of answering correctly based solely on chance. Therefore, even if the students had no shape matching skills, three or four correct answers would be reasonably expected from chance alone. The variability in the baseline trend lines does not go beyond what was expected by chance. If the intervention had little effect, the baseline trend line and the intervention trend line would be similar, as is the case with Miguel, Kyle, and Nathan. If the intervention was effective, an increase should be seen in the intervention trend line, as is seen with Carter. Within conditions analysis supports the conclusion that the intervention was sufficiently effective for only one student.

To estimate effect size, the percent of non-overlapping data (PND) was used (Table 13). This estimate involved calculating the percentage of data points in the treatment phase that did not overlap with data points in the baseline phase (Scruggs & Mastropieri, 2013). Effectiveness was determined from the guidelines provided by Scruggs and Mastropieri (1998): a PND of higher than 90% is highly effective, a PND from 70% to 90% is effective, and a PND from 50% to 70% is questionable. A PND of below 50% is ineffective. Table 13 shows each student's PND score. Miguel, Kyle, and Nathan's results had PNDs below 50%, which indicates the intervention was ineffective. The estimate of effect size for these students corroborates the results of the visual analysis. Carter's PND was 56% which is a questionable effect size. However, the last three data points in the intervention phase followed an absence of a week and a

Table 13

Estimated Effect Size

Student	Percent of Non-overlapping Data (PND)	Effectiveness
Miguel	42%	Ineffective
Carter	56%	Questionable
Kyle	24%	Ineffective
Nathan	15%	Ineffective

half due to illness and a dramatic decrease in his participation and engagement when he returned to school. This was his second extended absence, the previous one being due to an illness followed immediately by four snow days. Had there been consistent attendance, Carter's PND might have been higher.

Probes

During the matching identical shapes stage, random probes were taken of the other three target skills of (a) matching shapes with different sizes, (b) matching shapes with different orientations, and (c) identifying shapes. Minor variability was evident in the probes within a target skill for each student. As with the matching identical shapes baseline and intervention data variability, most of this variability can be explained by the fact that students have a one in three chance of guessing a correct answer. The exception is Nathan's probe of matching shapes with different sizes, wherein he correctly matched 6 of 10 shapes. However, because this rate is well above the rate from other probes within that target skill, the probe was viewed as an outlier. All of the other probes were at or below the three to four correct responses predicted by chance alone, indicating that students had not developed later target skills before mastering the first target skill.

Summary

In summary, two types of analyses were conducted, visual analysis and statistical analysis.   Visual analysis utilized a least-square regression to create a line of best fit for the baseline and intervention phase. PND was used for statistical analysis to estimate effect size. Both forms of analysis showed that while all students showed some improvement, this intervention was consistently effective for only one student and even for that student the effect size was questionable.

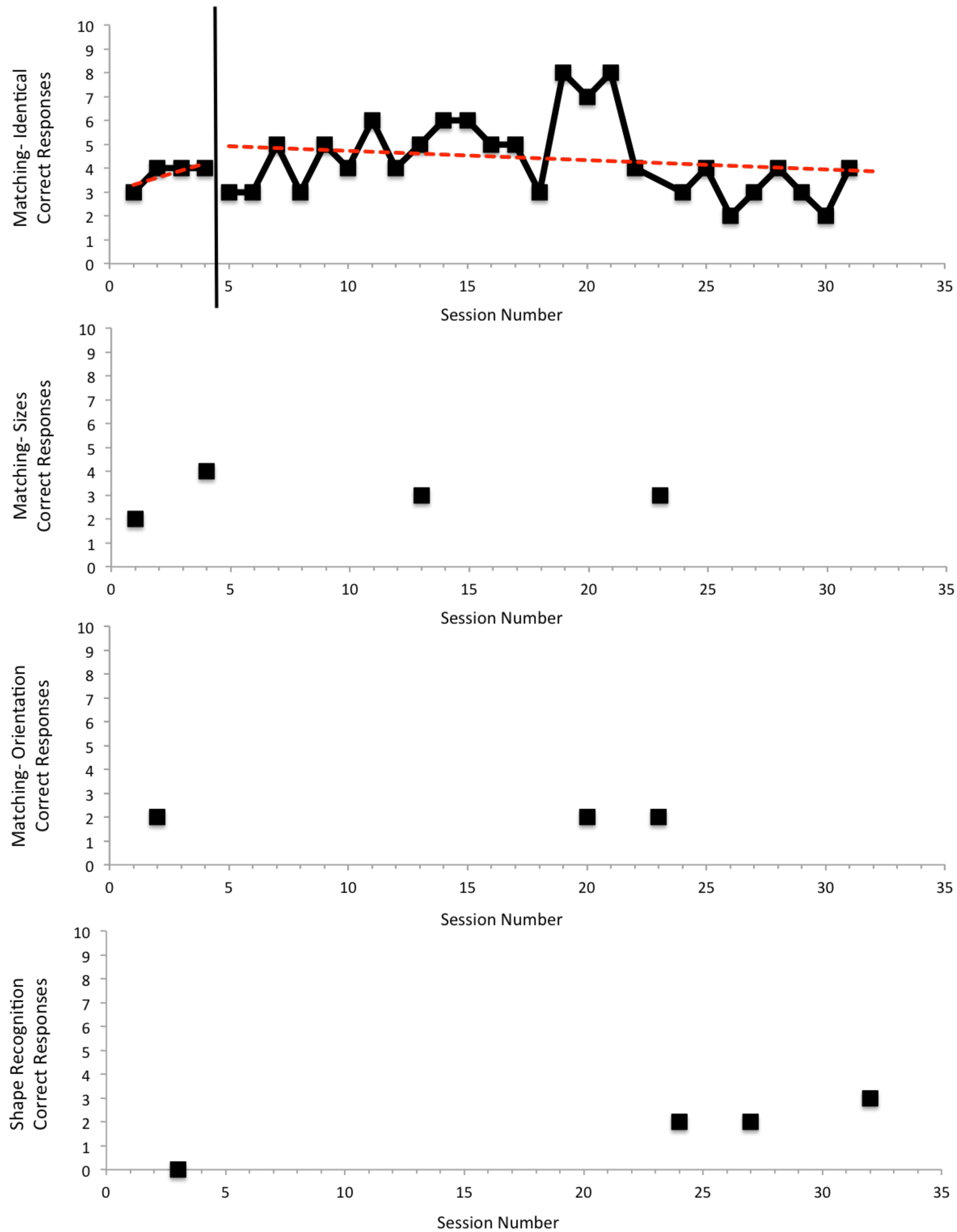


Figure 3. Student data: Miguel

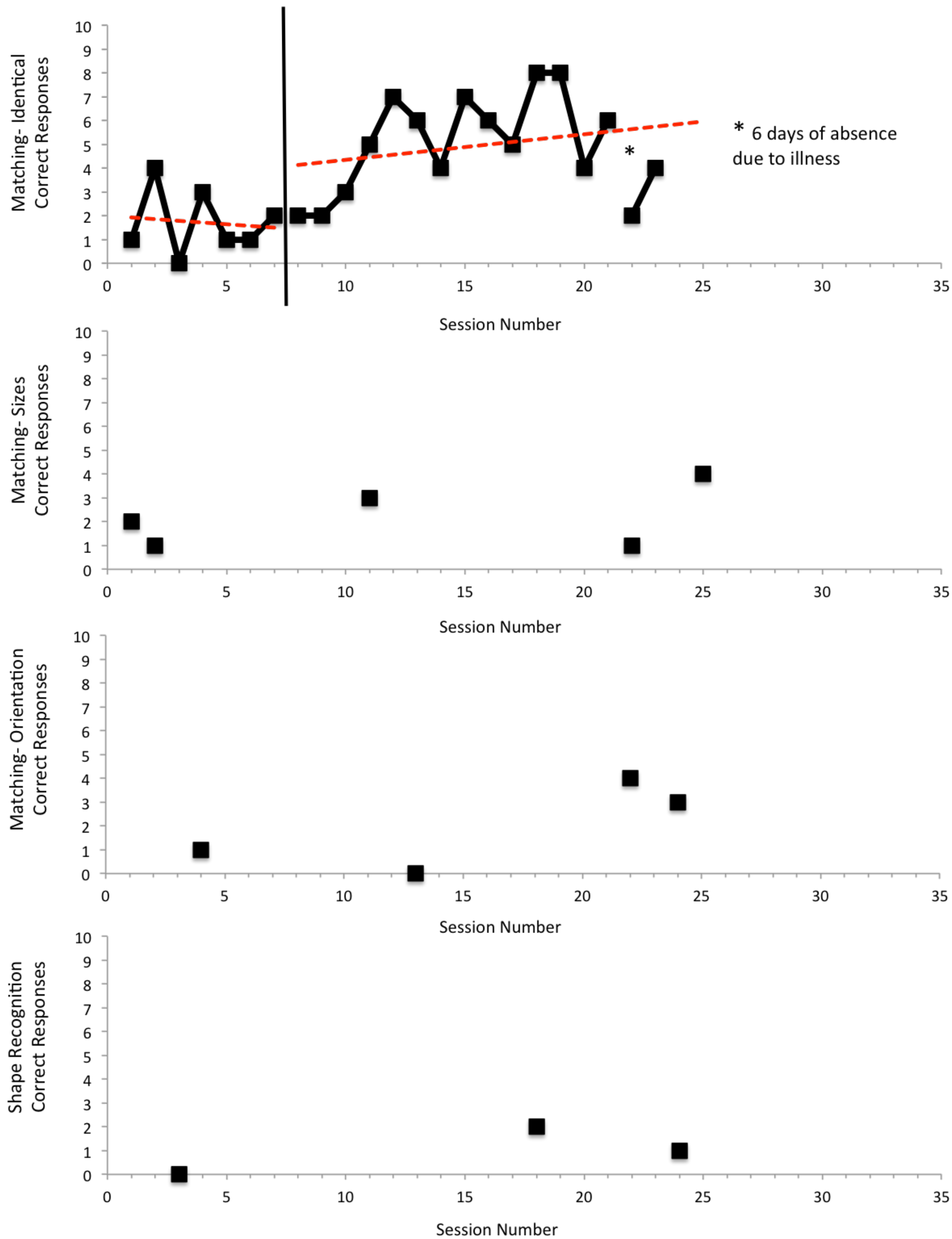


Figure 4. Student data: Carter

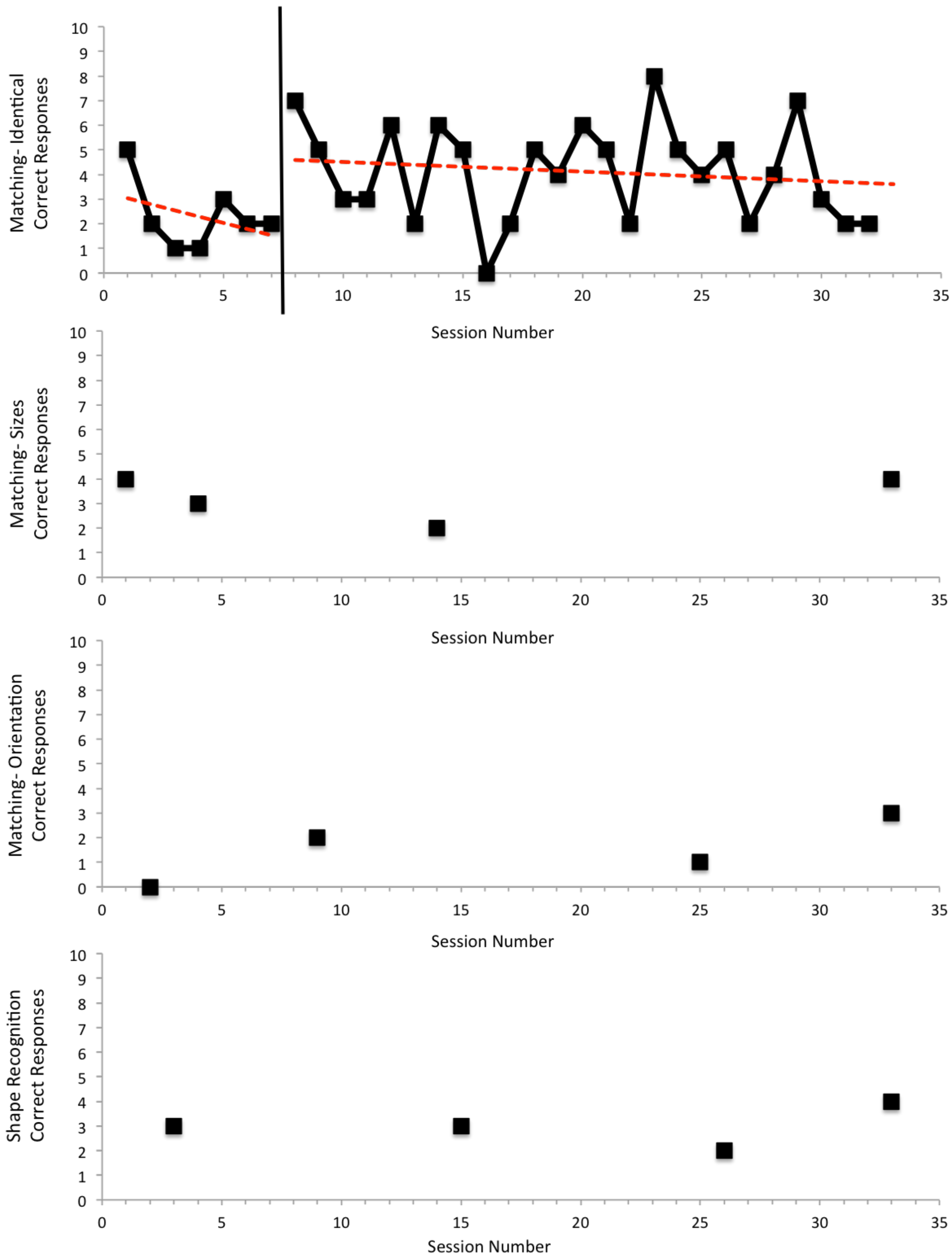


Figure 5. Student data: Kyle

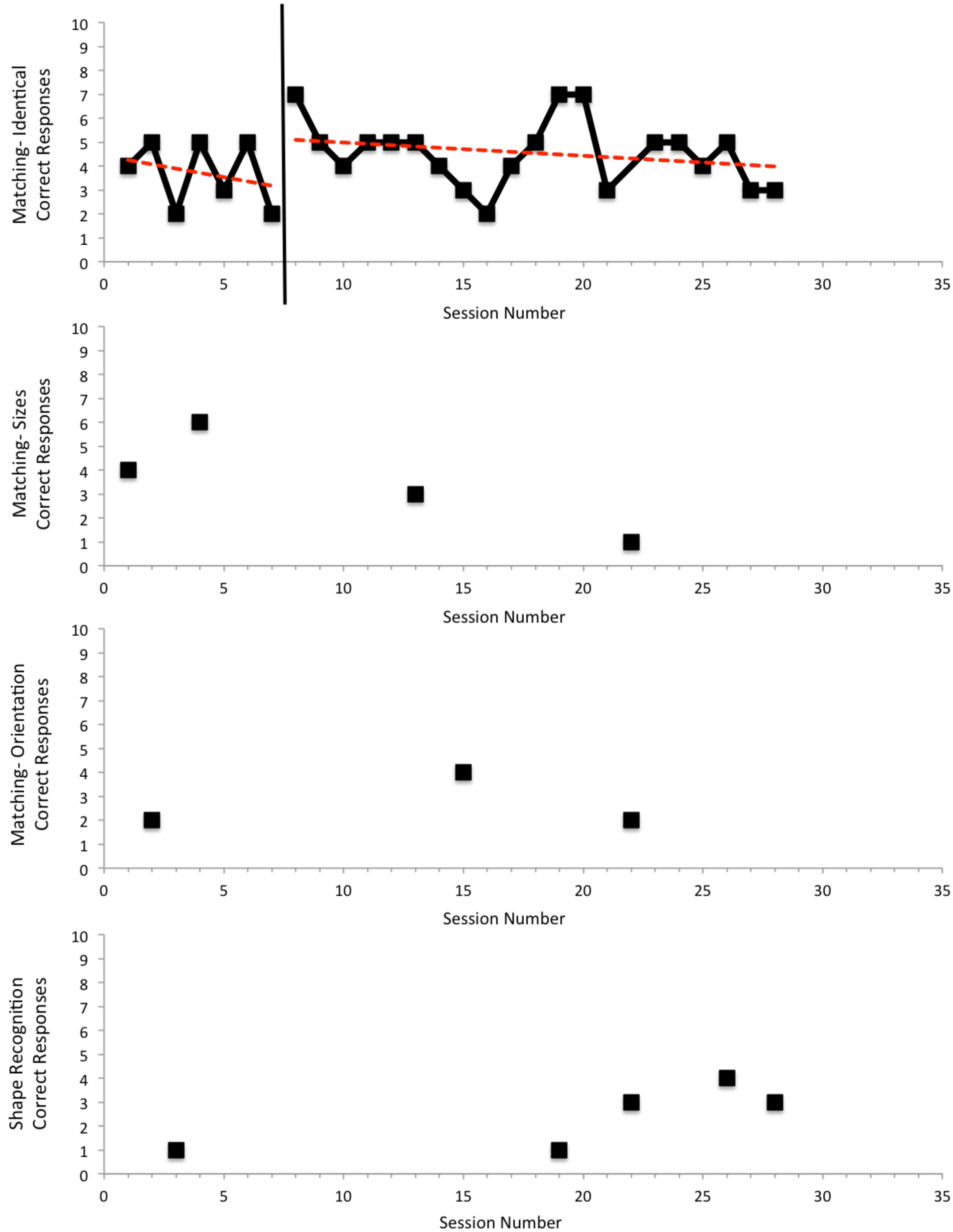


Figure 6. Student data: Nathan

CHAPTER IV—DISCUSSION AND SUMMARY

Conclusions

The purpose of this study was to evaluate the effectiveness of combining best practices in teaching mathematics to students with significant intellectual disabilities and evidence-based practices in teaching shape recognition. While all students approached the mastery criterion of 80% accuracy for three consecutive days in the matching identical shapes stage, no students fully reached it, preventing progression to the next three target skills of matching shapes with different sizes, matching shapes with different orientation, and shape recognition. The linear trend lines of the intervention data show only one student had a positive trend. Estimates for effect size indicate this student had a questionable effect size, while the other three students' effect sizes were in the ineffective range. Although the results of this study were not as positive as anticipated, all students showed some improvement from baseline. Even as the intervention was not as effective as desired, much can be learned from this study in regards to teaching mathematics to students with significant intellectual disabilities.

Two explanations are suggested for these results. First, performance issues are suspected as students became capable of performing the target skills through the intervention stage but were not sufficiently motivated to do so. All four students either reached the mastery level of 80% for at least one day or came close to reaching the mastery level, as is the case with Nathan who reached 70% for two days in row. Seventy and eighty percent accuracy is much higher than any of the students were performing at baseline and greatly exceeds the 33% accuracy that would be expected from chance alone. This finding suggests that during the intervention students learned to match identical shapes, but for some reason were not performing at the usual criterion for mastery of a learned skill, thus implying an issue with motivation.

The schedule of the each intervention session could have impacted motivation with a possible fatigue effect. Intervention sessions included an introductory activity, a work time activity, and a reflection prior to assessment. The cognitive demands on the student prior to testing may have created lower performance on assessments because the students were tired or no longer engaged. Frequently, students would do well on activities and reflections, correctly matching identical shapes with high accuracy, and then perform poorly on the assessment, suggesting fatigue. However, this finding calls in to question why students performed well on certain days and did not appear tired or less engaged at the point of assessment. One explanation could be the variation of activities on those low performance days. Although the structure of the intervention sessions was consistent, activities varied from day-to-day. Perhaps some activities were more cognitively demanding than others, increasing the likelihood of fatigue. One way to prevent this effect in follow-up studies would be to eliminate the assessment at the end of each intervention session and instead collect data on students' accuracy with the activities and reflections as a form of on-going progress monitoring.

The length of the intervention stage is an additional factor that may have affected motivation. The intervention stage followed four to seven baseline sessions and included approximately twenty to twenty-five daily sessions targeting matching identical shapes. Twenty-five days is a long time to address the same target skills with only minimal variations in the activities performed. Students may have become satiated with matching identical shapes toward the end of the intervention stage and therefore were no longer motivated to perform to their capabilities. This possibility seems especially likely in the case of Miguel. He performed close to or at criterion for three days in a row, after which his scores plummeted and remained low for rest of the study. At the point of his scores declining, he would choose the wrong answer and frequently start laughing when being assessed, suggesting that his inaccuracy may have been deliberate. This pattern was not the only suggestion that students had lost interest and motivation toward the end of the study; problem behavior such as physical aggression, avoidance, and distress rose to different degrees in all of the participants near the end of data collection. This disruptive behavior also indicates a drop in motivation.

One way to address the effects of study length might be to include multiple target skills at the same time. Although this approach would not shorten the length of the study and could quite possibly lengthen the study, it would introduce a variety of topics and activities instead of focusing solely one target skill with a small set of activities, thereby reducing the likelihood of satiation. Instead of focusing on one target in geometry, a study could be designed to simultaneously address target skills in multiple content area standards. Two studies have shown success targeting multiple mathematics skills in different content area standards at one time with students with significant intellectual disabilities (Browder, Jimenez, Spooner, et al., 2012; Jimenez & Kemmery, 2013). This multiple content approach may help maintain student interest and engagement by reducing the monotony of learning just one skill day after day.

Another way to address the length of the study would be to adjust the mastery criteria either by decreasing the level below 80% or shortening the days required at the level of mastery. Three days at mastery level was chosen because it has been shown to be a minimum requirement to promote maintenance of skills (Collins, 2012). However, the current study addressed four target skills that built on each other, so even if the students moved to the next target skills after one or two days at mastery, quite probably the skill would have been maintained through intervention for the next sequential target skill. Probes conducted after intervention would assess whether or not the skill had been maintained. Shortening the number of days required at criterion might have eliminated the sharp drop observed in the data from all four students after they approached mastery. Furthermore, requiring at least 80% accuracy, even though a generally accepted standard criterion, might have been too high for the population of students targeted in this study. Although students with significant intellectual disabilities can reach this level of performance, as shown by the students in this study, the process could take so long that students lose motivation and engagement. An alternative would be to move on to the next target skill once a student shows a steady trend of improvement, and then investigate whether the student can continue to show progress in the first target skill once instruction has moved to later skills.

Another possible explanation for the ineffective results is that something outside the study context was affecting students' capability to reach mastery. One possibility that may have affected their capabilities is the need for precursor skill instruction. In their learning trajectory for shape recognition, Sarama and Clements (2009) stated that before children can match identical shapes, they need to decide whether two things in their environment are the same of or different. This skill was not targeted in the study, but students may have needed instruction in matching familiar items before learning to match shapes. This instruction would also ensure that students understood the concepts of same and different prior to the study. One student in particular, Nathan, seemed to struggle with those terms throughout the study, often appearing unsure when he had to label two shapes as either same or different. His grasp on this key vocabulary did not improve through the matching identical shapes intervention, suggesting that he needed to learn the terms prior to applying them to shapes. Follow-up studies should screen participants for the precursor skill, that is, the ability to labeling familiar objects as same or different to identify.

Another factor that may have affected the students' capabilities in mastering target skills was their receptive communication skills. Difficulties understanding instruction or directions may have negatively impacted student achievement. The aided language input design was an attempt to address this issue, but special attention may be needed in future studies to decrease the language demands of instruction.

The intervention package may have also affected students' capabilities in reaching mastery. The hypothesis of this study stated that combining best practices in teaching mathematics to students with significant intellectual disabilities with evidence-based practices in teaching shape recognition would be effective for teaching shape recognition to students with significant intellectual disabilities. The results of this study fail to show that this combination was effective. A plausible explanation is that evidence-based practices in teaching shape recognition are not effective for students with significant intellectual disabilities. Specifically the evidence-based practices of using a variety of manipulatives, visuals, and models and multiple engaging activities may not be effective for students with significant intellectual disabilities. A wide variety of manipulatives and examples of shapes were used in this study including plastic shapes, foam shapes, pictures cards, drawings and photographs in books, and flash cards. Also a variety of activities were used such as matching shapes, shape memory, shape flipbook, and finding shapes in the real word examples. Students with significant intellectual disabilities often have difficulties transferring skills from one context to another (Collins, 2012). Instead of beginning with a wide variety of manipulatives and examples, this population of students may need to start with a smaller set that can be expanded as they master the target skills. A similar method of starting with a small set of activities and introducing new activities, while making explicit connections between activities, may be more effective for students with significant intellectual disabilities.

Limitations

Several limitations of the current study are noted. One limitation is inability of the researcher to deliver consistent, daily intervention. Due to snow days and extended absences, occasional significant gaps between sessions (up to four days due to snow and up to six days due to absences) resulted in an inconsistent teaching of shape matching. With more regular sessions, more positive results in student performance may have occurred.

A second limitation is that one-on-one instruction prevents students from learning through observation of peers, which has been shown to be effective for students with significant intellectual disabilities (Mechling, Gast, & Krupa, 2007; Smith, Collins, & Schuster, 1999; Werts, Caldwell, & Wolery, 1996). An alternative to one-on-one instruction is small group instruction, a procedure that has been utilized in several studies teaching mathematics to students with significant intellectual disabilities (Browder, Jimenez, Spooner, et al., 2012; Browder, Jimenez, & Trela, 2012; Browder, Trela, et al., 2012). Not only would small group instruction provide observational learning opportunities but is much more efficient for teachers.

A final limitation of this study is that all four students spent the majority of their school day in a self-contained setting with very little interaction with peers without disabilities. The purpose of this study was to investigate instructional methods that not only meet the significant support needs of this population but also promote access to the general curriculum and inclusion. However, these students had very few opportunities for inclusive educational experiences. Unfortunately, no opportunities to work with students in inclusive settings were able for this study.

Implications for Practice and Future Research

Earlier in this paper, the least dangerous assumption (Donnellan, 1984) was promoted, stating that in the absence of conclusive data it is far less dangerous to assume competence as opposed to incompetence. The current researcher assumed that students with significant disabilities could learn shape recognition skills. Although the students did not reach mastery criteria, the data provides evidence indicating that students with significant intellectual disabilities can learn these skills, therefore supporting the least dangerous assumption.

Recent legislation has required that all students, including students with significant intellectual disabilities, show progress in the general education curriculum. This policy implies a shift in the field of education, where curriculum for students with significant intellectual disabilities is no longer focused solely on functional skills such as daily living skills and vocational skills. Curriculum has now begun to include such academic skills as reading and math. Many educators struggle with how to teach content aligned with grade-level standards to students with significant intellectual disabilities (Browder, Trela, et al., 2012), highlighting a broader system capacity issue on how to prepare and support teachers in providing access to the general curriculum for students with significant intellectual disabilities.

The study calls into question whether adding disability-specific supports to content-specific, evidence-based practices for the general population is effective for students with significant intellectual disabilities. This population may not benefit from the evidence-based practices for the general education curriculum, even with such supports as systematic instruction, a system of prompting and AAC, suggesting that changes may need to be made to instructional methods typically used in the general education curriculum so that students with significant intellectual disabilities can succeed.

More research needs to be conducted to identify best practices for teaching shape recognition to students with significant intellectual disabilities. Combining best practices in teaching mathematics to students with significant disabilities and evidence-based practices in teaching shape recognition to the general population may not be effective. The best practice in teaching mathematics to students with significant intellectual disabilities, systematic instruction with a system of prompting, has been shown to be effective (Browder et al., 2008). Therefore, the element of this study in question is the evidence-based practices for teaching shape recognition. The results of this study suggest that the practices (avoiding common misconceptions; using multiple examples; creating engaging activities; using manipulatives, visuals, and models; and using computers) may need to be adjusted to meet the needs of all students. Future research needs to be done to find which evidence-based practices are not effective and how these practices can be modified to meet the needs of students with significant intellectual disabilities. As mentioned above, the use of a variety of manipulatives, visuals, and models and multiple engaging activities are two best practices the researcher suspects are not effective for students with significant intellectual disabilities and need to be modified in follow-up studies. Once follow-up studies that identify necessary modifications to best practices in shape recognition are completed, additional research is needed on whether these standards are applicable to later geometry topics such as three-dimensional figures, symmetry, and transformations.

While developing strategies that support access to the general curriculum is important, a limitation of this study is that instruction occurred in a self-contained setting instead of an inclusive classroom. An inclusive classroom would be a more natural environment for learning. Future research should investigate supporting access to the general curriculum for students with significant intellectual disabilities in the context of an inclusive educational setting. One direction to explore is whether early shape recognition skills can be embedded in a lesson addressing grade level standards, such as a third grade standard on partitioning shapes into equal parts. This approach will not only address access to an inclusive educational setting but will also evaluate how well these skills are generalized, another important area for future research.

Summary

In summary, combining best practices in teaching mathematics to students with significant intellectual disabilities and evidence-based practices was not as effective as desired for teaching basic shape recognition skills. However, although no students reached the goal of scores of 80% or higher for three days in a row, all students showed some improvement and approached mastery criterion. Future research is needed to address appropriate study methods and/or the relevance of evidence-based practices for teaching shape recognition to meet the needs of students with significant intellectual disabilities.

REFERENCES

American Association on Intellectual and Developmental Disabilities. (2013). AAIDD-Resources for Intellectual and Developmental Disabilities Professionals. from http://aaidd.org/

Aslan, D., & Arnas, Y. A. (2007). Three- to six-year-old children's recognition of geometric shapes. *International Journal of Early Years Education, 15*, 83-104. doi: 10.1080/09669760601106646

Battista, M.T. (2002). Learning geometry in a dynamic computer environment. *Teaching Children Mathematics, 8*, 333-339.

Berla, E. P., & Butterfield, L. H. (1977). Tactual distinctive features analysis: Training blind students in shape recognition and in locating shapes on a map. *The Journal of Special Education, 11*, 335-346. doi: 10.1177/002246697701100309

Beukelman, D. R. , & Mirenda, P. (2005). *Augmentative and alternative communication: Supporting children and adults with complex communication needs* (3rd ed.). Baltimore, MD: Brookes Publishing Co.

Browder, D. M., Jimenez, B. A., Spooner, F. , Saunders, A. , Hudson, M., & Bethune, K. S. (2012). Early numeracy instruction for students with moderate and severe developmental disabilities. *Research and Practice for Persons with Severe Disabilities, 37*, 308-320.

Browder, D. M., Jimenez, B. A., & Trela, K. (2012). Grade-aligned math instruction for secondary students with moderate intellectual disability. *Education and Training in Autism and Developmental Disabilities, 47*, 373-388.

Browder, D. M., Spooner, F., Ahlgrim-Delzell, L., Harris, A., & Wakeman, S. (2008). A meta-analysis on teaching mathematics to students with significant cognitive disabilities. *Exceptional Children, 74*, 407-432.

Browder, D. M., Trela, K., Courtade, G. R., Jimenez, B. A., Knight, V., & Flowers, C. (2012). Teaching mathematics and science standards to students with moderate and severe developmental disabilities. *The Journal of Special Education, 46*, 26-35. doi: 10.1177/0022466910369942

Brown, C.S. (2009). More than just number. *Teaching Children Mathematics, 15*, 474-479.

Burger, W. F. (1985). Geometry. *Arithmetic Teacher, 32*(6), 52-56.

Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education, 17*(1), 31-48. doi: 10.2307/749317

Burnett, S., Noble, A., & Wood, D. (n.d.). *Sing and read: Shapes* [Music CD]: Frog Street Press.

Burns, M. (1994). *The greedy triangle*. New York: Scholastic Press.

Clements, D. H. (2003). Teaching and learning geometry. In J. Kilpatrick, W. G. Martin & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 151-178). Reston, VA: National Council of Teachers of Mathematics.

Clements, D. H., & Battista, M. T. (1986). Geometry and geometric measurement. *Arithmetic Teacher, 33*(6), 29-32.

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of researach on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 420-464). New York: Macmillan Publishing Co, Inc.

Clements, D. H., Copple, C., & Hyson, M. (2010). *Early childhood mathematics: Promoting good beginnings*. Washington, DC: NAEYC/NCTM.

Clements, D. H., & Sarama, J. (2000). Young children's ideas about geometric shapes. *Teaching Children Mathematics, 6*, 482-488.

Clements, D. H., & Sarama, J. (2013). *Building Blocks*. Columbus, OH: McGraw-Hill Education.

Clements, D. H., Swaminathan, S., Hannibal, M. Z., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education, 30*(2), 192-212.

Collins, B. C. (2012). *Systematic instruction for students with moderate and severe disabilities*. Baltimore, MD: Brookes Publishing Co.

Collins, B. C., Hager, K. L., & Galloway, C. C. (2011). Addition of functional content during core content instruction with students with moderate disabilities. *Education and Training in Autism and Developmental Disabilities, 46*, 22-39.

Crowley, M. L. (1987). The van Hiele model of the development of geometric thought. In M. M. Lindquist (Ed.), *Learning and teaching geometry, K-12: 1987 yearbook of the National Council of Teachers of Mathematics* (pp. 1-16). Reston, VA: National Council of Teachers of Mathematics.

Darke, I. (1982). A review of research related to the topological primacy thesis. *Educational Studies in Mathematics, 13*, 119-142.

Diehl, D. (2007). *A circle here, a square there*. New York: Lark Books.

Dobbs, D. A. (1994). *The shape of things*. Somerville, MA: Candlewick Press.

Donnellan, A. M. (1984). The criterion of the least dangerous assumption. *Behavioral Disorders, 9*, 141-150.

Downing, J. E. (2005). *Teaching communication skills to students with severe disabilities* (Second ed.). Baltimore, MD: Brookes Publishing Co.

DynaVox Systems, LLC. (2011). Boardmaker plus! (Version 6.1.6) [Computer Software]. Pittsburgh, PA: DynaVox Mayer-Johnson.

Geeslin, W. E., & Shar, A. O. (1979). An alternative model describing children's spatial preferences. *Journal for Research in Mathematics Education, 10*(1), 57-68.

Gutiérrez, A., Jaime, A., & Fortuny, J. M. (1991). An alternative paradigm to evaluate the acquisition of the van Hiele levels. *Journal for Research in Mathematics Education, 22*(3), 237-251.

Hannibal, M. A. (1999). Young children's developing understanding of geometric shapes. *Teaching Children Mathematics, 5*, 353-357.

Herberg, K., McLaughlin, T. F., Derby, K. M., & Gilbert, V. (2011). The effects of direct instruction flashcards on shape recognition and recall for two preschool students with disabilities. *Academic Research International, 1*(3), 55-59.

Hitchcock, C. H. , & Noonan, M. J. (2000). Computer-assisted instruction of early academic skills. *Topics in Early Childhood Special Education, 20*, 145-158. doi: 10.1177/027112140002000303

Hoban, T. (1986). *Shapes, shapes, shapes*. New York: Mulberry Books.

Horn, C., Schuster, J. W., & Collins, B. C. (2006). Use of response cards to teach telling time to students with moderate and severe disabilities. *Education and Training in Developmental Disabilities, 41*, 382-391.

Horner, R. D., & Baer, D. M. . (1978). Multiple-probe technique: A variation of the multiple baseline. *Journal of Applied Behavior Analysis, 11*, 189-196.

Jahoda, G., Deregowski, J. B., & Sinha, D. (1974). Topological and Euclidean spatial features noted by children: A cross-cultural study. *International Journal of Psychology, 9*, 159-172.

Jaime, A., & Gutiérrez, A. (1994). *A model of test design to assess the van Hiele levels.* Paper presented at the 18th Conference of the International Group for the Psychology of Mathematics Education, Lisbon, Portugal.

Jimenez, B. A., Browder, D. M., & Courtade, G. R. (2008). Teaching an algebraic equation to high school students with moderate developmental disabilities. *Education and Training in Developmental Disabilities, 43*, 266-274.

Jimenez, B. A., & Kemmery, M. . (2013). Building the early numeracy skills of students with moderate intellectual disability. *Education and Training in Autism and Developmental Disabilities, 48*, 479-490.

Jorgensen, C. (2005). The least dangerous assumption: A challenge to create a new paradigm. *Disability Solutions, 6*(3), 1,5-9.

Mackay, H. A., Soraci, S. A., Carlin, M. T., Dennis, N. A., & Strawbridge, C. P. (2002). Guiding visual attention during acquisition of matching-to-sample. *American Journal on Mental Retardation, 107*, 445-454. doi: 10.1352/0895-8017

Martin, J. L. (1976a). An analysis of some of Piaget's topological tasks from a mathematical point of view. *Journal for Research in Mathematics Education, 7*(1), 8-24.

Martin, J. L. (1976b). A test with selected topological properties of Piaget's hypothesis concerning the spatial representation of the young child. *Journal for Research in Mathematics Education, 7*(1), 26-38.

Mechling, L. C., Gast, D. L., & Krupa, K. (2007). Impact of SMART Board technology: An investigation of sight word reading and observational learning. *Journal of Autism and Developmental Disorders, 37*, 1869-1882. doi: 10.1007/s10803-007-0361-9

National Council of Teachers of Mathematics. (2000a). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000b). *Principles and standards for school mathematics: An overview*. Reston, VA: Author.

Piaget, J., & Inhelder, B. (1967). *The child's conception of space*. New York: W. W. Norton & Company, Inc.

Sarama, J., & Clements, D. H. (2009). Shape *Early childhood mathematics education research: Learning trajectories for young children* (pp. 199-246). New York: Routledge.

Scruggs, T. E., & Mastropieri, M. A. (1998). Summarizing single-subject research: Issues and applications. *Behavior Modification, 22*, 221-242. doi: 10.1177/01454455980223001

Scruggs, T. E., & Mastropieri, M. A. . (2013). PND at 25: Past, present, and future trends in summarizing single-subject research. *Remedial and Special Education, 34*, 9-19. doi: 10.1177/0741932512440730

Shapes! (2012). Washington, DC: National Geographic Society.

Sinclair, N., & Moss, J. (2012). The more it changes, the more it becomes the same: The development of the routine of shape identification in dynamic geometry environment. *International Journal of Educational Research, 51-52*, 28-44. doi: 10.1016/j.ijer.2011.12.009

Skibo, H., Mims, P., & Spooner, F. (2011). Teaching number identification to students with severe disabilities using response cards. *Education and Training in Autism and Developmental Disabilities, 46*, 124-133.

Smith, R. L., Collins, B. C., & Schuster, J. W. (1999). Teaching table cleaning skills to secondary students with moderate/severe disabilities: Facilitating observational learning during instructional downtime. *Education & Training in Mental Retardation & Developmental Disabilities, 34*, 342-353.

van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. New York: Academic Press Inc.

Wehmeyer, M. L. , Lattin, D., & Agran, M. (2001). Achieving access to the general curriculum for students with mental retardation: A curriculum decision-making model. *Education and Training in Mental Retardation and Developmental Disabilities, 36*, 327-342.

Werts, M. G., Caldwell, N. K., & Wolery, M. (1996). Peer modeling of response chains: Observational learning by students with disabilities. *Journal of Applied Behavior Analysis, 29*, 53-66.

APPENDIX A

Informed Consents and Assent

Teaching Shape Recognition to Students with Significant Intellectual Disabilities

Dear Parent or Guardian,

The Department of Special Education at the University of Kansas supports the practice of protection for human subjects participating in research. The following information is provided for you to decide whether you wish your child to participate in the present study. You may refuse to sign this form and not allow your child to participate in this study. You should be aware that even if you agree to allow your child to participate, you are free to withdraw at any time. If you do withdraw your child from this study, it will not affect your relationship with this unit, the services it may provide to you, or the University of Kansas.

PURPOSE OF THE STUDY

We live in a mathematical world. From shopping to navigating around town, and much more, mathematics skills are needed to participate in society. Because of this all students need a high quality mathematics education. However, many students are not receiving this. One group in particular that needs better instruction in mathematics is students with severe intellectual disabilities. The majority of research available on teaching mathematics to this group of students is around number and operations and measurement skills. More studies are needed in other areas of mathematics. All children need a solid understanding of geometry. Geometry skills begin to develop very early as young children make sense of the world around them. An example of a beginning geometry skill is shape recognition. An understanding of shapes is critical to more advanced geometry study; however, research shows that many children struggle with shape recognition.

The purpose of the study is to establish research on effective instructional methods for teaching shape recognition skills to students with significant intellectual disabilities. The objective is to show that combining best practices in teaching shape recognition with best practices in teaching students with significant intellectual disabilities is effective in teaching shape recognition to students with significant intellectual disabilities.

PROCEDURES

Participants in this study will engage in a series of math lessons targeted at shape recognitions skills. These lessons will be approximately 30 minutes long and occur five days a week. Lessons will occur at your child's school during regular school hours. All lessons will be taught by the principal investigator, who is a licensed special education teacher with a long history of working with students with significant intellectual disabilities. The estimated length of this study is 6-8 weeks.

Lessons will target four shape recognition skills: (1) matching identical shapes; (2) matching shapes with different sizes but the same orientation; (3) matching shapes with different orientations but the same sizes; and (4) recognition of basic shapes. Lessons will include engaging and hands on activities designed around the Building Blocks Pre-K program (McGraw-Hill), a research based curriculum that was developed from a grant by the National Science

Foundation. This is a Pre-K curriculum but was chosen because it targets beginning shape recognition skills, which the students have not yet mastered. The curriculum will be adapted as necessary to make it appropriate for elementary students with significant intellectual disabilities. Changes may include adjusting the timing of the lesson (providing more time to master a topic), using assistive technology (such a switch that allows a student to operate a computer), and making changes for age appropriateness (i.e. changing the topic of the activity so that it is more appropriate for older students). After each lesson there will be a brief assessment to document your child's progress with shape recognition.

RISKS

Because this study involves learning a new skill (shape matching), mental stress is a possible risk. However this is a minimal risk because lessons will be short and frequent breaks will be given as needed.

As part of this study, the researchers would like access to private academic records- specifically, the student's Individualized Education Program (IEP). Access to your child's IEP is needed to get a general description of the student's disability and their IQ score, which helps determine if your child is a good match for this study. This risk is minimal because there will be no identifiable information connecting the students to the information. All identifiable information will be blacked out of the IEP and all participants will be given pseudonyms. IEPs will be kept in a locked file cabinet or on a password protected computer and will be deleted or destroyed after the completion of this study. If requested, you can be provided a copy of the IEP disclosed.

BENEFITS

Through participation in this study, your child will be learning beginning geometry skills. This will help him or her develop skills needed for more advanced mathematics topics. Developing math skills will have ongoing benefits in a variety of areas including school, work, and the community.

PAYMENT TO PARTICIPANTS

Participants will not be paid for participation in this study.

PARTICIPANT CONFIDENTIALITY

Your child's name will not be associated in any publication or presentation with the information collected about your child or with the research findings from this study. Instead, the researcher(s) will use a pseudonym rather than your child's name. Your child's identifiable information will not be shared unless (a) it is required by law or university policy, or (b) you give written permission.

All hard copies of information or data collected during this study will be stored in a locked file cabinet. Hard copies will be scanned and kept on the principal investigators password protected computer and then hard copies will be shredded. The principal investigator will be the only person with access to these documents.

Permission granted on this date to use and disclose your information remains in effect indefinitely. By signing this form you give permission for the use and disclosure of your child's information, excluding your child's name, for purposes of this study at any time in the future.

INSTITUTIONAL DISCLAIMER STATEMENT

In the event of injury, the Kansas Tort Claims Act provides for compensation if it can be demonstrated that the injury was caused by the negligent or wrongful act or omission of a state employee acting within the scope of his/her employment.

REFUSAL TO SIGN CONSENT AND AUTHORIZATION

You are not required to sign this Consent and Authorization form and you may refuse to do so without affecting your right to any services you are receiving or may receive from the University of Kansas or to participate in any programs or events of the University of Kansas. However, if you refuse to sign, your child cannot participate in this study.

CANCELLING THIS CONSENT AND AUTHORIZATION

Estimated length of the entire study is 8 weeks but may vary from child to child. You may withdraw your consent to allow participation of your child in this study at any time. You also have the right to cancel your permission to use and disclose further information collected about your child, in writing, at any time, by sending your written request to: Kristin Joannou Lyon; 1200 Sunnyside Avenue; 3150 Hayworth Hall; University of Kansas; Lawrence, KS 66045.

If you cancel permission to use your child's information, the researchers will stop collecting additional information about your child. However, the research team may use and disclose information that was gathered before they received your cancellation, as described above.

QUESTIONS ABOUT PARTICIPATION

Questions about procedures should be directed to the researcher(s) listed at the end of this consent form.

PARTICIPANT CERTIFICATION:

I have read this Consent and Authorization form. I have had the opportunity to ask, and I have received answers to, any questions I had regarding the study. I understand that if I have any additional questions about my child's rights as a research participant, I may call (785) 864-7429, write to the Human Subjects Committee Lawrence Campus (HSCL), University of Kansas, 2385 Irving Hill Road, Lawrence, Kansas 66045-7568, or email irb@ku.edu.

I agree to allow my child to take part in this study as a research participant. By my signature I affirm that I am at least 18 years old and that I have received a copy of this Consent and Authorization form.

_______________________________________ Date _______________________

Type/Print Participant's Name

____

Parent/Guardian Signature

Researcher Contact Information

Kristin Joannou Lyon	Wayne Sailor, Ph.D.
Principal Investigator	Faculty Supervisor
Department of Special Education	Department of Special Education
1200 Sunnyside Avenue	1122 W. Campus Rd.
3150 Hayworth Hall	Joseph R. Pearson Hall, Rm 541
University of Kansas	University of Kansas
Lawrence, KS 66045	Lawrence, KS 66045
785 864 -0594	785 864 -4950

Formulario de Consentimiento y Autorización para los Padres

Enseñando a estudiantes con discapacidad significativa cognoscitiva a reconocer formas geométricas

Estimado Padre, Custodio o Encargado,

El Departamento de Educación Especial de la Universidad de Kansas apoya la práctica de la protección de los sujetos humanos que participan en estudios de investigación. Le presentamos la siguiente información para que usted decida si desea que su hijo participe en el siguiente estudio. Usted puede negarse a firmar este formulario y prohibir que su hijo participe en este estudio. Queremos que este consciente de que aún si permite que su hijo participe del estudio usted puede terminar su participación en cualquier momento. El que usted decida retirar a su hijo de este estudio no afectará su relación con esta unidad, los servicios que esté recibiendo o su relación con la Universidad de Kansas.

PROPÓSITO DEL ESTUDIO DE INVESTIGACIÓN

Vivimos en un mundo matemático. Las destrezas matemáticas son necesarias para participar en la sociedad, desde hacer compras hasta recorrer el pueblo. Por esta razón todos los estudiantes necesitan una educación en matemáticas de alta calidad. Sin embargo, no todos los estudiantes la reciben. Existe un grupo particular de estudiantes que necesita recibir una mejor educación en matemáticas, lo es el de estudiantes con discapacidad cognoscitiva significativa. La mayoría de las investigaciones disponibles sobre la enseñanza de matemáticas para este grupo abordan los temas de números y operaciones, y destrezas de medición. Se necesitan más estudios en otras aéreas. Todos los niños necesitan tener un conocimiento sólido de geometría. Desde muy temprana edad, los niños comienzan a desarrollar destrezas en geometría al descubrir e interactuar con el mundo que les rodea. Un ejemplo del desarrollo inicial de destrezas de geometría es reconocer formas geométricas. Es necesario conocer estas formas para adquirir un conocimiento avanzado de geometría, sin embargo existen estudios de investigación que demuestran que a muchos niños se les dificulta el reconocer estas formas.

El propósito de este estudio es crear investigación sobre métodos instruccionales efectivos para enseñar destrezas de reconocimiento de formas a estudiantes con discapacidad cognoscitiva significativa. El objetivo es demostrar que, combinar las mejores prácticas en la enseñanza de reconocimiento de formas y las mejores prácticas en la enseñanza de estudiantes con discapacidad cognitiva significativa, resultan en la enseñanza efectiva de reconocimiento de de formas para estudiantes con una discapacidad intelectual significativa.

PROCEDIMIENTOS

En este investigación los estudiantes participaran de una serie de lecciones dirigidas al desarrollo de destrezas de reconocimiento de formas. Estas clases se ofrecen 5 días a la semana y duran 30 aproximadamente minutos. Las mismas se realizan en la escuela de su hijo durante el horario escolar regular. Todas las lecciones serán impartidas por la investigadora principal, quien es además, maestra de educación especial licenciada y tiene una vasta experiencia trabajando con estudiantes con discapacidad intelectual significativa. El tiempo aproximado de duración del estudio es de unas 6-8 semanas.

Las lecciones tienen como objetivo el desarrollar cuatro destrezas de reconocimiento de formas: (1) pareo de formas idénticas; (2) pareo de formas idénticas con diferentes tamaños y la misma orientación; (3) pareo de formas con diferente orientación y el mismo tamaño, y (4) reconocimiento de formas básicas. Estas lecciones incluyen la participación en actividades prácticas, diseñadas a partir del programa escalonado para preescolares *Building Blocks Pre-K* de Mac Graw Hill, que a su vez es un currículo basado en la investigación y desarrollado por la Fundación Nacional de Ciencias. Se escogió este currículo de preescolar porque tiene como objetivo el desarrollo de destrezas de reconocimiento de formas que los estudiantes aún no dominan. Se adaptará el currículo para hacerlo apropiado para estudiantes con discapacidad intelectual significativa de nivel elemental según sea necesario. Las adaptaciones pueden incluir el ajustar el tiempo de duración de la lección (proveer más tiempo para dominar un tema), utilizar asistencia tecnológica (por ejemplo un interruptor, o *switch* que permite al estudiante manejar una computadora), hacer ajustes apropiados a la edad (cambiar el tema de la actividad para que sea más apropiado para estudiantes que son un poco mayores). Al finalizar cada lección se realiza un avalúo para documentar el progreso de su hijo en cuanto al reconocimiento de formas.

RIESGOS

Debido a que este estudio envuelve aprender una nueva destreza (pareo de formas), existe un riesgo de tensión o estrés mental. Sin embargo, este riesgo es mínimo porque las lecciones son cortas y se proveerán recesos frecuentes según sea necesario.

Como parte de este estudio, a los investigadores les gustaría obtener acceso a archivos académicos privados –específicamente el Programa Educativo Individualizado, o *IEP*. Necesitamos permiso para acceder al *IEP* de su hijo y así obtener una descripción general de su discapacidad y los resultados de la prueba de Coeficiente Intelectual, o *IQ,* para determinar si su hijo es un buen candidato para este estudio. El riesgo también es mínimo porque no habrá información de identificación personal conectada a la información académica del estudiante. Toda la información de identificación personal se ennegrecerá para que no se pueda leer y el nombre de cada participante se reemplazara con un pseudónimo. Los documentos de *IEP* se guardaran en un archivo bajo llave, o, en formato electrónico y estarán protegidos en una computadora que requiera contraseña de acceso. Usted puede solicitar y ser provisto de una copia del *IEP* divulgado o utilizado.

BENEFICIOS

A través de la participación en este estudio su hijo estará aprendiendo destrezas básicas de geometría. Estas destrezas son fundamentales para aprender temas matemáticos más avanzados. El desarrollo de destrezas matemáticas resultará en beneficios continuos en diferentes áreas de su vida incluyendo la escuela, trabajo y comunidad.

PAGO A PARTICIPANTES

Los participantes no serán remunerados por su participación en este estudio.

CONFIDENCIALIDAD DEL PARTICIPANTE

El nombre de su hijo/a no estará asociado a ninguna publicación o presentación que contenga la información recogida sobre el/ella, o, los resultados de este estudio de investigación. El investigador utilizara un pseudónimo en vez del nombre de su hijo/a. La información de identificación de su hijo/a no se compartirá a menos que (a) sea un requisito legal o parte de la política de la universidad, o, (b) que usted otorgue su consentimiento por escrito.

Todos los documentos impresos con información o los datos recogidos durante el estudio serán debidamente almacenados en un archivo bajo llave. Los documentos impresos serán escaneados convirtiéndolos en documentos electrónicos que serán debidamente almacenados en la computadora del investigador principal que, a su vez, está asegurada por una contraseña. Los documentos impresos serán triturados una vez convertidos en documentos electrónicos. El investigador principal es la única persona que tiene acceso a los documentos.

El permiso de utilizar y divulgar su información que usted concede en la fecha de hoy, queda en efecto inmediatamente. Al firmar este formulario usted otorga permiso de utilizar y divulgar información sobre su hijo/a para propósitos de este estudio, y en cualquier momento en el futuro, a exclusión del nombre de su hijo/a.

DECLARACIÓN DE EXENCIÓN DE RESPONSABILIDAD INSTITUCIONAL

En caso de lesión, la Ley de Reclamaciones Tort de Kansas provee compensación si se puede demostrar que la lesión fue causada por un acto negligente, ilícito o por omisión de un empleado estatal que esté actuando dentro de los parámetros de su empleo.

DENEGACIÓN DE FIRMAR ESTE CONSENTIMIENTO Y AUTORIZACIÓN

Usted no está obligado a firmar este formulario de Consentimiento y Autorización y puede negarse a hacerlo sin que esto afecte su derecho a recibir servicios que esté recibiendo o pueda recibir de la Universidad de Kansas o de participar en programas o eventos de la misma Universidad. Sin embargo, si usted se niega a firmar su hijo no podrá participar de este estudio.

CANCELACIÓN DEL CONSENTIMIENTO Y AUTORIZACIÓN

El tiempo estimado de duración del estudio es de 8 semanas, pero esto varia de niño en niño. Usted puede retirar el consentimiento de participación de su hijo en el estudio en cualquier momento. Además tiene derecho de cancelar, en cualquier momento y por escrito, el permiso de divulgar la información recogida sobre su hijo/a enviando su solicitud escrita a: Kristin Joannou Lyon, 1200 Sunny Side Ave., 3150 Hayworth Hall, Universidad de Kansas, Lawerence, KS 66045

Si cancela el permiso de utilizar la información de su hijo/a los investigadores suspenderán el recogido de información adicional sobre su hijo. Sin embargo, el equipo de investigación puede utilizar la información recogida antes de recibir la cancelación como antes descrita.

PREGUNTAS SOBRE LA PARTICIPACIÓN

Cualquier pregunta sobre la participación puede ser dirigida a cualquiera de los investigadores que aparecen al final de este formulario de consentimiento.

CERTIFICADO DE PARTICIPACIÓN:

Yo he leído este formulario de Consentimiento y Autorización. He tenido la oportunidad de hacer preguntas, y he recibido respuestas a las preguntas relacionadas con el estudio. Entiendo que si tengo dudas adicionales sobre los derechos de mi hijo/a como participante en este estudio, puedo comunicarme con El Comité de Sujetos Humanos de Lawrence (HSCL por sus siglas en ingles) llamando por teléfono al (785) 864-7429, por correo al Human Subjects Committee Lawrence Campus (HSCL), University of Kansas, 2385 Irving Hill Road, Lawrence, Kansas 66045-7568, o a través del correo electrónico irb@ku.edu.

Yo autorizo a que mi hijo/a participe de este estudio de investigación. Al firmar este documento estoy confirmando que tengo al menos 18 años de edad y que he recibido copia de este formulario de Consentimiento y Autorización

_________________________________	___________________
Nombre de Participante en Letra de Molde	Fecha

____
Firma del Padre/Custodio/Persona Encargada

Información de Contacto de los Investigadores

Kristin Joannou Lyon	Wayne Sailor, Ph.D.
Principal Investigator	Faculty Supervisor
Department of Special Education	Department of Special Education
1200 Sunnyside Avenue	1122 W. Campus Rd.
3150 Hayworth Hall	Joseph R. Pearson Hall, Rm 541
University of Kansas	University of Kansas
Lawrence, KS 66045	Lawrence, KS 66045
785 864 -0594	785 864 -4950

Individual Assent Procedure

Teaching Shape Recognition to Students with Significant Intellectual Disabilities

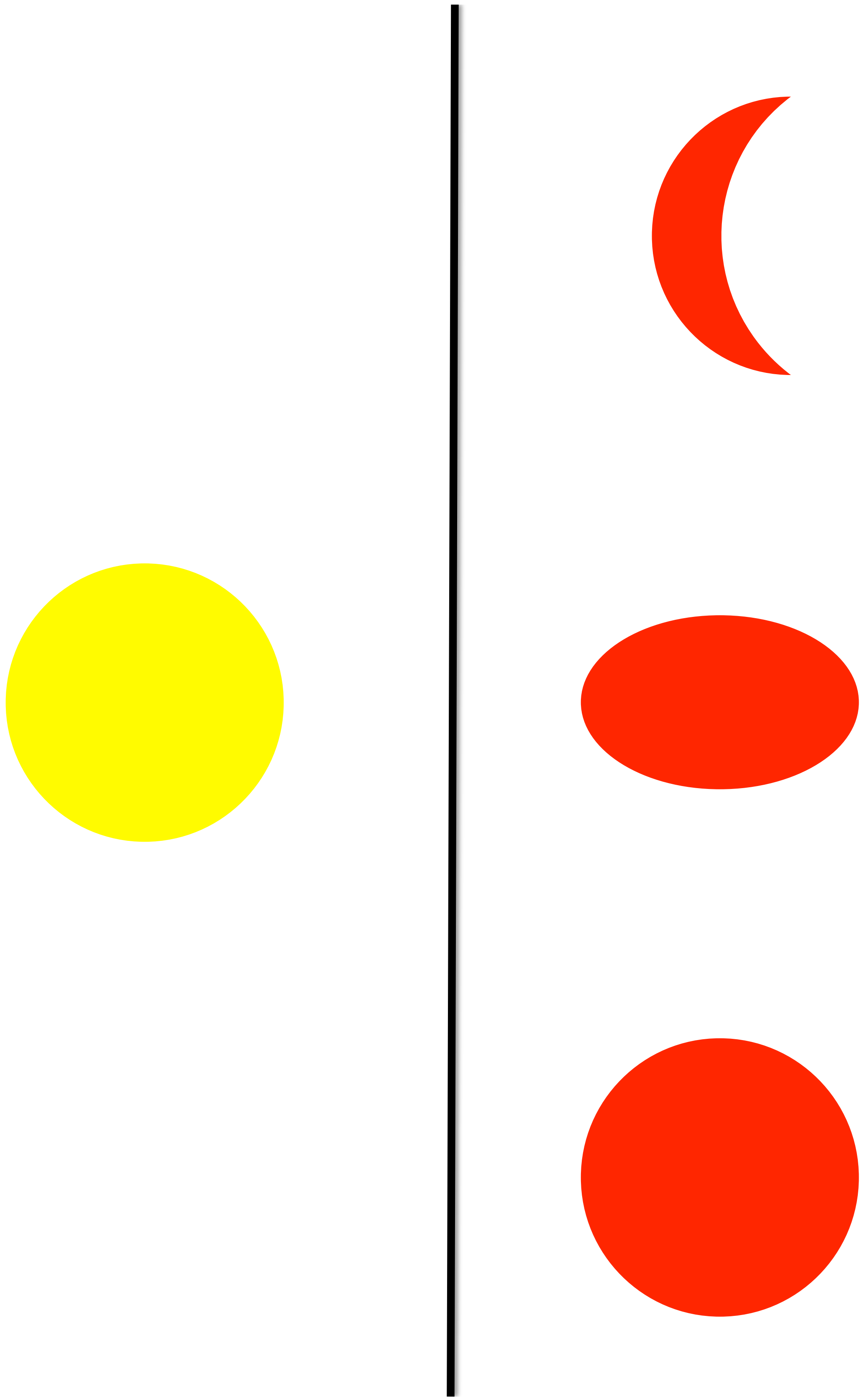
The participants of this research project are students with significant intellectual disabilities. The students may likely have limited verbal skills and communicate through a combination of gestures, utterances, pointing to picture, and/or using AAC devices. Students will respond using gestures, utterances and/or pointing to pictures indicating yes or no.

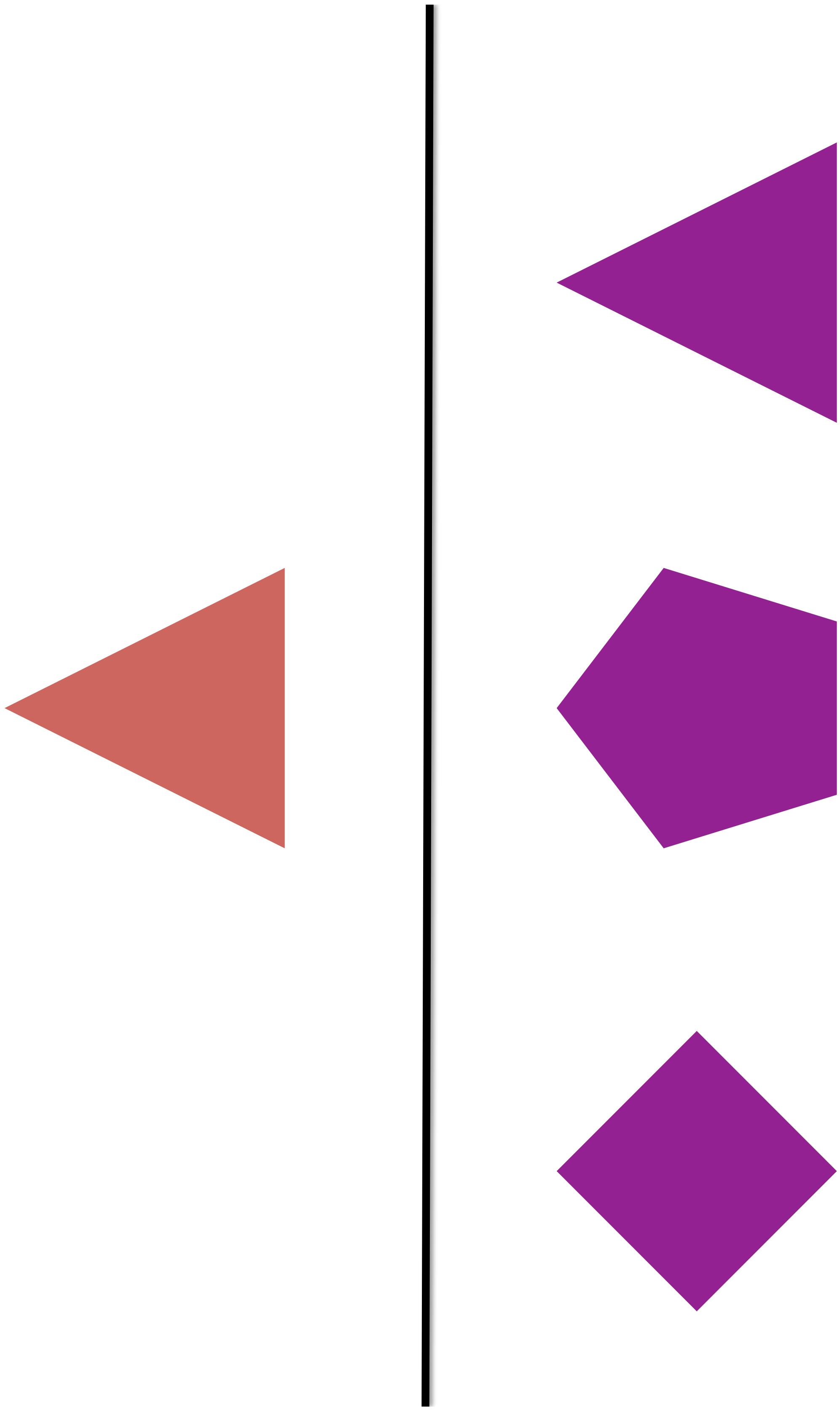
The following information will be read to the student:

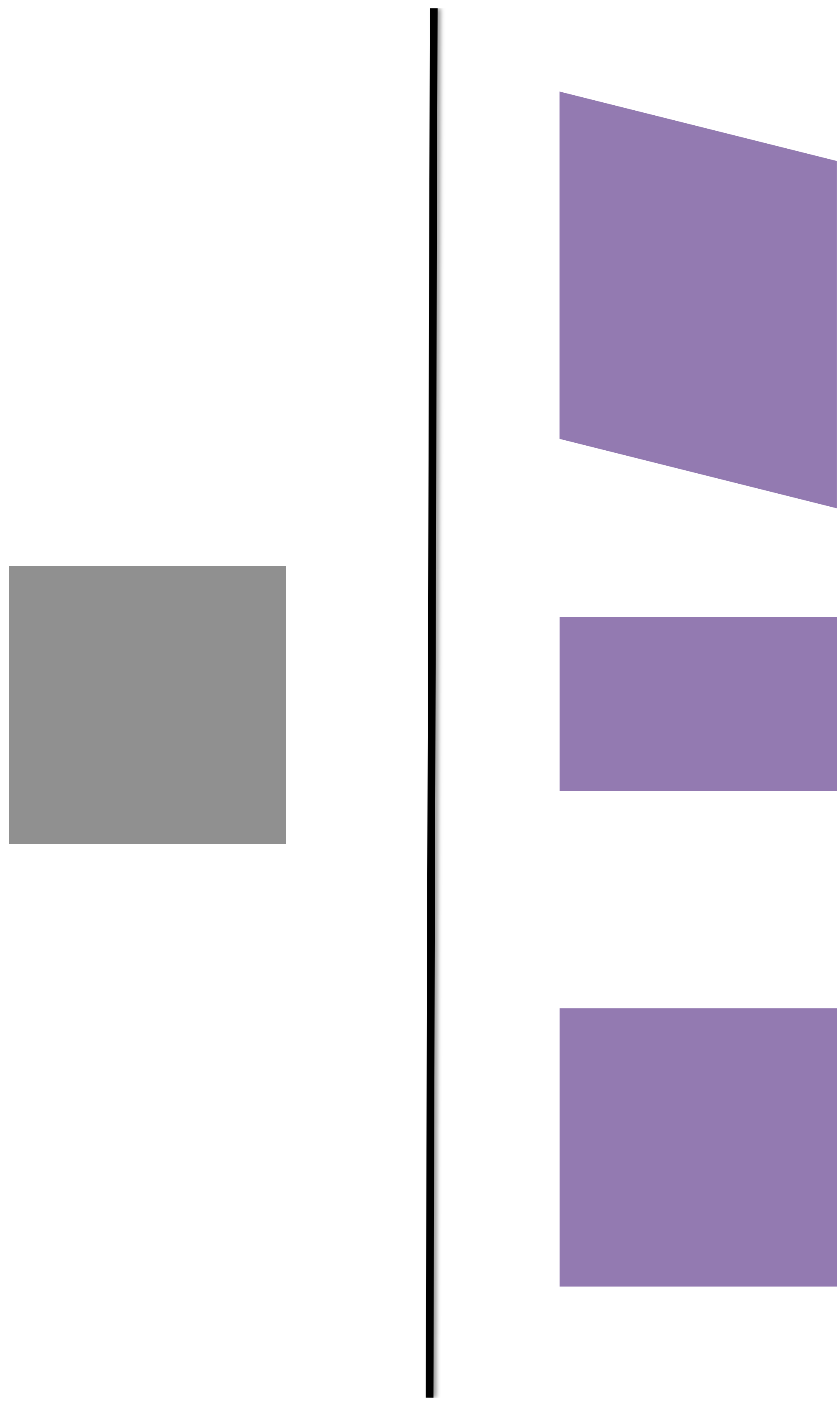
My name is Kristin, and I am learning about teaching shapes to students. I would like you to teach you about shapes. Lessons will take about 30 minutes. I will teach you every school day for several weeks. In each lesson I will do an activity with you and then give you a chance to show me what you've learned. If you don't feel like participating, you don't have to. You can stop at any time and that will be all right. Do you want to take part in these lessons?

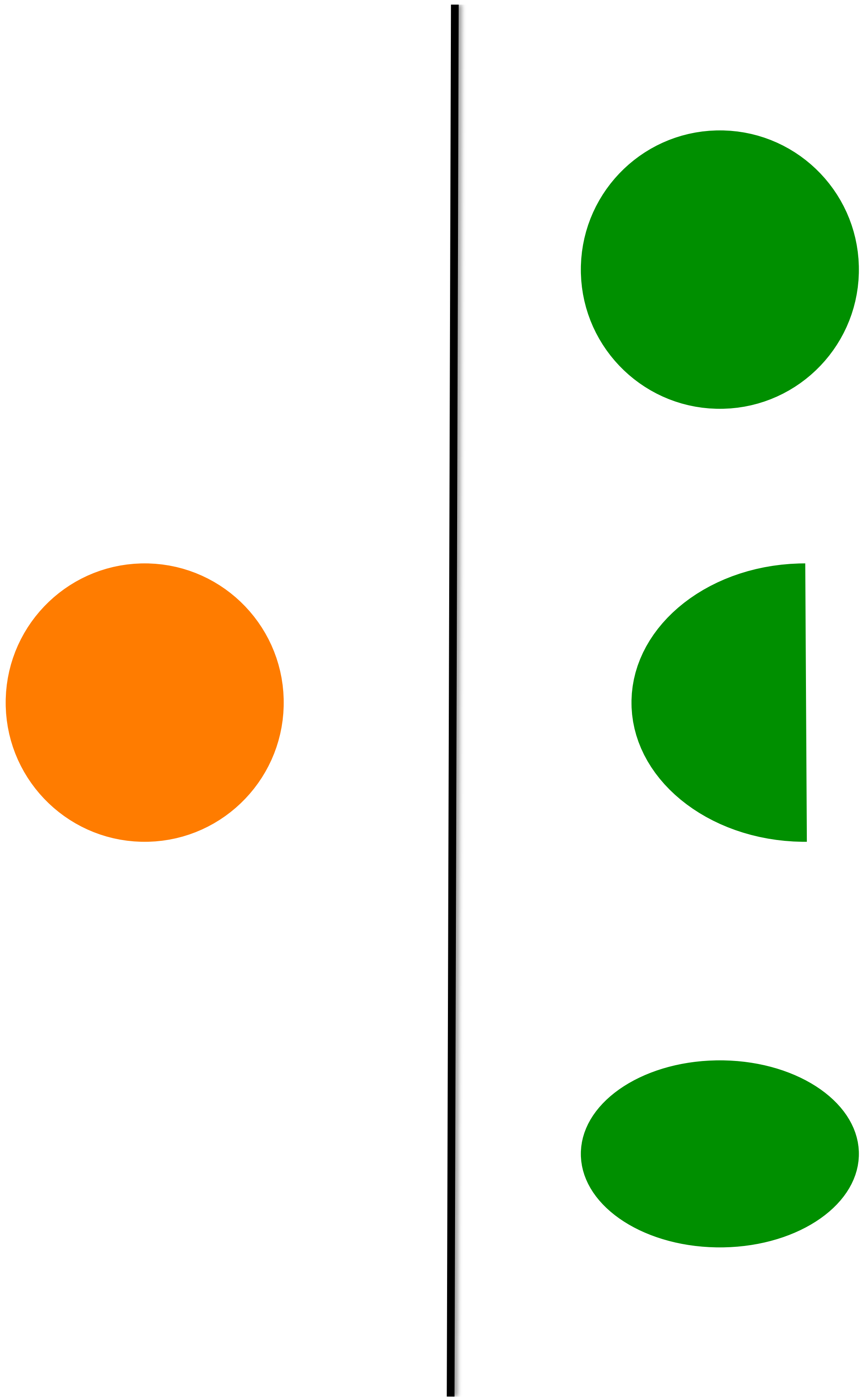
APPENDIX B

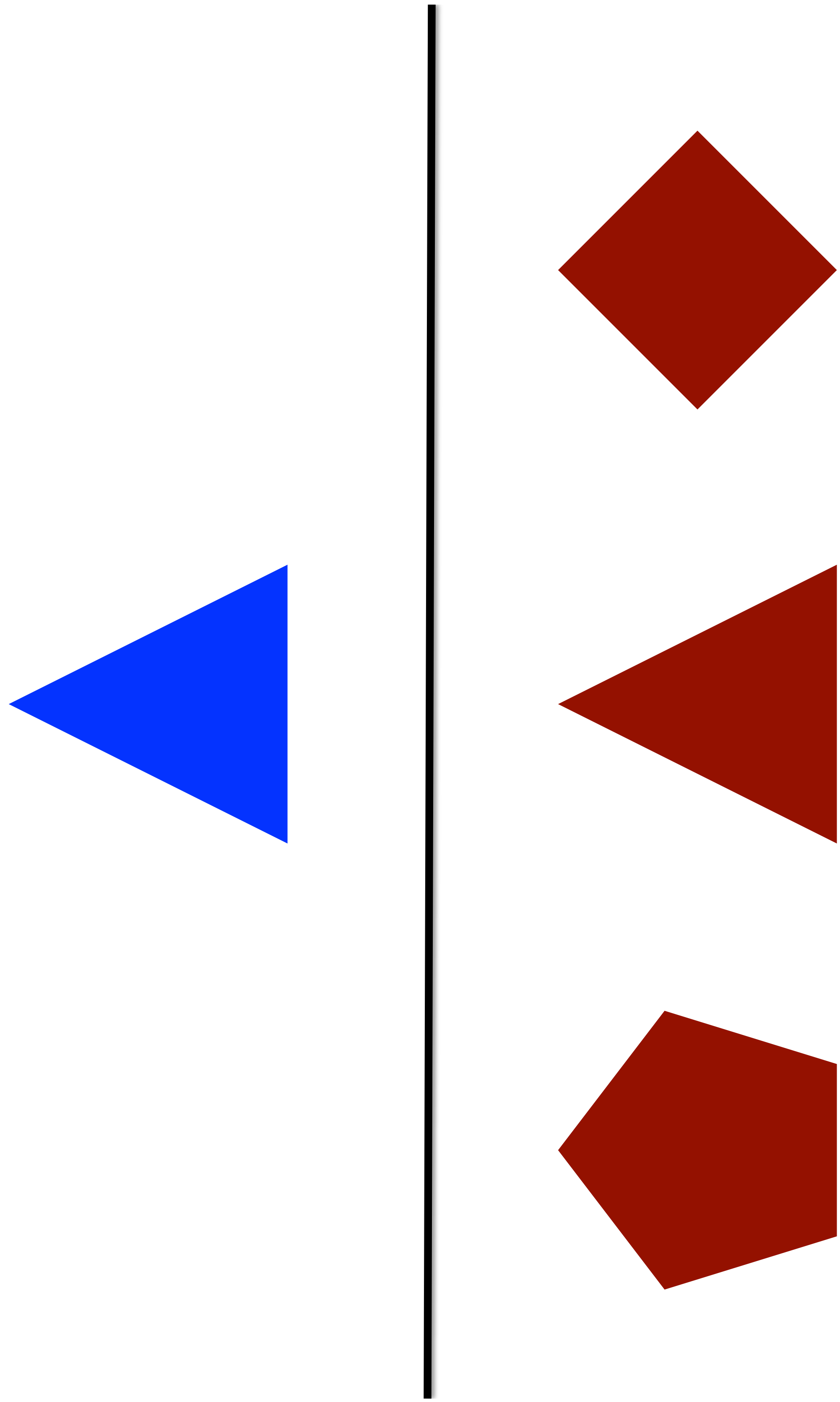
Matching Identical Shapes Assessment

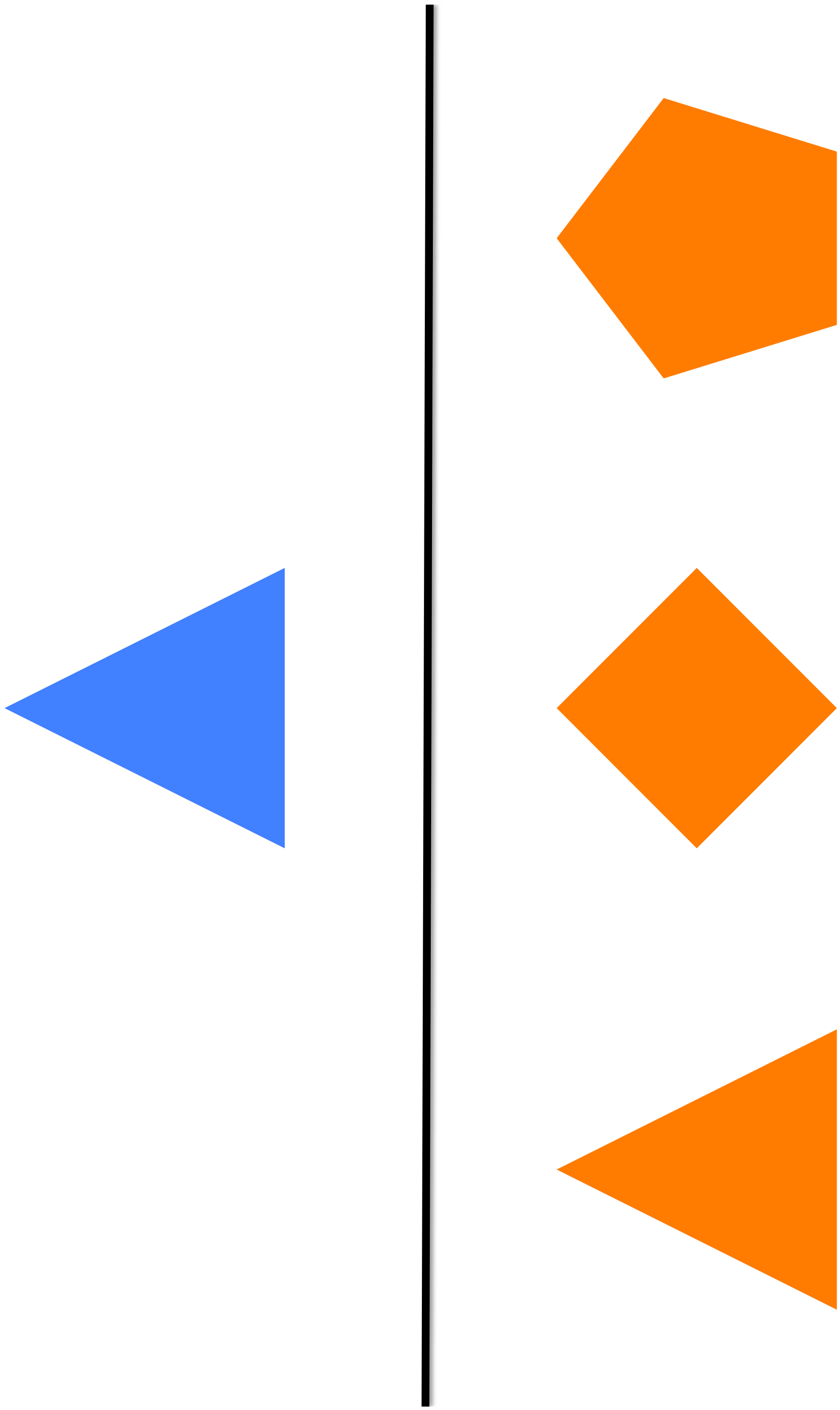


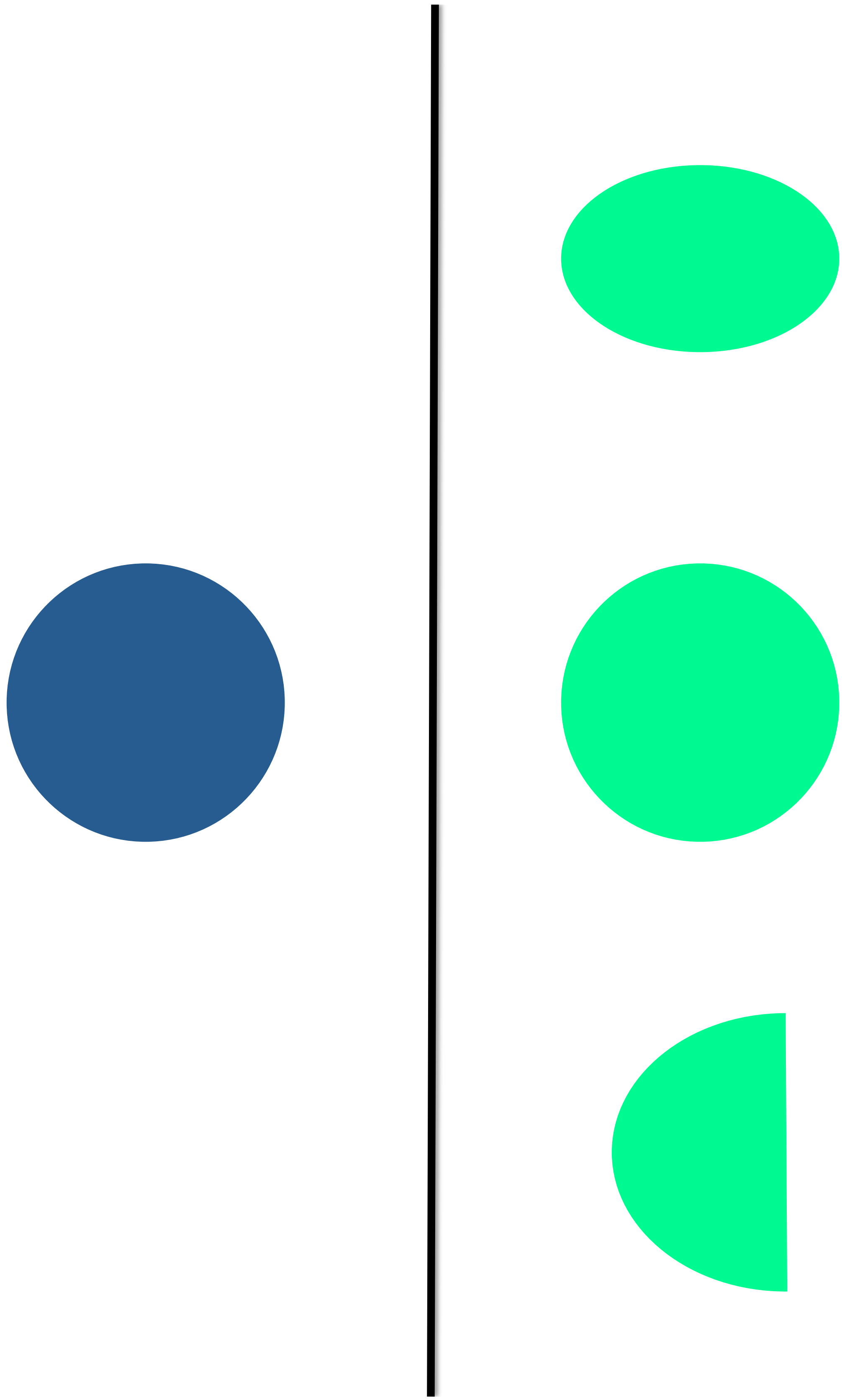


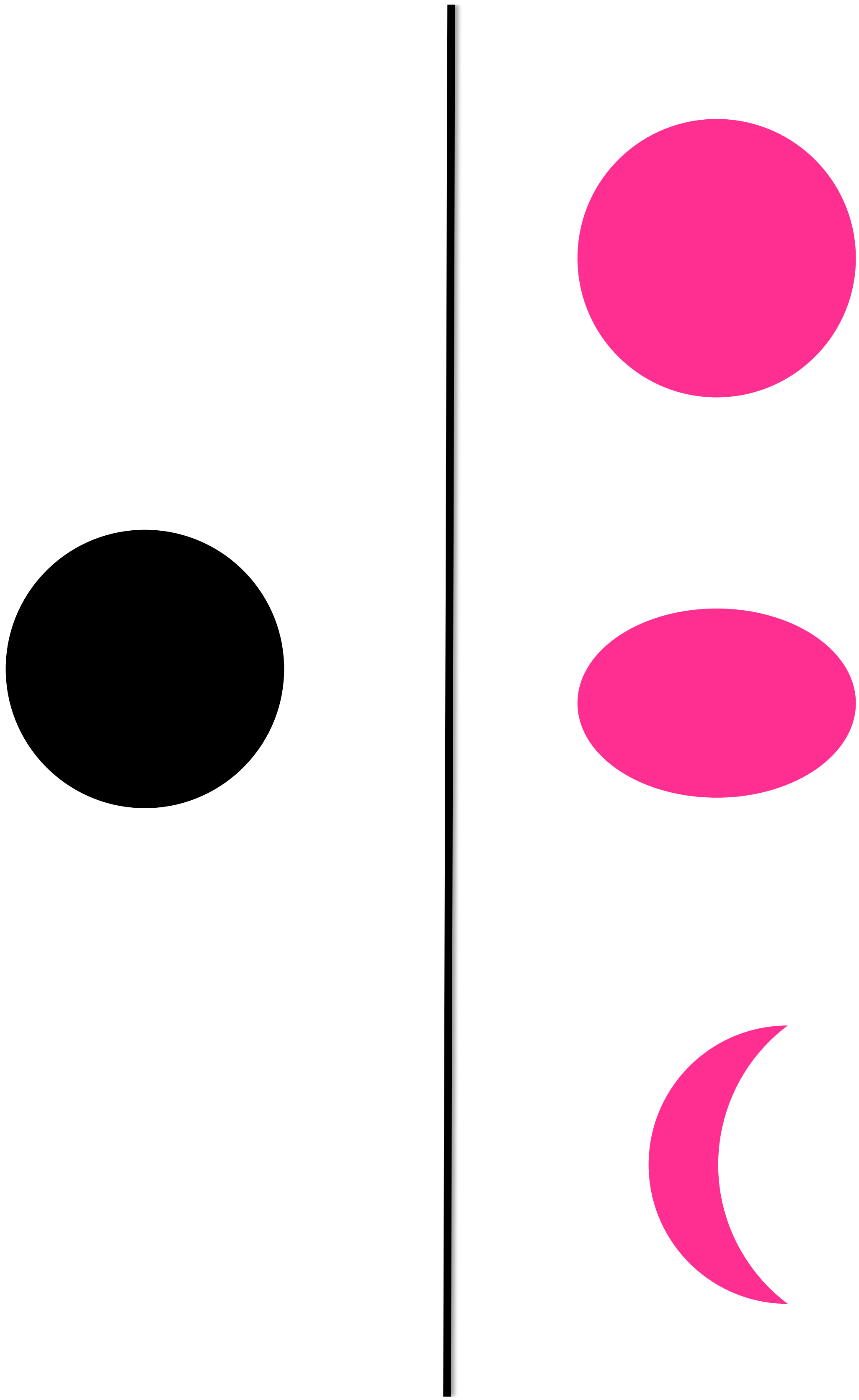


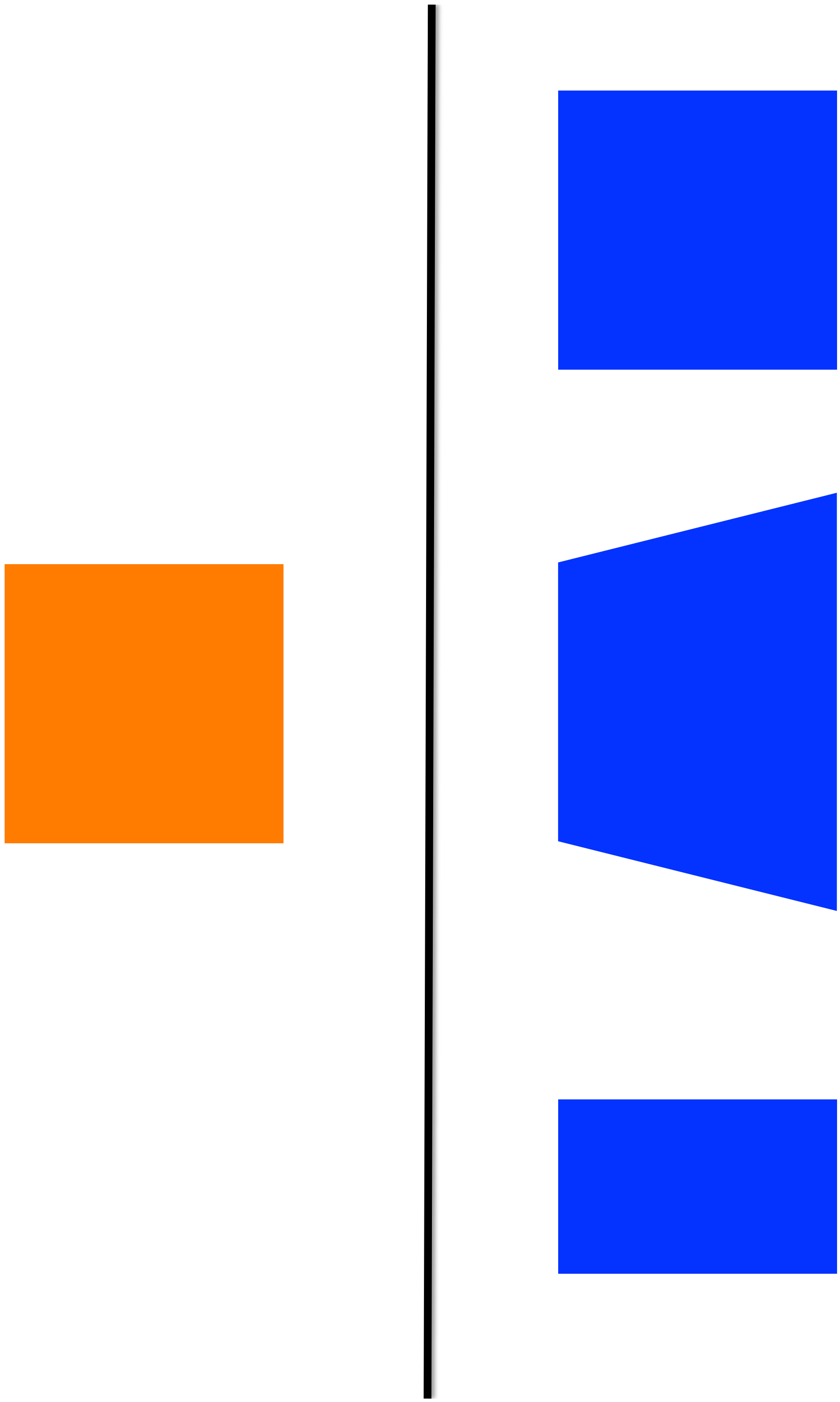






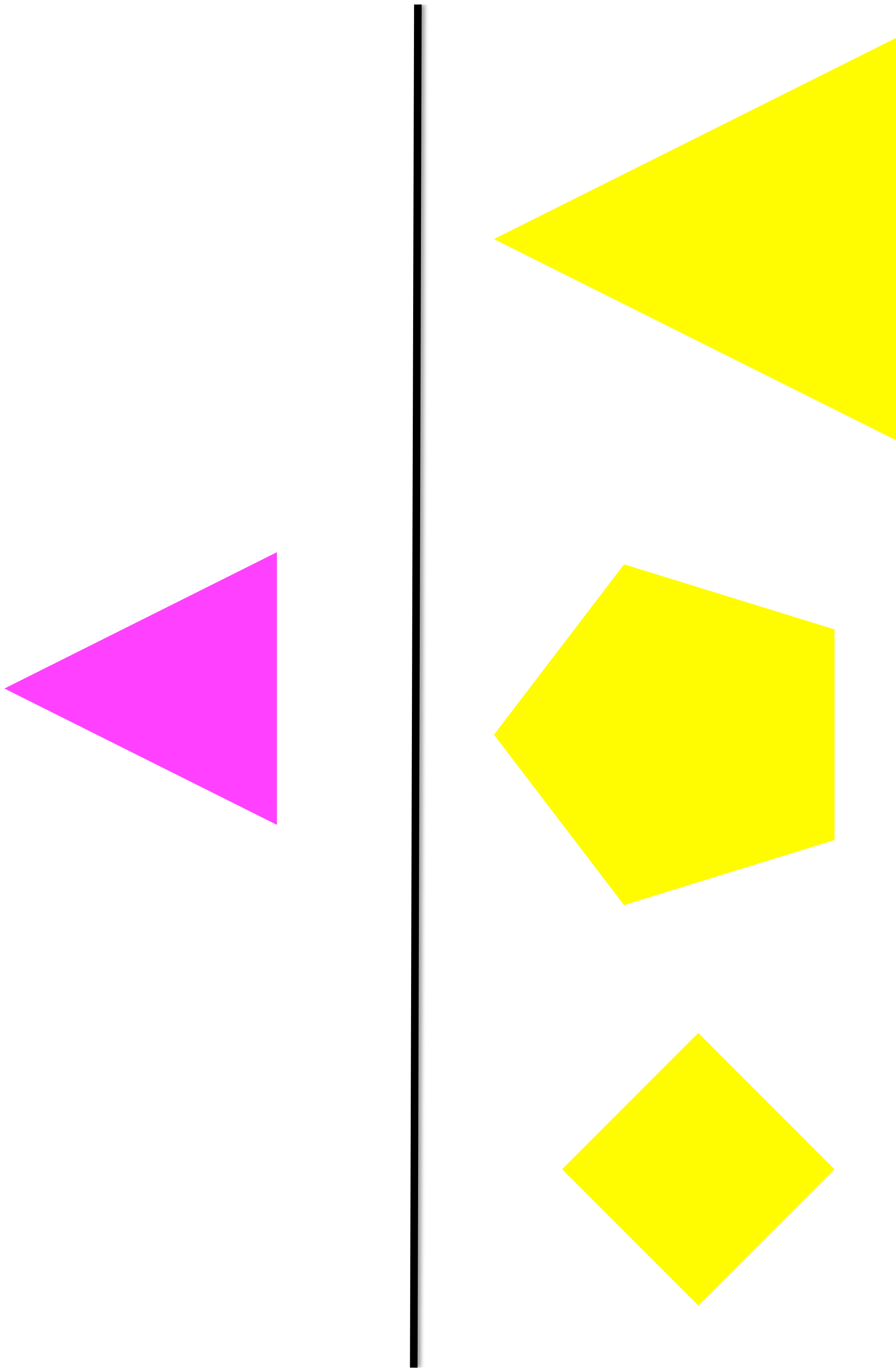


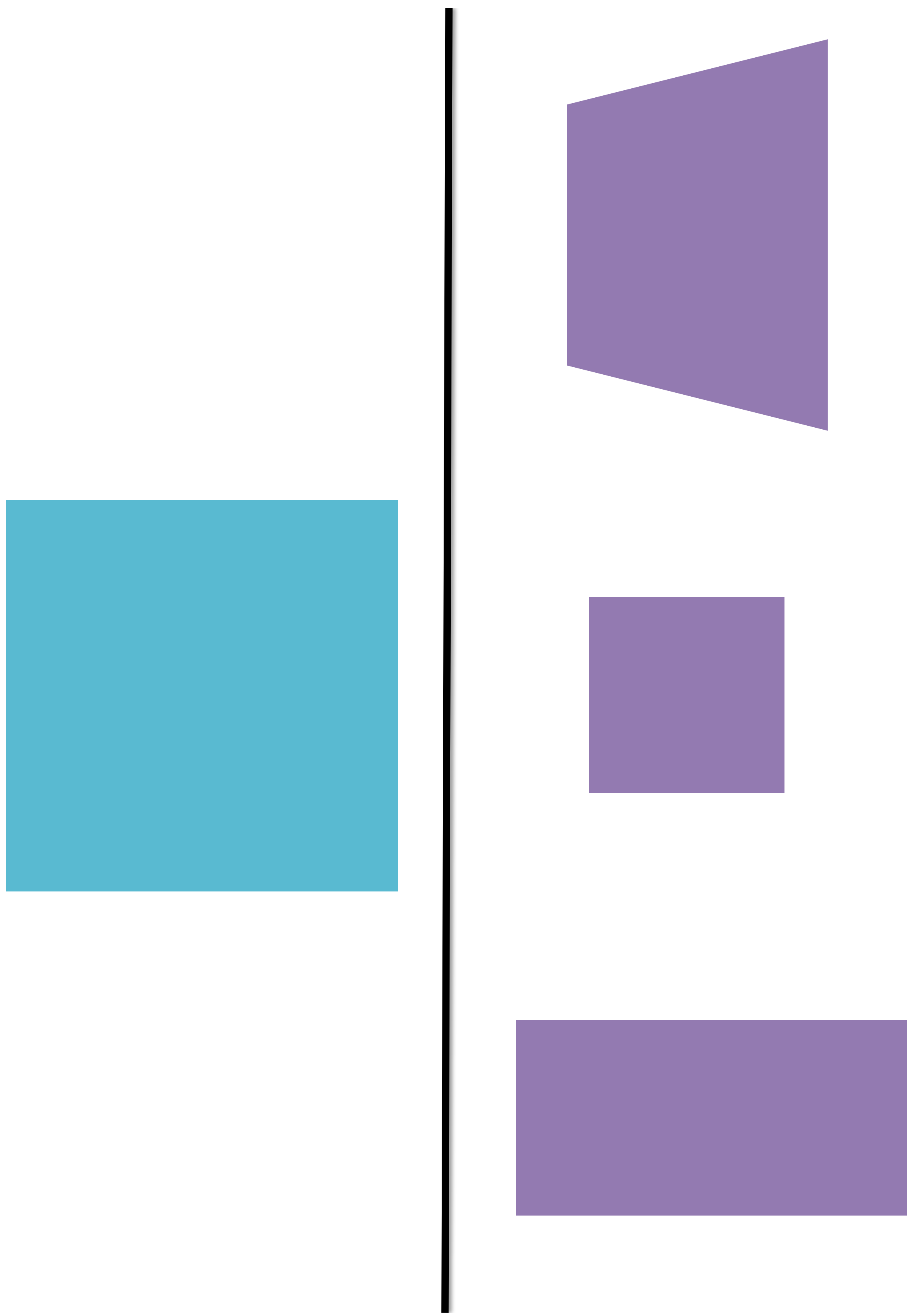


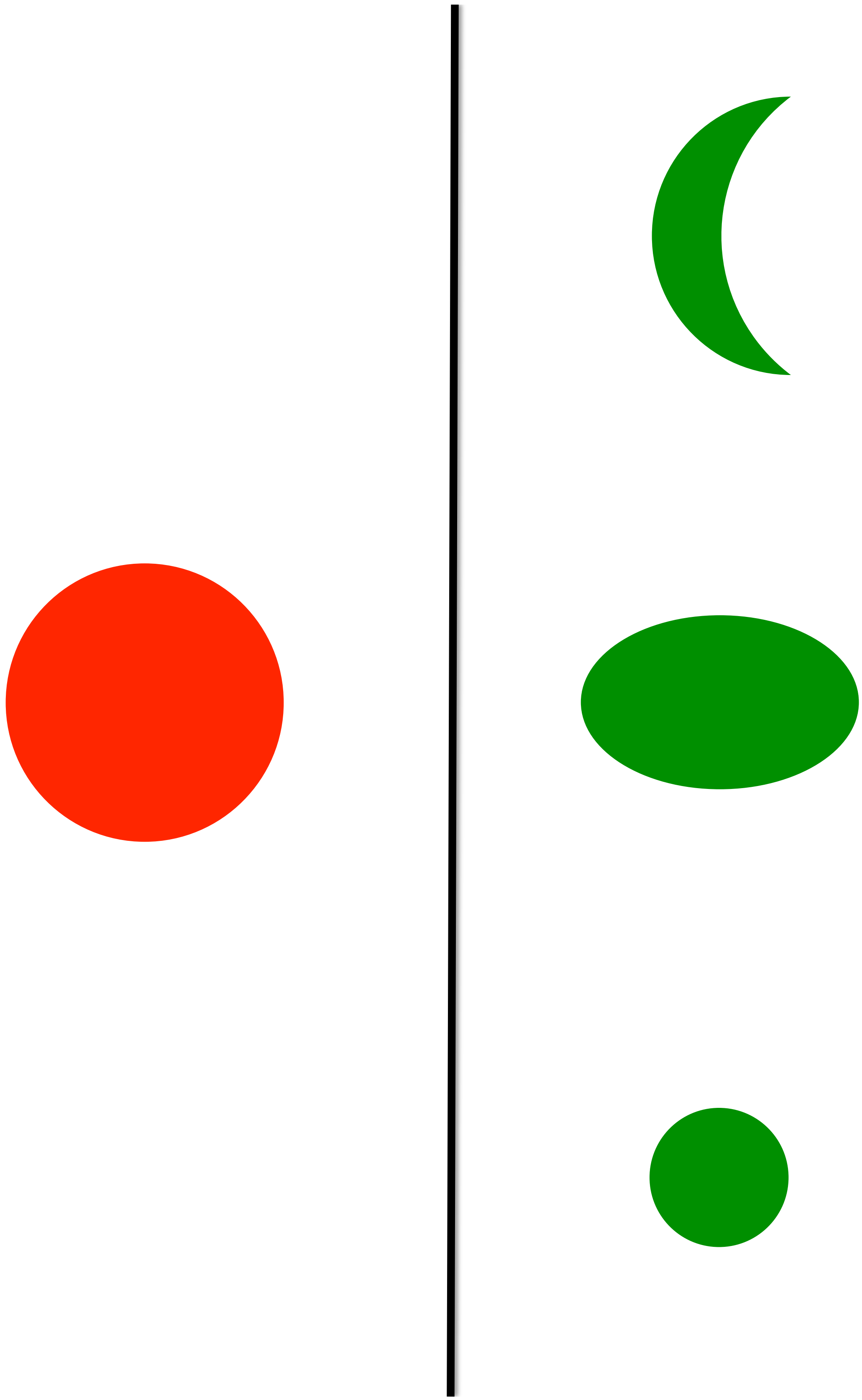


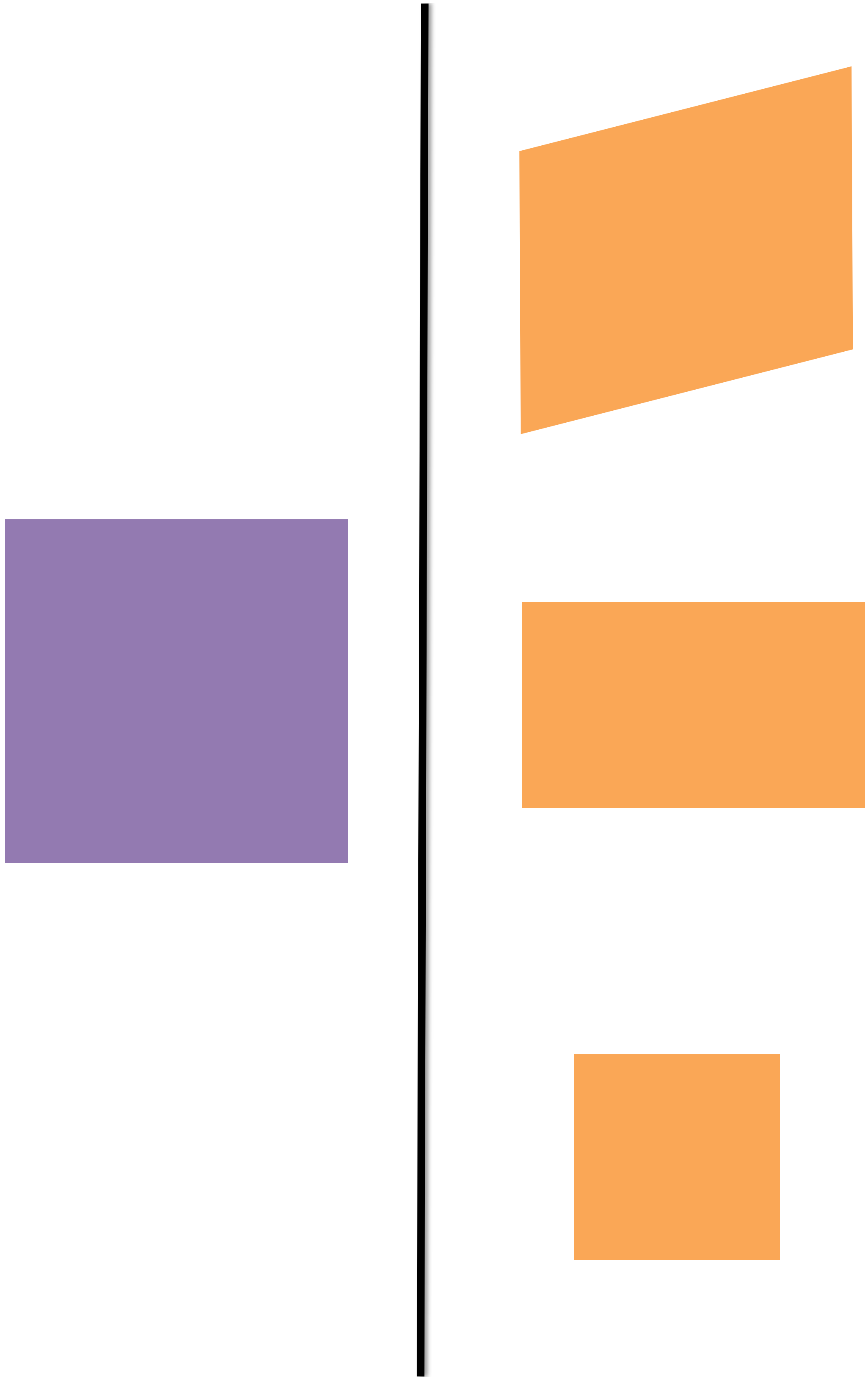
APPENDIX C

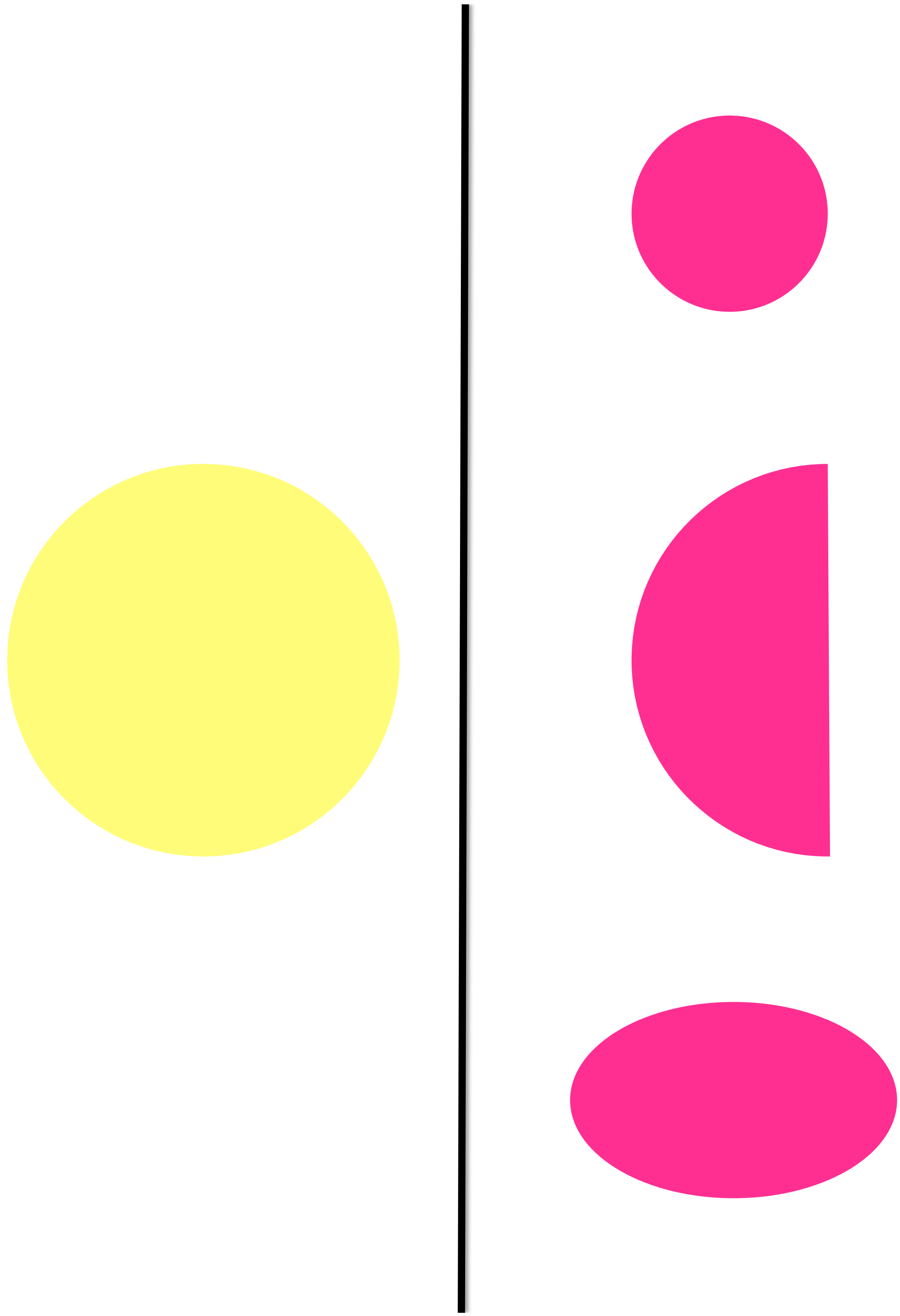
Matching Shapes with Different Sizes Assessment

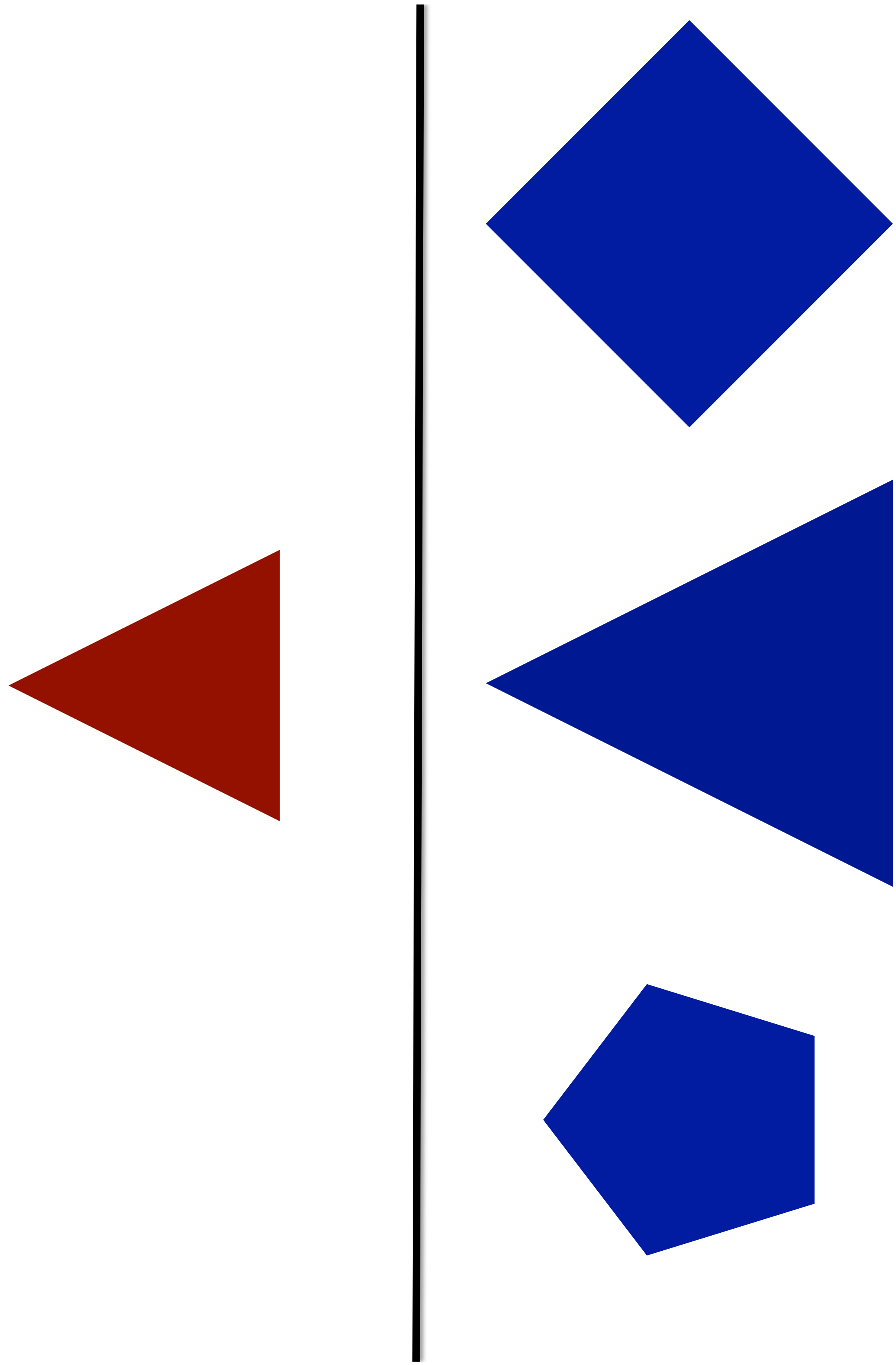


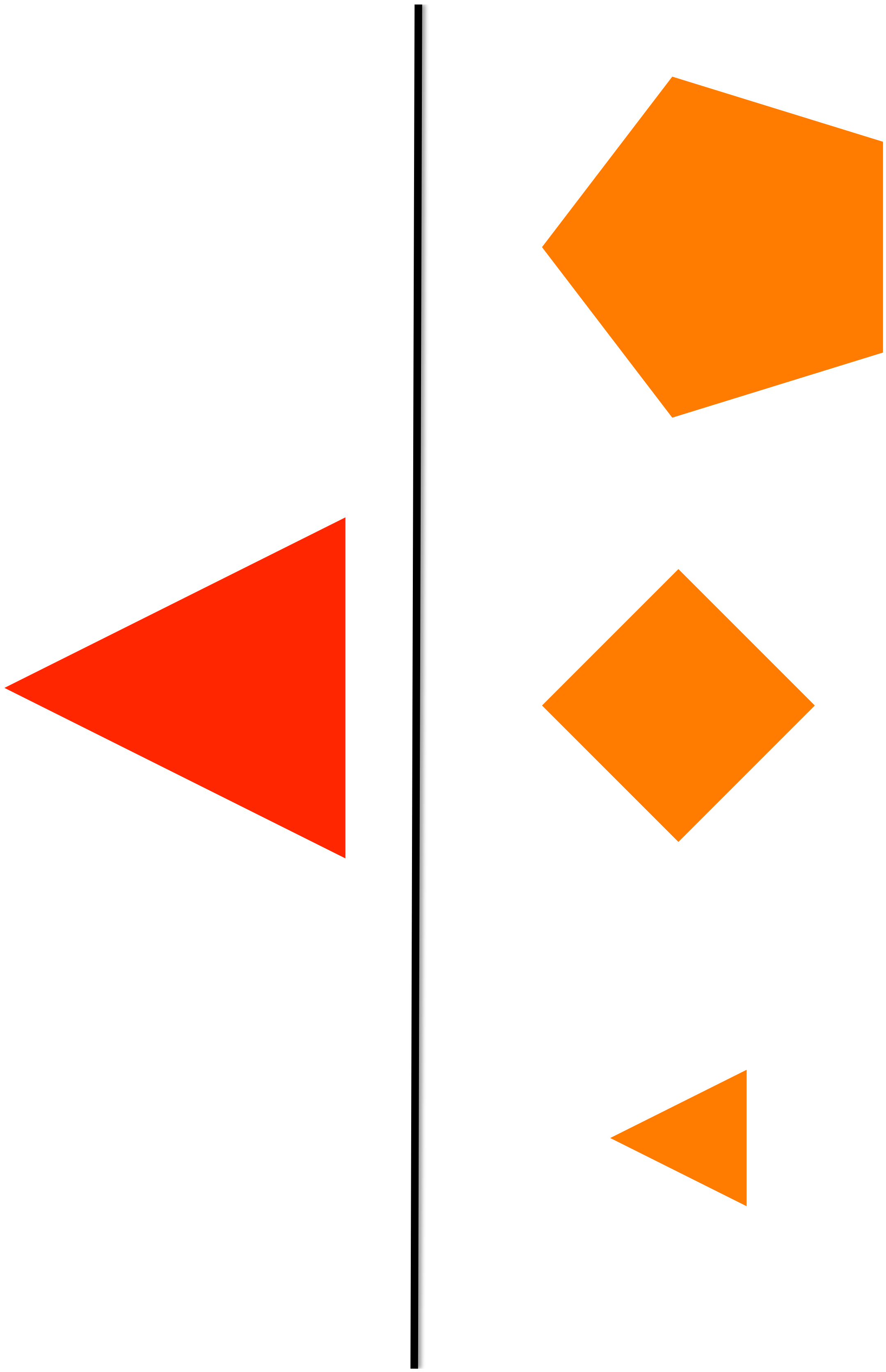


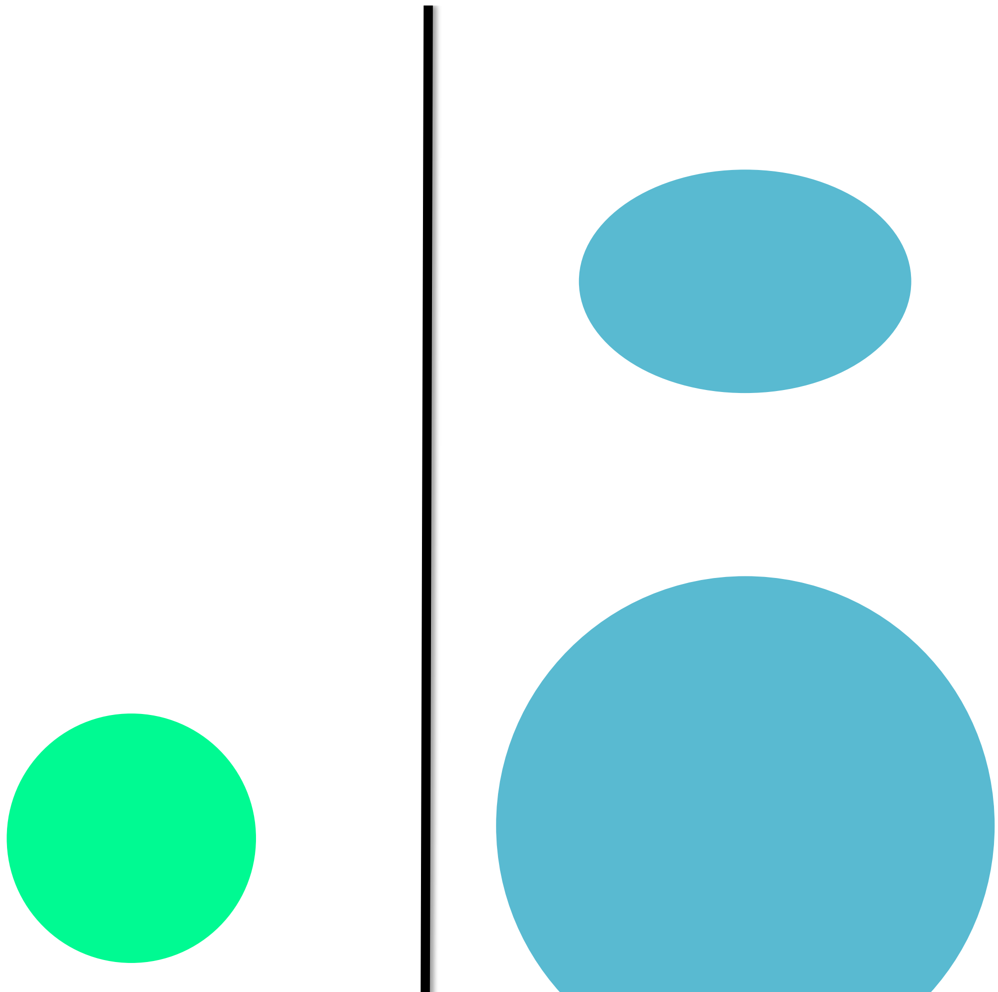


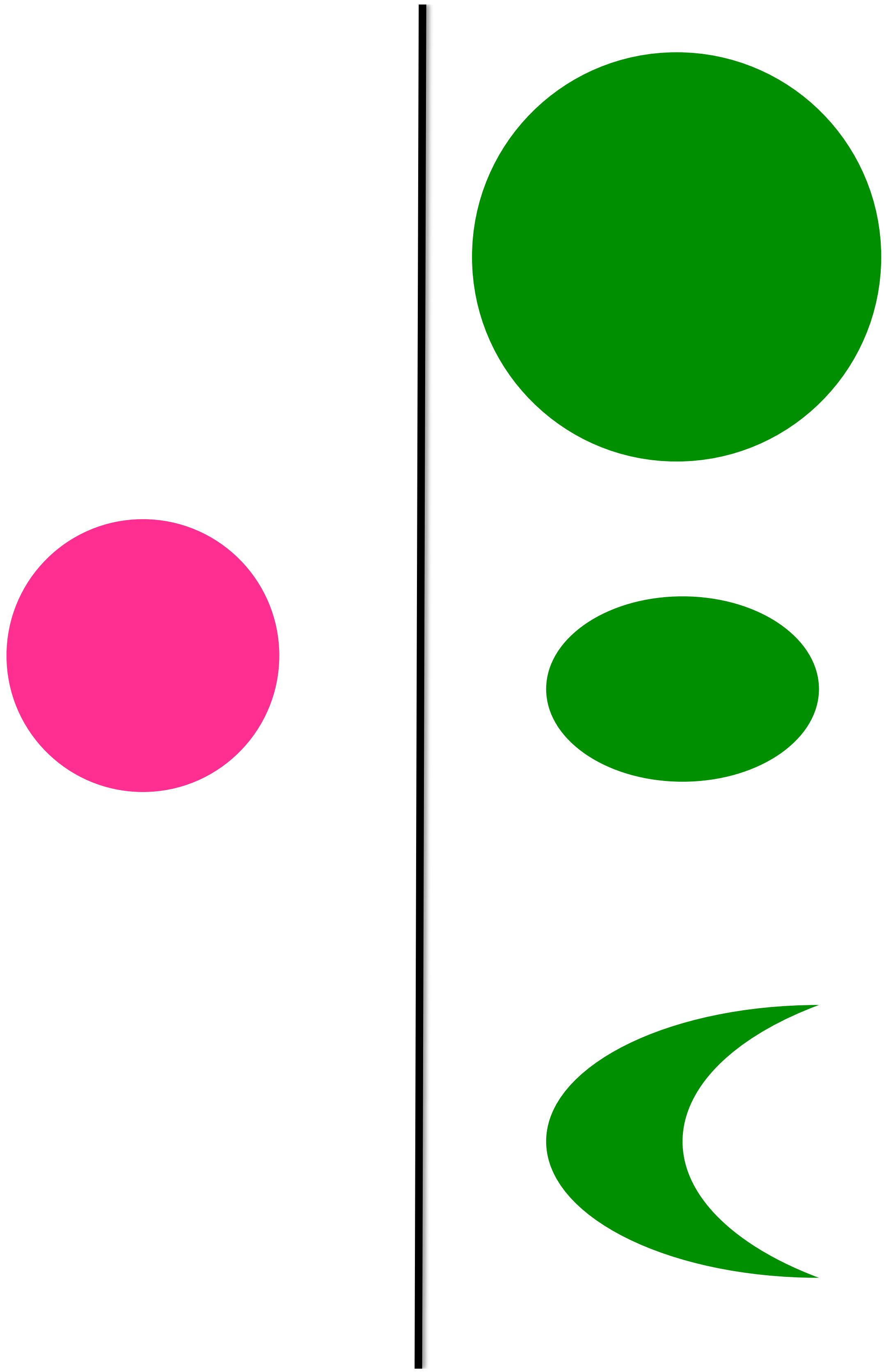


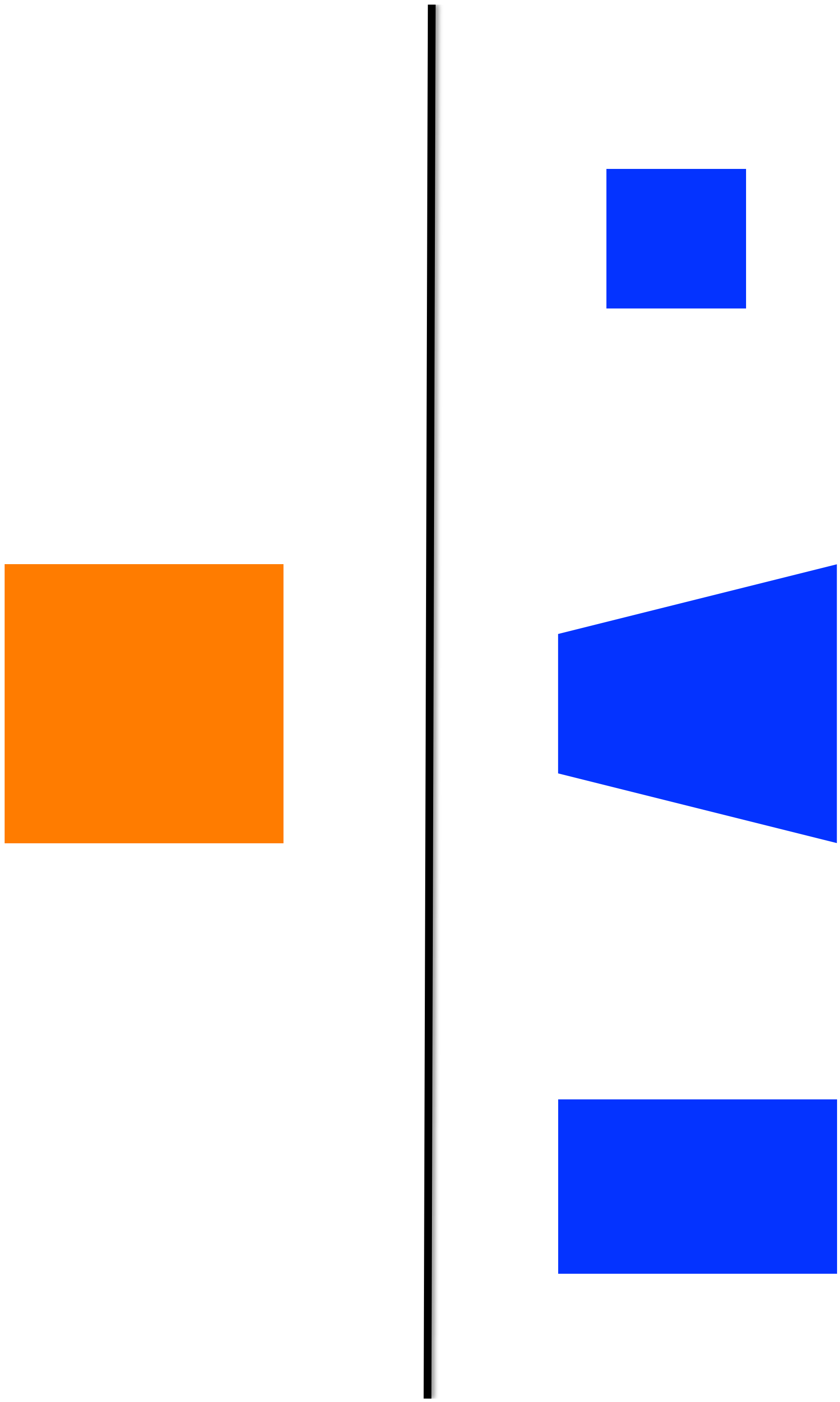






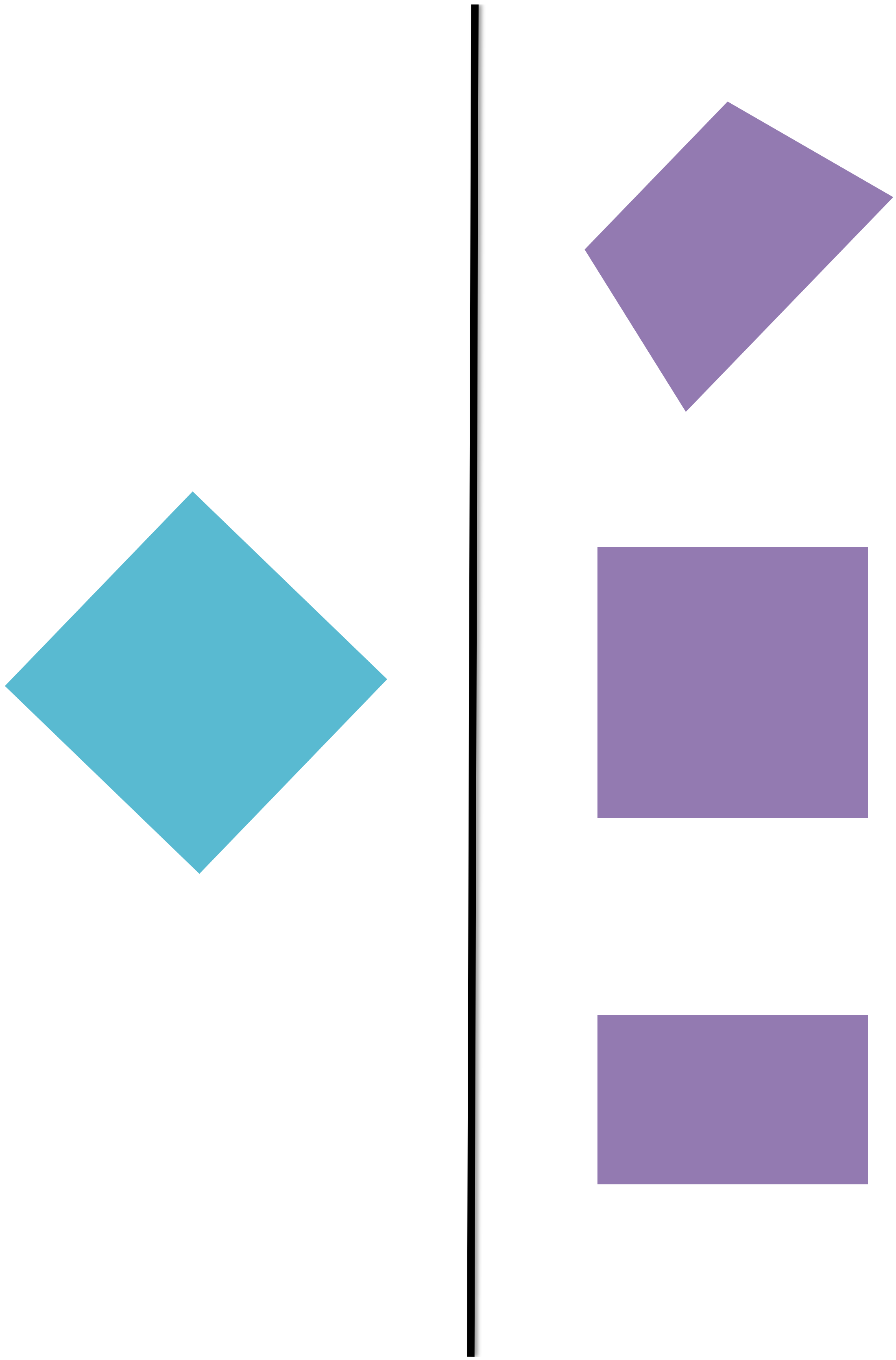


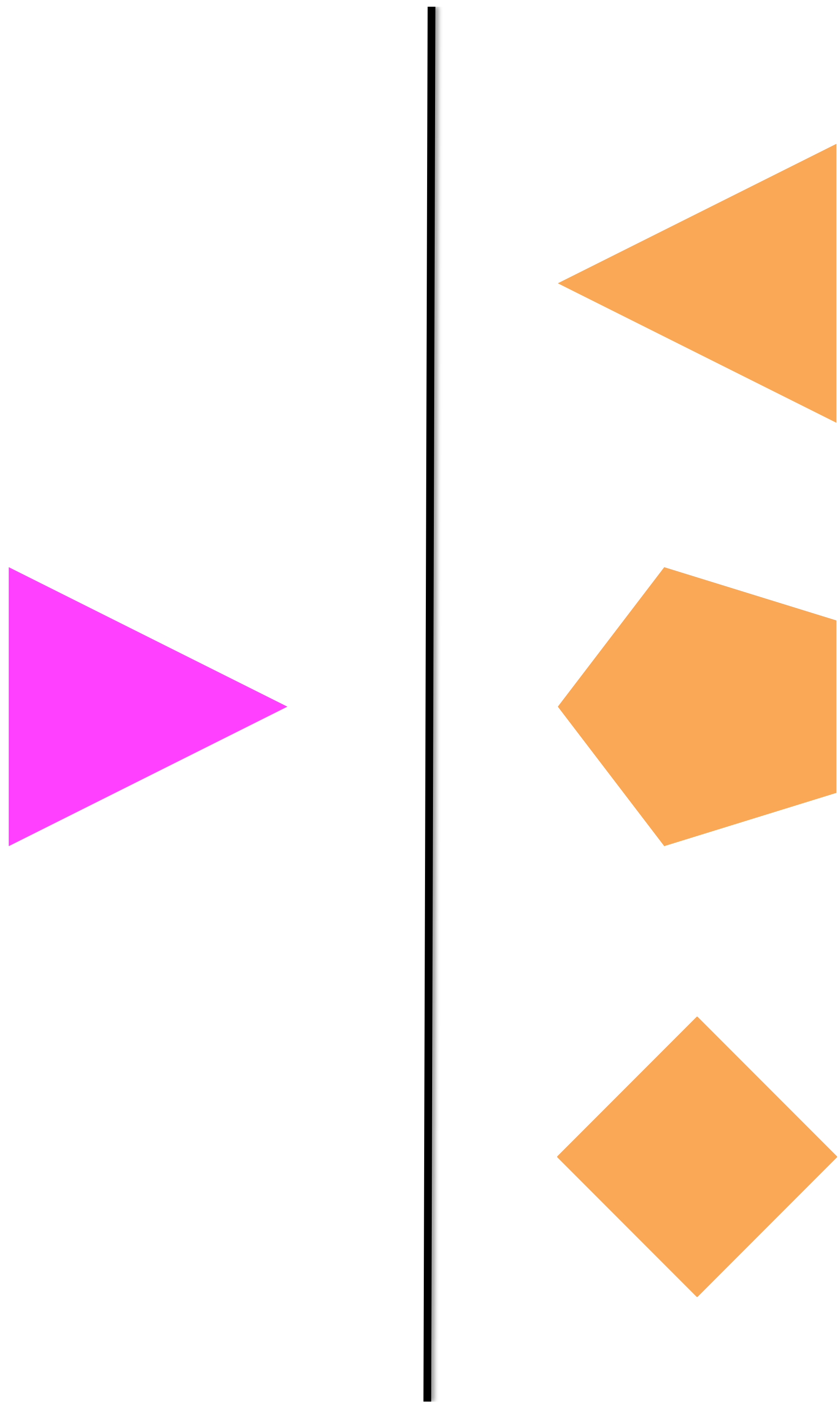


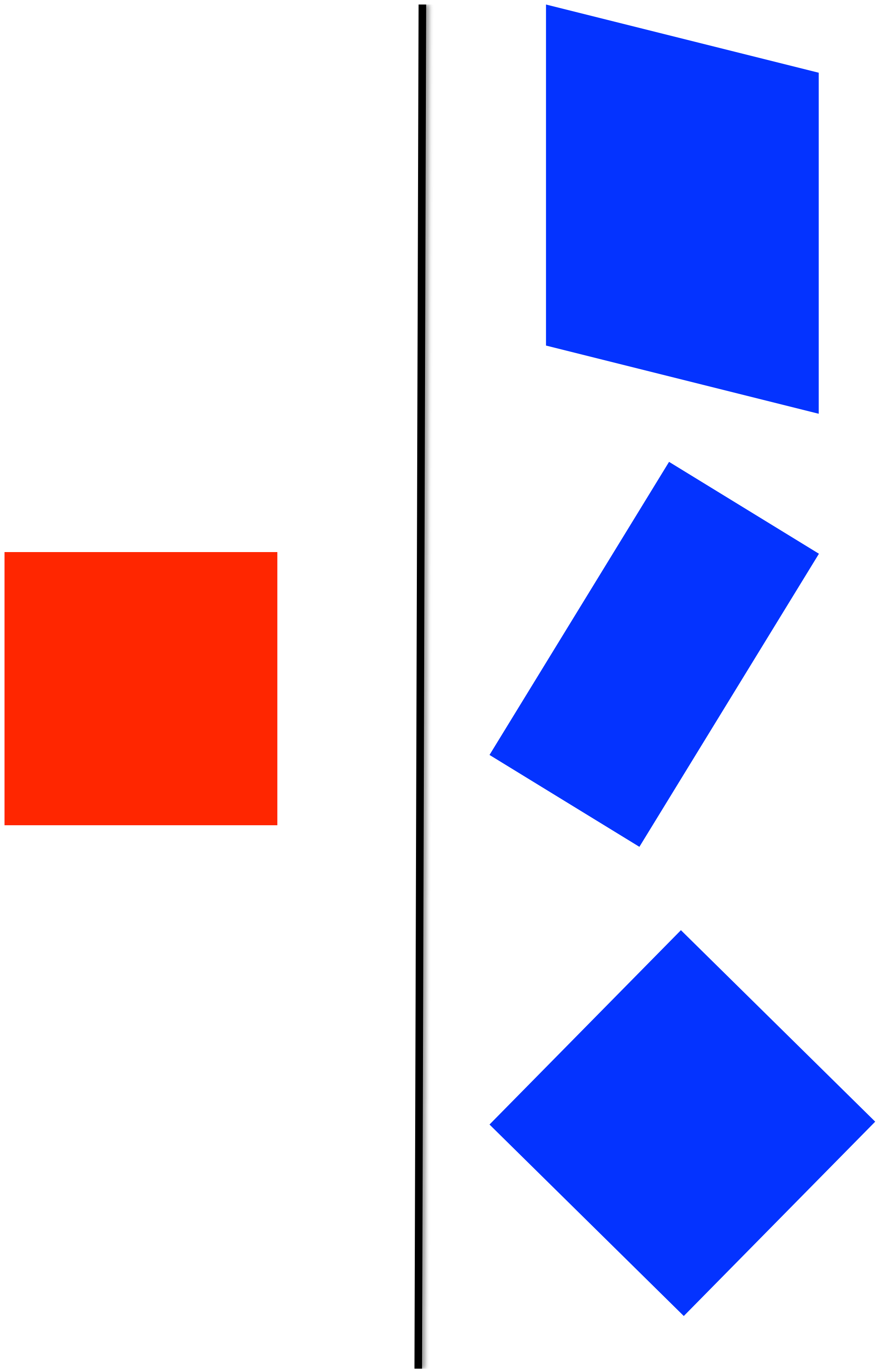


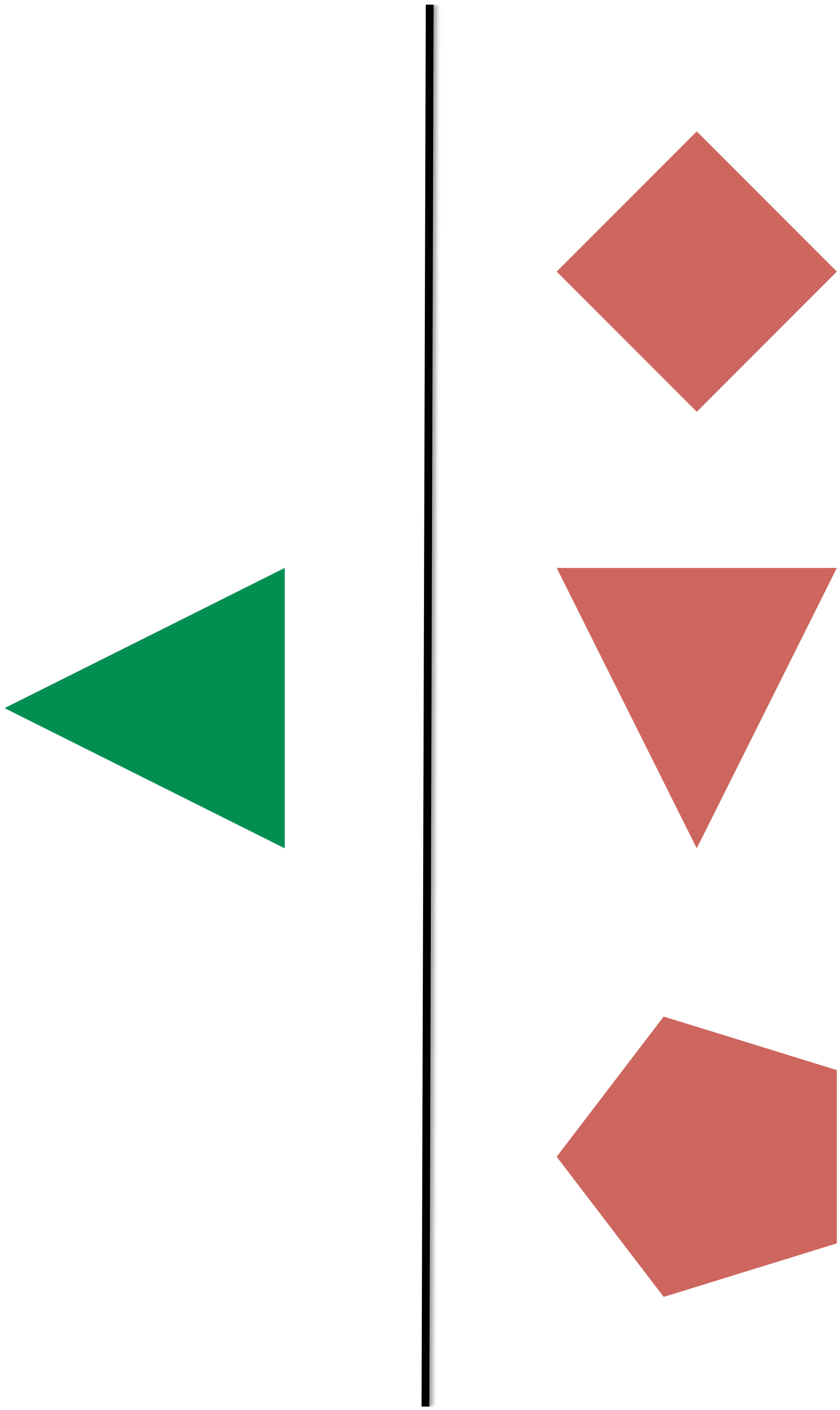
APPENDIX D

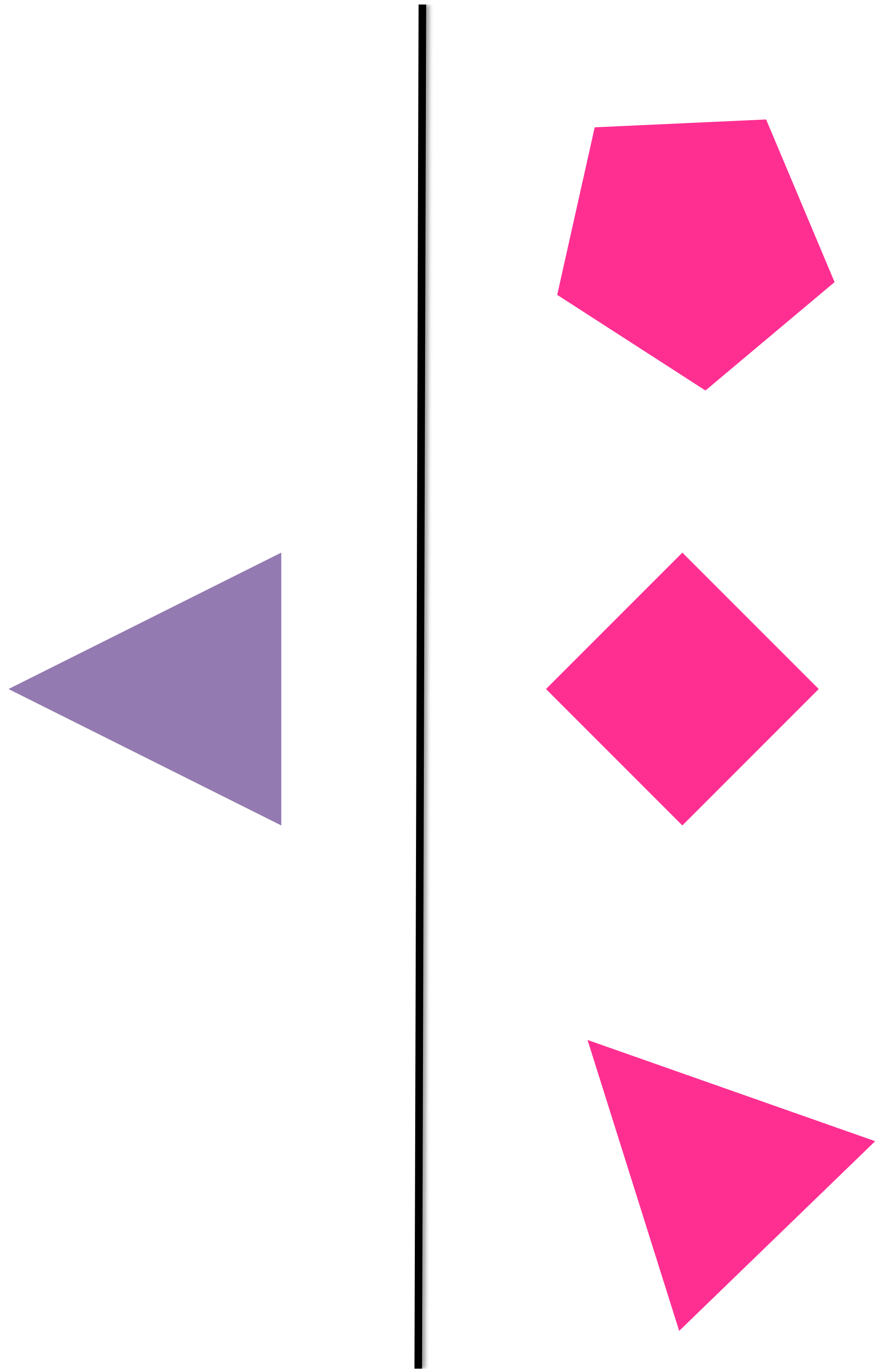
Matching Shapes with Different Orientations Assessment

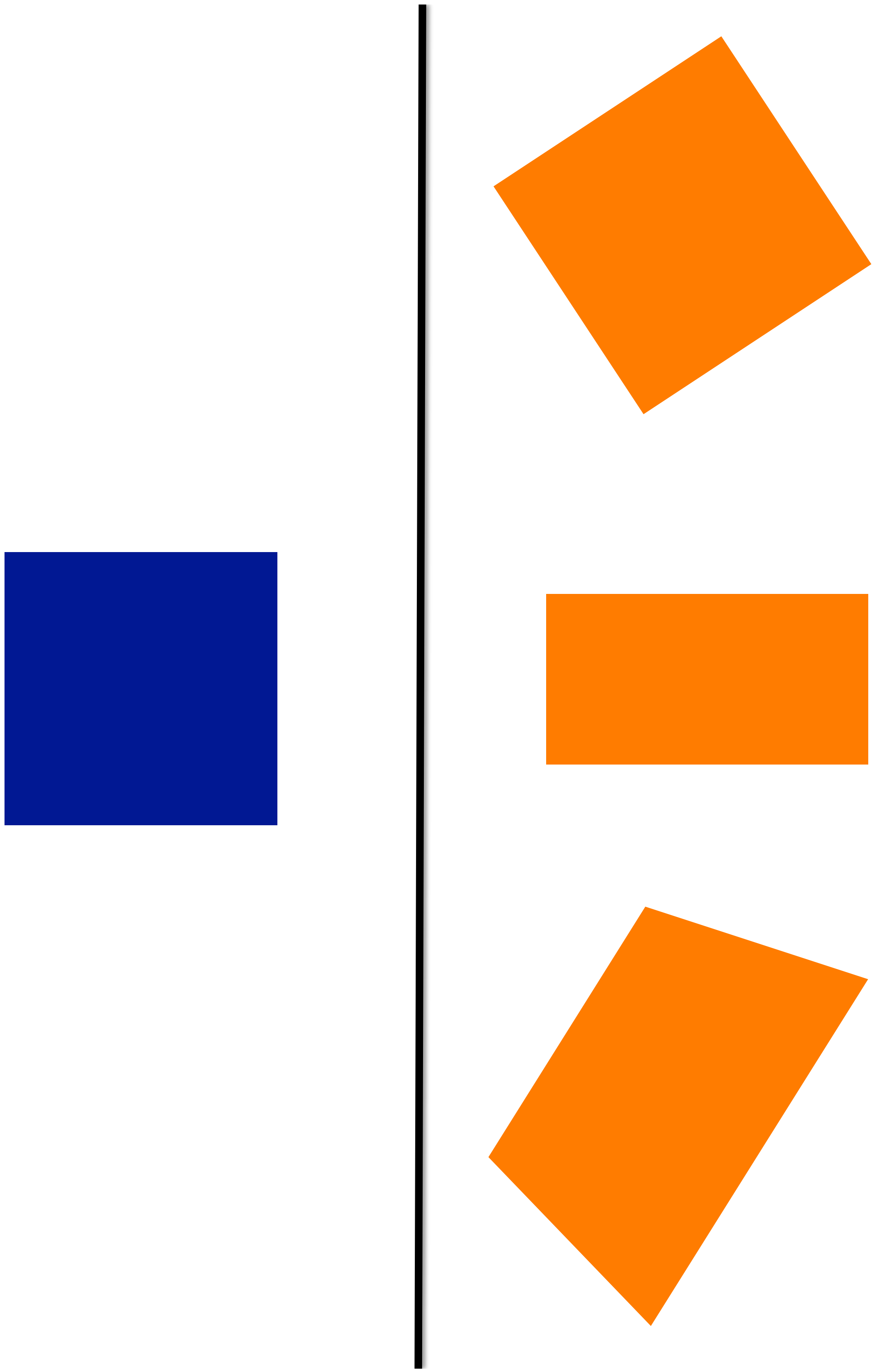


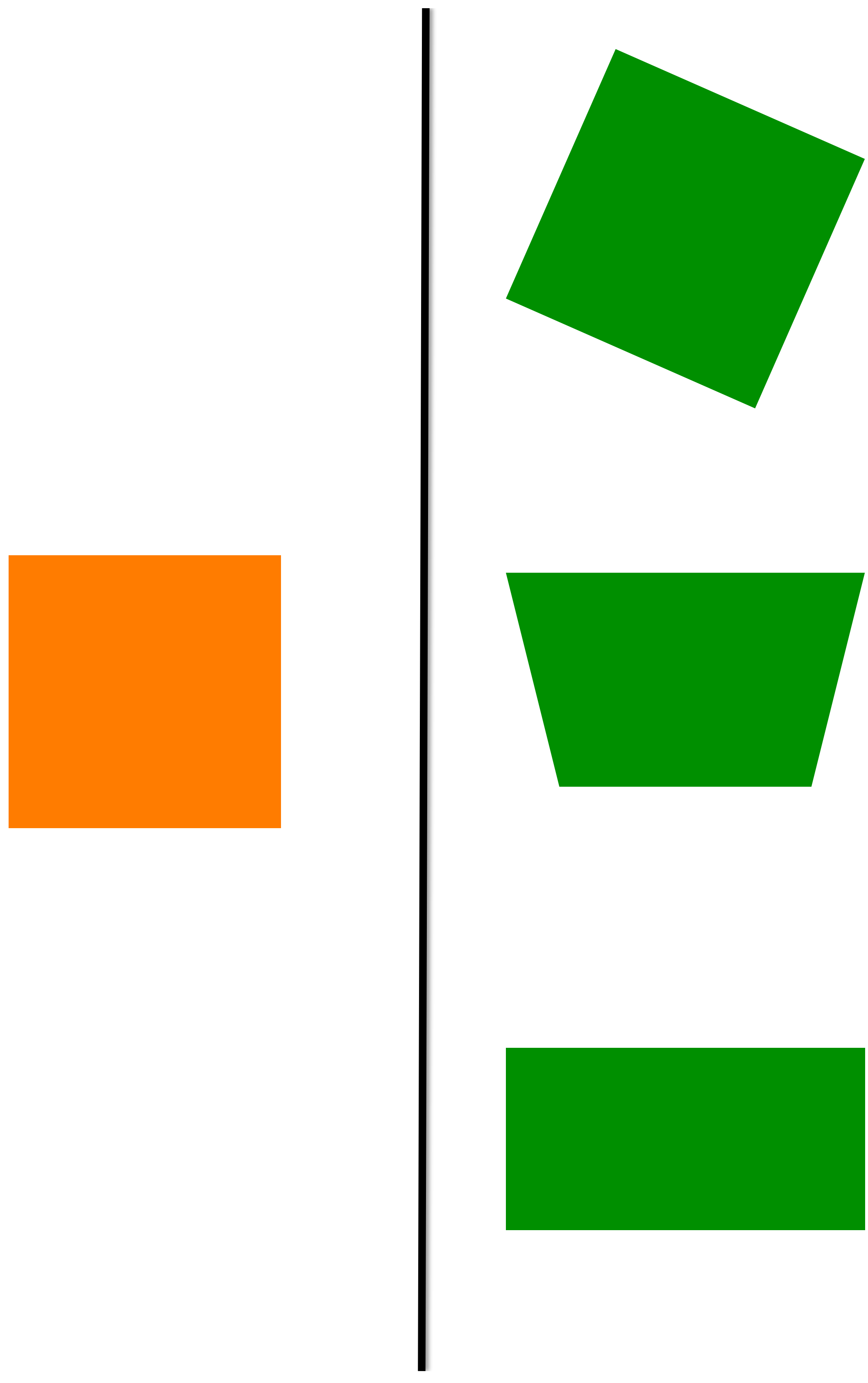


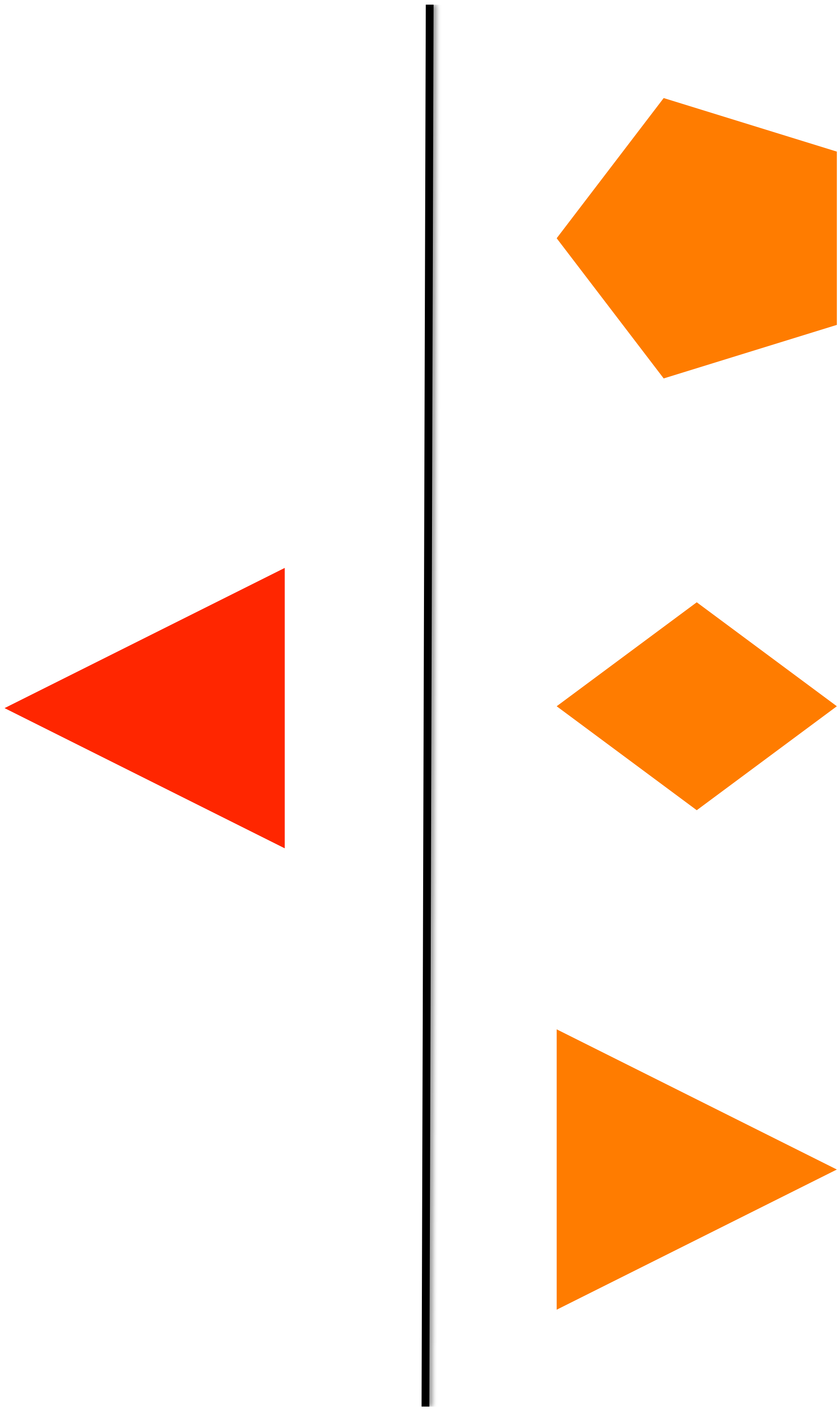


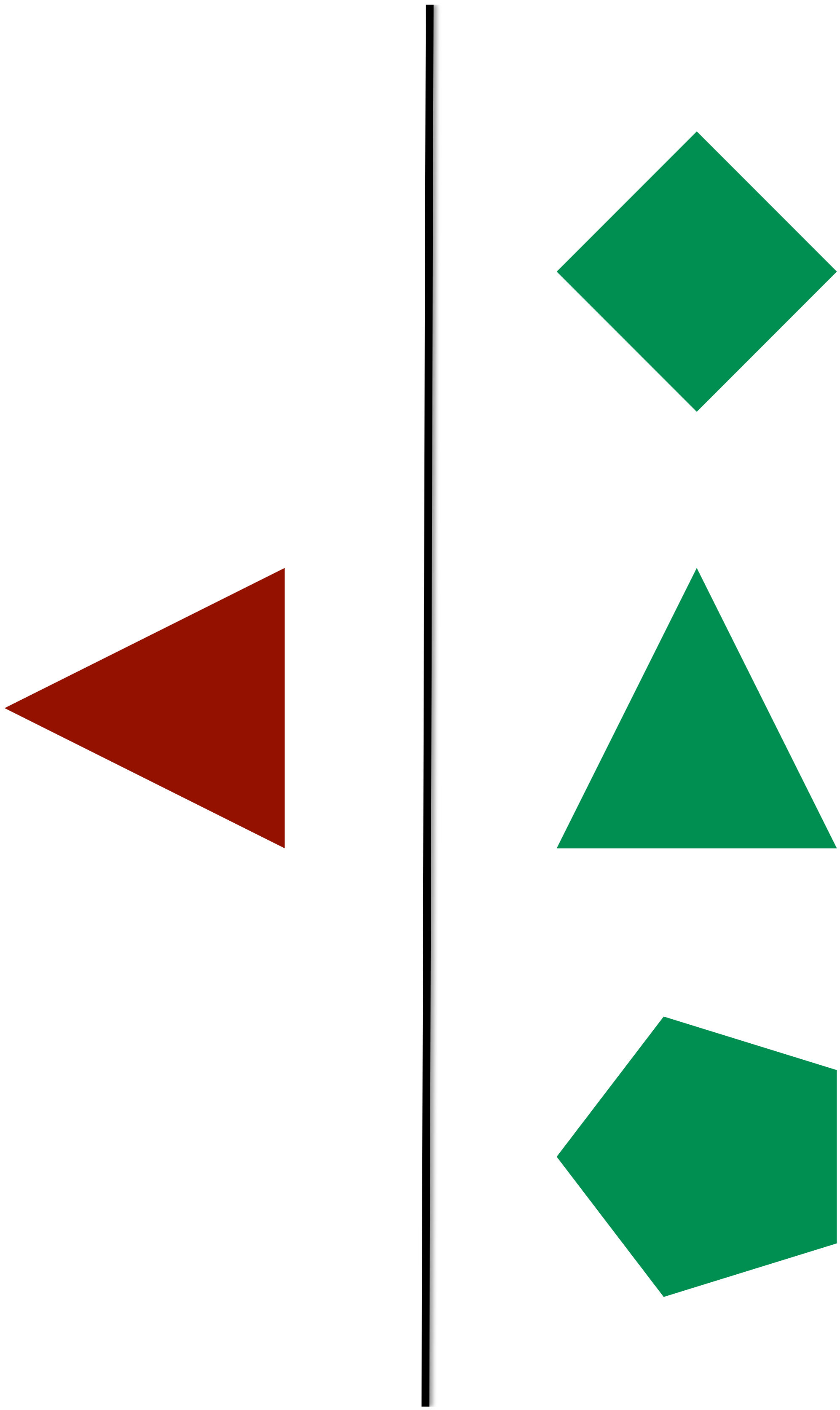


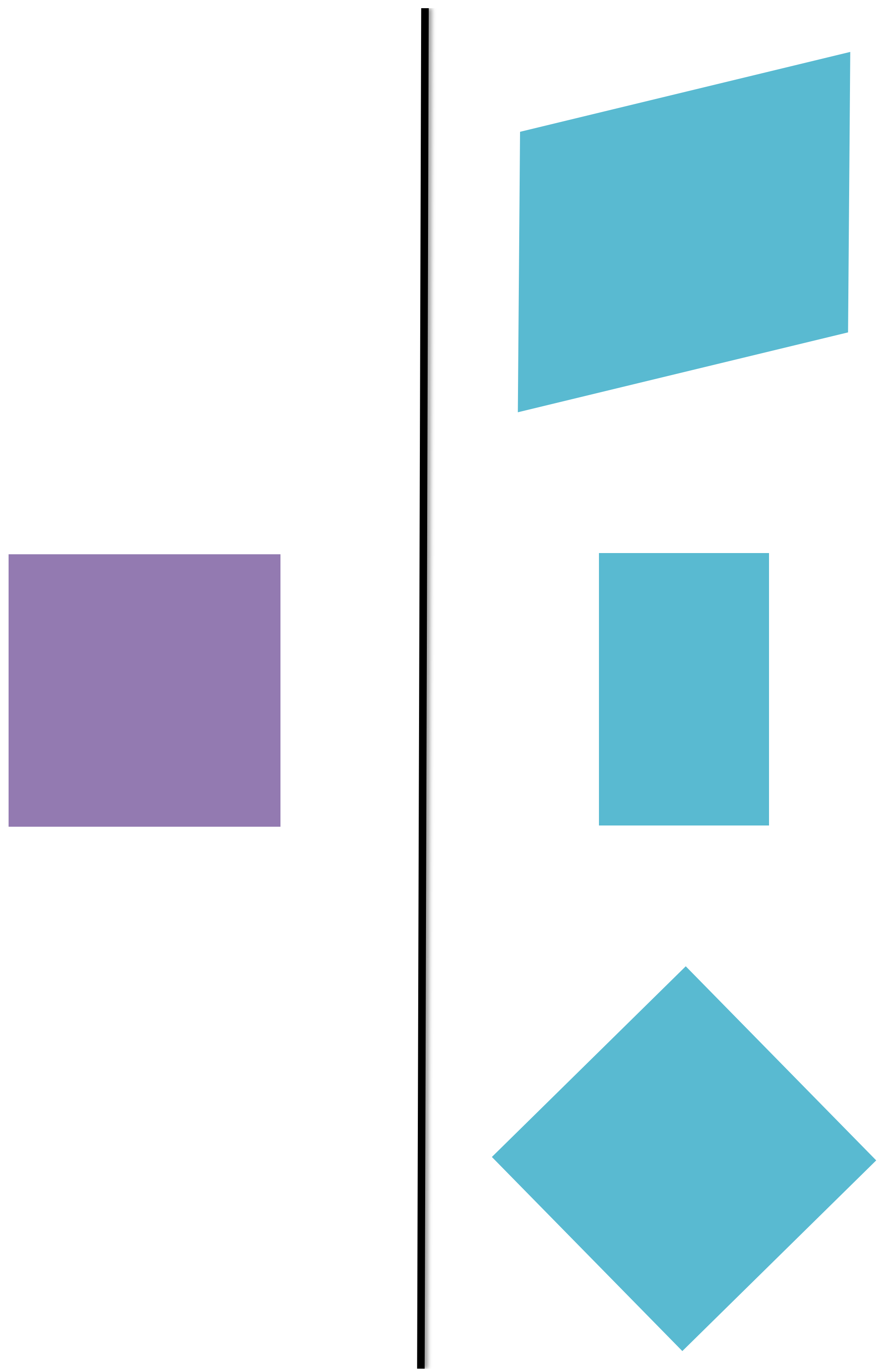












APPENDIX E

Shape Recognition Assessment